A Framework for Using Fused Image Data in Statistical Process Control

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A FRAMEWORK FOR USING FUSED IMAGE DATA IN STATISTICAL PROCESS CONTROL

by

Shengfeng Chen

A dissertation submitted to the Graduate College
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
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A FRAMEWORK FOR USING FUSED IMAGE DATA IN STATISTICAL PROCESS CONTROL

Shengfeng Chen, Ph.D.
Western Michigan University, 2020

The use of image data in manufacturing quality control systems has grown rapidly over the years. Technological advancements in imaging devices (e.g., digital cameras, x-ray scanners, infrared cameras) have provided convenient and cost-effective ways to obtain images. These images contain important quality features (e.g., textures and product dimensions) that can be used to monitor, diagnose, predict, and control manufacturing processes. A key tool for monitoring images is statistical process control charts. Integrating images of parts to detect process shifts can significantly improve a quality systems efficiency. Unfortunately, appropriate techniques that take full advantage of data-rich images for control charting are not well-developed, and more applications can be investigated by utilizing images in this area.

The overarching goal of this research is to investigate new opportunities of using high-dimensional image data with control charts and to increase their performances in real-world applications. This research addresses the need to extend current control charting approaches and proposes a new framework for using image data for manufacturing quality control. Specifically, a multi-image monitoring framework is proposed to monitor multiple images of the same part simultaneously in order to improve the overall system performance. In this dissertation, two approaches toward implementing the multi-image monitoring framework are introduced and the performance gains obtained with these approaches over monitoring single images are presented. In addition, in order to quantify these performance gains, a computationally efficient approach to simulate cross-correlated images was developed. Both simulation and experimental results justify the effectiveness of the proposed framework.
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Shengfeng Chen
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1. INTRODUCTION

Image-based statistical process control (SPC) has increasingly gained attention from researchers and practitioners to improve manufacturing quality. Two major factors make this area promising: 1) images are easy, quick, and inexpensive to obtain and 2) images are data-rich and informative, containing critical quality characteristics (e.g., appearance and color) and spatial information (e.g., locations and correlation) of a part. Current imaging technologies provide imaging devices (e.g., digital cameras) that are compact enough to be placed almost anywhere. Furthermore, with the integration of computers and sensors, cameras can be remotely controlled, allowing for cost-effective monitoring systems that can be continuously operated with minimal human interventions. Critical features within images collected from these systems can be extracted and monitored with a control chart to detect process shifts.

1.1. Background

Image-based control charts integrate image data with control charts for process monitoring. Generally, this process is comprised of four stages, as shown in Figure 1.1. Images are first acquired through a Machine Vision System (MVS) that uses imaging device (e.g., digital camera, X-ray scanner) to capture images. Acquired images need to be preprocessed to remove unnecessary information and noise. Common digital image processing techniques, such as cropping, resizing, noise removal, and edge enhancement, are performed depending on the usage and conditions of the acquired images. However, preprocessed images are often very large and contain significant amounts of data redundancy. Leveraging this redundant data, a series of techniques can be utilized to extract image features and reduce data dimensions as part of the multivariate image analysis (MIA) methodology, reviewed by Prats-Montalbán et al. (2011). The extracted features contain critical quality characteristics (e.g., surface, textures, patterns) from the
original images. Afterwards, a statistical process control chart is used to monitor these features over time to detect shifts.

Image-based statistical process control integrates machine vision systems with SPC approaches to monitoring images over time. Traditional SPC techniques are designed to monitor one or a few parameters, are relatively easy to implement, and are well understood after being successfully practiced for many decades. However, when image data is involved, monitoring becomes more challenging due to the dramatic increase in data size and complexity. Even though researchers have worked in this area for years, techniques that take full advantage of data-rich images for control charting are still in development and more applications can be explored in the area of image-based statistical process control.
1.2. Challenges

The major challenge of Image-based SPC stems from the complexity of image data as they are data-rich and consist of millions of pixels. Monitoring an image’s individual pixel levels is computationally exhaustive, and it is infeasible for multivariate control charts to consider each pixel as a variable. Therefore, before an image can be monitored, MIA techniques are required to reduce data dimensions and extract features. These extracted features can be monitored by control charts for detecting process shifts. Duchesne et al. (2012) provided a thorough review on MIA techniques. They enumerated a series of methods for feature extraction including textural analysis (Wavelet transforms, Grey Level Co-occurrence Matrix (GLCM)), Principle Component Analysis (PCA) for feature reduction, and Multi-resolution Multivariate Image Analysis (MR-MIA) for extracting both color and texture features. With respect to SPC, these methods are primarily used within a single image monitoring framework, where each observation of the system consists of one image. If multiple images are captured simultaneously, this framework would require multiple control charts, which is inconvenient, difficult to operate, and may be inefficient in detecting shifts. Therefore, a multi-image monitoring framework, where each observation of the system includes a fused image that draws important features from multiple images, is discussed in Chapter 4 to monitor two or more images in a single multivariate control chart.

The second challenge for Image-based SPC is the difficulty in simulating images for evaluating a control chart’s performance metrics. Evaluating a control chart’s performance for Image-based SPC requires a significantly large number of images that experience a variety of in-control and out-of-control conditions, which can only practically be achieved through simulations. In addition, the proposed new multi-image monitoring framework is based upon multiple images being monitored simultaneously. Therefore, analyzing the performance of a control chart under this
framework requires the ability to simulate multiple images. These multiple images should contain both spatial cross-correlation and spatial auto-correlation to accurately capture a control chart’s performance. Considering the image sizes and volumes needed, current image simulation techniques are inadequate. Therefore, a new fast and accurate cross-correlated image simulation approach is proposed and presented in Chapter 3 to handle this challenge.

1.3. Motivation and Significance

This research is motivated by recent technological advancement in computer vision technology, such as multivariate image analysis, a methodology to extract features from multi-dimensional images. For instance, the MIA tool, image fusion, could be applied to SPC to create control charts that efficiently monitor multiple simultaneous images of the same part. However, multiple images of a part not only contain more information regarding but also bring more data redundancy. If an image fusion approach can leverage redundant data and extract important features, it may provide a promising approach to increase Image-based control charts’ abilities to monitor features of a manufactured part, compared to monitoring a single image.

From this motivation, this dissertation will also investigate the effect cross-correlation across multiple images of the part has on control charting performances. It should be noted that multiple images of the same part should be spatially correlated as they share similar contents. Therefore, under the proposed framework, it is beneficial to understand how image cross-correlation can affect a control chart’s performance. For example, images captured with different camera settings of the same part are quite similar and spatially cross-correlated. It is important to know how the strength of this cross-correlation affects the ability to detect process shifts. Understanding the role of cross-correlation would provide a guidance for designing multi-image control charts and capturing images. Similar images may be more beneficial for process control than very different
images, and vice versa. The results may lead to future work by utilizing image spatial cross-correlation for more applications.

1.4. Research Objectives

Current introduction has mentioned the proposed multi-image monitoring framework, and how accomplishing this framework relies on successfully completing the following objectives.

1) Develop a simulation approach to assess the performance of the proposed framework. In SPC, Average Run Length (ARL) is typically used to measure control chart’s performance. For complex charts, such as Image-based control charts, ARLs are often estimated through simulation. In the field of Image-based SPC, current approaches assume random field are independent and identically distributed random variables that do not consider spatial correlation. For the multi-image monitoring framework, proposed in this dissertation, correlations between images must be considered. Not only would ignoring cross-correlations result in inaccurate ARL estimations, but it would completely defeat the purpose of considering multiple images within a single observation. Currently, there is no available approach to quickly generate cross-correlated images. Therefore, the aim of this objective is to develop a simulation approach that can generate cross-correlated images to evaluate the performance of multi-image control charts.

2) Demonstrate the performance benefit when implementing for the framework of using fused image in SPC. This objective aims to demonstrate the performance gains of using the proposed multi-image monitoring framework compared to using single images. In addition, two approaches for multi-image monitoring are developed and compared against each other. The aim of this study is to demonstrate the superiority of multi-image control charts that take advantage of cross-correlation (i.e., fused-image control charts). As a result, this study will assess how varying levels of cross-correlations affect the performance of fused-image control charts.
1.5. **Dissertation Outline**

This dissertation is comprised of five chapters. Chapter 2 provides a general literature review for the two identified objectives. It primarily concentrates on the background of Image-based SPC, discussing topics such as noise simulation, control charts, multivariate control charts, machine vision systems, and multivariate image analysis. It should be noted that more thorough literature reviews are provided within Chapters 3 to 4. Chapter 3 develops an image simulation approach to assess performance of the proposed multi-image monitoring framework, which is a version of the manuscript that will be submitted to Technometrics. Chapter 4 describes a control charting study that demonstrates the performance of the proposed framework, which a version of a manuscript that ready to be submitted to Journal of Quality Technology. Lastly, Chapter 5 summarizes the contributions of this dissertation and discusses future work.
2. LITERATURE REVIEW

In this chapter, a general literature review is provided focusing on four different research areas that are foundations to this dissertation. These research areas provide foundations for supporting the proposed method and framework that will be discussed in later chapters.

