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Problem Solving Like a Mathematician: Disciplinary Literacy Instruction in Elementary Mathematics

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Abstract

This study aims to grow the literature by examining the how of disciplinary literacies (DL) elementary mathematics instructional practices in teacher education, which is under-reported. Using qualitative case study methods, we examined how one cohort of elementary preservice teachers (PTs) enacted the DL instructional practices in their field practicum K–6 classrooms. Using a social constructivism perspective, we combined directed content analysis with real-time dialogue and the writing process as analytical tools to examine our data. Data sources included observation notes from the mathematics methods course, observation notes from the field practicum, and multiple artifacts (e.g., post observation oral and written reflections, lesson plans). Five findings emerged: The PTs (1) employed instructional practices to teach comprehension of the complex texts of mathematics, (2) provided instructional practices to build academic knowledge in mathematics, (3) taught the academic vocabulary unique to mathematics, (4) included instructional spaces for mathematical inquiry, and (5) used instructional strategies customized or adapted for the literacies of mathematics. While there were some limitations to our study, we found that elementary preservice teachers were able to enact the DL instructional approach when teaching mathematics to various levels of sophistication. We also found that the productive struggle approach emerged as a specialized instructional method specific to supporting the disciplinary literacies of mathematics, and it was enacted across several of the instructional tenets.

Key words: disciplinary literacies, mathematics, elementary preservice teacher education, social constructivism, and productive struggle
“Students who never expressed much interest in mathematics before were wholeheartedly putting their efforts in…. The problem-solving challenge changed my classroom for the better.” (Nicole)

We begin by highlighting a quote from Nicole (all names are pseudonyms), an elementary (K–6) preservice teacher (PT), as she reflected on the use of an instructional approach termed productive struggle in a fifth-grade mathematics lesson. As co-investigators, we observed not only Nicole, but also a cohort of 13 other PTs in the context of their mathematics methods course (taken in semester three of a four-semester/2-year program). We also followed them in their practicum field placements to understand how they enacted the disciplinary literacies (DL) of mathematics in their field placements (that occurred in semesters three and four).

**Review of Research on Disciplinary Literacies**

In order to forefront the concepts and terminology we refer to throughout this article, we introduce three perspectives surrounding (1) literacy, texts, and instructional strategies that support literacy; (2) DL in mathematics; and (3) elementary DL. All three of these perspectives inform our fourth perspective of DL instruction, which is the central focus of our study.

**Perspectives on Literacy, Texts, and Instructional Strategies for Literacy**

**Literacy**

We agree with scholars who describe literacy as a meaning-making process (National Council of Teachers of English [NCTE], 2019) that honors the broadest ways to make meaning using reading, writing, speaking, listening, viewing, computing, hand signing/gestures, gaming, performing, and so on as tools, processes, or experiences for thinking and learning in discipline-based practices while interacting with others (Draper et al., 2010; Moje, 2008). Draper and Siebert (2010) specifically defined literacy as “the ability to negotiate (e.g., read, view, listen, taste, smell, critique) and create (e.g., write, produce, sing, act, speak) texts” (p. 30).

**Texts**

In this same trajectory of thinking, we also embrace a broadened view of text (Draper & Siebert, 2010; New London Group, 1996) to “capture all of the objects that students need to make sense of in order to gain content knowledge” (Draper & Siebert, 2010, p. 29). In short, a text can include a broad array of representations and objects (Siebert & Draper, 2012) that are read/interpreted (Draper & Siebert, 2010). An object can be described as a novel for book enthusiasts, gestures for teachers (Wilson et al., 2014), an art piece for artists (Buelow et al., 2018), a body for dancers (Frambaugh-Kritzer et al., 2015), clothing for fashion stylists (Buelow, 2015), or a graduated cylinder for scientists (Draper & Siebert, 2010). Indeed, each discipline area presents unique texts to identify, understand, and interpret (C. Shanahan, 2015). For example, if an elementary teacher provides students a base ten block manipulative (as a text), we assert literacy (meaning-making) instruction is needed so students understand not only the mathematical knowledge, but that this base ten text represents a physical model of our place value system of numeration. Further instruction is also needed to help students comprehend what the single-unit cubes and rods represent.
In considering print-based texts students will encounter in mathematics classrooms, Fang and Chapman (2020) found they were “dense, abstract and complex, and the information in these texts is often packaged in ways different from how ideas are typically presented in the more mundane texts of everyday life” (p. 12). Additional mathematical texts may include charts, graphs, diagrams, tables, symbols, timetables, invoices, matrices, and math tools (e.g., ruler, protractor; de Lange, 2003). Moreover, mathematical equations that students encounter have their own unique language and have a particular organization and structure (C. Shanahan, 2015). Students must be able to read and comprehend mathematical texts to develop their mathematical DL (Picot, 2017).

**Instructional Strategies for Literacy**

For the purposes of this study, we use the term *instructional strategies for literacy* and include only instructional strategies that specifically support literacy growth. We further differentiate these instructional strategies for literacy to be more generic or discipline specific. Next, we describe three strategies to illustrate our definitions of these notions.

To begin, we argue the K-W-L chart (Ogle, 1986) is a generic instructional strategy for literacy. Ogle, the original creator of the K-W-L chart (K = What do I KNOW? W = What I WANT/need to know? L = What did I LEARN?) technically called it a “teaching model” that develops “active reading for expository text” (p. 564). This is why K-W-L is a popular strategy in the field of content area literacy (CAL; Fengjuan, 2010), which means it can be implemented across the discipline areas. In fact, many refer to K-W-L as a literacy strategy (Fisher et al., 2002; Tompkins, 2013). The reason these terms/strategies get used interchangeably is that K-W-L promotes the cognitive and metacognitive processes in which readers engage (Duffy et al., 1987; Keene & Zimmermann, 1997; Paris et al., 1984). Specifically, the K promotes activating a student’s prior knowledge, the W promotes asking questions, and the L promotes retelling or synthesizing what students have read.

When a teacher is employing K-W-L in a generic manner in mathematics, it may be to engage students in the inquiry of a mathematical concept (e.g., What do you know about fractions?). However, DL scholars argue for more nuanced and customized strategies specific to the discipline area (Buehl, 2011; T. Shanahan & Shanahan, 2008). This has resulted in generic instructional strategies such as K-W-L being greatly altered. For example, if a math teacher asks students to engage from a DL perspective, K-N-W-S (K = What facts do I KNOW from the information in the math problem? N = Which information do I NOT need? W = WHAT does the problem ask me to find? S = What STRATEGY/operation/tools will I use to solve the problem?; Barton & Heidema, 2002) will better guide students into discipline-specific ways of thinking like a mathematician and represents a stronger customized strategy.

We posit that games are another instructional strategy for literacy that can be adapted/customized to support elementary mathematics learning (Ferreira et al., 2012). In the generic sense, Gee (2008) explained, a well-designed game includes opportunities such as problem solving toward a specific goal and getting immediate feedback. Playing customized games for math and “analyzing optimal strategies to win a game” (DeLegge & Ziliak, 2021, p. 148) offer learners the ability to develop many of the mathematical habits of mind (MHM), such as challenging solutions, guessing, thinking abstractly, and looking for patterns (Levasseur & Cuoco, 2003).
Finally, we introduce the think-aloud strategy (Wilhelm, 2001) as an instructional strategy for literacy because it is not only popular within CAL, but widely suggested for math teachers to adapt in their practices (Bernadowski, 2016). In short, teachers can apply this strategy to specifically demonstrate how mathematicians read and think as they make sense (gain comprehension) of any given text or problem (Wilhelm, 2001). This metacognition, or “thinking about thinking,” is at the “heart of think alouds” (Bernadowski, 2016, p. 4). The think-aloud strategy is also fitting to engage in the MHM (Johnson et al., 2011) because it requires the teacher to be vulnerable as they demonstrate their internal thoughts (e.g., describe the patterns they found, share questions they have, express confusion, challenge solutions). Teachers can also invite students to think aloud (Baumann et al., 1993) in order for them to demonstrate how they solve and or struggle with mathematical problems once they gain understanding of the strategy.

