4-13-2018

Impacts of Representation on Increasing Flexibility in Understanding Number Composition

Abigail Carr
Western Michigan University, abby.carr43@gmail.com

Impacts of Representation on Increasing Flexibility in Understanding Number Composition

Abigail Carr

Honors College Thesis

2018

Introduction

Nearing the end of my studies in Elementary Education, I realized that I wanted to further my understanding of elementary mathematics by making it the subject of research for my Honors Thesis. The focus of my research was to discover the impacts of using representations on understanding the flexibility of number composition with focus groups of first and second grade children. My interest on this topic came from various readings recommended by my MATH 2650 and MATH 3520 instructor, and activities that were designed to help with the flexibility of number composition. The focus of my research was to have the children develop a better sense of number composition through various activities that involved careful teacher questioning and varied concrete representations to further their thinking and understanding.

As a future educator, I will be responsible for teaching children many new mathematical concepts and ideas, while making sure they have a deep understanding of what is being taught, rather than relying on rote memorization. A component of this work will be to help children develop fluency in mathematics. Fluency is described as "the ability to apply procedures accurately, efficiently, flexibly, and appropriately" (National Research Council, 2001). I wanted to focus my research on methods to develop fluency in mathematics with young children, specifically flexibility, because many children approach problems in only one way, when it might not be the most efficient or make the most sense.

My MATH 3520 professor, Gina Kling, approached me halfway through the course and asked if I would be interested in working on my Honors Thesis with her. I always enjoyed Mrs. Kling's classes and she challenged me to think deeper about elementary mathematics. I also knew that Mrs. Kling would be a great mentor for my Honors Thesis and challenge me to expand on what I knew about mathematical fluency. I accepted Mrs. Kling's offer to be my mentor for my Honors Thesis and we started working on a research study to answer the question: *After receiving small-group instruction on place value and the composition of numbers, do activities utilizing place value representations such as base-ten blocks increase flexibility in understanding number composition?* Mrs. Kling went down to part time instructor to complete other work and because of this, she could no longer be the sole chair of my Honors Thesis. Dr. Christine Browning agreed to step in as the co-chair for my thesis, although Mrs. Kling would still complete the research with me.

We selected St. Monica Catholic School, in Kalamazoo, Michigan as our research site. We chose one first grade classroom and one second grade classroom as the populations of students we would invite to participate in our study. We selected all of the eligible students from each grade to be a part of the study, a total of 8 students per grade. We worked with each grade after they had been taught the prerequisite content by their teachers so that no completely new information would be presented. After completing individual pre-assessment interviews on their current understanding of number composition, we divided the students into two groups. We met with each group four times and followed our group meetings with individual post-assessment interviews. Overall, there appeared to be improvement in the students' flexibility in understanding number composition. In the sections to follow, I will describe the readings that influenced my work, the methodology of the study, and the results of the study in greater detail.

Literature Review

The base-ten number system is the basis for all mathematics in the modern world. The base-ten system is relied upon by people in their everyday lives and is also used in commerce (National Research Council, 2001). In the base-ten system, there are 10 digits that can represent every quantity and the placement in a sequence of digits tells the value of the digit. “The positions of digits in numbers determine what they represent and which size group they count. This is the major organization principle of place-value numeration and is central for determining number sense” (Van de Walle, Karp, and Bay-Williams, 2013, 192). Place-value is one of the foundational concepts that children must learn in order to understand all basic operations in mathematics. The work that students do in counting and cardinality, addition, subtraction, multiplication, and division are all intertwined with the base-ten system. (Common Core Standards Writing Team, 2011). The Common Core State Standards (NGA and CCSSO, 2010) for Grades 1 and 2 both call for students to be able to “use place value understanding and properties of operations to add and subtract” (NGA and CCSSO, 2010). With the base-ten system being used so prevalently in today’s society, it is imperative that children have a solid understanding of place-value.

Language associated with place-value also has an effect on children’s ability to understand the concepts behind place-value. The language that we use to name numbers does not make the base-ten meanings of the numbers evident (Common Core Standards Writing Team, 2011). In contrast, in many other languages, the base-ten meaning of numbers is evident in the number names. For example, the Chinese name for 11 is “shi yi” which translates to “ten one” and the naming system goes on with “ten two,” “ten three,” etc. This language makes base-ten meaning very explicit, for it mirrors the structure of teen numbers (11, 12, 13, etc.) as 10 and 1, 10 and 2, 10 and 3, etc.. In English, the naming system in the teens is very unpredictable, especially with the numbers eleven and twelve, whose names do nothing to reveal their relationship to 10. There is more consistency in the remaining teen numbers (thir*teen*, four*teen*, fif*teen*, etc.), but compared to the Chinese names, the structure is obscured by English phonetic modifications (National Research Council, 2001). The words for the decades such as twenty, thirty, etc. also do not clearly express their relation to 10. The "-ty" in these names signifies ten, but this meaning is not made clear (Common Core Standards Writing Team, 2011). With the hidden base-ten meanings in the number names, children struggle to understand the connection to place-value. It has been shown through research on children's acquisition of number names, that they "fail to understand the base-10 structure underlying larger number names" (National Research Council, 2001). However, this effect can be mitigated by saying numbers in terms of tens and ones, known as using "Base-ten language". For example, the number 43 would be said as four tens and three ones (Van de Walle et al., 2013). This verbalization of the tens and ones reinforces the concept of understanding ten as a unit, and can help students see the relationship between tens and ones and the digits that represent each number.

