Angular Correlation Study of the 105 keV Level in Gd$^{155}$

John B. Gouvas

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ANGULAR CORRELATION STUDY
OF THE 105 keV LEVEL IN Gd$^{155}$

by

John B. Gouvas

A Thesis
Submitted to the
Faculty of the School of Graduate
Studies in partial fulfillment
of the
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John B. Gouvas
ANGULAR CORRELATION STUDY OF THE
105 keV LEVEL IN $\text{Gd}^{155}$. 

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INTRODUCTION

Gadolinium $^{155}$ is a stable isotope (abundance 14.73%) lying in the highly deformed nuclei region $153 \leq A \leq 187$. Its excited states can be populated by either the $\beta$-decay of $^{155}$Eu (1.81y) or electron capture in $^{63}$Tb $^{155}$ (5.6d). The nucleus of Gd$^{155}$ has been extensively investigated both experimentally and theoretically $^{1-21}$. The information accumulated on the decay of Eu$^{155}$ and some of the lower lying energy levels with their spin assignments is shown in Figure 1.

This energy level diagram represents the work of many investigators, and the results of these investigations were not always in agreement. It is seen that the spin assignments of the 105 keV level and the 87 keV level still remain undetermined. The spins seem to be either 3/2 or 5/2.

Experimental lifetime measurements $^{11,12,14}$ and g-factor measurements $^{16,17,18}$ favor the 5/2 spin assignment for the 105 keV level, while the lone angular correlation measurement $^{13}$ suggests a spin of 3/2 for this level. Since the spin assignment of 3/2 is reported only by a single angular correlation measurement by Subba-Rao$^{13}$, it seemed that a more accurate angular correlation measurement was necessary. This paper reports a measurement of the angular correlation of the 45-60 keV cascade, and the analysis of the angular correlation in an attempt to remove the ambiguity of the 105 keV level spin assignment.
Fig. 1. Levels of $^{155}\text{Gd}_{64}$ populated in the decay of $^{155}\text{Eu}_{63}$. 

$^{155}\text{Eu}_{63}$
HISTORICAL REVIEW

The ground state of the gadolinium 155 nucleus is known to have a spin and parity of $3/2^+$. Work on the excited states began in 1955 with Heydenberg and Temmer who measured the energy ratio of the second to the first rotational levels and reported the states to be at 145 keV and 60 keV respectively. Various simple decay schemes (Eu$^{155}$) proposed established low lying excited states at 60, 87, 105, 117, 145 keV. Further investigation of the excited states in 1959, by Vergues gave the lifetime of the 105 and 87 keV states as $\tau(87) = 5 \pm 1$ nsec, $\tau(105) \leq 1.2$ nsec. Another measurement of the 105 keV level lifetime was made by Deutsch et al. Their result, $\tau(105) = 0.6^{+0.2}_{-0.1}$ nsec, together with $\tau(87)$ given by Vergues was compared with the theoretical predictions of Nilsson's model. An agreement between experiment and theory was found for the spin assignment $87\rightarrow 3/2^+$, $105\rightarrow 5/2^+$ at a nuclear deformation $\delta = 0.26$.

Angular correlation measurements of the 45-60 keV cascade between the 105 and 60 keV levels by Subba-Rao, gave a function

$$W(\theta) = 1 + (0.155 \pm 0.016) P_2(\cos \theta) + (0.028 \pm 0.025) P_4(\cos \theta)$$

and deduced mixing ratios

$$\delta^2(45) = \frac{M_2}{M_1} = 0.0074 \pm 0.0011 \text{ and } \delta^2(60) = \frac{M_2}{M_1} = 0.04 \pm 0.01$$

According to Rao these ratios imply the $3/2^+\rightarrow 5/2^-\rightarrow 3/2^+$ spin sequence, and thus he assigned a $3/2^+$ spin to the 105 keV level in disagreement.
with the above mentioned work. Subsequent work, first by Vergues\textsuperscript{14} comparing B(E1) with theoretical predictions of Nilsson's model, and second Tb\textsuperscript{155} decay studies by Harmatz et al.\textsuperscript{15}, again report a spin of 5/2 for the 105 keV state. Also g-factor measurements of the 87 keV level\textsuperscript{17,18}, indicate that this level has a spin of 3/2, hence leaving the 5/2 assignment to the 105 keV level. The two most recent investigations: (1) study of Tb\textsuperscript{155} decay by P. Blickert-Toft et al.\textsuperscript{19}, and (2) study of internal conversion electrons in the decay of Tb\textsuperscript{155} by J. Kormicki et al.\textsuperscript{20}, are again in disagreement as to the 105 level spin, the first favoring the 5/2 while the second reports 3/2.

