Elastic Scattering of 1.33 MeV Gamma Rays from Uranium

David Rood Schwandt
Western Michigan University

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ELASTIC SCATTERING OF
1.33 MeV GAMMA RAYS
FROM URANIUM

by

David Rood Schwandt

A Thesis
Submitted to the
Faculty of the School of Graduate
Studies in partial fulfillment
of the
Degree of Master of Arts

Western Michigan University
Kalamazoo, Michigan
April 1969
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Discussions with Professors L. D. Oppliger and A. Dotson were very helpful. The interest and encouragement shown by the rest of the faculty of the Department of Physics has been greatly appreciated.

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David Rood Schwandt
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INTRODUCTION

The theory of quantum electrodynamics predicts the creation and annihilation of virtual electron-positron pairs in an electromagnetic field. This effect should be observable as a small contribution to the elastic cross section when gamma rays are scattered from a heavy nucleus. This effect is known as Delbrück scattering.

Delbrück scattering occurs when the photon enters the strong Coulomb field near the nucleus. The photon instantaneously creates a virtual electron-positron pair. During the lifetime of the virtual pair, the electron and positron interact with the potential field thus changing the momentum of the pair. In order to preserve momentum upon annihilation, the created photon goes off at some angle $\theta$ with respect to the direction of the initial photon.

Delbrück scattering is not the only nonclassical effect predicted by quantum electrodynamics. Other effects include vacuum polarization, scattering of light by light, and photon splitting.

Evidence for vacuum polarization has been provided by the famous Lamb shift experiment. The Lamb shift pertains to the energy level splitting between the $2S _{1/2}$ and $2P _{1/2}$ states of the hydrogen atom. The cause of the Lamb shift is essentially the shielding of the nucleus by production of virtual electron-positron pairs in the Coulomb field around the nucleus with a subsequent polarization effect due to attraction and repulsion by the nucleus of the electron and positron respectively.

1

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Evidence of photon splitting has recently been given by Adler and Cohen.\(^2\) Essentially, the effect is characterized by the simultaneous appearance of two photons whose energy sum is equal to that of the original incident photon.

The predicted cross section for the scattering of light by light is extremely small and the process has so far not been observed.\(^3\) However, the scattering of photons by a strong Coulomb field (Delbrück scattering) is closely related to the scattering of light by light and may be observable because the predicted cross sections are higher.

If Delbrück scattering can be established experimentally, it would furnish an independent check on the quantum electrodynamic concept of virtual pair production. Attempts to detect Delbrück scattering can be successful only if the effect is greater than the uncertainty in the experimental measurements, and accurate calculations can be made of other types of scattering coherent with Delbrück scattering.

The purpose of this thesis is to report on an experiment which shows that accurate cross section measurements can be made and to discuss the interpretation of these measurements.

The experiment was conducted using uranium as a scatterer, thus furnishing a strong nuclear potential field.

The elastic cross section was measured at various angles for 1.33 MeV gamma rays. To obtain higher energy resolution than in most previous experiments, a lithium-drifted, germanium, solid-state detector was used.
THEORY

Generally, the processes by which photons interact with matter can be separated into three categories. These three are absorption, inelastic scattering, and elastic scattering.

Absorption occurs when a photon is annihilated and all of its energy is given to the surrounding medium. This process is of little interest to our experiment except for possible secondary effects. Inelastic scattering arises from the photon losing some of its energy to the material with which it interacts. Elastic processes, which are of primary concern in the present work, are those in which the incoming photon gives up almost no energy to the scatterer.

Inelastic Scattering and Absorption

Inelastic scattering and absorption result from mainly three types of interaction; Compton scattering, photo-electric effect, and pair production.

Compton scattering is probably the best known of these interactions. It is the scattering of a gamma ray by a free or loosely bound electron.

The angular dependence of the energy \((\hbar \nu')\) of the scattered photon is

\[
\hbar \nu' = \frac{\hbar \nu}{1 + \alpha (1 - \cos \Theta)}
\]
where \((h \nu)\) is the energy of the incoming photon, \((\theta)\) is the scattering angle, and 
\[ a = \frac{h \nu}{m_0 c^2}, \]
with \(m_0 c^2\) representing the rest energy of the electron. Thus one can see that as \(\theta\) goes to zero \(h \nu'\) approaches \(h \nu\), and the energy loss approaches zero. In an elastic scattering experiment, one must be careful at small scattering angles to avoid confusing a Compton scattered photon with one that comes from an elastic process. Figure I shows the angular dependence of Compton scattering for 1.33 MeV photons.

There is also an effect due to Compton scattering from bound electrons of the atom.\(^5\) The photon collides with a bound electron, and although the electron is freed, some momentum may be taken up by the ionized atom. The scattered photon may then have any energy up to its original energy less the binding energy of the electron.

The photo-electric effect has been dealt with completely in many texts.\(^{(1,4)}\) A photon is completely absorbed by a bound electron. The electron is freed and has a kinetic energy which is the difference between incoming energy of the photon and binding energy of the electron.

