Reflective Teacher Change Processes in the Presence of a Mathematics Professional Development Intervention

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REFLECTIVE TEACHER CHANGE PROCESSES IN THE PRESENCE OF A MATHEMATICS PROFESSIONAL DEVELOPMENT INTERVENTION

by

Jason P. Gauthier

A dissertation submitted to the Graduate College
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
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REFLECTIVE TEACHER CHANGE PROCESSES IN THE PRESENCE OF A MATHEMATICS PROFESSIONAL DEVELOPMENT INTERVENTION

Jason P. Gauthier, Ph.D.
Western Michigan University, 2020

One of the most common methods employed to help teachers improve the teaching and learning of mathematics is professional development (PD). Research into professional development is a relatively new field, but still we know much about the features of effective PD with respect to such improvements. While the broad features of PD that are effective at creating change are relatively well-defined in the research, our understanding of the process of teacher change remains limited.

To begin to better understand this change process, this study examined the processes of teacher change associated with a specific PD intervention. The PD intervention in question focused on the development of research-informed, effective mathematics pedagogy PD for secondary teachers and was called the Secondary Mathematics Instructional Institute (SMII). Using a qualitative multiple case-study model, this study investigated the change processes of a sample of 3 teacher participants. Data gathered from participants included semi-structured interviews, classroom observations, and video recordings of PD sessions. Detailed case study reports were prepared for all participants and cross-case analyses were performed. Further, analysis of interview and PD session transcripts involved semi-open coding techniques using frameworks for teacher change and reflection.
The three subjects in this study experienced change processes that were inhibited and supported by multiple influences from multiple domains. Interestingly, subjects’ change processes were influenced by external factors both positively and negatively. The mediating processes of reflection and enaction appeared to drive the change processes of study participants—that is, subjects engaged in cycles of reflection followed by professional experimentation (or vice versa) and so incrementally changed their beliefs and practices. This was partially confirmed in some cases via classroom observation. All participants submitted examples of alternative patterns of practice for the lesson study sessions, thereby showing that, minimally, each could engage in practices consistent with those espoused by the PD intervention.

Change processes were mapped using Clarke and Hollingsworth’s Interconnected Model of Teacher Professional Growth (Teaching and Teacher Education 18(8), 947-967, 2002) and two types of pathways emerged: classical change pathways and alternative change pathways. Classical change pathways had significant initial involvement of subject belief systems, while this involvement was delayed in alternative change pathways. Reflective activity, while not linked strongly to time across the PD, was linked to engagement in lesson study activity—subjects reflected more often and at higher levels while collectively analyzing video examples of practice. Implications for future research and for PD providers are also discussed.

*Keywords:* teacher change, teacher learning, reflection, professional development, professional learning
I must gratefully acknowledge those without whom this project would never have been completed. First, my family, whose steadfast support and belief in me was a constant positive influence. To my wife, Lisa, whom I cannot thank enough for her support, encouragement, and love. To my son, Hayes, who learned to talk while I wrote this and, watching me, decided that he would work on his dissertation too. I can only hope that one day he will. Finally, to my parents, who instilled in me early a value for education. Without that, I would never have gotten this far. I am eternally grateful to you all.

Jason P. Gauthier
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CHAPTER 1

INTRODUCTION

The past half-century has seen multiple attempts to reform mathematics education in the United States. From the so-called “New Math” of the 1960s to the Back-to-Basics movement of the 1970s, there have often been pendulum swings in ideology and values. Subsequently, the focus and drive of professional organizations such as the National Council of Teachers of Mathematics (NCTM), NCSM: Leadership in Mathematics Education (formerly known as the National Council of Supervisors of Mathematics), and the Association of Mathematics Teacher Educators (AMTE) has served to stabilize the focus of national reform efforts. NCTM began what mathematics educators call the Standards era with the publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989 and followed with several subsequent publications designed to inform schools, teachers, and policy makers of an ambitious vision for mathematics education in the United States (National Council of Teachers of Mathematics, 1989; National Council of Teachers of Mathematics, 1991; National Council of Teachers of Mathematics, 2000; National Council of Teachers of Mathematics, 2014).

The NCTM standards documents were based on a then-growing research base which indicated that mathematics learning need not take the form found in most schools prior to 1989 (Cuban, 1984; Goodlad, 1984). Indeed, there was compelling evidence to suggest that learning could occur effectively without the aid of an instructor in the traditional sense as knowledge dispenser (Carraher, Carraher, & Schliemann, 1985; Lave & Wenger, 1991; Saxe, 1988). From studies such as these and from other areas of education and psychology, NCTM crafted a vision for school mathematics. The research community took up the challenge of studying and building a stronger case for the efficacy of this vision. Much of the research program of the 1990s was built on the foundation laid out by the NCTM *Standards* (1989). Today, the general consensus of the field of mathematics education is that the learning of mathematics is both an individual and a social, often collective, process of construction of knowledge based on experience. Mathematics instructional principles are derived from our knowledge of the learning process with the aim of providing students with experiences that aide in the construction process, both individually and socially. These teaching methods are based heavily in constructivist (Piaget, 1964; von Glasersfeld, 1995) and sociocultural theories (Brown, Collins, & Duguid, 1989; John-Steiner &
Mahn, 1996; Vygotsky, 1978) of learning. In all mathematics classrooms, NCTM envisions mathematics teachers making sense of mathematics with students and helping students learn how to collaborate to make sense of mathematical experiences as opposed to serving solely as a transmitter of information. In this vision, NCTM (2000; 2014) encourages teachers to develop mathematical experiences for students that

- involve engagement with cognitively demanding tasks (Henningsen & Stein, 1997; Stein, Smith, Henningsen, & Silver, 2009),
- encourage students to leverage multiple representations of mathematics (National Council of Teachers of Mathematics, 2000)
- develop connections between important mathematical ideas and between those representations (Lesh, Post, & Behr, 1987), and
- provide opportunities for students to individually and collaboratively construct and communicate their mathematical thinking to others (Stein, Engle, Smith, & Hughes, 2008)

Professional organizations such as NCTM, NCSM, and AMTE have mounted a continuous push to develop this ambitious vision into reality in every classroom. Unfortunately, despite these ardent efforts, the data paint a picture of a nation that has not seen the full realization of the hopes of the initiators of the standards movement (Stein, Remillard, & Smith, 2007). Given the evidence that teaching methods largely have not shifted, it seems likely that student learning would not have improved either. The data show the results of reform efforts to be mixed at best.

NCTM’s *Principles to Actions* (2014) summarized the current state of affairs quite well, beginning with national-level assessment data. In terms of success, NAEP proficiency numbers rose to an all-time high in Grades 4 and 8 (42 and 36, respectively) in 2013. SAT and ACT scores have also risen slightly as compared to 1990. Amid a long list of challenges, NCTM points out that the average National Assessment of Educational Progress (NAEP) scores for 17-year olds in mathematics have been flat since 1973. Further, the United States experienced a decline in mathematics scores on the Programme for International Student Assessment (PISA) exam from 2003 to 2012 while other countries’ scores increased (Organisation for Economic Co-operation and Development (OECD), 2013). Further, of those students who scored well on these assessments, those scores reflected large-scale positive performance on items requiring only
lower-level skills. This state of affairs has not improved significantly since 2014. The most recent NAEP results show almost no change in scale scores over the past decade—the national average scale score for Grade 8 students in 2009 was 283 as compared to 282 for the same group in 2019 (2019 NAEP Mathematics Assessment Highlights, 2019). NCTM’s (2014) notation of an uptick in scores in 2013 was correct, but any gains have since been lost. The results of the 2018 PISA assessment (taken by 15-year olds) offer little more: the United States ranked 31st out of the 41 countries that participated in the assessment (OECD, 2020). This ranking is unchanged compared to 2012.

These data are particularly striking given the educational context of the past decade. After nearly a decade of implementation of the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010)—a set of standards billed as comparable to those of high performing countries—we have seen virtually no change in relative standing world-wide (as measured by PISA). Some might point to the lack of uniform implementation across the nation to explain these data. However, it is worth pointing out that nearly every state has a set of standards that are identical to or closely mirror the original CCSSM document (Johnston, Stephens, & Ratway, 2018). Ultimately, these large trends are suggestive, even if they may not point to the problem directly. The achievement data across multiple assessments and measures suggest that we, as a country, have made limited progress in implementing the instructional changes called for by the professional organizations in mathematics education (Banilower, Boyd, Pasley, & Weiss, 2006). That this is a cause of our current challenges in test scores is certainly up for debate. Indeed, throughout the history of mathematics education in the United States there have been many debates over the causes and maladies of school mathematics (National Council of Teachers of Mathematics, 1970). I take the stance that our limited implementation of these reforms is a primary cause of our lack of success both nationally and internationally.

It is certainly possible that the national data mask pockets of excellence across the nation, and this is a notion worth addressing at this juncture. Examination of NAEP data encapsulates national-scale news and ideology. State-level data might paint a more favorable picture of student achievement and, by proxy, classroom practice. Unfortunately, this does not appear to be the case. Although changes in Michigan’s assessment systems and policies over the past 5 years
and the constant resetting of proficiency cut scores make longitudinal interpretations of the data difficult, the data remain suggestive.

In the 2018-2019 school year, 36.3 percent of Michigan Grade 11 students attained a proficient score on the SAT (Michigan Department of Education, 2019). SAT scores have remained virtually unchanged since Michigan began using it as the state assessment for 11th-graders in 2015. Prior to that, the test administered to high school students was the Michigan Student Test of Educational Progress (M-STEP), a test designed to assess the CCSSM standards. The M-STEP was in place for two years and saw no changes in proficiency levels (~29% proficient) as compared to the prior assessment, the Michigan Merit Exam (~29% proficient). Over the course of the past decade, by any measure used at the state level, Michigan’s high school students have seen little to no improvement in scores (Michigan Department of Education, 2019). Conversely, these data point to the fact that approximately 70 percent of Michigan’s 11th-grade students failed to meet the proficiency benchmarks prior to 2015 and approximately 65 percent of students have failed to qualify as college or career ready since 2015. The combination of low proficiency rates and lack of improvement paints an unfortunate picture of mathematics achievement in Michigan over the past decade.

Based on these data, there is evidence to suggest that the vision of NCTM has largely not been realized at any level or scale, within either the state or nation. That said, there are examples of classrooms in which the tenets of NCTM’s vision of ambitious mathematics instruction (Lampert & Ghousseini, 2012) have been implemented. Researchers have studied these kinds of classrooms, finding that students who learn in ways consistent with NCTM’s vision tend to perform on standardized assessments at least as well as (if not better than) students taught in a more traditional manner and that these same students show greater achievement on assessments measuring understanding and problem solving (Grouws, Tarr, Chavez, Sears, & Soria, 2013; Huntley, Rasmussen, Sangtong, & Fey, 2000; Tarr, Grouws, Chavez, & Soria, 2013). Lastly, there is also some evidence that student attitudes, motivation, and beliefs about mathematics can

1 Students must attain a Mathematics section score of 530 or higher to be considered “college and career ready” (College Board, 2019). As defined by College Board, “[t]he SAT Math benchmark is the section score associated with a 75% chance of earning at least a C in first-semester, credit-bearing, college-level courses in algebra, statistics, precalculus, or calculus” (p. 2)

2 Students’ M-STEP scale scores were broken down into four proficiency levels: Not Proficient, Partially Proficient, Proficient, and Advanced. The Michigan Merit Exam (MME) used the same proficiency levels.
be improved under this kind of instruction (Boaler, 1998; Boaler & Staples, 2008). These facts give rise to the need for a dramatic shift in the ways in which we conduct mathematics education. The research community continues to have a vital role to play if we are ever to make significant headway in achieving NCTM’s vision. In particular, there is a need to better understand the process of teacher learning and develop effective, supportive ways to influence that process. With a better understanding of the teacher learning process, there is hope that continued reform efforts aimed at changing the way students experience mathematics in this country might just succeed.

Background

One of the great challenges in the implementation of NCTM’s vision has been teacher preparation and professional development. This vision, as mentioned before, is based heavily in research and theory about the nature of student learning (National Council of Teachers of Mathematics, 2003), but very little of the research has made its way into classrooms and little of the NCTM’s (2000; 2014) vision has been implemented on a wide scale. One could argue that one of the most effective ways to increase student learning is to increase the quality of teaching (United States Department of Education, 2000). The beginning of a path to increased teacher quality requires an acknowledgement of the challenges facing teachers in this era of reform: teachers are being asked to teach mathematics content they have never experienced as students using methods with which they are unfamiliar. It is worth noting that many teacher preparation programs have implemented coursework for pre-service teachers that is consistent with NCTM’s vision; however, this implementation is far from universal. This is particularly true in teacher preparation programs that are housed in schools of education or mathematics departments with no mathematics educators as part of the faculty. Because of this inconsistency in program structure across the nation and the time since experienced educators’ pre-service training, in-service teachers’ lack of knowledge and skill in so-called ambitious teaching of mathematics is still extremely widespread (Fullan, 2009; Stigler & Hiebert, 2004).

Beyond any lack of skill or knowledge that might be attributed to teacher preparation, another reality about the nature of good mathematics teaching must be acknowledged: skill at teaching in the ways consistent with NCTM’s vision takes years, even decades, to develop (Berliner, 2001). This requires continuous support well beyond the preparation provided in
undergraduate teacher preparation programs. This support is necessary because teachers
oftentimes have conceptions of mathematics that run contrary to the vision of NCTM and other
mathematics education organizations. These conceptions of mathematics are not inherent—they
derive from a potent combination of previous experience, both mathematical and pedagogical
(Thompson, 1984). Not only that, these previous learning experiences create pedagogical
conceptions that are oftentimes in direct conflict with the values and pedagogy espoused by
many standards-era advocates. While systemic issues (e.g., collaborative structures, visions of
instruction, instructional coaching programs, etc.) are certainly vital to encouraging and
maintaining change (NCSM: Leadership in Mathematics Education, 2014), I have chosen to
conceptualize the problem of lack of implementation of high-quality mathematics instruction at a
broad scale as a support and empowerment problem. It is vital that teachers be given extensive
time, experiences, and systemic support if the education community expects them to teach in
ways consistent with those laid out by NCTM (2000; 2014) to address the mathematical content
and practices called for by many states (e.g., the CCSSM (National Governors Association
Center for Best Practice & Council of Chief State School Officers, 2010)).

Purpose

The research base on mathematics teacher professional development (PD) is solid and
nearly unanimous in its recommendations concerning the features of effective mathematics
professional development (see Chapter 2 for a more detailed discussion of this). However, this
research has focused, in the main, on program effectiveness as opposed to the nature of teacher
change (Goldsmith, Doerr, & Lewis, 2014). This distinction is vital and worth a moment of
consideration. After all, there are many metrics by which one might measure the “effectiveness”
of a given PD program. One might focus on measures of any of the following parameters to gain
insight into program effectiveness (Guskey & Sparks, 2000):

---

3 Professional literature, standards, and research studies refer to in-service teacher learning experiences
alternately as professional development (PD) and professional learning (PL). Professional learning is a term adopted
in recent years to highlight the fact that these experiences need not be about “fixing” teachers, but rather should be
about building off of existing strengths. Further, the idea of PL acknowledges that continued learning is a
professional expectation of all mathematics teachers. While I support this view and often refer to in-service teacher
learning as PL in my work, I will use the term professional development (PD) for the sake of consistency with the
research base. My use of the term does not indicate a departure from the view that teacher training should be a
supportive, empowering, and thoroughly asset-based experience.
1. Participants’ Reactions
2. Participants’ Learning
3. Organizational Support and Change
4. Participants’ Use of New Knowledge and Skills
5. Student Learning Outcomes

In Guskey and Sparks’ (2000) view, the list of metrics above is hierarchical, with each successive level becoming more difficult and complex to measure. While it might be tempting to define program effectiveness by resultant student learning outcomes, this metric is the most complex and difficult to effectively use. This is likely because many confounding variables exist in the space between a given PD intervention and students’ learning as measured by some sort of assessment. These confounding variables include such things as building culture (e.g., culture within a department or school district), administrative requirements of teachers, curriculum materials, and many more. Even such things as student absences can have a significant impact on student outcomes. This makes definitive connections between PD programs and student outcomes difficult to substantiate.

Instead of attempting to use student learning outcomes as a measure of program effectiveness, this study used a definition more closely aligned to levels 2 and 4 of Guskey and Sparks’ (2000) hierarchy. Goldsmith and colleagues (2014) note that “[t]ypically, teachers’ learning is treated as an indicator of the effectiveness of the program rather than as the primary object of inquiry” (p. 21). The authors go further and point out that while “many studies identified aspects of [teachers’ beliefs and practice] that professional development programs seek to affect” it remains true that “few studies focused on the processes or mechanisms of teachers’ learning” (p. 21). Thus, for the purposes of this discussion, program effectiveness refers to the ability of a given PD intervention to create changes in teacher knowledge, dispositions, and/or practice. Studies of program effectiveness attempt to measure teacher learning, as Goldsmith and colleagues (2014) noted.

This study aims to contribute to efforts aimed at understanding how mathematics teachers change in the presence of effective professional development, as opposed to simply studying whether they change. Study subjects participated in the Secondary Mathematics Instructional Institute (SMII), a year-long PD intervention aimed at shifting teacher beliefs and practices in mathematics instruction (a detailed description of the PD intervention can be found in Chapter
3). The locus of inquiry was the process of change experienced by participants these participants experienced over the duration of the intervention. The study also explored influences on those processes. Efforts to focus on and understand the “black box of teachers’ learning” have the potential to “allow us to develop more general understandings about how certain catalysts for change affect the pathways of teachers’ learning over time” (Goldsmith et al., 2014, p. 21). An implicit assumption in the discussion thus far has been that teacher change does result from participation in PD interventions. It is certainly possible that a teacher might participate in a PD and not change in any way detectable by the metrics employed. Worse, it is certainly possible that regression in teaching habits might result. If these are legitimate phenomena, then it is vital for the community to understand why no change occurred or why teachers regressed in practice—was it simply because the change was not measured or were there other factors that were measured which inhibited the change process? The research questions for this study, while phrased positively, allow for the possibility that no change (or regression) occurred and remain valid even in those circumstances:

1. What do teacher change processes associated with the SMII PD intervention look like?
2. What factors influence teachers’ change processes as they engage in the SMII PD intervention?
3. How does teacher reflection change during the SMII PD intervention?

Significance

This study has the potential to connect to research literature concerning teacher beliefs, teacher knowledge, reflection, effective professional development, lesson study, the use of video in PD (the SMII PD intervention features the use of video as a representation of practice), and teacher change in a meaningful way. There is significant support in the literature suggesting that SMII is a well-designed PD and lesson study is a firmly established method of changing teachers’ practice (this will be discussed in more detail in Chapter 3). From the base of a potentially effective PD, this study builds on previous work with the use of video as a learning tool in PD and teachers’ reflective practice. Further, this study adds to the growing research base involving the theoretical frameworks of teacher change reflection discussed in the following chapter. The teacher change framework (Clarke & Hollingsworth, 2002) has been used with some success in the realm of science education, but on a much more limited basis (until very
recently) in mathematics education. This study adds to the growing collection of mathematics education studies using the work of Clarke and Hollingsworth (2002) and, through that framework, develops our understanding of how in-service teachers’ reflection processes change over time. Previous work such as studies by Boston and Smith (2009) and Stockero (2008) studied pre-service teachers only. Additionally, this study provides some concrete links between the features of the PD intervention and aspects of teacher change, providing justification for future work with those features.
CHAPTER 2

REVIEW OF LITERATURE

This chapter contains two major components of this research study: a literature review followed by an explication of the theoretical framework used in the research process. I begin by summarizing the process used to undertake the literature review and move to reviewing the research literature related to the following areas of mathematics education:

- effective mathematics professional development (including lesson study)
- mathematics teacher knowledge
- mathematics teacher beliefs
- teacher reflection
- teacher change and learning.

Following this broad literature review is an explication of the theoretical framework used in this study. This framework consists of two parts: a framework for teacher change and a framework for examining reflective activity.

A Summary of the Review Process

The review of research for this project occurred in two distinct phases, each aligned with the phases of the project design. The first phase involved the planning, design, and preliminary implementation of SMII. During this phase, my research focused in two areas: 1) developing a comprehensive view of the features of effective professional development, and 2) developing an understanding of lesson study. Specifically, my searches involved the use of Google Scholar and the search terms teacher professional development, mathematics (and variants) and Japanese lesson study (with variants). Reference lists from articles found in Google Scholar expanded my search. The second phase of my literature review involved the search for studies involving the use of video as a tool in PD, an expanded search for literature on teacher change and teacher beliefs as well as a more limited search of the literature on reflection, using terms such as video, professional development, mathematics education (with variants), teacher change, mathematics education (with variants), and teacher reflection, teacher beliefs, mathematics. I found particular journals to be very useful in providing relevant resources, particularly though special issues.
Journals such as the *Journal of Mathematics Teacher Education, Teaching and Teacher Education, Journal for Research in Mathematics Education, the Mathematics Teacher Educator,* and *ZDM* provided a wealth of information from both individual articles and focus issues.

### Mathematics Professional Development and Teacher Learning

Like any complex skill, building masterful teaching skill and artistry is a lifelong quest. Previous studies have shown that developing such skill and artistry for mathematics teachers is aided by sustained access to and engagement with high quality professional development opportunities (Cordingley, Bell, Rundell, & Evans, 2005; Franke, Carpenter, Levi, & Fennema, 2001; Garet, Porter, Desimone, Birman, & Yoon, 2001; Ross & Bruce, 2007; Saxe, Gearhart, & Nasir, 2001).

While there is not a complete definition of what makes professional development for mathematics teachers high quality, there is strong support in the research literature for the idea that effective mathematics professional development has particular properties (Garet, Porter, Desimone, Birman, & Yoon, 2001; Loucks-Horsley, Stiles, & Hewson, 1996; Whitcomb, Borko, & Liston, 2009). Two informative views of these properties appear in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Two Views of Effective Mathematics Professional Development</th>
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<tbody>
<tr>
<td>Professional development programs should . . .</td>
<td>Professional development programs should . . .</td>
</tr>
<tr>
<td>Be driven by a clear, well-defined image of effective classroom learning and teaching (p. 3)</td>
<td>Be situated in [teaching] practice</td>
</tr>
<tr>
<td>Provide teachers with opportunities to develop knowledge and skills and broaden their teaching approaches (p. 3)</td>
<td>Be focused on student learning</td>
</tr>
<tr>
<td>Use instructional methods to promote learning for adults which mirror the methods to be used with students (p. 4)</td>
<td>Be embedded within professional [learning] communities</td>
</tr>
<tr>
<td>Build or strengthen the learning community of science and mathematics teachers (p. 4)</td>
<td>Be sustainable and scalable</td>
</tr>
<tr>
<td>Prepare and support teachers to serve in leadership roles if they are inclined to do so (p. 5)</td>
<td>Be supported and accompanied by carefully designed research</td>
</tr>
<tr>
<td>Consciously provide links to other parts of the educational system (p. 5)</td>
<td></td>
</tr>
<tr>
<td>Include continuous assessment (p. 5)</td>
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These two views provide some insight into the evolution of thinking in the field of mathematics professional development over the decade between them. In the mid-90s, there was agreement on a set of characteristics of effective PD that were specific and instructional in
nature. The focus on the kinds of instructional methods used (mirroring those the vision encouraged teachers to use with students) and on opportunities to build knowledge and skills, among other things, had a distinctly change as training (Clarke & Hollingsworth, 1994; 2002) feel. By 2009, the focus of the field had shifted to a more practice-based approach, with increased emphasis on studying student learning in a collaborative, supportive community. Doubtless, the features of effective PD outlined in 1996 live within those outlined in 2009; however, the notable differences include the situating of PD within teachers’ practice and a call for sustainability and scalability. The continued growth of lesson study as a form of professional development in the U.S. (Stigler & Hiebert, 2004) may have contributed in some small way to the shift in focus. Lesson study is, in many ways, the intensive study of teaching practice and so by its very nature is situated in that teaching practice. While the recommendations by Loucks-Horsely, Stiles, and Hewson (1996) called for linking PD experiences to other aspects of the educational system, the call by Whitcomb, Borko, and Liston (2009) was for PD to become situated in practice and embedded within teachers’ learning communities. The connection between PD and teachers practices and content needed to be much more robust and supported by a focus on PD experiences consistent with the idea of pedagogies of enactment (Cobb & Jackson, 2011).

Each of the recommendations in Table 1 was based on research findings relevant and current at the time of publication. However, a vital question to answer in this context is: what does research say about effective professional development today? To answer that question, I will appeal to several noteworthy reviews of literature and several notable examples of studies of the professional development of mathematics teachers.

Mathematics professional development research was well-treated in both the Second Handbook of Research on Mathematics Teaching and Learning (Lester, 2007) and the Compendium for Research in Mathematics Education (Cai, 2017). I mention these two resources together because they build from one another. In the Compendium chapter, Sztajn and colleagues (2017) pick up where Sowder (2007) left off in her review of research on mathematics professional development. I chose to focus, in the main, on Sowder’s work for this review because Sztajn and colleagues’ review focused largely on studies in Phases 2 and 3 of Borko’s (2004) PD research program. These studies focused on the scaling of PD and not on the content of the PD events themselves.
Sowder (2007) began with a list of ten questions which she attempted to answer as part of her discussion of mathematics teacher professional development. A full discussion of those questions is beyond the scope of this review. However, three of her questions bear directly on what research says about the nature of effective mathematics professional learning. These questions focus on 1) the goals of mathematics PD, 2) principles of design for mathematics PD, and 3) how teachers acquire knowledge of how to teach mathematics effectively.

In Sowder’s (2007) synthesis, the goals of mathematics PD should include 1) the development of a common vision of mathematics instruction, 2) the development of content knowledge of mathematics, 3) the development of understanding about how students think and learn mathematically, 4) the development of teachers’ pedagogical content knowledge, 5) the development of an awareness of equity in school mathematics, and 6) the development of mathematics teacher identities. Within these goals exists a synergistic combination of the two views of mathematics PD outlined in Table 1. A common vision of mathematics instruction should guide the development and enactment of practice-based learning experiences (both mathematical and pedagogical) for mathematics teachers, with continual emphasis on teacher identity and equity. Further, within these goals are connections to areas of mathematics education germane to this study. Sowder made the case in her discussion of Goal 1 that professional documents by NCTM and various national panels should be foundational to the creation of a shared vision of mathematics instruction. I have made a similar case in the introduction to this study, albeit limiting my reference to documents primarily attributed to NCTM. Goals 2, 3, and 4 have been developed significantly since the publication of Sowder’s work and have, in some sense, been combined in the construct of Mathematical Knowledge for Teaching [MKT] (Hill, Ball, & Schilling, 2008), which I will discuss in more detail below.

Sowder (2007) appealed to several different authors for a description of the features of effective professional development. These authors created lists similar to those in Table 1, often with large areas of overlap. As a result, one may use Sowder’s discussion to construct a consensus model of the features of effective mathematics professional development. Such a model might indicate that effective mathematics professional development should:

- be guided by theories of teaching and learning (including adult learning)
- include teachers in decisions about the focus of PD
- involve powerful, collaborative mathematical experiences
• focus on the important work of teaching mathematics
• develop teachers’ understanding of how students think about mathematics
• consider ways to help teachers navigate the tensions of the change process
• attend to teachers’ mathematical and mathematics teaching identities
• engage teachers in experiences over significant spans of time

One can see from the list above and Table 1 that there are broad areas of agreement about the features of effective mathematics professional development within the research community. Supporting and extending Sowder’s (2007) work, a study by Goldsmith, Doerr, and Lewis (2014) reviewing 106 research studies published between 1985 and 2008 considered what research had to say about the effects of mathematics PD on teacher learning. Specifically, the researchers defined teacher learning as changes in teachers’ knowledge, beliefs, and practices.

Goldsmith, Doerr, and Lewis (2014) found that research supported the potential of mathematics PD to affect six areas of teacher learning: 1) teacher beliefs, 2) teacher instructional practice, 3) teacher collaboration, 4) teacher attention to students’ mathematical thinking, 5) teachers mathematical content knowledge, and 6) teacher knowledge of curriculum and tasks. The review also revealed significant detail within each of these areas. I will review a selection of these areas in more detail below. Specifically, I will summarize the findings related to beliefs, instructional practice, and mathematics content knowledge, as each of these areas are relevant to the PD intervention related to this study.

Goldsmith and colleagues (2014) noted several ways in which mathematics PD can affect teacher beliefs about mathematics, teaching, and students. The researchers identified themes across the PD studies that showed changes in beliefs, noting that teacher beliefs affect the acceptance or uptake of PD. Further, observation of students and their thinking tends to move teachers’ beliefs about students’ mathematical competence in a more positive direction. The review also noted that PD activities featuring video analysis of classroom interactions shifts teachers’ stances over time away from evaluation, making them more likely to learn about how students think as opposed to whether students were thinking “correctly.” Lastly, effective mathematics PD affected teachers’ collegial attitudes and sense of efficacy.

In the area of instructional practices, the review found that PD could influence practice across three categories. First, PD could increase teachers’ mathematics content knowledge. Second, good mathematics PD was shown to increase teachers’ abilities to engage students in
mathematical discourse in their classrooms. Third, mathematics PD studies showed the ability to support teachers in increasing students’ mathematical autonomy.

Lastly, Goldsmith and colleagues (2014) noted that mathematics PD can affect the mathematical content knowledge of teachers in various ways. They justified the importance of this area by noting that some researchers found that “a lack of mathematics content knowledge can impede teachers’ abilities to notice and analyze students’ mathematical thinking (Goldsmith, Doerr, & Lewis, 2014, p. 17). Goldsmith and colleagues went on to identify and discuss several catalysts of change in mathematics content knowledge. Studies showed that teachers increased their content knowledge by studying students’ mathematical thinking, engaging in collaborative lesson planning, exploring new mathematics tasks or curriculum materials, engaging in professional conversations, and focusing PD activities on classroom mathematical discourse. Studies in the review also indicated that mathematics PD builds teachers’ agency with regard to mathematics content and instruction.

At the close of their review of research, Goldsmith and colleagues (2014) discuss three general findings: 1) teacher learning tends to be incremental and iterative, 2) PD intervention impact varies across individuals and contexts, and 3) many existing studies of PD focus on determining the effectiveness of the PD interventions rather than on the teacher learning processes present within them.

The question of teacher learning from PD takes the discussion away from the goals and features of effective mathematics PD to examples of such programs. Sowder (2007) laid out several broad areas of teacher learning from PD, including:

- learning from a focus on student thinking
- learning from curriculum
- learning from case studies
- formal course work

Here we see commonalities with the findings of Goldsmith and colleagues (2014). That review also indicated that teachers learn from PD focused on student thinking (in this case fostering increased content knowledge) as well as increasing knowledge of curriculum and tasks. The PD program associated with this study (see Chapter 3 for a detailed description of the program) falls broadly into the category of learning from case studies.
Learning from Video

Sowder (2007) highlighted video case studies as one potential example of a way teachers might learn from case studies. Primarily, video cases can provide windows into classroom instruction consistent with the vision of instruction which guided the PD. However, many programs avoid casting video cases as exemplars, believing that the idea of an exemplar classroom would be unrelatable to teachers and therefore make them less likely to understand and reason about the classroom practices present in a particular video case. With this precaution in place, it becomes more likely that teachers might learn from video in various ways. These include 1) learning mathematics content, 2) generalizing mathematical teaching knowledge across multiple video cases, 3) seeing real teaching as messy and complex work, and 4) developing new patterns and norms related to professional discourse. These four areas of learning also align with some of those outlined by Goldsmith and colleagues (2014) in their review. Specifically, PD activities which involve video cases support the development of alternative instructional practices, develop teachers’ collaborative habits and skills, as well as increasing teachers’ mathematical and pedagogical content knowledge. Research surrounding video cases has expanded significantly since the publication of Sowder’s (2007) discussion of the topic. Largely, this expanded research base has confirmed the potential of video as a highly useful tool in empowering teachers to make teaching practice an object of study with both in-service mathematics teachers (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Goldsmith, Doerr, & Lewis, 2014; Grant & Kline, 2010; Hennessy & Deaney, 2009; Hollingsworth & Clarke, 2017; Karsenty & Arcavi, 2017; Sherin & Dyer, 2017; van Es & Sherin, 2008) and pre-service mathematics leaders (Goldsmith, Doerr, & Lewis, 2014; Lesseig, et al., 2017). Additionally, research has highlighted the potential of video to enhance teacher learning (Schoenfeld, 2017) and best practices for using video in both pre-service and in-service teacher training (van Es, Stockero, Sherin, Van Zoest, & Dyer, 2015). These findings are relevant due to the video-based lesson study format of the SMII PD.

Schoenfeld (2017), in a discussion of some of his most recent work on characterizing powerful mathematics teaching, wrestled with one of the difficulties of using video examples for research and PD: “bandwidth” (p. 415). This refers to the fact that video can be challenging to use because of the sheer amount of information it conveys. In describing his pioneering work with video on problem-solving and decision-making, Schoenfeld made an unequivocal argument
that video is a vital tool in both research and PD, noting that video has the advantage of bringing the researcher or teacher as close to a phenomenon as can be without being present, “convey[ing] the immediacy of classroom actions in ways that support rich conversations about the nature of productive learning environments” (Schoenfeld, 2017, p. 431).

Related to Schoenfeld’s (2017) enthusiasm for the use of video, but from a more specific and pragmatic standpoint, van Es and colleagues (2015) discuss the affordances and issues associated with teacher self-captured video, providing recommendations for teachers and PD facilitators to make the process more productive. The recommendations focus in three areas: the location of the recording equipment in the classroom, deciding how much video to capture or use, and when facilitators might inform teachers about what to do with the video they have captured. Rather than pointing out the “do’s” and “don’ts” of engaging teachers in capturing video of their own classrooms, the researchers discuss the various possibilities in each space and make note of the affordances (both visual and auditory) of a particular choice. For example, in regard to where to place the recording equipment, the authors note that it can be placed in the front or back of the room in a stationary position, focused on a single group of students from a stationary position, or worn by the teacher. There are visual and auditory affordances to each choice, as well as visual and auditory limitations (e.g., a camera at the back of the room captures whole-class interactions well, but limits visibility of some students in favor of others and make some student easier to hear and others harder).

Of particular relevance here are the recommendations for facilitators concerning when to specify to teachers what they must do with the video they capture. Again, van Es and colleagues (2015) provide options and identify a challenge and opportunity associated with each (e.g., specifying the task prior to the collection allows lessons to be planned to capture specific ideas or actions but also requires teachers to decide to record during the cognitively complex act of teaching). The authors close with a focused set of recommendations for PD facilitators that include: 1) being clear about the purposes of capturing video at the outset, 2) ensuring that teachers have multiple opportunities to capture and analyze video of their classroom, and 3) helping teachers to plan for capturing video of events in their classroom related to the goals of the PD or the goals of the teachers. These recommendations connect to this study through the lesson study portion of the SMII PD. Study subjects captured video of their own classrooms in
various ways and made note of many of the same issues taken up in van Es and colleagues’ work.

Relatedly, work by van Es and Sherin (2008) points to the efficacy of a “video club” (p. 244) setting to enhance teachers’ abilities to notice student thinking and their abilities to engage in reflective activity. Video clubs were, in this context, simply a group of teachers that met together and collaboratively viewed and analyzed video episodes of their teaching (van Es & Sherin, 2008). The researchers found three different pathways through which teachers learned to notice. The direct pathway involved an abrupt qualitative shift in the focus and specificity of teachers’ discourse. A cyclical pathway was characterized by a repeated shifting from specific, interpretive noticing to broad, descriptive noticing. Lastly, an incremental pathway was characterized by consistent, small shifts toward more specificity and interpretive noticing. Along with these pathways, van Es and Sherin (2008) discussed factors that influenced teacher learning in their study. These included the focus of the video clips on student thinking and the multidimensional role of the facilitator in the video club meetings. These findings were added to Sowder’s (2007) list of tools that were effective at affecting teacher learning by Sztajn, Borko and Smith (2017) in their Compendium chapter.

The findings from over two decades of PD research point to the power of video as a way of tying PD closely to practice through the use of artifacts. Further, the role of video in PD was connected to a number of other areas of research on teacher learning, including noticing and reflection (van Es & Sherin, 2008), teacher learning (Sherin & Dyer, 2017), and lesson study (Stigler & Hiebert, 2016).

Lesson Study as Professional Development

Related to case study learning, and video case study learning in particular, Sowder (2007) also identified lesson study as a powerful form of professional development. The PD intervention associated with this study involves a form of lesson study (see Chapter 3 for more detail) and so considering the research on lesson study in the U.S. is necessary. In this space, I will consider not only Sowder’s commentary about lesson study, but also the larger body of research on the topic.

Sowder’s (2007) discussion of lesson study frames it as something more than what has become a common conception since its introduction into the United States in the late 1990s. A common (mis)conception of lesson study frames it as a way to collaboratively design and
accumulate a series of high-quality lesson plans for the collaborating teachers to use after lesson study is finished. In fact, lesson study is about the accumulation of collective knowledge about teaching (Sowder, 2007; Stigler & Hiebert, 2004; Stigler & Hiebert, 2016; Takahashi & McDougal, 2016), which never, in fact, ends. This distinction is vital and should not be overlooked when considering making lesson study a part of any PD offering.

Much of the initial research on lesson study was completed in Japanese classrooms, which is why the form is often called Japanese lesson study (Stigler & Hiebert, 2016). As such, there are some differences that the education field in the United States has had to consider carefully when implementing lesson study. Takahashi and McDougal (2016) take this discussion up and summarize the purposes, components, and necessary supports to implement lesson study effectively.

With regard to the purposes of lesson study (Jugyou kenkyuu in Japanese), Takahashi and McDougal (2016) use examples of what lesson study is not to, in effect, define what it is. In the end, the authors conclude that the purpose of lesson study is to generate new knowledge about teaching (contrasted with the conception of lesson study as perfecting a lesson). In fact, the authors note, repetition of a lesson as part of the lesson study process is rare in Japan.

One of the major factors influencing the effectiveness of lesson study is the presence of a particular set of structures. When lesson study goes awry, it is often due to the absence or misinterpretation of one of these structures. Specifically, effective lesson study processes ensure that

1. Participants engage in lesson study to build expertise and learn something new, not to refine a lesson.
2. It is part of a highly structured, school-wide or sometimes district-wide process.
3. It includes significant time spent on kyouzai kenkyuu [studying of research].
4. It is done over several weeks rather than a few hours.
5. Knowledgeable others contribute insights during the post-lesson discussion and during planning as well. (Takahashi & McDougal, 2016, pp. 515-516)

In many ways, the list of components of effective lesson study complements the authors’ suggestions for effective supports. In this space, Takahashi and McDougal found that it is vital that the school principal play a central role in the implementation of lesson study at the school level—which connects to the effect of subject context on the uptake of PD ideas discussed by Wilkie (2019). The main charge is to display a high level of enthusiasm for the process and
communicate that sentiment clearly and consistently. Further, the authors note that it is important to have another person who consistently advocates for the lesson study process. The lesson study process should be connected to a school-wide teaching and learning goal. Lastly, the school administration must provide teachers the time to engage in all aspects of the lesson study process, which should be supported by district-level professional development.

Despite the misinterpretations and false starts described in many accounts of importing lesson study (Fujii, 2014; Takahashi & McDougal, 2016; Takahashi, Watanabe, Yoshida, & Wand-Iverson, 2005), the practice is still widely considered to have great potential and research has borne this out in several cases (cf. Willems & van den Bossche, 2019). Several studies in recent years have examined lesson study in various forms and noted its impacts on teacher learning. Lewis, Perry, and Hurd (2009) posited a theoretical model of lesson study and examined a case of implementation. The model proposed by Lewis and colleagues involved four domains, which represent vital practices in the lesson study process: investigation, planning, implementing and observing a research lesson, and reflecting on that lesson to glean what was learned about teaching. Further, the model also identified three pathways through the model. These pathways proposed different ways teachers might change as they engaged in lesson study. Specifically, Lewis and colleagues proposed that lesson study had the potential to change teachers’ beliefs and knowledge, their collaborative professional communities, and the resources they used to support teaching and learning. After examining the case study, the researchers claimed that lesson study created effects along each of the three pathways. They also noted that changes in community and knowledge were integral mechanisms in the lesson study process.

Another study by Warwick, Vrikl, Vermut, Mercer, and van Halem (2016) examined the interactions of groups of teachers engaged in lesson study in the United Kingdom. The researchers were interested in how teachers’ collaborative conversations translated into pedagogical intentions—ideas about how to teach students. Video recordings of students engaged in mathematical activity served as the focus of the lesson study conversations. Researchers found that the focus on student learning was effective in supporting teacher collaboration about developing teaching moves that might address the needs of the students in a particular video clip. Ultimately, Warwick and colleagues conclude that collaborative dialogue is vital to the creation of changes in knowledge for all members of a teacher lesson study community.
In a similar study, Widjaja, Vale, Groves and Doig (2017) used the Interconnected Model of Teacher Professional Growth (Clarke & Hollingsworth, 2002) to study how teachers and coaches from three schools learned through participation in a lesson study project. The project focused on engaging teachers in implementing problem-solving lessons in a structured way. Widjaja and colleagues (2017) used the IMTPG to map subject change processes over time and relate them to the large-scale structures of the lesson study process at work in the project. The results of the study showed positive changes in the following areas:

- teachers’ abilities to plan collaboratively,
- an increase in teachers’ attention to students’ thinking about mathematics—similar to examples in Sowder’s (2007) review,
- an increase in the use of whole-class discussions using anticipatory techniques consistent with those advocated for by Smith and Stein (2018) (cf. Smith, Bill, & Hughes, 2008), and
- a general positive development in collaborative inquiry practices for teachers.

Through the lens of the IMTPG, the researchers noted the centrality of cycles of reflection and enactment to teachers’ professional growth. Further, they positioned the IMTPG as a theorization of the processes that underlie lesson study.

To summarize, lesson study, with its roots in traditional Japanese practice, has been shown to have the potential to support the teacher change process. As a form of PD, lesson study has been shown to affect large domains of teacher knowledge and practice (e.g., ability to anticipate student responses, the ability to implement problem-solving lessons, the ability to effectively collaborate, etc.). Lesson study as a form of PD has connections to both the acknowledged features of effective mathematics PD—it is embedded directly into teachers’ practice—and, potentially, the use of video as a tool for learning (if the lesson study cycles are conducted with video lessons as opposed to directly observed research lessons). Lesson study is a robust and supportive form of mathematics PD that has the potential to support the teacher change process.

This study will focus on teacher change processes as the result of interactions between study subjects and the PD intervention. Given the research reviewed above, it is clear that mathematics professional development can be effective at initiating and supporting teacher
change across a range of domains. As such, research findings about the features of effective PD have great relevance to this study. The professional development intervention in this study involves activities designed to impact teachers’ content knowledge, develop their abilities to implement problem-solving-based lessons effectively, and work collaboratively through lesson study to make instructional practice a focus of study using video recordings of practice. The research findings summarized above would suggest that, if designed appropriately, the PD in question in this study has the potential to support teacher change in each of the aforementioned areas. The connections between research on effective mathematics PD and the focus areas of this study make it necessary to consider what the research community knows about the areas that might reasonably be affected by the PD: teacher knowledge, teacher beliefs, teacher change, and teacher reflection.

Mathematics Teacher Knowledge

As discussed above, one of the major areas in which effective mathematics PD has the potential to affect teachers is their knowledge. Mathematics teacher knowledge is often assumed to be directly related to their knowledge of mathematics. While this is certainly true—and strong mathematical content knowledge is a vital component to effective mathematics teaching—it is not nearly the whole story. In some ways, the knowledge that teachers need to effectively teach mathematics is much broader in scope than the knowledge needed to do mathematics. For example, in his seminal work on the subject, Shulman (1986) characterized this knowledge into two spaces: Subject Matter Knowledge and Pedagogical Content Knowledge (PCK). Previously, he maintained, the ideas of content and pedagogy were treated separately by researchers and teaching programs. He proposed a combination of the two and spawned a new era in research on the knowledge necessary for teaching.

In mathematics education circles, perhaps the most well-known conceptualization that built on Shulman’s original ideas is that of Mathematical Knowledge for Teaching [MKT] (Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005). The research program associated with MKT elaborated on Shulman’s initial description and basic domains, creating a much more nuanced and complex framework for investigating mathematics teacher knowledge (see Figure 1).

The subject matter knowledge domains encompass three different kinds of knowledge: 1) the knowledge that might be expected from school mathematics experiences or mathematical knowledge that teachers use in a similar way as that knowledge is used in other careers, called Common Content Knowledge (CCK), 2) the very specific knowledge of mathematics content needed to teach it (e.g., knowledge of how to represent numbers in different ways), called Specialized Content Knowledge (SCK), and 3) the knowledge that exists at the mathematical horizon for teachers. The latter knowledge might be a knowledge of how ring theory encompasses the basic structures and rules of school arithmetic.

The domains of pedagogical content knowledge encompass knowledge that is in many ways distinct from knowledge of mathematics, but still intimately connected to it. The domain of Knowledge of Content and Students (KCS) includes the knowledge of how students typically attend to particular types of problems or how learning typically progresses along a continuum with respect to a given mathematical topic. Knowledge of Content and Teaching (KCT), on the other hand, involves knowledge of how to best deliver content to students or how to best facilitate learning with regard to a particular mathematical topic. Finally, Knowledge of
Curriculum encompasses teachers’ knowledge of the development, organization, content, and effective use of curriculum and curriculum materials in mathematics.

The developers of Mathematical Knowledge for Teaching have expanded their work to include a series of assessments designed to assess domains of teacher knowledge (cf. Hill, 2010). Others have expanded MKT and pedagogical content knowledge to include other areas of teacher knowledge such as knowledge of technology (cf. Niess, 2005). Still others have noted that the MKT framework and assessments focus at the elementary school level and developed instruments specific to the teaching of particular domains of secondary mathematics, such as algebra (e.g., McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012). From the wide acceptance and development of MKT, it is clear that this conceptualization of teacher knowledge has been a vital tool in developing our understanding of what teachers know about mathematics and teaching.

Mathematics Teacher Beliefs

In many, if not all, models of teacher change (see below for a more detailed discussion of these models), beliefs are present in some way. Generally, a useful definition of belief—from Borg (2001)—is:

- a proposition which may be consciously or unconsciously held, is evaluative in that it is accepted as true by the individual, and is therefore imbued with emotive commitment; further, it serves as a guide to thought and behavior (p. 186).

Additionally, Phillip (2007) offered a remarkably similar definition (with accompanying notations) in his review of research on mathematics teacher beliefs and affect:

- Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one’s view of some aspect of the world or dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual (p. 259).

While there is no single definition of the term, the aforementioned definitions of belief provide sufficient explanatory power to suffice as working definitions for the purposes of this discussion. However, significant, seminal work in the area of beliefs has elaborated and contextualized the notion of teacher beliefs. In particular, beliefs exist within systems (cf. Leatham, 2006) and these systems have dimensions that must be considered (cf. Green’s (1971)
three dimensions of a belief system). Green (1971) maintained that systems of beliefs involve quasi-logical relationships between different belief sets. Belief sets may be considered as either primary or derivative—derived from other sets of beliefs and experiences. And so, the quasi-logical relationships dimension involves the role of evidence in the formation and holding of beliefs. Beliefs, either primary or derivative, can be held “evidentially” or “nonevidentially” (Green, 1971, p. 48). For example, an individual might indicate a belief in A, citing belief B as evidence for A. In this case, belief A is held evidentially. However, it is also possible that, when presented with evidence contradicting belief A, that same person might not re-examine belief B. Thus, it is possible for a given person to “hold a belief because it is supported by the evidence, or he may accept the evidence because it happens to support a belief he already holds” (Green, 1971, p. 49). In any case, as Cooney, Shealy, and Arvold (1998) point out “it is important to recognize that one’s grounds for beliefs are of considerable importance when we contemplate how beliefs become modified” (p. 310).

Further, beliefs are held with differing strengths based on the individual and the context in which the belief was formed. Thus, beliefs are central (strongly held) or peripheral (weakly held). The dimension of psychological strength is somewhat simpler to illustrate. This dimension considers the strength of a given belief. Cooney and colleagues (1998) provide a useful example:

A person can believe that technology should be used to teach mathematics and therefore believe that students should be allowed to use calculators—a primary or derivative belief. But the psychological strength of this belief may not be strong. Thus, when faced with the usual classroom impediments, the teacher’s commitment to using calculators quickly dissipates (p. 309).

Thus, if a given belief is not held strongly enough, then it is unlikely to be seen enacted in complex or difficult settings.

Lastly, beliefs can, and often do, develop in clusters, some sets connected to other sets, some unconnected. The clustering of beliefs is affected by the context in which the beliefs are formed. Clustered beliefs are held in isolation from other beliefs, oftentimes rendering them immune to rational criticism. For example, “it is perfectly possible for teachers to simultaneously hold that problem solving is the essence of mathematics and that students best learn mathematics by taking copious notes and memorizing what is to be learned” (Cooney, Shealy, & Arvold, 1998, p. 310). Regarding these inconsistencies, Phillip’s (2007) review found that research explained them in different ways. Some research noted that teaching actions aligned more with a
pre-service teacher’s view of mathematics than of teaching or learning (Skott, 2001). Other research (Raymond (1997), as cited in Phillip (2007)) noted that contextual constraints and issues oftentimes caused a difference between belief and action for teachers and also found that alternative, non-mathematical goals in the classroom could lead to teachers enacting practices inconsistent with their beliefs (e.g., classroom management concerns might cause a teacher to structure the classroom in a way that was inconsistent with a belief in the value of small group work). Relatedly, clustered beliefs are also oftentimes not compared with one another and so teachers may hold contradictory beliefs without being conscious of it (Green, 1971; Phillip, 2007). Clustered beliefs can be immune to evidence contradicting them and they can also be both contradictory and central at the same time. These conditions are related to the contexts in which beliefs are formed, examined, and compared (Cooney, Shealy, & Arvold, 1998).

From the foundational work on teacher beliefs discussed above, researchers have been able to learn something about how PD can change teachers’ beliefs. In particular, Phillip (2007) gave several examples of the research paradox about changing beliefs. Teachers can change their beliefs and then change behaviors. However, this begs the question of how beliefs change? Phillip never offered a firm answer to that question. He appealed to Guskey (1986), discussed below, as an alternative to the “beliefs before actions” change sequence. Issues with that assumption as well as Guskey’s model are discussed below. Phillip (2007) noted that researchers maintained, based on research evidence, that beliefs did act as filters to what one can see and how one internalizes information and ideas. Further, he also discussed how reflection plays a role in the changing of belief structures. In this argument, he appealed mainly to Cooney and colleagues (1998).

Cooney, Shealy, and Arvold (1998) studied the cases of four undergraduate preservice mathematics teachers engaged in a final methods course and student teaching experience before graduation. The four students each had different reflective orientations and different set of beliefs. The case studies revealed changes in belief structures across the students’ experiences. For example, one subject experienced shifts in his beliefs through interactions with technology rich learning environments and problem-based investigations. He came to believe that technology was a vital tool in teaching and that investigations served to do much more than prevent boredom in students. Through the four case studies, the researches made the case that teacher beliefs can and do shift with exposure to the proper experiences.
A study by de Vries, Jansen, and van de Grift (2013) of over two hundred mathematics teachers in the Netherlands, found that teachers’ participation in continuing professional development (CPD) offerings tended to shift beliefs in a more student-centered direction. The rate of participation appeared modestly related (in the positive direction) to the strength of the shift in beliefs. The study in question used surveys to assess teacher beliefs about mathematics, teaching, and learning. Interestingly, the researchers found no correlation between participation in CPD offerings and shifts in belief about mathematics. The results of this study would seem to indicate that it is possible for PD to shift at least some belief sets of teachers as measured by pre-post survey data.

The Goldsmith et al. (2014) review of literature also considered the effectiveness of PD at shifting teacher beliefs. The review noted that, generally, PD could change beliefs. However, the breadth of the kinds of beliefs studied made the research base difficult to organize. Several themes did emerge, however. Of relevance here is that research has shown that when teachers are given opportunities to observe students closely, those teachers’ beliefs about students’ competence in mathematics tend to shift in the positive direction. Further, opportunities to collaborate with colleagues around teaching and learning activities can also positively shift teachers’ beliefs in students’ mathematical abilities. The Goldsmith and colleagues (2014) review indicated that professional learning could shift teachers’ beliefs about the value of inquiry-based teaching and teachers’ beliefs about their own efficacy. The PD offerings that produced these shifts used a variety of formats, including lesson study, video clubs, teacher research, study groups, and collaborative study of mathematics teaching. The evidence explored in Goldsmith and colleagues’ review would seem to indicate that there are multiple program formats that can produce changes in teacher beliefs. However, the program formats all share some common attributes: all were collaborative, all incorporated some form of artifacts of practice (e.g., classroom video or actual students), and all focused participants’ attention on mathematics teaching practice (Goldsmith, Doerr, & Lewis, 2014). These results indicate both what kinds of shifts in beliefs might be possible and suggest some common features of supporting PD that might help bring those shifts about.

In another study, Bobis, Way, Anderson and Martin (2016) investigated how fifth- and sixth-year teachers’ participation in a PD intervention focused on student engagement changed their beliefs about student engagement. The researchers used surveys, interviews, and video of
teachers’ participation in PD to establish a current state of teachers’ beliefs about student engagement and track changes in those beliefs over the course of the PD. The study found that teachers’ efficacy was an influential factor determining how subjects reacted to the content of the PD. Ultimately, only one of the three cases reported on in the study showed changes to beliefs about student engagement. The other two subjects developed an understanding of engagement as a general idea, but never actually translated that knowledge into a belief about how to engage students in mathematics, specifically. This study, while showing positive results, was also consistent with findings that suggest that the same PD intervention can affect individuals differently (Goldsmith, Doerr, & Lewis, 2014).

Finally, a study by Swan (2007) investigated changes in teachers’ beliefs and practices as they engaged in PD experiences designed around high-quality mathematics tasks. Swan used questionnaires, interviews, and subject statements to gain an understanding of subject beliefs at the outset of the study. He used a multidimensional framework to characterize the beliefs of study subjects as well. This framework, based in part on the work of Ernest (1989), considered teacher beliefs about mathematics as a school subject, beliefs about teaching mathematics, and beliefs about learning mathematics. A second set of descriptors (idealized and not mutually exclusive within an individual) were used to consider subjects’ orientations toward the three domains of teacher beliefs. These potential orientations were transmission, discovery, and connectionist (Swan, 2007, p. 226). Transmission-oriented individuals view mathematics as a large set of rules and procedures that must be learned via demonstration and repeated practice. Discovery-oriented teachers believe that mathematics is best learned through discovery and individual reflection, with the teacher playing a guiding, supportive role. Connectionists view mathematics as a giant web of interconnected ideas and concepts that is a co-construction between teachers and students.

Swan (2007) engaged 36 study subjects with PD activities involving novel mathematics tasks. Swan found that half of teachers who held a transmission orientation reported no changes in their beliefs. In fact, Swan noted that the some of the transmission-oriented teachers with the most extreme orientation in this regard believed that students could not learn in ways other than mimicking procedures they had observed. This group of teachers even disregarded experiences showing that students enjoyed such classroom events as mathematical discussions, citing inefficiency as an issue. Connectionist teachers, on the other hand, were supported and affirmed
by their experiences in the PD. Lastly, the entire group of teachers who began the PD with a
discovery orientation reported movement in the direction of a connectionist orientation by the
end. The second half of the transmission-oriented group reported varying results. Swan’s study
demonstrated the permeability of some belief systems to experience. The mathematical tasks and
facilitation methods successfully caused shifts in teacher beliefs in some cases. While there were
no strong positive correlations in the qualitative study, it does appear that—at minimum—a
teacher’s orientation toward mathematics teaching and learning has an effect on how they react
to and internalize PD experiences. There was also some evidence to suggest that particular
orientations were more resistant to change than others, specifically the transmission orientation
in this study.

Mathematics teacher beliefs have been the subject of research and debate for decades. The
field knows much about what constitutes mathematics teacher beliefs (Ernest, 1989;
Leatham, 2006; Phillip, 2007), how those beliefs influence practice (cf. Beswick, 2005; Cross,
2009), and how those beliefs can be challenged and changed (cf. Bobis, Way, Anderson, &
Martin, 2016; Cooney, Shealy, & Arvold, 1998; Thompson, 1984). In regard to the latter body of
knowledge, it would seem that there are many different options for ways to challenge beliefs,
many different kinds of experiences that can support changes in beliefs. Ultimately, though no
single format or combination of experiences can be guaranteed to produce change in every
participant in a PD, we can be reasonably sure that it is possible to support some changes in
some participants. Beliefs play an integral role in teachers’ instructional practice and, as such,
are an integral part of the teacher change process, which I take up next.

Mathematics Teacher Change

A number of studies have contributed to our understanding of teacher change. Clarke and
Hollingsworth (1994; 2002) engaged in thought-provoking discussions of the ways in which
teacher change can be conceptualized. They described six perspectives on teacher change: (1)
change as training, (2) change as adaptation, (3) change as personal development, (4) change as
local reform, (5) change as systemic restructuring, and (6) change as growth or learning. These
perspectives are not mutually exclusive, making it possible to hold multiple perspectives in mind
at a given time.
A Linear Change Model

Researchers have posited several models of teacher change over the past half-century. Guskey (1986), in a discussion linking teacher professional development and teacher change, noted that the majority of PD programs at the time were ineffective because of a failure to 1) account for teacher motivational factors and 2) accurately conceptualize the process by which teachers actually change classroom practice. He claimed that teachers are motivated to engage in PD because they believe it will help them become better teachers. There was little questioning of this state of affairs in his discussion. However, such was not the case for the process of teacher change. Guskey (1986) noted that typical staff development (PD) at the time aimed to create changes in teachers’ beliefs and attitudes as a way to initiate changes to classroom practices which would, in turn, influence student outcomes. This tacit logic model, linear in nature and based on a very particular set of assumptions about the order of change events in the change process, was the focus of Guskey’s critiques.

![Figure 2. An alternative model of the teacher change process. From “Staff Development and the Process of Teacher Change,” by T. R. Guskey, 1986, Educational Researcher, 15, p. 7. Adapted with permission.](image)

The assumption that teachers’ beliefs must change before they could change practice was, in Guskey’s (1986) view, not valid. Based on this, he posited an alternative to the traditional model. In Guskey’s new linear model (see Figure 2), practices changed before beliefs. Change was a learning process for teachers which emerged from experiences. He based this assumption on research findings suggesting that veteran teachers develop practices based on experiences in

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4 I will use the term change to represent a combined or interchangeable meaning for growth, change, or learning. Depending on one’s perspective about the nature of change (Clarke & Hollingsworth, 1994), one might use any of these terms.
the classroom—i.e., teachers adopt practices they have seen work (Lortie (1975), as cited in Guskey (1986)).

Guskey followed the discussion of his model of teacher change with three recommendations for PD providers. He called for 1) a recognition that change is a gradual process, and difficult, 2) teachers to receive frequent feedback on their students’ learning, and 3) initial trainings to be followed by continued support. Guskey argued there were many factors that cause teachers to be reluctant about making changes to their professional practice. Change meant risking failure. Further, changes that were too large were unlikely to be taken up, a factor related largely to the judgement of individual teachers about what constituted “too large” a change. Also, if teachers only changed beliefs after new practices showed efficacy in terms of student learning (as the model indicated), then naturally, teachers needed to be given more information about student outcomes. The third recommendation was related to the first. If the teacher change process was gradual and complex as Guskey (1986) maintained, then that process was unlikely to be supported by any single PD event. Hence the need for continued support after the initial training.

Guskey’s (1986) characterization of the teacher change process was, by his own admission, a simplified framework to be used in conceptualizing the “trends that appear to typify the dynamics of the teacher change process” (p. 7). He acknowledged that this simplification did not take into account many of the complexities of the process or the interrelationships among the components. Despite this, the model does appear to characterize the teacher change process as linear, a feature that Cobb, Wood and Yackel (1990) took issue with based on their own research experiences. Rather than a linear, temporally bound approach to characterizing the teacher change process, Cobb and colleagues (1990) maintained that beliefs and practices were “dialectically related” (p. 145). This relationship involved beliefs affecting and manifesting in practice and practice providing situations and experiences that could lead to a reorganization of beliefs. Ultimately, Cobb and colleagues (1990) did not say that Guskey’s (1986) model was incorrect; however, they did question the underlying assumption and attempted to clarify the relationship between the components in a way consistent with their research experiences.

Given the substantive critique of Guskey’s (1986) model of teacher change by Cobb, Wood, and Yackel (1990), there appeared to be a need to either reconceptualize the teacher change process or somehow accommodate the critique into the existing model. If the order of
change events was not strictly linear in a single direction and if the order of events was not static for any one individual, then a reconceptualization may have been inevitable.

The Interconnected Model of Teacher Professional Growth (IMTPG)

Clarke and Hollingsworth (2002) introduced and discussed the Interconnected Model of Teacher Professional Growth (IMTPG), shown in Figure 3, as a framework for describing teacher change. In their exposition, the researchers described several studies on which were used to develop an initial model from empirical data. The initial model was then reviewed and developed further by an international work group. This model builds on the earlier, linear model of teacher change of Guskey (1986), discussed above. The IMTPG eliminates the linearity of Guskey’s model in favor of an interconnected pattern which allows for multiple pathways of teacher change.

![Figure 3. The Interconnected Model of Teacher Professional Growth (IMTPG). From “Elaborating a Model of Teacher Professional Growth,” by D. Clarke and H. Hollingsworth, 2002, Teaching and Teacher Education, 18, p. 951. Copyright 2002 by Elsevier Science & Technology Journals. Reprinted with permission.]

Clarke and Hollingsworth (2002) consider the change environment to consist of four distinct domains: the External Domain, the Personal Domain, the Domain of Practice, and the Domain of Consequence. The External Domain contains all of the influences that come from
outside a teacher’s classroom (e.g., PD offerings, state assessment requirements, educator evaluation systems, school building context, etc.). The Personal domain encompasses, in many ways, the teacher as an educator, the teacher’s identity (e.g., the knowledge, beliefs, and attitudes held by the teacher). The Domain of Practice represents a teacher’s instructional practice, where he or she practices professional experimentation in the effort to learn which practices are more effective at supporting student learning (Guskey, 1986). Lastly, the Domain of Consequence encompasses all of the potential outcomes that might be relevant to a teacher’s practice. As Guskey noted, the content of this domain will vary by individual, but typically includes some form of increased learning by students.

One of the disadvantages of the simplicity of Guskey’s (1986) model was that it lacked detail in particular areas. While the areas of change were relatively intuitive and well-defined, the relationship between them was not. Although the domains of the IMTPG (Clarke & Hollingsworth, 2002) align closely with those outlined by Guskey, the IMTPG goes further and defines two so-called mediating processes: reflection and enaction. In the model, these two processes are represented by the dashed and solid arrows (see Figure 3) connecting the various domains in specific directions. Reflection and enaction are, in this case, the mechanisms that describe the relationships between the domains of the change process.

Specifically, Clarke and Hollingsworth (2002) note that “the term ‘enaction’ was chosen to distinguish the translation of a belief or a pedagogical model into action from simply ‘acting’, on the grounds that acting occurs in the domain of practice, and each action represents the enactment of something a teacher knows, believes or has experienced” (p. 951). This choice of terminology represents a way to bind the domains together, with change specified and evidenced by actions which originate in specific domains. Reflection, on the other hand, is a more internal process (Schön, 1983) than enactment, but no less vital to the process of teacher change. Both processes are necessary, but neither is sufficient for teacher change to occur (Cobb, Wood, & Yackel, 1990). The two exist in a dialectic relationship, specifying how change in a given domain is related to change in a second (Clarke & Hollingsworth, 2002)—it could be said that these processes, in fact, create the change process.

That reflection and enaction are integral to the change process can be seen from the visual representation of the model (see Figure 3). Some domains only have reflective links connecting them (e.g., the External Domain and the Personal Domain). Other domains have both
reflective and enactive links connecting them (e.g., the Personal Domain and the Domain of Practice). Lastly, there are instances where only enaction links two domains (e.g., the External Domain and the Domain of Practice). Thus, if one considers pathways through the model, any pathway must have at least one of the mediating processes within it. Any pathway of a complex nature, perhaps involving all four domains, would necessarily involve both mediating processes.

In detailing their model and the studies that went with it, Clarke and Hollingsworth (2002) also described the use of the IMTPG to identify and analyze patterns in teachers’ professional growth. They identified two kinds of patterns, each linked to the model: *change sequences* and *growth networks*. These constructs are rooted in patterns the researchers found in the empirical data and used to generate the model. *Change sequences* “[consist] of two or more domains together with the reflective or enactive links connecting these domains, where empirical data supports both the occurrence of change in each domain and their causal connection” (p. 958). It is clear from this definition that the authors intended for the IMTPG to be used empirically. Differentiating change sequences from growth networks seems to be a matter of identifying changes of a more lasting nature. Clarke and Hollingsworth (2002) indicate that “the term ‘growth’ is reserved for more lasting change” (p. 958), indicating that, to be termed a growth network, a given change must be shown (through empirical data) to be one that remains within a teacher’s professional world beyond a single instance or short series of instances. For example, a teacher’s experimentation with group work protocols for one week and the subsequent abandonment of them after a discovery that the complexities of group work can be difficult to manage might be considered a change sequence, but not a growth network. However, the case of a teacher who implements collaborative work protocols at the beginning of a school year and continually adjusts and improves those protocols throughout the year, enculturating students into this new manner of operating, might be considered evidence of a growth network.

Clarke and Hollingsworth (2002) give limited guidance on how to classify growth networks, merely noting that “[w]here data have demonstrated the occurrence of change is more than momentary, then this more lasting change is taken to signify professional growth. A change sequence associated with such professional growth is termed a ‘growth network’” (p. 958). This somewhat amorphous definition might be deliberate, as it allows researchers to choose the method by which they measure growth networks—essentially, any study wishing to identify
growth networks must carefully define what it means for change to be “more than momentary” (Clarke & Hollingsworth, 2002, p. 958).

The IMTPG is a flexible framework that describes a complex process of change. Its interconnected nature makes it more valuable as a research tool than previous, linear models. However, while Clarke and Hollingsworth (2002) pointed out several different ways the framework could be used, whether the model would be accepted by the research community remained to be seen.

Existing Research Using the IMTPG.

The IMTPG has been used with some success in Europe, specifically the Netherlands, with mathematics teachers (Witterholt, Goedhart, Suhre, & van Streun, 2012) and, more extensively, with science teachers (Eilks & Markic, 2011; Justi & van Driel, 2006; Voogt, et al., 2015). It has seen a more limited acceptance in the U.S. research community (Goldsmith, Doerr, & Lewis, 2014), although there are some recent exceptions (e.g., Perry & Boylan, 2018; Rubel & Stachelek, 2018).

European studies of science teachers’ change processes have shown valuable uses of the IMTPG as an analytic tool. Justi and van Driel (2006; 2005) examined five science teachers’ development over the course of engagement with a PD designed using the IMTPG as a framework. Justi and van Driel (2006) used the IMTPG in combination with a breakdown of the various aspects of Pedagogical Content Knowledge (PCK) (Schulman, 1987) related to models and modeling in science education. Further, the authors created a detailed description of the domains of the IMTPG and their meanings within the context of their project. This allowed them to be very specific as they identified the changes in PCK by teacher and mapped them onto the IMTPG. This mapping allowed them to identify change sequences and growth networks in their project data. Justi and van Driel (2006) defined change sequences in a manner similar to Clarke and Hollingsworth (2002), but were forced to modify the definition of growth network because they could not track changes over a sufficient length of time (the study lasted less than one year) to warrant using Clarke and Hollingsworth’s definition.

A separate study by Eilks and Markic (2011) focused on teacher change through engagement in a long-term participatory action research (PAR) project. The study involved fifteen science teachers over the course of six years and focused on changes in those teachers’ pedagogical content knowledge (PCK). Of particular relevance here is the use of the IMTPG in
combination with other frameworks, including Shulman’s (1987) PCK framework, and the methodology whereby subjects were asked to reflect on their experiences over the course of each year of the study. Not only were subjects asked to reflect on experiences, but they were also video recorded during their participation in the action research project.

In general terms, the Eilks and Markic (2011) study found changes in teachers’ pedagogical content knowledge and attitudes about teaching. Teachers also developed the habit and ability to examine their own and others’ teaching practices. The researchers were able to use the IMTPG as a tool to successfully track and describe the changes teachers experienced during the PAR project. This success is encouraging as it provides yet another example of effective use of the IMTPG as an analytical tool. Further findings from this study suggest that, over time, participation in action research—with its accompanying interactions between participants and researchers in a supportive setting—may have spurred the changes observed by researchers. Teachers’ reflections shifted over the course of the study from insecurity about sharing opinions and the purpose of the project (year 1) to a more reflective stance involving discussions of knowledge and meaning development (year 2) to confidence and the beginning of individual initiatives and critiquing of other colleagues within the schools (year 3 onward). The project showed that PAR can create both innovation and changes in teacher knowledge with respect to all five areas of the Pedagogical Content Knowledge portion of the theoretical framework. These five areas included teachers’ orientations toward teaching, curriculum knowledge, knowledge of assessment, knowledge of learners, and knowledge of teaching strategies (Eilks & Markic, 2011). This indicates that there is useful knowledge to be gained from participant interactions during PD of different forms. Also, the IMTPG played a prominent role in the analysis and findings. Eilks and Markic (2011) go so far as to note that “[t]his sustained interaction of the four domains taken from the IMTPG seems fundamental for the success and productivity of the project” (p. 157).

While the IMTPG has been used heavily in the Dutch science education community, it has seen relatively little uptake in the Dutch, or European, mathematics education community. There are two notable exceptions to this. First, a study by Witterholt, Goedhart, Suhre, and van Streun (2012) featured the IMTPG prominently. The centerpiece of their analysis was a modification of the IMTPG (shown in Figure 4) that allowed the authors to map specific change sequences and growth networks and display them in a compact, easy to read fashion.
Each of these pathways was supported by empirical evidence from their case study of one mathematics teacher. This teacher was part of a collaborative network consisting of three practitioners focused on creating classrooms in which students were able to engage in the statistical process from design to reporting. Witterholt and colleagues’ (2012) goal was to understand how the subject teacher’s practical knowledge changed over the course of participation in the network. An example might be of some assistance in understanding the use of the operationalized model.

The Witterholt et al. (2012) study indicated that the subject teacher underwent a cycle of change in the form of a chain $2 \rightarrow 5 \rightarrow 7 \rightarrow 9 \rightarrow 4$ (referring to the operationalization in Figure 4). In explaining this chain, the authors note that

Annet [the subject teacher] explored a new teaching strategy during the network meetings. She contributed to these meetings by using her practical knowledge that was partly based on her former experiences with the first teaching design (arrow 2). Arrow 5 represents Annet’s enactment of the design. Annet reflected on the consequences of that implementation, as represented by arrow 7, and decided that notable outcomes of the implementation of the design were improved student learning and motivation, and her increased satisfaction. Arrow 9 represents Annet’s reflection on the constituted outcomes of this exploration. This led to changes in practical knowledge regarding the value of this strategy (arrow 9), and consequently the inclusion of the strategy as a part of Annet’s practice. Arrow 4
represents the application of the new practical knowledge (via enactment) as a regular feature of Annet’s practice, to be further refined through Annet’s on-going experimentation and consequent reflection. In other words, arrow 4 means that decisions made by Annet during the lessons are based on her practical knowledge. (Witterholt, Goedhart, Suhre, & van Streun, 2012, p. 671)

Additionally, the authors were able to tie specific behaviors and comments to both the domains and the mediating processes of the IMTPG during their analysis.

Regarding results, Witterholt et al. (2012) were able to substantiate changes in the subject teacher’s practical knowledge. Not only that, but the authors were able to trace patterns involving both reflection and enaction throughout the project. They found that the subject teacher gained increased PCK, allowing her to become flexible in her implementation of a written lesson plan. The authors note that “[t]eacher change starts with the formulation of expectations and reflection on former experiences during the network meetings. The most important changes originate from the implementation of the teaching design in the classroom, by means of discrepancies between expectations and experiences” (Witterholt, Goedhart, Suhre, & van Streun, 2012, p. 671). This finding is consistent with the notion that PD which is effective at changing practice must be tied closely to that practice (Whitcomb, Borko, & Liston, 2009). It also provides a powerful example of the IMTPG’s usefulness and accuracy as a model of teacher change.

Another altogether different use of the IMTPG in mathematics appeared in a literature review by Goldsmith, Doerr, and Lewis (2014). Here we see another example of the flexible use of the IMTPG, where the authors used it as a conceptual framework to categorize the emergent codes developed in their analysis of the professional development literature. Beyond this initial use in the categorization process, the IMTPG does not appear prominently in the Goldsmith and colleagues (2014) literature review. However, the researchers position their review as contributing to the theorization of teacher change represented by the IMTPG. This use of the IMTPG was novel, allowing researchers to categorize results across multiple studies and formulate a set of recommendations for the field. The Goldsmith et al. (2014) study provided interesting and valuable insights which were detailed in the discussion of effective mathematics PD at the beginning of this chapter.

Finally, two studies published while this project was ongoing are worthy of note here, for different reasons. Rubel and Stachelek (2018) employed the IMTPG in the study of professional development, in the form of a “professional growth project (PGP) for secondary mathematics
teachers in urban schools” (p. 8). The PGP focused on “supporting mathematics teachers in increasing and diversifying classroom participation opportunities” (Rubel & Stachelek, 2018, p. 8). The authors used lesson observation narratives, interviews, teacher written reflections, and transcripts of teachers’ words from interviews and PGP sessions to develop understanding of how teachers learned to value diversity in student participation and create opportunities for student learning through diversifying student participation. The PGP “was initiated by a 40-hour summer institute and continued with 2-hour monthly meetings through the school year” (Rubel & Stachelek, 2018, p. 10) and focused on developing four dimensions of teachers’ practice:

(a) holding high expectations for Black and Latino/a students as learners of mathematics . . . (b) selecting cognitively demanding mathematical tasks . . . (c) connecting mathematics to students’ interests and experiences . . . and (d) providing opportunities for students to develop critical consciousness with mathematics . . . (Rubel & Stachelek, 2018, pp. 9-10).

The article in question featured case studies of two participants (one who showed significant change and one who did not) in which the IMTPG was used to track and visually map participants’ growth patterns in a very similar way to Witterholt and colleagues (2012). Ultimately, the authors conclude that the PD was effective and “supported teachers in making these changes [along the dimensions listed above]” (p. 20). Based on the case study findings, the authors concluded that “[t]his [PD] approach and these tools seem especially useful for school contexts dominated by passive classroom participation opportunities” (Rubel & Stachelek, 2018, p. 21).

Individual and Organizational Change

In reviewing and comparing the teacher change models of Guskey (1986) and Clarke and Hollingsworth (2002), I have focused on change models that define pathways of change. There are alternatives to this conceptualization of the process. One such alternative was leveraged by Kaasila and Lauriala (2010) to begin the work of developing a model of teacher change in which sociocultural and constructivist perspectives were combined. To accomplish this, the researchers drew on a number of theories of change. In the end, Kaasila and Lauriala (2010) derived a framework which consisted of three stages: unfreezing, moving, and refreezing. Further, each stage involved action at both the group level and the individual level.
The unfreezing stage at the group level involved three processes: 1) the accumulation of disconfirming data, 2) connecting that disconfirming data to the goals of change, and 3) creating a psychologically safe environment that prevented members of the community from becoming defensive and rejecting the disconfirming data. At the individual level, unfreezing required the presence of cognitive conflict or disequilibrium, which provides motivation to change.

After sufficient motivation has been created to ensure the change process begins (unfreezing), actual change (moving) must follow. At the group level, the moving stage involved persuasion and changes in beliefs. Specifically, teachers needed to be persuaded to take up new practices and that, by doing so, they were making themselves better. At the individual level, moving involves the creation of new conceptions and visions of mathematics teaching and learning.

