Spin-Flip Inelastic Scattering in $^{12}\text{C}(p,p')^{12}\text{C}$ Near the 9.14 Mev Doublet

Hsing-Tsuen Chen

Follow this and additional works at: https://scholarworks.wmich.edu/masters_theses

Part of the Physics Commons

Recommended Citation


Master's Theses. 2421.
https://scholarworks.wmich.edu/masters_theses/2421
SPIN-FLIP INELASTIC SCATTERING IN $^{12}\text{C}(p,p')^{12}\text{C}$
NEAR THE 9.14 MEV DOUBLET

by

Hsing-Tsuen Chen

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment
of the
Degree of Master of Arts

Western Michigan University
Kalamazoo, Michigan
December 1975
ACKNOWLEDGEMENTS

The author would like to thank Dr. Eugene M. Bernstein and Dr. Michitoshi Soga for their instruction, guidance and inspiration throughout the evolution of this work. Their willingness to help and answer questions is greatly appreciated. Thanks are also extended to Dr. Gerald Hardie for his valuable assistance during the course of this project.

Also, I have great appreciation for the Departments of Physics and Mathematics at Western Michigan University for providing the necessary hardware pertinent to this project.

Lastly, there is my great appreciation to many people I do not mention here, but I thank them in my heart.

Hsing-Tsuen Chen
INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in “sectioning” the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from “photographs” if essential to the understanding of the dissertation. Silver prints of “photographs” may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

Xerox University Microfilms
300 North Zeeb Road
Ann Arbor, Michigan 48106

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
MASTERS THESIS M-7783

CHEN, Hsing-Tsuen
SPIN-FLIP INELASTIC SCATTERING IN $^{12}$C(p,p)$^{12}$C
NEAR THE 9.14 MEV DOUBLET.

Western Michigan University, M.A., 1975
Physics, general

Xerox University Microfilms, Ann Arbor, Michigan 48106
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>INTRODUCTION</td>
</tr>
<tr>
<td>II</td>
<td>THEORY</td>
</tr>
<tr>
<td></td>
<td>Spin-Flip Measurements and Radiation Patterns</td>
</tr>
<tr>
<td></td>
<td>Conservation of Angular Momentum and Parity</td>
</tr>
<tr>
<td></td>
<td>Cross Sections</td>
</tr>
<tr>
<td>III</td>
<td>RESULTS AND CONCLUSIONS</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY</td>
</tr>
<tr>
<td></td>
<td>APPENDIX A</td>
</tr>
<tr>
<td></td>
<td>APPENDIX B</td>
</tr>
<tr>
<td>FIGURE</td>
<td>PAGE</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>I</td>
<td>4</td>
</tr>
<tr>
<td>II</td>
<td>6</td>
</tr>
<tr>
<td>III</td>
<td>10</td>
</tr>
<tr>
<td>IV</td>
<td>24</td>
</tr>
<tr>
<td>V</td>
<td>26</td>
</tr>
<tr>
<td>VI</td>
<td>28</td>
</tr>
<tr>
<td>VII</td>
<td>35</td>
</tr>
<tr>
<td>VIII</td>
<td>40</td>
</tr>
<tr>
<td>IX</td>
<td>42</td>
</tr>
</tbody>
</table>

FIGURES

I: Geometry for Spin-Flip Measurements Showing Quantization Axis
II: Energy Level Diagram for the $^{12}$C(p,$p'$)$^{12}$C Near 9.14 MeV Doublet
III: Radiation Pattern Diagram for an Electric Quadrupole Transition
IV: Angular Distributions for Inelastic Scattering and Spin-Flip Inelastic Scattering in $^{12}$C(p,$p'$)$^{12}$C Near 9.14 MeV Doublet
V: Energy Dependence of the Even-Order Legendre Coefficients for Inelastic Scattering
VI: Energy Dependence of the Even-Order Legendre Coefficients for Spin-Flip Inelastic Scattering
VII: Geometry for the New Quantization Axis $z'$
VIII: Energy Dependence of the Forward Spin-Nonflip Cross Section
IX: Energy Dependence of the Forward Spin-Flip Cross Section

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Legendre Coefficients for Inelastic Cross Section</td>
<td>20</td>
</tr>
<tr>
<td>II</td>
<td>Legendre Coefficients for Spin-Flip Cross Section</td>
<td>22</td>
</tr>
<tr>
<td>III</td>
<td>Parameters for Fits to the Energy Dependence of the Inelastic Legendre Coefficients</td>
<td>31</td>
</tr>
<tr>
<td>IV</td>
<td>Parameters for Fits to the Energy Dependence of the Spin-Flip Legendre Coefficients</td>
<td>33</td>
</tr>
<tr>
<td>V</td>
<td>Forward Spin-Nonflip Cross Sections</td>
<td>44</td>
</tr>
<tr>
<td>VI</td>
<td>Forward Spin-Flip Cross Sections</td>
<td>46</td>
</tr>
<tr>
<td>VII</td>
<td>Parameters for Fits to Forward Spin-Nonflip Cross Sections</td>
<td>48</td>
</tr>
<tr>
<td>VIII</td>
<td>Parameters for Fits to Forward Spin-Flip Cross Sections</td>
<td>50</td>
</tr>
<tr>
<td>IX</td>
<td>Values of the Parameters Corresponding to the Fits of Forward Spin-Nonflip Cross Sections to Equations (14) and (15)</td>
<td>53</td>
</tr>
</tbody>
</table>
CHAPTER I

Introduction

Many investigations into the structure of the nucleus come from the study of scattering. Most of this structure information is from elastic scattering; however, there has been recent interest in obtaining nuclear structure information from inelastic scattering.

Measuring proton spin-flip differential cross sections in inelastic scattering along with inelastic differential cross sections on and near a resonance has been developed by the Western Michigan University nuclear physics group. In the measurement of spin-flip probabilities and differential cross sections in $^{12}$C(p,p')$^{12}$C in the energy region of the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ doublet near $E_p = 9.14$ MeV, complete angular distributions were obtained at 13 energies (Mu 74). Preliminary analysis of the data is consistent with the following resonance parameters: (spin-parity, resonance energy in MeV, total width in KeV) $\frac{5}{2}^-$, 9.139, 33; $\frac{7}{2}^-$, 9.139, 88. These results are in good agreement with the determinations of these quantities from elastic scattering measurements (Be 68). In this thesis, the data are analyzed with a model independent theoretical formulation to obtain strengths for the inelastic decay partial level widths. Because there are interferences between the two resonating amplitudes, the analysis is very complex. Various methods have been tried to solve the complexities.
CHAPTER II

Theory

Spin-flip measurements and radiation patterns

For any two-body reaction in which parity and angular momentum are conserved, Aage Bohr (Bo 59) provided us a simple rule relating the polarizations of the particles involved in a collision process to their intrinsic parities:

\[ P_i e^{i\pi z} = P_f e^{i\pi z} \]  

(1)

where \( P_i \) = intrinsic parity of incident particle-target nucleus system, \( S_i \) = intrinsic spin projection of incident particle-target nucleus system, \( P_f \) = intrinsic parity of outgoing particle-residual nucleus system, and \( S_f \) = intrinsic spin projection of outgoing particle-residual nucleus system.

The quantization axis is defined (see Figure I) as:

\[ z = \frac{\bar{k}_i \times \bar{k}_f}{|\bar{k}_i \times \bar{k}_f|} \]

(2)

where \( \bar{k}_i \) = incident particle wave vector and \( \bar{k}_f \) = outgoing particle wave vector.

In the \( ^{12}\text{C}(0^+)(p,p')^{12}\text{C}(2^+) \) process (see Figure II), we know

\[ P_i = P_f. \]  

(3)

By Equation (2), Equation (1) can be written as:

\[ e^{i\pi (S_f - S_i)} = 1. \]  

(3)
Figure I

Geometry for spin-flip measurements showing quantization axis.
Figure II

Energy level diagram for the $^{12}\text{C}(p,p')^{12}\text{C}$ near 9.14 MeV doublet.
Equation (3) implies that

\[
S_f - S_i = 0, \pm 2, \pm 4, \pm 6, \ldots
\]

(4)

From the conservation of angular momentum, we know

\[
S_i = m_i + M_i \\
S_f = m_f + M_f
\]

where \( m_i \) = spin projection of incident proton, \( M_i \) = spin projection of target nucleus, \( m_f \) = spin projection of outgoing proton, and \( M_f \) = spin projection of residual nucleus.

In this process, \( M_i = 0 \) and Equation (4) becomes

\[
S_f - S_i = M_f + m_f - M_i = 0, \pm 2, \pm 4, \pm 6, \ldots
\]

(5)

We know the spin projection of proton is either 1/2 or -1/2, i.e.

\[
m_i = \pm \frac{1}{2}
\]

(6)

\[
m_f = \pm \frac{1}{2}
\]

and the spin projection of the residual nucleus is

\[
M_f = 0, \pm 1, \pm 2
\]

(7)

By means of Equations (5), (6) and (7), we can conclude that

- if \( m_i = m_f \) (spin-nonflip), then \( M_f = 0, \pm 2 \)
- if \( m_i = -m_f \) (spin-flip), then \( M_f = \pm 1 \).

The scattering leaves the residual nucleus in a \( 2^+ \) excited state, we may calculate the radiation pattern for the gamma-ray transition to the \( 0^+ \) ground state, this is no more than an electrical quadrupole radiation (\( l = 2, m = 0, \pm 1, \pm 2 \)).