The beauty of the base-ten number system is how efficiently numbers can be represented; even though the quantities of numbers may be large, they are relatively easy to write because the position of the digit indicates its value. Yet while this system is versatile and simple, it is not obvious to young learners or easily learned (National Research Council, 2001). Many children are able to name the ones and tens place, but do not *understand* place value. Young children may mistake the 1 in 18 as just 1 instead of 10. Children who make these mistakes may not understand that the digit in the tens place shows how many groups of ten are present in the number (Van de Walle et al., 2013). The Common Core State Standards call for children in Grade 1 to be able to "Understand that the two digits of a two-digit number represent the amounts of tens and ones" (NGA and CCSSO, 2010). Using representations to make the relationship between 1s and 10s, and 10s and 100s more apparent, can help with understanding the meaning of place-value. Using and discussing representations helps young children to make connections among the mathematical ideas that they are learning through the different forms that are presented. Representations allow for a "deeper mathematical understanding and enhanced problem-solving skills" (National Council of Teachers of Mathematics, 2014, 24). Representations also solidify ideas by showing the concept in a new way; "using these different representations is like examining the concept through a variety of lenses, with each lens providing a different prospective that make the picture (concept) richer and deeper" (National Council of Teachers of Mathematics, 2014, 25). One representation that has been shown to be very beneficial to students learning the place-value system is base-ten blocks. These blocks are a proportional physical model that show ones, tens, and hundreds and allow learners to construct and see the relationship that lies between 10 ones and 1 ten and 10 tens and 1 hundred. "Physical models for base-ten concepts play a key role in helping students develop the idea of "a ten" as both a single entity and as a set of 10 units" (Van de Walle et al., 2013, 195). Base-ten blocks allow children to manipulate quantities and develop a better sense of ten as a unit and as a set of tens.

A common goal in teaching mathematics is helping students become fluent in mathematics. One of the components to becoming fluent in mathematics is for students to be able to think flexibly and to "demonstrate flexibility in the computational methods they choose" (Gojak, 2012). Understanding place-value can help children become more flexible in their thinking about numbers and the way that they solve computations. Students may learn to decompose numbers into different arrangements of tens and ones by thinking flexibly about the composition of numbers and the base-ten place-value system. For example, the number 32 can be decomposed into 3 tens and 2 ones, but also into 2 tens and 12 ones, 1 ten and 22 ones, and finally 0 tens and 32 ones. Flexible thinkers recognize this, where as many, less flexible children may only see 32 as made of 3 tens and 2 ones. This observation was one of the driving forces behind the research study described in the next section.

Methodology

The purpose of this research was to devise a project that might (1) increase the researchers' understandings of how children comprehend number composition; (2) explore different activities designed to enhance children's understanding of place-value; and (3) evaluate the effectiveness of the work on increasing children's abilities to understand the flexibility of number composition using representations. This originated from the research question: *After receiving small-group instruction on place value and the composition of numbers, do activities utilizing place value representations such as base-ten blocks increase flexibility in understanding number composition?*

To begin this work, the researchers invited a class of second grade students and a class of first grade students at St. Monica Catholic School to participate in the study. Students who wanted to be a part of the study were chosen to participate. A total of seven second grade children and eight first grade children participated in the study. The children were assigned a number, 1-7 for second grade and 1-8 for first grade, in order to protect the identity of the participants. The study began after each grade had covered the prerequisite content, so that no completely new material would be presented. The second grade students participated in February 2017, and then work continued with the first graders in April 2017. Each student completed a pre-interview assessment of two questions pertaining to place value and composing numbers in multiple ways that Ms. Carr administered with Mrs. Kling assisting. A sample of each of the pre-interview assessments can be found in Appendix A. The students were prompted to explain their thinking on how they solved the place-value problem (Task 1) and were asked to use base-ten blocks to show all the ways they knew to represent a given number (Task 2). The purpose of this was to assess how each child was thinking about place-value and number composition in order to prepare subsequent small group activities that would elicit any misunderstandings and encourage discussion among the students. Pre- and post-test results were later informally compared to mark the growth each student made after multiple small group activities. The results of the pre-assessments were scored on the correctness of the answer given to each task (both tasks), how many different ways the student could represent the number (Task 2), and whether or not they used a system when making multiple representations (Task 2). When the student was able to correctly complete the first task, it was coded yellow, when they incorrectly completed the task, it was coded red, and when they were unable to complete the task, it was coded blue. The response from the student is also shown. For the second task the number of ways the student was able to represent the number was recorded and coded green if they used a system consistently, purple is they used a system for part of their response, and orange if they used no system. The number of ways they were able to make the number is also recorded. An example of the results can be seen below and a table of responses from all students can be found in the results section.

