Type 1a Supernova Models and Galactic Chemical Evolution

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Type Ia Supernova Models and Galactic Chemical Evolution
Spencer Henning
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Abstract

Multiple models of type Ia supernovae are discussed, as well as the properties of white dwarf formation, electron degeneracy pressure, nucleosynthesis, and galactic chemical evolution (GCE). A GCE computer model is used to produce nuclear abundance data corresponding to Chandra (W7) and sub-Chandra (WDD2) type Ia supernovae models for Ca, Sc, Ti, V, Cr, and Mn nuclei. Overall, a trend was found indicating that the WDD2 model had greater \([x/Fe]\) abundance values than the W7 model at the end time of the model. Additionally, each model fit within total observed data, but observed data filtered to highlight white dwarf stars tended to have greater \([x/Fe]\) values than those produced for each model. Overall, data was not sufficient to provide insight into whether one model fits better with observed data.

Introduction

Every heavy element in the periodic table exists due to the processes involved in stellar evolution. Studies into stellar nucleosynthesis and galactic chemical evolution (GCE) show that most of these elements are formed due to cataclysmic stellar explosions called supernovae. Type Ia supernovae occur in white dwarf stars, the core remnants of low and intermediate mass stars. By comparing galactic abundances produced by multiple type Ia supernova models, the role of type Ia supernova nucleosynthesis in galactic chemical evolution can be studied. GCE computer models exist for these processes, but no model on its own can produce data that completely and accurately reflects observed data. The purpose of this research is to compare
data from GCE computer models for two type Ia supernova models, as well as compare these data with observed data to see if either model fits better with observed data.

Chapter 1: White Dwarfs

White dwarfs are the setting of type Ia supernovae. Before discussing supernovae, it is important to understand how white dwarfs are formed, and supported. Unless otherwise noted, information in this chapter is referenced from [1].

Section 1.1: Origin of White Dwarf Stars

White dwarfs are the inert core remnants of intermediate stars with zero age main sequence (ZAMS) masses less than or equal to approximately 8 solar masses ($8M_{\text{sun}}$). These stars begin as large molecular gas clouds that populate galaxies like our Milky Way galaxy, which fragment and contract into hydrostatically stable objects known as protostars. A protostar contracts until thermal equilibrium is established by hydrogen fusion. Once the power outputted by hydrogen fusion is equal to the power lost at the surface of the star, the star is considered to have begun its lifetime on the “main sequence” of the Hertzsprung-Russell (HR) diagram. At this point, the star has a core temperature of $T \approx 10^7 \text{ K}$ [2].

When a star is on the main sequence, multiple independent hydrogen fusion processes convert hydrogen to helium in the core. The first process, known as the pp-chain (proton-proton chain), converts 4 protons into a helium nucleus. A second process, the CNO (Carbon Nitrogen Oxygen) cycle, does the same with carbon, nitrogen, and oxygen employed as catalysts. Each H burning process exists in the low and intermediate mass progenitor stars that become white dwarfs. However, the initial mass of the star determines which form of He production is more dominant. For low mass stars, with approximate initial masses $M < 1.5M_{\text{sun}}$,
the pp chain dominates He production. For intermediate stars, those with approximate initial masses $M > 1.5 M_{\odot}$, the CNO cycle dominates He production. This is because the core temperatures for intermediate stars are higher than those of low mass stars. These higher core temperatures provide enough kinetic energy for the particles to overcome the potential barriers between carbon nuclei and protons.

The HR diagram is shown in figure 1. Figure 2 shows the life cycle of a star with our sun’s initial mass and approximate initial composition: $\sim 70\%$ hydrogen, $\sim 28\%$ helium, $\sim 2\%$ heavy elements. The following discussion applies to the stellar evolution of such a star.

