12-8-2017

Cross-Curricular Writing in Mathematics for Comprehension

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Integrating Writing in a Secondary Mathematics Classroom to Increase Student Understanding

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Lee Honors College Honors Thesis

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When people consider a secondary mathematics classroom, writing is often not the first thing that comes to mind; instead, problem solving, critical thinking, and Pythagorean Theorem are terms that many recognize and recall. The National Council of Teachers of Mathematics, however, asserts that successful mathematical classrooms that benefit all students should involve students "revealing their mathematical understanding, reasoning, and methods in written work" so that teachers may elicit and gather evidence of student understanding of mathematical concepts as a use of formative assessment (National Council of Teachers of Mathematics, 2014, p. 56). As such, the idea of implementing writing in a mathematics classroom is far from new— as the concept of Writing to Learn Mathematics (WTLM) is referenced as early as 1989 (Teuscher, Kulinna, & Crooker, 2015)— and the benefits from doing so are hardly nonexistent (Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Williams, 2003; Williams & Wynne, 2000).

While writing not only allows students to organize and communicate their thinking, writing can also help students gain a better conceptual understanding of mathematical topics, develop a stronger sense of mathematical procedure, move beyond surface-level thinking, and place abstract ideas into context (Burns, 2004; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Sunstein, Liu, Hunsicker, & Baker, 2012; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003; Williams & Wynne, 2000). Moreover, writing implemented in a secondary mathematics classroom can be used to highlight misconceptions and understandings to the teacher in order to assess student learning, which in turn allows the teacher to adjust instruction as necessary (Burns, 2004; Cooper, 2012; Dundar, 2016; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Williams & Wynne, 2000).

Despite the benefits that writing in a mathematics classroom carries for both the students and the teacher, McIntosh & Draper (2001) found that, "although most teachers surveyed agree that writing is an important component of teaching mathematics, not all teachers use writing activities consistently" (p. 554). One reason listed for the lack of writing activities in a mathematics classroom is a lack of time (McIntosh & Draper, 2001; Teuscher, Kulinna, & Crooker, 2015; Williams & Wynne, 2000). Other concerns raised included lack of student self-efficacy and lack of teacher preparation (Burns, 2004; Cooper, 2012; Fukawa-Connelly & Buck, 2010). Furthermore, there "have been inconsistent reports regarding the benefits from [writing] for students and teachers; however, most [Writing to Learn Mathematics (WTLM)] studies were conducted over two decades ago and many things have changed in schools and in home life for students" (Teuscher, Kulinna, & Crooker, 2015, p. 61).

Although the idea of writing in a mathematics classroom has existed for several decades and researchers have debated whether or not there are advantages for students and teachers, more recent articles point to the overwhelming benefits writing activities can provide (Burns, 2004; Cooper, 2012; Dundar, 2016; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Sunstein, Liu, Hunsicker, & Baker, 2012; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003; Williams & Wynne, 2000). As such, one aim of this thesis is to pull together newer, relevant material on incorporating writing in a secondary mathematics classroom while considering the Common Core State Standards for Mathematics (CCSSM), which were implemented after much research on writing in secondary mathematics had been conducted. Moreover, the Michigan Department of Education (2015) lists four claims of student behavior, two of which relate to writing. In particular, Claim 1 asks students to explain mathematical concepts and Claim 4 asks students to "clearly and precisely construct viable arguments to support their own reasoning" (Michigan Department of Education, 2015).

Furthermore, as "learning more about WTLM is also useful since this method can benefit all students (regardless of ability levels or intention to enter a math-centric career)," the use of writing in a modern mathematics classroom is an idea that requires attention (Teuscher, Kulinna, & Crooker, 2015, p. 61). Thus, by examining pre-existing research and methods currently being used, this thesis aims to explore how writing can be implemented in a mathematics classroom. This thesis will discuss the benefits of writing in a secondary mathematics classroom for both the students and the teachers and will address concerns raised about this topic, such as the time constraint teachers face in the classroom. Finally, this thesis intends to demonstrate this idea by offering a unit plan that demonstrates what actual implementation of CCSSM would resemble in a high school Geometry.

Benefits for Students:

When it comes to implementing writing in a secondary mathematics classroom, there are numerous benefits for students. At the heart of promoting writing in mathematics is the thought that "...learning is an active process whereby students construct their own meaning about the content being studied rather than merely being passive recipients of information imparted by teachers" (Teuscher, Kulinna, & Crooker, 2015, p. 57). In a mathematics classroom, writing allows students to become active in the learning process by developing a better conceptual understanding of mathematical concepts, gaining a greater sense of process and procedure, moving beyond surface-level thinking, and placing abstract ideas into context (Burns, 2004; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Sunstein, Liu, Hunsicker, & Baker, 2012; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003; Williams & Wynne, 2000).

One benefit of implementing writing in a mathematics secondary classroom is that students can use writing to develop a deeper understanding of mathematical concepts. According to Pugalee (2004), "the ability to articulate one's ideas is seen as a benchmark of deep understanding, requiring reflection to identify and describe critical elements and concepts" (p. 24). Students who can articulate their own thinking and put their thoughts into words expressed on a page demonstrate a greater comprehension of concepts because they process ideas instead of simply repeating back what they have read/heard (Pugalee, 2004). When students are writing, they express concepts in their own words, describe their thinking, and illustrate their ideas to the teacher and their peers. While learning new mathematical concepts, students who can describe the material in their own words are able to better understand and then remember the new information (Pugalee, 2004; Fukawa-Connelly & Buck, 2010). Therefore, students who can respond to mathematics in writing—by expressing their own ideas or articulating concepts in their own words—are able to gain a better comprehension of mathematical concepts.

