The Bellringer Sequence: Investigating What and How Preservice Mathematics Teachers Learn Through Pedagogies of Enactment

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THE BELLRINGER SEQUENCE: INVESTIGATING WHAT AND HOW
PRESERVICE MATHEMATICS TEACHERS LEARN
THROUGH PEDAGOGIES OF ENACTMENT

by

Mary A. Ochieng

A dissertation submitted to the Graduate College
in partial fulfillment of the requirements
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THE BELLRINGER SEQUENCE: INVESTIGATING WHAT AND HOW PRESERVICE MATHEMATICS TEACHERS LEARN THROUGH PEDAGOGIES OF ENACTMENT

Mary A. Ochieng, Ph.D.
Western Michigan University, 2018

This study examines preservice teacher learning through pedagogies of enactment—approaches to teacher education that allow preservice teachers to learn by doing what teachers do. Preservice teacher (PST) learning is examined through the implementation of the Bellringer Sequence (BRS), a pedagogy of enactment conceptualized in the study. The BRS is centered around bellringers—brief mathematical tasks implemented as students arrive for class. The BRS is a sequence of four activities centered on a bellringer: preparation (for teaching a bellringer) implementation (of the bellringer with peers), debriefing (discussing the implementation as colleagues), and written reflection (about the effectiveness of the bellringer).

Practice-based approaches to teacher preparation have been emphasized as a way of addressing the disconnect between what goes on in teacher preparation programs and school classrooms. Pedagogies of enactment are considered to be practice-based approaches to teacher education because they focus on preservice teachers’ learning of what teachers do. However, little is known about how PST learning takes place through these pedagogies. This study investigates both what and how PSTs learn through the BRS in the context of a middle school mathematics methods course.
Data collected from 11 PSTs enrolled in a middle school mathematics methods course at a Midwestern university included audio recordings of PST-instructor preparation meetings, video recordings of the methods class sessions, interviews with each PST after implementation of their bellringer, written reflections by the PSTs, and artifacts from class related to the BRS. Instances of PST learning were identified and examined for what was learned. To understand how PSTs learned, events that prompted these instances of learning were identified and examined for the nature of the conversations within them. Constant comparative analysis was applied to extract common themes across PSTs’ expressions of the sources that they drew their learning from and also to characterize statements made in the learning prompts.

Learning prompts varied by location and directness in addressing the ideas learned. The examination of local explicit learning prompts revealed three stages: initiation—ideas learned are surfaced; precisification—the ideas are clarified; and, in some prompts, equilibration—PSTs assimilate and adapt the ideas to their knowledge and experiences. Precisification played an important role in shaping ideas in the learning process. The different foci of the BRS phases allowed for rich learning in an integrated way. PSTs learned content knowledge, pedagogical knowledge, pedagogical content knowledge, and high leverage practices. The dual role of PSTs as teachers and learners supported learning as they projected both the identity of a learner and a teacher in the learning prompt discussions.

The results of this study highlight the potential for supporting PST learning through pedagogies of enactment that have specific instructional goals and are centered around mathematical tasks. They also highlight the important role in PST learning, of classroom norms where PSTs feel responsible for one another’s learning and are free to share their ideas.
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Mary A. Ochieng
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CHAPTER 1

INTRODUCTION

Reforms in mathematics education, such as those advocated for and explicated in reform documents published by the National Council of Teachers of Mathematics (NCTM, 1989, 2000, 2014) and the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010), have resulted in higher expectations for teachers of mathematics. Some of the effective teaching practices outlined in NCTM (2014) include eliciting and using student thinking to inform instructional decisions, using purposeful questions, implementing tasks that allow students to engage in mathematical reasoning and problem solving, and allowing students to grapple with mathematical ideas. These effective practices are “necessary to promote deep learning of mathematics” (p. 9) and thus would be expected even of beginning teachers.

These higher expectations for beginning teachers are also echoed in the Standards for Preparing Teachers of Mathematics (Association of Mathematics Teacher Educators [AMTE], 2017). The bar is set even higher as these standards for teacher preparation consolidate ideas related to teaching and learning from various NCTM documents, CCSSM and ideas from research related to the teaching and learning of mathematics. The AMTE 2017 standards address four areas of knowledge that “well-prepared beginning teachers of mathematics” should acquire: “mathematics concepts, practices and curriculum; pedagogical knowledge and practices for teaching mathematics; students as learners of mathematics; and social contexts of mathematics teaching and learning” (p. 6). The effective teaching practices outlined in NCTM (2014), and
mentioned above, make up the indicators of knowledge of students as learners of mathematics in the AMTE standards. Therefore, teacher education programs need to support preservice teachers in learning how to elicit and probe student thinking through effective questioning and how to use prompts to extend their reasoning about mathematical ideas. Preservice teachers should have opportunities to select and implement tasks that engage students in thinking at a high level of cognitive demand (Smith & Stein, 1998) and to facilitate discourse around the tasks in ways that maintain the cognitive demand, thus allowing for productive struggle. Even practicing teachers need support to develop these effective teaching practices (NCTM, 2014); therefore, teacher preparation programs should deliberately incorporate instructional activities that will support preservice teachers’ development of such practices.

**Practice-Based Approaches to Teacher Education**

Concerns about the disconnect between what teachers learn in teacher preparation programs and what goes on in actual classrooms have led to calls for a greater emphasis on *practice* in teacher education (Ball & Cohen, 1999; Ball & Forzani, 2009). The word *practice* in the sense that is used here refers to doing the work of a particular profession or occupation; thus, for teaching, practice is the work that teachers do (Lampert, 2009). Approaches to teacher education that focus on learning the work that teachers carry out, without any implications for where that learning takes place, are referred to as *practice-based approaches* (Ball & Cohen, 1999). Note that this is in contrast to more recent use of the term *practice-based* to refer to programs that offer more extensive opportunities for preservice teachers to engage in actual classroom teaching (Forzani, 2014).

In this study I use the term *practice-based* in the way Ball and Cohen (1999) define it, as independent of location and focused on preservice teachers’ learning of the work entailed in the
occupation of teaching. Practice-based approaches in themselves may not be the panacea to all the challenges of teacher education. For instructional approaches in teacher education to be responsive to reforms, they need to facilitate preservice teachers’ development of “high-leverage practices”—practices that have an impact on student learning (Ball & Forzani, 2010, p. 45).

Therefore, irrespective of the location, whether in university or school classrooms, practice-based teacher education is distinguished by a “systematic focus on developing teacher candidates’ abilities to successfully enact high-leverage practices” (Zeichner, 2012, p. 378). For teacher education programs to influence mathematics instruction in schools, attention to student learning is critical (Forzani, 2014). The discussion in the next paragraph gives some insight on why a lack of attention to student learning may be one of the pitfalls of earlier practice-based approaches.

Early efforts to prepare teachers for practice include the Commonwealth Teacher Training Study in the early 20th century. The study surveyed teachers on the tasks and activities of their work and formed a list that could be used to inform the design of teacher training curriculum. However, the study did not have much influence on teacher education (Forzani, 2014; Zeichner, 2012). According to Forzani (2014), the project did not identify teaching practices that would be important for beginning teachers. A focus on practice simply based on what teachers do may not support preservice teachers to develop the ability to facilitate instruction that supports students’ learning in the ways articulated in standards documents (e.g., NCTM, 1989, 2000; NGA & CCSSO, 2010). Early practice-based approaches to teacher education were more focused on preservice teachers’ acquisition of teaching skills and lacked the kind of attention to student learning that is advocated for in the reforms in mathematics education (Forzani, 2014). Some of the more recent research related to practice-based
approaches has been more deliberate in relating teachers’ practice to student learning (e.g., Kazemi, Franke, & Lampert, 2009; Lampert et al., 2013). These studies have been more explicit about the interactions and processes involved in the implementation of practice-based approaches. However, little is known about how practice-based approaches actually support preservice teachers’ learning. Better understanding how practice-based approaches to teacher education support the development of teaching practice would help teacher education programs to be more responsive to reform expectations.

**Pedagogies in Teacher Education**

Insights into practice-based approaches may be facilitated by an examination of the pedagogies in which they are grounded. In the context of teacher education, Grossman (2005) defined pedagogy as classroom instruction and interaction, as well as tasks and assignments. She included tasks and assignments because they “represent a crucial ingredient in the pedagogy of teacher education, as they focus students’ attention on particular problems of practice and introduce them to ways of reasoning or performing” (p. 426). In her review of research on pedagogical approaches in teacher education, Grossman applied a broad categorization of pedagogies—micro-teaching and laboratory experiences, computer simulations, use of video technology and hypermedia, case methods, portfolios, and practitioner research—that she acknowledged was not exhaustive. Many of these pedagogies could be practice-based, but this is not true for all pedagogies in teacher education. For example, teacher education pedagogies based on lecture methods of instruction are not likely to be practice-based.

Another way to categorize pedagogies in teacher education is by describing the activities that preservice teachers are involved in as these pedagogies are implemented in teacher preparation programs. Examples include pedagogies of investigation (Lampert & Ball, 1998);
pedagogies of enactment (Grossman & McDonald, 2008; Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010); and pedagogies of reflection (Grossman & McDonald, 2008). These categories, however, are not mutually exclusive. For example, Sims and Walsh (2009) described reflective practice as a pedagogy of investigation, but it is also a pedagogy of reflection. Pedagogies of investigation and pedagogies of reflection may or may not be practice-based depending on their focus, but pedagogies of enactment are considered to be practice-based approaches to teacher education because they focus on preservice teachers’ learning of what teachers do (Lampert et al., 2010).

Ball and Cohen (1999) argued that knowledge is important, but by itself it cannot fully inform appropriate teaching practice; instead teachers must “learn in and from practice” (p. 10) as they implement what they know, adapt, and refine it in specific instructional contexts. Pedagogies of enactment provide preservice teachers with the opportunity to enact the practice of teaching (Grossman & McDonald, 2008) and to learn from that enactment. Therefore, they provide opportunities for preservice teachers to develop high-leverage practices—practices that, when enacted skillfully, have the effect of advancing students’ learning (Ball, Sleep, Boerst, & Bass, 2009). For these reasons, I focus on pedagogies of enactment in this study.

**Pedagogies of Enactment**

Pedagogies of enactment that have been clearly defined in the literature are micro-teaching (e.g., Allen, 1966; Bell, 2007), lesson study (e.g., Fernandez & Yoshida, 2004; Murata, 2011; Stigler & Hiebert, 1999) and rehearsals (e.g., Kazemi et al., 2009; Lampert et al., 2013). Research on these pedagogies has shown what preservice teachers learn through these pedagogies and explored the opportunities that the pedagogies provide for that learning (e.g., Cheng, 2017; Lampert et al., 2013; Sims & Walsh, 2009). However, little is known about how
preservice teachers’ learning takes place through the implementation of these pedagogies. Additionally, Desforges (1995) argued that research on learning to teach has produced findings on the changes in teachers’ skills, practices, and beliefs, but has not explained these changes in terms of a theory of learning or teaching. Explaining changes in teachers’ skills, practices, and beliefs may require examination of how these changes take place. Such examination has the potential to contribute to the development of a theory of preservice teacher learning.

This study may contribute the development of theory by addressing the following question: How does preservice teacher learning take place during implementation of a pedagogy of enactment in the context of a methods course? In chapter 2, I review the literature on micro-teaching, lesson study, and rehearsals to provide insight on what is known about pedagogies of enactment in teacher preparation and to explain how my study will both draw from and contribute to this body of knowledge.
CHAPTER 2

LITERATURE REVIEW

There are a plethora of pedagogies used in teacher education (Grossman, 2005, p. 429), all of which may impact preservice teacher learning in some way. Micro-teaching, lesson study, and rehearsals all involve enactment of the practice of teaching and have the potential to support preservice teachers’ development of such practice, characteristics that qualify them as “pedagogies of enactment” (Grossman & McDonald, 2008). In this study, preservice teacher learning is examined in the context of a pedagogy of enactment that draws from micro-teaching, lesson study, and rehearsals. Therefore this literature review is limited to a discussion of studies related to these pedagogies of enactment. In addition to reviewing studies on micro-teaching, lesson study, and rehearsals, this literature review introduces the pedagogy of enactment conceptualized for this study, the Bellringer Sequence, and discusses the research on which it is based. The chapter concludes with the specific research questions that the study addresses and the theoretical frameworks that inform the study.

Micro-teaching

Micro-teaching had its beginnings in the early sixties at Stanford University (Macleod, 1987). The use of micro-teaching is intended to reduce the complexity of teaching to facilitate preservice teachers’ learning. In micro-teaching, complexity is reduced by the brevity of preservice teacher’s enactment of a lesson (5 to 10 minutes), focusing on the preservice teacher’s development of a specific skill with a small group of students or peers. Additionally, micro-teaching conducted with peers in a university setting further reduces the complexity by removing
some of the commotion that sometimes comes with school classrooms and may interfere with PST learning (Ball & Cohen, 1999). Typically, in micro-teaching the preservice teacher’s enactment of the lesson would be videotaped to provide feedback for the preservice teacher, and to be used for evaluation by the teacher educator. Research on micro-teaching led to the realization of the value of the use of videotapes in instruction of preservice teachers in the way they capture classroom practices, allowing these practices to be revisited by replaying the videos (Grossman, 2005).

Earlier studies on micro-teaching were predominantly experimental studies that highlighted the effectiveness of micro-teaching (Hargie, 1977), but provided very little understanding of the process of micro-teaching in terms of skills, interactions, and how preservice teachers learn to become teachers. Specifically, more recent studies on micro-teaching, such as the study conducted by Bell (2007), have attempted to get inside the process of micro-teaching by examining the characteristics of the interactions that take place in micro-teaching and how preservice teachers approach the task of micro-teaching. Bell’s study involved the examination of 23 video tapes of micro-teaching: 14 involving math lessons, 2 English language arts, 2 ESL, and 5 on selected topics by preservice teachers on areas with which their peers may be unfamiliar. Data sources also included questionnaires and interviews with the preservice teachers; these sources provided insights about the preservice teachers’ perceptions of the micro-teaching activity and also helped in analysis of the videotapes. Bell used an interactional sociolinguistic perspective to explore preservice teachers’ construction of frames, and emerging and shifting frames.

The results revealed that preservice teachers framed micro-teaching as a course requirement or as performance rather than teaching. Additionally, preservice teachers did not see
themselves as teachers, but rather as students. The professor’s evaluative comments of the micro-
teaching enactment also positioned preservice teachers as students. This positioning was
emphasized further by fellow preservice teachers’ efforts to minimize errors to save their peer
acting in the role of teacher from a negative evaluation.

Qualitative data analysis allowed Bell’s (2007) study to get inside micro-teaching in a
way that the earlier studies in micro-teaching did not. However, Bell did not attend to preservice
teacher learning and how that learning may be supported.

The lack of knowledge of what goes on in micro-teaching may be because of its origins at
a time when experimental studies were the norm in research and most studies on micro-teaching
were experimental comparisons of various treatment combinations (Macleod, 1987). With the
use of qualitative research in mathematics education, the more recent studies on micro-teaching
have provided insights on micro-teaching that were not evident in previous studies.

Sezen-Barrie, Tran, McDonald, and Kelly (2014), in a study of 23 preservice science
teachers, applied a cultural historical activity theory (CHAT) perspective to analyze the
preservice teachers’ reflections on their microteaching experience. The study provided insight
into the social interactions in microteaching and revealed how preservice teachers focused more
in their reflection on mediating artifacts, scientific and educational rules and practices. The study
also revealed some of the challenges faced by preservice teachers during the microteaching
experience, for example, communicating ideas, keeping students focused, and coordinating
teaching roles while co-teaching. Teaching is relational and research methods that allow for
analysis of interactions have potential for greater leverage in preservice teacher education. Other
studies involving micro-teaching that also attend to interactions and are likely to provide insights
into the process involve a variation of micro-teaching called micro-teaching lesson study (e.g.,
Fernandez, 2005, 2010; Fernandez & Robinson, 2006), which I will discuss at the end of the Lesson Study section.

Summary

Micro-teaching has the advantage of reducing the complexity of teaching through a reduced class size and brevity of the lesson. According to Ball and Cohen (1999), not having to deal with a full class in a school setting also reduces interferences with PST learning. Video records of PSTs’ enactment of the lesson allow them to reflect on their teaching and may also be useful for their peers’ learning. However, the positioning of PSTs as students while they are enacting lessons reduces the authenticity of the teaching experience for PSTs. Additionally, there is little understanding of how micro-teaching supports preservice teachers’ learning, therefore limiting teacher educators’ opportunities to leverage micro-teaching for PST learning.

Lesson Study

Lesson study originated as a Japanese form of professional development that involves teachers working collaboratively in a cycle of enactment, reflection, and revision of a lesson, often with the goal of improving some identified aspect of instruction (Lewis & Tsuchida, 1998). The use of lesson study has spread to other countries, including the United States, where it has been used in professional development and is an emerging area of research (Murata, 2011). Its use and accompanying research in teacher preparation is still relatively new. Hence, few studies have been conducted on the use of lesson study with preservice teachers and even fewer have focused on development of the practice of teaching, particularly in mathematics.

Murata and Pothen (2011) provide a description of the use of lesson study with preservice teachers and illustrate its potential for teacher learning and improvement of instruction. They describe how they have used lesson study with preservice teachers in an elementary mathematics
methods course at Stanford University in which lesson study is implemented in the context of three methods courses, which make up a year-long program. During the program preservice teachers are assigned to elementary school classrooms and lesson study groups, which include the classroom teacher. Opportunities for preservice teacher learning are through cycles of enacting and refining the “centerpiece” (Lewis & Tsuchida, 1998), which is the research lesson.

Since the use of lesson study in teacher preparation is relatively new, most of the extant literature is related to exploration of its potential for preservice teachers’ learning. Therefore, research on the use of lesson study in teacher preparation has often examined the opportunities that aspects of lesson study provide for preservice teachers’ learning. For example, Fernandez and Zilliox (2011) investigated elements of lesson study approaches—learner centeredness, knowledge centeredness, assessment centeredness, and community centeredness—that would support preservice teachers’ development as teachers of mathematics. They investigated such elements with elementary and secondary preservice teachers at two universities. Their study found that lesson study approaches supported preservice teachers’ understanding and development of teaching practices aligned with reforms in mathematics. Additionally, the lesson study approaches gave preservice teachers a sense of ownership of the lessons and a view of lessons as “works in progress” (p. 100).

Sims and Walsh (2009) examined the process of integrating lesson study into a two-year early childhood course for preservice teachers. Taking the view of lesson study as providing an opportunity to learn from teaching, they examined preservice teachers’ learning against specific goals—analyzing lessons in light of lesson goals, looking beyond the surface features of a lesson, critiquing the lesson not the teacher, and collecting evidence of student learning. Preservice teacher learning in this study went beyond the articulated goals to supporting preservice teachers
to view their peers and others observing their lessons as people who could provide support to help them improve on what they were not able to do well, rather than as evaluators. The study revealed how lesson study can provide a framework that allows preservice teachers to learn from teaching. Sims and Walsh revealed preservice teachers’ overall gains in learning over the two-year period, despite the challenges faced in the first year of implementation, which was a learning experience for the researchers and informed the revision of goals for the second year of study. The preservice teachers involved in the lesson study did not see the research lesson as their own, but rather as belonging to the person who presented the lesson. Additionally, instead of focusing on the instructional details of the lesson, preservice teachers focused on evaluating the person who presented the lesson. These are just a few of the challenges Sims and Walsh faced in the first year of their study, forcing them to revise the structure of the program in the second year to provide preservice teachers with a proper lesson study experience. Some of the details they attended to in the revised structure included the timing of the lesson study process, composition of lesson study groups by attending to the individual strengths and personalities of the preservice teachers, and even delaying the choice of the lead teacher until the lesson plan was in place to give everyone in the group ownership of the lesson and to encourage collaboration. The lessons learned by Sims and Walsh during the first year of the study show how lesson study may not always give the desired learning outcomes with PSTs. Next, I discuss a study by Parks (2008) where some more undesired learning by PSTs took place.

Parks (2008) examined preservice teacher learning through lesson study in relation to specific mathematical content areas and development of specific pedagogy relating to equity. In alignment with the collaborative nature of lesson study, Parks framed the study using Lave and Wenger’s (1991) theory of learning as participation in a community of practice. The study
incorporated lesson study into a methods course to deepen preservice teachers’ understanding of mathematical content and also to prompt them to pay attention to students who had not been successful in mathematics. In alignment with lesson study’s focus on a particular instructional goal, preservice teachers were given goals to choose from; this was meant to encourage them to have conversations about culture, race, and gender, among other issues. Preservice teachers were to design lessons that addressed one out of three equity-related issues provided by the author (who was also the course instructor). One of the issues provided was “enable significant work for all children” (p. 1203). The data included audiotapes of whole class discussions, small group conversations of the PSTs, and field notes. The purpose of the study was to provide a description of the PSTs’ and researcher’s participation in the lesson study process.

The results revealed that relatively few (only 21 out of 181) episodes were related to mathematics. The groups that had relatively lower numbers of episodes related to mathematics tended to switch discussion to pedagogical issues when mathematical questions were raised. The ability of the groups to develop a mathematical lens was influenced by their dispositions toward mathematics and interactions with the district’s written curriculum. In some of the groups, conversations around one of the tasks revealed beliefs about rounding and estimation that were problematic and were further entrenched by their discussions. However, one group that had a positive disposition toward mathematics had meaningful conversations and even critiqued the way estimation was presented in the curriculum. With regard to equity, a group of preservice teachers attended to students’ ability positively by differentiating instruction in their lesson. However, their attention to students’ ability also reinforced assumptions they held before lesson study that low ability students should be grouped together and that they may not have much to contribute mathematically if grouped with high ability students.
Even though some of the PSTs had the opportunity to develop mathematics and ideas related to equity for teaching, Parks’ (2008) study reveals that some unintended learning that does not align with reform goals for instruction in mathematics may take place during lesson study. The reflection and refining of a lesson is beneficial only if problematic areas are identified and corrected. Thus, in a case where no one in the lesson study group is more knowledgeable than the others, the cycles of enactment and refining may not support PSTs’ development in areas where they have knowledge gaps. This is consistent with the finding of Fernandez and Zilliox (2011) that collaboration by itself did not support preservice teachers’ development in areas where they had knowledge gaps, particularly in situations where there was no interaction with knowledgeable others during the teaching cycles.

Studies examining the use of lesson study with preservice teachers have sometimes applied variations of lesson study for various reasons, including achieving specific learning goals for preservice teachers (e.g., Bieda, Cavanna, & Ji, 2015; Yu, 2011) and allowing for examination of aspects of preservice teacher learning (e.g., Myers, 2012). Micro-teaching Lesson Study (MLS) and Mentor Guided Lesson Study (MGLS) are recent distinctly defined variations of lesson study that I will briefly discuss here.

**Micro-teaching Lesson Study (MLS)**

Micro-teaching Lesson Study (MLS) is a variation of lesson study that integrates micro-teaching and lesson study. Studies on MLS reflect the cycle of planning, enactment, and reflection that is core to lesson study as a key feature in MLS (e.g., Fernandez, 2005, 2010; Fernandez & Robinson, 2006). It differs from micro-teaching because the collaborative nature of the process allows for preservice teachers’ cooperative learning while planning, enacting, and reflecting on lessons (Fernandez, 2005). Another difference is that MLS does not focus on one
particular teaching skill, as does micro-teaching (Fernandez, 2010); instead, it focuses on a learning goal. Similar to micro-teaching, in studies on MLS, mathematics topics chosen are those that preservice teachers have little familiarity with or cannot recall to allow for a more authentic learning experience with mathematical content during MLS as they take on the role of students.

The benefits of MLS are made explicit in studies by Cavin (2007, 2008) and Fernandez (Fernandez, 2005, 2010; Fernandez & Robinson, 2006). I first discuss Cavin’s (2007, 2008) study that examined changes in preservice teacher learning, aspects of MLS that allowed those changes to occur, and the factors and barriers that affected the preservice teacher learning. I then look at Fernandez’s body of work for further insight into the benefits of MLS. I conclude by discussing Sims and Walsh’s (2009) research, as it identified caveats in implementing MLS.

Cavin (2007, 2008) examined the development of the technological pedagogical content knowledge (TPCK) of six preservice teachers using a case study method. At the start of the study, data on preservice teachers’ personal histories, technological skills, and beliefs toward student learning were collected to assist forming the MLS groups and also to help with comparison during data analysis. The students were placed into two groups, the TI group which used the TI-83 calculator and the XL group which used the Excel spreadsheet program. The focus was on student learning goals; in this case, the overarching goal was engaging the students in exploring mathematical patterns and/or relationships using technological tools in order to develop a rich understanding of mathematical topics. Unlike other MLS lessons where the mathematics topics were those that preservice teachers were unfamiliar with or could not recall to make the teaching more authentic, in this study the researcher allowed preservice teachers to pick their own topics since in this case the focus was on use of the technological tool.
The data were analyzed for evidence of TPCK in preservice teachers’ observable behavior and verbal responses by applying a TPCK framework. The framework included adaptations of national and state standards related to technology and the following TPCK characteristics from Niess (2005), an overarching concept of what it means to teach a particular subject in which technology is integrated into learning; knowledge of instructional strategies and representations for teaching specific topics with technology; knowledge of students’ understandings, thinking, and learning with technology in a particular subject; and knowledge of curricula and curriculum materials that integrate technology with learning in the subject area. Changes in preservice teachers’ TPCK were identified by examining the progression of lessons and the changes in the design of the lessons during MLS.

Feedback and collaboration in the revision of lessons were found to provide opportunities to develop TPCK. The following example from the TI group shows some of the opportunities for learning TPCK and what was learned. When the TI group was addressing feedback from their first implemented lesson, they noticed that a recurring theme in the feedback was that the connection between the technology used and the content was weak. In the process of revising the lesson to address the connections between content and technology, the TI group realized the importance of using specific vocabulary in presentation of mathematical content. The evidence of this was in their grappling with students’ use of the terms experimental probability and empirical probability. They resolved this issue by tying both terms to theoretical probability in their revised lesson plan. Further evidence of TPCK in this first stage of revision was related to the TI group making changes to the technological tool commands to address the challenges faced by the students while using the calculators during the first lesson.
For the XL group, evidence of TPCK started with the choice of technology tool when they chose to use Excel instead of other options because Excel best supported realization of their lesson goals. Issues that were addressed by the group during revision of lessons that provided opportunities for the development of TPCK included the importance of written directions, the need for some direction from the teacher as students worked on a task with technology, and how to use aspects of technology to motivate students to continue working on a task. Evidence of TPCK included the XL group’s decisions to include “what if” questions that would allow students to explore by making changes in the Excel spreadsheet to the values of the variables related to the loan problem they were working. This provided evidence of TPCK in two ways: consideration of the use of technology to extend students’ mathematical thinking and creating tasks that leverage the features of technology. The study illustrates how situating learning on how to teach with technology in teacher preparation programs through use of MLS could be used as an intervention to disrupt Lortie’s (1975) “apprenticeship of observation” with regard to the use of technology. The collaborative process of lesson preparation, providing feedback, revising lessons, and enacting the improved lessons provided preservice teachers with opportunities to develop TPCK.

Fernandez and Robinson (2006) examined preservice teachers’ perspectives of MLS. Their study incorporated the simplified environment of micro-teaching by having 30-minute lessons taught by preservice teachers to groups of five to seven peers. However, the complexity of the task was not reduced because rather than focusing on a particular teaching skill, as in micro-teaching, the focus was derived from lesson study: meeting a particular student goal, in this case, promoting student reasoning. The purpose of maintaining the complexity of the tasks was to increase the authenticity of preservice teachers teaching to their peers, particularly since
the topics chosen were unfamiliar or those that preservice teachers could not recall on a beginning of semester survey. The groups of preservice teachers worked through the phases of lesson study—collaborative planning of the lesson, observation by colleagues and other experts, reflection, and revision. Preservice teachers wrote reflective reports of their MLS experience at the end and assessed the group members’ contributions for accountability of the group. Preservice teachers completed a micro-teaching feedback survey in which they responded to Likert scale items and, for each item, provided a reason for their rating.

Key elements of the preservice teachers’ learning were collaboration, reflection, and connecting theory and practice. Many preservice teachers expressed that they were able to put what they had been learning in the methods course into practice. Many valued the collaboration and found the different views and feedback from their peers helpful and also recognized the value of reflecting on their teaching as it supported them in improving the lessons. In addition to pedagogical learning, many preservice teachers also expressed that preparing and enacting the lessons gave them opportunities to engage with the mathematical content of the lessons, and that participation as students in their peers’ lessons broadened their mathematical knowledge. Video records of the lessons allowed preservice teachers to see some negative aspects of their teaching that their peers may not have had the insight or courage to point out to them. Overall, in examining preservice teachers’ perspectives on MLS, this study also highlighted the benefits of using MLS with preservice teachers and how the incorporation of aspects of micro-teaching and lesson study complement each other.

Some of the benefits of MLS from the perspective of preservice teachers highlighted by Fernandez and Robinson (2006) also represent aspects of MLS that support preservice teachers’ learning, as illustrated by Fernandez (2005, 2010). In these studies, collaboration, feedback, and
reflection presented opportunities for preservice teachers to learn about pedagogy and deepen their mathematical content knowledge. Additionally, the studies also reflected changes in preservice teachers’ teaching approaches that align with the kind of reforms reflected in NCTM documents (e.g., 1991, 2000, 2014). In addition to highlighting what preservice teachers learned through MLS, Fernandez (2010) examined how preservice teachers learned. In the extant literature addressing preservice teacher learning through pedagogies of enactment, this is the only study I found that explicitly addressed how preservice teacher learning takes place. It did so by using a situative perspective to provide a descriptive account of opportunities for the preservice teachers’ learning through the tasks related to the practice of teaching, iterative cycles of planning the lessons, support from a knowledgeable other, and reflection and collaborative “deliberation-in-process—repeated reframing, adjusting and implementing” (p. 360). This is important information for mathematics teacher educators; however, a more fine-grained account that looks at the structures inherent in the opportunities that influenced learning and the connections to the contexts in methods courses may provide deeper insights into how preservice teacher learning takes place. Such insight may also facilitate a more deliberate approach in presenting these opportunities in teacher education in ways that allow for greater leverage for preservice teacher learning.

Aspects of MLS, such as collaboration, that supported PST learning in the studies by Fernandez and colleagues (Fernandez, 2005, 2010; Fernandez & Robinson, 2006), however, may not always give the desired learning outcomes. As mentioned earlier, Sims and Walsh (2009) found that collaborative planning of lesson study was unproductive in the first year of their study. Purposeful planning around these aspects of MLS that have potential to support learning is necessary for any gains in preservice teacher learning to be realized. MLS draws from both
micro-teaching and lesson study, with micro-teaching contributing to limiting the complexity of teaching by reducing the class size and the lesson length. However, in the extant literature, the features of MLS that are salient in supporting preservice teacher learning are borrowed from lesson study: collaborative work on lesson planning, revision, and reflection—all guided by the focus on the student learning goals.