At this point it was felt that, in order to remove the ambiguity existing in the assignment of the 105 keV level spin, more angular correlation measurements were necessary. Since the 60 keV level and ground state are known to possess $I_\pi = 5/2^-$, $3/2^-$ respectively, such a measurement for the 45-60 keV cascade seems quite straightforward and unambiguous.
THEORETICAL CONSIDERATIONS

Multipolarity and Mixing Ratios

The selection rules for decay by γ-emission are well known, for example, see Burcham. They are given by the angular momentum and parity change,

\[ \Delta j = |j_1 - j_2| \leq L \leq j_1 + j_2, \quad \Delta \pi = (-1)^L \text{ for } EL \]

\[ (-1)^{L+1} \text{ for } ML. \]

Both rules applied to the 45 and 60 keV gamma rays show:

\[ \Delta \pi = -1 \text{ (change) and } \Delta j = 0,1: EL(M2) \text{ for } 45 \text{ keV}, \]

\[ \Delta \pi = +1 \text{ (no change) and } \Delta j = 1,2: ML(E2) \text{ for } 60 \text{ keV.} \]

However, while M2 can usually be discarded in an EL(M2) combination, the E2 of ML(E2) is able to compete with M1, so the resulting radiation is a mixture of the two. The observed intensity from such radiation has a mixing ratio defined by \[ \delta^2 = \frac{E_2}{M_1}. \]

Therefore, in the present investigation of the 45-60 keV cascade, \[ \gamma_1 \]

was assumed initially to arise from a pure EL transition and \[ \gamma_2 \]

was assumed to be a mixed M1, E2.

Gamma-Gamma Cascade

A nucleus may decay from an initial state of angular momentum \[ j_1 \]

to a final state \[ j_2 \]

through an intermediate state of angular momentum \[ j, \]

emitting two γ-rays, \[ \gamma_1, \gamma_2 \] having multipolarities \[ L_1, \]

\[ L_2 \]

respectively if there is no mixing.

Since the spin of the nuclei comprising a sample are usually
randomly oriented, the individual photons are emitted isotropically from individual transitions \((j_1 \rightarrow j, j \rightarrow j_2)\) will have no preferred direction of emission. However, in a double cascade transition \((j_1 \rightarrow j \rightarrow j_2)\) there is an angular dependence between the direction of the two successive \(\gamma\)-rays emitted from the same nucleus. The lifetime of the intermediate state being short compared to the reorientation time of the nucleus.

\[ W(\theta) = \sum_{\nu} A_{\nu} P_{\nu}(\cos \theta) \]

where \(P_{\nu}(\cos \theta)\) are the Legendre polynomials, \(A_{\nu}\) are coefficients of the series expansion with \(A_0 = 1\), \((W(\theta)\) normalized) and \(\nu\) is an

---

Fig. 2. Typical \(\gamma-\gamma\) cascade

Angular Correlation Function \(W(\theta)\)

The theory of angular correlation has been developed extensively in the literature for cases of both pure and mixed radiation transitions.
even integer ranging from $\nu = 0$ to $\nu = \min. 2(L_1, L_2, j)$. The 
$0 \leq \nu \leq \min. 2(L_1, L_2, j) $
selection rule for $\nu$ arises from the application of the triangle 
rule on the Racah coefficients, (associated with the coefficients $A_\nu$). In case of half integer spin, $j$ is replaced by $j - 1/2$ in the max. expression.

**Pure Cascade**

The angular correlation function for a double cascade, in 
which both $\gamma$-rays have pure multipole nature, is fairly simple and 
is given by $W(\theta) = \sum A_\nu P_\nu(\cos \theta)$, where the coefficients are 
defined by

$$A_\nu = F_\nu(L_1 j_1 j) F_\nu(L_2 j_2 j)$$

the new coefficients $F_\nu$ being calculated and tabulated.

