Three Essays on Asset Price Bubbles

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THREE ESSAYS ON ASSET PRICE BUBBLES

by

Frank Ofori-Acheampong

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This dissertation examines various issues associated with asset price bubbles. In the first essay, a Markov regime-switching model with time-varying transition probabilities is developed to identify asset price bubbles in the S&P 500 Index. The model nests two different methodologies; a state-dependent regime-switching model and a Markov regime-switching model. Three bubble regimes are identified; dormant, explosive, and collapsing. Time-varying transition probabilities are specified for each of the nine possible transitions in the Markov regime-switching model. Estimation of the model is done using conditional maximum likelihood with the Hamilton filter. Results show that transition probabilities depend significantly on trading volume and relative size of the bubble. Overall, the model works well in detecting multiple bubbles in the S&P 500 between January 1888 and May 2010.

In the second essay, a cross-market propagation of asset price bubbles is analyzed using a three-regime multivariate Markov switching model. The three bubble regimes identified are dormant (characterized by high returns and low volatility), explosive (characterized by high returns and high volatility), and collapse (characterized by low returns and high volatility). Results show that bubbles in the price of crude oil are influenced by bubble sizes in the S&P 500 Index and the price of gold. The bubble dynamics in gold price are driven by the bubble size in the S&P 500 Index. Lastly, bubbles in the S&P 500 Index tend to be driven largely by bubbles in crude oil price. Gold appears to be the most stable asset, having the least impact from the rest of the market. The
stability in gold price provides a case for gold serving as a safe haven asset in times of crisis or a hedge in normal times. The study uses monthly data from July 1989 to December 2014.

Finally, the third essay investigates the role of the Federal Reserve in the housing bubble between 2000 and 2006 as well as the eventual collapse of the bubble during the Great Recession. A mean group panel VAR is estimated for U.S states that experienced housing bubbles during the period. Two transmission channels are identified: an interest rate channel and a credit channel. The interest rate channel is traced with 30-year fixed mortgage rates whereas the credit channel is traced with real estate loans by all commercial banks in the U.S. Results show that the interest rate channel produces a greater impact on housing bubbles, following an expansionary monetary policy shock. The credit channel has a lower impact on housing bubbles following a monetary policy shock. The direct impact of a monetary policy shock on real estate loans gives evidence on the lending behavior of commercial banks in periods leading up to the recession. Overall, evidence shows that the Federal Reserve had a significant role in the housing bubble and the subsequent Great Recession. The date for the study spans 1998 to 2008.
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INTRODUCTION

This dissertation comprises three essays on asset price bubbles. Observed movements in asset prices can be difficult to explain by fundamental factors. The deviation of asset prices from their fundamental values have often been attributed to the presence of rational bubbles. A rational bubble is a large and persistent deviation of the market price of an asset from its fundamental value resulting from rational speculative behavior.

Asset price bubbles have been experienced by several economies, with varying degrees in times past. These bubbles have predominantly been followed by large-scale financial crisis. One of the earliest known bubbles was the Dutch Tulip Mania that started in 1634 and burst in 1637. This involved the establishment of futures contracts for the sale of tulip bulbs before the end of the growing season [Chang et al. (2016)]. The discovery of a rare species of tulips that were in short supply saw prices rising, eventually attracting speculators. At the peak of the crisis, a bulb of tulips was selling for an equivalent of $60,000 today [Scherbina (2013)]. Other historical bubbles include the South Sea bubble in England, the Mississippi bubble in France, and the Great Crash of October 1929 in the United States. The rapid rise in equity and real estate prices in Japan during the 1980s reached as high as 373% but fell by 50 percent over the next three years. Scherbina (2013) notes that at the peak of the crisis, the market value of all the lands in Japan was four times the land value in the United States. This changed by 1993 when Japanese land value fell by 50%. In very recent times, we experienced the technology bubble in the 1990s, bursting in March 2000. The 2006 housing bubble collapse and the stock market crash which led to the 2007/2009 Great Recession is the latest in the series of bubbles experienced in the past.
A common characteristic of all these bubbles is that they all go through three phases; dormant, explosive, and a collapse. The unique characteristics of each phase are described in subsequent sections.

The dormant regime of a bubble is mainly characterized by a steady growth in the asset’s price. It essentially sets the tone for the explosive regime to develop. Here, prices rise slowly until momentum is built as more investors enter the market. The influx of more investors creates excess demand for the asset, causing a sudden jump in prices. This ends the dormant regime, ushering in the explosive regime.

In the explosive regime, investors are in a state of euphoria due to profit-taking opportunities. Rational investors observe that the asset is over-valued, and its price will continue to increase in future. Thus, they buy the asset today for resale in future to the ‘greater fool.’ It is essential to understand here, why all investors do not harbor this assumption at the same time. Although all investors are deemed to have access to available information in an efficient market, they react to this information differently and at different times. Thus, there is always someone willing to buy an over-valued asset for a higher price. This phenomenon continues until demand exceeds supply, and the price of the asset explodes. In this regime, investors only care more about the capital gains and less about the quality of the asset since they are not holding to earn dividends.

The collapsing regime remains the most pervasive of the three regimes as investors are unable to predict when they are likely to occur. An investor may still be riding the explosive bubble at time $t$ going into $t+1$, only for the price to crash when time $t+1$ eventually arrives. The losses incurred here can be very huge and thus, investors are always concerned about when to exit the

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1 The greater fool theory argues that even though an asset may be over-valued, investors believe that there is always someone (the greater fool) willing to pay a higher price for it.
profit-taking strategy. As it turns out, this regime can be set in motion by this same uncertainty. Investors begin to heed some warning signs about the asset’s quality relating to a potential collapse and may exit the market too soon. This sends a signal to other market participants who follow suit. Large sell-offs occur, setting in motion a panic period. At this point, almost every investor dumps their asset at any price possible just to cut their losses and the asset’s price falls considerably. It is important to note that in the collapsing regime, all it takes is for one big investor to dump their assets and the rest of the market panics. A recent example characterizing the collapsing regime is the collapse of Lehman Brothers on September 15, 2008. On September 9, 2008, the share price of Lehman Brothers fell by 45%, dragging down the S&P 500 composite index by 3.4% after news of a failed takeover by Korean Development Bank emerged. Its share price continued to fall until the eventual collapse. This brings to light the effects of company news on share prices. In addition, investor sentiments can also set a bubble collapse in motion. Investors may simply decide that asset prices are too high and stop buying, with the rest of the market doing same.

In the first essay of this dissertation, a Markov regime-switching model with time-varying transition probabilities is developed to identify asset price bubbles in the S&P 500 Index. The model nests two different methodologies; a state-dependent regime-switching model and a Markov regime-switching model. Two strands of literature, often characterized as the indirect approach and direct approach, exist on empirically detecting the presence of asset price bubbles. The indirect approach was initiated by Diba and Grossman (1987, 1988) and recently revitalized by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b). In this approach, bubbles are detected through the time series properties of prices and dividends, specifically by showing that prices have explosive autoregressive roots and demonstrating that prices and dividends are not cointegrated. In the direct approach, an explicit model is specified for the latent periodically collapsing bubble
process and the parameters of the process are estimated by maximum likelihood or Bayesian methods. Within the direct approach, two sub-strands of the literature exist. The first approach follows Evans (1991) in which the bubble regime transition probabilities are governed by state variables such as the size of the bubble and trading volume. This line of research has been developed in a series of papers by van Norden and Schaller (1993, 2002), van Norden and Vigfusson (1998), Brooks and Katsaris (2005) and Anderson, Brooks and Katsaris (2010). In the second approach, bubble regime transitions are completely exogenous and are governed by a multiple state Markov process. Papers which employ this approach include Fulop and Yu (2017), Shi and Song (2014), and Balke and Wohar (2009).

This research follows the direct approach and estimates an explicit model for a periodically collapsing bubble in the S&P 500. The main contribution is that it combines the state-dependent approach with the Markov switching approach. A very general three-regime bubble process with dormant, explosive and collapsing states in which the regimes follow a Markov switching process with state-dependent transition probabilities is estimated. By combining the Markov and state-dependent models, the effect of a state variable can depend on the current bubble regime. Three bubble regimes are identified; dormant, explosive, and collapsing. Time-varying transition probabilities are specified for each of the nine possible transitions in the Markov regime-switching model. Estimation of the model is done using conditional maximum likelihood with the Hamilton filter.

In the second essay, a cross-market propagation of asset price bubbles is analyzed using a three-regime multivariate Markov switching model. The three bubble regimes identified are dormant (characterized by high returns and low volatility), explosive (characterized by high returns and high volatility), and collapse (characterized by low returns and high volatility). Studies on
cross-market linkages have predominantly ignored the role of asset price bubbles. The contribution of this study is to fill this gap by using the bubble components of asset prices rather than the actual prices of the assets as used in previous literature to study propagation effects. The second contribution is the inclusion of bubble components from other assets in the transition probabilities that drive the switching of an asset’s price between regimes.

Two approaches are followed in this research. First, asset prices are decomposed into a fundamental component and a bubble component, using the present value model. Second, a multivariate Markov regime-switching model is developed to trace the propagation of multiple bubbles across asset prices. A bubble in the price of asset A propagates through the price of asset B if there is a significant increase in cross-market linkage after a shock occurs in the price of asset A. Three asset markets are considered; gold, oil, and S&P 500. The inter-linkage between these assets results from the need to hedge in normal times against possible market downturns, or seek a safe haven in times of crisis. A critical distinction exists between a hedge and a safe haven asset. A safe haven asset is an asset that is uncorrelated or negatively correlated with another asset in crises periods. A hedge is an asset that is uncorrelated or negatively correlated with another asset not just in crises periods, but on average. This distinction turns out to be important as it provides information on portfolio formation. The study uses monthly data from July 1989 to December 2014.

The third essay investigates the role of the Federal Reserve in the housing bubble between 2000 and 2006 as well as the eventual collapse of the bubble during the Great Recession. The existence of housing bubbles became topical during the 2007-2009 financial crisis. The crisis has been blamed on high house prices starting in the early 2000’s leading up to the Great Recession. Although the effects of the bubbles in the U.S. housing market were felt nationally and globally,
the bubbles themselves did not occur throughout the country. Bubbles in the housing market can best be described as ‘localized’ given that they were largely concentrated in major urban areas in some specific states. An understanding of the financial crisis depends largely on detailed studies of the factors that led to the housing bubbles during the period. To what extent did Federal Reserve interest rate cuts contribute to the housing bubble? This is the main question of interest that is addressed in this study.

A mean group panel VAR is estimated for seven U.S states (California, Hawaii, Maryland, Massachusetts, New Jersey, New York, Rhode Island) and Washington D.C that experienced housing bubbles during the period. Two transmission channels are identified: an interest rate channel and a credit channel. The interest rate channel is traced with 30-year fixed mortgage rates whereas the credit channel is traced with real estate loans by all commercial banks in the U.S. Results show that the interest rate channel produces a greater impact on housing bubbles, following an expansionary monetary policy shock. The credit channel has a lower impact on housing bubbles following a monetary policy shock. The direct impact of a monetary policy shock on real estate loans gives evidence on the lending behavior of commercial banks in periods leading up to the recession. Overall, evidence shows that the Federal Reserve had a significant role in the housing bubble and the subsequent Great Recession. The date for the study spans 1998 to 2008.
REFERENCES


CHAPTER 1

A MARKOV REGIME-SWITCHING MODEL WITH TIME-VARYING TRANSITION PROBABILITIES FOR IDENTIFYING ASSET PRICE BUBBLES

1.1 Introduction

Observed movements in the S&P 500 Index can be difficult to explain by fundamental factors. The deviation of asset prices from their fundamental values have often been attributed to the presence of rational bubbles. A rational bubble is a large and persistent deviation of the market price of an asset from its fundamental value resulting from rational speculative behavior. Figure 1.1 shows the monthly S&P 500 price Index from January 1888 to May 2010 together with NBER recession dates over the same period. Stock market booms and busts are often seemingly linked to the business cycle. This is apparent in the period prior to the Great Depression in which the Index shows a steady growth up to 1922, then evolves into explosive growth from 1923 to 1929 and later collapses from 1929 to 1933. Similar patterns are observed in latter periods such as the late 1990’s and early 2000’s associated with the internet bubble and the Great Recession. In each of these cases, the stock price collapse coincided with a recession. The sequence of dormant, explosive and collapsing bubbles is not always consistent nor is it always associated with the business cycle. For example, periods of dormant bubbles appear to be followed immediately by a collapse in 1900-1902, 1905-1907 and 1978-1981. Similarly, a collapse can be followed by an apparently explosive bubble as in 1962, 1974, 2003 and 2009. Finally, an explosive bubble can be followed by a dormant bubble as in 1990-1994 and 2004-2006.

Two strands of literature, often characterized as the indirect approach and direct approach, exist on empirically detecting the presence of asset price bubbles. The indirect approach was
initiated by Diba and Grossman (1987, 1988) and recently revitalized by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b). In this approach, bubbles are detected through the time series properties of prices and dividends, specifically by showing that prices have explosive autoregressive roots and demonstrating that prices and dividends are not cointegrated. In the direct approach, an explicit model is specified for the latent periodically collapsing bubble process and the parameters of the process are estimated by maximum likelihood or Bayesian methods. Within the direct approach, two sub-strands of the literature exist. The first approach follows Evans (1991) in which the bubble regime transition probabilities are governed by state variables such as the size of the bubble and trading volume. This line of research has been developed in a series of papers by van Norden and Schaller (1993, 2002), van Norden and Vigfusson (1998), Brooks and Katsaris (2005) and Anderson, Brooks and Katsaris (2010). In the second approach, bubble regime transitions are completely exogenous and are governed by a multiple state Markov process. Papers which employ this approach include Fulop and Yu (2017), Shi and Song (2014), and Balke and Wohar (2009).

In this paper, the direct approach is followed and an explicit model for periodically collapsing bubbles in the S&P 500 Index is estimated. The contribution of this research is that it combines the state-dependent approach with the Markov switching approach. A very general three-regime bubble process with dormant, explosive and collapsing states in which the regimes follow a Markov switching process with state-dependent transition probabilities is estimated. By combining the Markov and state-dependent models, the effect of a state variable can depend on the current bubble regime. For example, it is observed that when the bubble is in the dormant regime, the probability of exiting the regime increases with the relative size of the bubble. In contrast, when the bubble is in the explosive regime, the probability of leaving the explosive
regime is independent of all the state variables. Moreover, when the bubble is in the collapsing regime, the probability of reverting to an explosive regime increases with the relative bubble size. These complex probabilities can explain why a sequential transition from dormant to explosive to collapsing regimes of bubbles in the S&P 500 Index is not always observed. Results show that the three-regime Markov switching speculative bubble model can identify most of the historical bubble phenomena, such as ‘The Great Depression,’ ‘Black Monday,’ ‘Friday the 13th,’ the Kennedy slide (flash crash) and the ‘Technology Bubble.’

