A Direct Measurement of the Ratio of the Reaction Cross Sections for Two-Photon and Three-Photon Annihilation of a Positron and an Electron in Aluminum

Jeffrey Ira Siegel

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A DIRECT MEASUREMENT OF
THE RATIO OF THE REACTION CROSS
SECTIONS FOR TWO-PHOTON AND THREE-PHOTON
ANNIHILATION OF A POSITRON AND AN ELECTRON IN ALUMINUM

by

Jeffrey Ira Siegel

A Thesis
Submitted to the
Faculty of the School of Graduate
Studies in partial fulfillment
of the
Degree of Master of Arts

Western Michigan University
Kalamazoo, Michigan
April 1970
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Thanks are due to Dr. John Kusmiss for his suggestions, and continued interest in the experiment. My thanks go to Dr. M. Soga, and Dr. J. McCully for their helpful assistance with the mathematics.

Thanks are also due to my wife, Janet, who somehow was able to type this thesis and still find time to offer encouragement.

Jeffrey Ira Siegel
MASTER'S THESIS

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REACTION CROSS SECTIONS FOR TWO-PHOTON AND
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Western Michigan University, M.A., 1970
Physics, solid state

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INTRODUCTION

The annihilation of a positron with its antiparticle, an electron, depends upon the relative orientation of the spins of the two particles. Since both are spin $\frac{1}{2}$ particles, there are two possible relative orientations of their spins. If the spins are aligned, $|s_2| = \frac{1}{2} + \frac{1}{2} = 1$, the system is in a triplet state and annihilation results in an odd number of photons. The predominant case is the creation of three photons. If the spins are antiparallel, $|s_2| = \frac{1}{2} - \frac{1}{2} = 0$, the system is in a singlet state and the annihilation results in an even number of photons, the most probable case being the production of two photons. This follows from the conservation of energy and angular momentum.

The present work reports a measurement of the ratio of the reaction cross sections for the two and three-photon annihilation processes in aluminum.

In the case of singlet state annihilation, considering the positron-electron center of
mass to be at rest, the photons, each carrying $m_0c^2$ energy, emerge colinearly with equal but opposite momenta ($m_0$ is the rest mass of the electron, and $c$ is the speed of light). The momenta must be equal and opposite because the total linear momentum before annihilation was zero, and so it must remain zero.

In the case of triplet state annihilation, the photons may have any combination of energies or orientation in a plane so long as energy and momentum are conserved. In the present work only the three-photon annihilations in which the photons share the energy equally and emerge $120^\circ$ apart are considered.

In 1949 Ore and Powell\textsuperscript{1} calculated the cross section for the annihilation of a positron and a free electron resulting in the emission of three photons, in the limit of small velocities, using time dependent perturbation theory. They neglected the effect of the Coulomb interaction on the forms of the wave functions of the

\footnotesize

\normalsize
positron and electron. In addition, the cross section for positron annihilation with the emission of three photons has been calculated by Lifshitz\(^1\) and by Ivaneko and Solokov\(^2\). The following values for the ratio \(\sigma_2/\sigma_3\) of the cross sections for two-photon and three-photon annihilations have been calculated:

<table>
<thead>
<tr>
<th>Source</th>
<th>Value</th>
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<tr>
<td>Ore and Powell</td>
<td>370</td>
</tr>
<tr>
<td>Lifshitz</td>
<td>235</td>
</tr>
<tr>
<td>Ivaneko and Solokov</td>
<td>1670</td>
</tr>
</tbody>
</table>

In 1954, Basson\(^3\) made the first direct measurement of \(\sigma_2/\sigma_3\) accurate enough to discriminate between the various theoretical results. Basson's experimental set-up followed conventional lines. Three NaI(Tl) scintillators, each 2.5 cm. x 4.0 cm., were placed around a Na-22 source sealed in an aluminum container thick enough to stop all of the positrons. The details of the sample were not given in the paper. The source strength was


0.436 rutherford (0.015 millicuries). The electronics was a triple coincidence system with 4.5 x 10^{-7} seconds resolving time. However, no reference could be found as to whether the attenuation of the photons in the aluminum sample was taken into consideration. Attenuation is the loss of primary photons due to scattering and absorption in the sample. Furthermore, it also appears that if he did use an extended source, the correlation between the solid angles subtended by the detector faces and the probability of having a three-photon annihilation with 120° symmetry was neglected. Basson’s measurement depends upon the evaluation of the absolute disintegration rate of the Na-22, the absolute efficiencies of each of the detectors, and the fraction of all three photon events which had the selected symmetry. In addition, the geometry used did not provide any shielding for the detectors from the 1.28 MeV gamma ray which accompanies the positron formation, or annihilations in the source. This led to a chance coincidence rate which was greater than the true coincidence rate.

The value for the ratio of the cross sections obtained by Basson was 402 \pm 50. This value is in
good agreement with the Ore and Powell calculation, and therefore the value of $\frac{\sigma_2}{\sigma_3}$ has long been accepted as 370.

In 1965 Bertolaccini et al.\(^1\) performed a triple coincidence experiment to measure the three-photon annihilation rate for several metals. They performed a relative measurement, that is, their values were obtained by comparing the three-photon yield in each substance with that of aluminum. The source was 40 microcuries of Na-22 deposited onto a thin moplifan foil. The values they obtained had to be corrected for annihilation taking place in the moplifan foil. In five cases (Ni, Ag, Cd, Pt, Pb) the three-photon rates differed from aluminum by more than one standard deviation. These elements exhibited slightly higher three-photon yields. The increase in the production of three-photon annihilations might be due to some positronium formation. In metals, however, positronium formation has long been considered

unlikely, since the large number of "free" electrons should tend to cause annihilation rapidly. There is also the possibility that the deviation resulted from neglecting the attenuation of the photons in the various samples used. Bertolaccini's work indicates that the interactions between positrons and electrons in metals should be studied further.

