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Math Curriculum (Algebra II) with Focus on Sports

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Math tends to be a subject that many students do not necessarily enjoy or find easy to learn. I believe that many high school students struggle with mathematics due to the structure of the textbooks used in many of their classes. Specifically, many of these textbooks do not use real-world contexts as a tool by which mathematics can be learned. Instead, contexts appear after students have been exposed to procedures and are used as a venue for practice of these procedures. I believe that the use of these contexts as a mathematical learning tool will enable students to, not only understand, but also, appreciate mathematics. Proponents of reform in mathematics believe that “‘knowing’ mathematics is ‘doing’ mathematics”. Therefore, reformers are pushing for classrooms that encourage students to act and think like mathematicians” (Draper, 2002, 521-522). This will greatly help the students better understand the mathematical concepts that they encounter in their classes.

When it comes to mathematics textbooks, many schools use what Stein, Remillard, and Smith (2007) refer to as conventional textbooks. Unfortunately, instruction following these textbooks does not correlate with reform-oriented instruction. As seen in a study conducted by Banilower, “classes with a greater emphasis on reform-oriented instructional objectives were more likely to experience reform-oriented instruction” (Banilower et al. 2013). The textbook and other instructional materials play a role in the instruction of the course. Conventional textbooks are set up differently. These texts normally follow a specific pattern for each lesson and unit. When a student opens their book to start chapter 1 lesson 1, they will immediately see the title of the lesson they are going to start. For example; Slope and Y-intercept. Here is a photo of a lesson on Slope and Y-intercept from a Prentice Hall Pre-Algebra textbook:

(Charles, McNemar, and Ramirez pages 415-419)

The text starts by giving the students a definition for all of the key terms and ideas for this lesson. They would see a clear definition of slope on the first page, followed by the formula to find slope. After that the students are provided with a few concrete step by step examples of how to find the slope of a function. Following these examples, the students then see a page of up to 100 similar problems for homework. While this is great information for the students to know before moving on to another lesson, this does not give the students any chance to explore the ideas of the lesson. They simply are given the definitions and then asked to copy the procedures from the examples for the homework. This seems very tedious to me and lacks of opportunities for the students to explore mathematical ideas and demonstrate creativity when learning mathematics. In the book The Schools Our Children Deserve, (Kohn, 1999), the author explains what traditional teaching styles would look like in a school. “In the Old School,... math classes emphaize basic facts and calculations. Academic fields (math, English, history) are taught seperately. Within each subject, big things are broken down into bits, which are then taught in a very specific sequence".
Will the information really stick with the students if they are just given definitions and guided step by step examples? What do the students really learn? Are the just regurgitating the content from the examples or are they really understanding the mathematics behind the problem? Repetition is very important for student learning. Over time, concepts might become more known to them, however, student may only recognize procedures and algorithms. It is very important to mix up the representations of the problems containing these concepts so that the students are not relying on procedures and actually understanding the reasoning behind the procedures that they saw previously.

A very common question that is asked in a mathematics classroom is “How will I use this later in my life?” Many students do not see how mathematics can be beneficial to their futures or see the connections with the materials and their everyday life. This is one of the factors that I believe pushes students away from wanting to learn math. In order to help get rid of this dilemma, I decided to create...
my own curriculum that is entirely centered around a context that is very prominent in our society and the lives of many middle school and high school students; sports. By using a common theme for this curriculum, the students will be able to think about and explore the mathematical ideas behind the concepts through something that is very prominent and familiar in their lives.

From here I decided that I wanted to create my own curriculum that would be different than the conventional textbook. Throughout my journey in becoming a mathematics educator, I have learned about the benefits of a more reform-oriented text and the teaching techniques that work hand in hand with it. As Jo Boaler (2016) claims in her book, *Mathematical Mindsets*, “Interestingly, I found that mathematics excitement looks exactly the same for struggling 11-year olds as it does for the high flying students in top universities – it combines curiosity, connection making, challenge, and creativity, and usually involves collaboration”. I wanted to create something that would be more helpful and relevant to my future students’ lives. I believe that this unit shares similarities with reform-oriented instruction (National Council of Teacher of Mathematics, 2014. My goal was to follow this structure of a more reform oriented model of teaching mathematics. In the book, *The Schools Our Children Deserve*, the author also describes the difference from a traditional classroom to a more reform oriented learning experience.

“Because learning is regarded as an active process, learners are given an active role. Their questions help to shape the curriculum and their capacity for thinking critically is honored even as it is honed. In such classrooms, facts and skills are important but in the end are not themselves. Rather, they are more likely to be organized around broad themes, connected to real issues, and seen as a part of the process of coming to understand ideas from the inside out” (Kohn, page 4).

This is how I wished to set up my curriculum. I believe that students will benefit more from a curriculum that is centered on these ideas. Not only is this something that I believe, this is clearly stated and supported by the National Council of Teachers of Mathematics (2014).

“The standards of the National Council of Teachers of Mathematics includes problem solving, reasoning and proof, communication, connections, and representations... The Working Mathematically strand incorporates five interrelated processes - questioning, applying strategies, communicating, reasoning, and reflecting. These processes underpin problem solving; a life skill that is universally considered center to the mathematics curriculum” (NCTM 2000).

