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Spectroscopic Determinations Using Resonance Spin-Flip in $^{12}$C (p,p$^1$) $^{12}$C and $^{50}$Ti (p,p$^1$) $^{50}$Ti

Lee Robert Baumann
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SPECTROSCOPIC DETERMINATIONS USING RESONANCE SPIN-FLIP IN

\[ ^{12}C (p,p') ^{12}C \text{ AND } ^{50}Ti (p,p') ^{50}Ti \]

by

Lee Robert Baumann

A Thesis Submitted to the Faculty of The Graduate College in partial fulfillment of the Degree of Master of Arts

Western Michigan University Kalamazoo, Michigan December, 1973

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Lee R. Baumann
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I. INTRODUCTION

Considerable information regarding nuclear structure has been attained as a result of scattering experiments. Most of this structure information is from elastic scattering measurements; however, there has been recent interest in attaining nuclear-structure information from inelastic proton scattering. Polarized beams of protons have been inelastically scattered for this purpose (Cl 71, Bo 71a, Bo 71b), and in other work p-γ coincidences have been used (Ab 70).

In the present work spectroscopic determinations for resonant compound nuclear states are made using a new technique of resonance spin-flip measurements. This method, which is applicable to excitation of a 2+ state by inelastic proton scattering from a 0+ target, was first used by Bernstein et. al. (Be 72) to ascertain the relative probability of decay by different partial waves of the 9.49 MeV 3/2− level in 13N to the 2+ first excited state of 12C. The method employs measurements of differential cross section angular distributions at various energies for inelastic scattering and for inelastic scattering with a spin-flip. These data are analyzed using model-independent theoretical calculations.
to obtain the partial wave amplitudes of interest. In the present work three compound nuclear levels have been investigated using this technique. The 8.9 MeV \(1/2^-\) level in \(^{13}\)N was found to decay to the 4.44 MeV \(2^+\) level in \(^{12}\)C by a mixture of 15\% \(P_{3/2}\)-waves and 85\% \(f_{5/2}\)-waves. The 12.28 MeV \(1/2^-\) level in \(^{51}\)V was adequately treated by the theory assuming pure \(P_{3/2}\)-wave decay to the 1.55 MeV \(2^+\) level of \(^{50}\)Ti; however, the possibility of \(f_{5/2}\)-wave decay cannot be eliminated. Similarly, the 12.54 MeV \(3/2^-\) level in \(^{51}\)V was adequately treated by the theory assuming pure \(p\)-wave decay to the 1.55 MeV level of \(^{50}\)Ti. It was found that a mixture of 97.0\% \(P_{1/2}\)-waves and 3.0\% \(P_{3/2}\)-waves gave the best fit to the data assuming the \(f\)-wave decay of this level negligible. The energy level schemes and reaction routes used to study the above compound nuclear states are shown in Figure I.
Figure I. Energy level diagrams for $^{12}\text{C}(p,p^1)^{12}\text{C}$, $^{50}\text{Ti}(p,p^1)^{50}\text{Ti}$. The levels of interest are shown with their spin and parity assignments. Excited states are labeled with the energy in MeV above the ground state.
II. THEORY

A. Angular Momentum and Parity Conservation

Shown in Figure II is an angular momentum diagram for a $^{0+}$ to $^{2+}$ excitation process of a target nucleus ($^{A}X$) by protons through an intermediate compound nuclear level ($^{A'}X'$) of spin and parity $^{J=\pi}$. The incident proton orbital and total angular momenta are denoted by $l, j$ respectively. The orbital and total angular momenta of the outgoing inelastically scattered proton are written $l', j'$. To conserve momentum and parity in this process one proceeds as follows.

The spin angular momentum of the compound nucleus is determined by the total angular momentum of the incident particle coupled to the spin angular momentum of the target nucleus:

$$j + 0 = J$$

$$j = J$$  \hspace{1cm} (1)

The total angular momentum of the incident proton is formed from the orbital and spin angular momentum, $S$, of the proton as follows:
Figure II. Angular momentum diagram.
\[ \vec{j} = \vec{l} + \vec{s} \]
\[ j = l \pm \frac{1}{2} \]  

(2)

From (1) and (2):
\[ \vec{J} = \vec{l} \pm \frac{1}{2} \]  

(3)

The parity of the system consisting of the target nucleus and incident proton must equal the parity, \( \Pi \), of the state formed in the compound nucleus.
\[ (-1)^l = \Pi \]  

(4)

The total angular momentum of the compound nuclear state must equal the total angular momentum of the outgoing proton coupled to the spin angular momentum of the residual nucleus.
\[ \vec{J} = \vec{j'} + \vec{s} \]  

(5)

The total angular momentum of the outgoing proton is formed from the orbital and spin angular momenta of that proton.
\[ \vec{j'} = \vec{l'} + \vec{s} \]
\[ j' = l' \pm \frac{1}{2} \]  

(6)

Substituting (6) in (5) yields:
\[ \vec{J} = (\vec{l'} \pm \frac{1}{2}) + \vec{s} \]  

(7)
The parity of the compound nucleus must equal the parity of the outgoing proton - residual nucleus system.

\[ (-1) \ell' = \bar{\ell} \]  
(8)

Comparing (4) and (8):

\[ (-1) \ell = (-1) \ell' \]  
(9)

Thus, \( \ell \) and \( \ell' \) are both even or both odd. To treat a \( \Sigma = \frac{1}{2}^- \) resonance:

\[ (-1) \ell = \bar{\ell} \]  
(4)

The possible values of \( \ell \) are then:

\[ \ell = 1, 3, 5 \ldots \]  

Therefore, only odd incident orbital angular momenta can conserve parity. To determine the possible incident orbital angular momenta:

\[ \Sigma = \ell \pm \frac{1}{2} \]  
(3)

\[ \frac{1}{2} = \ell \pm \frac{1}{2} \]

Since \( \ell \) must be odd the above equation implies,

\[ \ell = 1 \text{ only; } (P_{1/2} \text{ wave}). \]

From (9)

\[ (-1)^\ell = (-1)^\ell' \]
Thus, only odd \( \ell' \) can conserve parity, i.e.
\[
\ell' = 1, 3, 5, \ldots.
\]

The possible outgoing total angular momenta are found as follows:
\[
\frac{1}{2} = j' + \frac{1}{2}
\]
\[
\frac{1}{2} = j' + 2
\]

Solving this equation for \( j' \) yields:
\[
j' = \frac{3}{2}, \frac{5}{2}
\]

To determine the outgoing orbital angular momenta:
\[
\ell' = \ell' \pm \frac{1}{2}
\]

Substituting (10) in (6) yields two equations:
\[
\frac{3}{2} = \ell' \pm \frac{1}{2}
\]
\[
\frac{5}{2} = \ell' \pm \frac{1}{2}
\]

These equations yield the following results, respectively:
\[
\ell' = 1 \quad (P_{1/2} \text{ wave})
\]
\[
\ell' = 3 \quad (f_{5/2} \text{ wave})
\]

To summarize these results for \( \sqrt{m^2 - \frac{1}{2}} \):

Incident proton: \( P_{1/2} \)

Outgoing proton: \( P_{1/2}, f_{5/2} \).

To treat a \( \sqrt{m^2 - \frac{3}{2}} \) resonance:
\[
(-1)^{\ell} = \pi
\]
\[
(-1)^{\ell} = -
\]
The possible values of $l$ are again all odd, i.e.

$$l = 1, 3, 5, \ldots$$

To determine the possible incident orbital angular momenta,

$$\mathbf{S} = l \pm 1/2$$

$$3/2 = l \pm 1/2$$

Since $l$ must be odd the above equation implies:

$$l = 1$$ only \hspace{1cm} (P_{3/2} wave)

From (9)

$$(-1)^l = (-1)^{l'}$$

Thus, again only odd $l'$ can conserve parity, i.e.

$$l' = 1, 3, 5 \ldots$$

The possible outgoing total angular momenta are found as follows:

$$\mathbf{J} = \mathbf{j'} + \mathbf{j}$$

$$3/2 = j' + j$$

Solving this equation for $j'$ yields:

$$j' = 1/2, 3/2, 5/2, 7/2$$

(11)

To determine the outgoing orbital angular momenta:

$$j' = l' \pm 1/2$$

(6)
Substituting (11) in (6) yields four equations:

\[
\begin{align*}
1/2 &= \lambda' \mp 1/2 \\
3/2 &= \lambda' \mp 1/2 \\
5/2 &= \lambda' \mp 1/2 \\
7/2 &= \lambda' \mp 1/2
\end{align*}
\]

These equations yield the following results, respectively:

\[
\begin{align*}
\lambda' &= 1 \quad (P_{1/2} \text{ wave}) \\
\lambda' &= 1 \quad (P_{3/2} \text{ wave}) \\
\lambda' &= 3 \quad (f_{5/2} \text{ wave}) \\
\lambda' &= 3 \quad (f_{7/2} \text{ wave})
\end{align*}
\]

To summarize these results for \( \sqrt{\frac{\Omega}{\Lambda}} = \frac{3}{2} \):

Incident proton: \( P_{3/2} \)

Outgoing proton: \( P_{1/2}', P_{3/2}', f_{5/2}', f_{7/2} \)
B. Measuring Spin-Flip Cross-Sections

It has been shown by Schmidt et al. (Sc 64) that for a $0^+ \rightarrow 2^+$ excitation process by protons, the magnetic substate populations of the residual nucleus depend on the spin change of the incident and outgoing proton. This result is based on a theorem by A. Bohr (Bo 59) which is as follows:

$$ P_i e^{i\pi S_i} = P_f e^{i\pi S_f} $$

(12)