2.1. Random Field Generation

For Image-based SPC, image simulation is becoming crucially important as large amounts of images are required to evaluate a control chart’s performance (e.g., ARLs). In general, an image is simulated by adding noise to a nominal image. Noise is randomly generated with a certain distribution model. There are two commonly used methods to generate noise in an image: Gaussian Noise and Poisson noise. Gaussian noise is generated separately and independently from original noise and then added to nominal image. Gaussian Noise is usually defined by assigning a mean and standard deviations. See, for example, Koosha et al. (2017), Yan et al. (2014), and Wells et al. (2013). Poisson noise, on the other hand, assumes each noise variable is a function of pixel intensity value. It is typically generated by assigning Poisson distribution on every pixel with a distribution mean equal to the corresponding pixel value in the nominal image. In other words, Poisson noise is directly applied on images. Megahed et al. (2012) and He et al. (2016) used Poisson noise for their image simulations. While these two approaches generate noise in different manner, the common assumption is that noise is independently and identically distributed. This means the generated random field does not consider spatial correlation, and the effect of image correlation was not been thoroughly investigated in the SPC literature. When multiple images are taken simultaneously for the same products or processes, cross-correlation between images exists and should be considered during image analysis. Understanding image correlation may improve various industrial applications such as quality inspection, statistical process control, and other applications involving images.
In spatial statistics, spatial correlation is considered during random field generation. Spatial correlations are estimated by theoretical variogram with three terms: range, sill and nugget shown in Figure 2.1. The horizontal axis denotes the lag distance and the vertical axis denotes spatial variability. Range represents the lag distance at which data points are correlated. Beyond that range in lag distance, zero correlation is observed. Spatial variability increases as distance increases. Sill is the maximum variability when the curve reaches plateau. Nugget indicates spatial variability when lag distance is zero. For image data, lag distance is defined by the number of pixels, and nugget will be zero when there is no lag distance. More details regarding variogram can be seen from Bohling (2005) and Cressie (1993). The estimated range from theoretical variogram will be used for constructing spatial correlation functions. The constructed spatial correlation function approximates image spatial correlation that is utilized for generating a Gaussian random field. The generated random field will contain image spatial correlation as will be discussed in Section 3.

Figure 2.1 Illustration of Variogram
2.2. Statistical Process Control Chart

Control charts were first invented by Walter A. Shewhart in the 1920s, which brought statistical control into production system and developed the basis for what we know today as SPC. Shewhart discovered that a healthy process should only have common-cause variation, which is natural to the system. This process can be maintained in control by removing any special-cause variations which is often caused by process shifts. The well-known Shewhart control chart, well documented in the SPC literature, is effective in detecting moderate to large shifts in a single parameter. To improve upon the Shewhart control chart in detecting smaller process shifts, the Exponentially Weighted Moving Average (EWMA) chart and the Cumulative Sum (CUSUM) chart were created by Roberts (1959) and Page (1952), respectively. Researchers developed combined Shewhart-EWMA and Shewhart-CUSUM charts by monitoring both control charts simultaneously and recognized a shift when either chart signals. This combination takes advantage of both the Shewhart and EWMA/CUSUM charts to become sensitive to both large and small process shifts (e.g., Albin, 1997; Capizzi, 2010; and Abujiya, 2013). The disadvantages of combined control charts are that they 1) become more cumbersome as they require monitoring multiple control charts and 2) are harder to design since multiple parameters are required for the charts to operate it efficiently. To overcome these drawbacks, a Generalized Likelihood Ratio (GLR) control chart was introduced by Willsky and Jones (1976) for detecting a wide range of shift sizes. Reynolds and Lou (2010) compared a GLR chart with a combined Shewhart-CUSUM chart and proved that their overall performances are similar. The GLR chart does not require any control-chart parameters other than the size of the window and the control limit, which makes the GLR chart easier to implement than combined control charts. Megahed et al. (2012) introduced the spatiotemporal framework with a GLR control chart that provided a good estimate of the change.
point and the size/location of a process shifts in images. The GLR control chart have been investigated in a wide range of uses, for example Wang and Reynolds (2013), Lee et al. (2017), Reynolds et al. (2013), Koosha et al. (2017), and Huang (2011).

The multivariate $T^2$ control chart developed by Hotelling (1947) provides test statistics to measure variation across multiple variables. In essence, it extends single variable Shewhart control charts to monitor multiple variables and is widely used in industries (e.g., medical, chemical, and manufacturing) that requires monitoring multiple parameters together. The Hotelling $T^2$ chart provides a statistical measure to represent the variation across all variables. Tong et al. (2005) used Hotelling $T^2$ control charts to simultaneously monitor the number of defects and defect clusters on the wafer map in integrated circuit. Similarly, the Multivariate EWMA (MEWMA) and Multivariate CUSUM (MCUSUM) were developed by Lowry et al. (1992) and Crosier (1988), respectively, to detect small shifts in multivariate problems (see Figure 2.2 for MEWMA). A competitive alternative to the MEWMA and MCUSUM is the Multivariate GLR (MGLR) chart which is sensitive to a wide range of shift sizes. See, for example, Wang and Reynolds (2013), He et al. (2016) and Lee et al. (2017). The MGLR chart, adopting a moving window, calculates the moving window’s size that maximize the GLR statistics, which requires considerably more computations than the MEWMA or MCUSUM. Due to these additional computations, for processing large data, such as images, the MGLR may be impractical.
2.3. Image-based SPC Applications

In recent decades, machine vision systems (MVSs) have been widely implemented in industrial applications that utilize image capturing devices (e.g., digital camera) for system monitoring and defect inspection. A typical machine vision system, as shown in Figure 2.3, is used to reject defects. A central control unit controls the conveyor and communicates with the sensors that sense the incoming parts. A trigger signal is then sent to camera and lights that work in tandem to capture an image of the part. This image is processed in the control unit with pre-defined mathematical models to determine whether reject the part or not. Once a defect is found, a trigger signal is sent to ejector to reject the part correspondingly.
Zuech (2000) identified that MVSs are superior to human visual inspection for several reasons: 1) MVSs are capable of working at high production rate, 2) MVSs are adaptable to various environments, 3) MVSs are designed to be multi-tasking, and 4) MVSs are reliable for a long period of time. Furthermore, MVS system can be combined with control charts to monitor images for real-time process control. Lin (2007) monitored images of ceramic capacitors to inspect ripple defects. Image surface textures were extracted with wavelet transform and monitored by an Hotelling T2 control chart. Liu and MacGregor (2006) proposed an image monitoring framework for inspecting the appearance and aesthetics of manufactured products (e.g., stone countertop). They first extracted wavelet textural features, followed by a dimension reduction with Principal Component Analysis (PCA). Then, a PCA-based Hotelling T2 control chart was utilized for monitoring. Koosha et al. (2017) proposed using wavelet based nonparametric regression method to extract features from gray scale images and monitor the extracted features directly with a GLR.
control chart achieving suitable performance in detecting shifts. Images captured from MVS often requires extra processing steps before SPC monitoring. This is especially common for multivariate images (e.g., RGB images).

2.4. Multivariate Image Analysis

Multivariate Image Analysis (MIA) plays an important role in processing images in advance of SPC monitoring. It aims to extract important features from multivariate images. Multivariate images include millions of pixels and each pixel is a variable with intensity values ranging from 0 to 255. Pixel intensity values are highly correlated with neighboring pixels, which results in a significant amount of data redundancy. Leveraging this redundancy, dimension reduction techniques can be performed, from which features can be extracted. Duchesne et al. (2012) presented a thorough review paper regarding MIA in process monitoring. They proposed a series of steps to process images prior to SPC. For example, a raw image can be preprocessed by cropping it to a proper size, converting it from RGB to grayscale, removing noise, and/or enhancing edges. Gonzalez and Woods (2007) provided a more comprehensive discussion regarding image preprocessing techniques. Once preprocessed, features can be extracted from images. Textural features can be extracted by Wavelet analysis, as investigated by Mohanaiah et al. (2013). Another textural extraction approach is the Grey Level Co-occurrence Matrix (GLCM), as introduced by Ganesan et al. (2004). Spectrum features can be extracted by Fourier transform. Dimension reduction is as important as feature extraction and often seen in Multivariate image analysis handled by multivariate statistical framework, such as PCA or Partial Least-Square (PLS) regression. Yu and MacGregor (2003) implemented both PCA and PLS on color images for on-line monitoring in snack food industry.
For multivariate tensor data, Lu et al. (2008) introduced Multilinear Principal Component Analysis (MPCA) framework for extracting features and reducing dimensions on 2D/3D images. MPCA decomposes a multi-dimensional image in each mode (image dimension) to obtain eigentensors and make projection on eigentensors to obtain a tensor space retaining most of the original image variations. MPCA is superior to PCA because it avoids image vectorization which breaks image’s spatial correlation. MPCA is computationally more efficient than PCA because its covariance matrix created from each mode is significantly smaller for eigen decomposition. Yan et al. (2014) compared MPCA with several other low-rank decomposition techniques on color images (RGB three-dimensions) for process monitoring. MPCA works relatively well in detecting shifts through color images from results. However, Yan et al. (2014)’s work only focused on monitoring single image of a part. The multi-image monitoring and the cross-correlation in multiple images are yet to be investigated. MPCA is utilized for image fusion in this dissertation and will be discussed in Chapter 4.2.2.
3. FAST AND EXACT SIMULATION OF SPATIALLY CROSS-CORRELATED IMAGES VIA CIRCULANT EMBEDDING

This paper presents an efficient methodology for generating cross-correlated random fields as a Gaussian random process with cross-correlation and autocorrelation functions. These generated random fields can be used in engineering applications including cross-correlated image simulations for assessing image-based Statistical Process Control tools. The proposed approach can quickly simulate full-size images with exact auto/cross-correlation structures extracted from original random fields. The cross-correlated random fields are generated with the circulant embedding method. Here, a Toeplitz correlation matrix $R$ is embedded into a positive definite circulant matrix $S$. The properties of circulant matrix $S$ results in a computational complexity of $O(n \log n)$, which is significantly less than Karhunen–Loève (KL) expansion’s $O(n^3)$. Moreover, the circulant matrix $S$ contains the entire spatial correlation without information loss, which is better than eigen decomposition methods that only approximates correlations. This approach utilizes only a row matrix instead of a full-size correlation matrix saving considerable amount of computer memory and generating a pair of full-size images with millions of pixels simultaneously. The simulation results show that the proposed simulation methodology successfully generates large random fields containing exact spatial auto/cross-correlation.