Lastly, pair production must be considered at energies greater than 1.02 MeV. The photon disappears and an electron-positron pair is created. The cross section increases with increasing energy of the incoming photon.\(^4\)

Figure II shows the dependence of these cross sections on photon energy. From this figure it is seen that for 1.33 MeV gamma rays scattered by a uranium target the Compton scattering
Figure I. Energy of Compton scattered gamma ray plotted as a function of angle.

Incident Energy = 1.33 MeV

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Figure II. Cross section for inelastic processes plotted as a function of energy.
Elastic Scattering

There are four elastic processes which contribute to the differential cross section. These are called 1) nuclear resonant scattering, 2) nuclear Thomson scattering, 3) Rayleigh scattering and 4) Delbrück scattering. These processes all lead to one of two distinguishable final states in which the emerging photon has the same energy as the incident photon and either the same polarization (the non-spin-flip case, symbolized by NSF) or the opposite polarization (the spin-flip case, symbolized by SF). For a given final state, the four processes are indistinguishable, so their amplitudes must be added. Then, since the different polarization states are physically distinguishable, the elastic differential cross section is given by

\[
\frac{d\sigma}{d\Omega} \bigg|_{\text{el.}} = r_o^2 \left\{ |A_R + A_T + A_{NR} + A_D|_{SF}^2 + |A_R + A_T + A_{NR} + A_D|_{SF}^2 \right\}
\]

The subscripts on the amplitudes, R, T, NR and D represent Rayleigh, Thomson, nuclear resonant and Delbrück respectively. \( r_o \) is the classical electron radius.

Nuclear resonant scattering occurs by the excitation of a nucleus to a higher level by an incident photon, with subsequent re-emission of the excitation energy. It is difficult to achieve nuclear resonance because the incoming photon must have almost the exact excitation energy to excite the nucleus to the higher level.
The probability of this occurring in uranium is small at 1.33 MeV, so this process is neglected.

Nuclear Thomson scattering is the dominant part of Compton scattering by the nucleus. The nucleus is so massive that it carries away a negligible amount of energy in recoiling, thus leaving the energy of the photon essentially unaltered.

The two scattering amplitudes (spin flip and no spin flip) are

\[
(\mathcal{A}_T)_{\text{NSF}} = -\frac{(2e)^2}{M_n} \frac{(\cos \theta + 1)}{2}
\]

\[
(\mathcal{A}_T)_{\text{SF}} = -\frac{(2e)^2}{M_n} \frac{(\cos \theta - 1)}{2}
\]

where \(e\) is the charge of the electron, \(M_n\) is the mass of the nucleus, and \(\theta\) is the scattering angle. Figure III shows the angular dependence of the Thomson amplitudes for a uranium scatterer.

Although states of circular polarization (NSF and SF) have been used, transformation can easily be made to other states of polarization. The important point is to be consistent throughout the discussion.

Rayleigh scattering involves the interaction between the tightly bound electrons in heavy elements and the incoming photon. High probability of Rayleigh scattering occurs at small angles because the recoil imparted to the atom must not produce atomic excitation or ionization. Thus 60-70\% of the cross section is confined to angles smaller than 4° for lead at 1 MeV and a slightly higher percentage for uranium at 1.33 MeV. The cross section for this
Figure III. Thomson amplitudes (spin flip and no spin flip) are plotted as a function of scattering angle.
effect does not vary strictly as $Z^5$ as stated in most of the literature but varies as $Z^n$ where $(n)$ depends on the momentum transfer $(q)$ in the scattering process.  

Rayleigh scattering is the major contributor to the total elastic cross section, but its scattering amplitudes have been difficult to calculate. The first calculated values were given by Franz (1935) using the Fermi-Thomas electron distribution. It was later found that this form factor approximation is an extremely bad one for calculating the amplitude for no polarization change but gives accurate results for the amplitude with polarization change. More recently Brown and Mayers have calculated the Rayleigh amplitudes as a function of angle for 1.28 $m_o c^2$ and 2.56 $m_o c^2$ gamma rays scattered from mercury to an accuracy of about 10%. Brown and Mayers only considered scattering by the K-Shell electrons.

To completely specify the Rayleigh amplitudes one must include a L-Shell contribution. This is usually stated as a fraction of the K-Shell amplitude; for the case of 1.33 MeV gamma rays scattered by lead or uranium, it is usually taken as about 20% of the K-Shell amplitude for the SF case and zero for the NSF case.

Each Rayleigh amplitude of definite polarization consists of an imaginary part as well as a real part. The imaginary (absorptive) amplitude pertains to a process much like the photoelectric effect, but where the freed electron is recaptured into the initial state, thereby resulting in an elastic scattering. The imaginary part
can be neglected at small angles and low energy.\textsuperscript{8}

The Brown and Mayers amplitudes must be extrapolated to our specific situation; scattering of 1.33 MeV gamma rays by uranium. This will be discussed later when corrections and extrapolations are considered.