**Perspectives on Disciplinary Literacies in Mathematics**

To situate what we mean by DL in a general sense, we draw on C. Shanahan’s (2015) explanation: “Rather than focusing on the similarities of literacy in the content areas, disciplinary literacy focuses on the differences. And literacy in the various content area subjects is, indeed, different” (p. 1). DL examines the specialized ways of reading, writing, speaking, and so on determined by each discipline area (T. Shanahan & Shanahan, 2008). Thus, understanding how mathematicians read, write, think, and speak becomes a new reality for elementary teachers as they consider how to best support students to develop disciplinary content knowledge (Hillman, 2014; Moje, 2008) and disciplinary habits of mind (Fang & Coatoam, 2013).

Scholars have examined how experts from the field of mathematics think, read, write, and speak (Fang & Chapman, 2020; Hillman, 2014; National Council of Teachers of Mathematics [NCTM], 2000) in order to foster disciplinary content knowledge and habits of mind. For example, C. Shanahan (2015) explained, “Mathematicians read every word carefully. They know that a misinterpretation of one word can change the meaning of what they read. Any mistake will result in an incorrect answer” (p. 3). Fang and Chapman (2020, p. 12) agreed with others (Levasseur & Cuoco, 2003) that in order for students to think like mathematicians they must develop MHM such as “healthy skepticism, critical-mindedness, rigorous thinking, and close attention to logic and details—that enable them to succeed in mathematics reading and learning.” According to Levasseur and Cuoco (2003), MHM are best developed through problem solving, and they cite several habits such as conserving memory, using alternative representations, carefully classifying, and thinking algebraically. Moreover, as mathematicians engage in a “community of mathematical inquiry” (Goos, 2004, p. 259), they need to know how to model and formulate, transfer and manipulate, infer, and communicate (Kenney et al., 2005). When students engage in the practices of a mathematician, they make viable arguments (Common Core State Standards Initiative, 2010) and challenge solutions (Levasseur & Cuoco, 2003), both of which require the use of precise mathematical language and vocabulary.

**Productive Struggle**

One salient instructional approach that stands specifically tall in mathematics is productive struggle (Hiebert & Grouws, 2007), which engages students in mathematical problem solving, reasoning, and MHM (Hiebert & Grouws, 2007). Essentially, productive struggle guides students toward an understanding of the meaning of a mathematical variable before applying a formula or reading an equation (C. Shanahan, 2015). Productive struggle
allows students to engage with complex tasks and explore a problem prior to instruction (Kapur, 2010; Livy et al., 2018). In classrooms where productive struggle is used, students understand that “breakthroughs often emerge from confusion and struggle” and that they are “capable of doing well in mathematics with effort and perseverance in reasoning, sense making, and problem solving” (NCTM, 2014, p. 52). For educators, this means that they must allow sufficient intervals for students to make their own connections and understanding, engage in inquiry-based problem solving without too much prior explicit instruction on the content, and value persistence in learning over quick response time and shallow explanations (Hiebert & Grouws, 2007). Our synthesis of the literature on productive struggle has allowed us to see how this approach is relational to DL in mathematics and how it takes on many meanings depending on the instructional purpose or context. In sum, learners need time to struggle with mathematical ideas in order to make meaning.

Warrant for Disciplinary Literacies in Elementary Teacher Education

Multiple studies have determined the viability of DL in elementary school settings (Britt & Ming, 2017; Buelow et al., 2018; Frambaugh-Kritzer et al., 2015; Lemley & Hart, 2018; Lemley et al., 2019) as well as early childhood (Mongillo, 2017). Many cite the strong interest elementary children have in sophisticated topics in the discipline areas of science and social studies (Altieri, 2014; Brock et al., 2014) as well as math and engineering (L. E. Shanahan et al., 2016), which provide reason to support the concurrent teaching of basic, intermediate, and disciplinary literacies. Given this reality, we argue that elementary teacher education must also address DL.

In our specific effort to examine related literature in DL in elementary mathematics and in teacher education, we found that the contributions fall short. Kushner and Phillips (2020) acknowledged that there is an overall gap concerning how to prepare PTs for disciplinary literacy instruction. Hence, our specific inquiry is warranted as we seek to contribute more research in this area of elementary teacher education. Nevertheless, we take note of the research in DL secondary mathematics in teacher education (Fang, 2014; Siffrinn & Lew, 2017). While searching for additional perspectives, we appreciated Howell et al. (2021), who recently synthesized the DL literature as it relates to professional development for in-service teachers and found that the literature is more focused “on the what” of DL and less focused “on how to prepare teachers” to face the implementation of DL instruction (p. 14). We aim to fill a gap by examining the how, so our central focus surrounds the instructional practices of DL elementary mathematics. Next, we describe the DL instructional approach that is the central focus of our study.

Perspectives on Disciplinary Literacies Instruction

Ball et al. (2008) argued that teachers need to know mathematics in ways that are “useful for making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students” (p. 404). They must also have “pedagogical content knowledge” of mathematics (Thanheiser et al., 2010, p. 2). However, the standards of mathematical practices (SMP) do not offer overt DL instructional application in how to address the standards (Hillman, 2014). This is perfectly fine as that is not the goal of the standards. In our quest to understand the DL instruction, similar to previous work (Buelow et al., 2018; Frambaugh-Kritzer et al., 2015), we drew from the existing DL literature from elementary and secondary education (Chandler-Olcott et al., 2015; Doerr & Temple, 2016; Fang & Chapman, 2020; Vacaretu, 2008). Again, although we position our work with the understanding that DL focuses on the differences
for each discipline area (C. Shanahan, 2015), our synthesis led us to identify the same five DL instructional tenets (Buelow et al., 2018; Frambaugh-Kritzer et al., 2015) that are applicable to all grade levels and discipline areas. However, these practices are implemented in discipline-specific ways, which are described in Table 1 as we considered them through a disciplinary literacy lens specifically for mathematics.

We respect the many scholars (Ball, 1990; Baroody, 1989; Fuson & Briars, 1990; Izsák, 2008) who have contributed seminal research in the field of elementary mathematics instruction that greatly influences the way teachers instruct math today. Indeed, we noted how some examples from the field of mathematics instruction align with the recommended DL instructional approach we offer (Buelow et al., 2018; Frambaugh-Kritzer et al., 2015), thus we integrated examples into Table 1. We clarify each instructional tenet (left-hand column) in Table 1, followed by a brief explanation of the tenet (middle column) and provide one illustrative instructional example (right-hand column).

Table 1

The Five Instructional Tenets of Disciplinary Literacies (DL) in Mathematics

<table>
<thead>
<tr>
<th>Instructional tenet</th>
<th>Brief explanation</th>
<th>Instructional example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teach comprehension when reading complex discipline texts (Buehl, 2011; Fang, 2012).</td>
<td>Students need to know that rereading is required for mathematical comprehension because precision is necessary (C. Shanahan, 2015) when interpreting various complex texts (Siebert &amp; Draper, 2012; Vacaretu, 2008).</td>
<td>Demonstrate how to precisely comprehend an object like a ruler by specifically pointing out the direction to read (left to right), how to determine what the hash marks mean (inches on a ruler are subdivided into smaller equal units in the center of each inch of the ruler; Gómezescobar et al., 2020), and how to double-check (reread) the measurements to guarantee precise results.</td>
</tr>
<tr>
<td>Build academic knowledge of the discipline (Moje, 2011).</td>
<td>Students learn through firsthand experiences of hands-on manipulation, using models to represent concepts and engaging in authentic problem solving related to the natural world or by understanding the meaning of a mathematical variable before applying a formula or reading an equation (C. Shanahan, 2015).</td>
<td>Scaffold the mathematical academic concept of <em>addition up to 10</em> by presenting the following scenario: “There are 10 cars in the parking lot. Some of the cars are red and some of the cars are black. How many red cars and how many black cars could be in the parking lot? Think of as many different combinations of cars as you can. Show your solutions in as many ways as you can with cubes, drawings, or words, and write an equation for each solution” (NCTM, 2014, p. 21).</td>
</tr>
</tbody>
</table>
Teach the academic vocabulary unique to the discipline (T. Shanahan & Shanahan, 2014).

Students must learn accurate mathematical definitions, such as word families, associations, symbols, roots, and affixes (Hillman, 2014; Picot, 2017), so that they are able to understand the “grammar of a mathematics problem” (Vacaretu, 2008, p. 452).