1.2 A Three-Regime Markov Switching Rational Speculative Bubble Model

In this section, the three-regime model by Brooks and Katsaris (hereafter called BK model) is adapted to a Markov regime-switching model. To model the bubble behavior, the following present value model is used.

\[ P_t = (1 + R)^{-1}E_t(P_{t+1} + D_{t+1}), \]  

where \( P_t \) is the asset price at time \( t \), \( R \) is a constant rate of discount, \( D_{t+1} \) is the dividend payment at time \( t+1 \). By imposing the transversality condition, a particular solution to (1) is

\[ P_t^f \equiv \sum_{k=1}^{\infty} (1 + R)^{-k}E_tD_{t+k}, \]  

where \( P_t^f \) denotes the fundamental price of an asset. The solution to (1) may contain another component besides \( P_t^f \). Let \( B_t = P_t - P_t^f \). If

\[ B_t = (1 + R)^{-1}E_tB_{t+1}, \]  

then \( P_t = P_t^f + B_t \) is also a solution to (1). The component \( B_t \) is called a bubble.

Having identified the form of the bubble, the three-regime model for the bubble process in period \( t+1 \) is then formulated. Define an indicator variable \( S_t \) such that
\[ S_t = \begin{cases} 
1, & \text{if } B_t \text{ is dormant} \\
2, & \text{if } B_t \text{ is explosive} \\
3, & \text{if } B_t \text{ is collapsing.} 
\end{cases} \]

If the dormant bubble regime occurs in period \( t+1 \), the bubble will grow at a constant mean rate of \((1+R)\) such that

\[
E_t(B_{t+1}|S_{t+1} = 1) = (1 + R)B_t. \tag{4}
\]

In this regime, the bubble does not collapse and thus investors have no expectations of large deviations in asset returns. Denote the probability of being in the dormant regime by \( n_t \) which is state-dependent, and will also depend on the relative size of the bubble as well as the spread between fundamental returns and actual returns on the S&P 500 Index. Later, a Markov chain structure in which the regime probabilities also depend upon the previous regime is added. This is omitted for now to simplify notation. The probability of being in the non-dormant regime is given by \( 1 - n_t \). In the non-dormant state, two underlying regimes are defined: the explosive regime that occurs with probability \( q_t \) and the collapsing regime that occurs with probability \( 1 - q_t \). The probability \( q_t \) is specified as a function of the relative bubble size and volume traded of the S&P 500 Index. If the collapsing regime occurs

\[
E_t(B_{t+1}|S_{t+1} = 3) = g(b_t) \frac{R_t}{1 - q_t}, \tag{5}
\]

where \( g(b_t) \) is a continuous and everywhere differentiable function with \( g(b_t) > 0 \) and \( 0 < \frac{\partial g(b_t)}{\partial b_t} < 1 + R \). \( b_t \) is the relative bubble size defined by \( b_t = \frac{B_t}{P_t} \). The restriction on \( \frac{\partial g(b_t)}{\partial b_t} \) ensures that the bubble in the collapsing regime grows slower than that under the dormant regime.

One distinction between equation (5) and that specified in the BK model is the multiplication by \( \frac{1}{1 - q_t} \). As the probability of being in the explosive regime \( q_t \) increases, the collapsing regime is less likely to occur with probability \( 1 - q_t \). However, if the collapsing regime does occur, then the
collapse from a higher price level will be more severe than the collapse from a lower price level. Thus, \( \frac{1}{1-q_t} \) acts as a scale factor on the size of the collapse.

The expected size of the bubble under the explosive regime is given by

\[
E_t(B_{t+1}) = q_tE_t(B_{t+1}|S_{t+1} = 2) + (1-q_t)E_t(B_{t+1}|S_{t+1} = 3). \tag{6}
\]

Substituting (3) and rearranging yields

\[
E_t(B_{t+1}|S_{t+1} = 2) = \frac{1+R}{q_t}B_t - \frac{1}{q_t}g(b_t)P_t. \tag{7}
\]

Equation (7) shows that the expected bubble size in the explosive regime is a negative function of the probability \( q_t \) of being in that regime. This implies that as the probability of being in the explosive regime decreases, investors demand a higher return to compensate for the risk of a possible collapse in the asset price. Thus, in the explosive regime, the gross return \( \frac{1+R}{q_t} \) on the bubble exceeds the returns in the dormant and collapsing regimes.

In this paper, it is assumed that, conditional on the regime in period \( t+1 \), growth of the bubble is deterministic. Therefore, the bubble process evolves according to

\[
B_{t+1} = \begin{cases} 
(1+R)B_t, & \text{with probability } n_t \\
\frac{1+R}{q_t}B_t - \frac{1}{q_t}g(b_t)P_t, & \text{with probability } (1-n_t)q_t \\
g(b_t)\frac{P_t}{1-q_t}, & \text{with probability } (1-n_t)(1-q_t). 
\end{cases} \tag{8}
\]

The probabilities of being in the dormant and explosive regimes \( n_t \) and \( q_t \) are negative functions of the bubble size. As the bubble size continues to grow in the dormant regime, the probability of entering the explosive regime increases, hence \( n_t \) gets smaller. Likewise, as the bubble size continues to grow in the explosive regime the probability of the bubble collapsing increases, hence \( q_t \) gets smaller.

To generalize the BK model, the probabilities \( n_t \) and \( q_t \) are specified to follow a first-order Markov process. Specifically, the Markovian property requires that the current regime indicator

13
$S_{t+1}$ depends on its immediate past indicator $S_t$. As described above, in addition to the past state indicator, other conditioning driving variables $X_t$ are included in estimating the transition probabilities. Here, a probability matrix with a time-varying transition probability-generating function for each probability cell is specified. For the three-regime model, the time-varying transition probability function is written as

$$P[S_{t+1} = i | S_t = j, X_t] \equiv P_{ijt}$$

for $i, j = 1, 2, 3$.

For the three-regimes, the probabilities can be written out as

$$P_{1jt} \equiv P[S_{t+1} = 1 | S_t = j, X_t] = n_{jt}$$

$$P_{2jt} \equiv P[S_{t+1} = 2 | S_t = j, X_t] = (1 - n_{jt})q_{jt}$$

$$P_{3jt} \equiv P[S_{t+1} = 3 | S_t = j, X_t] = (1 - n_{jt})(1 - q_{jt})$$

with $\sum_{t=1}^{3} P_{ijt} = 1$ as required.

The state variables in $X_t$ include the spread between fundamental returns and actual returns of the S&P 500 Index $S_{t,f,a}$, volume traded of the S&P 500 Index $V_t$, and the relative bubble size of the S&P 500 Index $b_t$. To constrain the probabilities between 0 and 1, as well as ensuring that their sum equals 1, simultaneously, the probit model specification of Ding (2012) is adopted. Specifically,

$$n_{jt} = \Phi(\alpha_{n,0,j} + \alpha_{n,b,j}b_t + \alpha_{n,s,j}S_{t,f,a})$$

$$q_{jt} = \Phi(\alpha_{q,0,j} + \alpha_{q,b,j}b_t + \alpha_{q,v,j}V_t) \quad \text{for } j = 1, 2, 3.$$

$\Phi()$ is the cumulative normal density function, $S_{t,f,a}$ represents the spread of the absolute value of the average 6-month actual returns and the absolute value of the average 6-month returns of the estimated fundamental values, and $V_t$ is the trade volume of the asset. The inclusion of the spread is to separate bubble returns from fundamental returns. This ensures that the higher (lower) the spread, the lower (higher) the probability of the bubble continuing to be in the dormant regime. To achieve this switch between regimes, the spread is introduced in the equation for the probability
of being in a dormant regime. Doing so helps to identify the contribution of bubble returns in explaining average returns in period $t+1$.

Abnormal trade volume in the equation for the probability of being in an explosive regime serves as a signal to a possible bubble collapse. Based on the above, abnormal trade volume is added to the return equations in both explosive and collapsing regimes of the model, contrary to the BK model which only introduces abnormal trade volume in the explosive regime.

Trade volume ($V_t$) is omitted from the probability of being in a dormant regime ($n_{jt}$) because trade volumes do not experience large volatilities in the dormant regime of a bubble. Volatility in trade volume is negligible. With this identification structure, the spread and trade volume enter the probability equations separately.

The following signs are expected for the coefficient estimates of the probit models in equation (10).

\[
\begin{align*}
\alpha_{n,b,j} &< 0 \quad \text{for } j = 1 \\
\alpha_{n,S,j} &< 0
\end{align*}
\]

(10a)

\[
\begin{align*}
\alpha_{q,b,j} &< 0 \quad \text{for } j = 2 \\
\alpha_{q,v,j} &< 0
\end{align*}
\]

(10b)

\[
\begin{align*}
\alpha_{n,b,j} &> 0 \quad \text{for } j \neq 1 \\
\alpha_{n,S,j} &> 0
\end{align*}
\]

(10c)

\[
\begin{align*}
\alpha_{q,b,j} &> 0 \quad \text{for } j \neq 2 \\
\alpha_{q,v,j} &> 0
\end{align*}
\]

(10d)

Expressions (10a) and (10b) ensure that the probability of the bubble remaining dormant (explosive) decreases when the bubble size and spread (volume) increase. Similarly, (10c) and (10d) ensure that the probability of the bubble transitioning into a dormant (explosive) regime increases when the bubble size and spread (volume) increase.
1.3 Asset Returns

The bubble process above is applied to modelling the expected gross returns on an asset in each regime. The expected gross returns on an asset is given by

\[ E_t(r_{t+1}) = E_t \left[ \frac{P_{t+1} + D_{t+1}}{P_t} \right]. \]  

(11)

The expected gross returns\(^2\) in the dormant regime can be shown to be

\[ E_t(r_{t+1}|S_{t+1} = 1) = 1 + R. \]  

(12)

This implies that in the dormant regime, the expected returns to an asset are equivalent to the fundamental returns. In the explosive regime, the expected return on an asset is obtained as

\[ E_t(r_{t+1}|S_{t+1} = 2) = (1 + R) + \frac{1}{q_{jt}} [(1 - q_{jt})(1 + R)b_t - g(b_t)]. \]  

(13)

Equation (13) indicates that investors consider the probability of collapse once the bubble size continues to grow in the explosive regime. Given that they do not know when the bubble is likely to collapse they adjust their expectations of next period’s gross returns by considering the probability of collapse. In the collapsing regime, the expected gross returns equation is given by

\[ E_t(r_{t+1}|S_{t+1} = 3) = (1 + R) + g(b_t) \frac{1}{1-q_{jt}} - (1 + R)b_t. \]  

(14)

It is important to note that the expected bubble return is \(1+R\) in the dormant regime, greater than \(1+R\) in the explosive regime, and less than \(1+R\) in the collapsing regime. This helps investors to correctly identify which regime they are in based on their realized returns.

The expected returns equations above are highly non-linear, so to estimate them the approach of van Norden and Schaller (1999) is followed. Here, a first-order Taylor expansion is derived, around some arbitrary \(b_0\) and \(v_0\). This yields

\(^2\) A detailed derivation of model equations is available upon request from the authors.
where $b_t$ is the relative bubble size and $v_t$ is the abnormal share volume traded, and 1, 2, and 3 represent the dormant, explosive and collapsing regimes, respectively. Returns in the dormant regime are not affected by the bubble size and abnormal volume since they are equivalent to the fundamental returns. Thus, they can be treated as a constant with some unexpected deviations. However, the returns in both explosive and collapsing regimes are functions of the relative bubble size and abnormal share volume as explained in previous sections. Once investors have been able to correctly predict returns in the next period, they will know what regime they are likely to face. Large positive returns will imply a higher probability of being in an explosive regime. Likewise, significantly low returns will imply a higher probability of being in a collapsing regime. Lastly, steady returns will denote a higher probability of being in a dormant regime. Hence, instead of identifying the bubble regime directly, identification is done by inferencing from the returns equations.

As argued by Brooks et al. (2005), results of the Taylor series expansion yield some testable implications regarding the sign and magnitude of coefficient estimates from the three-regime model. These are required for the model to have explanatory power.

$$
\begin{align*}
\beta_{2,0} &> \beta_{1,0} > \beta_{3,0} \\
\beta_{2,b} &> \beta_{3,b} \\
\beta_{3,b} &< 0 \\
\beta_{2,v} &> 0 \\
\beta_{3,v} &< 0
\end{align*}
$$

Expression 15a is assumed to hold for the returns equations to yield the correct implications for the appropriate regime. On the other hand, 15b to 15e are required to hold based on the implications
of the Taylor series expansion. Expression (15a) implies that the returns in the explosive regime exceed those in the dormant and collapsing regimes, with the collapsing regime generating the least returns, in the absence of abnormal share volume and relative bubble size. Expression (15b) argues that as the bubble size increases, the expected returns in the explosive regime exceed the expected returns in the collapsing regime. Expression (15c) states that as the bubble collapses, the expected return on the asset should decrease. In (15d), expected returns in the explosive regime must increase if abnormal volume is observed, which signals an increase in the probability of a bubble collapse. Expression (15e) argues that an increase in abnormal share volume traded in the collapsing regime leads to negative returns on the asset.

It is important to note that the three-regime Markov switching model presented above is a generalization of the standard regime-switching BK model. By restricting the coefficients in the probabilities for the dormant and explosive regimes at time $t+1$ from varying with the regime existing at time $t$, as well as setting the coefficient of abnormal trading volume in the returns equation for the collapsing regime to zero, the Markov regime-switching model reduces to the BK model. The BK model can be tested against the more general Markov regime-switching model under the null hypothesis the following set of parameter restrictions

$$
\beta_{3,v} = 0
$$

$$
\alpha_{n,0,1} = \alpha_{n,0,2} = \alpha_{n,0,3}
$$

$$
\alpha_{n,b,1} = \alpha_{n,b,2} = \alpha_{n,b,3}
$$

$$
\alpha_{n,s,1} = \alpha_{n,s,2} = \alpha_{n,s,3}
$$

$$
\alpha_{q,0,1} = \alpha_{q,0,2} = \alpha_{q,0,3}
$$

$$
\alpha_{q,b,1} = \alpha_{q,b,2} = \alpha_{q,b,3}
$$

$$
\alpha_{q,v,1} = \alpha_{q,v,2} = \alpha_{q,v,3}
$$

against the alternative hypothesis that at least one of these restrictions is violated. A likelihood ratio test is used.
Under the assumption of disturbance normality, the above Markov regime switching model is estimated by conditional maximum likelihood. The conditional log-likelihood function for \( r_{t+1} \) is given by:

\[
\ln L(r_{t+1}|r_t, \theta, X_t) = \sum_{t=1}^{T} \ln \left[ \sum_{i=1}^{3} \sum_{j=1}^{3} f(r_{t+1}|S_{t+1} = i, S_t = j, r_t, \theta, X_t) P(S_{t+1} = i, S_t = j) \right].
\]  \tag{16}

where \( \theta \) is the vector of model parameters \((\alpha, \beta, \sigma)\), \( X_t \) is a vector of exogenous variables \((b_t, v_t, S_t^f, a)\) and \( f \) is the standard normal probability density function for each of the returns in (15). The log-likelihood function (16) represents a weighted average of the log-likelihood function in each regime with the weights being the transition probabilities. The latent transition probabilities are obtained using the Hamilton (1989) filter.