**Mixed Cascade**

The angular correlation function for the case in which one 
of the $\gamma$-rays of the cascade is not pure: $j_1 \xrightarrow{L_1} j \xrightarrow{L_2} j_2$ with 
$L_1' = L_1 + 1$, is more complex and is given by

$$W(\theta) = W^I(\theta) + \delta^2 W^{II}(\theta) + 25W^{III}(\theta)$$

where

$$W^I(\theta) = \sum A^I_\nu P_\nu(\cos \theta), \quad A^I_\nu = F_\nu(L_1 j_1 j) F_\nu(L_2 j_2 j)$$

$$W^{II}(\theta) = \sum A^{II}_\nu P_\nu(\cos \theta), \quad A^{II}_\nu = F_\nu(L_1' j_1 j) F_\nu(L_2 j_2 j)$$

are the functions resulting from two separate pure cascades

$\xrightarrow{L_1} j \xrightarrow{L_2} j_2$ and $\xrightarrow{L_1'} j \xrightarrow{L_2} j_2$
\[ W_{III}(\theta) = (-1)^{j-j_1} \left[ (2j + 1) (2L_1 + 1) (2L'_1 + 1) \right]^{1/2} \times \sum \nabla G_{\nabla} (L_1 L'_1 j_1 j_1) F_{\nabla} (L_2 j_2 j) P_{\nabla} (\cos \theta) \]

is the interference term. The coefficients \( G_{\nabla} \) are also tabulated. \( \delta^2 \) is the intensity ratio of the \( 2^L \) pole to that of the \( 2^L' \) pole, defined as:

\[
\delta^2 = \frac{\text{Intensity of } (L_1 + 1) \text{ pole}}{\text{Intensity of } (L_1) \text{ pole}}
\]

and \( \delta \) is defined as the (amplitude) mixing ratio.

Now \((-1)^{j-j_1} \left[ (2j + 1) (2L_1 + 1) (2L'_1 + 1) \right]^{1/2} \times \sum \nabla G_{\nabla} (L_1 L'_1 j_1 j_1) = F_{\nabla} (L_1 L'_1 j_1 j_1). \) The coefficients \( F_{\nabla} \) are tabulated and obey the condition \( F_{\nabla}(L_1 L'_1 j_1 j_1) = \delta_{L'_1} \). One also defines

\[ A_{III} = F_{\nabla}(L_1 L'_1 j_1 j_1) F_{\nabla}(L_2 j_2 j) \]

\[ W_{III}(\theta) = \sum A_{III} P_{\nabla}(\cos \theta) \]

\[ W(\theta) = \sum A_{I} P_{\nabla}(\cos \theta) + \delta^2 \sum A_{II} P_{\nabla}(\cos \theta) + 2\delta \sum A_{III} P_{\nabla}(\cos \theta) \]

The normalized coefficients \( A_{\nabla} \) of the general expansion are given by

\[
A_{\nabla} = \frac{A_{I} + \delta^2 A_{II} + 2\delta A_{III}}{1 + \delta^2}
\]

and can be calculated as a function of \( \delta \), since the coefficients necessary to calculate the \( A_{\nabla} \) have been tabulated.

In this case \( W(\theta) \) depends on six quantum numbers, \( L_1, L'_1, L_2, j_1, j, j_2, \delta \), (since \( L'_1 = L_1 + 1 \)), therefore with \( L L_j j_1 j_2 \delta \) and \( \delta \) value determined previously, one can predict the value of \( j_1 \) using

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the measured angular correlation function.