Model how to record definitions in vocabulary notebooks with specialized categories titled “Mathematics Definition” and “General Definition” (C. Shanahan, 2015, p. 5) with vocabulary like the word of (Carter & Dean, 2006). For example, one possible general definition for of is “distance or direction from.” The mathematics definition for of is “multiply.”

Provide instructional spaces for the inquiry process. Inquiry has many layers and should involve: the following

- students generating their own questions
- teacher role evolving from question asker to question modeler
- using a questioning the text and self-questioning taxonomy
- deep study of the discipline so inquiry can occur through the lens of the discipline or disciplinary perspective (Buehl, 2011)

Students can exercise perseverance in problem solving when engaged in inquiry-based mathematics (NCTM, 2000). Instruction should allow students to work in ways similar to the ways mathematicians think, read, write, and speak (Artigue & Blomhoj, 2013).

Begin inquiry requesting students to think like cartographers by asking them, “What makes the best map?” (Fry, 2013). When problem solving this inquiry, they should consider the mathematics (e.g., distance, time, map’s scale, area, coordinates, size, layout, elevation contours). Students should work collaboratively to generate valued qualities in a map (to negotiate what they meant by the ambiguity of “best”), design a map with these qualities, and seek to convince peers that it fits their criteria for the “best” map (Makar et al., 2015).
Use instructional strategies that are customized or adapted for the unique literacies of each discipline (Gillis, 2014; T. Shanahan & Shanahan, 2008).

Students engage with content more deeply when it is paired with instructional strategies that specifically support literacy growth. As previously discussed, these strategies are customized or adapted to demonstrate how mathematicians read, write, and think as they solve problems.

Use an instructional strategy for a mathematical word problem called RIDGES (Snyder, 1988, as cited in C. Shanahan, 2015, p. 5).

1. (R)ead the problem. If the problem is not understood, read it again.

2. (I)dentify all of the information given in the word problem. List the information separately. After listing all of the information, circle the information that is needed to solve the problem.

3. (D)raw a picture of the information in the problem. This may help a student pick out the relevant information.

4. (G)oal statements. The student should express, in their own words, the question the problem is asking.

5. (E)quation development. The student will write an equation to the problem (e.g., length + width + length + width = distance around the field).

6. (S)olve the equation. The given information is plugged into the equation (e.g., 10 + 6 + 10 + 6 = distance around the field).

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**Theoretical Perspectives**

**Social Constructivism**

We turned to the social constructivism perspective to analyze how the PTs enacted the DL instructional tenets. Social constructivism, largely drawn from Vygotsky’s (1978) sociocultural theory, argues that humans do not learn as isolated individuals. Au (1998) argued despite the many definitions, “at the heart of constructivism is a concern for lived experience, or the world as it is felt and understood by social actors” (p. 299). Hence, when we refer to social constructivism, we mean knowledge is constructed as a result of *doing* and through cultural experiences and interactions that takes place with others and the environment (Au, 1998; Bruner 1986). Since “social actors” (i.e., learners) are at the center of their meaning making in a sociocultural environment (Mitchell & Myles, 2004), they need time to generate ideas with others (Vygotsky, 1978) as they make
sense of the world. That is, through multiple interactions and social settings, learners are constantly reconstructing knowledge in their own minds (Buelow et al., 2018; Frambaugh-Kritzer et al., 2015). Learners construct new knowledge as they build on and from their personal and social backgrounds. Finally, learning is an active process in which learners are always using their existing knowledge and experiences to construct new meaning with others (Bruner, 1986).

The social constructivism perspective that informs learning assumptions is helpful in examining our study because we sought to understand how PTs experienced and learned the DL of mathematics while in Dr. Lowe’s class and how the PTs enacted their learning in their instructional practice in elementary classrooms. Adams and Pegg (2012) explained how a social constructivism perspective informs literacy practices that are centered on students’ “active engagement in knowledge construction” (p. 152). In large part, learners are socially constructing meaning in their context by interacting with others, and this is why, Smith (1992) argued, “we learn from the company we keep” (p. 432). The main pillars of social constructivism—learning by doing and learning from interaction with others—guided our analysis.

Methods

This qualitative case study is “an empirical inquiry that investigates a contemporary phenomenon within its real-life context…and multiple sources of evidence are used” (Yin, 1994, p. 23). We asked: How do elementary PTs enact disciplinary literacies instruction in mathematics?

Context

This study was conducted in the western United States at a research university that offers an undergraduate elementary education teacher preparation program. The PTs we observed were assigned a cohort, which means they took all of their courses together over two consecutive years (four semesters) and were supervised by a cohort coordinator (Stephanie) who observed them in the field and oversaw their progression through the program. Thus, we also collected data in the surrounding public K–6 elementary schools that our university partners with and serves.

To provide further context, we have conducted previous DL studies (Buelow et al., 2018; Frambaugh-Kritzer et al., 2015) with this cohort. We followed this same cohort of PTs in each of their methods courses (i.e., performing arts, visual arts, reading, writing, mathematics, social studies, and science) for a larger study examining DL practices in elementary education. In the first three semesters of the four-semester/2-year program, in addition to taking methods courses, the PTs spent two full days a week in local elementary (K–6) schools for field experiences, which included Stephanie’s weekly informal observations and three formally observed lessons per semester. In the fourth semester, the PTs completed a full-time (5-day-a-week) student teaching practicum for 16 weeks, which also included weekly informal and three formally observed lessons by Stephanie. For the purposes of this article, we focused only on the mathematics methods course (held in semester three) and mathematics field-based teaching experiences that occurred in semesters three and four.
The Elementary Mathematics Methods Course

The mathematics methods course provided PTs with the skills and experiences needed to plan, teach, and assess mathematics in an elementary classroom. The course instructor, Dr. Lowe, designed the course to engage the PTs in problem solving and modeling lessons where the PTs had opportunities to communicate like mathematicians. Dr. Lowe subscribed to a social constructivist philosophy and integrated the National Council of Teachers of Mathematics (2000) MHM, the K–6 Common Core State Standards (CCSS) mathematics standards (CCSS Initiative, 2010), and the CCSS SMP (CCSS Initiative, 2010), which were mandated in the state. This integration was carefully described in her syllabus and other course materials and fully implemented each week in the methods course. Dr. Lowe emphasized mathematical practices and content knowledge, and it was her stated goal to teach the PTs instructional approaches and habits of mind needed for learning mathematics.

Participants

A total of 14 PTs in an undergraduate elementary education program consented to our study. They all identified as female and their ages ranged from 20 to 29, with ethnic diversity representative of the local population (i.e., Pacific Islander, Asian, White, Latino, and African American). Although Dr. Lowe was not a main participant, we mention her involvement because she did agree for us to observe her methods course so that we could analyze the instruction PTs received, therefore providing us with more context to better triangulate our findings.

Researcher’s Role

We are teacher educators at the university where the study took place. Charlotte is a secondary literacy teacher educator (Grades 6–12), and Stephanie is an elementary literacy teacher educator (Grades K–6). Charlotte served purely as a researcher as she did not teach or supervise any of the PTs in the field. However, Stephanie’s relationships with the PTs as their cohort coordinator and literacy methods course instructor were more interdependent, which might have led to issues of bias or conflict (Nolen & Vander Putten, 2007). Charlotte counterbalanced this concern through her role because she was tied to neither the participants’ coursework nor their fieldwork. To attend to issues of power, we included on the consent forms the statement that there would be no penalty for nonparticipation and participants were given the option to opt out of the study at any time.

Data Sources and Collection

Qualitative case studies require a “rigorous methodological path” (Yin, 2011, p. 34) and multiple sources of data for triangulation. To conduct a rigorous case study with fidelity, we collected three data sources: (1) observation notes from the mathematics methods course, (2) observation notes from the field practicum when TCs taught math lessons (semesters three and four), and (3) multiple artifacts (e.g., postobservation oral and written reflections and lesson plans). To collect this empirical data, Charlotte observed and took detailed ethnographic notes and photos from the mathematics methods course for 15 weeks of the semester, totaling 37 hours. Stephanie observed and took ethnographic notes from the PTs in their field placements teaching 28 mathematics lessons over semesters three and four. Stephanie also engaged the PTs in postlesson reflective
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discussions. In addition, PTs’ artifacts were collected from both the methods and field course. In total, 28 mathematics lessons were developed, taught, and observed. Fourteen of these lessons occurred in semester three, and 15 occurred in semester four.