All teachers and educators should try to hit all of these standards in the classroom daily. With a more reform-oriented curriculum, these concepts are incorporated into the lessons and text, and can therefore benefit both the students and the teachers greatly.

**THE PROCESS OF PREPARATION**

I began this process by researching and looking through many different textbooks. I read many textbooks that were centered on Algebra and Algebra 2 content. I chose these subjects due to the fact that I am more familiar with content appearing in these courses. I previously had worked in a College Algebra 2 class and have helped out in an Algebra 1 class in my first pre-internship through the Western Michigan University Education Program. In order to do this I borrowed textbooks from my old high
school, and checked out textbooks from Western Michigan University Mathematics Education Department’s library. I was very shocked to see that there as so many different styles of textbooks and teaching mathematics. I was a little disappointed however, because out of all of the schools I had subbed for, gone through, and worked with, none of the math textbooks were reformed oriented. They were all traditional textbooks. The following are the textbooks that I used in order to help determine how I wanted to develop up my curriculum; Prentice Hall Mathematics, Prentice Hall Mathematics: Pre-Algebra, MATH 1110 Algebra 2, Algebra: Form and Function, Algebra: Form and Function 2nd Edition, and Pre-Calculus. Enhanced with Graphing Utilities. 5th ed.

After looking through these different textbooks, both teacher and student editions, I decided that I needed to focus on one unit and not an entire book. An entire textbook would be too long to complete for this project. With focusing on only one specific unit of my textbook, I would be able to dive deeper into my lesson planning and developing skills to really provide a good unit for teachers to use in their classrooms.

I then chose the topic of linear functions for my unit. I chose this topic because I had the confidence that I could dig deep into the concepts and procedures of which it comprised. Like I had previously mentioned, this is a subject that I have had a lot of experience with, not only as a student, but also from a teacher perspective. This was also a topic that I have created unit plans for a previous course I took at Western Michigan University. This way, I had confidence in my abilities to work with the material and hopefully to change the way the curriculum was designed so that students could really get a better and deeper understanding of the materials that would be presented in my unit.

For my unit I decided that I wanted to create not only the student textbook piece, but also a teacher edition of the text. This was I could implement tips for the teachers for how to guide their students’ thinking while going through these lessons and provide teachers with a break down for the lessons and answers to all the questions given throughout the different lessons.

From here it was necessary for me to determine what features of linear functions will be brought out throughout my unit. After sitting down and deeply thinking about this, I decided that the following were topics that I wanted to address in my unit.

- The concept of slope
  - This is a very important concept for students to learn when they begin working with linear function. Linear functions are defined by a unique characteristic that they possess; they always have a constant rate of change (slope). Slope is a concept that defines a linear function. All students need to understand this concept in order to understand the idea of a linear function. Therefore this is something that is very important for students to understand and comprehend when learning about linear functions in a math class.

- The concept of y-intercept
  - The y-intercept is another very important concept for students to begin to really understand. While added alongside of slope, students can understand how linear relationships work and where they start off. Again, this is a concept that defines a linear function. Therefore this is a very important concept when working with real life
examples. For most real world contexts students will not encounter negative inputs, and therefore should understand that they y-intercept is their beginning point for the data that represents a particular situation.

- Comparing the change in y-values and the change in corresponding x-values
  - This is the definition of slope within a tabular representation. Many students will see examples of slope in either a tabular or graphical form. However, a common misconception is that the slope only depends on the change in the y-values. This is due to textbook makers’ overuse of the change in x as a 1. What happens when the x-values jump up by 3? This can easily trick students, therefore I wanted to make sure that this was incorporated into my lessons so that the students will have to think about the slope in the correct form, $\frac{\text{change in } y}{\text{change in } x}$.

- Use of Technology
  - Technology use in the classroom is very important. With the help of technology students can visualize and manipulate things that they never could do in the classroom before. When the technology is incorporated into the lessons and the curriculum as a means of learning and investigating the concepts, students will begin to improve their skills. In their article, James J. Kaput and Patrick W. Thompson, express the different levels of “waves” of technology use in the classroom.

  “We characterize surface, wave-level studies as those that used calculators or computers as adjuncts to existing curricula and instructions. In such studies the computer or calculator either was used primarily as an aid to computation or the delivery of existing content” (Kaput, Thompson 1994, pg. 676-684).

As explained in the text, this use of technology is only hitting the surface level of the content that students are learning. The technology is an additional aid if needed, but is not used as a means to learn the new material. Later in their article they explain a deeper level of technology use in the classroom.

  “Studies at the “swell” level of change usually involved a closer look at the role of the technology in learning or cognition-- how it can be used to support problem solving or how it affected students’ learning of particular ideas” (Kaput, Thompson 1994, pg. 676-684).

Again, this is the level that we want the students to be at in terms of the technology use in the classroom.