	Assessment Task	
Student Number	1	2
1	Showed 10 pennies	3 ways

After all pre-assessments were completed, the students were placed into two groups of three or four students based on recommendations by the students' teacher for how the students would best work together. Each group met four times and various activities (Van de Walle, Lovin, Karp, & Bay-Williams, 2014 and Bell et al., 2016) were used by Ms. Carr to engage the students in thinking more deeply about number composition. These activities included Base-Ten Riddles, Finding Neighbors on the Hundreds Chart, and completing number-grid puzzles. During each meeting, the students were asked to explain how they knew different representations of a number still yielded the same number. The interviews were recorded with a video camera to allow the researchers to later examine more closely how students were thinking about the problems. A complete listing of the activities used for each group level can be found in Appendix B.

For second grade, Group 1 consisted of students #1, #4, and #7, and Group 2 consisted of students #2, #3, #5, and #6. For first grade, Group 1 consisted of students #1, #2, #3, and #4, and Group 2 consisted of students #5, #6, #7, and #8.

The first group meeting with the second grade students consisted of presenting the students with "base-ten riddles" (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Base-ten riddles require children to make a representation of a number using base-ten blocks to solve the riddle. The riddles range in difficulty and ask the students to use hundreds, tens, and ones, to compose the number. For example, a base-ten riddle presented was *I have 23 ones and 4 tens. Who am I?* Base-ten blocks are manipulatives consisting of cubes, longs, and flats used by students to help them understand place value and number sense. A cube is a single unit, a long is composed of 10 cubes, and a flat is composed of ten longs or a hundred cubes. An example of base ten blocks can be seen below.

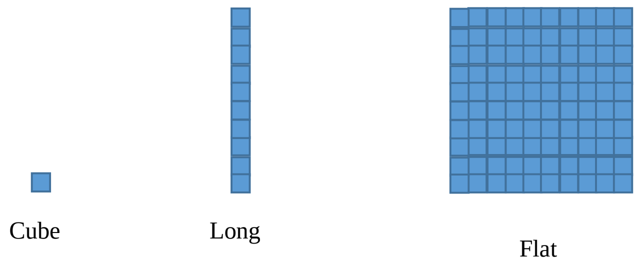

Figure 1. Images of base-ten blocks.

Ms. Carr began group meetings with the second grade focus groups by asking the students to solve the following riddle: *I have 23 ones and 4 tens. Who am I?* Each student was given a set of base-ten blocks and worked on their own to figure out the number. All students were able to correctly identify the number as 63. When solving this riddle, many of the students showed 6 longs and 3 cubes. Ms. Carr subsequently probed students as to why it works to show different representations of the same number. The students had a hard time answering this question and gave the response that it is easier to make the number that way and it would be a lot to use 23 cubes. The next riddle presented to the students was *I have 4 hundreds, 12 tens, and 6 ones. Who am I?*. After making their original representations, the students were asked to show 526 in a different way. After making their new representations, the students were asked how they knew their representations were equal. The students struggled to express how they knew they were equal and suggested they could count each representation to know they are equal. The students also responded that they were the same number.

On the second day of the second grade group meetings, Ms. Carr showed the students a hundreds chart with missing numbers (Van de Walle et al., 2014). A hundreds chart (see Figure 2) is an orderly way that the numbers 1 through 100 are shown and is a very common tool utilized in the early elementary grades. The numbers are in rows of 10, so that patterns among numbers that are related to Base-ten numeration can be explored. For example, if one starts at the number 43 on the hundreds chart, she can look down one row to find 53 or 10 more, up one row to find 33 or 10 less, to the right one column to find 44 or one more, and to the left one to find 42 or one less.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 2. The hundreds chart.

In their second small group meetings, students were presented with a hundreds chart in which some of the numbers were covered (See Figure 3). This task was designed to encourage students to use their knowledge about the structure of hundreds charts to solve the problem, as well as to increase their familiarity of place-value. The students in both groups were all able to identify the numbers that were missing based on the other numbers present on the hundreds chart. The participants used clues from the hundreds chart to find the numbers. For example, for one missing number, Student #1 noticed all the other numbers in the column end with 7 and all the other numbers in the row are in the 70s so the number is 77. This continued until all the missing numbers were identified.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46		48	49	50
51	52	53	54	55				59	60
61	62	63	64	65	66		68	69	70
71	72	73	74	75	76		78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 3. Finding Neighbors on the Hundreds Chart

Next students moved into a related number-grid activity (Bell et al, 2016) in which the participants were given pieces from a hundreds chart in which only one number was given. The students had to figure out which numbers belonged in the bolded boxes. An example is seen below.