$p_i \equiv$ Parity of incident particle-target nucleus system

$s_i \equiv$ Spin-projection of incident particle - target nucleus system

$p_f \equiv$ Parity of outgoing particle - residual nucleus system

$s_f \equiv$ Spin-projection of outgoing particle - residual nucleus system

The axis of quantization can be defined by a unit vector along that axis, which is (See Figure III):

$$ \mathbf{z} = \frac{\mathbf{k}_i \times \mathbf{k}_o}{|\mathbf{k}_i \times \mathbf{k}_o|} $$

$k_i \equiv$ Incident particle wavevector

$k_o \equiv$ Outgoing particle wavevector
For a $0^+$ to $2^+$ excitation by protons:

\[ P_i = P_f = \pm \]

\[ S_i = \pm \frac{1}{2} \]

\[ S_f = \pm \frac{1}{2} + M_n' \]

Substituting the above in (12) yields:

\[ \exp \left\{ i \hbar \left( \pm \frac{1}{2} \right) - \left( \pm \frac{1}{2} + M_n' \right) \right\} = 1 \]

The above equation yields the following results:

- \( M_n = 0, \pm 2 \) non spin-flip
- \( M_n = \pm 1 \) spin-flip

The spin-flip process then populates the \( M_n = \pm 1 \) magnetic substates uniquely. Since the residual nucleus is in the first excited state and is bound with respect to heavy particle emission, it will decay by emitting gamma rays of the appropriate angular momentum. Shown in Figure IV are polar diagrams of the relative intensities of the possible radiating nuclear substates. Only \( M_n' = \pm 1 \) radiation occurs along the previously defined axis of quantization. If the lifetime of the radiating nuclear state is short spin-flip events can be recorded by demanding a coincidence between the inelastically scattered protons and the de-excitation gamma rays emitted along the axis of quantization. In this way the spin-flip probability may be measured (see Chapter III, eq. 49).
Figure III. Geometry for spin-flip measurements showing the axis of quantization.
Figure IV. Radiation pattern diagrams.
\[ l = 2, m = \pm 2 \]

\[ l = 2, m = \pm 1 \]

\[ l = 2, m = 0 \]
C. General Discussion

One way of obtaining spectroscopic information about a state in a nucleus is to relate experimentally measured quantities to parametric theoretical expressions for those quantities. In the present case the experimentally determined quantities are the inelastic differential cross-section and the inelastic spin-flip differential cross-section, hereafter referred to respectively as the inelastic cross-section and spin-flip cross-section. In a region of center of mass bombarding energy which is near the energy required to form a state in the compound nucleus these cross-sections are explicit functions of the incident energy. They can be expanded in terms of Legendre polynomials and the series may be terminated by assuming only a finite number of partial waves contribute to the reaction. Since the cross-sections are functions of the energy so are the coefficients in the expansion. In the region of a narrow isolated resonance the energy dependence of the coefficients can be expressed by the well-known Breit-Wigner formula. In general, with background terms present, there will be a term which is anti-symmetric with respect to the resonance energy as well as a term which is symmetric with respect to the resonance energy. As will
be seen the coefficients are not, in general, all linearly independent, thereby allowing one to predict any coefficient in the linearly dependent set from the remaining ones. These same relations hold for the amplitudes of the energy dependent terms in the Breit-Wigner expression. The Legendre polynomial coefficients can be written as sums of products of transition matrix elements, which contain the spectroscopic parameters of interest.
D. Theoretical Basis

The differential cross section for scattering from a nucleus with spin angular momentum $I_n$ and spin projection $M_n$, by a particle with spin projection $m_s$, to a state composed of a residual nucleus with spin $I_n'$, spin projection $M_n'$, and an emitted particle with spin projection $m_s'$ is (Ch 72):

$$\frac{d\sigma}{d\Omega}(\Theta_p) = \frac{1}{2} \sum_{m_s M_n} \left| \sum_{m_s' M_n'} (\Theta = \frac{\pi}{2}, \Theta_p) \right|^2$$

Where the scattering amplitude is (Ch 72):

$$\sum_{m_s M_n} \sum_{m_s' M_n'} \left( \Theta = \frac{\pi}{2}, \Theta_p \right) = \frac{4\pi}{k_n} \sum_{\ell \ell' j j' s M} i^{(\ell - \ell')} \langle \ell \leq m_\ell m_s | j m_j \rangle \langle j I_n m_j M_n | J M \rangle \times$$

$$\langle l' s' m'_l m'_s | j' m_j' \rangle \langle j I_n m_j' M_n' | J M \rangle \times T_{\ell \ell' j j'} \gamma_{\ell m_\ell} (\Theta = \frac{\pi}{2}, \Theta_p = 0) \times$$

$$\gamma_{l' m'_l} (\Theta = \frac{\pi}{2}, \Theta_p).$$

(13)
The quantities appearing in (13) are:

- \( \Theta \) = scattering angle
- \( k_n \) = incident particle wave-number
- \( l \) = incident particle orbital angular momentum
- \( l' \) = outgoing particle orbital angular momentum
- \( j \) = incident particle total angular momentum
- \( j' \) = outgoing particle total angular momentum
- \( j_x \) = incident particle and target nucleus total angular momentum
- \( M \) = incident particle and target nucleus total angular momentum projection
- \( S \) = spin angular momentum of incident particle
- \( m_l \) = orbital angular momentum projection of the incident particle
- \( m_s \) = spin angular momentum projection of incident particle
- \( m_j \) = total angular momentum projection of incident particle
- \( S' \) = spin angular momentum of outgoing particle
- \( m_l' \) = orbital angular momentum projection of outgoing particle
- \( m_s' \) = spin angular momentum projection of outgoing particle
- \( m_j' \) = total angular momentum projection of outgoing particle.

The summation may be chosen to extend over whatever range is appropriate to reproduce the experimental data. Higher order partial waves may be neglected. For a \( 0^+ \) to \( 2^+ \) excitation process, \( I_n = M_n = 0 \) and \( I_n' = 2 \).
in that case the resonating transition matrices may be written (Ch 72):

\[
T_{\ell\ell'} = \sqrt{\frac{\Gamma_{\ell}}{E - E_R + i\frac{\Gamma}{2}}} e^{i\theta_{\ell\ell'}} R_{\ell\ell'} e^{i\beta_{\ell\ell'}}
\] (14)

The pure background transition matrices are (Ch 72):

\[
T_{\ell\ell'} = R_{\ell\ell'} e^{i\beta_{\ell\ell'}}
\] (15)

With the exception of the energy itself the quantities appearing in these transition matrices may, for a narrow resonance, be assumed to be energy independent, and may be treated as adjustable parameters.

\[\Gamma_e = \text{The elastic width of the resonating state in the compound nucleus,}\]
\[\Gamma_j = \text{Partial width of decay for emitting a particle of angular momentum } j'.\]
\[\Gamma = \text{The total width of the compound state}\]
\[E_R = \text{The resonance energy of the compound nuclear state.}\]
\[\alpha_{\ell\ell'} = \text{Resonating phase angle}\]
\[\beta_{\ell\ell'} = \text{Background phase angle}\]
\[R_{\ell\ell'} = \text{Background transition matrix amplitude.}\]

Certain restrictions can be made on these expressions if the initial and final nuclear states are known,
along with the incident particle (See Chapter II, Section B, pg. 14). They are:

- non-spin flip \( \Rightarrow \quad M_n' = 0, \pm 2 \)
- spin-flip \( \Rightarrow \quad M_n' = \pm 1 \)

In that case (Ch 72):

\[
\left[ \frac{d\sigma}{d\Omega} (\Theta_p) \right]^S = \frac{1}{2} \sum_{M_n' = \pm 1} \left| \sum_{m_s, m_s', M_s, M_s', I_n = 0} \right|^2
\]

(16)

\[
\left[ \frac{d\sigma}{d\Omega} (\Theta_p) \right] = \frac{1}{2} \sum_{M_n' = 0, \pm 2} \left| \sum_{m_s, m_s', M_s, M_s', I_n = 0} \right| + \left[ \frac{d\sigma}{d\Omega} (\Theta_p) \right]^S
\]

(17)

If one now limits the summation appearing in the scattering amplitude itself to include only a finite number of partial waves and expresses the \( \Theta_p \) dependent, spherical harmonic as (Me 70),
\[ \gamma_{lm}(\theta, \Theta_p) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi (l+m)!}} (-1)^m \int m e^{i \Theta_p \cdot m \cdot P_l(\cos \theta)} \]

the resulting expression will be a sum of \( \Theta_p \) dependent Legendre polynomials times angle independent constants, i.e.

\[
\left[ \frac{d\sigma}{d\Omega}(\Theta_p) \right]^S = \frac{1}{k_p^2} \left\{ A_0^S + A_1^S P_1(\cos \Theta_p) + A_2^S P_2(\cos \Theta_p) + \cdots \right\} 
\]

(18)

\[
\left[ \frac{d\sigma}{d\Omega}(\Theta_p) \right] = \frac{1}{k_p^2} \left\{ A_0 + A_1 P_1(\cos \Theta_p) + A_2 P_2(\cos \Theta_p) + \cdots \right\} 
\]

(19)

The highest order polynomial present in the above sum is determined by the number of partial waves assumed to be participating in the reaction. Each Legendre coefficient may be expressed in terms of the quantities appearing in the various transition matrices. These quantities are not all energy independent and in the case of an isolated single level resonance their energy dependence is expressed by the well-known Breit-Wigner formula (Ch 72, Be 72):
\[ A_n^{(s)} = B_n^{(s)} + \frac{C_n^{(s)}}{1 + X^2} + \frac{D_n^{(s)}X}{1 + X^2} \]  

\[ X = \frac{\lambda(E - E_R)}{\Gamma} \]

\[ \Gamma = \Pi^e + \sum_{j'} \Gamma_{j'}^o \]

In general, the resonating parts of the Legendre coefficients are not linearly independent. However, a relation of linear dependence does not always exist and different relations exist for different assumptions. All relations of linear dependence in this present work have been derived by M. Soga (So 72) or P. W. Cheng (Ch 72).
E. The Expressions for a $3/2^-$ Resonance

From the conservation laws of angular momentum and parity one would expect the following transition matrix elements for an excitation process from a $0^+$ state to a $2^+$ state through an intermediate $3/2^-$ resonance state:

$$\langle p^{3/2}, s \rangle \ , \ \langle p^{3/2}, p \rangle \ , \ \langle p^{3/2}, S \rangle \ , \ \langle p^{3/2}, S \rangle$$

In the present case both the resonance and background terms in the last two transition matrix elements have been assumed negligible i.e.

$$\langle p^{3/2}, S \rangle \ = \ \langle p^{3/2}, S \rangle \ = \ 0$$

Also, if the background or non-resonant partial waves are limited to s, p, and d waves for incident and outgoing protons, the following relation of linear dependence among the Legendre coefficients can be derived (Ch 72):

$$A_2^S = -0.15 A_0 + 0.75 A_2 + 0.5 A_0^S$$

The above assumptions were made in the analysis of the 12.54 MeV $3/2^-$ level in $^{51}$V.