While writing allows students to better understand concepts, it also allows secondary students to gain a stronger grasp on processes and procedures for solving problems (Fukawa-Connelly & Buck, 2010; Pugalee, 2004; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003). In particular, Williams (2003) discovered that students who wrote about their process increased their ability to "better organize their thought processes when they attempt to solve problems, and [this] may help students have less difficulty when they begin solving challenging problems" (p. 187). By putting thoughts down in written work instead of using only letters and numbers (or perhaps nothing at all), students can organize their ideas and track their progress through problems, allowing the students to easily review their thought progression. This can help students to better understand their own procedural thinking, because they can read their rationale for each step, as well as prompt them to consider the validity of their thinking. In particular, "students who wrote about their problem solving processes produced correct solutions at a statistically higher rate than when using think-aloud processes" (Pugalee, 2004, p. 43). Although think-alouds allow students to voice their thoughts, putting those ideas into writing mean that students must consider each step of the process and determine how to articulate their thoughts, prompting deeper consideration of the mathematical concepts and the procedural processes being used.

Because writing encourages students to develop a deeper conceptual understanding and a greater comprehension of mathematical processes, writing pushes students to go beyond surface-level thinking and beyond the notion of simply ‘plugging and chugging’ numbers into an equation. When students are writing about their understanding and procedures, they “do not merely execute the steps to arrive at an answer; they explain their thinking during the steps” (McIntosh & Draper, 2001, p. 556). Therefore, students examine the rationale of each step, highlighting their understanding of the concept and how they believe it relates to other ideas, moving beyond simply using arithmetic to solve equations (Burns, 2004; Teuscher, Kulinna, & Crooker, 2015). This allows students to surpass surface-level thinking as their writing must express ideas, communicate reasoning, provide support for statements, and/or reach conclusions. Thus, students reach a higher level of thinking over arithmetic that has a standard procedure; instead, students write about and explore ideas that they need to justify, explain, critique, etc.

Lastly, writing in a mathematics secondary classroom can aid students in placing math (which many view often as only an abstract concept) into context (Burns, 2004; Sunstein et al., 2012). For instance, students may be asked to explain in writing the relationship of a linear equation in a contextual example; this would involve describing rate of change in a real-world context and illustrating how the different variables interact instead of simply presenting an equation without understanding its applicability. When implementing writing in the classroom, Sunstein et al. (2012) discovered that “our students found ways to identify and define problems in their everyday lives, as well as communicate with precision” to their pen pal different mathematical problems they discovered and solved daily (p. 18). Instead of learning about area by talking about shapes drawn on a board and using letters to denote lengths, one student wrote to his pen pal in detail about the volume of silos in Iowa and how to determine how much grain one could hold (Sunstein et al., 2012). Thus, writing allows students to put abstract ideas into context by describing their applicability in real-world situations, which, in turn, can help students to gain a better conceptual understanding of the mathematical idea by relating it to their everyday lives.

Benefits for Teachers

By having students write about their understandings and rationale, a teacher can read and determine where her students are in the learning process in order to then adjust instruction as necessary (Burns, 2004; Cooper, 2012; Dundar, 2016; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Williams & Wynne, 2000). Although writing is far from the only form of formative assessment, student writing can provide explicitness that some formative assessment cannot (Burns, 2004; Cooper, 2012; Dundar, 2016). Having students write about mathematics in various ways allows for the teacher to determine if the student is struggling with conceptual understanding, procedural comprehension, or something else. Being able to determine the type of misconception a student has through writing would not be possible, for instance, through formative assessments during which students cannot highlight their thinking, only their ability to correctly recall information learned from a lesson. According to McIntosh and Draper (2010), the use of writing in their classroom "allowed [them] to know [their] students better, to understand their thinking better, to communicate individually with students through the written word, and to reevaluate [their] instruction on the basis of students' responses" (p. 556). Instead of a teacher looking over a student's arithmetic work and trying to decipher the rationale behind the student's steps, a teacher can use student writing to see what student thought processes are to then determine if adjustments in instruction are necessary.

Writing Practices Already Being Implemented

Since writing in a secondary mathematics classroom is far from a new idea, there are several existing different types and ways that writing has been implemented. The assignments and uses in the classroom differ slightly for each practice—as does the form of writing that students are using—

but each method involves students responding in their own words about something related to mathematics. Although listed are several instances where writing practices can be implemented, any of the methods could be adapted for use depending on the specific classroom, teacher, and students.

Writing to Learn Mathematics (WTLM)

When it comes to writing in academics, the "Writing to Learn Mathematics" (WTLM) process is often referenced regarding the idea of writing being something that students complete in order to comprehend information during the process of learning. The Writing to Learn process is "grounded in the constructivist belief that learning is an active process whereby students construct their own meaning about the content being studied rather than merely being passive recipients of information imparted by teachers" (Teuscher, Kulinna, & Crooker, 2015, p. 57). As such, WTLM can be implemented in a mathematics classroom to promote stronger student comprehension of mathematical concepts. This can occur with students synthesizing information, comparing and contrasting concepts, expressing theorems in their own words, or documenting personal reflections on what they have learned in class and how they feel about their level of understanding. According to Tuescher et al. (2015), WTLM involves the typical expository writing often found in secondary school, expressive writing as a type of think-aloud, and personal writing for reflection and personal commentary. In order to place this writing in a mathematics classroom, then, there are several types of writing prompts that can be utilized.