Delbrück first pointed out that quantum electrodynamics predicts the interaction of electromagnetic radiation with the nuclear Coulomb field. The result of the interaction, which is a small part of the Compton scattering by a nucleus, is a contribution to the elastic differential cross section.

Delbrück amplitudes have an imaginary part (absorptive) and a real part (dispersive). The imaginary part of the scattering amplitude arises from those transitions in which energy is conserved in the intermediate state. It is closely related to real pair production. Its amplitude is quite small compared to the real part at energies up to a few MeV. It is zero below the pair production threshold and increases roughly as the cube of the energy and has been verified at 9, 17, and 87 MeV.\textsuperscript{11}

The real part of the scattering amplitude accounts for the contribution from virtual (energy non-conserving intermediate state) pair production. The existence of these virtual electron pairs brings about the polarization of the vacuum through the inhomogeneity of the field in the neighborhood of a charge. Thus only the real part gives evidence for the existence of virtual pairs which results in the polarization of a vacuum.
Figure IV shows the Delbrück amplitudes as a function of photon energy of a scattering angle of zero degrees. At energies of a few MeV and lower, the real part predominates over the imaginary part.

The cross section for Delbrück scattering has been calculated by Ehlotzky to within 5-10%. The Delbrück amplitudes are given in Figure V as a function of angle. These amplitudes have been calculated for uranium at 1.33 MeV.

From the above discussion it can be concluded that accurate knowledge of the Rayleigh and nuclear Thomson amplitudes must be available to detect the presence of Delbrück contribution to the total elastic cross section.
Figure IV. Delbrück scattering amplitudes for forward scattering in units of $(\alpha Z)^2 r_0$ as a function of energy and scattering angle $\theta = 0$. 

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Figure V. Delbrück amplitudes plotted as a function of scattering angle.
EXPERIMENTAL PROCEDURES AND APPARATUS

The procedures are typical of an experiment to determine the angular distribution of a differential scattering cross section. Gamma rays from a 111 Curie Co⁶⁰ source are scattered by a uranium target and detected at a series of scattering angles. Although Co⁶⁰ emits two gamma rays in cascade, 1.17 MeV and 1.33 MeV, only the 1.33 MeV gamma ray are reduced to cross sections.

The differential scattering cross section, \((d\sigma/d\Omega)\) at a scattering angle \(\theta\) is given in terms of the number of events per second, \(n_a\), in the photopeak of the spectrum obtained with the detector by the following expression

\[
\eta_a = \frac{a}{\xi \pi r^2} N \left( \frac{d\sigma}{d\Omega} \right)_{el} \omega C
\]

where:
- \(a\) = the number of 1.33 MeV \(\gamma\)-rays per second emitted by the source;
- \(r\) = source-target distance;
- \(N\) = number of target atoms;
- \(\omega\) = the probability of the \(\gamma\)-ray elastically scattered within the solid angle \(\omega\) being registered by the counter (detector photopeak efficiency);
- \(C\) = a correction for absorption of gamma rays in the target and variation of gamma ray flux over the surface of the target.

Since \(\omega\) and \(\xi\) are difficult to determine accurately, a second measurement was made using an auxiliary Co⁶⁰ source made by spreading uniformly over a piece of cardboard with the same dimensions as the scatterer, a liquid containing Co⁶⁰. With the main
source blocked and this auxiliary source replacing the target, the
counting rate, \( n_b \), is given by

\[
[3] \quad n_b = \frac{b}{\gamma} \omega \epsilon
\]

with \( b \) representing the auxiliary source strength.

Since the geometrical arrangement is the same in both measure-
ments and since the gamma rays have the same energy in both cases,
\( \omega \) and \( \epsilon \), in equations [1] and [2] are the same and can be
eliminated to give

\[
[3] \quad \left( \frac{d\sigma}{d\Omega} \right)_\theta = \frac{n_b}{n_0} \times \frac{b}{\alpha} \times \frac{r^2}{N} \frac{1}{c}
\]

Before explaining the procedures in more detail the apparatus
will be described.

The high intensity source was housed in approximately 1.5 tons
of lead with a steel casing around the perimeter. The shutter
system is of the forced-mercury type. It consists of a mercury
filled cylinder in front of the beam. To open the shutter, the
mercury is forced out of the cylinder by compressed nitrogen gas
into a reservoir above the source, thus revealing the beam. Upon
release of the gas pressure, the mercury is gravity-fed back into
the cylinder, thus blocking the beam.

The target, or auxiliary source, was suspended from an aluminum
arm mounted well above the beam. This arm moved over an aluminum
disc which had degree markings on its periphery. Initial alignment
of this angular scale with the center of the beam was done with a
laser. To minimize the difference in scattering angle from differ-
ent parts of the target, the face of the target was not set perpendicular to the center of the beam, but at some angle \( \theta \). Figure VI is a scale diagram of the arrangement actually used for \( \theta = 75^\circ \). As a final check on the position of the beam, X-ray plates were placed behind the target and the shadow of the target observed on the developed plates. This check was performed at each angle.