**Mentor-Guided Lesson Study (MGLS)**

A more recent variation of lesson study that has been used in teacher preparation is Mentor-Guided Lesson Study (MGLS), developed by Bieda et al. (2015). In MGLS, a pair of preservice teachers and a mentor teacher make up a team that works collaboratively on enacting, refining, and re-enacting a lesson while rotating the role of the teacher among them. The teacher educator guides learning indirectly through the “reflection prompts” in the “collaborative learning logs” (p. 21) that preservice teachers have to complete online at every phase of MGLS. The preservice teachers’ responses to the prompts give the teacher educator insights into what the preservice teachers are attending to in classroom instruction. MGLS was part of a methods course intended to deepen preservice teachers’ learning and capacity to collaborate with their mentor teacher. Prior to the study, mentor teachers were familiarized with lesson study and the collaborative learning logs that preservice teachers were required to complete. The collaborative logs were the main source of data for their research study. Open coding led to identification of instances in which preservice teachers reflected on features of the lesson related to their knowledge of mathematics, of students and of teaching. A set of sub codes was generated for the three emergent themes: *analyzing mathematics*, *attending to student thinking*, and *analyzing teaching moves*. These categories were further refined using a constant comparative method.
The results of the study are based on preservice teachers’ responses to the Lesson Reflection and post-Lesson Discussion collaborative learning logs because they represented preservice teachers’ reflections on the research lesson. The findings included that instances of attending to student thinking occurred more frequently than other categories and far outnumbered codes for analyzing teaching moves. This finding suggests that MGLS focuses preservice teachers’ observations on their students’ thinking. The study also revealed that the mentors’ level of experience with lesson study influenced the frequencies of responses coded to the three observations categories—analyzing mathematics, attending to student thinking, and analyzing teaching moves. The frequencies increased across all the three categories of preservice teachers’ reflection as the mentors’ experience increased, with the highest frequencies recorded for the most experienced mentors. Additionally, there were patterned variations in the specificity of preservice teachers’ responses across the categories. Most responses coded attending to student engagement had low specificity compared to responses coded attending to student understanding, most of which had a higher specificity. This was attributed to the difference in grain size of the noticeable student behavior related to these codes, with the attending to student engagement involving noticing student behavior at a relatively large grain size. Bieda et al. (2015) highlight the influence of the more knowledgeable other on preservice teacher learning during implementation of the MGLS. The level of experience of the mentor teacher had an impact on what preservice teachers noticed in features of instruction relevant to developing knowledge for mathematics teaching. However, the interactions between the mentor teacher and the preservice teachers were implicit. Making these interactions explicit would provide insights not only into what preservice teachers noticed but how they were supported in the process.
Summary

The use of lesson study with preservice teachers is still relatively new and the studies discussed here mostly involve the examination of lesson study’s potential benefits for preservice teacher learning. Some of the challenges related to the implementation of lesson study with inservice teachers highlighted by Murata (2011), such as time and teachers’ mathematical content knowledge, are also relevant to preservice teachers. Time in methods courses is limited and it may be difficult to go through the necessary iterative cycles of lesson planning and enactment. Research has shown that preservice teachers come to the teacher preparation programs with gaps in mathematical content knowledge (Ball, 1990; Simon, 1993; Tirosh & Graeber, 1990) and preconceived ideas about teaching (Feiman-Nemser, 2012). Therefore, in the absence of a knowledgeable other in the lesson study group, such as a teacher educator or classroom teacher, preservice teachers’ learning of mathematical content and pedagogy would likely suffer.

Certain aspects of lesson study—collaboration, feedback, reflection, and revision of lessons, all guided by the articulated student learning goals for the lesson—were found to support preservice teacher learning across all the lesson studies discussed here, including the MLS and MGLS. An affordance of lesson study that was explicit in Fernandez and Robinson (2006) is the acknowledgement by preservice teachers of how lesson study enabled them to connect theory to practice. This is important because it may have the potential of addressing the problem of preservice teachers’ inability to transfer their knowledge and skills to actual classrooms.

The role of the knowledgeable other is critical in guiding preservice teacher learning during lesson study by ensuring the validity of the mathematical content and appropriate
pedagogy in addition to drawing preservice teachers’ attention to instructional details of the lesson to which they may not naturally attend. The consequences of not having someone in that role is evident in the study by Parks (2008), where collaboration served only to entrench mathematical misconceptions and inappropriate pedagogical approaches. Further studies with a more fine-grained focus on aspects of lesson study that support preservice teacher learning are needed to inform implementation of lesson study in teacher preparation programs to facilitate purposeful preservice teacher learning.

Rehearsal

Rehearsal, an emerging pedagogy in teacher education, is motivated by the need to be responsive to the high expectations of teachers as a result of reforms in mathematics education. The goal of rehearsal is to prepare preservice teachers for ambitious teaching. According to Lampert et al. (2013), ambitious teaching is teaching toward the ambitious goals of instruction consistent with those reflected in documents such as CCSSM (NGA & CCSSO, 2010). Various researchers have advocated for the identification of core practices of teaching and support of preservice teachers in learning them (e.g., Ball & Forzani, 2011; Zeichner, 2012). The means of achieving the goal of preparing preservice teachers for ambitious teaching is centered around instructional activities (IAs). Lampert et al. (2010) describe the conceptualization of IAs, which act as containers of core practices, and outline four IAs: choral counting, strategy sharing, strings, and solving word problems. The enactment of these IAs is intended to support preservice teachers’ development of core practices of teaching.

In this section I will start by discussing Kazemi, Franke, and Lampert (2009), which outlines what rehearsal entails, then move to studies in which rehearsal has been applied—starting with those in elementary school, then moving to studies that extended the use of IAs to
secondary mathematics content. I will then discuss studies that have focused on teacher educators’ learning, and, finally, look at studies outside of mathematics education that have applied rehearsal.

Lampert et al. (2013) and Kazemi, Ghousseini, Cunard, and Turrou (2016) are both part of a wider study for which Kazemi et al. (2009) describe the pedagogy of rehearsal of IAs; outlining what it entails both for teacher educators and preservice teachers. The preparation of the implementation of IAs with preservice teachers starts with the teacher educators’ preparation to enact the IA with preservice teachers. The teacher educators rehearse the IA, refine it, and prepare a protocol. The teacher educator then enacts the IA for the preservice teachers who are in the role of students; this is followed by a whole-class analysis of the enactment of the IA. Preservice teachers then have an opportunity to rehearse the IA with their peers. They prepare and then rehearse the IA with their peers in the role of students and the teacher educator in the role of a coach. The preservice teacher’s rehearsal of the IA is followed by a whole-class analysis of the enactment of the IA. During the rehearsal of the IA by the preservice teacher, the teacher educator provides feedback in the moment and the novice teachers can also stop and ask for direction from the teacher educator. Finally, the rehearsed IA is enacted by novice teachers in a school setting with a group of elementary school students. Analysis of this enactment for further refinement and rehearsal is carried out. This process is referred to as a cycle of enactment and investigation (Lampert et al., 2013).

Lampert and colleagues (2013) examined the interactions between preservice teachers and teacher educators during the enactment of the IA by novice teachers. The unit of analysis in this study was a teacher educator/novice teacher exchange during enactment of the IA. The data were 90 videos from methods courses at three universities. Lampert and colleagues used a mixed
methods approach to data analysis. They applied qualitative analysis to examine the interactions in terms of what was worked on (substance) and how it was worked on (structure), and quantitative analysis to paint a picture of the frequencies of the substantive focus. A significant finding of the study is that the most frequent work in the interaction of teacher educators and novice teachers was on eliciting and responding to student thinking. Unlike most studies on teacher education pedagogies that focus only on preservice teacher learning, Kazemi and colleagues (2016) build on Lampert and colleagues (2013), using the same data to highlight insights about teacher educators’ practice, based on the decisions they make during rehearsals.

The initial conceptualization of IAs by Lampert and colleagues (2010) and the study on rehearsal by Lampert and colleagues (2013) are all related to elementary school mathematical content. Other studies have extended the use of IAs to secondary mathematics content. Next, I discuss studies by Campbell and Elliot (2015) and Baldinger, Selling, and Virmani (2016) that have extended the use of IAs to secondary mathematics content.

Campbell and Elliot (2015) address concerns about the relevance of practice-based pedagogies of teacher education to school settings. The study draws on the cycles of investigation and enactment of IAs in methods courses and school settings to investigate the relevance of practice-based pedagogies. The study adapts the IAs to align with secondary mathematics content in the classrooms of the partner teacher, extending the potential for using the rehearsal framework developed by Lampert and colleagues (2013) and Kazemi and colleagues (2016) beyond elementary school mathematical content. Participants were three teacher candidates. The study involved three design cycles in one academic year with two partnering teachers, one in middle school and one in secondary school. In addition to all the events during the cycles being video recorded, one of the authors kept a reflexive journal
documenting all decisions that were made and the factors that influenced those decisions. Lessons of the teacher candidates were also video recorded during their student teaching after the third cycle. This was part of examining the relevance of practice-based pedagogies to see if the teacher candidates would replicate the structures of the IAs they had used during the three cycles.

Interviews with the teacher candidates at the end of the experience revealed information that was useful for the authors in considering breaking down and explicating the component parts of the IA. One of the teacher candidates admitted to avoiding the structures of the IAs because she was overwhelmed by their complexity. Yet she talked about how the planning protocols had allowed consideration of the different parts of the IA and what each of them would do for student learning. This prompted the teacher educators to begin to identify components of the IAs. They identified sequences of events that were common across IAs, which they called “episodes.” These episodes included things like launching a task and monitoring individual student or group work. In sum, Campbell and Elliot (2015) found that it is important for teacher educators to “make explicit the nested episodes, practices and moves within IAs—motivating their purposes, situating them relative to one another, and supporting their realization through concrete actions” (p. 160). This is a finding that provides useful guidance for all teacher educators considering the use of IAs. It also suggests that for all pedagogies of enactment, the tasks at the center of the enactment should be clearly articulated.

Other studies that have extended rehearsals to include secondary mathematical content include Baldinger et al. (2016). Their study was conducted with seven preservice teachers in the context of a secondary mathematics methods course. They used the cycle of investigation and enactment of IAs to examine how these cycles could support preservice teachers to learn how to lead discussions toward defining and clarifying a mathematical idea. They introduce an IA that
focuses on secondary mathematical content based on a card sort designed to refine the definition of a mathematical object. Preservice teachers had the opportunity to engage in a card sort to help them experience how the IA would support learners toward developing a definition of the mathematical object. After familiarization with the protocol for the IA, the preservice teachers had the option of designing their own card sorts or choosing from existing sets of cards, to be used for rehearsal with their peers and then to be enacted in a secondary school classroom. During rehearsal, the teacher educators coached preservice teachers on how to move the discussion toward defining the mathematical object. Sometimes during rehearsal, the teacher educators took on the role of students by voicing disagreement with ideas under consideration to highlight common misconceptions. Each rehearsal was followed by a debriefing. Each preservice teacher wrote a reflection on the rehearsal and how it would impact their enactment of the IA in a classroom.

The results showed that preservice teachers noticed three aspects of rehearsal related to guiding discussion toward a particular mathematical idea—the role of time, getting off track relative to the mathematical goal, and the role of sorting cards (Baldinger et al., 2016, p. 14). Two themes that emerged from the reflections of preservice teachers were the importance of having a mathematical idea toward which the discussion is directed and the importance of anticipating student thinking and planning appropriate responses.

The authors learned that the quality of the card sort had an impact on what preservice teachers learned through the rehearsal and enactment. They realized that designing a card sort was complex and that preservice teachers who chose to design their own cards needed more support than that which was provided by the teacher educators in this study. Additionally, they learned that for preservice teachers to learn from rehearsal, they need to have a clear
understanding of the mathematical idea that card sort is intended to highlight. From this study by Baldinger et al. (2016) and Campbell and Elliot’s (2015) study, which was discussed earlier, it is evident that teacher educators are also learning how to better support preservice teachers’ learning through rehearsal, even though that was not the initial focus of the studies. Next I discuss studies that specifically focus on mathematics teacher educators’ learning during rehearsals.

A few recent studies (Averill, Drake, Anderson & Anthony, 2016; Drake, 2016; Kazemi et al., 2016) have addressed mathematics teacher educator learning during rehearsals. Teacher educators who are in the role of a coach during rehearsals may have no prior training that prepares them for this role. This has prompted research such as the self-study by Drake (2016) on the author’s own learning as he supported preservice teachers during rehearsals in learning how to orchestrate whole-class discussions. Other research related to teacher educators has involved examination of techniques that may support them in facilitating preservice teachers’ learning. Averill, Drake, Anderson, and Anthony (2016) examined how the use of questions while coaching rehearsals supported preservice teachers’ learning of how to orchestrate mathematics discussions. Kazemi and colleagues (2016) provide insights into the decisions teacher educators make in their interactions with preservice teachers during rehearsal. Their study builds on the analysis of data from Lampert et al. (2013), focusing on the teacher educators’ interventions during rehearsal and the decisions that guided them. Three themes underlying the insights they gained were that teacher educators need to support preservice teachers in making their teaching public during rehearsal, preservice teachers’ experiences with IAs shape rehearsal, and the time interval between rehearsal and enactment with children shape the aspects of practice that teacher educators are likely to focus on during rehearsal. These
insights, according to Kazemi and colleagues, contribute toward breaking down the complexity of the work of mathematics teacher educators. The use of rehearsal has not only been confined to mathematics education.

A critical element of rehearsal in supporting PST learning is the interaction between teacher educators and preservice teachers. These interactions are often made explicit in studies on rehearsal, which gives these studies potential to go beyond investigating what preservice teachers learn to how that learning takes place. Exploiting this potential may require a shift in focus to incorporate the how of preservice teacher learning, and attention to other aspects of rehearsal and the contexts in which rehearsal takes place.

Summary

Studies on rehearsals contribute to addressing the call for preparing novice teachers’ for ambitious teaching. The conceptualization of IAs as “containers for learning the principles, practices, and knowledge of content that underlie ambitious elementary mathematics teaching” (Lampert et al., 2013, p. 240) allows for the delineation of and provides an avenue for novice teachers’ desired learning. Additionally, the studies address the concern expressed by McDonald, Kazemi, and Kavanagh (2013) about the need to have teacher education pedagogy accompany the identification of core practices and to support preservice teachers’ learning of those core practices.

The focus on core practices and their embodiment in an IA distinguishes rehearsals from the other pedagogies of enactment that I have discussed in this paper by focusing on preservice teacher learning. However the challenges in the use of IAs highlighted in Campbell and Elliot (2015), particularly the complexity, requires teacher educators to be skilled in designing IAs. Drake (2016) highlights the fact that teacher educators may not know how to support preservice
teachers during rehearsals. Therefore, there is need for more research like that in Kazemi and colleagues (2016) that would deepen teacher educators’ insights on how to use rehearsals effectively.

Equity in mathematics teaching has been advocated for (e.g., Battey, 2013; NCTM, 2014) and is addressed in the Standards for Preparing Teachers of Mathematics (AMTE, 2017). A focus on practice like that evident in these studies related to rehearsals—core practices like eliciting and responding, orienting students to one another’s thinking—aligns with equity concerns by allowing teachers to attend to assumptions about what students “can and cannot do and who to call on and why” (McDonald et al., 2013, p. 380). Still, it is important to better understand what supports preservice teachers’ development of such practices.

**The Bellringer Sequence: A Pedagogy of Enactment**

Studies that provide insight into the processes and interactions of pedagogies of enactment (e.g., Cavin, 2007, 2008; Fernandez, 2005, 2010; Fernandez & Robinson, 2006; Kazemi et al., 2016; Lampert et al., 2013) have the potential to support the refinement and improvement of design of such pedagogies to better support preservice teachers’ development of the practice of teaching. One component of these processes and interactions that is often implicit and could give leverage in the design of pedagogies of enactment is how preservice teachers learn through the implementation of pedagogies of enactment. Understanding preservice teachers’ learning through the implementation of pedagogies of enactment may allow teacher educators to leverage these pedagogies in the preparation of mathematics teachers. To contribute toward a better understanding of preservice teachers’ learning through pedagogies of enactment, my work examines the use of a particular pedagogy of enactment, the Bellringer Sequence (BRS), as an instructional tool in a mathematics methods course to support preservice teachers’
learning of high-leverage practices. The BRS is centered on the bellringer—a brief mathematical task implemented as students arrive for class.

**Bellringers**

Bellringers—also known as mind benders, parachute openers, warm ups, starters, and morning minutes—are tasks used to productively engage students as soon as they enter the classroom (Boettner, 2011). They provide a way of transitioning into class time quickly and smoothly (Romano, 2011) and help teachers maximize instructional time as they take care of administrative activities at the start of class (Paige, 2014). Bellringers are often used as a behavior management tool (Paige, 2014), but they have the potential to support the achievement of learning goals of the lesson when students see how the bellringers relate to what they are learning (Blackey, 2010). The centering of the BRS around a bellringer task allows for preservice teacher learning of aspects of the practice of teaching related to preparation and enactment of bellringers. Thus, the BRS may be viewed as a teacher education pedagogy. More specifically, it is a pedagogy of enactment since it provides preservice teachers with the opportunity to enact the practice of teaching.

Studies that have incorporated the use of tasks like bellringers include Boerst, Sleep, Ball, and Bass (2011), who refer to such a task as a mini-problem, and Tyminski, Zambak, Drake, and Land (2014), who refer to them as an opening routine. However, both Boerst and colleagues and Tyminski and colleagues focus their use of the mini-problem and opening routine, respectively, on preservice teachers’ learning of how to lead mathematics discussions. Desforges’ (1995) argument about paucity of research that explains how changes in teachers’ knowledge take place, is affirmed by Ball and Cohen’s (1999) assertion that we are missing theories of teacher learning that could inform preservice teacher education comparable to the way that cognitive psychology
has informed education of school children. My study investigates the use of the BRS, which was designed to address multiple goals with preservice teachers, including, but not limited to, leading mathematics discussions. In the next section I discuss the conceptualization of the BRS and the rationale for using its implementation to examine preservice teachers’ learning of the practice of teaching mathematics from a pedagogy of enactment.

**Conceptualization of the Bellringer Sequence**

The Bellringer Sequence (BRS) involved four phases centered on the bellringer task: (1) preparation, (2) implementation, (3) debriefing, and (4) written reflection. During a pilot study (Ochieng, 2017), characteristics of effective bellringers were identified from the literature (e.g., NCTM, 2014; Paige, 2014; Romano, 2011) and applied in developing a rubric that was used to guide the preservice teachers in designing high-quality bellringers and to support them in critiquing each other’s bellringer implementations. The rubric (see Appendix A) has four categories that reflect the characteristics of effective bellringers: appropriateness (of the mathematics), (focus on) student thinking, (effectiveness of the) implementation, and (quality of the) presentation. Each of these is described across four levels: unacceptable, tolerable, expected, and exemplary. The four phases of the BRS—preparation, implementation, debriefing, and written reflection—are described below.

**Preparation.** There are two parts to the preparation phase of the BRS, preparation of the preservice teachers for the BRS and preparation by preservice teachers for the implementation of the bellringer. Preparation of preservice teachers is done by the course instructor and starts with PSTs being assigned a reading that captures how to use a bellringer to further instructional goals of a lesson (Romano, 2011). The instructor implements a bellringer with the preservice teachers as students and leads them in a discussion about qualities of effective bellringers. The bellringer
rubric from the pilot study, which is used for evaluation of bellringer implementation, forms the basis of a discussion of bellringers and the bellringer assignment. Topics for the bellringers are drawn from Heid, Wilson, and Blume’s (2015) *Mathematical Understanding for Secondary Teaching*, which addresses topics that merit mathematical attention. Each topic is addressed in its own chapter starting with a description of an incident that provides a teacher with an opportunity to draw on their mathematical understanding. This is followed by an elaboration of mathematical ideas that the teacher could productively draw on in that context. Each preservice teacher is randomly assigned a chapter relevant to middle school learning goals to read and apply to the preparation of a bellringer. The second part of preparation involves preservice teachers’ preparation for their implementation of a bellringer.

In preparing (and implementing) the bellringer, preservice teachers are guided by the evaluation rubric and are also expected to apply what they have learned in the methods course about topics such as task analysis, effective questioning, and the *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011). In consultation with the instructor, each preservice teacher identifies a middle school mathematics lesson as the context for their bellringer and provides a brief description of the lesson, as well as what the hypothetical students have learned in the previous lesson, to set the stage for their fellow PSTs to engage with the bellringer as if they are students in that lesson. The bellringer task (see Appendix B) on their assigned topic is meant to be one where the mathematical ideas might be problematic to their peers so that it would give them an authentic learning experience. That some mathematical content is problematic for preservice teachers is not unusual since research has shown that preservice teachers come to teacher training with gaps in their mathematical knowledge (Ball, 1990; Simon, 1993; Tirosh & Graeber, 1990). The choice of topic based on problematic content
is similar to Fernandez’s (2005, 2010) selection of mathematical content that was not familiar so that PSTs would be teaching their peers authentically. Prior to implementing their bellringer, each preservice teacher had a one-on-one meeting with the instructor to discuss their bellringer task and plan for implementing it.

**Implementation.** Each preservice teacher implements a 5-to-7-minute bellringer of their own design at the beginning of a methods class. Implementation of the bellringer is expected to model instruction based on student thinking—a recurring theme throughout the middle school mathematics methods course. The preservice teacher implementing the bellringer takes on the role of a teacher—what we refer to as the teaching preservice teacher (TPST). The other preservice teachers in the class each take on the role of a student—what we refer to as a student preservice teacher (SPST). Implementation involves launching the bellringer task, monitoring students as they work on the bellringer, and facilitating whole class discussion of the bellringer. The course instructor takes on the role of a coach and observes the implementation, taking note of aspects of the implementation on which to provide feedback for the TPST and also observations that may be of benefit to all preservice teachers’ learning. While observing the implementation of the bellringer, the instructor also completes the bellringer task, like the SPSTs, but does not participate in the conversations during the discussion of the bellringer. Instead, the instructor monitors the discussion to better lead the debrief.

**Debriefing.** After implementation of the bellringer, everyone—the TPST, SPSTs and the instructor—completes the bellringer evaluation rubric. Completing the evaluation rubric allows the SPSTs to switch roles from students to colleagues of the TPST, providing an opportunity for them to reflect on the implementation. For all the preservice teachers, completing the bellringer evaluation rubric also prepares them to provide critique of the implementation during debriefing.
where the TPST receives feedback from their peers and the instructor. The debriefing phase is facilitated by the instructor and even though the focus is to provide feedback on the implementation, it is a collaborative effort of supporting learning not only for the TPST, but for all preservice teachers. This debriefing is used as an opportunity to review important mathematics and as a context to discuss aspects of teaching related to the bellringer implementation, for example, eliciting and responding to student thinking. The TPST is given the first opportunity to say something about their implementation of the bellringer. After that the conversation opens up for other preservice teachers to give their feedback or respond to statements made by their peers. The instructor facilitates the debriefing, sometimes pausing the conversation to highlight important mathematical or pedagogical ideas that arise from the discussion. The TPST is also given an opportunity at the end to express any final thoughts they may have related to their implementation and the conversations during the debrief.

**Written reflection.** Within 24-hours of implementing their bellringer, the TPST completes an initial reflection in the university electronic learning system. After all the preservice teachers have implemented a bellringer, each preservice teacher writes a reflection paper on bellringers. The BRS reflection paper is an opportunity for preservice teachers to use video tapes and artifacts from all the bellringer implementations to reflect across the different bellringers and their completed rubrics and demonstrate what they have learned about both bellringers and mathematics teaching in general (see Appendix C for the bellringer reflection assignment). In the writing of their reflection, preservice teachers have opportunity to demonstrate their understanding of the design of effective bellringers, and reflect on what they have learned about effective implementation of bellringers and effective teaching in general.
Rationale for Using the BRS to Examine PST Learning

The Bellringer Sequence (BRS) draws on the affordances of micro-teaching, lesson study and rehearsal, and adapts them for use in a middle school mathematics methods course. To reduce the complexity of teaching that may interfere with preservice teacher learning, such as having to deal directly with students in a school classroom (Ball & Cohen, 1999), the BRS draws from micro-teaching. However the complexity of the content is not reduced. The bellringer sequence can be implemented within the methods course without the need for a connection to student learning in a context outside the methods course, and instead focuses on preservice teachers’ learning of mathematical content addressed by the bellringer. This is an advantage of the BRS over micro-teaching, lesson study, and rehearsals because of the deliberate focus on preservice teachers’ learning of mathematics. Feedback and reflection, aspects common to all three pedagogies of enactment are also aspects of the BRS. Even though in the BRS each preservice teacher prepares and presents a bellringer individually, the collaborative aspects of lesson study and rehearsals are evident in the debriefing session where preservice teachers and the instructor collaboratively provide feedback and support one another’s learning. The centering of the BRS around a bellringer is parallel to the centering of rehearsal around the IA and lesson study around the lesson. This centering of the bellringer is particularly similar to the centering of rehearsal around IAs. However, unlike IAs that have specific articulated core practices embedded in them, the bellringer has broader practices embedded in it. The specificity of the core practices in rehearsal focuses preservice teacher learning, but may also limit that learning. The BRS leaves the design of the bellringer to the PST, which gives them a sense of ownership that parallels PSTs’ sense of ownership of the lesson in lesson study, by allowing them to make decisions about the lesson while revising it. In the BRS, leaving the choice of bellringer open
allows for more opportunities for preservice teachers to carry out components of teaching, such as choosing or designing appropriate tasks. By preparing and implementing the bellringer task—important components of teaching—the teaching preservice teacher (TPST) is positioned as a teacher with agency, unlike in micro-teaching and rehearsals where they are positioned as students even when teaching.

The BRS addresses multiple instructional goals by allowing preservice teachers to (1) practice components of teaching, (2) review important mathematics, and (3) learn how to effectively use bellringers. Even though the BRS as studied here has the limitation of not involving actual school classroom teaching\(^1\) therefore making it less authentic, the choice of bellringer based on mathematical content that is problematic for preservice teachers increases the authenticity of both their teaching and their learning experience.

The choice of the BRS as the medium for examining preservice teacher learning over micro-teaching, lesson study, and rehearsals is not to say that it is superior to the three pedagogies. Rather, by drawing on the affordances of these pedagogies, and avoiding their pitfalls where possible, the BRS optimizes the limited time and resources available in methods courses to support preservice teachers’ learning of mathematics and pedagogy. By examining preservice teacher learning through the BRS, this study seeks to answer the research questions outlined in the next section.

\(^1\)The preservice teachers were able to teach the bellringer in a school, though not necessarily the one they designed, but the additional component of the BRS is not part of the current study.
Research Questions

In an effort to better understand preservice teacher learning through pedagogies of enactment in the context of a methods course, this study seeks to answer the following research questions:

1. *What* do preservice teachers learn through the implementation of the Bellringer Sequence in a methods class?
2. *How* do preservice teachers learn through the implementation of the Bellringer Sequence in a methods class?

We conclude chapter 2 by discussing the theoretical frameworks that inform the study.

Theoretical Frameworks

The purpose of this study is to examine how preservice teacher learning takes place through the incorporation of the Bellringer Sequence (BRS) into a methods course. In order to do that, we first need to articulate what counts as learning. Sfard (2008) defines learning as a change in one’s discourse. Therefore, learning is reflected in a change in the way one communicates about an idea. For the purposes of this study we consider a preservice teacher to have learned something (e.g., an idea, concept, skill) if they *express a realization of having acquired the knowledge or demonstrate the knowledge in a way that they had not done before*. The study examines preservice teacher learning by applying three frameworks—a knowledge and practice framework that addresses *what* is learned, a learning theory framework that addresses *how* learning takes place, and a pedagogical framework that provides a *lens for viewing* preservice teacher learning in the context of a pedagogy of enactment. The knowledge and practice framework utilizes these existing constructs: *content knowledge* and *pedagogical content knowledge* (Shulman, 1986); *pedagogical knowledge* (Grossman, 1990); *mathematical knowledge* (Shulman, 1986); *pedagogical content knowledge* (Grossman, 1990); *mathematical knowledge* (Shulman, 1986).
knowledge for teaching (Ball, Thames, & Phelps, 2008); and high-leverage teaching practices (TeachingWorks, 2018). The learning theory framework integrates the emergent perspective (Cobb & Yackel, 1996) and the situative view (Putnam & Borko, 2000). The pedagogical framework is based on Grossman and colleagues’ (2009) pedagogies of practice related to professional education. In the following I will discuss each of the three frameworks, highlighting their underlying perspectives and how these perspectives will be applied to the examination of preservice teachers’ learning of the practice of teaching mathematics.

The Knowledge and Practice Framework

The practice of teaching is complex (Ball & Forzani, 2009), making it difficult to delineate all the knowledge and practices that teachers need for teaching. The attempt in this section to outline the knowledge teachers need for teaching is not expected to be comprehensive; rather, it identifies what I looked for in my study in terms of desired learning that has been outlined by researchers. I will start by addressing the knowledge aspect of the framework and then address the practice aspect.

Knowledge aspect of the framework. For the knowledge aspect of the framework I draw on the literature that takes into consideration both content and pedagogy as necessary for teaching, because, according to Shulman (1986), “mere content knowledge is likely to be as useless pedagogically as content-free skill” (p. 8). This study considers three categories of knowledge for teaching—content knowledge; pedagogical knowledge, and pedagogical content knowledge. Content knowledge refers to knowledge of concepts and facts in the discipline, and understanding of the structures of the discipline—how the field is organized and principles that underlie the validation of claims (Grossman, 1990; Shulman, 1986). Pedagogical knowledge includes knowledge and beliefs about learning and learners, general principles of instruction, and
the aims and purposes of education; it also includes knowledge and skills related to classroom management (Grossman, 1990). This category of knowledge is not specific to any discipline or subject, but is important for teaching across content areas (Shulman, 1986). Pedagogical content knowledge is a form of content knowledge that embodies aspects of content that are relevant to teaching the discipline or subject matter; it includes knowledge of common preconceptions, misconceptions, and ways of making a subject more comprehensible to students through the use of the most appropriate representations, illustrations, and examples (Shulman, 1986). Ball et al. (2008) have elaborated on the categories of content knowledge and pedagogical content knowledge specifically in relation to mathematics teaching. Although their subcategories are of a finer grain size than needed for this study and thus are not discussed here, their illustrations and elaborations will be useful in interpreting the learning that is observed.

Practice aspect of the framework. The high expectations of teachers prompted by reforms in mathematics education have led to calls for practice-based teacher education, with various researchers advocating for the identification of core practices of teaching that PSTs can be supported to learn (e.g., Ball & Forzani, 2011; McDonald et al., 2013; Zeichner, 2012). It is not possible to cover all teaching practices in the limited time available in teacher preparation programs; therefore, Ball et al. (2009) have focused on what they call “high-leverage practices” (p. 460). These are practices which allow teachers to significantly impact student learning. They have identified 19 such practices (see Appendix D). Even though these practices have been identified in relation to elementary education, they are applicable K-12. Two sets of criteria were used to identify these practices, one based on mathematics teaching and a second one based on the teacher education context. The criteria used for identifying these practices based on mathematics teaching were (1) supports work that is central to mathematics, (2) helps to improve
the learning and achievement of all students, (3) is done frequently when teaching mathematics, and (4) applies across different approaches to teaching mathematics. The criteria based on the teacher education context were (1) can be articulated and taught, (2) is accessible to learners of teaching, (3) can be revisited in increasingly sophisticated and integrated acts of teaching, and (4) is able to be practiced by beginners in their field-based settings (Ball et al., 2009, p. 461). Examples of these high-leverage practices include leading a group discussion; explaining and modeling content, practices and strategies; eliciting and interpreting individual students’ thinking; and coordinating and adjusting instruction during a lesson.