The normalized angular distribution function of the emitted energy for the electrical and magnetic multipole radiation is (Ch 72):

\[
E_{\alpha}(\theta, \phi) = \frac{1}{2} \left[ \frac{I_{2}^{(\text{magnetic})}}{I_{2}^{(\text{electric})}} \right] Y_{l,m} \left[ \theta, \phi \right] + \frac{I_{1}^{(\text{electric})}}{I_{1}^{(\text{electric})}} \left[ \frac{I_{2}^{(\text{electric})}}{I_{1}^{(\text{electric})}} \right] Y_{l,-m} \left[ \theta, \phi \right] + \frac{I_{2}^{(\text{electric})}}{I_{2}^{(\text{electric})}} \left[ \frac{I_{2}^{(\text{electric})}}{I_{2}^{(\text{electric})}} \right] Y_{l,m} \left[ \theta, \phi \right]
\]
For $l = 2$, $m = 0, \pm 1, \pm 2$:

\[
Z_{20}(\theta, \phi) = \frac{15}{8\hbar} \sin^2 \theta \cos^2 \theta
\]

\[
Z_{211}(\theta, \phi) = \frac{5}{16\hbar}(-3 \cos^2 \theta + 4 \cos^2 \theta)
\]

\[
Z_{212}(\theta, \phi) = \frac{5}{16\hbar}(1 - \cos^2 \theta)
\]

These radiation patterns are shown schematically as a polar diagram in Fig. III. Note that only $m = \pm 1$ radiation contribute to the intensity along the quantization axis. We may conclude that only the spin-flip scattering contributes to the gamma-ray de-excitation along the quantization axis. If the lifetime of the radiating nuclear state is short, spin-flip events can be recorded by demanding a coincidence between the inelastically scattered protons and the de-excitation gamma-rays emitted along the axis of quantization. In this way the spin-flip probability may be measured.

We may apply this technique to investigate $\frac{5}{2}^-$ and $\frac{7}{2}^-$ doublet in the $^{12}C$ and proton system.

**The conservation of angular momentum and parity**

The energy level schemes and reaction routes used to the study of the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ doublet in $^{13}N$ is shown in Figure II.

The angular momenta of the incident protons are $f_{5/2}$ and $f_{7/2}$.

The total angular momentum of the outgoing particle is $J_f = J_i - J_f$ where $J_f =$ angular momentum of the residual nucleus.
Figure III

Radiation pattern diagrams for electric quadrupole transitions.
$l = 2, m = \pm 2$

$1 = 2, m = \pm 1$

$1 = 2, m = 0$
\( j_f \) = angular momentum of the outgoing particle.
\( j_i \) = angular momentum of the incident particle.

For the \( \frac{7}{2}^- \) level, \( j_f \) ranged from \( \left( \frac{7}{2}^- - 2 \right) \) to \( \left( \frac{7}{2}^- + 2 \right) \); for \( \frac{5}{2}^- \), \( j_f \) ranged from \( \left( \frac{5}{2}^- - 2 \right) \) to \( \left( \frac{5}{2}^- + 2 \right) \).

By conservation of angular momentum, we know the orbital angular momentum of the outgoing particle is \( j_{f}^{+1} \); along with parity conservation, we summarize the results as:

(for \( \frac{5}{2}^- \) resonance) incident proton: \( f_{5/2} \)
outgoing proton: \( p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2}, h_{9/2} \)

(for \( \frac{7}{2}^- \) resonance) incident proton: \( f_{7/2} \)
outgoing proton: \( p_{3/2}, f_{5/2}, f_{7/2}, h_{9/2}, h_{11/2} \)

(This section follows closely that given by (Ba 73).)

**Cross sections**

The differential cross section for scattering from a nucleus with spin angular momentum \( I_n \) and spin projection \( M_n \) by a particle with spin projection \( m_s \) to a state composed of a residual nucleus with spin \( I_n' \), spin projection \( M_n' \) and an emitted particle with spin projection \( m_s' \) is (Ch 72):

\[
\frac{d\sigma(n)}{d\Omega} = \frac{1}{2} \sum_{M_n} \left| \sum_{M_n'} \left( I_n I_n' M_n M_n' \right) \right|^2 \left( \theta = \frac{\pi}{2}, \phi \right)
\]  

(8)
where the scattering amplitude is:

\[
\frac{iZ^2}{4\pi k_n} \sum_{l,l'j,j'lm} \sum_{j,j'lm'} a_{l,l'} \langle l,j|m_i \rangle \langle j'|m_i' \rangle T_{l,j,j'} Y_{l,j}(\theta, \phi) Y_{l',j'}(\frac{3}{2}, 0)
\]

and the other quantities in Equation (8) above are:

- \( \theta_p \) = scattering angle
- \( k_n \) = incident particle wave-number
- \( l \) = incident particle orbital angular momentum
- \( l' \) = outgoing particle orbital angular momentum
- \( j \) = incident particle total angular momentum
- \( j' \) = outgoing particle total angular momentum
- \( J \) = incident particle and target nucleus total angular momentum
- \( M \) = incident particle and target nucleus total angular momentum projection
- \( s \) = spin angular momentum of incident particle
- \( m_l \) = orbital angular momentum projection of incident particle
- \( m_s \) = spin angular momentum projection of incident particle
- \( m_j \) = total angular momentum projection of incident particle
- \( s' \) = spin angular momentum of outgoing particle
- \( m_l' \) = orbital angular momentum projection of outgoing particle
- \( m_j' \) = total angular momentum projection of outgoing particle

The summation may be chosen to extend over whatever range is appropriate to reproduce the experimental data. Higher order partial waves may be neglected. For a \( 0^+ \) to \( 2^+ \) excitation process \( I_n = M_n = 0 \),

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
and In' = 2, in this case, the resonating transition matrices may be written as (Ch 72, Ba 73):

\[
T_{\lambda j'j} = \frac{C}{E - E_R + i\gamma / 2} + R_{\lambda j'j} e^{i\beta_{j'j}}
\]

where

- \(C\) = elastic width of the resonating state in the compound nucleus
- \(C\) = partial width of decay for emitting a particle of angular momentum \(j'\)
- \(C\) = total width of the compound state
- \(E_R\) = resonance energy of the compound nucleus state
- \(\sigma_{\lambda j'j}\) = resonating phase angle
- \(R_{\lambda j'j}\) = background phase angle
- \(R_{\lambda j'j}\) = background transition matrix amplitude

The pure background transition matrices are:

\[
T_{\lambda j'j} = R_{\lambda j'j} e^{i\beta_{j'j}}
\]

with the exception of the energy itself the quantities appearing in these transition matrices may, for a narrow resonance, be assumed to be energy independent, and may be treated as adjustable parameters.

Certain restrictions can be made on these expressions if the initial and final nuclear states are known, along with the incident particle as discussed previously. They are

- Spin-nonflip → \(M_n' = 0, \pm 2\)
- Spin-flip → \(M_n' = \pm 1\)
In that case:

$$\left[ \frac{d\sigma(\theta_p)}{d\Omega} \right]^2 = \frac{1}{2} \sum_{M_\alpha = 0, \pm 1} \left| \sum_{j=1}^{2} \sum_{m=0}^{\pm 1} \sum_{M_j = M_\alpha \pm \frac{1}{2}} \int_{S_{j \rightarrow j}} (\eta_j, \theta_p) \right|^2 + \left[ \frac{d\sigma(\theta_p)}{d\Omega} \right]^2$$

If one now limits the summation appearing in the scattering amplitude itself to include only a finite number of partial waves and expresses the $\Theta_p$ dependent spherical harmonic as:

$$Y_{\lambda m}(\frac{\pi}{2}, \Theta_p) = \sum_{-\lambda}^\lambda \frac{(2\lambda+1)(\lambda-m)!}{4\pi(\lambda+m)!} (-1)^m e^{im\Theta_p} P^m_\lambda (\cos \frac{\pi}{2})$$

the resulting expression will be a sum of $\Theta_p$ dependent Legendre polynomials times angle independent constants, i.e.,

$$\left[ \frac{d\sigma(\theta_p)}{d\Omega} \right]^2 = \frac{1}{\lambda_p} \left\{ A^2_n + A^2_0 P^0_0 (\cos \Theta_p) + A^2_1 P^1_1 (\cos \Theta_p) + \cdots \right\}$$

$$\left[ \frac{d\sigma(\theta_p)}{d\Omega} \right]^2 = \frac{1}{\lambda_p} \left\{ A^0_n + A^0_0 P^0_0 (\cos \Theta_p) + A^0_1 P^1_1 (\cos \Theta_p) + \cdots \right\}$$

The highest order polynomial presented in the above sum is determined by the number of partial waves assumed to be participating in the reaction. Each Legendre coefficient may be expressed in terms of the quantities appearing in the various transition matrices. These quantities are not all energy independent and in the case of an isolated single level resonance their energy dependence is expressed by the well-known Breit-Wigner formulas. (Ch 72, Be 72)

$$A^0_n = B^0_n + C^0_n \frac{1}{1 + x^2} + D^0_n \frac{x^2}{1 + x^2}$$
In general, it should be noted that the Legendre coefficients which appear in the cross sections are not linearly independent.

For two resonances at the same energy dependence of the Legendre coefficients, it is more complicated than that for the single level case. For the $\frac{5}{2}^+$, $\frac{7}{2}^-$ doublet in $^{13}$N, which is of interest here, it was found from elastic scattering measurement (Be 68) that the two resonance energies are the same or within a few KeV of each other. Therefore, for the relation given below the two resonance energies are assumed to be the same.

If the $\frac{5}{2}^-$ level and the $\frac{7}{2}^-$ level transition matrices are designated as $T_1$ and $T_2$, respectively, and if background transition matrices are designated as $T_0$, it then follows (Mu 74) that:

$$T_1 = B_1 e^{i\theta_1} + C_1 \frac{1}{E - E_R + i\Gamma_1/2} e^{i\phi_1},$$

$$T_2 = B_2 e^{i\theta_2} + C_2 \frac{1}{E - E_R + i\Gamma_2/2} e^{i\phi_2}$$

$$T_0 = B_0 e^{i\theta_0}$$

where the first terms in $T_1$ and $T_2$ are the background terms and the second terms are the resonating terms.