**Figure 1:** The Hertzsprung-Russel (HR) Diagram, taken from figure 8.8 of [1]

The star will spend $\sim 11$ Gyr on the main sequence [2]. During this time, an increase in star’s density occurs, causing a release in gravitational potential energy. By the virial theorem, the thermal energy of the core then increases. Once the hydrogen supply in the core has been depleted, H fusion cannot continue, and the star’s life on the main sequence is over.
Figure 2: The life cycle of a M = 1 $M_{\text{sun}}$ star on the HR diagram, taken from [3]

The luminosity and radius of the star then increase, as the expansion of the star causes a decrease in surface temperature. This translates to the star moving toward the right on the HR diagram as it evolves off the main sequence. The rise in core temperatures will supply enough energy to allow hydrogen fusion to occur in a shell of hydrogen-rich gas surrounding the core, adding to the mass of helium in the core. The core then contracts as its density increases. During this time, fusion does not occur at the center of the star. Fusion in the hydrogen shell around the core provides the necessary outputted power to support the star. The star will move into the subgiant and red giant phases of its life as gravitational potential energy continues to be released from the contracting core, causing an expansion of the outer envelope. The star stays in the red giant phase of its life until the temperature and density in the degenerate core become sufficient for helium fusion. This happens at time $t = 12.23$ Gyr, at temperatures on the order of $\sim 10^8$ K [2].
A burst of helium fusion then occurs, known as the “helium flash.” Helium fusion uses the triple alpha process of nuclear fusion to create carbon and oxygen in the core of the star. During the helium flash, a thermonuclear runaway scenario occurs, where rapid fusion can increase core temperatures in an uncontrolled manner. The flash lasts merely seconds, until a stable temperature-pressure balance is once again reached. By the end of the helium flash, the effective temperature of the star has increased, and the luminosity of the star has decreased.

The next phase of the star’s life is known as the “horizontal branch,” where stable helium fusion occurs within the remaining helium in the core. The horizontal branch phase ends when the helium supply in the core of the star is depleted, leaving just carbon and oxygen in the core \( (t = 12.34 \text{Gyr}) \) [2].

The star then evolves to the asymmetric giant branch (AGB) of its evolution. Here, the star once again moves up and to the right on the HR diagram. Surrounding the core, the helium burning shell continues fusing helium into carbon, then to oxygen, so the mass of the carbon-oxygen core continues to increase. The combination of mass increase and subsequent core contraction raises the density of the core significantly, leading to a rise in electron degeneracy pressure, which is discussed in section 1.3.

While the core density increases, the outer envelope of the star loses large amounts of mass through superwind. This leads to the final stage of the star’s life, the planetary nebula. As a planetary nebula, the superwind causes the outer layers become gravitationally unbound from the star. This causes the visible surface of the star to move closer to the core. Eventually the visible surface of the star reaches the surface of the stellar remnant, where the optical depth of the star, \( \tau \), nears \( \tau \approx 1 \). This “photosphere,” or visible surface of the star, is made of
thin layers of helium and hydrogen that remain, forming a thin atmosphere and radiative blanket around the core. This movement of the visible surface of the star is why the star moves to the left nearly horizontally on the HR diagram. The observed luminosity decreases with a shallow slope as the optical surface moves into the hotter layers of the star [1].

The remaining stellar body is known as a white dwarf star, slowly cooling at constant radius over the course of billions of years. Energy is transferred to the outer layers of the white dwarf, where it is radiated away. This means that both the inner and surface temperatures decrease for the star. Stellar luminosity $L$ is defined as

$$L = 4\pi R^2 \sigma T_{eff}^4$$

where: $R$ is the radius of the star

$\sigma$ is the Stefan-Boltzmann constant, $\sigma = 5.670400 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

$T_{eff}$ is the surface (effective) temperature of the star.

Since the surface temperature of the white dwarf decreases, and the radius of the white dwarf is constant, the luminosity decreases. This results in the white dwarf moving down and to the right on the HR diagram.