Expository writing can be used to explain a mathematical concept. Expository writing typically occurs after students have already worked to some extent with a mathematical concept (Teuscher, Kulinna, & Crooker, 2015). When educators discuss how this could occur in a mathematics classroom, they suggest that students could be asked to write about how they would explain a topic to a struggling friend (Braun, 2014; Burns, 2004; Gillespie, Graham, Kiuhara, & Herbert, 2014). When asked to solve a problem step by step and provide a rationale for doing so, students may discover their own lack of comprehension that they may have not previously realized. Students can also be asked to complete personal writing–also referenced as expressive writing, as is the case for Teuscher, Kulinna, and Crooker (2015)–where they write on a more personal level regarding mathematics (Braun, 2014; Burns, 2004; Teuscher, Kulinna, & Crooker, 2015; Seto & Meel, 2006). This type of activity could involve students writing a mathematical autobiography, where they detail their experiences and opinions regarding previous mathematical experiences (Burns, 2014; Seto & Meel, 2006) or when a student describes in a journal at the end of the lesson their reactions to the day (Teuscher, Kulinna, & Crooker, 2015). The mathematical autobiography serves well as a first assignment for the teacher to gain a better understanding of their students' backgrounds and can help students become accustomed to writing in a mathematics class. Lastly, students could complete written reflections on their approach and result with a problem (Salend, 2016).

When teachers use WTLM, the form is not as essential as is the function. With WTLM, student writing can occur in various forms, such as in journals, on paper, in online discussion boards, in learning logs, or as formal papers (Teuscher, Kulinna, & Crooker, 2015). With the increase of technology in the classroom, the writing could also transition from typical paper-and-pencil to include online blogs or other forms of electronic communication. Regardless of how the writing is completed, the core of the WTLM idea is that writing prompts students to become engaged in the learning process, thus resulting in greater conceptual understanding and comprehension of new material.

Journal Writing:

There are various forms that journal writing can take and different ways to structure this type of writing in the classroom. According to Williams and Wynne (2000), journal writing is split into two types of writing prompts: mathematical and affective. With these two types of prompts, students could "communicate both their knowledge about mathematics and their feelings about the environment of the mathematics classroom" (p. 132). In one instance, students were asked to write one page twice a week, as one page was deemed enough for students to convey meaning but not too cumbersome for grading. Students were given ten minutes of class time to complete the journal but were not expected to finish it in class. When first implementing journal writing, students completed an affective assignment to allow them to become accustomed to writing as well as to signal the importance of student opinion (Williams & Wynne, 2000).

A slightly different type of journal writing can occur as impromptu writing prompts given in the classroom. These writing prompts can last 10-15 minutes and involve a variety of prompts for students to respond to, including asking students to reflect, explain, compare, and contrast (Seto & Meel, 2006). Although the prompts may differ, journal writing involves students writing in an informal setting that does not need to involve revision but asks students to write for a semi-extended length of time, typically at least one page or 10 minutes at a time.

Learning Logs:

Another type of writing can occur in learning logs, which are typically shorter than journals and are completed more often (Bahls, 2012; Burns, 2004; McIntosh & Draper, 2001). According to McIntosh and Draper (2001), the "purpose of writing in learning logs is to have students reflect on what they are learning and learn while they are reflecting on what they are learning (p. 554). While students need not write every single day, McIntosh and Draper (2001) recommend using learning logs several times a week, even if it is only for a short period of time. Entries in learning logs can be kept in a specific notebook, on pieces of paper to be handed in at the end of the day, or in any other form that fits the needs of instruction. The length of the entries can also vary depending on instruction. For instance, students might be asked to compose a few sentences every day in their learning log as they proceed through a unit and then asked to complete a longer entry after learning a big mathematical concept. Either way, learning logs can be read or skimmed quickly by a teacher to understand student understanding, and they can also serve as a reference tool for students for the future.

Portfolio Assignments

In general, portfolio assignments involve students creating a piece of work that spans multiple days or even an entire unit. Although there are multiple approaches and ways for students to complete portfolios, Fukawa-Connelly and Buck (2010) detail that there are several aspects of this type of assignment: cover letter, vocabulary, skill descriptions, unit questions, and reflections. For the cover letter, students "summarize what they learned during the unit and identify what was new knowledge, what was review material, and what questions remain" (Fukawa-Connelly & Buck, 2010, p. 650). In the vocabulary section, students define, paraphrase, and give examples of new terms from the unit. Under the skill descriptions, students write a step-by-step explanation of how to complete a skill by providing examples and a rationale of the procedural steps. For the unit questions section, students are asked to demonstrate their conceptual understanding. Lastly, the reflection allows for students to comment on more personal matters, such as how well they did in the unit and where they could look for improvement (Fukawa-Connelly & Buck, 2010). Regardless of the section, students are writing about their mathematical understanding and putting abstract concepts into their own words.

A less structured portfolio assignment is described by Sanders (2009). Students were asked to write reflections as well as other types of writing, including defining terms in their own words, answering problems by detailing their thought processes, and offering personal explanations of mathematical concepts using examples alongside written work (Sanders, 2009). By doing so, Sanders (2009) discovered that her students' understanding of the topics deepened and students understood *why* certain rules applied in Geometry. Although there is no time requirement for when a portfolio is created, many times portfolios coincide with a unit plan (Fukawa-Connelly & Buck, 2010).