The reason Basson studied positron annihilation in aluminum to check the $\frac{c_2}{c_3}$ ratio for annihilation with free electrons, and the reason Bertolaccini et al. used aluminum as a reference is clear; aluminum is a good example of a free electron metal. If one wants to check the Ore and Powell result for free electron and positron annihilation it still seems reasonable to use aluminum.

The purpose of the present work is to measure $\frac{c_2}{c_3}$ in aluminum using an improved geometry taking into account the attenuation of the photons in the sample, and the solid angle-probability relationship.
EXPERIMENTAL PROCEDURE

Geometry

To measure the singlet state annihilation cross section, two detectors were set along the same axis 15.0 ± 0.1 cm from the center of the aluminum target as shown in Figure 1. In the investigation of the triplet state annihilation cross section only annihilations where the photons emerged 120 degrees apart, and shared the total energy \(2m_0c^2\) equally, were measured. Thus in this measurement three detectors were set 120 degrees apart (as shown in Figure 2), again 15.0 ± 0.1 cm from the center of the aluminum target.
Figure 1: Geometry for Two-Photon Annihilation.
Sample

It was calculated that 0.030 inches of aluminum would be required to stop the maximum energy positrons emitted from the Na-22. A high-purity aluminum disc of this thickness and 1.0 inch in diameter was unavailable. Therefore, the sample was made from three 0.010 inch aluminum discs, each 1.0 inch in diameter. The target was heat treated for 24 hours at 550°C to remove strains and extended defects, which might cause appreciable deviation from the free electron case.

The surface of the aluminum rapidly oxidizes, forming a protective covering. An appreciable amount of aluminum oxide (insulator) could greatly increase the three-photon yield. However, this oxide thickness is small compared to the thickness of the aluminum and should have a negligible effect upon the results. The three discs were


fastened together using three small dots of Weldwood contact cement, manufactured by U.S. Plywood, between each surface at the edge. The disc sandwich was then attached to a plexiglass support rod and located directly over the source and equidistant from the detector faces in a plane as shown in Figure 2. The distance from the sample to the source was 4.4 ± 0.1 cm.
Source

The source consisted of 2.24 millicuries of Na-22 deposited in the center of a one-inch stainless steel mount and covered by a 0.0002 inch stainless steel foil. A decay scheme for Na-22 is shown in Figure 3. The source mount was recessed 1 cm into a 2.7 cm diameter hole, which had been drilled into a lead cylinder 16 cm high and 19 cm in diameter as shown in Figure 4. The lead cylinder absorbed most of the 511 keV photons resulting from annihilations in the source, and shielded the detectors against the 1.28 MeV photons which are emitted along with the positron.
Figure 3. Decay Scheme for Na-22.*

Figure 4. Position of Source in Lead Cylinder
Electronics

The electronics consisted of a fast coincidence system (2τ = 30nsec.), and two or three side channels. Each side channel consisted of a 1½" by 1" NaI(Tl) crystal and a R.C.A. photomultiplier tube (integrally mounted), a preamp, a linear amplifier and a single channel analyzer. The coincidence pulses were counted on two or more scalers (see Figure 5).

Since the detectors subtend a finite solid angle, there is a range in the energies of the photons corresponding to three-photon decay that can simultaneously initiate pulses in each of the side channels. For the triplet state case and the geometry employed in this work the energy range was 286keV to 386keV. The single channel analyzers were set to select only pulses corresponding to photons within this energy range. A 512 multichannel pulse height analyzer was used to set the windows on the single channel analyzers. The energy scale of the multichannel analyzer was calibrated using sources which decay by photons of known energy. The energy versus channel relationship was found to be linear within experimental
Figure 5. Schematic for the Electronics.
error as shown in Figure 6. Then, with the 511 keV annihilation peak set in channel 312 the channels corresponding to 286 keV and 386 keV were determined.

A pulser generating pulses with a pulse height corresponding to these energies was connected to the linear amplifiers. The windows were then adjusted to these energies. In the case of singlet state annihilation, the windows were set just above and below the 511 keV annihilation peak. Window settings for both cases are shown in Figure 7.

To insure that pulses originating from a true coincidence triggered the coincidence circuit, the times for the pulses to travel through the side channels had to be made equal. To accomplish this, first detectors one and two, then detectors one and three, and lastly detectors two and three were set 180° apart with the windows on each single channel analyzer set for singlet state annihilation (511 keV). The internal delay in each single channel analyzer was then systematically varied until the two-photon count rate reached a maximum. The largest delay required was 22 nanoseconds.

The present geometry seems to be an improvement
Figure 6. Energy versus Channel Number.
Figure 7. Two-Photon and Three-Photon Window Settings.
over the previous geometries used since the detectors are shielded from the 1.28 MeV gamma ray and annihilations in the source. The shielding reduces the chance rate, thus improving the ratio of the true to chance rate by 10 to 1 over the geometry used by Basson.
Procedure

If the activity of the source is $N_0$, and \( \frac{\sigma}{4\pi} \) is the solid angle subtended by the sample, then $N_0 \frac{\sigma}{4\pi}$ gives the number of positrons that reach the target per unit time. In the same manner, if \( \sigma_3 \) is the cross section for three-photon annihilation and $n_e$ is the total number of electrons available for annihilation per unit volume in the sample, it follows that $N_0 \frac{\sigma_3 n_e}{4\pi}$ will give the total number of three-photon annihilations which take place in the sample per unit time, or the rate of three-photon annihilation.

The actual number of photons detected depends upon the intrinsic photopeak efficiency of the detectors for 340 keV photons, the attenuation of the photons in the target, the solid angles subtended by the detectors, and the probability ($P_3$) of symmetric three-photon decay.

In the present geometry the aluminum sample acts as an extended source of photons. Since an annihilation can take place anywhere within the disc, the solid angles subtended by the detector faces depend upon where in the disc the annihilation occurs. Therefore, there is a
complex relationship between the solid angles subtended by the detector faces, where the annihilation occurs, and the probability \( P_3 \) that when a three-photon annihilation does occur, it will occur with the selected symmetry.