The cross sections are expanded as (Ch 72):

$$\left[ \frac{d\sigma}{d\Omega} (\theta_p) \right]^{(s)} = A_0^{(s)} + A_1^{(s)} P_1 (\cos \theta_p)$$

$$+ A_2^{(s)} P_2 (\cos \theta_p) \quad \quad (22)$$
The expressions for the Legendre coefficients in terms of transition matrix elements are (Ch 72):

\[ A_0 = 2 |T_{p\frac{3}{2}p\frac{3}{2}}|^2 + 2 |T_{p\frac{3}{2}p\frac{3}{2}}|^2 \]  
\[ A_1 = 2 \sqrt{2} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) + \frac{4 \sqrt{3}}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) 
+ \frac{4 \sqrt{3}}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) + \frac{4}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) 
- \frac{8}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) + \frac{24}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) 
- \frac{3}{5} \sqrt{2} R(T_{p\frac{3}{2}p\frac{3}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) \]  
\[ A_2 = -\frac{6}{5} |T_{p\frac{3}{2}p\frac{3}{2}}|^2 + \frac{2 \sqrt{3}}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{p\frac{1}{2}p\frac{3}{2}}}) 
- \frac{4}{5} \sqrt{2} R(T_{p\frac{3}{2}p\frac{3}{2}T_{p\frac{1}{2}p\frac{3}{2}}}) + \frac{8}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{p\frac{1}{2}p\frac{3}{2}}}) \]  
\[ A_0^S = \frac{3}{5} |T_{p\frac{3}{2}p\frac{3}{2}}|^2 + \frac{4}{5} |T_{p\frac{3}{2}p\frac{3}{2}}|^2 + \frac{1}{5} \sqrt{2} R(T_{p\frac{3}{2}p\frac{3}{2}T_{p\frac{1}{2}p\frac{3}{2}}}) + \frac{2 \sqrt{3}}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{p\frac{1}{2}p\frac{3}{2}}}) 
- \frac{4}{5} R(T_{p\frac{3}{2}p\frac{3}{2}T_{p\frac{1}{2}p\frac{3}{2}}}) \]  
\[ A_1^S = \frac{2 \sqrt{2}}{5} R(T_{p\frac{3}{2}p\frac{1}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) \]  
\[ - \frac{2 \sqrt{3}}{5} R(T_{p\frac{3}{2}p\frac{1}{2}T_{s\frac{1}{2}d\frac{3}{2}}}) \]  
\[ + \frac{6}{5} R(T_{p\frac{3}{2}p\frac{1}{2}T_{d\frac{3}{2}s\frac{1}{2}}}) + \frac{6}{5} R(T_{p\frac{3}{2}p\frac{1}{2}T_{d\frac{3}{2}s\frac{1}{2}}}) \]
$$A_{e}^{s} = -\frac{4}{s} \left| T_{p\frac{s}{2}p\frac{s}{2}} \right|^2 + \frac{12}{s} \left( T_{p\frac{s}{2}p\frac{s}{2}} T^{*}_{p\frac{s}{2}p\frac{s}{2}} \right)$$

$$- \frac{2\sqrt{2}}{s} \left( T_{p\frac{s}{2}p\frac{s}{2}} T^{*}_{p\frac{s}{2}p\frac{s}{2}} \right) + \frac{4}{s} \left( T_{p\frac{s}{2}p\frac{s}{2}} T^{*}_{p\frac{s}{2}p\frac{s}{2}} \right)$$

The transition matrix elements appearing in the even order coefficients are assumed to be:

$$T_{p\frac{s}{2}p\frac{s}{2}} = \frac{\sqrt{\pi e^{\frac{\pi^2}{4}}} e^{i\alpha p\frac{s}{2}p\frac{s}{2}}}{(E-E_{R}) + \frac{i\pi}{2}} + R_{p\frac{s}{2}p\frac{s}{2}} e^{i\beta p\frac{s}{2}p\frac{s}{2}}$$

$$T_{p\frac{s}{2}p\frac{s}{2}} = \frac{\sqrt{\pi e^{\frac{\pi^2}{4}}} e^{i\alpha p\frac{s}{2}p\frac{s}{2}}}{(E-E_{R}) + \frac{i\pi}{2}} + R_{p\frac{s}{2}p\frac{s}{2}} e^{i\beta p\frac{s}{2}p\frac{s}{2}}$$

$$T_{p\frac{s}{2}p\frac{s}{2}} = R_{p\frac{s}{2}p\frac{s}{2}} e^{i\beta p\frac{s}{2}p\frac{s}{2}}$$

Transition matrix elements for incident or outgoing s and d waves do not appear in the expressions for the even order coefficients. Before writing down the coefficients the following compact notation is defined:

$$\alpha_{p\frac{s}{2}p\frac{s}{2}} \equiv \alpha_{3} \quad \beta_{p\frac{s}{2}p\frac{s}{2}} \equiv \beta_{3} \quad \Gamma_{\frac{s}{2}} \equiv \Gamma_{1} \quad \beta_{p\frac{s}{2}p\frac{s}{2}} \equiv \beta_{1}$$

$$\Gamma_{\frac{s}{2}} = \Gamma_{1} \quad \chi \equiv \frac{2(E-E_{R})}{\Gamma}$$
\[ \beta \frac{p_{1/2} p_{3/2}}{p_{1/2}} = \beta_{13} = 0 \]
\[ R \frac{p_{3/2} p_{1/2}}{p_{3/2}} = R_{13} \]
\[ R \frac{p_{1/2} p_{3/2}}{p_{1/2}} = R_{13} \]

The transition matrices may be rewritten as:

\[ T p_{3/2} p_{3/2} = \frac{\sqrt{\frac{\text{ne} p_{3}^0}{\pi^2}} e^{i\alpha_3}}{E - E_R + \frac{i\eta}{2}} + R_{33} e^{i\beta_3} \]
\[ T p_{3/2} p_{1/2} = \frac{\sqrt{\frac{\text{ne} p_{3}^0}{\pi^2}} e^{i\alpha_1}}{E - E_R + \frac{i\eta}{2}} + R_{31} e^{i\beta_1} \]
\[ T p_{1/2} p_{3/2} = R_{13} \]

The even order Legendre coefficients are then:

\[ A_0 = A_0(B) + 2\left\{ \frac{\text{ne} p_{3}^0}{\pi^2} + R_{33} \sqrt{\frac{\text{ne} p_{3}^0}{\pi^2}} \sin(\alpha_3 - \beta_3) \right\} \frac{1}{1 + x^2} \]
\[ + 2\left\{ R_{31} \sqrt{\frac{\text{ne} p_{3}^0}{\pi^2}} \sin(\alpha_1 - \beta_1) \right\} \frac{x}{1 + x^2} \]
\[ + 2\left\{ R_{33} \sqrt{\frac{\text{ne} p_{3}^0}{\pi^2}} \cos(\alpha_3 - \beta_3) + R_{31} x \sqrt{\frac{\text{ne} p_{3}^0}{\pi^2}} \cos(\alpha_1 - \beta_1) \right\} \frac{1}{1 + x^2} \]

(32)
\[ A_2 = \frac{1}{4} \left\{ A_2^{\text{(B)}} + \frac{2\sqrt{2}}{5} R_{13} R_{33} \cos \beta_3 + \frac{\sqrt{2}}{5} R_{13} R_{31} \cos \beta_1 \right\} \]
\[ + \frac{1}{5} \left\{ \frac{6ne^{p_0}}{n^2} + \frac{3\sqrt{\beta}}{n} \sqrt{\frac{\beta}{n^2}} \cos (\alpha_3 - \alpha_1) + \sqrt{\frac{\beta e^{p_0}}{n^2}} \times \right\] 
\[ \left[ 4R_{33} \sin (\alpha_1 - \beta_3) + \sqrt{2} R_{13} \sin \alpha_1 \right] + \sqrt{\frac{\beta e^{p_0}}{n^2}} \times \]
\[ \left[ -6 R_{33} \sin (\alpha_3 - \beta_3) + 4R_{31} \sin (\alpha_3 - \beta_1) - 2\sqrt{2} R_{13} \times \sin \alpha_3 \right] \right\} \frac{1}{1 + x^2} + \frac{1}{5} \left\{ \sqrt{\frac{\beta e^{p_0}}{n^2}} \times \right\] 
\[ \left[ 4R_{33} \cos (\alpha_1 - \beta_3) \right. \]
\[ + \sqrt{2} R_{13} \cos \alpha_1 \left. \right] + \sqrt{\frac{\beta e^{p_0}}{n^2}} \left[ -6 R_{33} \cos (\alpha_3 - \beta_3) \right. \]
\[ + 4R_{31} \cos (\alpha_3 - \beta_1) - 2\sqrt{2} R_{13} \cos \alpha_3 \right\} \frac{x}{1 + x^2} \]

