A Comparison of Calculation Speed and Accuracy on Two Levels of Problem Difficulty Using the Conventional and Hutchings' "Low Stress" Addition Algorithms and the Pocket Calculator with High and Low Achieving Math Students

Leslie S. Zoref
A COMPARISON OF CALCULATION SPEED AND ACCURACY ON TWO LEVELS OF PROBLEM DIFFICULTY USING THE CONVENTIONAL AND HUTCHINGS' "LOW STRESS" ADDITION ALGORITHMS AND THE POCKET CALCULATOR WITH HIGH AND LOW ACHIEVING MATH STUDENTS

by

Leslie S. Zoref

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Leslie S. Zoref
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INTRODUCTION

As the content of school mathematics curricula continues to expand, the need to impart skills to students as efficiently as possible becomes evident. With respect to basic arithmetic, there has been a developing trend toward focusing on mathematical concepts while deemphasizing actual computational operations. This is the basic philosophy of "New Math" as it is now taught in the schools. However, the importance of being able to add, subtract, multiply, and divide has not diminished. One must be somewhat competent with these skills as a matter of survival as invariably these operations are frequently called for in today's world.

Keeping in mind the increasing volume of mathematical information that is being viewed as essential in primary education, the acquisition and maintenance of basic math skills must be learned quickly and accurately. The conventional algorithms taught to students often are not mastered, let alone used efficiently.

This state of affairs may be partially due to the negative reaction that is so often associated with mathematics in general. Skinner (1968) observed that "The figures and symbols of mathematics have become standard emotional stimuli. The glimpse of a column of figures...is likely to set off, not mathematical behavior, but a reaction of anxiety, guilt, or fear" (p. 18). Hutchings (1972), Gordon (1972) and Alessi (1974) share this viewpoint that the monotony and boredom generated from repeated drilling of math facts certainly
invites, if not conditions, a poor attitude toward the subject area being drilled. Hence, it appears that alternative methods for instructing basic arithmetic facts and operations need to be explored.

In 1972, Dr. Lloyd B. Hutchings developed a set of experimental algorithms for the four basic math operations at Syracuse University. According to Hutchings, the new algorithms "appear to permit easy mastery after brief training, to provide greater computational power than conventional algorithms, and to operate with much less stress on the user than conventional algorithms" (Hutchings, 1976, p. 219). Further advantages of Hutchings' algorithms appear to be: (a) easy identification of errors, (b) facility with locating error patterns and prescribing appropriate remediation, (c) effective drill in basic math, (d) a full written record that allows for specific feedback on accuracy, and (e) useful as a teaching tool to demonstrate carrying (regrouping). Several group design studies have been conducted in order to assess the computational power (speed plus accuracy) of the Hutchings' algorithms (Alessi, 1974; Boyle, 1975; Gordon, 1972; Dasiell, 1974; Hutchings, 1972). Recent single subject design studies have investigated whether the Hutchings' algorithm is preferred over the conventional algorithm in varying response cost and reinforcement conditions (Gillespie, 1976), and the computational power of the Hutchings' algorithm versus the conventional algorithm with normal and emotionally disturbed children in distracting and non-distracting environments (Rudolph, 1976).

Hutchings (1976) defines the addition algorithm as follows:
"The low-stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit" (p. 220).

Given a single column of figures to add, one would

"start at the top, add the first two digits, and record the sum in the new notation...The complete sum of each two digit addition is recorded in half-space notation, but only the ones portion of each sum is used in the next addition. The ones portion of the column sum is always the same as the ones portion of the last two digit sum. The tens portion of the column sum is always the same as the number of tens recorded at the left of the column... For a column in some multicolumn exercise, the last step - that is, counting the tens at the left of the columns - would be slightly changed...the total number of tens is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left... Note, however, that the column sum for the last column in a multicolumn example is recorded in exactly the same way as the sum of a single-column exercise" (Hutchings, pp. 221-223).

Example:

\[
\begin{array}{cccc}
2 & 4 & 9 & 6 \\
& 8 & 7 & 3 \\
& 3 & 6 \\
\hline
1 & 4 & 4 & 4 \\
\end{array}
\]

A somewhat similar, but less robust, algorithm was suggested by Sanders in 1971. His method consists of having the student silently compute the sum of the first two addends and then vocalize the ones portion aloud while holding up fingers to represent the number of tens. Like the Hutchings' algorithm, the ones portion of each sum
is used in the next addition, thus forming the subsequent pairs of addends. When the final pair of addends has been computed, the ones portion of the number verbalized is written as the ones portion of the column sum, and the number of fingers held up is counted and written as the tens portion of the column sum. While this method reduces the student's memory load (number of covert responses), it lacks the complete written record of computations afforded by Hutchings' algorithm. The advantages of such a written record have been previously enumerated with regard to their crucial value as a feedback system and a teaching tool.

O'Malley (1969) offers another algorithm that is identical to one component of the Hutchings' algorithm. Here the student writes down the tens, hundreds, etc. portion of the sums of addend pairs to the left of each column. Again, this method does not yield most of the advantages of Hutchings' full written record, as it is mainly a means to ensure proper carrying at a lower response effort for the user.

Fulkerson (1963) suggests yet another algorithm that closely approximates O'Malley's method. Here the student draws a line through the last digit used in obtaining a sum of ten or greater when adding binaries. Hence, the number of lines drawn per column represents the number of tens, hundreds, etc. to be carried to the next column. However, the student must remember the portion of the sum that is left over, after the ten has been indicated by a line, before he can start to compute the next binary. Since this method only furnishes one-half
of the overt cues granted by Hutchings' algorithm, it seems that it does not afford the many benefits of Hutchings' full written record.

Another recently viable alternative to efficient math instruction that does not involve any revised or novel algorithms is the electronic pocket calculator. The remarkable decrease in cost (from $100.00 in Spring, 1971 to $5.95 by Spring, 1976) of this item has resulted in an estimated one out of ten Americans owning one (Harrington, 1976). It is not surprising, then, that the increased availability of these calculators has engendered many debates about their use in the public schools.

However, at this time it appears that calculators are being accepted as supplementary teaching devices. Harrington (1976) cites endorsements from (a) the National Council of Teachers of Mathematics (NCTM), (b) the Conference Board of the Mathematical Sciences, and (c) the National Association of Secondary School Principals that attest to the legitimacy of calculators and the responsibility of schools to include calculators in their curriculum. The same author cautiously advises that, "it will be the responsibility of teachers and administrators to introduce the calculator to students and instruct them in its use - not as a modern miracle or as a crutch, but as a useful aid, as servant to (not a substitute for) the human brain" (p. 46). By "human brain" we might assume the author was referring to various calculation methods involving the use of algorithms.

The obvious advantages of using calculators are that they reduce tedium, can serve as a quick key, can be used to reinforce math facts
and concepts, and are generally fun (reinforcing) to use, (Hawthorne, 1973; Stultz, 1975). This last point is especially important when considering that math exercises often generate avoidance behaviors and complaints.

The NCTM Instructional Affairs Committee (1976) has identified still more advantages as justifications for using calculators in the schools. According to the committee, use of calculators, (a) promotes creative experimentation with mathematical problems, (b) helps one to become a well-advised consumer, (c) can be used to demonstrate the concept of repeated operations, and (d) encourages independent problem solving.