**Relationships within the framework.** The relationship among the knowledge and practice aspects of the Knowledge and Practice Framework is shown in Figure 1, which is influenced by Grossman’s (1990) Model of Teacher Knowledge. For the knowledge aspect of the framework, *pedagogical content knowledge* draws from both *content knowledge* and *pedagogical knowledge*. In mathematics, pedagogical content knowledge involves knowledge of a particular mathematical idea or procedure and knowledge of pedagogy for making that idea or procedure comprehensible for students (Ball et al., 2008). The practice aspect of the framework—*high-leverage practices*—both draws from the three categories of the knowledge aspect of the framework and contributes to them. As I conceptualize the relationship, the PSTs apply their knowledge as they implement high-leverage practices, and in turn their implementation of high-leverage practices increases their knowledge. This generative relationship is represented by the bi-directional arrows in Figure 1.
This attempt at outlining the knowledge teachers need for teaching does not imply that this knowledge is what they learn from teacher preparation; instead, it provides a framework for what to look for in terms of the field’s conception of desired learning. The four categories making up this framework will be used to identify what preservice teachers learn by examining instances of preservice teachers’ learning. Since the greater focus of this study is on how preservice teachers learn, identifying what is learned simply provides a focal point for examination of how learning takes place.

**The Learning Theory Framework**

Learning theories differ based on the perspectives and assumptions on which they are based and hence the contexts in which they are applicable. The various existing theories of learning were not developed specifically for mathematics education and each have limitations; hence, it is best to adapt them in ways that make them suitable for the learning contexts we seek to address (Cobb, 2007). Adapting learning theories to specific contexts may require the use of multiple theories, which has also been advocated for in the literature (e.g., Cobb, 2007; Sfard,
Additionally, the use of multiple theories allows one to work at different levels and also allows for broader interpretations of observations in research (Simon, 2009). In this study I apply the emergent perspective (Cobb & Yackel, 1996) and the situative view (Putnam & Borko, 2000) to allow for examination of preservice teachers’ learning of mathematics and pedagogy across the different components of the BRS. In this section I provide brief overviews of learning theories embodied in the emergent and situative perspectives and how they will facilitate my understanding of preservice teacher learning through the BRS in the context of the methods course.

The emergent perspective. Cognitive psychology focuses on how the individual reorganizes its activity and comes to act in a mathematical environment. Theories that have their foundations in cognitive psychology include constructivism, distributed cognition, and situated cognition (Cobb, 2007). Constructivism as a theory of learning holds that all knowledge is constructed from our experiences through our existing cognitive structures and that we have no access to objective reality (Confrey, 1990; Simon, 1995). Learning takes place through a process of equilibration as we adapt our experiences to what is expected or intended. Reflective abstraction—reflection on what has been adapted—then provides a base for modification of concepts and building of more advanced concepts (Piaget, 1964; Simon, 1995). Acknowledging the role of the negotiation of meaning that takes place through interactions nullifies the view of constructivist learning as a solitary affair that is independent of social and cultural influence (Voigt, 1995). The emergent perspective integrates interactionism and constructivism (Cobb & Yackel, 1996), thus allowing for the interpretation of observations of preservice teacher learning in the context of social interactions in the classroom. In this study I use the emergent perspective to make sense of preservice teachers’ experiences that lead to construction of knowledge.
The situative perspective. In the situated learning theory, learning is embedded within activity, context, and culture. According to situated learning theory, activity and situations are integral to cognition and learning, and are inseparable. Learning takes place in contextual situations and involves a process of enculturation (Brown, Collins, & Duguid, 1989). Therefore, learning takes place as the individual interacts with objects in the environment, which include tools and artifacts, and with other people. Gudjonsson (2007) presents a view of learning to teach as situated and involving interaction between the preservice teacher and the physical, social, and cultural environment of the school. A situative view of teacher learning—cognition as situated, social in nature and distributed across the individual, other persons and tools—allows for examination of the affordances and limitations across different contexts, communities, and available tools (Putnam & Borko, 2000). Additionally, in examining research on teacher learning through a situative lens, Putnam and Borko (2000) highlight how such a lens allows for examination of the relationships between knowing, learning, and the settings in which these occur.

Mathematics learning is situated with respect to the various means of support for learning in the classroom (Cobb, 2000). These supports include the motive for instruction, the structure of classroom activities, tools availed for student use, and the nature of classroom discourse. Cobb (2000) breaks down further the way mathematics learning is situated in these means of support, which constitute a “classroom activity system” (p. 57). Therefore, learning is situated with respect to students’ prior instruction, the classroom learning environment, the purpose of instruction, the conversations orchestrated by the teacher, students’ activity with tools, and the norms and culture in which learning takes place.
In this study I draw on Cobb’s view of mathematics learning as situated to make sense of preservice teacher learning of both content knowledge for teaching and teaching practices across the various means of support for learning in the methods course. Applying a situative view to examine preservice learning may address the challenge in research in teacher education pedagogy of how the pedagogy relates to other aspects of the course in which the pedagogy is applied (Grossman, 2005). Learning in this study would be situated with respect to preservice teachers’ prior learning experiences in K-12 classrooms, the learning environment in the methods course, the purpose of instruction in the methods course, conversations in the Bellringer Sequence, the bellringer task as an instructional tool, and norms and culture for supporting reform-based teaching. My hypothesis is that preservice teachers’ learning is supported first by a disposition toward learning to teach as learning from teaching (Ball & Cohen, 1999), and classroom norms where PSTs are free to critique one another’s teaching.

The Pedagogical Framework

Grossman and colleagues (2009) identified a framework for understanding the pedagogies of practice related to professional education: representations, decompositions, and approximations of practice. Representations are activities that demonstrate or model certain aspects of practice, therefore allowing preservice teachers to visualize practice in relation to that particular aspect of teaching. Decompositions are activities that break down and explicate the different components of the practice of teaching. Approximations of practice are those activities that preservice teachers engage in that are similar to real teaching. Studies in mathematics education that have applied Grossman’s framework have mostly been studies related to supporting preservice teachers learning on how to lead discussions. Ghousseini and Herbst (2016) illustrate how integration of the three pedagogies of practice provided and framed
opportunities to achieve learning goals for preservice teachers on how to lead discussions. Tyminski and colleagues (2014) used Grossman’s framework to organize activities that provided experiences for PSTs around representations, decomposition, and approximation of practice to support them in making sense of Smith and Stein’s (2011) *5 Practices for Orchestrating Classroom Discussions*. Boerst and colleagues (2011) use Grossman’s framework to describe and analyze work in their methods courses to develop preservice teachers’ skill in leading whole-class mathematics discussions.

My study integrates the three pedagogies of practice to examine teacher learning of practice in a context that incorporates multiple components of teaching, including the selection and implementation of tasks, orchestrating discussion, and reflection on practice. In my study the three pedagogies of practice will provide a lens for viewing preservice teacher learning of the practice of teaching across the phases of the Bellringer Sequence.

**Summary**

Pedagogies of enactment that have been identified in the literature are microteaching, lesson study, and rehearsal. Their use in relation to mathematics teacher preparation was discussed in this chapter. The Bellringer Sequence (BRS), a pedagogy of enactment that draws on the affordances of microteaching, lesson study, and rehearsal was conceptualized to provide a context through which preservice teacher learning was examined in the study. The study was designed to answer the following two research questions: What do preservice teachers learn through a pedagogy of enactment? and How do preservice teachers learn through a pedagogy of enactment? To answer these questions, the study applies three frameworks, the Knowledge and Practice Framework, a learning theory framework, and a pedagogical framework. In the next
chapter, the processes of data collection and analysis used to facilitate answering the research questions are described.
CHAPTER 3

METHODOLOGY

This chapter provides a description of the process used to obtain data for this study and how the data were analyzed. The chapter includes descriptions of the context of the study, the participants, the data collection, and the data analysis.

Context of Study

The study was conducted in a middle school mathematics methods course at a Midwestern university. This methods course is the first of three courses in the secondary mathematics education program that are devoted to the teaching of secondary school mathematics and is a requirement for both majors and minors. The course runs for 15 weeks with two 100-minute meetings a week. The course focuses on teaching mathematics for understanding, as articulated by NCTM (2014), and instruction that is based on student thinking is a recurring theme throughout the course. The course goals include acquiring mathematical knowledge for teaching; recognizing, valuing, and developing strategies for managing student mathematical learning; and developing skills and dispositions needed to access, interpret, and assess student thinking about mathematical ideas (see Appendix E for a complete list of the course goals). The course provides opportunities for preservice teachers (PSTs) to practice teaching (Lampert et al., 2010) by first implementing mathematical tasks with their peers and later implementing the tasks in a middle school classroom during the course field experience. The Bellringer Sequence (BRS) described in chapter 2 provided such an opportunity. In this study preservice teacher (PST) learning is examined as they take part in the BRS, which involves
preparing a bellringer, implementing it with their peers, debriefing the implementation, and reflecting on the entire process. As described in chapter 2, topics for the bellringers were drawn from Heid et al.’s (2015) *Mathematical Understanding for Secondary Teaching (MUST)*, which addresses topics that merit mathematical attention. Ten out the 11 PSTs implemented bellringers from the assigned topics. For one PST, it was not possible to identify a suitable bellringer for the assigned topic that fit with middle school standards, so she was assigned a task from Cooney, Sanchez, Leatham, and Mewborn’s (2001) *Open-ended Questions for Elementary, Middle, and High School Mathematics*.

**Participants**

The participants were 11 traditional PSTs enrolled in the methods course during the 2017 fall semester. Three of the PSTs were elementary education mathematics majors (Chloe, Oliver, and Yvonne), while the remaining eight were secondary education mathematics majors (Austin, Cameron, Evan, Isabelle, Ivy, Kylie, Layla, and Riley). The PSTs filled out consent forms at the start of the semester indicating their consent to participate or not, but the forms were not viewed by the instructor until after final course grades were submitted. The researcher knew who had given consent to participate in order to structure data collection in ways that did not focus on those who had not given consent but did not share that information with the instructor. This was done in order to ensure that the research does not introduce bias into the course. Because the instructor did not know if someone had withheld consent, there was no risk of the instructor penalizing a student for doing so through course grades. All 11 students taking the course gave their consent to be part of the study and were willing to be interviewed for the study.
Data Collection

Data sources for the study included a pre-course survey, audio records of bellringer preparation conversations, video records of the methods class sessions, interviews with the PSTs, and PSTs written reflections. The following paragraphs outline the data collection related to each of these sources and how the data facilitated addressing the research questions.

Survey

I used an existing survey that was created by the course instructor and given prior to the first class. The survey was intended to capture: PSTs’ perceptions about mathematics teaching; courses they had taken related to mathematical content and teaching; any prior teaching experience they had; and perceptions of their knowledge of mathematics, the middle school curriculum, and teaching and learning of middle school mathematics. This was important information because PSTs bring with them ideas, conceptions, and beliefs through which they filter their experiences and make sense of what they are learning in teacher preparation programs (Feiman-Nemser, 2012). Parts of these data that were relevant to the interview were used to probe how PSTs’ prior experiences may have influenced their learning during the BRS. The researcher incorporated the information from the survey into the interview questions. These data also supported the interpretation of evidence of learning that was useful for identifying changes in PSTs’ knowledge of mathematical content and pedagogy as the study progressed.

Instructor Meeting

Each PST met with the instructor at least a week before they implemented their bellringer to discuss their bellringer task and planned implementation. All the conversations during those meetings were audio recorded. The purpose of recording these conversations was to provide some insight into PST learning during the preparation phase of the bellringer since most of the
preparation took place privately and away from the class sessions. The audio records were transcribed and examined for evidence of instances of PST learning and later used to examine events that prompted learning that may have occurred during the bellringer preparation meetings.

**Course Sessions**

All the mathematics methods course sessions were video recorded to provide information on the context for PST learning and sources from which PSTs could be drawing their learning. In addition, these data provided information on the social and sociomathematical norms (Yackel & Cobb, 1996) of the methods course sessions. This assisted in making sense of the interactions in the class and how they supported PSTs’ construction of knowledge. The 499 minutes of the course directly related to the Bellringer Sequence (BRS)—bellringer implementation and debriefing sessions for all 11 PSTs—were transcribed for more detailed analysis. Evidence of PST learning was identified by watching the videos in Studio Code analysis software (Sportstec, 1997-2015) and referring to the transcriptions for verbal expressions that indicated learning. The bellringer videos enabled the identification of both evidence of learning that took place during the implementation and debriefing session, as well as occasionally provided a window into learning that took place during the bellringer preparation, which was not part of the video record. For example, during the debrief of Chloe’s bellringer, we see evidence of learning that took place during Chloe’s bellringer implementation when she asked Riley for his work and put it on the document camera:

Riley: Actually, when you pulled out my work like that, making me relook at my justification gave me my “aha.” As I was looking at it, then I considered that [Ivy] didn’t think of a case and I was like, “Oh, did I not think of a case?” Then I was looking at it, I was like, “Oh yeah, I didn't think of 0.” I had greater than 0, I had less than 0. I’m like, “What happens at 0?”
For Chloe, the realization of the need to consider zero came when she was driving, but she expressed it while implementing the bellringer immediately after Riley gave his response about the need to consider the case when \( x = 0 \). The video records allowed for the identification of Chloe’s instance of learning, which is illustrated in the following transcript:

TPST (Chloe): Yeah, you just brought up my next point. You beat me to it. I was gonna say that, for any proof or when you're providing reasoning, a way to disprove that is to provide a counter-example. I was going to bring up for [Ivy] and [Riley], they both- [Ivy] brought up that \( x + x \) is larger because when you're combining two \( x \)'s, so she brought up when \( x \) is positive. [Riley] added in the negative to it, but I don't think either of you guys considered the 0 in the moment, which is okay because when I first did this, it took me a day to think about it. I thought about it in the car one day as I was driving here. I was like, “Oh my gosh.” I didn’t even come across that one.

The transcribed video segments also supported identifying where possible of the conversation where the ideas learned surfaced. In the example of Chloe’s realization while she was driving, there was no such conversation. However, for Riley’s instance of learning, I was able to use what he said to trace the conversation where Chloe called Ivy to share her solution and then later called on Riley. This conversation happened during the bellringer implementation and was in the transcription for Chloe’s bellringer video. Therefore, the transcribed segments provided a context for PST learning in relation to the BRS and, more broadly, the entire collection of videos provided a context of PST learning in relation to the methods course.

Artifacts

The following artifacts were collected: completed bellringer evaluation rubrics, copies of bellringer tasks implemented, any handouts given to student preservice teachers (SPSTs) during bellringer implementation, and SPSTs’ work on the bellringer tasks. PSTs were given access to copies of completed bellringer evaluation rubrics and SPSTs’ work on bellringer tasks in addition to video recordings for all the bellringers implemented to support them in reflecting on
their own implementation and that of their peers. Collectively the artifacts provided a context for PST learning and supported interpretation of observations of PST learning.

**Interviews**

Interviews were conducted with each PST after they implemented their bellringer to provide insight into their learning from being a teaching preservice teacher (TPST). The interviews took about 20 to 30 minutes and were conducted after the TPST’s enactment at the earliest time that the TPST was available. Interview questions covered what the TPSTs learned from the first three phases of the BRS—preparation, implementation, and debriefing—and aspects of the methods course and other knowledge and experiences that supported their learning (see Appendix F for the interview protocol).

The interviews were intended to provide insights into PSTs’ learning as they enacted approximations of practice by taking on the role of a teacher as the TPST and as they participated in representations of practice by taking on the role of students (SPST). The interviews provided triangulation of the data with regard to observations of PSTs’ learning from the videos of the bellringer sessions and also brought to the fore instances of PST learning that were not identifiable from the video records of the bellringer sessions. Additionally, the interviews provided information on aspects of the methods course that supported TPSTs’ learning experiences and other knowledge and experiences TPSTs drew from in their learning.

**Written Reflection**

PSTs had two formal opportunities to document their reflection. The first one was a reflection written by the TPST within 24 hours after implementing their bellringer. The 24-hour reflection on the university’s online learning platform was designed to capture what they learned from the experience of implementing their bellringer. The prompt asked TPSTs to reflect on their
experience and include things that they had learned and what they would do differently. It was intended to capture their immediate thoughts.

After all PSTs had implemented bellringers, each PST wrote a reflection paper on bellringers. The paper was made up of two parts. The first part gave them an opportunity to reflect on their own bellringer by assessing their task, plan, and teaching, and revising the task and the plan and providing reasons for the changes. The second part of the reflection paper was an opportunity to reflect on their peers’ bellringers and what they learned from the BRS as a whole. The second part of the paper was intended to allow PSTs’ reflection across all the bellringers that were implemented so they could express what they had learned about effective bellringers and mathematics teaching in general. To support PSTs in writing the reflection papers, videos of all the bellringer implementations, copies of worksheets for the bellringer tasks, and the completed rubrics were made accessible to PSTs. The 24-hour reflection and the reflection papers allowed for identification of instances of learning that were not explicit in the videos and also provided triangulation for those instances that were explicit in the videos.

Data Analysis

The purpose of this study is to provide insight into PSTs’ learning. To provide this insight, analysis was conducted at two levels. The first level of analysis examined what was learned and the second level of analysis used the information around what was learned to examine how that learning took place. In this section I describe the units of analysis for each level, and how these units were examined, including a description of the codes applied to aid in their examination.
First Level of Analysis: Examining What Was Learned

As mentioned in the theoretical framework, for the purposes of this study I consider a PST to have learned something (e.g., an idea, concept, or skill) if they express a realization of having acquired the knowledge or demonstrate the knowledge in a way that they had not done before. Thus, I infer what PSTs have learned from evidence in what they say or do. The unit of analysis is an instance of evidence of PST learning—each occurrence of an expression or demonstration that they have learned something (e.g., an idea, concept, skill). Although I recognize that it is not possible to say with certainty that learning has occurred, for ease of communication I write “learning” to represent “evidence of learning.”

As an example of an instance of PST learning, consider Isabelle’s learning expressed during Riley’s bellringer implementation:

Isabelle: I liked how you connected everything and showing another way [that constructing polygons from another polygon] could be done and that it was thought about, really gave me that “aha” moment. In my mind, that was actually simpler than what- My mind’s a mess, but [pause] that was great.

Instructor: Can you articulate a little bit more about what made it great? What was it that made you have that “aha”?

Isabelle: I think just [pause] reminding that there’s multiple ways to go about it. To look at a shape, but see what else you can construct. I think we were, [pause] my mind was trying to make smaller shapes out of it. Sometimes if you look at the bigger picture, it could be simpler.

Grain size of what was learned. Each instance of learning was assigned to the relevant component of the Knowledge and Practice Framework—content knowledge, pedagogical knowledge, pedagogical content knowledge, and high-leverage practices (see the Theoretical Framework for more details). The different grain sizes used to identify the instances of PST learning for each component are described below.
The grain size I used for identifying content knowledge (CK) comes from Charles’ (2005) discussion of big ideas and mathematical understandings. He defines a big idea as “a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole.” A mathematical understanding (MU) is “an important idea students need to learn because it contributes to understanding [a] Big Idea” (p. 10). The grain size of the acquired content knowledge is at the level of a MU. An example of a MU is “Ratios give the relative sizes of the quantities being compared, not necessarily the actual sizes” (p. 15). The MU for Isabelle’s instance of learning from Riley’s bellringer implementation was “Polygons can be decomposed into other polygons.”

For pedagogical knowledge (PK), the grain size used was a pedagogical idea, such as “A teacher should project their voice so that all students in the class can hear them”; “Making eye contact with students when listening to them makes them feel valued”; “Clearly worded questions make it easy for students to know what they are responding to”; “Clearly labeled diagrams support students in making sense of the content in the diagram”; “Projecting an item on the board is one way of focusing students attention on the item”; and “Asking students to explain their answers even when they are correct, is one way of getting at their thinking.”

For pedagogical content knowledge (PCK), the grain size is at the level of the union of the MU underlying the instance and the relevant pedagogy. Consider a task on identifying a graph representing a proportional relationship having the only correct graph with the ratio of the corresponding terms as 1:1. The knowledge that this may lead to the misconception that for quantities to vary proportionally the ratio of corresponding terms must be 1:1 is an example of PCK. The MU for the content knowledge is, “If two quantities vary proportionally, the ratio of the corresponding terms is constant” (Charles, 2005, p. 18). The related pedagogical idea is,
“When designing tasks where students are to pick the correct response(s), one should not include responses that are likely to create or condone misconceptions.”

For high-leverage practices (HLPs), the grain size was how the high-leverage practices related to the Bellringer Sequence (BRS). For example, consider HLP 13: “Setting long and short term learning goals for students” (TeachingWorks, 2018). As part of the BRS, TPSTs had to identify a mathematical understanding as the learning goal for their bellringer task. Thus, the grain size for HLP 13 would be learning goals for the bellringer, such as Riley’s: “Polygons can be decomposed into other polygons.” Likewise, for HLP 1, “Leading a group discussion,” the grain size would be the discussion during the bellringer implementation.

**Coding instances of learning.** Instances of learning were first identified in interviews, the 24-hour reflection, bellringer reflection papers, and the bellringer videos. The coding of instances of learning from these four data sources was done by the researcher and another graduate student. After coding individually, the researcher and the other graduate student met to reconcile the coding. Instances of learning in a fifth data source, the bellringer preparation audios, were coded only by the researcher because they were added as a data source after coding of the first four data sources had been completed. The bellringer preparation conversations, interviews, 24-hour reflections, and bellringer reflection papers were coded using qualitative data analysis software (Dedoose 7.0.23, 2016). The bellringer videos were coded using video analysis software (Sportstec, 1997-2015). Only excerpts in which PSTs explicitly expressed that they had learned something were identified as instances of learning. This is not to say that there was no evidence of learning besides the explicit excerpts, but rather that a high bar was set for evidence of learning. The multiple data sources, however, highlighted instances of learning that were
implicit in some sources and explicit in others. For example, during the debrief in the video, Yvonne said,

I think, yeah, that would be a good thing in the lesson for sure, to talk about. What dilation exactly means and defining it. How it can be both bigger and smaller.

It is not easy to infer that this is an instance of learning, yet in the interview Yvonne said,

I think I would’ve done it [addressed the idea of that a dilation can result in a bigger or smaller object] in the recapping and I wish I’d done that.

The statement Yvonne made in the interview, that she had come to a realization of the need to address the idea that dilations can produce smaller or bigger objects, met the standard set for evidence of learning, while the statement in the debrief did not. Instances in which PSTs were simply rehashing ideas and or discussing ideas not related to the content of the methods course were not considered as instances of learning.

An instance of learning was coded as content knowledge (CK) if the PST’s expression embodied a correct or evolving perception of a mathematical idea and it could be inferred that the perception was new to the PST. Expressions of perceptions that may not appear brand new but seem to involve a deepening of understanding of a mathematical idea were also coded CK. An instance was coded pedagogical knowledge (PK) if the PST expressed an understanding or realization of an aspect of teaching. The grain size would be a pedagogical idea, for example, “It is important to engage all students in discussion.” An instance where the PST expressed an understanding or realization of an aspect of teaching related to mathematical content was coded pedagogical content knowledge (PCK). For example, Yvonne’s statement above was coded PCK because it is the union of the PK, “It is important to address key aspects of a concept,” and the MU, “Dilations produce smaller objects when their scale factor is less than one and bigger
objects when their scale factor is greater than one.” An instance where a PST expressed or demonstrated learning of a high-leverage practice was coded HLP.

When coding instances of learning for these four types of learning what was considered in inferring learning was the grain size related to the type of learning. For example, when a PST said, “I realize that it was difficult for students to make sense of the diagram because of the way I labeled it,” the PST learning inferred was, “Clearly labeled diagrams support students in making sense of the content in the diagram.” That instance focused on the teacher’s labeling on the board (rather than the way one labels graphs), thus the instance received the code PK.

Preparing for the second level of analysis. The goal of the study was to examine how PSTs’ learning takes place; therefore, in order to capture sources from which PSTs were drawing their learning, an instance of learning was also coded for what the PST was drawing their learning from. These codes included class readings, methods class discussions, bellringer preparation, bellringer debriefs, bellringer rubrics, and other bellringers. Some of these sources of learning were initially identified in the pilot study; additional codes emerged from analysis of the data in this study. The codes and their descriptions were revised to capture the data and the data were recorded to capture the revised descriptions. Revision included grouping and regrouping. For example, initially the five practices (Smith & Stein, 2011) was a source by itself because of how predominant it was in the pilot study, but as other class readings emerged in this study, all class readings were grouped together, such that reference to the five practices as a source of learning was coded class readings. Learning that drew from the conversations with the instructor during preparation of the bellringer and interactions with the MUST chapter readings (Heid et al., 2015) that guided the choice of topic for the bellringer were grouped under bellringer preparation. The following is an example of an instance of learning from the...
interview with Layla, which in addition to being coded as PK, had the source of learning coded as class readings.

Yeah, right now I’m with remedial kids, [non]mathematical kids, so I try to step into the shoes of what if they had absolutely zero understanding of what [pause] an exponential function is? The graph and the growth of it. How can I explain it in a way where I’m not giving too much information, but enough information to guide them? Which I think was in the Chen (2012) article from [this course].

Second Level of Analysis: Examining How Learning Takes Place

The second level of analysis involved the examination of what prompted learning—the context in which the ideas learned surfaced and were encountered by a PST. A prompt is a coherent collection of idea units (Stockero, 2008) that seem to work together. The statement or collection of statements that surface the ideas learned and interaction with those ideas make up the learning prompt. A learning prompt starts when the ideas related to the learning surface or emerge in discussion and ends when the conversation takes a different turn and the ideas related to the learning are no longer the focus of conversation. A learning prompt was the unit of analysis in this second level of analysis.

Identification of learning prompts. Learning prompts occurred in a variety of ways. Most learning prompts were separate from the instance of learning. In a few cases, the learning prompt coincided with the instance of learning. The following example illustrates a prompt that coincided with the PST expressing the instance of learning and how the prompt was identified. The instance of learning involves Riley’s connection that the multiplicative property of equality and the multiplicative identity property show why cross multiplication works, as illustrated in the following transcript:

Riley: 'Cause I had both these identities in my mind, like, “Okay. These are linked because of these two things.” But I never really put it together that this is why the cross multiplication works. I just knew these are facts and we can do these things.
To identify the prompt I looked back at the conversations before this for when the ideas about why cross multiplication works surfaced and identified the conversation in the following transcript as the learning prompt:

Chloe: I had noted that that’s something that you did really well, too. But my “aha” was the fact that there were so many methods to do it. Every time you brought up a different one, I was like, “Oh, wow. That’s something I didn’t consider.” That was kind of my “aha.”

I think too, having those different methods made cross multiplication have more meaning. You know what I mean? You kind of understood why that works, though I guess you didn’t explicitly state it. But, yeah.

Instructor: Do you think you could’ve had a mathematical understanding that would get at the cross multiplication? Like, “Cross multiplication works because...” That could’ve maybe incorporated some of these other things?

TPST (Oliver): Yeah, definitely.

Instructor: And give you some unified-ness across what you wanted to- ’cause you know, the [MUST chapter] was cross multiplication because people who use it don’t know how to use it. And people misuse it a lot.

Riley: Cross multiplication works because of those two properties.

TPST: Yes.

Riley: I just feel like I just ... I just want to make sure that’s what we were trying to get at.

TPST: Right, right.

Instructor: That little connection piece-

Riley: Yeah.

Instructor: Was missing.

TPST: Yeah.

Riley: ’Cause I had both these identities in my mind, like, “Okay. These are linked because of these two things.” But I never really put it together that this is why the cross multiplication works. I just knew these are facts and we can do these things.
This prompt for Riley’s instance of learning was inferred based on the idea Riley learned and was identified by looking back to where he would have encountered the idea. Some prompts were identified based on what was said explicitly in the instance of learning. Table 1 below shows two examples of instances of learning, one that made the learning prompt explicit and one where the learning prompt had to be inferred from the ideas learned.

Table 1

<table>
<thead>
<tr>
<th>Instance of learning</th>
<th>Data</th>
<th>Identification of prompt is based on...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ivy: I said something similar. I said that like, you made me have the “aha” when you presented that it could be a rectangle. Instead of breaking it up, you could just make it one big thing.</td>
<td>Riley bellringer debrief video</td>
<td>What was said explicitly: The underlined part of the statement indicates the learning prompt is in Riley’s bellringer discussion and can be traced back to the conversation in the video where Riley says, “One way I didn't see is does this whole polygon construct another polygon?”</td>
</tr>
<tr>
<td>Isabelle: ...Then my article, I feel like didn't really give me something so specifically. It said “Adding square roots”, but the whole article was just proving why [just adding the radicands] wasn't true. Interviewer: Mm-hmm (affirmative). Isabelle: So I was like, “Okay. What am I supposed to go off of?” I just learned that it can be kind of vague, I guess, and you really have to pick [your learning goal] out as a teacher, what you want the students to learn.</td>
<td>Isabelle’s interview</td>
<td>Idea(s) learned and where PST could have encountered the idea(s): The underlined part in the instance indicates the idea learned. This idea was traced to the bellringer preparation meeting. Note: In this case the context provided that, the article did not give Isabelle something specific indicates this learning was likely prompted by an event in the preparation phase of the BRS. Since Isabelle had a learning goal for her bellringer, we can infer that the learning took place during the bellringer preparation.</td>
</tr>
</tbody>
</table>
The prompts that were identified from the different data sources were referenced as follows: TPST name-data source-type of learning-numbering of the prompt in the data source. Therefore, a learning prompt that was identified in Evan’s bellringer video and was the second prompt related to CK learning would be referenced as Evan-video-CK-2. The first learning prompt related to pedagogy identified in the Chloe’s bellringer preparation meeting would be referenced as Chloe-prep-PK-1.

**Examination of learning prompts.** Learning prompts were examined in order to better understand how the conversations in the learning prompts contributed to learning. I drew from discourse analysis to examine the learning prompts identified in this study. Specifically I examined the conversations in the learning prompts by adapting the methodology Donath et al. (2005) used in a study that examined learning for a group of undergraduate engineering researchers. They provided descriptions of verbal interactions that they referred to as “speech events” (p. 408). Examples of the speech events from Donath et al. that I adapted in this study include *critique, instruction, and internalization.* I used Donath et al.’s idea of speech events to characterize the conversations in the learning prompts to gain insight into how the ideas that are learned emerge and how PSTs interact with those ideas. According to Gee (2014), language allows us to “say things, do things and to be things” (p. 2). Gee elaborates that in saying things we are also performing actions, and we are also projecting an identity. The characterization of conversation in this study was based on what the statement is likely to accomplish, that is, the “do things” to which Gee refers. The characterization of conversation facilitated further analysis, which provided insight into the language in the conversations that allowed PSTs to project certain identities that are discussed in the results chapter. The speech events were identified from bellringer preparation conversations and bellringer videos and were continually revised as
themes emerged, resulting in a total of 16 descriptive categories. Two additional categories, *cannot infer* and *not applicable (NA)*, were added to cover, respectively, conversations that were incomplete or incoherent because of crosstalk, and conversations that did not relate directly to PSTs’ learning. In consideration of how the different PST roles—as teacher (TPST) and as learner (SPST)—and the methods course may influence learning, conversations were also coded for who made the statement, a SPST, TPST or the instructor (I).

As already stated, the examination of the learning prompts is intended to get at how the ideas learned emerged and were interacted with by PSTs.

**Coding of learning prompts.** Table 2 shows the speech events, along with their definitions and example instances, that guided the coding of the prompts in this study.