(33)

\[ A_0^S = \frac{1}{4} \left\{ A_0^{\text{(B)}} + \frac{2\sqrt{2}}{5} R_{13} R_{33} \cos \beta_3 + \frac{\sqrt{2}}{5} R_{13} R_{31} \cos \beta_1 \right\} \]
\[ \cos \beta_1 \right\} + \frac{1}{5} \left\{ \frac{4ne^{p_0}}{n^2} + \frac{3e^{p_0}}{n^2} - \frac{4ne^{p_0}}{n} \times \right\] 
\[ \sqrt{\frac{\beta}{n^2}} \cos (\alpha_3 - \alpha_1) + \sqrt{\frac{\beta e^{p_0}}{n^2}} \left[ 3R_{31} \times \right. \]
\[ \sin (\alpha_1 - \beta_1) - 2R_{33} \sin (\alpha_1 - \beta_3) + \frac{1}{\sqrt{2}} R_{13} \sin \alpha_1 \]
\[ + \left. \sqrt{\frac{\beta e^{p_0}}{n^2}} \times \right\] 
\[ \left( 4R_{33} \sin (\alpha_3 - \beta_3) - 2R_{31} \sin (\alpha_3 - \beta_1) \right. \]
\[ + \sqrt{2} R_{13} \sin \alpha_3 \right\} \right\} \frac{1}{1 + x^2} + \frac{1}{5} \left\{ \sqrt{\frac{\beta e^{p_0}}{n^2}} \times \right\] 
\[ \left[ 3R_{31} \cos (\alpha_1 - \beta_1) - 2 R_{33} \cos (\alpha_1 - \beta_3) + \frac{1}{\sqrt{2}} \right. \]
\[ + \]
\[ A^S_2 = \frac{1}{4} \left\{ A^S_2(B) - \frac{2\sqrt{\pi}}{2} R_{13} R_{33} \cos \beta_3 + \frac{2\sqrt{\pi}}{5} R_{13} R_{33} \cos \beta_1 \right\} + \frac{1}{5} \left\{ -\frac{4\pi p_3^0}{n^2} + \frac{4\pi p}{n} \sqrt{\frac{\pi p_3^0}{n^2}} \cos (\alpha_3 - \alpha_1) \right. \\
+ \sqrt{\frac{\pi p_3^0}{n^2}} \left[ 2 R_{33} \sin (\alpha_1 - \beta_3) + \frac{1}{2} R_{13} \sin \alpha_1 \right] \\
+ \frac{\pi p_3^0}{n^2} \left[ -4 R_{33} \sin (\alpha_3 - \beta_3) + 2 R_{13} \sin (\alpha_3 - \beta_1) \\
- \frac{1}{2} R_{13} \sin \alpha_3 \right] \} \frac{1}{1 + X^2} + \frac{1}{5} \left\{ \sqrt{\frac{\pi p_3^0}{n^2}} \left[ 2 R_{33} \cos (\alpha_1 - \beta_3) + \frac{1}{2} R_{13} \cos \alpha_1 \right] + \sqrt{\frac{\pi p_3^0}{n^2}} \right. \\
\left. \left[ -4 R_{33} \cos (\alpha_3 - \beta_3) + 2 R_{13} \cos (\alpha_3 - \beta_1) \\
- \frac{1}{2} R_{13} \cos \alpha_3 \right] \} \frac{X}{1 + X^2} \right\} \]
F. The Expressions for a 1/2^- Resonance

From the conservation laws of angular momentum and parity one would expect the following transition matrix elements for an excitation process from a 0^+ state through an intermediate 3/2^- resonance state:

\[ T_{\ell 3/2}^p \ell^p, T_{\ell 5/2}^p \ell^p \]

In the present case the even order coefficients have been calculated assuming incident s, p, and d waves and outgoing s, p, d and f waves. Further, the resonance and background terms in both resonating transition matrix elements have been retained. With these assumptions no relation of linear dependence exists among the even order Legendre coefficients. The inelastic differential cross section and inelastic spin-flip differential cross section have been calculated by M. Soga (So 72):

\[
\left[ \frac{d\sigma}{d\Omega}(\theta_p) \right]^{(s)} = A_0^{(s)} + A_1^{(s)} P_1(\cos \theta_p) \\
+ A_2^{(s)} P_2(\cos \theta_p) + A_3^{(s)} P_3(\cos \theta_p)
\]

(36)
The expressions for the Legendre coefficients in terms of transition matrices are (So 72):

\[ A_0 = |TPy_p \ y_2 |^2 + |TPy_p \ y_2 |^2 \]  \hspace{1cm} (37)

\[ A_1 = \frac{2}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} d_{\hat{s}y_2}) - \frac{4}{5} \sqrt{6} R (TPy_p \ y_2 T_{\hat{s}y_2} d_{\hat{s}y_2}) \]
\[ + 2 \sqrt{2} R (TPy_p \ y_2 T_{\hat{d}y_2} s_{\hat{s}y_2}) \]  \hspace{1cm} (38)

\[ A_2 = \frac{2 \sqrt{2}}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{d}y_2}) - \frac{4 \sqrt{2}}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{d}y_2}) \]
\[ + \frac{6}{5} \sqrt{6} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{d}y_2}) + \frac{8 \sqrt{3}}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{d}y_2}) \]
\[ + \frac{4 \sqrt{3}}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{d}y_2}) + \frac{32}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{d}y_2}) \]  \hspace{1cm} (39)

\[ A_3 = 2 \sqrt{3} R (TPy_p \ y_2 T_{\hat{d}y_2} s_{\hat{s}y_2}) \]  \hspace{1cm} (40)

\[ A_0^s = \frac{3}{10} |TPy_p \ y_2 |^2 + \frac{1}{5} |TPy_p \ y_2 |^2 + \frac{16}{5} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) \]
\[ + \frac{2 \sqrt{2}}{5} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) - \frac{3}{5} \sqrt{6} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) \]
\[ + \frac{16}{5} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) + \frac{2}{5} \sqrt{3} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) \]
\[ + \frac{4}{5} \sqrt{3} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) - \frac{6}{5} \sqrt{11} R (TPy_p \ y_2 T_{\hat{d}y_2} T_{\hat{s}y_2}) \]  \hspace{1cm} (41)

\[ A_1^s = \frac{4 \sqrt{2}}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{s}y_2}) + \frac{2 \sqrt{3}}{5} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{s}y_2}) \]
\[ + \frac{3}{5} \sqrt{2} R (TPy_p \ y_2 T_{\hat{s}y_2} T_{\hat{s}y_2}) \]  \hspace{1cm} (42)
\[ A^S_{2} = \frac{2\sqrt{2}}{5} R (T_{P_{1/2}P_{3/2}'} T_{P_{3/2}P_{1/2}'}) + \frac{2\sqrt{2}}{5} R (T_{P_{1/2}P_{1/2}'} T_{P_{3/2}P_{3/2}'}) \]
\[ + \frac{2}{5} \sqrt{\gamma} R (T_{P_{1/2}P_{3/2}'} T_{P_{3/2}S_{1/2}'}) + \frac{4}{5} \frac{1}{\sqrt{3}} R (T_{P_{1/2}S_{1/2}'} T_{P_{3/2}'}) \]
\[ - \frac{4}{5} \frac{1}{\sqrt{3}} R (T_{P_{1/2}S_{1/2}'} T_{P_{3/2}'}) + \frac{16}{5} \frac{1}{\sqrt{7}} R (T_{P_{1/2}S_{1/2}'} T_{P_{3/2}'}) \]

The transition matrix elements are assumed to be:

\[ T_{P_{1/2}P_{1/2}'} = \frac{\sqrt{\gamma^2 R_{1/2}'}}{E - E_R + \frac{i\alpha}{2}} + R_{P_{1/2}P_{1/2}'} \]

\[ T_{P_{1/2}S_{1/2}'} = \frac{\sqrt{\gamma^2 R_{1/2}'}}{E - E_R + \frac{i\beta}{2}} + R_{P_{1/2}S_{1/2}'} \]

\[ T_{P_{3/2}P_{1/2}'} = R_{P_{3/2}P_{1/2}'} e^{i\alpha} \]

\[ T_{P_{3/2}P_{3/2}'} = R_{P_{3/2}P_{3/2}'} e^{i\beta} \]

\[ T_{P_{3/2}S_{1/2}'} = R_{P_{3/2}S_{1/2}'} e^{i\beta} \]