Data Analysis

We analyzed our data using a few methods. First, we used the directed content analysis tool (Hsieh & Shannon, 2005). Next, we engaged in “real-time dialogue” (Taylor & Coia, 2009, p. 177) to help us “process and discuss meaning” (p. 177) of the coding and findings. Finally, we employed the “writing” revision process as “a way of ‘knowing’—a method of discovery and analysis” (Richardson, 2000, p. 923). In total, each of these analytic tools served us in a particular way, leading to a deeper analysis of our findings.

Directed Content Analysis

This tool required the use of predetermined categories derived from existing theory and research. Together, we coded instances in each data source using our predetermined categories, which included the key words derived from the five DL instructional tenets found in Table 1. Our first step was to determine if we had enough evidence to substantiate whether the PTs experienced the five DL instructional tenets in their methods course and whether they enacted them in the K–6 field. Important to note, we did not tell Dr. Lowe or the PTs we used the five DL instructional tenets to analyze the data, as we wanted to determine if the tenets were part of this real-life context. Once we completed our coding, we found substantial evidence that the PTs not only experienced all five DL instructional tenets in each of Dr. Lowe’s class sessions, but they also enacted each tenet in the field.

Real-Time Dialogue

Our initial coding assigned instances in the data to a single predetermined category (i.e., the tenets). However, through “real-time dialogue” (Taylor & Coia, 2009, p. 177), we realized that some instances in a single data source were evidence of multiple tenets. For example, when we initially coded a part of Bella’s lesson that focused on telling time to the half hour on an analogue clock, we categorized her explanation of the words “minute hand” and “hour hand” to be listed under the vocabulary tenet. However, upon deeper examination and through our collaborative discussions, we realized she was also teaching the analogue clock as a complex text and the related academic vocabulary was supporting comprehension of the text. In this, we acknowledge that this instance in her lesson could also be coded under the tenet comprehension of complex text. It turned out all 28 of the PTs’ lessons enacted multiple tenets.

Writing/Revision

After we coded, we engaged in writing as a “way of ‘knowing’—a method of discovery and analysis” (Richardson, 2000, p. 923). That is, the majority of our analysis occurred through iterations of our back-and-forth writing process as we analyzed the data in deeper ways. However, for trustworthiness, we continued to employ real-time dialogue (Taylor & Coia, 2009) when more complex issues arose during our revision process. One issue we deliberated in real time surrounded the types of instructional strategies enact-
ed by the PTs because some of the same instructional strategies were enacted to meet different tenet goals. For instance, the think-aloud strategy was, at times, enacted to teach comprehension, while other times it served to build knowledge or learn new vocabulary. Through this conversation, we began to appreciate how important the think-aloud strategy can be for DL mathematics instruction.

While writing, we started to analyze the variance in the sophistication of how the tenets were enacted by the PTs; making us question whether our initial coding was accurate. In real-time we agreed to look for disconfirming examples (Creswell & Miller, 2000) to help us decipher whether we were being too generous in categorizing certain instances in a lesson as one of the instructional tenets. For example, Stephanie initially coded a part of Kate’s sixth-grade mathematics lesson as a data exemplar for the inquiry tenet. In brief, Kate provided students with knowledge (formula for finding the area of a rectangle) and asked them to apply this knowledge to determine the formula for finding the area of a triangle. With further scrutiny, we realized that Kate did not push students to inquire into the purpose for finding the area of a triangle or construct arguments as they critiqued and compared their formula to their peers, thus we no longer considered this an exemplar for inquiry.

Also, while writing, we came to the decision to highlight nine PTs (see Table 2) in the findings after we analyzed data from all 14 of the PTs. Although all 14 PTs demonstrated instances of the DL instructional tenets, we feature these specific examples from nine PTs whose 13 lessons offered the clearest examples of the DL tenets.

Table 2

<table>
<thead>
<tr>
<th>Preservice teacher</th>
<th>Field setting context</th>
<th>Number of lessons highlighted in findings (and semester taught)</th>
<th>DL instructional tenet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moriah</td>
<td>5th grade, rural Title I</td>
<td>1 (Semester 4)</td>
<td>Customized instructional strategy, built academic knowledge</td>
</tr>
<tr>
<td>Tatum</td>
<td>1st grade, rural Title I</td>
<td>1 (Semester 4)</td>
<td>Built academic knowledge</td>
</tr>
<tr>
<td>Nicole</td>
<td>5th grade, urban Title I</td>
<td>1 (Semester 4)</td>
<td>Inquiry</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>3rd grade, urban Title I</td>
<td>2 (Semesters 3 &amp; 4)</td>
<td>Complex text, built academic knowledge, academic vocabulary</td>
</tr>
<tr>
<td>Amy</td>
<td>1st grade, rural Title I</td>
<td>2 (Semesters 3 &amp; 4)</td>
<td>Inquiry, customized instructional strategy</td>
</tr>
<tr>
<td>Grace</td>
<td>2nd grade, suburban Title I</td>
<td>1 (Semester 4)</td>
<td>Academic vocabulary, built academic knowledge, customized instructional strategy</td>
</tr>
</tbody>
</table>
Notes. Title I is a federal program of the U.S. Department of Education that provides financial assistance to schools that serve high percentages of children from low-income families (Elementary and Secondary Education Act of 1965).

*In the state where this study took place, sixth grade is in the elementary building in some complex areas and in the middle school building in others.

Findings

Overall, our findings showed that the PTs constructed meaning with the ideas they learned in their methods course because they enacted each DL instructional tenet in their teaching. Accordingly, we present our findings organized by each instructional tenet. Given how influential the methods course was to the PTs’ learning, we also highlight our observations from Dr. Lowe’s methods course throughout the findings.

Preservice Teachers Taught Comprehension Using Complex Mathematical Texts

Our analysis of PTs’ lessons corroborated with their postlesson reflection showed that they internalized the importance of an expanded view of text while teaching comprehension of the mathematical texts they introduced to students. All 14 PTs taught elementary students how to comprehend manipulatives, graphs, rulers, word problems, or calculators. Interestingly, over the course of the semester, Dr. Lowe also introduced a variety of mathematical complex texts such as infographics, grid paper, number lines, multiplication charts, currency, and numerous types of manipulatives (e.g., place-value cards, tangram puzzle shapes, Cuisenaire Rods, base ten blocks). In addition, we observed her introducing many tools mathematics learners must comprehend and apply, such as thermometers, bar graphs, protractors, rulers, and calculators. Dr. Lowe explained to the PTs, “You can’t just hand a ruler to a first grader and assume they know how to read the hash marks and can denote what centimeters mean.” She also taught the MHM of attending to precision in a few ways, like “make sure to always have your students reread the math question” and “have them justify or explain their thinking aloud.” Important to note, we observed Dr. Lowe thinking aloud with an infographic on the Great Garbage Patch while she also demonstrated some of her struggles by asking, “I’m not exactly sure what the purpose of this infographic is, so how can I break down the data to better understand the purpose?”
Blakely embraced a broadened view of text and the necessity to provide comprehension instruction when she taught sixth-grade students the complex texts of base ten blocks to conceptually explore multiplication of decimals. Although base ten blocks were familiar to the sixth-grade students, Blakely first taught the nuanced differences in how the blocks are used to represent decimals. She used the think-aloud strategy when saying, “If I know flat equals 100 units and has the value of one, then I can figure out that a long is equal to 10 units and has the value of 10/100.”

She continued to guide students through the values of each base ten block in order to support the learners’ ability to use this complex text in solving multiplication problems involving decimals (e.g., a unit equals 1/100 or 0.01, a long equals 10/100 or 0.1). Students were also given a graphic organizer to record notes and support their comprehension of the base ten blocks’ decimal values. In a written reflection after the lesson, Blakely internalized the importance of the base ten blocks as a critical text in the lesson and the importance of access to this text in the lesson. She noted that she did not give all students enough blocks to solve the problem, and while some were able to translate the idea of the base ten blocks to pictorial representation, others needed the concrete objects.