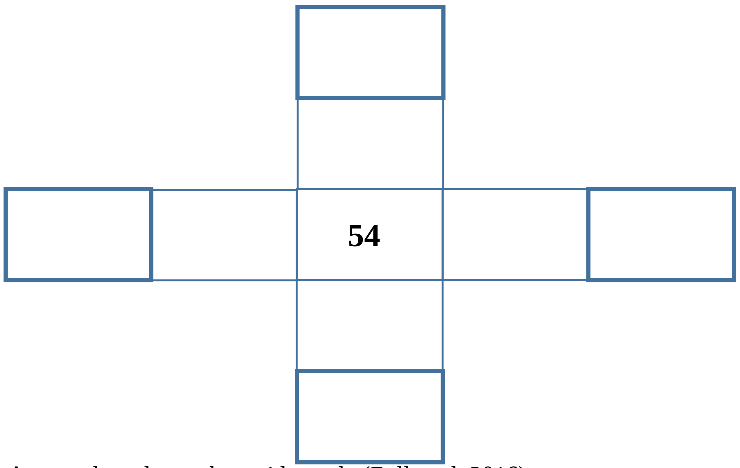

Figure 4. A second-grade number grid puzzle (Bell et al, 2016)

After the students completed this activity, they were asked to identify any patterns that they noticed. Student #1 in Group 1 and Student #6 in Group 2 noticed that the tens change when going up and down and Student #7 and Student #3 in Group 2 noticed that the the ones change when going across the rows. Students were asked to record any changes to their puzzles in a different color, and all samples were collected for later analysis

The final task of that meeting involved returning to base-ten riddles. The students were presented with the base-ten riddle: *I have 13 tens 2 hundreds and 21 ones. Who am I?* After everyone made their representations of the number 351, the students were asked to work together to make sure everyone had a different way to represent the number. The students were again asked how they could prove that all of the representations were equal without counting each representation, but again both groups struggled to answer the question.

On the third day of group meetings with second grade, we started with number-grid puzzles in the shape of a "t" and an "L" the numbers given were not centered in the grid which made the activity more difficult for the students because they have to figure out the intersecting number to find the rest instead of having that number given. An example of each grid is shown below.

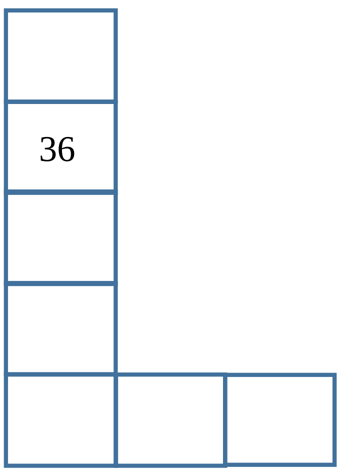

Figure 5. "L" shaped grid

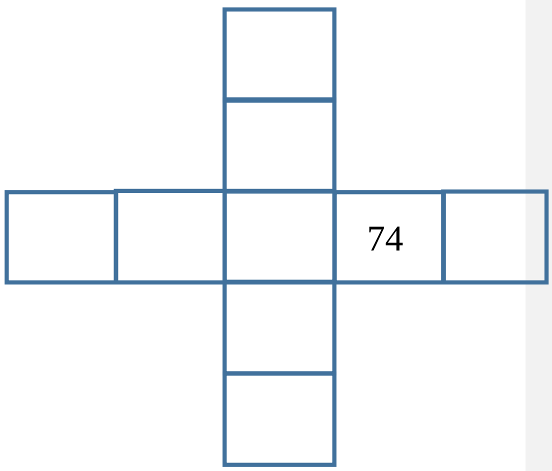

Figure 6. "t" shaped grid

Some of the students struggled filling in the missing numbers. When students presented different answers for their problems, the students were asked to explain how they solved the problem. Typically, during their explanation or another students' explanation, they were able to self-correct. Follow-up questions such as "What convinced you?" or "Why did you change your mind?" were used to help the student make sense of and articulate why they had changed their answer. After completing this task, the students were presented the number 233 represented by 2 flats, 3 longs, and 3 cubes and asked to change the representation by changing only one piece at a time. This was done so that the students could start to notice patterns and hopefully start to develop a system to flexibly change their own representations. The students were also asked how they could prove all of the representations they made were still the same number. Student #7 was the first to develop a way that could prove the numbers were equal. This was done by lining up 10 cubes next to a long and showing that they must be equal because they are the same size. The student said when trading a long for cubes "It's like subtracting a ten and adding a ten. It's still the same number." This student also did the same showing that 10 longs were the same as a flat by putting the 10 longs onto the flat and showing that they are the same.