Transition matrix elements for incident s and d waves do not appear in the expressions for the even order coefficients. Before writing down the coefficients the following compact notation is defined.
In the following expressions any phase angle may be set equal to zero (phase gauge). The transition matrices may be rewritten as:

\[
\begin{align*}
\alpha p_{1/2} p_{3/2} & = \alpha_{13} \\
\beta p_{1/2} p_{3/2} & = \beta_{13} \\
\alpha p_{1/2} s_{5/2} & = \alpha_{15} \\
\beta p_{1/2} s_{5/2} & = \beta_{15} \\
\beta p_{3/2} p_{1/2} & = \beta_{31} \\
R p_{1/2} p_{3/2} & = R_{13} \\
R p_{1/2} s_{5/2} & = R_{15} \\
R p_{3/2} p_{1/2} & = R_{33}
\end{align*}
\]

\[
\begin{align*}
\beta p_{3/2} p_{3/2} & = \beta_{33} \\
\beta p_{3/2} s_{5/2} & = \beta_{35} \\
\Gamma_{3/2} & = \Gamma_{3} \\
\Gamma_{5/2} & = \Gamma_{5} \\
X & = \frac{2 (E - E_R)}{\Gamma}
\end{align*}
\]

\[
\begin{align*}
\mathbf{T} p_{1/2} p_{3/2} & = \frac{\sqrt{\nu p_3^0} \ e^{i \alpha_{13}}}{(E - E_R) + \frac{i \Gamma}{2}} + R_{13} \ e^{i \beta_{13}} \\
\mathbf{T} p_{1/2} s_{5/2} & = \frac{\sqrt{\nu p_5^0} \ e^{i \alpha_{15}}}{(E - E_R) + \frac{i \Gamma}{2}} + R_{15} \ e^{i \beta_{15}} \\
\mathbf{T} p_{3/2} p_{1/2} & = R_{31} \ e^{i \beta_{31}}
\end{align*}
\]
The even order Legendre coefficients are then:

\[ T_{p_{3/2} p_{3/2}} = R_{33} e^{i \beta_{33}} \]
\[ T_{p_{3/2} s_{7/2}} = R_{35} e^{i \beta_{35}} \]

\[ A_0 = \frac{R_{13}^2}{4} + \frac{R_{15}^2}{4} + \left\{ \frac{\sqrt{n p_6}}{p_2} + \frac{r e p_6}{p_2} + R_{13} \sqrt{\frac{n e p_6}{p_2}} \right\} \sin (\alpha_{13} - \beta_{13}) + R_{15} \sqrt{\frac{n e p_6}{p_2}} \sin (\alpha_{15} - \beta_{15}) \]
\[ + \frac{1}{1 + x^2} + \left\{ R_{13} \sqrt{\frac{n e p_6}{p_2}} \cos (\alpha_{13} - \beta_{13}) + R_{15} \sqrt{\frac{n e p_6}{p_2}} \cos (\alpha_{15} - \beta_{15}) \right\} \frac{x}{1 + x^2} \]

\[ (49) \]

\[ A_2 = \frac{\sqrt{3}}{10} R_{13} R_{31} \cos (\beta_{13} - \beta_{31}) - \frac{\sqrt{27}}{5} R_{13} R_{23} \]
\[ \cos (\beta_{12} - \beta_{32}) + \frac{3}{10} \sqrt{\frac{3}{7}} R_{13} R_{35} \cos (\beta_{13} - \beta_{35}) + \frac{3 \sqrt{3}}{5} R_{15} R_{31} \cos (\beta_{15} - \beta_{31}) + \frac{\sqrt{3}}{5} R_{15} R_{33} \cos (\beta_{15} - \beta_{33}) + \frac{8}{5} \frac{1}{17} R_{15} R_{35} \cos (\beta_{15} - \beta_{35}) \]
\[ + \left\{ \frac{1}{2} \sqrt{\frac{n e p_3}{p_2}} \right\} \left[ \frac{2 \sqrt{2}}{5} R_{31} \sin (\alpha_{13} - \beta_{31}) \right. \]
\[ - \frac{4 \sqrt{2}}{5} R_{33} \sin (\alpha_{13} - \beta_{33}) + \frac{6}{5} \sqrt{\frac{6}{7}} R_{35} \sin (\alpha_{13} - \beta_{35}) \right\} + \frac{1}{2} \sqrt{\frac{n e p_6}{p_2}} \left[ \frac{8 \sqrt{3}}{5} R_{31} \sin (\alpha_{15} - \beta_{31}) \right. \]
\[ + \frac{4 \sqrt{3}}{5} R_{33} \sin (\alpha_{15} - \beta_{33}) + \frac{32}{5} \frac{1}{17} R_{35} \sin (\alpha_{15} - \beta_{35}) \right\} \frac{1}{1 + x^2} + \]
\[
A_0^s = \frac{3}{40} R_{33}^2 + \frac{1}{20} R_{15}^2 + \frac{\sqrt{6}}{20} R_{13} R_{15} \cos(\beta_{13} - \beta_{33}) \\
+ \frac{\sqrt{15}}{10} R_{13} R_{33} \cos(\beta_{13} - \beta_{33}) \left( R_{33} \cos(\alpha_{13} - \beta_{31}) + \frac{4 \sqrt{5}}{5} R_{33} \cos(\alpha_{15} - \beta_{33}) + \frac{22}{5} \frac{1}{17} R_{35} \cos(\alpha_{15} - \beta_{35}) \right) \\
+ \frac{1}{2} \sqrt{\frac{\mu e \rho_0}{\mu^2}} \left[ \frac{2 \sqrt{15}}{5} R_{31} \cos(\alpha_{13} - \beta_{31}) + \frac{4 \sqrt{5}}{5} \right] \\
\left( \frac{R_{33} \cos(\alpha_{15} - \beta_{33}) + \frac{22}{5} \frac{1}{17} R_{35} \cos(\alpha_{15} - \beta_{35})}{1 + x^2} \right) \]
\[
A_2^3 = \frac{\sqrt{2}}{10} R_{13} R_{31} \cos(\beta_{13} - \beta_{31}) - \frac{\sqrt{2}}{10} R_{13} R_{33} \cos(\beta_{13} - \beta_{33}) + \frac{2}{5} \sqrt{\frac{6}{7}} R_{13} R_{35} \cos(\beta_{35} - \beta_{33}) + \frac{2}{5} \sqrt{\frac{6}{7}} R_{13} R_{35} \cos(\beta_{35} - \beta_{35})
\]

\[
+ \left\{ \frac{1}{2} \sqrt{\frac{2}{5}} \left[ \frac{2}{5} \sqrt{\frac{6}{7}} R_{31} \sin(\alpha_{13} - \beta_{31}) - \frac{2}{5} \sqrt{\frac{6}{7}} R_{35} \sin(\alpha_{15} - \beta_{35}) \right] \right\} + \frac{1}{2} \sqrt{\frac{2}{5}} \left[ \frac{2}{5} \sqrt{\frac{6}{7}} R_{31} \sin(\alpha_{15} - \beta_{31}) + \frac{2}{5} \sqrt{\frac{6}{7}} R_{35} \sin(\alpha_{13} - \beta_{35}) \right] \frac{1}{1 + X^2}
\]

\[ + \frac{1}{2} \sqrt{\frac{2}{5}} \left[ \frac{2}{5} \sqrt{\frac{6}{7}} R_{31} \sin(\alpha_{15} - \beta_{31}) + \frac{2}{5} \sqrt{\frac{6}{7}} R_{35} \sin(\alpha_{13} - \beta_{35}) \right] \frac{1}{1 + X^2}
\]

\[ + \frac{1}{2} \sqrt{\frac{2}{5}} \left[ \frac{2}{5} \sqrt{\frac{6}{7}} R_{31} \sin(\alpha_{15} - \beta_{31}) + \frac{2}{5} \sqrt{\frac{6}{7}} R_{35} \sin(\alpha_{13} - \beta_{35}) \right] \frac{1}{1 + X^2}
\]
\[ R_{33} \sin (\alpha_{13} - \beta_{33}) + \frac{8}{5} \sqrt{\frac{6}{7}} R_{35} \sin (\alpha_{13} - \beta_{35}) \]
\[ + \frac{1}{2} \sqrt{\frac{\pi \hbar \gamma}{n^2}} \left[ \frac{4}{5} \frac{1}{\sqrt{3}} R_{31} \sin (\alpha_{15} - \beta_{31}) - \frac{4}{5} \frac{1}{\sqrt{3}} R_{33} \sin (\alpha_{15} - \beta_{33}) + \frac{16}{5} \sqrt{\frac{1}{7}} \sin (\alpha_{15} - \beta_{35}) \right] \]
\[ \times \frac{1}{1 + \chi^2} + \left\{ \frac{1}{2} \sqrt{\frac{\pi \hbar \gamma}{n^2}} \left[ \frac{2 \sqrt{2}}{5} R_{31} \cos (\alpha_{13} - \beta_{31}) \right. \right. \]
\[ - \frac{2 \sqrt{2}}{5} R_{33} \cos (\alpha_{13} - \beta_{33}) + \frac{9}{5} \sqrt{\frac{6}{7}} R_{35} x \cos (\alpha_{13} - \beta_{35}) \right\] + \frac{1}{2} \sqrt{\frac{\pi \hbar \gamma}{n^2}} \left[ \frac{4}{5} \frac{1}{\sqrt{3}} R_{31} \cos (\alpha_{15} - \beta_{31}) - \frac{4}{5} \frac{1}{\sqrt{3}} R_{33} \cos (\alpha_{15} - \beta_{33}) \right.
\[ + \frac{16}{5} \sqrt{\frac{1}{7}} R_{35} \cos (\alpha_{15} - \beta_{35}) \right\} \frac{X}{1 + \chi^2} \]
\[ (52) \]
G. Modification of the $1/2^-$ Expressions

If the background amplitudes in the $T_{p_{12} 5_{5/2}}, T_{p_{3/2} 5_{5/2}}$ matrix elements are small and can be assumed to be zero ($R_{15} = R_{35} = 0$) the only remaining f-wave contribution in the expressions will be due to the resonating part of the $T_{p_{12} 5_{5/2}}$ element. If that term also vanishes ($\Gamma_5^0 = 0$) there will be no f-waves present. In that case the same relation (eq. 21) holds among the Legendre coefficients as did for the $3/2^-$ pure p-wave resonance (this assumption was made in the analysis of the 12.28 MeV $1/2^-$ level in $^{51}$V). If the resonating f-wave term does not vanish, but the resonating p-wave term does ($\Gamma_3^0 = 0$) and $R_{35}$ is negligibly small a different relation exists among the Legendre coefficients. The relation is (So 72):

$$A_2^S = 0.40 A_0 + 0.33 A_2 - 2.00 A_0^S$$  \hspace{1cm} (53)

Again, if neither the f-wave or p-wave contributions vanish, no definite relation exists among the coefficients.
III. EXPERIMENTAL METHOD

The Western Michigan University En tandem Van de Graaff was used to produce beams of 4.0 to 8.0 MeV protons which impinged on 13 keV-thick (at 7.5 MeV) natural carbon and 50 keV-thick (at 4.5 MeV) $^{50}$Ti targets. The energy of the beam was determined by a 90° analyzing magnet calibrated by Parrot (Pa 71). Beam currents ranged from 50 to 450 nano-amps. The number of incident particles was determined by a beam current integrator.