**Keywords:** random field generation, circulant embedding, cross-correlated image simulation, spatial cross-correlation

3.1. Introduction

Spatially correlated images occur across a wide variety of fields, including industrial image-based statistical process control (SPC). For instance, in additive manufacturing, imaging systems capture part images over time (e.g., each layer) that are spatially correlated and monitored via control charts. A critical aspect of control charting design is understanding and assessing a chart’s
performance, which is often performed by simulating an enormous number (millions) of observations. Random field simulations are commonly used to mimic stochastic processes or systems, including spatially correlated images.

In the context of spatially correlated images, spatial correlation includes both cross-correlation and autocorrelation. Cross-correlation between images describes the statically dependency between pixels in one image and pixels in another image. Image autocorrelation describes how pixels within a single image are spatially correlated, which is most often function of spatial distance. Nearby pixels are often similar in color and brightness, while distant pixels are less similar sharing little information. In fact, image pixels typically have high autocorrelation with neighboring pixels and little to no correlation for pixels far away. For applications requiring the simulation of spatially correlated images, both cross-correlation and autocorrelation need to be considered to ensure high accuracy.

Considering both forms of correlations (cross and auto) requires a substantial amount of computational effort; especially for large images (millions of pixels). Simulating such large images would take days or even weeks using traditional approaches (e.g., K-L expansion, Chelosky Decomposition). Furthermore, simulating very large images may be infeasible as the size of the required covariance matrices may exceed computer memory capacity. Thus, a fast and exact approach for simulating spatially correlated images is critical for applications requiring massive amounts of simulated images, such as evaluating an image-based control chart’s performance. This paper presents a methodology for fast and exact generation of two cross-correlated random fields that can be overlaid onto nominal images for spatially correlated image simulation.

The subsequent sections are organized in the following order. Section 3.2 provides a literature review on generating spatially correlated random fields. Section 3.3 discusses the theoretical
model for spatial correlation, which are used as the basis for simulation. Section 3.4 discusses the single random field generation using circulant embedding method. Section 3.5 introduces the proposed method, cross-correlated image simulation with circulant embedding. Section 3.6 demonstrates an example of the proposed method by simulating cross-correlated random field based off two actual images. Section 3.7 conducts a performance comparison study to the KL expansion method. Finally, Section 3.8 concludes the paper and discusses future work.

3.2. Literature Review

In general, an image can be simulated by adding noise to an image’s mean surface. Noise is randomly generated with a certain distribution model. There are two commonly used noise models to generate noise in an image: Gaussian Noise and Poisson noise. Gaussian noise is generated separately and independently from original noise and then added to nominal image. Gaussian Noise is usually defined with zero mean and a standard deviation chosen from actual noise; See, for example, Koosha et al. (2017), Yan et al. (2014), and Wells et al. (2013). Poisson noise, on the other hand, assumes each noise variable is a function of pixel intensity value. It is typically generated by assigning a Poisson distribution to every pixel with a distribution mean equal to the corresponding pixel value in the nominal image. Poisson noise is directly applied on images, while Gaussian noise is added on images. Megahed et al. (2012) and He et al. (2016) used Poisson noise for their image simulations. While these two approaches generate noise in different ways, the assumption is in common that noise random variables are independent identically distributed. This means the generated random fields did not contain spatial cross-correlation. When multiple images are taken at the same time for the same products or processes, cross-correlation between images should be considered during image analysis. Understanding image spatial correlation may improve
various industrial applications such as quality inspection, statistical process control and other image-based applications.

To investigate spatial cross/auto correlation of images, semivariogram are often used for spatial correlation analysis according to Cressie (1993). Semivariogram’s value is evaluated at half of variogram’s value strictly. In this paper, the term variogram is referring to semivariogram for simplicity. In spatial statistics, variogram is used to display variability between data points in space as a function of distance. The empirical variogram measures the strength of statistical correlation of an image or random field and is regarded as an estimate of the theoretical variogram (Cressie (1993)). There are two types of variogram, isotropy and anisotropy variogram. Isotropy variogram consider distance as the solely factor to spatial variability. Anisotropy variogram evaluates spatial variability as a function of both distance and direction. To evaluate the spatial correlation of the random field, a theoretical variogram model is selected to approximate the empirical variogram. Zhu and Zhang (2013) characterized geotechnical anisotropic spatial field with various theoretical models (i.e., spherical, exponential, gaussian, etc.) with corresponding correlation functions. In this paper, the gaussian model was chosen for its good fit to the sample images.

One approach to generate a random field with correlation function is called Cholesky Decomposition. Decompose the corresponding covariance matrix (must be positive definite) into a lower triangle matrix. Then multiply the triangle matrix by a vector of independent Gaussian random variables to simulate a random field as proposed by Zhu et al. (2017). This method retains the exact covariance structure of the image, but it is more suitable for small image simulations (e.g. image size less than 40x40). Cholesky Decomposition method is computationally expensive with order $O(n^3)$ and is rarely used in applications with high dimensional images. Another method is the Karhunen-Loeve (KL) expansion used by Vořechovský (2008) and Venturi (2013), which
requires eigen-decomposition of the covariance matrix. Selecting proper number of principal components of the largest eigenvalues, followed by a multiplication with Gaussian random variables, a Gaussian random field is generated correspondingly. KL expansion can be expensive in computation when the corresponding covariance matrix is large. The reduced eigenspace, due to a portion of eigenvector being selected, causes simulation errors that the generated random field only approximates the prescribed covariance matrix.

This paper uses circulant embedding method for image simulations. Circulant embedding is a fast and exact method simulating large Gaussian random samples which has been widely used in various random field generations and signal processing. Since the spatial correlation matrix is a Toeplitz matrix, a Toeplitz matrix can be embedded in a circulant matrix. Consequently, a row vector of circulant matrix contains all information of the spatial correlation and can be used for Gaussian random process generation. The generated random fields contain exact spatial correlation as the prescribed ones. In addition, circulant embedding dramatically reduces the computational complexity to an order $O(n \log n)$, which is a significant improvement over KL expansion and Cholesky Decomposition approach. One restriction of circulant embedding method is that the estimated covariance matrix must be positive definite. More detail about circulant embedding can be seen from Dietrich (1997), Gneiting et al. (2006), Helgason et al. (2011) and Park and Tretyakov (2015).

A set of cross-correlated Gaussian random fields can be generated in different ways. Zhu et al. (2017) used Cholesky decomposition with copula conditional distribution to generate cross-correlated random fields. Copula function is excellent to correlate two joint distributions of random variables with a cross-correlation coefficients. It relies on probability distribution models to establish the cross-correlation, which may not always easily be found in actual images.
Vořechovský (2008) generated cross-correlated random fields by KL expansion. Zhao and Wang (2018) proposed Bayesian compressive sampling for estimating random field parameters along with KL expansion for generating cross-correlated random fields. As mentioned before, KL expansion does not generate an exact spatial correlation. In comparison, this work aims to provide a fast and exact simulation method that generates a pair of cross-correlated images containing both prescribed spatial auto/cross-correlation with circulant embedding.

3.3. Spatial Correlation

In this paper, anisotropy variogram was used with the assumption that industrial images, especially with clear patterns, often have different spatial variabilities at different directions. Thus, spatial correlation was analyzed by two variograms along horizontal x-axis and vertical y-axis respectively. Gaussian random fields are generated with the designated spatial correlation matrix estimated from image noise. Image noise’s spatial correlation can be interpreted by empirical variogram. The empirical variogram provides a description of how the data are related with distance. The empirical variogram’s mathematical representation is shown in Eq.1.

\[ \gamma(h) = \frac{1}{2|N(h)|} \sum_{N(h)} (z_i - z_j)^2 \]  

(1)

Where \(N(h)\) is the set of all pairwise Euclidean distances from location \(i\) to location \(j\). \(|N(h)|\) is the total number of pairs in \(N(h)\), and \(z_i\) and \(z_j\) are data values at location \(i\) and location \(j\) in field \(z\). \(h\) represents a lag distance, and \(h = i - j\). As two fields move away from each other, the lag distance increases along the direction shown in Figure 3.1. The overlapped region between top and bottom fields is the region evaluated by empirical variogram for spatial variance. As two fields move away from each other, the variance gradually reach the sill and maintain in sill.
Empirical variogram displays a random field’s spatial variability and can be estimated by a theoretical variogram. There are three common isotropic theoretical variogram models for approximating empirical variogram, which are listed in Eq.2-4.

**Spherical**

\[
\gamma(h) = \begin{cases} 
c \left[ 1.5 \left( \frac{h}{a} \right) - 0.5 \left( \frac{h}{a} \right)^3 \right] & \text{for } h \leq a \\
c & \text{for } h > a 
\end{cases} 
\]  
\hspace{1cm} (2)

**Gaussian**

\[
\gamma(h) = c \left[ 1 - \exp \left( -\frac{h^2}{a^2} \right) \right] 
\]  
\hspace{1cm} (3)

**Exponential**

\[
\gamma(h) = c \left[ 1 - \exp \left( -\frac{h}{a} \right) \right] 
\]  
\hspace{1cm} (4)

Where \( a \) represents range, \( c \) stands for sill, and \( h \) is the lag distance along one direction. These models can be chosen beforehand and plotted in variogram for the best fit for empirical variogram.