The fourth and last day of second grade group meetings, the students were presented with the base-ten riddle: *I am 45. I have 25 ones. How many tens do I have?* The students were asked to solve the problems without base-ten blocks, but they would be available if they needed to use them. The students were all able to answer this question and understood that two of the tens were already accounted for in the 25 ones. The next riddle was much more challenging for the students. They were presented with: *I am 341. I have 22 tens. How many hundreds do I have?* The students struggled as to whether the correct answer was 1, 2, or 3 hundreds. After a discussion in both groups about how the 22 tens already makes 200, all students came to agreement that there is 1 hundred. These discussions in both groups were mostly between the students with prompts by Ms. Carr to explain their thinking and why their thinking was correct. Examples of prompts used were "Why do you think that?", "What about the 22 tens?", and "What do the rest of you think about what this student is saying?"

Many of the same activities used for second grade were adapted and used for the first grade group meetings two months later. One main difference was that the first grade tasks utilized two-digit numbers instead of three-digit numbers in order to make them grade-appropriate. The first day of group meetings started with asking the students to solve the base-ten riddle: *I have 23 ones and 4 tens, who am I?* in order to nudge students to start thinking flexibly about numbers. The students all made their representations of the number and were then asked to make it in a different way. Students #3 and #8 both made multiple representations rather than just one. Students #3 and #5 explained to their groups that they could take away one long and replace the long with 10 cubes to make the same number. The next task for the first group meeting was to fill in missing pieces on a hundreds chart (see Figure 3 or do we need a different figure?). Like their second-grade counterparts, the first graders didn't have any problems solving the missing numbers and used the surrounding numbers as clues.

For the second group meeting with first grade, additional base ten riddles were explored to increase fluency with representing numbers. The students were given the riddle: *I am 45. I have 25 ones. How many tens do I have?* In Group 1, students #1, #2, and #4 responded with the correct answer, but were not able to explain how they got their answers. Student #3 was able to explain how she got the correct answer. She made 45, which has 4 tens, and took 2 away because they were included in the 25 ones and was left with 2 tens. In the second group, the students also struggled with this riddle. Some students ignored the 25 ones because they didn't know how to include them into the problem. The second task for the day was finding the missing numbers in the "L" and "T" shape on a hundreds chart (See Figure 7)The children were able to do this easily and used the patterns from the chart to find the missing numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27		29	30
31	32	33	34	35				39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56				60
61	62	63	64	65	66	67		69	70
71	72	73	74	75	76	77		79	80
81	82	83	84	85	86	87		89	90
91	92	93	94	95	96	97	98	99	100

Figure 7. First-grade Finding Neighbors on a Hundreds Chart task(Bell et al, 2016)

To start the third day of group meetings with first grade, children were presented with the problem: *Another first grader made 63 with 6 cubes in one pile, and 3 in another. What do you think? Is that 63? Why or why not?* This task was used because many of the students were unable to correctly identify the "1" in "16" as 10 units on the pre-interview. The students were right away able to tell what was wrong with the representation. They explained that instead of the pile of 6 cubes, there needed to be 6 longs. Otherwise, it was only showing 9. After discussing this task, the students were given a number-grid puzzle as shown above in Figure 4. The students were able to complete the horizontal numbers accurately, but many were unable to fill in the correct vertical numbers. The students were confused as to whether the numbers would increase by 1 or 10 when moving vertically. After a discussion, the students all agreed that moving up and down the column means 10 more or 10 less.

The fourth and final group meetings with the first grade students started with the base-ten riddle: *I have 4 tens and 1 one. What number am I?* The students were all easily able to identify the number as 41. The students were then asked to make a new representation of the number 41, but that each representation had to be different than what the other students were showing. The students each explained why what they had made was equal to 41, and checked to make sure that every student had a different way to make the number. The second task was completing number grid puzzles as seen in Figures 4 and 5. The students again made some of the same mistakes as they had in the prior meeting. Students #1, #2, #4, and #8 all filled in their blanks increasing by 1 as they went down the columns instead of 10. The students all discussed how they came to their answers and came to an agreement that they should increase by ten when moving vertically down the column. They noticed the pattern that in the columns they all have the same number in the ones place, and the tens place increases by 1 as you move down.

After each grade group finished their last group meeting, individual post-interview assessments were administered. This post-test was the same as the pre-interview assessment and was used to measure the growth that each student made after participating in the group meetings.

Results and Conclusions

The results from the pre and post interviews can be seen in the following tables. The exact interview questions can be found in Appendix A.

Key:

Question 1: yellow = correct answer, red = incorrect answer, and blue = unable to answer question.

Question 2: green = used system consistently, purple = used a system part of the time, and orange = used no system.