In third grade, Elizabeth distributed the complex texts of Cuisenaire rods and a number line for students to explore fractions that occur between 0 and 1. They learned to use these complex texts of mathematics strategically and to notice the structure of fractions in relation to the whole. To explore this, Elizabeth drew a number line on the board with the end points of zero and one. She asked students to discuss whether there are numbers between zero and one on the number line. As she circulated around the room to check students’ understanding, she asked them to “justify their thinking” as a way to reinforce the need to double-check their work, which supports both precision and the comprehension needed to problem solve this complex mathematical text. This experience gave students time to freely explore with the Cuisenaire rods prior to Elizabeth explicitly teaching them the nuanced ways to read this complex mathematics text (e.g., must use the same color as it represents equal parts, it must fit on the number line, the size of the Cuisenaire rod corresponds to the size of the denominator). Similar to how it was modeled in her methods course, Elizabeth used the think-aloud strategy to reveal how she read the complex text of Cuisenaire rods: “I know if I am sharing a cake with only one other person, we will both get bigger pieces than if I share the cake with 11 other people.” This think-aloud and connection to prior knowledge helped students consider the relationship between the size of the Cuisenaire rods and the size of the denominator (size of Cuisenaire rod is longer when the denominator of a fraction becomes smaller), as they learned to read this complex mathematical text with precision and understanding.

In other examples of the PTs’ comprehension instruction of the complex texts of mathematics, Bella instructed first-grade students to “reread the minute hand” as they learned to tell time to the hour and half hour, while Kate taught sixth graders to “explain how they got their answer” when using grid paper and cubic centimeter blocks to determine the total surface area of three-dimensional shapes. Overall, our data showed that the PTs embraced a broadened sense of text (Draper et al., 2010) to include the objects that students needed to make sense of as they engaged in the content learning. Although we observed Dr. Lowe model productive struggle as another approach for students to gain comprehension, we did not see the PTs enact this approach in the context of this tenet. We see this as a missed opportunity, yet PTs still enacted instructional practices such as think-alouds to model how they think like mathematicians to comprehend complex mathemat-
ical texts. And to further support students’ comprehension, the PTs often told students to justify and reread for precision.

Preservice Teachers Built Academic Knowledge

The importance of building knowledge was internalized by the PTs as evidenced in their mathematics lessons during their field practicum in elementary classrooms. Our analysis showed all 28 of the lesson plans designed by the PTs built knowledge on the mathematics content, again to various degrees of sophistication and through varied methods in the way in which it was implemented. At times, we observed knowledge building was the product of the lesson, and at other times, the knowledge building served as a scaffold to engage in problem solving. Moreover, PTs used a variety of approaches in their implementation (e.g., direct instruction, productive struggle, hands-on manipulation of models, bridging new knowledge through connections to prior experiences and personal/community assets), and they varied in the point in the lesson when this was accomplished (i.e., forefronted the knowledge or allowed it to be discovered in the lesson).

As previously shared, Dr. Lowe was observed emphasizing the pedagogical idea of productive struggle throughout the semester. In the context of this instructional tenet, Dr. Lowe explained how productive struggle allows students to explore a problem prior to instruction, which supports building academic knowledge. Elizabeth enacted this approach when she gave third-grade students time for productive struggle as they engaged in hands-on manipulation of sand and standard kilogram weights to build knowledge of how basic operations (e.g., division, addition) are used to measure and estimate volumes of masses using standard units of grams and kilograms. She asked, “How might we divide the kilogram of sand equally into the cups? How many ways can you divide the sand? Does the total mass change?” The third graders then explored measurement with volumes and masses of objects through addition, subtraction, multiplication, and division. After exploring with the sand, students were asked to discuss other ways (beyond those explored in the lesson) to divide 1,000 grams equally and to determine if the total mass changed when the sand was redistributed. During this time, Elizabeth circulated around the room to informally assess students’ understanding of the content and their use of alternative representations to solve the problem by asking them to explain and justify how they approached the task. This experience helped students build knowledge of how basic operations are applied to real-world applications such as measuring.

In another lesson Grace enacted a productive struggle while exploring polygons with unequal sides to build second graders’ knowledge. To begin the lesson, students were tasked to fold various cutouts of large odd-shaped polygons at the vertices in order to find the triangles within. Grace reviewed that the sides of a triangle do not need to be equal and reinforced that scalene triangles have unequal sides. As students explored this phenomenon with the paper manipulatives, Grace provided another manipulative to continue to deepen the content knowledge. Geoboards and rubber bands were distributed as students were tasked to make a quadrilateral of any shape and then use another rubber band to connect two vertices to split the quadrilateral into two parts. This work supported students’ problem solving, attention to precision, and reasoning with a variety of shapes and their attributes as they discussed questions surrounding the notions of whether every quadrilateral can be split into two triangles and whether there is a quadrilateral that you could make where this would not happen.
Reflecting on the lesson, Grace noted that although she offered opportunities for students to explore and engage in productive struggle to build new knowledge, clearer parameters and directions for their explorations were needed to build background knowledge around the notion that they needed to connect vertices to each other, rather than vertices to a line. She reflected, “By giving clearer, explicit directions and modeling what to do and what not to do, I think the students would have been more successful.” This reflection aligned with Grace’s learning in the methods class where Dr. Lowe explained that it is important for students to understand the problem-solving situation prior to starting. She suggested that teachers read the problem together as a class and then attend to any questions for clarification of the problem (without providing too much information about the steps that need to be taken). She added, “Let them struggle. However, you do not want them to be frustrated.”

In other lessons, PTs were more direct in how they built the mathematical content knowledge so that it was a scaffold for problem solving. For example, Kate explicitly provided the formula for finding the area of a rectangle before sixth-grade students applied the formula themselves. In another lesson, she reminded students of the commutative property of multiplication while using the think-aloud strategy. This served as a scaffold for solving volume problems with fractional measurements. With fifth-grade students, Moriah used the think-aloud strategy to model the box method and standard algorithm for multiplying multiple-digit whole numbers by one another prior to students participating in a game that required this computational knowledge.

Another point Dr. Lowe emphasized in the methods course was to give students firsthand real-world problem-solving experiences as a way to build academic knowledge. Dr. Lowe demonstrated this practice by inviting PTs to consider seeing their natural world through a mathematical lens. For example, it was observed throughout the semester that Dr. Lowe asked the PTs to think about patterns (in their clothing and gardens), measuring (as it related to cooking), reasoning (using statistics to think about how probable something will be), motion (how long will it take you to get to point A if you drive 25 mph), shape (is a square table better than a circle?), fractions (cutting pizza slices), and prediction (how much money will you need?).

Tatum followed suit by supporting first-grade students’ developing understanding of coin equivalency and strategic use of coins by having them engage in a shopping scenario. First, she explicitly told them the names and value of each coin and the equivalency of coins (e.g., five pennies equal a nickel). Then, students engaged in hands-on experiences as they were given set amounts of replica currency and shopped in the class store to purchase food items and classroom supplies. They took turns being the storekeeper and the shopper to deepen their understanding of the value of coins as well as basic addition and subtraction skills. In another lesson, we observed Elizabeth support students’ growing knowledge of equivalent fractions through simulations in sharing, fairness, and equity. Elizabeth supported students’ new knowledge by starting with a connection to what they already know (the concept of sharing) and provided experiences for hands-on manipulation of models. In these examples, we noted the PTs used relatable contexts (firsthand real-world problem-solving experiences) with learners to anchor their understanding of mathematical concepts while also providing students with a way to build academic knowledge.
Overall, building academic knowledge was a consistent finding among the PTs’ instruction. At times, knowledge building was the product of productive struggle and at other times, knowledge building was a scaffold for problem solving. Yet each approach provided elementary students with opportunities to reason abstractly and quantitatively and look for and make sense of structure (as noted in the SMP and CCSS), which is critical in DL instruction.

**Preservice Teachers Taught Mathematical Vocabulary**

All of the PTs’ lesson plans indicated the mathematical vocabulary and planned instructional supports required for successful participation in the lesson. The PTs utilized direct approaches for teaching the necessary vocabulary of the lesson. Interestingly, we found that each week, Dr. Lowe not only addressed vocabulary, she made it a priority. It was observed in every class session that mathematical definitions were important (e.g., percent, decimal). Her vocabulary instruction to the PTs was shared in a variety of ways. She used word walls to reference academic vocabulary as well as videos and hands-on activities to bring meaning to the vocabulary concepts. For example, Dr. Lowe was observed using hands-on activities with manipulatives in one lesson teaching two-dimensional (sides, angles, regular) and three-dimensional (edges, vertices, faces, regular) geometry vocabulary along with terms such as *platonic solids* and *vertices*. Dr. Lowe encouraged scaffolding mathematical concepts/vocabulary in creative and authentic ways with students.