1.2: Electron Degeneracy

One of the key concepts of the physics of white dwarfs is the principle of free electron degeneracy, which involves how its free electrons are arranged energetically. If the temperature of a gas cloud with temperature $T$ is decreased, the kinetic energy of the particles in system will also decrease. As the kinetic energy of the electrons in the system decreases, electrons will “fall” into the lowest available energy levels, limited by the Pauli exclusion principle. Per this principle, these quantized energy levels are determined (in part) by the fact
that each electron in a system must have a unique set of quantum numbers, thus a unique quantum state. This means only a certain number of electrons can fit in each available energy state. As the temperature of the gas approaches 0 K, electrons will continue to fall into the lowest available energy states. In the limiting case of T = 0 K, the maximum possible energy that an electron can have is referred to as the Fermi energy, \( \varepsilon_f \), defined as

\[
\varepsilon_f = \frac{\hbar^2}{2m_e} \left( \frac{3\pi^2 n_e}{2} \right)^{2/3}
\]

Where: \( \hbar = \frac{\hbar}{2\pi} \) and \( \hbar = 6.626 \times 10^{-34} \) J s (Planck's constant)

\( m_e \) is the electron mass

\( n_e \) is the number of electrons per unit volume.

When every electron in the system has energy less than or equal to the Fermi energy, the system is completely degenerate [1]. Figure 3.1 shows a relationship between the fraction of occupied states and \( kT/\varepsilon_f \).

![Figure 3: Fraction of energy states occupied vs the ratio of kT/\varepsilon_f. Taken from figure 16.5 of [1]](image-url)
A step function is seen for the system of complete degeneracy. When $T = 0$ K, all states below the Fermi energy are filled, and none above are filled. When $T > 0$, a curved line is seen, where the number of states filled above $\varepsilon_f$ equals the number of states empty below $\varepsilon_f$.

1.3 Electron Degeneracy Pressure

As the remnant cores of dead stars, white dwarfs are extremely dense. For example, Sirius B, historically one of the most studied white dwarf stars, has a mass of $1.018 \pm 0.011$ M$_{\text{sun}}$ and a radius of $0.008098 \pm 0.000046$ R$_{\text{sun}}$ [4]. This means that Sirius B has an average density of approximately $2.70 \times 10^6$ g/cm$^3$. Total pressure in the star can be thought of as the sum of ion, radiation, and electron degeneracy pressures inside the stars;

$$P_{\text{total}} = P_{\text{ion}} + P_{\text{rad}} + P_{\text{ED}}$$ (3)

The high densities of white dwarfs cause the contribution of electron degeneracy pressure to be so much greater than that of either ion or radiation pressure, that electron degeneracy pressure alone is effectively what maintains hydrostatic equilibrium in the star. Furthermore, electron degenerate equation of state is polytropic, defined as

$$P = K \rho^\gamma$$ (4)

where: $K$ is a constant which depends on the physical parameters of the system

$\rho$ is the density of the white dwarf

$\gamma$ is the polytropic exponent for the equation. This value has a range of $4/3 \leq \gamma \leq 5/3$, depending on if particles in the system are behaving fully classic manner ($4/3$), relativistic manner ($5/3$), or somewhere between either extreme case.

The electron degenerate arrangement of a white dwarf star counteracts the gravitational forces pushing inward because of the quantum mechanical properties of the
electrons. These systems allow for nearly complete degeneracy to occur, even when T >> 0 K, as
the condition of kT << ε_f is still true. In these systems, the increased momenta of the electrons
(determined by the quantum mechanical states of the particles) exert an outward pressure. As
the mass of the white dwarf increases, the momenta of the electrons must increase. Since the
velocity of the electrons is limited by the asymptotic limit of the speed of light, there is a mass
limit for white dwarfs. This is known as the Chandrasekhar limit, and has an approximate value
of ~1.4M_{\text{sun}}.

An important conclusion to mention from electron degeneracy pressure is that it does
not depend on temperature to contribute to total pressure, like the ion and radiation pressures
do. Temperature is only important to validate our assumption of complete degeneracy. As long
as the condition ε_f >> kT is true, the star can cool without having any significant effect on
degeneracy.