Grade 2 Pre-Assessment	**Assessment Task**	
Student Number	1	2
1	Showed 10 pennies	3 ways
2	Showed 10 pennies	3 ways
3	Showed 1 penny	3 ways
4	Showed 10 pennies	4 ways
5	Showed 1 penny	4 ways
6	Showed 10 pennies	1 way
7	Showed 10 pennies	4 ways

Commented [a1]: I can't seem to get the tables to line up without formatting issues. Not sure if this is a big deal.

Grade 2 Post-Assessment	**Assessment Task**	
Student Number	1	2
1	Showed 10 pennies	3 ways
2	Showed 10 pennies	10 ways
3	Showed 10 pennies	10 ways
4	Showed 10 pennies	5 ways
5	Showed 1 Penny	4 ways
6	Showed 10 pennies	3 ways
7	Showed 10 pennies	12 ways

Grade 1 Pre-Assessment	**Assessment Task**	
Student Number	1	2
1	Showed 1 penny	1 way
2	Showed 1 penny	4 ways
3	Showed 10 pennies	6 ways
4	Showed 1 penny	2 ways
5	Showed 1 penny	3 ways
6	Showed 1 penny	3 ways
7	Unable to complete task	2 ways
8	Showed 1 penny	6 ways

Grade 1 Post-Assessment	**Assessment Task**	
Student Number	1	2
1	Showed 1 penny	3 ways
2	Showed 1 penny	6 ways
3	Showed 10 pennies	6 ways
4	Showed 1 penny	6 ways
5	Showed 1 penny	6 ways
6	Showed 1 penny	6 ways
7	Unable to complete task	6 ways
8	Showed 10 pennies	6 ways

Below is a comparison of the students' results from the pre and post interviews. The pre-interview results are shown first and the post-interview results and shown second.

Key:
Question 1: yellow = correct answer, red = incorrect answer, and blue = unable to answer question.
Question 2: green = used system consistently, purple = used a system part of the time, and orange = used no system.

Grade 2 Pre/Post-Assessment	**Assessment Task**	
Student Number	1	2
1	Showed 10 pennies	3 ways
	Showed 10 pennies	3 ways
	No change ~ correct	No change
2	Showed 10 pennies	3 ways
	Showed 10 pennies	10 ways
	No change ~ correct	+7
3	Showed 1 penny	3 ways
	Showed 10 Pennies	10 ways
	Changed to correct	+7
4	Showed 10 pennies	4 ways
	Showed 10 pennies	5 ways
	No change ~ correct	+1
5	Showed 1 penny	4 ways
	Showed 1 penny	4 ways
	No change ~ incorrect	No Change
6	Showed 10 pennies	1 way

	Showed 10 pennies	3 ways
	No change ~ correct	+2
7	Showed 10 pennies	4 ways
	Showed 10 pennies	12 ways
	No change ~ correct	+8

Grade 1 Pre/Post-Assessment	**Assessment Task**	
Student Number	1	2
1	Showed 1 penny	1 ways
	Showed 1 penny	3 ways
	No change ~ incorrect	+2
2	Showed 1 penny	3 ways
	Showed 1 penny	6 ways
	No change ~ incorrect	+3
3	Showed 10 pennies	6 ways
	Showed 10 pennies	6 ways
	No change ~ correct	No change ~ all possible ways shown
4	Showed 1 penny	2 ways
	Showed 1 penny	6 ways
	No change ~ incorrect	+4
5	Showed 1 penny	3 ways
	Showed 1 penny	6 ways
	No change ~ incorrect	+3

6	Showed 1 penny	3 way
	Showed 1 penny	6 ways
	No change ~ incorrect	+3
7	Unable to complete task	2 ways
	Unable to complete task	6 ways
	No change ~ incorrect	+4
8	Showed 1 penny	6 ways
	Showed 10 pennies	6 ways
	Changed to correct	No change ~ all possible ways shown

After comparing and analyzing the results from the pre-interview and post-interview assessments, the following observations can be made regarding the data. However, no formal statistical tests were conducted to determine if the differences between pre- and post-assessment were statistically significant, as this was beyond the scope of this project.

Increased flexibility when composing numbers

- In the second task on the assessment, the students were asked to make a number as many ways as possible. 11 of the 15 students in both first and second grades showed more ways to compose numbers (First grade students #1, 2, 4, 5, 6, 7; second grade students #2, 3, 4, 6, 7). Four of the students showed no growth, but two of these students had already shown all of the possible ways to compose the number they were asked to represent, so showing growth was not possible.
- When adding together all the second grade student responses on the pre-interview assessment for the second task, there were a total of 22 responses (S1: 3 ways + S2: 3 ways + S3: 3 ways + S4: 4 ways + S5: 4 ways + S6: 1 way + S7: 4 ways = 22 total ways) of ways to represent the number 204. On the post-interview assessment when adding together all of the second grade student responses, there were a total of 47 responses of ways to make the number 204 (S1: 3 ways + S2: 10 ways + S3: 10 ways + S4: 5 ways + S5: 4 ways + S6: 3 way + S7: 12 ways = 47 total ways). 25 more responses were given on the post-interview assessment than were given on the pre-interview assessment. 5 of the 7 second grade students were able to show more ways to make the number 204 and 2 students showed no growth by making the same number of ways on both the pre- and post-interview assessment.