Defining $x_1 = \frac{2(E - E_R)}{\Gamma_1}$ and $x_2 = \frac{2(E - E_R)}{\Gamma_2}$, Equations (11) and (12) can be written as:

$$T_1 = B_1 e^{i\theta_1} + \frac{2C_1}{\Gamma_1} (\frac{1}{x_1 + 1}) e^{i\phi_1}$$

$$T_2 = B_2 e^{i\theta_2} + \frac{2C_2}{\Gamma_2} (\frac{1}{x_2 + 1}) e^{i\phi_2}$$
In these expressions $\text{Re}$ means to take the real part of the complex expression.

By making a linear combination of all of the terms in the above products of matrix elements which have the same energy dependence, one can write a general form for the Legendre coefficients. This general form can be written as:

$$A_n = B_n + \frac{E_n}{\delta x_1} + \frac{F_n}{\delta x_2} + \frac{C_n}{\delta x_1} \frac{X_1}{\delta x_2} + \frac{D_n}{\delta x_2} \frac{X_2}{\delta x_1} + \frac{1}{\delta x_1 \delta x_2} \left[ H_n (10x_1x_2) + G_n (x_1x_2) \right]$$  \hspace{1cm} (13)

The quantities $B$, $E$, $F$, $C$, $D$, $H$ and $G$ are complicated combinations of the amplitudes and phase angles of the $T$ matrix elements. For the narrow levels like the ones considered here, these quantities can be taken to be independent of energy, except the background term $B$ which is assumed to be linearly energy dependent. In this expression, all the energy dependence is contained in the parameter $x_1$ and $x_2$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Furthermore, the general form in Equation (13) can be reduced to a simpler one:

In a doublet,

\[ A_n = |T e^{i\beta} \frac{C_1}{e^{i\beta}} + \frac{C_2}{x_1} e^{i\theta} + \frac{C_3}{x_2} e^{i\phi}|^2 \]

\[ = T^2 + 2T C_1 \cos(\theta - \beta) \frac{x_1}{x_1} + \left\{ C_2^2 - 2T C_1 \sin(\theta - \beta) \frac{x_1}{x_1} \right\} + 2T C_3 \cos(\theta - \beta) \frac{x_2}{x_2} + \left\{ C_3^2 - 2T C_3 \sin(\theta - \beta) \frac{x_2}{x_2} \right\} \]

By defining \( x = x_1 \) and \( \eta = \frac{C_1}{C_2} \), the above expression becomes:

\[ A_n = B + B_0 \frac{x}{x_1} + B_1 \frac{1}{x_1} + C_1 \frac{\eta x}{1 + \eta x} + C_2 \frac{1}{1 + \eta x} \quad (14) \]

The quantities \( B, B_0, B_1, C_0 \) and \( C_1 \) are complicated combinations of the amplitudes and phase angles of the \( T \) matrix elements. Again the background term \( B \) is assumed to be linearly energy dependent and all other energy dependence is contained in \( x \).
CHAPTER III

Results and Discussion

In this thesis, all experimental data are taken from John Muhanji's work (Mu 74). As mentioned earlier (see Equations 9 and 10), the spin-flip and inelastic scattering cross sections can be expressed as Legendre polynomials. Since the highest partial wave of the outgoing particle is the f wave, the highest order of Legendre polynomials is six. The experimentally determined cross sections were individually fit to determine the Legendre coefficients at each energy point using a computer routine (We 72). Table I shows these coefficients, in units of millibarns per steradian, for the inelastic cross section. For spin-flip, we know, from theoretical calculation, that the highest order of the Legendre polynomials is five. Experimental data shows that it is very safe to take these polynomials up to the fourth order, for spin-flip. Table II shows these coefficients, in units of millibarns per steradian, for the spin-flip inelastic scattering cross section.

Figure IV shows typical angular distributions for inelastic scattering and spin-flip inelastic scattering. The solid curves are the Legendre polynomial fits to the data.

Now we are in the stage to fit these polynomials to Equation (14). This can be done linearly, since we know the resonant energies and widths of the doublet, from elastic scattering (Be 68), which again has been verified by inelastic scattering (Mu 74). Figures V and VI
Table I

Values of the Legendre coefficients for the inelastic cross sections as a function of energy. The coefficients are given in millibarns per steradian.
Table I
Inelastic

<table>
<thead>
<tr>
<th>Lab Energy</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>$A_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.901</td>
<td>19.949±.251</td>
<td>2.596±.655</td>
<td>15.662±.950</td>
<td>-0.941±1.205</td>
<td>-4.486±1.250</td>
<td>-0.609±1.073</td>
<td>0.241±.892</td>
</tr>
<tr>
<td>9.230</td>
<td>20.910±.236</td>
<td>-2.100±.571</td>
<td>8.807±.820</td>
<td>-3.375±1.084</td>
<td>-6.935±1.159</td>
<td>1.910±1.081</td>
<td>0.279±.941</td>
</tr>
<tr>
<td>9.292</td>
<td>18.682±.217</td>
<td>-1.146±.532</td>
<td>8.590±.770</td>
<td>-0.850±1.017</td>
<td>-7.605±1.057</td>
<td>0.860±.978</td>
<td>-0.405±.836</td>
</tr>
<tr>
<td>9.400</td>
<td>17.878±.215</td>
<td>0.052±.538</td>
<td>11.220±.776</td>
<td>2.970±1.002</td>
<td>5.729±1.012</td>
<td>1.721±.916</td>
<td>0.445±.781</td>
</tr>
</tbody>
</table>
Table II

Values of the Legendre coefficients for the spin-flip inelastic cross sections as a function of energy. The coefficients are given in millibarns per steradian.
Table II

Spin-flip

<table>
<thead>
<tr>
<th>Lab Energy</th>
<th>$A_0^s$</th>
<th>$A_1^s$</th>
<th>$A_2^s$</th>
<th>$A_3^s$</th>
<th>$A_4^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.901</td>
<td>$5.237\pm0.142$</td>
<td>$-2.501\pm0.365$</td>
<td>$4.894\pm0.452$</td>
<td>$1.490\pm0.460$</td>
<td>$-2.556\pm0.434$</td>
</tr>
<tr>
<td>8.997</td>
<td>$4.847\pm0.176$</td>
<td>$-3.224\pm0.466$</td>
<td>$3.252\pm0.578$</td>
<td>$-0.535\pm0.599$</td>
<td>$-4.768\pm0.538$</td>
</tr>
<tr>
<td>9.074</td>
<td>$4.599\pm0.142$</td>
<td>$-5.789\pm0.375$</td>
<td>$0.709\pm0.455$</td>
<td>$-1.021\pm0.483$</td>
<td>$-6.688\pm0.451$</td>
</tr>
<tr>
<td>9.100</td>
<td>$5.564\pm0.154$</td>
<td>$-5.730\pm0.410$</td>
<td>$0.452\pm0.514$</td>
<td>$-1.810\pm0.560$</td>
<td>$-7.410\pm0.511$</td>
</tr>
<tr>
<td>9.126</td>
<td>$8.457\pm0.200$</td>
<td>$-6.291\pm0.521$</td>
<td>$-5.110\pm0.717$</td>
<td>$-8.118\pm0.785$</td>
<td>$-7.824\pm0.698$</td>
</tr>
<tr>
<td>9.133</td>
<td>$10.353\pm0.269$</td>
<td>$-4.050\pm0.691$</td>
<td>$-7.506\pm0.975$</td>
<td>$-9.294\pm1.083$</td>
<td>$-8.946\pm0.826$</td>
</tr>
<tr>
<td>9.139</td>
<td>$12.227\pm0.239$</td>
<td>$-1.322\pm0.610$</td>
<td>$-7.163\pm0.848$</td>
<td>$-12.372\pm0.952$</td>
<td>$-9.826\pm0.780$</td>
</tr>
<tr>
<td>9.146</td>
<td>$10.122\pm0.273$</td>
<td>$-1.265\pm0.713$</td>
<td>$-3.803\pm0.988$</td>
<td>$-9.255\pm1.080$</td>
<td>$-8.277\pm0.807$</td>
</tr>
<tr>
<td>9.152</td>
<td>$8.779\pm0.208$</td>
<td>$-3.181\pm0.541$</td>
<td>$-3.933\pm0.756$</td>
<td>$-10.278\pm0.843$</td>
<td>$-9.228\pm0.679$</td>
</tr>
<tr>
<td>9.165</td>
<td>$6.137\pm0.168$</td>
<td>$-4.780\pm0.451$</td>
<td>$1.688\pm0.590$</td>
<td>$-6.321\pm0.628$</td>
<td>$-6.236\pm0.559$</td>
</tr>
<tr>
<td>9.230</td>
<td>$4.806\pm0.135$</td>
<td>$-4.855\pm0.363$</td>
<td>$4.297\pm0.448$</td>
<td>$-2.633\pm0.463$</td>
<td>$-4.269\pm0.444$</td>
</tr>
<tr>
<td>9.292</td>
<td>$4.870\pm0.137$</td>
<td>$-5.065\pm0.366$</td>
<td>$3.324\pm0.447$</td>
<td>$-2.356\pm0.461$</td>
<td>$-4.985\pm0.428$</td>
</tr>
<tr>
<td>9.400</td>
<td>$3.742\pm0.108$</td>
<td>$-6.555\pm0.297$</td>
<td>$1.993\pm0.355$</td>
<td>$-2.532\pm0.372$</td>
<td>$-5.721\pm0.377$</td>
</tr>
</tbody>
</table>
Figure IV

Angular distributions for inelastic scattering and spin-flip inelastic scattering near $\frac{5}{2}^-$ and $\frac{7}{2}^-$ doublet in $^{13}_N$. The solid curves represent Legendre polynomial fits to distribution is listed in MeV next to the curve.
Figure V

Energy dependence of the even-order Legendre Coefficients for inelastic scattering. The solid curves represent the fits to the experimental data.
Figure VI

Energy dependence of the even-order Legendre Coefficients for spin-flip inelastic scattering. The solid curves represent the fits to the experimental data.
show the energy dependence of the even order Legendre coefficients for inelastic and spin-flip cross sections, respectively. The solid curves shown in these figures are fits to the experimental data. The values of the parameters corresponding to the fits shown in Figs. V and VI are given in Tables III and IV.