**Chapter 2: Type Ia Supernovae**

Type Ia supernovae occur when carbon fusion suddenly occurs in a white dwarf star.
The condition for carbon fusion can be met if the mass of the white dwarf surpasses the
Chandrasekhar limiting mass, creating instability within the star [5]. From [6], carbon fusion in a
white dwarf can be triggered by “pycnonuclear reactions” resulting from extreme density, not
temperature, once the Chandrasekhar mass is exceeded. Since degeneracy pressure is also
independent of temperature, fusion in a white dwarf cannot be regulated by pressure. The
result is a thermonuclear runaway scenario, where a wave of uncontrolled fusion spreads,
creating cataclysmic explosion which destroys the white dwarf.
From [13], type Ia supernovae are classified as supernovae events which do not display hydrogen lines, while creating a distinct spectrum with prominent silicon lines in their spectra. White dwarf stars feature these spectroscopic qualities due to their carbon-oxygen (CO) composition. Since hydrogen is removed from these stellar remnants through hydrogen fusion and planetary nebula earlier in its lifetime, there is no hydrogen to be seen in their spectra, and a uniform carbon-oxygen composition can be assumed. In each case, the “flash” of carbon fusion occurs in the white dwarf, producing Si along with “intermediate mass elements such as Ca, S, Mg, and O; and the Fe-peak elements Fe, Ni, and Co” [7]. Multiple modes of type Ia supernovae exist, with different causes and levels of occurrence.

**Single Degenerate “W7” Model**

The single degenerate model is considered the more common of the two type Ia models. Reference [5] describes the single degenerate supernovae model, featuring a CO white dwarf in a binary system with a non-degenerate companion, such as a red giant. In such a system, the high density and intense gravity of the white dwarf draws matter from the outer shells of the companion star. This matter accretes to the surface of the white dwarf, leading to one of two possible single degenerate explosions. In the first single degenerate scenario, fusion occurs within the accreted material, contributing mass to the core of the white dwarf. The star’s mass, in turn, approaches and surpasses $M_{\text{ch}}$. Once this limit is exceeded, the star becomes unstable and a supernova occurs. The second single degenerate scenario is caused by helium fusion within accreted matter, which triggers thermonuclear runaway in the core of the white dwarf, before $M_{\text{ch}}$ is reached. This is known as a “sub-Chandrasekhar” type Ia supernova [5]. Although both Chandra and sub-Chandra scenarios may exist for each type Ia supernova
model used for this work, Chandra events are associated with the single degenerate W7 model, and sub-Chandra events are associated with the WDD2 model discussed below.

These single degenerate models display a “subsonic deflagration (flame) front” which spreads through the star [7]. The deflagration flame front spreads as a non-spherical front, with a large surface area due in part to folds in the front. One example of a spreading deflagration flame front is the ignition of a match head.

Double Degenerate “WDD2” Model

The other mode of type Ia supernovae is the double degenerate model. In this scenario, two carbon-oxygen white dwarfs collide. After the collision, a helium flash then occurs, creating a detonation front that spreads in all directions. When the detonation reaches the core of the star, a carbon flash occurs, creating another detonation front expanding outward radially in the star, creating a supernova [5].

Chapter 3: Type Ia Supernova Nucleosynthesis

Nucleosynthesis is the formation of heavy nuclei from preexisting nuclei. When nuclei have sufficient energy to collide, unstable massive nuclei can briefly form and decay into a nucleus heavier than either of the two original nuclei. Due to the endothermic nature of iron fusion, elements heavier than iron cannot be created by stellar fusion [5].

The high-energy conditions of supernovae, however, can allow synthesis of elements heavier than iron to occur. From [8], type Ia supernova events create nucleosynthesis temperatures ranging from $\sim 10^9$-$10^{10}$ K. The minimum nucleosynthesis temperature required for detonation fronts are $\sim 2*10^9$ K, as that for deflagration fronts ranges from $\sim 2-5*10^9$ K, depending on the physical parameters of the white dwarf.
Chapter 4: Introduction to Galactic Chemical Evolution

The field of galactic chemical evolution is central to this research. The goal of galactic chemical evolution (GCE) models is to accurately predict chemical abundances in the galaxy given a specific time and location [9]. This is done by calculating the abundances of elements created through nucleosynthesis during events such as the Big Bang, and stellar supernovae. Current models allow users to adjust parameter values to calculate nuclear abundances yielded by specific events.