Coding was done by the researcher and another mathematics education graduate student, who coded 30% of the prompts and then met with the researcher to reconcile the coding. Even though the second coder did not code all prompts, the conversations from the reconciling sessions led to refining of codes and revisions of coding for prompts that had been coded only by the researcher. Additionally, the coding on 10% of the instances that had been coded only by the researcher was discussed with a third graduate student. Any discrepancies identified in these discussions also led to refining of code definitions and revisions of coding for prompts already coded.
Table 2

*Definitions of Speech Events and Example From the Data*

<table>
<thead>
<tr>
<th>Speech event</th>
<th>Description (relates to what is likely to be accomplished)</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>Expressing agreement with an idea that has been shared</td>
<td>Yeah, right. Then that’s actually a reason why.</td>
</tr>
<tr>
<td>Cannot infer</td>
<td>The characterization cannot be inferred either because it is an incomplete statement or part of it is inaudible</td>
<td>So we’re not starting with-</td>
</tr>
<tr>
<td>Clarify</td>
<td>Making an idea clear or more comprehensible.</td>
<td>Are you saying that 3 years ago she had 21 less pieces of candy?</td>
</tr>
<tr>
<td>Classroom</td>
<td>The statement is intended to bring some order in the class by drawing students’ attention to an activity of focusing their attention on a task or idea. This includes providing information related to short and long term goals for the bellringer enactment or class.</td>
<td></td>
</tr>
<tr>
<td>Management</td>
<td></td>
<td>Okay, we got multiple things going on. Teacher, get control of your class please. Who do you want to talk?</td>
</tr>
<tr>
<td>Confusion</td>
<td>A verbal expression that one is uncertain, bewildered or lacks understanding of an idea.</td>
<td>I think that was my face at the end. I was just kind of like, “Well ... Why does it equal 1?”</td>
</tr>
<tr>
<td>Check-in</td>
<td>The statement elicits self-assessment of the class’ understanding or reaction to an idea that has been shared.</td>
<td>Does that make sense?</td>
</tr>
<tr>
<td>Critique</td>
<td>Seeking or providing feedback to an idea that has been presented. This includes giving an opinion, comment or analysis of the idea that has been presented.</td>
<td>Kind of, yeah. I mean, I still like the multiplication method, but I don’t think putting 5 boxes there and then putting 0 into each of them is gonna be reliable.</td>
</tr>
<tr>
<td>Information</td>
<td>A statement whose purpose is not to influence others but to share their perspective. It is not expressed as an opinion but rather a sharing of ideas or experience.</td>
<td>6th graders know what a tape diagram is. I deal with that every day.</td>
</tr>
</tbody>
</table>
Table 2—Continued

<table>
<thead>
<tr>
<th>Speech event</th>
<th>Description (relates to what is likely to be accomplished)</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elaboration</td>
<td>Providing for an expansion or explanation of an idea. May involve providing an example to illustrate the idea.</td>
<td>To find ( x ), we need to find a common denominator between the 2 expressions. Thus, ( \frac{2}{3} ) can be equal then to ( \frac{4}{6} ) because we’re trying to find the common denominator of 6. Which then leads to ( x ) equals 4 because ( x ) over 6 equals 4 over 6, so that has to equal 4. Since the denominators are the same, the numerators have to be the same.</td>
</tr>
<tr>
<td>Elicitation</td>
<td>The utterance is intended to draw out students’ ideas or make such ideas public.</td>
<td>I have a question about that. What kind of numbers would you use in your story problem? Would you do how we did ( \frac{15}{5} )?</td>
</tr>
<tr>
<td>Instruction</td>
<td>Providing new or additional knowledge or ideas, that were not already on the table, related to mathematical content or pedagogical.</td>
<td>So, mathematicians like to call that “the multiplication property of equality,” which basically just means if we multiply both sides of an equation with the same non-zero number, we’re gonna have an equivalent equation. Which then allows us to have nicer numbers that we can work with algebraically and get to our solutions.</td>
</tr>
<tr>
<td>Internalization</td>
<td>Responding to an idea in a way that suggests assimilating, taking ownership of the idea or adapting it for one’s own use.</td>
<td>Oh, okay. I get what you’re saying. You’re gonna have two 10s left and take those 10s away, there’s nothing.</td>
</tr>
<tr>
<td>Literal</td>
<td>Providing brief factual information related to an idea that has been shared.</td>
<td>It’s perfectly valid to add 0 five times.</td>
</tr>
<tr>
<td>Object</td>
<td>Express disagreement or disapproval of an idea.</td>
<td>It’s not necessarily that it’s PEMDAS. Let’s not talk about order of operations that way.</td>
</tr>
</tbody>
</table>
Table 2—Continued

<table>
<thead>
<tr>
<th>Speech event</th>
<th>Description (relates to what is likely to be accomplished)</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposition</td>
<td>Proposing a way, action or approach that could be or could have been taken in relation to an occurrence in the class or an idea that emerged.</td>
<td>You could even take Jolly Ranchers and be like, “Okay, here’s 15 and I want to give them to 0 people. Who do I give them to?”</td>
</tr>
<tr>
<td>Rationale</td>
<td>Providing or asking for a justification for a proposed action or an action that has already been implemented.</td>
<td>I think because their things were up there, I wanted them to correct themselves, rather than someone being like, “Oh, wrong.”</td>
</tr>
<tr>
<td>Self-reproach</td>
<td>An expression indicating a wish that one would have acted or not acted a certain way.</td>
<td>Yes, I definitely should’ve considered that and looking back, that’s a big thing.</td>
</tr>
<tr>
<td>NA</td>
<td>This code applies to all utterances that do not relate directly to the PST learning.</td>
<td>Put that on the doc[ument] cam[era].</td>
</tr>
</tbody>
</table>

Speech event codes were assigned to statements within the learning prompts by considering the context in which the utterance was made—what the utterance was a response to or what had been said previously that the speaker was reacting to—and the tone of voice. The coding was done by constantly referring back to the video to listen to the tone of voice to determine the most appropriate code, particularly where more than one code seemed applicable. This helped determine whether words like *yeah, right, and okay* were simply part of conversation convention or people were using them intentionally to indicate agreement. This means utterances were not coded in isolation, but rather in the context of ongoing conversation and class context in order to capture the flow of the ideas during the conversation.
As mentioned in the previous section, two codes, *cannot infer* and *not applicable (NA)*, were added to cover conversation that would likely not contribute to our understanding of PST learning. Reference to the context and tone of voice helped in determining utterances that did not relate directly to PST learning. Such utterances included statements that were simply meant to move the conversation along, such as “yeah,” “okay,” and “right,” which were coded *not applicable (NA)*. In situations where such utterances did not really change the flow of ideas of the preceding speaker, for example, when someone said “yeah” and the preceding speaker continued speaking without changing their train of thought, the “yeah” was not characterized separately and therefore did not get an NA code, but was subsumed into the entire utterance by the speaker. For example, in the excerpt below the “yeah” statement made by Riley is not coded because the instructor’s flow of ideas does not change and so the whole excerpt was coded as one and the code *critique* was applied to it.

Instructor: That little connection piece-

Riley: Yeah.

Instructor: Was missing.

In situations where the speaker continued with their train of thought after an interruption, but the statement made during the interruption had a code other than NA, the statements before and after the interruptions were coded separately even if they ended up with the same code.

Statements that were only likely to accomplish an action, but did not involve eliciting the action received only a code related to the action it was likely to accomplish. For example, the statement, “On the first one I multiplied, and then the second one they’re added,” is likely to accomplish the action of elaborating some mathematical work. It received the code *elaboration* and because it was made by a PST in the role of a teacher, it received an *elaboration (TS)* code.
However, a statement that involved eliciting some action such as, “Would you mind explaining what you did here,” received both the *elicitation* and *elaboration* codes. In this case, because the statement was made by a PST in the role of a teacher, the codes would be *elicitation (TS)* and *elaboration (TS)*. That is when the type of elicitation was inferable, the excerpt was assigned an elicitation code and a code which indicated the type of elicitation. All questions received an *elicitation* code. The codes *elaboration* and *instruction* had sub codes *mathematics* and *non-mathematics*, and so any excerpt coded *elaboration* or *instruction* had at least two codes. The two examples above which were all related to mathematics would additionally each receive the code mathematics.

After the characterization of the conversations in the learning prompts, two levels of analysis were conducted on the learning prompts. One involved quantifying codes and the other involved examining the structure of the prompts in relation to the ideas learned. As already stated, excerpts were coded both for the speech event and the speaker. Coding by the speaker allowed for comparison of quantities of speech events by speaker. This comparison was done to identify whether certain types of speech events were more common or rare among certain types of speakers. The second level of analysis involved identifying broad themes across the learning prompts related to how the idea that was learned surfaced and was engaged with by the PSTs. For each learning prompt, the speech events within each broad theme were identified and examined for how they collectively contributed to the broad theme. The examination of speech events within the broad themes provided insight into how PSTs interacted with the ideas learned.

**Summary**

This study was conducted in the context of a middle school mathematics methods course with 11 traditional PSTs. Multiple data sources—survey, audio, video, interview, and written
reflection—were used to identify instances of learning. The instances of learning provided a context for identifying learning prompts that were characterized using speech events to allow for examination of how PST learning took place. The results of the analysis of the instances of learning are the focus of the next chapter, to facilitate answering the first research question of what PSTs learned through the BRS.
CHAPTER 4
WHAT PRESERVICE TEACHERS LEARNED

This chapter addresses the first research question, What do preservice teachers learn through the implementation of the Bellringer Sequence in a methods class? Recall that a preservice teacher (PST) was considered to have learned something (e.g., an idea, concept, or skill) if they express a realization of having acquired the knowledge or demonstrate the knowledge in a way that they had not done before. The analysis identified 178 distinct instances of PST learning that fit into the four components of the Knowledge and Practice Framework: content knowledge, pedagogical knowledge, pedagogical content knowledge and high leverage practices. An additional eight instances of evidence of learning that did not fit into any of these four categories were put into a category called generic. Table 3 shows the number of distinct instances in each category. Pedagogical knowledge accounted for almost half of the total number of instances (92 out of a total of 186 instances, 49%). Content knowledge instances were second highest (41; 22%). Pedagogical content knowledge and high leverage practices made up 17% and 8% of the total number of instances, respectively. Table 4 shows the number of instances of learning by category broken down by the data sources that provided evidence for them. I begin by discussing some observations about the number of instances identified in the different data sources. I then illustrate what was learned in each of the categories.
Table 3

*Instances of Learning in Each Type of Learning Category*

<table>
<thead>
<tr>
<th>Category</th>
<th>Number of Distinct Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content knowledge</td>
<td>41</td>
</tr>
<tr>
<td>Pedagogical knowledge</td>
<td>92</td>
</tr>
<tr>
<td>Pedagogical content knowledge</td>
<td>31</td>
</tr>
<tr>
<td>High leverage practices</td>
<td>14&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Generic</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>186</strong></td>
</tr>
</tbody>
</table>

<sup>a</sup>This number includes only explicit instances of learning related to HLP. Further information about this distinction is provided in the section on learning related to HLPs.

As seen in Table 4, *Instances of Learning by Sources of Evidence and Type of Learning*, some instances of learning were identified in more than one data source. The number of distinct instances for each of the four learning types (shown in Table 3; excluding generic) is less than the corresponding totals in the last row of Table 4 because the total of sources of evidence for the learning instances reflects instances that had evidence in multiple sources. There were a relatively higher number of instances of evidence of learning identified in the bellringer reflection paper compared to the other four data sources. For instances of learning identified in only one data source, 65 out of a total of 143 instances (45%) were identified in the bellringer reflection paper. I now look at each of the five type of learning categories.
Table 4

Instances of Learning by Sources of Evidence and Type of Learning

<table>
<thead>
<tr>
<th>Sources of Evidence for Instances of Learning</th>
<th>CK</th>
<th>PK</th>
<th>PCK</th>
<th>HLP</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bellringer preparation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bellringer preparation (BR Prep) only</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>BR Prep and one other source</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>BR Prep and three or more other sources</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Bellringer implementation and debrief video</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Video only</td>
<td>16</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>Video and one other source</td>
<td>11</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Video and two other sources</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Video and three or more other sources</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
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Content Knowledge

As can be seen in Table 3, a total of 41 instances of evidence of learning related to mathematics were identified across the bellringer preparation, bellringer implementation and debriefing videos, 24-hour reflections, interviews, and bellringer reflection papers. Thirty-six instances were identified from only one data source and the remaining five instances were identified in two or three data sources. No instances of learning related to mathematics were identified in the 24-hour reflection. This may be because the 24-hour reflection was completed only by the preservice teacher who had implemented their bellringer on the particular day, and so their focus as they reflected may not have been on the mathematical ideas that they had been grappling with for some time.

The PSTs in this study had taken a lot of mathematics courses, particularly the secondary education majors who were the majority in the class (8 of the 11 PSTs), and I had initially thought there would not be much for them to learn in relation to mathematical content. However, the study provides evidence of learning mathematics for the PSTs that involved new mathematical ideas, deeper mathematical understanding, and uncovering (and resolving) misconceptions. Since the experience seemed different for PSTs depending on whether they were in the teaching (TPST) or the student (SPST) role during the bellringer implementation, the following subsections look at mathematics learning by role.

Content Knowledge Learning for TPSTs

Overall, as TPSTs grappled with mathematical concepts related to their bellringer topic, there was evidence of three types of mathematics learning: they learned new ideas (7), deepened their understanding of the topics (3), and had their misconceptions addressed (1). The description
provided here is not exhaustive of TPST mathematics learning but illustrates each of the three types of mathematics learning for TPSTs.

**New ideas.** Chloe realized that one needed to consider all cases when drawing conclusions. While preparing for her bellringer task, “Which is larger, $x$ or $x+x$?,” initially Chloe did not identify all three cases—when $x$ is greater than zero, less than zero, and equals zero—forgetting, like some of the SPSTs during her teaching, the case when $x$ equals zero. Similarly, Isabelle grappled with developing an explanation for why $\sqrt{2} + \sqrt{3} = \sqrt{5}$ is a false statement, but was able to articulate an explanation after back and forth email conversations with the instructor as she prepared for her implementation of the bellringer. Evan expressed in the bellringer preparation meeting that he had never thought of expressing monomial and binomial multiplication using area (something suggested in his preparation materials) and described how the use of area helps to highlight the difference between multiplication of monomials and binomials.

**Deepened understanding.** Riley deepened his understanding of the topic addressed in his preparation materials, area of plane figures: “It really gave me a grasp of the concept that I was trying to cover.” The material helped him know how to define a polygon and the correct terminology related to his topic. Ivy expressed during the bellringer preparation meeting that previously she just knew that a number raised to a zero power equals one was a math fact, but did not know why. After reading the materials for her bellringer, she said, “I was like, whoa, I guess that makes sense.”

**Addressing misconceptions.** For other TPSTs, preparing for the bellringer helped address misconceptions they had about their topic. For example, for Layla, the MUST (Heid et al., 2015) chapter that she was assigned for her bellringer addressed a misconception that she
had about linear interpolation. Layla said, “I never really understood linear interpolation until I read [the chapter]. I always thought you just squished the graph to make one line.” The chapter helped Layla understand that estimating the value of a function \( f(x) \) for \( x_n \), where \( x_n \) lies between \( x_1 \) and \( x_2 \), involved treating the function as if it was linear between the two points \( x_1 \) and \( x_2 \).

It is worth noting that the Bellringer Sequence allowed TPSTs to engage with the mathematical ideas in their assigned topics as they read the assigned material and as they communicated with the instructor during the preparation phase. This will be discussed further in chapter 5.

**Content Knowledge Learning for SPSTs**

Even though SPSTs did not have to make sense of the mathematical topics in the same way as the TPSTs, there was evidence of content knowledge learning from their engagement with mathematical ideas during the bellringer implementation and debriefing. This learning included new ideas (11), a deepening of understanding of content they already knew (17), and addressing misconceptions in their understanding (2).

Most of the content knowledge instances of learning came from the debriefing sessions because they gave the SPSTs the opportunity to reflect on the bellringer and, in some cases, the debriefing sessions involved further discussions of the mathematical ideas. This happened particularly in situations where at the end of the formal bellringer implementation it appeared that there were mathematical ideas that were not resolved. The bellringers on linear interpolation, appropriate measures of central tendency, multiplication of negative numbers, and division involving zero led to extended discussions of the mathematical ideas during the debrief.
New ideas. Ivy and Layla expressed during the debrief that Isabelle’s use of the order of operations to explain why $\sqrt{2} + \sqrt{3} = \sqrt{5}$ is a false statement, was an “aha” moment for them. For Cameron, learning during Kylie’s bellringer involved embracing additional approaches for explaining division involving zero. Cameron came into the division by zero bellringer with a clear algebraic conception (which he explained during the debrief) of how to show that a number divided by zero is undefined. Initially he was skeptical that the same could be done using the sharing and measurement meanings of division that were proposed by the elementary education PSTs in the class. As a result of the class discussion, Cameron finally accepted that the meanings of division can be used and elaborated the approach for $\frac{15}{0}$ as illustrated in the following transcript from the debrief for the bellringer on division involving zero:

Cameron: I take back what I said. You can equal it because it would make sense.

Instructor: Say more about that.

Cameron: You wanted to- How many times can you group 0 into 15? Or how many groups of 0 can you get from 15? Then you would say, “Oh, here’s one group of 0. It’s empty. Here’s another group of 0. It’s empty. Oh, keep adding more groups of 0, but I’m not getting anywhere closer to 15.” No matter how many groups of 0 you have, you’re never gonna get 15 things.

Instructor: So what does that say about the answer?

Cameron: There’s no way to do it. It doesn’t make sense logically, for this to ever work out. Yeah, it doesn’t make sense.

Instructor: And mathematicians call that “undefined.”

Cameron: Yeah, right. Then that’s actually a reason why.

Cameron was not the only SPST who benefited from the sharing and measurement meanings of division that were shared by the elementary education PSTs. Both Ivy and Layla also expressed that they gained a better understanding of division involving zero using the meanings of division.
**Deepened understanding.** Kylie’s bellringer on division involving zero also resulted in six SPSTs—Cameron, Evan, Isabelle, Layla, Riley, and Yvonne—expressing a deepening of their understanding of division by zero. Riley expressed in the interview how he had only known by rules how division by zero works and explaining the mathematical reasons for the rules was difficult, but he gained a lot from Kylie’s bellringer. He claimed that he learned a mathematically sound way of thinking about the three cases of division involving zero (0/5, 15/0 and 0/0) that were included in Kylie’s bellringer. Layla expressed that prior to the bellringer, she could not explain division with zero, particularly a number divided by zero and zero divided by zero, and she just accepted what she knew as truth. Layla said that the discussion during Kylie’s bellringer “opened my eyes to understand better.”

**Addressing misconceptions.** Because of the criteria provided for effective bellringers, a number of the bellringers addressed a common misconception; therefore, there were a number of instances of mathematics learning that involved addressing these misconceptions. For example, among the misconceptions addressed by Chloe’s bellringer around the task “Which is larger, $x$ or $x + x$?” was that students would not consider the case when $x = 0$. Riley exhibited this misconception during Chloe’s bellringer and he expressed it as an instance of learning during the debrief. Oliver had his misconception addressed during Evan’s bellringer on multiplication of monomials and binomials. He applied the “foil method” (multiply first, outer, inner, and last terms of two binomials) to the first task that involved multiplication of monomials and got an incorrect answer. Even though the first task was not discussed, Oliver explained, “[Evan’s bellringer] still led to an ‘aha’ moment for me personally because I realized I had misapplied the exponent rules.”
The instances of content knowledge learning described here show that despite the strong mathematical backgrounds of the PSTs in this study, mathematics learning still took place and there was mathematics for SPSTs to engage with even when the TPSTs thought otherwise during their bellringer preparation. For example, Chloe had expressed concern during her bellringer preparation meeting with the instructor that the SPSTs would identify all three cases and present a complete solution for her bellringer, and she had explored questions she would ask to facilitate discussion if all SPSTs gave a correct solution. However, during her bellringer implementation she was able to identify and have two SPSTs present incomplete solutions, which led to a rich discussion.

**Pedagogical Knowledge**

A total of 92 instances of evidence of learning related to pedagogical knowledge (PK) were identified across the five data sources. Seventy-three instances were identified from only one data source and the remaining 19 instances were identified in two or three data sources. The instances of learning related to PK broadly fell into two categories: effective bellringers (17) and effective teaching (75). This is partly due to the Bellringer Sequence (BRS) being centered around the bellringer task and its intention to support preservice teachers’ learning to use bellringers effectively and to develop effective teaching practices more broadly. Additionally, the bellringer reflection paper was structured to capture PST learning of features of effective bellringers and effective teaching (see Appendix C for the bellringer reflection assignment). However, it is worth noting that less than half of the PSTs explicitly addressed learning related specifically to features of effective bellringers in the bellringer reflection, despite being asked to do so. In the following, I first discuss learning related to effective bellringers and then learning related to effective teaching.
**Effective Bellringers**

The BRS was designed to both help PSTs learn about effective bellringers and to be a site for more general pedagogical learning. The features of effective bellringers were discussed in the course when bellringer implementation was modeled for the PSTs, and the bellringer evaluation rubric (see Appendix A) was introduced. However, PSTs’ expressions of evidence of learning related to effective bellringers seemed to be masked by ideas related to effective teaching, despite the fact that after each bellringer implementation, each SPST and the TPST completed the bellringer evaluation rubric. This could be because the debrief conversations may have focused more on ideas related to effective teaching. However, some PSTs precisely highlighted learning that was related to features of effective bellringers, as outlined in the bellringer evaluation rubric. They did this in reference either to their colleagues’ bellringers or their own bellringers. In this section I will focus on reporting only instances of learning that explicitly are related to features of effective bellringers as outlined in the bellringer evaluation rubric. That is not to say that expressions of evidence of learning attributed by PSTs to effective bellringers, which instead are features of effective teaching, do not represent evidence of learning. Rather, including such instances in this section would make it difficult to delineate what PSTs learned specifically about effective bellringers; thus, they are discussed separately.

Evidence of learning related to effective bellringers was mostly expressed in the bellringer reflection papers (14 out of 15 instances). This could be because the bellringer reflection paper had an explicit prompt that asked PSTs to write what they learned about features of effective bellringers. In the following, I describe instances of learning related to effective bellringers under each of the four criteria outlined in the bellringer evaluation rubric: reviews or previews important mathematical ideas related to the lesson (appropriateness); engages students
in thinking at a high level of cognitive demand about mathematical ideas (student thinking); identifies misconceptions and addresses them (implementation); and uses clear verbal and written instructions and appropriate mathematical notation and terms (presentation).

**Appropriateness.** Yvonne commented on Riley’s bellringer and highlighted the fact that an effective bellringer reviews or previews an important mathematical idea related to the day’s lesson, and she described how Riley’s bellringer did this. In a similar vein, Ivy commented on Yvonne’s bellringer saying it “helped enhance the day’s lesson because she was planning on talking about similar figures with dilations, rotations, and more.”

**Student thinking.** Austin and Yvonne both expressed evidence of learning related to an effective bellringer engaging students in thinking at a high level of cognitive demand. Both of them cited Chloe’s bellringer as having engaged students at a high level of cognitive demand, and Yvonne additionally cited Riley’s and Evan’s bellringers as having done the same. Austin said that Chloe’s bellringer required a good amount of thinking for students to come up with all three cases \((x = 0, x < 0, \text{ and } x > 0)\), as shown by the responses in the class where one person came up with one case and another one identified two cases. Yvonne cited the multiple approaches (numberline, tables, graphs and limits) in the worksheets that students used to solve Chloe’s bellringer. Yvonne talked about her own bellringer task as having involved students at a high level of cognitive demand, because the task had no numbers and pushed students to find information, rather than working with information that had been given to them.

**Implementation.** With regard to effective bellringers highlighting and addressing misconceptions, Yvonne expressed how Chloe’s bellringer highlighted and addressed the misconception that if a student tests an expression for one case of a variable, then they think it is true for all cases. She described how Chloe addressed the misconception by bringing up Ivy’s
solution of one case, Riley’s solution of two cases and, finally, Riley’s realization of the third case. While reflecting on her own bellringer, Ivy expressed learning related to how her bellringer may not have highlighted or addressed any misconceptions and proposed that if she would have had more discussion, misconceptions might have surfaced or she could have posed a wrong solution and asked the class to identify the error.

**Presentation.** Layla and Yvonne expressed evidence of learning related to using clear verbal and written instructions. Layla expressed that her own bellringer implementation had clear instructions on the overhead and the worksheets and also acknowledged that this is something that other TPSTs also did well. Yvonne expressed similar evidence of learning about her observation of Oliver’s bellringer, which she said did not only have clear instructions, but also involved the use of appropriate terminology—citing Oliver’s use of the “multiplication property of equality” and how Oliver expounded on the property.

**Effective Teaching**

Themes that emerged from the 75 instances of learning on effective teaching fell into two categories: those directly related to student thinking (33) and those not directly related to student thinking (42). I first describe the instances of learning that are directly related to student thinking, followed by those that are not directly related to student thinking. Note that the evidence of learning related to effective teaching discussed in this section does not include all the excerpts related to pedagogy that were identified from the data, but represents the salient ideas on pedagogy that reflect the themes that emerged from the excerpts.

**Themes directly related to student thinking.** Themes directly related to student thinking included creating space for students to think (13), using students’ ideas rather than focusing on one’s own ideas (17), and engaging more students in discussion (3). These three
themes revolve around student thinking and are consistent with the course goals (see Appendix E), which include developing an understanding of student thinking, accessing and interpreting student thinking, and using it to inform instruction.

Creating space for students to think. One instance of evidence of learning that was highlighted by four PSTs—Isabelle, Riley, Yvonne, and Chloe—was in relation to when to ask students to discuss with partners. This arose from a moment during Isabelle’s bellringer where SPSTs seemed to be struggling to figure out the reason why the equation given in the task was not true. Rather than stick to her plan for the SPSTs to develop an explanation themselves, Isabelle went ahead and explained it herself. Later in the debriefing, the discussion about this decision led to a claim that it would have been appropriate to ask SPSTs to talk to their partners. This claim resulted in the PSTs, guided by the instructor, explicating situations where it would be appropriate to ask students to talk to their partners. Three PSTs explicitly highlighted that they learned about when to use partners, including Isabelle, the TPST, who expressed her learning in her 24-hour reflection, interview, and bellringer reflection paper. Isabelle saw the use of partners as an opportunity for students to think through a problem. Riley described the use of partners as a way of allowing for “productive struggle.” He seemed to be drawing from the idea of productive struggle in NCTM (2014), which was one of the required class readings and had been discussed in an earlier class session. Yvonne applied her learning about use of partners immediately during her own bellringer, which took place on the same day right after Isabelle’s bellringer. Following is an excerpt of the transcript from Yvonne’s post-implementation interview where she talked about her use of partners during her bellringer:

Yvonne: ...that is definitely important and then also knowing when or when not to use partner work. I did not use [partner work] on purpose initially because I wanted to have that discussion of what students were thinking and to bring it out in a discussion together.
Interviewer: Mm-hmm (affirmative).

Yvonne: Then seeing, okay, I kind of used partner work for a simple question ’cause I saw I had a lot of looks where I was like, “I think they just need to talk it out.” So then I was like, “Okay, go for it.”

Ivy and Isabelle expressed instances of learning related to allowing students opportunities to think that were about giving students wait time. Ivy expressed in the 24-hour reflection and interview that she could have given students time to think when she posed the question “How do we get to $a^0 = 1$?” rather than just picking on the first person that raised their hand. Similarly, Isabelle expressed in the bellringer debrief, 24-hour reflection, the bellringer reflection paper that she could have allowed more wait time when she asked SPSTs how they would know that $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$.

Ivy later expressed similar learning about the need to allow students to think about questions, rather than answering questions for students, in the interview and the bellringer reflection paper. She elaborated on this by saying questions should not be explained to the point where it leaves little for students to think about. Austin echoed similar sentiments in the interview when talking about how in the bellringer he avoided doing what he had done in his prior teaching opportunities, which had led to situations that “[did not] create student thinking as much. Whereas, asking questions without giving away the answer does.” Another instance of evidence of learning related to the idea of allowing students to think—knowing how much information to give students to allow them to engage with an idea without giving too much so that it takes away thinking from the students—was expressed by Layla in the bellringer reflection paper. Drawing on the relevant class reading by Chen (2012), Layla described how this idea came up in several bellringer debriefing sessions.
The PSTs seemed to be at different points with regard to learning about the need to allow students to think through problems, with some still wrestling to make sense of the teacher’s role and others more certain about what that role should be. Isabelle’s expression in the interview about her learning of the need to give students the opportunity to make sense of ideas seemed to carry with it some dilemma as she said, “... feel[s] like the teacher should be giving the students the knowledge, but really we're learning in this class that the students should be getting it themselves.” Riley, on the other hand, sounded more certain about the teacher’s role in learning, as he expressed in the bellringer reflection paper that “good teaching is not doing the work or thinking for the student, but rather responding to their original thinking and asking the right questions to have them think through and solve problems on their own.”

Using students’ ideas. Evidence of learning related to the need to use students’ ideas rather than pursuing one’s own ideas was expressed by Chloe in her interview in relation to her bellringer implementation. Chloe highlighted how the instructor pointed out how easy it is to focus on one’s own thinking and not be aware of it. Chloe said,

I didn’t notice at all. You know what I mean? I just was teaching, so you’d think [focusing on the students’ ideas is] something that you’d think through, but until [the instructor] pointed that out and how easy it is to fall back into [telling the students your ideas]. So, I think [the importance of focusing on the students’ ideas] stuck with me.

Evan also expressed in the bellringer reflection paper how he had learned the need to pause to assess the class rather than pushing on with his own ideas. He wrote that he needs to take time thinking through what I wish to say, by doing so I will be more able to develop the mathematical understanding I wish to develop in the classroom for everyone, not just myself or the students I am calling upon.

Learning related to focusing on one’s own ideas was also expressed in relation to how such a focus influenced preparation for the bellringer, as illustrated in Cameron’s learning.
Cameron’s focus on his own way was reflected in how he talked about his bellringer preparation. He acknowledged in the interview that in his preparation he anticipated more about things from his perspective and was limited in his view of the task from a student’s perspective. He realized that he needs to work on his preparation and “expect everything that you can ‘cause for me, I think my expectations of what I wanted to do were different than the expectations of what the students and the task itself was trying to do.” Cameron further said in his interview, “If you don’t prepare yourself for the students’ different ways of thinking, then you are going to completely ignore that thinking and I think that hurts them.” Later in the interview, Cameron highlighted how his lack of consideration for how the SPSTs were likely to engage with the task led to unanticipated struggles by the SPSTs, some of which they were not able to resolve during the bellringer implementation. In contrast, Yvonne drew learning about preparing for students’ different ways of thinking from a positive experience with her bellringer. She briefly described how she anticipated different student responses, and even prepared questions that would extend students’ thinking while taking into consideration the nature of the class. She expressed, in the 24-hour reflection, “how important it was to anticipate student responses and to be prepared with questions that extend the students’ learning.” Additionally, she described in her bellringer reflection paper how she incorporated students’ ideas into discussion saying,

I listened to the responses of my students in what they put as their answers to the written prompt and used what they were thinking to guide the discussion, rather than just me discussing how I would approach and solve the problem.

Other evidence of learning related to using student thinking was expressed by Riley, as he cited ideas from the debrief of Chloe’s bellringer about acknowledging students’ ideas and allowing other students to build on those ideas. There also was an instance of learning that cautioned about spending too much time on a student idea if it is not likely to contribute to
learning for the class. This instance was about a computational error Austin made when calculating the area of the polygon in Riley’s bellringer. During the debrief the instructor highlighted the amount of time Riley spent on Austin’s error. Both Riley and Austin expressed evidence of learning related to this instance. In the interview, Austin said that one of the specific things he learned was, “if somebody understands where they’ve made a mistake, then it’s okay to move on because you want to keep the whole class in mind.” Similarly, Riley said during the debrief, “Once [Austin] confirmed to me that it wasn’t a conceptual thing and he understood what we were doing, then I should’ve moved on.”