1.4 Data

Data for the study comprises 1,469 monthly observations on the S&P 500 Index from January 1888 to May 2010. Data on S&P 500 Index is available in monthly frequency as provided on Robert Shiller’s website. It is worth noting here that, in cases where high frequency data such as weekly or daily data are available, they may not be ideal for the identification of an asset price bubble. A bubble phenomenon must be persistent, spanning a longer period. However, there are instances when asset prices undergo sharp price increases, which last for a few days or weeks. These events are characterized by a sudden ‘craze’ for a particular stock, especially during initial public offers (IPOs), with asset prices stabilizing after a few days or weeks. Such price increases represent fads and an attempt to identify bubbles using daily or weekly data will lead to a misclassification of such fads as bubbles. Therefore, the use of monthly data helps avoid this potential misclassification.
S&P 500 price Index and dividends are obtained from Robert Shiller’s website.\textsuperscript{3} Data on abnormal volume is provided by Brooks, C. [see Brooks and Katsaris (2005) for details on how abnormal volume is calculated] who obtained it from the New York Stock Exchange (NYSE). Monthly dividend and price series are converted into real variables using monthly U.S consumer price Index. All variables are seasonally adjusted. To calculate the spread, fundamental prices are first calculated using (17) and subsequently used to obtain fundamental returns. Actual returns are also calculated and the difference between the two returns are obtained using $S_t^{f,a} = |r_t^{a,6}| - |r_t^{f,6}|$, where $r_t^{a,6}$ is the average 6-month actual returns and $r_t^{f,6}$ is the average 6-month returns of the estimated fundamental values. Figure 1.2 shows the bubble deviations of S&P 500 price Index from January 1888 to May 2010. As it turns out, the largest deviation was observed around the period of the Great Depression (1929-1933). For the period of the Great Recession (2007-2009), a negative deviation of the bubble size is observed. It is observed from Figure 1.2 that large deviations are associated with low asset prices. In general, the bubble deviation and the S&P 500 price Index exhibit similar patterns over time.

To estimate the three-regime model, fundamental values of the S&P 500 Index are first determined. Under the assumption that the dividend process follows a random walk with drift, it can be shown that

$$ p_t^f = \rho D_t, $$

(17)

where $\rho$ is the sample mean of the price-dividend ratio. Having obtained the fundamental values, the relative bubble size $b_t$ is then calculated as $b_t = 1 - \rho \frac{D_t}{p_t}$.\\

\textsuperscript{3} \url{http://www.econ.yale.edu/~shiller/data.htm}
1.5 Results

In this section, results from the three-regime Markov switching model and the BK model are presented. A likelihood ratio (LR) test is conducted to examine the appropriateness of the BK regime-switching model against the more general Markov regime-switching model. The restrictions as noted above are the exclusion of abnormal share trading volume in the returns equation for the collapsing regime as well as the homogeneity of transition probability coefficients across the bubble regimes. Results are presented at the bottom of Table 1.1. The LR test strongly rejects the BK model against the Markov regime-switching model at a 1% level of significance. The coefficient estimates in Table 1.1 indicate that the rejection is due to both the Markov regime-switching structure and the inclusion of trade volume in both explosive and collapsing regimes.

Despite the rejection of the BK model against the Markov regime-switching, there are some points of agreement in the results from both models. The two models largely agree on the contribution of bubbles to the returns on an asset as shown by the beta coefficients in Table 1.1. The coefficients exhibit the same signs but different magnitudes in both models. One notable difference is the inclusion of trade volume in the returns equation corresponding to the Markov regime-switching model which turns out to be highly significant.

The major difference between the two models lies in the time-varying transition probabilities. Whereas the Markov regime-switching model finds a negative and significant effect, as indicated by $\alpha_{n,b,1}$, from relative bubble size on the probability of the S&P 500 Index remaining in the dormant regime, this is not the case in the BK model. In the original BK model however, this coefficient was significant. The likely reason for these results could be the difference in sample periods used. Results from the original BK model are based on a sample from 1888 to 2003 whereas this study uses a sample from 1888 to 2010. This represents the inclusion of seven more
years of monthly data. Although this may not be a wide gap, between 2003 and 2010 the Great Recession occurred, and the sheer magnitude of this recession could be a significant contributor to this difference in results from the two models. Another point of departure between the Markov regime-switching model and BK model is observed in the transition probabilities for the explosive regime. Here, results from the Markov regime-switching model show that once the bubble enters the explosive regime, the probability of remaining in the explosive regime does not depend on the relative bubble size. Once the explosive regime occurs the bubble sustains itself and grows with a fixed probability. Evidence in support of this is the prolonged period of growth in the S&P 500 Index from 1990 to 2000 (during the technology bubble) and again from 2002 to 2008 (during the housing bubble). The final distinction between the Markov regime-switching model and the BK model lies in the behavior of bubbles in the collapsing regime. Once in a collapsing regime, as the bubble size begins to grow the probability of transitioning into an explosive regime increases. This phenomenon of reverting from a collapsing regime immediately to an explosive regime was observed in the S&P 500 Index as recently as 2003 following the collapse of the technology bubble and in 2009 after the crash as observed in Figure 1.1. The remainder of this section provides a detailed discussion of the results.

First, the coefficients in the returns equation (15) are considered. From Table 1.1, the estimate of the intercept in the dormant regime \( \beta_{1,0} \) implies an average return of 0.52% per month (6.42% effective annual rate\(^4\)) whereas it is 0.86% per month in the BK model (10.82% effective annual rate). This magnitude is reasonable in the context of a bubble developing before transitioning into an explosive state. The initial realization of the presence of a bubble causes more investors to enter the market, bidding the price of the asset upwards. This leads to increased returns

\(^4\) Calculated using Effective Annual Rate = \((1 + i)^{12} - 1\)
in the dormant phase of the bubble. When the explosive regime is reached, the average returns $\beta_{2,0}$ on the asset independent of the bubble size and abnormal volume stands at 0.94% per month (11.88% effective annual rate). This is roughly twice what is realized in the dormant regime. In the BK model, average returns of 1.01% per month (12.68% effective annual rate) is realized. During this period, exponential growth in asset prices ensures that returns far exceed what is obtained in the dormant regime. More investors are attracted to the market to buy the asset and resell to another investor at a higher price in future. However, as several investors soon offer their asset holdings for sale, excess supply causes price to tumble. This ushers in the collapsing regime, where the return on the asset $\beta_{3,0}$ is -2.24% per month (23.80% effective annual rate). The average returns are -2.98% per month (30.44% effective annual rate) in the BK model. At this rate, the gains in the previous regime are soon wiped out. It is observed that the BK model consistently overstates the average returns on the asset. This could result from the model’s lack of consideration for what regime was experienced in the previous period.

Turning now to the slope coefficients, bubbles in the two regimes have the expected signs and are significant at 5%. It is observed that the size of the bubble contributes about 1.69% returns to the asset in the explosive regime $\beta_{2,b}$ for every 1% increase in the bubble size and -4.19% in the collapsing regime $\beta_{3,b}$ for every 1% decrease in the bubble size. The positive sign of the bubble in the explosive regime confirms the notion that as the bubble increases in size investors demand higher returns to compensate them for the risk of a potential bubble collapse.

It is also observed from the table that abnormal share volume has a significant impact on asset returns in both explosive and collapsing regimes. In the explosive regime, the average return on the asset $\beta_{2,V}$ increases by 0.11% per month for a 1% increase in trading volume. This increase agrees with the prediction of the rational speculative bubble model since investors make positive
gains from trading assets in the explosive regime. In the collapsing regime, average returns on the S&P 500 Index $\beta_{3,\nu}$ decrease by 0.07% per month for every 1% decrease in trading volume. Recall that the abnormal share volume traded was excluded from the returns equation in the collapsing regime in the BK model. The observed negative and significant impact of abnormal share volume traded on asset returns justifies its inclusion in the returns equation for the collapsing regime in the more general Markov regime-switching model. Inclusion of abnormal share volume traded in the collapsing regime is also consistent with the historical losses observed in the S&P 500 Index during a bubble collapse. On ‘Black Monday’ October 19, 1987, the S&P 500 lost$^5$ about 20.5% of its value which was the largest loss ever experienced by any Index in a day. Consequently, large average trading volume was experienced, standing at $277,026,455.45 in October 1987 compared to $177,318,727.14 in September 1987. Between January 20, 2009 and March 9, 2009 during the financial crisis, the S&P 500 Index lost about 15.90% of its value.$^6$ These market collapses are all characterized by abnormal share volumes traded during these periods. The S&P 500 saw average share volume traded increasing from $1.3trillion to $1.8trillion between January and March 2009.$^7$

Results of the time-varying transition probabilities (TVTPs) provide another point of departure of the Markov regime-switching model from the BK model. Whereas the Markov switching model estimates a different probability for transitioning between any two regimes or even the same regime from period $t$ to $t+1$, the BK model implicitly assumes that the transition probabilities depend only on the exogenous variables. This implies that $P_{11t} = P_{12t} = P_{13t} \equiv n_{jt}$ in the dormant regime, $P_{21t} = P_{22t} = P_{23t} \equiv q_{jt}$ in the explosive regime, and $P_{31t} = P_{32t} = P_{33t} \equiv 1 - q_{jt}$ in the collapsing regime. However, in the Markov regime-switching model

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$^7$ Based on data from the NYSE obtained by Chris Brooks
considered here, nine separate probabilities are obtained, regarding every possible transition between the three regimes.

The parameters in \( n_{jt} \), which determine the probability of being in the dormant regime are first considered. From Table 1, the intercept \( \alpha_{n,0,1} \) suggests that there is a 99.84% [obtained by \( \Phi(\alpha_{n,0,1}) \)] probability of the bubble remaining in a dormant regime when the bubble size and spread are zero. The negative and significant coefficient on the bubble size \( \alpha_{n,b,1} \) indicates that as the bubble size grows, the probability of being in the dormant (explosive) regime in period \( t+1 \) decreases (increases). Similarly, as the spread increases, the probability of remaining in the dormant regime decreases, as evident from the coefficient of the spread \( \alpha_{n,s,1} \). This is a reasonable outcome as an increase in the spread can only happen when the bubble size is increasing, thus, reducing the likelihood of being in the dormant regime. The probability of transitioning from an explosive regime to a dormant regime did not yield any significant coefficients. This may suggest that the possibility of a reverse transition from an explosive to a dormant regime is limited, although this phenomenon was observed between 1990 and 1994. The intercept \( \alpha_{n,0,3} \) in the equation for the probability of transitioning from a collapsing regime to a dormant regime is negative and significant. It signifies a 4.29% [obtained from 1- \( \Phi(\alpha_{n,0,3}) \)] probability of the bubble transitioning from the collapsing regime to the dormant regime.

The parameters in \( q_{jt} \), which determine the probability of an explosive regime occurring are considered next. It is observed that \( \alpha_{q,b,2} \) and \( \alpha_{q,v,2} \) are not significantly different from zero indicating that the bubble size and trading volume do not significantly impact the probability of an explosive regime continuing to the next period. The fact that \( \alpha_{n,b,2} = \alpha_{n,s,2} = \alpha_{q,b,2} = \alpha_{q,v,2} = 0 \) implies that once the bubble enters the explosive regime, the bubble dynamics are independent of the state variables. When the bubble is in the explosive regime, the probability that the bubble
remains in the explosive regime or transitions to the collapsing or dormant regime is constant. The coefficient of the bubble size in the probability of switching from a collapsing regime to an explosive regime $\alpha_{q,b,3}$ is positive and significant. This implies that as the bubble size grows in the collapsing regime, the probability of an explosive regime occurring the next period increases. This phenomenon was observed in the S&P 500 Index between 2002 and 2010 when bubble collapses were immediately followed by the emergence of an explosive bubble.

Lastly, as the rational speculative bubble model predicts, the standard deviations of residuals from the three-regime Markov switching model yield the desired results. The standard deviation of the residuals in the collapsing regime exceed that in the explosive regime which in turn exceeds the standard deviation in the dormant regime. From Table 1.1, the standard deviations for the residuals in the dormant, explosive and collapsing regimes are 0.0008%, 0.0010%, and 0.0035%, respectively. The high standard deviation in the collapsing regime is consistent with experience because collapsing bubbles are associated with highly volatile negative returns.

Figures 1.3, 1.4, and 1.5 plot the unconditional probabilities of being in a dormant regime, an explosive regime, and a collapsing regime, respectively, together with the S&P 500 price Index from January 1888 to May 2010. In general, the three-regime Markov switching process performs well at identifying periods of asset price bubbles.

Figure 1.3 shows that in periods when the S&P was relatively stable, the probability of observing a dormant bubble was high and remained so for an extended period from 1888 to about 1930. Except for some periods of brief collapse in the S&P 500 such as 1901, 1904, 1908, 1922, 1929, the Index was relatively stable up until the Great Depression. The year 1901 witnessed a stock market panic, lasting 3 years as well as the year-long panic of 1907.
Another significant observation here is that as the bubble size increases over time, the probability of being in a dormant regime decreases, signaling the onset of an explosive regime.

Figure 1.4 represents the probability of being in an explosive regime as the bubble size grows over time. It is observed that these probabilities remained low for an extended period from 1888 to 1898 and again from 1907 until the Great Depression. The observed behavior is characteristic of the dormant nature of the S&P 500 Index before the Great Depression as well as the collapse during the Great Depression. As expected, for the Great Depression period, the probability of an explosive bubble is close to zero as there was no possibility for an explosive bubble to occur during the period. Although the model correctly identifies this, it fails however, to identify the rising prices before the depression. It captures well the price rally from 1956 to 1966 in the S&P 500, following in that direction up to the 1990s when a price rally was more pronounced because of the technology bubble. With the bubble collapsing in 2000, a lower probability of an explosive bubble is observed until 2002. A price rally re-emerged during this period until its eventual collapse in 2007 during the Great Recession, evidenced by a low probability of being in an explosive regime.