Several precautions need to be considered regarding the use of calculators: (a) they do not illustrate mathematical operations; (b) they let students derive answers without having been taught the operations necessary to get those answers, (Hawthorne, 1973); and, (c) it has been suggested by Stultz (1975) that a minimum of one calculator per two students be available in order to avoid upsets and chaos associated with the logistics of using calculators in the classroom.

In light of the above discussion, this study was designed to investigate the differential calculation power (speed plus accuracy) with addition for the conventional algorithm (CA) compared with the Hutchings' low-stress algorithm (HA) as compared with electronic calculators (EC). The three methods are compared for two levels of problem difficulty, and for both low and high math achievers. A multi-
element baseline design was felt to be appropriate in that the study focused on acquisition of academic performance, in which unstable and/or changing baseline rates are expected (Ulman and Sulzer-Azaroff, 1975). Also, because of this acquisition function, a single-subject design was chosen rather than a group measure for the following reasons: (a) observation and analysis are afforded over time; (b) individual subject data are available, and (c) manipulation of variables is possible as well as the flexibility in manipulation needed to establish experimental control.

The present study attempts to examine the following questions:

1. Are calculation correct and error rates and accuracy percentages for the three methods different between "low" and "high" achievers in math?

2. Are calculation correct and error rates and accuracy percentages for the three methods affected by problem difficulty? (Difficulty is defined by the number of binary additions per problem.)
METHOD

Subjects

The subjects for this study were six fourth grade students, three male and three female, approximately 10 years of age. Half of the students (two female and one male) were identified as "low achievers" only in math by their teachers and by the fact that they were enrolled in the math skills improvement class. The remaining three students were identified as "high achievers" in math based on teacher reports of their previous classroom performances in this subject area. This dichotomy was confirmed by each student's Metropolitan Achievement Test scores in math. The percentiles based on national norms were the 10th, 11th, and 24th for the three "low" group subjects, and the 44th, 56th, and 94th for the three "high" group subjects.

Setting

The study took place at the elementary school the students regularly attended in Kalamazoo, Michigan. Sessions were held in a regular classroom before school began, from 9:00-9:20 a.m. A total of 29 sessions were held, with approximately three sessions per week.

Independent Variables

This study involved three independent variables: (a) three types of calculation procedures: conventional, Hutchings' low-stress
(Hutchings, 1976), and pocket calculators; (b) two problem array sizes: five columns by seven rows of digits and two columns by seven rows of digits (Appendix A); and (c) two types of children with respect to math skills: low and high achievers.

**Dependent Variables**

Three dependent variables were investigated: (a) rate of columns correct, (b) rate of columns incorrect, and (c) percent accuracy on attempted columns. Rate of columns correct and incorrect were calculated using the number of columns completed per five minute sample. Accuracy was calculated by dividing the number of columns correct by number of columns attempted.

**Reliability**

Reliability data were taken on correcting the students' papers for the number of columns correct and incorrect. A random sample of 15 papers was taken from each set of the students' worksheets collected over the entire study. Reliability data were collected on scoring worksheet responses by two independent scorers. Reliability was computed by dividing the total number of scorer agreements by the total number of agreements plus disagreements, multiplied by 100.

**Experimental Design**

A multielement baseline design was used (Ulman and Sulzer-Azaroff, 1975), varying type of calculation procedure within sessions and type
of problem array across sessions. When using this design, the results can be analyzed by examining the interaction between, as well as the main effects of, the variables manipulated.

Materials

For the present, six identical xeroxed sheets of 56 basic addition facts were used (Appendix B). For the three training sessions, two identical worksheets were used throughout to ensure equal practice effects for each calculation procedure. The worksheets consisted of addition problems of varying array formats and sizes (Appendix C).

Daily sessions required the following materials: (a) one stop-watch (AMF Hanhart Amigo); (b) one red pencil; (c) a maximum of six pennies; (d) 18 dittoed sheets (six sheets per algorithm), each with addition problems totaling 30 columns, printed with an IBM Selectric Orator typing element, and with double-spacing between columns and rows within each problem; (e) three answer sheets (one per type of algorithm); (f) two pocket calculators (Omron 86). (Note: When necessary, dittoed sheets were repeated at monthly intervals. It was felt that one month allowed sufficient time for possible "memory" effects to be minimal.)

Procedure

In order to ensure a satisfactory degree of competence with basic addition, each student was pretested on 56 basic math facts. A criterion of 92% correct within five minutes, using the conventional
algorithm, was set as necessary for participation in this study. Another requirement for selecting subjects was that they be able to get to school 20 minutes early in the morning.

After the six subjects were selected, a letter was sent to their parents explaining the purpose of the study, attendance requirements for the children and enclosing permission slips to be filled out and returned. All the subjects chosen were granted written parental permission. Each selected student was then told that he would be given a penny for every session that he arrived on time.

The first session consisted of teaching the Hutchings' "low-stress" algorithm according to a modified version of Hutchings' training sequence (Appendix D). The second session consisted of demonstrating how to perform addition on the calculator. The third session consisted of a review of the conventional algorithm. During each of the three 15 minute training sessions, worksheets were completed either at that time or before the next session.

Throughout the remainder of the study, sessions were run according to the following described format. The students were divided into three dyads, with one "high" and one "low" achiever per dyad. When the students entered the room on time, they received a penny. Dittoed sheets were already on the desks facedown, with the student's name at the top of the page.

Members of each dyad were seated back-to-back so as to minimize sharing of answers. Also, each of the three procedures was calculated in the same designated area each day. Specifically, the Hutchings'
"low-stress" algorithm was always done in the front, left corner; the conventional algorithm in the front, right corner; and the calculators in the center, back of the room. This was done to help the children remember which algorithm was to be used in which setting. The remaining sheets were arranged near their corresponding algorithm station such that the next day's papers were on top and facedown.

When all the students were seated appropriately, they were given the cue, "Ready...go". They were then timed for five minutes with a stopwatch. At the end of the five minutes, the students were told to stop, put down their pencils, bring the experimenter their papers, and then go to the next station. The students would then locate the station with papers bearing their name, be seated and wait for the cue to turn their papers over and start working. Each session consisted of three trials, each five minutes long, with one trial per calculation procedure. The order of trial procedures within sessions was selected on a random basis, using a random numbers table. The order of problem arrays across sessions was also random, using a coin toss. However, neither array was given for more than two consecutive sessions.

As the students began their second trial, the experimenter would correct the papers from the first trial by using a red pencil and putting a "C" next to the problem if it was completely correct. If part of a problem was wrong, then the correct numbers of the answer were circled individually. This procedure was also done during the third trial for papers completed during the second trial. At the
end of the session, the experimenter would go over the errors made by the students. At this time it was announced that if they weren't asked to remain, they had performed with a low error rate that day. However, all the papers were available for the students to check their own performance if they wished to do so. Because of limited time, the experimenter did not correct the final set of papers during the session. However, since the order of procedures used by each student was randomized, it is felt that feedback was given equally for each procedure over all experimental sessions.
RESULTS

Reliability

Reliability data yielded an index of 92% agreement for scoring papers for number of columns correct and incorrect.