- Slope intercept form: $y = mx + b$
  - This is one of the first types of equations that students will learn about in regards to linear functions. It is very helpful for students to understand and view the concepts in a different representation. Slope intercept form shows the students the information that they have already learned (slope, and y-intercepts) and puts it all together into one equation. This is a great tool to help the students continuously build on top of the information they have learned and add more to their knowledge of linear functions.
Recursive equations for a linear function

- When students are given the slope intercept form of a linear function's equation, they are working with an explicit form of an equation. This is great for students to learn about and work with, however, it is crucial that students learn about the concepts from multiple perspectives. A recursive form is another way for students to think about mathematical situations. It is also commonly the first way students think about representing linear relationships. Therefore it is important to incorporate this type of an equation as well as the explicit equation. Especially when relating real-world contexts to these concepts, some students might benefit greatly from a recursive form of an equation. This different type of equation can provide them with a different perspective and view of the information and hopefully understand the material better.

- Multiple representations (tabular, equation, real life context, graphical, and spoken)
  - Each and every student will have their own style of learning and absorbing information. With so many different types of learning styles out there, it is very important to try to cover all of them throughout instruction. There are so many ways for students to learn concepts, therefore it is important to use as many representations as possible in order to fulfill all the students' needs. Some students are visual learners, and will learn best from the help of graphs and tables. Other students might be more hands on and kinesthetic where they will learn best by doing the situation or re-creating it. Others will like to use pencil and paper to write notes and re-read them, and some students will benefit from hearing about situations verbally.

Obviously this does not cover all of the information students can learn about linear functions. There are some pieces about linear functions that I did not incorporate into this unit, however, my goal would be that this unit was used as an introductory level unit for linear functions and that the students would get to see this material later along in their educational journeys where they can begin to learn more applications of linear functions and dig deeper into the concepts. I believe that this material represents an introductory level of a linear functions unit since it covers all of the materials that students will learn when they are first learning about this type of functions. The materials presented throughout the unit represents the basics and foundations of linear functions, and is necessary for students to understand before they can move on to further depth in the subject. This is done by spending more time on concepts and thinking about them in multiple ways. Students get to experiment and work with this concepts before knowing their mathematical definitions. They get to create conclusions about the examples and context as a way to determine mathematical concepts. This unit is set up to play a part in a spiral approach style of curriculum in order to help the students really get a deep level of understanding of the concepts. The students will be given the fundamental concepts of linear functions in this lesson in hopes that the students can continue to build upon the materials that they learned in this unit to dig deeper into the subject in future classes. A goal of mine would be to continue to make future units so that students can continue increasing their knowledge about linear functions over multiple years.

While creating this unit it was necessary to consider the different principles that could be implemented into my unit in order to make it a more meaningful learning experience for my students. These components included ideas such as multiple representations, translations, transpositions,
repetition, high level cognitive demand, focusing on student thinking, connections between lessons, and real world context. All of these ideas and concepts were presented to me throughout my mathematics education courses at Western. These ideas all stood out to me throughout my studies and I knew that I needed to incorporate them into my unit.

### CURRICULUM SET UP

Instead of following the traditional pattern for my curriculum, I decided to make the lessons more interesting and engaging for the students. Instead of providing the students with the definitions of specific vocabulary terms, the students will use the questions in the lesson to determine these definitions.

Personally I think that this will help these mathematical ideas stick with the students for longer periods of time. The more reform-oriented idea of changing mathematics classrooms from learning strictly definitions and memorization of procedure towards a conceptualization of a “science of patterns” and a social learning experience, is a great movement within the last decade. This is described by Shoenfeld in his article *Learning to Think Mathematically; Problem Solving, Metacognition, and Sense Making in Mathematics*.

“Mathematics is an inherently social activity; in which a community of trainer practitioners (mathematical scientists) engages in the science of patterns- systematic attempts, based on observation, study, and experimentation, to determine the nature or principle of regularities in systems defined axiomatically or theoretically (“pure mathematics”) and models of systems abstracted from real world object (“applied mathematics”)... Learning to think mathematically means (a) developing a mathematical point of view- valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure- mathematical sense-making” (Schoenfield, 1992).

This text that I have created uses real world context and exploratory means for students to learn and absorb the new information provided to them throughout the lessons. I centered my lessons on the many ideas that I learned throughout my courses at Western Michigan University (MATH 3500, MATH 3510, and MATH 4500) in order to order to make this a meaningful experience for students. The curriculum is designed to have 8 different lessons with each of them having sub sections in them. I separated the unit into 8 different lessons because I believe that for an introductory unit on Linear Functions, there are 8 main ideas that are necessary to be covered throughout the unit which are the following:

1. Concept of Slope
2. Concept of a Y-Intercept
3. Moving from a Tabular Representation to other Mathematical Representations
4. Moving from a Graphical Representation to other Mathematical Representations
5. Moving from an Equation Representation to other Mathematical Representations
6. Moving within a Tabular Representation
7. Moving within a Graphical Representation
8. Moving within an Equation Representation
This order was chosen after analyzing other mathematics textbooks and their sequencing of lessons. As previously mentioned, I believe that one of the most foundational concept of linear functions is slope. Having this as one of the bases of the concepts from the unit, I knew that this had to be introduced first. From there, another very important concept is the y-intercept. To me, these were the 2 biggest concepts of linear functions that I wanted to present to students as they define linear functions. Therefore I needed to make them the first two concepts introduced throughout my unit. This way the students would get the most practice with these concepts as they continually see them throughout the different lessons of the units.