Dr. D.M. Rockmore, of the University of Maine, is studying this relationship and is calculating the parameter which expresses it, \( C_{123} \).

In addition, the measured three-photon rate must be corrected since some of the photons are not detected because of attenuation in the target. Thus, the number of true three-photon events detected per unit time will be

\[
N_0 \Omega_3 n_3 e_1(340) e_2(340) e_3(340) C_{123} A_{123},
\]

where \( e_i(340) \) is the intrinsic photopeak efficiency of the \( i \)th detector for 340 keV photons, \( C_{123} \) is the solid angle-probability relationship, and \( A_{123} \) is a correction term due to the attenuation of each of the 340 keV photons in the target.

With the two-photon geometry the number of true two-photon events detected is
\[
N_0 \sigma_2 \sigma_{2n} \leq_1 (511) \leq_2 (511) c_{12} \alpha_{12}.
\]

\( \leq_1 (511) \) is the intrinsic photopeak efficiency of the \( i \)th detector for 511 keV photons, \( c_{12} \) is the correlation factor connecting the solid angles subtended by detectors one and two, and \( \alpha_{12} \) is a correction term due to the attenuation of the 511 keV photons in the target. Taking the ratio and solving for \( \frac{\sigma_2}{\sigma_3} \) one obtains

\[
\frac{\sigma_2}{\sigma_3} = \frac{N_{28} \leq_1 (340) \leq_2 (340) \leq_3 (340) c_{123} \alpha_{123}}{N_{38} \leq_1 (511) \leq_2 (511) c_{12} \alpha_{12}}.
\]

The problem can be divided into several separate measurements and calculations: the measurement of the relative efficiencies \( \leq (340) / \leq (511) \) for two detectors; measurement of the two-photon and three-photon count rates; measurement of the absolute efficiency for one detector at 340 keV; the calculation of the attenuation terms for 340 keV and 511 keV photons, and the calculation of \( c_{123} / c_{12} \).
RESULTS

Determination of Efficiencies

Since the measurement of $\frac{\sigma_2}{\sigma_3}$ depends only upon the relative efficiencies of two detectors for 340 keV and 511 keV photons, the measurement of the absolute efficiencies at both energies was not necessary. A convenient way to measure the relative efficiencies of a detector for 340 keV and 511 keV photons is to use a source which decays by the emission of 340 keV and 511 keV photons of known relative intensities.

A search revealed that no convenient source was available which decayed with the emission of two photons having the appropriate energies. However, the decay of Hf-181 approaches the ideal. The Hf-181 decay scheme is shown in Figure 8. The intensity of the 482 keV photon relative to the 346 keV photon was measured by Alexander and Ryde\(^1\) as $(204.0 \pm 0.5) / (142.1 \pm 1.2)$. It was necessary to extrapolate from the

Figure 8. Decay scheme of $^{181}$Hf (energies in kev).
482/346 ratio to the 511/340 ratio.

A plexiglass container, shaped after the aluminum sample so that solid angle effects would be the same in both cases, was designed to hold the Hf-181 which was in solution. The container was thin-walled so that the photons would have a minimum attenuation. The Na-22 source was removed and the Hf-181 "disc" was placed in the same position as the aluminum sample, and counting runs were taken with each detector. The complex spectrum of Hf-181, less background is shown in Figure 9.

The lower energy peak contains a contribution from the Compton edge of the 482 keV photon. In order to separate the counts due to the two different photons, a monoenergetic photon spectrum (see Figure 7) was fitted\(^1\), using a 1620 computer, to the 482 keV peak. The fitted peak was then subtracted out of the complex spectrum leaving only the spectrum of the 346 keV photon in the area of interest.

\(^1\)See Appendix I
Figure 9. Typical Hf$^{181}$ Pulse Height Spectrum.
number of counts under each peak is proportional to the intensity of the source, the relative intensity of the two gamma rays, the solid angle subtended by the detector, and the efficiencies of the detector for photons of those energies. The ratio of the numbers of counts under each peak is then proportional to the relative intensity of the two photons and to the relative efficiency of the detectors for photons of those energies.

The number of counts under each peak was determined using two different methods. The first method consisted of simply summing the number of counts in the channels under each peak. The second method consisted of fitting a normal curve, in the least squares sense, to the top 1/3, top 1/2, and top 2/3 of each peak. The area under the normal curves was then computed. The various values obtained agreed to within 5%. The ratio of the efficiencies 482/346 was then determined. It was, however, necessary to extrapolate from the 482/346 ratio to 511/340.

1See Appendix II
2See Appendix III
Another method was also used to measure the ratio of the efficiencies of the detectors for 511 keV and 340 keV photons. Disc shaped sources of Cs-137 and Ba-133, calibrated to $\pm 3\%$ by Tracerlab, were used to find the efficiencies of the detectors for 662 keV and 357 keV photons. Disc shaped sources approximating the aluminum sample were used so that solid angle effects would be the same as in the case of the aluminum and Hf-181. The principle activity was $8.99 \times 10^5$ photons per minute for the 662 keV photon from Cs-137 and $5.35 \times 10^5$ photons per minute for the 357 keV photon from Ba-133. Counting runs were taken with both sources and the spectra obtained are shown in Figures 10 and 11. The number of counts under each peak was then determined in the same manner as with the Hf-181. The absolute efficiency of each detector for both energies was determined. When plotted on log-log paper, the photopeak efficiency versus energy is a straight line. The results of the two present

---

Figure 10. Typical Ba$^{133}$ Pulse Height Spectrum.
Figure 11. Typical Cs$^{137}$ Pulse Height Spectrum.
measurements are shown in Figure 12. The values of the efficiency for 346 keV, 482 keV, 340 keV, and 511 keV were then determined from the graph. The values for the ratio of the efficiencies thus obtained are in good agreement with the values obtained by the previous method, as reported in Table 1. The listed uncertainties are due to the uncertainty in determining the number of counts under each peak, the uncertainty in the intensity of the 482 keV photon relative to the 346 keV photon of Hf-181, and the uncertainty in the calibration of the Cs-137 and Ba-133.