The final stage, refreezing, aimed to institutionalize the changes that occurred during the moving phase. There is always a danger of backsliding and so the refreezing stage had to be structured to prevent that from happening. At the group level, refreezing involved the negotiation and establishment of new norms as ways of doing business. Refreezing at the individual level involved reflective analyses of the new approaches and an associated reorganization of beliefs. That is teachers needed to be convinced that the new beliefs and practices were worthwhile and made them better at their jobs.

Kaasila and Lauriala’s (2010) were able to successfully link the sociocultural and constructivist viewpoints into a multi-dimensional framework describing teacher change. This multi-dimensionality was seen as an advantage over other models by the researchers. The same is true of the IMTPG (Clarke & Hollingsworth, 2002) in that it describes change as a process involving change in multiple domains and through multiple mechanisms. While Kaasila and Lauriala’s (2010) framework is perhaps more comprehensive in that it accounts for group level change as well as individual change, the two are not entirely different. For example, reflection and enaction are both present in Kaasila and Lauriala’s (2010) model, reflection at the individual refreezing stage and enaction at the moving stage (as they explore and experiment with new practices). While the two models appear very different on the surface, because they describe the same process, they cannot be completely dissimilar. Indeed, they share at least one common feature: reflective activity.
Mathematics Teacher Reflection

Reflection has long been understood to be a vital part of the teaching process. Indeed, reflection is considered a vital component of expertise in general (Schön, 1983). Schön developed the ideas of reflection-on-action—reflection after action has occurred—and reflection-in-action—reflection that occurs during the action that is the object of reflection. Clarke (2000) leveraged and extended this idea to studying reflection in, on, and for practice. Reflection in practice, in Clarke’s view is quite similar to Schön’s (1983) conception: reflection and action, often simultaneously, in the context of a classroom or learning environment. Reflection on practice involves reflection on one’s own or others’ actions after those actions have occurred and, oftentimes, in contexts outside of those in which the actions took place. This idea is again similar to Schön’s (1983) conceptualization. Reflection for practice, on the other hand, is different. This kind of reflection involves reflection with consequences in practice, not simply for the sake of reflecting. For Clarke (2000), reflection for practice implies a change or refinement of practice as a consequence of reflection.

Ricks (2011), in a study of reflection during Japanese lesson study, extended the work of Schön (1983) by deriving two types of reflection: incident reflection and process reflection. Incident reflection refers to reflective activity that is specific, oftentimes spontaneous, and unconnected to concerns about future activity—that is, reflection that never results in action. Ricks (2011) defined process reflection based on Schön’s (1983) conception of reflection: reflective activity that is part of a cyclic process of building knowledge and changing ideas through experimentation. In short, incident reflection is passive and unconnected (to both action and other reflective events), while process reflection is a coherent set of reflective activities connected by actions. Ricks argued that incident reflection is the most common of the two forms. The reality of the complexities of teaching would seem to imply that he was correct. Reflection in the moment—reflection in action—involves very little time for the reflective act or for connected action afterward.

In his review of literature, Ricks (2011) noted that the field does not have a common conception of what reflection is or how or why teachers engage in it. In an attempt to address this concern, I have identified a definition of reflection that seems to plausibly fit reflection on action. Stockero (2008), in studying changes in reflective activity of pre-service teachers, used a definition of reflection related to teaching adapted from Rogers (2002): “analyzing classroom
events in order to identify often subtle differences in students’ mathematical understandings and the ways in which the teacher’s actions contributed to them” (Stockero, 2008, pp. 374-375). Further, we may then define a reflective stance as the ability to reflect in a manner consistent with the aforementioned definition (Stockero, 2008).

There is a significant body of literature on the use of video in teacher training (see the discussion above). Some of this literature has attempted to shed light on the interplay between video as a representation of practice and reflection by developing theory to explain and examine the construct in various contexts. For example, a study by Karsenty and Arcavi (2017) used a framework of six “viewing lenses” (p. 436) to structure and interrogate how teachers’ reflective activity occurred as they watched video lessons as part of a lesson study. The six lenses in the model captured large-scale mental orientations that shaped the focus of teachers as they viewed recorded lessons:

1. mathematical and meta-mathematical ideas around the lesson’s topic;
2. explicit and implicit goals that may be ascribed to the teacher;
3. the tasks selected by the teacher and their enactment in class;
4. the nature of the teacher–student interactions;
5. teacher dilemmas and decision-making processes; and
6. beliefs about mathematics, its learning and its teaching as inferable from the teacher’s actions and reactions (Karsenty & Arcavi, 2017, p. 437).

The researchers intended these lenses to become an internalized part of teachers’ reflective processes and a common language to discuss practice over the course of the lesson study PD. Ultimately, the study found several types of change in teachers. Participating teacher changed their perspectives about the value and use of video to learn about practice. Teachers also noted influences on their practice. Specifically, there were two types of influences: declared influences involved teachers making statements about how their practice had changed, while intended changes involved teachers discussing future actions they might take based on a given influence. The remaining effects were related to the evaluation of the PD program and do not bear relevance to this discussion.

While the two studies discussed above extend our understanding of the construct of reflection, other studies have developed theories with more fine-grained detail. For example, in the course of engaging teachers in examining practice during a video club, van Es and Sherin (2008) detailed five dimensions of teacher reflection. The first dimension identifies which Actor in the clip was the focus of teachers’ comments. In the case of the van Es and Sherin (2008)
study, the options for *Actor* were student, teacher, or other. *Topic* was the second dimension, detailing “what teachers noticed” (van Es & Sherin, 2008, p. 250) as they viewed the video clips during their interviews. Teachers might have noticed such things as pedagogical moves by the teacher, classroom climate and management, students’ mathematical thinking, or even other, unrelated issues. The third dimension, *Stance*, identified teachers’ orientations as they analyzed instances of practice. Specifically, stance describes whether teachers described, interpreted, or evaluated what they were seeing. The fourth dimension qualifies the level of *Specificity* (either general or specific) in teachers’ comments. Lastly, the fifth dimension indicated whether teachers commented on what they saw in the video or on “events outside of these segments” (van Es & Sherin, 2008, p. 250). The remainder of van Es and Sherin’s findings were discussed in detail in the review of PD literature above.

In another study focused on reflective activity, Stockero (2008) adapted the work of van Es and Sherin (2008). In her analysis of a video-based teacher preparation curriculum, Stockero (2008) used a framework for reflection that included four attributes of reflection: *agent, topic, grounding, and level* (p. 376). Instead of using the categories of Stance defined by van Es and Sherin (2008), Stockero (2008) introduced five levels of reflection as defined by Manouchehri (2002)—describe, explain, theorize, confront, and restructure. While similar to the levels defined by van Es and Sherin (2008), these five levels allowed her to develop a more detailed analysis of the data.

Manouchehri’s (2002) levels of reflection require some expansion. The two-fold goals of Manouchehri’s (2002) study involved first the investigation of the discussions of two prospective secondary mathematics teachers who were involved in a field experience. The second goal was the study of how those discussions changed (or not) the ways in which the PSTs analyzed mathematics teaching and student thinking issues. In this case study of two PSTs, Manouchehri (2002) derived five “layers of reflection” (p. 723) during data analysis. The descriptions of these five layers can be found in Table 2.

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5 I will use the terms *layers* of reflection and *levels* of reflection interchangeably in this study. The organization is hierarchical in the sense that, for example, *restructuring* is a more sophisticated kind of reflection than *describing*. However, one might make the argument that all have their place in the reflective process (cf. Stockero (2008)). Due to Manouchieri’s (2002) use of *layers*, this term appears more often than *levels* in my literature review.
Table 2
Layers of Reflection

<table>
<thead>
<tr>
<th>Level of reflection</th>
<th>Description</th>
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| Describing          | Story telling  
|                     | Recall of classroom events |
| Explaining          | Connecting interrelated events  
|                     | Exploring cause and effect issues related to teaching/learning actions |
| Theorizing          | An explanation of how one knows what she knows  
|                     | References to research on learning and teaching  
|                     | References to past experiences  
|                     | References to past course work and reading |
| Confronting         | Search for finding alternative theories to explain events and actions  
|                     | The individual challenges her own views and what she knows in the process of search  
|                     | Is there another way of looking at this?  
|                     | Is there another way of explaining this?  
|                     | Did I do it right?  
|                     | Am I right in my assumptions?  
| Restructuring       | Revisiting the event with the intent to re-organize teaching actions and/or curricular choices  
|                     | What can I do differently?  
|                     | How can I do it differently?  
|                     | What should I change?  
|                     | What else should I do? |


Manouchehri (2002) found that over the course of 11 weeks of discussion and interaction during the field experience, the two PSTs shifted their levels of reflection “from describing-explaining, to a theorizing and restructuring phase” (p. 723). Manouchehri’s (2002) study demonstrates that it is possible to track changes in teachers’ reflective processes and habits over time.

Teacher reflection, like teacher change is a complex, multifaceted process. Researchers have made significant progress in developing models to study reflection in, on, and for practice (Clarke, 2000). These models attempt to detail the complex workings of reflection, particularly of reflection on practice (Clarke, 2000; Schön, 1983). Much of the work of researching reflection has been done in the context of teachers reflecting on video examples of practice (e.g., Karsenty & Arcavi, 2017; Schoenfeld, 2017; Stockero, 2008; van Es & Sherin, 2008) This is only natural, as video is useful tool for studying the complexities of teaching practice, which inherently
involves reflection on that practice (Clarke, 2000). In the end, what we believe teaching to be influences how we attempt to study it. In particular, if teaching and reflection are intimately related, then studying them together is an idea with potential merit.

Theoretical Framework

This study began with a question that emerged from my professional practice: How do teachers change as they engage in professional development in mathematics? This question fundamentally drove both my inquiry and my decision-making process after the initial conception of the idea of studying teacher change in some detail. In deciding upon the content and structure of the theoretical framework for this study, several things were considered. Primarily, I considered the nature and focus of my intended inquiry: the teacher change process related to a professional development intervention. Based on this, I also considered my views on the nature of the change process. I chose to focus on teacher change as growth or learning (Clarke & Hollingsworth, 1994; 2002) for two reasons. First, this perspective aligns most closely with my orientation as a researcher (Schoenfeld, 2011) and professional developer. I fundamentally believe change to be a process of growth and learning and I believe that conceptualizing it in this way allows me to better study a process than another orientation such as change as training. Second, the Change as Growth or Learning perspective envisions change as an inevitable result of professional activity and participation in a learning community (Clarke & Hollingsworth, 1994), which aligns with the goals and intentions of the PD that is foundational to this study and, conveniently, the study itself.

With my views on the nature of teacher change solidified and supported in the research, I proceeded to consider which theoretical frameworks might best aid me in my inquiries.

A Framework for Teacher Change

The IMTPG is useful in studying teacher changes in relation to PD experiences. In particular, the SMII PD involved significant experiences for subjects in mathematics content and pedagogy, thus embedding it firmly in the External Domain of the IMTPG. The PD was designed to challenge teacher beliefs and support changes in instructional practice, elements of the Personal Domain and Domain of Practice, respectively. The PD also included several cycles of modified lesson study which, in addition to making teaching practice a focus of study, also allowed subjects to reflect on how their students reacted to any changes in instruction and how
those students may have learned or acted differently. The reactions of students, their differing participation patterns, and altered learning all reside in the Domain of Consequence. Further, the modified lesson study naturally engaged subjects in cycles of reflection and enactment, increasing the likelihood that my analysis would be able to both track changes in teachers throughout the study and map those changes in such a way as to allow change pathways—change sequences, growth networks, or even other categories of change—to be illuminated and interrogated in detail from the data. Because of the structure of the PD and the comprehensiveness of the teacher change framework, the IMTPG served well as an overarching theoretical framework for this study.

In considering the other two teacher change frameworks reviewed above, I noted some affordances, but ultimately too many concerns to warrant using either. The framework of Kaasila and Lauriala (2010), while in alignment with my social constructivist leanings as a researcher, involved a good deal of attention to organizational change, which I had no intention of studying. Further, the detail within the individual levels of change was also not sufficient for my needs. Guskey’s (1986) framework, given the arguments of Cobb and colleagues (1990), was also not suited to my purposes. Further, while Guskey (1986) and Cobb, Wood, and Yackel (1990) indicated that, despite the linearity of their models, change in practice happens in a more cyclical fashion, neither set of researchers made any systemic, specific attempt to describe the mechanism by which those changes happen. Clarke and Hollingsworth (2002) were much more specific in naming and describing those mechanisms, making their framework more useful for this study. Not only that, but the interconnected, non-linear nature of the IMTPG contained both of the patterns of change Guskey (1986) discussed and dealt with the concerns of Cobb and colleagues (1990) about the dialectic relationship between beliefs and practice.

For example, in looking at the visualization of the IMTPG in Figure 3, one can trace each of the pathways discussed and debated by Guskey (1986) and Cobb, Wood, and Yackel (1990). The change process might begin with an influence from a PD (in the External Domain), proceed reflectively to change teachers’ beliefs and knowledge (the Personal Domain), causing them to enact changes in their teaching practice (the Domain of Practice), reflection on that implementation might cause teachers to notice different—hopefully improved—outcomes for students (the Domain of Consequence). Conversely, the process might begin in the External Domain (e.g., with a PD or an article), proceed into the Domain of Practice (the teacher
experimenting with the new idea by enacting it, even if unsure), from which outcomes in the Domain of Consequence (e.g., reflection on practice leads to noticing of improved student learning) causes a change or reorganization of knowledge and beliefs (the Personal Domain). The fact that both pathways were contained within the IMTPG and both reflection and enaction linked the Personal Domain and the Domain of Practice made the IMTPG a solid choice.

Further, Witterholt and colleagues (2012) had taken up the IMTPG and operationalized it in such a way as to make it even more useful for the purposes of this study (recall Figure 4). The operationalization made it possible to describe change patterns in a compact way, which had the potential to make cross-case analysis easier. In particular, the ability to consider patterns in how the various change pathways looked, in addition to their content, would add a vital visual aspect to the analysis process. Consider the change pathway that Guskey criticized in his 1986 exposition and outlined first in the discussion above. Using Witterholt and colleagues’ notation, this pathway would appear as follows: $1 \rightarrow 4 \rightarrow 7$. Arrow 1 represents a reflective action connecting change in the External and Personal Domains. Arrow 4 represents enaction from the Personal Domain into the Domain of Practice. Arrow 7 represents reflection on the results of the implementation in the Domain of Practice that leads to noticing a change in the Domain of Consequence. From there, one might hope that further reflection would lead to a reorganization of knowledge and beliefs in the Personal Domain (Arrow 8). This hypothetical pathway would then represent a single cycle of reflection and enactment initiated by an external influence on the teacher in question. The combination of the IMTPG and Witterholt and colleagues’ (2012) operationalization allows for tracking and mapping very complex change patterns. Given the complexity of teacher change and its interactions with PD, this capability was vital to work in this study.

Finally, each of the studies using the IMTPG contributed to the field’s overall knowledge of teacher change, with respect to a particular content area (i.e., science or mathematics). This study aims to do the same. The main goal of this study is to gain an understanding of the teacher change process, with an additional focus on reflection on practice. As mentioned previously, Clarke and Hollingsworth integrated reflection into their model of teacher change, but did not explore or define its meaning. This was doubtless to increase the usefulness of their framework; researchers could integrate any additional frameworks to study reflection (or enaction) as they
wished. This made the framework more flexible and available in a broader range of contexts than if reflection had been specifically defined.

Further, Justi and van Driel’s (2006) approach served to inform some of the decisions in this study. Their careful definitions and mappings provided an informative lens through which to view my own analysis process. It seemed that pairing domains of the IMTPG with a framework for reflection (discussed below) might be a fruitful path to take during analysis and reporting. This would allow me to create a clearer picture of the change process writ large. After all, the domains of the IMTPG are only half of the description the model provides of the process of teacher change—reflection is also an integral component to the process. A reflective framework would allow me to begin to understand one of the mediating processes while still allowing for robust descriptions of changes within each domain of the IMTPG.

Justi and van Driel (2006) were able to substantiate strong findings indicating that the External Domain, in this case the PD they provided teachers, influenced, to varying degrees, the practices and knowledge of teachers. This finding is also consistent with Goldsmith and colleagues’ (2014) findings that the effects of PD programs generally vary by individual and by specific context. This connection pointed to the fact that I might do something similar in my own analysis. The goal of any PD is to influence teachers in some way (Guskey, 1986; Guskey & Sparks, 2000), but PD is not the only influence on teachers in these situations (Wilkie, 2019) and so an analysis of the influences on teachers’ change processes would also fit within the scope of my inquiry. The IMTPG, with its domains four of teacher change (External, Personal, Practice, and Consequence), would aid in this analysis.

This study aims to expand on the IMTPG by exploring the role of reflection in the teacher change process and how it mediates the transition between the domains of the IMTPG. As discussed above, the specific focus on reflection in this study requires the addition of a second theoretical framework in combination with the IMTPG, this one regarding teacher reflection on practice.

A Framework to Study Reflective Activity

As discussed in the literature review, reflection is an integral part of the teacher change process (Boylan, 2010; Clarke & Hollingsworth, 2002; Cobb, Wood, & Yackel, 1990) and the profession of teaching itself (Hollingsworth & Clarke, 2017; Karsenty & Arcavi, 2017). Further, consistent with those understandings, reflection is a mediating process in the IMTPG (Clarke &
Hollingsworth, 2002) which served as the overarching theoretical framework for this study. Given the centrality of reflection to the change process and teaching, I looked for frameworks that provided a significant level of detail about the nature of reflection as well as multiple categories that might allow for the characterization and categorization of reflective activity during analysis. Many studies I reviewed focused on reflective activity during the analysis of video examples of practice. This was a positive thing, as the PD intervention in this study involved exactly that sort of reflective activity. Studies by Stockero (2008), Ricks (2011), and Karsenty and Arcavi (2017) all provided options of useful frameworks.

Before diving too deeply into the frameworks, it is worth noting—as Ricks (2011) did—that there is no agreed upon definition of reflection in the literature. As such, I sought out studies that provided both a framework and a complementary definition that was consistent with the context of my study. Stockero (2008) used a definition adapted from Rodgers (2002) which was consistent with both the intent of this study and the intent of the lesson study portion of the SMII PD intervention (the features of the SMII PD are detailed in Chapter 3). Stockero’s definition, reviewed in the literature on reflection above, served as my definition as well.

Ultimately, I decided to use Stockero’s (2008) framework for studying reflective activity. It provided the necessary level of detail for my needs and was developed in a context similar to that of my study. The context of Ricks’ (2011) bore, perhaps, more similarity to mine, but his reflective framework did not have the detail that I desired. Similarly, Karesenty and Arcavi’s (2017) framework was designed to study and support teacher reflective activity during lesson study. However, their framework was too broad for my uses as the modified lesson study component of my study was focused on teachers supporting students’ explorations of a task. As such, teachers would be viewing portions of a lesson, not entire mathematics lessons.

The theoretical framework for reflection that will be used in this study is a combination of van Es and Sherin’s (2008) four attributes of reflection—agent, topic, grounding, and level—and Manouchehri’s (2002) five levels of reflection—describe, explain, theorize, confront, and restructure. This combination was devised and used successfully by Stockero (2008) as discussed in the literature review. The framework was useful in categorizing various aspects of reflective activity (e.g., identifying the focus and level of the reflection). The ability to categorize in this way would allow for an analysis of changes in reflection over time. This framework would allow for the identification of patterns in the development of teachers’ reflective activity connected to
the PD context. Further, this sort of analysis might give some insight into the potential of certain PD formats to increase reflective activity generally and, potentially, higher levels of reflective activity specifically.

Summary

The theoretical framework used in this study is complex, involving two distinct frameworks to study a complex process of teacher change. Table 3 attempts to provide a summary of the components and sub-components of the framework.

Table 3
A Framework for Studying the Teacher Change Process

<table>
<thead>
<tr>
<th>Teacher Change</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Domain</td>
<td>Influences and events from outside a teacher’s practice (e.g., PD events, research readings, school context, collegial relationships)</td>
</tr>
<tr>
<td>Personal Domain</td>
<td>The knowledge, beliefs, attitudes, and dispositions of the teacher.</td>
</tr>
<tr>
<td>Domain of Practice</td>
<td>Events within the teacher’s classroom instruction; professional experimentation</td>
</tr>
<tr>
<td>Domain of Consequence</td>
<td>Outcomes deemed important by the teacher (e.g., increased student learning or participation)</td>
</tr>
<tr>
<td>Enaction</td>
<td>The process of translating a belief into action or applying a piece of knowledge in practice</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Foci</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent</td>
<td>Teacher</td>
<td>Comment focused on teacher actions</td>
</tr>
<tr>
<td></td>
<td>Student</td>
<td>Comment focused on student actions</td>
</tr>
<tr>
<td>Topic</td>
<td>Pedagogical issue</td>
<td>Comment focused on pedagogical issues</td>
</tr>
<tr>
<td></td>
<td>Student thinking</td>
<td>Comment focused on student thinking</td>
</tr>
<tr>
<td>Grounding</td>
<td>Grounded</td>
<td>Comment supported by evidence from video or readings</td>
</tr>
<tr>
<td></td>
<td>Ungrounded</td>
<td>Comment not supported by evidence</td>
</tr>
<tr>
<td>Level</td>
<td>Describing</td>
<td>Storytelling</td>
</tr>
<tr>
<td></td>
<td>Explaining</td>
<td>Connecting interrelated events/exploring cause and effect</td>
</tr>
<tr>
<td></td>
<td>Theorizing</td>
<td>An explanation of the warrants for knowledge</td>
</tr>
<tr>
<td></td>
<td>Confronting</td>
<td>Searching for alternative theories to explain events and actions</td>
</tr>
<tr>
<td></td>
<td>Restructuring</td>
<td>Considering how to re-organize teaching actions</td>
</tr>
</tbody>
</table>

Note: The elements of the teacher change framework listed here are those in the Interconnected Model of Teacher Professional Growth (Clarke & Hollingsworth, 2002) and the framework for reflection is that used by Stockero (2008).

This study aimed to understand the change processes that teachers underwent while engaged in and supported by a particular PD intervention. Further, the study aimed to understand
the influences on those change processes and the ways in which teachers’ reflective habits changed over time. The PD intervention in question focused on changing teachers’ practice toward a more student-centered learning approach consistent with the recommendations of NCTM (2000; 2014). While the features of the PD are outlined in detail in Chapter 3, the fact that the PD involves a lesson study component and experiences designed to challenge teachers’ beliefs and practices as well as supporting them in implementing a robust exploration phase of a problem-based lesson in mathematics provides some indication of what types of changes might emerge from the analysis in this study.

The IMTPG is an umbrella framework intended to help encapsulate and describe, in detail, the change processes that subjects underwent. The various possible constructs that could involve change were contained within the IMTPG. For example, teacher knowledge resides within the Personal Domain, as do teacher beliefs. However, the focus of this study on understanding the change process implies the need for a different focus than many of the instruments that measure each of these constructs might allow. Changes in teacher knowledge can be qualitatively mapped and described without the use of pre- and post-assessments of domains of MKT. Changes in teacher beliefs emerged from the data without an *a priori* structure to measure those changes (which were outside the scope of this study).

The theoretical framework presented above, with its two components, is designed to aid and structure the qualitative description of the change processes of teachers in this study. From the research describing the effects of mathematics PD, it is reasonable to expect to see changes in the Personal Domain. Further, the modified lesson study component increased the likelihood of changes in the Domain of Practice and the Domain of Consequence would also appear. The component of the framework devoted to reflection was intended to take a more detailed initial look at one of the mediating processes of the model, a key component to the teacher change process. That enactment was not also included as an intensive focus was simply a logistical decision about the distribution of time, resources, and focus in this study—I chose to measure enactment as integrated into the larger process of teacher change.
CHAPTER 3
METHODOLOGY

Given the relative infancy of the field of research on teacher change during and through mathematics professional development, it seems wise to begin with what Borko (2004) recommended in outlining a research program for demonstrating the effectiveness of PD programs at scale. Borko (2004) noted that PD activities located in what she referred to as Phase 1 should aim to “create an existence proof; that is, to provide evidence that a professional development program can have a positive impact on teacher learning,” requiring that “[r]esearchers study a single professional development program at a single site, [exploring] the nature of the professional development program, teachers as learners, and the relationship between teachers’ participation in professional development and their learning.” (p. 5). She also noted that oftentimes studies in this phase of the research program focus on PD offerings that are small, designed by the researchers, and involve labor-intensive study (Borko, 2004). Borko (2004) also noted that in subsequent phases, the initial study of a particular program at a particular site has the potential to become part of a larger-scale series of studies which develop a case for a program’s effectiveness in more general ways. Given the nature of this study, it seems clear that it fits well into Borko’s (2004) Phase 1 space.

Given the conditions of this study, the research questions, and its placement within Phase 1 of Borko’s (2004) proposed research program, I chose a research design that would facilitate the study of teachers as learners while at the same time providing some insight into the relationship between potential teacher learning and the features of the PD.

Research Design

Given the recommendations by Borko (2004) and my focused interest in changes related to reflective processes, I chose to employ a multiple case study methodology (Creswell, 2013; Stake, 2006). The in-depth analysis and insight provided by a case study makes sense as the basis of further research. Creswell (2013) noted intent as a vital factor to consider when choosing a methodology. The intent of this study was to gain an understanding of the ways in which professional development affected mathematics teacher reflective processes and habits as well as how the PD influenced teacher beliefs and practices—a study with the aforementioned sort of
intent corresponds to what Creswell referred to as an *instrumental case* (Creswell, 2013, p. 98). The instrumental case (or cases) are chosen to “best understand the problem” (Creswell, 2013, p. 98), which in this situation was changing in-service teacher practice and reflective habits through professional development.

While Creswell referred to the focus of a multiple case study as “the problem,” Stake (2006) takes pains to build the multiple case study methodology out of individual cases. He notes that a certain similarity among the cases is necessary. However, Stake reminds his readers that the primary objective of case study work—both single case study and multicase—is to develop a deeper understanding of the case. This focus on the individual cases must be balanced against developing understanding of something larger. Here, the cases were three secondary mathematics teachers engaged in a common mathematics PD experience. This similarity helped enable an intense focus on the case: the change processes of the study subjects. Stake (2006) introduces a new word into the lexicon to describe the collection of similar cases that make up a multiple case study: “quintain (pronounced kwin’ton) . . . an object or phenomenon or condition to be studied” (p. 6). While I will not use this term frequently in the following pages—even Stake acknowledges that the word is unlikely to be taken up widely—I introduce it here to highlight the centrality of this idea to multiple case study research. While a researcher (or reader) may become engrossed within a particular case in a multiple case study, it is vital to remember that the purpose of every aspect of a multi-case study is *to better understand the quintain*—the process or phenomenon under study. It is important to note that, in this study, the aim was to create a deeper understanding of the process of teacher change related to a specific professional development intervention. Multiple case studies build this understanding through consideration not of the process itself, but rather a consideration of the cases as both individual and collective sources of information about the process. In Stake’s (2006) words “[a] multicase study of a program is not so much a study of the quintain as it is a study of cases for what they tell us about the quintain” (p. 7).

The idea of a multiple case study is appealing in this situation because of the complex nature of the study setting and the phenomenon being studied. A single case of a teacher changing as a result of being involved in a professional development program is not enough, simply because of the widely varying nature of teachers’ experiences. Further, single-subject studies already exist (see, for example studies by Chapman and Heater (2010) and Boylan
(2010)) and do not provide the contrast or detail that a multiple case study might. This is particularly true if we wish to develop ideas and applications beyond the individual case. The instrumentality (Creswell, 2013; Stake, 2006) of the cases allows for a certain amount of generalization or generation of theory. Thus, a multiple case study with three or more primary participants, each with varying degrees of teaching experience, had the potential to provide both a more complete description of the phenomenon and contrasting viewpoints. These contrasting viewpoints would, in addition to being informative, lay the groundwork for further studies. Additionally, any themes derived from the data of this study would be more robust than in a single-subject case study and have a greater level of transferability for the field, albeit still limited by the small number of teachers studied.

Lastly, there is a certain alignment between the multiple data sources needed for an effective case study and those that can be collected throughout the SMII program. Creswell (2013) points out that a hallmark of a case study is its presentation of an in-depth understanding of the cases in question. This is achieved through the use of multiple, detail-oriented data sources such as interviews, observations, documents, and other materials germane to the situation studied (Creswell, 2013). Each of these data types can be found within the SMII program, as most of them (with the exception of interviews) were integral parts of the professional development design.

Research Questions

In order to meet the dual purposes of investigating changes in teachers’ reflective processes and attempting to link these to program features, this study focused on the following research questions to drive the inquiry:

1. What do teacher change processes associated with the SMII PD intervention look like?
2. What factors influence teachers’ change processes as they engage in the SMII PD intervention?
3. How does teacher reflection change during the SMII PD intervention?
Setting

This study took place as part of a PD offering at a rural Intermediate School District (ISD) in a Midwestern state. The ISD is a county-level education agency that offers services (such as consultant and center-based educational services) that local districts cannot provide independently. As an employee of this ISD, it is my responsibility to provide PD and consultant services for 8 rural school districts within the state which range in size from 24 to approximately 3,000 students. SMII is a professional development offering provided under the auspices of that ISD. All PD sessions occurred at ISD facilities (permission was obtained to conduct research on the premises). The facilities are no more than 20 miles from any local district buildings.

Study Subject Recruitment

As an initial step, the SMII PD was advertised heavily across the ISD during the prior school year and during the month of August 2017. Unfortunately, no teachers signed up for the PD initially. Due to this lack of participants and subjects, I was forced to depart from the established recruitment procedures. Subjects were initially recruited via email both at the end of the 2016-2017 school year and again at the beginning of August 2017. When no participants or subjects presented themselves, I resorted to personal appeals at department meetings throughout my work area. These meetings were established through interaction with each district’s director of curriculum and with the building principal. Unfortunately, this meant that I was unable to attend these meetings until after the school year had begun. Therefore, the original design of SMII, with a 4-day Summer Academy, had to be modified.

After my meetings with area secondary mathematics departments, I gained five participants for the SMII PD (all from the same district), three of which agreed to take part in the study. These three subjects became the focus of this research project. Participants expressed interest in becoming study subjects via emails to me. After the initial contact, I worked with each potential subject to set up a date and time for an initial interview (again via email). Each study subject signed the consent paperwork and video release forms at the outset of the initial

6 Hereafter, I refer to those teachers taking part in SMII who agreed to participate in this study as subjects. The term participant will refer generally to those teachers who attended the PD intervention.
interview. No stipend was offered for any registrant; however, the PD sessions were held on school days, so participants were compensated via their district salary for their time.

Eligibility

In order to be eligible to take part in this study, subjects were required to meet all of the following criteria:

1. Attend a minimum of 6 of the 8 SMII PD sessions
2. Provide a signed consent form
3. Sign a video release form
4. Teach mathematics in grades 6-12

These criteria were in place to ensure the integrity of the study. In order to fully assess any changes that occur through participation in the PD offering, subject data from the full range of PD sessions and activities was necessary. There were no additional criteria for participation in this study beyond those mentioned above. Participants in SMII were strongly encouraged to sign the video release form to ensure that everyone (both students and teachers) was protected when viewing video of one another’s teaching. No subject or participant refused to sign the video release form. If a participant had refused to sign, then he or she would have been exempted from submitting video for the lesson study portion of the PD; however, said participants would still have been eligible to engage in video analysis of others’ practice. Further, as mentioned above, any participant who refused to sign a video release form would have been ineligible to participate in this study.

Prior to the study, I established criteria for determining removal of subjects. These were, again, designed to ensure the integrity of the study. I attempted to balance this need against an understanding that participants are humans, with lives outside of this work and with influences beyond their control. As such, any subject who met any of the following criteria would have been excluded from this study:

1. Refusal to sign Consent Form
2. Refusal to sign Video Release Form
3. Failure to attend at least six of the eight SMII PD sessions

In the end, no study subjects met the criteria for removal. There were two absences due to family emergencies, but no subject missed more than 1.5 PD sessions.
Subject Descriptions

As mentioned above, all three study subjects worked in the same mathematics department at a local high school. Two of the three had similar levels of experience and the third teacher was a veteran educator near the end of his career (he retired one year after the completion of this study). Table 4 contains a summary of study subject characteristics and experience. All three subjects were white, college educated teachers holding valid teaching certificates. More detail about each subject appears in the case study reports in Chapter 4.

Table 4

<table>
<thead>
<tr>
<th>Study Subject</th>
<th>Gender</th>
<th>Teaching Experience (years)</th>
<th>Courses taught in recent years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeremy</td>
<td>Male</td>
<td>7</td>
<td>Algebra 1, Geometry, Basic Algebra 1, Basic Geometry, Algebra Support</td>
</tr>
<tr>
<td>Rebecca</td>
<td>Female</td>
<td>8</td>
<td>Algebra 2, Precalculus, Trigonometry, Basic Algebra 2, Algebra 2 Support</td>
</tr>
<tr>
<td>Terry</td>
<td>Male</td>
<td>27</td>
<td>Geometry, Physics, Earth Science, Precalculus, Basic Geometry, Personal Finance</td>
</tr>
</tbody>
</table>

Subject Context

Jeremy, Rebecca, and Terry taught in a comprehensive high school that served approximately 680 students in grades 9 to 12. The school was also relatively racially homogeneous, with 91 percent of students identifying as white and no other racial subgroup containing more than 3 percent of the student population. The school was also nearly evenly split along gender lines, with 49.5% of students female and 50.5% of students male. Thirty percent of students qualified for the federal free and reduced-price lunch program during the 2017-2018 school year.

Secondary Mathematics Instructional Institute (SMII)

The Secondary Mathematics Instructional Institute (SMII) is a PD program designed to focus secondary mathematics teachers’ attention on pedagogical aspects of instructional practice. SMII is a collaborative effort between two mathematics PD specialists and myself as part of our duties as regional professional development providers. In preparation for the design phase of this

\(^7\) All subject names are pseudonyms.
project, I reviewed the literature on effective professional development. These recommendations were taken into account throughout the design phase.

The design of the Secondary Mathematics Instructional Institute (SMII) program is a combination of a professional development workshop and a form of modified lesson study (see Table 5 for a detailed description of the PD). This intentional dual-focus design is consistent with Cobb and Jackson’s (2011) call for teacher PD aimed at large scale instructional improvement to involve the study of *pedagogies of investigation*—“analyzing and critiquing representations of practice such as student work and video-cases of teaching”—and *pedagogies of enactment*—“planning for, rehearsing, and enacting high-leverage practices in a graduated sequence of increasingly complex settings” (p. 15). During the collaborative design phase of SMII, the designers decided that the PD needed, at minimum, the following goals:

- provide teachers with a vision of high-quality mathematics instruction
- provide teachers the tools and support necessary to enact that vision
- provide tasks and experiences that help begin to develop teachers who are reflective practitioners, confident and competent in examining their own practice as well as that of others.

Specifically, SMII consisted of 8 full days of PD (6 hours per day) delivered over the course of a single school year. The PD was delivered in two segments, two 2-day sessions in September\(^8\) followed by four 1-day meetings spread throughout the remainder of the year.

The first four days focused on building participant understanding of high-quality mathematics instruction particular to secondary mathematics, providing participants with the examples and tools necessary to support them in establishing and maintaining a collaborative classroom culture, and on developing the pedagogical content knowledge needed to instruct in a way consistent with best practice in mathematics. During this time, participants experienced mathematics instruction as students. I\(^9\) engaged participants with a mathematical task each day, modeling high leverage practices (Garet, Porter, Desimone, Birman, & Yoon, 2001; Loucks-

\(^8\) The original research plan called for 4 consecutive days of PD in August; however, a lack of participants forced a change in schedule. The content of the two 2-day sessions remained the same as that planned for the 4 days in August.

\(^9\) Three mathematics education consultants, of which I am one, collaborated in the creation of SMII. We designed the program over the course of 18 months. Due to scheduling conflicts and work demands, I facilitated all SMII sessions alone.
Horsley, Stiles, & Hewson, 1996). During this time, I also engaged participants with NCTM’s Mathematics Teaching Practices (National Council of Teachers of Mathematics, 2014) and with the chosen lesson plan format, the Thinking Through a Lesson Protocol (TTLP) (Smith, Bill, & Hughes, 2008). The major intended outcome of the first four days was that teachers leave those days with two things:

- a plan and a vision for establishing a collaborative, growth-mindset classroom culture
- a collaboratively planned lesson that they would use to begin their modified lesson study process in the subsequent PD session.

It is important to note that the mathematical content of the tasks was not the main focus of the work during this portion of each day. This was an intentional decision arrived at after much debate. My goal, as originator of the idea for the PD, was to make SMII a different kind of PD offering than those traditionally offered at both the national level and within Michigan itself. Many of the Mathematics and Science Partnership (MSP) grant-funded projects in Michigan focused on engaging and enhancing teachers’ content knowledge. These projects had relatively small pedagogical components compared to the PD offering I envisioned. My collaborators and I decided that the tasks we chose would be those that provided participants opportunities to expand their views of mathematics, develop understanding of alternative approaches to problem solving, and discuss potential student approaches to those same problems. This allowed me, as the facilitator, to develop and encourage reflection on instructional practices that were in alignment with the vision of NCTM (2000; 2014).

Each of the four remaining sessions was devoted to the lesson study process (Stigler & Hiebert, 2016). During each day, participants engaged with a mathematical task, studied relevant literature focused on an aspect of the exploration phase of a mathematics lesson (typically a resource concerned with discourse or questioning), viewed and reflected on classroom video (their own and those of colleagues), and engaged in lesson planning for the next round of lesson study. Participants recorded their enactment of the lesson planned during the previous PD session\(^\text{10}\) and chose relevant segments to study during the following session. The focus of the lesson study was learning how to effectively facilitate the explore phase of a lesson. This approach stands in contrast to more traditional interpretations of lesson study in the U.S., where

\(^\text{10}\) When video recording became a barrier to participation, I offered my services and equipment to assist participants in capturing video. I participated in the recording of one lesson during this study.
learning is focused on the creation and refinement of a particular lesson or series of lessons. However, this modified format is consistent Takahashi and McDougal’s (2016) view of lesson study (outlined in Chapter 2), framing it as a collaborative process aimed at creating collective knowledge of teaching mathematics.

The reality of most school systems and most PD offerings is that teachers’ schedules, the demands of curriculum, and school resources will not allow for multiple cycles of enactment and refinement of a single lesson. A focus on *learning about teaching* and *building expertise*, as Takahashi and McDougal (2016) advocate, avoids many situational and contextual barriers to effective lesson study.
## Table 5

**Summary of SMII PD Activities**\(^{11}\) by PD Session

<table>
<thead>
<tr>
<th>SMII Days 1 - 4</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth Mindset</td>
<td>Setting up a collaborative classroom – a practitioner example</td>
<td>Lesson Planning Process – Thinking Through a Lesson Protocol (Smith, Bill, &amp; Hughes, 2008)</td>
<td>Maintaining a collaborative classroom – a continuing practitioner example</td>
<td>Introduction to Lesson Study</td>
</tr>
<tr>
<td>SMII Framework Introduction</td>
<td>Example Lesson Plan Analysis</td>
<td>Introduction to Support Resources</td>
<td>Finalize list of classroom norms</td>
<td>Mathematical Task – Sunrise and Sunset (Schrock, Norris, Pugalee, Seitz, &amp; Hollingshead, 2013)</td>
</tr>
<tr>
<td>8 Teaching Practices (NCTM, 2014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Quality Instruction Video Example</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draft Classroom Norms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The goal of this day was to bring awareness of the importance of culture and mindset to teachers, both their own and students’. Participants engaged with a mathematical task, related working on it to the ideas of growth mindset. We reviewed the 8 Mathematical Practices and engaged with NCTM’s (2014) Teaching Practices to provide a vision of instruction. The group viewed examples of instruction consistent with the Teaching Practices. The final portion of the day was devoted to drafting a set of classroom norms for use during the school year.

To give participants a detailed view of our vision for a student-centered classroom, we enlisted the help of a practicing teacher to engage participants in her process of setting up and maintaining a classroom culture. After this, participants engaged with a proposed lesson planning process. I assumed an unfamiliarity with the TTLP (Smith, Bill, & Hughes, 2008) and allowed participants to engage with it in multiple stages, including through an analysis of a portion of an example lesson plan, specifically of the lesson (and task) they experienced the previous day.

Participants engaged in a follow-up discussion on how to maintain a collaborative culture in their classroom. Participants engaged with a mathematical task and reflectively created a lesson plan for implementing that same task. The goal of this was to build confidence and understanding of the lesson planning process in a supportive, reflective environment. I closed the day with the introduction of resources to support further lesson planning and provided structured exploration time with these resources.

The fourth day involved a great deal of lesson planning. The process unfolded in stages aligned to the three phases of the TTLP. Participants planned, received feedback, and modified in three stages, followed by an opportunity to finalize their list of classroom norms. The final mathematical task was a much more unstructured mathematical modeling task than the previous tasks. The day closed with a reflective activity and an evaluation component.

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\(^{11}\) In an effort to remain faithful to Goldsmith and colleagues’ (2014) call for the clear explication of PD program features in research literature on PD, I will give a “thorough and systematic description of the nature of the professional development approach . . . including the specific activities teachers engaged in and the amount of time devoted to each” (p. 22). Table 1 gives a summary of the activities in the PD by day.
Table 5 (continued)

*Summary of SMII PD Activities by PD Session*

<table>
<thead>
<tr>
<th>SMII Sustained Support Days</th>
<th>Day 5</th>
<th>Day 6</th>
<th>Day 7</th>
<th>Day 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 20, 2017</td>
<td></td>
<td>January 23, 2018</td>
<td>February 13, 2018</td>
<td>March 20, 2018</td>
</tr>
<tr>
<td>Reflection on opening the year or enactment between sessions</td>
<td></td>
<td></td>
<td></td>
<td>Practitioner mathematical task and support</td>
</tr>
<tr>
<td>Mathematical Task</td>
<td></td>
<td></td>
<td></td>
<td>Concept mapping activity</td>
</tr>
<tr>
<td>Research Reading</td>
<td></td>
<td></td>
<td></td>
<td>Deriving recommendations</td>
</tr>
<tr>
<td>Collaborative classroom video analysis and reflection</td>
<td></td>
<td></td>
<td></td>
<td>Reflective journaling</td>
</tr>
<tr>
<td>Collaborative lesson planning</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each of the first three Sustained Support days shared the same format. The days were designed to allow participants time to engage with a mathematical task as a model of instruction and engage with research on the explore phase of the TTLP. The research articles were chosen by the planners from both practitioner and research journals. Following this, the majority of each day was spent in collaboration as participants viewed, analyzed, and reflected on their chosen video clips of practice. Participants viewed clips in small groups, collaboratively debriefed what they saw, and helped each other plan for the next lesson. The final Sustained Support day was devoted to reflection and creating a plan for the future. I followed that up with a concept mapping activity focused on painting a picture of “high quality mathematics instruction.” The group spent time distilling what they had learned about facilitating the explore phase over the course of SMII. To facilitate this, I required the group to create a poster of recommendations that would help any colleagues who did not attend the PD understand the facilitation of the explore phase of a lesson. The poster had two components: lessons from research and lessons from practice. The day closed with a Reflective Journaling activity which served as both an encapsulating reflective activity and an evaluation.
While it was not possible to guarantee all of Takahashi and McDougal’s conditions (outlined in Chapter 2), I took pains to ensure that the PD

- focused participants on learning to carry out specific parts of a lesson,
- engaged participants in reading research about specific parts of a lesson,
- engaged participants over a significant duration and span of time (Loucks-Horsley et al., 1996), and
- I, as the facilitator and knowledgeable other, contributed to participants’ discussions during lesson planning and observation

Implementing the SMII PD was a large and complex undertaking. It required over a full year of advanced planning, with full day meetings of the facilitators once per month as well as significant work in between those days. I conducted a preliminary review of relevant research prior to engaging in planning and supplemented this review on an ongoing basis, particularly when it was decided that SMII would be the focus of a doctoral dissertation.

Justi and van Driel’s (2006) findings also bear suggestive relevance to the quality of the SMII PD intervention. Based on subject comments during interviews and PD, the authors were able to conclude that

... the external domain in the present project [the PD offering] had a strong influence on the teachers’ personal domain and the domain of practice. In our view, the most important elements of the design of the external domain were: (i) it was clearly and coherently related to teachers’ teaching practices, and simultaneously focused on perspectives that were distinct from those that teachers were used to; (ii) it encouraged teachers to think about both their previous teaching practices, and those which occurred during the development of their research projects; and (iii) it allowed teachers and researchers to interact not only during meetings but also during the whole process (p. 447-448).

The design of SMII, with its modified lesson study components, ties it closely to teachers’ practice. Further, the focus on pedagogy provides a perspective oftentimes vastly different from what many teachers typically encounter. Lastly, there are multiple opportunities built into the PD structure, both during lesson study and during the summer institute, for teachers to reflect on their previous and developing teaching.
Data Collection

Data collection began prior to the delivery of the first SMII PD session in September 2017. All three subjects participated in an initial interview prior to the first PD session. Data collection ended in June of 2018, shortly before the end of the school year. A full listing of study events appears in Table 6. Beyond the chronology of data collection events, it is vital to understand the relationship between the data sources in a multiple case study and the research questions. Each data source addresses at least one, but often both, research questions. Table 7 contains details about this intended alignment.

Semi-structured Interviews

Each subject took part in four semi-structured interviews (Corbin & Strauss, 2015). As can be seen in Table 6, these interviews were spaced throughout the school year. The initial interview occurred prior to the first PD session and the final interview occurred after the last PD session. The second interview occurred after the initial four PD sessions but before the first of the sustained support days focused on lesson study. I scheduled the interviews in this way to gain some insight into the effect of the initial four PD sessions. The third interview occurred after the second set of four PD sessions. This placement was to gain some insight into the effect of the lesson study. The placement of the final interview was also in hopes of gaining insight into if and how any change processes continued after the completion of the PD sessions. Given the theoretical framework in use, it seemed wise to allow for the maximal span of time between the first and fourth interviews. This would give the best chance of identifying any potential growth networks (Clarke & Hollingsworth, 2002) in subjects’ change processes. Appendices A, B, C and D contain the exact interview protocols. Each interview consisted of three distinct parts, each discussed below.
### Table 6

**Chronology of Study Data Collection Events**

<table>
<thead>
<tr>
<th></th>
<th>Interview 1</th>
<th>SMII PD Sessions 1-2</th>
<th>SMII PD Sessions 3-4</th>
<th>Interview 2</th>
<th>SMII PD Session 5</th>
<th>Observation 1</th>
<th>SMII PD Session 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terry</td>
<td>September 22, 2017</td>
<td>November 17, 2017</td>
<td></td>
<td></td>
<td></td>
<td>January 15, 2018</td>
<td></td>
</tr>
</tbody>
</table>

Table 6 (continued)

**Chronology of Study Data Collection Events**

<table>
<thead>
<tr>
<th></th>
<th>SMII PD Session 7</th>
<th>SMII PD Session 8</th>
<th>Interview 3</th>
<th>Observation 2</th>
<th>Observation 3</th>
<th>Interview 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeremy</td>
<td></td>
<td></td>
<td>May 14, 2018</td>
<td>March 29, 2018</td>
<td>May 29, 2018</td>
<td>June 4, 2018</td>
</tr>
<tr>
<td>Rebecca</td>
<td>February 13, 2018</td>
<td>March 20, 2018</td>
<td>March 30, 2018</td>
<td>March 29, 2018</td>
<td>May 29, 2018</td>
<td>May 29, 2018</td>
</tr>
<tr>
<td>Terry</td>
<td></td>
<td></td>
<td>May 14, 2018</td>
<td>March 28, 2018</td>
<td>May 29, 2018</td>
<td>June 4, 2018</td>
</tr>
</tbody>
</table>

### Table 7

**Study Data Sources Aligned to Research Questions**

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Research Question 1 (Change processes)</th>
<th>Research Question 2 (Influences)</th>
<th>Research Question 3 (Reflection)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-structured Interview (Mathematical Task)</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>Semi-structured Interview (Questions on Practice)</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>Semi-structured Interview (Concept Map)</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>Video of subject participation in PD</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>Classroom Observations</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
</tbody>
</table>
Mathematical Tasks

Each interview began with a mathematical task. These tasks were chosen based on the criteria for cognitively demanding tasks set out by Stein, Smith, Henningsen, and Silver (2009). Specifically, each task offered multiple pathways of solution while suggesting none, “require[d] complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the tasks, task instructions, or a worked-out example)” and also “[r]equire[d] students [here, teachers] to explore and understand the nature of mathematical concepts, processes, or relationships” (Stein, Smith, Henningsen, & Silver, 2009, p. 6), in this case mathematical growth (either linear or quadratic). The first three tasks involved visual representations of a growth pattern and required subjects to characterize, predict, and generalize this growth. The final task was more contextual, forcing subjects to derive meaning from a context to understand multiple patterns of growth and the relationship between them. Figure 5 gives the visual presentation (if any) of each task and its prompts.

The purpose of this section of each interview was to gain insight into two things:

1. Participants’ conceptions of mathematics and his or her ability to generate multiple solution methods for a given task.
2. Participants’ ability to predict the manner in which students might attack the same problem.

These are tasks that secondary teachers would potentially use in their own classrooms, focusing on mathematics that would be reasonable for the subjects to be familiar with even if they did not teach the content at that time. The mathematical tasks were chosen to give subjects multiple opportunities to show both their knowledge of mathematics (Specialized Content Knowledge) and their ability to anticipate how students might approach a problem (Knowledge of Content and Students). These aspects of Mathematical Knowledge for Teaching (MKT) (Hill, Ball, & Schilling, 2008) are intrinsically related to the work of the PD sessions as anticipation is part of the lesson planning process. This data was used to document any changes in the subjects’ abilities in anticipating multiple possible solution methods over the course of the PD
<table>
<thead>
<tr>
<th>Interview</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>S-Pattern Task</strong>[12]</td>
</tr>
<tr>
<td></td>
<td>![Diagram of S-pattern task figures]</td>
</tr>
</tbody>
</table>
|           | 1. What patterns do you notice in the progression of figures?  
|           | 2. Sketch the next two figures in the sequence.  
|           | 3. Determine a way to find the total number of tiles in a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.  
|           | 4. Characterize the relationship between the figure number and the total number of tiles. How do you know the relationship is as you describe? |
| **2**     | **The Hexagon Pattern Task**[13] |
|           | ![Diagram of hexagon figures] |
|           | 1. What patterns do you notice in the progression of figures?  
|           | 2. Determine the perimeter of each of the first four trains.  
|           | 3. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.  
|           | 4. Characterize the relationship between the figure number and the total number of tiles. How do you know the relationship is as you describe? |
| **3**     | **The Candy Bar Sale Task** |
|           | It’s the annual Freshman candy bar sale and you have 36 candy bars to sell. Your best friend only has 24 candy bars to sell. If you sell 2 candy bars per day and your friend sells 1 bar per day, how many days will it have been when you have fewer candy bars than your friend? |

*Figure 5. Mathematical tasks within the semi-structured interviews*

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Further, in an attempt to create contrast and measure this change more directly, the Hexagon Pattern Task appeared in both Interview 2 and Interview 3. Specifically, these consecutive attempts to solve an identical problem (separated by time and experiences in the PD sessions) were intended to probe subjects’ change processes further than two consecutive, distinct tasks might—one might expect different subject responses to prompts from the interviewer (e.g., about how students might attack the task or how he or she might facilitate this task with students) after engagement with further PD activities. Ultimately, the purpose of these tasks was to measure changes in teacher beliefs and knowledge (both aspects of the Personal Domain of the IMTPG). Performance and discussion of mathematical tasks have the potential to show changes in subjects’ beliefs about mathematics and their beliefs about students and student responses to mathematical challenges (cf. Swan, 2007), particularly when paired with appropriate prompts from an interviewer (e.g., asking the subject how he or she believes students might attack a given task or asking how he or she might respond to a student that was having trouble getting started on a task). These two ideas are linked closely to subject conceptions of practice, and thus have the potential to help answer the first of the research questions.

Questions about Teaching Practice

The second section of each interview consisted of questions aimed at gaining an understanding of subjects’ teaching practice. In Interview 1, questions focused on gaining detailed baseline data about typical and atypical lessons and classroom practices, with follow-up questions added to press for detail on particular issues. During Interviews 2 and 3, subjects answered similar questions, but were also pressed for detail about changes based on subjects’ PD experiences. During the final interview, I again asked about changes (both linked to PD experiences and not) but also added a final prompt aimed at gaining a summary description of subjects’ experiences. The interview prompts (see Appendices A, B, C, and D for further detail) were simple and linear, with some possible follow-up prompts listed as an interviewer resource. The vague nature of the protocol was deliberate, which is why it was classified as semi-structured (Corbin & Strauss, 2015). It was nearly impossible to anticipate with any accuracy the kinds of responses from subjects, and so the prompts provided aligned to the main purposes of the protocol outlined above. During each interview I constantly looked for opportunities to press for meaning and further detail. I pressed subjects to discuss how their experiences related to PD activities and for deeper meaning when any subject used terminology specific to their context.
While this did create some variation in subject responses and interview content, this variation was oftentimes in favor of more detail about particular classroom practices or teacher belief structures. Thus, variation in response was an asset to the data collection methodology, particularly given the intended outcome of detailed case study reports for each subject. Further, I anticipated that the reflective aspect of the questions (paired with data from the PD sessions) would help answer the second research question.

Concept Maps

The final section of each interview required subjects to create a concept map representing their current conception of high-quality mathematics instruction. It is worth noting here that Witterholt and colleagues (2012) used concept mapping activities to gain insight into the state of the subject teacher’s understandings and beliefs about the teaching and learning of research in the area of statistics. With the success Witterholt and colleagues experienced using concept mapping, I chose to incorporate it as a data source in my study, thereby reinforcing my inquiries into the evolution of subjects’ beliefs about teaching and learning mathematics. After each participant created a concept map, I prompted them to explain the diagram to me. During the explanations, I took advantage of opportunities to press for clarity, detail, and meaning. Any changes in concept maps that occurred over the course of the study have the potential to shed light on the first research question.

Video of Subject Participation in PD

The major data source related to the second research question (how teacher reflection on practice changes during participation in PD) was video recordings of subject participation in PD. This data source was used to study change in participants by Witterholt and colleagues (2012), Stockero (2008), and van Es and Sherin (2008), among others, making it a solid choice for my study. I recorded the entirety of each PD session using a high-definition video camera and attached microphone. Due to the parallel structure and content of the PD sessions, data collected from video of those sessions were intended to reinforce the data collected in the interviews. Recall that each day, participants engaged in a mathematical task, read research or studied professional recommendations, and engaged in collaborative video analysis. Many of these features mirror portions of the interview protocols. In addition, however, the lesson study format required participants to engage in sustained reflection on both their own practice and that of their colleagues. It is this reflective activity that was the focus of data collection during PD sessions.
Unfortunately, due to technical difficulties with recording equipment that were not discovered until after particular events, nearly three full sessions (the entirety of SMII PD sessions 2, 3, and half of session 8) of video recordings were rendered unusable due to poor audio quality. The implications of this loss will be discussed further in Chapters 5 and 6.

Classroom Observations

Subjects agreed to a minimum of three (3) classroom observations across the duration of the project. These three observations were structured and scheduled according to specific criteria to maintain the integrity of the study and to attempt to gain a picture of daily classroom instructional practice:

- observations occurred outside of participants’ planned lessons as part of the SMII lesson study process
- observations were dispersed throughout the second half of the school year so as to allow participants to grow as much as possible between observations
- no observation would occur on a test day
  - as a result, observations were scheduled in advance
- scheduling an observation on a review day was avoided as much as possible
- observations were scheduled to coincide with availability of subjects and the availability of the researcher
  - I maintained my employment status over the course of this study and so this placed some restrictions on my availability

Because all events were pre-scheduled, study subjects had a minimum of several days notice prior to an observation or an interview. This was unavoidable due to the unpredictable nature of the classroom schedule and the limited number of observations possible.

In any study, there always exists a set of constraints (physical, temporal, financial, structural, etc.) that influence methodological choices. Classroom observations are particularly time-consuming and difficult to schedule. Given the constraints of my duties to my employer and the contextual factors affecting subjects’ work (e.g., the structures of the school environment and school day), I believed that three classroom observations per subject was a number that would allow for enough classroom data to be collected while not proving too great a logistical challenge to overcome. These data, paired with the interview and PD session data, I believed would give a
somewhat clear picture of practice for each of the subjects. Further, the use of classroom observations was intentional as a verification of the self-report data from the interviews. This choice helped address concerns and findings of some researchers about the validity of teacher self-report data (Copur-Gencturk & Thacker, 2020).

Field notes served as the data from classroom observations. I used the Lesson Note© (Lesson Study Alliance, 2020) application installed on an iPad to capture my field notes. Lesson Note© allowed me to capture the physical layout of the classroom in a dynamic classroom map. I used that map to specify interactions (e.g., student(s)-to-teacher, student-to-student, and/or teacher-to-student(s)) that I observed. Further, the application captured the modality of classroom instruction (either whole group, individual, or small group modality), along with the specific time (from the beginning of the lesson) and duration (in seconds) of each interaction and modality. For each interaction I noted, I created a notation in which I captured activity, tasks, dialogue, questions, or recorded my evaluation of questioning patterns. I also captured photographic evidence of classroom activity (e.g., board work) at points I deemed necessary to provide contextual detail during the analysis process. Each of the aspects of classroom instruction that I could capture was linked to a portion of the PD intervention—that is, participants learned about and/or discussed content related to each of these aspects of the mathematics classroom during the initial four SMII PD sessions. I attempted to capture enough detail of the overall flow of the lesson to gain an understanding of what teaching practice looked like in this instance. I did focus specifically on teacher questioning patterns because that was a focus of the PD sessions, lesson study, and the research subjects read. The lesson observation field notes collected observational data only, avoiding, as much as possible, evaluative statements (any evaluation was completed during the analysis phase). The only exception to this was regarding patterns of questioning. Capturing the entirety of quick interactions between teachers and students was sometimes difficult. If I was unable to capture the full, or even partial, dialogue of an interaction, I applied a label consistent with the kind of interaction it was. The label I used most often was IRE referring to the Initiate-Respond-Evaluate questioning pattern (Herbel-Eisenmann & Breyfogle, 2005). I also used funneling (Wood, 1998) on several occasions.
After the completion of each observation, I used Lesson Note© to generate a report. This report contained a summary of all of the information I indicated above; I found the teaching modality diagrams particularly useful (see Chapter 4 and the case study reports).

Data Analysis

The goal of case study work is to create as detailed and honest a record of a subject and their circumstances as possible (Creswell, 2013; Stake, 2006). Further, as Stake reminds us, the ultimate goal of multiple case study analysis is to develop as complete an understanding of the phenomenon in question as possible. In the context of this study, the phenomenon of interest is the process of teacher change and, more specifically, changes in reflective habits. The analysis phases of this study held to the principles above as closely as possible. Data analysis focused in three areas:

- monitoring and describing teacher change processes related to the PD intervention
- describing influences on the change processes experienced by teachers in this study and
- describing changes in subjects’ reflective practices.

Phase 1: Case Study Reports and Cross-case Analysis

The central feature of a multiple case study analysis is the collection of case study reports along with any potential cross-case analyses that build an understanding of the phenomenon or process under study. This multiple case study is no different. In order to present and interpret the data from the interviews (including artifacts such as mathematical work and concept maps) and observations, individual case study reports were useful. The case study reports are detailed narrative descriptions of subject experiences and reports on experience during the SMII PD intervention. The case study reports have a two-fold organization. The primary organizational framework for each report mirrors the three-part structure of the interviews (outlined in Table 11). Within each of these parts, the case study report is organized chronologically. This choice allowed for a detailed answer to the first research question (what teacher change processes looked like over time) and also set up the cross-case analysis that followed.

At various times during the interviews, subjects discussed changes in their beliefs or practices (or changes could be inferred from subject commentary). When constructing the case study reports, I noted these instances and engaged in a short analysis in which I tied the changes
discussed to the IMTPG. This mid-phase analysis made the case study reports invaluable tools for constructing the cross-case analysis.

Change Sequences and Growth Networks

In order to frame my interpretations and develop my conclusions, I used the IMTPG as a mapping tool. I specified multiple change pathways\(^{14}\) for each participant within each case study report. I mapped each of these pathways using an operationalization of the IMTPG similar to that used by Witterholt and colleagues (2012), seen in Figure 6.

\[^{14}\text{When speaking of teacher change generally, I will use the term \textit{change pathway}. During more detailed analysis, I will distinguish between different types of change pathways as defined in the literature.}\]
professional experimentation, consisting of subjects’ teaching practices and their attempts at the implementation of the practices forwarded in the SMII PD (e.g., the Launch-Explore-Summary lesson structure or the questioning techniques within the TTLP). Finally, the Domain of Consequence, rather than focusing on teachers’ new conclusions, consisted mainly of observed (by the subjects) student responses to subjects’ professional experimentation.

by individual subjects. The operationalization offered a method of easily capturing and notating change pathways of nearly any length in a condensed form. For example, consider a hypothetical subject who learned about and saw a teaching technique in action in a PD session reported that the experience “opened her eyes to the possibilities” and then described how she tried to implement that teaching practice in her classroom shortly after the PD session. This change pathway began in the External Domain and followed a reflective pathway (1) into the Personal Domain and continued with an enactive pathway (4) into the Domain of Practice. In this hypothetical (and perhaps ideal) case, the complexity of this pathway could be notated in condensed form: 1 → 4. This notation, employed by Witterholt and colleagues (2012), features prominently in my analysis of the change pathways experienced by study subjects.

Mapping change pathways in this manner required an examination of the definitions of change sequence and growth network in order to gather insight into the complexity and significance of the change pathways identified. Clarke and Hollingsworth’s (2002) definition of change sequence was adapted: “two or more domains together with the reflective or enactive links connecting these domains, where empirical data supports both the occurrence of change in each domain and their causal connection” (p. 958). Given the sparse nature of my classroom observation data and the self-reported nature of my interview data, I wished to ensure that only the most complex and significant change pathways were substantiated in the cross-case analysis (where I outlined all change pathways I found in the case study reports). Therefore, I modified the definition of change sequence from the literature. For the purposes of the cross-case analysis in this study, a change sequence involves three domains and the reflective and enactive links among them.

For identifying growth networks, Clarke and Hollingsworth’s definition was also drawn upon. However, establishing sustained change over time in a one-year study was difficult. Given this and my altered definition of a change sequence, I employed a definition of growth network similar in some ways to that used by Justi and van Driel (2006): growth networks were defined
by more than three domains and the reflective or enactive links among them. I was able to establish limited evidence of sustained change over time from my data and so the distinction between a change sequence and a potential growth network in this study is more one of complexity than duration.

The alteration of established definitions can be cause for concern about the efficacy of the methodology and the conformity and reliability of the results of a study. In this case, the intensification of the literature-established definitions in only the cross-case analysis ensured that only the most complex change pathways were compared. Simpler change pathways involving two domains were still identified and explicated in each of the case study reports, so no data regarding change processes was ignored. However, examining change pathways of increased complexity has benefits. Primarily, the choice ensured that any interpretations based on these particular results would be robustly supported. Further, any claims made based on the number of change pathways identified in the cross-case analysis would be strengthened by the fact that those pathways were a subset of a larger set of change pathways within the case study reports. In short, the choice to intensify the definitions in this study served only to strengthen the results.

The final aspect of the multiple case study analysis was a cross-case comparison between all subjects. This cross-case comparison consisted of the identification and analysis of subjects’ change sequences and growth networks as well as the foci of those change pathways (e.g., a focus on implementing talk moves in the mathematics classroom). After subjects’ change pathways were identified and labeled as change pathways or growth networks, I considered patterns in those change pathways in order to identify categorizations alternative those provided by Clarke and Hollingsworth (2002)—change sequences and growth networks. Further, beyond the simple identification of the focus of each subjects’ various change pathways, the analysis included the identification and description of themes across those foci.

Phase 2: IMTPG Domain Analysis

In order to answer the second research question, I engaged in an analysis aimed at identifying the influences on subjects’ change processes. I used a semi-open coding scheme (Corbin & Strauss, 2015; Creswell, 2013) with two phases to code the interview data, derive categories of influence, and derive meaning from within each category. Specifically, I began by uploading all of the transcript data and artifacts from the interviews into Dedoose (2020) and setting up the coding scheme. I defined five a priori coding categories from the domains of the
IMTPG—i.e., External Domain, Personal Domain, Domain of Practice, and Domain of Consequence—along with an extra category for Reflection. I also included a code for each study subject.

After coding the entire data set using this scheme, I used Dedoose to output the collection of coding instances under each of the five *a priori* categories. Using these collections, I engaged in an open coding using a process of conceptual ordering (Corbin & Strauss, 2015) to generate emergent codes under each *a priori* category. These emergent categories were not defined prior to this phase of analysis—the defined categories emerged as a direct result of the conceptual ordering process. For emergent categories containing more than ten coding instances, I performed a second open coding to categorize within and provide a finer grain size for interpretation and derivation of meaning.

Phase 3: Reflective Activity Analysis

Phase 3 of the analysis process involved segmentation of the video recordings of subjects’ participation in PD activities. I reviewed the recording of each PD session and identified instances of reflective activity. I determined a segment of video containing each of these instances. The goal of this segmentation was to create “meaningful chunks” of video for analysis (Grant & Kline, 2010). These relevant portions were timeframes when participants engaged with classroom video and with research. Each segment contained a coherent exchange around a common topic or artifact. Table 8 summarizes the criteria I used to decide upon the transition between segments.

Table 8
A Summary List of Video Segmentation Criteria

<table>
<thead>
<tr>
<th>Video Segmentation Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversation involved at least one teacher discussing some aspect of practice</td>
</tr>
<tr>
<td>Conversation involved at least one teacher reflecting on teaching (classroom actions, planning, curriculum, etc.) in some way.</td>
</tr>
<tr>
<td>Segments included relevant participant responses to individual teacher reflections</td>
</tr>
<tr>
<td>Transitions between segments were defined (loosely) by</td>
</tr>
<tr>
<td>a shift in conversation topic and/or</td>
</tr>
<tr>
<td>a shift in speaker resulting in a connecting idea</td>
</tr>
<tr>
<td>Participants hypothesized about teaching practice</td>
</tr>
</tbody>
</table>

The goal of this analysis was to trace any changes in reflective practice and mathematical beliefs over the course of the PD. Again, I employed the use of a set of *a priori* codes in this phase of analysis. The *a priori* codes consisted of van Es and Sherin’s (2008) four attributes of
reflection—agent, topic, grounding, and level—and Manouchehri’s (2002) five levels of reflection—describe, explain, theorize, confront, and restructure. The framework describing teacher reflective activity is identical to that used by Stockero (2008)—specifically, the combination of the two frameworks is her creation.

In this analysis, change was measured by tracking participant interactions over the course of multiple PD sessions (or activities) and associating these interactions with the levels of the reflective framework. Each segment of conversation was assigned a code which linked it to the day of the PD intervention from which it was derived. For example, if a segment of conversation occurred on SMII Sustained Support Day 1, then it was assigned a code PD1Sx (where x represents the segment number for that day) code and a code identifying the participant who spoke. Codes related to the levels of reflection were added throughout the analysis. This, in effect, allowed me to characterize change as quantitative shifts in the various aspects of reflection over time. Witterholt and colleagues (2012) used a similar technique in their study, which allowed them to map changes at various points throughout the PD intervention. Once the coding had been completed, I used the capabilities of Dedoose to assist me in capturing quantitative counts of the code instances in each of the a priori categories.
CHAPTER 4

CASE STUDY REPORTS

Introduction

In order to contextualize the results of the data analysis phases (both the cross-case analysis and the IMTPG domain analysis) for readers, beginning with the case study reports seemed wisest. This will allow readers to gain some insights into the subjects as individuals and gain insights into their individual change processes. Beginning with the case study reports has an additional advantage: as a set, the case study reports generated as part of the analysis process begin to provide an answer to the first research question, namely,

*What do teacher change processes associated with the SMII PD intervention look like?*

Further, from a logistical and analysis standpoint, without generating the case study reports, it would have been both more difficult and less informative to generate the cross-case analysis laid out in Chapter 5. The results of the semi-open coding related to the IMTPG (presented in Chapter 5) will further illuminate this initial research question and provide a detailed, explanatory answer to the second research question:

*What factors influence teachers’ change processes as they engage in the SMII PD intervention?*

The analysis of changes in the reflective processes of subjects throughout the SMII PD intervention (also presented in Chapter 5) will attempt an answer to the third research question, namely,

*How does teacher reflection change during the SMII PD intervention?*

Generation of the case study reports was the initial act of analysis in this study. Each phase or step of analysis after that was informed by both the content of the case study reports and the process of generating them. As such, it may be helpful to consider the three case study reports that follow as a reporting of relevant data. While explicit connections to the IMTPG are drawn and change pathways noted where appropriate, the majority of the analysis will be presented in Chapter 5.
A Note on Context

One theme that recurred throughout the data and in the case studies was subject references to “notes” that were provided to students as part of the curriculum materials. It is helpful to describe these prior to engaging the detailed descriptive work of the case studies. Rather than present this same description at the beginning of each case study, I place it here so that any references to it hereafter (by subjects or me) can be better understood.

The mathematics curriculum materials used by Terry, Rebecca, and Jeremy (and their colleagues) at the time of the study had been designed by two of the teachers in the department. One of these teachers was also a study subject. These materials contained some excerpts from a traditional textbook series—that is, not one of the NSF-funded curriculum projects—by a major textbook publishing company. The excerpts in question mostly consisted of worksheets and problem sets. These excerpts were integrated into a more comprehensive set of materials that included daily notes, homework sets, quizzes, and unit assessments. The units of study and the pacing guide were set by the two department members who designed the materials.

In order to bring the appropriate context to some of the conversations that follow, it is vital to understand that when subjects refer to “notes” during commentary on instruction, they are referring to the daily notes included as part of those curriculum materials. To create this context, Figure 7 shows a sample set of these notes—in this case, notes on “graphing systems of equations.” These notes had particular features. First, each student received a copy of the day’s notes to fill in as the lesson progressed. This was made possible by the fact that the notes were formatted so that the organization and content were static and students simply had to fill in blank spaces at the appropriate moment. Typically, the daily notes followed a format beginning with definitions and technical mathematics vocabulary which might include such things as any general formulae students might encounter—e.g., a lesson on quadratics might define the word *quadratic* and provide students with the general form of a quadratic equation: \( ax^2 + bx + c = 0 \). Next, several example problems appeared, ostensibly so that the teacher could demonstrate the solution method for each, and students could record it in their notes for later reference. The final portion of daily notes was a modestly sized homework assignment consisting of approximately twenty exercises, often a worksheet from the textbook materials. Understanding this format is important as all of this is implicit in most of each subject’s commentary but appears often and has a profound effect on each of their thinking.
Graphing Systems of Equations

I can... identify a system of equations.
Graph a system of equations and identify the number of solutions, and what type of system the graph creates.

Vocabulary:
1. System of equations — Two or more equations that must be solved together to find the solution.

***The solution to a system is an ordered pair (x, y) that makes both parts of the system true.

***A system of two equations can have zero, one, or an infinite number of solutions.

Types of Systems:

<table>
<thead>
<tr>
<th>Type of System</th>
<th>Number of Solutions</th>
<th>What the Graph Looks Like</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consistent and Independent</td>
<td>One (x, y)</td>
<td>lines cross</td>
</tr>
<tr>
<td>Consistent and Dependent</td>
<td>Infinitely</td>
<td>2 equations but only 1 line</td>
</tr>
<tr>
<td>Inconsistent</td>
<td>Zero</td>
<td>parallel lines</td>
</tr>
</tbody>
</table>

Solving Systems by Graphing:
1. Make sure all equations are in $y = mx + b$ before graphing.
2. Graph both equations on the same coordinate plane.
3. The answer will be where the two lines cross.

Examples:
Directions: Solve each system by graphing. Tell how many solutions it has and what type of system it is.

1. \[x + y = 2\] \[y = -x + 2\]
2. \[x = 1\] \[2x + y = 4\]

\[(-1, 3)\] \[consistent/indup\]

\[(1, 2)\] \[consistent/indup\]
A Note on the Organization of the Case Study Reports

To gain an understanding of the complex series of events in this study, and of each participants’ role in those events, I took a two-layered approach. The three main sections of each case study report correspond to the three main sections of the interview protocols: participant mathematics, participant teaching practice, and participant vision of mathematics instruction. Within each of these sections, I have taken a chronological approach, treating each event in the order in which it occurred relative to the other events. My hope is that this will shed some light on any changes that may have occurred throughout this study. Each case study report begins with a brief introduction that contains a table which lays out a chronology of events for this study, relative to the subject of the case study report.
Case Study Report – Terry

“I know that's what I've become—somebody who's too quick to help when they struggle, and I've got to give them something. But that's not who I want to become as a teacher. I've been molded into that, and I guess I've given into that more than I should have.”

- Terry

Introduction

Terry

Of the participants in this study, Terry was by far the most experienced. He was in his twenty-seventh year of teaching during this study, many of those years at the same school. Terry is a white male from an upper middle-class background. He is a thoughtful, engaged professional who tends to be effusive in his storytelling. His commentary appears more often than any other in the transcripts of our time together.

Table 9 shows a chronology of events for this study.

Table 9
A Chronology of Terry's PD and Research Events

<table>
<thead>
<tr>
<th>Event</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interviews</td>
<td>Sept. 22</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
</tr>
</tbody>
</table>

Terry’s Mathematics

Terry’s mathematics, while relying a good deal on symbolic representations, was a flexible mix of leveraging geometric structure, abstracting to numerical representations, and generating symbolic forms, at least with regard to growing geometric patterns. The more familiar a context felt to Terry, the more readily the symbolic representations appeared. He seemed to draw on teaching experiences in his past to solve some of the kinds of problems presented to him in the interviews. He would often comment about how he used to teach using “stuff like this” but no longer did.
Interview 1 – September 22, 2017

When presented with the S-Pattern Task as part of his first interview (see Figure 8), Terry commented “. . . there’s a whole bunch of patterns. You want them all?” When I indicated that I did, Terry began to work.

![Figure 8. The S-Pattern Task](image)

Terry’s initial attempt involved taking advantage of the geometric structure of the pattern:

Terry I was saying, um, let’s take the groupings [motions to a figure]. So, I’ve got two groups of four, plus two. I’ve got three groups of five, plus two. Four groups of six, plus two.

Jason Okay. Nice.

Terry Which was, I was just about to go back and down here [points to question 3 about generating the number of tiles in a given figure], describe that. You telling me to do that?

Jason That’s wonderful, yeah. I was just curious.

Terry Let’s see, so that is . . . n plus one . . . and n minus one. [Inaudible problem-solving activity].

At this point, I expected Terry to simply leverage his geometric interpretation of the problem to get to a general formula. However, he did not do so. Instead, he commented on his sequence of numbers, saying “so now we’re talking about that kind of progression. So, we can
come up with a formula, an explicit formula for it.” As the second question asked him to generate the next two figures in the sequence, Terry did so and also calculated the number of tiles in each of the seven figures he now had. Interestingly, in drawing figures six and seven in his sequence, Terry did not draw individual tiles. Instead, he drew large rectangles and noted the dimensions of the sides, a method consistent with his initial thinking. In answering the prompt about generating an explicit formula for the nth figure, Terry expressed some unease:

Ah, since I haven’t done any explicit formula derivation in a while, this might take a little while . . . sequences and series, we don’t do that anymore and as you know, if you don’t do something for a while you get a little shaky.

This comment seems to indicate a certain amount of self-consciousness about his mathematics. The interview format and my presence likely contributed to that feeling. Terry and I were in his classroom and our conversation was recorded on my laptop for later analysis. In my experience, this tends to make participants apprehensive, particularly if they are not used to video recording themselves. Further, my presence as both the interviewer—Terry may have interpreted this as “interrogator” or “judge”—and as a perceived “expert” may have led to some anxiety on his part.

Despite his misgivings, Terry was able to generate an explicit formula by leveraging his geometric interpretations of the sequence of figures—his “groupings.” He had implicitly generated it previously when he drew the sixth and seventh figures—he drew large rectangles and noted the dimensions rather than drawing all of the individual tiles—and he brought this thinking to the surface again when he noted that “. . . n minus one, n plus one plus two should give the total number of tiles.”