A 10 cm active volume, lithium-drifted, germanium detector was used in the experiment. Figures VII and VIII are typical spectra of the auxiliary source and scattered radiation respectively. The scattering angle was set in the following way. The target was suspended by nylon string from each side of the disc at the proper angles. Initial alignment of the disc relative to the beam line was made with the laser, after this, subsequent alignments only had to be made between the target and the detector, again via the laser beam. Thus alignment can be made with an uncertainty of \( \pm 1 \) degree. This uncertainty was achieved through laser alignment and the ability to set precise angles on the machined disc mentioned above.

At each angle the detector was carefully shielded to minimize the detection of gamma rays not scattered by the uranium target.

To reduce pile-up effects due to the large number of Compton scattered gamma rays, a filter of lead was placed in front of the detector to discriminate against these lower energy gamma rays. For example, at \( \theta = 60^\circ \) a lead thickness of 0.5 inches was used which reduced the intensity of the Compton scattered gamma rays by about 70% and the intensity of the elastically scattered gamma...
Figure VI. Experimental arrangement for scattering angle $\theta = 75^0$. 

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Figure VII. Auxiliary Co$^{60}$ source spectrum at 90°. (800 minutes)
Figure VIII. Uranium target scattering spectrum at 90°. (2400 minutes)

- 1.17 MeV
- 1.33 MeV
rays by about 30%.

The small elastic scattering cross section and poor detection efficiency necessitated runs for a live time of 800 minutes. After each run with the auxiliary source or target in position, an 800-minute background run was made. At each angle the first run was made with the auxiliary source in position, then three runs were made with the target in position, and then another run was made with the auxiliary source.

As can be seen from equation three, it is also necessary to measure the ratio of the auxiliary source strength to the strength of the main source. Because this ratio is about $10^{-7}$, a direct measurement by relative counting rates at a fixed distance is not practical. To facilitate the determination of this ratio two Co sources of intermediate strength (nominally 1 mC and 100 mC) were used.

The ratio of the auxiliary source strength to the 1 mC source strength was determined by an inverse square procedure. Figure IX illustrates the electronic apparatus used for this and all other counting measurements. The counting rate as a function of distance was obtained for each source. Plotting the log of the count rate as a function of distance should yield a straight line with a slope of -2. Using a least squares method, lines were fit to both sets of data and the ratio was obtained by using these lines. These calculations were done on a computer, and the program is discussed and listed in Appendix B. Because of the large area of the auxiliary source, a small correction to the inverse square data was nec-
Figure IX. Lithium-Germanium Spectrometer
Absorption measurements were made to determine the ratios of the 1 mC to 100 mC sources and the 100 mC to $^{111}$C sources. The absorbers were lead, and the experimental arrangement is shown schematically in Figure X. Using points fit to the three absorption curves the two ratios were determined. To avoid systematic errors due to small angle scatterings, the same geometry was used for each source. This was accomplished by constructing a dummy shutter and housing for the 1 mC and 100 mC sources which would give the same small angle scattering.
Figure X. Experimental Arrangement for Absorption Measurement
CORRECTIONS

Only three corrections are significant in calculating the cross sections. The first, already discussed, is the small correction for the finite size of the auxiliary source necessary in the relative source strength measurement.

The second correction is for attenuation in the target of both the incident beam and the elastically scattered gamma rays. The actual counting rate, \( N_0 \), must be modified as follows to obtain the counting rate, \( N \), that would have been measured if no absorption had occurred:

\[
N = N_0 \frac{\sin \phi}{\mu} \left[ 1 - \exp \left( \frac{-\mu \omega}{\sin \phi} \right) \right]
\]

where \( \mu \) is the absorption coefficient, \( \omega \) is the target thickness, \( \phi \) is the angle the plane of the target makes with the beam direction, \( \delta = 1 + \left[ \frac{\sin \phi}{\sin (\theta - \phi)} \right] \), and \( \theta \) is the scattering angle.

The third correction, actually small in our arrangement, is for variation in the source to scattering center distance over the surface of the target. This correction is discussed in Appendix A.
RESULTS AND UNCERTAINTIES

The largest single uncertainty arises from the relative source strength measurement. The ratio of the auxiliary source to the nominally 1 mC source is $1.07 \times 10^{-2} \pm 3\%$. The ratio of the 1 mC to the 100 mC source is $7.40 \times 10^{-3} \pm 1.5\%$. The ratio of the 100 mC to the $^{111}$C source is $1.44 \times 10^{-3} \pm 1\%$. Hence the ratio of the auxiliary source to the main source is $1.141 \times 10^{-7} \pm 3.5\%$.

The dimensions of the target are $16.5 \pm 0.05$ cm and $14.2 \pm 0.05$ cm. The mass is $288.04 \pm 0.02$ gms. Spectroscopic analysis showed that there was less than 2% by weight of impurities. This dominates the uncertainty in the number of uranium atoms. The target contains $0.73 \times 10^{24} \pm 2\%$ uranium atoms.