Figure 3.1 Shifted Matrix for Computing Empirical Variogram
Figure 3.2 displays an empirical variogram (red squares) computed by increasing lag distance $h$ between two random fields. In there, a theoretical variogram (blue line) with Gaussian model is generated to approximate the empirical variogram. This theoretical variogram model provides an estimation for range, sill and nugget. Range is the lag distance where variogram reaches the plateau as shown in Figure 3.2. Sill is the maximum spatial variance where variogram is at plateau. Nugget is the initial spatial variance and is set to zero. Image data is a 2D plane which can shift along x-axis and y-axis. To account for spatial variability along both directions, anisotropic variogram was used for range and sill estimation. Thus, variograms were used for estimations for both directions. One was along the x-axis and another was along y-axis. More details regarding variogram can be seen from Bohling (2005) and Cressie (1993).
Spatial correlation function relies on theoretical variogram which provides parameters constructing correlation functions. It determines how the random field is spatially correlated and can be used for generating random fields directly. Likewise, there are three common anisotropic correlation functions according to Zhu and Zhang (2013) shown in Eq.5-7.

\[
Spherical \quad \rho = 1 - 1.5 \sqrt{\frac{\Delta x^2}{\theta_x^2} + \frac{\Delta y^2}{\theta_y^2} + 0.5 \left( \frac{\Delta x^2}{\theta_x^2} + \frac{\Delta y^2}{\theta_y^2} \right)^3} \tag{5}
\]

\[
Gaussian \quad \rho = \exp \left( -\frac{h^2}{\theta^2} \right) \rho = \exp \left( -\frac{\Delta x^2}{\theta_x^2} - \frac{\Delta y^2}{\theta_y^2} \right) \tag{6}
\]

\[
Exponential \quad \rho = \exp \left( -2 \sqrt{\frac{\Delta x^2}{\theta_x^2} + \frac{\Delta y^2}{\theta_y^2}} \right) \tag{7}
\]

Where \( \theta_x \) is the range of the theoretical variogram along the x-axis (horizontal axis), \( \theta_y \) is the range of the theoretical variogram along the y-axis (vertical axis), \( \theta_x \) and \( \theta_y \) can be estimated from theoretical variogram. \( \Delta x \) and \( \Delta y \) are the horizontal and vertical lag distances between two observations in the space. In this paper, Gaussian correlation function is chosen for estimating correlation matrix.

### 3.4. Generating Independent Anisotropic Random Field

Generating independent random field is the premise for generating cross-correlated random fields and should be discussed first. Dietrich and Newsam (1997) and Wood and Chan (1993) used the circulant embedding method to generate an accurate stationary Gaussian random field via circulant matrix with Fast Fourier Transform (FFT). This method is highly efficient and can be implemented in several steps as shown in Figure 3.3. A Random field \( S \) has a correlation matrix \( R \) estimated by Eq.6. The correlation matrix \( R \) is in fact a Toeplitz matrix. According to the definition of circulant embedding, a Toeplitz matrix can be embedded into a circulant matrix where every row vector is a cyclic shift of the preceding row. Therefore, correlation matrix \( R \) is embedded into a circulant
A special property of circulant matrix is that a row vector contains full information of entire circulant matrix. This row vector can be directly used for random field generations. The generated row vector contains duplicate information due to the circulant nature. This can be resolved by simply removing and shortening the row vector’s length. The reduced row vector is then reshaped into a random field \( S' \) that has the same size as the original random field \( S \).

![Random Field Generation Flow Chart](image)

Concretely, an example in Figure 3.4 illustrates the random field generation process with circulant embedding. Suppose a \( 4 \times 3 \) random field \( S \) that consists a total of 12 nodes. Defining the shortest distance between two nodes equal to \( h \), the correlation between node 1 and 2 can be evaluated by setting \( \Delta x = h \) and \( \Delta y = 0 \) in Eq.6. Likewise, the correlation between node 1 and 4, node 1 and 5, node 1 and 6 are calculated by choosing \( \Delta x = 0 \) and \( \Delta y = h \), \( \Delta x = h \) and \( \Delta y = h \), \( \Delta x = 2h \) and \( \Delta y = h \), respectively. Therefore, evaluating node 1 with all other nodes can generate a total of 12 correlation coefficients. This procedure can be repeated for every node that eventually construct a \( 12 \times 12 \) correlation matrix containing any pairwise correlation coefficients.
Figure 3.4 A Random Field $S$ with $4 \times 3$ grid points

Below Eq.8 is the correlation matrix $R$ constructed by evaluating any two nodes in random field $S$ Figure 3.4. To generalize the case, a $M \times N$ random field $S$ would generate a $MN \times MN$ correlation matrix $R$. As spatial correlation is a function of distance, the resulted correlation matrix $R$ is a Toeplitz matrix in which each descending diagonal from left to right is the same value and the correlation between two locations are symmetrical with respect to diagonal.

$$R = \begin{pmatrix}
C_0 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} \\
C_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} \\
C_2 & C_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \\
C_3 & C_2 & C_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\
C_4 & C_3 & C_2 & C_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \\
C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\
C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & C_1 & C_2 & C_3 & C_4 & C_5 \\
C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & C_1 & C_2 & C_3 & C_4 \\
C_8 & C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & C_1 & C_2 & C_3 \\
C_9 & C_8 & C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & C_1 & C_2 \\
C_{10} & C_9 & C_8 & C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & C_1 \\
C_{11} & C_{10} & C_9 & C_8 & C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 
\end{pmatrix} \quad (8)

This correlation matrix encompasses two important properties. First, it contains spatial correlation of the random field $S$ which can be used as the prescribed correlation matrix for random
field generation. Second, it is also a Toeplitz matrix which can be embedded into a circulant matrix whose property simplifies the computational complexity dramatically.

The general principal of circulant embedding is to embed a Toeplitz matrix into a circulant matrix where the first row is defined by \( r = (c_0, \ldots, c_N, c_{N-1}, \ldots, c_1) \) and the remaining rows are a cyclic shift by one space of the preceding row. As a result, the size of circulant matrix becomes \((2MN - 2) \times (2MN - 2)\), and each row’s length is \((2MN - 2)\).

\[
C = \begin{bmatrix}
c_0 & c_1 & \cdots & c_N & c_{N-1} & \cdots & c_1 \\
c_1 & c_0 & \cdots & c_{N-1} & c_N & \cdots & c_2 \\
c_2 & c_1 & \cdots & c_{N-2} & c_{N-1} & \cdots & c_3 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
c_N & c_{N-1} & \cdots & c_0 & c_1 & \cdots & c_{N-1} \\
c_{N-1} & c_N & \cdots & c_1 & c_0 & \cdots & c_{N-2} \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
c_1 & c_2 & \cdots & c_{N-1} & c_{N-2} & \cdots & c_0
\end{bmatrix}
\] (9)

Eq.9 is the symbolic block circulant matrix \( C \) embedded from a Toeplitz matrix. It should be noted that the structure of correlation matrix, also a Toeplitz matrix with size of \((MN \times MN)\), is intact and preserved at the upper left corner of the circulant matrix shown in bold text. This structure ensures the generated random fields have accurate correlation. Additionally, every row vector has the same information considering the cyclic shift, which means a single row vector completely determine the entire matrix. This is significantly important for generating large random field in which a matrix can be reduced to a \((2MN - 2)\) row vector. Generating the random variables rely on the decomposition of block circulant matrix \( C \) denoted by Eq.10.

\[
C = F \Lambda F^H = (F \sqrt{\Lambda}) \left( F \sqrt{\Lambda}^H \right)
\] (10)

\[
F = e^{\frac{2\pi i j k}{N}} \quad j, k = 0, \ldots, N - 1
\] (11)
\[
\Lambda = \text{diag}(\lambda_i) = \begin{bmatrix}
\lambda_1 & 0 & \ldots & 0 \\
0 & \lambda_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \lambda_N \\
\end{bmatrix}, i = 1,2,\ldots,N
\] (12)

Where \(F^H\) is the conjugate transpose of \(F\), and \(F\) denotes the square Discrete Fourier Transfer (DFT) matrix. The formula of DFT matrix is provided in Eq.11. Square matrix \(\Lambda\) is a diagonal matrix with eigenvalues \(\lambda_i\) on the diagonal indicated by Eq.12. The eigenvalue of the circulant matrix can be conveniently calculated by \(\lambda_i = \text{fft}(r_i)\), where \(r_i\) represents an element in the row vector of the circulant matrix.

Notice that eigenvalues \(\lambda\) is found by taking the Fast Fourier Transform (FFT) of the first row of circulant matrix, which is highly efficient in computational complexity. The circulant matrix \(C\) can be further converted to the form \(C = (F\sqrt{\Lambda})(F\sqrt{\Lambda}^H)\). According to Cholesky decomposition, random variables \(X\) can be generated by multiplying \((F\sqrt{\Lambda})\) with independent random variables as shown in Eq.13.

\[
X = (F\sqrt{\Lambda}) \times Z
\] (13)

Where \(Z\) denotes the independent complex Gaussian random variables \(Z = \text{randn}(2MN - 2) + j \times \text{randn}(2MN - 2)\). Because \(F\sqrt{\Lambda}\) is a set of complex number via FFT, the Gaussian random variables \(Z\) should be also generated in the form of complex numbers. Since only the real number is needed, the imagery part of \(X\) is thus removed, and the length of \(X\) is shorten to keep the first \(MN\) elements. Following by a reshaping process, a Gaussian random field \(S'\) is simulated to original size of \(M \times N\).