In hindsight, I regret not pressing Terry for other ways to generate an explicit formula. However, I changed tacks, going off script, and asked Terry how his students would likely attack the S-Pattern Task. Terry was emphatic in his answer:

Terry No. They would not know how to attack it.
Jason Okay.
Terry I’m being blunt with you. They would, because again we don’t, in our curriculum we’ve gotten away from much of this kind of stuff, so . . . probably, my guess would be they would count the number of tiles. We don’t talk about consecutive differences hardly ever
any more. [Inaudible] that’s a great skill. So, then what they would probably do is try to figure out a way to take this number [referring to the sequence index] and try to get this number [referring to the number of tiles in a figure] with it. They might, if they did that the right way, realize if they think about perfect squares at all that these are all one bigger than perfect squares and get that. Pictorially, I was trying to think if kids would try to rearrange the picture somehow to do something different with it. But I don’t think they’d, we just don’t do enough of that kind of thing. We don’t have manipulatives we play with. [inaudible] Okay rearrange the shapes. Can you do something different with it, so it stands out better? Use those same numbers but get a different shape? [Pause]

I would actually like a copy of this.

Terry’s belief in students’ lack of an approach to this problem stemmed from his knowledge of the curriculum used at his school. He indicated that his department no longer engaged students in problems of the kind he was working on. Because of this, he was also unable to anticipate what kind of responses students might give to this specific task.

His only guess as to how students might see the problem involved the use of consecutive differences in a sequence of numbers. He dismissed the idea that students might attempt to leverage the pattern visually. Seeing that Terry would likely not move further in his thinking, I moved to the next section of the interview.

Interview 2 – November 17, 2017

Terry’s second interview occurred after the initial four days of PD and prior to the first of the lesson study-based days. The Hexagon Pattern Task (see Appendix B) was a challenge for Terry, particularly in the area of predicting potential student responses. He dove into the problem, making note of several patterns, none of which involved the perimeter (recall that the second prompt to the task involved determining the perimeters of each figure).

Five additional segments, and then a hexagon, you have to have five segments long. Or if you're looking at angles, I'm looking at ways you can think about this thing. 720 degrees of additional, interior angle measurements. Or if you think
about this as a metric of distance from here to the other side, 1 over m, or something similar. I've got to think that one through.

It is interesting to note Terry’s varied thoughts here. The initial prompt asked him what patterns he noticed, and Terry was able to generate several in succession. “Five additional segments” and “a hexagon” seem to indicate that Terry conceived a growth pattern wherein a hexagon shape was attached to the right end of the figure to make the next. He moved on to thinking about interior angle sums in the same pattern of attaching a hexagon. His final suggestion was never fully realized, but it is interesting nonetheless. The notion of a distance metric is wholly outside the pure shape-based thinking he had displayed prior to that. Terry brought his mathematical knowledge to bear in a series of increasingly sophisticated (or perhaps abstracted) steps.

When asked about the perimeter of each figure, Terry initially counted all the sides of all the hexagons in each figure. It should be noted that I prompted him to explain his thinking several times during this exchange so that I could understand the connection between what he was writing and how he was thinking about the task. Again, he created a sequence of numbers first (see Figure 9):

2. **Determine the perimeter of each of the first four trains.**

   \[
   \begin{array}{cccc}
   1 & 2 & 3 & 4 \\
   6 & 10 & 14 & x \\
   \end{array}
   \]

   *Figure 9. Terry’s number sequence associated with the S-Pattern Task in Interview 1.*

Terry realized that he had miscounted during his initial work and corrected his totals, saying that he was “taking away one as I add five more. So that was the mistake . . . so we’re adding four each time, not five.” Terry was also able to move between the abstract sequence and the pictorial representation, albeit with prompting from me:

Jason  So you’re saying the four stands for . . .?
Terry  The additional four each time.
Jason  Okay.
Terry  Because in the zeroth case you’d have two.
Jason  Draw that for me.
Terry: Yeah, exactly. The zeroth case?
Jason: Draw it for me—seriously. Why would you only have two do you think?
Terry: Ah-ha, okay. So that’s not the way to look at it. So, we’re adding that each time. That’s what we’re really adding, those two lines. So, we’ve got the left and right ends, and then we are adding two on top and two on the bottom. That’s why there’s four.

![Figure 10. Terry's drawing of the "zeroth case" of the Hexagon Pattern Task in Interview 2.](image)

Terry’s instinctual language “So that’s not the way to look at it” gives some possible insight into his view of mathematics at this juncture. While other approaches were certainly possible, his language suggests there was still one “correct” way of looking at a problem. Beyond this, though, Terry was able to initiate a deeper analysis of the geometric nature of the pattern and how it related to the common increase he noted and to his explicit formula characterizing the relationship between the figure number and the perimeter of each figure.

Terry: Ah-ha, okay. So that's not the way to look at it. So, we're adding that each time. That's what we're really adding, those two lines. So, we've got the left and right ends, and then we're adding two on top and on the bottom. That's why there's four.
Jason: So what does the two stand for do you think?
Terry: Left and right side, endpoint, ends, edges.
Jason: How is that different than the way you were originally thinking about it?
Terry: Because I was thinking of taking what you had and adding to it with more hexagons, which is visually what you're doing. But to create those hexagons, especially if you're just talking about perimeter, this isn't relevant.
Jason The insides.
Terry So then you're really just bumping the edge out and extending two more tops and two more bottoms to them. That's kind of what that shows.

Terry was also aware of a compensatory method for counting, which involved “the plus one, minus one, but then that’s goofy.” This relied on taking into account the double counting of the interior edges as a new hexagon was added. His remark that this way of thinking was “goofy” further strengthens the contention that at that point, for Terry, there was a single “best” way to do a given problem. Other, more complicated ways were inferior.

As a final prompt, I asked Terry how he thought students might react to this problem. I did this because one of the foci of the SMII PD was helping participants develop the ability to anticipate student responses to a given mathematical task. Further, each time the participants engaged in a mathematics task as learners (this happened once per PD day), they had been encouraged to seek out and explain multiple solution methods and ways of conceptualizing the problem. I wanted to see if those activities had affected Terry’s mathematics or his Mathematical Knowledge for Teaching. Unfortunately, this prompt created an impasse for Terry. Ultimately, he was not able to overcome this. His inability to quickly derive an answer seemed to frustrate him and prevent him from being able to move on productively. However, he did express some confidence that he would be able to figure it out: “it hasn’t come to me yet.”

Interview 3 – May 14, 2018

The third interview took place near the end of the school year, after all of the PD days were completed. It provides an interesting opportunity to examine change in Terry’s mathematics since the task was the Hexagon Pattern Task, just as in the second interview. It is interesting to note that Terry made no mention of having done this problem before and I did not tell him he had. The time between interviews helped ensure that this was, in some ways, a new task altogether. In this new attempt, Terry took a similar initial approach to the problem:

I grabbed on that you add five each time. First thing I probably noticed was you're adding one hexagon, but I think what creates a hexagon is adding five more sides—so the equivalent. And then down here I got stuck on adding the perimeter because I was trying to tie it back into, well, I'm just adding five each time. So, I'm adding five, so that's where that came from. I'm thinking, oh, as soon as I do that, I'm getting rid of the interior sides. So, I can't be adding five each time. I'm really adding four each time. Because I'm adding five more sides, and I'm
covering up one. So, I went back and did that. And then down here, because I'm adding four each time, I have to figure out how to get n involved. So, I just put one n, and then too short. I put two n, still too short.

It is interesting to note that the issue that created his impasse in the second interview was dealt with almost immediately in this interview. His initial approach again used the “adding one hexagon each time” viewpoint, but he was able to adjust within his approach for the double counting of one interior side. He even went so far as to specify this when I asked him to relate the 4 in his formula (see below) to the pictures: “Well, it’s really plus five minus one. That’s where the four comes from.” His second approach, after a prompt from me, was to leverage “the symmetry of the thing with how many tops and bottoms I’m adding on. So, every time I’m adding one [hexagon] on, I’m adding one set of bottoms and one set of tops. You have to think about it that way.” He was also able to relate that interpretation to the symbolic expression he’d created (see Figure 11):

3. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

\[4 \cdot n + 2\]

*Figure 11. Terry's symbolic representation of the Hexagon Pattern Task in Interview 3.*

Jason And that explains which part?

Terry That would be back to this—adding four and the plus two is the ends.

He preempted my next prompt by saying that he wouldn’t think of it another way. He hypothesized a way of using the number of common sides, but that “seem[ed] like making it more difficult as opposed to more obvious.” After a moment or two of working on that approach, he noted “I don’t want to go there. I’m happy with that.” This struck me then, and now, as a remark that indicates his view of the interview as a test, as a place to perform, rather than a place to explore. Likely, this mirrors a bit of his view of mathematics as well. Terry repeatedly noted his value for efficiency and clarity.

Terry’s thoughts on how students might attack this problem were more significant, but limited to a single view, in this interview: “I think they would do what I started doing, which is counting hexagons. Because I think most kids think of things in terms of shapes, not the components of the shapes.” He followed this remark with an extended story that supported his
reasoning about students’ inability to conceptualize shape components—in this case finding the surface area of a cone attached to a hemisphere—and account for the common boundaries in their calculations. He was able to relate this to his compensatory, “plus five minus one” method of determining the perimeter in the Hexagon Pattern task.

Interview 4 – June 4, 2018

The final interview took place very near the last day of school. All the PD days were complete and nearly all of the teaching days were behind Terry at that point in the year. It seemed likely that any changes in Terry’s mathematics or ability to anticipate students’ thinking that were going to happen, would have happened by this time. The final task in the interview string was the Candy Bar Sale task (see Figure 12).

The Candy Bar Sale Task

It’s the annual Freshman Candy Sale and you have 36 candy bars to sell. Your best friend only has 24 candy bars to sell. If you sell 2 candy bars per day and your friend sells 1 bar per day, how many days will it have been when you have fewer candy bars than your friend?

This task was different because it did not include a visual representation of any kind. Further, it was a heavily contextual situation with many possible approaches. Terry’s initial approach was to create algebraic expressions to model the two parts of the scenario. At the start, he made a critical algebra error, but self-corrected almost immediately (see Figure 13).

Figure 13. Terry’s initial approaches to the Candy Bar Sale task in Interview 4.
Terry: I've got—I'm better with a pencil, because in my mind I can always fix a mistake faster. [Terry works on the problem.] Whoa, whoa, whoa! Man, it must be Monday.

Jason: What makes you think that doesn't seem right? I'm just curious.

Terry: Will you erase the tape?

Jason: Nope. That's okay. I'm fascinated by the fact that you got that far and then immediately—don't erase that for me. Leave it there. But I'm curious as to why? What made you think that that would be incorrect?

Terry: I'm thinking there's no way—there's a 12 bar discrepancy there, and there's no way you can make up that discrepancy even if this guy doesn't sell any in four days. So that's an algebra mistake there.

Terry had worked on the problem for approximately one minute before he realized his mistake. His reaction of asking me to “erase the tape” might still indicate his performance-oriented view of mathematics. He clearly possessed a strong verification routine embedded within his problem-solving cycle. He was able to diagnose his error with a few moments of thought. He had intuitions about the context, but his algebraic approach masked some of those intuitions. Terry was able to construct other strategies to solve this problem. This time he even linked those strategies to students without being prompted (see Figure 14).

Figure 14. Terry's alternative strategy to solving the Candy Bar Sale task in Interview 4.
Terry  Yeah. You could draw a picture and make it very visual. So, you'd make a [drawing]. This would probably occur to the lower level kids to do it this way, especially if their algebra skills are like mine. . . And then they'd make a table that goes with that . . . and bars left. I mean, if you really want to break it down, then cross off two here. So that would be—we'll start with this one. [Terry works on the task.] That would be a very, very visual way to kind of—if you didn't see that. [Motions to his previous attempt.].

Jason  Okay.

Terry  And depending on the kids you had, I suppose you might even start with that [tabular approach] and hopefully get to this [algebraic approach]. I guess it depends on who you're teaching this to. If this was an algebra A class, we'd probably go here. If it's an algebra C class, we'd hope by then they could do that.

Jason  But in algebra 1, you think this task would fit?

Terry  Yeah, it's just linear equations and solving.

Two features of this exchange are worthy of note. First, Terry seemed to characterize the tabular or visual approach as secondary to a symbolic approach—e.g., “if you didn’t see that.” This primacy of algebra was a near constant throughout his problem-solving. He saw algebraic approaches as more elegant and sophisticated than other, less abstract approaches—to Terry, succinct algebraic approaches to solving problems are best. Second, Terry placed the Candy Bar Sale Task firmly in the realm of algebra, but he was very specific about it—an “algebra A class” and “an algebra C class.” Like many high school teachers, Terry was heavily embedded in his context and viewed mathematical topics oftentimes in terms of the courses they fit in as opposed to viewing topics as part of a densely connected web of big mathematical ideas. Further complicating matters was his language about students. His statement that the “low kids” would only come up with the least sophisticated methods is telling about his beliefs about student learning.

Terry also developed a graphical approach to the Candy Bar Sale Task. He connected this approach to the tabular strategy, indicating that they are “the same idea.” When pressed for further diversity in approaches, Terry demurred, saying “I got the trifecta here, man. I got the
equations. I got the tables. And I’ve got the graphs.” He seemed to think that this was the limit of the mathematics to be gleaned from this task. At this point I took advantage of an opportunity I had seen earlier. I had hoped Terry would make use of his earlier reasoning to generate a strategy without my prompting. As this wasn’t the case, I decided to see if I could encourage him with some questions.

    Jason  I heard you say something earlier that I thought was interesting. When you were deciding that this was not correct, that 4 equals d is not correct, what did you say to yourself?

    Terry  I said there's no way that they could be equal after just losing a certain rate here. It wouldn't be equal to what this guy started with. It was barely even, barely less than what this guy started with there. It wouldn't be equal to. It wouldn't be the same. So, it would be 28, and this would be 24.

    Jason  Because what I heard you say was "there's a difference of . . ." and then you went on to explain. Think about that for a second.

    Terry  [Thinking.]. So, you're saying how could I have used my checking as a way to solve it? That’s what you’re asking me, right?

    Jason  Mm-hm.

    Terry  Well, I supposed you could turn it into guess and check.

    Jason  You could. That's always an option, but I think there's a more systematic way based on what you said. Because I heard two things that I found interesting. You said there's a difference of 12, and you said you couldn't make up that difference in four days. Which all of that is absolutely true. So, my question is, how can you take those two ideas of the difference and the making up idea and turn that into a solution?

    Terry  Well, I suppose you could say that the guy's gaining one a day on the other guy. Rate of change is one more than the other guy. So, if it's one more than the other guy, what's the difference between your starting values? Would that work? I don't know. [Inaudible.]. I'm thinking if this guy started with 48—no, it still would be the same. It would work out. Hm, that's an interesting idea. I never thought about it that way before. Is that always true? You know, I'm speculating here, that difference is 12, and
that's the number of days. Because the slope difference is 1, that's why that works. You could extend it. I don't think you'd monkey with algebra 1 kids too much with that idea about, okay, what's the difference between the slopes, and how do we use that? Going kind of with what you're asking.

So, if the difference was 3 instead of 1, it would take a third as much time.

Jason  Four days.

Terry  Four days. So, you could extend that with kids—the idea of differences and slopes. It gives us different rates of change. And when you talk about the differences and rates of change, that helps build in the total differences of values. We don't do anything like that. That would be an interesting way to go.

I was encouraged by this exchange. Terry was able to, with some questioning, build from his own thinking. He was able to develop an idea to fruition, ask himself questions about its validity and generalizability, and verify some of those ideas with reasoning. His comment, “Is that always true? You know, I’m speculating here, that difference is 12, and that’s the number of days. Because the slope difference is 1, that's why that works. You could extend it” supports this supposition. He followed this with a statement that hints at how his views of students impact his views of the mathematics they are capable of: “I don’t think you’d monkey with algebra 1 kids too much with that idea . . .”

Discussion

Terry displayed some consistent patterns in his mathematics. His initial approaches were nearly always numeric and symbolic. This did not change over the course of his engagement in the activities of SMII. However, over the course of the interviews, his ability to generate alternative approaches to problems grew. This was to be hoped, as the PD activities focused on generating multiple solution strategies and in anticipating student responses. The final interview provided the most opportunity for the generation of multiple solution pathways, so this ability is almost certainly tied to the nature of the task presented him. Terry thought in terms of numerical patterns and sequences when he analyzed growing patterns, both linear and not. His initial ideas tended to be geometric, but he tended to ignore these ideas in favor of attempting numerical and symbolic approaches. He was always able to relate features of his algebraic expressions back to
features of the geometric patterns he began with, but none of these remarks were spontaneous. He never relied on graphical representations to analyze patterns or sequences.

Terry’s predictions of potential student approaches were heavily influenced by his views of what those students were capable of. He tended to group students into categories of high, medium, and low ability or into levels of experience based on the courses taught in his context. Strategies Terry considered to be less sophisticated were expected from students he considered to be of low ability, with the inverse being true for those students Terry considered to be high ability. This pattern was repeated in his references to students’ experience levels. Terry could estimate the likelihood that groups of students (“high” or “low,” in his language) might take any of the solution pathways he had. However, he rarely mentioned the possibility of outlier students within each group—e.g., a student he considered to be “low” who was able to access or create an elegant solution such as the final method discussed in the Candy Bar Sale discussion.

Terry summed up his own beliefs about mathematics and student understanding quite well in the fourth interview:

That's what math is. It's a series of understanding this idea and saying, I don't have to do it that way anymore. I can build on that and do something a little more efficient. And then I can tie some things together and do something a little more—you know? It's kind of where we get kids to. How do we increase our efficiency without losing our conceptualization of that stuff? But there's a trade-off—the farther you get away from that initial entry point of understanding, the more likely you are to lose understanding of that entry point. I'm a firm believer in that after all these years. We have kids that really get good at manipulating formulas by the time they get to be juniors and seniors, but I'm not sure they can go back. And you say, why does that make sense? Why does that work? We don't ask enough of those kinds of questions at the higher levels.

Terry viewed mathematics as a process of refinement of ideas and procedures—specifically, refinement toward efficiency. He noted some outcomes for students, namely an increased ability to calculate and manipulate at the cost of a loss of conceptual understanding. Taken together, these comments may indicate a shift in Terry’s beliefs about teaching mathematics—he may have developed a deeper appreciation for the utility of multiple solution pathways in a mathematics task and saw this as a potential pathway to develop students’ understandings to the degree he desired. We will explore changes in Terry’s practice in more detail presently.
Terry’s Teaching Practice

Interview 1 – September 22, 2017

The initial interview served as an opportunity to gather a form of baseline data about practice (in the absence of classroom observation) and beliefs about mathematics teaching practice and mathematics. The interview began with a prompt about typical lessons in Terry’s classroom.

Typical lesson structure

Terry described a typical lesson as follows.

Terry

Well, you already know some of these answers, but I’m going to take it as a process here. Some kind of a warm-up activity which is one of three things. It’s either a preview of what’s coming, anticipatory set kind of thing. It’s a remediation/skills evaluation, Okay so we’re going to do something with fractions, I need to take a little inventory of what’s our fraction skills like, if it’s that kind of class. So, so some inventory of prerequisite skills kind of thing. Or it’s a review, entrance ticket type of thing for the previous day’s lesson. That’s probably eighty percent of what I do right there, because—.

Jason

One of those three?

Terry

The last, the latter of them.

Jason

Oh the last one [review of previous day’s lesson].

Terry

The first, all three of those are the menu I pick from, but more often than not I pick the “Here’s some problems from last night’s homework. Let’s look at those.” Because I’ve found as a time saver, um, that lets me know you’re processing that before we ever get around to looking at the homework, what issues the homework is going to have. [inaudible] so it lets me get my brain before we ever start talking about answers to the homework. Sometimes it’s even “We’re not going to do the homework today without [inaudible].” It may be a let’s put it back a day kind of thing. Or I’m going to give you a little more help and then we’ll correct
some problems. And then we get to the homework issues and the process and all that. Then invariably the new instruction, homework quiz, and whatever time’s left over would be practice slash homework time.

Terry envisioned choosing his lesson format from a mental menu that he had created. This menu contained ways of opening a lesson that were dependent upon a purpose for the day’s lesson and on the homework assignment from the evening before. The three categories of the menu were preview, review, and remediation. Terry seemed to value the opening of a lesson for its service to the topic of the lesson itself—that is, in most cases, the opening of a lesson was designed to support students’ understanding of the content of the upcoming lesson. Terry did not give detail about his decision-making mechanism for choosing when to use each of the options, but he did express that his choice of using homework problems as a warm-up allowed him to gain some insight into students’ mathematical understandings before the lesson began.

In closing, Terry referred to “new instruction” and so I asked him to elaborate. He indicated simply that new instruction consisted of “notes . . . notes, guided practice, modeling of process then they [students] practice, then feedback of some type via the [individual] whiteboards or via show of hands or any of those kind of classroom response things so I get some of do they know what’s going on.” Recall that “notes” refers to the department designed curriculum materials detailed in the introduction to the case study reports in this chapter.

At this point, Terry began to assume that I had an agenda to my questioning, that I was looking for a particular set of answers to my questions. While he was mistaken, as the prompt was designed to elicit descriptive details about current practice from study participants, his response to it provided detail about some of the elements of the External Domain of the IMTPG that were weighing on Terry’s mind:

I can see where this is going. It's okay. Because you and I had some conversations about—I keep trying to peel back what we're doing, and I wish somebody would just step to the podium and take the bullet and say, you know, you can't do all the Common Core. You just can't do justice to any of it well. You have to figure out what are the big—we'll go back to the power of standards, ideas—and give those kids those big weapons that they can then use on the little problems . . . So, we kind of do that without telling everybody sometimes . . . scale back the breadth of the content so it gives us some depth in the process of trying to do that with the geometry. You can't do all this stuff. The kids barely grab something before I move on. They really don't understand the connection between that and something
else. We never give them time, build on those lessons. At some point in time, the lessons need to be differently structured. Our current system, that model I just gave you, works fairly well for 80% of the kids... But the other twenty percent, they would benefit by a different instructional model. No doubt.

I believe that Terry believed I was aiming for a discussion of how his practice should be different. This was not true, but his view was likely due to my other role in his professional life at that time: that of a mathematics education consultant, whose job it had been to push for instructional changes for several years prior to this study. However, he mentioned the Common Core State Standards, which his state had adopted as the standards for school mathematics. The standards document and the training he had undergone to explore and unpack it remained as influences from the External Domain. His statements seem to indicate that he believed that there was too much material in those standards to cover in any way but through a method that many would describe as direct instruction.

In an attempt to close out the discussion of a typical lesson, I asked Terry about his homework assignments. He replied that the department had “scaled it back a lot... from thirty-plus problems to try to get around twenty problems focusing on quality over quantity. Trying to get more scaffolding of the thought process.” This statement indicated that Terry’s department was already involved in a change process, but of a limited sort focused on curriculum materials. It also indicated a reliance on large homework assignments in an attempt to serve both a practice function and a learning function.

I seized on an opportunity and prompted Terry about student engagement during class time:

Jason: But what are the strategies you use to engage kids with the mathematics?

Terry: Multiple things. So I'll do—I'll run through the menu. The cards—I have cards with kids names on it, random selection of cards if I want a response to a question or I want ideas. We'll do, for instance, the bell-ringer warm-up thing. We'll swap those so there is anonymity. And I'll say, how many of you missed number one?

Jason: You physically swap whiteboards? Or how does that work?

Terry: Well, if it's the warm-up thing, it's usually a paper and pencil warm-up. So, we'll swap those. I'll say, how many missed number
one? They go, oh. If I can identify types of errors or most likely—how many of them forgot to simplify the fraction? So, I get some sense of that. So, there's analysis of a mistake, and then a response to that. Sometimes it will be random selection done with numbers—random numbers or random selection. Sometimes it will be volunteers. I really don't like to do that very much, because you know how that works. It's the same six kids always putting their hands up, the ones who do the most, and they always have the right answer. Or the one kid who doesn't care if he's wrong, and he's wrong every single time he puts his hand up, but he still wants to volunteer.

Terry described his attempts to engage students as a whole group. He made no mention of small group work at this point. His engagement strategies involved differing ways of calling on individual students to provide the group with an answer. His description of the use of whiteboards and “swapping” answers was his only reference to student work and answers that were not strictly individual. He also showed an awareness of over-participation by certain students. I pressed Terry about his beliefs at this point, asking him why this over-participation was a bad thing. He said it was “because it disengages the rest of the students. The rest of them know, hey if I don’t put my hand up, I’m off the hook.” He elaborated further, indicating that he was in the process of professional experimentation with an alternative participation method:

Terry: So sometimes I've gotten more and more—especially since you and I already this year did our thing last year—did a little bit more think-pair-share kind of stuff. So, your answer to your neighbor, be ready to give me your neighbor's answer, and what do you think is right or wrong—so a little evaluation. If they had the same answer, I still want what's his answer. It can't be "because that's what I have." It has to be because he multiplied by two. We have twice as many of these. That's why two worked.

Jason: How do you feel that's working for you?

Terry: I like that. I do. It makes them listen to someone else, and it forces a little dialogue. If you're going to talk about why your neighbor
did it as opposed to what their answer is, you have to ask them, what did you do to get there? And if it's wrong, I don't care if it's wrong or not. I just want to know what their neighbor said and what their thinking was . . . we’ve got to get thinking and be able to explain things. Verbiage is really important. Choral response sometimes, especially after they’ve done something—sometimes we’ll do the think-pair-share. I’ve done it a couple of times in a group. So think-pair-share, talk your answer over now with the pair behind you. And you want to be sure there is some kind of consensus or response.

It is important to contextualize the previous interaction Terry referred to. This interaction occurred at the close of the previous school year and lasted approximately three weeks. I observed several of his classes and followed those observations with a reflective conversation about what I had seen and what his goals and intentions were during instruction. One of the main foci of our conversations was the value of student interaction and participation strategies during instruction. We focused on Think-Pair-Share as a strategy to engage all the students in a group as an alternative to the methods for choosing individual students that Terry was using then. It was encouraging to hear Terry explain how he had taken that experience to heart. It also gave an important indication as to Terry’s then-current state of change—he was already involved in some professional experimentation based on a previous experience in the External Domain.

Based on Terry’s explanation, I prompted him in a slightly different direction, focusing his thoughts on what happened when students talked to the whole class:

Especially early in the year here, not much yet. One of the big hitters I got from you last spring was the value in that [students talking to other students], because it gets more engagement, if nothing else, with students. It's very similar to when you model something for kids. You're the teacher. They're the student. There's a little bit of a disconnect, I think. When they see another student model something, there's more attention there. I'm not sure why. I don't know if it's because it's not myself, so they can poke fun of them. Or they say, hey, this one of my colleagues, one of my peers, doing it. It's not the teacher. Yeah, he's the expert, but I want to see how one of my buddies does this. But I've started, for instance, in Freshman Focus, not in math class, but you're supposed to come up with academic goals for themselves for the year. So, I put up a goal and say, let's evaluate this in terms of the smart goal criteria. What do you think? Well, he didn't say when he was going to study. Is that important? And it's different than if I say, do this, then this, then
this. Kids like to evaluate other kids' work. I don't think it's malicious. I just think it's more intriguing for them somehow and makes them think in a different way. So, I'm still messing with how to do that in the geometry class. I've gotten to know the kids better now after, you know, in week three here. I kind of know the clientele I've got, and I kind of know—because I think you have to find—if you pick on the wrong kids, I don't pick on them, but if you select the wrong kids, I think it can set more of not a positive tone to then participate. You want to build in a lot of success earlier on, I think. And for that, I think you almost have to stack the deck so the kids who are going to be the verbal ones, or are going to respond to somebody else's verbiage, it's going to be okay to take those risks. Because that's, I think, a big concern of kids. Taking risks in front of their peers, if it doesn't work out well, what happens? And those first few times as we start to build in—because we're starting a new unit next week. We're going to start talking about proofs. So, I'm starting to identify the kids. Okay, who are the people that I'm going to invite to share something with the class. And I'm going to be very careful of how I observe their responses to that . . . It's that peer acceptance thing. And if they don't see kids that are willing to take the risk being accepted, and I don't want to take that risk, I'm sure not going to say—I'm not going to get that engagement going. Anyways, that's where I'd like to get to. Where it's I monitor and facilitate discussion, whether it's questions about the new stuff or going over the old stuff. Where I'm just kind of the guy on the side way more.

While this explanation is long, it provides valuable insight into Terry’s beliefs about mathematics teaching and learning and his beliefs about students. Terry believed that he needed to get to know students well before he could effectively implement instructional routines that required students to engage in discourse with one another. He valued this discourse but was unsure as to how to develop it in his classroom. His value for the discourse developed, as he stated, from a series of conversations he and I had the previous spring. He echoed the substance of one of the pivotal conversations in the explanation above: we discussed the existence and implications of the different social contract that exists between students as compared to the social contract that exists between teacher and student. That set of experiences began a shift in his belief structure, and I hoped the SMII PD experience would continue this shift.

Further, Terry indicated that he was “messing around with” some of these new interaction patterns. I interpret this idea to represent a form of professional experimentation in the sense of the Domain of Practice from the IMTPG. It is interesting to note that Terry began this experimentation in classes that were not mathematics courses (in this case a Freshman Focus support course). He was more comfortable experimenting and using different instructional strategies in science and general courses than he was in the mathematics classes he taught. And
so the need to have things “go perfect the first time” contributed to Terry’s reluctance to begin professional experimentation in his math classes. This is an example of how influences in the Personal Domain (beliefs about the necessity of perfect lessons) can influence the Domain of Practice (Terry’s lack of professional experimentation in his math classes).

The explanation above also provides evidence for the contention that Terry was already in the process of change at the beginning of this study. His “messing around” in geometry class and his second mention of our previous interaction imply that he was at least reflecting on his practice and theorizing about how to implement different instructional practice in his classes. He also seemed aware of some of the affective aspects of the situation, as evidenced by his statements detailing how he believed students felt about social risk and about peer modeling as opposed to teacher modeling.

Physical room arrangement(s).

Terry indicated that he typically arranged his room in rows and columns of individual desks. However, he also elaborated on other structures he used occasionally.

Terry . . . When we do the pair-share or group stuff, we rearrange a little bit. But this is the basic structure.

Jason And when you do the pairs, you just slide the desks together?

Terry Front/back or left/right partners. If it's front/back, this kid will turn his chair around and sit here.

Jason So really, they might be in rows and columns, but there's kind of a quad thing going on.

Terry Yeah. And then if we do think-pair-share left to right, then to get the quad thing, it's front/back pair, turn your desks around, so you've got the double. That's more of a classroom management preference thing that anything else because when you put kids together in groups, some of them just can't handle it on a routine basis. I think they see it as a special treat that they don't want to abuse when they get to do it once in a while as opposed to just working on the desk before them.

Terry indicated that the only time he modified the arrangement of his room was on the occasion that he implemented a think-pair-share. He noted the cardinal direction options for
forming partners and discussed the ways he would have students reorient themselves. Interestingly, these reorientations were minimally disruptive to the classroom organization. Terry cited classroom management as justification for this—he believed that some students “just can’t handle” the rearrangement of the room. This is an example of how Terry’s beliefs (Personal Domain) effect his practice (Domain of Practice).

Atypical lesson(s).

When I asked Terry about atypical lessons, he was emphatic.

Terry: Not anymore.

Jason: Say more about that for me.

Terry: Since we've [his department] gotten on this treadmill of getting from point A to Z, the question I have to continue to ask is, how do I use the time most efficiently? And efficient isn't always the best. Back when I first started teaching, I don't know if I told you this or not, I loved projects. I love projects. And so, the flow then was we would do some math, and either at the end of the unit or the middle of the unit, we'd embark on a project of some kind that would utilize those math skills we got from that particular unit.

Terry continued with a detailed description of a project that he did prior to the changes he indicated. The project involved students designing a veterinary office. Terry described a near-daily “seminar” process by which he engaged students with contexts that they would need to understand (e.g., ceramic tile, paint, or various construction materials). His description was thorough and involved many elements that would fit in both the Domain of Practice and the Domain of Consequence. He lamented near the end of his description that the project-type instruction “doesn’t happen much anymore because we have this list of things we have to get to.”

Terry indicated that pressures from the External Domain—the “treadmill” problem—had forced his practice in a different direction. His description of his love for projects early in his career leads one to wonder specifically what happened in the intervening years to change his perspective. Terry provided his reason when he noted that the problem of content coverage forced him to focus on efficiency. Given the current state Terry described, I decided to press him about his belief in the value of projects, what he thought the benefits were for students. Terry was enthusiastic:
Yeah. The discussion, the dialogue, the grouping, the problems that you have to work through that are kind of non-routine-ish that make you think outside the step-by-step process . . . the learning and the excitement that went into that was very cool. Kids had to make commercials. They had to videotape them and present to the class. Some of the elaborate things those kids used to do to make a commercial was just . . .

Terry noted a difference in student engagement as well, saying that “there’s not that enthusiasm, not that excitement, not that—then [when he used to facilitate projects] it was ‘do we get to work on our projects today?’ Now it’s, ‘what are we doing today?’ Or maybe even, ‘what do we have to do today?’” He also noted that the response from students’ parents was markedly different, that “parents would come in and say, ‘hey, tell me about this cool project you’re doing. My kids tell me about it.’ I don’t get that.”

Discussion.

Terry described a professional practice that had shifted from a previous state. Influences from the External Domain—a different department focus and curriculum materials, new content standards, etc.—contributed to this shift and left Terry aware of key differences in student engagement and enthusiasm. He was also already engaged in a change process based on a set of experiences he had had with me (in my role as a mathematics education consultant) the previous year. According to Terry, these experiences left him with a heightened appreciation for student discourse but with less surety about how to implement instructional changes to bring that discourse about. He described professional experimentation in classes other than his mathematics courses, based on his beliefs about the required perfection of mathematics lessons. Terry entered the SMII PD with one foot already in the change space that the PD was designed to create.

Interview 2 – January 15, 2018

I engaged Terry in his second interview after approximately two-thirds of the PD sessions occurred. This means that by this interview, Terry had experienced two full PD sessions focused on lesson study and reflection.

Changes in teaching practice.

I opened the teaching practice section by asking about the changes in Terry’s teaching. He was blunt in his response:

No. For a couple reasons, I'm only teaching one math class this trimester. It's a geometry class. And we kind of had our—I want to say our trajectory we've already set without much wiggle room to throw in some new kind of bends and
twists that would slow things down. It hasn't changed my practice, but my thinking . . . this is maybe the best thing I've gotten so far from our four days together—is my thinking is now, how do I take what I’m already doing and make them more that type of format.

I asked Terry to elaborate on his latter statement. To Terry, “that type of format” meant the lesson plan format used in the SMII PD. Recall that this format was the Thinking Through a Lesson Protocol (Smith, Bill, & Hughes, 2008), which the participants and I referred to as “launch, explore, summarize.” Terry, however, referred to it in less positive terms: “Well, the lesson plan, what’s the official buzzword code for that?” So, while Terry claimed to be thinking about how to modify his current practice, he maintained a certain cynicism toward the specific resources and terminology explored in the PD sessions.

In the closing portion of his commentary on the lesson plan, Terry reiterated his commitment and his reasons for not moving forward: “So, it’s in the early stages for me, of how do I make this fit my classroom? I haven’t done anything—like I say, just the one class. With the few weeks we had, it just wasn’t going to work.” It was encouraging that Terry referred to a change process, as opposed to an event. He saw change as something that would take time, which might allow him to continue to reflect and move forward. He also brought back concerns from the External Domain, namely that he was only teaching one mathematics course at that point in the school year.

Despite his contention that he had not done anything, I asked Terry if his experiences had changed his thinking in any other ways. He indicated that he was changing “some of the questioning” he did in his classes. He shared an interesting effort he had engaged in.

I've actually created a chart for myself that I'm going to start next trimester with all my classes because I've got juniors in them. I couldn't do it with freshmen. But I'm going to find a capable kid to each day chart my questions, along with a little observation from that kid about, on a scale of 1 to 5, the effectiveness of the questions. And I'm going to tell them what effectiveness means—effectiveness means, is it promoting better understanding—before I go on someplace else. So, part of that is think-pair-share kind of stuff. So, I've got a wide variety of questions on that that I typically use right now, because I kind of lock into our favorite three or four style of questions. But other types of questions are valid also. So, it's going to give me some indication about the kind of patterns of questioning I have, along with the effectiveness.

Terry was looking for a way to get feedback on his questioning and he devised a method for utilizing some of his students to help him. Setting aside any other issues one might have with
the specifics of his plan, this represented a direct attempt to systematically study his practice. This was a spontaneous effort and not something that was discussed as part of the SMII PD. So, while Terry had not tried anything at that point in the year, he was still reflecting and planning. He maintained that “the old importance of asking good questions in whatever format . . . promoting that better questioning environment. That was one of my big takeaways.”

Interestingly, when I asked Terry what he used to make his chart, he indicated several elements from the External Domain. He used “some of the things that we experienced over the four days, some of the ways you [Jason] were asking questions, and some of the things I [Terry] did myself.” This represents a connection between the External Domain and the Personal Domain—Terry’s experiences in the PD (where participants learned about patterns of questioning) prompted him to contemplate and plan to get feedback on his questioning practices.

Discussion.

While Terry had made no true strides in changing his practice, his experiences in the SMII PD had begun to change his belief patterns. For example, he indicated some value for multiple solution pathways in mathematical tasks, a renewed enthusiasm concerning the role of questioning in promoting student thinking, and was considering classroom instructional routines such as think-pair-share. Terry’s change patterns were heavily influenced by factors from the External Domain that were not supportive: time constraints, teaching schedules, an unrelenting department-wide focus on direct instruction, and more. Despite this, he was able to engage in reflective activity and even make modest plans to study his practice in upcoming classes.

Observation 1 – January 15, 2018

The first lesson I observed was in January. It was a class called “Consumer Mathematics” as Terry’s schedule did not include any of the more traditional mathematics courses that trimester. Terry began class with a reflective activity requiring students to “choose one thing from the guest speaker on Friday that was an impact on your [students’] thinking. Also, tell me why you feel that way.” As students wrote their reflections, Terry collected the homework assignment from the evening before and passed out a reading for students to begin as they finished writing.

When all students had finished writing, Terry implemented a partner protocol. This protocol involved students forming groups of two with a student directly adjacent. Terry was very specific about which rows of students should form the pairs. He also indicated that “the
person with the longest hair goes first.” Students went right to work and began discussing what they had written.

After two minutes, Terry moderated a classroom summary by calling on individual students to share either their own or their partner’s responses. Students spoke to Terry as he called on them. At several points during this set of exchanges, Terry would enter a mode of storytelling, using analogy to drive home points that he considered important. For example, he used an analogy of his own alcoholic uncle to elaborate on a student’s short explanation of why getting a store-based credit card was potentially a bad idea. At the close of the discussion, Terry was very clear about the big ideas he wanted students to take from the comments preceding.

After summarizing the discussion, Terry began to guide students through a set of notes very similar in format and function to the example provided at the beginning of this case study report. Terry’s questions were for recall only and his patterns of interaction fit within an initiate-response-evaluate (IRE) pattern. Most often, individual students answered Terry’s questions with short answers of their own. On the occasions when students did ask questions, Terry also engaged in an IRE pattern. For more complex questions, Terry funneled students with procedural questions about the problem contexts and calculator inputs.

There were isolated examples of students assisting one another during the independent work time Terry assigned for the last sixteen minutes of the class period. During this time, Terry moved about the room helping students as they needed it. Generally speaking, Terry was the mathematical and content authority in the classroom at all times. He adhered to an IRE or funneling pattern exclusively during his lesson.

Discussion.

In this initial observation there was evidence of Terry implementing classroom structures designed to increase student engagement. The episode was short-lived, but provides a baseline about Terry’s abilities to implement classroom structures. It is impossible to identify the source of Terry’s motivation to use such structures. Under normal circumstances we might assume, based on initial interview data, that the first four PD sessions contributed to this effect. In this case, Terry and I had spent significant time the previous school year discussing just this sort of instructional move. Given this fact, and Terry’s commentary about our previous interactions, the two effects cannot be separated.
This initial observation provided data about Terry’s questioning techniques. At this point in his change process, Terry overwhelmingly used funneling or IRE patterns of questioning. There had been opportunities to consider changing these patterns during the first four PD sessions, but those opportunities did not appear to have made an impact on Terry’s practice at this point.

Observation 2 – March 28, 2018

My second observation of Terry’s teaching was in a class called “Basic Geometry.” This class was specifically for students the department had identified as low-achieving and in need of support. Terry began this lesson with students sitting in rows and a homework collection routine. However, the tenor shifted almost immediately. When students had arrived, Terry had asked them each to get a small, individual white board and dry-erase marker. Terry engaged the students in a whole-class formative assessment routine as the opener of his lesson. His directions to students were clear and concise. It was also clear that this was not the first time students had engaged in this way.

The prompt was simple: (3, 2) and (5, 2). Terry gave students twenty seconds to find the distance between the two points and write it on their white boards. He had them show the boards to him and he selected three of them. Terry mixed the boards up so that students have trouble remembering who each board came from.

Terry: Can you figure out what that person is doing? [Holds out the first example.]

Ss: Distance formula

Terry: What about this one? [A dot image of two general points diagonally from one another.] Did anyone figure out how to do this without a lot of work?

S: [Raises hand.]

Terry: What did you see?

S: I saw the points were in a straight line because the y’s were both two

Terry: [Evaluates response as correct and moves on.]

Terry gave students another problem to work on: (3, 2) and (6, 6). He repeated the white board work without taking examples this time. He made a note that he didn’t see anyone try to graph the points on a coordinate plane and using the horizontal and vertical directions to find the
distance. Since no student had tried this, Terry demonstrated, asking students for the calculations once he’d drawn in the vertical and horizontal lines. Figure 15 shows Terry’s demonstration board work.

Figure 15. Terry's board work for two example problems during Observation 2.

He attempted to prompt student to consider efficiency and representation:

How long did that take? This class is geometry, so we like seeing shapes to help us make sense. Tying into a picture as opposed to just some calculation.

Terry then asked students to find the area of the triangle formed from the previous work (note that Terry had used dashed vertical and horizontal lines to help students visualize those distances). He split the class into groups of three and asked them to work on that problem. As groups worked, Terry circulated the room, watching. He stopped and asked one student “How did you go about getting that?” As students ask questions, Terry responded with sentiments such as “I like the idea but . . . [explains].”

Terry closed the exploration of the distance formula concept by asking for a volunteer. After getting a volunteer, Terry asked “How’d you do it [the problem]?” The student provided a cogent explanation that leveraged the fact that the points in question were aligned vertically so all he had to do was figure out how far “down.” Terry pressed for meaning by asking “what was the idea with the down thing?” The student reiterated that the x-coordinates were identical, so the y-coordinates were the only numbers of concern. Terry pressed further, asking “did you use the distance formula?” When the student answered in the negative, Terry brought a question to the
rest of the class: “Can someone tell me why not?” Terry chose another volunteer student to answer. From there, Terry continued his string of problems with students working on whiteboards for the next five minutes, asking students for the perimeter of the triangle. When students gave their answers (all indicated 12), Terry asked “Everybody has twelve. What the story? Squared units or not?”

The remainder of the lesson was focused on a worksheet. Terry gave each group a specific problem to work on. Groups could work together or individually on the assigned problem. Terry moved through the room, assisting students as they asked questions. His interactions were focused on getting students to make sense of the problems they were given. He used guiding questions to activate students’ prior knowledge, asked assessing questions to get at student thinking (e.g., “What else did you do besides prove?” or “What way did you go to get the height?”), or more direct questions to help students get to the answers (e.g., “What did you need that you didn’t have to get the area?”). Students responded well to Terry’s questions, using academic vocabulary and precision in their explanations. They talked with each other in some groups as they worked. As groups finished their work, Terry paired groups with the same problem and asked them to compare their work in finding the lengths of the diagonals.

At this point, Terry transitioned the class back to a whole group format and led them through a series of examples involving finding the area of parallelograms of decreasing height. As he presented each parallelogram (see Figure 16 for examples), Terry asked the class to vote on whether they believed that there was enough information to get the area.

Terry  Do I have enough information to get the area?
Terry  What if I “mushed” it down? Could I still find the area?
Terry  What if I mushed it down even further? Same lengths . . .
[He noticed immediately when several students changed from a yes to a no vote.]
Terry  Someone who was a no vote, why did you change?
S  I thought we could cut it up to make a rectangle, but we don’t have the height.
At this point Terry agreed with the student’s explanation and explained what the problem was with the series of examples. He continued the discussion with his class, asking questions such as “What did you think? You had some interesting thoughts over there.” He had noticed a student working on a different approach in this case and decided to call it out to the class for examination. Later, he prompted a student to “Come up and draw it” and questioned the student through some incomplete thoughts as he/she worked. He transitioned the class to individual work to finish out the hour.

Discussion.

The lesson described above differs in some significant ways from the format and direction of the first lesson observation and from Terry’s initial descriptions of his practice. While he initially described his instruction as teacher-led and direct, and indicated in his second interview that he had not tried anything new, it appeared that things had changed by this point in the school year. In this lesson, I saw Terry use grouping structures and specific prompts and routines to shape student discussion. His use of groups was flexible and practiced, particularly his shift from groups of three to groups focused on like problems. His prompt asking students to compare strategies was different than any I had seen from him before (or heard him describe in any of his interactions). While not definitive, this observation would seem to confirm that Terry’s contemplative state concerning asking different types of questions in the Personal Domain in the first interview had shifted to one of enactment in the Domain of Practice. Certainly, it shows that Terry had the capacity to implement grouping structures in his classroom.

Further, Terry’s questioning patterns had shifted as well. Whereas before he had asked solely recall questions and used only funneling questioning patterns, Terry displayed the ability to ask more probing and focusing questions during this lesson. While he did not make full use of whole group participation structures to get students to process those questions, he nonetheless was able to ask them. This is indicative of another possible shift from contemplation in the Personal Domain (thinking about asking different questions) to enactment in the Domain of
Practice (actually asking those questions and attempting to structure student groups to engage all students with some of the questions).

Interview 3 – May 14, 2018

Near the end of the school year, Terry was making connections between his SMII PD experiences and his practice. Spontaneously, he linked his thinking about mathematics to his PD experiences.

A connection to the SMII PD activities.

. . . it comes back to the whole theme of this SMII stuff. Rather than just tell kids what to do and say, “Now go do it”, let's give them something that they can explore. And that's when they find it and they say, ah. And then we ask key questions along the way that makes them redirect their thinking to get them—instead of hitting a dead end here, let's head to an expansion idea. Say, where else can this go? In some things that's easy to do, but one of the things my brain has wrestled with now is with the results of the SMII thing. I can't do that with everything. For instance, I can't figure out a way to do that. But how do I take aspects of this thing and rather than saying, well, I'm just going to give them the formulas and away they go. How do I take aspects of it and create SMII type of environments where there's little mini moments of discovery as opposed to just me telling them “Oh, you don't have a base here, do you? So, you don't use it.”

Terry highlighted a connection between the External Domain and the Personal Domain when he talked about the “theme of this whole SMII thing” and when he confessed that he had “wrestled with” ideas as a consequence of the SMII PD activities. The PD appeared to have helped Terry develop value for the idea of giving students something to explore in the hopes of creating an Ah-hah! moment. However, this value was tempered by external influences such as time (“I can’t do that with everything.”). But Terry continued to reflect and tried to assimilate what he had learned into his practice, at least mentally.

When I asked Terry how this creation of “mini moments” was working for him, he replied that “it’s changed the kind of questions I ask.” Terry supported this statement with an example of an exchange he had with a student that day. His description was quite detailed. The exchange involved a context of a fish tank and a maximum number of guppies for a given volume. The task was to determine how many guppies could be in a tank in which each of the linear dimensions had been doubled.

So, this kid came up to me and he said, wouldn't I just double the number of guppies? And I thought about—I forgot you were coming in today. I'm not staging this for today. Until I saw you in the hall, I actually forgot you were
coming. I'm thinking, how do I make—I call it a mini-SMII moment, a mini-SMII moment for this kid.

The idea of a “mini-SMII moment” is an interesting one, particularly given that Terry spontaneously developed it as a way to prime his thought process before interacting with a student. Terry went on in great detail about his exchange with this student in which Terry “never told him a thing,” instead just “redirected him.” Ultimately, Terry was successful in supporting the student in reaching a correct answer and a correct conclusion on his own. However, at that point, Terry confessed, he “went back to old habits,” showing the student why the problem worked out the way it did. Despite this, the latter description points to a change pathway from the External Domain, through the Personal Domain, to the Domain of Practice. Terry described how his experiences in the SMII PD (External Domain) had led him to consider how to recreate some of those experiences with his students (Personal Domain) and to actually enact some of those ideas in practice (Domain of Practice).

Professional experimentation.

Not all of Terry’s experiences were as successful as the one above, however. Terry noted that he had “kids working together in volunteers, [groups of] two or three” and that he was able to use this structure to “just go in and throw in a question that doesn’t point them in the direction exactly, but it raises questions where they kind of create their own direction.” The latter description might be considered a definition of focusing questions (Wood, 1998), a topic addressed heavily in the SMII PD sessions. I pressed him further:

Jason: Is that volunteer groups thing a new thing for you?
Terry: Yeah, relatively.
Jason: Not something that would have happened a while ago?
Terry: No . . . It works and it doesn't work. I've done just the opposite in the . . . basic geometry class.

So volunteer grouping was relatively new for Terry in his current context. I wondered why, but Terry answered before I could ask the question:

And I got [Jeremy’s] kids [students] third trimester that he'd had for the full year, and I thought, man, this is going to be a culture shock for these kids because I teach so differently than Jeremy does. So, I got in and watched his room, and we did some of the stuff with SMII [Terry and Jeremy co-designed and co-taught a lesson, recorded it for study in the SMII lesson study] and that kind of thing. Okay, I'm going to try to emulate as much as I can Jeremy's style of teaching with
those kids so they're not all of a sudden thrown into a tailspin. And I gave it several weeks. And I noticed not good things for those kids.

Here Terry described a change pathway that began in the External Domain (with the knowledge that he was “getting [Jeremy’s] kids”), moved through the Personal Domain (Terry’s worry about the differing teaching styles), and into the Domain of Practice (modifying his own teaching style to match Jeremy’s more closely) with added influence from the External Domain (the co-teaching and SMII PD experiences). Further, his description moved directly into the Domain of Consequence (see below), making this a potential growth network (Clarke & Hollingsworth, 2002).

There were kids who would abdicate any thought to their group mate if they thought the person knew more than them, would just kind of listen to that person and write down what that person did. There were kids who got off task too easy. There were behavioral issues. And it became a constant management thing for me. And then when I was doing little assessments, they didn't know much.

So, Terry indicated that he “shifted gears.” He expressed his belief that in order for a collaborative, group-based instructional methodology to work, the group of students must have three things: “individual goals,” “the ability to communicate effectively,” and “the commitment to work using . . . time and resources as best as possible.” He believed that “for different reasons, some [students weren’t] on board with all three of those things. This [group-based instruction] isn’t working. So the other model is direct instruction.” Terry noted that some students expressed “relief to have that kind of structure provided for them.”

I asked Terry why he believed some students were relieved. He replied, “it’s because too many of them . . . don’t trust themselves.” He talked about how he had listed the benefits of direct instruction to the students. These benefits offer a window into Terry’s beliefs about mathematics instruction:

And I've told them that's one of the benefits to this . . . I will know each person what [they] know, and I'll know as a class what we know. And I can give you immediately the kind of help you need as opposed to finding out later there's a big issue. I don't call it handholding. It's not that because I'm not spoon-feeding them anything. But it's just they're more focused. I guess maybe that's the biggest thing. I see more focus because they don't get a whole lot of time between instruction and response. That sounds like a terrible thing to say, but for me it's a choice. It's a behavioral thing. It's not that they couldn't make the group thing work.
Terry believed that there were too many factors working against the success of the group work in this instance. He also noted that he was “getting more information,” that his teaching was “more pinpointed.”

In response to this, I asked Terry about his questions during direct instruction. He indicated that it was harder to find opportunities for mini-SMII moments with a large group. Because it's easier to take the pulse of a small group than a whole class. I'm going to go back to the thing that Jeremy and I did [the co-taught lesson for SMII]. Every time we'd move to a different station, it was a different set of questions we would ask of those particular kids because of where they were at in the process. Where in a class of 25 or 28 or whatever, it's not as easy to get that "where is everybody at right now" kind of thing. So, I think the questions aren't as precise there. They're more general or scripted.

Terry mentioned influences from the External Domain in this explanation as well. His experience in co-teaching a lesson for SMII with a colleague seemed to have had an effect on his thinking. In many ways it served as an anchor experience for him, allowing him to compare and contrast new experiences with it to gain perspective. This phenomenon is noteworthy and has implications for the design and enactment of professional learning (outlined in Chapter 6). It also provides an existence proof of a potential mechanism by which teacher change occurs—initiated by external experiences (External Domain), the successful enactment of new practices (Domain of Practice) provide comparative experiences for teachers to reflect upon (Personal Domain) and allow them to reformulate and refine those new practices (Domain of Practice). This phenomenon has the potential to both motivate and hinder the change process, as it appears to have done in Terry’s case.

Terry’s foray into volunteer groups was ultimately unsuccessful and led him to move back to a direct instruction model. This, too, is a change pathway. Terry wanted to shift his instruction for multiple reasons (Personal Domain), he tried to do so through professional experimentation (Domain of Practice) and discovered that it did not work for that group of students (Domain of Consequence). From a professional development design perspective, the danger here is that Terry might become unwilling to continue to experiment professionally. He might conclude, based on this one experience, that changing in the ways advocated for in the SMII PD is not effective. Ideally, the professional development would provide Terry multiple opportunities to reflect on the “unsuccessful” experience and modify his methods in an attempt to attain a more effective state of practice. Unfortunately, the structures necessary for that
reflection were no longer in place at this point as the PD was over. Terry was left to reflect and adapt on his own.

I asked Terry how he might change his approach if he were to move back to small group instruction with that same group. He noted that he did not know how he might do that.

I’ve thought about that. I’d have to come up with a different structure for them . . . almost like a science lab thing, where I can walk around in a lab and say, hey, you guys skipped this question. What’s going on? Or it looks like you’re struggling with this one. What's happening? Because it's too freeform . . . So, it's not structured enough that I can walk into a group and see where they're at and get them to where they're supposed to be.

The lack of structure likely contributed to the way Terry’s experience turned out. It was encouraging to see that he could make that connection on his own, even if he did not have solutions to the issue. In fact, Terry’s lack of solutions became a justification for him to continue to use a direct instructional model with that class: “But right now, to get the through the year with any measure of success, I don’t have that answer in place. So, I’m reverting back to something I know has worked and worked fairly well.” Terry acknowledged that this was, perhaps, not a preferred outcome, saying that “it’s not ideal, necessarily, but it’s not bad. It’s better than what they’ve had before . . . the way I was running it, it’s better than what they had before.”

Terry took responsibility on himself and acknowledged his lack of effective facilitation skills in this scenario. This was encouraging and led me to believe that he might not have given up entirely on attempting to shift his practice. I believed this because his reflection left him with the conclusion that he had control over the outcome of any attempt at professional experimentation—it was not the fault of the instructional paradigm and it was not that group work doesn’t work, as some might conclude.

Changes in teaching practice.

I closed the practice section of this interview with a question about changes in Terry’s practice. His initial comment encompassed changes in his thinking (the Personal Domain).

I think I've realized the importance of the structure of what we do in terms of what [does] it facilitate in their [students’] thought process. And I think there are lots of ways to get at that. I don’t think it always has to be a group thing. But I think it comes back to, if I don't create the right situation with the right questions and the right responses to those questions, it's just window dressing.
Terry linked these beliefs to practice by referring back to his conversation with a student about the guppy tank problem. He noted how there were many opportunities for him (Terry) to simply give an answer. He talked about his change process and what he was currently working on:

I'm working at pulling back a little bit more, especially for those kids that are willing to put that kind of time and energy in. The kid that gets really frustrated easy is a different ballgame. You have to [give] them a little more nudging . . . So, it's just, I think about instruction in terms of the interaction between me and the kids. I'm still not to the point—I may never get to the point where, like Jeremy can do so aptly it seems—where he gets the interaction of kid to kid and plays that off. Because I think that's the idea. I think that's where a lot of learning really happens is that kid to kid exchange. I'm getting better at it, but there's an awful lot of years of history here we're fighting.

Terry’s notion of “pulling back” is consistent with many of the conversations that occurred during the SMII PD. Participants spent significant amounts of time reflecting on questioning patterns and interactions that shift the mathematical load from the teacher to the students. Terry’s stated shift to considering instruction in terms of interactions was encouraging. It was a goal of the SMII PD to create such shifts in teacher thinking. Terry also continued to express the notion of change as a process. While he had misgivings about his ability to achieve a particular level of proficiency, he stated value for an instructional approach that prioritized interactions between students. This represented a shift from his original thinking, in which he acknowledged the role those interactions could play but maintained that direct instruction worked for a majority of students.

I closed by asking Terry what he thought it would take to attain the level of proficiency he sought. He indicated that he believed he could not do it himself, that he needed more support. That support took a particular form, one with connections to the SMII PD format:

So, for me, it would be getting in somebody's classroom that does this [reform-oriented instruction] on a regular basis and watching how it goes from day to day to day. I think that would be incredibly insightful for me to get that video as opposed to those little snapshots.

Terry had seen video clips of his colleagues’ teaching practice, some clips of teachers he did not know, he had co-taught and co-planned a lesson with Jeremy for the SMII lesson study component, and he had observed Jeremy’s practice enough to believe him to be a highly skilled practitioner. Despite all this, Terry wanted more. He wanted to spend extended time in someone
else’s classroom, someone proficient in the skills he wanted to obtain. This points, I think, to a gap in expectation. Terry wanted the PD to provide this extended time in an immersive environment and believed that is what he needed to complete the change. The former was not possible—indeed is rarely possible in any sustainable way—and the latter was simply untrue. But Terry’s belief was telling of how he conceptualized the change process. It was a process, but one he needed to see the final outcome of, in detail, before he could engage in it fully. However, Terry’s context lacked the systemic, supportive structures for him to continue the work of changing practice that might have been supported by examinations of his colleagues’ practice.

Observation 3 – May 29, 2018

The final lesson observation was quite different from the first two. I observed one of Terry’s Basic Geometry classes very near the end of the school year. The class was no longer arranged in rows; rather, Terry placed students in three groups of three and one group of four in separate corners of the room. He made it clear that the purpose of the class assignment that day was to prepare for an upcoming assessment using a review worksheet. The content of the worksheet was a set of contextual problems involving the application of surface area and volume of various solids such as cylinders, prisms, cones, and composite shapes. In addition to the group format, Terry had a very specific protocol for engaging students in the work. The groups were to work on one question at a time and call him over when they had reached a solution. The first group to reach a correct answer on a question would receive a bonus point. The members within a group numbered off by threes (fours for the group of four). When a group called Terry over, he rolled a die to determine which student would have to explain the work of the group. Terry was very careful to model this procedure with a group of students before the class set to work.

Terry’s demeanor during the remainder of the hour was different than it had been previously. He moved from group to group and asked questions and used prompts of varying levels throughout:

- What’s the base of a cylinder?
- Why does it make sense for this one?
- Tell me what you did.
- Think about a pipe. There is that much water inn that length per second. I don’t know how to say it clearer. Give it a shot.
- Tell me why you think that.
- Sounds like you talked yourself into something! Talk them into it and see what happens!

There were more student questions during this class period than the previous two as well.

Generally, the two groups at the back of the classroom spoke more animatedly with one another than the front two groups. About half way through the class period, Terry noticed that all of the groups were having trouble with a particular problem. His response was to create an example that might help the students understand the idea of a rate of flow as measured in gallons per second. That he took this option was surprising to me as I expected him to be much more direct with students as he had been in previous exchanges, explaining how they should move through that portion of the problem that had created difficulties for them. Further, this was a Basic course, filled with students perceived to be of low mathematical ability and Terry had expressed repeatedly that these students needed more structure, support, and direction.

Shortly after his example, Terry shifted the class back to whole group for several minutes as he used the whiteboard to create clarity about the final answer to the last question the groups were working on. He also explained phase two of the day to students at this point. The second half of the day would be focused on collaboratively correcting the work they had done on the problems that day. Terry offered extra credit again as an incentive.

There were several instances of interesting exchanges during phase two of the lesson. Terry continued to use questioning techniques similar to those he employed during the first half of the class period. The example below typifies the types of interactions I observed:

Terry  How many four-by-four squares fit into one square foot?
S    9
Terry  So each square foot gets multiplied by how many? [short wait, no response from student]
Terry  How many in one square foot?
S    9
Terry  In two square feet?
S    18
Terry  In three square feet?
S    36
Terry: How’d you get that?
S: I just continued the pattern of doubling.
Terry: You could add a nine each time, but what if I ask you for 40 square feet? What’s the shortcut for addition?
S: Multiply
Terry: [Continued to question students about the relationship between the number of square feet and the number of four-by-four tiles].

For the remainder of the class period, Terry moved from group to group, answering questions and peering over students’ shoulders to see what they were working on, occasionally asking clarifying questions. For the close of the lesson, Terry proposed to the class a process for attacking problems. He wrote each step on the whiteboard while explaining it to students (see Figure 17). He situated this as preparing them to succeed on the assessment.

![Figure 17](image)

Figure 17. Terry’s list of recommended steps in problem-solving during Observation 3.

Discussion

A comparison across lessons yields some interesting findings (see Figure 18). First, Terry’s use of group work changed over the course of the three lessons. He used more group work and incorporated more structure into the work students were asked to do. This is evidenced by, if nothing else, examining the seating charts for each lesson observation and a timeline for each lesson.

Caution should be used when viewing these data in this way. Recall that the lesson observations were three random days during the school year. They were scheduled prior to the day of the observation and they occurred in three different classes. A solid case for permanent change cannot be made based on what is seen here.
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<tr>
<th>Lesson 1</th>
<th>Lesson 2</th>
<th>Lesson 3</th>
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<tr>
<td>January</td>
<td>March</td>
<td>May</td>
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<tr>
<td>Consumer Math</td>
<td>Basic Geometry</td>
<td>Basic Geometry</td>
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Note: Orange indicates whole-class format. Green indicates individual work. Blue indicates group work.

*Figure 18.* Classroom layout and instructional modality charts for three of Terry’s lessons.

However, a solid case for a minimum of professional experimentation can be drawn from these data. Terry used significantly more group work as the year went on, at least in these observational instances. A Hawthorne Effect cannot be ruled out. Indeed, I suspect that my observations were responsible for some of this change. However, the fact remains that Terry
demonstrated that he was certainly capable of structuring class time collaboratively and that he had the ability to alter his patterns of questioning into more cognitively challenging spaces. While these patterns of questioning were not universal—indeed, Terry continued to engage in funneling patterns—the mere presence of alternative patterns of questioning and communication may indicate some shifts in beliefs and practice.

Terry’s Vision of High-Quality Mathematics Instruction

Terry struggled with the concept mapping portion of the interviews consistently. He noted almost immediately “I’ve never thought about it this way. . . I’m not an abstract thinker.” Despite this trepidation, Terry was able to discuss his vision extensively during our conversations.

Interview 1 – September 21, 2017

When Terry designed his first concept map he spoke at length about the model he had created (see Figure 19). Terry’s focus on the concept to be learned and the teacher’s intention about this concept was foundational in all his thinking. The fact that he addressed the two perspectives (student and teacher) indicated that he was aware of how what goes on in the minds of students determines what they learn. Terry valued student engagement and understood the necessity of creating that engagement for learning to happen.

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<td><img src="image_url" alt="Concept Map" /></td>
<td>I'm thinking about the components of a lesson and how they fit together from two perspectives at the same time—from the teacher's perspective and from the kids' perspective. So first of all, what's the concept? That's on the teacher to identify. And then I think along with that, what's the outcome? And it's that thing of—just because I teach it, doesn't mean that they're learning it. So what am I hoping that they will do after the concept has been taught? Or what are they hoping to do as it's being taught? So, the teacher has to identify the outcome, and then somehow, you've got to get the students to own it. If it's an outcome that they're not going to own or understand, good luck getting it.</td>
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*Figure 19. Terry's concept map and explanation from Interview 1 (September 19).*
Next, Terry moved on to “initial instruction” and provided a vague description of several options:

So now I'm saying, okay, initial instruction. And that's a variety of things. Without being too specific, that could be an activity like that where it's an explore thing. It could be a review of what was done the day before, but the teacher has to get the thing going, I think, in a direction. It can't be too open. And then the initial response—my definition of an initial response is when I'm watching the kids, and I'm starting to do something—because my radar is pretty good for the most part. Is there any kind of interest? Is there any kind of buy-in? Is there mass confusion just at the beginning? Because I think too many get started with something, then get too far along the way before you realize, uh-oh, I've lost them all. So now I'm working from that.

Terry’s initial view of mathematics instruction relied heavily on teacher control. His diagram indicated some student ownership during the lesson, but this did not appear until very late in his description. The initial portion of a good mathematics lesson belonged to the teacher, in Terry’s mind. He seemed to worry about losing students and students not understanding, which implies that his vision of mathematics instruction involved preventing ambiguity for students. He also addressed a formative assessment focus by calling it “evaluation.”

Terry And this is something I've got way better at through the years than I would have been twenty years ago—is that checks along the way. So my map's going to have this evaluation thing in it all over the place.

Jason And by evaluation, you mean evaluation of—?

Terry Are we where we need to be at this point interest-wise, understanding-wise, participation-wise, all of that stuff. I used to run down the trail behind me and say, where did everybody go? And now I take two steps in front of it. And you probably sensed that a little bit. That's the whiteboard thing. I can't get too far away from them, because then it becomes overwhelming to them, and it's too big a problem for me to fix. I've got too many knots down there.

Jason So when you say evaluation, you're talking about—?

Terry Teacher evaluation of my instruction, whatever it is, and their response to it. Is there a match there? My instruction is up here, and their response is down here, I've got to change.
In his first experience with concept mapping his vision of high-quality mathematics instruction, Terry used specific curricular examples extensively. As a matter of fact, while there was certainly a good deal of nuance to our initial conversation, an effective summary might be to present his example here in its entirety. It is worth noting that Terry became so engrossed in his example that he left off drawing his concept map for a short time.

...it's actually going to be the same cycle throughout the lesson. You just kind of keep doing it. So, okay, what's next? So, we're going to talk about complementary and supplementary angles... So, if that's the topic, the initial instruction is we have different angle relationships. And I would probably go back and say, we already know some. What are they? See what their response is, if they know anything about angle relationships. Was it learned about yesterday? Okay, great. So, there's another set of things. And then my next thing might be to say, okay, I'm going to draw a picture on the board of a right angle and say, what's this angle? Do we know it? That kind of thing...

The cycle Terry referred to seemed to be analogous to a formative assessment cycle (providing some initial instruction, eliciting evidence of students’ understandings, and modifying instruction based on that information). He focused on an example from geometry because he was teaching mathematics and science at the time. His two math courses were Consumer Mathematics and Basic Geometry. However, it is interesting to recall that Terry thought of teaching science differently than teaching mathematics. Mathematics held a special place for him, where lessons had to be “perfect” and so instruction would obviously have to emanate from the teacher. This was in contrast to science, where in laboratory exploration settings, Terry expressed that he felt much more comfortable letting students engage in exploratory and dialogic activity.

In this instance we see Terry’s value for the activation of students’ prior knowledge. He was still fully in charge of the classroom, but he began seeking information about what his students currently knew. He chose to do this through examples. He mentioned that he would ask students questions but based on his teacher-centered view of instruction, we might assume that he would either answer the question himself after students worked on it or he would solicit answers from individual students in the room using an IRE pattern. Terry continued to deepen his example.

...So, then the next level, now that we understand what the words mean, come up with definitions—that's the instruction. And then what I would probably do is go back to their response and say, with pictures of whatever, whiteboards, draw a
pair of complementary angles. Draw a pair of supplementary angles. Or put something up on screen and say, identify which of these, or what a counterexample would be, all that kind of stuff. But it’s a lot more—I mean, I’m still providing the bulk of the instruction at this point in time, but I’m getting, especially in the geometry class, I’m getting more like dialogue-ish.

Here again we see the example-focused nature of Terry’s view of mathematics. His use of the individual whiteboards was a recurring theme through his instruction and his vision. To Terry, at that moment, that was perhaps the best method for learning about what students knew and how they knew it. However, he seemed to be looking for classroom dialogue that was more student-to-student than student-to-teacher.

So, I’m just going to speed this up and say that this is the next thing here—it’s recycle the above, continuously looping through. And then at some point in time it leads to some kind of student activity, and that could be working on homework. It could be solving problems together. It could be trouble-shooting quizzes. But that’s where it culminates, I think, for me, is I’ve gotten them started. I’m pretty confident they know what they’re doing. And now it’s their turn to go out with whatever it is. And my role is from primary instructor to more like a guide/consultant.

At this point, I inquired about releasing responsibility for learning and exploration to students, but Terry disagreed. He was thinking about adding another layer to the lesson content, solving equations in this case. This was something Terry might call “chunking” and he mentioned it several times throughout his experiences in relation to his educator evaluation meetings. This was important to his administrator, an example of some of the teaching advice he had been given, and Terry modified his thoughts and lessons to take that into account.

Well, no, I cycle back, and the next level is let’s do some solving equations. And then I’ll model them for them, and then I get their response, engage them and stuff. And then at some point in time when I’m pretty convinced that we got this, we don't need another practice problem, it's time for you to go.

Terry referred to a release into independent work at the end of the latter statement. He had a value for individual work, even when students requested to be able to work together. He allowed his students to seek each other out “when you get stuck or need help or want to verify an answer” indicating that he thought they would be able to “have that short, little fifteen second burst of conversation.” Terry believed that it was possible for students to assist each other in that short an amount of time. It might be tempting to attribute this to his view of mathematics; however, he went on to explain that “it’s just a classroom management thing . . . Too many of
the kids just can’t handle it. As soon as they slide desks together, half of them are off task. And that’s just a risk I’m not willing to take.”

Terry’s view of mathematics teaching was heavily focused on teacher-led, example-driven instruction, with hints of formative assessment. He indicated that he wanted more student-to-student interaction but was unable to conceive of how to achieve this in his classroom. He confessed that “you're talking about "best I've taught or experienced," I don't know, to be honest, that I've taught or experienced that that much. It's kind of this little—I've had glimpses of it, but I can't say that's what I do or who I am.”

Interview 2 – November 17, 2017

Terry’s second attempt at a concept map representing his vision of high-quality mathematics instruction was little easier for him than his first. However, after some initial consternation, Terry began (see Figure 20). He constructed a pyramid structure, noting that the whole organization relied on “the concept” being taught on a given day. This could be interpreted as either a general concept such as “functions” or a more detailed, topical notion such as “solving one-step linear equations.” Given Terry’s context and his indications of the culture of his building (e.g., a focus on covering content, direct instruction focused on discrete mathematical topics) it is more likely that Terry used the latter interpretation than the former. Terry’s second level was devoted to allowing students time to engage in the task. He noted this with a title of “exploration/discovery” but provided few details as to what that might look like. It is interesting to note that Terry separated the exploration level from the “student interaction” level. Further, he separated it with a level involving teacher guidance. I think it likely this was Terry’s attempt to mirror the “launch, explore, summarize” format of the Thinking Through a Lesson Protocol (TTLP) mentioned earlier.

Seen through the lens of his engagement with the TTLP, Terry’s “teacher guidance” level becomes clearer. He elaborated on this, saying that “if you get a group that they get stuck at a certain spot, you may have to get in the middle of that thing and do a lot more guiding than with a group that’s on their own. They don’t need that guidance.” These statements may indicate Terry’s increasing value for responding to the needs of groups of students or individuals, as opposed to responding to the needs of the class as a whole. However, his lack of detail on what the guidance might look like may also indicate that he was simply attempting to mold his vision to what he believed I wanted to hear.
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<td><img src="image" alt="Concept Map Diagram" /></td>
<td>So a good foundation is the concept of whatever we're trying to teach. And I think this is, again, something I got from our days together. If you don't know what you're trying to teach, you're not going to find an effective way to get it taught no matter which way you approach it... the activity launch—because it has to get at that fundamental question. What we want kids to walk out the door with. If it doesn’t do that... then it’s just busy... and then giving students a chance to mess with the activity. And then I think part of the key is, along the way, is it going in the direction you want it to go in? Those carefully guided questions that... I don’t want to give them the answer. But if they’re experiencing too much frustration... find some little nudge to get them thinking. So, to me, it’s almost like a conversation thing. Teacher to students.</td>
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*Figure 20. Terry's concept map and explanation from Interview 2 (November 17).*

As he moved up the levels in his model, Terry discussed his belief that groups of students should share their learning with the whole group in some way (i.e., the summary phase of the TTLP). However, Terry had severe misgivings about how to do this effectively,

And I have trouble, to be honest with you... the whole class sharing out. Because I haven't seen that done—one of the videos you showed is one of the places I've seen it done the best. Because it was very—the teacher directed, the way it was figured out... the teacher... was asking, how many groups got—it was more like a moderated class-wide discussion with the sharing out of what happened in the groups. There wasn’t any one group that said now, you come up, and what did you do? It was somebody would say something, and she would probe a little deeper if she wanted to.

In this explanation, Terry referenced a video of teaching that the participants had watched together during the SMII PD. Viewing a video example of a teacher conducting a summary...
discussion of students’ group-based work\(^{15}\) was obviously powerful for him, particularly given his acknowledged lack of ideas about how to do that work himself. Terry linked this PD experience with his previous attempts at facilitating summary discussion in his classroom. He noted that his attempts “[were] very—okay, next group, what did you find? Show your results. And four kids came up here, one kid talk[ed], and three kids [stood] around . . .” He noted that discussions like those in the video example “[don’t] just happen by accident. You have to kind of know—and I think that’s maybe being in touch with the groups while they’re doing the work.” The latter quotation might indicate an internalization of some of the work of the PD, where we discussed practices for promoting mathematics discussion\(^{16}\).

Terry also indicated his value of discussion and student engagement in other portions of our conversation. After an extended example from his school days about a simulation in social studies class, Terry attempted to generalize to mathematics, saying that “it was the discussion. It was the dialogue. It was solving a common problem. Those components aren’t going to be different no matter what topic you’re talking about.” At this point, it appeared that Terry might be attempting to generalize his experiences in the PD sessions and combine them with his own practice. This seemed to indicate a shift in belief patterns, with enactment being heavily influenced by the curricular structure and building culture in which Terry worked.

Interview 3 – May 14, 2018

During his third interview, Terry created an unusual representation of his vision of mathematics instruction. He employed a metaphor of a sailboat to explain the features and components of high-quality mathematics instruction (see Figure 21).

In order to better understand the relationship between Terry’s diagram and his explanation, it is useful to begin with the process of drawing the picture. Terry drew the boat first, followed by people on the boat (labeled students), then there was one student in the water, then there was the surrounding weather and water and directionality. His explanation indicated that, in this metaphor, the boat was his classroom, Terry was the captain, and the students were

\(^{15}\) The video in question can be found at: [https://www.mathnic.org/tools/01_tru.html](https://www.mathnic.org/tools/01_tru.html) (Mathematics Assessment Resource Service, 2015). In this video, the teacher facilitates a summary discussion in which individual, volunteer students share their solution methods. As each student explains, the instructor creates notation on the board for the whole class, asks those students to tie their explanation to the original model on the overhead projector, and asks probing questions of students, getting them to tie their explanations to her notation.

\(^{16}\) Specifically, the group used a white paper (Kersaint, 2015) which summarized the *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2018)
the crew. “Port” seemed to refer to the successful attainment of the educational goal. Terry expressed that “the weather . . . that’s the toughest” which seemed to indicate that he used the unpredictability of weather to stand for the complexities of a large group of students in a classroom. It is important to note that Terry’s metaphor was not perfect—no such metaphors exists—but he was able to use it effectively to lay out his vision of mathematics instruction.