The values for the efficiencies obtained using the Ba-133 and the Cs-137 data should be corrected for attenuation in the sample. Since details of the source preparation are unknown, the effect of attenuation can only be estimated. The largest correction estimated is approximately 5%. Thus, since the 340 keV photon is attenuated more than the 511 keV photon, any correction made to the ratio of the efficiencies should bring the values into closer agreement with the Hf-181 results which were used in our measurement.
Figure 12. Intrinsic Photopeak Efficiency versus Energy of Photons as determined by the Cs-137 and Ba-133 data.
### Ratio of Intrinsic Photopeak Efficiency 482keV/346keV

<table>
<thead>
<tr>
<th>Source</th>
<th>Detector 1</th>
<th>Detector 2</th>
<th>Detector 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ba-133 and Cs-137</td>
<td>0.628±0.037</td>
<td>0.612±0.036</td>
<td>0.622±0.037</td>
</tr>
<tr>
<td>Hf-181</td>
<td>0.621±0.028</td>
<td>0.606±0.027</td>
<td>0.613±0.027</td>
</tr>
<tr>
<td>Theory*</td>
<td>0.630±0.040</td>
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</tr>
</tbody>
</table>

### Ratio of Intrinsic Photopeak Efficiency 511keV/340keV

<table>
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<th>Detector 1</th>
<th>Detector 2</th>
<th>Detector 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ba-133 and Cs-137</td>
<td>0.570±0.034</td>
<td>0.560±0.033</td>
<td>0.569±0.034</td>
</tr>
<tr>
<td>Hf-181</td>
<td>0.554±0.024</td>
<td>0.535±0.024</td>
<td>0.543±0.024</td>
</tr>
<tr>
<td>Theory*</td>
<td>0.542±0.040</td>
<td></td>
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</tbody>
</table>

Table 1. Ratio of Intrinsic Photopeak Efficiencies.

Attenuation

When a positron annihilates in the sample, the resulting photons must travel through some thickness of aluminum before being detected, hence some attenuation will occur. Since there is appreciable Compton scattering of the primary photons at angles larger than the $7\frac{1}{2}$ degrees subtended by each detector, the attenuation coefficient, and not the absorption coefficient, was used. The probability of attenuation of any photon is dependent upon the energy of the photon. The higher the energy of the photon, the more likely it is the photon will pass through the sample without interacting.

If in the case of two or three-photon annihilation each of the photons is considered as having to pass through some average thickness of the sample, the attenuation can be calculated. First all photons that exit through the sides of the sample are considered, then those which pass

---

through the top or bottom, headed toward the detectors.

We assumed that all annihilations occur in a "most probable" plane in the sample in order to avoid integrating over the sample thickness. The probability that a positron, emitted from a point source located at a distance \( H \) below the sample will annihilate with an electron in the sample at a point \((r, \theta)\) is

\[
\frac{r \, dr \, d\theta}{\pi b^2 (1 + (\frac{r}{H})^2)^{3/2}}
\]

where \( b \) is the radius of the sample. This can be expanded in \((\frac{r}{H})\) to

\[
\frac{r \, dr \, d\theta}{\pi b^2} (1 - \frac{3}{2} (\frac{r}{H})^2 + \frac{15}{8} (\frac{r}{H})^3 - \cdots \text{ terms < 1\%})
\]

The probability that a photon heads out of the sample in a solid angle subtended by an area on the edge is just the ratio of that solid angle to the total solid angle subtended by the edge. It is given by
The probability that a photon heads out in a solid angle subtended by an area on the top or bottom is the ratio of that solid angle to the total solid angle subtended by the top and bottom. It is given by

\[
\frac{dz \frac{d\phi}{d^3}}{\int_0^{2\pi} \int_{-\pi}^{\pi} \frac{br \cos(\phi - \theta) - b^2}{d^3} \, dz \, d\phi}
\]

where \(a\) is the constant \(z\) coordinate and \(d\) is the distance the photon must travel in the sample. In the case where the photon heads out the edge

\[
d = (z^2 + b^2 + r^2 - 2br \cos(\phi - \theta))^{\frac{1}{2}}.
\]
In the case where the photon exits through the top or bottom

\[ d = (a^2 + r^2 + a \cot^2 \alpha - 2ra \cot \alpha \cos(\phi - \theta))^\frac{1}{8}. \]

The probability that the photon gets out of the sample is \( e^{-\mu d} \), where \( \mu \) is the linear attenuation coefficient for the photon of interest. Thus the probability of attenuation of any photon that leaves the sample through an edge will be

\[
\iint_{r \leq 2b} \frac{r^2(r^2 + 15r^2 + 6)}{6(r^2 + 4)} (r \cos(\phi - \theta) - b) e^{-\mu d(r)} \, dr \, d\eta \, d\phi \, dz.
\]

\[
\pi b^2 \int_{r \leq d(r)} \frac{r^2(r^2 + 15r^2 + 6)}{6(r^2 + 4)} (r \cos(\phi - \theta) - b) \, dz \, d\phi.
\]

It then follows that the probability of attenuation of any photon that leaves through the top or bottom will be

\[
\iint_{r \leq 2b} \frac{r^2(r^2 + 15r^2 + 6)}{6(r^2 + 4)} (r^2 - d^2 - a^2 \cos^2 \alpha) e^{-\mu d(r)} \, dr \, d\phi \, d\eta \, d\phi \, dz.
\]

\[
\pi b^2 \int_{r \leq d(r)} \frac{r^2(r^2 + 15r^2 + 6)}{6(r^2 + 4)} (r^2 - d^2 - a^2 \cos^2 \alpha) \, dz \, d\phi \, d\eta \, d\phi.
\]
These integrals have been evaluated with the most helpful assistance of Dr. M. Soga, and Dr. J. McCully, of Western Michigan University.