Organization of Dependent Data Presented

The results of this study are presented in four topical sections: (a) Comparison of performances for 2x7 problems arrays, (b) Comparison of performances for 5x7 problem arrays, (c) Comparison of performances for low math achievers, and (d) Comparison of performances for high math achievers. Each of these performance comparisons for the three calculation procedures will be made in terms of the three study measures: (a) correct calculation rate, (b) incorrect calculation rate, and (c) percent accuracy for calculations. Summary data for these comparisons are presented in three tables and eight figures. Table 1 presents the means and standard deviations for each of the three dependent measures for each of the four topical areas listed above. All the eight figures presented supplement Table 1 by depicting data from that table under each of the four topical areas. Tables 2 and 3 present the means and standard deviations for individual students for each of the three dependent measures for each of the four topical areas listed above. Data from Tables 2 and 3 are cited only in cases where individual performances deviate markedly from the reported group performances. Since data are cited from the several
Comparison of performances on 2x7 problem arrays

Correct rate data for high and low achievers on 2x7 problem arrays.

Table 1 presents the means and standard deviations for the average session scores for the three calculation procedures. As shown by the left column of the left half of Table 1, the mean correct rates for the low achievers for the conventional (CA) and Hutchings' (HA) algorithms are much lower (4.1 and 8.5) than the same scores for the higher achievers (16.3 and 16.0). However, the mean correct rate for calculators (EC) is higher for the low achievers (13.5) than for the high achievers (12.5). For the total correct rate data regardless of type of student, EC was highest (13.0), HA was lower (12.3), and CA was lowest (10.2).

Table 2 presents the means and standard deviations of the average daily scores for individual low achievers for the three calculation methods. Table 3 presents the same data for individual high achievers. Group data are generally in agreement with individual subject data in that all the low achievers consistently performed highest with the EC and lowest with the CA. Of the high achievers (Table 3), only Subject 5 performed highest with the EC.

Figures 1 and 2 present the average daily scores of correct and error rates for the three calculation methods with 2x7 problem arrays.
Table 1: Means and standard deviations of the daily average session scores across the study for low and high achievers using the conventional and Hutchings' algorithms and the calculator with two different problem arrays.
TABLE 1

MEANS AND STANDARD DEVIATIONS OF THE DAILY AVERAGE SESSION SCORES ACROSS THE STUDY FOR LOW AND HIGH ACHIEVERS USING EACH OF THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
<thead>
<tr>
<th>Type of Student</th>
<th>Method of Calculation</th>
<th>2x7 problem arrays (13 binary operations)</th>
<th>5x7 problem arrays (34 binary operations)</th>
<th>MEAN TOTALS: HIGH AND LOW</th>
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<tr>
<td></td>
<td></td>
<td>Correct Rate</td>
<td>Error Rate</td>
<td>Percent Accuracy</td>
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<tr>
<td></td>
<td></td>
<td>X</td>
<td>SD</td>
<td>X</td>
</tr>
<tr>
<td>Low</td>
<td>CA*</td>
<td>4.1</td>
<td>1.4</td>
<td>3.4</td>
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<tr>
<td></td>
<td>HA**</td>
<td>8.5</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>EC***</td>
<td>13.5</td>
<td>3.0</td>
<td>1.5</td>
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<tr>
<td>High</td>
<td>CA*</td>
<td>16.3</td>
<td>3.2</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>HA**</td>
<td>16.0</td>
<td>3.0</td>
<td>.9</td>
</tr>
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<td></td>
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<td>12.5</td>
<td>2.2</td>
<td>.9</td>
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<td>CA*</td>
<td>10.2</td>
<td>2.3</td>
<td>2.9</td>
</tr>
<tr>
<td>HIGH + LOW</td>
<td>HA**</td>
<td>12.3</td>
<td>2.1</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>EC***</td>
<td>13.0</td>
<td>2.6</td>
<td>1.2</td>
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</table>

**NOTE:** As all three students in each group were not present for all daily sessions nor each trial within daily sessions, some daily session scores reflect the average of less than three individual scores.

*each score represents averages from 15 sessions

1. number of columns correct/5 minutes
2. number of columns incorrect/5 minutes
3. number of columns correct/number attempted

*conventional

**Hutchings

***calculator
Table 2: Means and standard deviations of the individual daily trial scores across the study for low achievers using the conventional and Hutchings' algorithms and the calculator with two different problem arrays.
TABLE 2

MEANS AND STANDARD DEVIATIONS OF THE INDIVIDUAL DAILY TRIAL SCORES ACROSS THE STUDY FOR LOW ACHIEVERS USING THE THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
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<th>Student</th>
<th>Method of Calculation</th>
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<th>5x7 problem arrays (34 binary operations)</th>
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<tr>
<td></td>
<td>Correct Rate</td>
<td>Error Rate</td>
<td>Percent Accuracy</td>
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<tr>
<td></td>
<td>N</td>
<td>x</td>
<td>SD</td>
</tr>
<tr>
<td>* CA</td>
<td>14</td>
<td>2.1</td>
<td>1.8</td>
</tr>
<tr>
<td>1 ** HA</td>
<td>14</td>
<td>7.6</td>
<td>1.0</td>
</tr>
<tr>
<td>*** EC</td>
<td>14</td>
<td>16.5</td>
<td>5.5</td>
</tr>
<tr>
<td>* CA</td>
<td>13</td>
<td>7.7</td>
<td>2.9</td>
</tr>
<tr>
<td>2 ** HA</td>
<td>13</td>
<td>10.4</td>
<td>1.8</td>
</tr>
<tr>
<td>*** EC</td>
<td>13</td>
<td>13.2</td>
<td>3.4</td>
</tr>
<tr>
<td>* CA</td>
<td>15</td>
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<td>3 ** HA</td>
<td>15</td>
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<td>1.4</td>
</tr>
<tr>
<td>*** EC</td>
<td>15</td>
<td>11.2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

NOTE: Since all students were not present for all daily session nor all trials within each daily session, scores for each method reflect averages of different numbers of trials (and sessions) per method per student.

*N = number of sessions attended

1 number of columns correct/5 minutes
2 number of columns incorrect/5 minutes
3 number of columns correct/number attempted

*Conventional
**Hutchings
***Calculator
Table 3: Means and standard deviations of the individual daily trial scores across the study for high achievers using the conventional and Hutchings' algorithms and the calculator with two different problem arrays.
# TABLE 3

MEANS AND STANDARD DEVIATIONS OF THE INDIVIDUAL DAILY TRIAL SCORES ACROSS THE STUDY FOR HIGH ACHIEVERS USING THE THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
<thead>
<tr>
<th>Student</th>
<th>Method of Calculation</th>
<th>2x7 problem arrays (13 binary operations)</th>
<th>5x7 problem arrays (34 binary operations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Correct Rate</td>
<td>Error Rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{x} )</td>
<td>SD</td>
</tr>
<tr>
<td>** CA</td>
<td>11 22.0 3.9</td>
<td>4.1</td>
<td>2.2</td>
</tr>
<tr>
<td>4</td>
<td>** HA</td>
<td>11 16.9 6.0</td>
<td>1.1</td>
</tr>
<tr>
<td>*** EC</td>
<td>12 9.9 2.5</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>** CA</td>
<td>13 14.3 3.8</td>
<td>2.4</td>
<td>2.1</td>
</tr>
<tr>
<td>5</td>
<td>** HA</td>
<td>13 15.1 4.0</td>
<td>.8</td>
</tr>
<tr>
<td>*** EC</td>
<td>12 16.6 2.5</td>
<td>.9</td>
<td>1.2</td>
</tr>
<tr>
<td>** CA</td>
<td>14 12.6 2.6</td>
<td>.8</td>
<td>1.1</td>
</tr>
<tr>
<td>6</td>
<td>** HA</td>
<td>14 16.1 3.1</td>
<td>.5</td>
</tr>
<tr>
<td>*** EC</td>
<td>14 11.6 2.2</td>
<td>.5</td>
<td>.9</td>
</tr>
</tbody>
</table>

**NOTE:** Since all students were not present for all daily sessions nor all trials within each daily session, scores for each method reflect averages of different numbers of trials (and sessions) per method for each student.