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<td>So the core of the thing, of course, is the boat, I guess. That's where the learning is supposed to take place. What do you have to work with? What's your content? What are you trying to teach? And that's the thing we have to get safely into port, but . . . the students . . . are a variable in and of themselves. So, some of them do what they're supposed to do, and some of them fall off the boat, but you got to get them all there. It's a crew that somehow has to work together to make that happen. It's not just a whole bunch of little rowboats. It's a ship that has a crew on it. And the captain's job is to make sure that crew knows how to work together to make that happen. Ultimately, the captain can't sail the ship . . . But a well-functioning crew doesn't need a captain much of the time to get done what it's got to do. And if you've got kids that can work together, they can get a lot more done than waiting for the captain to tell them what to do and his response to their particular task on the boat. And then you've got the weather—to me, that's the toughest. I say that's the toughest part about sailing is I can have a plan for where I'm going to go, but then the wind changes, I can't get there the way I wanted to go now. I've got to get there a different way. And it's still the same learning target. It's still the same crew. But now the path you're going to use to get there changes. And you can't sail in spite of the weather. You've got to sail with the weather, if that makes any sense. The wind changes, the rain comes up, you get a calm wind. And I think that's the biggest struggle of all is those unpredictables. You also have the crew is unpredictable. But whether you call it the environment, the bay, the learning target with more difficulty than I thought, whatever it is, I think that's the art of it.</td>
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*Figure 21. Terry's concept map and explanation from Interview 3 (May 14)*
I pressed Terry to connect this metaphor more directly with skillsets he might need in teaching mathematics. He responded, “I have to know the crew I’ve got, and I’ve got to know what different crew members are capable of. I think sometimes we just put kids together in groups. I think those groups need to be very carefully put together.” He expanded on his view of what “capability” means, in terms of students.

And I think if I was going to pick a single thing that I need to get better at it’s that—ascertaining those differences, especially for those struggling kids, earlier, so I know I can put together effective groups as opposed to just groups. I think that’s the biggest challenge if you’re going to do SMII kind of stuff. It's different if you have a pre-calculus class. It's different even if you have a regular geometry class. Because those kids are all fairly mathematically capable, all fairly committed to their own success, and all pretty willing to use the time efficiently. Lower ability kids—all those things kind of erode away a bit. . . You just don’t have as many crew chiefs to throw in the mix.

Terry’s vision of math instruction was still heavily based on his internal view of students’ capabilities. This was evidenced in his language; labels such as “lower ability kids” and his differentiation of “regular” courses from what his department calls basic courses. Those basic courses are filled with what Terry would consider “lower ability” students.

While unconventional as a concept map, Terry’s model centralizes the mathematics he wants students to learn. Part of this, though, includes the knowledge students already have related to the subject of a given lesson. Students appeared to be “variable[s] in and of themselves” in Terry’s model. He noted that “some of them do what they’re supposed to do, and some of them fall off the boat, but you got to get them all there.” His model forwarded the importance of reaching all students. How this is done, though, is worthy of note. Terry indicated that “the captain’s job is to make sure that crew knows how to work together . . . a well-functioning crew doesn’t need a captain much of the time to get done what it’s got to do.” The teacher is the captain in his model, with students being the crew. This choice of metaphor placed authority in the hands of the teacher, as with his other models. However, that authority now came with a different set of duties than in the previous model. The teacher’s job now was to help students learn to work together, allowing them to become somewhat independent of the full guidance of the teacher.
Interview 4 – June 4, 2018

In his final interview, Terry refused to design another concept map, saying “Can’t I just pull out the one from last time? It’s going to be the same thing. I can’t make it better than that. That was as good as it gets.” His discomfort with the format of the activity was the main reason for this, I believe. This is supported by the discussion that we had in the fourth interview, which was different than that of the third. Figure 22 recalls Terry’s concept map.

After his refusal to complete another concept map, Terry’s comments departed from his previous concept map quickly. He focused heavily on outside factors such as curriculum. He indicated that he thought the department taught too much material and did it in a way that was not as effective as it could be. He recommended that the department strip it [the department curriculum] down to the things we think are worth having . . . Now, I think some of the things that we need are on here—this is me—more of this kind of stuff. [Motions to the task sheet containing the Candy Bar Sale problem] . . . where there’s not just grabbing a formula and subst[ituting] a number in and say ‘oh, I got it. It’s 36.2. Did I round right? That’s the depth of the questions you get—is my rounding good here?

*Note: Terry received some personal news that required I end the interview before he could re-evaluate his concept map. I was not able to schedule a follow-up.

Figure 22. Terry’s concept map from Interview 4 (June 4).

Terry’s experiences with the mathematics in the SMII PD project seem to have had an effect. In addition to stating a belief that the curriculum needed more of the kinds of tasks he had experienced in the SMII PD, he reiterated an earlier thought about a potential subtitle for the PD itself: “how to get kids dynamically involved in their own learning.” The theme of motivation
and engagement related to student learning was present throughout our conversations. Terry went further, though, and began to lay out some difficulties of his circumstances.

After the problem solving, if the department were to add it, he said, “there’s nothing in place [currently] but ‘here’s a worksheet,’ which has nothing to do with what [students] just did.” Terry’s conclusion was that the department needed to find a curriculum that allowed teachers to teach in the ways they learned about in SMII PD sessions. One of Terry’s final comments showed his understanding of the complexity of mathematics instruction.

I think it's the careful structuring of prerequisite stuff to get here and then making sure that this is worthy of the time you're going to spend on it. It can't just be, okay, what's the slope of the second line, guys? Think-pair-share. Now what do we do for the next minute and a half? There's got to be more.

Unfortunately, Terry received some tragic personal news during the interview and was unable to finish. I was not able to schedule a time with him to complete a re-explanation of his sailboat metaphor. However, when I prompted him to re-draw it, he simply said, “It’s going to be my sailboat thing again.” While it is certainly possible that Terry’s explanation would have been substantively different than that in the third interview, the proximity of the two interviews makes this possibility less likely than it otherwise might have been.

Discussion

The changes in Terry’s vision for mathematics instruction were accompanied by some static elements as well. Throughout his experience, Terry continued to centralize the mathematical authority in the classroom in the teacher. However, the role of the teacher changed over the course of the year, moving from a directive figure to one that had a responsibility to train students in how to work together to solve problems (in addition to supporting them mathematically through questions). His value for students’ ideas and knowledge increased throughout our conversations. The latter two changes are encouraging, as the SMII PD was designed to instantiate such changes in its participants. However, the influence of outside factors on Terry’s vision remained heavy throughout the year. These factors included such things as building culture, new standards, school administration, and department views of the teaching and learning of mathematics.

In Terry’s comments, there is a sense that in the past, Terry had valued the kind of experiences forwarded in the SMII PD sessions. This value, interestingly, was for both himself
and for students. He mentioned it directly on at least one occasion, when lamenting the effects of his educator evaluations on his practice:

I know that’s what I’ve become—somebody who’s too quick to help when they [students] struggle and I’ve got to give them something. But that’s not who I want to become as a teacher. I’ve been molded into that, and I guess I’ve given into that more than I should have.

Terry’s experiences with teacher evaluations had not been positive in recent years. His administrator valued a different kind of teaching than that Terry had, at one time, been practicing. This fact came across in Terry’s evaluations and caused him to reduce the amount of cognitive demand in his teaching to accommodate the wishes of his administrator (who, according to Terry, believed that productive struggle was a sign of poor teaching). The experiences of SMII seemed to wake Terry’s knowledge of his previous practice, giving him permission to get back to that kind of thinking, giving him permission to value that which he once valued, and allowing him to become more of the teacher he once was and wanted to be. He still had his reservations, but he did begin to move in a more student-centered direction (recall his lesson observations). I choose to take two positive themes from Terry’s case: 1) even veteran teachers can overcome reticence and habit to begin the change process, and 2) there is power in positive, empowering PD experiences to overcome situational barriers.
“I’m a very traditional teacher in a nontraditional sense.”

- Rebecca

Introduction

Rebecca

During this study, Rebecca was in her ninth year of teaching secondary mathematics. Nearly all of those years were at the same school. Rebecca is a white female from an upper middle-class background. She displayed a no-nonsense attitude and often used sarcasm in her interactions with colleagues. She tended to be blunt, offering her opinions with little softening of tone or diplomatic word choice. In many ways this made her a subject that was easy to work with. While Rebecca was often blunt in her commentary, she tended to remain quiet during the PD sessions. She was very forthcoming during the interviews, but in the presence of her colleagues she tended to withdraw at times.

Table 10 below shows a chronology of events for this study.

Table 10
A Chronology of Rebecca’s PD and Research Events.

<table>
<thead>
<tr>
<th>Event</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interviews</td>
<td>Sept. 26</td>
</tr>
<tr>
<td>Observations</td>
<td>Jan. 4</td>
</tr>
</tbody>
</table>

Rebecca’s Mathematics

Recall that as part of each interview, each participant was asked to complete a mathematics task. These tasks involved either the analysis of a pattern of figures to predict an “nth” figure and generate a general formula (Interviews 1, 2, and 3) or making sense of a contextual problem that allowed multiple solution methods of varying degrees of sophistication (Interview 4).
Rebecca appeared to rely heavily on her memory of specific topics in mathematics as she reasoned. She readily made connections between what she was experiencing and formal mathematical labels such as “linear” or “arithmetic sequence.” This led me to characterize Rebecca’s mathematics as very technical in its nature, as may be seen below.

Interview 1 – September 26, 2017

Rebecca envisioned the S-Pattern Task (see Figure 23) as an additive sequence where she leveraged the geometric representation of the pattern to support her initial description.

they added a tile between each of the ends plus the one tile. Does that make sense? Like they moved it up one full tile and over one so they ended up with three tiles. So then like you just added one, or added, technically you added two, but then three, so like one over (inaudible). Does that make sense?

Figure 23. The S-Pattern Task.

Here Rebecca described the change between pattern 1 and pattern 2, where the “three” tiles represents the vertical column of three tiles in pattern 2. She visualized a transformation wherein the top of the original figure shifted up one row and over one row with a new tile appearing in the top row and bottom row and one tile in the middle to fill the open space. She repeatedly asked me if “that make[s] sense,” a potential indication of her uncertainty in her reasoning.

I encouraged Rebecca to draw how she saw the pattern in an attempt to elicit a generalization, but this seemed only to confuse her. In fact, she said that she could not draw how she saw the pattern growing. Instead, she tried to informally characterize the transformation as “getting taller and more boxy.” She continued to answer the questions, characterizing the next
two patterns as “there will be six on the bottom, seven up top, alright? [Writing]. Right? . . .
You’re making me feel stupid!” Rebecca expressed this in a joking manner, but it was an
outward manifestation of her discomfort with not knowing the answer to the questions quickly
enough. The nature of the task forced her into a period of extended reasoning and this made her
uncomfortable, likely because there was no direct connection to the mathematics with which she
was familiar.

As she engaged with another prompt about generalizing the pattern (see Appendix A for
the questions associated with the S-Pattern Task), she attempted to link the situation to her
existing knowledge:

Rebecca: So, I think this is arithmetic? Right?
Jason: What makes you think that?
Rebecca: So we are just adding to get the next set. I mean it could potentially
be geometric if we were raising it to a power . . .
Jason: Okay, but it doesn’t feel like we are doing that?
Rebecca: No. It feels like we are adding ‘cause this has two. This is two
tiles, this is five. [Writing] No, this is geometric, right?
Jason: I don’t know.
Rebecca: Alright, Jay [Jason]. [Sarcasm.]
Jason: How can you figure that out though?
Rebecca: Well this is only three, this is five, that’s seven, nine.
Jason: And that tells you that it is what?
Rebecca: Well it is not a common difference so it has to be geometric, right?
Jason: What defines the geometric sequence?
Rebecca: Well I always thought geometric difference was like, wasn’t it, I
haven’t taught geometric in two years because of my maternity
leave. You have your g-sub one and that’s times. . .
Jason: Give me an example of a geometric sequence.
Rebecca: You multiply to get to the next. Like. . .
Jason: So if that’s true, is that happening here?
Rebecca: No, we’re adding. Like it’s, we’re adding two to the one before.
Like this we added three, this we added two to get five, then we
add two more to get seven, and we add two more to get nine. So we would add two more, so we would be adding eleven to the next one.

Note that “arithmetic” is a reference to a kind of number sequence; Rebecca placed emphasis on the third syllable as she spoke—arithMETic. Rebecca’s attempts to connect to her existing knowledge appear to rely heavily on her recall of specific mathematical notations (e.g., a geometric sequence is “g-sub one”). Related to this, Rebecca’s mathematics appears to relate strongly to the mathematics content she currently teaches, as she noted she was rusty with geometric sequences because of her maternity leave. Despite this, she was able to recall the main relationship present in both arithmetic and geometric sequences, but was unable to reconcile those ideas with the fact that there was no common first difference or common factor in the sequence she had devised: 2, 5, 10, 17, 26, . . .

Rebecca expressed frustration as she attempted to fully make sense of the sequence: “What’s really screwing with me is that like it’s not a common difference, but we add two to each one [first difference] prior to it. Does that make sense?” At this point, I attempted to engage Rebecca in a different way, and we engaged in a flurried exchange of questions and answers:

Rebecca I think it’s arithmetic because we are adding, but like. . .
Jason But it is more complex than the simple arithmetic, isn’t it?
Rebecca Yeah, we just use simple arithmetic in our classroom so I haven’t really had to think about the more complex ones.
Jason What about writing a function?
Rebecca Like a linear function?
Jason Is this a linear relationship?
Rebecca No. It’s exponential.
Jason How do you know it isn’t a linear relationship?
Rebecca Because it doesn’t have, it doesn’t have a common rate. I’m kind of slow at that. Alright?
Jason If it’s not linear than it must be exponential is what you are implying?
Rebecca Well it doesn’t have to be exponential.
Jason What else could it be?
Rebecca: Geometric, right?

Jason: Geometric is a type of sequence.

Rebecca: Or quadratic.

Jason: How do you know it is quadratic?

Rebecca: I have no idea. See my thinking like on a dime like this I don’t do well.

Throughout this exchange, Rebecca repeatedly expressed uncertainty. It is almost as if she was guessing. Her thoughts focused mainly on linear and exponential relationships and only at the very end does the possibility of another type of relationship occur to her. Even then, though, she could not immediately test her hypothesis. She implied that she was not used to “thinking on a dime like this” which I took to mean that she was uncomfortable trying to make sense of an unfamiliar problem in the presence of someone else (or on camera, as the interviews were video recorded for later transcription). It is also likely that my presence contributed to this feeling. In my position as the county mathematics education consultant, I am often seen as having a certain level of expertise and mathematical authority. While I had developed what I considered a solid working relationship with Rebecca and her colleagues prior to this study, all of them continued to see me in a way that positioned me as an expert in all things mathematical.

I continued to question Rebecca as she attempted to reason. She even asked directly for my help at one point. Eventually, Rebecca went back to her original sequence and noticed that “the difference between all those [first differences] is two and that’s going to be common . . . So, for me, based on things I’ve done before, if they have a common second difference it’s quadratic. Right?” Despite this breakthrough, Rebecca immediately noted that she did not know how to create the equation for a quadratic pattern. She made no reference to the pictures in the task. She knew there was a particular procedure that she could use to convert a sequential representation of a pattern into an abstract, algebraic representation, but could not remember it.

At this point she asked for guidance again, which I declined. As our conversation progressed, Rebecca tried several different solution strategies. Once she identified the pattern as quadratic, she attempted to use “[v]ertex form, y equals a, x minus h, squared, minus k” to generate an abstract representation of a quadratic function to model the sequence. However, she noted that she did not “know what my vertex is, that’s the only problem.” So she attempted a guess and check strategy. She achieved limited success.
I prompted Rebecca to think about how she might help a student who was where she was at this point. She responded that she would encourage them to “try a different approach, but I don’t know what approach though.” She suggested that she might build the fiftieth figure with blocks, to which I inquired about a way to draw the fiftieth figure without drawing all of the individual boxes. This was perhaps the most successful prompt of our exchange, though I had to come back to it at least twice.

**Jason**  
We could physically build all those. Absolutely. So, I’m going to focus you back on a question I asked you earlier, right. Because you were looking at the picture and you said you could draw it. So, let’s build off that. Is there a way to draw the 50th figure without actually drawing every single box in the 50th figure?

**Rebecca**  
I think so because this has four columns plus one on either end and four columns. So, like the bottom row would have to have fifty and then it would have forty-nine columns and then it would have, it would have . . . it would have forty-nine center pieces with a 50th top. Does that make sense?

**Jason**  
Okay, so how many is that all together then?

**Rebecca**  
I have forty-nine times fifty and then add two.

**Jason**  
So you’re saying the 50th figure you can figure it out by doing the forty-nine times fifty plus two.

**Rebecca**  
Forty-nine times fifty plus two. Yes. Is that right?

After asking her to verify this, I prompted her to think about how the nth figure might look. Rebecca was able to reply immediately that “it would be n minus one times n minus one plus two times n.” This was encouraging, so I asked her to test her hypothesis.

**Rebecca**  
Okay, so that would be n squared minus 2n plus one plus 2n so n squared plus one. So, one squared is one plus one is two, two squared is four plus one is five. Son of a . . . Yeah that works!

**Jason**  
That works?

**Rebecca**  
Yeah. So, like five squared is twenty-five plus one is twenty-six. I hate you with so much burning.

**Jason**  
Why do you hate me?
Rebecca

Because it seems so simple, but so complex in the thought process. Like you, I’m telling my students all the time like think smarter not harder, don’t try to doctor it up. Try to think of it in the most simplest way you can. Like I was just telling my Trig students today if you over simplify sometimes you can’t see the big picture. And so I was . . .

Jason

And is that what happened here?

Rebecca

I think I was just, I was over complicating the situation, possibly. I don’t know.

Rebecca was surprised at her success with this solution strategy. She expressed this in her usual direct way, lashing out at me playfully and chiding herself for her perceived foolishness. Her frustration was understandable, particularly given that our full conversation about the S-Pattern Task took nearly twenty-seven minutes. Despite this, Rebecca was able to persevere through that time with very little in the way of negative feelings or unproductive frustration. In an effort to manage her frustration, I decided to forego the line of questioning about anticipating student responses to the task.

Discussion.

My initial impression of Rebecca’s mathematics is that it was tied heavily to the content that she taught. Further, her mathematical understanding consisted, in large portion, of memorized formulas and methods. It is interesting that in her derivation of the solution she made very few references to the diagrams—but in the end, a reference to the diagram and drawing figures helped her arrive at an answer. Rebecca struggled with this task, taking nearly twenty-seven minutes to finish, along with several separate attempts by me to help her build on her thinking. Despite this, she showed very little outward signs of frustration during this extended problem-solving episode. She remained calm and continued to try different avenues of thought; she reached out to me when she encountered a dead end in her thinking process. She showed no outward signs of embarrassment or reluctance to ask for help. So, while her mathematics may have felt very memory-driven and formulaic, her perseverance in the face of difficulty and her willingness to seek out thought partners speaks of her problem-solving ability.
Interview 2 – November 13, 2017

Rebecca’s second interview occurred after the initial four PD session but prior to the four lesson study-based follow-up sessions. The Hexagon Pattern Task (see Figure 24) formed the basis for the first section of the interview.

Each of the shape-trains below consists of regular hexagons.

![Hexagon Pattern Task](image)

*Figure 24. The Hexagon Pattern Task.*

Rebecca initially described the pattern as “just look[ing] linear.” She noted immediately that “the number of hexagons correlates with the number of term it is.” When she realized that the question prompts were asking about the perimeter of each figure, she effortlessly changed tactics, describing a growth pattern that was entirely new to me:

**Rebecca**

Okay. But if we're talking perimeter, like, you subtract one—well, would it technically be one? Like, two has one interior that won't be counted, and then three has two interiors, four has three interiors that you don't count. But I'm thinking that would be double. So, this one has a perimeter of six if we just call it a unit. And then this would be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. So like—well, five and five. But then six, four, and six. Is this Pascal's Triangle, in a way? Almost. Because this would be six, four, four, six.

**Jason**

Is that true?

**Rebecca**

Oh, no. Those are five's. This is a five. [Referring to the terminal hexagon in one pattern.]

**Jason**

I see where you're going though.

**Rebecca**

The outers [initial and terminal hexagons] will always be five's. So, the top would be six, and then five and five, and then five, four and five, and then five, five, four, four, five, four, four, four, five, four.

**Jason**

That's an interesting idea that it grows in that way.
Rebecca: Have you ever thought about it that way?
Jason: I have not ever thought about it that way.
Rebecca: So I keep bringing up the ones that you don't know yet.
Jason: That's good.
Rebecca: Because I have such an intriguing mind. Ha!

Rebecca characterized the pattern as growing by placing a hexagon in the middle of the pattern, so to speak. She envisioned keeping the first and last hexagons intact and inserting one in the middle, thereby adding four new unit sides to the perimeter. Once she had this worked out, she was able to generate the perimeter for each figure with ease. She also noted that “it’s going up by four, so that would be the common difference.” She said this with a good deal of certainty, and I was interested in where she would head next. When she read the prompt asking for a way to find the perimeter of a given sequence, she showed some hesitation: “Well, so do I want to go off the basis that I know arithmetic sequences? Because this is arithmetic, is it not?” When prompted, Rebecca was able to justify her classification, noting that the perimeter is “going up by a counted difference of four each time . . . so $a$ sub one would be six, $d$ is four. [Writing.] Right? That would be [the] sequence equation?”

Interestingly, Rebecca used recursive notation and a combination of recursive and explicit thinking to generate an explicit formula for the $n$th figure (see Figure 25). Her notation of $a_1 = 6$, is a hallmark of recursive thinking, as is her reference to the pattern “going up by four.”

3. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

\[
\begin{align*}
        a_1 &= 6 \\
        d &= 4 \\
        a_n &= a + 4(n - 1) \\
        a_n &= 6 + 4(n - 1) \\
        a_n &= 4n + 2
\end{align*}
\]

Figure 25. Rebecca’s representation of the Hexagon Pattern from Interview 2.

However, she then moved into explicit thinking by indicating that the increase would be $4(n - 1)$. She did not represent her sequence recursively even though she noted
an initial term and a constant difference. Our subsequent discussion focused on assigning
meaning to each of the parameters in her final formula.

Jason  So what's the "4" mean?
Rebecca  Four mean?
Jason  No, the "4" in your—
Rebecca  Oh, d is the—4 is the common difference. So, because you're
adding four each time, it's not geometric where you're multiplying
to get to the next step. You're adding the same number each time.

Jason  What about the 2?
Rebecca  Okay. So, I think what the 2 is representing is the fact that, like,
two share a wall. That's what I think that is. Because if it shares,
it's like 2 to 1. Does that make sense? So here, the two hexagons
share a wall, but then that gets negated because it's interior. It's not
exterior.

Rebecca was working through her conceptualizations as we spoke. Her initial thinking,
prompted by me, might have been incorrect, but with further conversation she was able to clarify
and articulate a more conventional answer to my prompt. Rebecca did not instinctively connect
her algebraic representations to the geometric representation on the page; I was forced to prompt
her to do so. This tendency supports my characterization of her mathematics as abstract and
formula-driven, relying on her memory.

Rebecca  So I know that this is three interior walls, but I think the two
represents, in my mind, that when two hexagons meet, that's the
two that it's talking about. Or you could think about it as—well,
see, then that wouldn't make sense—my thought process wouldn't
make sense on the first one because nothing's sharing. But if I look
at as a linear function, 2 is the starting, the 4 is the slope.

Jason  And it is—absolutely. The 2 is the starting point, and 4 is the slope.
So, let’s talk about that, I guess. 4 is the slope. What does that
mean in terms of the picture and what's happening?

Rebecca  Well, 4 to 1—every time it grows, it grows by 4.

Jason  Where are those 4's in these pictures?
Rebecca: Where are the 4's in these pictures?

Jason: Yeah, when you say growing by 4, where physically are the four that it's growing in each of those pictures?

Rebecca: Well, the interior, right? All the interior—it's growing by 4 because the next hexagon will have 4 walls that are exterior and 2 walls that are interior. So, is the 4 the exterior walls and the 2 is the interior walls that negate themselves?

Jason: Well, I think the 4—I agree with you that the 4 are the exterior walls. I think we have to do a little bit of work to figure out what the 2 represents at this point.

Rebecca: Because n minus 1—Okay, so maybe going back to the original arithmetic sequence that I came up with. 6 is the starting term. To get to the next term, which would be the first term minus 1—. So, like . . . So, if I thought about it, then that means that if this was 1, then 1 minus 1 is 0, times 4 is 0. So that's why it's 6. 2 minus 1 is 1. So, in this sense, any term is the term prior—the first term. Well, n minus 1 talks about the term you have. So, you use the term before it to get to the next one. Is that what this is saying? I can't remember.

Rebecca continued to struggle to connect her thinking with the various representations. I continued to prompt her, as I wanted to see if she was able to make a breakthrough in her thinking.

Figure 26. Rebecca's notations related to her justification of the meaning of parameters in her representations of the Hexagon Pattern in Interview 2.
Rebecca noted that if she “[thought] about it as a linear function, 2 [was] the starting, the 4 [was] the slope.” This was consistent with her drawing, as she created a “case zero” during her work (see Figure 26).

Rebecca So I'm giving you the answer how the students would answer—the 4 is the slope, the 2 is where you start. But you don't start at 2.

Jason Not really, do you?

Rebecca No.

Jason The 2 is where you'd start—what does start mean, I guess is the question?

Rebecca If you had to graph this, it's your y-intercept.

Jason Exactly. So, let's think about this for a second. If where you start is your y-intercept, what does that mean in terms of the pictures? What stage is that? [Pause for thinking]. Like, these are your stages, right? 1, 2, 3, 4? Your y-intercept, which stage is that?

Rebecca Your 0 stage.

Jason So what would your 0 stage look like then given what you know about the pattern?

Rebecca Your interior walls. Or your side walls—so two is the side walls.

Jason Does that seem to make sense for the pictures?

Rebecca Well, yeah. Because the 4 are your exterior walls that get added to the situation. So, this has 4 exterior walls, 2 interior walls.

Jason What about the next one?

Rebecca Well, 4, 4, 2, and 2. You have two sets of interior—ah, you have two sets of exterior walls of four, and two side walls. Hey! So 4n, four exterior walls and represents sides. Right? No. Because this wasn't—number of hexagons plus 2 would be your side walls. Right?

Jason I guess that's a good question. Is it your side wall—I know what you're trying to say with side walls. In terms of the picture, where are these two always located?

Rebecca On the outside walls.
Jason Yeah, kind of like the ends or whatever. Okay.

Rebecca But yeah, the end would be number of hexagons, 4 as in exterior walls per hexagon, and that's the number of hexagons.

Through continued prompting, Rebecca did indeed experience a breakthrough in her thinking. She was able to successfully connect her algebraic representation to the picture provided as the initial prompt. It is interesting to note that the pathway for this thinking led through what Rebecca knew about graphing linear functions. The reader may note that interior segments are circled in Rebecca’s work. This was a spontaneous development near the conclusion of the mathematics section of the interview. Rebecca developed a way to describe the pattern by counting the number of hexagons and then subtracting out the shared sides.

Well, just like this has 6, and 1, 2, 3—so this would be 12 minus the 2 that it shares. This would be 18 minus the 4 that it shares. This would 20 minus the 6 that it shares. So, you could come up with it that way. So, this would be 3 times 6 minus 2 times 2. This would be 2 times 6 minus 2. 4 times 6 minus 3 times 2. So, what that would be in terms of n—so like . . . 6n minus . . . 6n minus—I might not be able to start with that one. But . . . 2n minus 1?

Encouraged by this, I prompted Rebecca to justify her supposition. She did so by attempting to verify with another example, the next figure in the sequence. I prompted her further, albeit indirectly, to think about the necessary equivalence between her new expression and her previous one. She made the connection and justified the equivalence as seen in Figure 27.

![Figure 27](image)

Figure 27. Rebecca's representation for her alternative way of deriving an explicit formula to describe the Hexagon Pattern in Interview 2.

In closing the mathematics section of Interview 2, Rebecca offered a thought about how she would see students attacking this problem: “But I would see kids doing it this way prior to this way because half of them don't know what arithmetic is.” I asked her which way she thought
would be more accessible to students and why. She indicated that she believed students would find more access in the latter conception of the pattern because “[i]t goes more off like a visual pattern versus knowing the rule.”

Interview 3 – March 30, 2018

The third interview took place near the end of the school year after all of the SMII PD session were complete. This interview provided an interesting opportunity to study changes in Rebecca’s mathematics. The task in Interview 3 was identical to that in Interview 2. Rebecca did not recall working on the task before, and I did not tell her. As with Terry, the time between attempts on this task created a situation in which the task was nearly as novel as the first time Rebecca encountered it.

Rebecca’s approach to the Hexagon-Pattern Task in Interview 3 shared several features with her initial attempt in Interview 2 several months earlier. She immediately noted that the pattern involved “just adding a hexagon on the side” and, when pressed to speak more about that pattern, she characterized it in a fashion consistent with her final attempt from Interview 2:

Well, the perimeter in the first is just six sides, but then the perimeter of the second has one shared side, so two go away. And then three—there are two shared sides, so four go away.

From here, Rebecca began to construct a sequence of numbers as in her previous attempt. She predicted that the third figure would have a perimeter of fourteen and so I asked her to predict the fourth figure as well. She was able to do so, explaining that “this [figure 1 to 2] goes up by four, this [figure 2 to 3] goes up by four, so this [figure 3 to 4] will go up by four. So, it will have 18.” Figure 28 shows how she recorded her thinking:

![Figure 28](image)

*Figure 28. Rebecca's analysis of a number sequence associated with the Hexagon Pattern Task in Interview 3.*
Rebecca was able to generate another way to consider this pattern when I pressed her to do so. She thought of it in a similar way to one of her previous attempts: “instead of adding on six, you add on five sides.” Here she envisioned placing a new hexagon attached to the right-hand terminal end of an existing figure to generate the next. I pressed Rebecca to explain how this view was consistent with a common difference of four. With some difficulty, she was able to justify a connection:

Rebecca  \( n + 1 \).

Jason  Yeah, that's going to be the fifth pattern. But how does tacking one on the end only end up adding four?

Rebecca  Oh, I ended up adding five?

Jason  Four. Because this pattern is still true.

Rebecca  Well, yeah, I know that pattern is still true.

Jason  Which mean the increasing constant is always four.

Rebecca  Well, because we need to disregard that side because it's going to be a shared side at some point. Right? Because if I added on to the back, I lose two sides right there. Ah, I lose that side, which is the same as that side. Right? Because if I tag this onto this end, this end kind of like—you can imagine it shifts to this side.

Jason  You could do it that way, yes.

Rebecca  Because every time we add one, we're losing two sides.

Figure 29. Rebecca's notation of her thinking about her alternative method of describing the Hexagon Pattern in Interview 3.

As the dialogue and Figure 29 shows, Rebecca conceptualized the double count of the sides as new hexagons were placed (note the “2’s” written over each shared side on the interior
of the figure) and devised a way to think about this accounting when a new shape was added (which she represented as \( n + 1 \)).

Rebecca showed her first sign of genuine mathematical curiosity during Interview 3. In connection with the discussion above, she wondered about rearranging the figures and whether that would maintain or reduce the perimeter: “I'm curious if we were to take that away if we would have the same perimeter. Twelve? No. You'd be losing too many sides.” She went so far as to demonstrate on the third figure in the task (see Figure 30).

![Figure 30. Rebecca's spontaneous thinking about possible rearrangements of the shapes in the Hexagon Pattern Task during Interview 3.](image)

This curiosity was wholly unexpected. She engaged in a rapid analysis with little intervention from me. When I commented that I found her conceptualization interesting, she replied:

“Well, because I think of a honeycomb when I see hexagons. It just depends on how you set the problem up. So, this would not work. Because some of the things that you've had us do in the past, like that square problem, adding on the squares or the pyramid and whatnot. You could take this section, fold it over, and you could get that. That doesn't work for this.

Rebecca made an explicit reference to some of the work she and her colleagues engaged in as part of the SMII PD. Each day, participants engaged with a mathematical task, even on the lesson study days. Rebecca’s reference to “some of the things you’ve had us do in the past” was in connection with those experiences. This may indicate that the mathematics experiences she had as part of the PD (the External Domain) affected her mathematics. Simply the generation of the alternate scenario was a significant departure from her previous attempts where she showed no interest in thinking outside the bounds of the problem.

When she attempted to create an equation to model the growth of the pattern, Rebecca engaged in mathematics that was much more formal and less speculative than in the curiosity space she had just occupied.
Rebecca  [Reading the prompts out loud]. So, if the first one is six, and the next one is four. I'm trying to think of a student's perspective, not my perspective. So, if it's 6, 10, 14, 18, 22, this is term 1, term 2, term 3, term 4, term 5. Plus 1, plus 4. So, if I looked at it linearly, the rate of increase would be a 4, right?

Jason  Well, I'm obviously not going to tell you whether that's right or not.

Rebecca  Darn it, Jay [Jason]!

Jason  But I need you to justify that to yourself then.

Rebecca  Or—ha. 4 divided by 1. y divided by x. Hello! And then—but if we multiplied it, we would have to do subtraction . . . no addition.

Jason  Multiply it by what?

Rebecca  Well, I'm thinking—so if I have a slope. [Writes on her paper].

Jason  Point slope form?

Rebecca  Mmhmm. So y minus (I'll just use the first one) 1 equals 4x minus 6. [Whispered calculations]. So how about I put them in the right spot.

Figure 31 shows the full extent of Rebecca’s work on these prompts. Note that she characterized the relationship in her sequence by asking “if I looked at it linearly, the rate of increase would be four, right?” Her question at the end might appear rhetorical, but her response to my refusal to provide an answer tells the opposite story. Rebecca still sought me out as a mathematical authority, despite her growing comfort with the kinds of tasks she was exposed to. In fact, it is encouraging that she noted that she was “trying to think of a student’s perspective, not [her] perspective.” One of the kinds of activities involved in the SMII PD intervention was engaging teachers in anticipating how students would attack certain problems, both as a way of developing particular aspects of Mathematical Knowledge for Teaching (e.g., Knowledge of Content and Students) and as part of the lesson planning process (which enables a more successful explore and summary phase of a lesson). Here again we see potential influence on Rebecca’s beliefs and practices from the External Domain.
1. What patterns do you notice in the progression of figures?

2. Determine the perimeter of each of the first four figures.

3. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

Figure 31. Rebecca’s written work on the Hexagon Pattern Task in Interview 3.

Once again, I prompted Rebecca to link her algebraic representations to the progression of figures. She was able to do so, but as part of this discussion we encountered interesting questions of terminology. When I pressed Rebecca about connecting the $4x$ in her equation to the picture, we had the following exchange:

Jason: My original question was—I guess I'll modify my original question. We established what the 2 takes care of, basically, in the picture. What does the $4x$ term do?

Rebecca: Well, it talks about the four sides that you add each and every time.

Jason: So that's this 4 in the increasing pattern. But what does the $4x$ do? Is it different or is it the same?

Rebecca: Wouldn't it have to be different?

Jason: Why do you think that?

Rebecca: Well, here is a multiplication. Here is an addition.

Jason: Ah, okay.

Rebecca: So for me, like, for the term that you're on, you're on the first term, you have four exterior walls and two—you have four side walls and two front and back walls. Does that make sense? [Phone rings.]
Rebecca answers. So, what I was saying was this represents the row times the set of four per hexagon. I'm changing what I said. And then this is the two bases, if you want. Does that make sense?

Jason It seems to.

Rebecca If we look at it arithmetically, we add four each and every time. But if we want to think of it linearly—does that make sense?

Jason It does.

Rebecca Is arithmetic and linear the same thing?

While I answered in the affirmative to Rebecca’s last question, I had been inquiring after the difference between explicit and recursive thinking. Rebecca consistently described the growing pattern in a recursive way but just as consistently expressed it in an explicit way as a linear function. I pointed out this fact to her and she replied, “Oh, see, we don’t teach recursive and explicit.” I include this example to bolster my contention that, in many ways, Rebecca’s mathematics is tied very closely to the kind of mathematics that she teaches—for her, if she did not teach a particular piece of mathematics in her curriculum, then she believed that she would have difficulty with it. This was also borne out by the fact that when she attempted to use a “student’s perspective” to write an equation for the pattern, she chose to use the point-slope form of a linear function and moved to slope intercept form (note that both formulas appear prominently in her work—that is, she wrote the formulas down first, and then filled in the known variables to create the function, just as she would teach students to do).

Interview 3 involved a much more robust discussion of how students might attack this problem. I chose to engage Rebecca in a deeper discussion because I noted that she was able to solve the problem in multiple ways much more quickly than in her previous attempt. While this was to be expected given that she had engaged with this task before, Rebecca’s lack of recognition of the problem and lack of commentary on the familiarity or her previous work convinced me that her success may have been due to more than simple familiarity. With regard to anticipating which kinds of thinking students tend to take, Rebecca showed a good deal of certainty.

Jason . . . I was just curious whether you had it or not. Because the kids tend to go, I think, with that one first [points to recursive form].
Rebecca  Yeah, I would agree. They would go here first [recursive]. They wouldn't necessarily go here first [explicit]. And we don't teach this form [recursive].

Jason  So here's my question following up to that—how would you manage their thought process in making the transition from recursive thinking to— [explicit thinking]? 

Rebecca  I probably would ask them—you know, you can do it this way, what's another way you could represent the data? Here you found the pattern, but how else could we represent the data? And see if they could go to a table. But then I don't know if they could get from the table to here—well, they could. Because if they have the slope, they could do $y = mx + b$, solve for $b$. They would go this route versus this route.

Jason  So they would use the slope intercept form instead of the point slope form.

Rebecca  Yeah. Because we don't teach this [point-slope form] in algebra one. I teach it in my trig class.

Rebecca was able to envision various kinds of questions she might ask students to advance their thinking and she was able to predict how her students might proceed along those lines. This was encouraging, as the SMII PD sessions had focused to a large extent on participants’ skill in this area. Questioning techniques and patterns were a significant portion of the initial four sessions and were prominent in participants’ discussions of and reflections on examples of practice during lesson study. Once again there was evidence of a tight connection to the order in which content was taught in Rebecca’s context. I pressed Rebecca on how she might move students even further along and she again gave an answer that heavily relied on specific formulas and the order in which they were taught.

Rebecca  Well, we've taught them how to identify what kind of function it is. Whether it's linear or exponential or quadratic based on a table and then how to solve for those based on the table. We've taught them how to do that in algebra one. I can't remember how we do the exponential, but the quadratic we use vertex form.
And so basically what would they do to go from this table using that?

They would have found the rate of change, how I did it, and then just did it using slope-intercept versus point-slope. They would take a point and put it 6 equals 4 times 1 plus \( b \), and then \( b \) equals 2, and then they would say \( y \) equals \( 4x \) plus 2. They would have done it this way versus this way.

Okay.

And then I would have asked them, okay, so how can we check this?

Do you think one of those ways has inherent benefits over the other? Because I noticed that you would teach them to do this, but you chose to do it that way.

I feel that point slope form will get you any form of any linear equation that you want if you have a point and a slope. You don't have to solve for a variable and then rewrite the equation. I think there's less steps knowing this form.

So you chose this for its efficiency?

Yeah. We teach them this because this is what they need to use to graph, granted it's easier to graph \( x \)- and \( y \)-intercepts. But if we're graphing our system of equations, we do point-intercept because it will give us our slope points. And if they cross that exact slope points, then it's easier to see than two points and identifying the cross. Does that make sense?

In addition to giving some insight into Rebecca’s mathematics and, perhaps, her Knowledge of Content and Students (KCS) (Hill, Ball, & Schilling, 2008), this exchange gave a small amount of insight into the kind of mathematics that Rebecca taught. She described a very specific learning trajectory and set of skills students would—she would say “should”—have within an Algebra 1 course in her district. It is interesting to note that she chose a different method than she would have made available to her algebra students.
Discussion.

Rebecca’s two attempts at the Hexagon Pattern Task were quite similar in content. However, there were some notable differences as well. While her initial thinking was recursive in both attempts, she was able to generalize to an explicit formula much more quickly in her second attempt. Further, she made more connections between different methods in her second attempt than in her first. This fact, coupled with her unprompted connection to the PD activities, imply that her mathematics may have begun to shift slightly. This shift might be characterized as the preliminary development of the ability to bring her knowledge of content and students to bear on a problem and an initial propensity for “thinking like a student.” This is an influence from the External Domain into the Personal Domain through reflection. Rebecca leveraged her experiences with other rich mathematical tasks to make attempts at the Hexagon Pattern Task in Interview 3 that she might not otherwise have made. Further, she was more confident and fluent in her reasoning in her second attempt, although not without difficulties. Along with this, Rebecca’s mathematics remained very formula-driven and closely tied to the mathematics that she teaches.

Interview 4 – June 4, 2018

The final interview took place at nearly the end of the school year. Rebecca only had a few teaching days remaining. The timing is important simply because one might expect that if any changes were going to happen during this school year, they would have appeared prior to this interview.

In the final interview, Rebecca engaged with the Candy Bar Sale Task (see Figure 32). She appeared very comfortable and fluent in solving this problem.

**The Candy Bar Sale Task**

It’s the annual Freshman Candy Sale and you have 36 candy bars to sell. Your best friend only has 24 candy bars to sell. If you sell 2 candy bars per day and your friend sells 1 bar per day, how many days will it have been when you have fewer candy bars than your friend?

*Figure 32. The Candy Bar Sale Task.*

Rebecca was able to analyze the problem, develop a solution strategy, and obtain a solution. It took her less than three minutes. She was even able to change directions mid-thought. Note her initial attempt to label the two friends as $x$ and $y$ and how she implicitly dropped that
notation almost immediately (see Figure 33). The short time it took Rebecca to solve this problem is evidence of her surety in this arena, of her confidence in her mathematics (in this case a well-defined algebraic approach to a solution).

Rebecca

\[
\begin{align*}
    x & \text{ bars/\ day} \\
    y & \text{ total} \\
    36 - 2x &= y \\
    24 - x &= y
\end{align*}
\]

\[
36 - 2x < 24 - x + 2x + 2x
\]

\[
36 < 24 + x
\]

\[
-24 -24
\]

\[
12 < x
\]

\[
x > 12
\]

18 days to have fewer candy bars than my friend.

*Figure 33.* Rebecca's written work on the Candy Bar Sale Task during Interview 4.

Okay. So, we'll call friend number one, \(x\); and friend two, \(y\). Or maybe not. Friend one has 35, so that's what she starts with. And you sell two candy bars per day. So minus \(2x\), and that's going to give you a total number of bars you have left. And friend two has 24 candy bars, and she's only selling one per day. And that's going to give her a new total of bars she has. How many days will it have been when
you have fewer candies than your friend? So, 36 minus $2x$ less than 24 minus $x$. Keep going?

Jason Well, are you done yet?

Rebecca [Laughs.] Plus $2x$. 36 is less than 24 plus $x$. [Working on problem.] So, $x$ is greater than 12. So, when you have fewer candy bars, not the same amount. Bars per day. So how many days? 12 days to have fewer candies—I’m abbreviating—than my friend. You gave me an easy one.

Jason Did I?

Rebecca At least I think it was that easy.

It seems clear that Rebecca was significantly more comfortable with a context-based problem than with a more abstract pattern-characterization problem. She seamlessly applied her knowledge of linear functions and linear inequalities to create a system of linear inequalities. She solved said system using the “balance” approach—i.e., performing the same operation on each side of the equal sign to maintain equality—as Figure 33 shows.

Rebecca initially ended on the solution “$x$ is greater than twelve.” I pressed her for meaning and connection to the context and she revised her answer to be thirteen days. Rebecca was very comfortable with the algebraic manipulations, fluent even. However, when it came to re-interpreting her answer in terms of the context, she did not immediately do so. Her initial impression was that she was done after finding the algebraic inequality. This is consistent with the contention that her mathematics is formula-driven—she knew she could use a system of linear inequalities to solve, and that system had a well-defined, algebraic method for developing a solution.

When I asked her if she could think of another way to solve this problem, Rebecca jumped immediately to graphing. I prompted her to do so and Figure 34 shows her final graph and solution.
Rebecca did not, as students might, use the Trace function to estimate the solution. She fluently accessed the Intersection function on the calculator. While doing this, she adjusted her window to show the point of intersection, saying that she “just want[ed] to get the full picture.” She talked her way through the process as she carried it out.

I just want to make sure I get the full picture. But then as I'm graphing it, I'm realizing that I can't see the point of intersection. So, I'm going to change my window and my x to go to 20, scale by 2, graph it. And then I find my point of intersection by going second calculate intersection, estimate point 1, point 2, where I think the intersection is. And so, the intersection is (12, 12). So, where I have less candies than my friend is anything over that day. So more than 12, so 13, to the right of the intersection.

Notice Rebecca’s procedural description of her process. She knew the exact progression of buttons to push to get the calculator to generate the intersection point, as any experienced algebra teacher would. She was able to reconcile the solution point with the problem context by pointing out that the solution would be “to the right of the intersection.” She linked the graphical method to her previous method, characterizing it as “the same concept as solving it algebraically because I know I want to have less candies than them. So that’s why I did it with inequalities. Um, I mean . . . I don’t really know what else to do.”

When I asked Rebecca how she might expect students to attack this problem, she was able to add another potential method: “I guess I could have done just an xy table, possibly.” Her phrasing and tone here indicated that she thought of this method as inferior to her others, but she
created the table anyway when I asked her to do so. Afterward, she objected to the method, calling it “the long, drawn-out way.” Rebecca was not able to devise any other methods for solving this problem apart from guess-and-check.

While it is true that Rebecca was able to generate four distinct but related methods for solving the Candy Bar Sale Task, she appeared to place more value on the abstract, algebraic methods. This value for abstract methods was likely a manifestation of her previous training in mathematics and the nature of the curriculum materials her district used. I point out this valuation because it had strong implications for Rebecca’s teaching.

Discussion.

Rebecca’s mathematics remained formula-driven and highly dependent on the mathematics that she was currently teaching throughout the SMII PD intervention. Her mathematics tended to be formal and accompanied by specific mathematics vocabulary and terminology such as “arithmetic” or “geometric.” She showed evidence of a well-developed set of known formulas and procedures. She displayed discomfort with the pattern-characterization tasks such as the S-Pattern Task and the Hexagon Pattern Task and significantly more comfort with particular approaches to the context-based Candy Bar Sale task.

Rebecca’s mathematics did appear to develop in several interesting ways. She gained more comfort with pattern-characterization tasks over the course of the PD and was even able to link her PD experiences to those during her interview. She was also able to attempt more predictions of how students might attempt problems as the year went on. She spontaneously announced that she was attempting to “think like a student” and was able to produce more diverse solution strategies to the final problem than the first problem she encountered. However, Rebecca was not able to devise all of the mathematically distinct methods for solving the Candy Bar Sale task and, of those she did devise, she attributed more importance to the well-defined algebraic solutions than to the tabular or graphical solution pathways. This is of note because the SMII PD activities focused on developing teachers’ ability to anticipate possible solution strategies prior to engaging with students about the task in question.

This anticipation of solutions appeared in several places throughout the PD activities. Each time participants engaged with a mathematical task (and they encountered one each session), I facilitated a discussion focused on how students might approach the problem and which solution methods they were more (or less) likely to use. Further, the anticipatory activity
was an integral part of the lesson planning process which featured heavily in sessions 3 and 4. Lastly, discussions of student thinking and potential approaches were a part of the lesson study reflections and analysis.

At the end of the project, it appeared that Rebecca’s own mathematics was developing along with her conceptions of students’ mathematics. This connection between the External Domain (the SMII PD activities) and the Personal Domain (Rebecca’s beliefs about mathematics and students) is promising. The existence of such a link indicates that change was happening within Rebecca’s belief structures and that change, by her own admission, is at least partially due to her mathematical experiences in the SMII PD intervention.

Rebecca’s Teaching Practice

Generally, Rebecca’s teaching practice was very directive and driven heavily by the curriculum materials that her district required her to use (see the example set of “notes” at the opening to Chapter 4). The kind of mathematics taught using these materials in her courses was a very formal, symbolic mathematics that focused heavily on algebraic representation. Based on both observational data and Rebecca’s own commentary, it became clear that Rebecca was the mathematical authority in her classroom and many of her instructional decisions were designed to control behavior and provide clarity in content for students.

Interview 1 – September 26, 2017

Rebecca’s initial interview took place near the beginning of the school year, just before the first of the SMII PD days. The intention of this interview was to gather a form of baseline data about Rebecca’s practice. Her mind was fully engaged in her teaching work and she had had nearly three weeks to get her class up and running. While this timing was not perfect, it did ensure that Rebecca’s thoughts about teaching were as accurate as possible—if the interview had come before school started, her responses might have been more aspirational as opposed to being more specific to what she had already begun to do.

Typical lesson structure.

Rebecca was very frank in her descriptions of her teaching. She spared few details and oftentimes commented on what she thought that I thought about her teaching. Her initial description of her typical lesson structure was quite extensive. It is interesting to note that immediately concerns about the External Domain—namely, a lack of time—surfaced. These concerns remained constant throughout the project.
Rebecca: The students come in. I used to have warm-ups, but with our time constraints I don’t really do that.

Jason: Say more about time constraints for me.

Rebecca: You know when I first started teaching here it was like five periods a day and each period was like seventy-two minutes long.

Jason: Okay I get it, so you have shorter time now.

Rebecca: So we have like sixty minutes. So, cutting down those twelve minutes in each class period, that’s twelve minutes where I could have had like a warm-up or a quick, you know, check this, and make sure you understand this. You know, we could take a minute and we could talk about it, or it could be, you know, alright we are going to have fractions in this unit, let’s just talk about some basic fractions, rules, and what could we do. I used to have the time to do that . . .

Rebecca was clearly concerned about what she perceived as a lack of time to instruct students. The loss of the twelve minutes due to a schedule change increased her anxiety about the amount of content she needed to cover (see below) and altered her teaching. This is an example of a direct line of influence (via enaction) from the External Domain to the Domain of Practice.

Rebecca continued to describe her typical lesson structure by talking about how she “like[d] to spend time on helping them answer questions from the night before homework.” She indicated a tendency to support students extensively in this, saying that she would “spend really however much time on questions for the homework that the students need. . .” However, this support was predicated on whether or not she “[felt] like the students [had] put sufficient time into their homework.” This is the first indication of Rebecca’s belief about the importance of student effort (in the Personal Domain). Indeed, in her statement she implies that if she felt students had not tried hard enough, she would not spend time reviewing the homework assignment. Thus, we see a connection between Rebecca’s beliefs (in Personal Domain) and her enactment of those beliefs in the classroom (the Domain of Practice).

Rebecca continued her description, illustrating with a recent example of practice:

And then we grade it [the homework assignment], they pass it in, I do some notes. The notes might only take fifteen, twenty minutes, but I really try to let the bulk of the time be homework time, because I really feel that’s when they learn the
most . . . like for instance, my Algebra II class today: it was just review over solving systems with multiplication. I said, “Let’s just refresh a few things. This is how I would suggest doing it.” I’m like, “Work smarter not harder. There are so many different ways you can do it. Okay, on your own do two and three, check with your table mates, duke it out if you have to. I will come back and I’ll tell you what the answers are so if I need to do the work I will. And then like we get to the story problems and I’m like these are the story problems that we’ve been doing only the difference is xs and ys will all be different. Do you need me to do this with you guys? Yes or no? And everybody said no and one person goes, “Why don’t we just do one all the way through. Let’s just make sure.” I said, “Okay.” So even though one person wanted it done I did it and some of the students just continued on in their notes and then we checked our answers and we were good. I gave them thirty-five minutes to work in class just with each other and then coming up checking answers. That’s typically what my day is in all my classes. I mean it’s very direct instruction, there’s not a whole lot of, I don’t, and I haven’t been taught enough to do anything else. Sorry, it sounds . . . That’s my class.

In her description, Rebecca evokes a vision of a classroom that is teacher-directed. She is the mathematical authority nearly all of the time. Her comments about how she recommends doing problems and that she would tell students the answers support this supposition. However, she also attempted to be supportive, as in her example when she demonstrated an example problem for the single student who asked. Her final comments characterize her instruction well. Rebecca’s teaching practice consists mainly of “direct instruction.” Her apologetic tone is likely due to her extensive experience with me in my consultant role, where I push her thinking with a more student-centered model of instruction whenever I am called upon to work with her department.

I questioned Rebecca about grouping structures in her classroom and she indicated that students have the choice to work individually or in groups of two or more “as long as they’re using their time adequately.” She indicated that her role during this time is that of a task master, where she was “walking around and . . . listening and if they’re [students] not focused [she gives] them a warning. If they’re not focused a second time [she] separate[s] them.” She indicated that “the students really can do whatever they want to in my class” as long as she was able to “get [content] across to them.” She reiterated one of her beliefs in closing: “I feel like them doing the work is more beneficial.”

Rebecca then shifted directions spontaneously and commented on her mathematics and her ability to implement tasks with her students:
Rebecca: Like this as in you gave me this S-Pattern Task. And like I have enough, like content, like base that you could ask me directed questions and I could think about it, but I don’t know enough on how to do this with my students.

Jason: So you wouldn’t feel comfortable facilitating a task like this with your students?

Rebecca: I could and I would be willing to I just, with the resources I feel like I don’t have the resources to be able to provide my students with this kind of stuff. And if I provided my students with this kind of task I don’t think they would be able to handle it well. I would have to facilitate right off the get-go and like incorporate this into my lesson right away for me to have to delegate through my students. Where if I just threw this on them they would be like up in arms, like what the heck is this, I don’t want to do this. Like I would get a lot of, you know, pushback. And you know, a hard part that we are dealing with here at this school is not so much in like my Algebra II class or my Trig class, but the will versus the skill students. We have a lot of students who just don’t do it. They would sit there and wait for you to give them the answer than to attempt any type of understanding on their own. And I push my students to understand, you know, the content and they get ticked off at me. I’m like do it again. No, do it again. Like, and I push them, and I push them in the way that I think they can handle yet not get to the point that they are about to beat me up. Because some of them. . .

Jason: So student engagement is an issue and you’re, you try to monitor productive frustration, is what I’m hearing.

Rebecca: Yeah, yeah.

Rebecca’s commentary was filled with concerns and perceived (and real) impediments to her effective implementation of tasks like the S-Pattern Task. Interestingly, only one of her concerns came from the External Domain, namely her concern about her “resources”—by which
she meant her curriculum materials. The remainder of her concerns stemmed from her beliefs about students and their abilities: “if I provided my students with this kind of task, I don’t think they would be able to handle it well.” She hypothesized a connection between her practice (Domain of Practice) and students’ reactions (the Domain of Consequence): “if I just threw this [task] on them they would be like up in arms, like what the heck is this, I don’t want to do this. Like I would get a lot of, you know, pushback.” She justified her contention with an example in the latter portion of her comment. Rebecca expressed an experienced connection—as opposed to a hypothetical one—between the Domain of Practice and the Domain of Consequence. Her contention was that, in her current practice, when she pushed students, they expressed frustration: “And I push my students to understand . . . the content and they get ticked off at me.”

In addition to her description of her practice as “direct instruction,” Rebecca also indicated an alternative pattern that I would characterize as *gradual release*.

Like sometimes in my class like yesterday, um, the last example in the notes I said, “Okay I did this one with you. We went step-by-step on this one.” They did step one, I went over step one, did you get step one? “Okay, now do step two.” I give them two, three minutes. They do step two. Okay here is step two, did you get step two? Okay do step three on your own. Like, and so we do it step-by-step and then the last one I’m like this one’s all on you. Can you follow what we did in one and two? Can you do number three? When you are done, bring it up to me, I want to see it. And then when they come up to me, I said you made a mistake somewhere, go back and try to figure it out.

In this example, Rebecca remained the classroom mathematical authority. She made a limited concession to engender student thinking by allowing the class to try each step before she demonstrated it. Then she asked students to extrapolate their experiences by trying another step independently and checking with her for correctness. This example further corroborates the contention that Rebecca’s classroom is teacher-driven.

In the final few comments in Interview 1, Rebecca made a statement that caught my attention: “I know students need to be up, they need to be learning, and whatnot, but I just don’t know how to incorporate that [movement and active learning] in my classroom. I’m a very traditional teacher in a nontraditional sense.” I seized on this opportunity and asked her to elaborate. Rebecca indicated,

I want my class to be a very safe place for my students. I have students giving answers that are like totally not even near the realm of right . . . and we are able to laugh about it, but the students are comfortable enough to give me answers
because that’s their thought process and they’re not afraid of being wrong in my class.

This is how Rebecca characterized the “nontraditional” sense of her teaching. She believed that her classroom is a safe space for students. Her response to wildly incorrect answers was to attempt to make students comfortable through humor. Her beliefs about the importance of a safe space (in the Personal Domain) were enacted in the Domain of Practice in a very particular way. Her beliefs also led her to “hold [her] students accountable for everything they do in [her] class.” Rebecca even commented that her “mom says she would never want to be a student in my classroom . . . she thinks I’m a bitch.” So, Rebecca’s beliefs about a safe learning environment were tempered by her beliefs about the importance of student responsibility and accountability to her as the mathematical authority in the classroom. She enacted this last belief by creating an environment of control in her classroom. This, paired with her beliefs about students’ abilities and her curriculum materials, is likely the reason she found herself unable to create a more student-centered learning environment.

Interview 2 – November 13, 2017

The second interview occurred after the first four days of the SMII PD but before the first of the Sustained Support days where the modified lesson study activities really began. The first prompt after the mathematical task was about changes in practice.

Changes to teaching practice.

Rebecca indicated changes in her classroom practice centered on using “more mathematically appropriate words, like inverses.” While formal mathematical vocabulary was not a focus of the SMII PD activities, it had come up in discussions between the participants. For example, Terry brought up the formal name for what nearly everyone refers to as the Distributive Property—namely, “the Distributive Property of Multiplication over Addition.” Rebecca, who had never heard the formal name before, apparently took these conversations to heart. She elaborated on this and other areas, consistent with beliefs she expressed in Interview 1.

I'm trying to throw those key phrases in so that they know it's not just something I'm pulling out of thin air. I'm trying to incorporate more classwork time for them just to work in groups to help each other versus me help them. It's just really hard with my basic students because the want v. [versus] will. I have a lot of kids who don't want. They have nothing. So, it's hard for me because they don't—I'm having a hard time helping them because they're not doing the work. So, I don't know how to help them because they're not showing me what they're struggling
on. They just say they can't do it. So that's a struggle . . . as far as those classes are concerned. But with what we've talked about, you know, in terms of my regular algebra two class and my trig class—to be honest, not much has changed.

Here again we see Rebecca’s concern about students who do not want to participate. She labeled them the “will” students—that is, students who do not have the will to participate as opposed to those who do not have the skill to effectively contribute to the class. Her incorporation of more class time to work is consistent with her beliefs from Interview 1; however, her indication that the purpose of this time was for students to work together was a notable change. This small attempt at a shift may indicate a change pathway from the External Domain (the SMII PD discussions) to the Personal Domain (a change in beliefs about the value of student collaboration), through reflection, to enactment in the Domain of Practice. Alternatively, this shift may indicate a direct connection, through enactment, between the External Domain and the Domain of Practice. It is difficult to tell with the data available.

However, Rebecca’s subsequent comment may provide some support for the longer pathway: “I don't necessarily think a whole lot of outward changes are happening, but I'm questioning a lot of what I'm doing now. It just makes me think of how could I better teach this to my kids. And [Jeremy, a colleague] and I have had a lot of conversations.” Her indication of internal conflict indicates activity in the Personal Domain, lending support to the more complex change pathway that includes that domain.

As Rebecca elaborated, she continued to indicate the presence of at least one change pathway present at that moment.

. . . talking to [Jeremy], [Jeremy] was like, “yeah, the kids came up with the notes on their own, and then we actually did the notes. It seemed like they understood it more because they did the research, and they came up with that on their own.” But I'm like, how do I do that with logs [logarithms]? Because that's such a different way for me to think because I'm not used to it. So, I've been thinking of ways of how could I change what I do, and I don't feel like I can do it now because it would be such a drastic change for me in the middle of the trimester. So, something that I'm going to try next trimester with my basic kids is I'll teach a day, and then give them a whole in-class workday the next day just so I can bop around and have students helping students. Just start there because I don't really know what to do. I feel like I'm kind of at an impasse. I don't know what to do. There's so much new stuff that I don't know how to apply it.

It is interesting to note that an influence from the External Domain here came in the form of a conversation and reflection of a colleague, namely Jeremy. A conflicting influence from the
External Domain was the number of new stimuli—ideas, stories, conversations—that Rebecca experienced in close proximity to our conversation. Her expression of frustration in the face of all this was an indication of cognitive dissonance. This was compounded by the fact that Rebecca was forced to miss one and a half days of the initial four PD days due to a family emergency. She indicated that she felt “kind of behind the 8-ball because [she] wasn’t there.” I attempted to assuage her fears and reduce her anxiety before asking her if there was anything else she was thinking of changing during her next trimester of teaching.

Rebecca spoke about developing more “buy-in and them [students] wanting to be there.” She identified this as a central issue and adopted an inquisitive stance: “So how could I change instruction for them that they would get more from it? I just don't know what that looks like. I could try what [Jeremy] does, but where we start next trimester with basic algebra two is rational operations.” Her absence from the PD intervention appeared to be affecting her ability to enact any of the changes in her belief structures that might be occurring. She lacked a vision for what this new way of teaching might look like and her conversations with Jeremy were not enough. Influences (or lack thereof) from the External Domain left her feeling as though she didn’t know what to do, despite the activity in the Personal Domain (e.g., her shifting beliefs about the value of engaging students differently with mathematics).

Rebecca did not believe that operations on rational expressions was an appropriate place to begin her changes. She expressed that she feared students would “be like ‘nope’ . . . They won’t even try it.” Coupled with this fear was a guilt; Rebecca felt like she was “doing [her] students a disservice thinking they can’t be successful.” But she appealed to experience (the Domain of Consequence) to support her argument: “Because I give them in-class time to work all the time, and they don't use it. And so, to give them the freedom to try and learn it on their own or try to think in groups or try to make any type of connections, I just don't think they would do it on their own.”

From this position, Rebecca discussed several influences from the External Domain that weighed heavily on her mind as she contemplated potential changes in practice. These influences included available time and curriculum and teaching colleagues. About the latter she suggested

I teach with two other teachers right now who are just very “we have to get to point B, so we're all on the same page” . . . We have to end at the same place. How we get there isn't necessarily that big of a deal, but we have to make sure
that our kids are set up for the next trimester. And so, I'm afraid to touch those classes because of who I teach it with.

She referred to a scheduling phenomenon that was a consequence of the trimester schedule her district adopted—at the beginning of each trimester, each teacher gets a mix of students that he or she had taught in the course previous and those others had taught. This created a need for everyone to “end at the same place” at the close of each trimester. This influence created anxiety about the time required to teach material in a student-centered manner.

About the former influence, the lack of time, Rebecca asked “and then do I have the time? Like, I went to YouCubed and tried to look at all that stuff, but I don't feel like I have enough time during the school year to even attempt to try new things. If I don't get it done in the summer, it's not going to get done kind of thing.” Rebecca referenced a set of resources explored during the initial days of the SMII PD.

It seems clear that Rebecca’s perception of her change process thus far was heavily influenced by factors in the External Domain (mainly course schedules, curriculum, and other school system constraints). However, despite this limitation, there is evidence to suggest that she was experiencing changes in her belief systems (Personal Domain). This mainly occurred through her “questioning” of her current practice, often a powerful a reflective activity in its own right. She engaged in some small efforts at professional experimentation (e.g., changing the format of her independent practice time), both as part of the PD and independently, despite all of these limiting factors (Domain of Practice).

Observation 1 – January 4, 2018

My first observation occurred in a Trigonometry class midway through the school year, between the fifth and sixth SMII PD sessions. Rebecca had experienced the initial immersion in the first four days (of which she missed one and a half) and one day of modified lesson study prior to Observation 1. As Figure 35 indicates, Rebecca’s room was arranged in pairs of desks in rows and columns facing the front of the room. The majority of instructional time was spent in a whole group, teacher-led format.

The beginning of the lesson was dedicated to reviewing the homework from the night before. Rebecca led the activity, answering students’ questions directly. When Rebecca asked a

\[\text{As part of the SMII PD, participants engaged with several tasks, resources, and articles from the website https://www.youcubed.org/ (Standford Graduate School of Education, 2020).}\]
question of a student or of the class, there was little wait time. Most of the questioning patterns fit into the Initiate-Respond-Evaluate pattern, with Rebecca as the mathematical authority. She did ask some promising questions such as “How is this sine curve different from the parent function?” and “Does that make sense?” but these questions were never fully answered by students or were answered by at most one student.

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Seating arrangement was consistent with the previous observations.

Teaching modality in this lesson mirrored closely that of the first classroom observation in both type of format (i.e., whole group followed by individual) and time spent in each (all but the last 8 minutes of class were spent in whole group instruction).

Note: Orange indicates whole-class format. Green indicates individual work. Blue indicates group work.

Figure 35. Classroom layout and instructional modality charts for three of Rebecca’s lessons
In quick succession, Rebecca showed the answer key to the homework assignment which consisted of sixteen problems on a worksheet. She displayed each problem and discussed some important or relevant features of the solution as students checked the accuracy of their answers. The entire exchange took approximately seven minutes. Rebecca closed this section of the lesson with “Write the total number wrong on your paper and turn it in!” Students went along with this routine quietly and efficiently. It was clearly not the first time such a routine had been undertaken by the class.

Rebecca began the notes section of her lesson. The notes (recall the example at the beginning of Chapter 4) began with formal definitions, in this case of periodic “sinusoidal functions.” Rebecca wrote in the general formulas after each definition:

\[
y = A \sin(k\theta + C) + h
\]

\[
y = A \cos(k\theta + C) + h
\]

At this point, Rebecca asked the class “what are some things in nature that are periodic?” Three individual students offered answers such as “day and night,” “holidays,” and “phases of the sun.” She pressed students occasionally by asking “what about that [example] is periodic?” and individual students would elaborate on their examples. One student expressed skepticism about “months” as an example, asking “What about that is periodic? Aren’t months man-made?” Rebecca fielded this question herself, answering “I’d say it must be something that applies to all human beings.” The exchange continued, with Rebecca and the student elaborating and explaining further.

When Rebecca began the section of guided notes devoted to example problems, she set up the example for students first by filling in the general formula and then leading the class through a set of questions, which they answered chorally. She made the recommendation that students focus on how the example is or is not like the homework problems for that day. Then she asked the class “what does A mean again?” and accepted the choral answer of “amplitude.” She then asked, “How do we find A given [exact] values [for the other parameters]?” Several students offered tentative explanations. However, Rebecca chose not to take these up and answered the question herself, guiding students through the remaining coefficients in the same manner, with little wait time between her questions and her own answers.
For the parameter $C$ in the first example, Rebecca changed her facilitation strategy. She instructed students to “Take a minute and solve for $C$. If you need to work with a table mate, do it.” Students needed to solve the following equation:

$$20 = 24.5 \sin \left( \frac{\pi}{6} (1 + C) \right) + 44.5$$

After giving students approximately ninety seconds to work on the problem, Rebecca prompted the class, “What’s the first thing we do when solving for an unknown? . . . [short wait time] . . . Am I going to have to do this with you?” Just before Rebecca moved to demonstrate the problem, a student initiated an exchange:

S: Are we going to have to reverse sine this?
T: Use the correct term . . .
S: Inverse sine this?
T: We can input this into our calculators and get the most accurate answer possible. [Demonstrated the procedure she indicated and solved the problem for the students.]

The remainder of the lesson went in a similar fashion; when Rebecca asked a question of the class, individual students replied, or the class replied as a chorus. With approximately nine minutes left in the class period, Rebecca transitioned students to homework time. There was little academic interaction for those last minutes; only a small group of students chose to work on the homework assignment. The remainder of the class socialized quietly.

Discussion.

It is interesting to note that Rebecca generally asked questions at a mixture of levels. Some were clearly meant to elicit a pre-arranged choral response. These questions tended to be recall questions, such as “What does $A$ mean?” However, some of her questions were more insightful and had the potential to generate student thinking and discourse. Examples of these are:

- How is this curve different from the parent function?
- Does that make sense?
- We have this equation but what does it mean?
- How do we know when to use sine or cosine?
Each of these questions represented an opportunity for Rebecca to engage the class in a discussion of ideas and mathematics. However, Rebecca never chose to engage students in this way. This is likely because, as she commented in her opening interviews, she did not know how. Despite having her PD experiences behind her, Rebecca had many opportunities to learn about and enact strategies for engaging students with these kinds of discussions. Further, the data from this lesson observation confirm Rebecca’s assessments of her teaching from her initial interviews: she provided an example of how her instruction is “traditional.”

It is also possible that Rebecca’s claim that she is “questioning a lot of what [she does]” is in evidence here. Unfortunately, no baseline observational data exists about Rebecca’s teaching prior to PD intervention, but her contention that she was questioning her practice (from Interview 2) may have manifested itself here in the form of her questions. While Rebecca did not choose to facilitate a class discussion about questions that had the potential to generate student discourse, she did ask them. It is possible that she would not have even asked those kinds of questions prior to the SMII PD experience. I claim this because during the PD, she experienced mathematics as a learner in ways that involved her being pushed by me to think deeply. The press for meaning was something I did frequently as the group engaged in mathematics each day. Ultimately, however, the data are inconclusive on this point. In any case, while Rebecca’s facilitation of this lesson was what she called “traditional,” it was not without potential.

Observation 2 – March 29, 2018

The second lesson observation occurred in late spring, after all of the SMII PD sessions had been completed. At this point, Rebecca had engaged in several rounds of lesson study and had reflected on what she had learned about facilitating the explore phase of a mathematics lesson. This timing implies that one might reasonably expect to see some manifestation of the changes Rebecca had indicated were happening for her. This lesson was a Pre-calculus lesson. Rebecca’s room was arranged identically as in Observation 1, in rows with students’ desks paired. The overall distribution time during the lesson was slightly different, although there was only whole-class and individual work, consistent with Observation 1.

Rebecca began the class with the same kind of pre-formatted notes as in Observation 1. The class was continuing their study of hyperbolas. Rebecca directed the class to a specific example problem in their notes. This problem involved an algebraic equation of a hyperbola in two variables and students were tasked to solve said equation for y. Rebecca, writing on her own
copy of the notes and displaying her work using a document camera, demonstrated for students how to arrange the variables so that the equation resembled a standard quadratic equation in the variable y.

$$-2y^2 - 4xy + x^2 - 2 = 0$$

She identified the “coefficients” in the equation with those of a standard-form quadratic (namely $a$, $b$, and $c$) and asked the class “how might you want to solve for $y$?” Her intention was to clearly connect this representation to the Quadratic Formula, allowing students to solve it on their own. However, she decided to help students by filling in the quadratic formula for them and then set the class loose, individually, on simplifying the following equation:

$$y = \frac{4x \pm \sqrt{(-4x)^2 - 4(-2)(x^2 - 2)}}{2(-2)}$$

As students worked individually, Rebecca circulated the room, answering questions and pointing out errors she noticed. She noted these with comments such as “You can simplify further,” “Simplify further. Write the whole thing. That’s what the Discriminant should be,” and “You simplified your negatives wrong.” She was very direct, but not unkind in tone or temperament with her students.

After about five minutes of this, Rebecca brought the class back together and noted, “Some of you are doing okay. Some are getting tripped up by the algebra.” She then proceeded to demonstrate the simplification process on her copy of the notes. With the simplified equation in hand, Rebecca led the class into the next portion of the example. The goal of solving for $y$ was to make it possible to graph the function in question. Rebecca began this process by asking “how many equations will we punch in?” Several students answered chorally that there would need to be two equations. Rebecca physically and verbally walked through the process of entering the two equations into her graphing calculator, which she displayed on the document camera.

Students followed along silently.

Once the class had their graphs in hand, Rebecca asked a series of questions.

T: Why is there a gap? What do we know about hyperbolas?

Ss: [Various answers from individual students talking over each other.]

T: This one has a rotation, so what do the gaps represent? What axis are the vertices on? [wait time] Where can you look? [referring to students’ notes packets]

T: What axis are the vertices on?
S:  [mumbled, inaudible response]
T:  It’s on our transverse axis! Right!

Rebecca moved the class to the next example problem and asked, “Why is this a degenerate case?” A student replied with what was, apparently, a satisfactory answer, and Rebecca did something new. She evaluated the response as correct and made a facilitation move I had not seen her make before:

T:  Couldn’t have said it better myself! Can someone rephrase what was said?
S:  I didn’t understand.
T:  Can you rephrase?
S:  [Passable rephrasing of the previous explanation.]
T:  This is not how I did it, so how else might we do it?
S:  You could divide by three.
T:  [Evaluative response about the efficiency of the method being low.]
S:  Could you move the \((x - 1)\) binomial to the other side . . . and solve for y? Or you could use an x-y table.
T:  Is this a legitimate way?
Ss:  [Several student answer in the negative.]
T:  How could we figure out what kind of conic this is?
S:  Multiply it out.
T:  Exactly. If we do the box [method for distribution] on both terms we get [reads expanded form aloud].

At this point, Rebecca made a short reference to the notes page and related how this was, in fact, a degenerate case of a hyperbola. She then assigned the students approximately twenty homework problems and allowed the remainder of the class period (approximately 20 minutes) for homework time.

Discussion.

Interestingly, this classroom observation differed in one significant way from the previous observation. Namely, Rebecca employed a facilitation technique that was new to her practice. While the first portion of the lesson was very consistent with Rebecca’s teacher-led, “traditional” practice, she did incorporate a facilitation attempt into her lesson in the latter half. This new facilitation attempt was initiated by her asking students to rephrase what another
student said. This was significant because this talk move was one that featured in the SMII PD activities. It is possible that this evidence indicates the presence of an enactment in the Domain of Practice—Rebecca was engaging in professional experimentation. While she was not, perhaps, fully successful in implementing the facilitation move, she did make an attempt. Paired with her commentary in Interview 2 about questioning what she was doing, this may indicate a change pathway from the External Domain (talk moves in the SMII PD) through the Personal Domain (questioning current practice and finding value in alternative facilitation moves) to the Domain of Practice (professional experimentation with a specific talk move).

Another feature of interest is the fact that Rebecca asked for alternative methods of solution in this observation. She indicated in Interview 2 that she was doing so, but here we find the first direct evidence of it. While it is likely that her purpose in asking for other methods was that the one she used had not come up yet, the fact remains that she asked students for alternative methods, which she did not do during the first observation and did not report doing in her initial interview.

Rebecca showed evidence of attempting to integrate new instructional moves into her practice. She was not entirely successful at doing so and was not able to do so in a way consistent with her experiences in the SMII PD—but she made the attempts. This is promising evidence of a shift in both beliefs and practice. It gives insight into the role that professional experimentation plays in teacher change. These novice attempts at facilitation are a requisite for building more successful attempts and, more important, serve as grist for reflection.

Interview 3 – March 30, 2018

Interview 3 took place in late spring, ten days after the last SMII PD day for the year. As with prior interviews, the opening question of the teaching practice section focused on changes in practice.

Changes in Rebecca’s teaching practice.

Just prior to my opening question, Rebecca shared an example from her classroom. It is worth including here as it gives insight into her thought process and context to what follows. Rebecca spoke at length about engaging her precalculus students with a problem that consisted of a description of a square piece of material of a given area (486 square centimeters) that was then converted into a net which formed a box with a given volume (504 cubic centimeters) by
cutting a single square with side length three out of each corner. Students were tasked to use this verbal description to write a system of equations to describe this situation.

Rebecca began this story by saying that her class had “looked at [her] like [she] was crazy” when she told them they knew how to do this. Her response was to refresh students’ memories. The class “kind of went back, talked about what [they had] seen before, what [they had] done before . . . and they were fine. They just had to have a little refresher.” However, it turned out that students were not fine. They had, as one might suspect, “a hard time visualizing [a picture of the problem].” Rebecca described her solution: “I tried to relate it back to those blankets—those fleece blankets that people would make. You cut the corners out, you tie the things.” She indicated that then students were able to better visualize a diagram of the problem.