Since in the case of three-photon annihilation each of the photons is assumed to have an average attenuation $A(340)$, the correction factor is $(A(340))^3$. In the two-photon case the correction is $(A(511))^2$. The probability of attenuation of any photon leaving the target through an edge, and the probability of attenuation of any photon leaving through the top or bottom of the target were averaged. The results are reported in Table 2 along with the attenuation coefficients used, the total attenuation correction $A_{123}/A_{12} = (A(340))^3/(A(511))^2$, and the estimated uncertainty.
Table 2. Attenuation Coefficients.

<table>
<thead>
<tr>
<th>μ₀</th>
<th>Energy</th>
<th>A(E)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.257/cm</td>
<td>340 keV</td>
<td>0.815±0.040*</td>
</tr>
<tr>
<td>0.227/cm</td>
<td>511 keV</td>
<td>0.883±0.044*</td>
</tr>
</tbody>
</table>

\[ \frac{(A(340))^3}{(A(511))^2} = 0.698 ± 0.076* \]

*The deviations in the attenuation coefficients are due to the uncertainty in reading the values from the graph.

Three-Photon Coincidence Rate

The three-photon measurement consisted of several counting runs taken consecutively. The average counting rate is given by

$$\langle n \rangle = \sum_i \frac{T_i}{T} (N_i/T_i),$$

where $N_i/T_i$ is the rate during the $i$\textsuperscript{th} counting run, and $T$ is the total time of all the counting runs. The standard deviation in $n$ is $S_n$, where

$$S_n = \sqrt{\frac{\sum_i (N_i/T_i - \langle n \rangle)^2}{i_{\text{max}} - 1}}.$$ 

High precision measurements require observation of a large number of counts, which involves long observation times if the counting rates are low. In Table 3 the measured coincidence rate, together with the uncertainty in the measurement, the counts per run, and the time per run are reported. The times were recorded on an electric clock and a laboratory timer. The time read by the electric clock was regularly compared with the dial-the-time service.
Table 3. Summary of Three Photon Coincidence Rates.

<table>
<thead>
<tr>
<th>Run</th>
<th>Time (min.)</th>
<th>Counts</th>
<th>Rate/Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2299</td>
<td>1250</td>
<td>32.6 ± 1.0</td>
</tr>
<tr>
<td>2</td>
<td>1171</td>
<td>641</td>
<td>32.8 ± 1.4</td>
</tr>
<tr>
<td>3</td>
<td>352</td>
<td>200</td>
<td>34.1 ± 2.6</td>
</tr>
<tr>
<td>4</td>
<td>1123</td>
<td>646</td>
<td>34.5 ± 1.4</td>
</tr>
<tr>
<td>5</td>
<td>2480</td>
<td>1307</td>
<td>31.6 ± 0.9</td>
</tr>
<tr>
<td>6</td>
<td>1698</td>
<td>895</td>
<td>31.6 ± 0.9</td>
</tr>
<tr>
<td>7</td>
<td>935</td>
<td>471</td>
<td>30.2 ± 1.4</td>
</tr>
<tr>
<td>8</td>
<td>1056</td>
<td>597</td>
<td>33.9 ± 1.4</td>
</tr>
<tr>
<td>9</td>
<td>1368</td>
<td>743</td>
<td>32.6 ± 1.3</td>
</tr>
<tr>
<td>10</td>
<td>900</td>
<td>483</td>
<td>32.2 ± 1.5</td>
</tr>
<tr>
<td>11</td>
<td>586</td>
<td>288</td>
<td>29.5 ± 0.9</td>
</tr>
</tbody>
</table>

Average Rate per Hour = 32.4 ± 1.0
of the Michigan Bell Telephone System. The
time intervals were measured with such high
precision that timing errors may be neglected.
The final value for the three-photon coincidence
rate was $32.4 \pm 1.0$ counts per hour.
Three-Photon Chance Coincidence Rate

The experimental set-up for the three-photon chance coincidence measurement consisted of four different arrangements. First it was possible to get a chance coincidence by having a stray photon detected in detector one, while photons in true coincidence are detected by detectors two and three. The second possibility was to get a stray photon detected in detector two, with photons in true coincidence detected by detectors one and three. It was also possible to detect a stray photon in detector three with photons in true coincidence being detected by detectors one and two. Lastly, one could have stray photons detected by all three detectors.

The first three stray counting runs were made by alternately setting the internal delay in the single channel analyser of one of the detectors to $1\, \mu$ second, its maximum setting. This insured that a true coincidence could not trigger the coincidence circuit. In the last case, the delays were set at 0 for detector one, 0.5 $\mu$seconds for detector two and 1 $\mu$second
for detector three, so that it was impossible for a true coincidence to be counted. The final value for the real chance rate is the sum of the chance rates of each set-up. It is $7.0 \pm 0.6$ counts per hour. In Table 4 the measured chance rates, the counts per run, the time per run in minutes, the uncertainty in the measurement, and the type of experimental arrangement are reported.