Lastly, Figure 1.5 which reports the probabilities of being in a collapsing regime shows a large increase in the probability of a bubble collapsing during the Great Depression from 1929 to 1933. The mild Kennedy crash of 1962, also called the ‘flash crash’ is correctly identified by the model, as the probability of being in a collapsing regime increases sharply during the period. The stock market crash of 1970 and 1974 are also correctly identified with very high probabilities of a collapse. The most recent market crashes are seen in 1987, 1989, 2000 and 2007 on ‘Black Monday,’ ‘Friday the 13th,’ the collapse of the ‘technology bubble,’ and the ‘Great Recession,’ respectively.
1.6 Robustness Analysis

In this section, two main robustness analyses are conducted. First, the sample period is split into two parts; from January 1888 to December 1950 (756 observations) and from January 1951 to May 2010 (713 observations). This split is based on the fact that after World War II, stock markets were relatively stable with the evolution of Keynesian economic policies until the oil crisis in the 1970s. The choice of 1950 is to allow sufficient time after World War II so that the adverse economic effects of the war would have subsided, giving room for economic policies to function. Second, one-period lagged values of the spread between the actual returns and fundamental returns on the S&P 500 Index are used in estimating the model for the full sample. This change is based on the idea that the returns on an asset in the immediate past period will provide more information about the existing bubble regime in period $t$ than would returns for the past six months. Thus, one-period lagged values of the spread are expected to provide sharper identification for asset price bubbles in period $t$.

Results based on data splitting are shown in Figures 1.6 and 1.7, showing a plot of the smoothed probabilities of being in a dormant, explosive and collapsing bubble regime. It is observed that results based on the full sample are upheld by those based on the split data. Similarly, results based on one-period lagged spread values, reported in Figure 1.8 are consistent with those obtained in Figures 1.1, 1.2, and 1.3.

Based on the results obtained, it can be concluded that the Markov regime-switching model works well in identifying asset price bubbles. Results are not dependent on the sample period or how the spread is measured.
1.7 Conclusion

A three-regime Markov switching model was developed to identify asset price bubbles in the S&P 500 composite price Index, using data from January 1888 to May 2010. Regimes were classified into dormant, explosive, and collapsing, with each characterizing the observed behavior of the S&P 500 Index over the period. With the observed relationship between asset price bubbles and asset returns, return equations were subsequently formulated to provide a more flexible approach to identifying bubbles. Time-varying transition probabilities were formulated to account for every possible type of transition among the regimes. With a three-regime model, nine time-varying transition probabilities were formulated using a probit model to restrict the size between 0 and 1.

Two representations of the results from the three-regime Markov switching rational speculative bubble model are provided. First, coefficient estimates are provided in Table 1.1 and compared with those obtained in the BK model. Second, probability plots based on the coefficient estimates are presented for each regime in Figures 1.3, 1.4, and 1.5. Intercept terms show that on average returns on the S&P 500 Index increase by about 0.52% per month in the dormant bubble regime, 0.94% in the explosive regime and decrease by 2.22% in the collapsing regime. The bubble size as well as abnormal share volume traded impact asset returns significantly in the explosive and collapsing regimes. On the time-varying transition probabilities, it is observed that the spread between actual returns and fundamental returns negatively affects the probability of remaining in a dormant bubble regime. On the probability plots, it is observed from Figures 1.3, 1.4, and 1.5 that the three-regime rational speculative bubble model can identify most of the historical bubble phenomena, such as ‘The Great Depression,’ ‘Black Monday,’ ‘Friday the 13th,’ the ‘Kennedy slide (flash crash)’ and the ‘Internet Bubble.’
The above results show that the rational speculative bubble model accurately identifies multiple historical bubbles. It performs better than the BK model which fails to identify some historical bubble crashes such as observed in 1962 (the Kennedy slide), 1987 (Black Monday), 1989 (Friday the 13th), and 2000 (the technology bubble). The Markov regime switching model therefore has more explanatory power relative to the standard regime switching model. The likelihood ratio test also provides evidence that the Markov regime-switching model is a better specification for analyzing bubbles in the S&P 500 Index.
REFERENCES


### APPENDIX

Table 1.1: Results of the three-regime rational speculative bubble model

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Standard Error</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{1,0}$</td>
<td>0.0052*</td>
<td>0.0013</td>
<td>0.0086*</td>
<td>0.0011</td>
</tr>
<tr>
<td>$\beta_{2,0}$</td>
<td>0.0094*</td>
<td>0.0021</td>
<td>0.0101*</td>
<td>0.0049</td>
</tr>
<tr>
<td>$\beta_{3,0}$</td>
<td>-0.0224*</td>
<td>0.0098</td>
<td>-0.0298*</td>
<td>0.0127</td>
</tr>
<tr>
<td>$\beta_{2,b}$</td>
<td>1.6892*</td>
<td>0.5001</td>
<td>3.6805*</td>
<td>1.5160</td>
</tr>
<tr>
<td>$\beta_{3,b}$</td>
<td>-4.1854*</td>
<td>1.2159</td>
<td>-1.5555**</td>
<td>0.8069</td>
</tr>
<tr>
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Likelihood | 2801.9178 | 2685.2671
LR Test | 233.3014
Test statistic | 0.0000
P-value | 0.0000

All variables are in logarithms in both models. Both models are estimated using S&P 500 data from January 1888 to May 2010. The BK model is estimated with restrictions noted in Brooks and Katsaris (2005). *, ** represent 5% and 10% level of significance, respectively.
Figure 1.1: S&P 500 stock price Index and NBER recession periods between January 1888 and May 2010

Figure 1.2: Bubble deviations and S&P 500 price Index from January 1888 to May 2010
Figure 1.3: Smoothed Probabilities of a Dormant Bubble

Figure 1.4: Smoothed Probabilities of an Explosive Bubble
Figure 1.5: Smoothed Probabilities of a Collapsing Bubble

Figure 1.6: Smoothed Probabilities of a Dormant, Explosive, and Collapsing Bubble, 1888 to 1950
Figure 1.7: Smoothed Probabilities of a Dormant, Explosive, and Collapsing Bubble, 1951 to 2010

Figure 1.8: Smoothed Probabilities of a Dormant, Explosive, and Collapsing Bubble, 1888 to 2010
2.1 Introduction

Various assets have gone through periods of bubbles at some point in time, with varying severity. In several instances, these bubbles propagate through other markets or assets going through otherwise tranquil periods. These bubbles have usually manifested in three forms. A period of low return with high volatility (collapse), a period of high return and high volatility (explosive), and a period of high return with low volatility (dormant). In this regard, this study makes these distinctions between the types of bubbles in a three-regime Markov switching model. The goal of this research is to analyze the propagation effects of bubbles from one asset or market to the other.

Two approaches are followed in this paper. First, asset prices are decomposed into a fundamental component and a bubble component, using the present value model. Second, a multivariate Markov regime-switching model is developed to trace the propagation of multiple bubbles across asset prices.

In this study a bubble in the price of asset A ‘propagates’ through the price of asset B if there is a significant increase in cross-market linkages after a shock occurs in the price of asset A. Three asset markets are considered; gold, oil, and S&P 500. The inter-linkage between these assets results from the need to hedge in normal times against possible market downturns, or seek a safe haven in times of crisis. A critical distinction exists between a hedge and a safe haven asset. A safe haven asset

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8 The study uses ‘propagation’ instead of ‘contagion’ to address a common misconception about what contagion is. Contagion considers ‘nearest neighbor’ markets (such as geographic locations with a contiguous border, or assets in the S&P 500 Index) or assets of the same class (such as all metals), whereas propagation considers not only ‘nearest neighbors’ but also markets or assets not directly related or distant from each other (this includes geographic locations without a common border, different classes of assets, or assets on different stock exchanges).
is an asset that is uncorrelated or negatively correlated with another asset in crises periods. A hedge is an asset that is uncorrelated or negatively correlated with another asset not just in crises periods, but on average. This distinction turns out to be important as it provides information on portfolio formation.

Figure 2.1 shows the trend of price movements for the assets considered in the study from January 1985 to December 2016. There is enough evidence from these plots to suggest that the 2001 recession and the Great Recession of 2007-2009 had a simultaneous impact on all three assets considered. However, it is observed that during these periods, the price of gold was the least affected, perhaps investors turned to it as a safe haven. Over the years, gold has earned the reputation as a popular safe haven during market downturns. Its value has soared significantly in periods when other markets are experiencing a decline. During the Great Recession, gold price rose from an average of $631.17 per troy ounce in January 2007 to $1,134.72 per troy ounce in December 2009. At the same time, stock markets crashed, and crude oil prices reached an all-time high of $145.3 per barrel in July 2008. There appears to be some correlation between these markets, especially in crises periods.

The seeming positive correlation between gold and oil price movements have been linked to their impact on several economic variables as well as their attractiveness as investment assets. Hunt (2002) observes that a rise in the price of oil leads to a rise in the general price level. This provides a case for gold serving as an ideal asset to hedge against the rising inflation. Excess demand for gold eventually leads to increased gold price. This positive relationship between gold and oil has been established by several studies including Tully et al. (2007).

Another mechanism explaining this relationship is noted in Reboredo (2010). The author notes that oil price movements affect economic growth and asset values negatively. With an increasing price of oil, economic growth slows down, and asset values decline, making gold an alternative store of value in the capacity as a safe haven. Baur et al. (2010) have also studied the safe haven property of gold. Another reason for the observed positive correlation between oil and gold lies in the need for
portfolio rebalancing. To maintain fixed proportions of gold in their investment portfolios, oil exporting countries increase their gold reserves whenever the price of oil goes up. This leads to increased demand for gold, causing gold prices to rise. Baur et al. (2006) have found evidence to support gold serving as a safe haven from losses incurred on the stock and bond markets as well.

Tests for the presence of bubbles in gold price have included the unit roots test of Diba and Grossman (1988) and the present value model of Pindyck (1993) based on convenience yield.\(^9\) Following Pindyck (1993), several studies [including Bialkowski et al. (2011), Emekter et al. (2009)] have found evidence of bubbles in the price of gold during different periods in history. Baur et al. (2015) also use the Phillips et al. (2011) sup ADF test to identify bubbles in the price of gold. They observe that between 2002 and 2012 gold price was in a bubble except for 2008-2009 during the financial crisis. This finding points to the possibility of regime switching in the price of gold.

In January 2008, WTI crude oil was trading at $92 per barrel. On July 11\(^{th}\) of the same year, a barrel of oil was selling above $140 per barrel. However, this was not to last as the price of oil suffered a major dive below $40 per barrel by December 2008. By August 2014 the price of oil stood at about $102 per barrel, falling to about $53 per barrel by close of December 2016. Several studies [including Lammerding et al. (2013)] have shown that these dynamics represent a speculative bubble in the price of oil. Using a Bayesian Markov regime-switching state space approach with the convenience yield measure of Pindyck (1993), Lammerding et al. (2013) find a robust evidence in support of the existence of speculative bubbles in the price of oil. As noted in their study, the research on oil price bubbles appears inconclusive, given its macroeconomic influence.


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\(^9\) Convenience yield is the benefit derived from possessing the physical quantities of a commodity relative to holding a futures or forwards on the same commodity.
These papers have found evidence of bubbles in equity prices, in particular, the S&P 500 Index. Bubbles are noted to occur in two regimes [Van Norden and Schaller (1999), Evans (1991)] or three regimes [Brooks and Katsaris (2005)].

Several approaches have been adopted in the literature in analyzing the linkages across markets and assets, including GARCH models [Ciner et al. (2013), Creti et al. (2013), Holyst et al. (2009)], regime-switching models [Guo et al. (2011), Chan et al. (2011)], copula functions [Wen (2012)], wavelet approach [Barunik et al. (2016)], VAR [Qiu et al. (2016)], and Granger causality [Choudhry (2015)]. It must be noted that most of these studies combine two or more of these approaches, with conclusions mostly based on correlation estimates. The consensus among all these studies suggests the presence of some form of association between various assets during crisis periods. This study analyzes propagation effects through a multivariate Markov regime-switching model.

Studies on cross-market linkages have predominantly ignored the role of asset price bubbles. The contribution of this study is to fill this gap by using the bubble components of asset prices rather than the actual prices of the assets as used in previous literature to study propagation effects. The second contribution is the inclusion of bubble components from other assets in the transition probabilities that drive the switching of an asset’s price between regimes.

The importance of this study lies in the information it provides for investors. Knowing the extent of correlation across markets helps with risk diversification. Investors can decide on the optimal mix of assets that lowers their risk of loss in the event of market downturns.

2.2 Methodology
2.2.1 Extracting Asset Price Bubbles

To extract the bubble component of asset prices the present value formula is used. For a general class of assets, the following present value model is estimated.
\[ P_t = (1 + R)^{-1}E_t(P_{t+1} + D_{t+1}), \quad (1) \]

where \( P_t \) is the asset price at time \( t \), \( R \) is a constant rate of discount, \( D_{t+1} \) is the dividend payment at time \( t+1 \). The assumption of transversality after recursive substitutions yields a particular solution to (1) given by
\[ P_t^f \equiv \sum_{k=1}^{\infty} (1 + R)^{-k}E_tD_{t+k}, \quad (2) \]

where \( P_t^f \) is the fundamental price of an asset. Equation (1) may contain another solution besides \( P_t^f \).

Let \( B_t = P_t - P_t^f \). If
\[ B_t = (1 + R)^{-1}E_tB_{t+1}, \quad (3) \]

then \( P_t = P_t^f + B_t \) is also a solution to (1) and is valid for all asset types. The component \( B_t \) is called a bubble.

Given that gold and WTI crude oil do not pay dividends, \( D_t \) in the above formula is replaced with the ‘convenience yield’\(^{10} \) \( C_t \). Following Pindyck (1993) and Tsvetanov et al. (2016), the convenience yield is modelled to satisfy the no-arbitrage condition
\[ C_t = (1 + R)P_t - F_t, \quad (4) \]

where \( F_t \) is the futures price at time \( t \) for settlement at time \( t+1 \). Thus, the fundamental price of crude oil and gold can be written as
\[ P_t^f \equiv \sum_{k=1}^{\infty} (1 + R)^{-k}E_tC_{t+k}. \quad (5) \]

Then, \( P_t = P_t^f + B_t \), where \( B_t \) is as defined above and satisfies the rational bubble condition in (3).

It should be noted here that the fundamental price as well as the bubble component for each of the three assets are not necessarily the same, despite the use of the same notations \( P_t^f \) and \( B_t \), respectively.