A measurement of the distance from the main source to the center of the target yielded $125 \pm 1$ cm. This measurement is combined with the correction for the variation of $r$ over the target and given in Table I.

Counting times were sufficiently long so that statistical uncertainties were less than 3% for both the scattered radiation and the auxiliary source. However, additional uncertainties arise due to such factors as electronic channel drift, and a small variance of the ratio of the count rate for different choices of the number of channels included under the 1.33 MeV peak. This ratio and its associated uncertainty is listed in Table I for each angle.

The uncertainty in the scattering angle recorded in Table I
<table>
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<tr>
<th>SCATTERING ANGLE</th>
<th>RATIO OF COUNTS (Na/Nb)</th>
<th>r (cm)</th>
<th>ABSORPTION CORRECTION (%)</th>
<th>DIFFERENTIAL CROSS SECTION $X \times 10^{-28}$ (mb/sr)</th>
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<td>$60^\circ \pm 1.0^\circ$</td>
<td>.1750 + 4.3%</td>
<td>124.8 + 1%</td>
<td>12.8 + .35</td>
<td>4.29 + 6.0%</td>
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<tr>
<td>$75^\circ \pm 1.5^\circ$</td>
<td>.1107 + 4.3%</td>
<td>124.8 + 1%</td>
<td>10.7 + .23</td>
<td>2.70 + 6.1%</td>
</tr>
<tr>
<td>$90^\circ \pm 1.7^\circ$</td>
<td>.0921 + 4.3%</td>
<td>124.8 + 1%</td>
<td>8.5 + .22</td>
<td>2.26 + 6.1%</td>
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<tr>
<td>$105^\circ \pm 2.0^\circ$</td>
<td>.0862 + 4.3%</td>
<td>124.8 + 1%</td>
<td>7.7 + .23</td>
<td>2.11 + 6.0%</td>
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<tr>
<td>$120^\circ \pm 2.5^\circ$</td>
<td>.0840 + 4.5%</td>
<td>124.8 + 1%</td>
<td>7.5 + .32</td>
<td>2.05 + 6.2%</td>
</tr>
</tbody>
</table>
is the mean square variation of the scattering angle over the target.

Table I also includes the absorption corrections with their uncertainties and the measured differential cross sections.
COMPARISON WITH OTHER EXPERIMENTS

The only other work done at 1.33 MeV for a uranium scatterer is by Bernstein and Mann\textsuperscript{13} and Eberhard and Goldzahl\textsuperscript{7}. Their quoted uncertainties are about 20\% and hence much larger than our uncertainties.

Figure XI gives a summary of all the experimental work. The cross sections of the other workers are larger than ours. This is most likely due to some inelastic events under their elastic scattering photopeak, since their resolution was much poorer than in the present work.
Figure XI. Differential cross sections of other workers compared to our values.
THEORETICAL CALCULATIONS

To compare the measured cross sections with the theoretical cross sections, the amplitudes for Rayleigh, Thomson and Delbrück scattering must be known at a gamma-ray energy of 1.33 MeV for a uranium scatterer.

The Thomson amplitudes, easily calculated from the formula given in Chapter I, are listed in Table II.

The Delbrück amplitudes calculated by Ehlotzsky and Sheppy are in terms of \((\sim Z)^2r_0\). Hence, their amplitudes are multiplied by this factor for uranium and listed in Table III.

Rayleigh amplitudes for a gamma-ray energy of 1.31 MeV and for a Hg scatterer have been calculated by Brown and Mayers. It is difficult to extrapolate these amplitudes to our case, since their exact energy and Z dependence is not known. The form factor calculations give poor results for the amplitudes for the Rayleigh K-Shell scattering, especially the NSF amplitudes. However, the ratio of the uranium form factor at 1.33 MeV to the mercury form factor at 1.31 MeV should be more reliable. Thus the amplitudes obtained by Brown and Mayers can be multiplied by the ratio of the form factors to extrapolate to our particular case. The form of this ratio is as follows:

\[
\alpha(q_2, \theta, 1.33) = \alpha(q_0, \theta, 1.31) \frac{F(Q)(q_2, \theta, 1.33)}{F(Q)(q_0, \theta, 1.31)}
\]

Table IV has the extrapolated values listed.
Table II
Thomson Amplitudes

<table>
<thead>
<tr>
<th>SCATTERING ANGLE</th>
<th>AMPLITUDES (r₀)</th>
<th>Spin Flip</th>
<th>No Spin Flip</th>
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<tr>
<td>15</td>
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Table III