The protons were scattered from a target placed in an evacuated chamber and detected by a surface barrier detector. A 0.25 in. diameter Ta collimator was located 2.625 in. from the target. The effective half-angle subtended by the detector was then 2.7°. In between the target and collimator a 0.31 in. diameter anti-scattering collimator was located.

Gamma rays emitted perpendicular to the reaction plane were detected by a 2 in. diameter by 3 in. thick NaI crystal. The front face of the crystal was located 7.0625 in. above the target. The half-angle subtended by the crystal was approximately 6.7°.

Coincidences were detected between the inelastically scattered protons and the de-excitation
gamma rays using the electronics arrangement shown in Figure V.

Pulses from the solid state proton detector were used to generate fast logic pulses by the time pick-off unit (TPU) and time pick-off controller (TPC). These pulses triggered the start of the time to amplitude converter (TAC), and fast pulses from the NaI detector stopped the TAC. The fast pulses from the NaI detector were highly amplified and the fast discriminator level was set low so as to reduce time jitter due to different pulse heights. The pulse height spectrum from the TAC was then gated by amplified linear signals from the NaI crystal which were passed through a single channel analyzer (SCA). The window on the SCA was set on the part of the gamma ray spectrum of interest. In this manner the time jitter was reduced while allowing only those pulses from the TAC generated by a selected portion of the gamma ray spectrum to pass through the linear gate. A SCA was set on the TAC peak corresponding to true coincidences. The output of this SCA was used to gate the slow signals from the proton detector. The signals which passed through that gate were sent to the multi-channel analyzer (MCA). Ideally, with the gate in, only those pulses from the proton detector arising from spin-flip events would enter the MCA. In reality there were
Figure V. Block diagram of electronics used to detect coincidences between inelastically scattered protons and de-excitation gamma rays.
always a certain number of accidental coincidences across the entire time spectrum. With the final gate out all pulses from the proton detector were sent to the MCA. The MCA and a scaler were gated-on by the beam current integrator. The purpose of the scalar was to monitor the elastic or inelastic counts.

Shown in Figure VI are typical coincidence and non-coincidence spectra. The relatively small elastic peak in the coincidence spectrum is due solely to accidentals, Figure VII shows a typical TAC spectrum.

The inelastic and inelastic spin-flip differential cross sections for $^{12}$C $(P, P^1)$ $^{12}$C were measured at 10 to 12 angles ranging from about 45° to 165° (laboratory) at energies on and near the 1/2$^-$ resonance from 7 to 8 MeV (laboratory). Inelastic and spin-flip cross sections were also measured for $^{50}$Ti $(P, P^1)$ $^{50}$Ti in the energy region on and near the adjacent 1/2$^-$ and 3/2$^-$ resonances at energies ranging from about 4.2 to 4.8 MeV (laboratory) at 12 to 13 angles ranging from 50° to 165° (laboratory). The inelastic cross section was obtained by absolute normalization to cross sections measured by Barnard et. al. (Ba 66) in the case of $^{12}$C; and Cosman et. al. (Co 68) in the case of $^{50}$Ti.

The number of inelastic coincidences was corrected for accidentals by multiplying the inelastic counts by the ratio of the purely accidental elastic
Figure VI. Coincidence and non-coincidence spectra for $^n \text{C}(P, P) \text{n}_C$. 

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\[ ^{12} \text{C} + \text{protons} \]

Ungated Spectrum
\[ E_p = 7.654 \text{ MeV} \]
\[ \theta = 160^\circ \]

Coincidence Spectrum

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Figure VII. A typical TAC spectrum for $^{50}_{\text{Ti}} (p, p') ^{50}_{\text{Ti}}$. 

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$^{50}\text{Ti}^+$ protons

Counts

Channel Number

9 nano-sec

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counts in the coincidence spectrum to the total elastic counts and subtracting from the inelastic coincidences. The number of accidentals was generally less than 20% of the number of true coincidences.

The relation (Sc 64) between the number of true coincidences, \( N(\theta) \), and the spin-flip probability, \( S(\theta) \), is

\[
S(\theta) = \frac{8\pi}{5} \frac{N(\theta)}{\varepsilon \Omega}
\]

where \( \varepsilon \Omega \) is the product of the gamma ray detector efficiency and solid angle. The factor \( \frac{8\pi}{5} \) is present due to the fact that not all spin-flip events emit radiation perpendicular to the reaction plane. Corrections to the spin-flip probabilities due to finite solid angles were usually less than .02. These corrections were made using the results of Ko 69 and Hi 70.

The differential cross section for inelastic scattering with a spin-flip (called the spin-flip cross section) at a given angle and energy, is obtained from the measurements by multiplying the spin-flip probability at that angle and energy by the differential cross section for inelastic scattering at that angle and energy.

The proton detector was assumed to be 100% efficient. For the gamma ray detector the product of efficiency times solid angle was determined using
calibrated sources of $^{22}$Na and $^{60}$Co. These results were extrapolated to the gamma ray energies of interest in the present work using the efficiency curves in Marion and Young (Ma 68). The uncertainty in this determination is estimated to be $\pm 10\%$.

The uncertainties in the absolute values of the inelastic cross sections are 8% and 15% for $^{12}$C and $^{50}$Ti, respectively. The uncertainties in the relative values for different angles and energies are mostly less than 4%. The uncertainties in the relative spin-flip differential cross sections are usually determined by the statistical error in the number of coincidences.
IV. RESULTS AND DISCUSSION

A. Legendre Coefficients

It has been previously mentioned (see eqs. 18, 19) that both the spin-flip differential cross section and inelastic differential cross section can be expressed in a Legendre polynomial expansion. For the three resonances considered here, both experimentally determined cross sections were individually fit to determine the Legendre polynomial coefficients at each energy point with a computer search routine (We 72). The results of these fits to the center of mass cross sections are shown in Tables I and II.

Table I lists the values of the coefficients in units of mm/str for the spin-flip and inelastic cross sections for scattering to the $2^+$ first excited state of Ti on and near the adjacent $\not{J} = 1/2^-$ (at $E_R = 4.31$ MeV) and $\not{J} = 3/2^-$ (at $E_R = 4.57$ MeV) compound nuclear levels. These levels in $^{51}$V have been identified as analog states in measurements made by Cosman, Slater, and Spencer (Co 68). In this earlier work much finer resolution was used than in the present case, where the resolution (about 50 keV) exceeds the spacing of the fine structure.
Table I. Values for the Legendre coefficients in mb/sr at energies ranging over the adjacent 12.28 MeV $1/2^-$ and 12.54 MeV $3/2^-$ levels in $^{51}$V.
<table>
<thead>
<tr>
<th>Lab Energy</th>
<th>$A_0$</th>
<th>$A_0^S$</th>
<th>$A_1$</th>
<th>$A_1^S$</th>
<th>$A_2$</th>
<th>$A_2^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.168</td>
<td>0.598 ± 0.008</td>
<td>0.017 ± 0.017</td>
<td>-0.013 ± 0.017</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.268</td>
<td>2.059 ± 0.025</td>
<td>0.619 ± 0.020</td>
<td>0.335 ± 0.054</td>
<td>-0.058 ± 0.046</td>
<td>-0.175 ± 0.054</td>
<td>-0.090 ± 0.054</td>
</tr>
<tr>
<td>4.322</td>
<td>3.131 ± 0.023</td>
<td>0.873 ± 0.024</td>
<td>0.675 ± 0.072</td>
<td>-0.041 ± 0.053</td>
<td>0.054 ± 0.080</td>
<td>-0.040 ± 0.063</td>
</tr>
<tr>
<td>4.367</td>
<td>1.847 ± 0.019</td>
<td>0.438 ± 0.017</td>
<td>0.265 ± 0.042</td>
<td>-0.004 ± 0.038</td>
<td>0.088 ± 0.048</td>
<td>0.011 ± 0.044</td>
</tr>
<tr>
<td>4.445</td>
<td>0.961 ± 0.013</td>
<td>0.314 ± 0.012</td>
<td>-0.080 ± 0.028</td>
<td>-0.112 ± 0.026</td>
<td>0.096 ± 0.035</td>
<td>0.025 ± 0.035</td>
</tr>
<tr>
<td>4.503</td>
<td>1.581 ± 0.016</td>
<td>0.430 ± 0.014</td>
<td>-0.151 ± 0.033</td>
<td>-0.106 ± 0.030</td>
<td>0.241 ± 0.043</td>
<td>0.126 ± 0.041</td>
</tr>
<tr>
<td>4.450</td>
<td>3.935 ± 0.055</td>
<td>0.924 ± 0.024</td>
<td>0.396 ± 0.117</td>
<td>-0.196 ± 0.052</td>
<td>0.378 ± 0.101</td>
<td>0.231 ± 0.059</td>
</tr>
<tr>
<td>4.553</td>
<td>5.233 ± 0.056</td>
<td>1.119 ± 0.029</td>
<td>1.273 ± 0.121</td>
<td>-0.212 ± 0.062</td>
<td>0.774 ± 0.135</td>
<td>0.234 ± 0.069</td>
</tr>
<tr>
<td>4.577</td>
<td>5.960 ± 0.064</td>
<td>1.440 ± 0.032</td>
<td>1.722 ± 0.139</td>
<td>-0.146 ± 0.072</td>
<td>0.867 ± 0.140</td>
<td>0.352 ± 0.079</td>
</tr>
<tr>
<td>4.608</td>
<td>3.712 ± 0.040</td>
<td>0.820 ± 0.022</td>
<td>0.885 ± 0.086</td>
<td>-0.210 ± 0.049</td>
<td>0.532 ± 0.096</td>
<td>0.188 ± 0.062</td>
</tr>
<tr>
<td>4.623</td>
<td>2.544 ± 0.028</td>
<td>0.538 ± 0.018</td>
<td>0.751 ± 0.059</td>
<td>-0.144 ± 0.039</td>
<td>0.538 ± 0.065</td>
<td>-0.008 ± 0.050</td>
</tr>
<tr>
<td>4.660</td>
<td>1.151 ± 0.012</td>
<td>0.125 ± 0.026</td>
<td>0.182 ± 0.030</td>
<td>0.432 ± 0.045</td>
<td>0.332 ± 0.045</td>
<td>-0.052 ± 0.020</td>
</tr>
<tr>
<td>4.725</td>
<td>0.891 ± 0.023</td>
<td>0.190 ± 0.008</td>
<td>0.203 ± 0.048</td>
<td>-0.052 ± 0.017</td>
<td>0.332 ± 0.045</td>
<td>-0.052 ± 0.020</td>
</tr>
</tbody>
</table>
Table II lists the values in units of $\text{mb}/\text{sr}$ for the Legendre coefficients required to reproduce the spin-flip and inelastic cross sections for scattering to the $2^+$ first excited state of $^{12}\text{C}$ and on and near the $J^\pi = 1/2^-$ (at $E_r = 7.575 \text{ MeV}$) state in $^{13}\text{N}$. This level was identified in previous measurements by Barnard, Swint and Clegg (Ba 66).