Note Taking

Although many mathematics teachers might already use note taking in their classroom in some form or another, note taking is listed as one way to involve student writing in the classroom (Gillespie et al., 2014; Dundar 2016). In order to be most effective, however, note taking should be careful to not involve students simply filling in the blank from teacher-directed instruction. Instead, having student write in notebooks relates to student success (Dundar, 2016). Successful students write in notebooks where they can add illustrations, make connections to other material or concepts, and paraphrase ideas in their own words instead of writing them down verbatim from the teacher (Dundar, 2016). Although a different type of writing than used in journals, note taking that involves students becoming actively engaged in the learning instead of copying down verbatim prompts can result in higher comprehension.

Blogs/Other Technology Platforms

With the increase of technology and access to computers, students can take their writing online using platforms like blogs (Cooper, 2012). Instead of having students writing for a teacher, a blog can allow students to answer prompts online where they can be viewed and discussed by other classmates. Students can then write with an audience in mind and can comment on each other's thinking to point out misconceptions, reaffirm correct answers, or relate personally on the difficulty of a solution. This type of communication can not only allow for students to gain a greater comprehension of the mathematical concepts they are discussing, but can also allow them to work with classmates in order to reach conclusions and adjust misconceptions while building a classroom community.

Other instances of using technology to promote writing involve using emails (Seto & Meel, 2006). While other methods of communication have obviously been developed with the advancement of technology, emailing to the instructor or to other classmates also prompts students to put their thoughts in writing. So instead of simply repeating back what they have heard or read, students must move beyond regurgitation and instead express themselves and their ideas. Putting ideas into words requires students to have a grasp of the mathematical concepts they are using and the understanding to explain it to someone else (Seto & Meel, 2006).

Addressing Challenges of Writing Implementation

When educators are discussing the implementation of writing in a secondary mathematics classroom, there are several concerns raised, including lack of time in the classroom, student self-efficacy, and teacher preparation for teaching and using writing activities. Although there may be other difficulties depending on the type of the classroom, a common reason that writing is not implemented in the classroom is the time constraint teachers are constantly facing.

Time

Time is essential in the classroom and every minute counts. As a result, many teachers hesitate to implement writing in the mathematics classroom because they believe the activity takes up too much time that could be spent doing other things, especially with the pressure to complete a set curriculum (Gillespie et al., 2014; McIntosh & Draper, 2001; Teuscher, Kulinna, & Crooker, 2015). Moreover, many mathematics teachers believe that the time required to grade the writing assignments puts a heavy toll on them and can take up long hours after school. According to McIntosh and Draper (2001), however, “learning logs do not have to take much class time or grading time” (p. 555). Many writing assignments can be completed within 10 minutes—the length of time any classroom activity could take. Students could also complete the assignment at home, as

Williams and Wynne (2000) had their students do for journals. Although many teachers often believe that writing tasks involves students completing an entire essay—a process that occur over several days and involve multiple drafts—this is not often the case, as certain writing activities such as learning logs could be completed in a short amount of time every-other day (McIntosh & Draper, 2001).

While the in-class writing components can initially appear as a challenge for teachers, another aspect that many believe is a drawback of using writing in a mathematics class is the amount of time the teacher must spend grading or providing feedback on writing assignments (Gillespie et al., 2014; McIntosh & Draper, 2001; Teuscher, Kulinna, & Crooker, 2015). Many believe that a significant amount of time is required first to read the student responses and then respond to them. According to McIntosh and Draper (2001), however, "a set of learning logs takes about five to ten minutes to read" and can typically be scanned as students are leaving, at the end of the day, or during work time in class (p. 555). Longer assignments, such as journals or portfolios, will require more time on the teacher's part, but can also be scheduled so that teachers are not reviewing the entire assignment at the end of the unit (Williams & Wynne, 2000). Moreover, in-depth feedback is not required on every writing assignment; some writing may only need to be scanned by the teacher for her to understand where students are in their understanding.

Feedback and grading are both important for students to receive during the writing process, although they can differ depending on the teacher and the way writing is being implemented. Either way, it is important for students to not only know that their writing is being read and to receive feedback on it, but it is also essential for students to know whether they are correctly understanding a concept. As Williams and Wynne (2000) reported, "we were surprised to learn that most students in both of our classes wanted to continue with the journals [...] Their reasons included [...] that they learned to explain themselves better mathematically" as well as received feedback from their teachers about their understanding (p. 135). Furthermore, regardless of how a teacher determines how a grade is calculated on writing assignments, some sort of grade should eventually be given as a way for students to self-assess and to signal the importance of completing writing tasks (Fukawa-Connelly & Buck, 2010).Therefore, as something that can greatly benefit both the teacher and the students, writing in mathematics should be pursued, and there are multiple ways it can be implemented in order to best fit the time preferences of each teacher.

Student Self-Efficacy

When discussing the drawbacks of writing activities in a mathematics classroom, another aspect is the lack of student self-efficacy regarding their writing abilities (Teuscher, Kulinna, & Crooker, 2015; Williams & Wynne, 2000). According to Williams and Wynne (2000), at times "students said they did not know how to write what they wanted to say" (p. 134). Since writing is not often associated with mathematics, students may be reluctant at first to write because they either are not sure what to say or how to write what they are thinking (Teuscher, Kulinna, & Crooker, 2015). Furthermore, some students may have a lower writing level, which can prompt them to be even more hesitant to write in an unfamiliar setting (Teuscher, Kulinna, & Crooker, 2015).