**Engaging more students.** The need to engage more students in discussion came to the fore during Chloe’s bellringer and was addressed by the instructor during the debrief. After Riley realized that he had not considered the case when $x=0$ in answering the question, “Which is larger, $x$ or $x+x$?” Chloe had an extended exchange with Riley, leaving out many students in the class who seemed eager to make a contribution to the discussion. Both Riley and Chloe expressed learning about the need to engage more students in discussion, with Riley expressing it in the bellringer reflection paper and Chloe expressing it in the 24-hour reflection, the interview and the bellringer reflection paper. Chloe’s biggest take away from the bellringer experience, as expressed in the interview, was “to engage all students.” She continued saying “I didn’t realize then how much [the exchange with Riley] left everyone else out. I think it could’ve really helped to get them more involved. Just making sure that you’re including everyone.”

Oliver also expressed in the bellringer reflection paper the need to involve more students in discussion, rather than “the discussion being made up of single student solutions and the teacher response to those solutions.” Oliver further highlighted in the interview his learning of the challenge of using students’ ideas to drive discussion toward a mathematical idea and his
observation of how some TPSTs took over the discussion to drive home the mathematical understanding. The point raised by Oliver is a challenge even for practicing teachers (Peterson & Leatham, 2009; Scherrer & Stein, 2013).

**Themes not directly related to student thinking.** There were 42 instances of pedagogical knowledge learning that were not directly related to student thinking: the importance of good preparation and flexibility (23); task design that allows students to engage with mathematical ideas (7); importance of being clear and assertive in one’s language (3); efficient use of the board (3); ensuring students’ understanding of ideas (3); and strategies that allow monitoring of student work (3).

**Importance of good preparation and flexibility.** The importance of good preparation and flexibility made up more than half the instances of learning that were not directly related to student thinking. All 11 PSTs in the study expressed learning related this theme at least once. Austin, Layla, and Kylie expressed learning related to the need to be prepared for unexpected occurrences. Austin expressed in his 24-hour reflection that he was not expecting that a majority of the SPSTs would not model the problem in his task using negative numbers. Layla expressed in her interview the need to build up a repertoire of knowledge and skills to help her prepare for unexpected occurrences and cited some of the knowledge she had gained from her peers’ bellringer implementations as part of her growing repertoire. Kylie expressed in her 24-hour reflection how technology did not work for her and she was not able to access her PowerPoint slides that would have aided her implementation. On the other hand, Oliver credited his planning for the success of his bellringer, and thus underscored the need for preparation while expressing the wish that he could have prepared even better.
Yvonne expressed in her 24-hour reflection the importance of anticipating student responses and being “prepared with questions that extend students’ learning.” Cameron also expressed similar learning in his 24-hour reflection on the need to “anticipate through attempting to see the problem through the student’s eyes.” Yvonne’s take away from the BRS experience was that it is “good to be organized” and “being intentional in how you organize things. Intentional in how you ask questions. How you have students work together, the task you give them. Everything has to be very intentional.”

Learning with regard to the importance of setting up the class for the bellringer was expressed by Riley in his interview. He learned from the graduate student’s modeling of bellringer implementation, and liked how the set up directed them clearly to what they needed to do as SPSTs. He said, “We knew when we first walked in this is what we need to do. We can get straight to it.” Even though similar learning was not explicitly expressed by other PSTs, they demonstrated it implicitly in the way they set up their tasks at the beginning of class for their bellringers.

Finally, some TPSTs realized that even with a brief task like a bellringer, the preparation is demanding. Evan expressed in the 24-hour reflection that it “takes a lot of preparation, especially if you want to do it successfully.” Isabelle expressed learning that preparing a bellringer takes time and was shocked at how much time went into preparing the bellringer, yet it is not even a lesson. She further acknowledged learning “how much meaning goes behind a bellringer.” Austin expressed similar sentiments about how having to think about everything you have to do is a lot of work and how he wished he could have spent more time thinking about the wording for his bellringer. Chloe expressed in her 24-hour reflection that teaching the bellringer
was a positive learning experience for her but acknowledged that she “learned that bellringers are hard to execute in such a short amount of time.”

**Task design that allows students to engage with mathematical ideas.** Learning related to designing tasks that allow students to engage with mathematical ideas was expressed by five PSTs—Austin, Isabelle, Ivy, Riley, and Yvonne. Austin, Ivy and Yvonne expressed learning in relation to their own tasks. Austin and Ivy expressed learning in relation to what they could improve on their tasks to allow for greater SPST engagement with the mathematical ideas. Austin expressed during the bellringer debrief that he would have included only two questions rather than three to allow time for students to model the problem in different ways, rather than just using words. Ivy expressed in the bellringer reflection paper that she would have included more space for SPSTs to provide more ways of rewriting $\frac{2^3}{2^3}$, so that SPSTs who finished early could still be engaged productively. Similar learning about engaging SPSTs who finish early was expressed by Isabelle in relation to Kylie’s task. Isabelle expressed in her bellringer reflection paper appreciation for how Kylie’s bellringer worksheet had an additional piece at end to engage SPSTs who finished early.

Yvonne expressed learning in the bellringer reflection paper in relation to what worked for her task: having a task without numbers allowed SPSTs to engage at a high level of cognitive demand because they had to find information rather than working with information that is given to them. Riley also expressed learning in the bellringer reflection paper related to Yvonne’s task, about how “the use of tactile objects and a written task with prompts gave [SPSTs] a variety of ways to engage with the task.” Similar learning was expressed by Austin in the bellringer reflection paper about how the shapes provided in Yvonne’s bellringer made the task more
accessible for SPSTs and allowed more engagement with it. Austin also expressed learning in the bellringer reflection paper related to Oliver’s bellringer on how the different methods Oliver showed for solving the equation in the task increased accessibility for SPSTs, allowing for more of them to engage with the ideas.

**Importance of being clear and assertive in one’s language.** The importance of being clear and assertive in communicating the ideas one wants students to learn came up in the debrief for Austin’s and Cameron’s bellringer. Austin’s bellringer was on multiplication of negative numbers and he was using a context of someone giving away candy, but his wording during implementation did not support students well in making the connection to negative numbers. Austin said in his interview,

> I tried to get people to say what I wanted, but I think I didn’t do it very [pause] I’m gonna use the word “forcefully,” but I don’t mean to force them to do it. I just mean in a stronger way.

He later added,

> For me personally, I should say, to be more [pause], like I said, stronger in what I want to do because I think we touched on, I had touched on, what I wanted them to get, but I didn’t stay on it for very long.

Cameron’s bellringer was on choosing an appropriate measure of central tendency for a given data set and providing a justification for the choice. While responding to students’ solutions, Cameron did not commit to either mean or median as more appropriate and realized that his “biggest problem might have been my avoidance to say that a student’s method was wrong or that there were better answers out there.” Cameron was effective in eliciting student thinking but did not help the students to make sense of why mean or median might be a better measure. He went on to say, “As a teacher you have to be assertive, otherwise goals and expectations are not going to be met and the class will lose direction.” PST learning on clearly communicating ideas
also involved learning about clearly communicating expectations to students. Chloe expressed such learning in the bellringer reflection paper, saying that, “it is important to state expectations explicitly,” as she did early in her bellringer implementation, because doing so facilitated student engagement with the task.

**Efficient use of the board.** Learning related to the importance of presenting work on the board clearly came up during Riley’s bellringer when he was presenting students’ ideas on how to find the area of the polygon in the second question of his bellringer. He drew lines and assigned measurements in a way that would have made it difficult for students to follow. Both Riley and Isabelle expressed evidence of learning with regard to this occurrence. Riley said in the 24-hour reflection, “I should’ve [been] more precise with what I was writing down for the class to see. That way students could be able to interpret and follow along with the solution even if they weren’t a part of the discussion.” This was also an instance of learning for Isabelle, which she expressed in her interview. Isabelle referred to this instance in Riley’s bellringer and the reaction of the class when the instructor drew their attention to Riley’s writing on the board. Isabelle said, “We were all like, ‘Oh, that is confusing.’ Even in the sense of that, a lightbulb went off on my head like, ‘Wow. We have to pay attention to that, too.’” Other learning related to use of the board was not about clarity, but about appropriately using the board in terms of choosing what to display. Layla expressed in the bellringer reflection paper, interview, and 24-hour reflection how she learned that she did not have to write down everything SPSTs said, and that “it is okay to put up [SPSTs’] work [on the document camera].”

**Ensuring students’ understanding of ideas.** Learning related to ensuring students understand the ideas addressed in a bellringer was expressed by Isabelle, Ivy, and Yvonne. Isabelle expressed in the interview that she did not know whether the SPSTs understood the
justification she provided for why $\sqrt{2} + \sqrt{3} = \sqrt{5}$ was a false statement. She realized that she should have made a point of ensuring they understood it. Ivy expressed in her interview how toward the end of her bellringer the instructor brought to her attention the confused look on Isabelle’s face. Ivy expressed that she should have been more alert and noticed that and addressed the questions Isabelle may have had. Yvonne expressed in the interview the importance of recapping ideas at the end and that she should have done it.

**Strategies that allow monitoring of student work.** Learning related to strategies that allow monitoring of student work was expressed by Isabelle, Oliver, and Yvonne. Isabelle and Yvonne expressed learning related to designing the student worksheet for a task in a way that makes the ideas the teacher wants to monitor easy for the teacher to see. Isabelle expressed in her bellringer reflection paper that she would improve her task by removing the lines that she had put on the worksheet because they made it difficult for her to read SPSTs’ writing while she was monitoring. Yvonne expressed in her bellringer reflection paper and interview that she would “tweak the design to include ‘circle one: agree or disagree.’” Oliver expressed learning in his 24-hour reflection that some forms of student expression are easier to monitor than others. He found it easier to monitor algebraic strategies than justifications written in words because of the different paces at which SPSTs wrote, therefore requiring more time to monitor.

**Pedagogical Content Knowledge**

A total of 31 instances of learning related to pedagogical content knowledge (PCK) were identified from the bellringer preparation, bellringer implementation and debriefing videos, 24-hour reflections, interviews, and bellringer reflection papers. Eighteen instances of learning were identified from only one data source and the remaining 13 instances were identified in two or three data sources.
Instances of evidence of learning related to PCK fell broadly into three subcategories: the structure of the bellringer tasks (14), selecting and sequencing ideas (8), and making connections across ideas (9). For each of the three subcategories there was evidence of learning both from a perspective of what worked and what did not work.

**Structure of the Bellringer Tasks**

The 14 instances of learning of PCK related to the structure of the bellringer tasks were expressed by eight PSTs. In 12 of the instances, PSTs expressed learning in relation to their own bellringers. Some PSTs addressed problematic aspects of their tasks, others proposed improvements, and Yvonne expressed evidence of learning related to the success of her bellringer.

In the 24-hour reflection and the bellringer reflection paper, Austin highlighted how his task was read by SPSTs from the perspective of the person gaining candy and not the one giving away candy, and therefore did not support them in making the connection to multiplication of negative numbers. There were two instances of learning expressed by Cameron in relation to his own bellringer. In one instance, Cameron expressed in the 24-hour reflection, interview, and bellringer reflection paper, how some SPSTs found the table in his task confusing, and cited how Ivy had written on her rubric for his bellringer that the ordering of the table by number of students missing the points rather than by the number points missed made it difficult to read the table. In another instance, Cameron expressed in his bellringer reflection paper how the mean being close to the median in his task made it difficult for SPSTs to decide on which of the two was a better representation of central tendency for the data. He proposed that having a higher mean would have helped SPSTs realize that some high values, possibly outliers, were affecting the value of the mean.
Kylie’s bellringer was on division involving zero and she had posed the tasks 15/5, 15/0 and 0/5. Kylie expressed in the interview and 24-hour reflection how applying Oliver’s idea of using a real-life situation, rather than going straight into fractions in her task, might have made it easier for SPSTs to think about the situations involving zero.

There were two instances of learning expressed by Layla in relation to her bellringer. In one instance, Layla expressed how her bellringer’s use of decimal numbers in the context of growing mold was problematic for one SPST. In a second instance, Layla expressed how labeling the axes and including the exponential function might have avoided the confusion that arose from SPSTs filling out an x-y table for the situation using the information she had provided.

Unlike the instances of learning described so far, the next two instances of learning described are not about deficits in the tasks, but rather one is about learning related to how a task could have been enhanced and the other one is about features of a task that made it work well. Riley expressed in the interview and bellringer reflection paper learning related to an additional idea that would have enhanced his task. The idea of providing additional blank polygons was proposed by one of the SPSTs during the debrief. Riley elaborated on how this would have allowed students to better arrive at the mathematical understanding at the center of his bellringer. Finally, Yvonne expressed learning in the bellringer reflection paper about what made her bellringer support student learning. She had two polygons cut out of paper, and SPSTs were to use them to answer whether they agreed or disagreed with the statement, “The larger shape is similar to the smaller shape, but the smaller shape is not similar to the larger shape.” Yvonne described how her use of manipulatives engaged SPSTs in thinking about similarity. This is in
contrast to the instances of learning related to problematic aspects of tasks, which instead impeded learning.

In addition to the PSTs who expressed learning in relation to their own bellringers, two PSTs—Austin and Isabelle—expressed learning in relation to Riley’s bellringer. Isabelle expressed learning in the bellringer debrief, interview, and bellringer reflection paper about how Riley’s task had them break down a polygon into smaller polygons and how Riley then highlighted a different way of thinking about it, by constructing a bigger polygon. Austin expressed similar learning in the interview and said, “I didn’t even think about that. It’s trying to open your eyes to like, you can do it this way.”

**Selecting and Sequencing Student Ideas**

The eight instances of learning related to selecting and sequencing student ideas were expressed by six PSTs—Austin, Chloe, Isabelle, Ivy, Riley, Oliver, and Yvonne. Six of the instances were related to PSTs drawing that learning from their own bellringer and two involved Austin expressing his observations about Chloe and Riley’s bellringers. The first three of the instances that I will describe relate to expressions of learning about selecting and sequencing in ways that supported SPSTs’ learning. In his bellringer reflection paper, Austin described Chloe’s bellringer as making effective use of sequencing student ideas and explained how selecting and sequencing supported the development of the mathematical understanding. Austin observed Chloe’s intentionality in her selecting and sequencing of student ideas. Similarly, Chloe described in the bellringer reflection paper how she selected and sequenced SPSTs’ ideas and gave her reasons for each move she made to support them to arrive at the mathematical understanding. For example, she said,
I knew that I wanted to begin the whole-group discussion with an incorrect or incomplete answer in order to create an “aha” moment from the identified misconception. This is what prompted me to select Ivy’s solution to start the whole-group discussion since Ivy only identified one of the three cases in her response.

While referring to her own bellringer, Yvonne expressed how she sequenced student responses “to build a well-rounded definition of similarity.”

The other three instances of learning were either about selecting and sequencing that impeded learning or proposed ways of sequencing that may enhance learning. Isabelle expressed how her sequencing of ideas may have taken away the opportunity to highlight the misconception that the reason why $\sqrt{2} + \sqrt{3}$ is not equal to $\sqrt{2 + 3}$ is because $\sqrt{2}$ and $\sqrt{3}$ are not like terms. She proposed that she should have brought up the counterexample first then brought up Layla’s idea about $\sqrt{2} + \sqrt{2}$ being equal to $2\sqrt{2}$, and recognized that this case presented by Layla about like terms might have helped address the misconception.

Ivy’s bellringer addressed why a number raised to the zero power is 1. In the second part of the task, she asked TPSTs to write $\frac{2^3}{2^3}$ in at least two different ways. Ivy noticed while she was monitoring that Isabelle went straight to $\frac{2^3}{2^3} = 2^0$ and she wondered how Isabelle got there. Ivy avoided picking on the solutions where the SPST did not have all the work shown, like Isabelle’s, but later realized that asking Isabelle to share her solution and make her process transparent might have prompted more discussion toward the mathematical understanding of the bellringer.

**Connections Across Ideas**

The nine instances of PCK learning related to connections between ideas were expressed by eight PSTs. Seven of the instances involved PSTs’ expressed learning about their own bellringers. Austin and Riley expressed learning related to Evan’s and Kylie’s bellringers,
respectively. I briefly describe five of the instances of learning to illustrate PST learning related to how making connections across mathematical ideas would have supported development of the mathematical understandings at the center of the bellringers.

The goal of Evan’s bellringer was to show that multiplication of monomials differs from multiplication of binomials. Evan had a bonus question in his task that asked SPSTs to show a visual representation of multiplication of binomials. He expressed learning in the bellringer debrief and bellringer reflection paper that asking PSTs to consider an area representation of the first problem, which was a monomial, would have supported SPSTs’ learning of the difference between monomial multiplication and binomial multiplication.

Ivy also expressed learning involving a connection between ideas in her first and second questions. Ivy’s first question asked SPSTs why a number divided by itself equals 1 and asked, for example, why \( \frac{2}{2} = 1 \). During her bellringer implementation, Ivy did not refer to the first question and this was brought up by some SPSTs during the Ivy’s bellringer debrief. Ivy expressed in the 24-hour reflection and bellringer reflection paper how referencing the first question, on why a number divided by itself is 1, could have supported SPSTs to learn why a number raised to the zero power equals 1.

In Oliver’s bellringer he had posed the task \( \frac{x}{5} = \frac{2}{3} \), which SPSTs were to solve and then explain why their solution works. During his bellringer implementation, the goal for SPSTs to learn why cross multiplication works did not come through clearly. Oliver expressed in the bellringer reflection paper how improvements to his task implementation would include making the connection that cross multiplication works because of the multiplicative identity property and the multiplication property of equality.
The mathematical understanding for Yvonne’s bellringer was, “Shapes can be transformed to similar shapes, larger or smaller, with proportional corresponding sides and congruent corresponding angles.” During her bellringer implementation, her focus on similarity did not support SPST learning of the intended mathematical understanding. Yvonne expressed in the interview the reason for the lack of support for SPST learning of the mathematical understanding, how she emphasized similarity and did not talk about the actual shapes or sizes of the shapes SPSTs had worked with on the bellringer task. She referred to Ivy’s suggestion during the bellringer debrief that connecting similarity to dilation would have supported SPST learning of the mathematical understanding.

Kylie’s bellringer was on division involving zero. A considerable amount of time was spent during her bellringer debrief discussing the meaning of division. Riley expressed learning in the bellringer reflection paper on how getting SPSTs to think about the meaning of division was a good way of preparing them to provide justification for the different answers obtained for the different situations of division involving zero.

**High Leverage Practices**

Fourteen instances of HLP learning were identified in the bellringer implementation and debriefing videos, interviews, and bellringer reflection papers. Thirteen instances were identified only in one data source, while one instance was identified in two data sources. Even though most of the instances of learning categorized as High Leverage Practices (HLPs) also relate to another component of the Knowledge and Practice Framework, I have chosen to address them as a separate category of learning because of the recent emphasis on supporting PSTs’ development of HLPs as a way of improving teacher preparation. Addressing HLPs separately allows me to identify the extent to which the bellringer sequence supported their development.
In this study, I coded an instance of learning HLP if its primary focus was on developing a practice as opposed to developing knowledge. For example, if a PST said they learned about the importance of “monitoring” (Smith & Stein, 2011), the instance of learning would be coded PK. If instead they said they improved in their ability to monitor, it would be coded HLP. Instances of learning related to High Leverage Practices (TeachingWorks, 2018) were expressed for HLP 1—*Leading a group discussion* (6); HLP 6—*Coordinating and adjusting instruction during a lesson* (1); HLP 13—*Setting long- and short-term goals for students* (5); HLP 15—*Checking student understanding during and at the conclusion of lessons* (1); and HLP 19—*Analyzing instruction for the purpose of improving it* (1). HLP 19 was a special case; in addition to the one explicit instance, there were many implicit instances of PST learning of this practice, which are also discussed in the HLP 19 section below.

**HLP 1: Leading a Group Discussion**

Instances of learning related to leading a group discussion (HLP 1) were expressed by Chloe, Oliver, and Yvonne. They expressed aspects of leading a discussion, such as asking students to provide justification for their reasoning, orienting students to one another’s ideas, calling on different students to share their solutions, and making connections across students’ ideas. Chloe expressed in her bellringer reflection paper how she asked SPSTs to provide justification for their reasoning and how she also oriented the other SPSTs to Layla’s thinking by asking them to listen to Layla’s explanation and make sense of her reasoning. Similarly, Oliver also expressed in his bellringer reflection paper how he was able to address the multiplication property of equality by asking Cameron, “How did you know that you could multiply both sides by 6 and both sides by 3?” Yvonne expressed in the bellringer reflection paper how she built the definition of similarity with the SPSTs.
HLP 6: Coordinating and Adjusting Instruction During a Lesson

Learning related to coordinating and adjusting instruction during a lesson was expressed by Ivy during her bellringer debrief and interview. Ivy’s bellringer was about justifying why $a^0 = 1$. Ivy had just said a number divided by itself is 1 and was about to proceed and tell the SPSTs that therefore $\frac{2^3}{2^3} = 1$, when she stopped herself and instead posed a question to the SPSTs, asking them, “How do we get to $a^0 = 1$?” This was noticed by the instructor, who pointed it out to Ivy during the debrief and Ivy elaborated on her actions both in the debrief and in her post-implementation interview. Ivy realized that posing the question and allowing students to think about the answer would help them understand better than if she just told it to them, and so she adjusted and posed the question instead.

HLP 13: Setting Long- and Short-term Goals for Students

Instances of learning related to setting long- and short-term goals for students were expressed by Chloe, Kylie, Isabelle, Oliver, and Riley. Chloe expressed in the interview how, originally, she was thinking too broadly about the task and learned that she had to be specific in identifying her goal and designing the task to achieve it. Kylie expressed in her interview how the process of preparing her bellringer helped her understand how to use the Common Core State Standards for Mathematics (NGA & CCSSO, 2010). Kylie said that prior to her experience with the bellringer, she thought that standards could be referenced only to a whole lesson. She had not known that one could reference standards for a bellringer, but in preparing her bellringer she did so. Isabelle expressed in the interview how the MUST (Heid et al., 2015) chapter assigned to her did not give her a specific mathematical idea to focus her bellringer on, and so she learned she had to pick out what she wanted students to learn.
HLP 15: Checking Student Understanding During and at the Conclusion of Lessons

Learning related to checking student understanding during and at the conclusion of lessons was demonstrated by Cameron when he expressed in the bellringer reflection paper that looking at the SPSTs’ worksheets made him realize that “7 out of 10 [SPSTs] had a wrong answer at some point in their work” and that there was “room for them to learn to reach the designed [mathematical understanding].”

HLP 19: Analyzing Instruction for the Purpose of Improving It

There was only one explicit instance of HLP 19 in this study. During his interview, Austin expressed how in this course he had learned how to do more “self-reflecting” and to think about what “I’ve done and how I could’ve improved.” However, reflection is embedded in the BRS through the completion of evaluation rubrics immediately after each bellringer implementation, the debrief, the 24-hour reflection for the TSPTs, and the bellringer reflection paper. Therefore, learning of HLP 19 appeared to be woven into the other types of learning, where that learning was the result of a process of analyzing instruction for the purpose of improving it, particularly in the bellringer reflection papers.

For example, there is evidence of HLP 19 in Isabelle’s expressed learning of PCK, that if she was to do her bellringer again she would discuss the counter-example (false, because $\sqrt{4} + \sqrt{9}$ is not equal to $\sqrt{13}$ ) before Layla’s solution (false, because $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$ not $\sqrt{4}$) because that would allow her to address the misconception that the statement $\sqrt{2} + \sqrt{3} = \sqrt{5}$ was false because $\sqrt{2}$ and $\sqrt{3}$ are not like terms. Isabelle analyzed the way she sequenced ideas and came up with a way of improving it. This learning of HLP 19 is implicit because Isabelle did not say, “I learned how to reflect,” but from her statement we can infer that she did.
To address the implicit learning of HLP 19, this section describes that learning based on the bellringer reflection paper. This is because the bellringer reflection paper was assessed in a way that provided a measure of the extent to which preservice teachers learned the practice of analyzing instruction for the purpose of improving it. I will discuss the results of this assessment based on the reflection paper evaluation rubric. The discussion in this section is only about the first part of the bellringer reflection paper, which prompted PSTs to assess their bellringer task, plan and implementation, and propose what they could improve and provide reasons for doing so. The second part of the reflection paper prompted PSTs to reflect across all bellringers implemented and write about what they learned about effective bellringers and effective teaching, and so does not address HLP 19 as directly as the first part of the bellringer does.

Writing the bellringer reflection paper allowed PSTs to develop the ability to assess their bellringer tasks and implementation and identify how effective they were in meeting the goal supporting development of the mathematical understanding at the center of the bellringer. The extent to which PSTs developed this ability varied with a few PSTs that were able to assess their tasks and implementation very well and propose insightful changes. However, all 11 PSTs were able to at least address some aspects of the task and implementation fairly well.

For both the designed task and the task implementation, there were three criteria for assessment: (1) depth and accuracy of assessment of extent to which the design of the task/task implementation prompted development of the target mathematical understanding, (2) level of evidence provided for assessment, and (3) level of comprehensiveness and appropriateness of proposed strategies/ideas for improvement of the task. For each of the three criteria I will describe how PSTs fared in assessing task design and task implementation. Then I will discuss the evaluation of their revised task and plan that they turned in as part of the reflection paper.
Mathematical understanding. Ninety percent of the PSTs were able to accurately address at least some aspects of the task design and task implementation related to the development of the target mathematical understanding and provide some supporting details. Thirty-six percent of the PSTs addressed aspects of task design and task implementation related to the development of the target mathematical understanding particularly well.

Provided evidence. For both the task design and the task implementation, 82% of the PSTs at least supported some claims that they made in the assessment with evidence that they appropriately cited from videos, rubrics, student work, and readings. Twenty-seven percent of the PSTs supported all the claims they made in their assessment with evidence that they appropriately cited from videos, rubrics, student work, and readings.

Proposed strategies. With regard to proposed strategies for improvement, 82% of PSTs at least identified some aspects of the task and the implementation plan that could be improved to better support the development of the target mathematical understanding. Eighteen percent of the PSTs identified aspects of the task that could be improved in a particularly insightful way. An example of such insight is Isabelle’s proposal to improve her task implementation to address a time when SPSTs were stuck: beyond suggesting that she would ask the SPSTs to discuss with their partners, she also proposed a time when it would have been appropriate for her to do so. She wrote, “I think a great time to turn to partners would be when I proposed the question of, ‘What is happening to \(\sqrt{2} + \sqrt{3}\) to get \(\sqrt{5}\), first?’”

Revised task and plan. PSTs assessed their revised task and plan using two criteria: extent to which the revised task would prompt the development of the target mathematical understanding, and extent to which the revised lesson plan would support implementation in a way that engages the class in developing the targeted mathematical understanding. Sixty-four
percent of the PSTs revised the task at least in a way that would allow students to encounter and engage with the target mathematical idea. With regard to the revised plan, 55% of the PSTs at least clearly articulated most parts of the plan to collectively allow for the target mathematical idea to surface, and for students to make sense of one another’s ideas, make connections, and arrive at the targeted mathematical understanding.

**Summary.** The evidence suggests that learning of HLP 19 supported learning related to the other three components of the Knowledge and Practice Framework, with a majority of instances of that supportive learning expressed in the bellringer reflection paper. This significant contribution of reflection to PST learning is further discussed in chapter 5, where the results on how PST learning took place are reported.

**Generic Instances of Learning**

Eight instances of PST learning were categorized as *generic*. Other than two instances—one related to affect, and another about the need for teachers to be reflective, these instances were primarily expressions of value for aspects of the BRS. PSTs seemed to appreciate the BRS, particularly the opportunity to get feedback from their colleagues. Isabelle said in her interview, “I was almost excited to hear my feedback.” Additionally, Isabelle saw the giving of feedback as a way of teachers working as a team by being open about their teaching. For Oliver, the feedback allowed learning on how to take criticism well and not take it as an attack on one’s “personal style.” Austin valued the debrief not just for the feedback, but for the way people built on one another’s ideas. Other instances that reflected PSTs’ valuing of aspects of the BRS included expressions by Austin and Evan about how teaching the bellringer gave them an opportunity to improve their public speaking. Austin attributed his learning to be comfortable articulating what
he wants to say in front of people beyond the BRS to the nature of the methods course, describing it as “the environment in which sharing is so heavily relied upon.”

The two instances of learning that were not expressions of valuing of aspects of the BRS were expressed by Cameron and Riley. Cameron expressed in the interview that a teacher has to be willing to acknowledge things that do not work well in a lesson, so that they “can remedy them and make them better.” Riley cited Isabelle’s enthusiasm as she greeted them and how he learned that “when a teacher is excited about what we are about to learn, your students feed off that energy.” Greeting students at the door was a norm for the course and something the instructor did at beginning of every class. TPSTs also chose to greet SPSTs as they entered class at the start of their bellringers. It is interesting that the modeling of this practice by TPSTs also provided an opportunity for SPSTs’ learning about matters of affect that may influence learning (including theirs).

Summary

This chapter answered the first research question by highlighting what PSTs learned through the BRS. There were four types of PST learning: content knowledge (CK), pedagogical knowledge (PK), pedagogical content knowledge (PCK), and high leverage practices (HLP). There was evidence that both TPSTs and SPSTs attained CK learning from their engagement with mathematical ideas during the BRS. This learning included new mathematical ideas, a deepening of understanding of content they already knew, and resolved misconceptions.

PK was the most frequent type of learning, making up nearly half the total number of instances of learning. PK learning fell into two categories: effective bellringers and effective teaching. Most PSTs expressed learning related to effective bellringers that did not specifically address the criteria for effective bellringers and instead was related to aspects of effective
teaching in general, perhaps because of the focus on ideas related to teaching during the bellringer debriefing sessions. Almost half of the learning instances categorized as effective teaching were directly related to student thinking. These instances focused on creating space for students to think, using students’ ideas rather than focusing on one’s own ideas and engaging more students in discussion. The experience of preparing the bellringer also gave PSTs insight into the amount of work that goes into preparing for teaching, with many of them expressing learning related to the importance of preparation and the need to be ready for situations where things don’t go as planned.

The instances of PCK learning highlighted how the structure of the BRS supports PCK learning by requiring PSTs to identify a mathematical understanding for their bellringer and implement it in a way the supports the development of the mathematical understanding. HLP learning was supported by the BRS both implicitly and explicitly. Learning of HLP 19 was generative in supporting the learning of CK, PK and PCK.

The excerpts that represent instances of learning that are described in this chapter provide information that was used in this study to identify events that prompted the learning that is expressed. The results of the analysis of these events that prompted learning is the focus of chapter 5.

**Discussion**

The use of varied data sources in this study supported the identification of instances of learning. Sources of data where PSTs were explicitly prompted to express what they had learned like the 24-hour reflection, bellringer reflection paper, and the interview had comparatively more instances of learning identified from them. The 24-hour reflection, despite its brevity, in most cases half a page, still had an almost equivalent number of instances identified as the
implementation and debrief, which involved conversations that lasted at least 30 minutes for each bellringer. This highlights the importance, in studies like this that examine PST learning, for tools that make learning explicit either through their design or by explicitly eliciting expressions or demonstrations of learning.