Various approaches have been used in the literature to estimate the fundamental price of an asset from (2) and (5) [including Evans (1991); Campbell and Shiller (1987)]. In this study the VAR

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\(^{10}\) Convenience yield is the return derived from holding physical quantities of a good.
approach of Campbell and Shiller (1987) is used. In their seminal paper, the authors argue that the spread $S_t$ between stock prices and a constant multiple of current dividends is the optimal forecast of a multiple of the discounted value of all future dividend changes. Then, for a dividend-paying asset

$$S_t \equiv P_t^f - \frac{1+R}{R} D_t = \frac{1+R}{R} \left[ \sum_{i=1}^{\infty} \frac{1}{(1+R)^i} E_t(\Delta D_{t+i}) \right],$$

where $\Delta D_{t+i}$ is the change in dividends at time $t+i$.

The VAR methodology allows us to examine whether changes in dividends can be forecasted by the spread between prices and the multiple of current dividends. If investors include all relevant past information in their information set, this should be reflected in the spread which will make it a good predictor of changes in dividends. To investigate this, the following VAR model is specified

$$\begin{bmatrix} \Delta D_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta D_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} \mu_{1t} \\ \mu_{1t} \end{bmatrix},$$

where $a(L), b(L), c(L),$ and $d(L)$ are polynomials in lag operator $L$, $\Delta D_{t-1}$ and $S_{t-1}$ are the one-period lags of changes in dividends and stock prices, respectively, and $\mu_t$ is the residual term. If $b(L)$ is zero, then changes in dividends cannot be forecasted by the spread. Let $\overline{D}_t$ represent the VAR forecast of changes in dividends. Then,

$$P_t^f = \overline{D}_t + \frac{1+R}{R} D_t.$$

Having obtained the fundamental values of each asset, the bubble size $B_t$ is then calculated as $B_t = P_t - P_t^f$. The above argument is extended to crude oil and gold prices, by replacing $D_t$ with $C_t$.

### 2.2.2 Multivariate Bubble Model

In this section, a three-regime model for identifying asset price bubbles is developed. These are denoted as dormant, explosive, and collapsing regimes. The explosive and collapsing regimes

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11 In the sense that it minimizes the mean square forecast error.
constitute what is jointly referred to as a crisis regime. To model the form of the bubble, an indicator variable $S_t$ is defined such that

$$ S_t = \begin{cases} 
1, & \text{if } B_t \text{ is dormant} \\
2, & \text{if } B_t \text{ is explosive} \\
3, & \text{if } B_t \text{ is collapsing}.
\end{cases} $$

Suppose the dormant bubble regime occurs in period $t+1$, the bubble will grow at a constant mean rate of $I+R$ such that

$$ E_t(B_{t+1}|S_{t+1} = 1) = (1 + R) B_t. \quad (9) $$

Let the probability of being in the dormant regime be $n_t$ which depends on the relative size of the bubble at time $t$. Then, the probability of being in the non-dormant regime is given by $1 - n_t$. In the non-dormant regime, the asset price could be explosive with probability $q_t$ or collapsing with probability $1 - q_t$. If the collapsing regime occurs at time $t+1$, then

$$ E_t(B_{t+1}|S_{t+1} = 3) = g(b_t) \frac{P_t}{1-q_t}, \quad (10) $$

where $g(b_t)$ is assumed to be continuous and everywhere differentiable with $g(b_t) > 0$ and $0 < \frac{\partial g(b_t)}{\partial b_t} < 1 + R$. In addition, $b_t$ is the relative bubble size denoted by $b_t = \frac{B_t}{P_t}$. The restriction on $\frac{\partial g(b_t)}{\partial b_t}$ guarantees that the growth of the bubble in the collapsing regime is slower than that under the dormant regime. With a high $q_t$ (probability of being in the explosive regime) the probability of the collapsing regime $1 - q_t$ occurring is decreased. However, if the collapsing regime occurs, then the collapse from a higher price level will be more severe than the collapse from a lower price level. So, $\frac{1}{1-q_t}$ acts as a scale factor on the size of the collapse.

Under the explosive regime, the expected size of the bubble is given by

$$ E_t(B_{t+1}) = q_tE_t(B_{t+1}|S_{t+1} = 2) + (1 - q_t)E_t(B_{t+1}|S_{t+1} = 3). \quad (11) $$

Substituting (9) and (10) into (11) yields
\[
E_t(B_{t+1}|S_{t+1} = 2) = \frac{1+R}{q_t} B_t - \frac{1}{q_t} g(b_t)P_t.
\] (12)

Equation (12) suggests that the expected bubble size in the explosive regime is a negative function of the probability of being in that regime \(q_t\). As the probability of being in the explosive regime decreases, investors demand a higher return to compensate them for the risk of a possible collapse in the asset price. Thus, the gross return \(\frac{1+R}{q_t}\) on the bubble in the explosive regime exceeds the return in the dormant and collapsing regimes.

Conditional on the regime in period \(t+1\), growth of the bubble is deterministic. Therefore, the bubble process evolves according to

\[
B_{t+1} = \begin{cases} 
(1+R)B_t, & \text{with probability } n_t \\
\frac{1+R}{q_t} B_t - \frac{1}{q_t} g(b_t)P_t, & \text{with probability } (1-n_t)q_t \\
g(b_t)\frac{P_t}{1-q_t}, & \text{with probability } (1-n_t)(1-q_t).
\end{cases}
\] (13)

The probabilities of being in the dormant and explosive regimes \(n_t\) and \(q_t\) are negative functions of the bubble size. A continuous growth of the bubble size in the dormant regime increases the probability of entering the explosive regime, hence \(n_t\) gets smaller. Similarly, a continuous growth of the bubble size in the explosive regime increases the probability of the bubble collapsing, hence \(q_t\) gets smaller.

There is need to constrain the probabilities between 0 and 1. Thus, a probit function specification is employed. In this approach, a probability matrix with a recursive time-varying transition probability generating function for each probability cell is specified. For the three-regime model, the time-varying transition probability function is written as

\[
P[S_{t+1} = i|S_t = j, X_t] \equiv P_{ijt},
\] (14)
where $X_t$ is the set of factors that affect the transition probability over time, $\mathcal{P}_{ijt}$ is the probability of an asset transitioning from regime $j$ to $i$ at time $t$, and $\sum_{i=1}^{3} \mathcal{P}_{ijt} = 1$, with $i, j = 1,2,3$. The probabilities can be written out as

$$\mathcal{P}_{1jt} \equiv P[S_{t+1} = 1|S_t = j, X_t] = n_{jt}$$
$$\mathcal{P}_{2jt} \equiv P[S_{t+1} = 2|S_t = j, X_t] = (1 - n_{jt})q_{jt}$$
$$\mathcal{P}_{3jt} \equiv P[S_{t+1} = 3|S_t = j, X_t] = (1 - n_{jt})(1 - q_{jt}),$$

(15)

where

$$n_{jt} = \Phi(\alpha_{0,j} + \sum_{l=1}^{3} \alpha_{b,j}^l b_{lt}) ,$$

$$q_{jt} = \Phi(\omega_{0,j} + \sum_{l=1}^{3} \omega_{b,j}^l b_{lt} + \sum_{l=1}^{3} \psi_{b,j}^l b_{lt}^2) \quad \text{for } j = 1,2,3$$

and $q_{jt}$ is expressed as a quadratic function of $b_t$ to capture the effect of an explosive bubble and $l$ represents the number of assets. $\alpha_{0,j}$ is the intercept and $\alpha_{b,j}^l$ represents the coefficient of relative bubble size in the probability equation for being in a dormant bubble regime. On the other hand, $\omega_{0,j}$ is the intercept, $\omega_{b,j}^l$ is the coefficient of relative bubble size, and $\psi_{b,j}^l$ is the coefficient of the squared value of relative bubble size in the probability equation for the explosive bubble regime. $\Phi(\cdot)$ is the cumulative normal density function.

2.3 Asset Returns

With a positive relationship between bubbles and asset returns, the bubble process above is applied to modeling the expected gross returns to an investment in each regime. The expected gross returns equation can be written as

$$E_t(r_{t+1}) = E_t\left[ \ln \frac{P_{t+1} + D_{t+1}}{P_t} \right],$$

(16)

where $r_{t+1}$ measures the returns for all assets considered. The expected gross returns in the dormant regime can be shown to be

$$E_t(r_{t+1}|S_{t+1} = 1) = 1 + R.$$
This implies that in the dormant regime, the expected returns to an asset are equivalent to the fundamental returns. In the explosive regime, the expected returns to an asset is obtained as

\[
E_t(r_{t+1} | S_{t+1} = 2) = (1 + R) + \frac{1}{q_t} [(1 - q_t)(1 + R)b_t - g(b_t)].
\]  

Equation (18) suggests that investors take into account the probability of collapse once the bubble size continues to grow in the explosive regime. Given that they do not know when the bubble is likely to collapse they adjust their expectations of next period’s gross returns by considering the probability of collapse. In the collapsing regime, the expected gross returns equation is given by

\[
E_t(r_{t+1} | S_{t+1} = 3) = (1 + R) + g(b_t) \frac{1}{1 - q_t} - (1 + R)b_t.
\]  

It is important to note that the expected bubble returns are \((1 + R)\) in the dormant regime, greater than \((1 + R)\) in the explosive regime, and less than \((1 + R)\) in the collapsing regime. This helps investors to correctly identify which regime they are in based on their realized returns.

The expected returns equations above are highly non-linear; so, to estimate them the approach of Van Norden and Schaller (1999) is followed. Here, a first-order Taylor expansion is derived, around some arbitrary \(b_0\). This yields

\[
r_{t+1} = \begin{cases} 
\beta_{1,0} + \mu_{t+1} & \text{if } b_t \text{ is dormant} \\
\beta_{2,0} + \beta_{2,b} b_t + \mu_{t+1}^2 & \text{if } b_t \text{ is explosive} \\
\beta_{3,0} + \beta_{3,b} b_t + \mu_{t+1}^3 & \text{if } b_t \text{ is collapsing,}
\end{cases}
\]  

where \(\mu \sim N(0, \sigma^2)\), \(b_t\) is the relative bubble size, \(r_{t+1}\) is a measure of asset returns, and 1, 2, 3 represent dormant, explosive and collapsing regimes, respectively. Returns in the dormant regime are not affected by the bubble size and abnormal volume since they are equivalent to the fundamental returns. Thus, they can be treated as a constant with some unexpected deviations. However, the returns in both explosive and collapsing regimes are functions of the relative bubble size. Large positive returns will imply a higher probability of being in an explosive regime. Likewise, significantly low returns will imply a higher probability of being in a collapsing regime. Lastly, a high but steady rate of returns
will denote a higher probability of being in a dormant regime. Hence, rather than try to identify the bubble regime directly, it is inferred from the returns equations.

Under the assumption of disturbance normality, the above Markov regime-switching model is estimated by conditional maximum likelihood. The conditional likelihood function for $r_{t+1}$ is given by:

$$lnL(r_{t+1}|r_t, \theta, X_t) = \sum_{t=1}^{T} ln \left[ \sum_{i=1}^{3} \sum_{j=1}^{3} f(r_{t+1}|S_{t+1} = i, S_t = j, r_t, \theta, X_t) P(S_{t+1} = i, S_t = j| r_t, \theta, X_t) \right]$$

where $\theta$ is the vector of model parameters $(A, B, C, \beta, \sigma)$, $X_t$ is a vector of exogenous variables $(b_t, b_t^2)$ and $f$ is the standard normal probability density function for each of the returns in (20). Equation (21) represents a weighted average of the likelihood function in each state with the weights being the state transition probabilities. Given that these probabilities are not directly observable, (21) cannot be estimated directly. In this regard, the Hamilton (1989) filter is applied to obtain the latent transition probabilities. Equation (21) is estimated by maximum likelihood.

2.4 Data

The study uses monthly price data on spot WTI\textsuperscript{12} (WTISPLC) crude oil, gold\textsuperscript{13} (GOLDMGDB228NLBM), and S&P 500\textsuperscript{14} composite Index. The sample spans July 1989 to December 2014 with 306 observations. WTI crude oil price and gold price data are obtained from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. Monthly S&P 500 Index is obtained from Robert Shiller’s website.\textsuperscript{15} Table 1 provides details of all data used in the study with their respective sources. All data are converted into real values using the consumer price Index.

\textsuperscript{12} WTI crude oil was chosen since it serves as the benchmark instrument for oil price.
\textsuperscript{13} Gold is chosen for its attractiveness as a store of value.
\textsuperscript{14} S&P 500 is chosen to represent the U.S stock market due to its broad coverage and representativeness of the market.
\textsuperscript{15}http://www.econ.yale.edu/~shiller/data.htm
(CPI). The sample period encompasses recessions of the 1990s and early 2000s. Table 2 provides descriptive statistics of the asset prices considered in the study. It is observed that during the sample period from 1989 to 2014, the price of gold reached a low of $256.10 per troy ounce and a high of $1,771.80 per troy ounce. The price of oil reached a high of $133.93 per barrel from a low of $11.28 per barrel. The S&P 500 Index reached a high of $2,054.30 from $307.10. It is also observed that WTI crude oil price and gold price are positively skewed, with S&P 500 Index being negatively skewed. In addition, all asset prices exhibit platykurtosis relative to the normal distribution. Ljung-Box test at order 20 for autocorrelation rejects the null hypothesis, suggesting that asset prices are autocorrelated.

Because the goal of this paper is to analyze the market dynamics and interactions among assets going through bubbles (which is a non-stationary behavior), differencing variables to achieve stationarity will be self-defeating. Thus, the Markov regime-switching model is estimated in levels, using real variables.

2.5 Results

Results for the Markov regime-switching model are presented in two parts. Coefficient estimates are presented in table 2 and plots of time-varying transition probabilities are presented in Figures 2.2, 2.3, and 2.4.

Column two of Table 2.3 provides coefficient estimates on the returns equations for WTI crude oil price with corresponding transition probability coefficients. Results show that returns to WTI crude oil price increases by 0.0147% per month (0.1765% effective annual rate\(^\text{16}\)) in the dormant bubble regime \(\beta_{1,0}\), 0.0152% per month (0.1826 effective annual rate) in the explosive bubble regime \(\beta_{2,0}\), and decreases by 0.0043% per month (0.0516% effective annual rate) in the collapsing bubble regime.

\(^{16}\) Calculated using \textit{Effective Annual Rate} = \((1+i)^{12}-1\)
\( \beta_{3,0} \). In an explosive regime, returns from bubbles in WTI crude oil price yield 0.098\% increase in returns \( \beta_{2,b} \). However, returns decrease by 0.0071\% in the collapsing bubbles regime \( \beta_{3,b} \) per month.