\( ^aN = \) number of sessions attended

1. number of columns correct/5 minutes  
2. number of columns incorrect/5 minutes  
3. number of columns correct/number attempted  

*conventional  
**Hutchings  
***Calculator
Figure 1: Mean rate of columns correct and incorrect per session for low achievers using the three calculation methods with 2x7 problem arrays.
Mean rate of columns correct (number of columns/5 minutes) vs. sessions for Low Achievers:

- Conventional method
- Hutchings method
- Calculator method

Figure 1
Figure 2: Mean rate of columns correct and incorrect per session for high achievers using the three calculation methods with 2x7 problem arrays.
Figure 2

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Figure 3: Mean percent of columns correct per session for low achievers using the three calculation methods with 2x7 problem arrays.
Low Achievers: 2x7

Figure 3

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Figure 4: Mean percent of columns correct per session for high achievers using the three calculation methods with 2x7 problem arrays.
Figure 4

High Achievers: 2x7

PERCENT OF COLUMNS CORRECT (MEAN)

CONVENTIONAL
HUTCHINGS
CALCULATOR

SESSIONS

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Figure 5: Mean rate of columns correct and incorrect per session for low achievers using the three calculation methods with 3x7 problem arrays.
Low Achievers: 5x7

- CONVENTIONAL
- Hutchings
- CALCULATOR

MEAN RATE OF COLUMNS CORRECT (NUMBER OF COLUMNS/5 MINUTES)

MEAN RATE OF COLUMNS INCORRECT (NUMBER OF COLUMNS/5 MINUTES)

SESSIONS

Figure 5

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Figure 6: Mean rate of columns correct and incorrect per session for high achievers using the three calculation methods with 5x7 problem arrays.
HIGH ACHIEVERS: 5x7

- • • CONVENTIONAL
- □ □ Hutchings
- ○ ○ CALCULATOR

MEAN RATE OF COLUMNS CORRECT
(NUMBER OF COLUMNS/5 MINUTES)

MEAN RATE OF COLUMNS INCORRECT
(NUMBER OF COLUMNS/5 MINUTES)

SESSIONS

Figure 6

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Figure 7: Mean percent of columns correct per session for low achievers using the three calculation methods with 5x7 problem arrays.
Low Achievers: 5x7

Figure 7

PERCENT OF COLUNNS CORRECT (MEAN)

CONVENTIONAL
Hutchings
CALCULATOR

SESSIONS

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Figure 8: Mean percent of columns correct per session for high achievers using the three calculation methods with 5x7 problem arrays.
High Achievers: 5x7

Figure 8

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low and high achievers, respectively. Figure 1 shows that with low achievers there is an acquisition slope for the EC, while slopes for the CA and HA are relatively stable. There are distinct correct rates for each method, as virtually no overlapping of performance occurs. Figure 2 shows that there is an acquisition slope for the HA. While session performances for the CA and EC are somewhat variable, general performance over time remains at the same general level. Correct rates for the CA are generally distinctly greater than those for the EC. However, correct rates for the CA and HA are similar, with frequently overlapping performances.

Error rate data for high and low achievers on 2x7 problem arrays.
As shown by the middle column of the left half of Table 1, the mean error rates are largest with the CA for both low achievers (3.4) and high achievers (2.3), with low achievers making the greater number of errors. Mean error rates are relatively equal for the HA (1.2) and the EC (1.5) for low achievers, and identical (.9) for high achievers, with low achievers again making the greater number of errors. For the total error rate data regardless of type of student, HA was lowest (1.1), EC was higher (1.2) and CA was highest (2.9). Table 2 shows that individual low achievers consistently had the highest errors when using the CA. However, error rates for Subject 2 for the HA (.6) and EC (2.6) were not similar to group data. Regardless of the method being used, Table 3 indicates that individual high achievers consistently made fewer errors than individual low achievers (Table 2).

Figure 1 shows that error rates for low achievers were lowest
and most stable with the HA; slightly higher and more variable with the EC; and consistently highest with the CA. Figure 2 shows that error rates of high achievers were equally low for the HA and EC, and that error with the CA were consistently higher than for the other two methods.

Accuracy data for high and low achievers on 2x7 problem arrays. The right column of the left half of Table 1 shows that both low and high achievers were least accurate with the CA. However, low achievers were much less accurate (54.1) than high achievers (88.9). Accuracy rates were higher and similar for the HA and EC for both low achievers (87.7. and 90.5) and high achievers (95.0 and 93.0). For all methods, the high achievers were consistently more accurate than the low achievers. For the total accuracy data regardless of type of student, EC was highest (91.8), HA was lower (91.4) and CA was lowest (71.5). Individual subject data (Tables 2 and 3) are generally consistent with group data, except that Subject 2 (Table 2) performed more accurately with the HA (94.8) than with the EC (85.6).

Figures 3 and 4 present the average daily scores of percent correct for the three calculation methods with 2x7 problem arrays for low and high achievers, respectively. Figure 3 shows that for low achievers, trends for the CA and EC denote a gradual increase in performance. This acquisition is quite stable for the EC but rather variable for the CA. Performance is clearly the lowest for the CA. There is some overlapping of scores between the HA and the EC, but generally performance is the highest with the EC. Figure 4 shows that for high
achievers, performance for all three methods fluctuates moderately, with the HA as most stable and the EC as least stable. For the most part, performance is lowest with the CA. Also, there is a high degree of overlapping of accuracy levels between the HA and the EC.

Comparison of performances on 5x7 arrays

Correct rate data for high and low achievers on 5x7 problem arrays. As shown by the left column of the right half of Table 1, correct rates for low achievers are the lowest with the CA (4.4) and higher and approximately equal for both the HA (8.3) and the EC (8.4). For the total correct rate data regardless of type of student, HA was highest (12.0), CA was lower (10.0) and EC was lowest (7.9). As seen by Table 2 however, individual subject scores for the HA and the EC are not similar. Subjects 2 and 3 perform higher with the HA (10.1 and 7.5), while Subject 1 performs higher with the EC (10.9).

Table 1 shows that correct rates for high achievers are the lowest with the EC (7.4) and higher and approximately equal for both the CA (15.1) and the HA (15.6). Again, individual subject data shown on Table 3 indicate some variability in that Subject 4 performs much higher with the CA, while Subjects 5 and 6 perform higher with the HA. However, individual and group data are consistent for the EC.

High achievers consistently had higher correct rates than the low achievers for the CA and the HA, and consistently had lower correct rates than the low achievers for the EC. Individual subject data for the EC are in partial agreement with group findings. However,
of the low achievers (Table 2), Subject 3 scored lower than all the high achievers (Table 3). Also, Subject 2 (Table 2) of the low achievers scored about equally with (not higher than) Subject 6 (Table 3) of the high achievers.