From there I needed to incorporate multiple representations. As previously mentioned, students learn in many different ways and styles, and it is important to try to reach out to every student and their style. With a mix of representations for the unit, each student will have a lesson or a part of a lesson that focuses on their strengths in one representation and also helps them to improve their skills within other representations. With students thinking about mathematical concepts in multiple ways, they will become better mathematical thinkers, and better understand mathematics. This why the remaining lessons of the unit focus on each different representation along with using all of them at the same time.

Throughout the unit, the students will be learning about and diving into the ideas behind linear functions using a more non-traditional way of being presented new information. The students will get to explore linear functions in order to make conclusions about them throughout the unit.

While each of the lessons are separated to cover a specific math topic, each one also has a theme that fits along with the real world context that I chose for my overall theme for the unit. I thought that this was a good idea and could possibly help the students keep all the information straight and separated into lessons, if needed. It will be easier to have students organize their materials based on the “theme” for the lesson. The following section describes the different lesson titles and themes for the unit along with an explanation for why I believe that it was important to incorporate in an introductory unit to linear functions. I decided to stick to one real world context per lesson so that students could really dig into the mathematical concepts beneath the situation.

Another common theme that I focused on during the construction of my unit was the idea of a higher level of cognitive demand for the tasks provided. The goal was to create a unit with lessons that promoted a higher level of cognitive demand than those of a more traditional textbook. In order for students to be doing mathematical tasks at a higher level of cognitive demand, a few things must be done. From my studies, I have learn about the four different levels of cognitive demand. The first level, the lowest level, is memorization. This is something that is commonly seen in traditional textbooks where students are regurgitating answers that mimic the example problems in the text. The next level of demand is where students are performing procedures without connections. An example of this would be when students can solve a one-step equation, but do not know why the must go about it through the algebraic steps. They do not see the reasoning behind the procedure. Following that is the 3rd level of cognitive demand, procedures with connections. This is where students are following and using procedures and understand the reasoning behind the procedure. At this point the students understand what algebraic steps are required and why they are used. From there we lead on to the highest level of mathematics: doing the math. This is the stage that we want the students to be using in the classroom. The following is a list of requirements for a high level demand of a mathematical task
that I learned from a handout in my MATH 3500 course at WMU:
1. Require complex and non-algorithmic thinking
2. Require students to explore and understand the nature of mathematical concepts, processes, or relationships
3. Demand self-monitoring or self-regulation of one's own cognitive process
4. Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.


Also, the use of technology in a mathematics classroom is very important. The use of the TI-Nspire is promoted and used throughout this unit. The TI-Nspire is a handheld calculator that is very supportive of teaching strategies. The calculator provides students with multiple representations as well as a manipulation feature. This supports teachers when they want to show students multiple examples and non-examples. With the teacher software, teachers can create lessons that will be programmed into the students’ handhelds before the class has begun. The TI-Nspire used multiple representations that can be view simultaneously while working through a particular problem or lesson. This is clearly explained in by Texas Instruments in their notes about the TI Nspire calculator.

“Research reports on teachers’ classroom lessons reveal that the enhanced multiple representations in TI-Nspire are bringing the benefits of multiple representations to a wider range of mathematical topics. Graphing equations and data representations are used often, however, other researchers designed and observed learners using geometry and spreadsheet features” (TI notes, https://education.ti.com/sites/UK/downloads/pdf/Research%20Notes%20-%20TI-Nspire%20Navigator.pdf).

“Engaging Explorations via to Select and Drag (also commonly called “dragging”) enables learners to explore mathematical and scientific models interactively. The research literature shows that interactive exploration of models increases engagement, encourages learners to take responsibility for their own learning, and develops their understanding (Clark-Wilson, 2008; Aldon, et. al, 2008). Researchers examining classroom use of TI-Nspire find that teachers readily apply this new capability” (TI notes).

This piece of technology is very helpful for students. It is also very beneficial for the students because it allows the students to be efficient with some of the basic procedures that they might need to do while exploring a new idea. This helps the class get through the simple procedures faster, and focus on the main concepts for the lesson. Due to the many different functions of the TI-Nspire, I knew that I wanted to incorporate it into my unit.

LESSON LAYOUT

I created all of the lessons by starting the students off with a real-world context. This is a representation that students tend to see the least in traditional mathematics classes. However, real world contexts are the way that students experience mathematical contents most frequently in their lives. Due to this, I believe that this is a very powerful representation of mathematics and believe that students should be
exposed to this representation more often than in a traditional classroom. This is why I decided to start every lesson with a real world context every time.

At the beginning of the lessons, students are introduced to a proportional linear situation, and then later work with non-proportional relationships. I created this is this order to correlate with the order of mathematical concepts that I wished to introduce. In my opinion, it is easier to see and learn about the concept of slope through a proportional situation, since students are very familiar with situations that start at (0,0). When the lessons begin to move to the idea of a y-intercept, I introduced non-proportional linear situations so that students could clearly see how the initial value was not zero and that this change is represented by the y-intercept. After students receive practice with both proportional and non-proportional situations, along with the concepts of slope and a y-intercept, they are exposed to both types at a random order. At this point, it is crucial for students to understand the different between the types of relationships and apply them to the topics that they are studying.