To address this concern, there are several steps that teachers can take, including having the teacher model writing in front of the class before asking students to do a task (Gallagher, 2011; Salend, 2016). Before using a new writing task, a teacher can model for students how to complete the task so that students can know what the expectation is, can see what they are being asked to do, and have an example they can reference when they begin to craft their own pieces. For many students, the initial stage of writing can be daunting, but having a teacher process and produce a model to follow can prompt students into writing and build a better sense of confidence (Gallagher, 2011).

Another aspect that can help build student confidence in their writing is receiving feedback from their teachers. In particular, “students need to know that their learning logs are being read” (McIntosh & Draper, 2001, p. 555). As such, receiving feedback, even minimal, signals to students that what they are writing is important and it can build student self-efficacy with writing by knowing their efforts are not for nothing. Furthermore, “students are more likely to accept positive rather than negative feedback, and they can obtain one more positive connection with mathematics (McIntosh & Draper, 2001, p. 555). Since many students may not feel comfortable with writing or mathematics (or both), reaffirming what students are doing well can build confidence in students to continue to develop writing skills as they relate it to mathematics. Beginning the year with certain assignments, such as the mathematical autobiography, could also boost student confidence because the assignment may appear low-stakes and set on a more personal level instead of concentrating on mathematical concepts students are still grappling with (Burns, 2014; Seto & Meel, 2006).

Teacher Preparation

A third aspect discussed regarding the minimal writing implementation currently in mathematic classrooms is the lack of teacher preparation for preparing and teaching writing activities in a mathematics classroom. According to a study conducted by Gillespie et al. (2014), “most teachers reported they received minimal or no formal inservice preparation” for using writing tasks in a mathematics classroom (p. 1051). Furthermore, “teachers did not receive adequate preparation during college or from their school districts in how to use writing to support student learning” (Gillespie et al., 2014, p. 1074). As a result, many mathematics teachers may be hesitant to implement writing tasks in their classroom because they are not sure of the best practices associated with writing or how to structure their use in the classroom.

When looking specifically at the Writing to Learn Mathematics (WTLM) movement, Teuscher, Kulinna, & Crooker (2015) found that many of the teachers surveyed lacked both the knowledge of how to properly use WTLM in their classroom and confidence in its results. Yet, the "majority of secondary mathematics teachers who were familiar with WTLM reported positive effects on student achievement" (Teuscher, Kulinna, & Crooker, 2015, p. 71). Although lack of preparation results in many mathematics teachers hesitating to use writing strategies in the classroom, there are resources available for those who do not receive the proper training from their university or district, including those listed in this thesis. Overall, the positive impacts of writing in a secondary mathematics classroom greatly overshadow the initial drawbacks, drawbacks that can be addressed in order to encourage students to benefit from writing-related tasks in a mathematics setting.

Feedback

As with other aspects of the classroom, understanding and planning for feedback and assessment are also necessary when using writing in a mathematics classroom. Although there are various ways of providing feedback, as feedback may differ depending on the type of the assignment and the preference of the teacher, in general feedback on writing should be returned quickly, be given consistently, and be positive (Ehrenworth, Minor, Federman, Jennings, Messer, & McClou, 2015; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001).

Returning feedback in a timely manner provides benefits for both the teacher and the student. For the teacher, it could mean that the end of the unit is not overloaded with assignments to be graded and handed back (Fukawa-Connelly & Buck, 2010). For the student, timely feedback allows for the student to be making changes as they occur (Ehrenworth et al., 2015; Williams & Wynne, 2000). According to Ehrenworth et al. (2015), writing feedback is more effective when given in context so that the student can make adjustments, not handed back weeks later when the assignment is no longer on the student's mind. When students receive timely feedback, they are also able to revise as they continue working instead of after they have already completed and handed in an assignment (Williams & Wynne, 2000). Timely feedback can be verbal, as the teacher rotates during class work time and points out to students the things she is noticing so that students can address it before handing in the assignment, or can be written after students hand in the assignment and quickly returned.

Not only should feedback be returned efficiently, but it should also be given consistently (Fukawa-Connelly & Buck, 2010). By receiving feedback consistently, students can know if they are on the right track or if they are having misconceptions that need to be addressed from reoccurring remarks from the teacher. According to Williams and Wynne (2010), their students enjoyed receiving feedback because the "comments on the journals gave them immediate feedback on their understanding" (p. 135). In order for students to know whether they are on the right track and comprehending the material correctly, it is important for them to not only receive timely feedback, but to receive feedback consistently throughout the unit plan or the semester.

Lastly, feedback should be framed positively and respectfully (Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001). According to McIntosh and Draper (2001), "students are more likely to accept positive rather than negative feedback, and they obtain one more positive connection with mathematics" (p. 555). To do so, teachers should offer feedback that is constructive, descriptive, and phrased in a manner that is respectful and aimed at ensuring the student understands what he did well and what specifically he may need to improve on. This does not, however, mean that teacher should not point out flaws in mathematical thinking. While pointing out misconceptions or errors, however, the feedback should be phrased to signal to the student the incorrect material while also providing ways the student can find the correct information; then, instead of simply pointing out flaws, the teacher is offering the student a way to find a solution. It is important, however, to keep from only listing out student errors; if this is the only feedback a student receives, then they do not know what they are doing well and may hold a perception that they only made mistakes.