At this point I asked about notes, because she had shown me a copy of the notes for the day she indicated above. She indicated that these notes were “just a directional,” a way of providing students “an example of what they could see or how they could use it.” She expressed concern about the problems being “so long-winded that [she didn’t] want to have a student spend time writing [the problem] down.” She referred back to her example and said, “were these notes, if I would have taught, like, bam bam bam, I would have been done in twenty minutes.” I asked her what she did differently.

Rebecca I gave them time to work as a group to see if they could figure out what this picture was and what this was. I gave them 10 minutes to work on it.

Jason Is that the same or different than you would have said last year when you taught this?

Rebecca Well, I didn't teach this last year, but it's different than what I would have had I taught it last year. Because I would have just gone into, okay, let's talk about what the picture looks like. I wanted them to see if they could figure out what the picture looked like based on the description of the problem and come up with generic equations for what that would look like over here.

Jason So you would have run that conversation yourself as opposed to giving them time to do it as a group first and come back to it.

Rebecca Yeah.
Jason Interesting. Were they successful?
Rebecca Some of them were. Some of them were like—.
Jason How did you leverage the ones who were successful? Did you leverage their thinking at all?
Rebecca Probably not like I should have. I could have had them lead the discussion versus I just asked them what it looked like, and then I just drew it.
Jason But you did ask the students who—.
Rebecca Yeah, I asked them what it would look like, and then they told me that these were all three. And then I asked them what would this look like based on this picture. I didn't give this to the students. They gave this to me. Well, I helped them change it to $x$ and $y$ because they wanted to say $l$ and $w$. But it wouldn't have helped them when it comes to solving for $x$ or $y$ or whatnot.

Jason Consistency with variables.
Rebecca But then this question said find the dimension of the piece of [inaudible] where you would have to solve the system together. And since there is an $xy$ term and 486 is what $xy$ equals, one of the students is just like, oh, just replace that. But one of the other students said, but you would still have two variables in your equation and you couldn't solve for one variable. I said, can anybody resay that? And then I said, [Student], does that make sense why that wouldn't work? And he said, oh, yeah. That makes sense. So what else could we do then, [Student]? So I put it back on him, and he's like, well, we could solve for one of the variables. I'm like, exactly. And so I left it there. And then this one, use a graphing calculator to check your solution—so the equation, again, would have to equal $y$. I had the students do that one on their own.

Rebecca described a situation in which she had every opportunity to simply demonstrate the solution process to students, as she had done in her classroom observations. However, she chose not to. Instead of using the pre-formatted notes as a template for her to fill out while
students watched (as in Observations 1 and 2), Rebecca made the choice to give students time to work on the problem prior to the whole class engaging with it. This is different than what she described in her initial interviews and what I saw in Observation 1.

It is interesting to consider the possibility that this is an extension of what she did at the beginning of Observation 2 (she gave students time to attempt to simplify the Quadratic Formula before demonstrating). Here she gave students the entire problem to work on for a significant amount of time, where before she only gave part of the problem for a short time. Rebecca described a set of intentional teacher moves that were different from her instinct, different than she would have done before her SMII PD experiences. Coupled with the fact that the PD participants had had an extended discussion of how to use the pre-formatted notes in ways that supported student inquiry and understanding, her description here may indicate an instance of more advanced professional experimentation. This lends support to the idea that the change pathway described above, from External Domain through the Personal Domain to the Domain of Practice (and centered around facilitation moves), might have been developing more fully.

Further changes in Rebecca’s teaching practice.

Rebecca also indicated that she was “feeling more comfortable being willing to pose questions that I’m not comfortable with, knowing that I might not be able to answer what they’re [students] asking.” She said that she was more comfortable because it didn’t “really feel like [she was] on a time crunch with these students [precalculus].” I asked her if it would be different were she teaching a different course (say an algebra course) and she noted that she “would feel more on a time crunch because I have other colleagues that teach it with me . . . [she] would feel like [she] would have to get to end point A at the end of first [trimester] simply because at the end of first all our kids are getting mixed up.” She further expressed concerns about her colleagues, saying “Now, if I know that I can get to point A in first trimester teaching however I want to teach, I would be comfortable with that even though I would get flak from some of my colleagues.” This influence from the External Domain remains a recurring phenomenon, permeating all of Rebecca’s conversations. However, it is interesting to note that she was able to engage in more professional experimentation in a space where this external influence was not present.

Another exchange focused on Rebecca’s use of the phrase “how I want to teach”: 
Jason You used the phrase a couple different times here—"how I want to teach it." Say more about that for me. Would you do it this way if left to your own devices or would you depart from those?

Rebecca I might not necessarily depart from these notes. But something that [Jeremy] did that I didn't even think to do to get my students up and moving—we could have made this picture with my desks and said, okay, make a rectangle, okay, now take a square out, which would take a desk from each side so that they could visually see what the representation would look like.

Jason And he did that with his kids?

Rebecca He did that with his basic geometry kids when they were trying to talk about, like, area and perimeter, like one desk was one square.

Jason So were you in there for that, or was that just a conversation you two had?

Rebecca I had to ask him a question about a student in my fifth hour that was his student in his algebra one class. And I just stood there and watched him teach and explain.

Rebecca benefitted from a spontaneous opportunity to view one of her colleagues teaching. She was able to reflect on that and make a connection during our conversation in Interview 3. She later elaborated on her treatment of the notes, saying that while she would still use the notes, “how we talk about it and explain it might look different, and then we could come and put our final thoughts on here.” Her comment seemed to indicate an exploration phase followed by a summary phase during which the notes would be filled out. This represented a reversal of her previous practice.

Interestingly, the idea of getting students “up and moving” was not part of the SMII PD activities in any way. Rebecca attributed her value for this idea to another influence from the External Domain:

Rebecca If they can see it and touch it, it makes it more real for the students. I could even have them—like, take a sheet of paper, take a piece of paper, and say, okay, here's your rectangle. Okay, now make a
cube, the corners have to be 3x3. And have them actually cut it out and make it.

Jason  So you've seen more of that over the course of this year, is that what I'm hearing from that conversation?

Rebecca  Not from me, but from what [Terry] and [Jeremy] have done. They've made it more hands-on. The first time through this particular lesson, I don't know what it looks like. So, I don't think I'm willing to do as many out-of-the-box things because I don't know what the questions are and whatnot because I haven't done it before. But as far as my algebra two class, this is something that I could do in my sleep. I think I would be more open to changing how teach it—if that makes sense.

Jason  So what I'm hearing is your comfort level with the content determines how free you feel to depart from this more structured way of doing things.

Rebecca  Oh, absolutely. I'm not afraid to not know how to answer a student. I think the timeframe constraint is what hinders me more.

Rebecca’s values were influenced by the activities of two other teachers and their collaborative work on a geometry course (External Domain). She internalized some value for physical, “hands on” activity in learning. Additionally, Rebecca noted that she was more likely to try new facilitation methods if she was comfortable with the curriculum materials. This is an interesting loop between the Personal Domain and the Domain of Practice—as Rebecca’s knowledge of the curriculum (knowledge of content, pacing, lesson detail such as example problems) increased (the Personal Domain), her comfort and ability to professionally experiment in the Domain of Practice increased. This may indicate that, for Rebecca, her ability to change was influenced by her knowledge of the content and her experiences in professional experimentation.

Later in our conversation, I asked Rebecca if things would continue to change and this led to a conversation about some successes she had experienced:
Rebecca: I think so. I think things will continue to change because that's just the natural progression of things, honestly. I'm seeing success in some of the things that I've changed.

Jason: Like what? Give me a couple of examples.

Rebecca: More buy-in from the students, more ah-ha's.

Jason: And those you think are related to what kind of instructional moves?

Rebecca: Putting it back on them. Making them think about the problem. Trying it out on their own. Working through the frustration. Working with each other versus me up here dictating the conversation and being a drill sergeant. And some of the things that you've taught me—take it back to concrete things that they know. Saying, okay, how could we relate this? Okay. Let's change this to an x-y concept. Now let's apply it to what the question is asking us to do. That I would never have done before because simply I didn't think to do it before. And now I will continue to do it because I think it's really helped, especially with my trig students, to see it and just rewrite it in a different format for the time being.

In this exchange, Rebecca highlighted several elements from the Domain of Practice (e.g., “putting it back on them”) and coupled them with elements from the Domain of Consequence (e.g., “more ah-ha’s”). This may indicate the presence of a change pathway involving the Domain of Practice, the Domain of Consequence, and the Personal Domain.

A connection to the SMII PD activities.

In closing the teaching practice section of the interview, I asked Rebecca which experiences from the SMII PD she thought were the most influential for her. She gave an answer typical of teachers in PD studies but justified it in an interesting way.

Rebecca: I think just the collaboration with my colleagues and the problems that we did together and just asking for help. Asking them how they would look at it, what they would do differently. The videos really helped, just hearing their feedback based on what I've done
to see if there's any suggestions that they could give. I wasn't there for the lesson planning that you guys did, so I can't really speak to that. Yeah. I've really enjoyed the conversation and the suggestions between colleagues.

Jason  So the collaboration kind of tops your list then.

Rebecca  Pretty much for me, yeah.

Jason  Good to know.

Rebecca  Because without this—we’ve always talked about getting in each other's classes, but it's become so much more real for us.

Jason  With the time actually set aside to do that.

Rebecca  Mhm. Yeah.

Jason  Okay.

Rebecca  And that's something that I think at least the three of us are going to continue to do.

Jason  Why do you feel that the collaboration has been so beneficial to you?

Rebecca  Because I'm not on my own little island trying to figure out how to do it on my own. [Jeremy] is really good at turning the questions around and answering the question with a question, putting it back on the students. [Terry]'s really good at asking questions that I wouldn't even think to ask. I'm just really good at giving the kids a swift kick in the butt to get them to do the work. I don't know.

Jason  So you can use their strengths to supplement your own then in those conversations.

Rebecca  Mhm.

Rebecca made note of some interesting things in her explanation. She began by speaking about her value for the collaboration time during the PD but quickly moved on to other features of the PD including studying teaching practice through video. This value is clear, particularly when one considers that in her interviews Rebecca referred to multiple instances of colleagues’ work—and she was not the only one (see Terry’s and Jeremy’s case study reports for similar references). She made specific reference to the skills of her colleagues and implied that those
skills had somehow benefitted her in their collaborative work. More specifically, “collaboration” was an overarching term for Rebecca. Though she did not say it directly, she used collaboration as an umbrella to encompass all of the collaborative experiences she valued. These included doing mathematics with her colleagues, studying teaching, and generally talking about teaching practice. She indicated that this was something the department had wanted to do but was unable to due to time constraints. This group-level value for observing one another’s teaching was satisfied by some of the SMII PD activities (the lesson study offered extended time to collaborate and discuss teaching practice as a group, the mathematical tasks provided time to discuss mathematics and student thinking as a group, etc.). Further, Rebecca was able to take ideas and skills away from those experiences she would not otherwise have.

Observation 3 – May 29, 2018

The third and final classroom observation occurred near the end of the school year. This was deliberate, as this allowed for two things: a maximum amount of time for study participants to integrate changes into their practice and sufficient time for reflective change to occur after the final SMII PD sessions. It must be noted that the record of this final observation for Rebecca differs from the previous records. The first two observations were recorded using the Lesson Note software on an iPad. Technical difficulties precluded this approach for the last observation, so in lieu of Lesson Note statistics and visuals, I created a classroom observation narrative as the lesson progressed. I sat at the back of the room and wrote the narrative on a laptop. This final observation took place in a precalculus class. The full narrative appears below:

Rebecca’s class was organized into rows and columns of pairs of students. She used an interactive white board (IWB) for her teaching presentations. The IWB was surrounded by dry-erase style boards filled with announcements and schedules for her classes. Rebecca’s desk was located at the front left of the room (from the students’ views).

The beginning of class was dedicated to reviewing and assigning grades to the homework assignment. Several students asked questions about the homework assignment, mostly noting that they needed help with a particular number. Sometimes students were specific about the issues related to the problem in question. In all cases, Rebecca provided direct answers to student questions either in the form of verbal responses, or a combination of verbal and written work
demonstrating how to complete various aspects of the problem under consideration.

As the lesson progressed, Rebecca guided students through a set of preformatted notes. Each student had a copy and Rebecca filled in her own copy, using a document camera to display her work. The content of the notes consisted of an opening vocabulary section and a series of example problems related to the day’s content (exponential and logarithmic functions, as well as linearization of data). Exchanges with students were few during this process. If an exchange occurred, it was between teacher and student and followed a tradition IRE pattern. Rebecca responded to student questions with explanations of content. The questions Rebecca asked tended to be surface level, asking for answers to calculation problems (which she checked on her own calculator), or asking students to relate the context-specific form of a function to the general form of that function (e.g., matching coefficients with parameters, etc.). Occasionally, students were given short periods of time to work on aspects of a particular problem individually. The classroom organization did not encourage students to work in groups and so none did. Students rarely spoke to one another.

Rebecca made attempts to get at deeper knowledge, but these attempts were stymied by a lack of wait time. One such attempt involved moving through a procedure for converting the general form of an exponential function into a linear function through the use of the natural logarithm. Rebecca began with the general form for an exponential function and guided students through the process of conversion. She did so in demonstration fashion, from the front of the room using a document camera to project her work. During this time, Rebecca asked questions that pushed students to make meaning beyond the simple memorization of the process. However, after each question there was very little wait time and so students were deprived of the time to think about answers to those questions.

The lesson closed with approximately eight minutes of independent work time (for a homework assignment of sixteen problems). Most students did not take advantage of this time to complete the homework problems.
Generally, this observation was very similar to the first observation in both format and content. The only notable difference involved Rebecca’s questions. She continued her pattern of asking questions of the class that pushed for deeper meaning; however, she did not facilitate students’ answers to those questions in ways that allowed for higher quality responses. She relied heavily on the pre-formatted notes and these seemed to drive her teaching and her facilitation choices. She did give students time to work on problems individually before she demonstrated the solutions for the class, but the time was unstructured, and students’ collaborative efforts were prevented by the arrangement of the room. Rebecca also continued to make regular use of the Initiate-Response-Evaluate questioning pattern. This was despite the fact that the SMII PD intervention dealt specifically with IRE and funnelling questions and provided both resources and planning time to establish and maintain alternative patterns of questioning. It seems clear that, at least with regard to precalculus, while Rebecca was able to bring some of the changes she discussed to her practice in a temporary fashion, she was not able to integrate them into her practice so that they became a regular part of her everyday teaching.

Interview 4 – May 29, 2018

The final interview of the project took place two weeks before the end of the school year. This timing was mainly to ensure that all classroom observations had been completed prior to the final interview.

Changes in Rebecca’s teaching practice.

I opened this section of the final interview with a question about changes in Rebecca’s teaching practice. Her initial example was that she was “trying to have students reiterate what [she had] said more” which she believed “kind of shows [her] if they actually heard what [she] said.” She elaborated when I asked her if she felt this approach was working,

Yeah, because I think listening to the students try to explain what I've said gives me an idea if I'm saying it in a way that they can understand, or the way I said it, are they interpreting it the right way if they can explain it. So, it's giving me an insight on if they're actually able to replicate what I've taught them, conceptually at least.

Rebecca referred to an instance in her previous class period where she had applied a facilitation strategy to strengthen and reiterate an explanation posited by a student. One student indicated a lack of understanding and so Rebecca asked another student to reiterate the explanation. She indicated that this was a new approach for her.
Yeah, he had asked, why are we doing this? And I had just explained it to another student who asked the same question. So, I had another student explain to them what I just explained, which I had never done before. Some other things—just really asking more thought-provoking questions. What I've noticed I've done more this year because of SMII is they [students] would ask a question that I might not have done the problem that way . . .

It is worth noting here that despite Rebecca’s claim of this facilitation move (i.e., rephrasing) being new, it was not. She made a similar facilitation move during Observation 2. The fact that she also reported using it in other lessons is promising and lends support to her claim that she was trying to create more of those kinds of opportunities for students.

Rebecca also gave a specific example of how she handled it when a student asked about a solution method that Rebecca had not done herself, as she referred to in the excerpt above (Figure 36 shows Rebecca’s solution). Rebecca indicated that she would have taken the more conventional approach of “taking the log of both sides.” However, the student in question did not. In fact, she “actually did it very differently where she used the change of base formula.” I asked Rebecca how she dealt with this.

Figure 36. Rebecca’s example of a student's proposed solution method.

Rebecca This is how I did—or what I did in class, I said—she suggested something. And I said, well, you know, I didn't do it that way, but explain to me what you want to do. And so I wrote it down. And I'm like, okay, what would be your next step? What would be your next step? And she walked me through it. And I said, okay. I don't know if it's right—I knew it was right—but I told her that. And I said, what's a different way we could do it? And I was trying to get them to come here, using our rules of logs. And both methods
worked. So, I went outside my comfort zone. Instead of just saying, no, that's not how I suggest you do it.

Jason So is that what you would have in past years is to just—.

Rebecca Possibly. Like, no, just use our rules of logs. And I would have kind of shut her down because it's not what I'm used to doing, but she suggested something that worked. So yeah . . .

Indicating that in the past she would have “shut [the student] down” but that now she handled it differently lends support to the idea that Rebecca had shifted some of her beliefs about the value of students’ ideas in her teaching and that she had enacted those beliefs in her classroom by accepting students’ thinking and attempting to leverage it for learning. This tied back to the SMII PD experiences as many of the readings featured similar situations and the discussions of the group around mathematical tasks focused largely on multiple solution methods. So, there is support for a change pathway from the External Domain, through the Personal Domain, to the Domain of Practice with regard to the use of student thinking.

Interestingly, Rebecca commented further on issues in the Domain of Consequence, saying that students “don’t really like it when I put the thoughts back into their own pocket. They really hate it, actually.” She said this was because, as precalculus students, students “just want me to give them the notes already.” Despite this, Rebecca confessed that she was continuing in “just asking different questions . . . allowing [herself] to go off on a whim.” Previously, she indicated some anxiety about covering material, but this time she took a more expansive approach:

if I don't get through the material, I don't get through the material. I'm becoming like, okay, let's try this. You suggest this, so let's go this way. You explain to me what your thought processes are, and I'll say if mathematically it's legitimate or not. You know, because sometimes they want to do this, but, no, mathematically we're not allowed to do that.

While Rebecca continued to place herself as the mathematical authority, she indicated a process she had developed for dealing with solution methods she had not anticipated. However, this approach seemed to be limited to content that students had had some experience with before. For new content, Rebecca indicated that she did not give students time to work on problems individually before her demonstration.
It appears that Rebecca took a very specific facilitation move from the SMII PD (namely, the rephrasing talk move) and worked at implementing it in her classroom. She did not indicate any other different facilitation moves from the PD; rather, she indicated a process for dealing with unanticipated student thinking. It is worth noting that the SMII PD activities focused on anticipating student thinking as part of the planning process. Rebecca’s interpretation of those experiences was to begin to accept student answers that did not conform to her pre-conceived notions of what a solution method should be. She did not appear to use anticipation as a strategy to prepare for these exchanges, rather, she reported that she began to accept alternative student methods and attempted to deal with them on the fly, so to speak. These examples are interesting in that they both are the result of professional experimentation, but the development process for each was vastly different and connected to different sets of experiences. The use of the rephrasing talk move likely came from observing me and her colleagues using it in the SMII PD sessions and classroom videos. Rebecca’s process for accepting and processing students’ ideas was something she developed on her own—no such process was laid out or discussed explicitly in the SMII PD. However, the value for student thinking and multiple approaches was stressed in the SMII PD.

Rebecca’s plans for the future.

Rebecca indicated that she planned to shift the way she used the pre-formatted notes in the future:

here are the notes for today, a lot of it is stuff that you've seen before, do what you can. Work with your partners, see what you can remember. And then giving them 10, 15 minutes to do that on their own, as long as they're focused on it, and then bring them back together and say, okay, what did you guys get for this one? Did anyone get anything different? Let's talk. Let's compare. Why does this one work versus this one not work?

More generally, she indicated that she planned to be “more Q and A versus sit and drill . . . now that [she] kind of know[s] what that looks like from the practice [she had] done.” Again, she referred to a rephrasing talk move: “Just, hey, he said this. Can you reiterate that? Or just having students answer the questions more and trying not to funnel my questioning”—Rebecca had confessed during a PD session that she was “a huge funneler.”

Rebecca’s plans for the future appeared to focus on those things that she knew how to do or had spent time considering. She had said from the second interview forward that she was
constantly thinking about the kinds of questions she was asking students. She also took up a specific facilitation move that she had witnessed and experienced and was in the process of practicing it in her classroom. Generally, her plans indicated a possible shift in beliefs about teaching (the Personal Domain) as a result of her PD experiences (the External Domain) and her practice (the Domain of Practice). The connections along this pathway appear to be both reflective and enactive.

Connections to the SMII PD experiences.
Rebecca noted several features of the SMII PD that she found helpful.

Rebecca I think more of the conversation, the activities that we did that you had given us as a group.

Jason Like the actual math problems?

Rebecca The actual math problems to talk about it and explain what the individual pieces mean. When you asked the question today in math, “What's the slope [mean]? What's the y-intercept [mean]?” [I had observed her classroom that same day and asked those questions of her students as an example of how to connect a final representation back to an initial problem context]. I wouldn't have thought to ask those questions, but those are some things that are thought-provoking that I can look towards. But it's the group work and the thought process of others to help identify how my students might think. That was really beneficial. You asking, okay, we could do it this way. Well, what are some other ways that we could do it? I try to do that with my notes, like the question she [the student who proposed an alternative solution method] asked, just that kind of stuff . . . Watching the videos was really good. Hearing what other teachers are doing, how they're asking the questions. A couple things that I've really taken is just using my mathematical words appropriately versus making sure we're the same across the board kind of thing.

Rebecca indicated that she valued the conversations with her colleagues, the mathematical experiences focused around multiple solution methods and watching classroom
video. These experiences in the External Domain translated into changes in Rebecca’s beliefs and knowledge in the Personal Domain, as well as her teaching practice in the Domain of Practice.

When asked what advice she might give to other teachers trying to make these shifts through a PD like SMII, Rebecca echoed her colleagues’ recommendation to just do “a little bit at a time . . . you’re not trying to change the world all in one day. Pick a class and focus on that class . . .” This incremental, small changes approach might come via doing “one lesson a week. Just start small. Try not to change everything right away because it will be too much. And you don’t have to change everything . . . just start doing it. Get your feet wet. And if you don’t know how, that’s when you ask your colleagues.” Rebecca also noted that even with this “small changes” approach, more support was needed.

Discussion.

Rebecca was able to make some small changes to her practice over the course of this study. These changes were slow to manifest, and she had not fully established the new practices as part of her daily instruction. In the main, the shifts that occurred for Rebecca were in the Personal Domain—where she shifted her beliefs about the value of students’ ideas and her understanding of both mathematics and how students were likely to attack problems—and in the Domain of Practice—where she was able to implement a rephrasing talk move on multiple occasions and experience enough success that she planned to continue to use the facilitation move. Several change pathways involving three of the four domains (the Domain of Consequence rarely came up) manifested through observational and interview data. More specifically, Rebecca’s changes appeared to be heavily context-dependent—that is, she was more likely to use the “new” skills in situations where she was familiar with the mathematics and the materials used to teach it. Further, she used one new facilitation move only—she did not develop facility with others. This is consistent with her advice to others: start small and change a little bit at a time.

Rebecca’s Vision of High-Quality Mathematics Instruction

Rebecca, like her colleagues, struggled to create a concept map that described her vision of mathematics instruction. However, also like her colleagues, she was able to speak about her vision of mathematics instruction in far more detail than any diagram that she was able to produce.
Interview 1 – September 26, 2017

Prior to the SMII PD intervention, Rebecca described mathematics instruction as “direct instruction . . . student guided notes with vocab, step-by-step instruction . . . I give all the definitions, but like teacher-led question, then step-by-step question, student-led question. Like I do, we do, they do.” She also described a progression of lessons devoted to a single concept:

**Concept Map**

```
  Basic Alg 2
  \[ a_1 + a_2 \text{ new} \] \[ \text{I teach direct instruction} \]
  \[ \downarrow \]
  \[ \text{I do review - white boards} \]
  \[ \downarrow \]
  \[ \text{student lead} \]

  I do - teacher lead question
  we do the step-by-step question.
  you do student lead question.
```

**Explanation**

. . . usually day one in my basic class will be like concept one like writing a linear equation and give a slope and point. Day two writing a linear equation given two points. Day three write a linear equation and I don’t tell them, I just say, “Write a linear equation with this given information, with this,” and then I throw it up on the board and the students have white boards and they write the equation, they hold up the white board and if they are good they erase, they put it down and they sit tight until everybody is done. Like you can phone a friend if you need some explanation and then I always give the answer at the end. And that’s the white boards. White boards are student led. I don’t teach I just say, “do this problem.” It’s another way instead of me just sitting at my desk teaching. They fill up the white boards, it breaks up the monotony of direct instruction. It just double checks for understanding.

*Figure 37.* Rebecca’s concept map and explanation from Interview 1 (September 26).

Generally, Rebecca envisioned the first day as a simple introduction to the topic, day two as an extension of day one, and day three as a review day (see Figure 37). She described the first two days as direct instruction, with the third day devoted to “white boards.” While she said that this activity was usually student-led, her description indicated differently. The teacher was still the one leading the class, but there appeared to be more freedom for students to work
independently after problems had been posed. Rebecca indicated that the third day is structured this way because she wanted to “teach them to be problem solvers. That’s really what I want out of my class.”

Rebecca made an extended comment after the interview was over that I captured (with her permission). She noted that “people say that direct instruction isn’t the best instruction or the best way to teach math or whatnot, but you know right now it is the only thing that I know.” So her vision of mathematics instruction was rooted firmly in what she knew how to do.

She knew there were other approaches out there—we had discussed several in our interactions in my position as a consultant—but did not know enough about them for that knowledge to influence her vision. She was rather harsh in her description of her teaching, saying that it was “very straight laced, narrow minded, kind of like old school in a way.” While the tone was perhaps a bit unfair, the sentiment aligned with her then-current vision of mathematics instruction.

Interview 2 – November 13, 2017

After the initial four PD sessions, Rebecca created a concept map representing her vision of mathematics instruction (see Figure 38). Her vision for mathematics instruction was radically different in this case than in her first attempt. In particular, she described a method for introducing students to content that was different than her previous thoughts.

Rebecca’s vision of mathematics instruction had shifted from simple direct instruction to something more complex. In this case it appeared to be more student-centered. Her vision took her out of the role as the sole mathematical authority, at least initially. Her notation of “Teacher” for the last step in the process—matching the output with the desired outcome—indicated that she now envisioned her role as more prominent at the close of a lesson than in the beginning. Rebecca was not particularly forthcoming with details about how she might facilitate this new way of teaching, but that, perhaps, is due to the fact that I did not press her for those details. Ultimately, her vision conformed to her thoughts about how to use the structured notes provided to her as curriculum materials—she noted that she would facilitate the class to “organize our thoughts with the notes that I’ve provided.

I asked Rebecca if this vision was one of every day classroom instruction and she indicated that this was not necessarily the case,
It might be just an overarching, okay, we're going to get into a unit of rationals. What do we know about rationals? What are things that might be needed? When would we need them? Or like, we're going to be talking about exponents. What about exponents would we need to know, and what would be important? And then, like, just talking about the basics so that we can build on a base knowledge.

Figure 38. Rebecca's concept map and explanation from Interview 2 (November 13).

Her examples indicate that her ideas about mathematics instruction were still heavily influenced by the content she was teaching at that time. This is consistent with her mathematics at that time as well. However, her description of the process of mathematics instruction in this instance is more open than in her first interview, with more room for students’ ideas and more focus on mathematical topics (e.g., rational functions) as opposed to discrete mathematical skills (e.g., linear equation from two points). It is possible that this shift is a result of her PD experiences, as the SMII program focused on providing a vision of mathematics instruction that was more open and more student-centered. This shift was encouraging, as one might hope to see such a shift after three months of PD and reflection.
During the third interview, Rebecca created a more complex concept map to describe her vision of mathematics teaching (see Figure 39). In this attempt, she embedded her vision in a progression of lessons.

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**Concept Map**

**Explanation**

I always start with questions from the night before, but I don't always collect the homework. Sometimes students are like, can we get an extra night because it was just too hard? So, notes as a roadmap is typically just how I use them. Depending on the class, it's teacher-led. Depending on the content, is it new versus previously learned? Because usually it's just me teaching it. Like my algebra two right now, they've never seen trig. So, it's pretty much me leading the conversation, talking about what the parent functions look like. And I actually showed them the graphs on Desmos yesterday. That was pretty nifty. But you know, the notes, if it's previously learned knowledge, I'll open up the floor to students thinking about the concept. And then either from there it's like students asking questions from students and students answering, or teacher helping ask guided questions to help students thoughts, answer questions with a question. And then bringing it back with something concrete for the notes to write down on the notes for the examples. So here is typically where I'm at for algebra two. Here I would say it's 50/50 for trig.

*Figure 39. Rebecca's concept map and explanation from Interview 3 (March 30).*
Rebecca did not elaborate on the conditions necessary for each branch of her map at this point. She did, however, indicate that how a lesson was taught should depend on the class being taught, in terms of both content and the teacher’s perceptions of that group of students. She indicated that this was because

if they've never seen it before, I don't know if they would know what to do. I don't know if they would feel comfortable doing it in front of their peers . . . [b]ecause it's not something that we do enough of. Like, when they get to my trig/pre-calc class, we haven't done enough of it leading up to this point for them to feel comfortable.

Rebecca seemed to believe that because students had not had experience in exploring mathematics in small groups, they would not feel comfortable doing it in her upper level course and so would not cooperate.

Interview 4 – May 29, 2018

In her final interview, Rebecca created a concept map that was similar to her previous map. While it might be tempting to claim that her final map was more complex than the previous maps, this may not be so. First, the final map was not embedded in a series of lessons and it was focused on ways to fill in the notes, as were the others. In reality, the three paths available to a teacher conform roughly to the two pathways and the accompanying branches from Rebecca’s previous map. Rebecca’s description bears this supposition out (see Figure 40).

These descriptions bear remarkable similarity to those in Interview 3. This may indicate that Rebecca’s vision of mathematics instruction had solidified somewhat. She also, again, indicated that she thought “it just depends on what you’re teaching, when you’re teaching, and how much prior knowledge the students have going into it.”

Discussion

The development of Rebecca’s vision of high-quality mathematics instruction followed a trajectory of increasing sophistication and became increasingly student-centered—albeit in a limited fashion—over the course of this project. The SMII PD intervention promoted a vision of mathematics instruction that was inherently student-centered, task-based, structured, and discourse-rich. While there was not a specific vision statement that participants were to conform to, there were multiple aspects of high-quality mathematics instruction highlighted and advanced. Further, there were no evaluative mechanisms built into the program—the teacher change process is fragile under the best of circumstances and the PD design was one focused on
support and empowerment, not evaluation and conformity. So, while her vision did not come to conform generally to that promoted in the SMII PD, some aspects of the vision in the PD did appear in subsequent concept maps.

**Figure 40.** Rebecca's concept map and explanation from Interview 4 (May 29).

First, in terms of sophistication, consider that Rebecca’s initial description can be summed up in a few short words: “direct instruction” and “I do, we do, you do” (also known as gradual release of responsibility (Fisher & Frey, 2008)). However, her subsequent conceptions are not so easily summarized. Her map from Interview 2 involved a linear process of developing student ideas and then structuring those ideas to fit into the pre-formatted notes. Interviews 3 and
4, however, involved a somewhat more nuanced vision of mathematics instruction. In these last two versions, the student-centeredness of a lesson was dependent on several factors, including the content, the level of prior knowledge students possessed, and the placement of the lesson within a sequence of lessons or within the school year.

Second, several ideas promoted in the SMII PD surfaced in Rebecca’s descriptions. The idea of student discussion made an appearance, and while it was not as all-permeating as otherwise might have been the case, it was a promising start. The idea of different kinds of questions, questions that push students to think more deeply, also appeared in Rebecca’s descriptions. While discourse and questioning are closely related, Rebecca did not make this connection explicit. The idea that questions were important was unquestionably present, questions from both teacher and students, but how these questions (and the answers to those questions) play out in the classroom remained unclear.

A constant in each of the versions of Rebecca’s vision was the prevalence of the pre-formatted student notes. This is an indication of how great the pressure can be from the External Domain—so great that Rebecca’s entire vision of mathematics instruction revolved around those curriculum materials. This is the case for two likely reasons. First, Rebecca was one of the two teachers who designed and wrote the curriculum materials. Second, those materials are the main method by which the department ensures that “everyone ends in the same spot.” These pressures permeated all of Rebecca’s ideas and conceptions throughout this project.
Case Study Report – Jeremy

“That’s how we hone our craft, I guess. It’s never a waste of time if we’re constantly reflecting on what we are doing and trying to find different ways to engage students in the material. Sometimes it’s a hit, sometimes it’s not.”

- Jeremy

Introduction

Jeremy

Jeremy is a laid back, thoughtful teacher with seven years of teaching experience (at the time of this study). His professional life is an interesting collection of events. He entered teaching late, having worked in the house construction business for years prior to going back to get his bachelor’s degree and teaching certificate. This allowed him to bring an interesting mix of ideas to any table at which he sat. Jeremy related very well to his students, readily forming positive professional relationships with them through an easy-going manner, a quick wit, and a tendency to patiently listen to each of them. In professional conversation, Jeremy was thoughtful and always willing to challenge the status quo. He was fond of interrogating why things were done the way they always had been. He thought broadly about teaching, trying to reach for big ideas.

Jeremy taught math to mostly ninth and tenth grade students in courses such as Algebra I, Basic Algebra 1, Geometry, Basic Geometry, and Algebra Support. To begin, Table 15 illustrates a timeline of data-gathering events.

Table 11
A Chronology of Jeremy’s PD and Research Events.

<table>
<thead>
<tr>
<th>Event</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interviews</td>
<td>Sept. 26</td>
</tr>
<tr>
<td>Observations</td>
<td>Feb. 15</td>
</tr>
</tbody>
</table>
Jeremy’s Mathematics

Jeremy’s mathematics consisted generally of properly applied algebraic procedures with largely symbolic representations. In pattern analysis tasks, he generally leveraged the pictorial representation to begin his thought processes and to generate data in numerical form. After he had a tabular representation of the pattern, he applied a well-known procedure to develop an explicit formula from that data. Jeremy was also able to connect geometric representations to his symbolic representations, albeit with prompting from me. It is interesting to note that Jeremy’s mathematical activity, while heavily symbolic, was also punctuated by attempts to connect to mathematical big ideas.

Interview 1 – September 26, 2017

Jeremy attacked the S-Pattern Task (see Figure 41) in the first interview with surprising quickness. In less than two minutes, he had noticed and leveraged vital structural details in the progression of figures.

Jeremy: So other than . . . the interior being a square and the tops and bottom being multiples of the . . . progression.

Jason: Say more about that for me.

Jeremy: So on the inside of it I see a square shape, so one by one, or two by two or three by three or four by four.

Jason: Okay.

Jeremy: And ahh across the bottom and the top it corresponds with the sequence that you’re in.

Jason: Okay.

Jeremy: Double those.

Jason: How does the square correspond with the sequence that you’re in?

Jeremy: One less.
This tendency to think visually and geometrically was a strength that would serve Jeremy to varying degrees over the course of our conversations. When I pressed him for another way to visualize the pattern, he was less articulate while trying to leverage the row and column structure within each figure: “I’m trying to make a connection to the columns . . . other than being one less and one more . . . one more high, one less column.” He was able to connect the height of each column to the sequence number, as he called it, but was unable to generalize the number of columns as one less than that number, at least at first (see Figure 42 for Jeremy’s work). Note that Jeremy was very close to a generalization, but his focus on representing the individual squares within each figure appeared to hinder his generalization process. He noted the columns and saw a leverage point but was never able to integrate the columns and the rows into an abstract area model, which might have allowed him to see the general pattern.

It was a problem-solving strength of Jeremy’s that he was comfortable voicing his incomplete thoughts. He was not overly self-conscious about being “wrong” or getting incorrect answers. This was likely due to his generally laid-back nature; he attacked mathematics with interest, but not as much pressure for performance as the other subjects. I pressed him further regarding his rows and columns thinking and he was able to articulate a connection, saying he would “do the sequence minus one to get the column number, times the sequence plus one? Plus two?” As noted above, even with more pressing from me, Jeremy never visualized this connection as a generalized rectangular array within each figure.
1. What patterns do you notice in the progression of figures?

2. Sketch the next two figures in the sequence.

Figure 42. Jeremy's work on the S-Pattern Task during Interview 1.

When he and I moved on to characterizing the pattern with a family of functions, Jeremy again began by sharing his initial thinking freely. When I asked him what type of relationship this problem contained and how he might speak about it with his students, he expressed himself tentatively.

Jason Describe it to me. What is, what kind of relationship is it? How would you talk about it if you were talking to your students?

Jeremy Mmm, yeah I would want to push them, I think, maybe towards an exponential function as it’s growing, something being . . . square.

Jason So you’d describe this as exponential?

Jeremy Potentially.
Jason: How do you know it’s exponential?

Jeremy: [Long pause]. Other than the fact that it’s growing really fast and I have limited data?

Jason: Okay. [Pause]. Say more about growing really fast for me. How do we define growing really fast in regards to exponential functions?

Jeremy: Well, I’m trying to decide between that and quadratics because you have to be careful with quadratics.

Later in our discussion of this task, Jeremy indicated that he would identify a quadratic pattern by determining the second difference between consecutive terms and an exponential pattern by seeing “a straight multiplication pattern,” to which I am assuming he referred to a common multiplier. I closed out the task section of the interview, unintentionally leaving off the line of questioning about anticipating student responses.

Interview 2 – November 13, 2017

Initially, Jeremy thought of the Hexagon Pattern (see Figure 43) as growing by one hexagon each time. I gleaned this from his very first calculations where he said it would “be, minus one every time? So . . . three times six minus one, minus the stage.” Here he was attempting to use the number of hexagons as a starting point for his pattern.

Each of the shape-trains below consists of regular hexagons.

![Hexagon Pattern Task](image)

Figure 43. The Hexagon Pattern Task.

The adaptation of “minus one, minus the stage” was an initial attempt to subtract out the inner, shared sides within each figure. Jeremy wrote very little down (see Figure 44) while working in this vein of thought. He was able to move directly from the pictorial representation to the abstract, algebraic representation. He made no notations on the actual figures themselves, as one might when puzzling through a complex, compensatory method. Further, a common pattern for many algebra teachers is to generate an explicit formula directly from the numerical progression rather than from the figures themselves. Jeremy did neither of these.
1. What patterns do you notice in the progression of figures?

2. Determine the perimeter of each of the first four trains.

3. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

Figure 44. Jeremy's work on the Hexagon Pattern Task in Interview 2.

He initially recorded his expression of $4n + 2$ without explaining it. I pressed him for meaning and he easily connected each of the factors and terms in his expression to the context of the problem. Jeremy was also able to confidently identify this as a linear relationship using the form of his expression rather than the properties of the sequence he generated. His comfort level with linear functions and contextual situations was much higher than with non-linear situations. This was evidenced by his reliance on abstract, algebraic thought processes and his relative speed in reaching his conclusions (he did all his thinking, writing, and justifying in just short of six minutes, even with my questioning). He only resorted to using the figure explicitly when I prompted him to make connections.

When I pressed him for other ways to think about this pattern, Jeremy was able to generate, with some effort, the expression $6n - (2n - 2)$. This was somewhat of a challenge for him and he said so:

Oh my goodness, now you’re gonna make me do like some takeaway, something so I can subtract these [points to diagrams] every time. Or I can take a case . . . so you can do it, probably two different ways. You can do the case number behind it, which is not really the, that would be the tougher one.
With several more minutes of work, Jeremy was able to connect his thinking with the diagram and justify the validity of $6n - (2n - 2)$ in predicting the perimeter of subsequent figures. He justified the validity of the latter expression by noting that $6n$ represented the total number of hexagons in the $n$th figure and that it was necessary to subtract the number of shared sides within the figure. Implicit in his final notation was the fact that there were $n - 1$ instances of shared sides, each representing one side from an adjacent hexagon. He had mentally expanded the expression $2(n - 1)$, which represented the total number of internal, shared sides, to $2n - 2$.

**Interview 3 – May 14, 2018**

Jeremy’s second attempt at the Hexagon Pattern Task showed an example of his forethought and his willingness to challenge himself. He initially thought of the pattern as “increasing by one [hexagon] every time.” He engaged in approximately six minutes of independent problem solving before giving up in confusion, asking “[w]hy can’t I think straight?” I supported him in his efforts to move forward by asking him questions about what he had created. He had miscounted at some point during his initial construction of the number sequence. I asked him what he counted differently the second time, but he simply replied that he was “probably just rushing through.” I thought different. I had noticed a difference in his counting and was curious about it.

Up to this point in the interview, Jeremy had been attempting to solve the Hexagon Pattern Task in a way with which he was not entirely comfortable. He thought about adding a single hexagon in each step the entire time, as opposed to his previous attempt (in Interview 2) where he had abandoned that reasoning when he had created his sequence of numbers. This was evidenced by his initial creation of the expressions $6 \cdot (n - 1)$, $6(n) - (n - 1)$, and $6n + 4$ in several attempts to symbolize the underlying relationship as he saw it (see Figure 45).
1. What patterns do you notice in the progression of figures?

\[ \text{Shape} + 1 \]

2. Determine the perimeter of each of the first four trains.

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
6 & 10 & 16 & 18 \\
\end{array}
\]

3. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

\[
x_2 = (n-1) \\
x_3 = (n-1) + 2 \\
x_4 = (n-1) + 2
\]

Figure 45. Jeremy's work on the Hexagon Pattern Task in Interview 3.

He clarified his counting method when pressed, noting that he “took the number of sides . . . So, minusing that \[2(n - 1)\] instead of minusing that \[(n - 1)\], counting these as doubles. But it’s only doubles for those ones in the middle.” His discomfiture is evident from his use of the word “minusing,” a term he would never normally use as he focuses heavily on academic vocabulary with his students. He also confessed his frustration after several more minutes of questioning:

Jason: So you found the common differences of four down there. Okay, so that’s where they are. So the questions is how do you abstract that pattern now?

Jeremy: Yeah . . . and that’s why I’m drawing a blank. This is way easy. I’ve already done this one before. I know, it’s driving me nuts. I’ve done this one a hundred times probably. Um, and I know this is as easy as it is. That’s the frustrating part.

Despite his mounting frustration, Jeremy continued to engage in the problem. His explanation of his final two answers is clear and quite revealing about his geometric
visualization. He noted that “[i]f you move that end over [referring to the right-hand vertical side of the hexagon in the second image] right, to make the original. Then we have four left and this being a sequence . . . minus one gets me the addition of the one shape.” This was an interesting way to view the growth of the sequence, and one that I had not seen before. I had pressed him hoping he would articulate it in exactly this way.

His explanation of the second (note that his initial attempt was the expression $6 \cdot (n - 1)$, which he crossed out) of his answers, $4n + 2$, was interesting as well, although not for the mathematics. He said that he thought “this is where I want to go with the original one. I think I can do that here. Just got to look at the pattern [the number sequence]. Plus two. Right, four times sequence . . . that’s the easy one.” I took this to mean that he had seen the latter option earlier but chose to pursue the more complex constructive reasoning instead. Given that it took Jeremy approximately 30 seconds to derive the latter solution and his use of the term “original one,” I feel I am justified in my conclusion that he chose a less familiar approach after some initial thinking about the relationship described by $4n + 2$. Jeremy was also, with little effort, able to relate each factor and term in his expressions to his interpretation of the context.

Interview 4 – June 4, 2018

In the final interview, Jeremy engaged with the Candy Bar Sale Task (see Figure 46) with a great deal of confidence and surety.

**The Candy Bar Sale Task**

It’s the annual Freshman Candy Sale and you have 36 candy bars to sell. Your best friend only has 24 candy bars to sell. If you sell 2 candy bars per day and your friend sells 1 bar per day, how many days will it have been when you have fewer candy bars than your friend?

*Figure 46. The Candy Bar Sale Task.*

After about two minutes of work, he requested the use of a calculator, the first time he had done so in any of his work. Although he requested a calculator relatively quickly, his initial solution was to use a table to find the answer (see Figure 47). This table did not appear as a traditional $xy$-table, with dependent and independent variables labeled. Rather, his tabular approach was simply a way of organizing a counting-based approach to solving the problem. Jeremy’s initial answer of 14 days is interesting. I hypothesize that it came from his count of every entry in his table, as he did not initially label the days with numbers.
Jeremy’s second solution method involved the use of a calculator to graph and solve the system of linear equations in Figure 48. His comfort with this method was obvious. He spoke little and did not hesitate when he converted the contextual situation into a set of linear equations. Interestingly, Jeremy did not use a traditional calculator in this episode. Rather, he used an emulator app on his smartphone. This indicates a certain level of comfort with the use of technology beyond more traditional graphing calculator technology. As one can see from Figure 38, Jeremy also used the emulator’s ability to find the intersections of graphs to get his answer. He did not use the trace function to estimate the answer (note that if he had used the trace function, the display would not indicate “Intersection” in the same way).
The third solution Jeremy provided was unprompted by me. He spoke it out loud as he wrote it down: “Thirty-six minus two n equals twenty-four minus n.” He solved it in a traditional algebraic fashion (see Figure 49). We had very little discussion about this solution method, and when I prompted Jeremy for another, he was not able to generate a fourth.

![Figure 49](image)

*Figure 49. Jeremy's third solution to the Candy Bar Sale Task.*

Due to an unanticipated time constraint in Jeremy’s schedule, I was forced to proceed to the next section of the interview.

**Discussion**

Jeremy’s mathematics, while being heavily symbolic in nature, also had subtle hints of geometric interpretation. Jeremy reasoned about contexts fluidly, in most cases, and attended to representations and solutions. When prompted, he flexibly connected his symbolic representations to contextual features and could move between the two representations well. His initial thinking tended to be geometric or pictorial, with algebraic generalization coming after the generation of some data in (loose) tabular form.

When he encountered difficulty, Jeremy did persevere, but tended to do so in silence until he had at least a partially developed idea to share. He verbalized his thoughts while solving mathematics problems but did not seek discussion with others until his thoughts were more fully formed or he reached an impasse. Those impasses tended to happen with content outside of linear functions, as Jeremy appeared much more comfortable, flexible, and fluent in that domain than in others.

**Jeremy’s Teaching Practice**

**Interview 1 – September 26, 2017**

In his initial interview prior to the first SMII PD session, Jeremy was very direct and descriptive when he spoke about his teaching practice. He described his activities in terms of
large-scale paradigms and followed that with more detailed descriptions of how those paradigms might vary from day to day.

Typical lesson structure.

Jeremy described his typical day as follows:

Jason  Typical, every day. . . not every day, but a typical lesson in your classroom. How does that look? What does it look like, what does it feel like?

Jeremy  Typical lesson, a typical lesson?

Jason  Just . . . yeah.

Jeremy  So, umm . . . it is probably we’re split. Right now, I like to do fifty-fifty between day one is more teacher-led and day two is more student-led.

Jason  Okay.

Jeremy  So day one would be teacher-led notes with ummm . . . student questioning. Umm, try to throw in as many real-world examples and/or real-world scenarios that I can to lead up to what it is we’re going to be covering so at least they have some kind of background knowledge of why or where they would see this kind of stuff.

Jason  Okay.

Jeremy  Then we get into the nuts and bolts of whatever it is that we’re doing. Umm, and then the last twenty minutes of the hour is student work.

Jason  That’s day one? And that’s introducing new content? That’s how we do it? Okay.

Jeremy  And then day, well, sometimes, but yeah most of the time that would be our typical day. Ahh, for me like a day 2 is, is the student-led version of that where they work together in groups. Umm, similar items from the day before, but they have the conversations. I answer questions only after redirection several times. And then, even then it’s more of a hint in the right direction than an answer. . .
Jeremy . . . If we extended into a day three, I like to try to do a two or three question “mini quiz,” just at the beginning of the hour just to see if we can extend some stuff, some concepts. And that can lead to some good discussions. Other than that, if we do a day three it’s usually, um, come right in and we do any homework questions, they have the rest of the hour to work on whatever the assignment happens to be and then we typically do the, we grade that assignment right then that day too. And/or take the quiz.

Jeremy thought of his teaching in two- to three-day “chunks,” with each chunk focused on a different set of activities related to a given idea or topic. His initial day of instruction focused on a prepared set of student notes provided as part of the district-designed curriculum materials. Jeremy described these in some detail:

Jeremy On a typical day . . . uhhh, most of them are set up so that at the very top we cover what is the expectation. So the I can statements. So we’ll cover a couple of different specifications with introduction of terminology coming right after that. Whatever happens to be vocab for that unit. And then depending on if it’s a single topic being covered or multiple topics being covered then it’s scaffolded for each topic.

Jason Say more about scaffolding for me.

Jeremy So first example would be something single, maybe like a single-step equation. Alright it’s gonna be a single-step equation, but it’s not gonna be an addition-subtraction, it’ll going to be a multiplication-division.

Jason Oh, so it’s example problems is what you’re talking about.

Jeremy Yeah, yeah-yeah. We do have some notes, and I think this is what you were getting at too, so some scaffolding . . . Let’s just say we get into quadratic formula. So we start out with just the Discriminant first. And that’s all we focus on that day, just identifying a, b, and c. Then, we don’t even get into the Quadratic Formula, we just identify pieces and then day two okay now we’re
gonna do just the Discriminant. And now on day three we’re gonna plug the Discriminant in and everything else and start working on the whole thing as a whole versus jumping right in and trying to do, you know, swimming with rocks.

As can be seen above (and in the opening of Chapter 4), student notes often contained pre-structured vocabulary sections followed by specific examples that teachers demonstrated to students. Once students had a correctly completed set of notes, day two focused on students doing more example problems, with less teacher support. This gradual release of responsibility potentially continued into day three, with an initial gathering of assessment data and a review of the homework assignment. The examples within the notes were carefully chosen to scaffold student understanding in small steps toward more abstract and complex thinking.

I wanted to check my understanding of Jeremy’s “chunking by topic” view of instruction and recalled the question I wanted to ask earlier in the interview, which was specifically about a one- to three-day per topic pattern. Jeremy confirmed this, but with a caveat:

Jeremy: And on those days, with a topic that’s a little more intensive like those can be, um, it can stretch out to four. But it depends on your class, it um, some teachers are “this is what the syllabus says, or this is what my outline for the class says, so on day [x] I gotta be here.”

Jason: Uh-huh. And is that you, or . . .?

Jeremy: That’s not me.

Jason: That’s not you. You’re more . . .?

Jeremy: If we need more time, we take more time. So it’s, for me it’s the feel of the class. Now if it’s single, you know, if it’s just a few individuals in that class then that’s a session that I can have either after a quiz is taken or before a quiz I can pull those guys aside, like at lunch or in between classes or after school, before school, something like that. But for the most part if I extend something it’s because it’s a . . . almost total class breakdown of something.

An independent streak appeared here for the first time (and continued through the remainder of Jeremy’s thinking). He valued student understanding, as he perceived it, and was
willing to take time to make sure a majority of his students understood the material before moving on, something he indicated was different from some of his colleagues. The latter commentary was something that Jeremy brought up several times during our conversations. It appeared that Jeremy believed that some of his colleagues adhered strictly to the timeline outlined by the curriculum materials. He also appeared uncomfortable with this choice and described his optional day 4 as a way of meeting his students’ needs.

Jeremy also placed some value on student discussion. That notion appeared several times in his initial description of his practice (e.g., “they [students] have the conversations,” “[a]nd that can lead to some good discussions”). However, despite its appearance in his dialogue, it was unclear what Jeremy meant when he said, “good discussion.” This could have been remedied by a question from me, but I did not take advantage of the opportunity to ask one.

Physical room arrangement(s).

Jeremy described his initial room arrangement as “single desks spacing in between everybody” saying that it was “just to kind of establish the working order of the classroom . . .” He did indicate that he intended to change this arrangement as the year progressed.

. . . when it’s work time, they can move desks, they can move chairs, they can group threes, fours I don’t . . . appropriate about four, for me so they can move around and work wherever they want and however they want. Later on in the trimester we shift to, I shift to groups of two, and so that they’ve always got somebody next to them as the content gets a little harder so they’ve got somebody to work with.

Here one can see Jeremy’s intentional grouping of students to support those he perceives as struggling. From a management perspective, he preferred groups of no more than four, shifting to teacher assigned groups of two as the year progressed. This is another indication of Jeremy’s value for student discussion and collaboration.

Another arrangement of Jeremy’s classroom was a double U-shape, facing the front of the room. Jeremy filled the inner, smaller row with students he perceived as struggling so he “can spend more of [his] time” with them. The outer row was filled with students that he “know[s] can handle the content fairly decently.”

I asked Jeremy what precipitated a change in his classroom arrangement, and he gave an interesting justification:
Jeremy . . . But a lot of times I want them working together because that’s real-world to me. That’s real-world scenario, I mean there’s not a job that I’ve done in the past that I haven’t worked with somebody and had to communicate or pass papers or, you know what I’m saying, do something. There’s nothing that I’ve hardly, that I’ve ever done that I’ve done by myself. So, it facilitates work, conversation, if I can get them to use math vocabulary, the same.

Jason Super. I see.

Jeremy Those kinds of things. And then you point that out, you go point that out: group two is on task, I hear them using the right vocab, they’re doing awesome. And you can point out positive things.

Jeremy’s background of professional work outside of the education sphere appeared here. His statements indicated that he valued preparation for the “real world” as a primary purpose of school mathematics and education in general. His last statement in the exchange above indicated a secondary motive, namely, that of providing examples to students of how to work in ways that he valued as a teacher. When Jeremy noted that he, as the teacher, would “point that [good conversations, good vocabulary use] out” he implicitly indicated his value for those kinds of student behaviors and interactions.

Atypical lesson(s).

While my initial questions focused on the structures of a typical lesson, I also inquired about lessons that were atypical—i.e., lessons that did not share the same structure as most day-to-day lessons. In many cases these examples included what Jeremy called “projects” or “labs” or any lesson format more “hands on” than those structured by the curriculum materials. Here Jeremy provided an example: “We start exponential functions with the Skittle Project . . . it’s a nice way to kind of introduce the concept.” I prompted him for more detail and Jeremy described an experimental data collection routine using a collection of Skittles candies. I was familiar with this project, as I had used something similar (with M&Ms candies) when I was in the classroom.

. . . everybody gets a cup of Skittles and you start out with one Skittle on your plate and you take that one Skittle, that’s trial number one, you put it in your cup, flip it out, and if the S is up, then you add a Skittle. So for every S you add more, so it produces exponential functions. And then we do it backwards, so now you take all the Skittles you have, you put them in your cup, you dump them out and every one [with an S up] you remove.
He described the way the project unfolded with a different structure than his typical lessons. “. . . we come in, we have a discussion about exponential functions, we talk about how the project’s run, they do the project, at the end of the hour we talk about outcomes.” This structure mirrored, in some ways, a Launch-Explore-Summarize format. I was encouraged by this as I believed it gave Jeremy a foundation to build upon with the work of SMII.

Jeremy also gave another example of another atypical lesson (also around exponential functions), but this one focused on student loan debt. An interesting outcome of this example was the revelation that the department does not use that project any longer. When I asked why, Jeremy indicated that it was “[d]ue to time constraints” partially because this project required students to “dig into it a little bit deeper than the Skittle Project . . .”

Discussion.

Jeremy’s instruction was heavily influenced by several factors. First, his beliefs about the purposes of school and school mathematics implied that he valued student discourse and collaboration. This value was in tension, always, with external influences such as the curriculum format, the curriculum calendar, the time constraints of the school year, and the amount of required material. Despite the fact that he expressed value for student collaboration, he only invited that collaboration during homework time or he structured the collaboration specifically to support students he perceived as struggling (as with his room arrangement). This interplay of the Personal and External Domains of the IMTPG, in Jeremy’s case, provided some explanation for the way he structured his classroom and his lessons.

Interview 2 – November 13, 2017

Recall that Interview 2 occurred after the initial four SMII PD sessions, which ended on October 25. The format of the interview followed the same basic structure as Interview 1, but now with a focus on changes in practice as opposed to simply descriptions of teaching practice.

Changes to teaching practice.

I began this section of the interview with a simple question: what has changed in your teaching practice? Jeremy indicated that there were several changes to his practice. He illustrated with examples.

. . . I have, as we enter new concepts or if I think they’re a review concept, I’ll let the students come up with, um, their definitions. So like we just did absolute value. I have never in the past, so like we just cover the notes. Here we go! This is
what absolute value is. This time I let the students tell me what they thought absolute value was. And for some, who thought that they knew what it was, to explain it was difficult. And then others could explain it to the degree of what they thought it was and it was completely not right. Which got some of those misconceptions out there too. So that was, that was the nice thing about that was we kind of hit it from all different views and I could figure out where a lot of different students were at versus me just saying “here’s what it is! I don’t know where you were at, but this is what it is! Adjust.”

This example indicated that Jeremy had begun to experiment with a more student-centered approach to teaching. Instead of just “cover[ing] the notes,” now he began with what students thought about a given topic. This professional experimentation is what the SMII PD activities were designed to encourage, and so one can see evidence of a potential pathway from the External Domain to the Domain of Practice. I was concerned that this was an isolated incident, but the next exchange indicated both that it was not isolated and that the change pathway was more complex than my analysis above indicated:

Jeremy
Right, right. You know, so not only did we address the misconceptions, but then they built it. And it was kind of cool watching some of the other, because I have other kids that kind of participate in that stuff that usually don’t. And so they’ll just start throwing out what they think, because they are not shy to do that. Right or wrong, but then it . . . the discussion back and forth between some of them will get us to where we need to be.

Jason
Were you able to achieve that discussion back and forth between students?

Jeremy
Yeah. Yeah, yeah.

Jason
Interesting.

Jeremy
It wasn’t . . . I didn’t think that one went bad, because I’ve done this several times now and sometimes it gets ugly and not anywhere close to where I want it to be [inaudible]. And some days, like so I did it again with my, with my Basic Geometry class. We had to, I just put one problem on the board, and we took the entire hour talking about the relationship between the other angles. It was parallel line properties. We did one problem the entire hour.
But I had goosebumps the entire hour. It was awesome. It was one of those days where I said “Man I wish [my principal] were here to see this!” And then he came in the very next hour. We did it again with my next Basic—so we did it again. That one was like phenomenal. Those kids were . . . they basically wrote a proof without knowing they were writing proofs.

Jeremy’s enthusiasm during this exchange was heartening. He noticed that some students who were not previously likely to participate began to do so (a connection to the Domain of Consequence). He was enthusiastic about the idea that students built the definitions, instead of simply receiving them from him. Jeremy’s descriptions indicated a change pathway, indeed a potential growth network, from the External Domain (the SMII activities focused on discourse) to the Domain of Practice (where Jeremy experimented with a different lesson format), into the Domain of Consequence (where Jeremy noticed different participation patterns from some students) and lastly into the Personal Domain (Jeremy’s enthusiasm likely increased his value for this kind of instructional interaction). Even his acknowledgement of the challenges and failures did not seem to dampen his pride in what he had done. He noted that students

. . . were using all the vocab words and some people were saying “Why can’t you do that?” and so they were talking and again, the biggest push for me, coming from the stuff that we’ve been talking about too, is that constant use of vocab. You can’t explain something unless you’re using our vocab. You can’t just say that it goes from angle two to seven without saying why.

This comment is interesting for two reasons. First, it again highlights Jeremy’s value of student discourse. Second, it is a direct comment about the effect of the PD intervention activities (i.e., “coming from the stuff that we’ve been talking about too”). The SMII PD focused on promoting student discourse in a classroom environment with norms that supported students in learning how to communicate mathematically. This resulted in Jeremy’s interpretation of that as a need for rich academic vocabulary, despite that not being a focus of the PD activities. I think this is likely because of the format of the district curriculum materials. Given that those materials begin with laying out specific formal academic vocabulary, it is only natural that Jeremy might begin to experiment with different ways to engage students with that vocabulary. This is another interplay between the External Domain and the Domain of Practice through enactment.
I realized that I needed some clarity on what Jeremy meant when he said “good discussion” (in Interview 1 and other commentary in Interview 2). So, I prompted him to describe how he facilitated a conversation he referenced about developing the definition of absolute value with students. His reply was matter of fact and simple:

Jeremy  I just asked them what absolute value was.
Jason  And then . . . let them?
Jeremy  They . . . just went at it!
Jason  Really? Okay.
Jeremy  Yup. What do you guys think, what is the meaning of absolute value? And . . . so okay go ahead what do you think of this [pantomimining a motion around the room]. And so they just kind of went around the room and whoever wanted to pitch in an idea or concept or what they thought, they just did and there was discussion on that sometimes and sometimes it was “okay well I don’t think, to me that’s not the way I see absolute value.” So, it was nice . . . the misconceptions were handled very well by the students that hour.

Jason  Okay.
Jeremy  So it wasn’t . . .
Jason  They handled them [the misconceptions] themselves then.
Jeremy  Well it wasn’t like, they didn’t shoot each other down, like. You know what I’m saying? They didn’t beat each other up because I thought absolute value was this or you thought it was that . . .

Jason  So it was a positive experience then?
Jeremy  Yeah. It was positive.
Jason  Okay.
Jeremy  But they ended up . . . between the group, everybody kind of putting stuff together, they ended up with a really good definition of absolute value on their own. Now it took us . . . probably fifteen minutes. And that was one of those things I’d said to in those questions [SMII Day 4 Survey] was it’s tough to give up fifteen,
even ten minutes, or fifteen to twenty minutes or that hour that I gave up, that kind of time to hope that you get to where you want to be.

This relatively unstructured discussion format was a stark contrast to Jeremy’s previous description of a typical lesson. While neither he nor I made a claim that this had become his new normal, it is certainly indicative of an amount of professional experimentation in the Domain of Practice.

Even here, though, we see influences from the External Domain encroaching. Jeremy’s closing comment about time is telling. As open as he was to the possibilities, he still worried about taking too much time with student discussion or with the alternative, student-centered formatted lessons. The amount of time he worried about tells its own story. His reluctance to “give up fifteen, even ten minutes” indicates a relatively serious pressure to cover content and curriculum. This pressure comes from some of his colleagues. On the opening day of the SMII PD, Jeremy commented on this, saying “Every time [department chair] comes around, am I doing the right thing at the right time? It doesn’t matter because I’m doing my thing anyway.” During the second PD session it came up again in a discussion about using graphing calculators: “And I would say, totally, and it’s definitely worth the time. And the fight would be, where does the time come from.” This issue arose again on the fifth PD session, where Jeremy commented that

[i]t’s uncomfortable. When I’m giving up, that was only ten and a half minutes or something, but it was probably twenty, twenty-five minutes of the hour by the time we got everything out . . . I’m giving up a lot of time, and I have no idea where it’s going, if we’re going to get anything out of it. With that class, we are already a week and a half behind.

Despite this pressure, Jeremy repeatedly affirmed that he considered the time well spent, both the fifteen minutes developing a definition of absolute value and an entire hour spent on a single problem. When asked why he felt this way, Jeremy replied

I think it’s well-spent because they have to do the thinking first. It’s not me doing the thinking for them, dumping it in, right? They have to recall prior knowledge [holds thumb up for list] . . . they have to build the definition or what they think the definition is [holds up another finger] . . . the group has to come to that conclusion that that’s what they’re going to say it is. And then, you know, the nice thing was when I put my notes up there, we had already covered almost
everything on there. But they did it on their own . . . I’ve used that approach again and again and again. Just did it the other day.

Jeremy believed that there was value in students doing the work, that students are not empty vessels for him to dump knowledge into. He indicated this again in an exchange a short time later about finding “labs,” or what I would call quality tasks, online. He complained that many were “supposed to be an investigation-type thing” but that they really amounted to a guided tour “the same way I’d cover notes. So, are they really exploring?”

Connections between SMII and teaching practice.

In his second interview, Jeremy made mention of a particular geometry lesson that he recorded. He intended to use this video as the subject of the lesson study portion of the next PD meeting (SMII Day 5). I choose to include it here because it appears in both the interview and in an extended conversation during the fifth day of the SMII PD. During his second interview, Jeremy lamented the process of finding quality “labs” online. He was vague in his description during the interview but was much clearer on day five. The entire exchange on day five (Segment PD5S26) follows:

Participant: I hope that wasn't your bad lesson.
Jeremy: It was.
Participant: That was your bad lesson?
Terry: What was bad about it?
Jason: Yeah, that was where I was going to go next. What might you consider changing if you had to do this again? And feel free to jump in, but let's hear from [Jeremy] first.
Jeremy: Well, after hours of scouring the internet, I'm trying to find an actual project that wasn't guided. I took all the scaffolding away from a project, and it left me with nothing. So there was no—I mean, every project that I came across literally said, okay, you're going to be doing side-side-side, so please measure the sides. What do you notice from one triangle to the other? Oh, they're the same. Well, that's not really an investigation. And I did the same thing drawing it. So I just—this one, for me, was—I had zero plans. I had a general concept from a project that I had come across that I
wasn't sure how it was going to work out. I had no question strategies ready to go. I had no idea how it was going to work out. And this, to me, I was actually sweating bullets throughout this whole—.

Participant We couldn't tell. And my thought was, if you wanted them to, you could have given them a protractor and a ruler if you wanted them to do the measuring—I.

Jeremy Yeah, that's where I don't have any idea of how to make that better.

Participant I mean, if you wanted to not have to make them do the, like, making a triangle out of a square with paper and comparing it. But I don't think you did a bad job.

Rebecca Was the paper their idea, though?

Jeremy Mhm. The only instructions I gave them was we covered what congruence meant, and then it was how can you copy a triangle, basically.

Terry So I still don't know why you felt it was bad.

Jeremy I just—for all those reasons. I was unprepared. I didn't know where it was going to go. I didn't know how it was going to end up. I didn't know if we'd even get—.

Terry Was the result bad?

Jeremy I don't think so now that I'm looking at it.

Terry If the result is good, the activity had to somehow feed that.

Jeremy Like I said, all three groups did get side-side-side. We might not have got as far as I thought maybe we would get. But I think just me being unprepared for anything—.

Terry What would you do different the next time?

Jeremy I would probably—like, the same thing that we did when we created those lesson plans. Have a bank of questions that I can have at my disposal to maybe funnel or guide.

Terry You surprised me. Because the questions you came up with were—you never repeated a question. Every question you asked
had a little different phrasing. It looked to me like you kind of had all the dead ends kind of covered I thought.

Jeremy  I was nervous. I was way nervous.

Jeremy had prepped the entire group by sharing that he had brought a good lesson and a bad lesson. His “bad lesson” was a short video of him engaged with a group of students around an exploratory activity with straws of different lengths. As he indicated above, his goal was for students to make sense of the triangle congruence theorems through exploration. Specifically, he was working with a group of four students on developing the SSS congruence criteria.

In his second interview, Jeremy had raised some of the same concerns that he did in the day five exchange: lack of preparation, want of a list of pre-prepared questions to support students, lack of collaborative planning time. It is important to note that despite this, Jeremy went ahead with the lesson, recorded it, and was willing to share it with the group. His revelation came from the fact that none of his colleagues or myself agreed that it was a “bad” lesson. The group focused on his questioning techniques, an area in which he expressed some confidence during interview two, saying that he was “usually pretty good with flying by the seat of my pants questioning.” I inquired further:

Jason  So are you feeling more comfortable with your role in this different way of doing things?

Jeremy  Oh yeah. I was comfortable with it beforehand.

Jason  Even though you were sweating bullets?

Jeremy  Yeah. Well that, again, that’s just I feel I’m wasting time. I hate that part of it. But as far as being, as far as, for me I’ve always thought of myself as a tour guide. Here are the sights and I’ll tell you some interesting facts and history behind them, but you’re the one that has to, you know, enjoy it or work with it, take pictures of it, whatever else. But . . . or I’m the guide on the camping trip. We’re going to go into the mountains, I’m looking out for you but you have to handle everything else. So bringing labs in and me stepping back, that’s not a problem for me at all. That’s what I’ve wanted to do for years. So . . . uh, and I think it’s more enjoyable for the kids. So, instead of me talking at them, they are talking to
each other and they are asking me “are my ideas right?” You know, I’d rather answer that question or give them a guiding question towards something if they’re not right than just say “two plus two is four. Remember that.”

Discussion.

There are several items worthy of note here, with respect to characterizations of Jeremy’s teaching practice and with respect to connections to the PD activities. First, one can find some evidence of Jeremy’s view of his role as a mathematics educator. He claimed to see himself as a “tour guide” by which he means that he was “looking out for [students], but [they] have to handle everything else.” This sentiment seemed to align well with his earlier statements about valuing interactions where students are the primary reasoners and sense-makers, not just empty vessels waiting to be filled. It is vital to note that the video example he shared bore out that Jeremy could actualize these sentiments in his teaching practice. The latter evidence, coupled with his earlier statements from multiple portions of conversation, indicates a firm connection between the External Domain (the ideas about facilitation of student thinking presented in the SMII PD sessions), the Personal Domain (Jeremy’s assimilation of the PD ideas), and the Domain of Practice (his efforts to implement different instructional practices in his classroom). These connections were formed through sequences of enactment.

Of particular interest here is a mismatch of perception. Jeremy believed that his enactment of an open, inquiry-based, student-centered lesson was poor (Domain of Practice). His discomfort with his lack of preparation and the lack of clarity during his interactions with students (the Domain of Consequence) caused him to believe that he had not properly enacted the principles he believed in (the Personal Domain) and those that had developed from the SMII PD experiences (the External Domain). The act of watching the video with his colleagues seemed to cause a good deal of reflection and revision of his view about his teaching in this instance (in the Personal Domain).

Here we also see the diversity of influences from the External Domain. Jeremy’s experiences in the SMII PD built upon his already-existing beliefs about teaching, telling him that enacting his beliefs in this way was productive and valuable. However, his comments about feeling as though he was “wasting time” show that concerns from the External Domain were conflicted, telling him that this kind of learning took too long and that it is not valuable enough
to implement on a daily basis because of the amount of material he had to teach and the pressure of his colleagues being in a different place in the curriculum. As a matter of fact, this influence was so great that Jeremy (in his second interview) confessed “So, I mean at least we got the side-side-yard out of it, but I scrapped that lab after that hour. We didn’t do it the next hour. There was just no way.” It is these kinds of conflicting influences that make the change process both extremely personal and extremely complex. The SMII PD attempted to deal with these kinds of conflicts through collaborative structures within the lesson study process. We spent a good deal of time discussing how to balance the pressures of time against the need for professional experimentation to learn how to teach differently.

Observation 1 – February 15, 2018

The first class period I observed was for a class called “Algebra Plus.” This course was a support course, meant to be taken concurrently with an Algebra 1 course. The students in the room were those that the department and the school counselors felt needed the extra support to pass a high school algebra course.

Jeremy’s room was never in well-ordered rows and columns. Even when giving a test, he simply made sure that his students are not sitting right next to one another by arranging desks an appropriate distance apart. This day saw his students in groups of two or three, arranged generally in rows (see Figure 50 for detail). The main assignment for this class period was an opportunity to re-take a quiz for those who needed it. This was an unfortunate coincidence, as I had tried to avoid those kinds of scheduling conflicts. However, this instance was, I believe, more due to the nature of the course than the timing of the observation. The supportive nature of the course meant that the time was used differently than in a regular class. Because of this, Jeremy spent approximately the first twenty minutes circulating the room, helping students who requested his assistance. Most students worked independently, with at least one pair of students quietly working together. Student questions ranged from conceptual, content-based questions to more mundane procedural questions about the directions for taking the quiz.

Occasionally, Jeremy would sit down next to a student he was helping. This was usually an indication that he intended to stay in that position for an extended period of time. He only did this with some students, those he knew to need more support. For example, he sat with one particular student for nearly five minutes. During this interaction, Jeremy scaffolded the student
heavily to help with comprehension of the problem on the quiz. The content of the quiz related to quadratics in different forms. Jeremy followed a pattern when working with this student.

<table>
<thead>
<tr>
<th>Lesson 1</th>
<th>Lesson 2</th>
<th>Lesson 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 15, 2018</td>
<td>March 29, 2018</td>
<td>May 29, 2018</td>
</tr>
<tr>
<td>Algebra Plus</td>
<td>Algebra 1</td>
<td>Basic Geometry</td>
</tr>
</tbody>
</table>

Note: Orange indicates whole-class format. Green indicates individual work. Blue indicates group work.

*Figure 50.* Classroom organization and instructional modality charts for Jeremy's teaching.
First, he would pre-explain the question and any context associated with it. Jeremy followed this by re-phrasing the question, indicating which information the student could ignore, and helping the student think about how to get more comfortable by changing the variables in the problem from \(x\) and \(y\) to \(t\) and \(h\). He linked these choices to the context (a projectile motion situation).

He stopped short of giving the student the answer, though. Instead, he simply said “Calculate the vertex point and apply.” Then he moved on to explaining the next question on the quiz.

Next, Jeremy fielded a question from a student about the second quiz problem.

Student: Did we have to do anything after finding \(x\)?

Jeremy: You need \(a\), \(h\), and \(k\). You have \(a\) and \(h\) from here. Find \(k\). Just like linear, \(y = mx + b\), this was \(m\) and this was \(b\). [Points to paper].

Same here.

Jeremy was careful not to tell the student exactly how to go about finding the parameter required by the problem, but he did heavily guide the student in that direction. Another example of this occurred moments later, when Jeremy helped another student with a question. His initial remark was evaluative, “Good. Except the 1 doesn’t belong.” He then referred the student to their notes. After the student found the correct set of notes, Jeremy noticed that the notes were incomplete or incorrect and indicated that the student should fix that by adding a specific note about finding the three parameters in question (\(a\), \(h\), and \(k\)). Moments after this, Jeremy engaged in a two-minute procedural explanation with a student. The two examined a graph on a computer. Jeremy provided heavy guidance such as “I just picked \(x\) positive one. Any random number I want; one is easy for me.” The student “drove” on the computer while Jeremy gave directions.

Approximately twenty-three minutes into the lesson, Jeremy shifted the class from individual work on a re-quiz to whole group discussion format. This would become, in effect, a mini-lesson for the day. He began with an example,

\[
gcf(8, 24, 16)
\]

As he began teaching, Jeremy asked questions of the students. Most of these questions were simple recall questions or questions which required an answer of yes or no. He gave minimal wait time and did not structure student conversation in any significant way. His questions were asked of the group as a whole and answers were accepted from the quickest of volunteers.
Jeremy Can the G.C.F. be bigger than 8?
Students No.
Jeremy Why not?
Students [Various responses, some incorrect, some correct]
Jeremy The G.C.F *has to come* from the smallest number. [Emphasis]
Jeremy’s] You are wasting your time with the prime factorization method, in my opinion.

Jeremy then proceeded to demonstrate a method he considered to be more efficient to the class. His method involved generating the factor pairs of the smallest number, beginning with the largest factor, and comparing it to the other two. In this case it worked out nicely.

\[
\begin{array}{ccc}
8 & 24 & 16 \\
1 \cdot 8 & 8 \cdot 3 & 8 \cdot 2 \\
2 \cdot 4 & & \\
\end{array}
\]

For his next example, Jeremy chose the greatest common factor of 30 and 35. For this example he began by listing the factor pairs of 30 and then asking a volunteer a series of questions such as “Does 30 go into 35?” After each answer of “no” he would cross off the largest factor in the pair and move to the next one down. Jeremy provided another example to the whole class, that of \( \text{gcf}(16, 24, 48) \). His pattern of interaction was the same as the previous example. His intended purpose appeared to be to support students’ understanding of the worksheet by outlining a process by which they could find greatest common factors. The challenge for students, then, would be to take that process and apply it to the rest of the worksheet. Some of these additional problems involved variables as part of the terms in question. The only guidance Jeremy provided in this regard was to say that “[w]hen you get to variables it’s the same concept as numbers.”

Throughout this process, Jeremy referred students often to multiplication tables that each had at their desk. The remainder of the class was devoted to individual work time (approximately 8 minutes).

Discussion.