The results of this study show that half of the distinct instances of learning were related to pedagogical knowledge, which is not surprising given that this was a methods course. The relatively low number of distinct instances related to HLP learning, at only 8% of the total number of distinct instances, is also not surprising. There are several plausible explanations for this relatively low number. First, this study considered only instances of learning that were explicitly expressed by PSTs. Second, an instance of learning was coded as HLP only if its primary focus was on developing a practice as opposed to developing knowledge. Finally, the fact that each PST implemented the bellringer only once may have also limited observations of development of practice. Observations of learning of HLP may require extended periods of implementation of the BRS possibly with each PST implementing a bellringer more than once.

The one observation that was surprising was the relatively low number of distinct instances of PCK learning, at 17% of the total number of distinct instances of learning. This is surprising because the BRS involved PSTs designing a task and implementing it in a way that supports the development of a mathematical understanding. Thus, for TPSTs the process of preparing and implementing the bellringer involved consideration of pedagogical ideas that would support SPSTs’ development of the mathematical understanding. It would therefore be expected that evidence of the development of PCK could have been more prominent in the study. The low number of instances identified could be because of the methods used to identify learning which depended on PSTs expressions of learning. There could have been instances of PCK
learning that may have not been captured because in the PSTs’ expression the connection of pedagogy and content may not have been clear. Additionally, the interviews and the opportunities for reflection in the BRS elicited for learning generically and did not push for PSTs to share specific types of learning. To identify PCK learning, one may need to use activities specifically designed to make PCK learning explicit, as some studies have done (e.g., Aguirre, Zavala, & Katanyoutanant, 2012).

It is worth noting that although this group of PSTs was strong mathematically, instances of learning of CK still made up 22% of the total number of distinct instances of learning. The skepticism expressed by most TPSTs during the bellringer preparation meetings on whether there was any mathematical idea for their peers to learn from their bellringer was often challenged during bellringer implementations. This confirms that the topics assigned from Heid et al.’s (2015) MUST book were indeed topics that merit attention. Structuring bellringers around topics that are problematic allowed for engagement of mathematical ideas for PSTs and hence supported learning. Additionally, having each TPST design a bellringer task allowed them to engage with the ideas in the MUST chapters assigned to them in ways that supported learning of CK. While assigning TPSTs topics constrained them, picking a mathematical idea they wanted SPSTs to learn through the bellringer allowed them some choice within the assigned topic. Furthermore, in order to choose a single mathematical understanding for their bellringer, TPSTs had to make sense of the larger set of mathematical ideas in the topic.

The goal of the BRS was for PSTs to learn how to use bellringers effectively and also to learn about effective teaching. Even though PSTs expressed learning related to effective bellringers in 18% of the instances of PK learning, less than half the PSTs were explicit about how the learning was related to the criteria for effective bellringers in the rubric. Rather, most of
the expressions of learning that PSTs attributed to effective bellringers were ideas related to effective teaching. This suggests that most PSTs were not clear on features of effective bellringers despite their involvement with the rubric, which had all the criteria outlined, at each bellringer implementation. This may be the result of the focus during the debrief on ideas related to effective teaching that may have obscured ideas related to effective bellringers. This highlights the need to be deliberate in drawing PSTs attention to the ideas one wants them to learn, particularly in situations where there is a lot of information for them to process.

The emphasis on ideas related to effective teaching is evidenced by 82% of the instances of PK being expressions of learning related to effective teaching. It is worth noting that half of these instances were directly related to student thinking. This is consistent with the fact that use of student thinking was a recurring theme in the course. A prominent subcategory of ideas not directly related to student thinking was the importance of preparation and flexibility in dealing with unexpected occurrences. This suggests that even with the reduced complexity in the BRS—a brief task implemented for 10-15 minutes with a small group of SPSTs, PSTs still had the opportunity to experience the enormity of the practice of teaching. Pedagogies of enactment may be a way of giving PSTs a foretaste of the practice of teaching.

The four types of learning identified in this study had a total of 20 subcategories—CK (3), PK (9), PCK (3), and HLP (5). This variation in evidence of learning observed in this study highlights the potential in pedagogies of enactment for supporting integrated PST learning. Even though pedagogies of enactment tend to have a specific focus—microteaching focuses on PSTs development of a particular skill, lesson study focuses on the improvement of an identified aspect of instruction, and rehearsals focus on the development of core practices—they have the potential to support integrated PST learning. It is difficult to address everything that PSTs need
to learn in the limited time in methods courses. My conjecture is that pedagogies of enactment can be used to optimize the limited instructional time in methods courses by simultaneously addressing all four components of the Knowledge and Practice Framework.
CHAPTER 5
HOW PRESERVICE TEACHER LEARNING TOOK PLACE

The purpose of this chapter is to answer the second research question: How do preservice teachers learn through the implementation of the Bellringer Sequence in a methods class? To provide insight into preservice teacher (PST) learning, this study examined the events that prompted learning. These events were the conversations during which PSTs encountered and engaged with the ideas learned. I refer to such segments of conversation as learning prompts. The results reported in this chapter show that the learning prompts varied in relation to their location and directness in addressing the ideas learned. They also highlight how the ideas learned surfaced and where engaged with by PSTs. In the following, I first explore these results by describing the different types of prompts and illustrating the nature of conversation in a learning prompt. I then report on supportive aspects of the BRS and the context of the methods course that were highlighted by the examination of learning prompts.

Unpacking Learning Prompts

The results reported in this section address types of learning prompts and the nature of the conversations that occur within them.

Types of Learning Prompts

The study revealed variations in learning prompts based on the possible location of the prompts and how the ideas learned surfaced in the prompts. The process of locating learning prompts resulted in two types of learning prompts, local and global. Local learning prompts were made up of a segment of continuous conversation entirely located within a specific phase of the
Bellringer Sequence (BRS): preparation, implementation, debriefing, or written reflection phase. *Global* learning prompts were identified primarily in the written reflection of the BRS, and could not be narrowed down to a specific continuous segment of the BRS, or were located outside of the BRS but took place during the time when the BRS was going on in the methods course. I first discuss global learning prompts and then local learning prompts.

**Global learning prompts.** There were 113 identified instances of learning that had a global learning prompt. An example of an instance of learning that had a global prompt is Isabelle’s first instance of pedagogical knowledge learning, which was identified in her bellringer implementation and debrief video. At the beginning of the debriefing session Isabelle said:

> I should’ve given way more wait time on how \(\sqrt{2} + \sqrt{2} = 2\sqrt{2}\). When I asked that question, “How do we know \(\sqrt{2} + \sqrt{2}\) equals \(2\sqrt{2}\)?” I wish I would’ve given a lot more wait time.

This instance of learning seemed to have been prompted by Isabelle’s reflection on the point in her bellringer when the SPSTs were stuck when she posed the question and she went ahead and answered the question herself. It is not possible to locate a continuous segment of conversation in the BRS related to Isabelle’s learning prior to her expressing it at the beginning of the debrief, but what prompted her learning was clearly within the BRS. In general, global learning prompts—responsible for the majority of learning—appeared to involve substantial reflection. The global learning prompts provide important evidence of learning taking place, but do not provide insight into how that learning took place.

**Local learning prompts.** I identified 54 local learning prompts (see Appendix G for a complete list), which accounted for 71 out of a total of 186 instances of learning (38%). A single learning prompt could be related either to an instance of learning for one preservice teacher or to
instances of learning for more than one preservice teacher. There were four situations where the
same learning prompt resulted in different types of PST learning. An example of this involves an
instance during Oliver’s bellringer (described in chapter 3 as an illustration of how learning
prompts were identified) when Riley made the connection that the multiplicative property of
equality and the multiplicative identity property show why cross multiplication works, leading to
content knowledge learning for Riley. From the same segment of conversation, Oliver expressed
learning in the bellringer reflection paper, related to pedagogical content knowledge. Oliver’s
expressed learning was on how his structuring of ideas during the bellringer implementation did
dnot support SPSTs’ making the connection explicitly to why cross multiplication works. PSTs
drawing different learning from the same experience resulted in learning prompts with identical
collection segments. Since these learning prompts were made up of the same segment of
conversation, they were coded for speech events only once.

Two categories emerged in these local learning prompts: explicit and implicit. These are
discussed in the following subsections.

Implicit learning prompts. For implicit learning prompts, the ideas learned did not
surface directly in the prompts and were instead inferred by the learner through their own
interpretations of the bellringer circumstances. Most of the implicit prompts involved
mathematical ideas, while the instance of learning related to the prompt was learning of
pedagogical knowledge (PK), pedagogical content knowledge (PCK) or high leverage practices
(HLP). For example, Table 5 illustrates two implicit learning prompts by describing them in
relation to the ideas learned and tracing the possible connections to those ideas in the bellringer
sequence and the methods course context.
Table 5

Examples of Implicit Learning Prompts

<table>
<thead>
<tr>
<th>Learning prompt (full prompt if brief or description if prompt is long)</th>
<th>Description of instance of learning</th>
<th>Possible connection PST is making to learned ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yvonne-video-PK-1 (full prompt): “What would you need to know to know for certain if these two shapes were similar? Go ahead and think about it. Go ahead and turn to your partner and talk about it for just a moment. What tools would you need to know for certain if they were similar?</td>
<td>Isabelle’s learning of the need to allow students to grapple with mathematical ideas.</td>
<td>Isabelle could be making connections to the idea of allowing students to grapple with mathematical ideas from NCTM (2014) which had been discussed in the course and had also come up in the debriefing conversations.</td>
</tr>
<tr>
<td>Evan-video-PK-1 (description): Evan starts by saying “Okay, everyone has it as equivalent so I’m not gonna cover this question first. I’m gonna go onto the second one since everyone had the same thinking.” He discusses Chloe’s solution and then makes the connection between the first and second question by asking, “I feel like that’s the difference between the first and the second one. Can anyone explain what I’m meaning by that? That there’s a difference?”</td>
<td>Austin’s learning as he reflects in the bellringer reflection paper on Evan creating more time by not discussing the first question yet briefly acknowledging students’ responses to it and yet still using it to connect to the second question later on in the discussion.</td>
<td>Ivy’s bellringer implemented before Evan’s bellringer had two questions and Ivy went straight to the second question and did not make the connection of the first question to the second question. This came up in the debrief and it was discussed how making the connection would have been useful. Austin could be connecting what he is observing in Evan’s bellringer to Ivy’s bellringer.</td>
</tr>
</tbody>
</table>

Six of the 54 local prompts were implicit prompts. It was not possible to analyze these six prompts in the same way that instances with explicit learning prompts were analyzed since the ideas addressed in the conversation did not relate directly to the ideas learned from the implicit prompts. The implicit prompts, however, highlight an interesting dimension of PST learning
where learning seems to be triggered by PSTs making connections between the actions they are observing in the bellringer sessions to ideas that are not directly stated during those sessions.

**Explicit learning prompts.** The ideas that surfaced in the conversations in explicit learning prompts were directly related to the ideas expressed in the instance of learning, for example, Cameron’s instance of learning, described in chapter 2, that the sharing and measurement meanings of division could be used to explain why a number divided by zero is undefined. The ideas he learned had been explicitly discussed in the segment of debriefing conversation that was identified as the learning prompt for this instance. The explicit prompts were further characterized and analyzed for speech events, as discussed in the following section.

**Nature of Conversations in Learning Prompts**

This section reports the results of examining the nature of learning prompts at two levels: the statements in the learning prompts and broad themes across each learning prompt. The first part reports on the characterization of conversations in the learning prompts, and the second part reports on the stages of the learning prompts that were identified.

**Characterizations of conversations.** To better understand how the learning prompts contributed to PST learning, the 16 speech events defined in Table 3 (in chapter 3) were used to characterize the conversations in the explicit learning prompts. A total of 1,669 codes were applied to excerpts in the learning prompts. As already described in chapter 3, learning prompt excerpts were coded by both the speech event and the speaker. In this section, I report on the results of this characterization of the conversations, focusing on codes related to 10 of the 16 speech events: check-in, clarify, confusion, critique, elaboration, elicitation, internalization, instruction, literal, and proposition. I have chosen to report on these 10 speech events either because they were relatively common or because there were interesting patterns across speakers.
Table 6 provides frequencies for the codes related to the 10 speech events for the total number of excerpts, as well as a breakdown of the numbers for content knowledge (CK) excerpts and excerpts related to teaching (PK, PCK, and HLP).

Table 6

*Frequencies of Speech Events by Type of Learning Prompted and Speaker*

<table>
<thead>
<tr>
<th>Speech event</th>
<th>Learning prompts for content knowledge</th>
<th>Learning prompts for pedagogical knowledge, pedagogical content knowledge, and high leverage practices</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Instructor</td>
<td>TPST</td>
<td>SPST</td>
</tr>
<tr>
<td>Check-in</td>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Clarify</td>
<td>23</td>
<td>53</td>
<td>43</td>
</tr>
<tr>
<td>Confusion</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Critique</td>
<td>18</td>
<td>12</td>
<td>54</td>
</tr>
<tr>
<td>Elaboration (mathematical)</td>
<td>20</td>
<td>29</td>
<td>51</td>
</tr>
<tr>
<td>Elaboration (non-mathematical)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Elicitation</td>
<td>58</td>
<td>51</td>
<td>18</td>
</tr>
<tr>
<td>Internalization</td>
<td>2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Instruction (mathematical)</td>
<td>4</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Instruction (non-mathematical)</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Literal</td>
<td>16</td>
<td>42</td>
<td>71</td>
</tr>
<tr>
<td>Proposition</td>
<td>14</td>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>Totals</td>
<td>167</td>
<td>210</td>
<td>297</td>
</tr>
</tbody>
</table>

*Note.* TPST: Teaching Preservice Teaching; SPST: Student Preservice Teacher.
Table 6 shows how frequencies of speech events varied across speakers and types of learning. Some of the variations highlight aspects of the BRS and the course context that supported PST learning and are discussed in the relevant sections later in this chapter.

Some speech event codes were more common than others. The most frequent speech events were *elicitation* and *critique*, each at 13%, followed by *literal* (12%), *clarify* (11%), and *elaboration* (10%). The higher frequencies of these speech event codes suggest that PST learning took place through rich generative conversations. The next section on stages of learning prompts provides insight into these conversations through which PST learning took place.

**Stages of learning prompts.** The events that prompted learning when analyzed further for the substance of content in relation to the ideas learned revealed three stages in the conversations: *initiation*—the ideas learned surfaced or were made public, *precisification*—the ideas surfaced were made clear, and *equilibration* (Piaget, 1964)—PSTs related the ideas to their knowledge or experiences. Not all learning prompts had all three stages, but all of them had at least initiation and precisification stages. Eleven of the 48 local explicit learning prompts had all three stages. For the remaining learning prompts, the equilibration stage was not identified before the prompt ended, which does not necessarily mean that equilibration did not take place. Given the fundamental role of equilibration in the construction of knowledge (Piaget,1964), my assumption in these cases is that equilibration occurred at an individual level for those PSTs who expressed learning that was attributed to such prompts. Finally, the stages of learning prompts were not necessarily linear, and, in some cases, learning prompts contained multiple cycles of these stages. Three learning prompts are briefly discussed to illustrate these special cases.

There was one brief learning prompt, Riley-video-PCK-2, where initiation and precisification occurred simultaneously. The learning prompt is related to Riley’s learning
expressed in the interview and bellringer reflection paper, that he could have given additional blank polygons to allow SPSTs to express different ways of constructing polygons and how that would have supported SPSTs in seeing that “all polygons can be decomposed into other polygons, not just the one polygon given in [Riley’s] original design.” The prompt is made up entirely of Kylie’s statement, where she makes a proposition shown in the following transcript:

I was gonna say, if you were wanting more variety in the first one, I think we should’ve been given more squares for [pause]Yeah, that way we [crosstalk 35:30]. Yeah, that’s what I did too, or else I would only expect a lazy student to just do one polygon.

The learning prompt does not point to suggestions for SPST learning, but Riley makes a connection is his expressed instance of learning how giving more squares would have influenced SPST learning. This suggests that in some situations PSTs were making connections for their learning to ideas beyond the conversations in the learning prompts.

There were two long learning prompts, Kylie-video-CK-1 and Isabelle-video-PK-3, that had repeated cycles of initiation, precisification, and equilibration. Both learning prompts involved conversations on multiple ideas that were related, hence the repeated cycles of the stages of the learning prompts; as the different ideas surfaced, PSTs engaged with them and other related ideas emerged. For example, in Kylie-video-CK-1, the instances of learning were related to division involving zero. Kylie had posed the following problems in her bellringer: $\frac{15}{5}, \frac{0}{5},$ and $\frac{15}{0}$. As PSTs tried to adapt the ideas that emerged to what they knew (equilibration), they shared their perspectives and approaches. Three approaches were proposed: division as the opposite of multiplication, a grouping strategy using diagrams, and then a sharing strategy. Each approach proposed a process of making those ideas clear (precisification) in applying them to the three situations occurred. In the second long learning prompt, Isabelle-video-PK-3, the idea
learned was when it is appropriate to use partners. The related ideas that resulted in repeated cycles of the stages of the learning prompts were the different situations when it would be appropriate to ask students to discuss with their partners.

According to Gee (2014), language allows us to say things, perform actions, and project an identity. This section illustrates the first two things that language allows us to do: say things and perform actions. Projecting an identity will be illustrated in the section on PSTs’ ability to be both learner and teacher. In the following, I use a learning prompt from each the four types of learning—CK, PK, PCK, HLP—to illustrate the stages of learning prompts. The purpose is to show how the ideas learned evolved in the stages of the learning prompt.

**Content knowledge.** This section illustrates the stages of the CK learning prompt, Evan-video-CK-3. The prompt was related to Evan’s learning that the equals sign should not be placed between two expressions that are not equivalent because the equals sign represents equality of the expressions on each side of it. This idea surfaced through Cameron providing *critique* by saying, “I had a little heart attack when you wrote that these two things were equal at the very beginning. If we’re to show that they’re equivalent.” This was followed the precisification stage, to make clear what was problematic with Evan’s use of the equals sign. This stage involved an exchange between Cameron and Evan, with Cameron providing *critique* and Evan responding by giving a *rationale* for what he did as illustrated in the following transcript:

TPST (Evan): Well I asked, "Are they equivalent?"

Cameron: Yeah, but then you wrote an equal sign between ’em, but we don’t know that they’re equal.

TPST: That’s what you’re showing.

Cameron: It’s a stickler thing. I’m just saying-

TPST: Oh, okay.
The equilibration stage was provoked by the instructor eliciting Evan’s thoughts on what he could have done instead. Chloe made a proposition that Evan could have used a question mark and Evan said he could have “just wrote the two expressions.” The instructor summed up the propositions made and said she had reacted in a way similar to Cameron. This led to Evan, Layla, and Ivy expressing that they had not realized that the way the task was written was problematic. They seemed to be adapting what they saw in the task to what they now know after the conversation that has taken place. Evan’s learning was demonstrated in the way he revised his task in the bellringer reflection paper, where he did not use the equals sign but instead put the two expressions separately and added the question “Are these expressions equivalent?

**Pedagogical knowledge.** This section illustrates the stages of the PK learning prompt, Austin-video-PK-4. This learning prompt was related to Austin’s learning of the need to use clear language when giving explanations and that his wording during the bellringer implementation was not strong enough to allow students to make the connection that the task involved negative numbers. Austin expressed this learning in his 24-hour reflection, the bellringer reflection paper, and the interview. The learning prompt for this instance had all the three stages: initiation, precisification, and equilibration. During the initiation stage, the ideas learned by Austin were surfaced by Isabelle, highlighting the problem of the lack of clear language. Isabelle made a proposition that Austin could have been more direct in asking the SPSTs to think about the number of years as a negative number and doing that would have contributed to her learning. This is shown in the following transcript:

> I think in your explanation ’cause we all solved it using positive numbers. I think when you explained it, you should’ve straight said like, “Well what about thinking it this way?” ’Cause you were touching on that. You said, “Years ago, how do we know that’s not -3?” You started to say that, but I think maybe just pointing out like, “What about this way?” I think would’ve been my “aha” moment. You know? It would be okay to say that.
The next stage of the conversation involved precisification—clarifying what the problem highlighted by Isabelle was about. It started with Austin seeking clarification (*eliciton, clarify*) in the next statement. Isabelle responded by providing clarification (*clarify*) on what Austin would have said and how the SPSTs would have responded. This is shown in the following transcript:

TPST (Austin): So being more like, forward with what I wanted right there?

Isabelle: Yeah, ‘cause if we- You obviously didn’t get what you wanted on the paper, so if you were just to spark that, “Well, it’s 3 years ago. Wouldn’t that be a negative number?” We’d be like, “Essentially, yeah.” Then you could show how you do it that way, you know?

The instructor then contributed to making clear the issue on the table by providing a *critique* of what happened during the implementation of the bellringer in relation to the issue of Austin not being direct and providing *instruction* on what is pedagogically an appropriate approach in the following statement:

Instructor: I’m gonna just add a little bit to that. A lot of the language that I heard you using was more like you just said: “Would that be?,” “Wouldn’t that be?,” “Could that be?” I think discerning the things that you are not wanting their opinion on, but to say, “Mathematicians use negative numbers to represent things that go backwards in time.” If the class isn’t understanding. If you have something like the situation here, and then you ask for their opinions or you make it kind of ... Like, “Oh, this is open to discussion,” I think we saw that yesterday or last time with Cameron’s, too. If you leave it open, you often miss the opportunity to drive home a point.

We’ve seen a few examples of that where you’re kind of, “You know, we could think of it like this,” or, “Can’t we do this?” or “Might we do this?” As a teacher, we know that people don’t learn very well when they’re just told things. They need to experience them. They need to somehow have a relationship with what they’re experiencing. That doesn’t always mean that we are wishy-washy, for lack of a better word.

TPST (Austin): Mm-hmm (affirmative).

Instructor: What it means is, we know clearly what we’re trying to get them to think about and we create situations that give them the opportunity to think about that. By asking a well-directed question, or by giving a piece of information and then saying-
if you had said, “Let’s use negative numbers to represent going back in time. How does that change your equation? How does that change?”

In the equilibration stage, PSTs tried to relate this clarified idea, that it is important for the teacher to be direct in the language they use to guide students to intended learning, to their knowledge and experience. Chloe elicited an elaboration (elicitation, elaboration) on how as a teacher one can be direct without giving away everything to the students. This is shown in the following transcript:

Chloe: Does it create a relationship with the material? You know what I mean? I feel like that’s the fine line, is I think that’s why he didn’t want to say it is ’cause he doesn’t want to just tell us that. We’re not gonna learn it, unless we’re really using it or creating that relationship. So how do you do it in a way without just [pause], yeah, and expecting them to get it without doing it.

This started a discussion in which PSTs brought their experiences as they shared events related to wait time in classrooms in which they had been a part. There was a back and forth of information from PSTs and instruction by the instructor as she brought in the idea of “judicious telling.” Some of the information provided by PSTs included ways of addressing wait time that are not pedagogically appropriate, such as one that suggested giving student hints, which according to the instructor could compromise students’ engagement with the ideas to be learned. The transcript below shows how Ivy provided critique—her opinion that is okay to let students to struggle and then she provided information on how she had seen teachers give hints and how she has also done so sometimes, resulting in the instructor giving some caution about use of hints in the following transcript:

Ivy: I think letting them have that struggle is okay, too, but if you’re giving that minute or minute and a half or whatever much time, you can- And they’re still not really having the answer, what I’ve noticed a couple teachers do is give a hint. I’ve even done it in situations where I’m like, “Okay. We’ve gone over this vocab word before and talk about it with your partner, what it is. Or we talked about it last year,” or something. Or, “You should’ve talked about it last year. Go over with your shoulder partner what it is.”
There’s not very many hands going up, I’ll be like, “Okay, starts with an ‘A.’” Then maybe that gets their juices flowing. “Oh yeah, that’s this word.”

Instructor: Let’s talk about that in relationship to the article that you read that talked about focusing and funneling. Because what I’m hearing you saying, that hint thing, it depends on the nature of the hints that you’re giving. That guessing game like, “I’ll tell you the first letter. Still don’t have it. Let me tell you what it rhymes with.” That’s probably not the type of engagement we want.

Ivy’s suggestion confirms that PSTs bring into teacher preparation preconceived ideas about teaching (Feiman-Nemser, 2012) and highlights the fact that some of the ideas they bring may not be productive. Ivy’s idea of hints allowed the discussion to address the need for supporting students even as they are given the opportunity to grapple with ideas. Supports suggested included allowing students access to ideas by scaffolding their thinking and reminding them of ideas that they need to be able to grapple productively.

**Pedagogical content knowledge.** This section illustrates the stages of the PCK learning prompt, Yvonne-video-PCK-1. This learning prompt was related to learning expressed by Yvonne in the interview on the need for her to have referenced dilations, rather than her focus on similarity, to bring out the mathematical understanding for her bellringer task. The mathematical understanding for her task was, “Shapes can be transformed into similar, smaller or larger, shapes with proportional corresponding sides and congruent corresponding angles.”

This learning prompt had only two stages, initiation and precisification. Kylie set off the initiation stage with her expression of critique, as she wondered what the “planned aha moment” for the bellringer was. Yvonne then provided an elaboration of the mathematical understanding that she had intended for her bellringer task. The conversation that followed, which was the precisification stage of the prompt, involved unpacking what Yvonne’s focus was and the cause of the mathematical understanding not coming through as expected. The conversation included...
critique on how she brought out the ideas on similarity clearly, elicitation of clarification on whether the intention of the bellringer was to review similarity, and proposition on how she could have referenced dilation. In the following transcript, Oliver provides an elaboration of a point during the bellringer where it would have been appropriate for Yvonne to reference dilations:

Yeah, in my explanation when I was saying that I agreed that the larger shape is similar to the smaller shape. Then because that is true, the smaller shape must also be similar to the larger shape. Yvonne did comment on that, that if [pause]. If one thing is true, then the other way must be true. But we didn’t really discuss that in depth. That’s where we could have [talked about dilation].

The instructor then provided critique, validating Yvonne for having done a good job of highlighting the ideas on corresponding angles and sides, and whether that was the mathematical understanding that was on Yvonne’s mind. The following transcript shows Yvonne’s acknowledgement that the mathematical understanding on her mind was what was presumed by the instructor and that her focus was on similarity. The following transcript also shows how the instructor used the opportunity to provide instruction on how what is foremost on a teacher’s mind is what they are likely to direct student learning to:

TPST (Yvonne): It was, and I think I focused more on the similarity part between the two shapes, rather than talking about the dilation part or the size.

Instructor: I think that’s a really interesting object lesson and how that happens to us. The thing that’s foremost in our mind is gonna be the thing that we naturally orient the students to. So, there were different ways this Bellringer could’ve gone and it was oriented towards the part that was review, which is important because they do need that to engage in this lesson. But it left out that more “aha” piece.

This learning prompt highlights an interesting dimension of PST learning through the BRS, where PSTs seem to be monitoring their own learning. They have an anticipation that they should learn something through the bellringer implementation, so as they participate in it they
are looking for that “aha” moment. They are also cognizant of what they have learned and in the
debrief, when they get to see the intended mathematical understanding, they are able to make
comparisons as to whether or not it aligns with their learning. The following two examples
illustrate similar situations where SPSTs made explicit comparisons between what they
experienced during a bellringer implementation and the intended goal by the TPST. The first one
was in Oliver-PCK-video-2, where Riley expressed critique by saying that he thought the
mathematical understanding for Oliver’s bellringer “was on ways to solve for $x$ using fractions”
and “[he] didn’t know the point was cross multiplication.” The second one was in Riley-video-
PCK-1, where Isabelle expressed critique, saying she liked how “the mathematical
understanding, it was almost word for word in my mind before [the instructor] put it up there.”
These examples highlight a feature of the BRS that is consistent with the specificity of
pedagogies of enactment with regard to the focus of learning and show how that specificity
supports PST learning.

**High leverage practices.** This section illustrates the stages of HLP learning prompt,
Oliver-preparation-HLP-1. It is related to Oliver’s learning expressed in his interview, on how he
arrived at the mathematical understanding for his bellringer. Unlike other learning prompts that
involve conversation among SPSTs, TPST, and the instructor, this prompt, like other preparation
conversations, involved only the TPST and the instructor.

This learning prompt had initiation, precisification, and equilibration stages. The ideas
that surfaced in the initiation stage, and then went through precisification, were Oliver’s thoughts
on the important mathematical ideas he wants SPSTs to learn through his bellringer. In the
initiation stage, the instructor surfaced the ideas by eliciting from Oliver what SPSTs need to
better understand about his assigned topic on cross multiplication. Oliver’s thoughts on what he
wanted SPSTs to learn came from what he was trying to make sense of in his assigned chapter. Oliver started off thinking broadly, “Yeah, well I mean, the main understanding is that, is the finding common factors, so that the fraction can be simplified.” This comes from the following equation given in the chapter as a prompt that resulted in a hypothetical student asking whether cross multiplication could be used in this situation: \[ \sqrt{\frac{12}{81}} = \sqrt{\frac{3}{27}} = \sqrt{\frac{4}{27}}. \]

The instructor offered *critique* that Oliver’s idea would be something that would be covered in a lesson. Precisification started with making sense of the mathematical idea from the chapter that would be appropriate for a bellringer for the SPSTs. This began an exchange that involved *critique, information, and elaboration* between Oliver and the instructor as they assessed what was in the chapter to pick out something that would highlight a misconception SPSTs may have. This is shown in the following transcript:

Oliver: Mm-hmm (affirmative). Well ’cause, yeah. Well, the misconception I was given was that they cross-multiplied-

Instructor: Mm-hmm (affirmative).

Oliver: When they factored out the number, but that really ... I thought that was really kind of off-the-wall thinking. (laughs) Like, it wasn’t a common misconception.

Instructor: Yeah, I agree. It’s a, it’s an example of an over-application of-

Oliver: Right.

Instructor: Cross-multiplication, but I don’t think it’s that common. Another um, over, generalization of, cross-multiplication is in that same article on things that expire; the butterfly rule.

The conversation on things that expire, allowed the instructor to elaborate on how comparing \( \frac{1}{2} \) and \( \frac{2}{3} \) by multiplying each of them by a number that is equivalent to one like \( \frac{3}{3} \) and \( \frac{2}{2} \), respectively, allows one to compare only numerators. Through *elaboration* and
literal statements, the instructor and Oliver continued precisification by applying the idea of multiplying a fraction by a number equivalent to 1, to an equation involving fractions and, in the process, highlighted why cross multiplication works.

In the equilibration stage, Oliver tried to figure out how the idea of cross multiplication would work during his bellringer implementation and said, “I guess I am having a hard time seeing past, just like setting up a proportion like that with a variable and then explaining why cross multiplication works.” The conversation shifted to pedagogical considerations as Oliver considered how SPSTs were likely to respond to the task of solving the equation \( \frac{x}{6} = \frac{2}{3} \) given the goal that he now had in mind.

Oliver: Oh, right. I think some people might use the justification that, just that \( x \) equals 4 because 2/3 is 4/6.

Instructor: Mm-hmm (affirmative).

Oliver: And then we’re not cross-multiplying, but ... Maybe that’s-

Instructor: Somebody’s gonna cross-multiply.

Oliver: Yeah, yeah.

Instructor: I can almost guarantee that-

Oliver: Right, it’s just-

Instructor: Somebody’s gonna cross-multiply.

Oliver: It’s so engrained. Yeah.

Instructor: So, the fact that you want to talk about cross-multiplying doesn’t mean that you have to tell everybody to-

Oliver: Right.

Instructor: Cross-multiply.
Oliver came into this conversation thinking broadly about the goal of his bellringer and the process of clarifying a goal involved assessing his chapter for what would be appropriate, identifying the mathematical idea, and considering pedagogical issues related to how SPSTs would respond to a bellringer with a goal like the one Oliver now had in mind.