Assessment

Student writing can be used both in a formative and summative (as defined by Wiggins and McTighe (2011)) nature to assess student understanding, but two components are involved in assessment: content assessment and assessment of critical reflection in writing. In order words, assessing student written work involves determining whether the student has correctly grasped the mathematical content knowledge being explored as well as assessing the student's critical written reflection involving explanation and justification. As already emphasized, a plethora of benefits have been documented for students using writing in a mathematics classroom to boost conceptual and procedural understanding (Burns, 2004; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Sunstein, Liu, Hunsicker, & Baker, 2012; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003; Williams & Wynne, 2000). Writing, however, can also be effectively used as a tool for assessing, both formatively, summatively and as a tool for self-assessment.

When focusing specifically at assessment of content, assessment often involves determining whether students are grasping key content-specific understandings or highlighting misconceptions that they may have. In WTLM, students may often be in the process of learning as they are writing; they may not have fully grasped the material yet to accurately write about it when first learning the new concept (Teuscher, Kulinna, & Crooker, 2015). As a result, this stage of WTLM allows teachers a chance to use writing as a formative assessment. When having students work with learning logs, McIntosh and Draper (2001) point out that these are useful tools for formative assessments as they "prompts force students to go beyond a surface-level examination of their thinking and problem solving. Students do not merely execute the steps to arrive at an answer; they explain their thinking during the steps" (p. 556). Thus, learning logs push students to move beyond recitation of learned information because the students are making explicit connections through their written work.

Furthermore, the use of learning logs in a mathematics classroom allowed for the authors to "know our students better, to understand their thinking better, to communicate individually with students through the written word, and to reevaluate our instruction on the basis of students' responses" (McIntosh & Draper, 2001, p. 556). In this instance, assessment occurs as formative assessment, as students are documenting their learning while working through new concepts. Their writing, however, allows for teachers to assess whether students are grasping material, if their line of thinking is correct, or if students are carrying misconceptions. As writing often highlights student thinking while traditional problems (i.e., "solve the equation for x") do not, this allows for teachers to assess whether students are on track or if adjustments in instruction are needed.

Although the explicitness of writing easily lends itself to formative assessment, writing tasks in a mathematics classroom can also be used as a summative assessment. For instance, the use of portfolios due at the end of a unit can lend itself toward a summative assessment as instruction on material is already over. When using the portfolio design, students are asked to "demonstrate mastery of essential unit skills, explain procedures, and communicate conceptual understanding" (Fukawa-Connelly & Buck, 2010, p. 650). As students are expected to have already learned, practiced, and refined skills in the unit, assessment on writing about content may adhere more strictly to whether students are accurately discussing the key mathematical concepts. In this instance, writing becomes a summative assessment by having students explicitly detail their understanding of concepts covered in the unit.

Writing also provides students with a chance to self-assess. According to Dungan and Mundhenk (2006), "writing has a prominent place in the use of [student self-assessment], particularly as a means to promote reflection" (p. 57). When placed in a mathematics classroom, this reflection could range from students grasp on concepts, their understanding of how and why a procedure works, to their overall disposition towards a mathematics classroom. While not the only medium of writing, journals could offer a place for this type of reflection. By writing, students "examine what is working for them in order to build on their successes. They also assess their difficulties and consider strategies and steps to overcome them" (Dungan & Mundhenk, 2006, p. 57). In this instance, students gain a greater sense of self-awareness of their own understandings of mathematics through their written work, granting them opportunities to self-assess. This could occur as part of the WTLM process, as students are writing as they are working through new concepts (Teuscher, Kulinna, & Crooker, 2015). Thus, one aspect of this writing could pose a question to students to prompt them self-assess their own work by examining what they have been writing through and responding on whether they believe they are succeeding, what they could do to improve their understanding, and what obstacles may be in their way.

Thesis Rationale

Given the research that details the benefits that writing in a mathematics classroom has for students (Burns, 2004; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Sunstein, Liu, Hunsicker, & Baker, 2012; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003; Williams & Wynne, 2000) and for teachers (Burns, 2004; Cooper, 2012; Dundar, 2016; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Williams & Wynne, 2000) as well as the ability of writing to allow for both formative and summative assessment of student understanding (McIntosh & Draper, 2001; Teuscher, Kulinna, & Crooker, 2015), it is clear that writing in a secondary mathematics classroom should be integrated in the curriculum. To demonstrate how writing could be completed in a secondary mathematics classroom, this thesis includes a unit plan (see below) adapted from Wiggins and McTighe (2011).

While the unit plan does not discuss all genres of writing that could aid in learning in a mathematics classroom (as there would simply be too much writing content included for a realistic classroom), the unit plan offers a perspective on what writing integration could resemble using content standards from the Common Core State Standards of Mathematics. While the unit below offers only one perspective, there are many ways that mathematics teachers could effectively incorporate writing into their classrooms to increase student conceptual and procedural understanding. To highlight the validity and practicality of including writing in the curriculum, the unit plan serves as a tool to illustrate a unit that mathematics teachers could adopt for their specific classroom. As the research has clearly identified the numerous advantages writing in a secondary mathematics classroom has to offer (Burns, 2004; Fukawa-Connelly & Buck, 2010; McIntosh & Draper, 2001; Pugalee, 2004; Sunstein, Liu, Hunsicker, & Baker, 2012; Teuscher, Kulinna, & Crooker, 2015; Williams, 2003; Williams & Wynne, 2000), this unit plan marks one of many instances when students and teachers alike can benefit from the combination of writing and mathematics.