The case in which both $\gamma$-rays are of mixed multipolarity is described by the function:

$$W(\theta) = \sum_{\gamma} F_\gamma(\omega) \left\{ F_\gamma(L_1 j_1 j) + \delta_1^2 F_\gamma(L_1^' j_1 j) \right\}$$

$$+ 2\delta_1 (-1)^{j_1-j_1} \left[ (2j + 1) (2L_1 + 1) (2L_1^' + 1) \right]^{1/2} G_\gamma(L_1 L_1^' j_1 j) \right\}$$

$$x \left\{ F_\gamma(L_2 j_2 j) + \delta_2^2 F_\gamma(L_2^' j_2 j) +$$

$$2\delta_2 (-1)^{j_2-j_2} \left[ (2j + 1) (2L_2 + 1) (2L_2^' + 1) \right]^{1/2} G_\gamma(L_2 L_2^' j_2 j) \right\}^{25},$$

which is normalized to $(1 + \delta_1^2) (1 + \delta_2^2)$. Following the convention given above we see that the coefficients of the general expansion are given by $A_\gamma = A_\gamma(L_1 L_1^' j_1 j) A_\gamma(L_2 L_2^' j_2 j)$, where

$$A_\gamma(L_1 L_1^' j_1 j) = \frac{F_\gamma(L_1 L_1^' j_1 j) + 2\delta_1 F_\gamma(L_1 L_1^' j_1 j) + \delta_1^2 F_\gamma(L_1 L_1^' j_1 j)}{1 + \delta_1^2}$$

$$A_\gamma(L_2 L_2^' j_2 j) = \frac{F_\gamma(L_2 L_2^' j_2 j) + 2\delta_2 F_\gamma(L_2 L_2^' j_2 j) + \delta_2^2 F_\gamma(L_2 L_2^' j_2 j)}{1 + \delta_2^2}$$

**Theoretical Predictions**

The 105 keV level is known by other means to have spin either 5/2$^+$ or 3/2$^+$ while the 60 keV level and ground state are known to be 5/2$^-$ and 3/2$^-$ respectively. The two possibilities for the cascade transition are shown in Figure 3.
In this case we have given

\[ j_1 = 3/2 \text{ or } 5/2, \quad j = 5/2, \quad j_2 = 3/2 \]
\[ L_1 = 1, \quad L_2 = 1, \quad L' = 2 \]
and \( \gamma_{\text{max}} = \min(2, 1, 5/2 - 1) = 2 \)

Therefore, expressing the angular correlation function as function of \( \delta \) we have:

\[ j_1 = 5/2: \quad W(\theta) = 1 + \delta^2 + \{-0.16 + 0.08165^2 + 0.81325\} P_2(\cos\theta) \]
\[ j_1 = 3/2: \quad W(\theta) = 1 + \delta^2 + \{0.14 - 0.07145^2 - 0.71066\} P_2(\cos\theta) \]

Since the purpose of this investigation is to determine the spin

---

Fig. 3. Possible \( \gamma-\gamma \) cascades between 105-60 keV levels of Gd\(^{155}\).
of the 105 keV level \((j_1)\), we take the best value of 5, obtained from previous experimental measurements, and compare the measured angular correlation function \(W(\theta)\) with the above \(W(\theta)\) for \(j_1 = 5/2\) and \(j_1 = 3/2\).

The mixing ratio of the 60 keV transition has been measured by several people using two different methods (for example see H. Harmatz et al.\(^{15}\), D. Ashery et al.\(^{22}\)). The results of these measurements are in reasonably good agreement, being:

\[
\begin{align*}
\delta^2 &= 0.0385^{15} \\
\delta^2 &= 0.0400^{21} \\
\delta^2 &= 0.0345^{19} \\
\delta &= -0.228^{22}
\end{align*}
\]

Taking therefore the average of these values,

\[
\delta^2 = 0.0412 \quad \text{and} \quad \delta = -0.203,
\]

and using Rose's formulae\(^{25}\), the theoretical expressions for the angular correlation \(W(\theta)\) are calculated for both possible spin assignments of the 105 keV state: \(j_1 = 3/2, j_1 = 5/2\).

The results of these calculations are shown in Figure 4.
Fig. 4. Theoretical possibilities for $W(\theta) = 1 + A_2 P_2(\cos \theta)$ with $\langle \phi \rangle = -0.203$. 

$\frac{3}{2}, A_2 = 0.27$ and $\frac{5}{2}, A_2 = 0.31$. 