In summary, given a field’s spatial correlation \(S\) with its corresponding correlation matrix \(R\) who must be positive definite, the algorithm that generates the Gaussian random field with circulant embedding method require following steps:
a) Vectorize \( S \) into a row vector \( r = (c_1, ..., c_N) \) that has \( N \) elements.

b) Extend the row vector to a circulant row \( r = (c_1, ..., c_N, c_N, c_{N-1}, ..., c_2) \).

c) Compute eigenvalue \( \lambda = \text{fft}(r) \)

d) Generate a vector \( Z = Z_1 + iZ_2 \) of dimension \( 2N - 1 \) with \( Z_1 \) and \( Z_2 \) being independent Gaussian random variables.

e) Compute random vector \( w = \frac{\text{fft}(\sqrt{\lambda})}{\sqrt{2N-1}} \times Z \) and keep the real part only.

f) Reduce vector \( w \) by taking the first \( N \) elements \( w = w(1:N) \)

g) Reshape vector \( w \) to the same size of \( S \) for a generated random field

3.5. Cross-correlated Random Fields Generation

Extending independent random field generations, the cross-correlated random fields generation should consider the cross-correlation between fields and autocorrelation within each field. Several researchers have contributed to this topic. Vořechovský (2008) simulated correlated random fields using Karhunen-Loève (KL) expansion. Zhu et al. (2017) generated cross-correlated random fields through Cholesky decomposition conditionally sampled with joint distribution using copula function. Zhao and Wang (2018) developed a cross-correlated random field generator based on Bayesian compressive sampling and KL expansion. This section aims to simulate cross-correlated random fields with circulant embedding method.

Discussing the cross-correlated random fields generations, the simplest case is to simulate two cross-correlated random fields simultaneously. Assume two sample images are correlated. The autocorrelation for each image and cross-correlation between two images can be derived from empirical variogram and modeled by theoretical Gaussian variogram. The theoretical variogram helps estimating the range of fluctuation \( \theta_x \) along horizontal direction and \( \theta_y \) along vertical direction. Assuming the random fields are anisotropic, it results a spatial correlation matrix for
each image field and cross-correlation matrix between two image fields. The combined spatial correlation matrix is shown in Eq.14, where $R_{11}$ and $R_{22}$ denote the autocorrelation matrix for random field 1 and random field 2, $R_{12}$ represents the cross-correlation between field 1 and 2.

$$R' = \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{22} \end{bmatrix}$$ (14)

The spatial correlation matrix can be further circulant embedded, using the approach described in Section 3.4, to form a circulant matrices in Eq.15, where $C_{11}, C_{22},$ and $C_{12}$ are corresponding circulant matrices to $R_{11}, R_{22},$ and $R_{12}$.

$$C' = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix}$$ (15)

Based on the property of circulant matrix in Eq.10, $C'$ can be decomposed into the form in Eq.16, where each circulant matrix is represented by $F\sqrt{\Lambda}F$. $F$ is the DFT matrix and $\Lambda$ is a diagonal matrix with eigenvalues in the diagonal. $\Lambda$ can be fast computed via FFT of the row vector from circulant matrix by $\Lambda = \text{diag}(\text{fft}(r))$.

$$C' = \begin{bmatrix} F\sqrt{\Lambda_{11}}F & F\sqrt{\Lambda_{12}}F \\ F\sqrt{\Lambda_{12}}F & F\sqrt{\Lambda_{22}}F \end{bmatrix}$$ (16)

With eigenvalue decomposition, the two random fields can be generated simultaneously with three independent random row vectors ($Z_{11}, Z_{12},$ and $Z_{22}$) shown in Eq.17.

$$\begin{bmatrix} X_{11} \\ X_{12} \\ X_{22} \end{bmatrix} = \begin{bmatrix} F\sqrt{\Lambda_{11}}F & F\sqrt{\Lambda_{12}}F \\ F\sqrt{\Lambda_{12}}F & F\sqrt{\Lambda_{22}}F \end{bmatrix} \odot \begin{bmatrix} Z_{11}' \\ Z_{12}' \\ Z_{22}' \end{bmatrix}$$ (17)

Where $Z_{11} = \text{randn}(n) + j \ast \text{randn}(n)$, $Z_{12} = \text{randn}(n) + j \ast \text{randn}(n)$ and $Z_{22} = \text{randn}(n) + j \ast \text{randn}(n)$ are three vectors of independent standard random variables including both real and imaginary parts. Moreover, the original standard deviation can be added to standard
normal variables by an element-wise multiplication in Eq. 18, where $\sigma_{11}$, $\sigma_{22}$, and $\sigma_{12}$ are standard deviation of field 1, field 2, and between both fields.

$$\begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{12}' & Z_{22}' \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \odot \begin{bmatrix} \sqrt{\sigma_{11}^2 - \sigma_{12}^2} & \sqrt{\sigma_{12}^2} \\ \sqrt{\sigma_{12}^2} & \sqrt{\sigma_{22}^2 - \sigma_{12}^2} \end{bmatrix}$$ (18)

The $X_{11}$, $X_{12}$ and $X_{22}$ in Eq. 17. are calculated by an element-wise multiplication between eigen components and Gaussian random variables, which result Gaussian random row vectors with the same size as the circulant row vector. Let $X_1 = X_{11} + X_{12}$ and $X_2 = X_{12} + X_{22}$ Keep the first $N$ variables from $X_1$ and $X_2$, and reshape $X_1$ and $X_2$ to the size of original image noise. In this way, two cross-correlated random fields $X_1$ and $X_2$ are simulated simultaneously with circulant embedding method that contain the prescribed auto/cross-correlation. One advantage of this approach over KL expansion is that due to no reduction in eigen components the generated random fields contain the exact original spatial correlation.

### 3.6. Simulations Study

This simulation study examines the circulant embedding method to generate two cross-correlated random fields based off two prescribed image noise fields. The simulation performance is evaluated by comparing the spatial auto/cross-correlation of the generated random fields with the original noise fields. It should be noted that three or more cross-correlated random fields can be generated simultaneously in this approach. However, for the sake of reducing computational burden, only the case of simultaneously generating two random fields is demonstrated in this study.

#### 3.6.1. Preprocessing

Two sample images of size of $1956 \times 1936$ pixels in Figure 3.5 (a) and (b) were taken by camera with different camera settings for a ceramic tile. Figure 3.5a was taken under high exposure setting making ceramic tile brighter, while Figure 3.5b was taken under low exposure setting resulting
darker ceramic tile. These two images are spatially correlated due to their similarity and can be simulated together with their auto/cross-correlation of each image.

![Image](image1.png) ![Image](image2.png)

Figure 3.5 Nominal Image and Noise

Every image contains random noise. Figure 3.5c is the extracted noise for image 1 and Figure 3.5d is the extracted noise for image 2. A common approach for image simulation is to simulate noise. The simulated noise is then added to the corresponding nominal image to form a new image. Simulating a pair of cross-correlated images requires both noises to be extracted for understanding their spatial auto/cross-correlations. Figure 3.5 (c) and (d) displays both noises extracted from two images with median filter.

The procedures to simulate an image are illustrated in Figure 3.6 image simulation flow chart. The original image first needs to be preprocessed including cropping and RGB-to-Grayscale. This is followed by extracting noise from original image. According to Gonzalez and Woods (2007), median filter is widely used in image processing for noise removal. A 5x5 median filter was used
to filter noise out to obtain nominal image. The nominal image has mean surface preserved but noise removed. The noise can be extracted by subtracting nominal image from original image. Subsequently, a random field simulation method is utilized to generate noise based off the spatial information from the original noise. Integrating simulated noise with nominal image, a new image is then simulated.

Figure 3.6 Image Simulation Flow Chart

Before modeling spatial correlation of images, it is necessary to investigate how the random noises are distributed in order to decide which noise model to choose. Noise 1’s histogram is shown in Figure 3.7a. Likewise, noise 2’s histogram is displayed in Figure 3.7b. Comparing noise 1 and 2’s histogram, it appears that they both are normally distributed and centered around zero. A Gaussian model for theoretical variogram is suitable for this case. However, noise 1 has less
variation than noise 2. This can be seen from Figure 3.7a where distribution is narrower and taller than the distribution from Figure 3.7b.

The noise autocorrelation and cross-correlation must be evaluated to construct correlation matrix. To do that, empirical variogram is plotted with respect to the lag distance. Then theoretical variogram with Gaussian model is drawn to estimate variogram’s range $\theta$. Figure 3.8 (a) and (b) illustrates the variogram of noise 1 along x-axis and y-axis. Likewise, (c) and (d) are variograms of noise 2. Variograms (e) and (f) are cross-variograms between noise 1 and 2 on both axes. These variograms illustrates spatial autocorrelation for each image and spatial cross-correlation between two images.
Figure 3.8 Original Noise Field Analysis at Horizontal and Vertical Direction
Spatial auto/cross correlation from variograms in Figure 3.8 can be quantified by their respective range and sill. Table 3.1 shows ranges and sills for field 1 and 2, and Cross variogram on both x-axis and y-axis. The displayed range and sill estimate the original noise fields’ auto/cross correlation. These ranges and sills will be used as the benchmark to compare against the generated random fields.