Figures 5 and 6 present the average daily scores of correct and error rates for the three calculation methods with 5x7 problem arrays for low and high achievers, respectively. Figure 5 shows that for correct rates for low achievers, there is a gradual upward trend for the EC. Correct rate trends for the CA and the HA are fairly stable across sessions. Correct rates for the CA are the lowest, and distinct from, the performances for the HA and the EC. Correct rates for the HA and the EC are highly similar, having a high degree of overlapping of session performances between the two methods. However, of the two methods, the HA is clearly more stable than the EC. Figure 6 shows that for correct rates for high achievers, there are definite upward slopes for both the CA and the HA, and a more gradual upward slope for the EC. However, performance is clearly lowest for the EC. There is a high degree of overlapping performances between the CA and the HA, but no overlap of performances with the EC over either of the other methods.

Error rate data for high and low achievers on 5x7 problem arrays. As shown by the middle column of the right half of Table 1, the mean error rates are the lowest with the HA for both low achievers (.9) and high achievers (1.1). For the total error rate data regardless of type of student, HA was lowest (1.0), EC was higher (2.3) and
CA was highest (3.0). This is generally consistent with individual subject data, except that one low achiever, Subject 3 (Table 2), had an even lower error rate with the EC (.4) and one high achiever, Subject 6 (Table 3), had a slightly lower error rate with the EC (.2), whereas their error rates for the HA were 1.5 and .5, respectively.

Error rates for low achievers were highest with the CA (3.4) and second highest for the EC (2.0). This is generally not consistent with individual subject data (Table 2). For Subject 1, error rates were equally low for the HA and the EC at 1.4 and 1.5, respectively. For Subject 2, error rates were highest with the EC (3.1) and second highest for the CA (2.3). For Subject 3, as stated previously, error rates were lower for the EC than for the HA.

Error rates for high achievers were very similar for both the CA (2.5) and the EC (2.6). This is consistent with individual data with the exception already noted for Subject 6.

Figure 5 shows that for low achievers, error rates are the lowest and most stable for the HA; higher for the EC; and highest for the CA. Error rates for the CA and the EC have similar degrees of fluctuation across sessions. Error rates for the HA are clearly lower than for the CA, as there is no overlapping of scores except for one session. Error rates for the EC are most variable, overlapping with scores from the other two methods.

Figure 6 shows that for high achievers, error rates are the lowest for the HA, and higher and very similar for the CA and the EC. Error rates for the EC are the most variable. Error rates for the
HA are clearly lower than for both the CA and the EC, as there is minimal overlapping of scores with these other two methods. However, there is a high degree of overlap between the other two methods.

**Accuracy data for high and low achievers on 5x7 problem arrays.**

The right column of the right half of Table 1 shows that for the HA, both low achievers (88.1) and high achievers (93.5) were most accurate. For the total accuracy data regardless of type of student, HA was highest (90.8), EC was lower (75.8) and CA was lowest (70.1). This is in agreement with individual subject data with the exception of Subject 1 (Table 2) who performed equally as well with the EC. The low achievers were least accurate with the CA (54.3), more accurate with the EC (74.5) and most accurate with the HA (88.1). Only Subject 2 (Table 2) was less accurate with the EC (73.6) than with the CA (76.9). The high achievers were least accurate with the EC (77.0), more accurate with the CA (85.9) and most accurate with the HA (93.5).

Figures 7 and 8 present the average daily scores of percent accuracy for the three calculation methods with 5x7 problem arrays for low and high achievers, respectively. Figure 7 shows that for low achievers, performance is the highest and most stable for the HA. Performance is clearly the lowest for the CA and also somewhat variable. Accuracy is the most variable for the EC, with scores overlapping those of both the HA and the EC.

Figure 8 shows that for high achievers, there is a definite upward trend indicative of skill acquisition for both the HA and the EC. Accuracy for the CA is the least variable. In general, for the high
achievers, accuracy is highest for the HA; somewhat lower and less variable for the CA; and the lowest and most variable for the EC.

Comparison of performances for low achievers

Correct rate data for low achievers on 2x7 and 5x7 problem arrays.
The upper half of Table 1 presents the means and standard deviations for correct and error rates and for percent accuracy for low achievers on 2x7 and 5x7 problem arrays. The left-hand column of both quadrants show that correct rates for the CA and the HA remain basically unchanged for the two different arrays. However, there is a decrease from 13.5 to 8.4 for the EC when solving 2x7 and 5x7 arrays, respectively. For the total correct rate data regardless of problem difficulty, EC was highest (11.0), HA was lower (8.4) and CA was lowest (4.3). Figure 5 verifies that correct rates for the HA and the EC are about equal by the high degree of overlapping between the scores for these methods. When comparing Figure 1 with Figure 5, it can be seen that correct rates for the EC are higher and distinct from the HA only with the 2x7 arrays.

Error rate data for low achievers on 2x7 and 5x7 problem arrays.
The middle columns of the upper two quadrants of Table 1 show that there are similar error rates for all three calculation methods, with a slight decrease with the larger problems. Error rates for the CA remain the same and also the highest of the three methods. Error rates for the EC have increased from 1.5 for 2x7 arrays to 2.0 for 5x7 arrays. For the total error rate data regardless of problem difficulty,
HA was the lowest (1.1), EC was higher (1.8) and CA was highest (3.4). According to Table 3, individual subject data are generally not consistent with group findings. Error rates for Subject 1 decreased for the larger problems with the CA and the HA and remained stable for the EC. Error rates for Subject 2 also decreased for the larger problems with the CA and the HA. Error rates for Subject 3 increased for the larger problems with the CA and the HA and remained stable for the EC.

Figures 1 and 5 confirm the likeness of the three error rates across problem size. Overall, the HA has the greatest stability, while the EC has the greatest variability of the three methods.

Accuracy data for low achievers on 2x7 and 5x7 problem arrays. The right-hand column of the upper two quadrants of Table 1 show that accuracy levels are similar for the CA and the HA across problem size. However, there is a large decrease in accuracy for the EC from 90.5 to 74.5 with increased problem size. For the total accuracy data regardless of problem difficulty, HA was highest (87.9), EC was lower (82.5) and CA was lowest (54.2). Individual subject data (Table 2) are somewhat consistent with group data. Accuracy for Subject 1 for the HA increased from 81.0 to 85.1 with the larger problems and remained very similar for the EC. Accuracy for Subject 2 increased from 72.1 to 76.9 for the CA and from 94.8 to 98.3 for the HA with the larger problems. Accuracy with the HA for Subject 3 decreases as problem size increases from 87.7 to 80.9.

Figures 3 and 7 show that there is far less variability, as
well as overall higher accuracy level, for the EC with the smaller
problems than with the larger ones. When comparing Figure 1 with
Figure 4, it can be seen that performances for both the CA and the
HA are relatively equal with regard to level of accuracy and vari­
ability across problem size. However, performance for the EC is much
more variable and less accurate with the larger problems.

Comparison of performance for high achievers

Correct rate data for high achievers on 2x7 and 5x7 problem arrays.
The lower half of Table 1 presents the means and standard deviations
for correct and error rates and for percent accuracy for high achievers
on 2x7 and 5x7 problem arrays. The left-hand columns of both quadrants
show that correct rates for the CA and the HA have slightly decreased
with the 5x7 problems, from 16.3 to 15.1 and from 16.0 to 15.6, re­
spectively. There is a much larger decrease in mean correct rates
for the EC when problem size increases, from 12.5 to 7.4. For the
total correct rate data regardless of problem difficulty, HA was
highest (15.8), CA was lower (15.7) and EC was lowest (10.0). In­
dividual subject data (Table 3) are consistent with group data with the
following exceptions: correct rates for Subject 5 increase slightly
with the HA on the larger problems, and correct rates for Subject
6 did not decrease as largely for the EC with the 5x7 problems (from
11.6 to 9.8).