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THE EXPERIMENT

Coincidence Method

The combination of high counting efficiency, short pulse duration, at a moderate energy resolution, makes the NaI-Tl crystals suitable for coincidence measurements. Two photomultipliers receive the light emitted by the scintillating crystals optically coupled to them and produce voltage pulses. These pulses usually require amplification before they are suitable for analysis. The amplification is accomplished by linear amplifiers, which deliver shortened pulses with a fast rise-time. The pulses representing the two γ-rays are then selected by single channel analyzers and then fed to the coincidence circuit, where they are either counted as a coincidence or are rejected, depending on the resolving time (2τ) of the coincidence circuit.

Resolving Time

The simultaneity of the arriving pulses is determined by the resolving time (2τ) of the coincidence circuit. The resolving time is the minimum time interval between two pulses, at which the coincidence circuit senses both pulses separated. Thus, if two pulses are separated by a time interval t, such that t<2τ they will be counted as a single pulse, while if t>2τ they will be rejected. In the present investigation of the 45-60 keV cascade of Gd\textsuperscript{155}, it had been previously determined that the mean lifetime of the intermediate state (60 keV) is 9.7 × 10\textsuperscript{-11} sec\textsuperscript{19}. Hence the resolving
time of the coincidence circuit should be greater than $9.7 \times 10^{-11}$ sec to produce a coincidence pulse from each rapid succession of the two $\gamma$-rays, $(\gamma_1, \gamma_2)$. A first attempt was made with a resolving time of 110 nsec. However, the number of accidental coincidences was far too large in comparison with the number of true coincidences. Thus a smaller setting of the resolving time had to be used. The ultimate resolving time of the equipment used is $2\tau = 10$ nsec.

Considerable amount of data have also been taken at a setting of $2\tau = 26$ nsec, which is the resolving time used by Subba-Rao, for the purpose of comparison and also at $2\tau = 15$ nsec. The angles at which coincidence counting rates have been determined ranged from $90^\circ$ to $270^\circ$, in $30^\circ$ steps.

**Counting Rates**

A source of $\gamma$-rays undergoing $N_0$ disintegrations per second and subtending solid angles $\omega_1$, $\omega_2$ at detectors with efficiencies $\epsilon_1$, $\epsilon_2$, will produce a total coincidence rate

$$N = N_{\gamma\gamma} + N_{\gamma\gamma}$$

where $N_{\gamma\gamma}$ is the rate of true coincidences and $N_{\gamma\gamma}$ is the rate of accidental coincidences. The true coincidence rate is given by

$$N_{\gamma\gamma} \propto N_0 \omega_1 \omega_2 \epsilon_1 \epsilon_2$$

where the proportionality constant is $W(\Theta)$, and the accidental coincidence rate is given by

$$N_{\gamma\gamma} = 2\tau N_1 N_2$$

where $N_1$, $N_2$ are the rates due to the single $\gamma$-rays given respectively by $N_1 = N_0 \omega_1 \epsilon_1$, and $N_2 = N_0 \omega_2 \epsilon_2$. 

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A coincidence measurement then, consists of determination of the counting rates $N_1, N_2, N_3$ which by the relations

$$N_{sc} = 2\gamma N_1 N_2$$

and

$$N_t = N_{\gamma\gamma} + N_{sc}$$

furnish the rate of true coincidences.

The Apparatus

Source

The source, 4mc of Eu$^{155}$ in a normal solution (0.86 acid) of HCl, was obtained from Oak Ridge National Laboratory (Oak Ridge, Tennessee) where it was prepared by irradiation of Sm$^{154}$.

A small sample of the source was placed in a Lucite cylindrical container with inner diameter of 2 mm and maximum source length of 1 mm. Such a source is, to a good approximation, a point source. The distance from the source to the face of the detectors was 20 cm.