Lesson 1: World Record Speeds

This lesson was created to get the students introduced and familiarized with the mathematical concept of slope. There are 4 separate parts of this lesson (1.1, 1.2, 1.3, 1.4) in order to give the students multiple examples and uses of slope in real world contexts. The first three of these lessons contain real world contexts that are centered on proportional linear relationships. I believe that it is very important to have to students be introduced to the concept of slope while using proportional linear functions. To me, this is a clearer way to visually see, and comprehend the idea of slope. They get to think about the mathematical concept in a real world context that they might have experienced in their lives. This brings a feeling of familiarity to the mathematical concepts and helps the students to better understand them. Using real world contexts is a great way to implement tasks that promote reasoning and problem solving. The students must use what they have in order to build their thoughts and ideas about a new mathematical concept. The National Council of Teachers of Mathematics believe that, “Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and involve multiple entry points and varied solution strategies” (National Council of Teachers of Mathematics,, 2014, pg 17). I have shown this throughout the many lessons that I have created with the help of given real world contexts rather than definitions. The last part of this lesson was structured as a summary and conclusion of the three previous sections. This was implemented so that they students can pull all the information they received from the first 3 parts of the lesson to make comparisons across the mini lessons and then make a general consensus of the concepts.

Lesson 2: High Expectations

This lesson was created to get the students introduced and familiarized with the mathematical concept of a y-intercept. There are 4 separate parts of this lesson (2.1, 2.2, 2.3, 1.4) just like in lesson 1. In order help the students have a good understanding of a y-intercept, these three lessons use multiple examples and uses of a y-intercept in real world contexts. The first of these lessons contain real world contexts that are centered on a non-proportional linear relationships. I believe that this is a great way to learn the ideas of a y-intercept. Just like the last mini lesson of lesson 1, last part of this lesson was structured as a summary and conclusion of the three previous sections. This was implemented so that they students can pull all the information they received from the first 3 parts of the lesson to make
comparisons across the mini lessons and then make a general consensus of the concepts. On top of all of this, Lesson 2 all pushes students to continue to practice utilizing their skills and understanding of slope from Lesson 1. By continuing to use the material from the previous lesson, the students will better understand the concepts from the unit as a whole.

Lesson 3: Getting Pricey
This lesson was created to get the students accustomed to using and seeing multiple representations of linear functions. This lesson mainly focused on the use of a tabular representation. I decided that this would be the first representation to focus on since it is one of the representations that students are most familiar with. Students begin working with tables at a very young age, so by this point, they should understand how they are set up and function. The students will learn to navigate through this representation and then learn how to apply that information to another mathematical representations. Again, it is clearly stated by the National Council for Teachers of Mathematics that “Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedure and as tools for problem solving (Principles to Actions, Page 24). Again, one of the main components of this curriculum is using and moving between multiple representations in order to help the students better understand the concepts being taught. This lesson incorporates 2 parts so that students can see the information about linear functions that are both proportional and non-proportional. This way that can compare the two different types of linear functions and then take information from the tabular perspective and then apply it to other types of mathematical representations.

Lesson 4: Watching the Weight Fall Right Off
This lesson was created for the same reasons as lesson 3. The goal is to get the students accustomed to using and seeing multiple representations of linear functions. This lesson mainly focused on the use of a graphical representation. The students will learn to navigate through this representation and then learn how to apply that information to another mathematical representations, similar to the previous lesson. This lesson also incorporates both proportional and non-proportional linear functions. The students get the opportunity to compare and contrast them and see how these linear functions are translated from a graphical representation to another function representations.

Lesson 5: A Professional’s Opinion
This lesson was created for the same reasons as lessons 3 and 4. This lesson provides students with opportunities to translate across multiple representations of linear functions. This lesson focused on the use of the equation representation for a linear function. The students will learn to navigate through this representation and then learn how to apply that information to another mathematical representations, just like in the previous 2 lessons. This lesson also incorporates both proportional and non-proportional linear functions. The students get the opportunity to compare and contrast them and see how these linear functions will be transferred from an equation representation to other mathematical representations. With translating from one representation to another, the students will get the opportunity to review the materials from lesson 3-5 within each other. I believe that connections across lessons is very beneficial for students’ deep understanding of mathematics. I believe that if students are making comparisons and connections between different exercises and between different lessons, students are really utilizing their skills from the seventh Mathematical Practice.
Standard. This standard states that students must be able to look for and make use of structure. By recognizing patterns and similarities between concepts given throughout different lessons, students are really hitting this standard to a tee.

Lesson 6: Reaching New Heights

This lesson is designed so that students only get to use and work within one type of representation. This lesson consists of two parts. One part is focused on a proportional linear function and the other is about a non-proportional linear function. For both of these parts, the focus is on a table representation only. The students must use a portion of a tabular representation in order to complete the rest of it and then think about different possible coordinate pairs to that table.

Lesson 7: Whoa, that’s Hot!

Just like with the previous lesson, this lesson is designed so that students only get to use and work within one type of representation. This lesson consists of two parts. One part is focused on a proportional linear function and the other is about a non-proportional linear function. For both of these parts, the focus is on a graphical representation only. The students must use a graph with only one coordinate plotted in order to create a graph that could represent the function. This gets at the fact that there are multiple lines going through one point and that all students may approach and think about this problem in different ways, and they all can be correct. This is an idea that I have been able to promote to my students during my student teaching. Through a program that is called “Week of Inspirational Math” (Jo Boaler, Stanford University, https://www.youcubed.org/week-of-inspirational-math/).