Figures 2 and 6 show that correct rate performance with the CA
had slightly less variability on the 5x7 problems than on the 2x7
problems.

**Error rate data for high achievers on 2x7 and 5x7 problem arrays.** The middle columns of the two lower quadrants of Table 1 show that there are similar error rates for the CA and the HA, regardless of problem size, with identically slight increases (.2) for the larger problems. However, errors for the EC increased from .9 to 2.6 with increased problem size, an increase of almost 400%. For the total error rate data regardless of problem difficulty, HA was lowest (1.0), EC was higher (1.8) and CA was highest (2.3).

Figure 2 shows that for 2x7 problem arrays, error rates are equally low with the HA and the EC and generally highest with the CA. Figure 6 shows that for 5x7 problem arrays, error rates are equally high with the CA and the EC and generally lowest with the HA.

**Accuracy data for high achievers on 2x7 and 5x7 problem arrays.** The right-hand columns of the two lower quadrants of Table 1 show that the HA is the most accurate method, regardless of problem size; however, there is a small decrease from 95.0 to 93.5 with increased problem size. For the CA, accuracy decreases from 88.9 to 85.3 with increased problem size. For the EC, accuracy markedly decreases from 93.0 to 74.1 with increased problem size. For the total accuracy data regardless of problem difficulty, HA was highest (94.3), CA was lower (87.4) and EC was lowest (85.0). These findings are in accord with individual subject data with the following exceptions for Subject 6 (Table 3); accuracy for the CA and the HA remain about equal across problem size, and accuracy for the EC only decreased
from 95.9 to 87.7 with increased problem size.

Figure 4 shows that there were similar accuracy levels with 2x7 problem arrays for the HA and the EC as indicated by the high amount of overlapping scores. Performance with the CA was generally lower than the other two methods. Figure 8 shows that there were distinct accuracy levels among the three methods, with the HA generally at the highest, the CA lower and the EC at the lowest accuracy levels. Also, performance for the EC on 5x7 problems was much more variable than on 2x7 problems (Figure 4). This increase in variability with increase in problem size was not evident for the other two methods.
Summary of Results by Calculation Method

The Hutchings' "low-stress" algorithm was generally the most stable method (i.e., had the lowest standard deviations) across all measures (correct rate, error rate, and percent accuracy), regardless of type of student or problem difficulty. This method also had the lowest error rate over the other two procedures, across type of student as well as level of difficulty. Finally, the Hutchings' "low-stress" algorithm was generally the most accurate method used.

The calculator produced higher correct rates for the low achievers over the high achievers, regardless of problem difficulty. This method also produced the lowest accuracy scores of the three procedures for the high achievers on 5x7 problem arrays. However, the calculator showed the highest accuracy scores of the three procedures for the low achievers on 2x7 problem arrays. A large decrease in accuracy was seen with increased problem difficulty. This did not occur with the other two methods. Lastly, performances for the calculator were the most variable across all measures for both types of students and for both problems arrays.

The conventional algorithm produced the worst performances on all measures for the low achievers regardless of problem difficulty. This method was faster and more accurate for high achievers on 5x7 problem arrays.
DISCUSSION

The results of the present study indicate that high achievers had higher correct rates and lower error rates, and thus performed more accurately than low achievers when using the conventional algorithm, regardless of problem size. These findings were expected. Consistent accuracy with basic addition facts and regrouping are essential for proper use of this algorithm. It is precisely this factor that differentiated the low achievers from the high achievers selected for the purposes of this study, and these results can be considered an operational confirmation of both teachers' opinion and results of standardized test scores used for the initial selection.

High achievers also consistently had higher correct rates and performed more accurately than low achievers when using the Hutchings' algorithm, regardless of problem size. However, error rates for both groups of students were virtually equal with this algorithm. These results further confirm earlier and more recent findings (Hutchings, 1972; Gordon, 1972; Alessi, 1974; Dashiell, 1974; Boyle, 1975; Rudolph, 1976; Gillespie, 1976) that the Hutchings' algorithm is a more accurate calculation method than the conventional algorithm, and especially effective in reducing calculation errors.

When using the pocket calculator, the low achievers had higher correct rates than the high achievers with the less difficult problems. However, the low achievers had higher error rates also, resulting in slightly less accurate overall performances than the high achievers.
An error analysis for the calculators suggests the following three patterns: (a) failure to clear the calculator before starting a new problem, (b) repeating and/or omitting a row of numbers, and (c) punching recording the wrong number(s). The low achievers worked with the calculators more quickly than the high achievers did. This holds true for working the more difficult problems also. Novelty effects may be an operating variable here, having a more durable effect on the low achievers. However, it is the author's opinion that novelty only plays a partial role. It is suggested that low achievers in math have had a history of repeated failure with basic math skills. Yet such a history does not apply to the use of the calculator, this machine does not demand an adequate arithmetic repertoire for speed.

Likewise, assuming that the high achievers have had a history of general success with mathematics (which implies that they are skilled in this area), it appears plausible that their rate is slowed down by the actual mechanics of using a calculator. Specifically, perhaps the high achievers are faster at computing sums covertly (from memory) rather than overtly (from a machine). Only when the low achievers were using the calculators were they able to work faster than the high achievers. When the low achievers were using either the conventional or the Hutchings' algorithms, they were working at a maximum of approximately one-half the speed of the high achievers.

Why then, when using the calculators with the less difficult problems, did the low achievers make more errors than the high
achievers, resulting in lower accuracy scores? The following are offered as potential explanations: (a) the minor excitement of using a calculator mildly detracted from the concentration needed for the academic task at hand, and (b) these students' math histories would suggest that they had a high probability of making errors when calculating problems, and (c) low achieving students generally are less careful in executing academic tasks.

When using the calculators with the more difficult problems, the low achievers again had higher correct rates than the high achievers. They also had a lower error rate. However, accuracy for both low and high achievers was virtually equal. One can speculate that the high achievers' increased errors can be attributed to the fact that it was their slowest method of calculation, rendering them even slower than the low achievers. A possible reason for this reduced speed could be that the calculator was not established as a cue for computation behavior. That is, the calculator did not serve as a discriminative stimulus for the increased attention needed for adding a large number of figures. Hence, the higher probability of distraction resulted in increasing the chances of making mistakes. Or, perhaps high achievers have stronger algorithm repertoires established which compete with using other mechanical calculation methods. The low achievers, on the other hand, were reinforced by the use of the calculators, by virtue of the academic success it afforded them. Therefore, perhaps they were more likely to be on-task when using this method. These findings indicate that perhaps calculators should be used as an enrichment
tool with high achievers, as a part of their math curriculum. However, with low achievers, it appears that calculators could be integrated with their instructional program over the conventional algorithm, but in conjunction with the Hutchings' algorithm.

The present study also indicates that clear differences exist with regard to calculation rates and accuracy when problem difficulty increases. Yet those differences only occur with the use of the calculators, whereas it appears that problem difficulty does not significantly affect performance with the conventional and Hutchings' algorithms. Performance always decreased (correct and error rates plus accuracy) as difficulty increased, for both low and high achievers when using the calculator.