<table>
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<th>Y-axis</th>
<th>X-axis</th>
<th>Y-axis</th>
</tr>
</thead>
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<tr>
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<td>0.7281</td>
<td>2.3254</td>
<td>2.3154</td>
</tr>
<tr>
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<td>0.8227</td>
<td>4.2046</td>
<td>4.2273</td>
</tr>
<tr>
<td>12</td>
<td>0.7757</td>
<td>0.7926</td>
<td>0.1016</td>
<td>0.1142</td>
</tr>
</tbody>
</table>

Generating two cross-correlated random fields rely on the range and sill values in Table 3.1. Those ranges can be utilized as the parameters for constructing anisotropic correlation function which creates auto/cross correlation matrix for random field 1 and 2. As described in Section 3.5, these correlation matrices are Toeplitz matrices showing spatial correlation between any two points at any locations. Combining these auto/cross correlation matrices together creates a large combined correlation matrix as indicated in Eq.14. This matrix is then embedded into a circulant matrix. Continue the process mentioned in Eq.15-18 to generate two cross-correlated Gaussian random fields. Finally, the generated two random fields should have the same spatial correlation as the original noise field 1 and 2.

3.6.2. Simulation Results

This subsection discusses the performance of the proposed method that generates cross-correlated random fields. The key performance metric is the spatial auto/cross correlations that evaluates the similarity between the generated random fields and original noise fields. Figure 3.9a is the generated random field for noise 1 and Figure 3.9b is the generated random field for noise 2. They are expected to have the similar auto/cross correlation between the noise field 1 and 2.
To understand how similar the spatial correlations between original and generated random fields are, 100 samples were generated for statistical analysis in evaluating auto/cross correlations. Every sample includes two cross-correlated random fields whose correlations are evaluated by auto/cross variogram’s range and sill across both x-axis and y-axis. Table 3.2 enumerates the mean value of auto/cross variogram’s range between the original and simulated random fields. Based on the average of the 100 samples, the simulated random field has the similar anisotropic variogram’s range to the original noise fields.

Table 3.2 Range of Auto/cross Anisotropic Variogram

<table>
<thead>
<tr>
<th>VARIOGRAHM</th>
<th>DIRECTION</th>
<th>ORIGINAL MEAN</th>
<th>SIMULATION MEAN</th>
<th>RSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIELD 1</td>
<td>x-axis</td>
<td>0.4793</td>
<td>0.5093</td>
<td>0.0303</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.7281</td>
<td>0.7311</td>
<td>0.0031</td>
</tr>
<tr>
<td>FIELD 1&amp;2</td>
<td>x-axis</td>
<td>0.7757</td>
<td>0.7742</td>
<td>0.0161</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.7926</td>
<td>0.7933</td>
<td>0.0123</td>
</tr>
<tr>
<td>FIELD 2</td>
<td>x-axis</td>
<td>0.3313</td>
<td>0.4302</td>
<td>0.1088</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.8227</td>
<td>0.8220</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

Another metric commonly used to measure the error of simulations is the Residual Standard Deviation (RSD) that can be found in Eq.19.

\[ S_{res} = \sqrt{\frac{\sum(Y - Y_{sim})^2}{n - 2}} \]  

(19)
where $S_{res}$ is the residual standard deviation, $Y$ is the noise field values, $Y_{sim}$ is the simulated field values, and $n$ is the total number of samples. RSD in Table 3.2 indicates small range errors from both auto/cross variograms. Overall, the simulated random fields have accurate spatial auto/cross correlations in terms of the small errors in variogram’s range.

Auto/cross variogram’s sill is another important metric evaluating spatial correlation. The sill comparison between the original and simulated fields is shown in Table 3.3 based on 100 simulation samples. The average sill values of field 1, field 2, and cross-variogram are nearly identical to the originals. The RSD for simulated sill indicates small difference between original sill and simulated sill.

Table 3.3 Sill (Original vs. Simulation)

<table>
<thead>
<tr>
<th>VARIOGRAM</th>
<th>DIRECTION</th>
<th>ORIGINAL</th>
<th>SIMULATION MEAN</th>
<th>RSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>SILL 1</td>
<td>x-axis</td>
<td>2.3254</td>
<td>2.3202</td>
<td>0.0055</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>2.3154</td>
<td>2.32</td>
<td>0.0049</td>
</tr>
<tr>
<td>SILL 1&amp;2</td>
<td>x-axis</td>
<td>0.1016</td>
<td>0.1077</td>
<td>0.0065</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.1142</td>
<td>0.1078</td>
<td>0.0068</td>
</tr>
<tr>
<td>SILL 2</td>
<td>x-axis</td>
<td>4.2046</td>
<td>4.2163</td>
<td>0.0123</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>4.2273</td>
<td>4.2155</td>
<td>0.0126</td>
</tr>
</tbody>
</table>

The simulation results can also be illustrated in the form of variograms. 30 samples were generated and their theoretical variograms are displayed in one figure for comparison. Although only the cross-variograms are discussed in this comparison, the similar results also occur to variogram 1 and 2 respectively at both directions. Figure 3.10a displays the cross-variogram on x-axis between original noise field and simulated random fields, and Figure 3.10b displays the same comparison of cross-variogram on y-axis. The blue dash line in center is the original cross variogram, and the solid lines are simulated cross variograms. It can be seen that the original cross variogram is well centered among the simulated 30 examples, which indicate an accurate approximation of original cross variogram with adequate randomness. This result further validates
that the simulated random fields are cross-correlated on both directions and their cross-correlation is distributed around the original noise fields.

![Cross Variogram Comparison on X-axis and Y-axis](image)

Figure 3.10 Cross Variogram Comparison on X-axis and Y-axis

Finally, adding the simulated random fields to nominal images, a pair of cross-correlated images are simulated including spatial auto and cross correlations of the original two images. This image generation process can be repeated for producing mass images.

### 3.7. Comparison to KL Expansion

This paper has proposed an approach to generate cross-correlated random fields with circulant embedding method. In this section, the proposed approach is compared against KL expansion method to verify the proposed approach’s performance benefits. KL expansion has been a well-known method used for random field generations considering spatial correlation. As investigated by Vořechovský (2008), KL expansion method can be used to generate cross-correlated random fields that are in relatively small size. Mathematically, circulant embedding method’s $O(n \log n)$ is more efficient than KL expansion method $O(n^3)$, which allow circulant embedding method to generate very large random fields with less computational penalty. However, this section aims to compare the two methods’ accuracy in approximating spatial correlation. The discrepancy of computational efficiency is minimized by generating relatively small random fields. It should be
noted that another method Cholesky Decomposition is not included in this comparison because it is computationally too intensive and suitable for generating very small random fields.

KL expansion method employs eigen decomposition on a spatial correlation matrix generated from a random field. As the random field’s size increases, the corresponding correlation matrix become significantly large. It would be difficult to eigen decompose a large correlation matrix with regular computer. Therefore, this comparison used two original random fields in a relatively small size with $97 \times 97$ pixels respectively in Figure 3.11, which output a correlation matrix with an manageable size of $9409 \times 9409$ for eigen decomposition. Although circulant embedding method does not have such computational burden and can process much larger random fields, the same $97 \times 97$ random field was used for a fair comparison.

To illustrate the simulation results, each simulated random field is computed to show its variogram. A total of 30 examples of random field 1 and 2 were simulated for each generation method. The resulted variograms (i.e., field 1, field 2 and cross field) along y-axis are shown in Figure 3.12. The resulted variograms along x-axis are similar and omitted in this comparison. The first column of figures are with KL expansion method, and the second column of figures are with circulant embedding method. Figure 3.12 (a) and (d) is for random field 1 variograms, Figure 3.12
(d) and (e) is for random field 2 variograms, and Figure 3.12 (c) and (f) is for cross variograms. Within each figure, the blue dash line represents the variogram of original random fields, and the remaining solid lines are 30 random simulations.

The 95% KL expansion method only accounts for 95% data variance so that the simulated variograms is vertically shifted away from original variogram with noticeable smaller sill. This can be seen from Figure 3.12 (a) and (b). In contrast, for circulant embedding method, the variograms for simulated random field 1 and 2 are well centered around original variogram due to no reduction in data variance indicating a well approximation in spatial autocorrelation. This can be seen from Figure 3.12 (d) and (e) respectively. Moreover, the simulated cross-variograms for KL expansion and circulant embedding methods, in Figure 3.12 (c) and (f) respectively, are both centered around originals. However, the simulated variograms for KL expansion method appears to have more noise in approximating original variogram than circulant embedding method.
The resulted variogram’s range and sill can be estimated to evaluate spatial correlation quantitatively. Table 3.4 provides range comparisons between the two simulation approaches. Both RSD and Residual Percentage Error (RPE) are calculated to estimate the simulation’s range accuracy, and the better values are emphasized in bold text. The results are based on 100 simulated random fields. In addition to 95% KL expansion, the 75% KL expansion is also included in this comparison to investigate the effect of data variance to the performance.

It can be seen from Table 3.4 that circulant embedding method outperforms KL expansions with smaller RSD for the variogram’s ranges except at field 2 x-axis where 95% KL expansion performs better. This can be caused by statistical errors in the simulation but does not change the fact that circulant embedding method is more accurate in generating cross-correlated random fields.
with similar variogram’s range. Additionally, 95% KL expansion performed better than 75% KL expansion with smaller RPE due to the greater percentage of data variance the PCA explains.

Furthermore, Table 3.5 presented the sill comparisons for the same 100 simulated random fields. This comparison is based on the combined sill, where sills along x-axis and y-axis are combined together with equation $\text{sill} = \sqrt{\text{sill}_x} \times \sqrt{\text{sill}_y}$, to explain the overall data variance along both directions. It can be observed that circulant embedding outperformed KL expansions with dramatic smaller RPE at both field 1 and 2 sills. For cross-variogram, 75% KL expansion has the smallest RPE value but is only marginally better than others. This is due to simulation errors which may be fixed with larger sample size. Overall, circulant embedding method shows advantage over KL expansion with significantly higher accuracy in approximating spatial auto/cross correlations in terms of variogram’s range and sill. The results of range and sill comparison are aligned with the results of variograms providing stronger evidence that the proposed approach can simulate cross-correlated random fields fast and accurately.