Jeremy’s approach to teaching this class is a stark contrast to the kinds of teaching he discussed in his interviews. This is likely for two reasons. First, the class is intended to support a core algebra class, so other instruction had already occurred by the time students entered this
class session. This makes this observation, in my opinion, less than informative about Jeremy’s typical teaching methods. Second, I believe Jeremy altered his style of instruction to accommodate his perceptions of the abilities of students in this class. The class was populated by students who the department and the district believed were in need of extra support to pass the Algebra 1 course. And so the interactions I observed during this class period were tailored specifically for the population of students Jeremy knew to be in his classroom. He might not have, for instance, been so explicit in his attempts to assist students in a class that he thought was more mathematically skilled. This sort of behavior is not surprising, given what the field knows about the effects of tracking on both teacher and student mindsets (NCTM, 2018). I also argue that this idea is supported by the way Jeremy arranges his room. He paired students based on his perceptions of their abilities or he arranged his room with the students he believes will struggle placed close to him in a group. Because of these factors, I can make no claims about any particular changes in Jeremy’s practice based on this specific observational data.

Observation 2 – March 29, 2018

The second observation of Jeremy’s classroom was in an Algebra 1 class. Jeremy’s room was arranged in groups of two spread throughout the space and a single U-shape formed by five desks directly in front of the main board. All students faced the front of the room except two. Jeremy began the lesson by reviewing the answers to the homework, from which it was clear that solving quadratics was the general topic, with factoring a particular focus.

Homework correction began with a question from Jeremy: “If I pick factoring [as a solution method] . . . don’t I have another method to get the same answer?” Individual students suggested both completing the square and the quadratic formula as alternate solution methods. Jeremy provided an analogy for students, saying that “These are like tools. If one doesn’t work, choose another. It’s like you chose a flathead screwdriver but needed a Phillips. Both do the same job.” From here, Jeremy proceeded to review the answers to the homework problems with the whole class. He provided editorial commentary about the answers as he went (e.g., indicating which methods might be the most efficient or useful for a specific problem). As he reviewed the answers to the homework, Jeremy consistently referred to a diagram he had created on the board. The diagram was a generalized quadratic function that opened downward with the location of the vertex and the intercepts noted prominently and notated. This diagram was intended to anchor
students’ understandings of the answers to questions about the maxima and intercepts of quadratic functions.

For the last problem on the homework assignment, Jeremy solicited an answer from a student (8.72 and 3.72 were intercepts of the quadratic function in question). Jeremy requested confirmation or disagreement from the rest of the class but received no answers. It was clear to Jeremy that the students had not done the assignment and he said as much to them. He asked about their confidence level given that there was a quiz scheduled for later that hour. All of Jeremy’s questions were asked of the class as a whole, not to individual students and he did not use any kind of routines to engage students with his questions. Jeremy received sporadic responses indicating levels of comfort, so he decided to push the quiz to the next lesson.

From there, Jeremy moved into a section of the lesson where students filled in the notes pages they all possessed, which lasted for nearly the entire class period. The structure of the notes dictated Jeremy’s treatment of the topic of the lesson: graphing quadratic inequalities. Jeremy prompted students to read through the first and second sections of the notes pages they had. While they read, he wrote an example problem on the board, a quadratic equation in two variables. He asked students what they noticed and Student 12 initiated an exchange:

S12 You put the equal sign when it says greater than or equal to.

J So you’re saying I’m wrong in putting the equal sign up there? Good. As you look through the notes, what else do you see?

From here, Jeremy consistently interacted with Student 12. As a matter of fact, of all the interactions I noted in my field notes, the overwhelming majority were either with Student 12 or questions asked to the class as a whole. Jeremy interacted briefly with three other students throughout the lesson.

The remainder of the lesson took on a call and response format, with Jeremy providing information for students to use to fill out their notes pages. There were only three deviations from this pattern. During the first, Jeremy spoke with Student 3 about how to locate the vertex of a parabola when graphing it from the equation. Student 3 noted that the value of the x-coordinate can be found by the formula “negative b over two a.” Jeremy responded by evaluating and elaborating, “Says it in your notes, negative b over two a. I’m gonna go back further . . .” He continues to lecture students, reviewing their prior knowledge about graphing linear inequalities in two variables using dashed and solid lines.
The second deviation occurred when Jeremy asked students how they might test their answer. Student 12 answered that they “did the V-thing” and went on to explain how to choose a point in the shaded area and test it in the original inequality. Jeremy elaborated on this point, saying that he “could choose any point as long as it is not on the line. We choose zero-zero because it’s easy!” [emphasis Jeremy’s]. He went on to verbally walk students through testing the origin in the original equation.

Deviation three involved Jeremy facilitating the group creation of a table of values for a function. He gave students a few moments to generate their tables and then produced a table on the board using choral answers from the students. Shortly thereafter, Jeremy transitioned the group to individual work for the remaining six minutes of the class period.

Discussion.

This lesson was in near-perfect alignment with Jeremy’s original description of his practice (homework correction, followed by notes, followed by independent homework time), but with more time spent in a whole-class setting. His questioning was limited, and his questions were low level. The structure of the notes guided nearly his every move. What is clear from this lesson is that Jeremy had not transformed his practice. His comments during the PD session and interviews indicated that he was in a space of professional experimentation and transformation or confirmation of beliefs. Despite this, I found no confirmation of these things during this lesson.

Interview 3 – May 14, 2018

Interview three was later, perhaps, in the school year than I wished. However, Jeremy had much to say about his teaching practice. He continued his pattern of describing his practice in terms of the content he was covering at the time and the large-scale instructional ideas, supported by story-like details.

Jason . . . So let’s talk about class time. What’s going on? What do your lessons look like now?

Jeremy Umm, my lessons typically . . . right now it’s easier because we’re doing survey questions, so it’s a lot of survey-type stuff and we’re making graphs. Bar graphs, pie charts, those kinds of things. But with Basic Geometry class it is concepts, so concepts, trying to do a nice mix of real-world applications and conversions of stuff. Where they are generally, they have the idea so we can talk about
anything, we can do anything and some days are, we do, try to do, just go outside and do anything that ties into what we're covering. Volume-wise it's cylinders. So today it was Pringles cans and we started tossing around the idea of we haven't covered cones and spheres yet. So, we started tossing around those ideas as we were looking at shapes and some of the kids were like "well can't we do, you know two-liter bottles?" and we started talking about how that end is not flat across the top, which shape does it kind of sort of look like?

Jason Absolutely.

Jeremy And they were, someone came up with cones on their own. And it's close enough for where we're, for what we're looking at doing, if you treat them like cones. And then like footballs. Cones, for the most part. . . like I said, the whole trying to prep them for real application kind of stuff, with the final exam being a box of randomness. Tell me what it is.

Jason So what does that look like though? In terms of—.

Jeremy In class?

Jason Yeah, in class. What's going on?

Jeremy Uh, everybody's kind of working all over the place on the problems that they have. So it is a mix, depending on the day, of either worksheet kind of stuff, but it's all application, most of it is application-based. Very few problems right now that we're working on are here's a box and here's the length and the width, or here's a can and here's the, you know, the radius and the height or something like that. Most of them are "here's a scenario, here's a couple of components," sometimes those measurements are not even the same unit of measurement, so they've got to do conversions right off the bat and eventually we're looking at what's the total weight? Or based off of the volume, how long is it going to take to fill something? Things like that, so pretty good
extension questions. And on the other days when we’re not doing that, like I said, I’ll throw out some cans . . . what’s the volume, what’s the surface area? What if it’s just a label? What if it doesn’t have a lid on it? Um, go outside, do the football field. We did the football field, the track. Just trying to get everything . . . just rectangles and circles. And how to apply that.

Jeremy relied heavily on specific examples to speak about and describe his teaching. I had to press him to talk about his instruction in terms that were not content-focused. This may indicate that his primary focus when planning for, enacting, and reflecting on instruction was mathematical content. His descriptions of the activities of the class were also content-laden: “Pringles cans,” “footballs,” etc. He did mention worksheets but hurried to add that the worksheets consisted of “application-based” problems. His notion of “tossing around those ideas” seemed promising on the surface, because the SMII PD emphasized this kind of student discourse and student-driven ideation. The exchange above provides some evidence for a change pathway beginning in the External Domain (the demands of curriculum) through the Personal Domain and into the Domain of Practice (Jeremy’s enactment of that content in a way consistent with his beliefs about student learning) and into the Domain of Consequence (where Jeremy noted students’ independently generated ideas and reacted to them).

His description continued to show his value for interacting with student ideas and discourse; however, his description lacked detail, so I pushed him further. I wanted to know what it looked like when his students were engaged in this way. He replied that “[t]hey are definitely, some of them are working independently. It totally depends on the kid. Some of them like to work independently, they don’t want any help. Um, others, most of them are working together in groups . . .” This led me to believe that Jeremy was facilitating his classroom discussions as a whole group and letting students choose how they wished to complete the worksheet material. He did go into more detail about what students had to do.

[W]e are doing the actual measurements, they are doing all those measurements. And they have to use and reference charts, tables for correct formulas, conversions and everything else. They are teaching one another. They are double-checking each other’s work. As far as the measurements go, they’ll measure it two or three times. Especially if it’s something they have to do on their own. It was . . . eye-opening to see how many kids had never used a tape measure or measured. I’ve got a hundred-foot tape and a two-hundred-foot tape and a
measuring wheel and we brought all that stuff in. It was a day just kind of for some of them figuring out how to use that kind of stuff, which was cool. But it’s very independent and I am really just a resource where if they are unsure, they ask me.

Here again one can see Jeremy’s focus on content and student process. He did not think in terms of teaching moves or large-scale collaborative structures in a classroom. His focus was on the students’ actions as they engaged with the relevant content, typically not on their interactions. This subtle difference was a focus of work during the PD sessions. However, there was evidence of Jeremy valuing students’ ideas and supporting them as they make sense of problematic situations (another focus of the PD). He saw himself as a “resource,” not simply the mathematical authority (something consistent with the instructional model espoused in the PD). I pressed him on this point as well.

Jason So if a kid comes up to you, if a kid comes up to you and says “Am I doing this right?” what do you say?
Jeremy So if they say am I doing that right, my first question is “what are you doing?”
Jason Okay.
Jeremy And so they’ve got to tell me what their whole plan is, the whole outline of whatever it is the question is asking them to do. Usually if it’s about a formula, I say I don’t know, I don’t have a formula sheet on me. So, they have to go get it, and do that, and tie it back to the plan. That’s been my original thing is, is . . . I try to make it as easy as possible for them to set up a plan. So, when we were just doing composite shapes the other day if it was a circle on a square it would be literally just a circle plus a square. And then they would go get those formulas and then they would show their work and do any kinds of conversions they’d have to do for them also, but that’s always, that’s always my go back to, I go all the way back to the beginning every time. Does the plan match the concept? Because if that’s not good, then we don’t even care about any of that other stuff. So, if the plan is good, then we check parts and pieces. Are you using the right parts and pieces? Right units of
measurement? Consistent use of measurement? And I just build them from there.

From this exchange, it was clear that Jeremy was still the mathematical authority in the classroom. He had the answers. However, he did have a strategy in his questioning of students. While there was a focus on formulas in his description, there were also indications of Jeremy focusing students on higher-level structures for problem solving.

Changes in Jeremy’s teaching practice.

I asked Jeremy what had changed in his instruction since the beginning of his SMII experiences. He indicated that he was “still doing very little guided instruction” with most of the time being “student work time . . . they are responsible for interacting together and working with one another . . .” He indicated that after students had engaged with the material for most of the hour, “we [the class] come back together for ten minutes in an hour. That’s about it.” I pressed Jeremy on how students responded to this type of instruction and on what the content of those ten minutes was. To the latter question, he characterized it as “a comparison of ideas, really.” He alluded to ceding some of his authority at this point:

So, I might have my answers, so when we went out to do the field, I have my version of what the answers are, but it’s a back and forth of “Uh, I have this as a length and a width.” Okay well, you’re four feet off from what I am. What’s that going to do to everything else? So, it’s less, so everything you’re going to have is going to be smaller. We talk about those kinds of concepts or “you missed four feet” so now I’m going to actually go around that football field, cut off four feet all the way down there. You just missed a ton of area, right? It’s this big, giant thing now. So hopefully they’re seeing the, the . . . attention to detail and the accuracy probably necessary for big projects like that if they were to re-turf our football field and that attention to detail. So . . . and it’s funny because we’ve got some kids that use three point one four, we’ve got some kids that use the pi key, and so there’s always been this fluctuation of answers to begin with. And they’ll talk about it. “Well I used pi, my answer’s going to be a little bit bigger, I have this” versus “I used three point one four so I’m like four fifty-eight you had four sixty-two, is that okay?” It’s a lot of interaction. Most of the time if there are questions, I try to answer the questions before we go through the work so that person can make an attempt at it real quick. And like I said before, some of them that don’t have them done try to finish them and turn them in anyways. So . . . for the most part I’m happy with the production we’re getting.

Jeremy described an approach to content here that was in contrast with his initial description of his practice. Jeremy’s initial description of his teaching painted a picture of a specifically structured lesson with homework correction, notes, and independent practice times;
the description above was much more consistent with the student-centered descriptions Jeremy
gave during Interview 2, although the experiential, outdoor setting was also a new development.
I believe that part of this difference comes from the content (geometry) and the time of year (late
spring). There are multiple opportunities to apply geometric concepts within and around a school
building. His approach was to have students apply their knowledge by going outside to actually
measure spaces and objects as if students were engaged in a project. The content was not grade-
level content, but the format of instruction was different than what Jeremy described in the
beginning. His ten-minute conference with the class had the feel of a summary discussion, but
with little of the structure that might ensure that multiple student ideas were heard and analyzed.
He remained the mathematical authority and the conversation was likely mainly between him
and the students.

Despite the lack of a shift in the locus of mathematical authority, there is still evidence of
a change pathway to be had in the latter description. Jeremy chose to engage students with
particular content based on the time of year (the External Domain) and in a way that
decentralized him as a provisioner of examples and notes (the Personal Domain) and allowed
him to create opportunities for students to engage in discourse about measurement and
calculation methods (the Domain of Practice). He even noted that he was satisfied with the way
the lessons turned out (the Domain of Consequence).

I asked Jeremy what he might do differently next year, and we had an interesting
exchange. His first comment was that he would “cut out quite a bit of stuff that we have covered
in the—.” But I cut him off and refocused him on instruction. He, however, maintained a heavy
connection to content,

That’s why, though. I would cut out that stuff so I could do a lot more hands-on,
you guys [students] are coming up with the numbers, you’re comparing—I’m not
making answer keys—you guys . . . are the answer keys. You come to an
agreement or disagreement and somebody prove why you’re right and the other
person is wrong.

Clearly, Jeremy’s mind was in what might be termed a “project-based” space at this time.
But one message is clear: he valued students’ work and he preferred that they engage in
mathematical sense-making.
A connection to the SMII PD activities.

The final exchange Jeremy and I engaged in about his instruction during Interview 3 focused on connecting his current thinking and planning to his learning from SMII.

Jason: So . . . is this something you wanted to do before or is this something that SMII has kind of helped you develop?

Jeremy: I think it’s both. It’s something that we’ve [his department] talked about always doing and trying to incorporate, but having this time with SMII and making those attempts in the classroom, being kind of, I don’t want to say “forced to,” but signing up to do that, to make those attempts—not forced, but actively seeking opportunities to do those things . . .

Jason: [Laughs] Well said.

Jeremy: It just shows . . . again it just proves to me that if they can’t apply what they are learning in the classroom outside in the real world, then what’s the point? There’s a huge disconnect. Especially for the kids that I’m working with. They’re going to be . . . you know, hanging pipes, doing electrical, construction, who knows? But at least they’ll have some exposure to evidence.

Jason: So it sounds like the lesson study part of SMII was particularly useful to you.

Jeremy: Yeah, I wish I had hours a week to work on that kind of stuff. Totally. To build good, quality lessons, material, concepts, questions. You know, we worked with you for an entire day. We got four hours, three hours of that to work on one lesson and we only get, we walk away with an outline of a concept. And that’s in three hours.

Jason: It’s hard.

Jeremy: Yeah! Yeah, it’s not like a lesson just falls out of the sky. It’s the, you know, it’s things that we’ve tried to, I’ve tried to get across to all the others is that some of the projects and things we have put together in the past . . . it is, I probably have a hundred hours into a
certain project. It didn’t just happen. And it’s on iteration number seven, or whatever it happens to be and then okay I just put all that and we don’t even do that project anymore. But . . . trying to find the time.

This exchange was particularly encouraging for me as the designer and facilitator of the SMII PD. Jeremy indicated his value for several aspects of the SMII program, from simply the allocation of time to collaborate to the collaborative planning time for him and his colleagues to the “forced” opportunities to enact new classroom practices. Interestingly, when I brought up lesson study, Jeremy focused on the planning time as opposed to the reading of research or the viewing of lessons. This indicated a very particular link between the Personal Domain and the External Domain. Jeremy valued what he perceived he needed given his context, and that was separate from what I had intended or what I would have argued he needed. Once again, the influence of the External Domain was conflicted. Time appeared as both a positive and negative factor in Jeremy’s explanation. He needed and valued the time to plan and collaborate, but in taking that time it pushed him back from where he needed to be according to the calendar.

Observation 3 – May 29, 2018

The third and final observation was of one of Jeremy’s “Basic Geometry” classes. These courses were populated by students similar to those in the Algebra Plus classes: students who the staff and counselors perceived would struggle with the pace and/or content of a regular geometry course. The “basic” aspect of these courses was that only the major focus areas of geometry were studied, in effect creating a “stripped down” version of the standard high school geometry course at this school. In this class, Jeremy arranged the desks in pairs or threes spread throughout the workspace. All students faced the front of the room.

When Jeremy had scheduled this observation with me, he indicated that this was a regular class period. However, upon arriving and listening to his objectives for the day, it became clear that the plan for the day had changed to a focus on making corrections to or re-taking an assessment. Under normal circumstances I would have rescheduled the observation at that point, but due to the proximity of the date to the end of the school year, I decided against rescheduling. Despite my hesitance to schedule observations on days scheduled for the kind of work Jeremy had indicated, there were some insights to be gained from the time.
Jeremy distributed assessments to individual students, both completed assessments for corrections and blank assessments for students who needed to re-take. He then gave a set of directions which split the classroom into three sections and assigned each section to a particular student (e.g., “If you are over here, then she [indicating student 5] is your assistant.”). Jeremy indicated that he would assist the five students in the middle of the classroom. After this, he began to circulate the room, answering questions from students about their tests.

Jeremy had several interactions with students during this time that were non-routine and involved more lengthy exchanges of questions and answers. While some students asked procedural questions about the test correction options, others asked content-oriented questions. The latter questions are of interest. The first example of this is a two-minute exchange between Student 19 and Jeremy, about a question on that student’s worksheet.

Student 19: I don’t understand this.
Jeremy: Which is it? What is a rotation is what it [the question] is asking. What formula do you need?
Student 19: I don’t know, which?
Jeremy: You tell me.
Student 19: [Provides the formula.]
Jeremy: Once we figure this out [indicating a variable], we can figure that out [indicating a different variable].

Jeremy assisted another student for approximately one minute before he was called back by Student 19. Another two-minute exchange followed.

Student 19: This one is right here.
Jeremy: Let’s do this the easy way: What is this length? Times three. So how is that possible?
Student 19: I don’t know.
Jeremy: Exactly. So, what about this [indicating a figure]? It’s not a straight line, right? So what is all this together? [Jeremy continued his funneling pattern until he and the student reached a resolution to the problem.]

Later, Jeremy engaged in a two-and-a-half-minute exchange with Student 4 concerning an area problem.

Student 4: I can’t find the area.
Jeremy: What do you have?

Student 4: I have three there.

Jeremy: So what are your calculated ones?

Student 4: [Uses a calculator and explains the work done so far.]

Jeremy: Good. Now keep going.

Jeremy’s patterns of questioning continued in this way for the remainder of the class period. With some students he was very direct and funneled thinking down to a specific answer (as he did with Student 19). With other students he was more circumspect, using questions that might be considered more guiding than funneling (as he did with Student 4 above) and would leave the student to complete the work. This difference in treatment of students was potentially linked to Jeremy’s beliefs about the needs and abilities of the particular student in question—he was more direct with students he believed needed that direction and more guiding with those he believed could continue to work independently. Jeremy would often answer students’ questions with a question of his own, at least initially. These initial questions often focused on features of the problem (e.g., “Is this an area of perimeter problem?” or “How much of this do I need?”) or on getting students to explain their work thus far (similar to his approach with Student 4 above). However, beyond these initial questions, Jeremy’s patterns diverged for different students. Some students received feedback and guiding questions while others received more direction and funneling questions.

Jeremy kept the class in an individual work format for the remainder of the hour. During this time, he continued to circulate the room, answering students’ questions as they arose. While this observation was not as fruitful as I would have hoped, it is possible to gain some insight into Jeremy’s practice from his exchanges with students during this class period. Here we saw evidence that Jeremy did have alternative patterns of questioning beyond funneling. However, these alternative patterns were inconsistently applied. It is entirely possible that this inconsistency was a result of Jeremy’s perceptions of students and their mathematical abilities. Further, the student population of the Basic courses at Jeremy’s school may have influenced his thinking and his questioning patterns, making him less likely to use focusing questions generally for that subset of his students.
Interview 4 – June 4, 2018

The final interview followed a similar structure to the first three, with questions focused on change in practice but with one extra focus: how did the SMII experiences influence any changes that were made.

Changes in Jeremy’s teaching practice.

Jeremy summarized changes to his practice nicely in his opening response:

[P]robably trying to find things like what we just did to give kids different entry points in the types of questions versus how we typically teach which is teacher-led instruction, um, with example problems for any and all of the types of things we might be covering so that they have had some guidance in some way or another. Versus what I, where we are pushing with this is give them something that they can use prior knowledge on and try to come up with a method, regardless of that method, and then have those types of conversations about why your method worked or didn’t work... That is something I’m trying to incorporate more is, is trying to introduce topics with teacher guidance second versus right up front. And then... I don’t know if you want to call them labs, but I would call that right there [the problem he just finished] basically a lab, say... intro concepts with labs, with minimal guided examples maybe? So that would be it.

Jeremy equated problems like those he was asked to solve in the interview process with “labs,” an interesting choice of words. But his summary of the goals of this type of instruction was telling about his beliefs at this point in the process (Personal Domain): letting students do the thinking and choose their methods based on that. Instruction became a discussion about ways of making sense and solving as opposed to a delivery of a list of examples.

Jeremy readily acknowledged, however, that things were not perfect. He indicated that “[w]e [he and his colleagues] still do guided notes for a lot of the... for the majority of the material that we cover. Again, that’s just, that’s slow change from what we have, where we’re at to maybe where we want to end up or where we want to be.” This acknowledgement was consistent with messages from the SMII PD and from various conversations that we had had as a group (we spent several segments of time discussing how to manage the time conflicts inherent in the change to more student-centered instruction). The acknowledgement that changes is slow was vital to getting participants to experiment with the new, student-centered methods in their classrooms—they needed permission to take it slow or they would have been overwhelmed by the enormity of the change.
Jeremy also hinted at how he had attempted to work within the framework of the curriculum materials he had been given:

I have students do a lot of homework answering of questions over the assignment as of late or lately versus me just reading off the answers. So, we’ll have students provide answers and basically, it’s a majority rules and they explain why. If somebody got an answer wrong, they can explain what they did and then if somebody has the, what we are assuming is the correct answer can offer up their reasoning why. So, more student interaction-based on that front. Which I’ve liked.

Once again, Jeremy indicated his value for student interaction, which he reiterated when I pressed him on the reasons he liked it: “Student interaction. Student discourse. They are the ones doing the . . . I don’t know if you want to call it instruction, but I would . . . Providing reasoning for their own lines of thinking and how they approached the problem.” This statement is interesting in that it may indicate a shift in Jeremy’s view of the locus of instruction. Instead of residing within him, it now seems to reside, at least in part, within the class of students he is teaching. While the data are not definitive, the consistency with which Jeremy displayed value for student ideas and commented positively about the outcomes of his professional experimentation with student-centered methods indicate the strong possibility of a change in his belief structures (the Personal Domain) at a minimum.

I shifted the conversation to how students respond to this kind of teaching and Jeremy expressed some surprise at how some of his students responded. First, we clarified what it means for students to “respond well” to this type of instruction: “I guess for me it’s their interactions are positive . . .” He noted that some of his students did not like “having to be responsible for providing answers . . . but when called on they still did.” I inquired about students’ choice in those interactions. Jeremy’s response was thoughtful and detailed, using a couple of different examples.

Jeremy So, when we started I kind of threw that out there that that was something I’d like to do and you could hear some of them hemming and hawing, so I kind of would the first couple days through, um, would not pick those kind of people but if they wanted to reply to something being wrong then I would pick on them. And then like within the third or fourth day I would kind of let them know “hey I’m going to call on you for one of these today
just to give them a heads-up that they were, that they were okay with that.

Jason and you say the responded well to the heads-up?

Jeremy Yeah, yeah. So, they were pretty fine with that and even, this goes, I know I’ve got one girl that doesn’t particularly like to do a whole lot. But this is one area where she just wanted to speak up and it was kind of exactly like this. I did not see that coming from this person and it was nice to get her involved in some of the ideas. And it might just be because we’re in statistics right now and she might just . . . like statistics for whatever reason because it’s more, I guess, interactive if you will. But it was kind of nice. I was pleasantly surprised with that. But they’ve all seemed to like it.

Here Jeremy outlined a process by which he began to acclimate reluctant students to the new norms of the classroom. He also noted that he was surprised by the increased participation of particular students. This is an example of a connection between the Personal Domain, the Domain of Practice, and the Domain of Consequence—engaging students in that way (which Jeremy valued) encouraged some students to participate more than they otherwise might have. We might hope to see this realization connect to the Personal Domain by influencing or reinforcing Jeremy’s beliefs about students and the value of engaging students in this structured way.

Jeremy’s plans for the future.

As part of the last interview, I asked Jeremy what changes he might be planning to make in the future. His answer was somewhat off-topic, but in line with his content-focused view of instruction.

. . . it’s going to be quick turnaround response time with homework and quizzes, that immediate feedback . . . I am actually pushing for all of the homework answers to be out in the classroom . . . one of the questions on [the SMII Teacher Belief Survey] was “Do students have to come to me for answers?” . . . I was a yes and a no, I was torn in between. No, I don’t want them to, but some students want to have that reassurance. And unfortunately, I’m the only one that has the answers and why is that? Just because they are not out there in the classroom, so . . . I’ve done a flipped classroom before in the past where I’ve had the answers out in the classroom and the dramatic improvement I’ve seen out of . . . students that
typically were in the lower percentile range. Those were the students that benefitted the most out of that.

My intention behind the question was to elicit Jeremy’s plans for changing his instruction—I hoped for detailed descriptions about how he might plan to facilitate students’ explorations, discussions, and work time differently. It would appear that Jeremy was not ready to think about such detailed changes at this point in his change trajectory. That is not to say that providing all students with the answers to the homework sets was not a large step for him, given the pressures from the External Domain as they have already been laid out. Indeed, I was very happy to hear about a change that was specifically designed to support all students and potentially make different use of class time. We might be able to infer that if students had the answers to the homework exercises, then extended time spent checking homework as a class (as in Observation 2) would no longer be necessary.

This example also provides insight into Jeremy’s reflective process. He indicated an experience he had in the past and extrapolated from that to make his plans for the future. This indicates a reflective connection between the Domain of Consequence and the Domain of Practice. Jeremy also elaborated on his hopes for this new homework-related strategy:

. . . I’m hoping, right, that as these groups start to develop themselves that they will have that communication back and forth on how they are solving problems, how they’re approaching problems. And they’ll have that conversation before they go to check answers . . . that’s how I would want to guide them to that, is . . . make the attempts, compare attempts, check answers, so . . . we have multiple ways we can plug stuff back in and see if our answers are right before I go checking an answer key or moving on to the next problem . . . I would love to find a lot of problems like the ones you keep giving us.

While on the surface, Jeremy’s homework-related changes may have seemed unrelated to the work we had engaged in throughout the school year, the reality is somewhat more complicated. The provision of answers to the homework exercises is, in fact, a strategy to increase student engagement and discourse during the “homework time” portion of Jeremy’s lessons. This may indicate Jeremy’s reflection on and modification of his beliefs (Personal Domain) and the structure of his current materials and lessons (External Domain and Domain of Practice, respectively)—his plan represented an attempt to work within the confines of the material he was given to teach while still striving to achieve some of the goals of the work of our PD that year.
Connections to the SMII PD experiences.

To close the teaching practice portion of the final interview, I asked Jeremy two questions. Those questions and Jeremy’s responses follow.

1. “What experiences have you had that you found most valuable in helping you in making those kinds of changes [the kinds of changes pushed for in the SMII PD]?

Probably the biggest one is working together in our math group with the same problem and looking at everybody’s entry into those types of problems. And it’s, it’s no different from what you see in the classroom, which is the funny part is that everybody comes at it from their perceived strength, maybe. And then to have those conversations about why and which direction we’re going and how that gets us all to the same answer in the end. That and then working with, obviously working with my colleagues on that whole front. Just listening to the way that they introduce stuff, the way that they teach things, the order in which they teach things in, their ideas in general. Those are easily the top two things, for me.

2. “. . . what advice would you give to [colleagues who did not participate in SMII] about making those shifts themselves? What would you tell them?

. . . my advice would be start small. It’s okay to spend time doing something you’re not comfortable with and getting, or I should say not getting the results you hoped for at the beginning. And then adjusting those ideas of what results actually are. Like anything you try, some things are going to work out nice and you’re going to make a nice cake. Other times you’re not going to make anything at all. So, being okay with that, and it’s not, it’s not a waste of time. That’s how we hone our craft, I guess. It’s never a waste of time if we’re constantly reflecting on what we are doing and trying to find different ways to engage students in the material. Sometimes it’s a hit, sometimes it’s not. So that . . . work with your colleagues. You’re not alone. Develop things together. If somebody’s already got something developed, use it, turn it into your own if you don’t like the way it’s set up. But if it’s in the same vein, so if it’s an algebra concept, because we’d want to have everybody on the same page, have those discussions within that component of your department. And maybe your idea is worthwhile for somebody to implement so don’t, don’t . . . you’re not on your own.

Jeremy valued the collaborative setting in which he and his colleagues worked during SMII. His remarks about the different approaches that he and his colleagues used in tackling and making sense of problems and the way he linked them to his experiences in the classroom may indicate a confirmation of his beliefs about mathematics (e.g., that there is value in exploring problems with multiple entry points and solution pathways, an increased focus on understanding and justifying is valuable). Further, his value in listening to his colleagues discuss teaching and
describe different approaches indicated a desire for support in his professional experimentation and work. This supposition is supported by his answer to the second question, which focused on advice such as “work with your colleagues. You’re not alone. Develop things together.” He and Terry worked extensively together during the course of the PD. They even submitted a pair of videos of a class project that they developed and facilitated together.

Discussion

Jeremy continued to struggle with the pressures of the External Domain to the end. He spoke about how professional experimentation is “never a waste of time if we’re constantly reflecting on what we are doing.” He continued to justify this to both himself and his colleagues throughout his experiences. However, these justifications were always tempered by an acknowledgement of the very real constraints of his situation: curriculum materials, time, evaluation structures, his perceptions of students’ abilities. Despite all of this, Jeremy reported instances of professional experimentation much more often than any of his colleagues. He pushed the boundaries of his experience, leveraging his beliefs to create different environments for students to engage with mathematics.

Ultimately, the classroom observation data showed that Jeremy was not successful in habituating instructional practices consistent with those espoused in the SMII PD or developing systematic methods for implementing them in his classroom. However, despite the external challenges present in his work, Jeremy was able to make space for professional experimentation. He spent valuable time honing his craft in the way he believed he must, balancing external influences against his own desire to learn to teach differently. In Jeremy’s case, this meant both small and large attempts at professional experimentation. He and Terry worked together to design a custom project for a Geometry course and co-facilitated that project with Jeremy’s students. This was largely to allow them to participate in the lesson study sessions of the SMII PD, but Jeremy indicated value for similar kinds of projects on multiple occasions.

Beyond his collaboration with Terry, Jeremy also experimented with group work and alternative patterns of questioning. This showed through prominently in his video sharing during lesson study. Of particular note was the instance of Jeremy’s realization that the lesson he thought was “bad” was actually not so. In that instance, Jeremy was able to establish and maintain collaborative exploration of a task in his classroom. Further, he was able to use alternative patterns of questioning to support a group of students in exploring that same task.
While none of the data indicate that Jeremy permanently changed his instructional practice, there is much in the way of data suggesting that he was in the process of learning how to teach mathematics differently.

Jeremy’s Vision of High-Quality Mathematics Instruction

Interview 1 – September 26, 2017

Concept mapping was a struggle for Jeremy throughout the year. He spoke in some detail about his beliefs about good mathematics instruction, but he was not able to convert those details into a visual representation of any kind.

In lieu of a concept map, during the first interview I asked Jeremy to discuss how he thought math instruction should be (see Figure 51). There appeared to be some conflict within Jeremy’s conception of mathematics instruction. His notion of the “tossup” between beginning with prior knowledge or to begin with more open, lab-like activity is evidence of that conflict. Here, again, Jeremy referred to “labs” by which he meant tasks of the sort he and his colleagues engaged in during SMII and the interviews for this study. He went further, indicating a plan for the results of those labs. He wanted to “start to break those [results] down into the concepts we would cover. So that it’s more hands-on interactive.” He contrasted this with an alternative approach, which was to “go the other way around, kind of like we do with the Skittles Lab.”

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<td>Jeremy was unable to construct a concept map for Interview 1.</td>
<td>So I guess for me, it’s a tossup between do you start with trying to talk about prior knowledge and skills that they should have and that expectation that you have these items at your disposal to use and then I would love to do more investigative work, labs, and then take those labs and results and start to break those down into the concepts that we would cover. So that it’s more hands-on interactive. Or go the other way around, kind of like we do with the Skittles Lab. We start out with something that’s interactive, but if you don’t use it after that . . . what was the point? Where if we could do more labs and collect data and information. So in our first unit right now we’re identifying between those concepts, linear, quadratic, and exponential. Why aren’t we doing a bunch of labs and take our own results and decide what they are? Versus okay here’s a given list of data that I just randomly made up, now you tell me what it is with zero interest or zero input. So, I would like to see that.</td>
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*Figure 51. Jeremy's explanation of good math teaching from Interview 1 (September 26).*
This referred to a conversation he and I had earlier in the first interview (see above). He introduced exponential functions through a lab experience for students and followed this with a breakdown of the requisite learning into smaller, lesson-sized pieces. He made the case that this should be done more often.

When I pressed him to think about what he might say about facilitating those kinds of conversations, Jeremy asked “are we talking about how to get those ideas to come from students?” Clearly, he was guessing at what I wanted to hear, and his answer was a challenge for him: “I really don’t know. [Long pause]. I have no good answer for how to do a . . .” It seems clear that there was a mismatch between Jeremy’s beliefs in the Personal Domain with his knowledge of how to enact those beliefs in the Domain of Practice. He knew what he wanted but was unable to make it a reality in the classroom. This may explain why Jeremy’s typical practice looked nothing like his professed vision.

I asked Jeremy about the best lessons he had ever given. He focused solely on student interaction,

A lot of interaction between teacher [and] student. A lot of good questioning, so where we would maybe spend three or four minutes on a single concept or idea and bat around all of the different viewpoints that students may have. Why they think something is what it is or how they got to an answer. Have that open atmosphere, that, I guess that class where students feel they can share answers, first of all. [Pause]. Total interaction. I love when we do, we have flip cards, sometimes we do flip cards. And then they, so everybody’s gotta reply. And then eventually we get to the point where students are willing to share their work and their methods up in front of everybody else. We start out doing that with small groups. And then just talk about answers. And then we’ll get into sometimes methods. [Pause]. Student [to] student interaction, where if that’s done enough where you’ve redirected them so that they’re taking responsibility for being on task and on topic and they’re guiding the other person sitting next to them or within their group, that’s pretty awesome. Without me having to intervene at all. I love that.

Jeremy’s final statement in Interview 1 was, perhaps, most telling of all: “I say my best lessons are when there’s always a lot of interaction and back-and-forth.”

Interview 2 – November 13, 2017

The second interview saw Jeremy create a very basic, linear vision of mathematics instruction. His diagram was simple and lacked specific details. However, Jeremy provided more detail in his explanation of that diagram (see Figure 52).
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<td>Alright, so for me it’s and these are all generic, I guess, for me. So, you’ll have this which is good: Intro the concept. Depending on whether it’s a new concept, a brand-new concept or one that is an extension of prior knowledge you would want to approach that differently. So, if it’s, a lot of the time mine is an extension of a concept so I generically just introduce the inequality sign [motions that this is an example referred to previously]. You guys have experience, obviously this changes something. Start thinking. So, I introduce some things very vaguely and others a little more concretely. Then I would, a lot of times it’s students are working together in groups, brainstorming ideas, using prior knowledge . . . anything they have at their disposal they can use. They are creating their own lists of instructions or making observations and sharing them with each other. Then we bring that stuff together and share together as a class. We address misconceptions and we create a final class concept that we can either agree on or disagree on and then we can compare those two at the end to achieve the desired outcome for whatever the original concept was. But from here [draws on paper] to the end, it’s all student-driven. And the last one . . . is teacher.</td>
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*Figure 52. Jeremy’s concept map and explanation of mathematics instruction from Interview 2 (November 13).*

Jeremy indicated he would begin by “intro[ducing] the concept . . .” This introduction did not appear in his diagram. Based on his explanation, it appeared that Jeremy saw group work as “brainstorming” rather than a purposeful space to collaboratively reason about a situation. He was beginning to actualize his beliefs about teaching but used vague words to describe what this enactment looked like. Jeremy’s description here began to show evidence of a more student-centered classroom, with an increased sharing of mathematical authority, at least in theory. This was borne out in his next comment, describing the remainder of his concept map:

Jeremy Then we bring that stuff together and share together as a class. We address misconceptions and we create a final class concept that we can either agree on or disagree on and then we can compare those two at the end to achieve the desired outcome for whatever the original concept was. But from here [draws on paper] to the end, it’s all student-driven. And the last one . . . is teacher.
Jason: Got you.

Jeremy: Which is different. Completely backwards than what I had done in the past. Where it was all this one [refers to top of map] and they’d better agree with me when we go through there.

Jeremy appeared to indicate a shift in his views on instruction here. In Interview 1, he indicated that in the brainstorming phase students had freedom but that was curtailed in the end by his direction and his mathematical authority as the teacher. His description now told a different story. He indicated that the “come together as class for discussion” stage was student driven, where before it was not.

Jeremy also indicated that it was harder to teach in this new way. He implicitly brought his experiences from SMII into the discussion by talking about planning time. He had integrated the ideas of the complexity of good mathematics instruction and the need for comprehensive planning from his SMII experiences (the External Domain) into his beliefs about teaching (the Personal Domain) and, at least somewhat, into his practice (the Domain of Practice). The difficulty he said, was in finding time to do the prep work,

. . . but only harder, once again it’s no different than the flipped class. Once all the prep work is done, right? So once I have my nice lesson plan with all my questioning techniques and expected questions from the students and if it’s a quality lab and I’ve got all the resources and I have at least some experience running it before and tweaking it, it’s going to be no different than, you know, it’s just notes. But it’s them making and creating the notes. So . . . work to get prepared, after that not so much. And I guess fair to say there too I’m very . . . oh, what’s the right word? [Pause]. My classroom management technique is very open . . . for ideas.

These are direct references to the planning components we discussed in the SMII PD. “Questioning techniques” refers to the list of pre-prepared questions based on anticipated student thinking and “quality lab” refers to the quality of the task students were to engage in. Jeremy believed, at this point, that if a teacher had prepared in this way, then the lesson would be easy to facilitate, similar to how easy it was for him to lecture while students take notes.

Interview 3 – May 14, 2018

For the third interview, Jeremy created his first cyclical vision of high-quality mathematics instruction. Again, his concept map was simple, with few entries and no details.
Jeremy’s conflicted beliefs came up again in his explanation. He displayed uncertainty about where and how to begin.

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<td><img src="image" alt="Concept Map" /></td>
<td>See, now I don’t know where to start. I go back and forth. Do I start with prior knowledge, do I just throw some stuff out there and have kids explore and come up with their own ideas? There’s two different, you know what I mean, there’s two different routes to go with that . . . It’s circular, man! It’s circular, entry points all over the place. So, it’ll be like, vocab, right? [drawing on paper] . . . labs, I don’t know, direct instruction maybe . . . what am I thinking? . . . So, all three of these, all four of these give me different entry points into any type of instruction I’m going to do. Three of the four are student-based, one of them is teacher-based. Depending on, for me it would depend on the material. If it was something brand-spanking new . . . even there I could hesitate and say, split. Student research of some kind, something like that . . . So if I wanted to labs, there would probably be some reading or some prior knowledge that would have to be kind of tied into that at the same time along with some common vocabulary. So, I think about how all of these kind of hit together in the same vein if you will. Some labs I’ve done . . . with zero exposure, I guess, to prior knowledge, just to see what the kids come up with. But you can enter this at any point.</td>
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*Figure 53. Jeremy's concept map and explanation from Interview 3 (May 14).*

As can be seen in Figure 53, Jeremy’s model had at least two new features. First was the introduction of the idea that mathematics instruction might be cyclical, switching between modalities based on the content and the students’ understanding. Second, Jeremy differentiated between direct instruction and other forms of instruction. He made this differentiation based on the student-centeredness of the instructional mode. In reality, his first diagram is contained within his new one. The “prior knowledge extensions” and “vocab” sections might involve
aspects of his prior diagram. “Labs” was more involved, and new here, while “direct instruction” referred to giving students notes and lecturing. This development is only natural. As Jeremy’s conceptions of teaching began to change, as he experienced different ways of thinking about mathematics and about facilitating student learning, his conceptions of teaching shifted from simple to more complex—in this case, from linear to cyclical. This transition is continued in the fourth interview.

Interview 4 – June 4, 2018

Jeremy’s fourth attempt at a concept map was the most complex and sophisticated of the three (see Figure 54). It contained elements of his first three attempts and much more detail. He did, however, struggle to improve his diagram. He still conceptualized mathematics teaching strongly as cyclical, “it’s the same thing in my head, it’s like this circular thing now where we can just go in here . . .” I commented that he had not seemed happy with his diagram from last time (he agreed) and I prompted him to see if he could improve upon it, creating “circle two-point-oh.”

Jeremy began with “what [he] . . . would want kids to do with the exploration piece first.” But immediately Jeremy wondered “is an exploration different than an investigation? I think exploration is more like a lab where investigation is maybe I [students] go and do some research on a topic . . .” As he wondered this, he began to construct his new concept map.

Jeremy displayed a remarkable frankness in this explanation, sharing his wonderings and his conflicting thoughts about teaching. His model introduced the idea of assessment through the “quick check” which informed him about which modality to use in subsequent lessons. To him, mathematics teaching had become a series of decisions about how to facilitate student learning as opposed to a series of decisions about which content to tackle next. This stands in contrast to his initial description, which involved a lot of the same ideas, but in a more vague and unconnected way.

Jeremy’s models showed a consistent development of a different vision of mathematics instruction, from simple linear, teacher-led instruction to a much more robust, often student-centered kind of instruction. Unfortunately, there is a lack of data about specific instructional moves that Jeremy might employ. This is likely a function of my lack of asking pressing questions in these areas and of Jeremy’s growing conception of teaching—he had not reached the point in his development where he thought about specific kinds of facilitation moves he might
make on a daily basis. This broad focus is likely a function of the task setting as well—conceptualizing the entirety of mathematics instruction may have prevented Jeremy from thinking in specific terms.

<table>
<thead>
<tr>
<th>Concept Map</th>
</tr>
</thead>
</table>

![Concept Map Diagram]

<table>
<thead>
<tr>
<th>Explanation</th>
</tr>
</thead>
</table>

So is an exploration different than an investigation? I think exploration is more like a lab where investigation is maybe I [student] go and do some research on a topic, complete the square, find some videos and some funky way somebody else did it online. That’s what we’re looking at . . . So, depending on the concept, . . . maybe it’s a previous skill that they should have had some exposure in, whenever, in the past. So then I can have, I can come in with any three of these type places [debate, investigation, or exploration] . . . and then from there if we develop, I don’t know, a set of standards or ideas of what it [the mathematical concept to be learned] is, then we can get into some of this other stuff. We can go around any direction that we want to go around. If it’s at least we had some conversation, then for me I always go back to like a complete the square which is a tough concept, to where that would be one of those ones where, you know, maybe, I have always thought it was always best to do teacher-led instruction on that but maybe it’s not. Maybe the students go out and find some videos and try to figure out and make sense of what it is that we’re doing first. And then we go back into that teacher-led stuff and then into some of that other, into some of those other things. Because, really, what do I know? Unless I’ve tried this other stuff, how do I know that’s actually the best that there is? That just is the thing that I’ve always done. So, I don’t know. But from any point in time we don’t necessarily have to go through all four of them. If you wanted to we can . . . go right into a quick check depending on the concept. So if it’s coming from prior knowledge and it’s an extension maybe one exploration or a lab was good enough and we can go to a quick check and if we’re good we can move on, if not maybe that points us into one of the other, other directions. And then boom, we can come back and do whatever you want! And eventually test, somewhere, in any of those.

Figure 54. Jeremy’s concept map and explanation from Interview 4 (June 4).
Discussion

Over the course of his experiences in the SMII PD, Jeremy experienced several changes. His conceptions of mathematics teaching and learning began to shift from a vague, teacher-centered approach to more student-centered approaches. His teaching practice shifted in similar ways, bounded by the constraints of his department and curriculum. Jeremy was very open about his professional experimentation and spoke about the results of his experiments often and at length. His initial beliefs were the most supportive of the kinds of changes advocated for in the SMII PD intervention, hence he was the most prolific of his colleagues in producing video in which he was trying pieces of new instruction. He maintained a belief in the value of student discourse and interaction throughout his experiences. Despite his continual battle with the pressures of time, Jeremy seemed to show an increased value in more student-centered approaches to learning. One might hope that with continued support and collaboration, Jeremy would move further in the journey he began during the SMII PD experiences.
CHAPTER 5

SUBSEQUENT RESULTS: FURTHER EXAMINATION OF CHANGE

Introduction

While the presentation of the individual case study reports is certainly a central feature of a multiple case study analysis, further elaborative analyses are both helpful and necessary. To that end, as discussed in Chapter 3, this chapter includes the results of several analyses subsequent to the creation of the individual case study reports. The presentation of results will proceed in the following way:

1. Cross-case analysis
2. Change processes related to the IMTPG
3. Changes in reflective processes
4. Summary and discussion

Cross-case Analysis

The main substance of the case study reports provides a great deal of detail in answer to the first research question:

*What do teacher change processes associated with the SMII PD intervention look like?*

However, a deeper analysis is possible. By studying commonalities across cases, a certain level of generalization is possible, at least with respect to this particular group of subjects and their associated context. Within each case study, potential change sequences and growth networks were identified and elaborated upon. A comparison of these by focus may be revealing. To do this, I adopted an operationalization of the IMTPG similar to that of Witterholt and colleagues (2012), laid out in Chapter 3 (Figure 6).

Change Sequences and Growth Networks

The case study reports identified multiple change sequences and potential growth networks for each participant. Table 12 contains a summary list of the identified and explicated change pathways for each participant. Each subject reported evidence of both change sequences and growth networks during this study. Recall that a change sequence is defined as a mapping on
the operationalized IMTPG involving up to three of the four domains (with any associated reflective or enactive links) and a growth network is a change pathway that involves more than three domains and relevant reflective or enactive links.

Table 12
Change Pathways Identified for Each Subject within Case Study Reports.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Change Pathways</th>
<th>Category</th>
<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terry</td>
<td>1 → 4</td>
<td>CS</td>
<td>Implementing grouping structures</td>
</tr>
<tr>
<td></td>
<td>1 → 4</td>
<td>CS</td>
<td>Using probing and focusing questions</td>
</tr>
<tr>
<td></td>
<td>1 → 4</td>
<td>CS</td>
<td>Creating “mini-SMII moments” for students</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 7</td>
<td>GN</td>
<td>Matching teaching style to students’ experience</td>
</tr>
<tr>
<td></td>
<td>5 → 3 → 4</td>
<td>GN</td>
<td>Melding co-teaching experiences with grouping structures</td>
</tr>
<tr>
<td></td>
<td>4 → 7</td>
<td>CS</td>
<td>A failed attempt at implementing “volunteer groups”</td>
</tr>
<tr>
<td>Rebecca</td>
<td>1 → 4</td>
<td>CS</td>
<td>Incorporation of key academic vocabulary</td>
</tr>
<tr>
<td></td>
<td>7 → 9 → 1 → 4</td>
<td>GN</td>
<td>Questioning current practice and planning future changes</td>
</tr>
<tr>
<td></td>
<td>1 → 4</td>
<td>CS</td>
<td>Questioning strategies</td>
</tr>
<tr>
<td></td>
<td>1 → 4</td>
<td>CS</td>
<td>Implementing specific talk move</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 3</td>
<td>GN</td>
<td>Implementing alternative facilitation moves</td>
</tr>
<tr>
<td></td>
<td>4 → 7 → 9</td>
<td>GN</td>
<td>Actualizing value for “getting [students] up and moving”</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 7 → 6</td>
<td>GN</td>
<td>Increased understanding in students after changes</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 3</td>
<td>CS</td>
<td>Change in beliefs about teaching</td>
</tr>
<tr>
<td></td>
<td>5 → 7 → 9</td>
<td>GN</td>
<td>Alternative participation patterns</td>
</tr>
<tr>
<td></td>
<td>1 → 4</td>
<td>CS</td>
<td>Changing role as an instructor</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 7 → 9</td>
<td>GN</td>
<td>Enactment of alternative lesson format</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 7</td>
<td>GN</td>
<td>Noticing and using students’ independent ideas</td>
</tr>
<tr>
<td></td>
<td>1 → 4 → 7</td>
<td>GN</td>
<td>Creating opportunities for student discourse</td>
</tr>
<tr>
<td></td>
<td>4 → 7</td>
<td>CS</td>
<td>Questioning strategies</td>
</tr>
</tbody>
</table>

Note: CS = change sequence, GN = growth network.

The most common example of a change sequence identified in the case study reports was the 1 → 4 change sequence (expanded in Figure 55). The prevalence of this change sequence in the data indicates that the teacher change process was often influenced by factors from the External Domain. In the cases outlined in this study, subjects’ change sequences oftentimes began with empowering influences from the SMII PD sessions. However, in many other cases, the change sequences began with a combination of empowering and inhibiting influences from subjects’ contexts. Further, many of the growth networks outlined in Table 12 began in a manner consistent with the 1 → 4 change sequence but continued onward into other domains or cycles of enaction and reflection.

The influence of the SMII PD is evident in the focus of many of the change sequences and growth networks. Some of these influences were straightforward, as when Terry indicated that he was constantly thinking about how to “create mini-SMII moments” for his students.
Other examples were more subtle, involving a focus on questioning techniques or grouping structures, which were both a focus of the SMII PD. Interestingly, most of the change pathways (both change sequences and growth networks) that began with the SMII PD in the External Domain followed a reflective pathway into the Personal Domain (Arrow 1) and then an enactive pathway into the Domain of Practice (Arrow 4). I choose to label change pathways that begin in this way classical change patterns, due to the way in which they mirror the classic logic model associated with a “PD as training” viewpoint (Clarke & Hollingsworth, 2002) represented in Guskey’s (1986) original change model. This may indicate that the SMII PD was effective at challenging subjects’ knowledge, beliefs, and conceptions. There were only two instances in which change pathways began with a direct connection between the External Domain and the Domain of Practice (Arrow 5). One of these examples appears in expanded form in Figure 56.

Figure 55. The expanded 1 → 4 change sequence.

Figure 56. The expanded 5 → 3 → 4 growth network Terry described.
In this case, the SMII PD and Terry’s context combined to encourage him to partner with Jeremy to co-plan and co-teach a lesson. Terry needed an opportunity to record a lesson for the lesson study portion of the SMII PD sessions but was not teaching a math course that trimester. To solve this problem, Terry worked with Jeremy on a new set of lessons focused on similarity in Jeremy’s geometry course. To fulfill the requirement of video recording himself teaching, Terry co-facilitated the lesson with Jeremy during his planning time. The growth network identified in Figure 56 involved Terry’s attempts to make sense of his experiences with co-teaching and grouping students effectively in the Domain of Practice (5)—that is, he applied his experience from the PD directly into his classroom practice (enactive pathway 5). As he did so, Terry engaged in reflection on his work, attempting to reconcile his experiences in professional experimentation with his knowledge and beliefs about grouping students (reflective pathway 3). Over time, Terry re-enacted his management of student groups in his classroom (enactive pathway 4). In many ways, his initial experience within the context of the SMII PD stayed with him for several months, forming a kind of anchor experience that he could attempt to recreate or modify based on his new context. Generally, I grouped all change pathways that did not begin with a reflective connection between the External and Personal Domain (Arrow 4) into a single collective. This category of change patterns I labeled as alternative change patterns.

In the context of this study, most of the change patterns observed were of classical change patterns. Only six of the twenty-one change patterns that were supported in the data were alternative change patterns. In many ways, this is an unsurprising result given the nature of the SMII PD intervention and the dynamics of the study subjects. The SMII PD was designed specifically to change beliefs through experiences that challenged participants’ beliefs both directly and indirectly. Thus, it was to be expected that many change pathways might begin with influences from the External Domain (the SMII PD) and move into the Personal Domain (changes in subjects’ beliefs and knowledge). However, it is interesting to note that the SMII PD did not constrain the emergence of other potential change pathways. The lesson study portion of the PD provided opportunities for subjects to choose particular practices to enact directly in their classroom, even if that subject did not have a strong belief in the efficacy of that practice. Further, the SMII PD did not prevent participants from engaging in professional experimentation independently of the PD sessions. While the instrumentation used to measure teacher change in this study was not designed to fully capture change pathways of this independent nature, it is
likely that such pathways manifested and that the data in this study represent portions of such pathways.

Patterns of Focus Across Cases

As I alluded to previously, there are some noteworthy patterns to be found in looking across the foci of subjects’ change patterns, both classical and alternative. Many of these focus areas can be linked to aspects of the SMII PD intervention.

Questioning

Six of the change pathways identified within the case study reports involved subjects focusing on changing the kinds of questions in use in their classrooms. Some of these involved changing patterns of questioning from funneling to focusing and others involved simply asking questions that required more thought on the part of students. For example, all three participants engaged in one change pathway focused on questioning strategies. Terry focused on using probing and focusing questions specifically, while Rebecca’s and Jeremy’s pathways focused on questioning strategies more broadly. Further, Terry’s pattern focused on the creation of “mini-SMII moments” for students also included a great deal of questioning work, as did Jeremy’s pattern focused on creating opportunities for student discourse. In any case, the idea that the kinds of questions teachers ask students during instruction greatly influences the learning experiences of students was a central focus of the SMII PD.

Facilitation Strategies

Connected to the idea of questioning, four of the change pathways involved a focus on implementing different facilitation strategies in the classroom environment. Invariably, these facilitation strategies relied on the use of quality questions. For example, Rebecca engaged in patterns focused on implementing the rephrasing talk move, specifically, and another pattern focused on using alternative facilitation moves to increase student discourse. Jeremy engaged in patterns focused on changing his role as an instructor from guide to facilitator and enacting the Launch-Explore-Summarize lesson format supported by the TTLP. The SMII PD offered many opportunities to observe alternative facilitation strategies in practice, both in classroom video and when I used them in facilitating mathematics tasks with participants. Further, the PD offered opportunities for participants to practice those facilitation strategies, reflect on their enactment, and receive feedback from colleagues.
Grouping Students

Four of the pathways identified in the case study reports focused on methods and logistics for grouping students. In these pathways, subjects were struggling with justifying the use of student groups, the structures of those groups (e.g., how many students should be in each group), and the classroom structures necessary to manage students’ work in those groups. Terry, in particular, struggled with the use of groups in both the Personal Domain and the Domain of Practice. He engaged in change patterns where he experimented with grouping students successfully and also unsuccessfully. Even his change pattern involving matching his teaching style to Jeremy’s involved the use of groups in some way. Further, Jeremy’s pathway focused on enacting the alternative lesson format involved grouping students differently. Rebecca, interestingly, was the most hesitant to experiment with grouping structures. She tended to partner students only during portions of her lessons; however, she made more concerted efforts when submitting video for collaborative analysis and submitted one example of her use of small group work. These change pathways were supported by activity within the SMII PD, where there were multiple opportunities to observe group work in the classroom, discuss and problem solve group work logistics, and practice the enactment of grouping students in their classrooms.

Student Thinking

Lastly, five of the change pathways focused on student thinking in some way. For example, both Rebecca and Jeremy reported change pathways that involved an increased appreciation for and use of student thinking in their teaching. Rebecca gave several specific examples of how she had modified how she dealt with situations in which students devised methods for solving problems with which she was unfamiliar. She ultimately indicated that she was more readily able to accept and make use of students’ thinking in her classroom, where prior to the SMII PD she would have simply “shut them down” and pushed her own thinking onto the group. Again, this work was supported by experiences within the SMII PD, specifically the lesson analysis portions of the lesson study sessions and collaborative problem-solving, which tended to have more explicit focus on student thinking than other activities such as reading of research.

Discussion

In examining both the nature of the change pathways that manifested in this study and associated patterns in the focus of those change patterns, the influence of the SMII PD on the
change process is clear. Study subjects experienced change that was supported and empowered by the SMII PD intervention. The design of the SMII PD encouraged professional experimentation without truly limiting subjects in their work. The lesson study component of the PD connected the work of the PD closely to participants’ practice. The various foci of the PD (e.g., questioning techniques, value for student discourse, value for student thinking, etc.) showed clearly in the data collected. While the study design focused on capturing both the change process and the mechanisms by which it was connected to the PD activities, the presence of the alternative change pathways showed that the PD did not limit subjects’ change processes and the instrumentation was sensitive enough to capture change pathways that were not specifically targeted within the SMII PD.

While the cross-case analysis provided an answer to the first research question, there is much more to consider about the change processes experienced by the three study subjects. Some of the findings thus far indicate a connection between the change processes experienced by study subjects and the SMII PD intervention as well as a number of different influences outside of the SMII PD. However, much more detail about those change processes can be obtained from a deeper analysis of the factors that influenced the change processes of subjects in this study.

Change Processes Related to the IMTPG: Domain Coding Analysis

In an effort to effectively combine and leverage the data from the subject interviews and the recording of subject PD participation, I engaged in the two-phase coding analysis process outlined in Chapter 3. This second phase of coding generated a set of emergent codes and sub-codes for each domain of the IMPTG. This phase of analysis was primarily focused on answering the second research question.

*What factors influence teachers’ change processes as they engage in the SMII PD intervention?*

The reader will note that not all emergent codes have a set of sub-codes associated with them. This was an intentional decision based on the thickness of the data associated with each emergent code. If a given emergent code did not have a significant number of coding instances associated with it, or I was not able to effectively categorize those coding instances into a sensible set of sub-codes, then no sub-codes were created.
External Domain Analysis

I shall begin with the External Domain. Table 13 lists the emergent codes, sub-codes and examples that resulted from the open-coding process. From the data it is clear that the External Domain was a complex combination of factors and influences for study subjects. Generally, these influences were of two types: empowering and inhibiting.

Empowering Influences

Participants reported influences from the External Domain which helped them change their practice or had positive effects on their change processes. These empowering influences included influences from me in my role as a mathematics education consultant outside of the SMII PD study, influences from other practitioners, and influences from the SMII PD itself. It is important to understand that prior to this study, I had developed professional working relationships with each of the subjects. I had engaged in curriculum work with them and I had engaged in consultation on classroom instructional and leadership issues with them. There were also some limited coaching interactions between myself and Terry, which featured prominently in some of the data. These influences showed as a theme in the External Domain data.

Study subjects also mentioned influences from other practitioners in their commentary. These other practitioners included each other, colleagues from their staff, and even colleagues from other school districts who they had encountered on site visits and similar experiences. Subjects held value in these influences, with individual subjects referring to specific influences in comparison to his or her current context and practice. These influences from other practitioners were without exception positive, even if some of them caused consternation and cognitive conflict within the subject who referenced the experiences.

Lastly, whether because of the nature of the SMII PD or the nature of the study instrumentation (likely both), subjects reported a significant set of influences from their SMII PD experiences. Some of these influences correlated directly with elements of the SMII PD, namely video experiences, lesson planning, and collaboration and problem solving. Participants expressed value in their experiences with classroom video, indicating that they believed that they had learned from making their practice an object of study and from examining and reflecting on examples of real practice that were not their own (both of which were part of the SMII PD). Further, the SMII PD included lesson planning time, which subjects expressed value for but also indicated was not sufficient for their needs. Lastly, subjects commented extensively on their
Table 13
Emergent codes and associated sub-codes generated from the *a priori* External Domain code.

<table>
<thead>
<tr>
<th>Emergent Code</th>
<th>Sub-code</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influences from the author outside of SMII</td>
<td></td>
<td>One of the big hitters I got from you last spring was the value in that [student talk], because it gets more engagement, if nothing else, with students. So sometimes I've gotten more and more—especially since you and I... Did our thing last year--did a little bit more think-pair-share kind of stuff. Show your answer to a neighbor, be ready to give me your neighbor's answer, and what do you think is right or wrong. And some of the things you've taught me--take it back to concrete things that they know.</td>
</tr>
<tr>
<td>Influences from other Practitioners</td>
<td>Within school building</td>
<td>But we will get apprehension from our department chair because it's so far above and beyond [her] comfort level with everything else going on that she will fight us tooth and nail. Not from me, but from what [Terry] and [Jeremy] have done. They've made it more hands-on.</td>
</tr>
<tr>
<td></td>
<td>Exterior to school building</td>
<td>If they're [another school] doing something completely different, and their test scores are compatible to ours, maybe a little better, then we're not going to lose by trying something different.</td>
</tr>
<tr>
<td>SMII PD Influences</td>
<td>Video experiences</td>
<td>I wished we would have had more time to do more video analysis or get into each other's classes... That was insightful, the different ways that people work with kids, especially in group situations, and that kind of questioning styles, and the kind of responses to kids working together. Well, the one video that you showed us with the teacher that was asking. How many groups got--it was more like a moderated class-wide discussion with the sharing out of what happened in the groups. There wasn't any one group that said now, you come up, and what did you do? It was somebody would say something and she would probe a little deeper if she wanted to.</td>
</tr>
<tr>
<td></td>
<td>Collaboration and problem solving</td>
<td>I think just the collaboration with my colleagues and the problems that we did together and just asking for help. Asking them how they would look at it, what they would do differently. So I thought it was really cool that as fellow math educators we experienced the same thing we would probably see our students experience. And that's we don't see it [math] the same way.</td>
</tr>
<tr>
<td></td>
<td>Implications for practice</td>
<td>So this kid came up to me and he said, wouldn't I just double the number of guppies? And I thought about... How do I make--I call it a mini-SMII moment, a mini-SMII moment for this kid?... And I never told him a thing. I just redirected him. I wouldn't have thought about doing something like that until our four days. Because that was one of the things I walked away from those four days with was--it's the old importance of asking good questions in whatever format. I'm trying to push, more so because of our conversations, I'm trying to push experience. It's experience, it's not &quot;I did eighteen assignments.&quot; It's you have x hours of experience.</td>
</tr>
</tbody>
</table>
Lesson planning

I think the lesson planning is huge, which we didn't get enough time for I don't think.

Pushing thinking

If I'm going to summarize what I'm getting out of this whole SMII thing, is how do we get kids to talk more about--if you're going to do a little subtitle for SMII, that would be the subtitle. How to get kids to dialogue more about the math.
My first thought was this [SMII PD ideas]. This is what I want--I want to do something different than what I've always done.
But in many ways, it took me in directions I wasn't fully expecting, and I think they were good places to go . . . So, thank you for that.

<table>
<thead>
<tr>
<th>Systemic and Structural Constraints</th>
<th>Teaching assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>I haven't taught the Algebra I curriculum in a while . . . And so, I know to look for the second difference and I know that it's quadratic . . . So, I don't know what I'm teaching next year . . .</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>School schedule</th>
</tr>
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<tbody>
<tr>
<td>So, we have like sixty minutes. So, cutting down those twelve minutes each class period [in a shift from 5 periods per day to 6], that's twelve minutes where I could have had like a warm-up or a quick . . . check this and make sure you understand this . . . I used to have time to do that. When we had more time, before they shortened our schedule again . . .</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Collaboration time</th>
</tr>
</thead>
<tbody>
<tr>
<td>We don't get together [as a staff] and say, &quot;hey, what question techniques are you using and what kind of structure is this?&quot; It's all data driven stuff . . . It should be that stuff [collaborative work around instructional practice]. And so at least with geometry I can still have or should be having conversations with [Rebecca] or [Terry] . . . If I'm stuck with my algebra 1 class and support class all day long . . . Nobody wants to have conversations with me about those classes.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cultural Influences</th>
<th>Student culture</th>
</tr>
</thead>
<tbody>
<tr>
<td>It's just really hard with my basic students because [of] the want [versus] will. I have a lot of kids that don't want . . . I'm having a hard time helping them because they're not doing the work. So, I don't know how to help them because they're not showing me what they're struggling on. Isn't that sad that in our culture, if we don't give kids a worksheet of 20 problems to look at and say &quot;Oh, I can do this&quot; they walk out the door feeling like something was wrong. . .</td>
<td></td>
</tr>
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<table>
<thead>
<tr>
<th>School building culture</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have high fliers [students] and I have low fliers [students] But that, to me, is a symptom of our system, not you as a teacher, but our system. When kids aren't used to as part of the learning process--I have to explain my reasoning.</td>
</tr>
</tbody>
</table>
Curriculum Constraints  Pacing directives  Since we've gotten on this treadmill of getting from point A to Z, the question I have to continue to ask is, how do I use the time most efficiently? And efficiently isn't always the best. So, I'm going back to the pacing guide days when we were ripped if we weren't on track with the pacing guide.

Instructional consistency directives  So, I try to be as consistent with how everybody teaches the lesson, but it really restricts it for me . . . But I go into, like, conceptual understanding. We have to end at the same place . . . So, I'm afraid to touch those classes because of who I teach it with.

Content and focus  We should look at the content we are covering, which ones do we want to cover in depth that provide the leverage that we think is important. But it's our mile wide, inch deep thing. We're still floundering with that even after our ELT [Essential Learning Target] times and strictly curriculum bound.

Time Constraints  It's uncomfortable. When I'm giving up, that was only 10 and a half minutes or something, but it was probably 20, 25 minutes of the hour by the time we got everything out . . . With that class, we're already a week and a half behind.

I know Common Core is supposed to be project-based instruction or the intent of it was, but we don't do projects in our curriculum. We created them, but we don't have time for them. That's what's hard.

I feel I'm wasting time. I hate that part of it.

The biggest obstacle for me doing this [instruction consistent with NCTM's vision], other than just not being familiar with it and being set in my ways, is time.

Administrator and Educator Evaluation Influences  I have seen that come through on a couple of my evals, too . . . I'm not chunking things enough, or I'm not . . . Guidance, lack of guidance. And even when it was state up front . . . The purpose [of this lesson] is for them to struggle . . . The very next two days later, when I sent in my post-observation, [an administrator says] "Well, they seem to struggle . . . shouldn't you have chunked it differently . . .?"

I'm not teaching pre-calculus anymore because two sets of parents called and said, "My kid's having to do too much teaching herself in that class." Because they viewed, if they didn't have an example just like it, that's what they had to do. They had to teach themselves.

I've always thought I've asked good questions. I'm a good questioner. And [administrator] has mentioned that before.

District pressure for scores  The driver for us [district] is those scores--we're doing good here; we're not doing good here . . . We said we want to be on the opposite end. We want to do teaching and let the scores be what they may.

Bullcrap.

Well, there is a vision [purposes of instruction]. This is what it is said it is [focus on rigor, understanding], but this is what it actually is [get better test scores]. There's that tension.
<table>
<thead>
<tr>
<th>Desire for Resources and Training</th>
<th>I would be more than happy to do whatever I would need to do to get to that point, but I just don't feel like right now with where I'm at I don't have the means or the resources to get there. I've done a few things different but not a whole lot because of lack of ideas.</th>
</tr>
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<tbody>
<tr>
<td>Previous Experiences</td>
<td>I'm getting better at it, but there's an awful lot of history we're fighting. I can tell that was one of the outcomes with the flipped classroom and having the answer [to the homework] out there. Those kids at the bottom that would not have known whether they were doing something right or wrong, they were the ones checking their work for every single problem. I just think I don't know how to. I want to. I just don't feel I have--like, through college, they don't really teach you how to teach or teach you to come up with ideas to promote student learning. They don't do that.</td>
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</table>
collaborative problem-solving experiences. They indicated that they learned much from seeing how their colleagues approached mathematical problems and conceptualized teaching. They expressed appreciation for the time spent solving problems together and studying practice together, indicating that they valued one another as resources in their search for ways to think about instructional practice differently.

Other empowering influences from the SMII PD were not aligned as closely with aspects of the PD experiences. Study subjects were able to derive implications for instruction from their experiences, in particular those experiences within the SMII PD that provided them with ideas that they had not considered before. For example, they saw examples of facilitation moves that they had never thought to use with students and examples of problem approaches that they themselves had not considered before. Subjects also expressed how their PD experiences had pushed their thinking in directions they had not expected, for instance bringing back old values about teaching and learning (in Terry’s case from his early career) and forcing them to conceptualize the purpose of teaching differently.

Inhibiting Influences

Unfortunately, in the case of the subjects of this study, there were a large number of influences from the External Domain that inhibited the change process. Specifically, they included time constraints, systemic and structural constraints, curriculum constraints, administrator and educator evaluation influences, and cultural influences.

A strong recurring theme throughout the data was the inhibiting influence of a lack of time. Study subjects reported that they did not feel they had enough time to enact some of the practices discussed and displayed as part of the SMII PD. They lamented that they did not have enough time during individual class periods and within the school year to make room for more student-centered experiences. Some subjects also felt that when they did engage students in ways consistent with the SMII PD they were wasting valuable instructional time. This appeared to be because they were not able, in many cases, to see immediate, obvious results with students.

Combining with the perceived lack of time were a set of systemic and structural constraints that included teaching assignments, the school schedule, and a lack of collaborative work time. Due to the scheduling necessities of the trimester schedule, subjects oftentimes did not know what they would be teaching the next trimester (even if said trimester started the next day). Further, a lack of knowledge about teaching assignments the next year also created some
inhibiting uncertainty (e.g., neither Rebecca nor Terry knew how many mathematics courses he or she would be teaching from trimester to trimester or the following school year). In this case, subjects did not want to expend large amounts of effort in redesigning a course that they would no longer teach the following year, when they would have to do all of that work again for a different course. Along with the teaching assignment uncertainty came an uncertainty about the make-up of individual classes from trimester to trimester. The entire cohort of students taking a given course, such as Algebra 1, were shuffled between different instructors at the outset of each trimester. This led the department to believe that, for consistency’s sake, each instructor needed to end each trimester “at the same spot” in terms of the lessons in the curriculum. This need weighed heavily on subjects’ minds when they considered the time required for professional experimentation, making them reluctant to attempt any major changes. Lastly, study subjects indicated that their professional experimentation was not supported by collaborative time to help each other think about practice. Of course, the SMII PD offered opportunities for this, but the general feeling of participants was that more collaboration time was needed.

Another inhibiting factor was the district curriculum materials. The curriculum, designed by two members of the department, included some stringent requirements in terms of pacing, instructional consistency, and content and focus. The pacing requirements were most closely related to the time constraint factor discussed previously. The department focused almost exclusively on coverage and ensuring that each instructor in a course was in the same place as the other instructors at various important times. Most often these included the end of trimesters, but also included the timing of unit assessments. Along with these pacing requirements came an implicit and explicit demand that the materials provided—i.e., the notes described at the beginning of the case study reports—were to be used as often as possible, again to ensure consistency for students between trimesters and instructors. Those curriculum materials also imposed requirements of content and focus which forced some subjects to reconsider making instructional changes. For example, the way in which the materials treated quadratic equations and functions was heavily algebraic with little regard for multiple representations. The SMII PD experiences of study subjects strongly implied a different instructional format for content such as factoring quadratic expressions. However, subjects were reluctant to change the treatment of such content for fear that it would disadvantage students if they had a different instructor the
following semester. It is interesting to note that most of this pressure came from the department chairperson instead of from some sort of administrative directive.

However, building and district administration still had some deep and enduring effects on the change processes of study subjects. A recurring theme throughout the SMII PD was a set of stories told about conversations and results of educator evaluation interactions. Subjects told these stories to explain why their practice looked as it currently did and as ways to explain their apprehension about making some of the changes called for in the SMII PD sessions. Legitimately, subjects were oftentimes worried that if they implemented changes similar to those they experienced in the SMII PD, they would be evaluated negatively by their administration. Indeed, some shared stories where this had already occurred. Further exacerbating this problem was a district-wide demand for increased scores on state assessments. Subjects had little faith that any new changes would result immediately in higher scores. As a matter of fact, some were concerned that changing instructional practice would result in scores slipping. These worries caused some reluctance to begin any major changes in practice.

Finally, study subjects noted some cultural influences on their change processes. These influences stemmed from the culture of students and the culture of the building in which subjects worked. Many times, subjects commented on a lack of willingness on the part of students to participate and associated this with their own apprehensions about and difficulties in implementing new instructional practices. In addition to a lack of “will” in students, subjects also noted that students carried a specific set of expectations about the nature of school, work, and mathematics that created friction when new instructional practices made their way into the classroom. Further, there were instances of subject commentary which pointed to the culture of the building in which they worked. The building culture was an inhibiting influence because there was a tendency to label students as “low” or “struggling,” resulting in depressed expectations and beliefs about the abilities of those students. Subjects also made note of the fact that the culture of the building, administrators and teachers alike, did not value some of the experiences subjects encountered in the SMII PD. This made implementation of these new practices feel unsupported and unwelcome, inhibiting subjects’ willingness to make changes to instructional practice.

One set of influences acted in both an empowering and an inhibiting capacity. Subjects’ previous experiences were influential in their considerations of how to implement new practices.
In many cases there was a significant history of practice that was contrary to instruction as portrayed in the SMII PD, leading teachers to have to fight against old habits to implement new practices. However, there were also instances of empowering experiences that subjects could leverage to support their work in changing practice (e.g., previous experiences with providing the answers to homework problems while students worked on the assignment). The empowering and inhibiting nature of subjects’ previous experiences only added to the complexity of the influences from the External Domain on the change processes experienced by subjects in this study.

Personal Domain Analysis

The analysis of the Personal Domain codes produced a number of primary codes, some with sub-codes and some without. The primary codes can be broken into two groups: sets of beliefs and sets of personal concerns. I take up each of these in turn below. Table 14 lays out these results.

Sets of Beliefs

The Personal Domain data revealed sets of beliefs held by subjects about a variety of things, including the use of mathematical tools and representations, the need for structure, the change process, the effectiveness of subjects’ prior practices, mathematics, learning, students, and PD. I begin with the belief sets that did not generate sub-codes.

Subjects expressed beliefs about the necessity and usefulness of mathematical representations and tools, such as visual representations of mathematics (as opposed to purely symbolic representations). Tools such as calculators were also topics of conversation, with subjects noting the advantages of those tools, particularly in creating representations efficiently. Discussions of PD and next steps for participants also developed as a theme in the data for this study. Subjects considered the varying options for PD experiences that they had and had not experienced, noting that an increased number of visits to classrooms would have been helpful. In particular, subjects wished to see more examples of the practices they were trying to develop in their own classrooms. Further, subjects expressed desire for more opportunities to do things like lesson planning as part of the PD.

The data revealed a set of beliefs about the change process from subjects’ conversations as well. In the final interview, when asked about advice they might give to a hypothetical group of PD participants the following year, subjects noted that initial changes needed to be small to
avoid becoming overwhelmed in the change process. Subjects also expressed beliefs about the effectiveness of their prior practices throughout the PD. For example, subjects believed that their current, teacher-centered practices were working for the vast majority of students. Subjects expressed beliefs about mathematics and what it means to know mathematics. Many of these instances were associated with problem-solving activity, but some also accompanied attempts to summarize and process the aims of the PD (encouraging subjects to recognize that multiple approaches to a given mathematical problem are possible and worthy of recognition and learning).