We observed that some PTs also used common instructional strategies for literacy, such as word walls, while others named the concept/vocabulary as it was presented in the lesson. For example, Bella presented first-grade students with a list of the relevant vocabulary required of exploration of time (e.g., analogue, digital, minute hand) and, as the concept arose in the lesson, showed students the word from a reference chart. Whereas Grace overtly taught second graders the vocabulary of shapes and their attributes via a variety of complex mathematical texts and concrete objects. She purposely connected students’ prior knowledge with everyday language and referred to the Latin and Greek roots of the words (e.g., tri = three like tricycle and triangle). This process was repeated for quad-, penta-, and hexa-. As the vocabulary of the lesson was being explicitly taught, Grace added these words to a mathematics word wall of the names of two-dimensional shapes for student reference. The word wall was displayed in three columns: one containing the word, a second containing the number of sides, and the third containing visual images of the shape. Grace intentionally demonstrated how to utilize this word wall by thinking aloud and then posing questions to students (e.g., If I use the word wall root words, I can figure out how many sides a pentagon has. Can you determine how many sides a hexagon has? Polygon?).

In another example, Bella explicitly taught first graders the mathematical vocabulary related to building composite shapes from two- and three-dimensional shapes. She created a word wall containing relevant geometry-based terms (e.g., vertices, face, surface, flat, curved), and next to each word she used an image to illustrate the concept. Similar to Grace, she explicitly taught the language students needed to engage in the content at the onset of the lesson. Elizabeth also utilized a direct approach to teach third graders the academic language at the onset of the lesson. She began the lesson by writing the fraction $\frac{1}{2}$ on the board. Next, she wrote the words *fraction*, *numerator*, and *denominator* on the board and asked students to discuss how they would label the number $\frac{1}{2}$.
using the words written on the board. This knowledge-building task served as a support for students’ use of the mathematical vocabulary throughout the lessons as they explored fractions on a number line.

Finally, reading complex mathematics texts may also require explicit instruction in the nuanced vocabulary of mathematics that differs from everyday meanings of common words. For example, in Blakely’s lesson with sixth graders, she needed to forefront experiences and instruction with the language of algebra. She reviewed ideas such as the multiplication symbol $\times$ often can represent the word of in a sentence, which means to multiply. Similar to Dr. Lowe, the PTs’ vocabulary instruction presented a variety of approaches, with the two most common being word walls and hands-on activities or images to help bring meaning to the vocabulary concepts. We did not observe any of the PTs simply sharing a list of words and definitions as an approach to teaching the mathematical vocabulary of the lesson; the PTs understood that vocabulary instruction was critical to supporting communication in the classroom and deepening learners’ conceptual understanding.

**Preservice Teachers Included Instructional Spaces for Mathematical Inquiry**

Our findings show that creating space for mathematical inquiry is critical for problem solving. While Dr. Lowe encouraged student-led mathematical inquiries, due to time limits she often posed the essential question for the inquiry in the methods course. For example, Dr. Lowe took the PTs to a local botanical garden and asked: (1) Are patterns consistent in nature? (2) Can you find a connection between patterns in nature and the Fibonacci sequence? PTs then examined nature to engage in the inquiry as they were engaged in gathering evidence and explaining their findings. This experience provided opportunities for the PTs to develop and apply MHM that encouraged them to formulate questions about evidence, patterns, and connections, all of which are fundamental to the inquiry process. Once again, we noted how Dr. Lowe modeled productive struggle, yet for this tenet it was taken up to engage PTs in inquiry-based problem solving without too much prior explicit instruction on the content so that the PTs could appreciate the value of persistence in learning.

From their experiences in the methods course, all 14 of the PTs took up inquiry in the form of problem solving in their lessons to some degree; however, the level of sophistication of these inquiries was not at the same depth as Dr. Lowe’s and showed more variance among the PTs. For example, 11 of the 28 observed lessons placed problem solving at the core of the lesson through productive struggle, whereas the other 17 lessons used more direct approaches to learning the mathematics content but included problem solving in the lesson. In addition, while the PTs’ lessons did provide opportunities for elementary students to reason abstractly and quantitatively, the PTs’ inquiries were all teacher directed and lacked opportunities for elementary students to critique the reasoning of others or compare solutions. The lessons too often focused on the answer to the problem over the problem-solving process. Nonetheless, our findings highlight the strongest exemplars of inquiry where problem solving was predominant and where productive struggle was a central focus of the lesson.

Nicole took up a teacher-led inquiry with fifth graders as they engaged in a productive struggle that was focused on basic operations involving decimals and division. Students were given a restaurant menu and tasked with the problem of determining what
might be ordered from the menu given a set budget and the total number of people in the group. Students needed to account for tax and tip in addition to their food and then compare their order to those of other groups. The complexity of the problem was appropriately challenging, and the mathematical content knowledge required to solve the problem was fitting to the grade level. In reflecting on these experiences, Nicole was pleased with the “welcomed change the lesson brought” to mathematics instruction. She noted in using an inquiry-based approach to mathematics instruction that “students who had never expressed much interest in mathematics before were wholeheartedly putting their efforts in.... The problem-solving challenge changed my classroom for the better.” However, she also realized the importance of her role as a facilitator during mathematical inquiries. She admitted to struggling with knowing when she should step in to offer guidance or redirection. She reflected how one group estimated the cost of the menu items rather than calculating with the exact costs. “I noticed their mistake immediately and waited for them to realize it, but it took much longer than I expected. I wish I would have just asked them why they were estimating sooner.” In addition, Nicole reflected on the importance of discussing and extending solutions to mathematical problems as a critical component that was missing in her lessons, demonstrating her deepening knowledge of mathematical inquiry in elementary classrooms.

We observed Amy use an inquiry-based approach when she posed the following question to first graders: “How can you use an addition fact to find the answer to a subtraction problem?” Although the question leads to a single correct answer, Amy’s instructional approach represented productive struggle as students considered the properties of operations. She provided students with the equation $11 - 4 =$ and tasked them to collaboratively determine how to solve the equation using addition facts. The objective was that students would figure out how to use the “count-on” strategy (start at four and count on to 11) or draw upon “fact family” knowledge ($11 - 4 = 7, 4 + 7 = 11$, etc.) to solve the problem. After students engaged in a productive struggle to solve the problem, Amy introduced the count-on strategy and how to use fact families to solve the problem. Although students struggled with this open-ended approach, Amy recognized the importance of providing opportunities for problem solving that requires quantitative reasoning, yet later reflected that this approach was not commonly used in the classroom where she was placed. She offered specific ways to support these young learners as they engaged in productive struggle. For example, she reflected that the students would benefit from recalling the strategies they know for solving addition problems as resources to support them in solving the essential question. In her written reflection on the experience, Amy also reflected on the importance, when engaging young learners in inquiry and productive struggle, of teachers being able to recognize struggle versus frustration:

I saw that the students hit a frustration point, which was when I brought them back together as a class. I liked the idea of struggle because it was different and the students had a good attitude about it. I would definitely use struggle to teach the students but scaffold it since they aren’t used to being thrown in the deep end.

Blakely led sixth-grade students through teacher-led inquiry focused on multiplication of decimals. She posed the following to students: “Using base ten blocks, figure out an equation if the product is 4.2.” To scaffold students’ experiences, Blakely provided guiding questions as they worked collaboratively to first solve problems where they were given the equation and then use the guiding questions and base ten blocks to solve the
problem where they were given the product (e.g., Is the product less than one half? More than one half but less than one whole? What is the closest integer to which you could round the answer? And what is the place value of the answer?). In addition, she required students to discuss how they approached the problem, rather than focusing solely on the solution. Blakely discussed the outcomes of the lesson in a written reflection:

I would have changed how I introduced the chart [of guiding questions] for their productive struggle. Instead of doing the first line of the chart with them, I would have just given the chart and the ten blocks to the students and told them to try their best with their partner to see if they could figure it out on their own. My students do not usually get the opportunity to “struggle” or attempt a problem before being told how to do it so this concept was very difficult for them.