Table 3.4 Range Comparison

<table>
<thead>
<tr>
<th>Variogram</th>
<th>Directions</th>
<th>Residual Standard Deviation</th>
<th>Residual Percentage Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Circulant</td>
<td>95% KL</td>
</tr>
<tr>
<td><strong>Field 1</strong></td>
<td>x-axis</td>
<td>0.0432</td>
<td>0.1148</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.015</td>
<td>0.1122</td>
</tr>
<tr>
<td><strong>Field 2</strong></td>
<td>x-axis</td>
<td>0.806</td>
<td>0.6699</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.4768</td>
<td>13.7599</td>
</tr>
<tr>
<td><strong>Cross</strong></td>
<td>x-axis</td>
<td>0.037</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td>y-axis</td>
<td>0.0173</td>
<td>0.0794</td>
</tr>
</tbody>
</table>

Table 3.5 Sill Comparison

<table>
<thead>
<tr>
<th>Variogram</th>
<th>RESIDUAL STANDARD DEVIATION</th>
<th>RESIDUAL PERCENTAGE ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Circulant</td>
<td>95% KL</td>
</tr>
<tr>
<td><strong>FIELD 1</strong></td>
<td>0.1101</td>
<td>0.4816</td>
</tr>
<tr>
<td><strong>FIELD 2</strong></td>
<td>0.0969</td>
<td>0.3497</td>
</tr>
<tr>
<td><strong>CROSS</strong></td>
<td>0.0742</td>
<td>0.0866</td>
</tr>
</tbody>
</table>
3.8. Conclusion

The proposed approach had been proved to be effective to fast generate multiple random fields with exact spatial correlations. The inclusion of circulant embedding method in generating cross-correlated random fields can bring three benefits. First, it allows large random fields to be generated due to the low memory occupancy. Computer memory is saved by reducing a large correlation matrix into a row vector, which allow more data to be simulated in a limited memory space. Second, this approach is fast and achieved computational efficiency to $O(n \log n)$, which is a significant improvement over KL expansion’s $O(n^3)$. The benefits of computational efficiency allow the proposed approach to quickly generate several large (e.g., $1956 \times 1936$) cross-correlated random fields simultaneously. Third, the proposed approach generates random fields with exact spatial auto- and cross-correlation. Despite correlation matrix is reduced to a row vector, the information of spatial correlation is entirely retained in the circulant row. As a result, the simulated random fields have no data variance reduction and the generated random fields contain the exact auto- and cross- correlation of the original random fields. This framework can be effectively utilized in engineering applications such as Image-based statistical process control that requires simulating numerous images to evaluate control chart’s performance metric.
3.9. References


Cressie, N., 1993, Statistics for spatial data, Wiley Interscience


4. MULTI-IMAGE MONITORING WITH MPCA FOR STATISTICAL QUALITY CONTROL

Image-based statistical process control is increasingly gaining attention for use in advanced manufacturing systems. Images contain information-rich data regarding a product’s critical quality characteristics (e.g., dimensions, texture, color) and spatiotemporal information (e.g., defect locations and occurrence times). As a result, monitoring images in a mass production environment has profound impact on manufacturing quality. This paper proposes a new framework for image monitoring with control charts that aims to improve the overall performance in detecting a variety of surface-related process shifts. Specifically, multiple images are acquired under different capturing parameters for the same part at the same time. To illustrate this framework, this paper considers two multi-image control chart approaches: 1) Fusing multiple images together with multilinear principal component analysis and monitoring with a single-image control chart and 2) Using a combined single-image control chart. A rigorous two-image simulation study is conducted, based upon real cross-correlated images, to compare the performances of these two multi-image control charting approaches to single-image control charts. The results indicate that multi-image control charts outperform individual control charts when multiple shifts are considered. Another two-image simulation study investigates the effect cross-correlations levels have on the two multi-image control charts’ performances. This study indicates that at low levels of cross-correlation the two multi-image charts have equivalent performances and at high levels of cross-correlation the fused-image chart outperforms the combined image chart. In addition, a case study with real images is conducted to demonstrate the proposed multi-image monitoring framework’s ability in detecting shifts.

Keywords: Fused image with MPCA, ARL, MEWMA control chart, Spatial cross-correlation
4.1. Introduction

Machine Vision Systems (MVSs) are often implemented in advanced manufacturing systems for product inspection. These systems consist of imaging devices (e.g., digital cameras, X-ray cameras), sensors, and light sources used for acquiring images of manufacturing parts. Computer software processes, analyzes, and measures pre-defined characteristics within the images to make decisions regarding the part’s quality. This image-based approach has become increasingly popular because of its ability to capture information such as surface defects, product finish/colors, and geometric features without physical contact. Furthermore, modern computer systems make this process highly efficient and cost effective.

Incorporating MVSs with statistical process control (SPC) allows for the acquired images to be monitored through a control chart to detect process shifts. Image-based control charts can be very sensitive to small changes within an image and can quickly signal that a process shift has occurred. In addition, image-based control charts can provide valuable diagnostic information regarding a shift’s magnitude, location, and occurrence time. However, an image-based control chart’s ability to detect specific shift types/locations relies heavily on image capturing parameters. These parameters can be divided into two categories: 1) environmental conditions (e.g., lighting) and 2) imaging device parameters (e.g., sensor type, settings, locations). For instance, high exposure images enhance an object’s brightness, while low exposure images emphasize small surface details. Depending on the nature of a shift, a higher exposure image may be more sensitive to a shift than a lower exposure image, and vice-versa.

Optimal capturing parameters for image-based SPC are not constant and vary as a function of shift types and locations. For example, Figure 4.1 shows two images of a part with different contrast levels. This part contains shifts at two different locations, identified as shift A and shift B.
in the figure. The image in Figure 4.1a is sensitive to shift A and less sensitive to shift B, while the image in Figure 4.1b is sensitive to shift B and less sensitive to shift A. This example illustrates that information regarding a shift within an image is affected by the aforementioned capturing parameters. Since numerous shift types/locations can occur within a process, it becomes a challenge to configure capturing parameters for image-based SPC that are robust to all shifts.

To overcome this challenge, this paper proposes a multi-image monitoring framework, that incorporates multiple images acquired under different capturing parameters. In this paper, this framework is illustrated using two multi-image control chart approaches: 1) Fusing multiple images together into one image with multilinear principal component analysis (MPCA) and monitoring with a single-image control chart and 2) Using combined single-image control charts. Both control charting approaches are demonstrated using multivariate exponential weighted moving average (MEWMA) charts.

The remainder of this paper is organized as follows. Section 4.2 provides a background on the modelling approaches that constitute the two control charting approaches and their performance analysis. This includes a discussion on multivariate control charts, combined control charts, image
fusion with MPCA, and simulating cross-correlated images to acquire control charting performance metrics. Section 4.3 conducts a simulation study that evaluates and compares performances between single image MEWMAs, single-image combined MEWMA (simultaneously monitoring multiple single-image MEWMAs), and fused-image MEWMA control charts. Section 4.4 provides an experimental study to examine the applicability of the proposed control chart in monitoring a manufactured part. Lastly, conclusions and a discussion for future research is provided in Section 4.5.

4.2. Background

This section introduces the key modelling concepts used to develop and analyze the control charts considered in this paper. First an overview of multivariate and combined control charts provides the basis for multi-image monitoring and the single-image combined MEWMA, respectively. Second the image processing methodologies are introduced for the proposed framework that are used to extract image features for SPC. The third subsection introduces image simulation approaches to generate cross-correlated images to estimate image-based control chart performance metrics.

4.2.1. Multivariate and Combined Control Charts

The $T^2$ control chart, developed by Hotelling (1947), provides a statistical measure to represent variations of multivariate data. Relying on correlations within and across multiple variables, $T^2$ control charts are implemented in manufacturing QC systems to detect large shifts. $T^2$ control charts has been applied to image data, for example Tong et al. (2005) used them to simultaneously monitor the number of defects and defect clusters on a wafer map in an integrated circuit. Similarly, the multivariate exponential weighted moving average (MEWMA) chart and multivariate cumulative sum (MCUSUM) chart were developed by Lowry et al. (1992) and Crosier (1988),
respectively, to detect medium to small sustained shifts. A competitive alternative is the multivariate generalized likelihood ratio (MGLR) chart that is sensitive to both small and large shifts, as discussed by Wang and Reynolds (2013), He et al. (2016), and Lee et al. (2017). However, the MGLR chart is more computationally exhaustive than an MEWMA or MCUSUM chart. In this paper, an MEWMA chart is used to monitor images due to its computational efficiency and good performance in detecting small shifts.

Control charts are often monitored simultaneously, resulting in a combined chart, which has a performance that is a mixture of its “parent” charts’ performances. For example, Capizzi (2010) took advantage of the performances of Shewhart (sensitive to large shifts) and EWMA (sensitive to small shifts) charts by monitoring them simultaneously, creating a combined Shewhart-EWMA chart that is sensitive to both small and large shifts. Wang and Reynolds (2013) proposed a combined MEWMA chart to monitor multivariate data. In their work, two individual MEWMA charts were utilized with different weight parameters. Their combined MEWMA demonstrated a wider range of sensitivity to detect process shifts than individual MEWMAs.