In order to determine the strengths of the partial waves involved in the scattering, we have to find the amplitudes and phase angles of the T matrix elements, from the theoretical expressions (calculated by the computer routine in Appendix A), that are consistent with the parameters shown in Figs. V and VI. This is extremely complex work. In order to simplify the fitting, we may define the new quantization axis as the direction of incident particles (see Figure VII), and fit the forward (i.e., scattering angle is zero) cross section to Equation (14).

The forward differential cross sections in the new coordinates can be expressed as those in the old coordinates. The derivation is as follows:

The scattering amplitude for \((1, j, m_s)\) incoming particle to \(0^+\) target and \((1', j', m_s')\) outgoing particle leaving residual nucleus in the \((2^+, M)\) state is

\[
\hat f_M(1, j, m_s, 1', j', m_s') = \frac{i}{\hbar} g_m T_{j j'}
\]

where

\[
g_m = \sum_{m' m''} <1 m m_j j'> <1' m' m'' j'> \tilde{Y}_{j m'}(\alpha') Y_{j m''}(\alpha_k)
\]

Denote the differential cross section for a substate \(M\) of the final \(2^+\) state as \(\frac{d\sigma_M}{d\Omega}\) and define

\[
\frac{d\sigma_{M,\Omega}}{d\Omega} = \frac{d\sigma_M}{d\Omega} + (1 - \delta_{M,0}) \frac{d\sigma'_M}{d\Omega}
\]

\[
= \frac{1}{\hbar} \sum_{j j'} \sum_{j'' j'''} T_{j j'} T_{j'' j'''}^* \frac{C_m^{IM}}{n} P_n (\cos \Theta)
\]
Table III

Parameters for fits of Equation 14 to the experimentally determined energy dependence of the Legendre polynomial coefficients for inelastic scattering. Values are given in millibarns per steradian.
Table III

<table>
<thead>
<tr>
<th>Inelastic</th>
<th>$J_1 = \frac{5}{2}$</th>
<th>$J_2 = \frac{7}{2}$</th>
<th>$E_{R_1} = E_{R_2} = 9.139$ MeV</th>
<th>$\Gamma_1 = 0.033$ MeV</th>
<th>$\Gamma_2 = 0.088$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legendre Coefficient</td>
<td>$B_0$</td>
<td>$B_1$</td>
<td>$C_0$</td>
<td>$C_1$</td>
<td>$B$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>14.502±0.749</td>
<td>0.519±0.745</td>
<td>12.347±0.632</td>
<td>0.479±0.693</td>
<td>(-0.094±0.013)x+(18.583±0.130)</td>
</tr>
<tr>
<td>$A_1$</td>
<td>3.721±1.706</td>
<td>10.291±1.669</td>
<td>-9.560±1.486</td>
<td>-8.159±1.629</td>
<td>(-0.069±0.033)x+(1.711±0.327)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>-13.742±2.420</td>
<td>6.181±2.365</td>
<td>-9.217±2.115</td>
<td>-9.363±2.328</td>
<td>(-0.099±0.047)x+(12.836±0.474)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>-4.490±3.255</td>
<td>10.836±3.174</td>
<td>-23.986±2.820</td>
<td>-6.960±3.089</td>
<td>(0.153±0.061)x+(1.180±0.612)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>-2.833±3.655</td>
<td>-4.770±3.602</td>
<td>-3.137±3.116</td>
<td>0.641±3.387</td>
<td>(-0.039±0.063)x-(5.515±0.640)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>-1.484±3.497</td>
<td>0.708±3.491</td>
<td>7.366±2.906</td>
<td>0.970±3.177</td>
<td>(0.065±0.057)x-(0.119±0.568)</td>
</tr>
<tr>
<td>$A_6$</td>
<td>5.769±3.062</td>
<td>5.664±3.067</td>
<td>3.388±2.531</td>
<td>-5.206±2.758</td>
<td>(0.042±0.048)x+(0.036±0.477)</td>
</tr>
</tbody>
</table>
Table IV

Parameters for fits of Equation 14 to the experimentally determined energy dependence of the Legendre polynomial coefficients for spin-flip inelastic scattering. Values are given in millibarns per steradian.
Table IV

<table>
<thead>
<tr>
<th>Spin-Flip</th>
<th>$J_1 = \frac{5}{2}$</th>
<th>$J_2 = \frac{7}{2}$</th>
<th>$E_{R_1} = E_{R_2} = 9.139$ MeV</th>
<th>$\Gamma_1 = 0.033$ MeV</th>
<th>$\Gamma_2 = 0.088$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legendre Coefficient</td>
<td>$B_o^S$</td>
<td>$B_1^S$</td>
<td>$C_0^S$</td>
<td>$C_1^S$</td>
<td>$B^S$</td>
</tr>
<tr>
<td>$A_0^S$</td>
<td>$7.819\pm0.391$</td>
<td>$-0.613\pm0.390$</td>
<td>$-0.871\pm0.326$</td>
<td>$1.075\pm0.372$</td>
<td>$(-0.054\pm0.007)x+(4.670\pm0.074)$</td>
</tr>
<tr>
<td>$A_1^S$</td>
<td>$7.097\pm1.027$</td>
<td>$3.316\pm1.023$</td>
<td>$-4.964\pm0.864$</td>
<td>$-0.206\pm0.986$</td>
<td>$(-0.145\pm0.019)x-(4.074\pm0.194)$</td>
</tr>
<tr>
<td>$A_2^S$</td>
<td>$-10.387\pm1.355$</td>
<td>$-0.536\pm1.372$</td>
<td>$-0.917\pm1.113$</td>
<td>$4.852\pm1.264$</td>
<td>$(-0.146\pm0.023)x+(3.635\pm0.240)$</td>
</tr>
<tr>
<td>$A_3^S$</td>
<td>$-7.988\pm1.470$</td>
<td>$-1.238\pm1.489$</td>
<td>$-3.153\pm1.190$</td>
<td>$-1.016\pm1.346$</td>
<td>$(-0.106\pm0.024)x-(0.573\pm0.249)$</td>
</tr>
<tr>
<td>$A_4^S$</td>
<td>$-1.862\pm1.267$</td>
<td>$-3.274\pm1.277$</td>
<td>$-3.296\pm1.069$</td>
<td>$5.493\pm1.214$</td>
<td>$(-0.150\pm0.023)x-(4.182\pm0.234)$</td>
</tr>
</tbody>
</table>
Figure VII

Geometry for the new quantization axis $z'$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
incident beam  tgt.  new quantization axis $z'$

The diagram illustrates the relationship between the incident beam and the scattered particle. The old quantization axis $z$ is shown, along with the new quantization axis $z'$. The angle between the two axes is denoted by $\theta_p$. Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
where $C_n^{[M]}$ is the resulting product of the Clebsch-Gordan coefficients and the converting factors of spherical harmonics to Legendre polynomials.

Now consider the forward scattering.

$$\sum_n C_n^{[M]} P_n(\cos \theta) = \sum_n C_n^{[M]}$$

(since $\theta = 0$)

In the new coordinate system (quantization axis $z'$)

$$g_M^* = \langle \ell \alpha \ell \beta | j m_I \rangle \langle j' m_0 | j m_R \rangle \langle j' m_0 | m_0 \rangle \sum_{s=\pm \frac{1}{2}} \frac{\sqrt{2s+1}}{\sqrt{2s'+1}}$$

since $Y_{s+1}(\theta, \phi) = \frac{\sqrt{2s+1}}{\sqrt{2s'+1}}$

For even $M$ (spin-nonflip):

$$g_M = \delta_{M,0} \langle \ell \alpha \ell \beta | j m_I \rangle \langle j' m_0 | j m_R \rangle \langle j' m_0 | m_0 \rangle \sum_{s=\pm \frac{1}{2}} \frac{\sqrt{2s+1}}{\sqrt{2s'+1}}$$

For odd $M$ (spin-flip):

$$g_M = \delta_{M,\pm \frac{1}{2}} \langle \ell \alpha \ell \beta | j m_I \rangle \langle j' m_0 | j m_R \rangle \langle j' m_0 | m_0 \rangle \sum_{s=\pm \frac{1}{2}} \frac{\sqrt{2s+1}}{\sqrt{2s'+1}}$$

and if $j' = j$ then $\langle j -2 \ell | j m_0 \rangle = 0$ for $s \neq \frac{3}{2}$.

In the old coordinate system (quantization axis $z$):

Define $K_M = \sum_n C_n^{[M]}$ and let the superscripts $n$ and $o$ represent the new and old coordinate systems, respectively. From the coordinate transformation we obtained (So 75):

$$K_2^o = \frac{3}{4} K_2 + \frac{1}{2} K_1^o$$

$$K_1^o = \frac{1}{2} K_1^o$$

$$K_0^o = \frac{1}{4} K_0^o$$
Therefore we obtained the two following rules:

1. $K_2^o - K_1^o = 3K_0^o$

2. If the incident and outgoing particles have the same orbital and total angular momenta, then $K_1^o = 0$.

The transformation of the spherical harmonics from the new coordinates to the old coordinates is

$$
\begin{align*}
Y_{12}^\text{new} & = \begin{pmatrix}
-\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} \\
-\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\
0 & \frac{1}{2} & -\frac{1}{2} & 0 \\
-\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\
\end{pmatrix}
Y_{12}^\text{old},
\end{align*}
$$

The corresponding transformation of the differential cross section is

$$
\begin{align*}
\left[ \frac{d\sigma}{dn} \right]_{\text{new}} & = \begin{pmatrix}
-\frac{1}{2} & \frac{3}{2} & 2 \\
\frac{3}{2} & 4/3 & -2 \\
1 & -1 & 1 \\
\end{pmatrix}
\left[ \frac{d\sigma}{dn} \right]_{\text{old}},
\end{align*}
$$

From the rules we mentioned and the above transformation, we found

$$
\begin{align*}
\left[ \frac{d\sigma}{d\alpha} \right]_{\text{new}} & = 0 \\
\left[ \frac{d\sigma}{d\alpha} \right]_{\text{new}} & = 2 \left[ \frac{d\sigma}{d\alpha} \right]_{\text{old}} \\
\left[ \frac{d\sigma}{d\alpha} \right]_{\text{new}} & = \left[ \frac{d\sigma}{d\alpha} \right]_{\text{old}} - \left[ \frac{d\sigma}{d\alpha} \right]_{\text{old}} + \left[ \frac{d\sigma}{d\alpha} \right]_{\text{old}} \\
& = \left[ \frac{d\sigma}{d\alpha} \right]_{\text{old}} - 2 \left[ \frac{d\sigma}{d\alpha} \right]_{\text{old}}
\end{align*}
$$
The energy dependences of the forward cross sections in the new coordinate system are shown in Figures VIII and IX for spin-nonflip and spin-flip inelastic scattering. The solid curves shown in these figures are fits of Equation (14) to the experimental data. The experimental values corresponding to Figs. VIII and IX are given in Tables V and VI. The values of the parameters corresponding to the fits shown in Figs. VIII and IX are given in Tables VII and VIII.