The true three-photon coincidence rate is the real rate less the chance rate, or $25.4 \pm 1.1$ counts per hour.
<table>
<thead>
<tr>
<th>Run</th>
<th>Time</th>
<th>Counts</th>
<th>Rate</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1440</td>
<td>57</td>
<td>2.4±0.4</td>
<td>(det.1) at 1μsec.</td>
</tr>
<tr>
<td>2</td>
<td>791</td>
<td>21</td>
<td>1.6±0.4</td>
<td>(det.1) at 1μsec.</td>
</tr>
<tr>
<td>3</td>
<td>1530</td>
<td>45</td>
<td>1.8±0.3</td>
<td>(det.2) at 1μsec.</td>
</tr>
<tr>
<td>4</td>
<td>633</td>
<td>24</td>
<td>2.3±0.5</td>
<td>(det.2) at 1μsec.</td>
</tr>
<tr>
<td>5</td>
<td>1500</td>
<td>50</td>
<td>2.0±0.3</td>
<td>(det.3) at 1μsec.</td>
</tr>
<tr>
<td>6</td>
<td>660</td>
<td>20</td>
<td>1.8±0.4</td>
<td>(det.3) at 1μsec.</td>
</tr>
<tr>
<td>7</td>
<td>965</td>
<td>32</td>
<td>2.0±0.3</td>
<td>(det.3) at 1μsec.</td>
</tr>
<tr>
<td>8</td>
<td>1103</td>
<td>20</td>
<td>1.1±0.3</td>
<td>delays 0.0-0.5-1.0μsec.</td>
</tr>
<tr>
<td>9</td>
<td>700</td>
<td>10</td>
<td>0.9±0.1</td>
<td>delays 0.0-0.5-1.0μsec.</td>
</tr>
</tbody>
</table>

Average Rate = 7.0 ± 0.6 counts/hour

Table 4. Summary of Three Photon Chance Rates.
Two-Photon Coincidence Rates

The two-photon coincidence rates were measured using detectors one and two, detectors one and three, and detectors two and three. The chance coincidence rate was obtained by either inserting an electronic delay into one of the single channel analyzers (as above), or by moving one detector 30 degrees off the coincidence axis. In Table 5 the singles rates, the coincidence rates, the calculated chance rates and both measured chance rates are reported. The disagreement between the measured chance rates is not as yet understood so an average value was used in the present work.
Table 5. Two Photon Coincidence Data.

* \( N = N_1 N_2 (2\gamma) \)
Consistency of Data

One may use the singles rates together with the two-photon coincidence rates and efficiencies to check the consistency of this data. The true two-photon rate for the present set-up is

\[ N_t = N \omega_{ij} \varepsilon_i \varepsilon_j \]

where \( N_t \) is the true two-photon rate, \( \omega_{ij} \) is the solid angle subtended by detectors \( i \) and \( j \), \( \varepsilon_i \) and \( \varepsilon_j \) are the intrinsic photopeak efficiencies of the \( i^{th} \) and \( j^{th} \) detectors for 511 keV photons, and \( N \) is the total number of 511 keV photons emitted per second. For the present work, \( \omega_{12} = \omega_{23} = \omega_{31} \) within experimental uncertainty. The singles rate for the \( i^{th} \) detector is \( N_i = N \omega_i \varepsilon_i \). Thus the ratio of the singles rates from two different detectors is proportional to the ratio of the efficiencies of the detectors for 511 keV photons. In a similar manner the ratio of the coincidence rates of the different pairs of detectors is also proportional to the ratio of the efficiencies of the detectors for 511 keV photons.

In Table 6 a comparison of the ratios of the singles rates, the ratios of the coincidence
### From Singles Rates

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Efficiency Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{N_1}{N_2} )</td>
<td>( \frac{\epsilon_1(511)}{\epsilon_2(511)} ) = 0.91 ± 0.02</td>
</tr>
<tr>
<td>( \frac{N_1}{N_3} )</td>
<td>( \frac{\epsilon_1(511)}{\epsilon_3(511)} ) = 1.01 ± 0.02</td>
</tr>
<tr>
<td>( \frac{N_2}{N_3} )</td>
<td>( \frac{\epsilon_2(511)}{\epsilon_3(511)} ) = 1.11 ± 0.02</td>
</tr>
</tbody>
</table>

### From the Absolute Efficiencies

<table>
<thead>
<tr>
<th>Efficiency Ratio</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\epsilon_1(511)}{\epsilon_2(511)} )</td>
<td>( \frac{\epsilon_1(511)}{\epsilon_2(511)} ) = 0.95 ± 0.03</td>
</tr>
<tr>
<td>( \frac{\epsilon_1(511)}{\epsilon_3(511)} )</td>
<td>( \frac{\epsilon_1(511)}{\epsilon_3(511)} ) = 1.04 ± 0.04</td>
</tr>
<tr>
<td>( \frac{\epsilon_2(511)}{\epsilon_3(511)} )</td>
<td>( \frac{\epsilon_2(511)}{\epsilon_3(511)} ) = 1.06 ± 0.04</td>
</tr>
</tbody>
</table>

### From Two Photon Coincidence Rates

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Efficiency Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{N_{13}}{N_{23}} )</td>
<td>( \frac{\epsilon_1(511)}{\epsilon_2(511)} ) = 0.93 ± 0.02</td>
</tr>
<tr>
<td>( \frac{N_{12}}{N_{23}} )</td>
<td>( \frac{\epsilon_1(511)}{\epsilon_3(511)} ) = 1.01 ± 0.02</td>
</tr>
<tr>
<td>( \frac{N_{12}}{N_{13}} )</td>
<td>( \frac{\epsilon_2(511)}{\epsilon_3(511)} ) = 1.09 ± 0.02</td>
</tr>
</tbody>
</table>

Table 6. A comparison of the Ratio of the Efficiencies.
rates, and the ratios of the absolute efficiencies are reported. As can be seen, the results are in excellent agreement. The uncertainties listed are due to counting statistics.