Chapter 5: Experimental Procedure

The goal of this research was to map experimental stellar chemical abundance data gathered from one GCE model to determine how to most accurately represent observed data. This was done by comparing calculated nuclear abundance for two types of type Ia supernovae, Chandra and sub-Chandra type Ia events. Ideally, comparing the results of the simulation with observed data would allow for greater understanding of the levels of occurrence of each of these modes of type Ia supernova.

The galactic chemical evolution model used for this research corresponds to the code listed as reference [10], with corresponding paper [9]. The following information for the rest of this section is referenced to [9].

The GCE code calculates nuclear abundances of elements through time for selected supernova models using GCE mass fractions of various isotopes as inputs. Nuclear abundance values are calculated for “radial zones” of the exponential galactic disk. The abundance values of these zones are added together, yielding galactic nuclear abundance values for isotopes at time t. The timescale used for this program moves forward, with present day being t = 0, and
starting time $t_0$ set as the moment of the Big Bang, approximately at time $t = \text{-}14 \, \text{Gyr}$. These calculations use the following equation, given in a general, qualitative form, as the rate at which isotope abundances change in time

$$\frac{d\sigma_i}{dt} = \text{stellar death} - \text{stellar birth} + \text{infall} + \text{decay}$$

where $\sigma_i$ is the surface mass density of isotope i in a radial zone. The four terms (expanded as mathematical terms in an integral) respectively represent the nuclear abundances created during stellar death, depleted during stellar formation, gained through accretion during various galactic processes, and the abundance that is formed through radioactive decay of heavier nuclei. Additional mathematical terms are added to account for the effects of type Ia supernovae [9].

For this research, single-degenerate “Chandra” (W7) and double-degenerate “sub-Chandra” (WDD2) models are studied and compared to one another. For each, the abundances produced assume that all white dwarfs are identical in size, composition, and explosion mechanism. Data is plotted as $[x/Fe]$ vs. $[Ni/Fe]$ through time, where $x$ is the abundance of a given element, and

$$[\frac{x}{Fe}] = \log \left( \frac{x_{Fe}}{x_{sol}/Fe_{sol}} \right) = \log \left( \frac{x}{Fe} \right) - \log \left( \frac{x_{sol}}{Fe_{sol}} \right)$$

The elements in focus for this study are Ca, Sc, Ti, V, Cr, and Mn. Their isotopic GCE mass fraction (mass of isotope i/total ejected mass) values for the W7 and WDD2 models, provided by [11], are included in appendix A. These elements were chosen for this study due to the significant difference between their GCE mass fraction values between the two models. These
data lines produced by the GCE program are also compared to data points from the SAGA
database of observed nuclear abundance data [12].

Chapter 6: Results

The graphs for [x/Fe] vs. [Ni/Fe] compared to observed stellar metallicities are included
as figures 4-15 below. Each shows the values of these ratios as they move forward in time, with
the end of the lines corresponding to initial time labeled as \( t_0 \). Even numbered figures show the
curves for each model compared to SAGA data with [Fe/H] > -0.5 used as a filter. This was done
to highlight SAGA data points that most likely correspond to type Ia supernova events. Since
type Ia supernovae occur in white dwarf stars, and white dwarfs are the remnants of stars with
potential lifetimes on the order of \( \sim 10 \) Gyr, stars satisfying the [Fe/H] > -0.5 criterion correspond
to more recent events, and thus are more likely to be associated with white dwarfs.

Odd numbered figures show these lines superimposed with all observed data points
from the SAGA database for [x/Fe] vs. [Ni/Fe]. The SAGA data produces a “cloud” of data due to
nuclei being formed by a variety of stellar processes. The uncertainty lines also contribute to
the cloud-like appearance of the data. The computer program data, on the other hand,
produces curves. This is due to the extremely low uncertainty from the model, as well as the
fact that complete uniformity is assumed. The model produces data only based on the selected
supernovae event. Additionally, the program assumes that all white dwarfs and supernovae
events are identical, which produces a series of points forming a smooth curve, instead of a
cloud of data.