The forward scattering amplitude $F$ for a doublet is (So 75):

$$F = B^0 e^{i\omega_0} + B^e e^{i\omega_e} + \frac{c_{R_1}}{\sqrt{2/3}} e^{i\omega_{R_1}} + \frac{c_{R_2}}{\sqrt{2/3}} e^{i\omega_{R_2}}$$

$$= e^{i\omega_e} [B^0 e^{i(\omega_0 - \omega_e)} + B^e e^{i(\omega_e - \omega_0)} + \frac{c_{R_1}}{\sqrt{2/3}} e^{i(\omega_{R_1} - \omega_e)} + \frac{c_{R_2}}{\sqrt{2/3}}]$$

The forward differential cross section $A$ becomes:

$$A^e = \left[ B^0 + B^e \right] + \frac{\pi}{\sqrt{3/1}} \left[ 2B^e C^R \sin(\omega_e - \omega_0) - \frac{1}{\sqrt{3/1}} C^R C^R \sin(\omega_R - \omega_R) \right]$$

$$+ \frac{\pi}{\sqrt{3/1}} \left[ 2B^e C^R \sin(\omega_e - \omega_0) + \frac{1}{\sqrt{3/1}} C^R C^R \sin(\omega_R - \omega_R) \right]$$

$$+ \frac{\pi}{\sqrt{3/1}} \left[ 2B^e C^R \sin(\omega_e - \omega_0) + \frac{1}{\sqrt{3/1}} C^R C^R \sin(\omega_R - \omega_R) \right]$$

$$+ \frac{\pi}{\sqrt{3/1}} \left[ 2B^e C^R \sin(\omega_e - \omega_0) + \frac{1}{\sqrt{3/1}} C^R C^R \sin(\omega_R - \omega_R) \right]$$

(15)

$$A^o = \left[ 2B^0 B^e \cos(\omega_e - \omega_0) \right] + \frac{\pi}{\sqrt{3/1}} \left[ 2B^0 C^R \cos(\omega_e - \omega_0) \right] + \frac{\pi}{\sqrt{3/1}} \left[ 2B^e C^R \cos(\omega_e - \omega_0) \right]$$

(16)

where $e$, $o$, $R_1$, and $R_2$ stand for even, odd, resonance 1 and resonance 2, respectively. $B^0$, $B^e$, $C^R_1$, $C^R_2$, $(\omega_e - \omega_{R_1})$, $(\omega_o - \omega_{R_2})$, and $(\omega_{R_1} - \omega_{R_2})$ are parameters to be determined.
Figure VIII

Energy dependence of the forward differential cross sections for spin-nonflip inelastic scattering. The solid curves represent the fits to the experimental data.
Figure IX

Energy dependence of the forward differential cross section for spin-flip inelastic scattering. The solid curves represent fits to the experimental data.
Table V

Values of the forward spin-nonflip inelastic cross sections as a function of energy. Values are given in millibarns per steradian.
Table V

Forward differential cross section for spin-nonflip inelastic scattering

<table>
<thead>
<tr>
<th>Lab Energy</th>
<th>Even $(A_o^0+A_2^0+A_4^0)^2(A_o^0+A_2^0)$</th>
<th>Odd $(A_1^0+A_3^0)^2(A_1^0+A_3^0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.901</td>
<td>16.216±2.231</td>
<td>3.068±2.100</td>
</tr>
<tr>
<td>8.997</td>
<td>25.665±2.565</td>
<td>7.809±2.438</td>
</tr>
<tr>
<td>9.074</td>
<td>40.338±2.407</td>
<td>11.494±2.267</td>
</tr>
<tr>
<td>9.100</td>
<td>36.274±2.647</td>
<td>-1.386±2.460</td>
</tr>
<tr>
<td>9.126</td>
<td>49.800±3.638</td>
<td>-0.757±3.342</td>
</tr>
<tr>
<td>9.133</td>
<td>56.896±4.181</td>
<td>0.017±3.978</td>
</tr>
<tr>
<td>9.139</td>
<td>49.855±4.084</td>
<td>0.186±3.860</td>
</tr>
<tr>
<td>9.146</td>
<td>42.306±4.186</td>
<td>1.358±4.037</td>
</tr>
<tr>
<td>9.152</td>
<td>47.716±3.711</td>
<td>8.633±3.528</td>
</tr>
<tr>
<td>9.165</td>
<td>34.053±3.080</td>
<td>12.328±2.893</td>
</tr>
<tr>
<td>9.230</td>
<td>13.393±2.150</td>
<td>11.411±2.013</td>
</tr>
<tr>
<td>9.292</td>
<td>12.844±2.016</td>
<td>13.706±1.914</td>
</tr>
<tr>
<td>9.400</td>
<td>23.786±1.845</td>
<td>22.917±1.743</td>
</tr>
</tbody>
</table>
Table VI

Values of the forward spin-flip inelastic cross sections as a function of energy. Values are given in millibarns per steradian.
Table VI
Forward Differential Cross Section
for Spin-Flip Inelastic Scattering

<table>
<thead>
<tr>
<th>Lab Energy</th>
<th>Even $2(A_0^s+A_2^s+A_4^s)$</th>
<th>Odd $2(A_1^s+A_3^s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.901</td>
<td>15.150±0.909</td>
<td>-2.022±0.830</td>
</tr>
<tr>
<td>8.997</td>
<td>6.662±1.144</td>
<td>-7.518±1.073</td>
</tr>
<tr>
<td>9.074</td>
<td>-2.760±0.928</td>
<td>-13.620±0.864</td>
</tr>
<tr>
<td>9.100</td>
<td>-2.788±1.048</td>
<td>-15.080±0.982</td>
</tr>
<tr>
<td>9.126</td>
<td>-8.954±1.443</td>
<td>-28.818±1.332</td>
</tr>
<tr>
<td>9.133</td>
<td>-12.198±1.847</td>
<td>-26.688±1.817</td>
</tr>
<tr>
<td>9.139</td>
<td>-9.524±1.665</td>
<td>-27.388±1.600</td>
</tr>
<tr>
<td>9.146</td>
<td>-3.916±1.846</td>
<td>-21.040±1.830</td>
</tr>
<tr>
<td>9.230</td>
<td>9.688±0.912</td>
<td>-14.976±0.832</td>
</tr>
<tr>
<td>9.292</td>
<td>6.418±0.897</td>
<td>-14.842±0.833</td>
</tr>
<tr>
<td>9.400</td>
<td>0.028±0.748</td>
<td>-18.174±0.673</td>
</tr>
</tbody>
</table>
Table VII

Parameters for fits of equation 14 to the experimentally determined energy dependence of the forward spin-nonflip inelastic cross sections. Values are given in millibarns per steradian.


Table VII

<table>
<thead>
<tr>
<th>Spin-Nonflip</th>
<th>(J_1 = \frac{5^+}{2})</th>
<th>(J_2 = \frac{7^+}{2})</th>
<th>(E_{R1} = E_{R2} = 9.139) MeV</th>
<th>(\Gamma_1 = 0.033) MeV</th>
<th>(\Gamma_2 = 0.088) MeV</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Forward Differential Cross Section</th>
<th>(B_0)</th>
<th>(B_1)</th>
<th>(C_0)</th>
<th>(C_1)</th>
<th>(B)</th>
</tr>
</thead>
</table>

Even

\((A_0 + A_2^G + A_4^G + A_6^G) - 2(A_0^G + A_2^G + A_4^G)\)

12.632±6.616 16.523±6.600 22.244±5.79 -36.342±6.173 (0.509±0.115)x+(17.831±1.165)

Odd

\((A_1^G + A_3^G + A_5^G) - 2(A_1^G + A_3^G)\)

-3.400±6.235 15.787±6.205 -6.484±5.238 -9.537±5.805 (0.705±0.108)x+(10.793±1.100)
Table VIII

Parameters for fits of equation 14 to the experimentally determined energy dependence of the forward spin-flip inelastic cross sections. Values are given in millibarns per steradian.
Table VIII