Subjects spent significant amounts of time discussing their belief in the need for structures in various aspects of the classroom. These included facilitating discourse, group work, learning objectives, task design, managing participation, and the launch portion of a lesson. Subjects noted that student discourse was valuable, but that it needed to be carefully structured to ensure that learning happened. Further, subjects noted the importance of teacher questions as part of the structure of discourse in the classroom. Related to discourse was the use of group structures. Subjects expressed beliefs that engaging students in group work was effective, but that it needed specific structures to be so. In particular, subjects noted that students do not instinctively know how to work in groups and so needed to be taught to do so. Further, subjects noted a belief that groups should be carefully constructed (i.e., the teacher carefully deciding which students were placed in each group) in order to be effective. Teachers’ knowledge of the lesson objectives was also a set of beliefs that emerged from the data. Subjects expressed belief in the idea that teachers focusing on the main mathematical idea of a proposed lesson allowed for more effective delivery. Related to this, subjects also noted that task design was key in supporting student learning—specifically, tasks need to be carefully designed to focus on key concepts in ways that allow students to access and learn from the task effectively.

Subjects also noted that there were inherent difficulties in managing student participation in classroom activities. In particular, subjects noted trends such as over participation of some students and under participation of others, particularly in whole group discussions and results sharing sessions. Structuring change in the classroom emerged as a sub-code as well. In these instances, subjects oftentimes reflected on events and planned alternative methods for future work. Lastly, subjects discussed the need to carefully structure the opening phase of a lesson,
Table 14
Emergent Codes and Associated Sub-codes Generated from the *a priori* Personal Domain Code

<table>
<thead>
<tr>
<th>Emergent Code</th>
<th>Sub-code</th>
<th>Examples</th>
</tr>
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<tbody>
<tr>
<td>Beliefs about Representations and Tools</td>
<td></td>
<td>If they can see it and touch it, it makes it more real for the students. And I think that's the biggest thing this has done in terms of helping me format things for kids is you have to have some kind of visual. I think to get to the level of thinking you want kids to get to is you can't do just a bunch of numbers. There's got to be something there that you can see, and see how things move or change, and how it affects something else.</td>
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<tr>
<td>Beliefs about PD and Next Steps</td>
<td></td>
<td>So, it almost has to be—and again, this is one of the things that's exciting about the training next year—it has to be departmental. Where we agree, here's our content, and we understand it's this, and it used to be this, but we're happy with this. And we can justify just this instead of the other thing. We're not there yet, but I think there's a need for us to get there. I think the lesson planning is huge, which we didn't get enough time for I don't think. That would be the biggest thing, I would say, is you've got include some more classroom observations of people who are doing this on a frequent basis and maybe even in those classrooms somehow. Or bringing those people in or showing videos of those people.</td>
</tr>
<tr>
<td>Beliefs about the Change Process</td>
<td></td>
<td>Do one lesson a week. Just start small. Try not to change everything right away because it will be too much. And you don't have to change everything. It will naturally just come the more you do it. So just start doing it. Get your feet wet. And if you don't know how, that's when you ask your colleagues. You're not trying to change the world all in one day. Pick a class and focus on that class and go over the notes for the day.</td>
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<tr>
<td>Beliefs about the Effectiveness of Prior Practices</td>
<td></td>
<td>Our current system, that model I just gave you, works fairly well for 80% of the kids. But the other twenty percent, they would benefit by a different instructional model. I feel like my students are getting what they need in my regular classes. Do I think that some students aren't getting what they need? Possibly, yeah, and it kills me to say that.</td>
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<tr>
<td>Beliefs and Knowledge of Mathematics</td>
<td></td>
<td>I'm better with a pencil, because in my mind I can always fix a mistake faster. Sequences and series, we don’t do that anymore and as you know, if you don’t do something for a while you get a little shaky. It underscores one of the premises you have of this thing [SMII PD]. Everybody thinks about math differently.</td>
</tr>
<tr>
<td>Belief in the Need for Structure</td>
<td>Facilitating discourse</td>
<td>I believe it's [student discourse] valuable, but I think it takes some careful structuring to make it work. I think those have to be the right questions at the right time. That's why I'm calling them magic moments. The magic gets created by the magician and not by the act itself.</td>
</tr>
<tr>
<td>Beliefs about Learning</td>
<td>Student thinking is central to learning</td>
<td>But then if you talk to [Principal] or whatever, he'll say, well, you should be modeling each one, giving them examples to do, and basically leading them by the hand. My thing is that's taking all the student thinking out of it . . . But I'm like, when you struggle through it, that's when the most learning happens.</td>
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<td></td>
<td>Students must learn to be problem-solvers</td>
<td>Problem solving skills. You have a good set of problem-solving skills, you can do a lot of things with that. Do they need it to become better thinkers and problem solvers? Absolutely. Yeah. That's how you measure success.</td>
</tr>
<tr>
<td></td>
<td>Student discourse is central to learning</td>
<td>So, it's just I think about instruction in terms of the interaction between me and the kids . . . Because I think that's the idea. I think that's where a lot of learning really happens is that kid to kid exchange. I think if we're able to engage the students in more of that stuff, they might take ownership of the math more.</td>
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Using group structures

When it works, I don't think there's anything better than having student groups. I think we tend to think as teachers, we put them in a group, and they just automatically should know what to do. And then we get upset when they don't. I think sometimes we just put kids together in groups. I think those groups need to be very carefully put together. And there's all sorts of ways to do that, of course.

Learning objectives

And I think this is, again, something I got from our days together. If you don't know what you're trying to teach, you're not going to find an effective way to get it taught no matter which way you approach it.

Task design

that's why I said "carefully designed." Because if it [the task] doesn't get you the concepts with a better understanding, it's just an activity, and you don't dress it up to it more than it is. And that's why the task structure is so important.

Managing participation

I really don't like to do that very much, because you know how that works. It's the same six kids always putting their hands up, the ones who do the most, and they always have the right answer.

Launch

But if you take them here without doing that prerequisite stuff, which I think too many of us are guilty of—we launch them into some cool activity, and they sit there for 20 minutes saying, I don't know what I'm supposed to do. I don't know what this means. We haven't gotten to that point properly and have some assurance that they're going to have a chance to succeed.
Beliefs about Students

| Doubts about students’ abilities | if I provided my students with this kind of task, I don’t think they would be able to handle it well. I would have to facilitate right off the get-go and like incorporate this into my lesson right away for me to have to delegate through my students. Where if I just threw this on them, they would be like up in arms, like what the heck is this, I don’t want to do this. Like I would get a lot of . . . pushback. and do they have the tools-- and that's my biggest reluctance to do it. I don't know if they have those tools. |
| “High” and “low” students | And you know for my high fliers like my regular Algebra II kids and my Trigonometric kids, like you know, for me if they have a question, I expect them to come to me and like if they don’t come to me and ask a question, then that’s on them. Like what are you willing to put forth to get out what you need? And they’ll say the right kind of things and do the right kind of things. But the lower ability groups you have, the more there's terrorists in these little groups And I think we forget this sometimes is kids are, especially lower ability kids, they're so quick to give up on themselves. |

Fears about students’ reactions

| And so, to give them the freedom to try and learn it on their own or try to think in groups or try to make any type of connections, I just don’t think they would do it on their own. I almost feel like that might be how students feel with us sometimes, and they're afraid to answer the question because they don't want to take that intellectual risk. Because they think we'll judge them, kind of like how we felt when he would judge us in a judge-free zone. |

Students respond differently to students than teachers

| It's very similar to when you model something for kids. You're the teacher. They're the student. There's a little bit of a disconnect, I think. When they see another student model something, there's more attention there. I'm not sure why. I don't know if it's because it's not myself, so they can poke fun of them. Or they say, hey, this one of my colleagues, one of my peers, doing it. It's not the teacher. Yeah, he's the expert, but I want to see how one of my buddies does this. Kids like to evaluate other kids' work. I don't think it's malicious. I just think it's more intriguing for them somehow and makes them think in a different way. |

Expectations for students’ futures

| again, it just proves to me that if they can’t apply what they are learning in the classroom outside in the real world, then what’s the point? There’s a huge disconnect. Especially for the kids that I’m working with. They’re going to be . . . you know, hanging pipes, doing electrical, construction, who knows? But at least they’ll have some exposure to evidence. like half of my basic students when they go out into the real world, they’re not going to university they might go to a vocational school, but that’s it. |

Students’ habits of mind

| But the interesting thing for those kids was—because we’ve not done it quite this way before. It showed me how formula driven they are. They really are. Because I think most kids think of things in terms of shapes, not the components of the shapes. |
Students who are insecure especially the kids who are very insecure with their math skills, they're getting verification instantly that they have it, or they're getting instant help if they don't have it . . . I think it's because too many of them don't either trust themselves, behavior-wise or analogy-wise, there's instance feedback from me on everything we do now up until the point where it comes time to work on homework.

Changes in Thinking I think I would be more prone to do that next year now that I kind of know what that looks like from the practices that we've done. Just, hey, he said this. Can you reiterate that? Or just having the students answer the questions more and not trying to funnel my questioning. I'm feeling more comfortable being willing to pose questions that I'm not comfortable with, knowing that I might not be able to answer what they're asking. I don't necessarily think a whole lot of outward changes are happening, but I'm questioning a lot of what I'm doing now. It just makes me think of how could I better teach this to my kids.

Personal Struggles That's the biggest thing I still struggle with is throwing it back on the kids. I'm too quick to say, great [points to an imaginary student]. What do you guys think about that? [inaudible]. You're doing great. That's great. And I think if I was going to pick a single thing that I need to get better at it's that—ascertaining those differences, especially for those struggling kids, earlier, so I know I can put together effective groups as opposed to just groups.

Teacher Identity Aspirations My first thought was this [SMII PD ideas]. This is what I want—I want to do something different than what I've always done . . . I would much rather do what you're doing when you work with us and say, you've got to come up with three different ways to solve this problem. One way won't work . . .

Too quick to help I know that's what I've become—somebody who's too quick to help when they struggle, and I've got to give them something. But that's not who I want to become as a teacher. I've been molded into that, and I guess I've given into that more than I should have. I'm pretty patient in general. I'm pretty reluctant to just spoon-feed answers. But I'm all too quick, if the time is ticking—like the group you saw me with last week—to just at some point in time draw the line and say, now, doesn't that mean, and kind of summarize for them what they've just done instead of letting them put the final pieces together.

Knowledge and teaching So, it's like I can teach what I know how to teach and how I know how to teach it . . . I don't have a problem showing that I don't know all the answers, but as far as teaching methods, it's hard for me to teach in a way that I'm not used to teaching. All these questions make me feel narrow-minded in my teaching just because it, you know, people say that direct instruction isn't the best instruction or the best way to teach math or whatnot, but you know right now it is the only thing that I know.
Good questioner  I've always thought I've asked questions. I'm a good questioner. And [administrator] has mentioned that before.  
   Because I think I do a better job—I've always thought I've asked questions. I'm a good questioner.

Teacher as learner  I think back to the things that I learned the most from when I was in school, they were a lot of simulation kind of stuff.  
   I was that student that annoyed the entire class because I asked the questions until I understood.

Expressing Ignorance  When it works, I don't think there's anything better than having student groups. But there's problems that I don't know how to work through.  
   Just start there because I don't really know what to do. I feel like I'm kind of at an impasse. I don't know what to do. There's so much new stuff that I don't know how to apply it.  
   I know I need to do something different. I just don't know what that different looks like.
ensuring that students are clear on what to do and that prior knowledge had been activated effectively.

A second primary code was a set of beliefs about the nature of learning mathematics. Subjects discussed beliefs about the centrality of student thinking to effective learning. For example, subjects noted that productive struggle enhances learning and questioned the veracity of approaches to instruction that reduced the amount of student thinking required. Subjects also expressed value for problem solving skills and noted that one of the purposes of teaching is to develop students’ skills in this area. Lastly, subjects spent time discussing their beliefs about the centrality of student discourse to learning.

Subjects also expressed a great many beliefs about students during their conversations. In particular, subjects expressed doubts about students’ abilities in the mathematics classroom. These most often took the form of beliefs that students would not be able to engage with tasks presented to them or beliefs that students lacked the skills to effectively engage with concepts. Further complicating this picture were subjects’ beliefs about “high” and “low” students. This deficit language permeated all aspects of subjects’ discussions. Students in advanced courses such as Trigonometry or Pre-calculus were often labeled as “high,” while students in classes labeled “Basic” were most often referred to as “low.” These references were often accompanied by preconceived beliefs about students’ mathematical abilities—that is, “low” students were inherently less mathematically capable than “high” students. Subjects also expressed fears about how students would react to new classroom practices. These fears most often manifested as beliefs that students would simply refuse to play along if presented a task with minimal initial instruction. Subjects also discussed how students perceived social risk in classroom interactions, noting that some students would feel nervous about participating in a class discussion. Another set of codes revealed an interesting set of beliefs about how students respond differently to other students than to the teacher in problem-solving and discourse situations. The general belief was that students tend to be more willing to talk with each other about mathematics than with a teacher. This also extended to evaluating other students’ work—subjects believed that students were more willing to critique the work of other students than to engage with work the teacher had done. Subjects appeared to hold beliefs about particular groups of students and their future jobs as well. In these cases, some subjects believed that the “low” students would not attend college or did not aspire to do so. Instead, those subjects believed that those “low” students
would attend vocational schools and work at menial tasks and jobs. There also appeared to be an implicit hierarchy of rigor associated with these different careers—that is, vocational schools and physical labor were assumed to be less academically demanding than college coursework and white-collar careers. Subjects also used specific experiences to form general conceptions about students’ habits of mind—for instance, concluding that all students tend to think a certain way based on a single classroom event. Lastly, subjects expressed beliefs about student insecurities and how those insecurities affected classroom interactions, both positively and negatively.

Sets of Personal Concerns

Through the data analysis, sets of personal concerns emerged as primary codes. These personal concerns were not beliefs, but still resided within the Personal Domain. They included changes in thinking, personal struggles, teacher identity, and instances of expressing ignorance.

Subjects discussed the ways in which their thinking had changed over the course of their PD experiences. They noted practices they would feel more comfortable applying in the classroom moving forward and discussed how their thinking processes had changed (e.g., questioning current practices and considering alternative ways to engage students). Along with these changes in thinking, subjects noted areas in which they struggled to make change—for instance, struggling to engage students in deeper discourse or struggling to use formative assessment strategies to gauge students’ levels of understanding. Subjects also expressed ignorance at certain points, noting where their knowledge fell short and how this affected their efforts to change. For instance, subjects often noted that they knew they needed to do something different but lacked the knowledge of what that looked like or acknowledged that there were logistical issues associated with particular classroom practices (e.g., implementing group work) that they did not know how to work through. This was despite the flexible PD agenda and supportive setting of the lesson study format. Subjects had plenty of opportunity to engage with ideas problematic to them individually. However, even these opportunities appeared not to be enough to overcome their concerns.

Lastly, subjects discussed aspects of their identity as mathematics educators. They discussed the kinds of teachers they wanted to become, oftentimes noting that they wanted to do something different than they had always done or noting that they approved of some of the experiences they engaged in as mathematics learners during the SMII PD and indicating that they wanted to develop those kinds of experiences in their practice. Subjects also noted that they
believed they were oftentimes too quick to help students who were struggling. This help often took the form of hints and explanations instead of the focusing questions discussed in the SMII PD. Subjects also noted difficulties associated with teaching in ways they were unfamiliar with or indicated that their current practice looked as it did because that was all they knew. Some subjects noted that they believed they were good questioners as well. Lastly, subjects discussed how their own learning experiences related to their current and future teaching practice.

Domain of Practice Analysis

The analysis of the Domain of Practice, outlined in Table 15, uncovered a significant number of emergent codes. Some of these emergent codes resulted in sub-codes in the second phase of analysis. I begin with the group of emergent codes with no associated sub-codes.

Domain of Practice Emergent Codes Lacking Sub-Codes

From the data, it appears that change processes involving the Domain of Practice rely on sharing examples of teaching practice, both general and specific. These storytelling moments were very context-specific and emerged organically from conversation. As such, I was not able to generate sensible sub-codes for either the general or specific teaching example emergent codes. There were other emergent codes that did not generate sub-codes. Subjects sometimes noted instances where they had engaged in professional experimentation that took them “out of their comfort zone,” indicating that change processes may have an element of discomfort associated with them, that participants must engage in practices that make them intellectually or emotionally uncomfortable (or both) in order to change practice.

In addition to working outside of their comfort zones, subjects also indicated an appreciation for how “messy” experimentation in teaching can be. Some subjects indicated a belief that mathematics lessons should be very neat and well-structured that was challenged by the SMII PD. Subjects readily acknowledged that in order for change to happen, there would be mistakes and that not every lesson would be a success. Terry, in particular, noted a dichotomy for himself. He indicated that he had little difficulty with uncertainty and “messiness” in his science lessons; however, he expected his mathematics lessons to be nearly perfect on the first try. This was a curious dichotomy given its subject-specific tones. Jeremy, on the other hand, was very accepting of messiness, readily acknowledging that some lessons would inevitably not go well and that the only thing to be done was to build off of the failure and improve.
Subjects also mentioned the importance and benefits of formative assessment as they spoke about instructional practice and their efforts to improve. Some spoke of formative assessment in the theoretical sense and others spoke about actual classroom examples. These classroom examples oftentimes included instances of gathering information about students’ thinking from the work or discussions in which those students engaged.

Along with formative assessment, there was significant mention of the importance of activating students’ prior knowledge. Again, there was a theoretical/practical divide, with some comments of a theoretical or aspirational nature and others of a more experience-based or practical nature. Some subjects described their efforts to activate and build upon students’ prior knowledge, while others spoke about the foundational nature of activating prior knowledge and how lessons that did not effectively do so were doomed to specific types of failures.

Another theme that emerged from the data was that subjects would oftentimes fall back on “bad habits” as they attempted to implement changes in practice. Interestingly, the notion of “bad habits” was defined by subjects’ PD experiences. Subjects labeled instructional practices such as low-level questions or demonstrations in response to student questions as bad habits after the SMII PD, whereas they may not have labeled those actions as such before the PD experience. A reversion to bad habits was most often described in relation to an instance of trying to change practice. For example, Terry mentioned a specific example of his interactions with a student in which he redirected and questioned the student as he engaged in a mathematical task. These interactions were consistent with the SMII PD until a point when Terry noticed that too much time had passed. After this, Terry reverted to a demonstration to explain the mathematical phenomenon instead of continuing his questions. In particular, this emergent theme appears related to influences from the External Domain such as a perceived lack of time. These instances of interconnectedness between domains paints an increasingly complex picture of the teacher change process.
Table 15
Emergent Codes and Associated Sub-codes Generated from the a priori Domain of Practice Code

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<tr>
<th>Emergent Code</th>
<th>Sub-code</th>
<th>Example</th>
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<tr>
<td><strong>Specific Teaching Examples</strong></td>
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<td>The one thing that we found was they weren't even using a scale factor at all. They were trying to find other pieces in the image. They were like measuring stuff and then trying to produce the scale factor based off of the image instead of using the original scale factor, manipulating an image, and making a comparison to other items in the pictures. And again, that all comes back to the way we worded the question and the tables that we thought were going to be helpful in going in and trying to guide them, but they weren't so much. Like, we did a half-life problem today in class. And they're just like, no, the fact that a is a, and y is 0.5a—nope, mm-mm, don't want to do it. They see something that's new and different, and they just shut down. And this is the group I'm dealing with.</td>
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<tr>
<td><strong>General Descriptions of Teaching Practice</strong></td>
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<td>the teacher has to identify the outcome, and then somehow, you've got to get the students to own it. If it's an outcome that they're not going to own or understand, good luck getting it. Some days are like today. It was five minutes on a warm up. We'd corrected homework the day before so it was fifty-five minutes of homework time with the, um, assignment corrected at the end of the hour. So really on a typical day, if we're doing notes, that's about the timeframe. Some notes are more, some notes are less, so that time changes.</td>
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<td><strong>Departing from Subject’s Comfort Level</strong></td>
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<td>. . .[T]his was going way out of my comfort zone . . . I said, okay. You can move around. You can work. And nobody was going anywhere. I go, if you want to phone a friend who's on the other side of the class and get up out your chair and walk over to them and start working with them—. And I said, okay. I don't know if it's right—I knew it was right—but I told her that. And I said, what's a different way we could do it? And I was trying to get them to come here, using our rules of logs. And both methods worked. So, I went outside my comfort zone. Instead of just saying, no, that's not how I suggest you do it.</td>
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<td><strong>Experimentation is “messy”</strong></td>
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<td>Just the other day I had the worst trig class I ever had . . . [and am] going to build on that or not make those same mistakes again. I've done several things where . . . It was terrible. And then the next hour I've got to redo the whole thing over. And I need to adjust immediately . . . I need to approach from more of a science perspective when I'm teaching math sometimes . . . Because teaching itself is kind of a trial and error experimentation.</td>
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<td><strong>Formative Assessment</strong></td>
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<td>It's unpredictable, because you don't know unless you start asking those kinds of questions, especially if it's early in the lesson. Or worse, you get halfway through a topic . . . and you find out they don't have it at all. Now you've got to back up.</td>
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So that was, that was the nice thing about that was we kind of hit it from all different views and I could figure out where a lot of different students were at versus me just saying “here’s what it is! I don’t know where you were at, but this is what it is! Adjust.”

<p>| Activation of Students’ Prior Knowledge | I’ve been trying to do that as often as I possibly can—to go back to all that prior knowledge that they have and just do one example, and then make an extension of that example, and then make an extension of that example, and lead up to what it is that I’m trying to teach. But if you take them here without doing that prerequisite stuff, which I think too many of us are guilty of—we launch them into some cool activity, and they sit there for 20 minutes saying, I don’t know what I’m supposed to do. I don’t know what this means. We haven't gotten to that point properly and have some assurance that they're going to have a chance to succeed. |
| Reversion to “Bad Habits” | the difference now would be I didn't have anything to fall back on as a new teacher. I don't know what I would have done. But now, after 27 years, I have bad habits that I'll fall back on. And I asked him, why does that make sense? And he said, it just works out that way. And then I backtracked and went to old bad habits. I went to show him the 2L, 2W, 2H thing just for the sake of time. |
| Scaffolding | I go all the way back to the beginning every time. Does the plan match the concept? Because if that’s not good, then we don’t even care about any of that other stuff. So, if the plan is good, then we check parts and pieces. Are you using the right parts and pieces? Right units of measurement? Consistent use of measurement? And I just build them from there. Teaching them the concepts they need to be successful at the projects, that's what takes the time. |
| Areas of Growth | Better Questioning | I have made an effort on my part to make sure that I'm asking—I'm not guiding. I'm asking questions to their questions. I never really answer a question. |
| Better Facilitation | . . . so, when I asked a question and he would give an answer to a group. I didn't then say, what do you guys think about it? Can somebody say it in a different sort of way? I didn't go there next. I was too quick to, okay, now what are you guys going to do now that you know that? . . . one of my things I want to do next year is create, intentionally, in my lesson plans, what I'm calling magic moments. I've got to have a magic moment, at least one think-pair-share question a day, and then see what that spawns . . . But to force myself to get more kid-to-kid interaction. |
| Getting Students to Think | Putting it back on them. Making them think about the problem. Trying it out on their own. Working through the frustration. Working with each other versus me up here dictating the conversation and being a drill sergeant. That's the biggest thing I still struggle with is throwing it back on the kids. I'm too quick to say, great [points to an imaginary student]. What do you guys think about that? [inaudible]. You're doing great. That’s great. |</p>
<table>
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<tr>
<th>Topic</th>
<th>Description</th>
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<tr>
<td>Shift to Instructor-as-Guide</td>
<td>So, I'm working at pulling back a little bit more, especially for those kids that are willing to put that kind of time and energy in. So, the thing of putting a student's work up on the document camera and saying, what do you think? Is there a mistake here? If there is a mistake, can you find it? And getting them in that kind of critique working with each other, listening to each other. Anything that we can do to get something, that's one of my building points, personally, is getting more student involvement with other students. What do you think about his idea? But I'm still directing it. It's still all from me. My hard part is you get those classes where you know students won't participate. And it's like pulling teeth to get someone to speak. Kids like to evaluate other kids' work. I don't think it's malicious. I just think it's more intriguing for them somehow and makes them think in a different way. So, I'm still messing with how to do that in the geometry class. For me, for especially, and I've tried, you know, that's been a big thing in Geometry anyways is Geometry is vocab. If they're not using it, it's hard to learn. And I can't force them to . . . spend any extra time on it than what they want to. So, the only thing I can do is in class we have to use vocab. And I fight that. I'm an efficiency guy. Let's get this done and not waste whatever. So, I have trouble holding back with my students, going back and saying—well, that's an interesting idea, but it's not going to any place. It's uncomfortable. When I'm giving up, that was only 10 and 1/2 minutes or something, but it was probably 20, 25 minutes of the hour by the time we got everything out . . . I'm giving up a lot of time, and I have no idea where it's going, if we're going to get anything out of it.</td>
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<tr>
<td>Increasing Student-to-Student Interaction</td>
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<tr>
<td>Adapting Methods to Practice</td>
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<tr>
<td>Discomfort with Time</td>
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<td>Expressing Ignorance</td>
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<tr>
<td>Lack of Ideas about Teaching</td>
<td>So, something that I'm going to trying next trimester with my basic kids is I'll teach a day, and then give them a whole in-class workday the next day just so I can bop around and have students helping students. Just start there because I don't really know what to do. Because that's the issue—they don't want to be there. So how could I change instruction for them that they would get more from it? I just don't know what that looks like. I could try what [Jeremy] does, but where we start next trimester with basic algebra two is rational operations. Because if they've never seen it before, I don't know if they would know what to do. I don't know if they would feel comfortable doing it in front of their peers . . . Because it's not something that we do enough of.</td>
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<tr>
<td>Knowing How Students React</td>
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So again, the deal is letting go of that control and knowing that they're getting done what they need to do, and do they have the tools— and that's my biggest reluctance to do it. I don't know if they have those tools.

Student Solution Methods
I go, I have never thought to do it that way. But you could definitely do it that way. And so, we went along with what he said, and it worked. And I'm like, good job. And I let him lead it while I wrote things down on the board. And I have no problem admitting to my students that I might not necessarily know the best way to answer a problem.

Modifying Tasks
So, take that existing kind of not-real-great activity, how would you fix that? Would you create those kinds of questions like, how many ways are there—find a different way to count the squares. You say, what formula? If I give you a bigger piece of graph paper, do you begin to take that core thing and add to it? Or do you completely redesign it?

Teaching Conceptually
I create the formula as opposed to just getting one. But that's tough to do, especially in geometry. How is a kid supposed to figure out the lateral area for a cone, that formula? I don't know if I've ever seen it taught where you derive that at a geometry level for sophomores. Maybe there's something out there. I don't know.

Structure and Planning
Maintaining Expectations
I think depending on the teacher, like, when my students come into my room, they know exactly what's expected of them, and what they can and cannot do as far as communicative and attitude. They know if they step over the line . . . [motions cracking a whip]. But we let it slide. We let it slide on a daily, sometimes minute-by-minute, basis. We don't say, no, here's our expectations of whatever we're doing. Here's the standard, guys, and we're not here. And if you do it when it's just a little off, it's a minor adjustment. It's like your car. When your car gets a little out of whack, you get it fixed. Don't wait for it to become this junk-heap going down the road. That's what we do as teachers. And then this junk-heap, it's a pile of junk. What happened?

Lack of Planning
I didn't have that kind of preparation for that lab either . . . Like ten minutes of preparedness. And again, that just goes to show that if you're going to do something with quality you have to have all these things ready to go too. Even in spite of the SMII training, even in spite of me knowing discussion, and having been in all your wonderful sessions, and seeing, hey, watch these videos, and the kid-to-kid interaction, how little of that I structure.

Planning for Discourse
I can put a block of time in my lesson plan for presenting the non-linears, and then 2 minutes of student response time, and then 3 minutes of guided practice. And I can block it out and know that I can get that in 10 minutes.
So that's one takeaway is that types of questions I need to ask and the types of structures that those questions come from needs to intentionally designed. You can't just have them willy-nilly, and they can't be just by accident. It's got to be designed.

Task Structure
But the way that that kid used the problem is equally as valid as hers, and somehow you have to incorporate that. That's why the task structure is so important.

Planning for Group Work
We brought strings and stuff. And in these things, if you're going to do it right, there are some underpinnings you have to make happen. If it's in bits and pieces, it's not going to work. And I think like a lot of things, we do bits and pieces, and then say it didn't work.

Lesson Planning as a Necessity
You can't do it all at once. I'd be crazy if I think I can. So, what's the focus in that lesson plan? Where am I going to focus my energies to make sure that part of the lesson is what it needs to be? The whole lesson itself may not be a great thing, but that part of the lesson I can plan for specifically. Is it going to be getting launched properly? Is it going to be, at the end, getting the kids to process the learning? Is it in the middle if they struggle? How am I going to know if they struggle?

| Grouping Structures | Purposes of Grouping | But a lot of times I want them working together because that’s real-world to me . . . I mean there’s not a job that I’ve done in the past that I haven’t worked with somebody and had to communicate or pass papers . . . There’s nothing that I’ve hardly, that I’ve ever done that I’ve done by myself. So, it facilitates work, conversation, if I can get them to use math vocabulary, the same.

| Purposeful Non-random Grouping | . . . it became more evident when I redid this project. I went to [a colleague’s] room, and we did it with the other geometry classes. And it was more relevant there, again, just because of population. That’s a really good class. And I separated my groups into really good students and then mixed them up. So, every table had at least one to two leaders in there to push the conversation. So, I was very, I guess, specific on how I set those groups up in particular.

| Teacher Expectations | I think we tend to think as teachers, we put them in a group, and they just automatically should know what to do. And then we get upset when they don’t.

| Flexibility in Grouping | Yeah, and then I’ll do rows of three. So that, but they’re ten per row . . . and then on occasion I’ll split that up so that it’s groups of five, in the three rows. I’ve done tables of four to six, smashed together.

| Mixed Group and Independent Work | . . . some of them are working independently. It totally depends on the kid. Some of them like to work independently, they don’t want any help . . . others . . . are working together in groups
Selection Methods
Sometimes it will be random selection done with numbers—random numbers or random selection. Sometimes it will be volunteers. I really don't like to do that very much, because you know how that works. It's the same six kids always putting their hands up, the ones who do the most, and they always have the right answer. Or the one kid who doesn't care if he's wrong, and he's wrong every single time he puts his hand up, but he still wants to volunteer.

Independent Work
But it's very independent and I am really just a resource where if they are unsure, they ask me. Or they'll, I've got one kid that struggles where for him, his first step is just to try to put together the concepts. Like what is the question even asking me to do? And so, he's just trying to put those concepts in the right order and wrap his head around the question. And then he'll ask whether he's headed down the path, you know, the right path.

Instructional Routines and Grouping
Choral response sometimes, especially after they've done something—sometimes we'll do the think-pair-share. I've done it a couple of times in a group. So, think-pair-share, talk your answer over now with the pair behind you. And you want to be sure there's some kind of consensus or a response.

Questioning Current Practices
So, we don’t graph on the number line first which is the x-axis. I mean that’s what I tell them it is. It’s the x-axis, so that they, some of the kids I’ve had in my plus class and then on the IXL stuff, they’ve seen that because they’ve had to graph those things and the open dots and the closed dots, so they know that’s included or not included.

Teacher’s Role
But that's why I say it's a lot more—I actually used the word "interactive." Then direct instruction, guided practice, individual stuff—that's the biggest change, I think, in what I've got than what I maybe had before.

Teacher Expectations
I think as teachers we have to realize everybody has different, our kids, we all have different gauges. And that's why the idea of norms, we minimize the importance of that. Here's what people are going to say and do, and here's what people are not going to say and do. For the sake of—and convince kids why that's—use the gauges we're all going to agree to look at. I know I don't do a good enough job of that.

Offering Alternatives
instead of me talking at them, they are talking to each other and they are asking me “are my ideas right?” You know, I’d rather answer that question or give them a guiding question towards something if they’re not right than just say “two plus two is four. Remember that.” So, in our first unit right now we’re identifying between those concepts, linear, quadratic, and exponential. Why aren’t we doing a bunch of labs and take our own results and decide what they are? Versus okay here’s a given list of data that I just randomly made up, now you tell me what it is with zero interest or zero input. So, I would like to see that.
| Comparing Instructional Approaches | I’m trying to push, more so because of our conversations, I’m trying to push experience. It’s experience, it’s not “I did eighteen assignments.” It’s you have x hours of experience. That’s going to show when it’s time to . . . and those, and I think my kids, for the most part, they’re getting that. I have not used my whiteboards for this kind of thing. I use them for funneling, basically. Everybody do this. What’s the next step? Hold them up. Everybody have 2? Okay. |
| Lack of Conceptual Teaching | But we just identified a lot of things that have a missing link in what we teach. We focus on the what and not the why. So, for kids it becomes a guessing game. But even the kids who experience success at a deeper level of understanding . . . this is a very superficial level of what we’re asking kids, for most kids. But I’m honest enough with myself to know that this is a bit of a departure from what I normally do. |
| What Should Teaching Look Like? | If that makes sense to you, help me understand it better. We don’t invest that time, for whatever reason. We just say, you’re not thinking how I want you to think. This is a better way to go. In the end, it needs to be, how are we going to get kids more dynamically involved in their own learning? |
| Questioning and Discourse | Funneling or Focusing? Especially with something like this project that is new, I feel like I was . . . very funneling. Part of the hard part for me is, like, if we want our kids to think this way, spending the time to expand on questions like this and knowing how to ask the questions so that we’re not funneling for them. And then, you know, at what point—like, how hard do you want to push them before everybody just gets frustrated and just gives up? |
| Lack of Discourse | Because I don’t want to—because when I first started teaching, I did some of that. It was very—okay, next group, what did you find? Show your results. And four kids come up here, and one kid talks, and three kids stand around kind of with their hands in their pockets. |
| Strategies for Increasing Discourse | I love when we do, we have flip cards, sometimes we do flip cards. And then they, so everybody’s gotta reply. And then eventually we get to the point where students are willing to share their work and their methods up in front of everybody else. We start out doing that with small groups. And then just talk about answers. And then we’ll get into sometimes methods. But there’s a fine line there. The tricky part to me is I don’t want kids to feel like I’m looking down on them necessarily. Because I don’t want to get to a place where kids aren’t willing to stick up their hand and ask a question. So, I just say, does anybody have an answer for that? |
| Asking Deeper Questions | I use Photomath just like that. Or I’m not even going to ask you what the answer is. I’m going to ask you, what would you do on step number 4? Why would you use whatever operations? What method are you going to use to solve this? A method isn’t going to answer that question. So, then we can get into the why, not the how. |
I just did it with this kid yesterday. I asked him the why questions—I’ve been working with him this whole month. Why do you think that that graph is going to be quadratic? And he went to his notes and said, because it goes up and down. That’s not really a why because you’re matching. I want to know why you think that’s going to be quadratic.

Facilitating Discourse

I think I would be more prone to do that next year now that I kind of know what that looks like from the practices that we’ve done. Just, hey, he said this. Can you reiterate that? Or just having the students answer the questions more and not trying to funnel my questioning.

For instance, I’ll go back to that kid’s conversation. There were many opportunities I had to say, yes, you got it, or isn't it really this? And that holding back and letting that kid work his way through that—that’s probably the biggest thing I’ve done is I’ve always been a first is answer these questions.

Increasing Student-to-Student Interaction

. . . we leave them alone for 10 minutes, and they’re somewhere else. So, you have to keep them there. And my way of doing it, for me, is the whiteboards. They get the whiteboards out and they know they’re going to be interacting with those things. So, as I’m thinking about this great discussion, how do I keep it everybody and not just Lana and Tom and Jennifer? The teacher does it. In my plus class, my algebra students helping students, that’s what I tell them. If you’re giving guidance, don’t answer. Question. Who’s doing the thinking? Are you doing the thinking or is the person?

Expressing Value for Discourse

I say my best lessons are when there’s always a lot of interaction and back-and-forth.

Self-evaluation of Questioning

Because I think I do a better job—I’ve always thought I’ve asked questions. I’m a good questioner. I don't know how it gets propagated, but I would bet most of us, if were to chart the questions that kids ask, a huge percent fall into the procedural. What's next? Or, I didn't what you did then.

Discourse Dynamics

. . . giving the students a chance to mess with the activity. And then I think part of the key is, along the way, is it going in the direction you want it to go in? Those carefully guided questions that, Okay, I can't give them the answer. I don't want to give them the answer. But if they’re experiencing too much frustration, or they’re going in a direction that's going to be counter-productive to where you've got to get them to, find some little nudge to get them thinking. So, to me, it’s almost like a conversation thing. Teacher to the students.
Study subjects also spent time discussing the idea of scaffolding student understanding. They shared how such scaffolding manifested in their practice, their reasons for providing such scaffolding, and the necessity of scaffolding in relation to complex tasks. Sometimes these discussions took the form of storytelling, as with the general and specific descriptions of practice, and other times took the form of justifications or explanations. The level of specificity of the stories and examples varied widely. Some examples were specifically tied to instances of classroom practice and particular content while others were more general in nature, describing an overall philosophy of, or approach to, scaffolding.

Domain of Practice Sub-code Groups

While the previous emergent codes were perhaps more general in scope, there were a set of emergent codes which provided significant detail and a number of sub-codes. The first of these codes emerged from a set of instances in which study subjects described areas in which they believed they needed to grow or were currently working to change their practice. Subjects indicated that they needed to become better questioners, by which they meant gaining facility with questioning techniques that more closely followed a focusing pattern and pushed students for meaning as opposed to simply pushing for answers and recall. This was a main focus of the SMII PD activities. Coupled with an interest in questioning, subjects also indicated that they needed to become better at facilitating discourse between students in small and whole group settings. A related sub-theme emerged as well, this one involving getting students to do more of the thinking in the classroom. Subjects oftentimes referred to this as “putting it back on the kids,” meaning pushing students to do the work of answering their own questions as opposed to waiting for an explanation from the teacher. As subjects considered how best to create student ownership of learning, they were forced to consider a shift in the role of the instructor. This shift was from a view of the instructor as the provider of knowledge to one of the instructor as a guide—a questioner first, and a dispenser of knowledge second. A more general sub-theme emerged, perhaps related to the previous sub-themes as well: subjects expressed struggle in promoting student-to-student interaction. They universally expressed value for those kinds of interactions, but also universally acknowledged that it was an area in which they needed to improve. The final two sub-themes involved subjects’ acknowledgements that there were things they were working to translate into their own classrooms (e.g., grouping methods and structures) and a general discomfort with the amount of time required for the new practices. Subjects indicated that they
were working to overcome negative feelings associated with time. Those negative feelings seemed to be associated with an uncertainty about the outcome of instructional interactions—that is, not knowing if students will gain any knowledge out of a given set of interactions during a lesson.

In addition to acknowledging areas of growth, subjects also spent time expressing ignorance. The fact that “expressing ignorance” emerged as a theme is important. Oftentimes, the change process is short circuited by a reluctance to admit that we don’t know things. This was not the case for the three teachers in this study. They admitted to a lack of ideas about how to teach differently, even after the PD experiences. Some acknowledged that change is possible, but that a lack of vision prevented such changes while others were at a loss as to how to proceed in particular areas (e.g., student motivation). Subjects also discussed how a lack of knowledge about how students would react to instructional changes created hesitancy to implement changes. Subjects worried that students would not respond well to the changes we were discussing and this worry made them reluctant to try anything new in their classrooms. Two sub-themes associated with tasks emerged from the Expressing Ignorance data: a lack of knowledge about how to modify tasks and a lack of knowledge about how students would approach particular mathematical problems. Given the curricular situation in which the subjects worked, it is only natural that some might wonder about how to modify tasks they have (or they find) to create opportunities for students to engage in mathematical sense-making and discourse. Further, participants wondered about how to respond when students produced a solution method that the teacher was unable to anticipate. Lastly, subjects spent time wondering how to teach conceptually for content that they traditionally thought of as abstract, such as formulas for volume or factoring quadratic expressions.

Structure and planning appeared as an emergent theme as well. Subjects discussed the necessity and difficulty in maintaining high expectations of students, both behaviorally and mathematically. Subjects also shared instances in which there was a lack of planning, discussing the difficulties inherent in responding to students “on the fly” with no resources to fall back on. Some subjects also noted how, even with the SMII PD experiences, some aspects of classroom practice remained unstructured, particularly with regard to discourse. Associated with the emergence of instances of lack of planning were comments about how teachers might go about planning for discourse in their classrooms. There were specific and general recommendations in
these discussions ranging from the need to design structures for discussion around questions to specific time intervals associated with the phases of a think-pair-share instructional routine. The SMII PD also appeared to give subjects an appreciation for the necessity of task structure. In these instances, subjects discussed how the nature of a task (e.g., low-floor, high-ceiling) impacted both the quality and quantity of discourse in the classroom. Subjects also acknowledged the need to plan for group work in particular and the integral role of lesson planning in general. Subjects worried that if group work was not planned for and structured properly, then it would fail and could then be dismissed as ineffective. When subjects discussed lesson planning, they oftentimes focused on one particular aspect of a lesson (e.g., the launch, explore, or summary phase), noting that a full plan for the entire process was beyond their capabilities given the constraints of their work environments.

Grouping structures were also a topic of extending conversations in the data. Subjects discussed reasons or purposes for grouping, ranging from concerns about mirroring the world outside of school to creating teams for the purposes of engagement with specific aspects of a project and increasing student engagement. A large sub-set of these conversations revolved around strategies and reasons for grouping students by ability or heterogeneously so that different ability levels were distributed in each group. These non-random grouping strategies were most often based on teachers’ perceptions of student ability rather than on data that supported a particular level of proficiency. Subjects discussed the pitfalls and failures in applying grouping structures in the classroom. Oftentimes, teachers neglected to spend time instructing students in how to work in a group, instead simply expecting that students would know. Some subjects described how they arranged their rooms so that they could change group structures flexibly based on need. Subjects indicated that they would oftentimes employ a mix of group and independent work during their lessons, with some lessons consisting of a good deal of independent work with the teacher circulating and helping individual students for extended periods of time. Lastly, study subjects also discussed methods they used to select students and form groups (if groups were to be random) and how they applied instructional routines to groups of students.

Study subjects also engaged in frequent and extended bouts of questioning their own current instructional practices. They questioned their perception of the teacher’s role in instruction, noting the differences between direct instruction and more student-centered
instruction. Further, subjects spoke at length about the effect that teacher expectations can have on classroom instruction. Subjects offered each other alternatives to sets of instructional moves and in response to instructional situations they observed in lesson study. Comparing instructional approaches was also common, with discussions ranging from differing philosophies about teaching to direct comparison of current practice with more reform-oriented approaches discussed as part of the SMII PD experiences. Also, subjects often lamented the lack of conceptual teaching in their current context, discussing, for example, how the focus of teaching tended to be procedural instead of conceptual. Lastly, subjects’ PD experiences led them to ask larger questions like “What should teaching look like?” This more philosophical set of discussions tended to focus on the group’s beliefs about the nature of teaching and learning and how the SMII PD experiences interacted with those beliefs.

Questioning and discourse was another emergent theme in the data. Subjects took up the vocabulary and conceptual framework of funneling and focusing questioning patterns readily and discussed them often, both in relation to their own practice and in relation to others’ practice. Subjects lamented the lack of discourse in their classrooms as well, noting their difficulties in promoting student mathematical talk. Oftentimes the latter sorts of comments led to a discussion of strategies for increasing student discourse in the classroom. The strategies were both general (e.g., use flip cards) and specific (e.g., suggesting a different question to ask when viewing classroom video). Subjects also discussed the need to ask deeper questions of students and also gave specific examples of when and how each had done so. Related to this, subjects also discussed how to facilitate discourse in the classroom, giving specific examples of instructional actions such as applications of talk moves. Repeatedly, subjects discussed the need for and value in increasing student-to-student interaction in their classrooms. They oftentimes lacked solid ideas as to how to proceed or wondered how to modify well-established routines (e.g., the use of individual white boards) to ensure more participation from more students. Subjects also engaged in self-evaluation of their own questioning techniques and discussed the varying dynamics of mathematical discourse in the classroom, noting such things as the necessity of monitoring students’ frustration levels to ensure that students were not pushed into an unproductive space.

Domain of Consequence Analysis

The emergent codes and sub-codes that I derived from the Domain of Consequence data are summarized in Table 16. Again, there are some emergent codes which have associated sub-
codes and some that do not. However, in this case it may be helpful to organize the emergent influences of the Domain of Consequence into three categories: empowering influences, neutral influences, and inhibiting influences.

Inhibiting Influences

Inhibiting influences are those that caused study subjects to struggle with implementation of new classroom instructional practices. Subjects noticed instances of students’ struggle with concepts and attempted to explain these as caused by the curriculum pacing and treatment of content. This struggle was perceived as a negative thing and contributed in some ways to subjects’ resistance to continuing to implement changes in their classrooms. Along with the conceptual difficulties experienced by students, subjects identified and expounded on various inherent student traits that caused them (the teachers) frustration when attempting to implement any changes in classroom practice. These inherent student traits were formed from subjects’ perceptions of students’ ability levels and work ethic. Subjects noted a lack of willingness on students’ parts to dig deep for understanding—many students simply wanted the procedures laid out for them so that they could practice and gain proficiency in producing correct answers. Throughout the conversations, subjects used ability labeling language to talk about students, noting which classes had “high” students and how classes with more “low” students were more difficult. “Low” and “reluctant” appeared to be used interchangeably during these conversations. Lastly, subjects noted at length that students lacked time management skills and discussed how that problem might be addressed.

Related to “reluctant learners” above, subjects also noted that a lack of student effort sometimes made their attempts to change difficult. These complaints were in most cases generic but point to a perception that a lack of effort was inherent to students and could not be influenced by changing instructional approaches.

The final and most complex of the inhibiting influences that emerged from the data was a set of subject observations about students’ difficulties and negative reactions to some of the changes in questioning and classroom practice. Subjects noted that students often expressed frustration when pushed for meaning or when subjects would answer a question with another question. This sub-theme is likely related to some of the influences from the External Domain (e.g., students’ expectations of what mathematics learning should be).
<table>
<thead>
<tr>
<th>Emergent Code</th>
<th>Sub-code</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Conceptual Difficulties</td>
<td></td>
<td>But we just identified a lot of the things that have a missing link in what we teach. We focus on the what and not the why. So, for kids it becomes a guessing game. The kids barely grab something before I move on. They really don't understand the connection between that and something else. We never give them time, build on those lessons. At some point in time, the lessons need to be differently structured. But as we're coming down to the end of the trimesters . . . some of the things that I thought kids should get by now, they ain't got. And it's because there's not enough slowing down, enough real deal process.</td>
</tr>
<tr>
<td>Inherent Student Traits</td>
<td>Students don’t want to dig deep for understanding</td>
<td>Again, maybe it's our culture, but there would be kids who, after all that stuff, as good as it is, would say . . . so what are we supposed to do? Why can't you just tell us what we're supposed to do to get the answer? Why does it have to be so complicated? I don’t think all of them feel that way. I think the vast majority does. Simply because at this level they're just like, give me the notes already. Let me just do it.</td>
</tr>
<tr>
<td>“Top” or “high” students</td>
<td></td>
<td>My geometry class is really strong right now. So those kids don't get lost too fast, which is good. But again, we're talking—same pre-calc, those top kids, for the most part, you point them in the direction, and they start going. And almost sometimes in spite of what we do, they find a way to get there.</td>
</tr>
<tr>
<td>“Reluctant” or “low” students</td>
<td></td>
<td>I think that's what sets up in kids' brains. If they aren't forced in some way, shape, or form to be involved, because the whole thing depends on their involvement, some of those reluctant learners will just sit back and say, not interested. So, I think some of it is tricking them into learning And they'll say the right kind of things and do the right kind of things. But the lower ability groups you have, the more there's terrorists in these little groups.</td>
</tr>
<tr>
<td>Lack of time management skills</td>
<td></td>
<td>So, for instance, freshman focus, that thing is to work on all week long. We start on Monday and okay. And even within a day, there are kids who can't manage 20 minutes of their own time constructively. They just can't at this point. That's who they are. As freshmen coming in, I know they get a lot better by the time they get to you. You almost have to walk around and say you’re gonna have to speed up or you won’t get problem three done. You almost have to micromanage their time because they can't or won't manage their own.</td>
</tr>
<tr>
<td>Lack of Student Effort</td>
<td>So, my basic algebra 2 students this trimester—lack of skill, no, the lack of will. They do it but—so I just graded my first hour exams. Probably 75% of my students failed the test. I get students who don’t want to try, they will just sit back quietly and not do anything and then when I give them the answers, they write things down. They don't do the homework.</td>
<td></td>
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<tr>
<td>-----------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Observed Student Difficulties and Negative Reactions</td>
<td>Student frustration</td>
<td>I push my students to understand, you know, the content and they get ticked off at me. I’m like do it again. No, do it again. Like, and I push them, and I push them in the way that I think they can handle yet not get to the point that they are about to beat me up. Because the number one question you get from kids who are frustrated, I don't even know how to start this problem. How many kids say that? Even my algebra two class, I asked them one time to go outside their comfort zone, and they really hated me.</td>
</tr>
<tr>
<td>Lack of student transfer of learning</td>
<td>So, it's like, wait a minute, we talked about this sort of. It should have been enough for them. You know what I'm saying? But they can't extend it because we haven't practiced that. And they came away with it . . . all the groups got side-side-side, but they could not figure out how to use an angle. It was like . . . pulling teeth And they couldn’t transfer from the physical, concrete thing to the algebra with it. Some kids could but very few, which was really surprising.</td>
<td></td>
</tr>
<tr>
<td>Cooperation instead of collaboration</td>
<td>Because typically what we do is worksheets with blanks, something like that. Well, the easy way, let the smart kid do them all or give him the easy two problems to do if he's got to have his thing. Because I've watched kids do that. If you let them divide into tasks, they'll say, how wants to do this? Then I'll do this. And four kids come up here, and one kid talks, and three kids stand around kind of with their hands in their pockets.</td>
<td></td>
</tr>
<tr>
<td>Gaming the system</td>
<td>And just sit here and hopefully it's not my name. And if he pulls my card, I'll just say, I don't understand this stuff. And even this, if I pull cards out—well, there's 20 of us in here. I've got a 1 out of 20 chance of pulling a card out of the deck with my name on it, so I'm not going to worry about this too much.</td>
<td></td>
</tr>
<tr>
<td>Curriculum Insights</td>
<td>It was . . . eye-opening to see how many kids had never used a tape measure or measured. The one thing that we found was they [students] weren't even using a scale factor at all. They were trying to find other pieces in the image. They were like measuring stuff and then trying to produce the scale factor based off of the image instead of using the original scale factor, manipulating an image, and making a comparison to other items in the pictures. And again, that all comes back to the way we worded the question and the tables that we thought were going to be helpful in going in and trying to guide them, but they weren't so much.</td>
<td></td>
</tr>
</tbody>
</table>
Table 16 - continued

| Subject Hypotheses about Student Motivation and Attitudes | Those kids who did it over and over again, you get really good procedure, but they don't get the concepts nearly as well as the kids who talk about it, why this doesn't work, all that kind of stuff. And so, at the high school there is a little bit more . . . social insecurity, don’t want to look bad, don’t want to be too conformative with my peers. |
| Success Stories about Collaboration and Discourse | Increased levels of student thinking But a lot of compare and contrast was with that, so they had to dig into it a little bit deeper than what the Skittle Project was. So, there was a lot of follow up questions. They actually had to produce the graphs and they could use those as artifacts for comparing and contrasting. Those kids were . . . they basically wrote a proof without knowing they were writing proofs. . . So, we spend way more time on the credit chapter . . . than I ever have before . . . I turned it into a three-week. It wasn't until I got to week two that I started getting insightful questions from kids. It dawned on me—I think sometimes we go at such a fast clip the kids haven't even processed it to a point where they know what kinds of questions to ask until they've had to monkey with it for a while. You know what I'm saying? And it was that third week where a lot of the pieces came together for kids and they started, oh, that's why this—and I got a lot of, “oh,” kind of things from kids. That's why your credit score will go down. Yeah! We go so fast, especially in math. |
| Student “light-bulb” moments | See, we know when we see an x-intercept it's 0, but when they see that 0 there, I have kids say, oh, look, there's a 0 there. And that's why they call it a 0. It's like the light clicks off. I had a student do a similar kind of thing this year that had just an a-ha moment, and she tied it back to last year. Now that makes a whole lot more sense why they were doing that last year. And it didn't click until now. I can't remember what it was. |
| Increased student engagement | And so, they just kind of went around the room and whoever wanted to pitch in an idea or concept or what they thought, they just did and there was discussion on that sometimes and sometimes it was “okay well I don’t think, to me that’s not the way I see absolute value.” So it was nice . . . the misconceptions were handled very well by the students that hour. it’s funny because we’ve got some kids that use three point one four, we’ve got some kids that use the pi key, and so there’s always been this fluctuation of answers to begin with. And they’ll talk about it. “Well I used pi, my answer’s going to be a little bit bigger, I have this” versus “I used three point one four so I’m like four fifty-eight you had four sixty-two, is that okay?” |
| Non-traditional participation patterns | So those kids, they’re the ones because they have that experience, they were the ones sharing that. And again, nice to see because those are the kids that normally struggle. The plus class. And now they had a little extra opportunity and they are sharing that with the other kids. So, I thought that was pretty cool, um, that they could, that they were the ones with the knowledge. |
I know I’ve got one girl that doesn’t particularly like to do a whole lot. But this is one area where she just wanted to speak up and it was kind of exactly like this. I did not see that coming from this person and it was nice to get her involved in some of the ideas. And it might just be because we’re in statistics right now and she might just . . . like statistics for whatever reason because it’s more, I guess, interactive if you will. But it was kind of nice. I was pleasantly surprised with that. But they’ve all seemed to like it.

Well it wasn’t like, they didn’t shoot each other down, like . . . They didn’t beat each other up because I thought absolute value was this or your thought it was that . . . It was [a] positive [experience]. But they ended up . . . between the group, everybody kind of putting stuff together, they ended up with a really good definition of absolute value on their own.

<table>
<thead>
<tr>
<th>Positive student experiences</th>
<th>Subjects’ Positive Perceptions of Students’ Reactions to Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I think the buy-in is a little bit more when they have to do it themselves.</td>
</tr>
<tr>
<td></td>
<td>I love it when students get frustrated. I don't know if that's a good thing or a bad thing. But when they actually have to do get out and figure it out on their own, I love it.</td>
</tr>
<tr>
<td></td>
<td>I think things will continue to change because that's just the natural progression of things, honestly. I'm seeing success in some of the things that I've changed.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subject Perceptions about Student Participation</th>
<th>There are still those [students] that . . . that’s the last thing they want to do is be put in the spotlight, so they tend to shy away from that. Now if it’s working in their groups, they are more prone to work at least in their little groups of four or five than they are to share with the entire class, yeah.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Student [to] student interaction, where if that’s done enough where you’ve redirected them so that they’re taking responsibility for being on task and on topic and they’re guiding the other person sitting next to them or within their group, that’s pretty awesome. Without me having to intervene at all. I love that.</td>
</tr>
</tbody>
</table>
Further, subjects noted that oftentimes students had difficulty in transferring experiences in one setting to another, more abstract, setting. Along with the frustration, subjects noted that students oftentimes engaged in cooperative activity instead of true collaboration. This was particularly pointed in examples of the explore and summary phases of a lesson. In these instances, students simply divided up the different parts of a task and combined their answers in the end or, when asked to share their results with the class, a group might stand in front of the class while only one of their number spoke about the results. Lastly, students who were used to more traditional teacher to student interactions would attempt to take advantage of those patterns of activity. Specifically, subjects expressed frustration with the fact that when a teacher asked the class a question, students would wait for certain students to be called on, knowing that the teacher traditionally called on those students first.

Neutral Influences

Two themes emerged from the data that I could not classify as either empowering or inhibiting. First, subjects noted students’ reactions to the curriculum design of specific activities, in particular the co-facilitated lesson that Jeremy and Terry devised. The two subjects noted that students responded to the design of the activity in very particular ways, many of which were unanticipated (e.g., students did not use the scale factor to derive results as Jeremy and Terry had envisioned). These insights did not appear to dismay either Terry or Jeremy. Instead, the two simply made note of them and proceeded to consider how they would modify the activity when they implemented it again the following year.

The second neutral influence involved subjects hypothesizing about students’ motivation and attitudes toward learning. These hypotheses were often part of discussions that were philosophical in nature and attempted to explain certain behaviors that subjects noted in students (e.g., students’ lack of willingness to take academic risks in the mathematics classroom or students’ unwillingness to attempt an answer to a teacher’s question). I classified these instances as neutral because of the hypothetical nature of the commentary and the fact that subjects appeared to consider these hypotheses neither a positive nor negative factor in their discussions, or appeared to consider them both positive and negative influences at different times and under differing circumstances.
Empowering Influences

Three major empowering themes emerged from the data. First, subjects were quite willing to share success stories with each other. These stories involved instances where teachers had successfully engaged students in collaborative activities or successfully facilitated classroom discourse. Some of these stories were related to classroom activity that the group studied as part of the lesson study component of the SMII PD and others were of attempts subjects made to independently implement student-centered discourse practices in their classrooms. In particular, subjects made note of increased levels of student thinking resulting from these efforts, oftentimes expressing surprise at what students were able to do. Further, subjects celebrated moments when students experienced conceptual breakthroughs, so-called “light-bulb” moments or “ah-ha” moments. Increased efforts to center student thinking and discourse in the classroom also resulted in increased student engagement, as noted by study subjects. Related to this, subjects also noted that some students who typically chose not to actively participate in the classroom began to do so when more student-centered instructional practices were attempted. Lastly, subjects noted general patterns of positive experiences for students associated with the “new” classroom practices.

Another empowering influence was subjects’ general positive perception of students’ reactions to the instructional changes. For example, some subjects noted that they felt good when students were frustrated, likely because this meant that they (the subjects) were successfully implementing the intended instructional practice. Further, subjects hypothesized about student engagement increasing when mathematical authority was shared among the teacher and students.

Lastly, subjects discussed their perceptions of students’ participation in classroom activities that were more student-centered than subjects’ traditional practice. They noted that while there were still some students who chose not to participate, they still saw increased participation from more students and that this participation sometimes depended on the size of the group in question. For example, some students still refused to participate in whole class discussions but were quite willing to engage in small group conversation and work.

Changes in Reflective Processes: Reflective Activity Analysis

To gain some insight into how subjects’ reflective activity changed over the course of the SMII PD, I leveraged the segmented PD session video. I coded each segment or portion of a
segment in Dedoose using the a priori coding scheme devised by Stockero (2008). Table 17 summarizes the a priori coding scheme. The main focus of the reflective analysis was changes in subjects’ levels of reflection over time. The remaining attributes of reflection serve to provide context to findings focused on levels of reflection.

Table 17
A Framework for Reflection

<table>
<thead>
<tr>
<th>Attribute of Reflection</th>
<th>Codes within attributes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent</td>
<td>Teacher</td>
<td>Comment focused on teacher actions</td>
</tr>
<tr>
<td></td>
<td>Student</td>
<td>Comment focused on student actions</td>
</tr>
<tr>
<td>Topic</td>
<td>Pedagogical issue</td>
<td>Comment focused on pedagogical issues</td>
</tr>
<tr>
<td></td>
<td>Student thinking</td>
<td>Comment focused on student thinking</td>
</tr>
<tr>
<td>Grounding</td>
<td>Grounded</td>
<td>Comment supported by evidence from video or readings</td>
</tr>
<tr>
<td></td>
<td>Ungrounded</td>
<td>Comment not supported by evidence</td>
</tr>
<tr>
<td>Level</td>
<td>Describing</td>
<td>Storytelling</td>
</tr>
<tr>
<td></td>
<td>Explaining</td>
<td>Connecting interrelated events/exploring cause and effect</td>
</tr>
<tr>
<td></td>
<td>Theorizing</td>
<td>An explanation of the warrants for knowledge</td>
</tr>
<tr>
<td></td>
<td>Confronting</td>
<td>Searching for alternative theories to explain events and actions</td>
</tr>
<tr>
<td></td>
<td>Restructuring</td>
<td>Considering how to re-organize teaching actions</td>
</tr>
</tbody>
</table>


Patterns of Reflective Activity by Attribute

To gain an understanding of how subjects’ reflective habits changed over time, I employed Dedoose to track the number of code applications by PD session. These results appear in Tables 18 and 19. There are some general and specific trends in the data which are worthy of note. First, generally speaking, subjects spent more time talking about teacher actions than student actions (see Table 18). In every PD session, there were more instances of the teacher as agent in a comment than students. However, with the beginning of the lesson study portion of the PD—sessions five and six—came an increase in the levels of commentary focused on both teacher and students. Sessions seven and eight, on the other hand, had levels similar to sessions one and two. The increase observed in sessions 5 and 6 is likely due to an increase in the number of opportunities to reflect provided by the lesson study format.

Trends similar to those seen in the Agent attribute exist within the Topics of subjects’ reflections. Generally, subjects commented on issues of pedagogy with much greater frequency
than student thinking. Once again, with the initiation of lesson study came a corresponding increase in the number of reflections in both categories. These increases were most likely due to the affordances of the lesson study format. This format created many more opportunities for reflection on practice than the initial four days simply because those initial sessions were focused on changing knowledge and beliefs about teaching through experiences with mathematics and with research. The lesson study format focused in much greater detail on classroom practice.

Table 18

<table>
<thead>
<tr>
<th>PD Session</th>
<th>Agent</th>
<th>Topic</th>
<th>Grounding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Student</td>
<td>Teacher</td>
<td>Pedagogical Issue</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>3*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>23</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>8†</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

* Video data from SMII PD days 3 and 4 was lost due to technical difficulties
† Video data from SMII PD day 8 was incomplete due to technical difficulties.

Subjects began the SMII PD with ungrounded reflections, which was only natural considering the lack of experience, research, or evidence with which to ground those reflections. However, grounded reflections increased sharply during PD sessions five and six, coinciding with the beginning of the lesson study format. Accompanying these sharp increases was a shallow decrease in the number of ungrounded reflections. Again, these trends can be explained by the structure of the PD sessions. Lesson study is a PD format fruitful with opportunities for reflection on practice, particularly in contrast to the knowledge and belief building focus of the initial four sessions.

PD session 7 is an interesting case. Instances of reflection were fewer in number during this session, while the ratio of codes assigned to each option remained relatively stable. This was likely due to two factors. First, there was inconsistent participation in the video reflections during the PD sessions. Session 7 was one session where only two participants brought new classroom video to share with the group. This resulted in a decrease in the number of opportunities for reflection in general. Second, my facilitation of the group discussions was by no means perfect and/or perfectly consistent. It is likely that due to the lower number of new instances of
classroom practice available during session 7, I made facilitation decisions that lowered the number of opportunities for reflection.

Related to the issues with PD session 7 were issues with PD session 8. These issues were two-fold: a lack of usable data from session 8 and a differing focus for that session. Due to technical difficulties the majority of the video data from PD session 8 was unusable. This obviously decreased the number of observed instances of reflection. Second, the focus of the final PD session was for subjects to summarize what they had learned about effectively facilitating the explore phase of a mathematics lesson. This was a fundamental shift away from the study of specific classroom activity into a more general summary reflective event. Therefore, it is likely that even if the data had not been lost there may have been a sharp decline in the number of reflective instances coded for that PD session.

Table 19 details the distribution of the Level of Reflection codes by PD session. Similar trends to those identified in the attributes of Agent, Topic, and Grounding exist within the attribute of Level as well. Generally, the total number of instances of reflection increased beginning with PD session 5. This was true across all levels of reflection. Subjects engaged in explanation more often than any other kind of reflective activity, followed by description. This might be expected given the lesson study format of the PD days in question. Part of the protocol for lesson study is an introduction to the video example about to be studied. This doubtless contributed to the larger incidence of description and explanation observed.

Table 19

<table>
<thead>
<tr>
<th>Code Occurrence by PD Session for Levels of Reflection</th>
<th>Describing</th>
<th>Explaining</th>
<th>Theorizing</th>
<th>Confronting</th>
<th>Restructuring</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD Session</td>
<td>1</td>
<td>2</td>
<td>3*</td>
<td>4*</td>
<td>5*</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>3</td>
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<tr>
<td>2</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>2</td>
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<td>3*</td>
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<tr>
<td>4*</td>
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<tr>
<td>5</td>
<td>7</td>
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<tr>
<td>7</td>
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<td>0</td>
<td>6</td>
</tr>
<tr>
<td>8†</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

* Video data from SMII PD days 3 and 4 was lost due to technical difficulties
† Video data from SMII PD day 8 was incomplete due to technical difficulties.

Study subjects engaged in Theorizing and Confronting less often than reflection at other levels. Reflection at the level of Restructuring occurred most often during the lesson study portions of the PD. Further, instances of Restructuring remained relatively stable across the
lesson study PD sessions. This may imply an advantage of the lesson study format in creating and sustaining these opportunities.

Patterns in Levels of Reflective Activity by Study Subject

It is also useful to examine the distribution of instances of levels of reflection among study subjects. Table 20 lays out the aggregate Level of Reflection data in this way. Jeremy and Terry engaged in a larger number of instances of reflection than Rebecca over the course of the SMII PD program. Interestingly, Rebecca engaged in more description of classroom practice than either Jeremy or Terry. She engaged in significantly less restructuring activity than both Jeremy and Terry. Jeremy engaged in more explanatory activity than either Rebecca or Terry.

Table 20
Code Occurrence by Study Subject for Levels of Reflection

<table>
<thead>
<tr>
<th>Subject</th>
<th>Describing</th>
<th>Explaining</th>
<th>Theorizing</th>
<th>Confronting</th>
<th>Restructuring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeremy</td>
<td>13</td>
<td>23</td>
<td>0</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Rebecca</td>
<td>19</td>
<td>11</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Terry</td>
<td>7</td>
<td>11</td>
<td>6</td>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>

The aggregate data leaves many questions unanswered. In particular, Table 20 provides no information about how levels of reflection changed over time. To achieve this, Tables 21, 22, and 23 provide a breakdown of the aggregate data by study subject, level of reflection, and PD day. In this way, one can examine an individual subject’s reflective activity during a single PD session and can examine instances of that subject’s reflective activity, by level, across multiple PD sessions. The latter manner of viewing the data provides the most insight into changes in reflective activity over time.

Jeremy

During the first PD session, Jeremy did not engage in any reflective activity at any level. During subsequent PD sessions, Jeremy did begin to engage in reflective activity at different levels. Beginning in session five, Jeremy engaged in significantly more reflective activity, likely because session five was the first of the lesson study-focused sessions and Jeremy submitted two video clips for group analysis that day. In this instance, each of Jeremy’s video clips required significant contextualization for the other participants. This explains the elevated description and explanation statistics. Session six in particular saw Jeremy engage in the largest number of instances of restructuring, in addition to a large number of description and explanation instances. This was likely because the lesson study for that day was focused on Jeremy’s own teaching.
practice; other days were focused on others’ practice. Overall, Jeremy engaged in explaining more often than other forms of reflection, particularly during the first two lesson study sessions.

Table 21

<table>
<thead>
<tr>
<th>PD Session</th>
<th>Describing</th>
<th>Explaining</th>
<th>Confronting</th>
<th>Theorizing</th>
<th>Restructuring</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
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<td>1</td>
</tr>
<tr>
<td>3*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<td>5</td>
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<td>1</td>
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<td>6</td>
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<td>8</td>
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<td>0</td>
<td>7</td>
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<tr>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8†</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

* Video data from SMII PD days 3 and 4 was lost due to technical difficulties
† Video data from SMII PD day 8 was incomplete due to technical difficulties.

Rebecca

Rebecca engaged in reflection during nearly every SMII PD session. The exception appears to be session eight, but due to a lack of data this appearance may be deceptive. Generally, Rebecca engaged more in lower levels of reflection than higher levels. Sessions five and six show the greatest diversity in Rebecca’s reflective activity. Session six in particular showed an increased level of reflective activity, likely due to the fact that the lesson study activity on that day focused on Rebecca’s practice more than others’.

Table 22

<table>
<thead>
<tr>
<th>PD Session</th>
<th>Describing</th>
<th>Explaining</th>
<th>Confronting</th>
<th>Theorizing</th>
<th>Restructuring</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
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<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4*</td>
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<td>-</td>
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<td>1</td>
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<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
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<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>1</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
</tbody>
</table>

* Video data from SMII PD days 3 and 4 was lost due to technical difficulties
† Video data from SMII PD day 8 was incomplete due to technical difficulties.

Terry

Terry engaged in reflective activity consistently throughout the PD sessions he attended (he did not attend session six). Of the participants, Terry’s instances of reflection were more evenly distributed across different levels. Terry’s restructuring activity was highest during sessions five and seven. This was likely true because Terry’s practice was under intense study
during those sessions. Of all the participants, Terry engaged in the most restructuring during the early PD sessions.

Table 23
Terry - Instances of Reflective Activity by Level of Reflection and PD Session

<table>
<thead>
<tr>
<th>PD Session</th>
<th>Describing</th>
<th>Explaining</th>
<th>Confronting</th>
<th>Theorizing</th>
<th>Restructuring</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
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<td>1</td>
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<td>3*</td>
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<td>4*</td>
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</tr>
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<td>7</td>
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</tr>
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<td>8†</td>
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<td>0</td>
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<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

* Video data from SMII PD days 3 and 4 was lost due to technical difficulties
† Video data from SMII PD day 8 was incomplete due to technical difficulties.

Summary

This study examined subjects’ change processes from several different angles and presented those results in several, interrelated ways. The main presentation of the data occurred through the development of the individual case study reports in Chapter 4. These provide the most detailed data about subjects’ PD experiences (obtained through semi-structured interviews) and also triangulation of findings using data sources such as classroom observation and PD session transcripts. Further analysis of these data sources resulted in cross-case analyses, IMPTG domain coding analyses, and a reflective process analysis, all presented here in Chapter 5.

The cross-case analysis process provided insight into the kinds of change sequences and growth networks experienced by study subjects. The analysis produced two types of change pathways (whether change sequences or growth networks): classical change pathways and alternative change pathways. Classical change pathways—those beginning in the External Domain and proceeding through the Personal Domain and into the Domain of Practice (and possibly continuing onward)—were the most common type of change pathway experienced by study subjects. Alternative change pathways—those beginning with patterns different than the classical pathways—were present also, but in lesser numbers. Further, analysis of the change pathway data revealed patterns in the focus of those change pathways. The change pathways experienced by subjects in this study fell into four categories: pathways focused on questioning, facilitation strategies, grouping students, and student thinking.
The IMPTG domain analysis consisted of a two-phase semi-open coding analysis involving the four domains of the IMPTG as priori coding categories (phase one) and emergent codes and sub-codes within each domain (phase two). This analysis provided some insight into the influences from each domain of the IMPTG on subjects’ change processes. Additionally, the IMTPG served well as an organizing framework for the deeper, qualitative analysis that followed, just as in other studies (e.g., Justi and van Driel (2006) and Witterholt and colleagues (2012)). Generally speaking, subjects experienced change influenced heavily by the External Domain in the form of a set of empowering influences and inhibiting influences. Influences from the Personal Domain included sets of teacher beliefs and concerns associated with such things as mathematics, classroom practice, and teacher identity. The analysis of the Domain of Practice produced sets of emergent codes describing the classroom experiences of study subjects, both in terms of everyday experience and in terms of professional experimentation associated with the SMII PD intervention. Lastly, the analysis of the Domain of Consequence codes produced sets of empowering, inhibiting, and neutral influences—that is, influences which supported the changes subjects were trying to make in their classrooms, those that did not support efforts to make those changes, and influences that had the potential to do both (or neither).

Data concerning changes in subjects’ reflective processes over time revealed several patterns. The analysis considered the aggregate number of instances of reflection, by level and by PD session. Further, an analysis of each individual subject’s reflective activity, by level and by PD session, was also reported. Generally speaking, subjects’ reflective activity increased during the lesson study-focused PD sessions. Subjects tended to focus more on teacher activity and pedagogical questions than on student activity and student thinking. Further, subjects’ commentary tended to be grounded more often than ungrounded, particularly during lesson study. Subjects engaged in describing and explaining during reflection more than other levels of reflection. However, instances of restructuring rose sharply during lesson study.

The case study reports in Chapter 4 provide a detailed account of subjects’ experiences, in their own words and from their own classrooms, across the SMII PD intervention. The data in Chapter 4, coupled with the results of the three subsequent analyses provided in Chapter 5, create a highly detailed, analytical account of Jeremy, Rebecca, and Terry’s experiences over the course of the SMII PD. A detailed interpretation of these results and a contextualization in the field of research on teacher PD and teacher change follows in Chapter 6.
CHAPTER 6

DISCUSSION

Introduction

This chapter contains a detailed discussion and interpretation of the results of this study organized around several large-scale, structural features. I begin with a discussion of how the results of my analysis answered the research questions and how the different data sources within this study related to those answers. Following that is a general interpretation and discussion of the results presented in Chapter 5. I will also attempt to situate the results of my study within the broader landscape of the teacher change and mathematics professional development literature. I will discuss the limitations of this study, followed by the implications of my work for PD providers, and close with a short discussion of future directions for research based on my results.

Answering the Research Questions

Ultimately, the goal of any study is to, as fully as practicable, answer the research questions that guided the research. As such, no discussion or interpretation of the results of a study is complete without explicit attention to the research questions. In the case of this study, the guiding research questions were:

1. What do teacher change processes associated with the SMII PD intervention look like?
2. What factors influence teachers’ change processes as they engage in the SMII PD intervention?
3. How does teacher reflection change during the SMII PD intervention?

Research Question 1

The theoretical framework for this study is the lens through which the answers to the research questions can be effectively viewed. Due to this fact, any descriptive results obtained in answer to the first research question were heavily shaped by the nature of the theoretical framework. Therefore, a general answer to the question of what teacher change processes looked like during the SMII PD is as follows:

Based on the data collected from the subjects in this study, these three high school mathematics teachers experienced change in multiple areas through repeated
engagement in PD experiences (both individual and collective) and professional experimentation in their own classrooms.

Put more simply, in this study teachers changed by engaging in and reflecting on the work of teaching, supported by collaborative PD experiences and their own beliefs and inhibited by external factors and their own beliefs. The case study reports lay out extended narratives consistent with this description of the change process.

Beyond the case study reports, the cross-case analysis revealed the presence of classical and alternative change pathways for each participant. The existence of these two distinct types of change pathways points to a range of different experiences for the study subjects. Classical change pathways involved the changing of beliefs, knowledge, or attitudes before changing classroom instructional practices. In many ways, this seems like a natural progression. After all, if it is expected that a teacher implement a new instructional practice in the classroom, then it is reasonable to assume that some knowledge of the new instructional practice (its existence, at a minimum) is requisite to its implementation. However, knowledge is not enough to ensure that a new practice is taken up in the classroom, particularly regarding the complex interplay between knowledge and beliefs (cf. Phillip, 2007). It is natural to assume that a teacher that has beliefs consistent with the potential new practice would be more likely to implement it well. In many cases, this is exactly how subjects’ change processes unfolded.

In other cases, however, there was not a significant change in knowledge or beliefs before new practices made their way into the classroom. The alternative change pathways involved, in many cases, changing practice prior to changing beliefs or attitudes. These pathways are in some ways counter-intuitive. Their existence shows that it is possible to build knowledge and change beliefs by direct experimentation in the classroom based on experiences in a PD setting. A teacher engaged in change in this way may not be sure about the efficacy or feasibility of an instructional practice but might still be willing to attempt to implement it in her classroom. From this initial implementation, reflection on how it felt to teach in that way or how students responded to the experience is possible. It is this reflective activity—a result of enaction based on external experience—that forms the basis for the alternative change process.