Delbrück Amplitudes

<table>
<thead>
<tr>
<th>SCATTERING ANGLE</th>
<th>AMPLITUDES (r_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spin Flip</td>
</tr>
<tr>
<td>15</td>
<td>.007 - .0000i</td>
</tr>
<tr>
<td>30</td>
<td>.007 + .0001i</td>
</tr>
<tr>
<td>45</td>
<td>.005 + .0004i</td>
</tr>
<tr>
<td>60</td>
<td>.005 + .0007i</td>
</tr>
<tr>
<td>75</td>
<td>.004 + .0009i</td>
</tr>
<tr>
<td>90</td>
<td>.002 + .0011i</td>
</tr>
<tr>
<td>105</td>
<td>.001 + .0013i</td>
</tr>
<tr>
<td>120</td>
<td>.001 + .0015i</td>
</tr>
<tr>
<td>SCATTERING ANGLE</td>
<td>AMPLITUDE ($r_0$)</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------</td>
</tr>
<tr>
<td>15</td>
<td>.0262 - .0045i</td>
</tr>
<tr>
<td>30</td>
<td>.0530 - .0106i</td>
</tr>
<tr>
<td>45</td>
<td>.0530 - .0106i</td>
</tr>
<tr>
<td>60</td>
<td>.0498 - .0087i</td>
</tr>
<tr>
<td>75</td>
<td>.0460 - .0072i</td>
</tr>
<tr>
<td>90</td>
<td>.0439 - .0055i</td>
</tr>
<tr>
<td>105</td>
<td>.0454 - .0055i</td>
</tr>
<tr>
<td>120</td>
<td>.0431 - .0048i</td>
</tr>
<tr>
<td>135</td>
<td>.0444 - .0049i</td>
</tr>
<tr>
<td>150</td>
<td>.0441 - .0045i</td>
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</tbody>
</table>
COMPARISON WITH THE THEORETICAL CALCULATIONS

The coherent addition of Rayleigh K-Shell and nuclear Thomson amplitudes is given in Figure XII. Our experimental results are also shown in the figure. Clearly, a discrepancy exists between the experimental points and the addition of these two types of scattering. If one adds in the Delbrück amplitudes, the dashed curve in the figure is obtained. This obviously cannot account for the discrepancy. Nor does the simple addition of 20% Rayleigh K-Shell amplitude bring the theoretical values into agreement with experimental results.

Two possible ways of obtaining agreement between theory and experiment have been suggested by Dixon and Storey. The first explanation is that Rayleigh and Thomson scattering add incoherently. The theoretical curve which results from the incoherent sum of Rayleigh and Thomson scattering is shown by the solid line in Figure XIII. This curve includes a contribution for L-Shell Rayleigh scattering, taken to be 20% of the Rayleigh K-Shell amplitude.

The second possible explanation by Dixon and Storey is that there is a 180° phase shift between the components, $A_R^{\parallel}$ (parallel polarized component of the Rayleigh amplitude) and $A_T^{\parallel}$ (parallel polarized component of the Thomson amplitude), while there is no corresponding shift for the components polarized perpendicular. As can be seen from Figure XIV, the agreement obtained with these
Figure XII. Theoretical differential cross sections compared to experimental results.
Figure XIII. Theoretical differential cross section compared to experimental results.

- Rayleigh & Thomson (added incoherently) & 20% K-Shell

- Our experimental values
Figure XIV. Theoretical differential cross section compared to our experimental results.

Thomson amplitude (parallel) added destructively to Rayleigh. Our experimental values.
changes (plus a Rayleigh L-Shell correction) is good when compared with the experimental results.

Another explanation, which does not require the arbitrary changing of the sign of one amplitude or postulating incoherence, is that the Rayleigh L-Shell amplitude for SF changes sign by crossing the $A_L = 0$ axis at about $90^\circ$ and neglecting the NSF L-Shell amplitude. Agreement between theory and experiment is obtained using the L-Shell contribution given in Figure XV. Figure XV also shows the relative values of the two methods of L-Shell correction as compared to the K-Shell amplitude. Of course, no significance is to be attached to the agreement shown in Figure XVI; this was assured by the choice of L-Shell amplitudes. The point is that the discrepancy between theory and experiment can be accounted for by a reasonable amount of L-Shell amplitude, with reasonable dependence upon scattering angle.
Scattering Amplitude (in units of $r_0$)

Rayleigh K-Shell (Spin Flip)

L-Shell contribution (20% Rayleigh K-Shell - Spin Flip)

L-Shell contribution (experimental fit)

Figure XV. Amplitude of the L-Shell contribution vs scattering angle.
Rayleigh & Thomson & L-Shell (according to experimental fit - Figure XV)

Our experimental values

Figure XVI. Experimental curve - (L-Shell contribution changes sign at 90°).
CONCLUSIONS

The differential cross sections for scattering of 1.33 MeV gamma rays by uranium have been measured as a function of scattering angle, from $60^\circ$ to $120^\circ$. Using a detector with good energy resolution, our measurements have a significantly smaller uncertainty than previous methods and are less susceptible to systematic errors.