Why is this method far less durable and more variable than the others? The author suggests that the answer is inherent in the mechanics of using the calculator. When using the calculator, the students were instructed to work horizontally, i.e., row by row. Thus, the number of figures to be visually scanned and recorded per row more than doubled (from two to five) with the larger problems. Moreover, any errors made on the calculator required the students to redo the entire problem, regardless of how much of the problem had been completed (recorded).

On the other hand, the students were instructed to work the conventional and Hutchings' algorithms vertically. Hence, the larger problems merely demanded a longer repetition of the procedures the students had already been using with the smaller, less difficult
problems. Furthermore, although one might assume that the probability of making errors will increase with greater problem size, the consequences of making errors with these two algorithms are far less time consuming than ones made with the calculator. It might than be induced that it is also more punishing to make errors with the calculator, especially when the problems are large. This last point can be subjectively verified by the author via anecdotal data; the highest frequency of groaning, complaining, and hitting the desk came from the part of the room where the calculators were being used.

It should also be noted that as problem difficulty increased, the Hutchings' algorithm clearly became the most accurate calculation method for all of the students. These results again verify those of other studies previously mentioned, with respect to the superior calculation power of the Hutchings' algorithm over the conventional one.

In sum, it can be reported that high achievers perform better than low achievers when using either the conventional or Hutchings' algorithms. However, both high and low achievers were generally equally as accurate when using the calculators. Finally, it can be proposed that increased problem difficulty lowered performance markedly only when the students used the calculators.

Generalization based on this study's findings should be made cautiously. All students were white, lower middle class fourth grade children attending an urban elementary school including kindergarten and grades four through six. They had all been taught with the Holt,
Rinehart and Winston "Exploring Elementary Mathematics" curriculum during the fourth grade. Results of the study might be generalized to other fourth grade students of similar socio-economic, cultural and achievement (as measured by the Metropolitan Achievement Test scores) backgrounds, and taught in a small group setting.

It appears that an efficient elementary math curriculum might use the Hutchings' algorithm for the initial instruction of basic arithmetic. This algorithm also affords an excellent means of determining how well students are maintaining the newly taught operations, via the full written record of each binary sum. In essence, the Hutchings' algorithm is the most accurate and stable method available that can illustrate the components necessary for success with addition. Therefore, it seems that this study offers further support to the stipulation that the electronic calculator's place in the elementary classroom is only as a supplementary device to basic math instruction, and not as primary source for imparting these skills.

Of even greater importance, however, is the increasing evidence suggesting that perhaps the conventional algorithm has been, at long last, outdated and is due to be replaced, by either pocket calculators or alternative algorithms. Future research might investigate student performance on the three calculation methods with the remaining basic operations of subtraction, multiplication, and division. It would be expected that for multiplication and division the pocket calculator would achieve its greatest superiority in performance speed and accuracy over any procedure using algorithms.
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APPENDIX A
SAMPLE DAILY WORKSHEETS
TWO-COLUMN, SEVEN ROW PROBLEM ARRAYS
AND
FIVE-COLUMN, SEVEN ROW PROBLEM ARRAYS
<p>| | | | | | |</p>
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APPENDIX B
INVENTORY OF BASIC ADDITION FACTS
(Adapted from Otto, McMenemy, and Smith, 1973, p. 221;
taken from Alessi, 1974.)
INVENTORY OF BASIC ADDITION FACTS

\[
\begin{align*}
+2 & +6 & +3 & +9 & +4 & +6 & +5 & +7 \\
+5 & +4 & +8 & +6 & +5 & +7 & +4 & +9 \\
+3 & +8 & +2 & +4 & +6 & +8 & +9 & +6 \\
+2 & +2 & +7 & +9 & +3 & +5 & +4 & +6 \\
+3 & +9 & +4 & +5 & +6 & +8 & +9 & +5 \\
+8 & +4 & +3 & +6 & +3 & +3 & +8 & +9 \\
+4 & +8 & +5 & +4 & +8 & +8 & +2 & +5 \\
+7 & +8 & +6 & +7 & +2 & +7 & +2 & +5 \\
\end{align*}
\]

Score: ________
APPENDIX C
PRACTICE WORKSHEETS FOR TRAINING SESSIONS
Practice Sheet #1

\[
\begin{array}{cccccc}
6 & 8 & 7 &  &  &  \\
4 & 8 & 3 &  &  &  \\
6 & 9 & 5 &  &  &  \\
\hline
+ 8 & 7 & 4 &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccc}
6 & 8 & 7 &  &  &  \\
4 & 8 & 3 &  &  &  \\
6 & 9 & 5 &  &  &  \\
\hline
+ 8 & 7 & 4 &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccc}
6 & 8 & 7 & 6 & 9 & 5 \\
4 & 8 & 7 & 6 & 4 & 2 \\
\hline
+ 8 & 7 & 6 & 9 & 8 & 3 \\
\end{array}
\]

\[
\begin{array}{cccccc}
7 &  &  &  &  &  \\
9 &  &  &  &  &  \\
9 &  &  &  &  &  \\
\hline
6 &  &  &  &  &  \\
\end{array}
\]

\[
\begin{array}{cccccc}
4 & 8 & 7 & 6 & 8 & 7 & 6 \\
9 & 8 & 7 & 6 & 8 & 7 & 6 \\
9 & 8 & 7 & 6 & 8 & 5 & 6 \\
\hline
7 & 9 & 5 & 6 & 7 & 9 & 3 \\
8 & 5 & 2 & 7 & 4 & 9 & 8 \\
\hline
+ 6 & 7 & 8 & 5 & 6 & 7 & 8 \\
\end{array}
\]
Practice Sheet #2

\[
\begin{array}{cccc}
3 & 5 & 2 \\
8 & 7 & 8 & 9 \\
5 & 9 & 2 & 2 \\
3 & 8 & 9 & 3 \\
9 & 6 & 5 & 9 \\
7 & 8 & 6 & 7 \\
9 & 5 & 5 & 8 \\
\end{array}
\]

\[
\begin{array}{cccc}
 & 5 & 8 & 6 \\
5 & 9 & 3 & 8 \\
7 & 6 & 5 & 8 \\
2 & 6 & 7 & 8 \\
9 & 6 & 7 & 5 \\
5 & 8 & 7 & 4 \\
\end{array}
\]

\[
\begin{array}{cccc}
6 \\
8 \\
7 \\
3 \\
3 \\
5 \\
8 \\
4 \\
9 \\
1 \\
7 \\
\end{array}
\]

\[
\begin{array}{cccc}
6 & 7 & 8 & 6 \\
6 & 7 & 8 & 6 \\
6 & 5 & 8 & 6 \\
3 & 9 & 7 & 6 \\
8 & 9 & 4 & 7 \\
8 & 7 & 6 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
6 & 7 & 8 & 4 \\
6 & 7 & 8 & 9 \\
6 & 5 & 8 & 9 \\
3 & 9 & 7 & 9 \\
8 & 9 & 4 & 7 \\
8 & 7 & 6 & 6 \\
\end{array}
\]
APPENDIX D
HUTCHINGS' ADDITION ALGORITHM, LESSON
(Adapted from Hutchings, 1972)
I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

\[ \frac{7}{15} \]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

\[ 7 + 3 \]

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? (Point)

\[ \frac{7}{15} \]

Let's look at another one. I can write "9 plus 5 is 14" like this or like this.

\[ \frac{9 + 5}{14} \]

Both of these say "9 plus 5 is 14."