Transitions in WTI crude oil price are governed by probabilities reported in the second section of column two, Table 2.3. As expected, a dormant bubble in WTI crude oil price increases the probability of it staying dormant next period \( \alpha_{b,1} \) (WTI). As the bubble becomes larger, the probability of switching from an explosive regime to a dormant regime diminishes \( \alpha_{b,2} \) (WTI). A dormant bubble in gold price \( \alpha_{b,1} \) (GOLD), on the other hand, increases the probability of crude oil price bubble remaining dormant. This confirms the observed positive relationship between the prices of oil and gold as noted by Tully et al. (2007). An explosive bubble in the S&P 500 Index tends to increase the probability of gold price transitioning from a dormant regime to an explosive regime \( \psi_{q,1} \) (S&P). The reason behind this could be that since the S&P 500 Index comprises some energy stocks, as the stock market enters an explosive bubble regime crude oil prices also enter an explosive phase, thus reducing the probability of remaining dormant. Overall, bubbles in crude oil price tend to be impacted by bubbles in gold price and the S&P 500 Index.

Column 3 of Table 2.3 also provides results on the returns equation for gold with the associated transition probability coefficients. Results show that returns on gold experience a mild monthly growth of 0.0315\% per month (0.3787\% effective annual rate) during the dormant bubble regime \( (\beta_{1,0}) \) and 0.0537\% per month (0.6463\% effective annual rate) in the explosive regime \( (\beta_{2,0}) \). However, in the collapsing regime \( (\beta_{3,0}) \), returns on gold decrease by about 0.0233\% per month (0.2792\% effective annual rate). In the explosive bubble regime \( (\beta_{2,b}) \), returns on gold increase by 0.014\% per month (0.1681\% effective annual rate). In the collapsing regime, a decline of 0.0067\% per month is observed in the return on gold.
The dynamics in gold price are governed by time-varying transition probabilities from period $t$ to $t+1$. The coefficients of these probabilities are shown in Table 2.3. In general, not much effect is produced by WTI crude price and S&P 500 Index on the probability of gold price transitioning between regimes. Most of the impact emanates from an explosive growth in gold bubble itself. The exception here is that an explosive bubble growth in the S&P 500 Index increases the probability of gold price bubble transitioning from a dormant regime to an explosive regime $\psi_{b,1}(S&P)$. Results show that gold price is not hugely impacted by the dynamics in other assets. This provides support for why gold is a preferred safe haven asset or a hedge.

In column 3 of Table 2.3, results for bubble dynamics in S&P 500 Index are presented. Results show that in the dormant regime ($\beta_{1,0}$), the S&P 500 Index experiences a monthly average return of 0.0208% (0.2499% effective annual rate) and 0.0654% per month (0.7876% effective annual rate) in the explosive regime ($\beta_{2,0}$). In the collapsing regime ($\beta_{3,0}$), the returns to S&P 50 Index reduce by about 0.0049% per month (0.0588% effective annual rate). The explosive bubble coefficient ($\beta_{2,b}$) contributes about 0.0345% increase to the returns on S&P 500 Index per month whereas in the collapsing regime, the bubble component ($\beta_{3,b}$) leads to a 0.0015% decrease in returns.

Time-varying transition probabilities governing the dynamics in S&P 500 Index are also shown in column three of Table 2.3. A bubble in gold price increases the probability of S&P 500 Index transitioning from a collapsing bubble regime to a dormant bubble regime $\alpha_{b,3}(GOLD)$. On the other hand, bubbles in either WTI crude oil price $\omega_{b,1}(WTI)$ or gold price $\omega_{b,1}(GOLD)$ decrease the probability of S&P 500 Index transitioning from a dormant bubble regime to an explosive bubble regime. As WTI crude oil price bubbles enter an explosive regime $\psi_{b,1}(WTI)$, they tend to have a similar negative effect on bubbles in the S&P 500 Index, but increase the probability of bubbles in the S&P 500 Index remaining in the explosive regime $\psi_{b,2}(WTI)$. However, explosive bubbles in gold have no significant impact on bubbles in the S&P 500 Index transitioning from a dormant regime to
an explosive regime $\psi_{b.1}(\text{GOLD})$. Lastly, the probability of transitioning from a collapsing to an explosive bubble regime in the S&P 500 Index is increased as bubbles in crude oil price $\psi_{b.3}(\text{WTI})$ increase and decrease as gold price bubbles increase $\omega_{b.3}(\text{GOLD})$.

It is observed from the above results that bubbles in the S&P 500 Index do not cause much variation in other assets’ prices. However, crude oil prices seem to be a major driver of bubbles in other assets. This realization supports the idea of Lammerding et al. (2013) that crude oil prices have a far-reaching macroeconomic impact than other asset prices. Thus, it is no surprise that it tends to drive the bubble behavior of gold price and S&P 500 Index. Although gold price bubbles are impacted by bubbles in the S&P 500 price Index, they tend to be more robust to collapses and are the most stable over time among the three assets considered in this study. This points to the universal importance investors attach to gold as a safe haven or hedge given that it is able to withstand shocks from other markets. Results show that there is no significant impact from bubbles in crude oil price at all on gold.

The lower section of Table 2.3 shows volatilities in the three regimes for all three assets. As expected, in all three cases, the least volatility is observed in the dormant regime and the highest is observed in the collapsing regime.

The second set of results borders on plots of smooth transition probabilities obtained from estimating the Markov regime-switching model whose coefficients are reported in Table 2.3. Smooth transition probabilities are shown in Figures 2.2, 2.3, and 2.4 for WTI crude oil price, gold price, and S&P 500 Index, respectively. It is observed from Figure 2.2 that the transition probabilities in the dormant bubble regime capture periods when oil price was relatively stable especially between 1990 and 2000. In this period, very high probabilities of being in a dormant regime are observed, confirming the dormant state of oil prices existing at the time. Beyond that, when oil prices rise as observed during the 2007/2009 recession, the probability of being in a dormant regime is low, close to zero. Thus, the model incorporating the effects of bubble sizes in WTI crude oil price and the S&P 500 Index helps
explain the observed dynamics in crude oil prices between 1989 and 2015. The probability of crude oil price being explosive is also well captured in the second diagram of Figure 2.2. Here, significant periods of oil price hikes such as 1989/1990, 1999/2000, and 2008 when oil price reached its all-time high are well captured by the high transition probabilities. It is worth noting that these periods also coincide with observed recession dates. In the collapsing bubble regime, high transition probabilities are observed during the crude oil price collapse between 1985 and 1987, 2008/2009, and as recent as 2014 to 2016. Besides these periods of significantly large declines in crude oil price, mild declines are observed in periods such as 1989 and 1998/1999. Overall, the model performs well when the effects of S&P 500 Index and gold price bubbles are accounted for.

Figure 2.3 looks at the transition probabilities for gold price bubbles. Transition probabilities in the dormant regime are highly persistent denoting the extended periods of stability in gold price especially over the 1990/2001 period. Beyond 2001 gold prices began to rise. Thus, the probability of being in a dormant regime is low at this point. Periods of price increases are interspersed with brief periods of collapsing prices. Explosive behavior in gold price is observed during the 2007/2009 recession and immediately after, until the price collapsed from 2013 onward. Finally, in the collapsing regime of gold price bubbles, the high transition probabilities capture periods of collapse such as those during the 2001 recession, 2007/2009 recession period, and the 2013/2016 period.

Time-varying transition probabilities depict large but clustered variations in the S&P 500 Index as shown in Figure 2.4. In both the dormant and explosive regimes, high transition probabilities correctly capture a mix of these two regimes. The S&P 500 Index starts to rise after the 1990/1991 recession, captured by the high transition probabilities in the graph for the explosive regime until 1999/2000, when it begins to collapse. In periods when a collapse is observed such as in 2000/2001 and 2007, collapsing regime probabilities are high, as expected. Thus, the model that captures the
The effect of WTI crude oil price bubbles and gold price bubbles on S&P 500 Index dynamics is able to explain the observed volatilities in the S&P 500 Index over the sample.

2.6 Conclusion

This study examined the interaction between various assets to understand the effect of bubble propagation from one market to another, as well as analyzing which assets matter most in driving bubbles in other assets’ prices. Results show that bubbles in crude oil price are influenced by the bubble sizes in S&P 500 Index, gold price, and crude oil price itself. For gold, the dynamics in the three regimes are driven by bubble sizes in S&P 500 Index and gold price itself. Lastly, S&P 500 Index tends to be driven jointly by bubbles in all assets, including the growth of bubbles in S&P 500 Index itself.

The findings of this paper suggest that gold is the least affected asset from bubbles in other markets. Given the impact of crude oil price on the stock market, it is important for policy makers to prioritize the implementation of policies that will stabilize crude oil prices in crisis periods to halt their negative impact on the stock market. By so doing, recovery of the stock market from a crash will be faster than it would be if policy makers implemented policies that neglected the underlying source of the crisis.

Lastly, investors should use gold hedges to protect their portfolios from price downturns. This is because gold price has proven to be more stable even when the rest of the market is not doing well. Using gold hedges will potentially help minimize losses to improve profitability. The stability in gold price also provides a case for gold serving as a safe haven asset in times of crisis.
REFERENCES


APPENDIX

Table 2.1: Summary of model variables. Sample period is from June 1989 to December 2014.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot price of WTI crude oil</td>
<td>FRED</td>
</tr>
<tr>
<td>Spot price of gold</td>
<td>FRED</td>
</tr>
<tr>
<td>S&amp;P 500 Index</td>
<td>Robert Shiller’s website</td>
</tr>
<tr>
<td>S&amp;P 500 dividends</td>
<td>Robert Shiller’s website</td>
</tr>
<tr>
<td>Convenience yield for gold*</td>
<td>Own calculation</td>
</tr>
<tr>
<td>Convenience yield for crude oil*</td>
<td>Own calculation</td>
</tr>
<tr>
<td>Crude oil futures price</td>
<td>Investing.com</td>
</tr>
<tr>
<td>Gold futures price</td>
<td>Investing.com</td>
</tr>
<tr>
<td>3-month Treasury Bill Rate</td>
<td>FRED</td>
</tr>
</tbody>
</table>

*Authors’ own calculation. FRED denotes Federal Reserve Economic Data, St. Louis FED.

Table 2.2: Descriptive Statistics for Asset Prices from June 1989 to December 2014

<table>
<thead>
<tr>
<th></th>
<th>WTI CRUDE</th>
<th>GOLD</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>11.28</td>
<td>256.10</td>
<td>307.10</td>
</tr>
<tr>
<td>Maximum</td>
<td>133.93</td>
<td>1771.8</td>
<td>2054.30</td>
</tr>
<tr>
<td>Median</td>
<td>29.58</td>
<td>386.00</td>
<td>1108.40</td>
</tr>
<tr>
<td>Mean</td>
<td>46.11</td>
<td>629.00</td>
<td>1014.00</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>29.79</td>
<td>428.38</td>
<td>547.49</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.79</td>
<td>1.26</td>
<td>-0.04</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.23</td>
<td>3.16</td>
<td>2.20</td>
</tr>
<tr>
<td>Q(20)</td>
<td>4449.40</td>
<td>5344.20</td>
<td>4101.80</td>
</tr>
</tbody>
</table>

*J-B refers to Jarque-Bera statistic, Q(20) refers to Ljung-Box statistic at order 20*
Table 2.3: Results of the three-regime speculative bubble model with time-varying transition probabilities

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>WTI Crude</th>
<th>GOLD</th>
<th>S&amp;P 500 Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{1,0}$</td>
<td>0.0147* (0.0051)</td>
<td>0.0315* (0.0014)</td>
<td>0.0208* (0.0042)</td>
</tr>
<tr>
<td>$\beta_{2,0}$</td>
<td>0.0152* (0.0049)</td>
<td>0.0537* (0.0060)</td>
<td>0.0654* (0.0014)</td>
</tr>
<tr>
<td>$\beta_{3,0}$</td>
<td>-0.0043* (0.0012)</td>
<td>-0.0233* (0.0061)</td>
<td>-0.0049* (0.0020)</td>
</tr>
<tr>
<td>$\beta_{2,b}$</td>
<td>0.0980* (0.0047)</td>
<td>0.0140* (0.0052)</td>
<td>0.0345* (0.0061)</td>
</tr>
<tr>
<td>$\beta_{3,b}$</td>
<td>0.0071* (0.0005)</td>
<td>-0.0067* (0.0005)</td>
<td>-0.0015* (0.0004)</td>
</tr>
<tr>
<td>$\omega_{0,1}$</td>
<td>-0.1684 (0.0152)</td>
<td>72.3641 (60.7803)</td>
<td>-22.4143 (18.4301)</td>
</tr>
<tr>
<td>$\omega_{1,1}(WTI)$</td>
<td>0.0548* (0.0036)</td>
<td>-0.4639 (0.5000)</td>
<td>120.7045 (80.7505)</td>
</tr>
<tr>
<td>$\omega_{1,1}(GOLD)$</td>
<td>5.0001* (1.7520)</td>
<td>-79.0388 (66.8144)</td>
<td>19.7458 (10.8542)</td>
</tr>
<tr>
<td>$\omega_{1,1}(S&amp;P)$</td>
<td>-</td>
<td>-15.7284 (14.5255)</td>
<td>-14.4164 (11.1005)</td>
</tr>
<tr>
<td>$\omega_{0,2}$</td>
<td>-6.5619 (7.2631)</td>
<td>0.3048 (0.6665)</td>
<td>0.8692* (0.0012)</td>
</tr>
<tr>
<td>$\omega_{2,2}(WTI)$</td>
<td>4.2065* (1.8461)</td>
<td>-2.0509 (8.4727)</td>
<td>-16.6530 (15.8905)</td>
</tr>
<tr>
<td>$\omega_{2,2}(GOLD)$</td>
<td>1.4801 (0.9820)</td>
<td>-1.4485* (0.6107)</td>
<td>-20.7753 (18.2015)</td>
</tr>
<tr>
<td>$\omega_{2,2}(S&amp;P)$</td>
<td>-</td>
<td>1.4485 (0.9162)</td>
<td>18.3793 (15.9706)</td>
</tr>
<tr>
<td>$\omega_{0,3}$</td>
<td>-18.4834 (10.5003)</td>
<td>-2.9320* (1.4658)</td>
<td>0.8506 (1.9568)</td>
</tr>
<tr>
<td>$\omega_{3,3}(WTI)$</td>
<td>-7.7619* (3.4260)</td>
<td>0.5784 (8.1157)</td>
<td>-13.5357 (10.8920)</td>
</tr>
<tr>
<td>$\omega_{3,3}(GOLD)$</td>
<td>-3.4983 (0.9597)</td>
<td>0.3034 (1.1043)</td>
<td>0.1385* (0.0527)</td>
</tr>
<tr>
<td>$\omega_{3,3}(S&amp;P)$</td>
<td>-</td>
<td>-1.2654 (1.5938)</td>
<td>1.8017* (0.7004)</td>
</tr>
</tbody>
</table>

All variables are in logarithms. Standard errors in bracket. * represents 5% level of significance.
Figure 2.1: Trend of Price Series for Gold, WTI Crude, and S&P 500 from June 1989 to December 2014
Figure 2.2: Smoothed Transition Probabilities of Dormant, Explosive, and Collapsing Bubble Regimes for WTI Crude Oil Price

Figure 2.3: Smoothed Transition Probabilities of Dormant, Explosive, and Collapsing Bubble Regimes for Gold Price
Figure 2.4: Smoothed Transition Probabilities of Dormant, Explosive, and Collapsing Bubble Regimes for S&P 500 Index
CHAPTER 3

MONETARY POLICY AND HOUSING BUBBLES

3.1 Introduction

The existence of housing bubbles became topical during the 2007-2009 financial crisis. The 2007-2009 financial crisis has been blamed on high house prices starting in the early 2000s. Although the effects of the bubbles in the U.S. housing market were felt nationally and globally, the bubbles themselves did not occur throughout the country. Bubbles in the housing market can best be described as ‘localized’ given that they were largely concentrated in major urban areas in some specific states. An understanding of the financial crisis depends largely on a detailed study of the factors that led to the housing bubbles during the period. To what extent did Federal Reserve interest rate cuts contribute to the housing bubble? This is the main question of interest that is addressed in this study.