The present geometry allows the calculation of the ratio of the cross sections to be made three different ways:

\[
\frac{\sigma_2}{\sigma_3} = \frac{N(12) \epsilon_1(340) \epsilon_2(340) \epsilon_3(340) c_{123} A_{123}}{N_3 \epsilon_1(511) \epsilon_2(511) c_{12} A_{12}}
\]

\[
\frac{\sigma_2}{\sigma_3} = \frac{N(13) \epsilon_1(340) \epsilon_2(340) \epsilon_3(340) c_{123} A_{123}}{N_3 \epsilon_1(511) \epsilon_2(511) c_{12} A_{12}}
\]

\[
\frac{\sigma_2}{\sigma_3} = \frac{N(23) \epsilon_2(340) \epsilon_3(340) \epsilon_1(340) c_{123} A_{123}}{N_3 \epsilon_2(511) \epsilon_3(511) c_{12} A_{12}}
\]
Approximation of $C_{123}/C_{12}$ and Results

From the preceding results all of the information needed to evaluate $\sigma_2/\sigma_3$ can be determined except $C_{123}/C_{12}$. A comparison of the three values $\sigma_2/\sigma_3$ in terms of $C_{123}/C_{12}$ follows;

\[
\frac{\sigma_2}{\sigma_3}(12) = (4.47 \pm .41) \times 10^4 \frac{C_{123}}{C_{12}},
\]

\[
\frac{\sigma_2}{\sigma_3}(13) = (4.35 \pm .39) \times 10^4 \frac{C_{123}}{C_{12}},
\]

\[
\frac{\sigma_2}{\sigma_3}(23) = (4.74 \pm .42) \times 10^4 \frac{C_{123}}{C_{12}},
\]

average $\frac{\sigma_2}{\sigma_3} = (4.52 \pm .41) \times 10^4 \frac{C_{123}}{C_{12}}$.

$C_{123}/C_{12}$ is a function of the probability ($P_3$) that if a three-photon annihilation occurs the photons will be emitted at 120 degrees from each other, and the solid angles subtended by the detectors for three-photon and two-photon processes.

At the present time Dr. Rockmore has not calculated the final result. It is possible, however, to estimate the value of $C_{123}/C_{12}$ if the
correlation between $P_3$, the solid angles, and the position of annihilation in the sample is neglected. This may introduce an error as large as 15\% for both $C_{123}$ and $C_{12}$.

In the case of three-photon annihilation, the number of photons emitted equals $3N_3$, where $N_3$ is the number of three-photon annihilations in the target. In the case of two-photon annihilation, the number of photons emitted is $2N_2$, where $N_2$ equals the number of two-photon annihilations in the target. The probability that one photon with the correct energy strikes detector one, then another with the correct energy will hit detector two or three is $\pi a^2/2\pi rl$, where $a$ is the radius of a detector face, $r$ is half the distance from the center of one detector to the center of the next, and $l$ is the diameter of the detector face.

It is also possible to determine upper and lower limits for the value of $P_3$. A function which is proportional to the probability that one photon has energy between $E$ and $E + \Delta E$ is reported by Ore and Powell. A graph of this function normalized to the total number of
photons is shown in Figure 13. The probability that one photon has an energy within the present window width set on the single channel analyser is equal to one-third the ratio of the area under the curve between the window settings, to the entire area under the curve. The probability that a second photon having this energy will strike a detector face must be equal or greater than that of the first photon. The probability that a third photon strikes a detector face is then one. Hence, the lower limit of $P_3$ is

$$P_3 = \left(\frac{1}{3} \frac{\Delta A}{A}\right) \left(\frac{1}{3} \frac{\Delta A}{A}\right)(1),$$

and the upper limit is

$$P_3 = \left(\frac{1}{3} \frac{\Delta A}{A}\right)(1)(1).$$

The approximation for $C_{123}/C_{12}$ is equal to $3/2w_{123}P_3$. An analysis of this data in the previous equations gives us as an upper limit to the ratio of the cross sections $\sigma_2/\sigma_3 = 600 \pm 150$ and as a lower limit $\sigma_2/\sigma_3 = 46 \pm 12$. 

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Figure 13. Energy Spectrum of Individual Photons from the Three Photon Annihilation of an Electron and Positron - Ore and Powell.
CONCLUSION

The value for the ratio of the reaction cross sections measured in the present work seems to be in agreement with the theoretical value calculated by Ore and Powell, although final judgement must be delayed until $C_{123}/C_{12}$ is calculated.

If the assumption is made that the shape of the sample used in Basson's work was either an ideal point source surrounded by a spherical shell of aluminum thick enough to stop all positrons, or more reasonably a small disc sandwiched between two 0.030 inch thick aluminum foils 0.5 inches in diameter, the attenuation of the photons in his experiment can be estimated. Performing the appropriate calculations, the estimation for the attenuation correction is from 10% to 30%. Furthermore, since the actual shape of his source is unknown, there is a possibility that if he did have an extended source the correlation between the solid angles subtended by his detectors was also neglected, which as in the present work introduces a
correction as large as 15%. These corrections would change Basson's result for \( \sigma_2/\sigma_3 \) to a maximum value of 360 \(-\) 70, which would still be in agreement with the theory of Ore and Powell.

In conclusion, the geometry used in the present work is an improvement over the geometry used by Basson. In taking the attenuation of the photons and the correlation between the solid angles into account the present work seems to have reconfirmed the results of Ore and Powell for the ratio of two-photon to three-photon annihilation of a positron and electron.
BIBLIOGRAPHY

BOOKS


PERIODICALS


APPENDIX I

Stripping of the Hf-181 Spectrum

The following program was used to fit a monoenergetic photon spectrum to the 482 keV peak of Hf-181.

```
DIMENSION A(520), B(520), CRTN(520), SEP(520)
1 FORMAT(8F8.2)
READ1, (A(I), I=1, 512)
READ1, (B(I), I=1, 512)
SUM=0.0
DO4  I=345, 424
SUM=SUM+A(I)
4 CONTINUE
SUMA=SUM+A(I)
DO7  I=350, 424
SUMA=SUMA+B(I)
7 CONTINUE
DO22  I=1, 512
CRTN(I)=(SUM/SUMA)*B(I)
SEP(I)=A(I)-CRTN(I)
22 CONTINUE
PUNCH10, (I, SEP(I), I, CRTN(I), I=1, 512)
10 FORMAT(4HSEP(, I3, 2H)=, F10.2, 5HCRTN(, I3, 2H)=,
     F10.2)
PUNCH99, SUM, SUMA
99 FORMAT(F14.0/F14.0)
END
```