**Supportive Aspects of the Bellringer Sequence**

This study highlighted aspects of the BRS that supported PST learning. In this section the results on the following aspects of the BRS that supported PST learning are reported: focus of the BRS phases, instructor’s role, and relationship to course content.

**Focus of the BRS Phases**

Recall that a total of 54 local learning prompts (see Appendix G for a complete list) were identified collectively from the five data sources. The number of learning prompts identified for each TPST bellringer across all the BRS phases ranged from 2 to 7, with a mean of 4.9 learning prompts. Most of the 54 learning prompts were located in the bellringer debriefing phase (54%), or the bellringer implementation phase (37%). Only 9% were located in the bellringer preparation phase. Table 7 shows the distribution of the 54 local learning prompts by the four learning types and three phases of the BRS. The reflection phase is not included in Table 7 because, although theoretically an instance of learning could have been prompted by reflection, there was no explicit evidence of this.
Table 7

*Distribution of Learning Prompts by Type of Learning and BRS Phase*

<table>
<thead>
<tr>
<th></th>
<th>Preparation</th>
<th>Implementation</th>
<th>Debrief</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content knowledge</td>
<td>1</td>
<td>10</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>Pedagogical knowledge</td>
<td>0</td>
<td>5</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>Pedagogical content</td>
<td>0</td>
<td>3</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>knowledge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HLP</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Totals</td>
<td>5</td>
<td>20</td>
<td>29</td>
<td>54</td>
</tr>
</tbody>
</table>

The frequencies of the types of learning in each of the phases of the BRS reflected the focus of conversation of each of those phases. In the preparation phase, conversations were mostly about identifying the mathematical understanding for the bellringer task, refining the task, and identifying the relevant standards for the bellringer and the lesson that would follow the bellringer. Four out of the five learning prompts in the preparation phase were related to HLP 13—setting long- and short-term goals for students. In the implementation phase, the focus of conversation was on the mathematical ideas related to the bellringer. Consistent with this focus, the highest number of learning prompts in the implementation phase was related to CK. In the debriefing phase, the greater focus was on ideas related to teaching, and even though mathematical ideas were discussed, they tended to be discussed in the context of teaching. Therefore, it seems that the different foci of the bellringer phases allowed the different types of learning, providing an opportunity for PSTs to develop in all areas during the BRS.
Ability to be Both Learners and Teachers

In this section I draw on Gee’s (2014) third use of language—project an identity—to illustrate how PSTs projected multiple identities through the statements that they made in the learning prompts. The BRS structure allowed PSTs to take on both the role of teacher (TPST) and that of learner (SPST). The topics assigned from the MUST chapters (Heid et al., 2015) were topics that merit attention because they are known to be problematic; thus, the TPSTs were challenged to make sense of the mathematics during their preparation, and the SPSTs could authentically take on the role of learners of mathematics that is relevant to teaching.

Additionally, when preparing their bellringers, TPSTs were expected to consider the criteria in the bellringer evaluation rubric of engaging SPSTs at a high level of cognitive demand.

Since, the bellringer implementation was a representation of practice, it was an opportunity for SPSTs to learn ideas related to teaching at the same time as they engaged as learners with the mathematical ideas. TPSTs were also learning as they approximated practice, learning from preparing the bellringer, their own implementation, and from the feedback that the instructor and their colleagues provided. One reflection of the dual roles was in excerpts of instruction where the speakers were SPSTs and TPSTs. The dual roles allowed them to project more than one identity during discussion, that of a learner and/or teacher-evaluator—a teacher evaluating a colleague. In addition to projecting the role of a teacher when they were the TPST, PSTs also took on the role of teacher-evaluator when they were SPSTs. The identity projected was not tied to the speech event but was based on what was said. In this section I illustrate how these identities were projected using examples of excerpts from learning prompts.

**PST as learner.** In this section I illustrate where PSTs projected the identity of a learner using statements whose speech events were confusion, critique, elaboration, information, and
propositions. The choice of these speech events is simply based on the fact that these were some
of the more common speech events. The examples described here are all drawn from the learning
prompts that were illustrated in the stages of learning prompts section to make it easier to
understand the contexts in which the statements were made.

In Cameron-video-PK-1, Isabelle expressed confusion about Cameron picking on the
mean as the more appropriate measure of central tendency for the data in the second question on
his bellringer task:

Isabelle: Knowing that there’s a lot of points on the left side in the second graph, I still
don’t really understand why you’d go with the mean on that. You know what I mean?
I’m still a little bit confused and I’m not even a 6th grade student. I don’t know.

As Isabelle expressed confusion, she projected an identity of a learner trying to make sense of
why the mean would be more appropriate than the median when there were so many data points
clustered on the left side of the graph.

In the Evan-video-CK-1, Chloe made a proposition that Evan could have used a question
mark but did so in a tone that suggested that she was seeking validation for what she is
proposing: “Question mark?” Chloe was therefore projecting the identity of a learner. In the
same learning prompt, Evan, Ivy, and Layla, by providing information together, expressed their
realization that they had not seen the use of the equals sign as problematic, also projecting the
image of a learner.

In Austin-video-PK-4, after it had been clarified that Austin had not been clear and
assertive in his language, some PSTs grappled with what being direct would look like in a
classroom focused on basing instruction on student thinking. The following excerpt shows
Chloe’s elicitation for an elaboration on how Austin could have brought to the SPSTs’ attention
that his task involved negative numbers, without giving away too much information.
Does [asking a well-directed question, or by giving a piece of information] create a relationship with the material? You know what I mean? I feel like that’s the fine line, is I think that’s why [Austin] didn’t want to say it is ’cause he doesn’t want to just tell us that. We’re not gonna learn it, unless we’re really using it or creating that relationship. So how do you do it in a way without just [pause], yeah, and expecting them to get it without doing it.

In Oliver-preparation-HLP-1, Oliver, a TPST preparing a bellringer task, projected an identity as a learner, while trying to process how SPSTs may respond to a task on solving an equation involving fractions when the goal is to show why cross multiplication works. This is shown in the transcript below where Oliver seems to critique the ideas he is discussing with the instructor in light of how SPSTs would respond to the task:

Oliver: Oh, right. I think some people might use the justification that, just that \( x \) equals 4 because \( \frac{2}{3} \) is \( \frac{4}{6} \).

Instructor: Mm-hmm (affirmative).

Oliver: And then we’re not cross-multiplying.

The examples shown here of situations where PSTs projected the identity of a learner within the learning prompts indicate that PSTs were contributing to their own learning and that of their peers as they projected the identity of learner in the statements they made. Similarly, PSTs also contributed to their own learning and that of their peers when they projected the teacher-evaluator identity. This is illustrated with examples in the next section.

**PST as teacher-evaluator.** The contribution of teacher-evaluator identity to learning was salient in Austin-video-PK-4, Evan-video-CK-3, and Yvonne-video-HLP-1, where the ideas learned were surfaced by statements in which the PST projected an identity of a teacher-evaluator. In Evan-video-CK-3, Cameron provided critique in his observation of the inappropriate use of the equals sign by Evan. It took some discussion in the learning prompt for
Evan (TPST) to realize the error and the TPST and two other SPSTs expressed that they had not even noticed the error until Cameron pointed it out.

Another situation where the teacher-evaluator identity’s contribution to learning was salient was in Austin-video-PK-4. Isabelle made a *proposition* to Austin, identifying what Austin could have done differently and how that would have supported her learning. This is shown in the following transcript:

I think in your explanation ‘cause we all solved it using positive numbers. I think when you explained it, you should’ve straight said like, “Well what about thinking it this way?” ’Cause you were touching on that. You said, “Years ago, how do we know that’s not -3?” You started to say that, but I think maybe just pointing out like, “What about this way?” I think would’ve been my “aha” moment. You know? It would be okay to say that.

This statement by Isabelle contributed to the initiation of Austin’s learning that he should have been more direct and assertive in his language during his bellringer implementation.

In most situations, an identity was projected singularly. However, there were situations where a PST projected the identity of both learner and teacher-evaluator in a single conversational turn. An example of this was in Cameron-video-PK-1, as Riley expressed *critique*, about Cameron not being direct about which was a better answer, mean or median for the data sets in the bellringer task. As Riley expressed *critique*, he projected an identity of an teacher-evaluator as he said,

I feel like you weren’t explicit with [pause] ’Cause as an instructor, you kind of have to make a point to say, “Okay. As an instructor in this situation, this is the better answer.” I understand you’re trying to respect everyone’s ideas and everything, but at the end of the day, one of these is better than the other and here are the reasons why.

As he continued, Riley’s projected identity shifted to that of a learner as he expressed how during the bellringer implementation he was trying to figure out which measure of central tendency was better and was not able to. This is shown in the following transcript:
During your lesson I was kind of thinking like, “Okay. So, which one of these is better than the other?” I still didn’t know which one is better than the other. Like [Isabelle] said, I’m not even a 6th grade student.

The statement in which Riley projected the identity of a teacher-evaluator seems to have significantly contributed to precisification of the ideas learned by Cameron, that he should have been more direct about which was a better answer—mean or median—for the data sets in the bellringer task.

**Instructor’s Role**

The frequencies of the speech events in Table 6 highlight the role of the instructor in terms of the situations in which she spoke and the content that was addressed. The instructor accounted for only 30% of the 10 speech events discussed here. During the bellringer implementation, the TPST and the SPSTs were the ones directly involved in conversation while the instructor observed the implementation. During the debriefing, the instructor guided the conversation and so there was more talk from the TPST and SPSTs than from the instructor. The only phase of the BRS where the amount of talk by the instructor was comparable to that of the TPST was during the bellringer preparation meetings, where there was back and forth conversation between the instructor and the TPST. The fact that the two most frequent speech events for the instructor were *elicitation* and *critique* suggests the role of the instructor as coach—a feature of the BRS drawn from rehearsal.

There were only three codes—*check-in*, *elicitation*, and *instruction*—where the instructor had higher frequencies than either the TPSTs or SPSTs, further suggesting an instructor role in guiding the content and process of PST learning. That the bulk of excerpts coded *instruction* (31 out of 35) with the instructor as the speaker were on issues related to teaching indicated the opportunities that the BRS provided for the instructor to attend to teaching issues, some of which
had not been formally addressed in the course in other ways. An example of such an opportunity came up during Isabelle’s bellringer when the SPSTs seemed stuck when Isabelle posed a question. Discussion about that incident during the bellringer debrief provided an opportunity for the instructor to address the issue about when it is appropriate to use partners.

*Check-in* statements made by the instructor were almost evenly distributed between ideas related to teaching and those related to mathematics. There were some interesting observations on the number of excerpts related to mathematics and those related to teaching (PK, PCK, and HLP) for some speech events. The frequencies for *check-in* and *clarify* speech events suggest the instructor’s responsibility for assessing understanding of ideas and clarifying ideas for both mathematics and ideas related to teaching statements. This is shown in the way these codes were almost evenly distributed between mathematics and ideas related to teaching in situations where the speaker was the instructor. For SPSTs and TPSTs, these speech events had higher frequencies for ideas related to mathematics (TPSTs: *check-in* 88%, *clarify* 73%; SPSTs: *clarify* 68%; there were no excerpts were coded *check-in* for SPSTs).

The instructor role of guiding the content and process of learning is a BRS aspect that is related to the course context and is revisited in that section later in this chapter.

**Utility of the Artifacts**

In this study artifacts supported PST learning by structuring discussion and aiding reflection. In the following I report first on results related to how artifacts structured discussion and then how they supported reflection.

**Structured the discussions.** The bellringer evaluation rubrics, task worksheets, and lesson plans provided structure during the debriefing sessions as they helped focus the discussion on the immediate bellringer. The rubrics allowed PSTs to put down their thoughts in terms of
feedback and therefore influenced the agenda for the debrief, while the worksheets and lesson plans provided reference points during the discussion to highlight some of the ideas being discussed. In this section I illustrate how the artifacts contributed to structuring the debrief.

PSTs used the evaluation rubrics to highlight their evaluation for each of the different criteria, but also added notes next to their identified level, which they referred to as they provided feedback during the debrief. For example, Ivy had a note in the rubric that she completed for Riley’s bellringer—“made me have an aha when showed the whole rectangle”—that reminded her to bring up the idea during the debrief. In the rubrics for Yvonne’s bellringer, Kylie had noted, “I don’t feel I was thinking that hard,” and “not sure there were misconceptions.” Kylie’s feedback at the beginning of the debrief seemed to draw from these notes as she asked Yvonne what her planned “aha” moment was for the bellringer and said how she found the task really simple. Kylie’s feedback led to a discussion that made up a learning prompt related to PCK on how the lack of connection between similarity and dilation may have impeded the development of the mathematical understanding for Yvonne’s bellringer. Similarly, Kylie had a note in her completed rubric for Isabelle’s bellringer, “I think we could have benefitted from a pair/share,” and she brought that up in the debrief. Kylie’s proposal for use of partners led to the class explicating when it is appropriate to use partners and was expressed as an instance of learning by four PSTs, including Isabelle, the TPST.

PSTs also referenced the task worksheets during the debrief. An example of this happened during the debrief for Cameron’s bellringer, when Ivy pointed out that the ordering of the table by number of students who missed a particular number of points rather than by number of points missed. This was part of a learning prompt and contributed to Cameron’s learning. There were instances during the debrief where some PSTs’ completed worksheets were put on
the document camera to highlight ideas during discussion. This happened during the debrief for Riley’s bellringer, when the instructor highlighted Riley’s lack of clarity in the way he recorded SPSTs’ contributions on the blank worksheet he had shared on the document camera during his bellringer implementation. The instructor put Riley’s worksheet on the document camera and it helped highlight what was problematic about Riley’s representation. Isabelle expressed in the interview that as soon as it was put up, “We were all like, oh, that is confusing.” Isabelle and Riley expressed an instance of PK learning related to this—the need for clarity when representing student work. During Kylie’s bellringer debrief, which involved an extended discussion of mathematical ideas related to division involving zero, there was reference to worksheets, like one from Ivy, to help illustrate their thinking on grouping as an approach to explaining division involving zero. Another reference to a worksheet was made during the debrief for Layla’s bellringer, when it was highlighted that some SPSTs had trouble filling out the table. Putting the worksheet on the document camera and referencing it allowed discussion of what made it difficult for some SPSTs to fill out the table and led to an instance of PCK learning that Layla expressed in her 24-hour reflection.

The bellringer lesson plans also helped structure the conversation during the debriefs by allowing SPSTs to check the mathematical understanding for the bellringer, the relevant standard, and how the bellringer would have enhanced the day’s lesson. This helped them in filling out the evaluation forms, but also supported them in providing feedback related to the mathematical understanding and how it related to the standards and the day’s lesson. An example of this occurred during Riley’s bellringer where Isabelle, in reference to the mathematical understanding, said it was exactly what she had in her mind as a possible mathematical understanding for the bellringer. Another example is when Kylie questioned Layla on why she
did not write Oliver’s solution in which he had used exponent rules to simplify $2^{2.5}$ yet her topic was on exponents. Kylie referred to Layla’s bellringer lesson saying, “For Layla your lesson topic being all around exponents and using them, I thought it was kind of odd that you didn’t write out Oliver’s solution.”

The examples I have given here are all related to instances of learning that were expressed by PSTs. They provide evidence that using artifacts to focus the debrief agenda and referencing sources for highlighting ideas aided PST learning.

**Supported reflection.** The following examples illustrate how the bellringer task worksheets, rubrics, and videos—all which became part of the BRS artifacts to which PSTs had access—supported PSTs’ reflection. In an instance of PCK learning for Cameron, Ivy had pointed out during the debrief that the table in the first question in Cameron’s bellringer task could have been ordered by points missed rather than by number of students. In addition to this, Ivy wrote the same on her worksheet. Cameron expressed learning in the bellringer reflection paper about what Ivy pointed out and the following excerpt from the reflection paper shows that he was also drawing on the worksheets for his learning:

> Ivy essentially writes that on her worksheet (Sheet 5, Student Worksheets) and it is definitely something I should have done. This more than anything else in the task places an unnecessary cognitive demand/struggle on the student which diverts time and attention from the actual task.

Another excerpt from Cameron’s bellringer reflection paper also shows him drawing learning from the SPSTs’ worksheets:

> Now with the worksheets in hand, I see that 7 out of the 10 students had some version of a “wrong” answer at some point during their work. (Sheets 2-5 & 8-10, Students Worksheets). So clearly there was room for the students to learn to reach my designed mathematical understanding, “Data sets can be skewed which could result in inaccurate representations by the mean and median.”
In addition to the worksheets, TPSTs drew learning from the feedback they got from the completed bellringer evaluation rubrics. The following excerpt from Ivy’s bellringer reflection paper shows her drawing learning from the completed rubrics:

On a few of the rubrics, my peers had said that I didn’t really address misconceptions very well or weren’t sure if there even was any. So if I would’ve discussed more, I could’ve found those misconceptions and addressed them.

The videos also supported PSTs’ reflection on their implementation. For some of them it clarified problematic issues raised during the debrief, while for others it highlighted things that they did well. For Riley, it helped confirm some of the issues that had been addressed during the debrief, as he expressed, “After watching the video lesson I can clearly see how my writing on the board would be very difficult to follow if a student wasn’t part of the group discussion.” For Oliver, the video highlighted how well he addressed the multiplication property of equality by the way he questioned Cameron.

Access to the artifacts supported PSTs’ reflection and contributed significantly to their learning. In this study only 38% of the instances of learning were accounted for by the local learning prompts. The majority of the instances of learning were the result of reflection; therefore, the contribution of artifacts to the written reflection phase of the bellringer highlights their potential in supporting PST learning.

**Relationship to the Course Context**

Some of the aspects of the BRS that supported PST learning in this study were related to the context of the course. Table 8 gives the frequencies of the two sources of learning, interview and the bellringer reflection paper, related to the course context that were mentioned explicitly in instances of learning.
Table 8

Frequencies for Sources of Learning Related to Course Context

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<thead>
<tr>
<th></th>
<th>Interview</th>
<th>Bellringer reflection paper</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Class discussions</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Class readings</td>
<td>8</td>
<td>13</td>
<td>21</td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>14</td>
<td>25</td>
</tr>
</tbody>
</table>

The sources of learning related to the course context and corresponding frequencies in Table 8 may not be the only sources of learning that PSTs drew from in their learning, but rather these are the ones that PSTs explicitly expressed in their instances of learning. The frequencies in the table are a factor of the structure of sources from which they were identified. For example, the high frequency of sources identified in the interview is because the interview included questions that asked PSTs what they were drawing their learning from (see Appendix F for interview protocol). Similarly, the bellringer reflection paper required PSTs to provide evidence for the conclusions they made in paper. Lack of similar prompting could be the reason that no instances were identified from the 24-hour reflection and video. The high frequency for class readings as a source of learning (84%), in comparison to class discussion, is consistent with the emphasis that was placed on class readings in the course and how class readings were incorporated into class discussions and assignments.

The identities projected by PSTs in the statements they made suggest social norms in the course that allowed PSTs to be vulnerable by openly acknowledging their struggles to make sense of the ideas discussed and also to have the courage to provide critique directly on aspects of teaching that did not work well. The willingness to be vulnerable was shown in Cameron-
video-PCK-I where Isabelle and Riley expressed *confusion* about the mean or the median being a better measure of central tendency for the data sets in Cameron’s bellringer. In the same learning prompt, Riley was courageous enough to remind Cameron of his responsibility as the teacher to have ensured it was clear whether the mean or median was a better measure for the data sets.

*Proposition* statements were one of the five frequent codes suggesting class norms where PSTs were comfortable sharing their perspectives on emerging ideas in the class and contributing to one another’s learning. Additionally, they seemed to be comfortable sharing their perspectives both on mathematical ideas and ideas related to teaching. This is reflected in way the frequencies of *proposition* statements made by the instructor, TPSTs, and SPSTs were evenly distributed between ideas related to teaching and those related to mathematics.

Overall, the learning prompt conversations reflected collaboration in learning as PSTs contributed and engaged with one another’s ideas. The role of the instructor in guiding the process was critical as closer scrutiny of PSTs’ contributions revealed that sometimes they may be based on experiences and preferences that are not grounded in knowledge of the relevant content. An example of this was discussed in Austin-video-P-4, described earlier, where Ivy proposed giving students hints when they are stuck. As Ivy elaborated on the idea of giving hints, it appeared that what she had in mind would likely result in taking away the thinking from students. Another example occurred in Isabelle-video-P-3, where Kylie initiated the prompt by proposing that Isabelle should have had SPSTs to share with their partners when they were stuck. In the ensuing conversation, it emerged that Kylie “loved” the use of partners, but she was not clear when it is appropriate. Therefore, PST statements should not be taken at face value, in the same way that a correct answer does not always indicate one has the correct understanding of an
idea. Getting at PSTs’ underlying reasoning was something the instructor did often in this course. One of her common phrases was, “Say more about that.”

**Summary**

This study identified two types of learning prompts, *local* and *global*. Examination of the identified 54 *local* learning prompts showed that PST learning through the BRS took place through conversations which highlighted the ideas learned through a process of initiation, precisification, and, in some cases, equilibration. In these conversations, PSTs contributed to learning by surfacing the ideas learned, making those ideas clear, and, in some situations, assimilating and adapting those ideas to their own knowledge and experiences. The frequent speech events identified in these learning prompts show that PST learning took place through rich generative conversations.

This study highlights how the BRS supported PST learning with the four foci of the different phases of the BRS supporting the four different types of learning. For example, the focus on mathematics during bellringer implementation supported more CK learning in that phase.

Aspects of the BRS like the dual roles of the PSTs—TPST and SPST—supported learning in the different identities PSTs projected. As PSTs made statements, they projected the identity of a *learner*, but some also projected the identity of a teacher-evaluator. Artifacts supported PST learning by guiding the debrief conversations, providing reference points for highlighting ideas, and supporting reflection. The use of artifacts also highlighted the specificity of pedagogies of enactment in their focus of what is to be learned, and this study shows how the use of artifacts can be leveraged to further PST learning.
This study also highlights how the role of the instructor in the BRS—guiding the content and process of PST learning—was consistent with the role of the instructor in the course context. Class norms of being responsible for one’s learning and that of one’s peers and being comfortable sharing ideas were reflected in PSTs’ contributions in the learning prompts.

The local learning prompts examined in this study accounted for only 38% of the instances of PST learning. The majority of instances of learning in this study were prompted by reflection highlighting the important role of reflection in PST learning.

**Discussion**

The learning prompts in this study varied by location and directness in addressing the ideas learned, thus highlighting the complexity of PST learning. For 62% of the instances of PST learning, no continuous segment of conversation could be traced within the same phase of the BRS and they were attributed to global learning prompts. Most of these instances appeared to be the result of reflection, particularly those expressed in the bellringer reflection paper. Reflection is one of the aspects of the BRS that was drawn from microteaching, lesson study, and rehearsal. The finding that the majority of instances of learning could be attributed to reflection highlights the important role of reflection in PST learning. This also corroborates what has been expressed in the literature on the important role of reflection in teacher education (e.g., Loughran, 2002). The embedding of reflection in pedagogies of enactment is an aspect that enhances their support for PST learning.

Six of the 54 learning prompts where a continuous segment of conversation could be identified were implicit learning prompts, that is, the ideas in the conversations were not directly related to the ideas learned. Implicit local learning prompts revealed an interesting dimension of PST learning, where PSTs connected what they were observing to ideas not directly expressed in
the conversations. Specifically, PSTs made connections between mathematical conversations and ideas related to teaching. They drew the connection to teaching from ideas that had been discussed in the course, ideas that had come up during the bellringer debrief, and ideas from the course readings. This suggests that PSTs’ learning is supported by access to ideas that allow them to make connections to their observations of teaching. It highlights the complexity of their learning situation as they pay attention to mathematical ideas while simultaneously making connections to learning of ideas related to teaching. This also suggests that the representation of practice implemented by the TPSTs could be providing SPSTs with alternative teaching to observe that may help to disrupt Lortie’s (1975) apprenticeship of observation.

Examination of learning prompts by characterizing them and identifying broad themes provided insight into the nature of conversations that support PST learning. The speech events that were frequent in the learning prompts, for example, elicitation, elaboration, critique, clarify, literal, proposition and information, suggest that PSTs engaged with the ideas learned through generative conversations where ideas were elicited, elaborated, critiqued, clarified, proposed, and information and factual ideas provided. Collectively these frequent speech events in the learning prompts also suggest that PSTs engaged with the ideas learned through a process of negotiation that is consistent with interactionism. The content and process of learning was guided by the instructor as suggested by the relatively higher number of statements made by the instructor that were coded check-in, elicitation, and instruction compared to those made by TPSTs and SPSTs. Additionally, the two most frequent speech events for the instructor were elicitation and critique, highlighting the instructor’s role as a coach, which is a feature of the BRS that was drawn from rehearsal. Grossman, Hammerness, and McDonald (2009) emphasize coaching as a necessary
aspect of pedagogies of enactment for supporting PSTs to make sense of the complexity of teaching practice.

In rehearsals, the teacher educator’s role as coach allows them to intervene at any point during PSTs’ implementation of the instructional activity with their peers (Lampert et al., 2013). In the BRS, the teacher educator did not intervene during implementation but did so during the debrief, and that aspect of the structure of the BRS still supported PST learning as evidenced by the frequency of instruction codes for the instructor in the learning prompts. Whether or not one form of situating the teacher educator as coach is more effective cannot be determined from this study. However, the fact that PST learning was supported in both situations may allow for flexibility in situating the teacher educator as coach while designing pedagogies of enactment to specific contexts and PST learning needs. Additionally, the roles of teacher educators and PSTs in pedagogies of enactment appear to have an impact on PST learning.

In this study the positioning of PSTs as both learners and teachers supported PST learning. This positioning was a feature of the BRS that was influenced by the course context. Class norms that allowed PSTs to be vulnerable in acknowledging what they did not know or were making sense about, and the courage to candidly give feedback on their peers’ bellringer implementations led to expressions of ideas that supported PST learning. Therefore, positioning of PSTs may be influenced by contextual factors which, in turn, could impact the extent to which a pedagogy of enactment supports PST learning. For example, Bell (2007) showed how the evaluative role of the teacher educator in microteaching positioned PSTs as students and affected their responses during microteaching as they avoided responses that would make their peer who was implementing microteaching be evaluated negatively.
The BRS was designed to provide PSTs with many opportunities to contribute to, and to shape, class discussion. During implementation, the instructor did not intervene, and during the bellringer debrief, the instructor intervened only when she felt it necessary. Thus, during these two phases of the BRS, the TPSTs and SPSTs had more opportunities to contribute to the conversations than the instructor. It was only in the bellringer preparation meetings where the instructor and TPST may have had comparable opportunities to speak. This allowing of space for TPSTs’ and SPSTs’ contributions was reflected by higher frequencies for the majority of speech events for statements made by TPSTs and SPSTs in the context of explicit learning prompts. This feature of the BRS appeared to give PSTs ownership of the learning process and was consistent with the expectations of the course where PSTs were responsible for their own learning and that of their peers. Additionally, this feature allowed PSTs to share the knowledge and experiences that they brought into teacher preparation and hence provided opportunities for the instructor to address knowledge and experiences that were not consistent with reforms in mathematics education. Three broad themes were identified from the conversations in the learning prompts in this study.

The broad themes identified were the stages of the learning prompts—initiation, precisification, and equilibration. The initiation stage is critical because ideas have to be surfaced for PSTs to engage with them. However, the fact that precisification was present in all learning prompts suggests that it was an important part of shaping ideas in the learning process for PSTs. In the one learning prompt where the ideas surfaced were clear, Riley-video-PCK-2, initiation and precisification occurred simultaneously. All the explicit learning prompts had at least an initiation and a precisification stage. My conjecture is that the precisification stage in the learning prompts mediated learning by shaping the ideas to a form that PSTs could assimilate
and adapt to their own knowledge and experiences. This suggests that precisification is a stage that needs to be guided carefully and skillfully to optimize opportunities when important ideas are surfaced in conversations.

Videos have been used in microteaching, lesson study, and rehearsal to support reflection. In this study, rubrics, bellringer lesson plans, and worksheets were also used to support reflection, and they also supported learning in the way they structured the debrief conversations and provided reference for some of the ideas PSTs expressed. PSTs added notes to rubrics, which they referred to during the debrief, even though space had not been provided for such notes. Use of artifacts in this way provided easy access for PSTs to information as they processed ideas, therefore reducing cognitive load. This suggests that the use of artifacts in pedagogies of enactment can be leveraged for further support of PST learning by using them intentionally as they were used in this study.
CHAPTER 6
DISCUSSION AND IMPLICATIONS

Practice-based approaches in teacher preparation have been emphasized as a means of addressing the disconnect between what goes on in teacher preparation programs and school classrooms. Additionally, expectations articulated in reform documents, such as *Principles to Action: Ensuring Mathematical Success for All* (NCTM, 2014), *Common Core State Standards for Mathematics* (NGA & CCSSO, 2010) and *Standards for Preparing Teachers of Mathematics* (AMTE, 2017) have set a high bar for mathematics teaching. Pedagogies of enactment provide opportunities for preservice teachers (PSTs) to learn through practice and have recently been the focus of study with a view to finding ways of leveraging such pedagogies to support PSTs’ learning of knowledge and skills for ambitious teaching (e.g., Baldinger et al., 2016; Campbell & Elliot, 2015; Lampert et al., 2013). My study sought to contribute to the field’s knowledge of pedagogies of enactment by providing insight into what PSTs learn through a pedagogy enactment and how that learning takes place. The Bellringer Sequence (BRS), a pedagogy of enactment conceptualized in this study, facilitated examination of PST learning in the context of a middle school mathematics methods course with the aim of answering the following two research questions:

1. *What* do preservice teachers learn through the implementation of the Bellringer Sequence (BRS) in a methods class?

2. *How* do preservice teachers learn through the implementation of the Bellringer Sequence (BRS) in a methods class?
In the following I answer these two questions and discuss additional contributions of the study, implications for teacher preparation, and limitations and further research.

**What Preservice Teachers Learned**

This study showed that pedagogies of enactment can support preservice teacher (PST) learning not only of knowledge and skills related to teaching but also of content knowledge. Reforms in mathematics education have emphasized approaches to teaching that support conceptual understanding. However, their own experiences in elementary and secondary school mathematics classes do not adequately equip PSTs with the kind of mathematical understanding that would enable them to teach for conceptual understanding (Ball, 1990). Additionally, various studies have shown that school mathematics content is not simple, and PSTs have challenges in their understanding of school mathematical content (e.g., Kaasila, Pehkonen & Hellinen, 2010; Lo, Grant, & Flowers, 2008; Thanheiser, 2009).

The mathematical content addressed in the bellringers in this study was school mathematical content that is considered problematic (Heid et al., 2015). Other studies have also shown how addressing topics that PSTs are not familiar with or are problematic allows PSTs to engage authentically with the content (e.g., Bell, 2007; Fernandez, 2010). Despite the strong mathematical backgrounds of the PSTs in this study, the Bellringer Sequence (BRS) allowed them to learn new ideas, deepen their mathematical understanding, and uncover (and resolve) their misconceptions. The preparation, implementation, and debriefing phases of the BRS all provided opportunities for PSTs’ development of mathematical ideas. In the preparation phase, teaching preservice teachers (TPSTs) developed mathematical knowledge as they engaged with the mathematical ideas in their assigned topics. SPSTs developed their mathematical knowledge as they engaged with the mathematical ideas during the bellringer implementation. The debrief
provided an opportunity for both TPSTs and SPSTs to engage with the mathematical ideas as they looked at the ideas from the perspective of teaching and as they grappled with ideas that did not get resolved during implementation. The opportunity for PSTs to approximate practice by preparing and implementing a task that allowed their peers to engage authentically with the mathematical ideas supported PSTs’ learning of the mathematical knowledge needed for teaching. This shows the potential that pedagogies of enactment have for developing this type of mathematical knowledge.