<table>
<thead>
<tr>
<th>Spin-Flip</th>
<th>$J_1 = \frac{5}{2}^-$</th>
<th>$J_2 = \frac{7}{2}^-$</th>
<th>$E_{R1} = E_{R2} = 9.139 \text{ MeV}$</th>
<th>$\Gamma_1 = 0.033 \text{ MeV}$</th>
<th>$\Gamma_2 = 0.088 \text{ MeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward Differential Cross Section</td>
<td>$B_0$</td>
<td>$B_1$</td>
<td>$C_0$</td>
<td>$C_1$</td>
<td>$B$</td>
</tr>
<tr>
<td>Even</td>
<td>$2(A_o^S + A_2^S + A_4^S)$</td>
<td>-8.793±2.684</td>
<td>-8.923±2.713</td>
<td>-10.248±2.232</td>
<td>22.878±2.537</td>
</tr>
<tr>
<td>Odd</td>
<td>$2(A_1^S + A_3^S)$</td>
<td>-2.020±2.540</td>
<td>3.847±2.558</td>
<td>-16.025±2.082</td>
<td>-2.145±2.363</td>
</tr>
</tbody>
</table>
For $\frac{5}{2}^-$ and $\frac{7}{2}^-$ doublet, we found from the theoretical calculation that

\[
\begin{align*}
\text{For } \frac{5}{2}^- \\
C_{\phi_5} e^{i\phi_{5\phi_5}} &= -\frac{\sqrt{5}}{16} C_{\phi_5} e^{i\phi_{5\phi_5}} - \frac{\sqrt{15}}{16} C_{\phi_5} e^{i\phi_{5\phi_5}} + \frac{5}{16} C_{\phi_5} e^{i\phi_{5\phi_5}} \\
C_{\phi_5} e^{i\phi_{5\phi_5}} &= \frac{\sqrt{15}}{16} C_{\phi_5} e^{i\phi_{5\phi_5}} - \frac{\sqrt{5}}{16} C_{\phi_5} e^{i\phi_{5\phi_5}} + \frac{5}{16} C_{\phi_5} e^{i\phi_{5\phi_5}}
\end{align*}
\]

(17)

\[
\begin{align*}
\text{For } \frac{7}{2}^- \\
C_{\phi_7} e^{i\phi_{7\phi_7}} &= \frac{\sqrt{7}}{16} C_{\phi_7} e^{i\phi_{7\phi_7}} - \frac{\sqrt{5}}{16} C_{\phi_7} e^{i\phi_{7\phi_7}} + \frac{5}{16} C_{\phi_7} e^{i\phi_{7\phi_7}} \\
C_{\phi_7} e^{i\phi_{7\phi_7}} &= \frac{\sqrt{5}}{16} C_{\phi_7} e^{i\phi_{7\phi_7}} - \frac{\sqrt{7}}{16} C_{\phi_7} e^{i\phi_{7\phi_7}} + \frac{5}{16} C_{\phi_7} e^{i\phi_{7\phi_7}}
\end{align*}
\]

where $n_f$, $f$, $C_{\phi_\phi_\phi}$, and $\beta_{\phi_\phi}$ stand for spin-nonflip, spin-flip amplitude of the T matrix element, and phase angle of T matrix element, respectively. Once we can find the parameters in Equations (15) and (16), we can determine the values of the amplitude and phase angle of the T matrix element from Equation (17). Then by meshing the T matrix elements to the coefficients in Equation (14), we can tell the strengths of the partial waves involved in the scattering.

The values of the parameters corresponding to the fits of Table VII for spin-nonflip inelastic scattering to Equations (15) and (16) are shown in Table IX. The fits were obtained using a computer search routine (given in Appendix B). Unfortunately, it seems impossible to find fits for spin-flip inelastic scattering. Nor can we tell the relative intensities for the various partial waves in the inelastic decay of the doublet.
Table IX

Values of the parameters corresponding to the fits of forward spin-nonflip inelastic scattering cross sections to equations 15 and 16.
### Table IX

<table>
<thead>
<tr>
<th>Be</th>
<th>Bo</th>
<th>C^R1</th>
<th>C^R2</th>
<th>(α^e - β^e)</th>
<th>(α^o - β^o)</th>
<th>(ϕ^e - ϕ^e)</th>
</tr>
</thead>
</table>
In summary, the present results indicate that the inelastic decays of the $\frac{7}{2}^-$ and $\frac{5}{2}^-$ levels are by both p and f waves.
BIBLIOGRAPHY


So 75  M. Soga, private communication.

We 72  Computer routine obtained from J. J. Ramirez; written by H. R. Weller, University of Florida.
APPENDIX A

This appendix gives a listing of the computer routine which makes theoretical expressions for a partial wave analysis of inelastic proton scattering and spin-flip inelastic proton scattering in $^{12}_c(P,p')^{12}_c$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
MAIN PROGRAM

PURPOSE:
    TO GENERATE THEORETICAL EXPRESSIONS FOR A PARTIAL
    WAVE ANALYSIS OF INELASTIC PROTON SCATTERING AND
    SPIN-FLIP INELASTIC PROTON SCATTERING IN 12C(P,p')12C

SUBROUTINES TO BE USED:
    AI
    B
    CP
    PC
    CG
    MCMDI
    IPR

DIMENSION II(2000,3),INDEX(2000),A(4)
COMMON II,INDEX
DATA A('A0','A2','A4','A6')
DO 100 MM=2,3,-1
DO 1 MM=-MM,MM,MM
DO 2 IS1=1,-1,-2
IS2=IS1
IF(MM.EQ.0) IS2=-IS1
CALL AI(M,IS1,IS2)
CALL B
CALL CP
CONTINUE
IF(MM.EQ.0) GO TO 3
CONTINUE
1
END FILE 12
END FILE 12
END FILE 14
END FILE 16
10 FORMAT('I',1X,'42','ABS(M)='11)
WRITE(17,10) A(1),MM
CALL IFILE(24,'FOR10')
CALL PC
CALL IFILE(24,'FOR12')
WRITE(17,10) A(2),MM
CALL PC
CALL IFILE(24,'FOR14')
WRITE(17,10) A(3),MM
CALL PC
CALL IFILE(24,'FOR16')
WRITE(17,10) A(4),MM
CALL PC
END FILE 10
END FILE 12
END FILE 14
END FILE 16
CALL OFILE(10,'FOR10')
CALL OFILE(12,'FOR12')
CALL OFILE(14,'FOR14')
100 CALL OFILE(16,'FOR16')
END

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
SUBROUTINE AI

PURPOSE:
TO CALCULATE THE SCATTERING AMPLITUDE IN TERMS OF SPHERICAL HARMONICS,

SUBROUTINES TO BE USED:
CG
NCOMD1

SUBROUTINE AI(M1,MJ,MSF)

DIMENSION NN(3), ND(3), J1(3), M1(3), J2(3), M2(3), MMA(3),
1 M(3), JJJ(3), MM(3), NYN(0/5,-5/5), NYD(0/5,0/5)

DATA NYN/5=0,693,4=0,315,4=0,35,0=385,0=0,15,0=45,0,
10=45,4=0=35,0=355,4=0,315,6=0=693/
DATA NYD/1,0,4,0,64,0,0,2,16,0,128,0,0,8,32,4=0,
116,0,256,4=0,128,0,256/
J2(1)=1
J2(2)=1
J1(3)=4

CALL IFILE(20,'DATA1')
READ (20,12,END=160) J1(1), J(1), J1(2), J(2)
J1=J1(1)/2
J12=J1(2)/2
J(3)=J(1)
J2(3)=J(2)
M1(3)=2*MSF
M2(3)=MSF

DO 120 MK=J2(3),-J2(3),-2
M2(3)=MK
M(3)=M1(3)+M2(3)

IF (IABS(M(3)),GT,J(3)) GO TO 120
M(2)=M2(3)
IF (IABS(M(2)),GT,J(2)) GO TO 120
M(1)=M3(3)
IF (IABS(M(1)),GT,J(1)) GO TO 120
M1(2)=M(2)-M2(2)
IF (IABS(M1(2)),GT,J1(2)) GO TO 120
M1(1)=M(1)-M2(1)
IF (IABS(M1(1)),GT,J1(1)) GO TO 120
MT=J1(2)*M1(2)*((J1(2)+M1(2))/4)**4
IF (MT,NE,0) GO TO 120
MT=J1(1)*M1(1)*((J1(1)+M1(1))/4)**4
IF (MT,NE,0) GO TO 120

DO 2 I=1,3
CALL CG(J1(1),M1(1),J2(1),M2(1),J(1),M(1),NN(1),ND(1))
IF(NN(1),EQ,2,OR,ND(1),EQ,2) GO TO 120
CONTINUE

DO 3 I=1,3
JJJ(I)=J1(I)/2
MM(I)=M1(I)/2

MMA(I)=IABS(MM(I))
NN1=NN(I)NN(2)*NN(3)*NYN(JJJ(I),MM(I))*NYN(JJJ(2),MM(2))
ND1=ND(I)ND(2)*ND(3)*NYD(JJJ(I),MM(I))*NYD(JJJ(2),MM(2))

CALL NCOMD1(NN1,ND1)
WRITE(21,15)NN1,ND1,MM(2),J1(2),J1(I)
WRITE(22,15)NN1,ND1,MM(2),J1(2),J1(I)

120 CONTINUE

160 CONTINUE

3

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
120 CONTINUE
   GO TO 1
9   FORMAT(3I)
10   FORMAT(4I)
15   FORMAT(1X,2I7,2(1,1,12))
160 END FILE 20
END FILE 21
END FILE 22
RETURN
END
SUBROUTINE B

PURPOSE:
   TO CALCULATE THE DIFFERENTIAL CROSS SECTIONS BY
   SQUARING THE SCATTERING AMPLITUDE.

SUBROUTINES TO BE USED:
   NCOMDI

SUBROUTINE B

READ(21,10,END=1000)NN1,ND1,NE1,AT1,NT1,AT11,NT11
READ(22,10,END=999,NN2,ND2,NE2,AT2,NT2,AT22,NT22
NN=NN1*NN2
ND=ND1*ND2
NE=NE1*NE2
CALL NCOMDI(NN,ND)
WRITE(23,11)NN,ND,NE,AT1,NT1,AT11,NT11,
   AT2,NT2,AT22,NT22
GO TO 2

999  END FILE 22
GO TO 1

10  FORMAT(1X,217,I2,2(A1,I2))
11  FORMAT(1X,2110,I2,4(A1,I2))
1000 END FILE 23
END FILE 22
END FILE 21
RETURN
END
SUBROUTINE CP

PURPOSE:
    TO EXPRESS THE DIFFERENTIAL CROSS SECTIONS IN TERMS OF LEGENDRE POLYNOMIALS (ORDER UP TO 6).