Lesson 8: Family Fun Time

Again, this lesson is designed so that students only get to use and work within one type of representation. This lesson consists of two parts just like the previous two lessons. One part is focused on a proportional linear function and the other is about a non-proportional linear function. For both of these parts, the focus is on an equation representation only. From here students will need to come up with other possible equations to also represent this situation.
Throughout the unit, the following Common Core State Standards will be covered along with the following Standards for Mathematical Practice. By hitting all of these standards, the students will be well prepared to move on to the next level dealing with this material and will have developed the skills of becoming good mathematical thinkers. Each standard is stated and accompanied by a direct example of its presence in the lessons from my curriculum (shown by a snip of the student edition of the lessons).

**Common Core State Standards for Mathematics**

Mathematics – High School – Algebra

★ A-CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**Example: LESSON 1.1**

**Task 4**

Before they are given these questions students are provided with the average speed that Usain Bolt runs for the 100m dash. From here students are asked to think about how far Usain Bolt would travel based on how long he was running. The questions build as the students are asked to find that distance for many different values of time. At the end of task 4 students are asked to generalize their findings to create an equation that will represent the distance that Usain traveled based on any time that he was running “N”

Student Edition (page7)

**TASK 4: BUILDING AN EQUATION**

How far would Usain travel if he ran his speed for 1 hour?

For 2 hours?

For 10 hours?

Supposed Usain runs for “n” hours, how far would he have traveled?
★A-REI ★ Reasoning with Equations and Inequalities
Represent and solve equations and inequalities graphically
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Example: LESSON 5.2
Task 3
Before task 3, the students were provided with an equation representation of the linear relationship between price to pay for private baseball lessons and the number of sessions attended. Now the students are asked to take the information that they gathered from the equation representation and translate it to a graphical representation. By creating a graph for this situation, the students are representing the equation in another form. They are also asked to compare the graph’s features and the equation, so that they can show their understanding of a graphical representation is.

Student Edition (page 55)

<table>
<thead>
<tr>
<th>TASK 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>What would a graph for this function look like?</td>
</tr>
<tr>
<td>Graph this situation by hand.</td>
</tr>
</tbody>
</table>

What similarities do you notice with your graph and the equation that you started with?
Mathematics – High School – Functions

**F-IF Interpreting Functions**

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**Example: LESSON 8.1**

**Introduction**

Students are asked to observe a given equation that represents a linear relationship between the price for a game of mini golf and the number of holes that are played. They are given the equation in terms of function notation and then must distinguish the difference between the independent and dependent variables which represent the domain and range of the function.

Student Edition (Page 69)

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The following is an equation that represents the price for a game of mini golf per hole that is played.

$$P(x) = \frac{1}{2}x$$

What is represented by the independent variable? Which variable is it? Justify.

What is represented by the dependent variable? Which variable is it? Justify.

Write one equation that is equivalent to this equation that could also represent this situation.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Example: LESSON 5.1
Introduction
Students are asked to observe a given equation that represents a linear relationship between the pay a professional tennis player will make per hour worked as a trainer for country club members. They are given the equation in terms of function notation and are expected to understand that given notation. They must then consider its domain.

Student Edition (Page 50)

The following is an equation that describes the price a professional tennis player will receive \( P(x) \) based on the number of hours they work and train members from the community club.

\[ P(x) = 40x \]

What conclusions about this linear situation can you conclude from the equation?

Can we plug in any number for \( x \) into the equation? Can we have any value of \( x \) for this situation? Justify your answers.
Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Example: LESSON 2.4
Entire Lesson
Common Core claims that interpreting functions includes having students determine characteristics about a given function. In this task, students are asked to observe three given tables along with the graphs that go along with the tables. From there the students must make inferences about the key features of these linear functions based upon the similarities and differences that they see among the three tables and the three graphs.

Student Edition (Page 32)
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

Example: LESSON 4.1
Beginning of lesson and Task 1
Students are asked to observe a given graph that represents a linear relationship between a person’s weight and the number of calories that they will burn while biking at 12 mph. They must decide, based on the context, which variable is independent, represented along the $x$-axis, and which variable is dependent, represented along the $y$-axis. They must also consider if there are possible $x$-values that are not plausible for this particular given situation. All of these questions are asking students to utilize the graph and its features to determine a possible domain and range for the linear situation.

Student Edition (Page 43)

The following is a graph that shows the relationship between a person’s weight (in pounds) and the number of calories that they will burn while biking at 12 mph.

What variable is along the bottom of the graph ($x$-axis) and what variable goes along the $y$-axis? Justify your answers.

What conclusions about this linear situation can you conclude from the graph?
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Example: LESSON 2.3
Students are asked to think about a linear situation that represents sky diving. They have to think about this situation and use the given information in order to calculate different rates of change for this situation. Students then will have to develop a conclusion from those separate rates of changes to determine the average rate of change for the situation. They will then create a table for this information, draw a graph to represent this information and show how slope is illustrated within each of these representations.