It has been found that a discrepancy exists between these measurements and the theory for Rayleigh, Thomson and Delbrück scattering. One possible way of resolving this discrepancy is to arbitrarily change the sign of one of the states for Thomson scattering. Since the relative signs of the Thomson amplitudes are certainly given by quantum electrodynamics, there is no theoretical justification for this procedure. This conclusion is justified by Figure XIV. A second explanation to resolve the discrepancy is to assume that Rayleigh and Thomson scattering became incoherent, at least for the scattering angles investigated in the present work. But the final quantum states of the system for Thomson and Rayleigh scattering are the same, so it is difficult to see how this incoherence could occur. Since an exact calculation has never been made to determine the Rayleigh L-Shell amplitudes, it is inviting to use this process to resolve this discrepancy. We have shown that only L-Shell amplitudes of a reasonable magnitude are needed,
provided the sign of the amplitude changes around a scattering angle of $90^\circ$.

It is clear that no definite statement can presently be made regarding Delbrück scattering. There are two reasons for this situation. The first is that the uncertainties in the experimental cross sections are larger than the Delbrück contributions. Secondly, the exact L-Shell Rayleigh amplitudes, which are at least as important as the Delbrück amplitudes at these angles and energy, have not been calculated.
APPENDIX A

Calculation of the Average Flux Over the Target Surface

The average photon flux may be written as

\[ \langle \psi \rangle = \langle \frac{E}{4\pi r^2} \rangle = \frac{E}{4\pi} \langle \frac{1}{r^2} \rangle \]

where \( A \) is the source strength, and \( r \) is the distance from the source to the target. Thus the average value of \( \langle 1/r^2 \rangle \) must be found over the surface of a target which is at some angle \( \theta \) with respect to the source beam.

\[
\begin{align*}
    r &= \left( z^2 + y^2 + r_0^2 - 2r_0 y \cos \theta \right)^{1/2} \\
    \tan^{-1} x &= x - \frac{1}{3} x^3 + \frac{1}{5} x^5 + \cdots
\end{align*}
\]

where \( r_0 \) is the distance to the center of the target.

Thus, performing the integration over the rectangular surface

\[
\langle \psi \rangle = \frac{1}{A^2} \int_{y_0}^{y_b} \int_{z_0}^{z_b} \frac{dy \, dz}{(z^2 + y^2 + r_0^2 - 2r_0 y \cos \Theta)^{1/2}}
\]

using the approximation,
for the first integral solution and the binomial expansion,

\[
(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3
\]

with \( x = \frac{y^2 - 2r_o \cos \theta}{r_o^2} \)

we arrive at the final result for \( \langle 1/r^2 \rangle \)

\[
\langle \frac{1}{r^2} \rangle = \frac{1}{r_o^2} \left( 1 - \frac{b^2}{3r_o^2} + \frac{a^2(y \cos^2 \phi - 1)}{3r_o^2} \right)
\]

where terms higher than \( b^2/r_o^2 \) have been dropped because \( r_o \gg b \).
APPENDIX B

Fortran Computer Program

The calculations for the relative source strength considerations were entirely done by computer. The Fortran III program included in this appendix is the one that was used.

The program calculates the following:

(a) Least square fit of data, both inverse square and absorption type.

(b) A chi-square on data points to the least square fit.

(c) Slopes and absorption coefficients.

(d) Ratio of points on the least square fit graph for the auxiliary source to 1 mC, 1 mC to 100 mC, and 100 mC to 111 Curie source.

(e) The final, relative source ratio.
DIMENSION X(10), Y(10), DEV(10), COUN(10), SAVE(10), USE(8)
DIMENSION SUMRAT(6), RATIO(6, 20), AVERAT(6)
DIMENSION ALSO(8), ELEM(6, 20), ERROR(6, 20), SUMER(6)
L=0
JCOUNT=0
J=0.
B=0
ABSAV=0
NUM=6
READ 1, N
JCOUNT=JCOUNT+1
DO 3 I=1, N
  C THIS CALCULATES LEAST SQUARE FIT
  3 READ 2, X(I), Y(I)
      CHISQ=0
      SUMX=0
      SUMY=0
      SUMXO=0
      DO 4 I=1, N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      SUMXY=SUMXY+X(I)*Y(I)
      SUMXQ=SUMXQ+X(I)**2
  4 U=N
     DETA=U*SUMXO-SUMX**2
     DETB=SUMY*SUMXQ-SUMX*SUMXY
     DETC=U*SUMXY-SUMY*SUMX
     COEA=DETB/DETA
     COEB=DETC/DETA
     L=L+1
     ALSO(L)=COEB
     USE(L)=COEA
DO 80 I=1,N
C FINDS DEVIATION FROM FIT
   DEV(I)=Y(I)-(COEA+COEB*X(I))
80   COUN(I)=10.**DEV(I)
C THIS CALCULATES CHI SQUARE
DO 88 I=1,N
88   CHISQ=CHISQ+(10.**COEA+COEB*X(I))-10.**Y(I))**2/10.**Y(I))
   ABCOE=2.303*COEB
   IF(N-4)>17.17.16
   J=J+1
   SAVE(J)=ABCOE
   ABSAV=ABSAV+ABCOE
   B=B+1.
16   PUNCH 20
7   DO 7 I=1,N
7    PUNCH 21,X(I),Y(I),DEV(I),COUN(I)
7    PUNCH 22, SUMX, SUMY, SUMXY, SUMXQ
7    PUNCH 23, COEA, COEB
7    PUNCH 24,N,ABCOE
7    PUNCH 82,CHISQ
   IF(JCOUN-NUM)15,87,87
C CALCULATES AVERAGE MU AND STANDARD DEVIATION
87   AVECOE=ABSAV/B
   K=B
   J=0
   VARSQ=0
99   DO 99 J=1,K
99      VARSQ=VARSQ+(SAVE(J)-AVECOE)**2/(B-1.)
   VAR=VARSQ**.5
   PUNCH 98,AVECOE,VAR
   L=0
C THIS FINDS VARIOUS Y VALUES
43   M=0
   A=0
Z = 0
L = L + 1
IF (L - 2) 41, 41, 42