Unit Plan

Adapted from Wiggins and McTighe (2011) Understanding by Design unit framework

Stage 1 -- Desired Results

<table>
<tr>
<td rowspan="3">CCSSM:
G.CO.2 Represent transformations in the plane using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (translation vs. horizontal stretch).
G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.5 Given a geometric figure and a rotation, reflection, or translations, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Big Idea(s):
Make logical conclusions</td>
<td>Transfer
Students will be able to independently use their learning to . . .
Approach situations of any kind (mathematics or not) and devise a logical approach to solving it.
Determine whether the conclusion is logical based upon contextual clues.
Justify conclusions based upon evidence and logical reasoning</td>
</tr>
<tr>
<td>Meaning
UNDERSTANDINGS
Students will understand that . . .
Rigid transformations retain properties associated with the original shape, while other transformations may alter properties.
A function is a single-variable mapping that can be applied to a domain consisting of elements other than real numbers.
Geometric constructions (constructing shapes, segments, etc.) is different than drawing.
A figure may be represented in multiple ways but will always have the same properties.</td>
</tr>
<tr>
<td>ESSENTIAL QUESTIONS
Students will keep considering . . .
When can mathematicians determine that a stated relationship will always be true?
How much information is necessary to solve for an unknown?
How do different representations highlight information, and which is the most useful?
Why is it important for mathematicians to have universal language?
How accurate do representations or descriptions need to be?</td>
</tr>
</table>

Stage 1 (Continued)

Acquisition

Students will know . .	*Students will be skilled at . . .*
The definitions of angle, circle, perpendicular line, parallel line, and line segment. Different ways to represent transformations. The definition of different transformations (rotation, reflection, translation) and the way it changes the graphical representation. A shape can be mapped to itself from a series of transformations. The definition of a function. The definition of point, line, distance along a line, and distance around a circular arc.	Using GeoGebra to make constructions. Writing transformations as functions. Identifying a series of transformations based on a graph or a function. Drawing graphical representations of transformations. Describing the changes in properties that a transformation has a line or figure.

Stage 2 -- Evidence

Evaluative Criteria *What criteria will be used in each assessment to evaluate attainment of the desired results?* 1) Supported by evidence, conjectures are specific and logical, 2) Clear definition, explanation supported by evidence in lesson 3, 5, 6, 10) Conjectures are specific and logical, justifications apply to general case instead of specific problems, clear evidence backs claims 4) Justification applies to general cases, reasoning is logical and developed, argument includes definition of a function 7) Clear evidence 8, 9) Correctness of answer, appropriateness of approach, 11) Logical response, correctness of answer, specific details are highlighted (i.e., examples) 12) Correctness of answer, evidence included citing specific transformations 13) Justifications provided, clear response 14) Correctness of answer, justifications included and logical and apply to general case	Label each with the appropriate code that ties the task/evidence to one or more Stage 1 goals (A= Acquisition; M= meaning making; T=transfer). Performance Tasks: *Students will show that they really understand by evidence of:* 1. Pre-assessment activity on definition of functions, A, M 2. Student-defined definition of a function. A, M 3. Short-answer response on properties of translations. A, T 4. Student response and justifications to whether transformations are functions. A,M, T 5. Short-answer response on properties of reflections. A, T 6. Short-answer response on properties of rotations. A, T 7. Written response on how transformations are similar and how they differ. M, T 8. Practice problems on writing transformations as functions. A, M 9. Practice problems on identifying types of transformations. A, M 10. Short-answer response on properties of dilations. A, M 11. Response on difference between drawing and constructing. M 12. Summary of which transformations change properties of a shape. A, M 13. Student written response to series of transformations that return an image to its pre-image. A 14. Summative Assessment. A, M, T

Stage 2 (Continued)

Other Evidence: Responses are thorough, honest, detailed, supported by evidence, reference classwork,	OTHER EVIDENCE: *Students will show they have achieved Stage 1 goals by* ● Learning log entries ○ Reasoning behind pre-assessment ○ Definition of a function ○ Transformations as functions or not ○ Difference between constructing and drawing ● Journal entries ○ Describe transformations to a friend ○ Reflection on learnings of week ○ Reflection on preparation for test ● Take-home work

Stage 3 -- Learning Plan

Learning Events

Student success at transfer, meaning, and acquisition depends upon . . .