SUBROUTINES TO BE USED:
   ONE

SUBROUTINE CP
<22N=4
<22D=3
<23N=1
<23D=3
<44N=64
<44D=35
<42N=16
<42D=21
<41N=1
<41D=15
K66N=512
K66D=231
K64N=1152
K64D=1155
K62N=4
K62D=21
K62D=1
K62D=35
I11=1
I12=2
98 REA0(23,72,END=1000)I1, I2, IE, A1, J1, A2, J2, A3, J3, A4, J4
99 GO TO (98, 98, 98, 98, 98, 98, 98, 98, 200, 98, 402, 98, 700), I1
228 WRITE(12,73)I11, I12, I1, I2, A1, J1, A2, J2, A3, J3, A4, J4
228 GO TO 98
400 WRITE(14,73)K44N, K44D, I1, I2, A1, J1, A2, J2, A3, J3, A4, J4
400 GO TO 98
700 WRITE(16,73)K66N, K66D, I1, I2, A1, J1, A2, J2, A3, J3, A4, J4
700 GO TO 98
72 FORMAT(1X,2I10, I2, 1, A1, I2)
73 FORMAT(1X,2I15, 2I10, 4(A1, I2))
1000 END FILE 23
RETURN
END
SUBROUTINE CG

PURPOSE:
TO CALCULATE THE VALUE OF CLEBSCH-GORDAN COEFFICIENT.

SUBROUTINES TO BE USED:
NCOIMDI

SUBROUTINE CG(JJ1,MM1, JJ2, MM2, J, M, NN, ND)
J1=JJ1
M1=MM1
J2=JJ2
M2=MM2
NSIGN=1
IF(J2.LE.4) GO TO 99
J1=JJ1
M1=MM1
M2=MM2
MS=J1*J2-(J1+J2-J)/4.*4
IF (MS.NE.0) NSIGN=-1
99 GO TO (100,200,300,400),J2
100 IF(M2) 120,110,110
110 IF(J=J1) 112,111,111
111 NN=J1+M+1
ND=2*J1+2
GO TO 500
112 NSIGN=NSIGN
NN=J1+M+1
ND=2*J1+2
GO TO 500
120 IF(J1) 121,121,121
121 NN=J1+M+1
ND=2*J1+2
GO TO 500
200 IF (M2) 230,220,210
210 IF (J=J1) 213,212,211
211 NN=(J1+M)*J1+M*2
ND=(2*J1+2)*(2*J1+4)
GO TO 500
212 NSIGN=NSIGN
NN=(J1+M)*J1+M*2
ND=2*J1+2
GO TO 500
213 NN=(J1+M)*J1+M*2
ND=2*J1+2
GO TO 500
220 IF (J=J1) 223,222,221
221 NN=(J1+M2)*(J1+M2)
ND=(2*J1+2)*(J1+2)
GO TO 500
222 IF (M2) 230,220,210
223 NSIGN=NSIGN
NN=(J1+M)*J1+M
ND=J1+M
GO TO 500
Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
230 IF (J=J1) 233,232,231
231 NN=(J1=M)*(J1=M+2).
ND=(2*J1+2)*(2*J1+4)
GO TO 500
232 NN=(J1=M)*(J1=M+2)
ND=2*J1*(J1+2)
GO TO 500
233 NN=(J1=M+2)*(J1=M)
ND=2*J1*(2*J1+2)
GO TO 500
300 JJ=(J1+5)/2
MM=(M+2)/2
GO TO (340,330,320,310), JJ
310 GO TO (314,313,312,311), JJ
311 NN=(J1=M+1)*(J1=M+1)*(J1=M+3)
ND=(2*J1+2)*(2*J1+4)*(2*J1+6)
GO TO 500
312 NSIGN=NSIGN
NN=3*(J1=M+1)*(J1=M+1)*(J1=M+3)
ND=2*J1*(2*J1+2)*(2*J1+6)
GO TO 500
313 NN=3*(J1=M+1)*(J1=M+1)*(J1=M+3)
ND=(2*J1+2)*(2*J1+4)*(2*J1+6)
GO TO 500
314 NSIGN=NSIGN
NN=(J1=M+1)*(J1=M+1)*(J1=M+3)
ND=2*J1*(2*J1+2)*(2*J1+6)
GO TO 500
320 GO TO (324,323,322,321), JJ
321 NN=3*(J1=M+1)*(J1=M+3)*(J1=M+3)
ND=(2*J1+2)*(2*J1+4)*(2*J1+6)
GO TO 500
322 IF ((J1=3*M+3);GT,0) NSIGN=NSIGN
NN=(J1=3*M+3)*(J1=3*M+3)*(J1=M+1)
ND=2*J1*(2*J1+2)*(2*J1+6)
GO TO 500
323 IF ((J1=3*M+3);LT,0) NSIGN=NSIGN
NN=(J1=3*M+3)*(J1=3*M+3)*(J1=M+1)
ND=(2*J1+2)*(2*J1+4)*(2*J1+6)
GO TO 500
324 NN=3*(J1=M+1)*(J1=M+1)*(J1=M+1)
ND=2*J1*(2*J1+2)*(2*J1+6)
GO TO 500
330 GO TO (334,333,332,331), JJ
331 NN=3*(J1=M+3)*(J1=M+1)*(J1=M+3)
ND=(2*J1+2)*(2*J1+4)*(2*J1+6)
GO TO 500
332 IF ((J1=3*M+3);LT,0) NSIGN=NSIGN
NN=(J1=3*M+3)*(J1=3*M+3)*(J1=M+1)
ND=2*J1*(2*J1+2)*(2*J1+6)
GO TO 500
333 IF ((J1=3*M+3);GT,0) NSIGN=NSIGN
NN=(J1=3*M+3)*(J1=3*M+3)*(J1=M+1)
ND=(2*J1+2)*(2*J1+4)*(2*J1+6)
GO TO 500
334 NSIGN=NSIGN
NN=3*(J1=M+1)*(J1=M+1)*(J1=M+1)
ND=2*J1*(2*J1+2)*(2*J1+6)
GO TO 500
340 GO TO (344,343,342,341), JJ

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
433 GO TO 500
IF (3*JM*H+1*J1+2) LT 0 NSIGN = NSIGN
NN = (3*JM*H+1*J1+2)*2
ND = (2*J1+2)*J1*(J1+1)/(2*J1+1)
GO TO 500

434 IF (J1+H,GT,0) NSIGN = NSIGN
NN = (J1+H)*J1*(J1+1)/(J1+2)
ND = (2*J1+1)/(2*J1+2)
GO TO 500

435 IF (J1-H,GT,0) NSIGN = NSIGN
NN = (J1-H)*J1*(J1+1)/(J1+2)
ND = (2*J1+1)/(2*J1+2)
GO TO 500

440 GO TO (444,444;443;442/441) JJ

441 NN = (J1-H)*J1*(J1+1)/(J1+2)
ND = (2*J1+1)/(2*J1+2)
GO TO 500

442 IF ((J1-H+2)*M*4,LT,0) NSIGN = NSIGN
NN = (J1-H+2)*M*4*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

443 IF (((J1-H+2)*M+2,LT,0) NSIGN = NSIGN
NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

444 IF (((J1-H+2)*M+2,LT,0) NSIGN = NSIGN
NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

445 NN = NSIGN
NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

450 GO TO (455,454;453;452/451) JJ

451 NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

452 NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

453 NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

454 NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

455 NN = (J1-H+2)*M+2*(J1-H+2)*M
ND = 1*(2*J1+1)/(2*J1+2)
GO TO 500

500 CALL NCOMDI(NN;ND)
NN = NSIGN
RETURN
END
SUBROUTINE NCOMDI

PURPOSE:
   TO REDUCE THE RATIO OF TWO INTEGERS TO THE SIMPLEST FRACTION,

SUBROUTINES TO BE USED:
   NONE

SUBROUTINE NCOMDI(N1,N2)

   IF (N1.EQ.0 OR N2.EQ.0) GO TO 5
   IF (N2.LT.0) N1=-N1
   N2=ABS(N2)

   IF (N1.NE.N2) 1,2,3
   N2=N2/(N2/N1)*N1
   IF (N2.EQ.0) GO TO 2
   GO TO 4

3   N1=N1/(N1/N2)*N2
   IF (N1.EQ.0) N1=N2
   GO TO 4

2   N10=N10/N1
   N20=N20/N1

5   RETURN
END
SUBROUTINE IPR

PURPOSE:
   TO REDUCE A SQUARE ROOT TO ITS SIMPLEST FORM AND
   MULTIPLY THE RESULTING COEFFICIENT WITH THE INITIAL
   COEFFICIENT OF THE SQUARE ROOT.

SUBROUTINES TO BE USED:
   NONE

SUBROUTINE IPR(IN,IS)
   IF(IS,LT,0) IN=-IN
   IS=ABS(IS)
   ISS=SQRT(FLOAT(IS))
   DO 1 J=2,ISS
   IF(I,LT,J) GO TO 4
   IA=I
   3 CONTINUE
   IA=IA/J
   IF(IA,NE,(IA/J)*J) GO TO 1
   1 CONTINUE
   4 F=IS/I
   IB=SQRT(F)*0.5
   IN=IN*IB
   IS=I
   RETURN
END
SUBROUTINE PC

PURPOSE
    TO COMBINE AND TO SORT THE TERMS IN EACH COEFFICIENT
    OF LEGENDRE POLYNOMIAL TO MAKE A COMPACT FORM.