Although the PTs took up inquiry through problem solving, the data examples did not yield opportunities for elementary students to critique the reasoning of others and construct arguments as to their individual approaches to solving problems. A common sentiment brought up by the PTs was the notion that productive struggle in mathematics was unfamiliar to elementary students.

**Preservice Teachers Used Instructional Strategies Customized or Adapted for the Literacies of Mathematics**

When we examined the instructional strategies that support literacy taken up by the PTs, there was much to unpack, hence we organized this section in the following way. Our analysis shows that the PTs enacted many types of instructional strategies for literacy. Of these, PTs mostly enacted generic instructional strategies to teach the mathematical content. However, when the PTs did customize/adapt instructional strategies for math, they were enacted as games. According to our data, some interesting phenomena surrounding think-alouds and productive struggle emerged. Finally, all of the PTs enacted strategies that mirrored their learning experiences in the mathematics methods course, which are weaved throughout.

**Generic**

Our analysis showed the PTs frequently enacted instructional strategies for literacy, yet they were mostly generic versus customized, as explained in our literature review. For example, PTs used graphic organizers to support and scaffold estimation, vocabulary development, and problem solving. Figure 1 shows Blakely’s teacher-made graphic organizer used during an opportunity for productive struggle when sixth-grade students were learning to multiply decimals using base ten manipulative blocks. The graphic organizer provided a place for students to scaffold the process of moving from a given product (far right column) to an equation (far left column) when multiplying decimals using base manipulatives.
# Figure 1

**Blakely's Teacher-Made Graphic Organizer**

<table>
<thead>
<tr>
<th>Equation</th>
<th><strong>Estimate:</strong> Less than one half?</th>
<th><strong>Estimate:</strong> More than one half, but Less than closest integer?</th>
<th>Answer will be to which place?</th>
<th>Draw a diagram</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3 x 0.5 =</td>
<td>Yes</td>
<td>No</td>
<td>0 hundredth</td>
<td><img src="https://example.com/diagram1.png" alt="Diagram" /></td>
<td>0.15</td>
</tr>
<tr>
<td>3.2 x 2.1 =</td>
<td>No</td>
<td>No</td>
<td>6 hundredth</td>
<td><img src="https://example.com/diagram2.png" alt="Diagram" /></td>
<td>6.72</td>
</tr>
</tbody>
</table>

**Game:** Guess the teacher’s equation
Using the product, and base ten blocks, find my Equation!

<table>
<thead>
<tr>
<th>Equation</th>
<th><strong>Estimate:</strong> Less than one half?</th>
<th><strong>Estimate:</strong> More than one half, but Less than closest integer?</th>
<th>Answer will be to which place?</th>
<th>Draw a Diagram</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.6</td>
</tr>
</tbody>
</table>
Other common graphic organizers we observed the PTs use were two- and three-column notes. For example, in another math lesson, Blakely utilized two-column notes to support vocabulary development related to the concepts of area and perimeter, while Kate utilized three-column notes to support students’ conceptual understanding of determining the area of a triangle, and Grace used three-column notes to support second-grade students’ vocabulary development. Although all these instructional strategies for literacy found in our data were not customized, they still provided opportunities for students to make meaning of the mathematical content. When examining the data from Dr. Lowe’s course, we saw that she applied mostly generic instructional strategies for literacy (e.g., read-alouds, graphic organizers, think-pair-share, Frayer models, place mats) and adapted them to the mathematical content.

**Games**

When analyzing instructional strategies for literacy that were customized and/or adapted, we noted seven of the 28 lesson plans presented strong examples of using games to engage with mathematics content and the MHM. We also noted the many different kinds of mathematics games Dr. Lowe introduced. For example, she taught the PTs the games Fraction Decimal Matching Game, the Race for the Whole, Jumping Frog Jubilee, and The How Many Handshakes Problem. These customized games supported the PTs in engaging in the MHM and SMP, such as modeling with mathematics, using appropriate tools strategically, attending to precision, and looking for patterns.

In turn, the PTs enacted games as an instructional strategy for literacy in the elementary classroom. Moriah designed a card game for fifth-grade students’ practice with multidigit addition. In the card game, each student worked with a deck of cards containing the numbers 11–22. At the signal, players flipped over their cards to reveal the number, and the first of the two players to correctly calculate the product of the two numbers won the round and received both cards. After the entire deck of cards was played, the player with the most cards won from the multiplication rounds was declared the winner.

In first-grade classes, Amy and Bella collaboratively planned and independently implemented mathematics games around the use of telling time and clocks. Students played a matching game where they needed to match the time on a digital clock to the time displayed on an analogue clock. In another game to develop their time-telling skills, students played Race Against Time, in which they used a game board containing clocks showing analogue time to the hour and half hour (see Figure 2). Each player flipped a coin and moved one or two spaces (heads = move one space, tails = move two spaces). If they read the time correctly, the player remained on the clock game board until their next turn. If they did not read the time correctly, the player moved back one clock space. The first player to reach the end line won the game.
Productive Struggle and Think-Alouds

An interesting phenomenon that emerged from our analysis showed that productive struggle (Hiebert & Grouws, 2007) and the think-aloud strategy (Wilhelm, 2001; both modeled by Dr. Lowe) were enacted by the PTs the most, making these the go-to instructional strategies for this cohort. In total, 11 PTs enacted productive struggle and 16 PTs enacted the think-aloud strategy. In our coding process, we realized that these two instructional approaches could achieve different instructional purposes, which is why they are highlighted across several of the DL tenets. Again, a productive struggle was enacted to support inquiry (see Nicole’s, Amy’s, and Blakely’s examples) and build knowledge (see Elizabeth’s and Grace’s examples). This was also the case for the think-aloud strategy. Sometimes PTs enacted it to teach comprehension of complex texts (see Elizabeth’s and Blakely’s examples), while other times it served to build knowledge (see Moriah’s and Kate’s examples) or it was used to learn new vocabulary (see Grace’s example).

While our data showed that both of these instructional tools shared similarities as a go-to for the PTs, upon further analysis, we started to note that productive struggle stood out as a specialized DL instructional approach unique to mathematics. This came through in not only our observations of PTs, but in Dr. Lowe’s instruction as well. She explained in one class session, “Please don’t tell the students…. Make a space for discovery….for them to ask questions….to wonder….to imagine what it is like to learn for the

**Figure 2**

*Race Against Time Game Board*
first time…. That is your teacher role.” Dr. Lowe referred to the mathematical discourse of productive struggle to describe this process.

As for the think-aloud strategy, we acknowledge it was also enacted across the tenets like productive struggle, but we would not refer to this strategy as specialized in the same way we positioned productive struggle. Instead, we acknowledge that the think-aloud strategy is a generic instructional strategy for literacy, yet due to our findings we posit that it may be one of the more potent and versatile strategies for math teachers compared to other generic Content Area Literacy strategies. Its potency was demonstrated multiple times when the PTs enacted think-alouds to demonstrate how mathematicians (the teacher) think as they solve problems, engage in inquiry, and grapple with new vocabulary and content. Interestingly, we also observed PTs enacting think-alouds to model mathematical tasks. For example, when Amy and Bella planned and taught a lesson focusing on attributes of two- and three-dimensional shapes, they utilized learning stations. To prepare first-grade students for the stations, Amy and Bella both used the think-aloud strategy while modeling how to engage in the mathematical content at each station. Finally, we began to see some transfer of the responsibility of thinking aloud over to elementary students when the PTs asked them to justify and/or explain their thinking and/or problem-solving approach.

Discussion and Implications

The central focus of this case study was analyzing how the PTs enacted the DL instructional approach in mathematics with the K–6 students in their practicum field placements. Next, we use a social constructivist perspective to support our analysis of why we think this happened. We have identified two discussion points and implications that add to the literature in elementary DL mathematics.

All DL Instructional Tenets Were Enacted

First, the PTs enacted all five instructional tenets of DL, which supported the K–6 students in learning how to read, write, and think like mathematicians, developing their disciplinary content knowledge (Hillman, 2014; Moje, 2008), and developing their disciplinary habits of mind (Fang & Coatoam, 2013). Yet each of the PTs constructed meaning in unique ways (Mitchell & Myles, 2004), which is why we noted various degrees of sophistication in how they were enacted. This is not surprising given their novice status; nevertheless the PTs still achieved these DL instructional tenets, and this confirms previous studies we conducted in other discipline areas (Buelow et al., 2018; Frambaugh-Kritzer et al., 2015).