Detectors

The detector system consisted of NaI(Tl) crystals of right cylindrical shape (radius = 2.54 cm, height = 1.27 cm) coupled to EMI - 9536 photomultiplier tubes (EMI Electronics LTD. Middlesex, G. B.), as shown in Figure 6. The measured resolution of the present crystals is about 11% at 662 keV. A schematic diagram of the angular correlation table is shown in Figure 7. It consists of a ring which serves both as a base for the detectors and as a guiding rail for the sliding detector, and a centered source holder.
Electronic equipment and arrangement

A block diagram of the experimental arrangement is shown in Figure 8. As preamplifiers two White-type cathode followers were used. The linear amplifiers, single channel analyzers and coincidence unit were ORTEC models: 410, 420, and 414A respectively, while the electronic scalers were borrowed from Argonne National Laboratory.
Fig. 6. Detector system.
Fig. 7. Angular Correlation Table.
Fig. 8. Arrangement of electronic apparatus.
Measurement

Resolving time verification

Since the accidental coincidence rate is given by $N_{sc} = 2\tau N_1 N_2$ and since the rate of the true coincidences ($N_{\gamma\gamma}$) is given by $N_{\gamma\gamma} = N_t - N_{sc}$, the precise determination of the true coincidences will depend on obtaining $N_{sc}$ accurately. Therefore, one makes sure that $N_{sc}$ is the same at all angles.

In this experiment the values of $N_1, N_2$ at each position was constant to within 1%. This being done, the accidental coincidence rate will be constant with adequate accuracy, provided the actual value of the resolving time is known. Therefore a verification of the resolving time is necessary before one proceeds with the measurement. This has been accomplished in the present experiment as follows: two Eu$^{155}$ samples of roughly the same strength were placed in front of the detectors, the two systems being separated by lead shielding. This assured that all the coincidence counts were accidental. The single rates $N_1, N_2$ and accidental rate $N_{sc}$ were determined from counts collected over 12 hour periods. The resolving time was found to be $2\tau = \frac{N_{sc}}{N_1 N_2} = 10 \pm 1$ nsec which is in agreement with the value given by ORTEC.

Test of the equipment

The operation of the electronic equipment was tested by performing an angular correlation of the well-known 1.33 - 1.17 Mev Co$^{60}$ cascade. A source of roughly 10 $\mu$C strength was placed at a distance
Due to the thickness of the present crystals, the resolution of such high energy peaks was not good (11\%). This in turn made the window settings difficult. A measurement of \( W(\theta) \) was first made by setting the windows such that each accepted both \( \gamma \)-rays (1.33 and 1.17). This setting is as effective and much more efficient, especially in this case where there are only 2 \( \gamma \)-rays present. A second measurement was made by separating the windows so one was accepting the 1.17 MeV \( \gamma \)-ray while the other was accepting the 1.33 MeV \( \gamma \)-ray. Although this second setting was somewhat uncertain (due to resolution, the measurements were in good agreement.

The measured angular correlation function was
\[
W(\theta) = 1 + (0.1037 \pm 0.0030) P_2(\cos \theta) + (0.0101 \pm 0.0020) P_4(\cos \theta)
\]
and the anisotropy \( A = 0.1633 \pm 0.0065 \), which are in good agreement with the theoretical values
\[
W(\theta) = 1 + 0.1020P_2(\cos \theta) + 0.0091P_4(\cos \theta)
\]
and \( A = 0.1667 \).

**Window settings**

The windows of the single channel analyzers were set utilizing the anticoincidence and coincidence circuits of the 512 Nuclear Data multichannel analyzer. The necessary connections for the operation are shown in Figure 9. A free spectrum was first taken with coincidence off and the energy scale was calibrated. Due to the relatively poor resolution of the crystals (11\%), and to the fact that both the 45 and 60 keV \( \gamma \)-rays have low intensity, only
Fig. 9. Coincidence connection for window setting.
three peaks were visible in the spectrum. One in the vicinity of
the 45 keV $\gamma$-ray mostly due to the $K_{x}$, $K_{p}$ x-rays (42, 50 keV respecti-
vely), the second due to the 87 keV $\gamma$-ray and the third due to
the 105 keV $\gamma$-ray, not quite resolved. The calibration of the
energy scale was accomplished using the single $\gamma$-ray of $^{57}$Co 122 keV
and the 25 keV x-ray of Sn$^{119m}$. The position of these peaks again
due to the resolution, was not without uncertainty. However, because
of the great care taken, it is estimated that this calibration
located the desired peaks with an uncertainty of about $\pm 1$ channel.