Tell me what these say:

\[ \begin{array}{cccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 6 & 6 \\
+8 & +7 & +7 & +5 & +5 & +6 & +6 & +2 \\
\frac{17}{13} & \frac{18}{9} & \frac{17}{12} & \frac{16}{12} & \frac{16}{7} & \frac{16}{7} & \frac{16}{7} & \frac{16}{7} \\
\end{array} \]

(Call on students, point to the full notation form when asking.)
The little number on the right* is understood to be in the one's place, as are 9 and 8.

The little number on the left* is understood to be in the ten's place.

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big one" to "little one").

Now watch me write the following facts both ways.

\[
\begin{array}{cccccc}
9 & 9 & 8 & 8 & 4 & 4 \\
+7 & 17 & 5 & 5 & 9 & 9 \\
\hline
16 & 13 & 15 & 5 & 9 & 9 \\
\end{array}
\]

Look at the last pair. Are they different from the others? Note that there is no ten's place number and (do not draw box until after saying this) there is no "little one" on the left.

Let's look at another.

a) 4 Is there any ten's number here? (Do not draw box until after asking question.)

b) NO!! (repeat)

c) So will there be any little number on the left?

d) 3 NOT (Do not draw box until after asking question.)

NO!! (repeat)
If there is no ten's place number there is no "little number" on the left.

Now watch me write the rest of these.

Notice

\[
\begin{align*}
4 &+ 3 \\
7 &
\end{align*}
\]

If there is no ten's place number there is no "little number" on the left.

Again,

\[
\begin{align*}
4 &+ 3 \\
7 &
\end{align*}
\]

If there is no ten's place number there is no "little number" on the left.

Notice

\[
\begin{align*}
3 &+ 1 \\
4 &
\end{align*}
\]

There is a ten's number here so there is a "little number" here.

but

\[
\begin{align*}
7 &+ 8 \\
15 &
\end{align*}
\]

There is a ten's number here so there is a "little number" here.

Again, notice

\[
\begin{align*}
5 &+ 1 \\
6 &
\end{align*}
\]

There is no ten's number here so there is no "little number" here.

but

\[
\begin{align*}
8 &+ 5 \\
13 &
\end{align*}
\]

There is a ten's number here so there is a "little number" here.
Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again (repeat previous statement).

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

First, do you see that an example can be just number facts piled one atop the other? (Do not point with this question.)

OK! Here we go, starting at the top, writing facts as you learned and using only numbers at the right for addition.

a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

c) Say, "Now, eight plus five is thirteen."

d) Write the thirteen, i.e., $\frac{13}{4}$ in the example.

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw arrow $\frac{7}{4}$.

c) Say, "Three plus seven is ten."

d) Write the 10, i.e., $\frac{7}{4}$ in the example.
a) Say, "We've written the ten but we'll use only the 0."

b) Draw arrow \( 9_{d} \).

c) Say, "Zero plus nine is nine."

d) Write the 9, i.e., \( 9_{t} \) in the example.

---

a) Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow \( 8_{a} \).

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., \( 8_{e} \) in the example.

---

a) Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow \( 6_{c} \).

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., \( 6_{e} \) in the example.

---

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow \( 3_{e} \).

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., \( 3_{e} \) in the example.
a) Say, "We've written the eleven but we'll use only the one."

b) Draw arrow \( \frac{7}{2} \).

c) Say, "One plus seven is eight."

d) Write the eight, i.e., \( \frac{7}{2} \), in the example.

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.

The last little number on the right is the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of them, so the first half of the answer is five. The answer is 58.

Now watch me do another. Remember we use only the right side "little numbers." We will not bother to write the arrows anymore, just say

Now the last number on the right is a 2, so the right half of the answer is a 2! We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is 5. The answer is 52.
Now say the work for these with me as I do them at the board. (Children do not copy this.)

\[
\begin{array}{ccc}
8 & 9 & 4 \\
3 & 4 & 1 \\
6 & 7 & 2 \\
7 & 6 & 1 \\
5 & 8 & 0 \\
3 & 1 & 7 \\
2 & 9 & 4 \\
52 & 49 & 43
\end{array}
\]

Now copy these examples and do them by yourself. If you have any questions, ask me.

\[
\begin{array}{cccc}
6 & 8 & 5 & 9 \\
5 & 2 & 4 & 8 \\
9 & 7 & 9 & 3 \\
8 & 6 & 8 & 2 \\
5 & 9 & 7 & 7 \\
6 & 8 & 9 & 6 \\
+9 & +5 & +8 & +9
\end{array}
\]

After most have finished, say, "Check your work with mine as I do them at the board."

After doing the examples, say, "Now let's review."
I'll write the work for another one on the board. I want someone to raise his hand and tell me what the answer is.

\[
\begin{align*}
6 &\quad 6 \text{ plus } 8 \text{ is } 14 \\
9 &\quad 9 \text{ plus } 0 \text{ is } 9 \\
5 &\quad 5 \text{ plus } 5 \text{ is } 10 \\
3 &\quad 3 \text{ plus } 3 \text{ is } 6 \\
\end{align*}
\]

(Point to box.) Who will tell me what the right side of the answer is and how he got it.

(Point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it.

(Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this

\[
\begin{align*}
4 &\quad 6 \\
7 &\quad 8 \\
6 &\quad 7 \\
8 &\quad 6 \\
7 &\quad 3 \\
\end{align*}
\]

Can we still write our left-hand answer number at the bottom if there is more than one column? No, we can't?

When there's more than one column, each column can have only one number at the bottom (except for the very last column which does have the usual two).
So the single number that we put at the bottom is always the right-hand number.

(Write and point) 7

What can we do with the left-hand number? 8

Would it make sense to throw it away? No, it's part of the problem. So we will put it at the very top of the next column on the left. That way we don't lose it and it's still on the left side.

Watch! (Write on board.)

Count the little number on the left with me.
One, two, three, four.
There are four of them,
so we write a 4 at the top of the next column.

Now, when I start adding that column I will start with the four (4) first. Let's be sure you understand. (Repeat twice from the ♦.)

This is called carrying, some of you already understand it. Good.
Carrying is very easy.

But carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)

(Do with volunteers from class at board.) Good, we write the left-hand answer number at the top of the next column. (Repeat three times.)

Remember though that for the last column only, the left-hand answer number is at the bottom as though it were a single column.
Now, copy these examples and do them with me.

<table>
<thead>
<tr>
<th>7 6 9 8</th>
<th>7 6 9 8</th>
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</thead>
<tbody>
<tr>
<td>5 9 6 3</td>
<td>6 7 8 5</td>
</tr>
<tr>
<td>4 7 6 3</td>
<td>5 7 6 9</td>
</tr>
<tr>
<td>6 9 5 9</td>
<td>8 7 6 2</td>
</tr>
<tr>
<td>8 3 9 8</td>
<td>8 5 7 6</td>
</tr>
<tr>
<td>9 5 4 2</td>
<td>8 3 9 5</td>
</tr>
</tbody>
</table>

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good Are there are any questions?

Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help.

<table>
<thead>
<tr>
<th>7 6 8 7 9</th>
<th>7 6 5</th>
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<tbody>
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<td>4 8 3</td>
<td>5 3 9</td>
</tr>
<tr>
<td>6 9 5</td>
<td>1 8 1</td>
</tr>
<tr>
<td>+ 8 7 4</td>
<td>+ 5 8</td>
</tr>
<tr>
<td></td>
<td>+ 3  + 4 2</td>
</tr>
</tbody>
</table>

Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.