Student Edition (Page 28)

<table>
<thead>
<tr>
<th>TASK 1</th>
</tr>
</thead>
</table>

Thinking about this rate, you probably won’t be falling for hours out of the plane. Convert this rate to feet per second.

What altitude will you be at after 30 seconds into the fall?

What time will you need to pull the parachute cord?

What is the slope to represent this situation?

What information about this linear situation can you find out from this given table?
Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Example: LESSON 1.3
Tasks 4-5
Students are asked to determine how fast Andy’s car would go for “n” hours. This pushes students to think about the situation in terms of an equation representation. After that they are asked to graph their equation to see if it matches with the one graphed from their table.

Student Edition (Page 15)

<table>
<thead>
<tr>
<th>TASK 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>How far could Andy and his car travel in 1 hour?</td>
</tr>
<tr>
<td>In 7 hours?</td>
</tr>
<tr>
<td>In 4 hours?</td>
</tr>
<tr>
<td>Suppose Andy drives his crazy fast car for “h” hours, how far would he travel?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TASK 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>On your TI-Nspire, plot your hypothesis function on your graph page. If your graph doesn’t match the data change it so that it fits.</td>
</tr>
</tbody>
</table>
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

**Example: LESSON 8.2**

**Beginning of lesson**

Students are asked to create their own equation that represents the linear relationship between the number of bowling alleys in the United States since the year 1997. This is a skill that is important for the students to practice understanding the concept of equivalent equations.

**Student Edition (Page 72)**

The following is an equation that represents the number of bowling alleys in the United States since the year 1997.

\[ B(x) = -21.5x + 7611 \]

What is represented by the independent variable? Which variable is it? Justify.

What is represented by the dependent variable? Which variable is it? Justify.

Write one equation that is equivalent to this equation that could also represent this situation.
★ F-BF Building Functions
Build a function that models a relationship between two quantities
1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Example: LESSON 3.1
Task 1 and Task 3
At the beginning of this lesson students are given information about a linear relationship between the price of tickets for a hockey game and the number of ticket bought. Students are then asked to create both a recursive and an explicit equation to represent the linear situation they were given in a tabular representation.

Student Edition (Page 36)

<table>
<thead>
<tr>
<th>TASK 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.</td>
</tr>
<tr>
<td>Remember: A now-next formula uses the previous term to find the next term.</td>
</tr>
<tr>
<td>For example: How much would you have to pay for 4 tickets, based on the price for 3 tickets?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TASK 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write an equation to represent this data.</td>
</tr>
<tr>
<td>How did you come up with this equation?</td>
</tr>
<tr>
<td>Show that your equation works for this situation.</td>
</tr>
</tbody>
</table>
**F-LE Linear, Quadratic, and Exponential Models**

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   
a. Prove that linear functions grow by equal differences over equal intervals.

**Example: LESSON 2.1**

**Task 2**

Students are asked to determine whether the given situation is linear or not by proving if there is a constant rate of change or not, and explaining why they believe that conjecture to be true.

Student Edition (Page 11)

<table>
<thead>
<tr>
<th>Time (years since 1997)</th>
<th>Female Athletes (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

What do you notice about this table?

Is there anything that seems to be different from the tables and information from the last lesson on “World Record Speeds”?

Do you think that this situation has a constant slope?
If yes, how did you decide that and what is the slope?
If no, how did you decide that?
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Example: LESSON 1.2  
Task 2  
Students are asked to compute different rates of change for a given situation. After this they are asked to determine a conclusion about all of the different slopes.

Remember, in the last lesson we mentioned the idea of slope as $\frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}}$  
Calculate the slopes for the following:

- From 0 hours to 1 hour
- From 2 hours to 3 hours
- From 1 hour to 7 hours
- From 3 hours to 9 hours

What do you notice with these different slopes?
2. Construct linear functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Example: LESSON 3.2  
Tasks 1 and 3  
Students are asked to come up with both a recursive and an explicit formula to represent the given linear situation. This pushes the students to create and construct linear functions in the form of both recursive and explicit equations.

Student Edition (Page 40)

**TASK 1**

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.

Remember: A now next formula uses the previous term to find the next term.  
*For example:* How much would you have to pay for 4 tickets, based on the price for 3 tickets?

**TASK 3**

Write an equation to represent this data.

How did you come up with this equation?
3. Observe a quantity increasing by a constant rate using graphs and tables.

Example: LESSON 1.4

Entire Lesson

Students are asked to observe three given tables along with the graphs that go along with the tables that represent linear functions. They then have to look and compare multiple rates of changes and other characteristics of linear functions throughout the three different tables and graphs.

Student Edition (Page 17)
Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.

   Example: LESSON 1.1

   Task 1

   Students are given a real life situation where the rate of the runner is given in units that are uncommon when considering speed. Students must then try to figure out a better way to represent his speed in units that are more familiar to them. This is a task that involves students in making sense of speed in terms that they see more normally. This is a task that involves a lot of perseverance, as this could be a challenging task that students may have not seen before.

Student Edition (Page 5)

1.1 ON YOUR MARK... GET SET... GO!

THE FASTEST MAN ON THE EARTH

The fastest human footspeed on record was seen during the final 100 meters sprint of the World Championships in Berlin on 16 August 2009 by Usain Bolt. He won the race in 9.58 seconds.