A verification of the 45-60 keV positions was made by taking a
spectrum with a Ge-Li detector which, on account of its high resolu-
tion, after sufficient counting times, produces a visible 60 keV
peak(Figure 10). With the gains adjusted such that all peaks used
(122, 105, 87, 25 keV) coincided in the two spectra (NaI-Tl & Ge-Li),
the relative position of the 60 keV was precisely calculated from the
Ge-Li spectrum and then compared with that of the original calibration.
The baseline and width of the single channel analyzers were next
set so the resulting coincidence spectrum, in comparison with the
free spectrum, positioned the windows at the required peaks. Such
an operation produces on the oscilloscope readout of the multi-
channel analyzer only the peaks accepted by the windows, while the
rest of the spectrum is not visible. Comparing then the coincidence
spectrum with the free spectrum one locates the window positions.
It was found that both anticoincidence and coincidence spectra
located the windows at the same positions. The stationary detector
Fig. 10. Gd^{155} Spectrum taken with Ge-Li detector. The 122 keV peak is interpreted as being due to long lived isotopes Eu^{152} and Eu^{154}.
was arranged to accept the 45 keV γ-ray with a window width of about 6 keV, while the movable detector accepted the 60 keV γ-ray with a window width of about 10 keV. The free spectrum of Gd\textsuperscript{155} and the positions of the windows are shown in Figure 11.

The ratio of the total coincidences to the true coincidences was found to be about 3:1. Our attempts to improve the ratio by making the necessary arrangements so that the experimental parameters \( T \) (total counting time), \( \epsilon_1, \epsilon_2, \omega_1, \omega_2 \) are as large as possible, proved futile. A way of explaining the poor ratio of 3:1 is to consider the γ-rays of Gd\textsuperscript{155}. Both the 45 and 60 keV γ-rays, being very weak, do not give pronounced peaks. Thus a window of 10 keV width, accepting the 60 keV γ-ray, will also certainly accept the 57 keV γ-ray (117-60) thus adding accidental coincidences, while a window of 6 keV accepting the 45 keV γ-ray does not eliminate interference of so many closely spaced γ-rays under one visible peak (30, 41, 42, 45 keV).

Furthermore, since some of these γ-rays come from higher lying levels, as for example the 57 keV γ-ray from the 117 keV level to the 60 keV level, pile-up affects might be present, i.e. coincidences might be recorded which are in fact true coincidences but are not desirable in the present measurement.

However this kind of coincidences was checked, by placing both windows first at the 60 keV peak and then at the 45 keV peak. It was found that possible coincidences between 57-60 keV beyond stray coincidences was almost negligible within experimental error, while
Fig. 11. Spectrum taken with NaI-Tl detector and the window positions.
some coincidences were recorded at the 45 keV position. This may be interpreted as coincidences between the 42.5 keV x-ray and the 45 keV γ-ray.

The 42.5 keV γ-ray is also in coincidence with the 60 keV (actually it is in coincidence with almost all the γ-rays in the spectrum). However any possible contribution to the angular correlation from these coincidences was discarded after counts at different angles showed them to be isotropic.
RESULTS AND ANALYSIS

The interpretation of the low lying structure of Gd$^{155}$, given by several authors is that the 60 keV (5/2-) and 146 keV (7/2-) levels are the first and second rotational members of the ground state band built on the $[521] 3/2^-$ Nilsson orbital, and that the 87 keV and 117 keV levels may belong to a band built on the orbital of the 87 keV level. The E1 character of transitions from the 105 keV and 87 keV to the ground state $^{11,15}$ settles the positive parity of these two levels and from the systematics of low lying neighboring nuclei, possible spins are 3/2 and 5/2 alternatively. The theoretical analysis of the electromagnetic lifetimes$^{12}$ is most consistent with the assignment $^{651} 3/2^+$ and $^{642} 5/2^+$ for the 87 and 105 keV levels respectively. In the above analysis the interpretation is valid only with the assumption that the 87 keV state is a pure Nilsson state, which may not be the case considering the Coriolis interaction and the possibility of considerable admixtures of vibrational states into single particle states. The possibility of admixtures is indicated by the g-factor measurements$^{18}$ where the experimental result $g = 0.384 \pm 0.030$ compared with values derived from theory, shows that in no case can the 87 keV level be interpreted as a pure Nilsson state. An inspection of the Nilsson diagram for the intrinsic single-neutron states of $A = 155$, shows that low lying states with spins 3/2 and 5/2 and positive parity may correspond to the Nilsson orbitals $^{402} 3/2^+$,
[651] 3/2+, and [642] 5/2+. However, if the 87 keV level is of predominant [402] 3/2+ character, no sensible admixtures of other states can explain the experimental results. From the two possibilities [651] 3/2+ and [642] 5/2+ left the [651] 3/2+ is discarded after a calculation of the Coriolis interaction, which is considerable in odd-nuclei, so the 87 keV state is assigned to the [642] 5/2+ Nilsson orbital, thus relinquishing the [651] 3/2+ orbital for the 105 keV state. The angular correlation measurement by Subba-Rao, as it was pointed out earlier also reported a spin of 3/2 for the 105 keV level. The analysis of the results of this investigation are in agreement with such an assignment.