SUBROUTINES TO BE USED
    IPR
    NCOMDI

SUBROUTINE PC
COMMON II(2020,6), INDEX(2020)
I=1
READ(24,11,END=2)(II(I,J),J=1,9)
FORMAT(1X,2I5,2I10,4(A3))
INDEX(I)=1
CALL IPR(II(I,1),II(I,3))
CALL IPR(II(I,2),II(I,4))
CALL NCOMDI(II(I,1),II(I,2))
I=I+1
GO TO 1
2
IF (N.EQ.0) GO TO 7
J=1
5
IF(J.GE.N) GO TO 71
DO 3 J=1,N
IF((II(J,5),EQ,II(I,7),AND;
II(J,6),EQ,II(I,8),AND;
II(J,7),EQ,II(I,5),AND;
II(J,8),EQ,II(I,6),OR;
II(J,5),EQ,II(I,5),AND;
II(J,6),EQ,II(I,6),AND;
II(J,7),EQ,II(I,7),AND;
II(J,8),EQ,II(I,8))) GO TO 4
3 CONTINUE
J=J+1
GO TO 5
4
IF(II(J,3),EQ,II(I,3),AND;
II(J,4),EQ,II(I,4)) GO TO 8
II(J,3)=II(J,3)+II(J,4)
CALL IPR(II(J,1),II(J,3))
II(J,2)=II(J,2)+II(J,4)
CALL NCOMDI(II(J,1),II(J,2))
II(J,4)=1
II(J,3)=II(J,3)+II(J,4)
CALL IPR(II(J,1),II(J,3))
II(J,2)=II(J,2)+II(J,4)
CALL NCOMDI(II(J,1),II(J,2))
II(J,4)=1
II(J,1)=II(J,1)+II(J,2)+II(I,1)+II(J,2)
II(J,2)=II(J,2)+II(I,2)
CALL NCOMDI(II(J,1),II(J,2))
71
DO 6 L=1,N
IF(II(I,6)=II(I,8)) 83,62,63
6
DO 6 M=1,8
II(L,M)=II(L,M)
GO TO 5
71
DO 63 I=1,N
IF(II(I,6)=II(I,8)) 83,62,63
63

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>IF(J(J1,5)=J(J1,7)) 83,83,63</td>
</tr>
<tr>
<td>63</td>
<td>J1=INDEX(I)</td>
</tr>
<tr>
<td>64</td>
<td>J2=INDEX(I+1)</td>
</tr>
<tr>
<td>65</td>
<td>IF(J(J1,6)=I(J2,5)) 76,78,79</td>
</tr>
<tr>
<td>66</td>
<td>INDEX(I)=J2</td>
</tr>
<tr>
<td>67</td>
<td>INDEX(I+1)=J1</td>
</tr>
<tr>
<td>68</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>69</td>
<td>DO 73 L=1,N</td>
</tr>
<tr>
<td>70</td>
<td>DO 76 I=1,N,L</td>
</tr>
<tr>
<td>71</td>
<td>J1=INDEX(I)</td>
</tr>
<tr>
<td>72</td>
<td>J2=INDEX(I+1)</td>
</tr>
<tr>
<td>73</td>
<td>IF(J(J1,6)=I(J2,5)) 76,78,79</td>
</tr>
<tr>
<td>74</td>
<td>INDEX(I)=J2</td>
</tr>
<tr>
<td>75</td>
<td>INDEX(I+1)=J1</td>
</tr>
<tr>
<td>76</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>77</td>
<td>GO TO 76</td>
</tr>
<tr>
<td>78</td>
<td>IF(J(J1,5)=J(J2,5)) 76,80,79</td>
</tr>
<tr>
<td>79</td>
<td>IF(J(J1,8)=J(J2,8)) 76,81,79</td>
</tr>
<tr>
<td>80</td>
<td>IF(J(J1,7)=J(J2,7)) 76,78,79</td>
</tr>
<tr>
<td>81</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>82</td>
<td>DO 75 I=1,N</td>
</tr>
<tr>
<td>83</td>
<td>WRITE(17,12) (INDEX(I),J),J=1,8</td>
</tr>
<tr>
<td>84</td>
<td>FORMAT(1X,'(I6,I6,'/'I6,'/'I4,'/'I4,')',A3</td>
</tr>
<tr>
<td>85</td>
<td>1,1/21,'A3,'/2,1,'A3,'/2)</td>
</tr>
<tr>
<td>86</td>
<td>END FILE 24</td>
</tr>
<tr>
<td>87</td>
<td>RETURN</td>
</tr>
<tr>
<td>88</td>
<td>END</td>
</tr>
</tbody>
</table>
APPENDIX B

This appendix gives a listing of the computer search routine which was used to obtain the fits of forward cross sections to Equations (15) and (16).
MAIN PROGRAM

PURPOSE:
TO FIT FORWARD CROSS SECTIONS TO Eqs. 15 AND 16.
(LEAST CHI-SQUARE BY THE GRADIENT METHOD)

SUBROUTINES TO BE USED
BC

DIMENSION X(0:7),CH(7),M(7),Y(10),YE(10),YD(10),YDE(10),
1N(7),XT(7),OUT(5)
COMMON X,Y,YE,PHI,YD,YDE,DUM
DATA OUT/'BO', 'B1', 'C0', 'Ci', 'Dk'/
DELQ=2.0
TYPE 41
INPUT=NOFP
ACCEPT 42,YES
IF(YES.EQ.'Y') INPUT='FLP'
CALL IFILEC20,INPUT)
READ(20,43)YE,YDE,DUM

TYPE 44
ACCEPT 45,M

ACCEPT 47,(X(I),I=1,7)

ACCEPT 47,(X(I),I=5,7)

999 0=1.0
N=1
PHIL=1000000000.

TYPE 49
N=1

XY='1'

1000 CALL BC

TYPE 50,PHI,Q,(X(I),I=1,7)

1001 IF(PHI.LT.PHIL) GO TO 3000

2000 DO 2001 I=1,7
IF(M(I),EQ.0) GO TO 2001
X(I)=X(I)-Q*CH(I)/XNORM

2001 CONTINUE
IF(XY.EQ.'F') GO TO 5000

Q=Q/DELQ

3000 XNORM=0.0

PHIL=AMIN1(PHI,PHIL)

DO 3001 I=1,7
IF(M(I),EQ.0) GO TO 3001
X(I)=X(I)+.0005

CALL BC

X(I)=X(I)+.0005

CH(I)=PHI-PHIL

XNORM=XNORM*CH(I)**2

3001 CONTINUE

XNORM=SQRT(XNORM)

DO 3002 I=1,7
IF(M(I),EQ.0) GO TO 3002
X(I)=X(I)-Q*CH(I)/XNORM

3002 CONTINUE

N=N-1

IF(N=5) 1000,8000,7000

8000 CALL BC

TYPE 51,PHI,Q,(X(I),I=1,7)
GO TO 1001

ACCEPT 42, XY
IF(XY,EQ,'S') GO TO 5001
IF(XY,EQ,'F') GO TO 2000
GO TO 1000

CALL BC
TYPE 52: PHI, Q, (X(1), I=1,7):
1((OUT(I), Y(I), YE(I), YD(I), Y(I+5), YE(I+5), YD(I+5)), I=1,5)

TYPE 93
ACCEPT 42, YES
IF(YES, NE, 'Y') CALL EXIT
TYPE 54
ACCEPT 42, YES
IF(YES, NE, 'Y') GO TO 5002
TYPE 44
ACCEPT 45, M

TYPE 55
ACCEPT 42, YES
IF(YES, NE, 'Y') GO TO 999
TYPE 56
ACCEPT 45, IN
TYPE 57
ACCEPT 58, XT
GO 5004 I=1,7

TYPE 93
ACCEPT 42, YES
IF(YES, NE, 'Y') GO TO 999
TYPE 56
ACCEPT 45, IN
TYPE 57
ACCEPT 58, XT
GO 5004 I=1,7

FORMAT(1X, 'SPIN-FIIP?YES OR NO :', S)
FORMAT(A1)
FORMAT(5F)
FORMAT(1X, 'SEARCHI:', S)
FORMAT(7I1)
FORMAT(1X, 'TYPE X1 X2 X3 X4 X5')
FORMAT(4F)
FORMAT(1X, 'TYPE X5 X6 X7 X8')
FORMAT(6X, 'PHI X1', 'Q1 X7 X1', '6 X2', '6 X3'
16 X', 'X4', 'X5', '6 X6', 'X6', 'X7')
FORMAT(1X, 2F8.3, /1X, 7F8.3)
FORMAT(1X, 2F8.3, /1X, 7F8.3, S)
FORMAT(1X, 'FINAL PARAMETERS 1 1 X', 'PHI=', 'F10.5'
13X, 'Q1', 'F10.5', '4(2X, F10.5)', '3(2X, F10.5)', '7X', 'CA
1LEVEN EXP=EVEN DIF=EVEN CAL=ODD Exp=ODD
1DIF=ODD', 5(2X, A2, 1X, 5, F10.5, ))
FORMAT(1X, 'TRY AGAIN!', S)
FORMAT(1X, 'CHANGE SEARCH?', S)
FORMAT(1X, 'CHANGE X?', S)
FORMAT(1X, 'X TO BE CHANGED?', S)
FORMAT(1X, 'TYPE THEM!')
FORMAT(7F)
END
SUBROUTINE BC

PURPOSE:
TO EVALUATE THE COEFFICIENTS IN Eqs. 15 AND 16.
TO DEFINE THE CHI-SQUARE.

SUBROUTINES TO BE USED:
NONE

SUBROUTINE BC
DIMENSION X(0:7),Y(10),YE(10),YD(10),YDE(10)
COMMON X,Y,YE,PHI,YD,YDE,DSUM

Y(1)=X(3)*X(2)*X(3)*SIN(X(6)-X(7))*2.7273
Y(2)=2.*X(2)*X(3)*COS(X(6)-X(7))
Y(3)=X(3)*X(4)*SIN(X(7))
Y(4)=2.*X(2)*X(4)*COS(X(6))+2.7273*X(3)*X(4)*
SIN(X(7))
Y(5)=X(1)*X(2)*X(3)*X(4)*SIN(X(5)-X(7))
Y(6)=2.*X(1)*X(3)*SIN(X(5)-X(7))
Y(7)=2.*X(1)*X(4)*COS(X(5)-X(7))
Y(8)=2.*X(1)*X(4)*COS(X(5))
Y(9)=2.*X(1)*X(2)*COS(X(5)-X(6))

DO 1 I=1,10

YD(I)=Y(I)-YE(I)

PHI=PHI*(YD(I)/YDE(I))**2

RETURN
END