Instruction that focuses on student thinking is advocated for in reform documents, and some of the effective teaching practices outlined by NCTM (2014) include eliciting and using student thinking to inform instructional decisions. However, these kinds of practices are not easy to enact, even for practicing teachers (Scherrer & Stein, 2013). Areas in which PSTs expressed learning included student thinking related to creating space for students to think, using students’ ideas rather than focusing on one’s own ideas, and engaging more students in discussion. Most of the instances of learning expressed with regard to student thinking were realizations of how their practice did not align with the three themes above. The BRS provided an opportunity for PSTs to come to the realization of the need to work on getting better at creating space for students to think, using student ideas, and engaging more students in discussion, but it cannot be said that they perfected their practice. However, the realization in itself is useful, considering the role of student thinking in supporting mathematics learning and how this realization positions PSTs to capitalize on opportunities to develop this practice.

In this study I chose to address PST learning of high leverage practices (HLPs) separately, even though most HLPs are related to other components of the Knowledge and Practice Framework. I wanted to explore how the BRS supported learning of HLPs because of
their role in ambitious teaching. Because HLPs are by nature learning of practice, it may not be easy to identify evidence of learning within limited time frames; thus, I was not sure that the time allotted to the BRS in the course would allow PSTs to develop HLPs. However, some learning took place for the following HLPs: HLP 1—Leading a group discussion; HLP 6—Coordinating and adjusting instruction during a lesson; HLP 13—Setting long- and short-term goals for students; HLP 15—Checking student understanding during and at the conclusion of lessons; and HLP 19—Analyzing instruction for the purpose of improving it. This learning highlights the potential of the BRS to support PST learning of HLP. Additionally, learning of HLP 19 supported other types of learning and was responsible for generating the instances of learning expressed in the bellringer reflection paper. HLP 19 plays an important role in the development of effective teaching practices. According to NCTM (2000), effective teaching requires that teachers have “opportunities to reflect on and refine their instructional practice—during class and outside class, alone and with others” (p. 19).

The results of this study show that the BRS allowed for different types of learning for PSTs. The different foci of the BRS phases allowed for rich learning in an integrated way. Having multiple phases is not unique to the BRS but is often a feature of pedagogies of enactment. This study highlights how attention to the foci in the different phases of pedagogies of enactment may be leveraged for PST learning, particularly with a view to optimizing the limited instructional time in methods courses.

**How Preservice Teachers Learned**

In this study PST learning was prompted by events that varied by location and directness with regard to how they addressed the ideas learned. The global learning prompts highlighted the significant role of reflection in PST learning.
Analysis of the explicit local learning prompts revealed how the nature of conversations in the learning prompts allowed PSTs to engage with the ideas learned. The results highlight rich generative conversations characterized by PSTs and the instructor clarifying, critiquing, elaborating, eliciting, proposing ideas, and making factual statements related to mathematics and teaching. The stages of precisification and equilibration within the learning prompts that embody this generative process suggest collaborative negotiation of ideas that requires a repertoire of knowledge for PSTs to draw from, class norms that allow PSTs to freely share their ideas, and instructor guidance of the process to ensure learning of appropriate content. This underscores the role of the course context in PST learning through pedagogies of enactment—equipping PSTs with sources from which to draw learning, creating an environment conducive for PSTs to share freely, and monitoring the content in the ideas PSTs surface.

Aspects of the BRS also supported the generative conversations, particularly the structure of the debrief sessions. Artifacts provided structure for the debrief by defining the agenda and providing points of reference during the discussion, contributing to the focused and productive conversations in the learning prompts. PSTs’ dual roles as teachers and learners in the BRS, highlighted by statements coded instruction coming from PSTs in both roles, supported their learning of all three types of knowledge. The role of the instructor was that of coach guiding the process and intervening when necessary. These results about the structure of the BRS join with other studies (e.g., Bell, 2007; Sims & Walsh, 2009) in suggesting that the way pedagogies of enactment are structured in their use in teacher preparation has an impact on PST learning.

The importance of the teacher educator’s role in guiding the content and process of learning was highlighted in this study by instances in the learning prompts where PSTs proposed ideas or acted in ways that were not pedagogically appropriate. It was further corroborated by the
higher number of excerpts coded check-in, elicitation and instruction, in situations where the speaker was the instructor. This role is important in pedagogies of enactment because of the need of a more knowledgeable other during conversations where feedback is provided, because those conversations provide opportunities for PSTs to consolidate their learning. This is consistent with the findings of Parks (2008), where the lack of a more knowledgeable other resulted in unintended learning that did not align with reform goals for mathematics instruction.

The analysis of learning prompts in this study showed how PSTs were cognizant of what they should be learning and seemed to monitor their own learning as they compared what they had learned with the intended learning goals for bellringers. These actions were supported by the BRS being centered around a mathematical task and having specific instructional goals. This kind of specificity for PST learning around instructional tasks is evident in the use of instructional activities in rehearsal such as choral counting (Kazemi et al., 2009; Lampert et al., 2013) and Contemplate then Calculate (Kelemanik & Lucenta, 2015). This specificity of instructional goals helps to focus PST learning on the intended ideas. However, in this study the specificity of instructional goals also generated additional learning for TPSTs when SPSTs explicitly compared what they had actually learned with the intended learning goals, and the reasons for any differences were discussed. This suggests the potential of leveraging this specificity in pedagogies of enactment to support a broad range of PST learning.

My hypothesis at the beginning of this study was that preservice teachers’ learning is supported first by a disposition toward learning to teach as learning from teaching (Ball & Cohen, 1999), and classroom norms where PSTs are free to critique one another’s teaching. The BRS as an instance of a pedagogy of enactment in this study has shown how opportunities to
enact teaching can support PST learning and how classroom norms in this study supported PST learning as was reflected by the conversations in the learning prompts.

**Additional Contributions**

In addition to answering the research questions, this study made several contributions to the field. The literature review identified and critiqued pedagogies of enactment from microteaching to lesson study and rehearsal and the more recent variations in microteaching and lesson study. Using this information, I gleaned best practices for pedagogies of enactment and used them to conceptualize the BRS. The BRS itself is a contribution to the field as it may be used by other teacher educators in their methods courses. The study also pulled together existing constructs of knowledge that teachers need for teaching into the Knowledge and Practice Framework. Finally, the methods used to identify preservice teacher learning and learning prompts may be applicable to other studies.

**Implications for Teacher Preparation**

This study shows that intentional use of pedagogies of enactment could support PST learning of ambitious teaching. The potential of pedagogies of enactment in supporting PST learning lies in the way they are structured for use in teacher preparation programs and in their specificity with regard to instructional goals for PSTs. To capitalize on the limited instructional time in methods courses, pedagogies of enactment need to be structured with varied foci to allow for more integrated PST learning. In this study the varied foci of the BRS phases supported integrated PST learning of all the components of the Knowledge and Practice Framework. Additionally, aspects of pedagogies of enactment like artifacts can be used intentionally, beyond their current role in reflection, to guide conversation and to reference ideas to support PST learning, as was done in this study. Another area where intentionality is required is drawing
PSTs’ attention to important ideas one wants them to learn. The varied instances of learning in this study show how a lot of ideas emerge during the implementation of pedagogies of enactment, and so highlighting important ideas can help to focus PSTs’ attention on them. In this study, despite being provided with criteria for effective bellringers, less than half the PSTs precisely expressed learning related to effective bellringers. This study has shown how the specific instructional goals for PSTs in pedagogies of enactment support PST learning. This specificity of instructional goals in pedagogies of enactment can be leveraged for PST learning by making instructional goals for PSTs explicit.

This study and other studies on pedagogies of enactment (e.g., Fernandez, 2010) have shown how using problematic topics or topics with which PSTs are not familiar can support PST learning of content knowledge. Use of problematic topics and centering pedagogies of enactment around a mathematical task would allow for mathematics learning that one may not have the opportunity to address elsewhere in the teacher education program. This study also highlighted the important role of reflection in PST learning. Microteaching, lesson study, and rehearsal all have reflection as a feature. However, being deliberate in structuring opportunities for reflection as was done in the BRS with the debrief, 24-hour reflection, and the bellringer reflection paper would provide better support for PST learning.

The important role of precisification in PST learning highlighted in this study suggests that there is need for teacher educators to develop skills that would enable them guide conversations in which important ideas emerge, in ways that would support PST learning of those ideas. One such skill would be to recognize an important idea the moment it surfaces in conversation, even in its unclear form, so that they can guide conversation toward making it precise.
Limitations and Further Research

In this study the learning prompts that were able to be examined in depth accounted for only 38% of instances of learning. The remaining instances were prompted by global and implicit events that could not be captured given the study design. Additionally, instances of learning were identified only through explicit verbal communication or demonstration. Therefore, this study likely captured only a small subset of PST learning. Further studies are needed to explore the larger subset of PST learning that was beyond the scope of this study. For example, the methods of identifying learning in this study may have limited the number of instances of PCK that were identified. This highlights the need for using more than one method to identify instances of learning and particularly for PCK, using methods that make evidence of PST learning explicit.

This study identified precisification as an important part of PST learning that needs to be guided skillfully. I have proposed here that teacher educators need to recognize important ideas that emerge even in their unclear form in order to guide the conversation toward making the idea precise. Further studies are needed to unpack what it would take for teacher educators to recognize these important ideas for the different types of learning and how to guide conversation successfully to the precise idea.

In this study each PST enacted a bellringer once within the context of the methods course. Thus, the results may provide a limited view of PST learning during pedagogies of enactment. Later in this course, the PSTs partnered to rehearse and teach a subset of the bellringers in a middle school classroom. The data from that field experience were not part of this study. It is likely that more, and perhaps different, learning took place during the field experience, both because PSTs were given a second opportunity to implement a bellringer, and
because they were doing so with a group of middle school students this time, instead of their peers. Researching multiple opportunities to implement a bellringer would allow one to consider the impact of expressed learning from the first bellringer experience on the next experience, both in terms of changes in teaching practice and additional instances of PST learning. Doing such research might also shed light on the issues arising when PSTs use bellringers with middle school students rather than their peers, thus helping teacher educators to bridge the gap between preservice education and classroom teaching.
REFERENCES


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Appendix A

Evaluation Rubric for Bellringer
[Course name] Evaluation Rubric for Bellringers

Bellringer Name ___________________________________________________ Teacher: ___________________ Evaluator: ___________________

Mathematical Understanding:

<table>
<thead>
<tr>
<th></th>
<th>Unacceptable</th>
<th>Tolerable</th>
<th>Expected</th>
<th>Exemplary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriateness</td>
<td>Not accessible or reasonable.</td>
<td>Addresses a mathematical topic that is accessible but not necessarily relevant or important.</td>
<td>Reviews or previews an important mathematical understanding.</td>
<td>Reviews or previews an important mathematical understanding in a way that enhances the lesson.</td>
</tr>
<tr>
<td>Student thinking</td>
<td>Does not engage students in thinking about mathematics.</td>
<td>Engages students in thinking about mathematics.</td>
<td>Engages students in thinking at a high level of cognitive demand about an important mathematical understanding.</td>
<td>Engages students in thinking at a high level of cognitive demand about an important mathematical understanding related to the day’s lesson.</td>
</tr>
<tr>
<td>Implementation</td>
<td>Unprepared. Doing the work for the class.</td>
<td>Mathematical ideas are implicit or vaguely interpreted.</td>
<td>Identifies obvious misconceptions and address them. Highlights an important mathematical understanding.</td>
<td>Identifies subtle misconceptions and address them in a skillful way. Highlights an important mathematical understanding in a way that generates “aha’s”.</td>
</tr>
<tr>
<td>Presentation</td>
<td>Unclear or difficult to read. Disorganized.</td>
<td>Interpretable instructions. Spoken words are mostly audible and include some eye contact. Notation and terms are often appropriately used.</td>
<td>Clear instructions. Spoken words are audible and include eye contact. Appropriate use of notation and terms.</td>
<td>Clear instructions and appropriate use of notation and terms presented in a coherent, precise and enthusiastic manner that engages the class.</td>
</tr>
</tbody>
</table>

The target time is 3-5 minutes for the task and 4-10 minutes for the discussion. This bellringer took ___ minutes.
Appendix B

Bellringer Tasks
Teaching preservice teacher: Austin

Name____________________________

Elijah goes trick or treating at Mr. and Mrs. Smith’s house each year for Halloween. Each year Mr. and Mrs. Smith give Elijah five pieces of candy. Elijah also goes trick-or-treating at Ms King’s house each year and each year Ms. King gives Elijah seven pieces of candy.

Please answer the following questions about the situation you just read. Be able to explain your answer. You may use any method that is helpful to you.

1. How much more or less candy will Elijah have after two years?

2. How many years have passed if the Smiths have given away fifty pieces of candy?

3. How much more or less candy did Ms King have three years ago?
Mrs. Skewness’s Algebra I classes took a test last week and after grading all the tests, she found both the mean (average) and the median (middle number) for each class period. She notices that the mean and the median are not the same for each of the classes and wonders to herself, whether she should use the mean or the median to represent “central tendency” for each class.

Based on what you know about mean and median, should Mrs. Skewness use the mean or the median for each of her Algebra I classes to best represent the “central tendency”?

1st hour: The___________is a better representation of central tendency because_________

3rd hour: The___________is a better representation of central tendency because_________
Teaching Preservice teacher: Chloe

Name_________________ Period____________ Teacher________________

Which is larger, $x$ or $x + x$? Explain your reasoning
Teaching preservice teacher: Evan

Name__________________________________________

1) \((xy^5)^2 = x^2y^{10}\)

Are these expressions equivalent? Explain your reasoning.

2) \((x + y)^2 = x^2 + y^2\)

Are these expressions equivalent? Explain your reasoning.

Bonus: Can you explain why the expressions in the second question are equivalent or not equivalent visually? (Finish after you have completed the two questions above, use the back of the paper if you need to)
Teaching preservice teacher: Isabelle

Name_______________________________

Identify whether the equation is true or false and provide a justification for your choice

\[ \sqrt{2} + \sqrt{3} = \sqrt{5} \]

Circle: True or False

Justification:
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
Teaching preservice teacher: Ivy:

Why does a number divided by itself equal 1? For example, why does \( \frac{2}{2} = 1 \)?

Rewrite the expression below in at least 2 ways and explain why the value you obtained is the same. One of your rewrites should use exponent rules.

\[
\frac{2^3}{2^3} = \quad \frac{2^3}{2^3} =
\]
Teaching preservice teacher: Kylie

Use what you know about fractions, to explain what the shown fraction equals and why the fraction represents that.

The fraction \(\frac{15}{5}\) equals \______\_. I know this because \______________________________\.

Work space

If you finish see if you can think of another way to solve the problem

Work with your shoulder partner to describe what the fraction equals when the denominator is changed to 0. Again you can use pictures, words, numbers or a combination of them to explain your thinking.

The fraction \(\frac{15}{0}\) equals \______\_. I know this because \______________________________\.

Work space

If you finish see if you can think of another way to solve the problem

Work with your shoulder partner to describe what the fraction equals when the numerator is changed to 0. Again you can use pictures, words, numbers or a combination of them to explain your thinking.

The fraction \(\frac{0}{5}\) equals \______\_. I know this because \______________________________\.

Work space

If you finish see if you can think of another way to solve the problem
Teaching preservice teacher: Layla

Name_____________________________

A science experiment involves periodically measuring the number of mold cells present on a piece of bread. At the start of the experiment there was one mold cell. They noticed that at each weekly observation, the number of mold cells had doubled from the week prior. The graph of the mold growth is shown here.

1. Without a calculator, complete the following chart about the growth of the mold cells after each weekly observation.

<table>
<thead>
<tr>
<th>Number of Weeks</th>
<th>Number of Mold Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

2. What is the mold growth at 2.5 weeks? How did you come to that answer?
Teaching preservice teacher: Oliver

Name________________________

Solve for x, then provide a written justification for your solution below.

\[
\frac{x}{6} = \frac{2}{3}
\]

My solution works because
Pictured below is a rectangle. What other polygons could construct the given rectangles?
(Please draw out your thinking)

What is the area of this polygon?
(Please draw and write out your solution)
Teaching preservice teacher: Yvonne

Lacey was asked the following question:
“Are these shapes similar to one another?” [students were provided with two similar shapes cut out of paper]

Lacey thinks that the larger shape is similar to the smaller shape, but she does not think that the smaller shape is similar to the larger shape. Do you agree or disagree with Lacey’s thinking? Explain your reasoning below.
Appendix C

Bellringer Reflection Paper Assignment
Bellringer Reflection Paper

This assignment is an opportunity to reflect on both your and your peers’ bellringers as implemented in our classroom. Rather than being on the success of the bellringers, the focus of this assignment is on what you have learned about bellringers, and teaching in general, as a result of the in-class bellringer component of [course name].

Part I: Analyzing your own bellringer implementation

The goal of each bellringer was to generate a mathematical understanding. Watch the video of your bellringer implementation in the methods class and assess the extent to which you:

1. Designed a task that prompted the development of your target mathematical understanding.
2. Implemented the task in a way that engaged the class in developing this mathematical understanding.

Make sure to document the evidence that led you to your conclusions (see Documentation below). Submit this to Elearning within a week of your teaching.

Part II: Reflecting across all the bellringer implementations

1. Criteria for effective bellringer tasks

Use at least three bellringer tasks from our class to illustrate features of an effective bellringer task. Describe the features that each task you selected illustrates in a way that demonstrates your understanding of the design of an effective bellringer task.

2. Implementation of bellringers

Reflect on what you have learned about effective implementation of bellringers from your own experience and from watching your peers. Make at least three observations about effective implementation of bellringers and support them with specific details from the class’s bellringers.

3. Effective teaching

Reflect on what you have learned about effective teaching in general from our discussions about the bellringers. Make at least three observations and support them with details from the class’s discussion.

Documentation

Use the video, the completed Bellringer Rubrics, and student work to provide evidence for your statements. Cite each video segment you refer to by timestamp or provide a transcript of it and refer to the transcript. Use the format (Bellringer#, timestamp [or transcript numbers]). Other evidence (student work, rubric) and readings (include page number of the reading) should also be cited.
Appendix D

High Leverage Practices
High Leverage Practices

1. Leading a group discussion
2. Explaining and modeling content, practices, and strategies
3. Eliciting and interpreting individual students’ thinking
4. Diagnosing particular common patterns of student thinking and development in a subject-matter domain
5. Implementing norms and routines for classroom discourse and work
6. Coordinating and adjusting instruction during a lesson
7. Specifying and reinforcing productive student behavior
8. Implementing organizational routines
9. Setting up and managing small group work
10. Building respectful relationships with students
11. Talking about a student with parents or other caregivers
12. Learning about students’ cultural, religious, family, intellectual, and personal experiences and resources for use in instruction
13. Setting long- and short-term learning goals for students
14. Designing single lessons and sequences of lessons
15. Checking student understanding during and at the conclusion of lessons
16. Selecting and designing formal assessments of student learning
17. Interpreting the results of student work, including routine assignments, quizzes, tests, projects, and standardized assessments
18. Providing oral and written feedback to students
19. Analyzing instruction for the purpose of improving it

Appendix E

Course Goals
Course Goals

Within the middle school context, students will:

1. Acquire mathematical knowledge for teaching;
2. Recognize, value, and develop strategies for managing student mathematical learning;
3. Develop an understanding of student thinking about mathematics and the relationship between student thinking, learning, and teaching;
4. Develop skills and dispositions needed to access, interpret and assess student thinking, learning and teaching;
5. Use student thinking and students prior knowledge as a basis for instructional planning and implementation of those plans;
6. Analyze classroom events in order to identify often-subtle differences in students’ mathematical understandings and the ways in which the teacher’ actions contributed to them; and
7. Develop critical knowledge of frameworks, curricular materials and supporting resources.
Appendix F

Interview Protocol for Teaching Preservice Teacher (TPST)
Interview Protocol for Teaching Preservice Teacher (TPST)

The purpose of this interview is to provide insight into your learning experience as you prepared and implemented the bellringer, and took part in the debriefing session. I am going to ask you some questions with follow up questions to clarify any responses that you give me.

1. What did you learn from preparing the bellringer?
   a. What aspects of the methods course supported your learning experiences?
   b. What other knowledge and experiences did you draw from during the preparation of your bellringer?

2. What did you learn from implementing the bellringer?
   a. What aspects of the methods course supported your learning experiences?
   b. What other knowledge and experiences did you draw from during the implementation of your bellringer?

3. What did you learn from the debriefing session?
   a. What aspects of the conversation during debriefing influenced your learning?
   b. Are there any specific statements or ideas that came up during the debriefing, that particularly influenced your learning? If so please share them.
   c. What aspects of the methods course supported your learning experiences?
   d. What other knowledge and experiences did you draw from in making your contributions during the debriefing?

4. What did you learn from the implementation of bellringers by your peers? (this question will not apply to the first TPST to implement the bellringer)
Appendix G

Local Learning Prompts
## Local Learning Prompts

<table>
<thead>
<tr>
<th>Learning Prompt</th>
<th>Related learning instance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layla-video CK-1</td>
<td>Learning of linear interpolation for Cameron and Yvonne.</td>
</tr>
<tr>
<td>Layla-video-PK-1</td>
<td>Layla’s realization that she did not have to write everything students were saying rather she could have asked them to bring their papers so that she could put them on the document camera.</td>
</tr>
<tr>
<td>Layla-video- PCK-1</td>
<td>Layla's realization that she should have used a context for the bellringer task that would not likely result in confusion for students and distract from the mathematical ideas.</td>
</tr>
<tr>
<td>Layla-video-PCK-2</td>
<td>Layla's realization expressed that she should have made sure the chart had the exponential function on it as well as the labels used on the axis and that would have avoided some of the confusion that was associated with the graph and filling out the chart.</td>
</tr>
<tr>
<td>Yvonne-video-CK-1</td>
<td>Chloe’s learning of the fact that one could check equivalence of the angles of the tow shapes provided by placing one on top of the other.</td>
</tr>
<tr>
<td>Yvonne-video-PK-1</td>
<td>Isabelle’s learning about the need to provide opportunities for students to grapple with mathematical ideas.</td>
</tr>
<tr>
<td>Yvonne-video-PCK-1</td>
<td>Yvonne’s learning of the need for her to reference dilations rather than her focus on similarity to bring out the mathematical understanding.</td>
</tr>
<tr>
<td>Austin-video-CK-1 and PCK Instance 1</td>
<td>Involves both learning of CK and PCK. Learning related to PCK is implicit since Chloe inferred that learning from the kind of questions Austin used to get at the mathematical understanding.</td>
</tr>
<tr>
<td>Austin-video-CK-2 and Austin-video-PK-1</td>
<td>Austin’s realization his wording was not clear and resulted in confusion and the discussion going off on a tangent and Yvonne’s learning of the mathematics in Austin’s bellringer.</td>
</tr>
<tr>
<td>Austin-video-PK-2</td>
<td>Austin’s learning about asking questions without giving away the answer i.e. allowing students to think about the problem.</td>
</tr>
<tr>
<td>Austin-video-PK-3</td>
<td>Austin’s realization that he should have used fewer questions and modeled using other graphs or number lines rather than just words.</td>
</tr>
<tr>
<td>Austin-video-PK-4</td>
<td>Austin’s learning of the need to use clear language when giving explanations and his wording was not strong enough to allow students to make the connection that the task involved negative numbers.</td>
</tr>
<tr>
<td>Ivy-video-PK-1</td>
<td>Ivy realization that she would have noticed the expression on Isabelle’s face and asked her a question that would have allowed some discussion.</td>
</tr>
<tr>
<td>Ivy-video-PK-2</td>
<td>Ivy’s learning having students do a question and not refer to it would make them wonder about the value of having spent time on the question. This is in reference to her failure to refer to question one on her bellringer task.</td>
</tr>
<tr>
<td>Ivy-video-PK-3</td>
<td>Ivy’s learning that she should have had discussions related to students’ justifications for their answers</td>
</tr>
<tr>
<td>Video/PCK</td>
<td>Description</td>
</tr>
<tr>
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</tr>
<tr>
<td>Ivy-video-PCK-1</td>
<td>This instance is related to Ivy’s learning in relation to how referencing question one in the task (why a number divided by itself is one) and using it to make a connection to why $a^0 = 1$, would have supported SPST learning.</td>
</tr>
<tr>
<td>Ivy-video-PCK-2</td>
<td>Ivy’s learning that selecting Isabelle to share her response would have prompted more discussion related to why a number divided by itself is one.</td>
</tr>
<tr>
<td>Isabelle-video-CK-1</td>
<td>Learning for Riley and Ivy, that considering order of operations was the reason why the expression in the task was not correct.</td>
</tr>
<tr>
<td>Isabelle-video-PK-1</td>
<td>Riley’s learning that the excitement of a teacher can make students look forward to learning.</td>
</tr>
<tr>
<td>Isabelle-video-PK-2</td>
<td>Isabelle’s learning that making the goal of the lesson public can sometimes distract learning.</td>
</tr>
<tr>
<td>Isabelle-video-PK-3</td>
<td>Learning for Chloe, Riley, Isabelle, and Yvonne on when it is appropriate to ask students to discuss with the partners or pair share.</td>
</tr>
<tr>
<td>Isabelle-prep-CK-1</td>
<td>Isabelle identifying the mathematical understanding for her bellringer—the reason why square root of 2 plus square root of 3 is not equal to square root of 2 plus 3.</td>
</tr>
<tr>
<td>Isabelle-prep-HLP-1</td>
<td>Isabelle’s learning that a teacher has to pick out what they want students to learn.</td>
</tr>
<tr>
<td>Chloe-video-CK-1</td>
<td>Riley’s realization that he had not thought about zero as one of the cases to consider in the task.</td>
</tr>
<tr>
<td>Chloe-video-PK-1</td>
<td>Chloe’s learning the importance of engaging all students when there is an important mathematical idea that they are ready to grapple with.</td>
</tr>
<tr>
<td>Chloe-video-PCK-1</td>
<td>Learning for Austin, Chloe, and Riley of how sequencing in the purposeful choices Chloe made supported learning of the task.</td>
</tr>
<tr>
<td>Chloe-video-PCK-2</td>
<td>Layla’s learning of how Chloe’s bellringer implementation reviewed the mathematical understanding.</td>
</tr>
<tr>
<td>Chloe-prep-HLP-1</td>
<td>Chloe’s learning about how specific you have to be in your goal with the bellringer because originally, she was thinking too broad.</td>
</tr>
<tr>
<td>Evan-video-CK-1</td>
<td>Learning for Oliver, Layla, and Chloe on why order of operations is not the reason why the expressions $(x + y)^2$ and $x^2 + y^2$ are not equivalent and the connection between the first and second problem and the difference in multiplication of monomials and binomials.</td>
</tr>
<tr>
<td>Evan-video-CK-2</td>
<td>Learning for Layla on the connection between area and multiplication of binomials.</td>
</tr>
<tr>
<td>Evan-video-CK-3</td>
<td>Learning for Evan on his incorrect use of the equal sign between the two expressions whose equivalence was supposed to be determined.</td>
</tr>
<tr>
<td>Evan-video-PCK-1</td>
<td>Oliver’s learning from Evan’s deliberate support for students’ learning of the mathematical understanding in the way he implemented the bellringer.</td>
</tr>
<tr>
<td>Evan-video-PCK-2</td>
<td>Evan’s realization that it would have been useful to show the area representation of multiplication of monomials to bring out the differences between monomial and binomial multiplication.</td>
</tr>
<tr>
<td>Instance Type</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Riley-video-CK-1</td>
<td>Learning for Isabelle, Ivy and Layla that one could construct another polygon from the one given in the task by making a bigger polygon.</td>
</tr>
<tr>
<td>Riley-video-PK-1</td>
<td>Instance for Isabelle and Riley’s on the need to be clear in when one is representing student work on the board.</td>
</tr>
<tr>
<td>Riley-video-PK-2</td>
<td>Learning for Austin and Riley that if a students’ error is not conceptual then he should not spend too much time on it.</td>
</tr>
<tr>
<td>Riley-video-PCK-1</td>
<td>Learning for Austin and Isabelle, that one can manipulate the way students think in the way they sequence mathematical ideas in a task to get them to learn a particular idea.</td>
</tr>
<tr>
<td>Riley-video-PCK-2</td>
<td>Instance is related to Riley’s learning that he could have provided more shapes for students in the first question to bring out the variety in the ways one can construct polygons from other polygons.</td>
</tr>
<tr>
<td>Riley-video-HLP-1</td>
<td>This instance is related to Riley’s learning about focusing the bellringer on the mathematical understanding.</td>
</tr>
<tr>
<td>Oliver-CK Instance 1</td>
<td>This instance is related to Chloe’s learning of the different methods of cross multiplication.</td>
</tr>
<tr>
<td>Oliver-video-CK-2and</td>
<td>Riley’s learning of the connection that the multiplicative property of equality and the multiplicative identity property show why cross multiplication works and Oliver’s learning of the need to make a connection between the different methods shared and why cross multiplication works. CK learning for Riley and PCK learning for Oliver.</td>
</tr>
<tr>
<td>Oliver-PCK Instance 1</td>
<td>Oliver’s learning from the clarifying of his mathematical understanding the weaknesses of his bellringer and what he would need to do to revise his bellringer.</td>
</tr>
<tr>
<td>Oliver-prep-HLP-1</td>
<td>Oliver’s arrival at the mathematical understanding for his bellringer.</td>
</tr>
<tr>
<td>Cameron-video- CK-1</td>
<td>Austin and Kylie’s learning on when to use mean and median.</td>
</tr>
<tr>
<td>Cameron-video- PK-1</td>
<td>Cameron’s learning that he should have been more direct and assertive in the way he handled student responses so that it should have been clear which was a better measure of central tendency for each of the two questions on his bellringer.</td>
</tr>
<tr>
<td>Cameron-video- PCK-1</td>
<td>Learning for Cameron that the table was a bit challenging to read due to the fact the order was based off of number of students rather than points missed.</td>
</tr>
<tr>
<td>Kylie-video-CK-1</td>
<td>Learning for Cameron, Layla, Kylie, Isabelle and Riley on how to explain division involving zero for the situations 0/5, 15/0 and 0/0.</td>
</tr>
<tr>
<td>Kylie-video-PCK-1</td>
<td>Learning for Kylie that using real life context as proposed by Oliver would have made it a lot easier for students than going straight to numbers.</td>
</tr>
</tbody>
</table>
Appendix H

Human Subjects Institutional Review Board
Letter of Approval
Date: August 22, 2017

To: Laura Van Zoest, Principal Investigator
   Mary Ochieng, Student Investigator for dissertation

From: Amy Naugle, Ph.D., Chair

Re: HSIRB Project Number 14-08-19

This letter will serve as confirmation that your research project titled “Prospective Teachers’ Developing Ideas about Teaching and Learning Mathematics” has been approved under the expedited category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note: This research may only be conducted exactly in the form it was approved. You must seek specific board approval for any changes in this project (e.g., you must request a post approval change to enroll subjects beyond the number stated in your application under “Number of subjects you want to complete the study”). Failure to obtain approval for changes will result in a protocol deviation. In addition, if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

Reapproval of the project is required if it extends beyond the termination date stated below.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: September 1, 2018