The results of the calculation for the theoretical expressions of the angular correlation function for the two possible cases of the 105 keV spin have been plotted in Figure 4. It can be seen from this graph that the angular correlation function of the j_1 = 5/2 case is quite different from that of the j_1 = 3/2, and it gives a negative coefficient A_2 in the expansion W(θ)=\sum_A A_r P_r(cosθ), while the j_1 = 3/2 case gives a positive A_2.

The experimental data, coincidence counts N_{γγ}, were least-square fitted to the Legendre polynominal expansion and the measured angular correlation function was represented by:

\[ W(θ) = 1 + (0.15 \pm 0.06)P_2(cosθ) \]

This function together with the experimental points is plotted in Figure 12. This representation giving a positive coefficient A_2,
Fig. 12. Experimental data (error bars are purely statistical) and the representation of the fitted function $W(\theta) = 1 + (0.15 \pm 0.06)P_2(\cos\theta)$. 
would determine the spin of the 105 keV level as 3/2, however as it can be seen from this plot, the 180° count is somewhat low. It was noticed therefore that the experimental points could be fitted better by the admixture of a small amount of $P_4(\cos\theta)$ term, whose presence indicates a mixed 45 keV transition with a ratio $\delta^2_1 = \frac{M2}{El}$.

Up to this point such competing multipoles have been discarded as being negligible, however cases of considerable M2, El mixture have been reported. The least-square fit was therefore carried to the $A_4$ term, and the resulting angular correlation function was represented by

$$W(\theta) = 1 + (0.15 \pm 0.6)P_2(\cos\theta) - (0.03 \pm 0.03)P_4(\cos\theta)$$

This function together with the experimental points is shown in Figure 13, where it can be seen that the data are somewhat better fitted.

The coefficients $A_2$, $A_4$ expressed as a function of $\delta_1$, $\delta_2$ in terms of the general correlation function, given previously for both transitions mixed, can supply information about the mixing ratio $\delta_1$.

In this case, the known value $\delta^2_2 = 0.0412$ together with the measured coefficients $A_2 = 0.15$, $A_4 = -0.03$ and the use of tables gave a mixing ratio of $\delta^2_1$ about 1% for the $j = 3/2$ case. The data could also be consistent with the $j = 5/2$ case, that is the 5/2 case could also produce a coefficient $A_2$ positive, but a calculation showed that such a case would require a very large mixing ratio of...
Fig. 13. Experimental points and the representation of the fitted function.

\[ W(\theta) = 1 + (0.15 \pm 0.6)P_2(\cos \theta) - (0.03 \pm 0.03)P_4(\cos \theta) \]
about 50% M2, E1 for the 45 keV transition and such ratio is highly unlikely. Therefore this experiment favors the $j = 3/2$ spin assignment of the 105 keV level, with a possible mixed 45 keV transition M2 + E1, of mixing ratio about 1%.

It is felt that this experiment can be greatly improved by the use of higher resolution detectors, since then the windows could be set with adequate precision thus eliminating contributions to the angular correlation from coincidences due to the 42.5 keV x-ray. One possibility of such improvement could be the use of two Ge-Li detectors.
BIBLIOGRAPHY