Figure VIII shows angular distribution data points for both spin-flip and inelastic cross sections at several energies on and near the adjacent $J^\pi = 1/2^-$ (4.322 MeV) and $J^\pi = 3/2^-$ (4.575 MeV) resonances in $^{51}\text{V}$. The solid curves are the Legendre polynomial fits to the data.

Figure IX shows angular distribution data points and the corresponding Legendre polynomial fits to the data for the $J^\pi = 1/2^-$ (7.575 MeV) resonance in $^{13}\text{N}$, for typical energies on and near the resonance.
Table II. Values for the Legendre coefficients in mb/sr at energies ranging over the 8.90 MeV $1/2^-$ level in $^{13}$N.
\[ ^{12}\text{C}(p,p')^{12}\text{C} \]

<table>
<thead>
<tr>
<th>Lab Energy</th>
<th>( A_0 )</th>
<th>( A_0^s )</th>
<th>( A_1 )</th>
<th>( A_1^s )</th>
<th>( A_2 )</th>
<th>( A_2^s )</th>
<th>( A_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.542</td>
<td>14.817±.251</td>
<td>3.396±.067</td>
<td>7.788±.587</td>
<td>.555±.143</td>
<td>5.301±.671</td>
<td>1.534±.144</td>
<td>-1.036±.469</td>
</tr>
<tr>
<td>7.648</td>
<td>15.668±.239</td>
<td>3.053±.060</td>
<td>2.668±.546</td>
<td>-1.889±.130</td>
<td>10.880±.625</td>
<td>2.358±.148</td>
<td>.567±.520</td>
</tr>
<tr>
<td>7.719</td>
<td>15.303±.223</td>
<td>2.885±.061</td>
<td>.709±.505</td>
<td>-2.564±.123</td>
<td>11.102±.579</td>
<td>2.623±.163</td>
<td>.898±.509</td>
</tr>
</tbody>
</table>
Figure VIII. Angular distributions for inelastic scattering and spin-flip inelastic scattering near the center of the $\Sigma^N=1/2^-$ (4.3220 MeV) and $\Sigma^N=3/2^-$ (4.577 MeV) resonances in $^{\text{40}}\text{V}$. Off resonance energies (4.445 MeV, 4.725 MeV) are located in between and well above both resonances, respectively. The solid curves represent Legendre polynomial fits to the experimental data.
Figure IX. Angular distributions for inelastic scattering and spin-flip inelastic scattering near the center of the \( ^3\Sigma^+_1/2^- \) (7.575 MeV) resonance in \(^{13}\)N. Off resonance energies (6.998 MeV, 7.797 MeV) are located below and above the resonance, respectively. The solid curves represent Legendre polynomial fits to the experimental data. Note that the energy of each angular distribution is listed in MeV adjacent to the curve.
B. Energy Dependence of the Legendre Coefficients

The equation describing the energy dependence of each Legendre coefficient (see eq. 20) is:

\[ A_n^{(s)} = B_n^{(s)} + \frac{C_n^{(s)}}{1 + x^2} + \frac{D_n^{(s)} x}{1 + x^2} \]

For the three resonances considered here, all but the \( \frac{31}{2}^- \) resonance in \(^{13}\text{N}\) were treated assuming the \( C_n^{(s)} \) and \( D_n^{(s)} \) to be constant. The background term was assumed to be a linear function of the energy i.e.

\[ B_n^{(s)} = a_n^{(s)} E + b_n^{(s)} \]  

(55)

The values for the \( a_n^{(s)}, b_n^{(s)}, C_n^{(s)} \) and \( D_n^{(s)} \) were attained using a computer search routine (Ra 72); they are shown in Tables III, IV and V. Only the even order coefficients were fit since the odd order coefficients arise from interference between partial waves of different parity and are not of interest for the considerations made here. Also appearing in each table are the resonance energy \( (E_R) \) and total width \( (\Gamma) \) of the state, which were attained simultaneously with the other parameters.

Since the \( \frac{31}{2}^- \) resonance in \(^{13}\text{N}\) has a relatively large total width \( (\Gamma = 250 \text{ keV}) \) the barrier penetration may change significantly over the resonance.
Table III. The symmetric \( C_n^{(s)} \) and antisymmetric \( D_n^{(s)} \) amplitudes from eq. 20 are given in \( \text{mb/sr} \) for the \( J^P = 1/2^- \) 12.28 MeV level in \( ^8\text{B} \) for all even order coefficients required to fit the data. The total width \( (\Gamma) \) and resonance energy \( (E_r) \) are given in MeV. The slope \( (\alpha_n^{(s)}) \) and constant \( (b_n^{(s)}) \) of the linearly varying background are given in units of \( \text{mb/sr-MeV} \) and \( \text{mb/sr} \), respectively.
\[ ^{50}\text{Ti} \left( p, p' \right) ^{50}\text{Ti} \quad ^{30}\text{P} = \frac{1}{2}^- \text{ Resonance} \]

\[ \Gamma_1 = 0.110, \quad E_R = 4.312 \]

<table>
<thead>
<tr>
<th>Legendre Coefficient</th>
<th>( C_n^{(s)} )</th>
<th>( D_n^{(s)} )</th>
<th>( a_n^{(s)} )</th>
<th>( b_n^{(s)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 )</td>
<td>3.00</td>
<td>0.0</td>
<td>0.523</td>
<td>-2.033</td>
</tr>
<tr>
<td>( A_0^s )</td>
<td>0.80</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.067</td>
<td>0.0</td>
<td>0.355</td>
<td>-1.561</td>
</tr>
<tr>
<td>( A_2^s )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

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Table IV. The symmetric \( C_{n}^{(0)} \) and antisymmetric amplitudes from eq. (20) are given in \( \text{mb/str} \) for the \( \Sigma^+ = 3/2^- \) 12.54 MeV level in \( ^{31}\text{V} \) for all even order coefficients required to fit the data. The total width \( (\Gamma) \) and resonance energy \( (E_R) \) are given in MeV. The slope \( \left( a_{n}^{(1)} \right) \) and constant \( \left( b_{n}^{(1)} \right) \) of the linearly varying background are given in units of \( \frac{\text{mb}}{\text{str} \cdot \text{MeV}} \) and \( \frac{\text{mb}}{\text{str}} \) respectively.
\[ \text{Ti}(\text{P}, \text{P}) \text{Ti}, \quad J^\pi = 3/2^- \text{ Resonance} \]

\[ \Pi = 0.073, \quad E_R = 4.5725 \]

<table>
<thead>
<tr>
<th>Legendre Coefficient</th>
<th>( C_n^{(s)} )</th>
<th>( D_n^{(s)} )</th>
<th>( a_n^{(s)} )</th>
<th>( b_n^{(s)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 )</td>
<td>6.091</td>
<td>0.06</td>
<td>0.523</td>
<td>-2.033</td>
</tr>
<tr>
<td>( A_0^{s} )</td>
<td>1.370</td>
<td>-0.05</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.842</td>
<td>-0.03</td>
<td>0.279</td>
<td>-1.210</td>
</tr>
<tr>
<td>( A_2^{s} )</td>
<td>0.4024</td>
<td>-0.057</td>
<td>0.0</td>
<td>-0.03</td>
</tr>
</tbody>
</table>
Table V. The symmetric ($C_n^{(s)}$) and antisymmetric ($D_n^{(s)}$) amplitudes from eq. (20) are given in $\text{mb/sr}$ for the $^{14}$N = 1/2− 8.90 MeV level in $^{15}$N for all even order coefficients required to fit the data. The total width ($\Gamma$) and resonance energy ($E_R$) are given in MeV. The slope ($A_n^{(s)}$) and constant ($b_n^{(s)}$) of the linearly varying background are given in units of $\text{mb/sr MeV}$ and $\text{mb/sr}$, respectively.
\(^{12}\text{C} (p,p')^{12}\text{C}, \quad \frac{\hbar}{\hbar} = \frac{1}{2}^- \text{ Resonance} \quad \Gamma = 0.250, \quad E_R = 7.575