TASK 1

We normally don’t think about speed per 100 meters. When you think of speed what do you think of?

Convert his speed into units that are more commonly used.
2. Reason abstractly and quantitatively.

Example: LESSON 4.2
Tasks 1, 2, and 3
Common Core claims that mathematically proficient students make sense of quantities and their relationships in problem situations. The skills of reasoning both abstractly and quantitatively involves students making connections between situations and the mathematics behind it. Throughout this lesson, students are given a linear situation in the form of a graphical representation for a given linear situation. They must then think about what they were given by the graph, and use that to create both a table and two different types of equations to represent the situation. By doing this students must think about the mathematics behind the situation that they see in the graphical representation, and apply them to a different representation.

Student Edition (Page 47)

TASK 1

Based on this, create a table that represents the data from the graph.

| | What variable goes in the left column? Justify |
| | | |
| | | |

| | What variable goes in the right column? Justify |
| | | |
| | | |

TASK 2

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.

TASK 3

Write an equation to represent this data.

How did you come up with this equation?
3. Construct viable arguments and critique the reasoning of others.

Example: LESSON 6.1
End of Lesson
At the end of this lesson, students are asked to observe another student’s idea of a possible point for the given situation. They must then determine whether this is a correct or false point to go along with the situation that was given. They must also prove why or why not. This is a task that involves a high level of cognitive demand since students are asked to critique others’ work and then explain why they believe their findings.

Student Edition (Page 58)

A student claimed that this was another point represented in their table from the information given at the beginning on the lesson:

<table>
<thead>
<tr>
<th>2000</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>75</td>
</tr>
</tbody>
</table>

Does this point make sense to be a possible point on our table? Why or why not?
4. Model with mathematics

Example: LESSON 7.1
End of the Lesson
Modeling with math involves students creating representations for a given mathematical concept or idea. In this lesson, students are given a graph to determine whether it is a viable graph to represent the situation given previously. They must use their modeling skills to determine whether this graph could represent the situation. For this problem, students may create their own model for this linear situation in order to determine whether the given one is a possible representation or not.

Student Edition (Page 64)

A student claimed that this was another graph that could represent the information given at the beginning on the lesson:

Does this make sense to be a possible graph for this situation? Why or why not?
5. Use appropriate tools strategically.

Example: LESSON 2.3  
Task 3  
Students are asked to use a TI-Nspire as a means to plot the points given from a table and then examine key features from this graph. Throughout the unit, the TI-Nspire is used frequently as a mathematical tool for the students to get a better understanding of the mathematics that they are working with. This is a much faster way to get students to examining the features of the graph. This is also more efficient than having the students take the time draw the graphs by hand individually.

Student Edition (Page 30)

**TASK 3**

On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.

Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down? Justify your answer.

Looking at this graph, how does it differ from the ones from previous situations in previous lessons?
6. Attend to precision.

**Example: LESSON 2.4**

End of Lesson

Attending to precision involves students focusing on specific details in their work so that they can continue to progress in their understanding. Students are asked to observe three given tables along with the graphs that go along with the tables that represent linear functions. Through the previous 3 lessons students will experience encounters with y-intercepts but will not be exposed to the definition until the end of the last lesson.

Student Edition (Page 34)

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From this...

\[ Y = mx + b \]

is called **Slope Intercept Form** for a linear equation. Where \( m \) represents the slope of the line and \( b \) represents the y-intercept, or initial value of the function.
7. Look for and make use of structure.

Example: LESSON 6.2
Beginning of Lesson
Looking for and making use of structure is a very common theme within mathematics classrooms. In this lesson, students are asked to make conclusions about linear functions from the patterns and structures that they have observed in previous lessons. They then are to use the conclusions of patterns that they have seen and must apply them to fill in the rest of a table with only one pair of \( x \) and \( y \) values given.

Student Edition (Page 59)

Let the following table represent the men’s world record for pole vault over the years since 1900.

Complete the table with 6 other possible points that could fulfill this situation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Height in meters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>4.78</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Is this a proportional or non-proportional linear relationship? Justify your answer.
8. Look for and express regularity in repeated reasoning.

Example: LESSON 2.2
Entire Lesson
The entire unit is set up so that the students are looking for patterns and regularity between different examples and different lessons. Students are asked to reflect back on ideas from other lessons in order to keep them repeating and reviewing the materials and ideas throughout the entire unit, rather than just one lesson or just one given problem.

Student Edition (Pages 24-27)

Is there anything that seems to be different from the tables and information from the last lesson on “World Record Speeds”?
In sum, the goal of this curriculum is to provide a new style of textbook for students to learn through. My goal was to create a more reform-oriented text that would promote higher levels of mathematical thinking and classroom engagement. I believe that this style can really make a positive impact on the students’ experiences with math classes. Through the exploratory process of this text and the common context of sports and games, I believe that this will be a very helpful and successful unit if and when used in a classroom. This curriculum is set up to provide the students a deeper understanding of linear functions while hitting the Common Core State Standards and Mathematical Practice Standards and will really help students to really understand and comprehend the materials presented in the unit.
Works Cited