Hf-181 data
Monoenergetic photon spectrum
Fitted peak (482 keV)
346 keV spectrum

ZZFORX
APPENDIX II

Fitting of a Normal Curve

The following program was used to fit, in the least squares sense, a normal curve to the top $\frac{1}{3}$, top $\frac{1}{2}$, and top $\frac{2}{3}$ of the 346 keV and 482 keV peaks of Hf-181, the 357 keV peak from Ba-133, and the 662 keV peak from Cs-137. This program, number 938, is on file at the Western Michigan University Computer Center.
C

PROGRAM 938
DIMENSION X(65),Y(65)
DIMENSION T(65),T1(65),T2(65),T3(65)
100 READ 1,IX,N,X(1),U,W
1 FORMAT(2I2,3F6.0)
XN=N
DO 101 I=2,N
J=I-1
101 X(I)=X(J)+1.
READ 3,(Y(I),I=1,N)
3 FORMAT(13F6.0)
PUNCH 15,IX
15 FORMAT(/12HDATA SET NO. , I3)
XK1=W
S1=SQRT(2./(X(N)-U)**2*LOG(XK1/Y(N))
PUNCH 4,XK1,S1
4 FORMAT(3HK1=,E20.14,5X,3HS1=,E20.14)
XK=XK1
S=S1
5 DO 6 J=1,N
T(J)=EXP((X(J)-U)**2/(-2.)*S**2)
T1(J)=Y(J)-XK*T(J)
T2(J)=T(J)
6 T3(J)=-XK*(X(J)-U)**2*S*T(J)
SSY=0.
SSK=0.
SSS=0.
SPKS=0.
SPKY=0.
DO 7 J=1,N
SSY=SSY+T1(J)**2
SSK=SSK+T2(J)**2
SSS=SSS+T3(J)**2
SPKS=SPKS+T2(J)*T3(J)
SPKY=SPKY+T2(J)*T1(J)
7 SPSY=SPSY+T1(J)*T3(J)
D=SSK*SSS-SPKS**2
XK2=(SSS*SPKY-SPKS*SPSY)/D
S2=(SSK*SPSY-SPKS*SPKY)/D
XK2=XK+XK2
S2=S+S2
PUNCH 8,XK2,S2
8 FORMAT(3HK2=,E20.14,5X,3HS2=,E20.14)
IF(XK2/XK-.999)11,20,20
IF(S2/S-.999)12,20,20
IF(S2/S-1.001)10,20,20
IF(XK2/XK-1.001)9,20,20
62
20  XK=XK2
   S=S2
   GO TO 5
12  SSR=(SSY-XK2*SPKY-S2*SPSY)/(XN-2.)
   C22=(SSK*SSR)/D
   C12=(-SPKS*SSR)/D
   C11=(SSS*SSR)/D
   V22=SSK/D
   V11=SSS/D
   A=SQRT(SSR)
   B=SQRT(C11)
   C=SQRT(C12)
   D=SQRT(C22)
   PUNCH 13,A,B,C,D
13  FORMAT(16HSTANDARD ERRORS ,4F14.6)
    Z=XK2/S2
    G=(2.576**2*C22)/(S2**2)
    H=V11-V12**2/V22
    H1=SQRT(V11-2.*Z*V12+Z**2*V22-G*H)
    ZL=(Z-G*V12/V22-2.576*A*H1/S2)/(1.-G)
    ZU=(Z-G*V12/V22+2.576*A*H1/S2)/(1.-G)
    SR6=SQRT(6.2832)
    AE=SR6*Z
    AL1=SR6*ZL
    AL2=SR6*ZU
   PUNCH 14,AE,AL1,AL2
14  FORMAT(14HAREA ESTIMATE=E20.14/12HAREA LIMITS,
            E20.14,2X,E20.14)
   GO TO 100
END
APPENDIX III

Extrapolation from the 482/346 ratio to 511/340

From the Hf-181 data

\[ \frac{\varepsilon_1(482)}{\varepsilon_1(346)} = 0.621, \quad \frac{\varepsilon_2(482)}{\varepsilon_2(346)} = 0.606, \quad \frac{\varepsilon_3(482)}{\varepsilon_3(346)} = 0.613. \]

Furthermore, it can be shown that

\[ \log \varepsilon(482) - \log \varepsilon(346) = m (\log 482 - \log 346) \]

or,

\[ \log \frac{\varepsilon(482)}{\varepsilon(346)} = m \log \frac{482}{346}, \]

where \( m \) is the slope of the efficiency versus energy graph.

For detector 1

\[ \log(0.621) = m \log(1.39) \]

\[ m = -1.45. \]

For detector 2

\[ \log(0.606) = m \log(1.39) \]

\[ m = -1.53. \]
For detector 3

\[ \log(.613) = m \log(1.39) \]

\[ m = -1.50 \]

Since the efficiencies for 511 keV and 340 keV also lie on the graph.

\[ \log \varepsilon(511) - \log \varepsilon(340) = m(\log 511 - \log 340) \]

or,

\[ \log \frac{\varepsilon(511)}{\varepsilon(340)} = m \log \frac{511}{340} \]

Thus, for detector 1

\[ \log \frac{\varepsilon(511)}{\varepsilon(340)} = -1.45 \log(1.5) \]

\[ \frac{\varepsilon_1(511)}{\varepsilon_1(340)} = .554. \]

For detector 2

\[ \log \frac{\varepsilon(511)}{\varepsilon(340)} = -1.53 \log(1.5) \]

\[ \frac{\varepsilon_2(511)}{\varepsilon_2(340)} = .535. \]
For detector 3

\[
\log \frac{\varepsilon(511)}{\varepsilon(340)} = -1.50 \log(1.5)
\]

\[
\frac{\varepsilon_3(511)}{\varepsilon_3(340)} = 0.543
\]