- When adding together all of the first grade student responses of the pre-interview assessment for the second task, there were a total of 27 responses of ways to represent the number 52 (S1: 1 way + S2: 3 ways + S3: 6 ways + S4: 2 ways + S5: 3 ways + S6: 3 way + S7: 2 ways + S8 6 ways = 27 total ways) . On the post-interview assessment when adding together all of the first grade responses, there were a total of 45 responses of ways to make the number 52 (S1: 3 ways + S2: 6 ways + S3: 6 ways + S4: 6 ways + S5: 6 ways + S6: 6 way + S7: 6 ways + S8 6 ways = 45 total ways). 18 more responses were given on the post-interview assessment than were given on the pre-interview assessment. 6 of the 8 first grade students showed growth on the post assessment by providing more ways to make the number 52. The other 2 students had already made all of the possible ways to make 52, so there wasn't a way for them to show more responses. 7 of the 8 students were able to show all possible ways to compose the number 52.

More complex ways to compose numbers were used

- On the pre-assessment most students composed the number using the maximum number of rods and flats and the minimum number of cubes. For example, 204 was composed with 2 flats and 4 cubes, and 52 was composed with 5 longs and 2 cubes. For their second representation, many students composed the number using only cubes (e.g., 52 as 52 cubes). Several of the second grade students who were given the three-digit number 204 also broke down the flats into longs (e.g.,1 flat and 10 longs, and 20 longs). However, not many students in either grade attempted to break the longs into cubes on the pre-interview assessment. In contrast, on the post-interview assessment, four of the seven second grade students and all first grade students were more flexible in decomposing their numbers into more complex variations. The students used an assortment of both rods and cubes for first grade, and flats, longs, and cubes, for second grade. The students included the same combinations on the post-interview assessment as they did on the pre-interview assessment, but expanded the ways they composed their numbers. For example, Students 3, 4, and 7 composed the number 204 with 1 flat, 9 longs, and 14 cubes in second grade, and Students 2-8 composed the number 52 with 3 longs and 22 cubes.

More students used systems to compose the number as many ways as possible

- Many of the students on the pre-interview assessment hadn't developed a way to decompose and compose the number in a systematic way. By systematic way, we mean an orderly system to know if all possible ways to compose the number have been found. For example, trading in tens one at a time for 10 ones until all tens have been traded in. In second grade on the pre-interview assessment, no students had a system for composing numbers. On the post-interview assessment for second grade, one student used a system throughout the task, three used a system part of the time, and three students did not use a system. In first grade on the pre-interview assessment, two students used a system, and six students had no system for composing numbers. On the post-interview assessment for second grade, five students used a system throughout the task, two used a system part of the time, and one students did not use a system. The students that used a system were more efficient in composing numbers and were able to compose more combinations.

Little improvement in identifying the place-value of numbers in the tens-place.

- The first task on the assessment was designed to gauge students' abilities to accurately show the value of a number in the tens place. Ms. Carr pointed to the "1" in the number 16 and said "Show this part of the number with pennies." For this task, only a response of 10 was coded as correct, while a response of anything else, including 1, was coded as incorrect. Five of seven second grade students were able to correctly answer this question on the pre-interview assessment, and six of seven on the post-interview assessment. Most students already had a good understanding of the value of a digit in the tens place in this age group and only one student improved after the group meetings.
- One of eight first grade students were able to successfully answer that the digit in the tens place is equal to 10, while 6 other students answered incorrectly, and one student was unable to complete the task on the pre-assessment (no response). On the post assessment, two of eight students could successfully complete the task, while 5 students answered incorrectly and one students was unable to complete the task.
- During the group meetings with the first graders, the students were presented with an incorrect way to represent the number 63. They were shown a pile of 6 cubes and a pile of 3 cubes and asked if they agreed that this representation showed 63. All students were able to recognize that it was incorrect because the pile of 6 cubes should be 6 longs since the number is in the 60s. The students all proved during this task that they understood the place-value of the "6" in 63. Because of this, it was surprising that the post-interviews showed so little growth on Task 1.
- The lack of growth in the first grade students might possibly be attributed to the way that the question was phrased to the students. The students were asked to "show this

part of the number" while Ms. Carr circled the one in the number 16. It is possible that many students took this too literally, seeing and taking the "1" completely out of context instead of viewing it as representing the 10 part of the number 16. Results may have differed if the question was phrased more clearly by asking to "show the value of this part of the number 16."