These two types of change processes might have been theorized from an examination of the IMTPG. However, this study derived them from data about the experiences of real teachers in real classrooms. The fact that these change pathways are possible and do, in fact, actually
occur, implies that the teacher change process is indeed more complex than any linear model can account for. Further, the IMTPG was a useful tool in visualizing and extracting subjects’ change processes from the data. In a way, the identification of these two types of change patterns is an expansion of Clarke and Hollingsworth’s (2002) model.

Returning to focus on answering the first research question, the combination of the case study reports and the cross-case analysis provided a great deal of detail about the look (and feel) of subjects’ change processes. Each subject experienced both classical and alternative change pathways, but pathways that followed a similar—or even identical—pattern were unique to the individual in terms of content and focus. This points to a likelihood that, as previous research has indicated, PD interventions affect different people in different ways (Goldsmith, Doerr, & Lewis, 2014). While this is true, that is not to say there are not connections between the foci of individual change patterns and the foci of a PD intervention. That appeared to be the case here. And so it appears that it is possible for PD to influence teachers’ change processes even if they take very different pathways through the change process.

Research Question 2

As the SMII PD proceeded, it became clear that numerous influences on subjects’ change processes were evident. This was unsurprising, as the study design had anticipated that, as had the PD. The domains of the IMTPG allowed for a convenient categorization influences. The emergent coding phase provided much richer and more explanatory detail. Based on those results (presented in Tables 18, 19, 20, and 21), an answer to the second research question follows:

Teachers’ change processes were empowered and inhibited by multiple influences from each domain of the IMTPG. In particular, inhibiting influences from the External Domain were prominent for the three teachers in this study. Further, change processes were empowered by SMII PD experiences and collegial collaboration on the study of teaching and learning.

While the statements above answer the research question in very general terms, the IMTPG Domain analysis provided a great deal more detail about the influences on teachers change processes in this study.

External Domain

It appears that, generally, any changes the subjects in this study experienced were influenced by elements of the External and Personal Domains. This analysis revealed External
Domain influences (see Table 13) on teacher change processes that inhibited those processes (e.g., the school schedule, pacing directives, educator evaluation concerns, etc.). Conversely, empowering influences also emerged from the External Domain data (e.g., video experiences, collaborative problem solving, influences from other staff members, etc.). These somewhat conflicting results paint a complex picture of the general nature of the teacher change processes uncovered in this study.

Personal Domain

The complexity of the change process becomes more apparent as we layer on additional domain analysis results. The Personal Domain analysis (see Table 14) also revealed complexities, some in sync with those of the External Domain and some in conflict. The data from the interviews and PD sessions revealed much about subjects’ beliefs. In particular, the analysis revealed categories or sets of beliefs which impacted subjects’ change processes in different ways. Subjects’ beliefs about students were a particularly influential factor. These beliefs were, at least initially, grounded in a deficit view of students—that is, the beliefs were focused on what subjects believed that students could not or would not do. Through engagement in professional experimentation and reflection on aspects of the Domain of Consequence (detailed in a later section), subjects’ beliefs began to shift as each realized students’ capabilities and how students interacted with new instructional techniques.

Subjects’ beliefs about learning also impacted the change process. Over the course of the SMII PD, subjects came to believe that student thinking and student discourse are central aspects of learning mathematics. For example, Rebecca’s initial set of beliefs about tips and tricks and demonstration before engagement inhibited her change process because these beliefs caused a good deal of skepticism about the ideas presented in the PD. Rebecca had to overcome this skepticism before she could even experiment professionally with some of those ideas. Further, while subjects may have believed in ideas such as “student thinking is central to learning,” those beliefs did not show in the current practice described by any of the subjects in the first interview. However, by the end of the school year, elements consistent with this belief had appeared in subjects’ practice, even if only on an intermittent basis.

Related to subjects’ beliefs about learning were their beliefs about the efficacy of their current practice. Terry and Rebecca in particular held the belief that their current (at the outset of the PD) practice benefitted a majority of students. While Jeremy did not hold as strongly to this
belief, his practice was consistent with it nonetheless, simply because of influences from the External Domain (e.g., pressure from colleagues and curriculum influences). This belief created a certain inertia in the practice of both Terry and Rebecca, but Rebecca in particular. Terry was able to overcome this inertia, mostly due to his previous experiences and because of his reflective nature. Rebecca, on the other hand, had a difficult time overcoming this belief. Rebecca repeatedly claimed that while no overt changes in her practice had occurred, she was changing the way she was thinking about her teaching. This shortened change pathway, beginning in the External Domain and terminating in the Personal Domain was common throughout Rebecca’s case study report. Terry also had many instances of such a shortened change pathway.

Domain of Practice

Themes emergent in the Domain of Practice data (see Table 15) add another layer of complexity to the change processes experienced by subjects in this study. The Domain of Practice generated the largest number of both coding instances and themes during coding analysis, evidence of its importance to the change processes of study subjects. While some of this preponderance is due to the nature of the instrumentation used (each interview asked about current practice), the number of themes is more telling than simply the number of coding instances (although this was the largest as well).

The emergent theme of areas of growth gives some insight into the change processes experienced by study subjects. Their experiences in the SMII PD and with professional experimentation in their own classrooms led them to conclude that they needed to grow in particular areas of practice. The nature of those growth areas is particularly telling. That subjects self-identified areas of growth such as better questioning and better facilitation indicated that their conceptions of the nature of teaching mathematics were changed (the case study reports support this supposition). The new conceptions of teaching were formed on the central concepts of questioning and facilitation as opposed to more traditional delivery or transmission paradigms. Professional experimentation and reflection (both in and, presumably, out of PD) had led them to identify these growth areas.

Part of the change process involved an identification of areas in which participants lacked knowledge and experience. For the three subjects, there were instances where it was necessary and helpful for them to admit that they did not know. Sometimes these instances formed
explanations for why the subject had not shifted practice in ways he or she thought were “required” by the PD. However, despite the fact that expressing ignorance often manifested as excuses, it provided vital information for me as a facilitator about which messages and experiences from the PD had been effective. For example, when Rebecca noted that she didn’t know “what different look[ed] like,” it told me that she had not internalized some of the critical experiences focused on instructional moves and providing vision in the PD. This was due in part to her absence for one and a half of the initial four PD sessions, but she had had multiple opportunities to engage with what “different” might look like. This formative facilitation data was vital in helping me plan further experiences and focus the work of the group as we moved forward.

The fact that subjects expressed ignorance about such things as a lack of ideas about teaching, student solution methods, and teaching conceptually, all of which were dealt with heavily within the PD, point to a reality of the change process for these subjects: knowledge builds over time and sometimes unequally between specific areas. Some of the code segments within the expressing ignorance theme were from early in the PD sequence and some were from later. This supports the supposition that knowledge and beliefs change over time with exposure to multiple experiences because the PD was designed to provide experiences in each of the areas subjects mentioned from the outset. Over time, the PD experiences were designed to reinforce and elaborate on the initial messages and experiences. This would seem to imply either that some experiences were more effective than others at engendering change or, more likely, that it took multiple experiences to initiate the change process in subjects.

Subjects also expressed ignorance about modifying tasks to be more cognitively demanding and knowing how students would react to new instructional practices. These two areas were telling in that they point to two different knowledge spaces. Ignorance about how student would react to instructional changes manifested in a similar fashion to knowing what different looked like, as an explanation for not engaging in professional experimentation. It is possible that this reticence was rooted in subjects’ identities—perhaps they did not see themselves as capable of effectively managing student reactions they were not prepared for. The modifying tasks knowledge space is related to the curriculum influence from the External Domain. Subjects’ change process was inhibited because they did not have supportive curriculum materials and they did not know how to alter their curriculum materials to engage in
more student-centered instruction. I did use this information to alter the focus of several of the planning activities and discussions during the PD. I wanted to support subjects in using their materials more effectively during those times when they had not had time to find or develop a new task. Generally speaking, I tried to facilitate the PD in a way that was as flexible and responsive to participants’ needs as possible while still maintaining a focus on the overall goals set out at the beginning.

The emergent theme of structure and planning offers additional insight into the complexity of the change process. Subjects experienced difficulties in dealing with the complex nature of the mathematics classroom and reported struggling with such things as maintaining expectations and a lack of planning time. These struggles, along with the results of reflection on professional experimentation, appeared to lead some subjects to question beliefs and, in some cases, change those beliefs. Specifically, subjects came to realize the importance of planning in the teaching process. Prior to the PD experiences, subjects’ need for planning was minimal due to the highly structured format of the curriculum materials—most of the planning, such as the choice of examples to demonstrate, was done for them already. During their PD experiences subjects came to realize that the complexity of using tasks with multiple solution pathways required more extensive planning. Further, subjects also came to realize that planning for discourse in the classroom and planning for group work were both vital considerations in any mathematics lesson based on a rich task (which also needed to be well-structured). This shift in beliefs was promising as the SMII PD was designed to bring about such changes and to support teachers in implementing those changes (e.g., through the use of the TTLP, discourse supports, research readings, etc.).

Related to the structural considerations in the previous emergent theme, subjects also spent considerable time discussing and reflecting on classroom grouping structures. The emergent codes within this theme ran the gamut from the philosophical (e.g., the purposes of grouping students) to the practical (e.g., how does one effectively select students for a group?). While two of the participants, Terry and Jeremy, reported using grouping structures in their classrooms previously, many of the emergent sub-codes appeared to develop spontaneously during the course of the PD sessions. This spontaneous development often appeared to coincide with cycles of reflection and enaction during professional experimentation. Many of the philosophical conversations occurred prior to attempts at implementation, although some did
continue afterward. Conversely, the more practical considerations emerged as subjects collectively and individually reflected on efforts to plan for and use grouping structures in their classrooms. If one considers the entire list of sub-codes for the grouping structures theme, it might be reasonable to conclude that the list consists of nothing more than one might expect from a group of teachers learning a new skill.

**Domain of Consequence**

The emergent themes within the Domain of Consequence (see Table 16) formed a more compact list than the other domains, but no less informative. Generally speaking, the data within the Domain of Consequence codes consisted of stories about the results of professional experimentation in the Domain of Practice. This linked the Domain of Practice and the Domain of Consequence data closely together as many of the coding instances from each domain appeared close to one another as different parts of the same teaching story told by a study subject.

Subjects’ stories of teaching represent episodes of professional experimentation and the resultant reactions of both teachers and students, filtered through the subjects’ lenses of experience and beliefs. In chapter 5, I noted inhibiting, neutral, and positive influences from the Domain of Consequence data. It is worth noting that several of the themes derived from the data have connections to subject beliefs and conceptions of teaching. For example, the assignment of traits inherent (in subjects’ views) to students was closely related to beliefs about students in the Personal Domain. Similarly, subjects’ hypotheses about student motivation and attitudes is connected to subjects’ beliefs about students and associated doubts about students’ abilities. This connection between the Personal Domain and the Domain of Consequence has implications for researchers and PD providers. These will be discussed in a later section.

Sharing success stories appeared to be an important aspect of the teacher change process as well. Participants were enthusiastic in their sharing of success stories, describing in detail student reactions and interactions as well as useful and effective strategies employed to achieve that success. Terry in particular seemed to gain motivation from seeing what he called “light bulb moments.” The more instances of student conceptual breakthroughs he noted in association with new teaching practices, the more motivated he became to continue implementing those practices. Jeremy was also motivated by similar moments of conceptual breakthrough in students; however, he also gained motivation from noticing different, non-traditional participation patterns
in his classroom when he implemented new practices. Jeremy was very aware that some students who traditionally chose not to participate in his classroom were choosing to do so under the new instructional conditions. While Rebecca also shared in storytelling, her stories were oftentimes more general than either Jeremy’s or Terry’s. Her details tended to be general statements about whole group performance or her feelings about the class period under discussion. When Rebecca did share specific stories, she always shared instances in her practice where she responded differently to an individual student as opposed to a group of students. This was consistent with Rebecca’s limited range of change processes and her indication that she was not changing a great deal in her practice. Rebecca’s contributions oftentimes focused on her doubts about her students’ abilities to respond to new teaching practices.

The themes in the Domain of Consequence served as catalysts for change in subjects’ beliefs and practices. Even inhibiting influences such as subjects’ observed student difficulties and negative reactions served as sources of information about how to move forward. Some change pathways doubtless moved from the Domain of Consequence into the Personal Domain, resulting in shifting or reinforcing subject beliefs. Other pathways, though, may have moved back into the Domain of Practice, informing further efforts at professional experimentation.

The Role of Professional Experimentation

In the IMTPG, the Domain of Practice is the locus of professional experimentation. However, the Personal Domain and the External Domain are connected to this domain through enactive links. Hence the theoretical model (Clarke & Hollingsworth, 2002) of the teacher change process posits that enaction from other domains into the Domain of Practice is a central driver of that process. The data in this study would seem to confirm that theoretical connection. Enactment and professional experimentation played central roles in each of the subject’s change processes. Indeed, one might argue that the more professional experimentation a subject engaged in, the deeper and more extensive his or her change process. This pattern is borne out in the cross-case analysis data—Jeremy and Terry had much richer professional experimentation experiences during the SMII PD than Rebecca. The classroom observation data (at least for Terry) and interview data indicate the truth of this, particularly Rebecca’s continued insistence that her practice did not change substantially. Jeremy and Terry sought each other out to create an opportunity to collaborate and participate fully in the PD activities. Rebecca did not seek out such professional collaboration. This was likely due to her teaching assignment (recall that
Rebecca taught Algebra 2, trigonometry, and Pre-calculus, which neither Jeremy nor Terry taught). All the same, when Terry’s teaching assignment made it difficult for him to participate in the PD activities, he sought Jeremy out and the two collaborated to give Terry an opportunity to engage in professional experimentation.

The centrality of professional experimentation to the teacher change process in this study is also demonstrated by the robustness of the Domain of Practice data. This domain had the largest number of coding instances overall and the major themes developed out of those data appear as the result of efforts at professional experimentation in many cases. Consider emergent themes such as areas of growth, structure and planning, grouping structures, and even reversion to “bad habits.” The data in each of these emergent codes (and any sub-codes) all derive from subjects’ reports of professional experimentation. These codes would seem to indicate that professional experimentation and enaction are central to the Domain of Practice and the change process generally. In short, the results of this study are consistent with Clarke and Hollingsworth’s (2002) conceptualization of the teacher change process, with respect to mathematics and the three study subjects. Without the addition of the mediating process of enaction, the explanatory and interrogatory power of the Interconnected Model of Teacher Professional Growth [IMTPG] (Clarke & Hollingsworth, 2002) would be much more limited.

Research Question 3

Reflective activity is an assumed, integral mechanism within the teacher change framework of this study (Clarke & Hollingsworth, 2002). Reflective links exist between multiple domains of the IMTPG. In particular, some connections between domains are only reflective (e.g., the connection between the Domain of Consequence and the Personal Domain). This dependence heightens the importance of reflective activity as part of the change process. The data reveal a general answer to the third research question, which concerned how subjects’ reflective activity changed over the course of the SMII PD.

The three high school mathematics teachers in this study engaged in varying levels of reflective activity during most of the PD sessions. Subjects engaged in higher levels of reflective activity (in this case restructuring) during engagement in lesson study activity over the course of the final four PD sessions. Subjects also tended to engage in more descriptive and explanatory activity during these episodes, particularly when a subject’s own teaching was under scrutiny.
The analysis of subjects’ reflective activity revealed that while each engaged in some form of reflection during nearly every PD session, only the later, lesson study-focused PD sessions included higher level reflective activity. This might be true for several reasons. First, there were significantly more opportunities to engage in reflective activity during lesson study. Second, participants engaged in sustained analysis of classroom video in excess of one hour during each of those sessions, allowing subjects to gain comfort and confidence in their ability to reflect effectively on artifacts of practice. Further, subjects’ analysis of classroom video was facilitated toward restructuring activity during each of these PD sessions. Lastly, subjects spent considerable time critically analyzing their own practice during these sessions.

That reflective activity in general, and restructuring activity in particular, were concentrated in the final four PD sessions was a function of the format of the PD itself and, to a lesser extent, the facilitation of those sessions and the collective reflection process itself. Prompted reflective activity was common during these sessions, but this prompting was as often from the participants themselves as from the facilitator. Lesson study is a form of PD almost exclusively devoted to reflective analysis of teaching practice, whether it be one’s own practice or that of a colleague in the process of developing an ever more effective lesson or teaching technique. Given this focus and form of PD, it is only natural that more reflective activity should appear during sessions with this format than in sessions without.

Also, the process of analyzing video of practice involved structures and routines that encouraged descriptive and explanatory activity. These structures and routines were mostly focused on contextualizing a given video for other participants. When a PD participant provided a video for the group’s analysis, he or she would first provide an explanation to the group of the context of the video clip we were all about to watch and analyze. Participants would talk about the course the video was taken from (e.g., Geometry), the mathematical focus of the lesson, and any lesson activity that occurred prior to the beginning of the video clip. Further, some participants chose (or were prompted) to contextualize the clip further by describing how the lesson continued after the group had watched it. This necessary contextualization activity likely led to increased instances of descriptive and explanatory activity during the final four PD sessions.

I wish to avoid the appearance of an implicit assumption in my theoretical framework and so I must point out that while there is an increased value placed on restructuring activity in my
work, I am by no means advocating that activity such as description and explanation is useless or unnecessary. On the contrary, reflection by storytelling was an incredibly common occurrence during the PD sessions and interviews in this study. Stockero (2008) similarly found that descriptive activity failed to disappear completely, even after significant intervention through video-based curriculum materials. Stockero considered the question of whether it is possible, or even desirable, for reflective practitioners to avoid description and explanation altogether when analyzing practice. This study supports the claim that it may not. In this study, subjects reported learning things from these stories and descriptions such as new techniques and possibilities. General and specific descriptions of practice were major emergent themes in the Domain of Practice analysis. This indicates that storytelling and describing are natural parts of the change process and so have their own value, which in some ways is equal to that of higher-level reflective activity such as restructuring. While restructuring is a reflective activity with great potential to result in changing practice, particularly if engaged in on a regular basis, descriptions and storytelling are experiences that can form a basis for more effective restructuring activity.

Consider the nature of restructuring: examining an event or events with the intention of changing the teaching actions one undertook during that event. The intention is not enough, in the end. For true change to happen, intention must be carried through to action—or at least hypothesis, as most reflective activity is “reflection on action” as opposed to “reflection in action” (Schön, 1983). But where do the ideas for those actions or hypotheses come from? As Rebecca noted, and is likely true for many practitioners, “I don’t know what different looks like.” A more common aphorism might be “You can’t know what you don’t know.” So, if this lack of knowledge or vision exists for a given practitioner, then how might we expect him or her to effectively engage in restructuring activity? The storytelling and describing that happen as part of descriptive and explanatory activity can, in effect, seed practitioners’ minds with different ideas that become grist for restructuring activity.

This “seeding” effect was apparent at many different points during this study. The External Domain data showed this clearly: influences from other practitioners within the building was an emergent theme in the data. Further, each subject noted that he or she valued the time to collaborate with colleagues because they could see how others might approach a given situation, among other reasons. This gradual, collective accumulation of knowledge and experience was the engine which drove the reflective activity of the teacher change process during this study.
The Role of Reflection

Just as with professional experimentation and enaction, reflection played a central role in the change processes of the subjects of this study. However, unlike professional experimentation and enaction, reflective activity is much harder to measure. The theoretical framework of the study provided some insight into the existence of reflective activity, but ultimately, the majority of reflective activity undertaken by study subjects was not measured as it appeared nowhere in the data. Consider that the reflective activity measured in this study was associated only with reflection during PD activity. Reflection during teaching—reflection in action (Schön, 1983)—was something that the methodology and instruments employed in this study were unable to measure. However, the instrumentation did collect data on reflection on action (Schön, 1983) in the form of reflective activity captured during PD sessions.

Due largely to the lesson study format of the PD, study subjects reflected not only on their own teaching but on others’ teaching as well. It is this fact that lends importance to the descriptive and explanatory levels of reflection in the theoretical framework. When reflecting on one’s own practice, there is less need for describing and explaining the teaching activity—one is already familiar with the context. However, in order to reflect on someone else’s practice, one needs a primer on the context of the instance of teaching one is about to view and reflect on. This difference between reflection on one’s own teaching and reflection on another’s teaching helped drive the need for and importance of all levels of reflection examined in this study.

Subjects’ reflective activity in this study focused in two main areas: reflection on SMII PD experiences and reflection on professional experimentation. There were exceptions to this generalization, to be sure. However, the locus of the majority of reflective activity points not only to the central role of reflection in subjects’ change processes, but also to the close connection between the two mediating processes in the IMTPG. Reflection on professional experimentation led in two possible directions, as defined by the model: back into the Personal Domain, affecting beliefs and knowledge or into the Domain of Consequence, allowing subjects to look for and notice the fruits of their efforts at enaction. Further, recall that subjects tended to focus more on teacher activity and pedagogical questions than on student activity and student thinking. This finding may indicate a stronger connection between the professional experimentation and subjects’ beliefs about teaching as opposed to their beliefs about students. While reflection focused more on student thinking is preferable in many cases, that is not to say
that reflection on teacher activity without its uses. Indeed, if restructuring activity is a vital aspect of the change process, possibly indicating deeper changes through deeper reflection, then perhaps a focus on teacher activity is necessary. After all, it does not seem possible to consider how to restructure classroom events to achieve different outcomes—i.e., the definition of restructuring—without some element of focus on teacher activity.

Further, the fact that subjects tended to reflect at higher levels when analyzing examples of teaching practice, both their own and others’, points to the effectiveness of the lesson study format at creating opportunities for reflection in general and high-level reflection in particular. The restructuring activity that occurred during the lesson study sessions provided a great deal of learning and experience for subjects, leading to changes in beliefs and practice. Recall the example of Rebecca, who noted several times that she would never have thought to make particular instructional moves if she had not seen them enacted by another. Also, Jeremy, who brought a lesson he considered to be “bad” to the group and left with a realization that his efforts at maintaining cognitive demand in students were, in fact, quite the opposite of “bad.” Examples such as these abound in the case study reports in Chapter 4. Some are dramatic, others are not. Few are as dramatic as Jeremy’s revelation, but that is not to be lamented. Reflection does not appear to need drama to be vital to the change process. Rather, a steady stream of more modest reflections (at all levels) on practice were effective at supporting change in the participants of this study.

Considering Themes Across Domains.

In answering the research questions at both the general and specific levels, themes began to emerge. The analysis and interpretation of the data forced a kind of synthesis of the data and analyses of individual IMTPG domains. Patterns began to emerge across the various phases of analysis in this study. These patterns might be explanatory and might help support the arguments presented in answer to the research questions. I will refer to the patterns which emerged across domain themes as super-themes, indicating the fact that they contain emergent codes from the study data (which are themes in and of themselves). Ultimately, the purpose of this study was to gain a better understanding of the change processes associated with a particular PD intervention. These so-called super-themes provide some of that insight.
Storytelling.

Episodes of storytelling punctuated each of the PD sessions. Participants shared teaching experiences on a frequent basis. On the surface, this activity at first seemed inconsequential. However, during the analysis of the reflective activity data it became increasingly clear that the storytelling served at least three different purposes: contextualization, vicarious experience, and evidence of success.

Recall that subjects often shared stories to contextualize experiences for those that did not have first-hand knowledge of events. At several points, this contextualization allowed subjects to offer alternative instructional moves for consideration by the group. In short, this contextualization storytelling allowed subjects to engage in restructuring reflective activity associated with teaching episodes that were not their own. Storytelling also served as a fertile bed of ideas and vicarious experience for participants. These stories became, for study subjects, a set of ideas and experiences they could draw upon to conceptualize teaching in a different way. Hearing the stories of others’ attempts at instruction created a set of ideas that tended to stick with subjects over time. They would often refer to stories in their interviews, attributing the associated teaching idea to the person who told the story. While there was no way to predict which stories would stay in the minds of particular subjects, each of them had some subset of the group’s stories to draw upon when considering how to restructure instructional episodes and in creating plans for the future.

Lastly, and related to the second use of storytelling, subjects’ stories of teaching served to show both that success was possible (from stories in the Domain of Consequence) and that there were certain common pitfalls to be avoided in the process of implementation. This effect of storytelling served to focus subjects’ thinking and conversations on specific ideas from the SMII PD. This supposition is borne out by the presence of stories as emergent codes in both the Domain of Practice and the Domain of Consequence as well as by the nature of the emergent codes in the Domain of Practice data (e.g., structure and planning, grouping structures, etc.—which were foci of the SMII PD).

The prevalence of storytelling in the data indicate that it is a phenomenon intimately connected to the change processes of the subjects of this study. The fact that
storytelling appeared in multiple domains and served multiple purposes may indicate that it is an important, supportive part of the change process generally. In the case of lesson study-based PD specifically, such storytelling may a necessary component of the teacher change process. Storytelling certainly played a vital role for the subjects in this study.

Beliefs.

Specifically, the results of this study indicate that beliefs are central to the change process because of how they act and interact with the mediating processes of reflection and enaction. Subject beliefs acted as a filter or lens through which they could interpret experiences of all types (SMII PD experiences, professional experimentation, results in the Domain of Consequence, etc.), consistent with the definitions of belief from Borg (2001) and Phillip (2007). Two sets of beliefs acted in tandem to influence subjects’ reactions to information during this study: beliefs about teaching and beliefs about students.

The beliefs super-theme crossed the domains in a particular way. Beliefs arose prominently in the Personal Domain, as was to be expected from the content of this domain—in particular, the two belief sets of concern for this theme are subjects’ beliefs about students and beliefs about learning. In many ways the sub-codes under each of these emergent belief sets were contradictory. Subjects professed beliefs in the importance of student thinking and discourse to learning and beliefs consistent with the idea that students must learn to be problem-solvers. However, this belief set was balanced against doubts about students’ abilities and a permeating belief that some students are “high” while others are “low” and associated expectations of each of those artificial groups. These associations are, perhaps, unsurprising given that prior work has found that subjects oftentimes hold conflicting belief sets (Green, 1971; Phillip, 2007) concurrently. However, these beliefs appeared in and appeared to influence the content of other domain data as well.

Teacher beliefs about teaching.

The beliefs about teaching super-theme is actually a combination of belief sets from the data, including beliefs about representations and tools, beliefs and knowledge of mathematics, belief in the need for structure, and beliefs about learning. A case can be made for the inclusion of beliefs about students within this theme as well; however, I have chosen not to include it simply because subject beliefs about students emerged as a theme across domains in its own right. This separation increases clarity and amplifies explanatory power.
The initial set of beliefs about learning manifested in the Domain of Practice through emergent codes such as activation of students’ prior knowledge, scaffolding, and expressing ignorance about teaching conceptually. In these areas, subjects’ beliefs about learning can be traced into the individual codes within each emergent code (e.g., activation of prior knowledge is vital because students cannot attack new tasks without it, low students need more scaffolding than high students, etc.). Ultimately, the beliefs of study subjects, to a greater or lesser degree based on the person in question, influenced how the ideas about mathematics instruction presented in the SMII PD were taken up (if at all). Subjects who held teaching beliefs (particularly those based on past experience) that were in direct contradiction to ideas in the SMII PD were less likely to take up those ideas in professional experimentation. The supportive setting of the PD was intended to mitigate some of this challenge, but the level of success of that mitigation is difficult to determine. These difficulties point to a need to understand teacher belief systems prior to engaging them in PD experiences. That way those experiences might be specifically tailored to chip away at or overcome reluctance or resistance based on particular beliefs about teaching.

Teacher beliefs about students.

The second beliefs super-theme focused on subjects’ beliefs about students. The Domain of Practice data saw beliefs about students manifest when subjects expressed ignorance of how students would react to novel (for them) instructional techniques and tasks as well as within the sub-codes for grouping structures (e.g., avoiding random grouping to ensure that students would work nicely together, limiting the use of group work because some students “just can’t handle it,” etc.) and structure and planning (e.g., planning for discourse and planning for group work are necessary to ensure that some students do not misbehave or other students will not be able to avoid participating). One might argue that it is only natural for beliefs to influence the Domain of Practice (cf. Ernest (1989)), and that contention has been supported in research (e.g., Skott (2001), as cited in Phillip (2007)). The latter contention is undoubtedly debatable—however, the fact that subjects’ beliefs about students appeared in much less subtle ways in the Domain of Consequence creates the conditions for a more interesting discussion, particularly with regard to students’ mathematical potential or abilities. For example, an emergent code dealing with “high” and “low” students appeared prominently in the Domain of Consequence data. The fact that this code appeared with a similar form and language as in the Personal Domain points to the power
this set of beliefs held for subjects. More specifically, subjects commented on inherent student
traits, labeling students again as “high” or “top” students and “low” or “reluctant learners.”
These labels formed a lens through which subjects viewed student actions, interactions, and
learning. This lens colored subjects’ thoughts about those students and influenced subjects’
interpretations of those students’ actions and thoughts. Further, sub-codes such as gaming the
system indicated a somewhat cynical belief set concerning students and their motivations.

In reality, the super-theme of beliefs about students is a theme of belief in inherent
mathematical ability. For these subjects, some students are born “better” at mathematics, while
other students will inevitably “struggle” or will be “reluctant learners” because of their lesser
mathematical ability. Further, subjects’ belief sets concerning the nature of learning was, in
reality, about learning in ways consistent with those advocated for in the SMII PD. This was
evident in the nature of subjects’ concerns about students’ responses and in the various ways in
which subjects expressed those concerns (e.g., “I’m afraid that students will refuse to [engage in
the task].”). In this study, subjects’ professed beliefs were not always consistent with how those
beliefs were enacted in the classroom. For example, Rebecca indicated that she understood that
students learn better when they could engage in discourse about mathematics. However, her
practice remained very teacher-centered all through the PD intervention. Her belief in the value
of student discourse was not sufficient to overcome her concerns about classroom control and
structure. That teachers can hold contradictory beliefs is not a new idea (cf. Green (1971), nor is
the idea that teacher beliefs influence teaching practice (cf. Ernest (1989) and Beswick (2005)).
Studies have examined the connection between teacher beliefs and teaching practice (Bobis,
Way, Anderson, & Martin, 2016; Cross, 2009; Swan, 2007) and noted that the connection
between the two, while complex, is virtually undeniable.

Discussion

In examining the data from this study and the super-themes across the domain
data, there are several conclusions that can be drawn (at least for the three subjects of this
study). The teacher change processes in this study were influenced by the context in
which the teachers worked. Further, the teacher change processes observed in this study
appeared to be influenced by the beliefs that those teachers held about various aspects of
teaching and learning. Lastly, the teacher change processes in this study were driven by
cycles of reflection and enaction (professional experimentation).
Contextual factors in subjects’ professional lives featured prominently in the External Domain data for this study. Besides the influences of the SMII PD, which were to be expected, subjects’ expressed concerns and stories about how their context negatively influenced their practice and their attempts to take new practices from the PD into their classrooms. The two main influences for the subjects of this study appeared to be systemic structural constraints and curriculum constraints. A secondary, but more long-term influence appeared to be administrator and educator evaluation. These influences prevented teachers from readily taking up the ideas in the SMII PD (as in the case of the curriculum constraints). Further, some external influences (such as the administrator and educator evaluation influences) inhibited changes in subjects’ beliefs. This distinction is mostly due to the physical, temporal, and psychological differences between the two sets of influences. For example, study subjects may have held beliefs about teaching that were not manifest in their teaching due to a perception that there “was not enough time.” However, subjects were also reluctant to take up the PD ideas because of how their administrator would react under educator evaluation conditions. The latter was a mainly psychological reaction to previous experiences with the administrator in question, while the former was a reaction to a knowledge of the curriculum and scheduling constraints of the school system. The varying ways in which subject context can influence change processes adds yet another layer of complexity to a process already fraught with myriad, often conflicting, influences from multiple domains.

Subjects’ beliefs about students and about teaching appeared to influence their change processes as well. This aspect of the study findings was explicated in detail above so I will not belabor the point here. However, it is worth mentioning that the foundational nature of beliefs can be “circumvented” at least to a degree through engaging teachers in alternative change pathways. While classical change pathways involve a direct pathway through the Personal Domain, alternative change pathways offer a way to (temporarily) circumvent beliefs and thereby gain experience that might change those beliefs either through direct reflection or through a reflection loop containing the Domain of Consequence. These pathways also promote the possibility of experiences that create dissonance in those beliefs. This finding has important implications for the design and enactment of PD programs in mathematics education, which will be discussed below.
Finally, the mediating processes of reflection and enaction in the IMTPG were the driving force behind the teacher change processes of study subjects. The IMTPG is more than a collection of domains. Without the two mediating processes, the model would be much less powerful in tracking and explaining the change process. More than this, though, was the fact that continued engagement in cycles of reflection and professional experimentation was what made teacher change possible, at least for the subjects of this study. The features of the PD that encouraged subjects to engage in cycles of reflection and enaction (even those not captured by the study metrics) offer tantalizing examples of useful methods for changing teacher beliefs, practice, and knowledge.

Relationships between this Study and Major Segments of Literature

Studying something as complex as the teacher change process involved bringing together many different segments of mathematics education research literature. In my literature review I went into some detail on each of the following topics: frameworks for teacher change, teacher reflection, effective mathematics professional development, the use of video to examine practice, and the lesson study form of professional development. As beliefs became increasingly central to the results, I also began a preliminary literature review in that area. While the main “findings” of this study align strictly with the research questions, there are many consistencies between this study and those topics covered in my literature reviews. In this section, I will elaborate on a selection of those consistencies in an effort to situate and contextualize this study within a larger body of research. Table 24 lists several areas of consistency and connection between this study and the larger field of mathematics education research. The table also makes note of where this study is consistent with literature and where it adds to the knowledge base in particular areas.

This study is consistent with other studies that have used the IMTPG as a theoretical framework for investigating teacher change. The IMTPG played a vital role in allowing me to map the change pathways that subjects moved through. This usefulness is consistent across multiple studies, both within mathematics education (Perry & Boylan, 2018; Rubel & Stachelek, 2018; Widjaja, Vale, Groves, & Doig, 2017; Wilkie, 2019; Witterholt, Goedhart, Suhre, & van Streun, 2012) and outside of mathematics education (Eilks & Markic, 2011; Justi & Van Driel, 2005; Justi & van Driel, 2006). Each of these studies used the IMTPG in a fashion very similar
to that employed in this study and each was successful in deriving unique, useful results from their analysis.

Table 24

Connections Between Study Findings and Other Mathematics Education Research

<table>
<thead>
<tr>
<th>Findings</th>
<th>Previous Research</th>
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<tbody>
<tr>
<td>The IMTPG is a useful analysis tool in investigating and mapping teacher change processes</td>
<td>Adds to and consistent with: Use of IMTPG in science education (Eilks &amp; Markic, 2011; Justi &amp; Van Driel, 2005; Justi &amp; van Driel, 2006) and mathematics education (Perry &amp; Boylan, 2018; Rubel &amp; Stachelek, 2018; Widjaja, Vale, Groves, &amp; Doig, 2017; Wilkie, 2019; Witterholt, Goedhart, Suhre, &amp; van Streun, 2012)</td>
</tr>
<tr>
<td>Reflection plays a vital role in the teacher change process.</td>
<td>Adds to and consistent with: Teacher change literature (Boylan, 2010; Chapman &amp; Heater, 2010; Clarke &amp; Hollingsworth, 2002; Hollingsworth &amp; Clarke, 2017), teacher reflection literature (Manouchehri, 2002; Rodgers, 2002; Schön, 1983; Stockero, 2008)</td>
</tr>
<tr>
<td>Professional experimentation (enaction) plays a vital role in the teacher change process.</td>
<td>Adds to: Uses of the IMTPG as an interrogatory tool in mathematics education (Clarke &amp; Hollingsworth, 2002; Justi &amp; van Driel, 2006; Witterholt, Goedhart, Suhre, &amp; van Streun, 2012), uses of the IMTPG as a mapping tool (Justi &amp; van Driel, 2006; Witterholt, Goedhart, Suhre, &amp; van Streun, 2012) Consistent with: Use of IMTPG in science education (Eilks &amp; Markic, 2011; Justi &amp; Van Driel, 2005; Justi &amp; van Driel, 2006; Witterholt, Goedhart, Suhre, &amp; van Streun, 2012)</td>
</tr>
<tr>
<td>Subjects’ change processes are influenced by external, contextual factors.</td>
<td>Adds to and consistent with: Teachers’ change processes are influenced by various institutional factors (Wilkie, 2019)</td>
</tr>
<tr>
<td>Subject beliefs influence the individual change processes.</td>
<td>Adds to: There is a relationship between teacher beliefs, PD, and teaching practice (Cooney, Shealy, &amp; Arvold, 1998; de Vries, Jansen, &amp; van de Grift, 2013) Consistent with: Teacher beliefs influence classroom practice (Bobis, Way, Anderson, &amp; Martin, 2016; Cross, 2009; Swan, 2007)</td>
</tr>
<tr>
<td>Lesson Study was an effective PD format for initiating reflective activity and teacher change.</td>
<td>Adds to and is consistent with: Lesson study is a useful format for effective PD (Lewis, Perry, &amp; Hurd, 2009; Stigler &amp; Hiebert, 2004; Stigler &amp; Hiebert, 2016; Takahashi &amp; McDougal, Collaborative Lesson Research: Maximizing the Impact of Lesson Study, 2016; Warwick, Vrikkii, Vermunt, Mercer, &amp; van Halem, 2016; Widjaja, Vale, Groves, &amp; Doig, 2017; Willems &amp; van den Bossche, 2019)</td>
</tr>
<tr>
<td>Video of classroom instruction was an effective tool for initiating reflective activity and teacher change.</td>
<td>Adds to and is consistent with: Video is a useful tool in examining teaching practice (Grant &amp; Kline, 2010; Hollingsworth &amp; Clarke, 2017; Sherin &amp; van Es, Using video to support teachers’ ability to notice classroom interactions, 2005; Stockero, 2008; van Es &amp; Sherin, 2008)</td>
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</table>
The idea that reflection plays a role in the teacher change process was also made clear in the results of this study. Schön’s (1983) seminal work exploring the reflective practitioner is, of course, a primary source of the understanding of reflection’s role in expertise. Further, reflection is one of the mediating processes in the IMTPG (Clarke & Hollingsworth, 2002) and so this study is obviously consistent with any literature employing the IMTPG as a tool. Beyond these consistencies, other case studies of teacher change exist and feature reflection prominently (e.g., Boylan (2010), Chapman and Heater (2010)).

The results of this study also made clear the importance of professional experimentation to the teacher change process. Clarke and Hollingsworth (2002) call this “enaction” and indicate that the Domain of Practice in the IMTPG is the domain of professional experimentation. Related to this, each of the studies that has used the IMTPG as a tool (for example, those mentioned above) has also taken serious note of the role of professional experimentation. Any attempt to make use of the IMTPG as a mapping tool, as Witterholt and colleagues (2012) did, must inevitably result in accounting for and making note of professional experimentation and enaction. This study was no different.

The fact that the subjects in this study experienced change as influenced by external factors other than the PD intervention is interesting with regard to consistency with large bodies of research. While this type of influence is assumed to exist as it is part of the IMTPG (i.e., the connections between the External Domain and the Personal Domain and Domain of Practice), much of the teacher change and professional development literature does not take up this issue in detail. Two exceptions are worthy of note. First, the seminal text on the features of effective professional development for mathematics and science teachers by Loucks-Horsely, Stiles, Mundry, Love, and Hewson (2009) does make note of context as an input into the planning process. The authors acknowledge that, in order for PD to be effective, an understanding of participants’ contexts is vital. Second, Wilkie’s (2019) study involved the use of the IMTPG and an additional framework which included dimensionalities that accounted for external influences in the teacher change process. Wilkie’s study is similar to this study in that an additional framework was used in conjunction with the IMTPG to focus on a particular aspect of the teacher change process.

Interestingly, while the Personal Domain of the IMTPG did include beliefs, dispositions, and attitudes, it was not until the analysis phase of this project that the scale of the importance of
teacher beliefs to the change process became clear—not the existence of a relationship between beliefs and practice, to be sure; but rather the degree to which beliefs influence teacher change processes. This finding—that teacher beliefs influence practice (Bobis, Way, Anderson, & Martin, 2016; Cross, 2009; Swan, 2007) and the uptake of ideas shared in PD (Cooney, Shealy, & Arvold, 1998; de Vries, Jansen, & van de Grift, 2013)—was certainly an intuitively agreeable one; however, it is also consistent with a wide range of teacher belief literature.

Lesson study was a prominent feature of this project, particularly in the second half of the PD sessions. The fact that subjects’ reflective activity was heightened in both frequency and level during the lesson study sessions is largely consistent with the idea of lesson study as an effective format for teacher PD. There is a vast body of research around the effectiveness of lesson study (Lewis, Perry, & Hurd, 2009; Stigler & Hiebert, 2004; Stigler & Hiebert, 2016; Takahashi & McDougal, Collaborative Lesson Research: Maximizing the Impact of Lesson Study, 2016; Warwick, Vrikk, Vermunt, Mercer, & van Halem, 2016; Widjaja, Vale, Groves, & Doig, 2017; Willems & van den Bossche, 2019) and my study contributes to this body by showing that even a modified lesson study format can be effective at encouraging teacher change.

Finally, this study is consistent with the body of literature focusing on the usefulness of video of classroom instruction as a tool in PD. Examples of studies consistent with this finding are van Es and Sherin (2008), Sherin and van Es (2005), Grant and Klein (2010), and Hollingsworth and Clarke (2017). In each of these studies, professional growth in some area was fostered by collective analysis of classroom video. This consistency is synergistic with the use of lesson study as well. The modified lesson study process employed in this study used video recordings of classroom practice as opposed to the more traditional observation of a research lesson. The combination of these two methods of producing teacher change felt productive from the outset and the results of this study would seem to indicate that this was the case.

Contributions to the Knowledge of the Field

This study contributed to the knowledge base of mathematics education research in more significant ways than discussed above. Specifically, this study:

- Increases the number of case studies of mathematics teachers’ change processes
- Contributes a novel combination of the IMTPG and a framework for studying reflective activity
• Provides a supplementary way to consider change pathways within the IMTPG
• Provides a PD format that has the potential to foster mathematics teacher change.

While in the previous section I outlined consistencies between this study and other research, here I make the case that this study can also be distinguished from those mentioned above. I will treat each of my contentions in more detail below.

One might argue that there can never be too many case studies of a particular phenomenon. In that spirit, this study contributes to the overall number of case studies of teacher change. The descriptive detail provided by this study supplements the range of teacher case studies already available. This contribution may, on its face, seem superfluous. However, if one considers an analogy with case law, the contribution becomes more significant. In case law, the number of cases established as precedent create a more powerful argument for the veracity of a given legal position. In a similar way, the larger the number of existing case studies of teacher change, the more powerful our understanding of the process becomes. In this way, this study bolsters the research base on and our understanding of teacher change.

As our understanding of the teacher change process develops, so too does the need for more effective tools in studying that process. This study contributes a novel theoretical framework to the set of frameworks and tools available to study this complex process. While the two frameworks used in this study are not novel creations of mine, the combination of the two in this way is novel. Further, from the results of this study, I maintain that the combined framework is a useful one. While the findings of this study did not contradict any previously reported results, the use of this novel framework to study teacher change processes and the reflective activity within them represents a first step toward a more detailed understanding of how teacher change occurs. Certainly, there is more work to be done, more detail to be developed, and additional lenses to apply, but this study is a proof of concept that it is possible to study the teacher change process writ large and the role of reflection within that process specifically.

Related to the novelty of the theoretical framework used in this study is a portion of the results regarding classical and alternative change pathways. In their original work, Clarke and Hollingsworth established change sequences and growth networks as two types of change pathways. Their work differentiated these pathways based on significance of change and the length of the pathway. This study contributes a second way to conceptualize and categorize change pathways within the IMTPG, namely based on path taken from the External Domain into
other domains. The existence of these two types of change pathways could easily have been theorized from simply looking at the IMTPG, however this study takes the additional step of verifying that these two types of pathways actually exist in teacher change processes. Not only that, but this supplementary conceptualization has implications for PD design and further research which are discussed below.

Lastly, this study provides evidence that the PD format and combination of activities that subjects experienced can encourage and support teacher change in beliefs and practice. The usefulness and effectiveness of lesson study is not widely questioned; however, this study shows that a modified form of lesson study, with a focus on learning something about teaching as opposed to the creation of a single lesson, can be effective in encouraging teacher change. Further, this modified lesson study format, with its focus on pedagogies of enactment, tends to increase teachers’ reflective activity as compared to a PD format focused on pedagogies of investigation.

These are modest contributions to the field of mathematics education research. Despite this modesty, this study has effectively increased our knowledge of the teacher change process, at least with respect to mathematics and the particular PD in question. And, as with any study, these contributions come with a set of specific conditions and limitations. These are taken up in the next section.

Limitations

The limitations particular to this study fall into three main areas:

- Study sample
- Instrumentation and data collection
- Researcher positioning

I will treat each of these areas in turn, but prior to that another discussion is of import.

Methodology was not an area included in the list of limitations above—a deliberate choice on my part. Case study research may be seen by some as a limiting methodology as most case studies, even multiple case studies, do not have sample sizes large enough to warrant robust statistical treatments or to warrant broad-scale generalizations of findings. I do not see this as the case. The methodology was chosen with deliberate care to robustly address the research questions for this study. In order to deeply understand any aspect of the teacher change process, a
great deal of detail and description was required. Case study was an option that provided the means for gathering, studying, analyzing, and delivering the kind of rich, deep, descriptive findings that would contribute to the understanding of the teacher change process. I put forth similar arguments in Chapter 3 and so I will not belabor them here; however, I believe it important to note that methodology is not an *a priori* limitation in this study.

Study Sample

The sample of subjects in this study possessed several characteristics that make it, if not unique, then certainly uncommon. First, the size of the sample was small, which can be seen as a limitation even in multiple case study research. Three subjects may not have generated enough data to allow the analysis process to reveal underlying themes foundational to understanding the quintain (Stake, 2006), the process under study—in this case, the teacher change process. Despite this possibility, I believe the robustness of my findings in this study support its importance and contribution.

Second, the sample consisted of subjects drawn from the same school and the same mathematics department. Such would not have been the case had I any choice in the matter. Unfortunately, enrollment in the SMII PD program was limited during the study. If the participants had been drawn from different school settings and different mathematics departments, the results of this study might well have looked different. Further, with regard to the participants in the SMII PD, the data may have looked different if participants from other schools had been part of the discussions and lesson study cycles. This limitation is one that flows in both directions, however. For example, the fact that the study subjects were comfortable with one another made them more willing to share specific, personal stories and reflections they might not have in less familiar company. Specifically, confessions regarding the administrator and educator evaluation constraints might not have been as robust within a larger, mixed group. Conversely, the fact that there were no participants from other schools may have led to an impoverished idea environment during PD discussions and reflections. The study subjects knew each other well and were obviously familiar with one another’s ideas and so no “new” ideas were injected into the conversations that did not come from my facilitation.

Finally, the small sample size and the small number of participants in the PD may have altered my facilitation in unforeseen ways. Facilitating for a small group, particularly one that has a great deal of internal familiarity, is a much different task than facilitating a large group that
has low internal familiarity. This change in task may have created an PD experience for participants that was significantly different than the intended, planned-for experience of the SMII PD. I took pains to ensure that the PD experience did not depart significantly from the original plan for the SMII PD, but that was not always possible while balancing the need to be a responsive, compassionate facilitator.

While the study sample provided interesting challenges in a number of areas, the robustness of the data and the detail of the case study reports and cross-case analyses is still compelling. I believe that this study, despite the uncommon sample, produced an understanding of the teacher change process that, in addition to being linked to the three study subjects, transcends the sample, contributing to a more general understanding of the teacher change process.

Instrumentation and Data Collection

Issues of data collection and instrumentation choices also created limitations for this study. Specifically, the incomplete data set and the limited number of classroom observations are areas of concern. The loss of the data from two full PD sessions and partial data loss from a third was concerning from the moment I discovered the problem. The loss was due to malfunctioning recording technology that was not discovered until after the sessions had been completed. A mediating factor with regard to the incomplete data is the content of the sessions in question. Sessions three and four contained no examples of classroom practice beyond those practices consistent with high-quality instruction embedded within the facilitation of the sessions. Therefore, the number of instances of reflection missed during those sessions was likely minimal. However, there was a good deal of time devoted to planning for the establishment of classroom culture and of initial lesson planning during those sessions and so it is likely that some reflective activity was missed. The missing data for the final session, unfortunately, likely contained a good deal of reflective activity. Not only did participants engage in a lesson study session during that time, but I also facilitated a reflective activity focused on encapsulating what the group had learned about facilitating the explore phase of a mathematics lesson over the course of the year.

Despite the loss of this data, the results of the study were robust in many ways. The case study reports did not suffer greatly as the main source of data for them was the interview data, which was complete. The loss of the PD session data may have created weak triangulation points
within the case studies themselves, however any deficiencies were ameliorated by the robustness of the PD session data that was successfully collected.

Another limitation to this study was the limited number of classroom observations used to gather evidence of changes in practice. The classroom observations were of limited use in establishing changes in practice. In the best cases, the observations provided existence proofs of subjects’ attempts to change instruction, but ultimately, they did not provide enough data to allow any definitive conclusions about changes in teacher practice to be drawn.

Researcher Positioning

During the course of this study I functioned as both the researcher and the facilitator of the PD sessions. On its face, this fact seems problematic. However, I was able to take steps to ensure that my roles remained as separate as possible over the course of this study. For example, I adhered to strict protocols while observing classrooms. I did not insert myself into the flow of classroom events any more than was necessary to gain entrance and assuage any curiosity on the part of the students. While facilitating the PD sessions, I attempted to remain centered on my role as facilitator, keeping that frame of reference in tension with the knowledge that I was engaging in research and so needed to be cautious about how I interacted with subjects and participants.

Further, I video recorded the sessions using technology that allowed me to move and interact freely, with minimal need for maintenance or interaction with the technology during each session (e.g., I only had to turn the camera off and on before and after lunch breaks). Generally, I was able to keep my two “selves”—the researcher and the PD facilitator—separate by means of mental framing before every event in the study. For example, before each interview I reviewed the protocol for that interview and briefly re-entered the mathematical task to ensure that I was prepared to be agile in my questioning. These activities helped center my mind on my work as the researcher. Similarly, prior to each PD session I reviewed my detailed plan for the session, including goals, outcomes, the mathematical task, any research articles or chapters, and refreshed my memory of the supporting slide show. These activities helped ensure that I was in the mental space of the facilitator prior to beginning a PD session.

While these actions by no means guaranteed that no interaction between my “selves” occurred, undertaking them helped to minimize any interactions that might have influenced the data in this study. Further, in qualitative research generally it is not considered problematic that the researcher enters the worlds of the subjects and interacts with them, often significantly.
Indeed, it is often the only way to gather the kind of data required for the exceedingly detailed descriptions that result from qualitative research.

I have discussed several potential limitations of my work in this section and, I believe, have addressed each of them as completely as possible. I noted the particular peculiarities of my sample for this study, the incompleteness of my data set, and my dual roles as researcher and facilitator. Keeping these limitations in mind when considering the results of my study will ensure that any conclusions drawn will be based in a contextualized reality and not pure, hopeful speculation. Further, despite these limitations and issues of generalizability of qualitative research, I believe that this study does contain findings that move beyond the particular cases presented here (as was the intent of the multiple case study methodology). Indeed, I believe that the field can consider some of my findings as explanatory and informative in conducting further research and designing professional development for mathematics teachers.

Implications for PD Providers

The findings in this study have implications for those individuals and institutions dedicated to providing high-quality professional development for teachers of mathematics. Specifically, there are five areas for which this study provides insight:

- A framework for teacher change is vital in planning effective mathematics PD
- Mathematics PD design should intentionally plan for both classical and alternative change pathways.
- Effective mathematics PD should provide multiple opportunities for reflection and enactment.
- Understanding and planning for participants’ context is vital to effective mathematics PD
- An understanding of participants’ belief systems is vital to managing effective mathematics PD

How we conceptualize the nature of teacher change determines, both broadly and specifically, how we attempt to foster that change. This study involved the use of a relatively complex model of teacher change and the data painted a picture of a process that was even more complex than the model itself implied. If the teacher change process is as complex as the results of this study indicate (it is most likely more complex yet), then any attempt to provide professional development that encourages and supports mathematics teachers in changing
practice without some guiding framework is almost certainly doomed to failure. Professional development designers should use their guiding framework to conceptualize how teachers might respond to the specific PD activities planned. In fact, PD designers should engage in a planning process analogous to the initial stages of that outlined by Stein and Smith (2018) for orchestrating productive mathematics discussions. Related to the use of a guiding framework for teacher change, PD designers should intentionally design PD to allow for and encourage teachers to engage in both classical and alternative change pathways. Primarily, this multiple-pathway design accounts for the deeply personal nature of teacher change and the fact that it is impossible to predict with any certainty which type of pathway a given teacher might take under the influence of a PD intervention. Further, PD designed in this way allows for a much broader range of interactions and change processes for individuals and groups. For example, given that PD should be intended to engage each participant in multiple change pathways across its duration, a given teacher might take a classical change pathway for a particular aspect of a PD, but might require an alternative change pathway for another aspect, perhaps one that challenges a primary belief. For mathematics PD to be truly effective, its design must allow teachers to move through cycles of change in as many different ways as possible.

Regardless of the model of teacher change chosen, the results of this study indicate that reflection and enaction are vital drivers of the change process. As such, effective mathematics professional development should provide teachers with opportunities to engage in multiple cycles of reflection and enaction. Just as we acknowledge that student knowledge builds over the course of time and multiple experiences, so too must teacher knowledge. New or different practices do not manifest directly from PD into practice—rather, those practices are taken up in fits and starts, with the teacher engaging in experimentation followed by reflection followed by modification or re-enactment. Only through cycles of professional experimentation and reflection can teachers develop the knowledge and experiences necessary to support them in making continued changes to practice. Many traditional PD formats only include pedagogies of investigation—facilitated opportunities for participants to read about or observe particular practices. In order to provide opportunities for participants to engage in cycles of reflection and enaction, mathematics PD design should include the use of pedagogies of enactment—facilitated experiences in which participants enact particular practices in a supportive environment (e.g., through role playing scenarios). Other researchers have argued similarly for pre-service teacher
education (see, for example, Ghousseini (2009)). Put simply, if PD designers expect teachers to adopt new instructional practices based on PD experiences, then those teachers must be given multiple opportunities for “deliberate practice” in which they engage in “repeated experiences in which the individual can attend to the critical aspects of the situation and incrementally improve her or his performance in response to knowledge of results, feedback, or both . . .” (Ericsson, 2002, p. 368).

Another implication of this study for PD designers concerns context. In this study, subjects’ responses to the PD activities were significantly affected by the context in which they worked. A deeper understanding of this context during the design phase of the PD may have allowed my PD design to be more responsive to participants’ needs. Reducing these barriers will increase the probability that teachers will engage in a positive change process aligned with the intentions of the PD designers. This idea, too, is not new. The framework laid out in the seminal text, *Designing Professional Development for Teachers of Science and Mathematics* (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2009), includes context as a vital part of the planning process. This study simply reinforces the importance of context as a vital input to the planning process.

Lastly, given the prominent role that beliefs played in this study, it is vital that PD designers make efforts to understand and account for teachers’ belief systems as part of the design of any mathematics PD. Teacher beliefs in mathematics education are particularly complex when one considers that it is not just beliefs about teaching that must be accounted for. A significant understanding of teachers’ beliefs about learning and about mathematics must also be developed. Indeed, teachers’ beliefs about mathematics can be said to have a nearly direct relationship to particular aspects of their practice. Designers of mathematics PD must understand the complex interplay of teacher beliefs, both generally and specific to a given set of teachers. If such an understanding is not developed, then PD is likely to be less effective than it otherwise could be.

The complexity of the task of engaging in high-quality mathematics teaching cannot be overstated. Given this reality, it is only natural that the development, design, and implementation of mathematics professional development should account for and share as much of that complexity as possible. This parity of complexity ensures that mathematics PD is responsive, supportive, and empowering for participating teachers. A deep understanding of the teacher
change process, including belief systems, potential change pathways, and teacher context is vital to ensuring that mathematics teachers are supported in honing their incredibly complex craft.

Directions for Future Research

The related fields of mathematics professional development and mathematics teacher change are fertile areas for further research. This study points to several potential areas for further research; however, I propose a different conceptualization of the task at hand. Given that there is no prominent candidate for a widely accepted model of teacher change in mathematics, I propose the design of a research program centered on the Interconnected Model of Teacher Professional Growth. The IMTPG is an empirically derived and supported model of the teacher change process that goes further than many others in explicating the complex interactions of the teacher change process. If the IMTPG is, metaphorically, the bones of a research program designed to investigate teacher change in mathematics, then this study begins to form the connective tissue attached to those bones. The IMTPG indicated that reflection was a mediating process in the overarching landscape of teacher change. Considering reflection as a mediating process connects the IMTPG to a vast body of research specific to reflection. This study adds a level of detail to that specification—the inclusion of a framework for the analysis of reflective activity added a new level of detail to the study of the teacher change process. Similar approaches might be taken to create a more comprehensive framework for studying teacher change.

A consideration of each of the four domains of the IMTPG offers up many options for further research. From the Personal Domain, researchers may wish to develop more effective metrics for measuring changes in teacher beliefs. Work in this area has already begun (cf. Copur-Gencturk & Thacker, 2020). Related to this, the development of robust measures of Mathematical Knowledge for Teaching (Hill, Ball, & Schilling, 2008) beyond the elementary level would also be beneficial. From the External Domain, further studies relating lesson study and other PD formats to the IMTPG would also contribute to a larger mathematics teacher change research program. Also, studies focused on the interplay between teacher context and teacher change would continue to flesh out the field’s understanding of the teacher change process. The Domain of Practice has been deeply researched in many areas. However, there is still significant work to be done in the areas of pedagogies of enactment of specific instructional
moves that are effective. In short, the field still needs to learn how teachers learn to implement particular sets of instructional moves. This study provided hints about a potential process for this—or perhaps it provided specific examples of how specific teachers chose to learn (e.g., Rebecca’s focus on one particular talk move). The overall research program, though, remains skeletal at best.

To remedy this, one might consider Lakatos’ (1976) metaphor for describing paradigms. Lakatos described paradigms in terms of a central core of ideas that are largely theoretical. These core ideas could then be surrounded by a belt of subsidiary supporting theories and, in turn, by another layer of empirical studies that investigate and support the core and subsidiary theories. This framework allows for a growing, adaptive research paradigm. I would pose the following question related to this idea: what might the field of teacher change research consider to be its core theoretical ideas? The IMTPG is empirical in that it was derived from actual PD studies and it is theoretical in that it aims to describe a process that is, at best, partially understood (or perhaps partially understood in pieces). Could the IMTPG form the core of a comprehensive research program designed to systematically investigate the teacher change process? It is certainly possible. However, a requisite element does not currently exist: large-scale commitment to the formation of such a research program. I hold hope that such commitment will one day be possible.

Summary and Closing

This study attempted to plumb the depths of secondary mathematics teacher change processes associated with a specific mathematics professional development intervention—a small island in a sea of potential PD interventions. Using a multiple case study methodology and a theoretical framework that integrated a model of teacher change and a model of levels of reflection, this study revealed a complex interplay of influences on mathematics teachers’ change processes. The sources of these influences ranged from the external (e.g., the PD intervention and building culture factors) to the personal (e.g., teacher beliefs) to the practice-based (e.g., results of professional experimentation in the classroom). These influences served to either empower subjects to make changes or inhibit subjects’ change processes. Further, the mediating processes of reflection and enaction appeared to drive the change processes of teachers in this study. As teachers engaged in cycles of enactment and reflection, they confronted inhibiting
influences and leveraged empowering influences. These cycles of reflection and enaction often took the form of mappable change pathways through the theoretical model.

In studying the change pathways that study subjects engaged in, it became clear that two different types of change pathways were in evidence. Classical change pathways involved the internalization of ideas from the PD sessions and the resolution of conflict with personal beliefs and attitudes that allowed teachers to attempt to implement specific classroom instructional practices during professional experimentation. Alternative change pathways, on the other hand, involved subjects taking ideas directly from the PD sessions and experimenting with them in their classrooms, regardless of whether they believed those ideas or practices had merit. The alternative change pathways provide existence proofs of ways in which beliefs can be influenced through professional experimentation primarily, rather than secondarily through reflective activity.

Despite the existence of alternative change pathways, reflection still played a central role in subjects’ change processes. As one of the mediating processes in the theoretical model, it was important to gain an understanding of the role that reflection played in the teacher change process. While not definitive in any way, the results of this study indicated that subjects engaged in reflective activity often during the PD sessions and that the level of that reflection was influenced by the format of the PD activities. While I was not able to draw conclusions about how subjects’ reflective activity changed with regard to time, I was able to make a connection between the format of the PD and reflective activity. Specifically, the modified lesson study structure in which teachers collectively viewed and analyzed video recordings of practice encouraged higher levels of reflective activity in study subjects. This finding indicates that lesson study is a promising format for mathematics PD focused on creating change in classroom practices.

In considering the data as a whole, several super-themes emerged as indicators of areas of importance. First, subjects engaged in storytelling often and for varied purposes throughout this study. While the reflective framework used in this study relegates storytelling or “describing” as a low-level reflective activity, the results of this study indicate that this kind of reflective action is important to the teacher change process. As part of the lesson study process, subjects used storytelling to contextualize personal video clips for other participants, allowing for a more nuanced and productive discussion of practice. Further, participants shared success stories with
one another often. This sharing of success empowered and supported the community of PD participants. It is important that PD providers not dismiss the utility of teacher storytelling to promoting and supporting change.

A second super-theme that emerged from the data was one focused on teacher beliefs. Teacher beliefs played a deeply significant role in the change processes in this study. Specifically, teachers’ beliefs about teaching and their beliefs about students influenced the change processes experienced by study subjects. While the theoretical framework places beliefs firmly in one domain (the Personal Domain), the data indicated that those beliefs permeated the other domains as well. The fact that beliefs were influential across multiple domains of teacher change indicates their importance to the teacher change process.

Returning to the larger picture in closing, I will note that this study brought together many different segments of research (e.g., research on effective professional development, the use of video, lesson study, reflection, beliefs, etc.). This complexity was required given the highly personal and complex nature of teacher change. A thorough, deep, and nuanced understanding of the teacher change process is necessary to the vital work of reform, empowerment, and support of mathematics teachers. The field currently does not have such an understanding. This study, I hope, contributed in some tiny way to a collective endeavor to develop that understanding. While I hope this, I also wonder if there really is such a collective endeavor. If there is not, then perhaps it is (finally) time to create one. Reform initiatives never really cease, particularly given the cultural influences on school mathematics, and so I am left to wonder: what can we do now to prepare for the next wave of teaching reforms? What can we do now to ensure that mathematics teachers are systematically and systemically supported and empowered in their incredibly complex work? My answer? Create a sustainable, comprehensive research program focused on understanding the mathematics teacher change process. The work has begun, and I believe I have contributed usefully to it. Now, to continue.
REFERENCES


APPENDIX A

SMII Project Interview Protocol – Interview 1
SMII Project Interview Protocol – Interview 1

Interviewer Name: Date:

Subject Name:

Note: Words in italics are meant to be spoken aloud by the interviewer. Deviation from the script is permitted in form, but not in general meaning. Further, clarifying questions are encouraged, provided they are not suggestive of potential answers.

Thank you again for agreeing to work with me. As part of this study, I would like to talk with you about some mathematics and ask you some questions about your teaching. With your permission, I will video record our conversation so that I can transcribe it later. Let’s start with some math and then move on to teaching.

Mathematics

1. Present the subject with the S-Pattern Task provided below.
   a. Look at this task for me.

2. Prompt subject to solve the task.
   a. Find a solution to this problem for me. As you work, or after you’re done, please talk about or write down how you thought about your solution. Feel free to use pictures, expressions, or any other method you choose.
   b. Make it clear that s/he should show relevant thinking processes in writing, with pictures, or speaking out loud. DO NOT provide assistance to subject as s/he completes the task.
   c. Question the subject to clarify any areas of the solution that you do not understand.
3. If/when the subject completes the task, prompt him/her to think about another solution method.
   a. *How might you think about this a different way? Is there another solution method that comes to mind?*
   b. Prompt subject for as many solution methods as s/he can provide using language similar to Part a.

4. After subject has provided as many solution methods as s/he can, ask
   a. *How might you expect a student to solve this problem?*
   b. *What makes you think that a students might respond in that way?*

**Teaching Practice**

1. Think about a typical lesson in your classroom.
   a. *How do you begin this lesson?*
   b. *Describe the sequence of instruction in this lesson.*
      i. If subject has difficulty answering, ask
         1. *How do you introduce students to new content?*
         2. *What does classroom talk look and feel like? Estimate the ratio of teacher talk to student talk?*
         3. *How do students spend their time?*
         4. *How often do students speak with you or with each other about the mathematics?*
   c. *How do you close this lesson?*
   d. *What strategies do you use to get all students to engage with mathematics?*
   e. *How do students get to speak to the class?*
2. **Think about the physical environment of your classroom. What does it look like?**
   a. If subject has difficulty answering, ask
      i. *How are the desks arranged? How does this impact the way students work?*
      ii. *What do the walls of your classroom look like? What is on your bulletin boards?*

3. **Talk about lessons that might be “atypical” in purpose or form. What do these lessons entail?**
   a. As subject discusses, be sure to prompt about the following
      i. *Instructional goals*
      ii. *Reasoning behind instructional choices*

4. **How do you feel that your students respond to your teaching?**
   a. *Are they engaged in the work? Or do they have trouble maintaining focus?*

**Concept Mapping** – Present subject with the Concept Mapping Activity

*The goal of this next activity is for you to create a concept map that characterizes and relates the components of “high quality mathematics instruction.” You may use whatever format you would like so long as it conveys relationships between concepts. I will ask you to explain your concept map after you’ve completed it.*
The S-Pattern Task\textsuperscript{18}

1. What patterns do you notice in the progression of figures?

2. Sketch the next two figures in the sequence.

3. Determine a way to find the total number of tiles in a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

4. Characterize the relationship between the figure number and the total number of tiles. How do you know the relationship is as you describe?

Concept Mapping Activity

Think about the best mathematics lessons you’ve taught or experienced. Combine this with what you know about high quality mathematics instruction to create a concept map detailing the characteristics and components of good math instruction. The concept map need not stand on its own. You’ll be asked to explain your concept map when you’ve finished.
APPENDIX B

SMII Project Interview Protocol – Interviews 2 and 3
SMII Project Interview Protocol – Interviews 2 and 3

Interviewer Name: Date:

Subject Name: Interview Number:

Note: Words in italics are meant to be spoken aloud by the interviewer. Deviation from the script is permitted in form, but not in general meaning. Further, clarifying questions are encouraged, provided they are not suggestive of potential answers.

Thank you again for agreeing to work with me. As a continuing part of this study, I would like to talk with you about some mathematics and ask you some questions about your teaching. With your permission, I will video record our conversation so that I can transcribe it later. Let's start with some math and then move on to teaching.

Mathematics

1. Present the subject with the Hexagon-Pattern Task provided below.
   a. Look at this task for me.

2. Prompt subject to solve the task.
   a. Find a solution to this problem for me. As you work, or after you're done, please talk about or write down how you thought about your solution. Feel free to use pictures, expressions, or any other method you choose.
   b. Make it clear that s/he should show relevant thinking processes in writing, with pictures, or speaking out loud. DO NOT provide assistance to subject as s/he completes the task.
   c. Question the subject to clarify any areas of the solution that you do not understand.
3. If/when the subject completes the task, prompt him/her to think about another solution method.
   a. *How might you think about this a different way? Is there another solution method that comes to mind?*
   b. Prompt subject for as many solution methods as s/he can provide using language similar to Part a.

4. After subject has provided as many solution methods as s/he can, ask
   a. *How might you expect a student to solve this problem?*
   b. *What makes you think that a student might respond in that way?*

**Teaching Practice**

1. *Think about a typical lesson in your classroom.*
   a. *How do you begin this lesson?*
   b. *Describe the sequence of instruction in this lesson.*
      i. If subject has difficulty answering, ask
         1. *How do you introduce students to new content?*
         2. *What does classroom talk look and feel like? Estimate the ratio of teacher talk to student talk?*
         3. *How do students spend their time?*
         4. *How often do students speak with you or with each other about the mathematics?*
   c. *How do you close this lesson?*
   d. *What strategies do you use to get all students to engage with mathematics?*
   e. *How do students get to speak to the class?*
2. **Talk about the physical environment of your classroom. What does it look like?**
   
   a. If subject has difficulty answering, ask
      
      i. How are the desks arranged? How does this impact the way students work?
      
      ii. What do the walls of your classroom look like? What is on your bulletin boards?

3. **Based on your experiences in this project so far, how has your teaching shifted from what you at the beginning of the project?**
   
   a. Optional prompts
      
      i. How, if at all, have your patterns of communication changed?
      
      ii. How, if at all, have student interactions with you and each other changed?
      
      iii. How, if at all, have your planning habits changed?

4. **How do you feel that your students respond to your teaching?**
   
   a. Are they engaged in the work? Or do they have trouble maintaining focus?

5. **As you think about your practice moving forward, which practices do you feel you’ll keep the same? Which are you thinking of changing for next year?**

6. **Think about the changes you’ve identified in your practice thus far. What do you feel are the experiences you’ve had in SMII that have had the most impact on your thinking, beliefs, and teaching? Why were these so effective for you?**

**Concept Mapping**

Present the subject with the Concept Mapping Activity

The goal of this next activity is for you to create a concept map that characterizes and relates the components of “high quality mathematics instruction.” You may use whatever format
you would like so long as it conveys relationships between concepts. I will ask you to explain your concept map after you've completed it.

The Hexagon-Pattern Task

Each of the shape-trains below consists of regular hexagons.

5. What patterns do you notice in the progression of figures?

6. Determine the perimeter of each of the first four trains.

7. Determine a way to find the perimeter of a given figure in the sequence. Explain your method and how it relates to the visual diagram of the figures.

8. Characterize the relationship between the figure number and the total number of tiles. How do you know the relationship is as you describe?

Concept Mapping Activity

Think about the best mathematics lessons you’ve taught or experienced. Combine this with what you know about high quality mathematics instruction to create a concept map detailing the characteristics and components of good math instruction. The concept map need not stand on its own. You’ll be asked to explain your concept map when you’ve finished.
APPENDIX C

SMII Project Interview Protocol – Interview 4
SMII Project Interview Protocol – Interview 4

Interviewer Name: 

Subject Name: 

Note: Words in italics are meant to be spoken aloud by the interviewer. Deviation from the script is permitted in form, but not in general meaning. Further, clarifying questions are encouraged, provided they are not suggestive of potential answers.

Thank you again for agreeing to work with me. As a continuing part of this study, I would like to talk with you about some mathematics and ask you some questions about your teaching.

With your permission, I will video record our conversation so that I can transcribe it later. Let’s start with some math and then move on to teaching.

Mathematics

1. Present the subject with the Candy Bar Sale Task provided below.
   a. Look at this task for me.

2. Prompt subject to solve the task.
   a. Find a solution to this problem for me. As you work, or after you’re done, please talk about or write down how you thought about your solution. Feel free to use pictures, expressions, or any other method you choose.
   b. Make it clear that s/he should show relevant thinking processes in writing, with pictures, or speaking out loud. DO NOT provide assistance to subject as s/he completes the task.
   c. Question the subject to clarify any areas of the solution that you do not understand.
3. If/when the subject completes the task, prompt him/her to think about another solution method.
   a. *How might you think about this a different way? Is there another solution method that comes to mind?*
   b. Prompt subject for as many solution methods as s/he can provide using language similar to Part a.

4. After subject has provided as many solution methods as s/he can, ask
   a. *How might you expect a student to solve this problem?*
   b. *What makes you think that a student might respond in that way?*

**Teaching Practice**

1. *Think about a typical lesson in your classroom.*
   a. *How do you begin this lesson?*
   b. *Describe the sequence of instruction in this lesson.*
      i. If subject has difficulty answering, ask
         1. *How do you introduce students to new content?*
         2. *What does classroom talk look and feel like? Estimate the ratio of teacher talk to student talk?*
         3. *How do students spend their time?*
         4. *How often do students speak with you or with each other about the mathematics?*
   c. *How do you close this lesson?*
   d. *What strategies do you use to get all students to engage with mathematics?*
   e. *How do students get to speak to the class?*
2. Talk about the physical environment of your classroom. What does it look like?
   
a. If subject has difficulty answering, ask
   
i. How are the desks arranged? How does this impact the way students work?
   
ii. What do the walls of your classroom look like? What is on your bulletin boards?
   
3. Based on your experiences in this project so far, how has your teaching shifted from what you at the beginning of the project?
   
a. Optional prompts
   
i. How, if at all, have your patterns of communication changed?
   
ii. How, if at all, have student interactions with you and each other changed?
   
iii. How, if at all, have your planning habits changed?
   
4. How do you feel that your students respond to your teaching?
   
a. Are they engaged in the work? Or do they have trouble maintaining focus?
   
5. As you think about your practice moving forward, which practices do you feel you’ll keep the same? Which are you thinking of changing for next year?
   
6. Think about the changes you’ve identified in your practice thus far. What do you feel are the experiences you’ve had in SMII that have had the most impact on your thinking, beliefs, and teaching? Why were these so effective for you?
   
7. What advice might you give to another teacher who wants to make the shift toward an instructional model consistent with what we have discussed in SMII?
Concept Mapping

Present the subject with the Concept Mapping Activity.

*The goal of this next activity is for you to create a concept map that characterizes and relates the components of “high quality mathematics instruction.” You may use whatever format you would like so long as it conveys relationships between concepts. I will ask you to explain your concept map after you’ve completed it.*
The Candy Bar Sale Task

It’s the annual Freshman Candy Sale and you have 36 candy bars to sell. Your best friend only has 24 candy bars to sell. If you sell 2 candy bars per day and your friend sells 1 bar per day, how many days will it have been when you have fewer candy bars than your friend?
Concept Mapping Activity

Think about the best mathematics lessons you’ve taught or experienced. Combine this with what you know about high quality mathematics instruction to create a concept map detailing the characteristics and components of good math instruction. The concept map need not stand on its own. You’ll be asked to explain your concept map when you’ve finished.
APPENDIX D

Lesson Study Research Notes Matrix
# Lesson Study Research Notes Matrix

<table>
<thead>
<tr>
<th>Name of Article</th>
<th>Please list the author and title of the article you were asked you to examine.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise/hypothesis of the article</td>
<td>What is the premise of this article?</td>
</tr>
<tr>
<td></td>
<td>How well do you believe the author(s) have made a case for their viewpoint? What evidence from the article makes you feel that way?</td>
</tr>
<tr>
<td>Recommendations from the article</td>
<td>What recommendations do(es) the author(s) make that seems relevant to the Explore phase of the TTLP? What support do they give for these notions?</td>
</tr>
<tr>
<td>Content influencing thinking/planning</td>
<td>How do the recommendations/findings in this article change how you think about the Explore phase of the TTLP? What implications might this have for your continued planning and work? Why do you feel that way?</td>
</tr>
<tr>
<td>---------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Discussion Notes</td>
<td>Write down any notes about your discussion with your group. How did your groupmates’ comments influence your thinking?</td>
</tr>
</tbody>
</table>
APPENDIX E

Classroom Video Observation Notes Protocol
## Classroom Video Observation Notes Protocol

<table>
<thead>
<tr>
<th>Name: __________________________________________</th>
<th>Date: ____________</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical Goal(s) of the Lesson</strong></td>
<td>What are the proposed mathematical goals of the lesson from the lesson plan? What goals did your colleague think were achieved?</td>
</tr>
<tr>
<td><strong>Potential Key Points in Video Clip</strong></td>
<td>As you watch, write down key observations you make. Be sure to focus on questioning patterns throughout the video clip. How did the teacher respond to student difficulties? What kinds of questions were asked?</td>
</tr>
<tr>
<td>Key Points as Stated by Participant</td>
<td>What were the key points that served as reasons why your colleague chose this video clip?</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>--------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Discussion Notes</td>
<td>Write down any notes about your discussion after watching the clip together. Try to keep the discussion focused on questioning and discourse patterns that you observed. What might have been done differently? How did your thinking shift about your own practice as you watched this clip?</td>
</tr>
</tbody>
</table>