Moreover, our systematic examination allowed us to see that Dr. Lowe’s instruction influenced the PTs’ enactments of each of the DL instructional tenets. In each tenet, we observed that the PTs moved from dependent learners in their methods classes to independently enact mathematical DL instructional tenets in their field practicum. This aligns with Vygotsky’s (1978) belief that we learn first “on a social level, and later, on an individual level” (p. 57). In fact, we noted multiple social learning experiences that Dr. Lowe created for the PTs to learn by doing (Au, 1998) and to learn with others (Smith, 1992; Vygotsky, 1978) while engaging in the MHM, which in turn led to DL instructional development (Picot, 2017). Next, we hone in on some particular DL instructional tenets that best grow the literature in DL elementary mathematical instruction.
Expanded View of Mathematical Texts and Comprehension

One DL instructional tenet that stuck out to us was how quickly the PTs seemed to embrace an expanded view of literacy and mathematical text (de Lange, 2003; Draper & Siebert, 2010) after they learned this in the methods class. We observed PTs enacting multiple ways to represent concepts, such as print-based texts to using manipulatives (base ten blocks), as they encouraged K–6 learners to make sense of their understanding (Siebert & Hendrickson, 2010). It is through this finding that we continue to support those who push the boundary of what constitutes text (Draper & Siebert, 2010), especially in a mathematics classroom where students will encounter many complex texts and tools that require explicit comprehension instruction (Siebert & Draper, 2012; Vacaretu, 2008). The PTs followed Dr. Lowe’s lead when they prioritized comprehension by having students reread or justify the answer. Yet we want to note that PTs fell short in enacting productive struggle as a comprehension method for complex text even though Dr. Lowe modeled it. Nevertheless, the PTs followed Dr. Lowe’s direction as they embraced diverse and expanded views of mathematical texts.

Instructional Strategies for Literacy

Another finding that provided us with great insight was noting the different types of instructional strategies for literacy that were enacted by the PTs. Again, our data showed PTs largely enacted instructional strategies for literacy that were generic, which aligns to CAL as we described in our literature review (e.g., K-W-L vs. K-N-W-S). However, when the PTs customized their instructional strategies, they came in the form of games. Evidenced in our data, Dr. Lowe exposed the PTs to many games that reinforced MHM (i.e., predicting, looking for patterns), making us once again appreciate the social constructivist (Vygotsky, 1978) perspective to why the PTs enacted these games in their field experiences. Due to this finding, we assert that customized mathematical games are an appropriate instructional strategy to implement for supporting mathematical literacy, and this aligns with others who made this case (Ferreira et al., 2012).

Finally, the most common instructional strategy we observed across several tenets was the think-aloud strategy. We recognize this is a generic instructional strategy for literacy, but our data showed that think-alouds allow math teachers to demonstrate the MHM (Bernadowski, 2016). A common phrase heard in the classrooms in our study (mathematics methods course and K–6 classrooms) was to “explain or justify your thinking.” When asking this of students, teachers are asking learners to think aloud as they demonstrate how they solved or struggled with the problem (Baumann et al., 1993). Although the think-aloud strategy is associated with CAL, we assert that it is quintessential to mathematics DL instruction as a way for students to understand the teachers’ problem-solving processes and vice versa.

Inquiry

The last instructional tenet that is worthy to mention is inquiry. Although the PTs’ lesson reflections indicated that they did not often witness mathematical inquiry in their field experiences (from their mentors), we observed evidence of them working to provide a problem-solving environment where productive struggle could occur as that is how they were learning to teach mathematics in their methods course. From these
findings, we assert that, because of their positive, engaging, and scaffolded experiences in
the mathematics methods courses, supported by compelling DL instructional strategies,
PTs were able to take up these authentic and progressive classroom experiences with
their own students. This finding reminds us that in a sociocultural environment (Mitchell
& Myles, 2004) learners are at the center of their meaning-making and they need time
to generate ideas with others (Vygotsky, 1978) as the PTs made sense of how to include
inquiry in their instructional practice which was continually encouraged by Dr. Lowe.

Finally, while the PTs provided instructional space for inquiry, the sophistication
at which it occurred fell short of true student-centered mathematical problem solving and
inquiry. For example, in Amy’s lesson with first graders, the complexity of the problem
was appropriately challenging and the mathematical content knowledge required to solve
the problem was fitting to the grade level; however, the approach was lacking as true
mathematical inquiry because the activity failed to provide opportunities for students to
discuss and critique alternative solutions and approaches to solve the problem. Preservice
teachers are often caught short of time during their emerging implementation of ideas and
practices; however, our data showed that PTs tried new instructional approaches as they
oriented themselves as educators in K–6 classrooms.

Productive Struggle

For our second discussion point, we assert that productive struggle is a special-
ized DL instructional practice in mathematics. Per our literature review, we explained that
productive struggle could serve many purposes depending on the instructional outcomes
or context. To that point, this came to fruition in our data as productive struggle was en-
acted by both the PTs and Dr. Lowe across multiple tenets. PTs enacted productive strug-
gle within the DL inquiry and building knowledge tenets, which makes sense because
productive struggle allows students to engage with complex tasks and explore a problem
prior to instruction (Kapur, 2010; Livy et al., 2018).

We assert the PTs enacted this approach because Dr. Lowe provided a space for
them to learn from one another and by doing (Au, 1998) in the weekly methods classes
while they engaged in productive struggle. We also assert the PTs enacted productive
struggle in their lesson plans due to their own “breakthroughs” in the power of the strat-
edy that “emerged from confusion and struggle” (NCTM, 2014, p. 52). During one class,
Dr. Lowe warned the PTs to “trust that letting them struggle allows them to make sense
of the problem.” The PTs followed her lead despite the field placement school culture not
being fully familiar with this approach. Indeed, we noted a common sentiment that “the
students [K–6] weren’t used to productive struggle, and were used to more direct instruc-
tion approaches” in the PTs’ reflections when they attempted to provide opportunities for
productive struggle. It was courageous of the PTs to work against the socially constructed
cultural expectations of their field placements (Au, 1998; Bruner 1986).

In the end, our data provide evidence that the PTs’ mathematics methods course
socially set them up to enact productive struggle. And while they were in the field,
they discovered that K–6 students needed more opportunities to grapple with complex
problems and engage in productive struggle before being provided with an algorithm or
a procedure (Hiebert & Grouws, 2007). This finding confirms that students acting like
mathematicians need time to struggle in order to make meaning. Anyone outside math
education or those not familiar with productive struggle can hopefully take away the
importance of this instructional approach as it is highly relational to the DL of mathematics, which is why we think it is specialized. To us, this is at the heart of meaning making/literacy. Due to our findings, we can’t think of another discipline other than mathematics and English language arts (specifically writing) where productive struggle is most critical.

Conclusion

This study examined how PTs enacted the DL of mathematics in their field practicum classrooms, which contributes to an underserved area in the DL instruction literature (Howell et al., 2021). Our findings are promising as the DL field continues to emphasize the need to support elementary children developing MHM by teaching them how to solve problems like mathematicians (T. Shanahan & Shanahan, 2008). To be clear, we acknowledge the great work that has already been uncovered in elementary mathematics instruction (Ball, 1990; Baroody, 1989; Fuson & Briars, 1990; Izsák, 2008) and the literature surrounding productive struggle (Hiebert & Grouws, 2007; Kapur, 2010; Livy et al., 2018); however, when framing our study from the perspective of the five DL instructional tenets coupled with social constructivism, we could ascertain a strong link between the learning context of the mathematics methods course and the way the PTs enacted the DL instructional tenets in their own practice. Our most interesting finding was how powerful productive struggle is in relation to DL mathematics instruction, making it specialized. And although think-alouds are recognized as a generic CAL strategy, our data evidenced how important it was for math DL instruction.

It was clear from our analysis that the DL instructional tenets can be realized in the elementary mathematics classroom. We also want to point out that we think the examples in the findings meet the benchmark for mathematics DL instruction, especially in preservice teacher education. Yet we suggest that more studies be conducted in the field of DL for the elementary mathematics classroom to add examples at the in-service teacher level of expertise. Hence, we encourage the DL instructional approach to be more overtly taken up.

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