Concluding Thoughts

After the completion of the study, I believe that the students benefitted from the tasks that they were presented with in the group meetings. Most of the students showed a positive growth in the complexity and number of responses of ways to compose a given number with flats, rods, and cubes from the pre-interview assessment to the post-interview assessment. In the future, I would like to see if there are other activities and tools that also effectively increase children's' flexibility in composing numbers. I also think it would be interesting to see if children are better able to answer the first question on the assessment if the wording was changed to make the meaning clearer, and to see if the same results would occur with a larger sample size.

The focus of my research was to answer the following research question: *After receiving small-group instruction on place value and the composition of numbers, do activities utilizing place value representations such as base-ten blocks increase flexibility in understanding number composition?* After completing the research, I believe that the activities used in group meetings do increase flexibility in understanding number composition. Students were much more flexible when composing numbers on the post-assessment than in the pre-assessment and were much more comfortable manipulating the base-ten blocks.

References

Bell, M., Bell, J., Bretzlauf, J., Dillard, A., Hartfield, R., Isaacs, A., McBride, J., Moran, C. G., Pitvorec, K., & Saecker, P. (2016). *Everyday Mathematics, Grade 2. Teacher's Lesson Guide, Volume 1*. Columbus, OH: McGraw-Hill Education.

Common Core Standards Writing Team (2011). *Progressions Documents for the Common Core Math Standards*: *Draft K–5 Progression on Numeration in Base Ten*. http://ime.math.arizona.edu/progressions/

Gojak, L. M. (2012). Fluency: Simply fast and accurate? I think not! NCTM *Summing Up*. Retrieved from https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/Fluency_-Simply-Fast-and-Accurate_-I-Think-Not!/

National Council of Teachers of Mathematics. (n.d.). Procedural Fluency in Mathematics. Retrieved July, 2017, from http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices and Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics*. Washington, D. C.: NGA Center and CCSSO.

National Research Council (2001). Kilpatrick, J., Swafford, J., & Findell, B. (Eds.), *Adding it up: Helping children learn mathematics* (pp. 181-230). Washington D.C.: National Academy Press.

Van de Walle, J. A., Karp, K. S., and Bay-Williams, J. M. (2013). *Elementary and Middle School Mathematics: Teaching Developmentally (8th edition)*. New York, NY: Pearson Education, Inc.

Van de Walle, J. A., Lovin, L. H., Karp, K. S., and Bay-Williams, J. M. (2014). *Teaching student-centered mathematics*. New York, NY: Pearson Education, Inc.

Appendix A:
Pre/Post-Interview Forms (Second and First Grade)

Second Grade Pre/Post-Interview ID #:_________

TASK ONE

* Circle the 6 with the eraser of a pencil and say, "Show me that part of the number with the pennies."

* Then circle the 1 and say, "Show me that part of the number with the pennies."

Showed 1 penny	Showed 10 pennies	Unable to complete the task

TASK TWO

Provide the child with base-ten blocks and ask the child to represent the number 204. Then ask the child to represent it another way, another way, etc.

First Grade Pre/Post-Interview ID #:_________

TASK ONE

* Circle the 6 with the eraser of a pencil and say, "Show me that part of the number with the pennies."

* Then circle the 1 and say, "Show me that part of the number with the pennies."

Showed 1 penny	Showed 10 pennies	Unable to complete the task

TASK TWO

Provide the child with base-ten blocks and ask the child to represent the number 52. Then ask the child to represent it another way, another way, etc.

Appendix B:
Group Meeting Tasks (Second and First Grade)

Second Grade

Day One:

- I have 23 ones and 4 tens. Who am I?
- I have 4 hundreds, 12 tens, and 6 ones. Who am I?
- I have 30 ones and 3 hundreds. Who am I?

Day Two:

- Hundreds Chart with missing numbers
- Number-Grid Puzzle (Figure 4)
- I have 13 tens, 2 hundreds, and 21 ones.

Day Three:

- Number Grid Puzzles (Figures 5 & 6)
- Given 2 hundreds, 3 longs, 3 ones. Change only one piece at a time

Day Four:

- Fill in selected pieces from a nearly blank hundreds chart (Group 1 only)
- I am 45. I have 25 ones. How many tens do I have?
- I am 341. I have 22 tens. How many hundreds do I have?
- I have 17 ones. I am between 40 and 50. Who am I?

First Grade

Day One:

- I have 23 ones and 4 tens, who am I?
- Hundreds chart: L on 5, 15, 25, 35, 36.

Day Two:

- I am 45. I have 25 ones. How many tens do I have?
- I am 57. I have 37 ones. How many tens do I have?
- Hundreds chart "L" and "t" (Figure 7)

Day Three:

- Another first grader made 63 this way (show 6 cubes in one pile, 3 in another). What do you think? Is that 63? Why or why not?
- Number Grid Puzzle (Figure 4)

Day Four:

- I have 4 tens and 1 one. What number am I?
- Number Grid Puzzles (Figures 5 & 6)