Observed data points without error bars correspond to points from the SAGA database
with no available uncertainty values.
Figure 4: [Ca/Fe] vs. [Ni/Fe] nuclear abundance values in time, compared to filtered SAGA data

Figure 5: Curves from figure 4 with total observed data from the SAGA database
Figure 6: [Sc/Fe] vs. [Ni/Fe] nuclear abundance values in time, compared to filtered SAGA data

Figure 7: Curves from figure 6 with total observed data from the SAGA database
Figure 8: [Ti/Fe] vs. [Ni/Fe] nuclear abundance values in time, compared to filtered SAGA data

Figure 9: Curves from figure 8 with total observed data from the SAGA database
Figure 10: [V/Fe] vs. [Ni/Fe] nuclear abundance values in time, compared to filtered SAGA data

Figure 11: Curves from figure 10 with total observed data from the SAGA database
Figure 12: [Cr/Fe] vs. [Ni/Fe] nuclear abundance values in time, compared to filtered SAGA data

Figure 13: Curves from figure 12 with total observed data from the SAGA database
**Figure 14:** [Mn/Fe] vs. [Ni/Fe] Nuclear Abundance Values in Time

**Figure 15:** Curves from figure 14 with total observed data from the SAGA database
Conclusions:

The nuclear abundance lines for vanadium showed the most significant difference between the two models. Other than brief overlap between [Ni/Fe] values ranging from -0.12 ± 0.01 to -0.04 ± 0.01, the V lines didn’t lay over one another as they did for the rest of the elements shown. The nuclear abundance lines for Ca, Sc, Ti, Cr, and Mn had close agreement through most of the curves, spreading further apart as time approached t_f. One consistent trend seen in these figures is that at t_{WDD2} = t_f, the W7 model produced smaller values of [x/Fe].

The second objective of this research was to see if data from one model fit better with observed data from the SAGA database. If SAGA data showed consistently better matching with either of the two models, it may have indicated that one model is more prevalent than the other.

For the nuclear abundance lines for each model when compared to the filtered SAGA data (even numbered figures), only Ca and Cr showed significant overlap with the “cloud” of observed data points. The lines for the other elements sat below the cloud of data, only overlapping with error bars for some of the points (where uncertainty was available).

The abundance lines produced by this model were much smaller in size than the cloud of the total observed data for each element. The fact that each set of lines sits within these clouds indicates that type Ia supernovae contribute to observed nuclear abundances. However, the lines from each model become nearly indistinguishable when superimposed over the cloud.

Overall, conclusions cannot be drawn as to which model fits better with observed data. The small differences between the curves in relation to the size of the clouds of observed data indicated that mathematical fitting would not be useful to analyze which model agrees better.
with the observed data. If uncertainty values were available for the filtered data, mathematical fitting could be used to compare the models with filtered data.

A trend was seen indicating that the WDD2 model had greater $[x/Fe]$ values at $t_f$ than the W7 model. Additionally, each model fit within total observed data, indicating potential agreement between calculated and observed data. However, this idea of agreement is disputed by the lack of overlap between calculated and filtered data. This model will need to be altered, or more observations will be needed to allow for studies into which model fits best with observed data. Doing so will allow future computer modeling to produce original data with increased confidence in accuracy.
**Appendix A: GCE Values**


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<tr>
<th>Isotope</th>
<th>W7</th>
<th>WDD2</th>
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</thead>
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<td>$^{40}$Ca</td>
<td>$1.134 \times 10^{-2}$</td>
<td>$2.477 \times 10^{-2}$</td>
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<td>$^{42}$Ca</td>
<td>$2.469 \times 10^{-5}$</td>
<td>$2.901 \times 10^{-5}$</td>
</tr>
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<td>$^{43}$Ca</td>
<td>$7.083 \times 10^{-8}$</td>
<td>$4.318 \times 10^{-8}$</td>
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<td>$^{44}$Ca</td>
<td>$7.910 \times 10^{-6}$</td>
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<td>$^{46}$Ca</td>
<td>$4.303 \times 10^{-11}$</td>
<td>$4.952 \times 10^{-10}$</td>
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<td>$^{48}$Ca</td>
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<td>$2.018 \times 10^{-10}$</td>
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<td>$^{45}$Sc</td>
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<td>$^{55}$Mn</td>
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