1. Pre-Assessment and introduction to unit where students are asked to complete a high-cognitive task that introduces transformations as functions by having students explore how a series of movements allows products to get from one place to another. This activity will highlight what previous knowledge students have about functions and transformations based upon their ability to interact with the task. After the task, students will complete a learning log entry (1 page) describing their reasoning behind the task and why they believe their solution works (writing about procedure and conceptual understanding of how a function works). G.CO.2
2. Revisiting previous lesson, students will develop the definition of a function and record their understanding of it in their learning logs (writing about conceptual understanding of what a function is). Sample prompt: *How would you define a mathematical function? What characteristics does a mathematical function have? How did you arrive at this conclusion? What evidence or details do you have to support this conclusion?* G.CO.2
3. Lesson on translations. Students will use GeoGebra as a mathematical action technology and guided questions to write observations, conjectures, and evidence they have on how each transformation changes the shape (WTLM on how translations are viewed in the graph and how it impacts points on graph, expository writing). Sample prompt: *Write a conjecture for how the graph of the function will change. Why do you think this is? After working with GeoGebra, what conclusions can you make from your observations? Was your conjecture correct or do you need to make adjustments, and if so, why?* G.CO.4
4. Lesson on writing transformations as functions. Students will have a chance to agree/disagree that transformations are function and then explore how transformations are considered a function (i.e., every point of original image is mapped to only one new point). At end of lesson, students will write in learning logs why they think transformations are/are not functions (conceptual understanding of the definition of a function, opportunity for formative assessment). Sample prompt: *Are transformations functions? Use evidence from class, your prior knowledge, or you own reasoning to support your answer.*
5. Lesson on reflections. Students will use GeoGebra as a mathematical action technology and guided questions to write observations, conjectures, and evidence they have on how each transformation changes the shape (WTLM on how reflections are viewed in the graph and how it impacts points on graph, exploratory and expository writing). Sample prompt: *Write a conjecture for how the graph of the function will change. Why do you think this is? After working with GeoGebra, what conclusions can you make from your observations? Was your conjecture correct or do you need to make adjustments, and if so, why?*
6. Lesson on rotations. Again, students will use GeoGebra as a mathematical action technology and guided questions to write observations, conjectures, and evidence they have on how each transformation changes the shape (WTLM process on how transformation changes the shape on the graph, exploratory and expository writing). Sample prompt: *Write a conjecture for how the graph of*

the function will change. Why do you think this is? After working with GeoGebra, what conclusions can you make from your observations? Was your conjecture correct or do you need to make adjustments, and if so, why? G.CO.4

7. Revisiting transformations as functions. Have students respond in writing about the three covered transformations. Potential prompt: *In what ways are the three transformations similar? How are they different? How would you describe the 'change' done by each transformation in your own words?*
8. GeoGebra lesson on dilations using GeoGebra as a mathematical action technology. Journal prompt: *How would you describe transformations in your own words to a friend?* (mathematical topic) G.CO.4
9. Using GeoGebra, students will explore and discuss the difference between *drawing* and *constructing* by examining cases of both and determining whether the properties of a shape are maintained. Record in learning log their understanding of difference between drawing and constructing (chance for formative assessment). Sample prompt: *Discuss how drawing and constructing differ. Feel free to relate back to examples from our GeoGebra work in class.* G.CO.2, G.CO.5
10. Practice problems using GeoGebra where students are given a shape (line, point, etc.) and a series of transformations and asked to construct the new image. Throughout, students will write justification for each step (writing about procedure) Journal Prompt: *Was this week difficult for you at all? Why or why not? What helped you succeed this week and what kept you from succeeding?* (affective journal topic) G.CO.2, G.CO.5
11. Using GeoGebra and resources from class (such as previous writing), students will participate in small group discussions as they consider: *What transformations retain properties such as angle measure and length and which transformations alter properties?* Students will write a short summary individually afterward of findings (chance for formative assessment, writing about conceptual understanding of how properties are changed in dilations but unchanged in translations, rotations, and reflections) G.CO.2
12. Problems where students are using paper and pencil to draw a series of translations. Students will respond to: *Given this shape, what series of transformations will return the image to the original place? How many ways are there to do this? Justify your response.* (WTLM process of how a shape is returned to original image) G.CO.2, G.CO.3, G.CO.5
13. In groups, students will use technology software such as GeoGebra to consider how reflections map an image to itself. Sample prompt: *How can you use only reflections to map an image to itself? How many ways are there to do so? Justify your response.* Journal prompt: *How are you feeling about the upcoming test? What do you need to do in order to prepare?* (WTLM of rotations returning image to preimage, affective journal topic) G.CO.3, G.CO.5
14. Summative Assessment: Students will respond individually in writing to the prompt *Given these two shapes on the graph (both an image and its preimage), what series of transformations would carry the preimage to its image? How would you describe to a friend how you determined this series of transformations?*

Stage 3 (Continued)

Monitoring Student Progress:

- Formative assessment highlighted in learning assessment (additional formative assessment included on detailed lesson breakdown)
- Student justifications and conjectures are developed over multiple days that signal level of student understanding of mathematical concepts
- Learning logs allow for formative assessment to determine learning progression
- Journal writing used for determining student perception about course (with affective topics highlighting student self-assessment and perceptions of class, such as how prepared they feel for a test)

Potential Student Misunderstandings:

- Transformations are not functions because the domain is not x-values and the range is not y-values
- Students may have misconceptions about transformations, such as a horizontal 'stretch' is the same as a vertical 'stretch', having (x-3) moves the graph 3 to the left, ½*x is a vertical stretch, etc.
- Constructing is the same as drawing
- Transformations are used on polynomial functions and are not applied to shapes, points, curves.
- Students may have heard/used the Vertical Line Test before and assume it works in all cases presented on a graph

Giving Student Feedback:

- In-class discussions will highlight important content to students and determine whether their conjectures were correct or incorrect
- Students with incorrect conjectures can use discussions with partners and whole class to reconsider their thinking to reconcile with findings of others
- Students receive feedback from the instructor as well as peers during the class discussion, as students are asked to weigh in on each other's conjectures, approaches, and findings
- Students receive written feedback from the instructor on all journal entries
- Verbal feedback given from instructor during student exploration time
- Written feedback would also be given on practice problems students

Using Feedback

- Students are expected to use feedback as they continue to explore different transformations
- As the lessons continue to build on one another, students will need to use what they have already done to consider new material, so they will need to integrate feedback into future explorations
- Feedback gained during whole class discussions should be considered as students then need to refine or adjust initial conjectures

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