An important factor attributed to causing or exacerbating the housing bubble is the direction of Federal Reserve monetary policy that maintained low interest rates [see McDonald et al. (2013b), Shiller (2009), Taylor (2007)], especially during 2001-2004. In this period, the Federal Reserve cut the Federal funds rate from 6.50% to 1.75% during the 2001 recession, and further reduced it to 1% by June of 2003. House prices experienced significant growth during the period until 2006.

Figure 3.1 shows a plot of Federal funds rates from 1998 to 2008. It is observed that the Federal Reserve consistently maintained a low Federal funds rate in each of the recession periods, in particular, from 2001-2004. House prices reacted to these rate cuts with high growth until 2006.
when the housing market started showing signs of distress. It can be argued that policy makers did not take into account the lagged impact of monetary policy on house prices. The sudden rates increase between 2004 and 2006 may have had the opposite effect on house prices as would rates cut. The Great Recession followed.

Taylor (2007) argues that rather than setting low rates, a higher Federal funds rate would have prevented much of the boom, and the eventual collapse would not have been so acute. The low interest rates continued to exist in periods leading up to and during the crisis until they hit the zero lower bound (ZLB) in December 2008. Bryant et al. (2013) suggest that the Federal Reserve’s expansionary monetary policy at the time had no role in the housing bubble and its eventual collapse. The exact role of the Federal Reserve in the housing bubble and thus the Great Recession therefore remains an issue of intense debate.

House prices increased sharply in some U.S. states in the period leading up to 2006. Mention can be made of the high growth in house prices in California, Hawaii, Maryland, Massachusetts, New Jersey, New York, Rhode Island, and Washington, DC. These dramatic price surges were concentrated largely in areas in the East and West coasts of the country. Former Federal Reserve Chairman, Alan Greenspan terms this as a collection of “local bubbles.” Given the localization of these bubbles, it is possible for some state-specific factors to have contributed to their existence in the face of monetary policy changes. Del Negro et al. (2007) argue that some states are affected more by common cycles than others. Thus, a monetary policy change may affect states differently. They continue to argue that for many states that experienced large house price growth, a substantial share of this growth is attributable to national factors.17

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17 These national factors include monetary policy.
McDonald et al. (2015) posit that the cause of the housing bubble can be attributed to the unsound lending practices of financial institutions as well as large capital inflows resulting from increased trade deficit. In addition, they find evidence that adjustable mortgage rates and Federal funds rate affect house prices. According to Miles (2009), growth in the secondary mortgage market led to the housing market commanding a larger role in the economy than in the past. Miles (2014) also finds evidence of long-term mortgage rates having a high predictive power on house prices. Greenspan (2009), on the other hand, notes that the interest rate that matters for the housing bubble is a long-term, fixed mortgage rate. Payne (2006) points out that there is no relationship between fixed mortgage rates and Federal funds rate in the short run. In the long run, however, a unidirectional relationship exists in which fixed mortgage rates adjust to changes in the Federal funds rate asymmetrically.\textsuperscript{18} If so, then Federal funds rates are expected to have an effect on house prices through long-term fixed mortgage rates.

Although, Federal government fiscal policies ensured this growth by making it easy for people to own homes\textsuperscript{19}, little was done to shield the macroeconomy from negative shocks to the housing market. These policies augmented existing ones aimed at increasing home ownership such as the deductibility of mortgage interest payments from tax payments, the channeling of funds from capital markets into housing by the Federal Home Loan Bank System (FHLBS) and Federal National Mortgage Association (NFMA) [see Sellon (1990)]. In fact, inadvertently augmenting the effects of the fiscal policies, the Federal Reserve conducted expansionary monetary policies

\textsuperscript{18} This result is obtained from an asymmetric error correction model (AECM) that explores the dynamics between fixed mortgage rates and the Federal funds rate. The AECM allows negative and positive shocks to have different impacts.

\textsuperscript{19} The New York times reported that the housing policies of President George W. Bush and his hands-off approach to regulation encouraged easy credit behavior by financial institutions.

which further ensured growth in the housing market as more investments were made by homeowners in house purchases.

Figure 3.2 shows house price indexes from 1975 to 2016, using 1980 as the base period. The sample comprises seven U.S states (California, Hawaii, Maryland, Massachusetts, New Jersey, New York, Rhode Island) and Washington, DC as shown in Table 3.1, whose house price indexes exceeded the national average by about 2 standard deviations from 1998 to 2008. As noted by Del Negro et al. (2005), house prices for many U.S states were very noisy before the mid-eighties, with sharp fluctuations. Prices became less noisy starting from the mid-eighties. It is observed that house prices for all seven U.S states and Washington, DC began rising in 1997 and peaked around 2006. Beyond 2006, house prices went through an extended period of collapse, reaching a trough in 2011.

Given the observed relationships between Federal Reserve policy and house price movements across the seven U.S states and Washington, DC, this study analyzes the role of monetary policy in driving house prices. Two main contributions are made to the existing literature. First, the study uses state-level data as opposed to previous studies [see McDonald et al. (2015), Bryant et al. (2009)] that mostly relied on national-level housing data. Having been suggested that the origin of the housing bubble was local rather than national, these studies tend to produce biased results on the impact of monetary policy on house prices when national level data is used. Thus, a more appropriate remedy is to use state-level housing data to analyze the role of monetary policy in the housing bubble. Second, a mean group panel vector autoregressive (VAR) model in the spirit of Gambacorta et al. (2014) is estimated using state-specific variables and the Federal funds rates.
This methodology is appropriate when faced with a short time-series as used in this study. To increase the efficiency of the VAR results, the analysis is done over several cross-sections involving seven U.S states (California, Hawaii, Maryland, Massachusetts, New Jersey, New York, Rhode Island) and Washington, DC that experienced house price bubbles over the sample period. As noted by Canova et al. (2013), panel VAR methods are more suitable for analyzing the effect of idiosyncratic shocks across different units and time. The model allows for cross-sectional heterogeneity in slope parameters to account for the possibility that different states respond to monetary policy differently, as argued by Del Negro et al. (2007). The mean group panel VAR provides a more efficient way of analyzing the impact of monetary policy on house price bubbles across seven states in the U.S and Washington, DC between 1998 and 2008.

3.2 Extracting Bubble Components of House Prices

To extract the bubble components of asset prices the present value formula is used such that

\[ P_t = (1 + R)^{-1} E_t (P_{t+1} + C_{t+1}) , \]  

(1)

where \( P_t \) represents house price at time \( t \), \( R \) is a constant rate of discount, \( C_{t+1} \) is the total rental cost at time \( t+1 \). Transversality is assumed after recursive substitution to yield a particular solution to (1), given by

\[ P_t^f = \sum_{k=1}^{\infty} (1 + R)^{-k} E_t C_{t+k} , \]  

(2)

where \( P_t^f \) denotes the fundamental price of a house. Besides \( P_t^f \), the solution to (1) may contain another component \( P_t^f \). Let \( B_t = P_t - P_t^f \). If

\[ B_t = (1 + R)^{-1} E_t B_{t+1} , \]  

(3)
then \( P_t = P_t^f + B_t \) is also a solution to (1). The component \( B_t \) is called a bubble.

Various approaches have been used in the literature to estimate the fundamental price of an asset from (2) [including Evans (1991); Campbell and Shiller (1987)]. In this study, a variant of the VAR approach of Campbell and Shiller (1987) is used. In line with their original proposition, it is argued that the spread \( (S_t) \) between house prices and a constant multiple of current rental costs \( (C_t) \) is the optimal forecast of a multiple of the discounted value of all future changes in rental cost.

\[
S_t \equiv P_t^f - \frac{1+R}{R} C_t = \frac{1+R}{R} \left[ \sum_{i=1}^{\infty} \frac{1}{(1+R)^i} E_t(\Delta C_{t+i}) \right],
\]

(4)

where \( \Delta C_{t+i} \) is the change in rental cost at time \( t+i \).

The VAR methodology makes it possible to examine whether changes in rental costs can be forecast by the spread between prices and the multiple of current rental cost. If home owners include all relevant past information in their information set, this should be reflected in the spread which will make it a good predictor of changes in rental costs. To investigate this, the following VAR model is specified

\[
\begin{bmatrix}
\Delta C_t \\
S_t
\end{bmatrix}
= \begin{bmatrix}
a(L) & b(L) \\
c(L) & d(L)
\end{bmatrix}
\begin{bmatrix}
\Delta C_{t-1} \\
S_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\mu_{1t} \\
\mu_{2t}
\end{bmatrix},
\]

(5)

where \( a(L) \), \( b(L) \), \( c(L) \), and \( d(L) \) are polynomials in lag operator \( L \), \( \Delta C_{t-1} \) and \( S_{t-1} \) are the one-period lags of changes in rental cost and spread, respectively, and \( \mu_t \) is the residual term. If \( b(L) \) is zero, then changes in rental costs cannot be forecast by the spread. Let \( \bar{C}_t \) represent the VAR forecast of changes in rental costs. Then,

\[
P_t^f = \bar{C}_t + \frac{1+R}{R} C_t.
\]

(6)

Having obtained the fundamental price of houses, the bubble size \( B_t \) is then calculated as

\[
B_t = P_t - P_t^f.
\]
3.3 Panel VAR Estimation

A mean group panel VAR approach, following Gambacorta et al. (2014), is used to analyze the impact of monetary policy on house price bubbles across seven U.S states (California, Hawaii, Maryland, Massachusetts, New Jersey, New York, Rhode Island) and Washington, DC between 1998 and 2008. Four variables (house price bubble, unemployment rate, 30-year fixed mortgage rate, and Federal funds rate) are considered for each state $i$ in the benchmark model. The benchmark panel VAR specification is then given as

$$Y_{i,t} = A_{0,i} + A_i(L)Y_{i,t-1} + \mu_{it} \quad i = 1, \ldots, N \quad t = i, \ldots, T$$

where in the benchmark model $Y_{it}$ is a $4 \times 1$ vector of endogenous variables, $A_{0,i}$ is a vector of constants, $A_i(L)$ is a matrix polynomial in lag operator $L$, $\mu_{it}$ is a vector of $4 \times 1$ random disturbances correlated across states, and $i$ represents the individual states. $Y_{i,t-1}$ represents the dynamic interdependencies among the endogenous variables across the states. Cross-sectional heterogeneity in slope parameters ($A_i \neq A_j$) is allowed to account for the fact that house prices in different states may respond to monetary policy differently. This specification is analogous to treating the states as though they were countries in a monetary union, faced with a common monetary policy. As Canova et al. (2013) argue, in such a situation it may be more important to account for slope heterogeneity.

The use of panel VAR estimation technique provides more accurate estimates relative to a state-by-state estimation due to the cross-sectional heterogeneity that is accounted for. The mean group estimator, originally developed by Pesaran and Smith (1995), makes it possible to account for cross-state heterogeneity with regard to monetary policy response. The cross-state heterogeneity results from individual state characteristics that account for the state-specific
response to a common monetary policy shock. Thus, as rightly noted by Gambacorta et al. (2014), the economic structures and dynamics of the states are not required to be the same. The mean group panel VAR first estimates individual cross-sectional reduced-form VAR models and then averages them across states. Impulse response functions are also obtained in an analogous manner.

The study uses both zero and sign restrictions on the variance-covariance matrix to identify exogenous shocks in the panel VAR model. The zero restrictions have the advantage of isolating the impact of monetary policy shocks on house price bubbles from shocks coming from other model variables. This helps to obtain the impact emanating from Federal Reserve monetary policy without diminishing its true impact on housing bubbles. Consequently, zero restrictions provide the additional advantage of reducing the number of impulse response functions needed to explain the model results. The advantage of using sign restrictions is that they help avoid abnormal interactions among economic variables.

The following zero restrictions are implemented in the estimation process. Federal Reserve monetary policy shocks are assumed to have only a lagged impact on house price bubbles and output. In turn, house price bubbles have only a lagged impact on the 30-year fixed mortgage rates and output, but a contemporaneous impact on the Federal funds rate. It is assumed that 30-year fixed mortgage rates do not have a contemporaneous impact on Federal funds rate. On the other hand, Federal funds rates are assumed to have a contemporaneous impact on 30-year fixed mortgage rates.

Two sign restrictions are used in this study. First, an expansionary monetary policy shock leads to a decrease in 30-year fixed mortgage rates. This leads to increased demand for mortgage

---

20 This is the zero restriction that differentiates the identification scheme used here from a Choleski decomposition.
loans and subsequently house purchases begin to rise. With increased demand for houses, prices eventually rise as long as policy direction remains the same. In such a situation, as argued by McDonald et al. (2015), financial institutions engage in unsound lending practices which lead to house price bubbles. Therefore, an expansionary monetary policy increases house prices and thus house price bubbles with a lag. Reductions in the Federal funds rate first affect mortgage rates and then house prices in the benchmark model. It is not surprising that policy makers failed to recognize the immediate impact of continuous reductions in the funds rate on house prices in the early 2000’s. Second, an expansionary monetary policy shock is assumed to lead to an increase in house price bubbles. This restriction follows from the first sign restriction above. This helps limit policy outcome to only those that satisfy the above sign restrictions. The above sign restrictions are imposed on impact of the monetary policy shock. Several possible impulse responses are then drawn and those that do not satisfy the sign restrictions are discarded. Equation (8) below provides details of the identification assumptions used in the study.

\[
\begin{align*}
hpb_i &= e_{1i}unemprte + e_{2i}fmtg30 + \mu_{1i} \\
unemprte_i &= \mu_{2i} \\
fmtg30_i &= e_{3i}unemprte + e_{4i}ffr + \mu_{3i} \\
ffri &= e_{5i}unemprte + e_{6i}hpb + \mu_{4i}
\end{align*}
\]  

(8)

where small letters denote reduced-form shocks of model variables.