<table>
<thead>
<tr>
<th>Legendre Coefficient</th>
<th>(C_n^{(S)})</th>
<th>(D_n^{(S)})</th>
<th>(A_n^{(S)})</th>
<th>(b_n^{(S)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_0)</td>
<td>5.642</td>
<td>0.0</td>
<td>-1.5</td>
<td>21.9</td>
</tr>
<tr>
<td>(A_0^s)</td>
<td>1.712</td>
<td>-0.2123</td>
<td>-0.330</td>
<td>4.340</td>
</tr>
<tr>
<td>(A_2)</td>
<td>4.170</td>
<td>4.768</td>
<td>0.700</td>
<td>-1.0</td>
</tr>
<tr>
<td>(A_2^s)</td>
<td>3.140</td>
<td>3.470</td>
<td>-0.770</td>
<td>7.060</td>
</tr>
</tbody>
</table>

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Further, the presence of $A_3$ indicates that f-waves are involved in the decay of that level. It was decided that the effect of the barrier penetration could be investigated by calculating a curve for $A_0$ using eq. 20 with the barrier penetration factor present, and then fitting this calculated curve with the same equation except with no penetration factor present. The maximum effect of the barrier penetration on the shape of this curve occurs for pure f-wave decay of the $1/2^-$ resonating level. Thus, a purely symmetric background free curve was calculated for $A_0$ and multiplied by the normalized f-wave barrier penetration. The parameters $\Gamma, E_R$ and $C_0$ were chosen to be close to those of the experimental $A_0$. This curve was then fit using the previously mentioned computer search routine so as to obtain a new set of parameters. The new parameters are those which would be attained if a pure f-wave resonance were treated neglecting the barrier penetration. The parameters obtained from the fits to this "mocked-up" data did not differ from the original parameters by more than 5%. Thus it was concluded that for this resonance the barrier penetration is not a large effect even in the "worst" possible case of pure f-waves. However, a compromise barrier penetration factor, between the f-wave and p-wave penetrabilities, was introduced into eq. 20 so as to take into account the effect of the combined
penetrabilities in an approximate manner.

Figure X shows the Legendre coefficients as functions of energy for the $\tilde{J}^\pi = 1/2^-$ resonance in $^{3}\Lambda N$. The solid curves drawn through the data points represent fits to the experimental data. The dashed curve drawn through $A^S_L$ is the curve predicted from the previously mentioned (see eq. 21) linear relation among the coefficients, which holds for a pure p-wave resonance. The resonance clearly does not decay by pure p-waves.

Figure XI shows all Legendre coefficients required to reproduce both the $\tilde{J}^\pi = 1/2^-$ (at $E_R = 4.31$ MeV) and $\tilde{J}^\pi = 3/2^-$ (at $E_R = 4.573$ MeV) resonances in $^{51}V$. The solid curves represent fits to the data except in the case of $A^S_L$, in which case they represent predicted curves. These predicted curves were calculated using eq. 21, which holds for a pure p-wave resonance. The dashed lines represent the background ($B_n^{(SS)}$). Again, fits to $A_L, A^S_L$ are not necessary for the considerations made here.
Figure X. Energy dependence of Legendre coefficients for $J^P = 1/2^-$ ($E_R = 7.575$ MeV) resonance in $^{13}$N. The solid curves represent fits to the experimental data in the case of $A_2, A_4$ and $A_6$. The dashed curve for $A_4$ is the curve predicted by the theory for pure p-wave decay. The solid curve for $A_2$ was calculated assuming a mixture of 15% p-wave decay and 85% f-wave decay.
Figure XI. Energy dependence of Legendre coefficients for the adjacent $J^T=1/2^-$, $J^P=3/2^-$ levels in $^{61}$V. The solid curves represent fits to the data in the case of $A_0$, $A_2$, and $A_4$. For $A_2$, the solid curve represents the prediction of the theory for a pure p-wave resonance. The dashed lines represent the slowly varying background.
C. Spectroscopic Parameters

As previously mentioned the symmetric ($D^{(s)}_n$) and asymmetric ($F^{(s)}_n$) amplitudes in eq. 20 may be associated with the coefficients of the energy dependent factors in the previously presented expressions for the Legendre coefficients (eqns. 32-35, 49-52). Thus, for each resonance the previously determined symmetric and asymmetric amplitudes may be expressed in terms of the various phase angles ($\alpha_{ij}'$, $\beta_{ij}'$) background amplitudes ($R_{ij}'$) and partial widths of decay ($\Gamma_{ij}'$). The essence of this experiment lies in the fact that both the spin-flip and inelastic Legendre coefficients for the same resonance can be expressed in terms of the same $\alpha'$s, $\beta'$s, $R'$s and $\Gamma'_{ij}$s. Thus, a determination of the spin-flip coefficients provides powerful constraints on the regions of parameter space which will reproduce the symmetric and asymmetric amplitudes for a given resonance. A computer search routine has been written by J. J. Ramirez and E. M. Bernstein (Ra 72) which calculates the set of even-order symmetric and asymmetric amplitudes from a set of starting parameters and compares them with the previously determined values. A compatible set of parameters is arrived at by repeatedly calculating the gradient with respect to all
parameters and changing those to which the amplitudes are most sensitive. Thus, finally arriving at a set of parameters for which small incremental changes produce no better reproduction of the amplitudes.

Compatible sets of phase angles, background amplitudes and partial widths were attained for the $3/2^{-} S_{1/2}$ and $1/2^{-} N_{1/2}$ resonances using this computer search routine. The two sets are:

<table>
<thead>
<tr>
<th>$3/2^{-}, V$</th>
<th>$1/2^{-}, N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 180.0$</td>
<td>$\alpha_{13} = 0$</td>
</tr>
<tr>
<td>$\alpha_3 = 180.0$</td>
<td>$\alpha_{15} = -105.6$</td>
</tr>
<tr>
<td>$\beta_1 = 180.0$</td>
<td>$\beta_{31} = -121.3$</td>
</tr>
<tr>
<td>$\beta_3 = 180.0$</td>
<td>$\beta_{33} = -213.7$</td>
</tr>
<tr>
<td>$\beta_{13} = 0$ (phase gauge)</td>
<td></td>
</tr>
<tr>
<td>$R_{31} = .002$</td>
<td>$R_{31} = .306$</td>
</tr>
<tr>
<td>$R_{33} = .002$</td>
<td>$R_{33} = .240$</td>
</tr>
<tr>
<td>$R_{13} = .021$</td>
<td>$R_{13} = 0$</td>
</tr>
<tr>
<td>$R_{15} = 0$</td>
<td>$R_{15} = 0$</td>
</tr>
<tr>
<td>$R_{35} = 0$</td>
<td></td>
</tr>
</tbody>
</table>

Assuming pure p-wave decay of the $3/2^{-} S_{1/2}$ V resonance, the above results indicate that the inelastic width for scattering to the $2^+$ state in $^{50}$Ti is composed of
about 97% $P_{7/2}$-waves and 3% $P_{3/2}$-waves. In the case of the $1/2^−$ $^{13}$N level the results indicate that the inelastic width for scattering to the $2^+$ state in $^{12}$C is composed of about 15% $P_{7/2}$-waves and 85% $f_{5/2}$-waves. The uncertainty in these percentages was investigated in a separate set of calculations and found to be ± 15%.

The $1/2^−$ $^{51}$V resonance was not investigated in the above manner since the amplitudes in eq. 20 ($c_n^{(s)}$, $d_n^{(s)}$) listed in Table III (Chapter IV, Section B) are consistent with pure p-wave decay of that level. The possibility of $f_{5/2}$-wave decay cannot be eliminated within the experimental error. It is interesting to note that the very small values for the $d_n^{(s)}$ in Table III indicate that there is very little background present to interfere with the resonating amplitudes. In essence this lessens the effective number of constraints on those amplitudes, since fewer relations must be satisfied by them. In the case of the $1/2^−$ $^{13}$N resonance the presence of large interference amplitudes (see Table V, Chapter IV, Section B) is consistent with the fact that non-resonating background amplitudes are present. These non-zero values for the $d_n^{(s)}$ provide constraints on the resonating parameters as well as the background parameters, thus aiding in limiting the allowed range of the resonating transition matrix amplitudes. For the limited number of cases studied it
appears that the presence of background partial waves which interfere with the resonating partial waves, does help in the analysis.

A somewhat different situation is present in the case of the $3/2^-$ $^5l_1 V$ resonance. As in the case of the $1/2^- \ ^3l_1 V$ resonance the $D^{(s)}_n$ are all negligibly small (see Chapter IV, Section B, Table IV), however there exist four non-zero $C^{(s)}_n$ as opposed to two in the $1/2^-$ case. These non-zero values constrain the regions of parameter space which will adequately represent the data. Thus, the consistency in the fit for $3/2^- \ ^5l_1 V$ level with no f-wave decay included is more significant than for the $1/2^- \ ^3l_1 V$ level.

In conclusion, the results found indicate that the 8.90 MeV $1/2^-$ level in $^{13}N$ is adequately treated by the theory assuming 15% $p_{3/2}^-$ -wave decay and 85% $f_{3/2}^-$ -wave decay to the $2^+$ first excited state of $^{12}C$. The data for the adjacent 12.28 MeV $1/2^-$ and 12.54 MeV $3/2^-$ levels in $^5l_1 V$ were found to be consistent with decay by pure p-waves to the $2^+$ first excited state of $^{50}Ti$, with the $3/2^-$ level decaying by almost pure $p_{3/2}^-$ waves.
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Ramirez; written by H. R. Weller, University 
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