41 A = A + 2
B = USE (L) + ALSO (L) * (LOG F (A) / 2.303)
M = M + 1
ELEM (L * M) = 10 * B
ERROR (L * M) = ELEM (L * M) ** 5
IF (A - 4) 41, 43, 43

42 Z = Z + 10.
D = USE (L) + ALSO (L) * Z
M = M + 1
ELEM (L * M) = 10 * D
ERROR (L * M) = ELEM (L * M) ** 5
IF (Z - 200) 42, 44, 44

44 IF (L - 6) 43, 45, 45
L = 0
M = 0
J = 1
DO 62 L = 1 * 5
SUMER (L) = 0
SUMRAT (L) = 0
J = J + 1
DO 62 M = 1 * 20
C THIS CALCULATES THE RATIO
RATIO (L * M) = ELEM (L * M) / ELEM (J * M)
SUMRAT (L) = SUMRAT (L) + RATIO (L * M)
C CALCULATION OF ERROR
ERROR (L * M) = RATIO (L * M) * (ERROR (L * M) / ELEM (L * M) + ERROR (J * M) / ELEM (J * M))
62 SUMER (L) = SUMER (L) + ERROR (L * M)
L = 0
C THIS FINDS THE AVERAGE RATIO
DO 63 L = 1 * 5
AVERAT (L) = SUMRAT (L) / 20.
63    AVEER(L) = SUMER(L)/20.
C FINDS FINAL RATIO
   FINRAT = 1.
   FINERR = 0
   L = 0
   DO 70 L = 1, 5
      IF(L-2)68, 69, 68
      IF(L-4)73, 69, 73
      68    AVERAT(L) = 1.
      69    AVEER(L) = 0
      73    FINRAT = FINRAT*AVERAT(L)
      70    FINERR = FINRAT*(AVEER(L)/AVERAT(L)+FINERR/FINRAT)
      J = 1
      DO 38 L = 1, 5
         J = J + 1
      PUNCH 53, L, J
      PUNCH 54
      A = 0
      M = 0
21    FORMAT(F11.3, 5X, F10.4, 12X, F15.4, 8X, F14.4)
   DO 32 M = 1, 20
      IF(L-2)51, 51, 52
      A = A + 2
      52    IF(L-2)32, 32, 52
         A = A + 10.
38    PUNCH 55, A, ELEM(L, M), ELEM(J, M), RATIO(L, M), ERRGR(L, M)
      PUNCH 56, FINRAT, FINERR
      PUNCH 50, A, ELEM(L, M), ELEM(J, M), RATIO(L, M), ERRGR(L, M)
      PUNCH 55, A, ELEM(L, M), ELEM(J, M), RATIO(L, M), ERRGR(L, M)
      PUNCH 56, FINRAT, FINERR
20    FORMAT(3X, 7HXVALUE5, 8X, 13HVALUE OF LOGY, 9X, 12HDEV FROM FIT, 8X,
      110HACTUAL DEV)
22    FORMAT(/6HSUMX = , F12.3, /6HSUMY = , F12.4,
      17HSUMXY = , F15.7, /7HSUMXG = , F16.6)
23    FORMAT(/13HCOEFFUENT A = , F16.6, /13HCOEFFUENT B = , F16.6)
24    FORMAT(/7HNO = , 12, /7HABCOE = , F20.6)
82 FORMAT(19HCHI-SQUARE VALUE = ,F16.5)
1 FORMAT(I2)
2 FORMAT(F6.3,F5.4)
98 FORMAT(26HAVERAGE ABSORPTION COEF = ,F11.8,7H+ OR - ,F11.8)
54 FORMAT(6HXVALUE,10X,10HCOUNTS(L),6X,10HCOUNTS(J),
16X,10HRATIO(L/J),5X,12H+ OR - ERROR)
53 FORMAT(15HRATIO DATA L = ,12,1X,8HAND J = ,12)
50 FORMAT(F7.2,4X,F14.1,3X,F22.1,1X,F14.5,1X,F13.5)
55 FORMAT(15HAVERAGE RATIO = ,F13.8,8H + OR - ,F13.8)
56 FORMAT(26HFINAL RATIO OF EXTREMES = ,F13.8,8H + OR - ,F13.8)
END
BIBLIOGRAPHY