Estimation of the benchmark mean group panel VAR model is done for seven U.S states (California, Hawaii, Maryland, Massachusetts, New Jersey, New York, Rhode Island) and Washington, DC from 1998 to 2008. An infinite number of orthogonal impact matrices can be obtained for a one standard deviation shock to Federal funds rate. Impulse response functions that do not satisfy the sign restrictions for all states simultaneously are discarded. This is repeated to
obtain 1,000 mean group impulse response functions via bootstrapping. The 16\textsuperscript{th} and 84\textsuperscript{th} percentiles of state-specific impulse response functions from individual state VAR models are also reported. The 16\textsuperscript{th} and 84\textsuperscript{th} percentiles thus represent the range of all possible state-specific impulse response functions as the model varies with different orthogonalizations of the impact matrix. As noted by Fry et al. (2011), these percentiles are not quite the same as confidence intervals (because the distribution is across models, and thus represent model uncertainty, not sampling uncertainty), so referring to the range as confidence intervals is quite false. This study uses the median of these impulse response functions to explain the impact of a monetary policy shock on other model variables.

To ascertain the robustness of the benchmark specification, two alternative variables are considered; including 1-year adjustable mortgage rates and real estate loans by all commercial banks in the U.S.

3.4 Data

Data for the panel comprises a cross-section of seven U.S states and Washington, DC ($N=8$) as shown in Table 3.1, spanning 1998:Q1 to 2008:Q4 with $T=44$. The sample is chosen from 1998 due to data availability and ends in 2008 to acknowledge the zero lower bound in Federal funds rate which is the monetary policy variable in this study. The cross-sections selected are those that consistently had their house price indexes exceed the national average by about 2 standard deviations from 1998 to 2008. The number of cross-sections selected is also constrained by the requirement that for a mean group panel VAR estimation, $T$ should be ‘sufficiently’\textsuperscript{21} larger

\textsuperscript{21} Although there is no consensus on how large T should be, it should be large enough to reduce any estimation bias.
than $N$. So, with $T$ constrained between 1998 and 2008 inclusive, $N$ is kept as small as possible, while maintaining the integrity of the model results to convey the key idea of the study.

Data used in the study comes in two parts. First, the benchmark model uses quarterly data on state-level house price bubbles, state-level gross unemployment rate, 30-year fixed mortgage rates, and Federal funds rate. House price bubbles are obtained according to the procedure in section 2 using information on state-level house price index and rental cost. Rental cost is measured with imputed rent which is available in annual frequency from the Bureau of Economic Analysis. Annual imputed rent is converted to quarterly frequency using quadratic interpolation. The house price index is obtained from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. State-level unemployment rate is used as a proxy for the level of economic activity. Figure 3.3 shows a plot of the average values of all four model variables from 1998 to 2008 across all seven U.S states and Washington, DC. It is observed that the unemployment rate and house price bubbles are positively correlated throughout the sample period. Similarly, Federal funds rate and 30-year fixed mortgage rate move in the same direction. The Federal funds rate and house price bubbles appear to move in opposite directions, especially between 2000 and 2006. The relationship between the unemployment rate and the Federal funds rates is not so clear from the graph given that unemployment rates have remained relatively stable with Federal funds rate experiencing large volatilities over the same sample period.

Second, two main robustness analyses are undertaken. Quarterly data on 1-year adjustable mortgage rates is used in place of 30-year fixed mortgage rates in the benchmark model. This change is motivated by suggestions in the literature regarding the impact of adjustable mortgage rates on house price bubbles. Similarly, total real estate loans by all commercial banks in the U.S are used in place of 30-year fixed mortgage rates in the benchmark model to capture the behavior
of banks during the Federal reserve policy rate cuts and accompanying house price bubbles, prior to the Great Recession. This change is done to capture the direct role of commercial banks in house price bubbles following monetary policy shocks. Potentially, this provides an alternative explanation of how monetary policy shocks impact house prices via the credit channel. The role of commercial banks in the crisis period mainly centers around their unsound lending behavior as argued by McDonald et al. (2015).

House price bubbles are extracted from house price index as explained in section 2. The house price index is measured as ‘all-transactions house price index’ obtained from FRED. Table 3.2 provides information on variables used in the study, their definitions, and sources of data. Model variables, except Federal funds rate and mortgage rates, are seasonally adjusted and in real values. The conversion is done using the consumer price Index (CPI) for all urban consumers, excluding food and energy. Acknowledging that some states are producers of various forms of energy products such as crude oil, natural gas, and electricity, their expenditures on energy will vary considerably from other non-energy producing states. Not accounting for this will result in biased estimates. The CPI excluding food and energy has less variation and is therefore more ideal in the present study to avoid the bias. Additionally, logarithmic transformations of house price bubbles and real estate loans are done. Because the goal of the study is to analyze house price bubbles, which is a non-stationary phenomenon, variables are used in their levels without differencing to achieve stationarity.

3.5 Results of the Panel VAR Model

In this section, results of the mean group panel VAR model are presented. Impulse responses from an expansionary monetary policy shock (represented by a decrease in Federal funds
rate) are presented in Figures 3.4 and 3.5 for the benchmark model. The 16th and 84th bootstrap percentile intervals represent the range of all possible state-specific impulse response functions from individual state VAR models based on different orthogonalizations of the impact matrix. Figure 3.4 gives results of the mean group impulse response functions from the panel VAR model involving all seven U.S states and Washington, DC that are consistent with the sign restrictions. A 10 basis-point (bp) reduction in the Federal funds rate causes 30-year fixed mortgage rates to fall to about 20bp and rises gradually after that, as shown in Figure 3.4.

The fall in fixed mortgage rates makes mortgage loans attractive to potential homeowners. In response, banks increase their lending activities for mortgages. This leads to increased demand for homes, causing house prices to increase. The role of banks’ lending practices in house price bubbles as argued by McDonald et al. (2015) is seen from the above. House price bubbles peak after about 6 quarters and decline thereafter. The response of unemployment rate to a decrease in Federal funds rate is unexpected. The unemployment rate increases in response to an expansionary monetary policy shock, peaking at 15bp after 5 quarters. Figure 3.3 shows time-series plots of the Federal funds rates, unemployment rates, 30-year fixed mortgage rates, and house price bubbles. It is observed that decreasing Federal funds rates between 2000 and 2005 coincided with relatively stable unemployment rates during the period.

The impact of an expansionary monetary policy shock on house price bubbles appears to have the largest magnitude compared to other model variables. Explosive bubbles in house prices last for 6 quarters for every 10 basis-points decrease in the Federal funds rate. Results show the unintended consequence of Federal Reserve expansionary monetary policy action on house prices.
The 16th and 84th percentile impulse response functions also provide an avenue for policy analysis. They give information on the minimum and maximum possible responses of model variables to a monetary policy shock.

Figure 3.5 shows results of individual state impulse response functions based on the individual VAR models. Overall, states exhibit similar directional responses to an expansionary monetary policy shock, with very little differences in magnitude. The two outer solid bands (blue lines) represent 16th and 84th percentiles of the range of all possible state-specific impulse response functions obtained from different orthogonalizations of the impact matrix. The middle solid line (black line) represents the median point estimate for a response to an expansionary monetary policy shock. The shaded area (green region) shows the mean group impulse response function produced from the panel VAR model and is consistent with the sign restrictions. It provides a means of comparing how well the individual state VAR models fit the data, in explaining the response of model variables to a monetary policy shock. It is observed that the results of the mean group impulse response functions mostly match the individual state impulse response functions. The mean group impulse response functions mostly lie within the 16th and 84th percentile range of individual state impulse response functions.

Contrary to arguments in the literature [such as Bryan et al. (2013)] that the Federal Reserve had no role in the housing bubbles between 2000 and 2006, the results in this study show otherwise.

3.6 Robustness Analyses

In this section, results of robustness analyses conducted with 30-year fixed mortgage rate replaced with 1-year adjustable mortgage rate and real estate loans are presented. Because Federal Reserve monetary policy shocks have only an indirect impact on the housing market through the
mortgage lending rate, it is important to verify the type of mortgage rate that has a larger impact on house prices and house price bubbles. The use of 1-year adjustable mortgage rate is necessitated by some arguments in the literature [such as McDonald et al. (2015)] that suggest that adjustable mortgage rates have an impact on house prices. On the other hand, Bryant et al. (2013) argue that 1-year adjustable mortgage rates had no significant impact on house price bubbles during the Great Recession. The use of 1-year adjustable mortgage rate provides another perspective on the interest rate channel of monetary policy propagation. Real estate loans serve as the credit channel for analyzing the impact of monetary policy shock on house price bubbles.

Figure 3.6 shows the results for an expansionary monetary policy shock and its impact on other model variables. As noted, the effect on house price bubbles is transmitted through 1-year adjustable mortgage rates. It is seen that, following a shock to the Federal funds rate, 1-year adjustable mortgage rates initially decrease and rise gradually, peaking after 16 quarters. This pattern follows closely the behavior of Federal funds rate itself. Similarly, house price bubbles are also impacted by a monetary policy shock. Here, house price bubbles initially increase and then begins to decrease after peaking in 6 quarters. Although the initial response of house price bubbles to the shock is the same as the case where 30-year fixed mortgage rate is used as the transmission channel, the duration of positive impact in the two cases are different. The positive impact lasts longer by 1 quarter in the case of adjustable mortgage rates. This is expected as lenders are free to vary contractually adjustable mortgage lending rates whenever economic conditions change. In general, the main conclusions from the impact of a monetary policy shock on house price bubbles remain robust based on the two transmission channels, with adjustable mortgage rates leading to a longer positive impact by an extra quarter.
Figure 3.7 shows results associated with the role of real estate loans in causing the housing price bubbles. It is observed that following an expansionary monetary policy shock, real estate loans by commercial banks in the U.S immediately increase and remain positive well over 40 quarters. This outcome corroborates the argument by McDonald et al. (2015) regarding the lending behavior of banks towards real estate in periods when the Federal funds rate declined. Following this channel, a monetary policy shock leads to an increase in house prices, peaking after 5 quarters. Relative to the interest rate channel, the credit channel produces a shorter duration of impact on house prices following a monetary policy shock.

In general, the above robustness analyses show that an expansionary monetary policy shock eventually leads to several periods of house price bubbles.

### 3.7 Conclusion

This research sought to investigate the role of Federal Reserve monetary policy on house price bubbles between 2000 and 2006 and their subsequent collapse. Opposing arguments in the literature became more paramount in the wake of the housing bubble collapse and ensuing Great Recession. The above analyses provide evidence on the role of Federal Reserve monetary policy on house price bubbles. However, the magnitude of impact is somewhat affected by the transmission channel of a monetary policy shock to house prices.

The above results show that a monetary policy shock transmitted through 1-year adjustable mortgage rates have a longer duration of impact on house price bubbles. Real estate loans yield a shorter duration of impact on house price bubbles from a monetary policy shock. This result is due to the flexibility of adjustable rate mortgage contracts based on the performance of other macroeconomic variables. These differences notwithstanding, the main conclusion here is that the
Federal Reserve played a role in the housing bubble between 2000 and 2006 as well as its eventual collapse, resulting from its monetary policy stance over the period.
REFERENCES


APPENDIX

Figure 3.1: Federal Funds Rate and NBER Recession Periods from 1998 to 2008

Figure 3.2: House Price Index (1980=100) for seven U.S States and Washington, DC from 1975 to 2016
Figure 3.3: Average values of model variables across all seven U.S states and Washington, DC from 1998 to 2008

Figure 3.4: Mean Group Impulse Response Functions for a One-standard Deviation shock to Federal Funds Rate. 1000 mean group impulse response functions produced from the mean group panel VAR model and consistent with the sign restrictions are produced and the 16th and 84th bootstrap percentile intervals are reported above. Horizons are in quarters.
Figure 3.5: State Level Impulse Response Functions for a One Standard-deviation Shock to Federal Funds Rate. The two outer solid bands (blue lines) represent 16th and 84th percentiles of the range of all possible state-specific impulse response functions obtained from individual state VAR models based on different orthogonalizations of the impact matrix. The middle solid line (black line) represents the median impulse response function from the 16th and 84th percentiles. The shaded area (green region) shows the 16th and 84th bootstrap percentile intervals of the mean group impulse response functions produced from the mean group panel VAR model and consistent with the sign restrictions. Horizons are in quarters.

Figure 3.6: Mean Group Impulse Response Functions for a One-standard Deviation shock to Federal Funds Rate (model with 1-year adjustable mortgage rates). 1000 mean group impulse response functions produced from the mean group panel VAR model and consistent with the sign restrictions are produced and the 16th and 84th bootstrap percentile intervals are reported above. Horizons are in quarters.
Figure 3.7: Mean Group Impulse Response Functions for a One-standard Deviation shock to Federal Funds Rate (model with real estate loans). 1000 mean group impulse response functions produced from the mean group panel VAR model and consistent with the sign restrictions are produced and the 16th and 84th bootstrap percentile intervals are reported above. Horizons are in quarters.
Table 3.1: List of U.S States with High House Prices between 1998 and 2008 used in this study

<table>
<thead>
<tr>
<th>State</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>Maryland</td>
</tr>
<tr>
<td>Washington, DC*</td>
<td>New Jersey</td>
</tr>
<tr>
<td>Hawaii</td>
<td>New York</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>Rhode Island</td>
</tr>
</tbody>
</table>

*Washington, DC is not a state, but included in the analysis due to the severity of house price bubbles it experienced.

Table 3.2: Definitions of Model Variables and Sources of Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPB</td>
<td>House Price Bubble</td>
<td>Own Calculation</td>
</tr>
<tr>
<td>RNTCSTQ</td>
<td>Quarterly Rental Cost</td>
<td>Own Calculation</td>
</tr>
<tr>
<td>UNEMPRTE</td>
<td>Unemployment Rate</td>
<td>FRED</td>
</tr>
<tr>
<td>FFR</td>
<td>Federal Funds Rate</td>
<td>FRED</td>
</tr>
<tr>
<td>FRMTG30</td>
<td>Fixed Rate Mortgage (30-year)</td>
<td>FRED</td>
</tr>
<tr>
<td>AMRTG1</td>
<td>Adjustable Rate Mortgage (1-year)</td>
<td>FRED</td>
</tr>
<tr>
<td>RELNS</td>
<td>Real Estate Loans</td>
<td>FRED</td>
</tr>
</tbody>
</table>

‘FRED’ denotes the United States ‘Federal Reserve Economic Data” by the St. Louis Federal Reserve.