In this paper, a single-image combined MEWMA (referred to herein as simply a combined-image MEWMA) is implemented to monitor multiple images of the same part at the same time. Each image is monitored by a single-image MEWMA chart, which are all given the same weight parameter. When any single-image MEWMA chart signals, the combined MEWMA identifies a process shift.

4.2.2. Fusing Images with MPCA

The proposed fused-image MEWMA monitors multiple images of the same part simultaneously by using an individual MEWMA chart. This is achieved by fusing two images into one image with the implementation of MPCA. Image data generally consist of millions of variables (i.e., pixels)
whose intensity values range from 0 to 255. Dealing with this many variables becomes a challenge for multivariate control charts. However, there exists significant amounts of data redundancy within an image, which can be exploited to reduce to the amount of data that needs to be considered. Duchesne et al. (2012) thoroughly reviewed approaches to reduce the redundancy within data by extracting features for dimension reduction. Appropriate image feature extraction techniques are problem dependent. For example, textural features can be extracted by using the grey level co-occurrence matrix (GLCM), as investigated by Ganesan et al. (2004). Niemi et al. (1999) utilized the power spectrum obtained from a 2D Fourier transform to extract textural features of the froth flotation of minerals. Mohanaiah et al. (2013) extracted textural features from image through a wavelet transform.

Yu and MacGregor (2003) implemented both principal component analysis (PCA) and partial least-square (PLS) regression on color images for feature extraction and employed the extracted data for on-line monitoring in the snack food industry. Lu et al. (2008) introduced an MPCA framework to extract features and reduce dimensions of 2D/3D images. MPCA decomposes a multi-dimensional image (e.g., color image) to obtain eigentensors and projects the image onto the reduced tensor space for dimension reduction. MPCA is superior to PCA because it avoids image vectorization that disrupts an image’s spatial information. Furthermore, the computational complexity of eigen-decomposition in MPCA is only $O(2n)$ compared to PCA’s $O(n^2)$, which is a significant improvement in efficiency. Yan et al. (2014) demonstrated color image monitoring with MPCA. They implemented MPCA to extract features from color images and monitored the extracted data with $T^2$ and $Q$ (MPCA model residuals) charts to detect process shifts. However, the uses of MPCA for cross-correlated images have yet to be investigated. In this paper, to
demonstrate the multi-image monitoring framework, MPCA is used for dimension reduction and image fusion.

For multiple image monitoring, MPCA can fuse two or more images together while preserving most the original images’ information. For multi-dimensional image data, MPCA has been proven to effectively reduce the number of data dimensions by extracting key features from high dimensional images. It should be noted that this paper does not focus on comparing different techniques for dimension reduction. MPCA was chosen to demonstrate the proposed multi-image monitoring framework because its proven ability to handle high dimensional image data efficiently and preserve spatial information.

Figure 4.2 Third-order Tensor Decomposition

An example of using MPCA for extracting features from high dimensional image data is illustrated in Figure 4.2. A $1000 \times 900 \times 3$ image (i.e., 3rd-order tensor space) is decomposed and projected onto a 2nd-order tensor space. Mode-1 of this 3rd-order tensor space has 1000 slices (i.e., matrices), mode-2 has 900 slices, and mode-3 has 3 slices. To decompose Mode-1, this image is unfolded into a $1000 \times 2700$ tensor space. Suppose that the image in Figure 4.2 is one observation from a set of $m$ images. Stacking all $m$ unfolded mode-1 tensor spaces allows for a $1000 \times 1000$ covariance matrix to be estimated. Performing an eigen-decomposition on the $1000 \times 1000$
covariance matrix obtains the corresponding eigentensors. By selecting the $k$ (e.g., $k = 100$) eigentensors with the largest eigenvalues, the first mode is reduced to $100 \times 2700$ eigentensors.

The same procedures can be executed for the 2nd and 3rd modes, reducing them to a $90 \times 3000$ eigentensors and $1 \times 900000$ eigentensors, respectively. Subsequently, a full projection onto the eigentensors produces an $100 \times 90 \times 1$ image from the original $1000 \times 900 \times 3$ image. Depending on the number of eigentensors selected for each mode, the projected tensor space retains different amounts of information regarding the variation and spatial correlation of the original tensor space.

Figure 4.2 illustrates an MPCA application for image data that is a 3rd-order tensor space. Since a grayscale image is only a 2nd-order tensor space, MPCA can fuse two or more grayscale images by “stacking” them together. This technique offers immense benefits for multi-image monitoring where multiple images can be stacked together and subsequently monitored as one image.

4.2.3. Cross-correlated Image Simulation

In this paper, the performances of a fused-image control chart and a combined-image control chart are compared using the well-known average run length (ARL) performance metric. Given the complexity of the control charts being implemented in this study, ARL estimates are acquired through simulation. However, to estimate ARLs that accurately represent the performances of the control charts considered in this paper requires the ability to simulate cross-correlated images efficiently and accurately.

The major principal for simulating images, as mentioned by Gonzalez and Woods (2007), is to simulate noise and add the simulated noise to an image’s mean surface. Noise is always present in digital images and can occur during image acquisition and/or transmission. Boyat and Joshi (2015) reviewed common types of image noise and described their unique characteristics with
corresponding mathematical models. For control charting, Megahed et al. (2012) simulated noise by assigning a Poisson distribution to every pixel and used the simulated images to calculate the median run length (MRL) control chart performance metric. Koosha et al. (2017) used a Gaussian distribution for noise generation to compute ARLs for their control charts. Gaussian and Poisson models are the two popular distribution models for generating random noise; however, spatial correlation is not formally considered through these noise generation processes.

To assess the performance of the multi-image monitoring framework, noise is generated to produce simulated images that are spatially cross-correlated, where each individual image is spatially auto-correlated. Spatial-statistics models can be used to generate spatially auto- and cross-correlated data. In spatial-statistics, variograms (Cressie, 1992) are often used to estimate spatial correlations of random fields. A variogram’s range represents the distance that a random field is correlated and a variogram’s sill estimates the maximum variability within a random field. Zhu and Zhang (2013) summarized various theoretical variogram models (spherical, exponential, gaussian, etc.) to approximate a variogram’s range and sill to estimate spatial auto- and cross-correlation of random fields.

Unlike conventional image simulation, the images generated in this paper need to satisfy the following requirements: 1) They are large enough to mimic high definition images used in manufacturing systems; 2) They contain spatial auto- and cross-correlation; and 3) They are generated reasonably fast with minimal computational effort. To achieve these requirements, a fast and accurate cross-correlated image simulation using the circulant embedding method is used in this paper. More specifically, this simulation method is used to simultaneously generate two images with accurate spatial auto- and cross-correlation. Due to the adoption of circulant embedding, this method is very efficient as its computational complexity is $O(n \log n)$. This
method also requires less computer memory than other approaches, as it only uses a row of the circulant embedded covariance matrix instead of the entire covariance matrix, which is beneficial for simulating large images. The fast speed and low memory requirements make this method ideal for simulating massive amounts of large cross-correlated images, which is essential for estimating a multi-image control chart’s performance. Detailed algorithms for circulant embedding can be found in Dietrich and Newsam (1997), Chan and Wood (1999), Gneiting et al. (2010), and Park and Tretyakov (2015).

4.3. Simulation Study

In this paper, an MEWMA chart was chosen for its good performance to detect small shifts in multivariate SPC as indicated by Prabhu and Runger (1997). The goal of this simulation study is to assess the performance gains, if any, when applying a multi-image monitoring control chart. To accomplish this goal, a set of simulation studies were created to examine these control charts. These simulation studies are based upon two real-world images of a part taken under different image device parameters. As a result, these two images contain different (yet correlated) information regarding the part. Using these images, two out-of-control fault scenarios were simulated to accomplish the following three objectives.

Objective 1 – Verify that a single-image control chart’s ability to detect shifts within images is affected by capturing parameters (i.e., discussion revolving around Figure 4.1). This is accomplished by monitoring the images with two separate single-image MEWMAs and comparing their performances in detecting two different process shifts.

Objective 2 – Demonstrate the advantages of the proposed multi-image monitoring framework. This demonstration is achieved by comparing the performances of the previous two single-image MEWMAs to the combined-image and fused-image MEWMAs.
Objective 3 – Determine the effect of spatial cross-correlation for the multi-image monitoring control charts. This objective is accomplished by modifying the real-world images to create two different cross-correlation cases: 1) Case 1: High cross-correlation and 2) Case 2: Low cross-correlation. The performances of the combined-image and fused-image MEWMAs are compared under these two cases.

The following subsection describes the design and implementation procedures for the simulation studies used to accomplish these three objectives. The results of these simulation studies to address the first two objectives and the third objective are provided in Sections 4.3.2 and 4.3.3, respectively.

4.3.1. Simulation Design and Implementation

As mentioned previously, three different types of image-based control charts will be studied in this simulation. Figure 4.3 presents a visual summary of these charts, which are: a) Single-image MEWMA: An image is monitored by a single-image MEWMA chart; b) Combined-image MEWMA: Two single-image MEWMAs are monitored simultaneously; and c) Fused-image MEWMA: Two images were fused into one image by MPCA that is monitored by an MEWMA chart. It should be noted that MPCA is used for all three control charts because it can effectively reduce the dimensionality for both single- and multi-dimensional images (e.g., fused images). In addition, having all three control charts rely on MPCA allows for a fair comparison between the use of single-image, combined-image, and fused-image control charts.
Figure 4.3 Summary of Image-based Control Charts Considered in this Paper

The image-based MEWMA simulation design and implementation strategy, used in this paper, is displayed in Figure 4.4 and consists of three stages: random field generator, historical data analysis, and Phase II analysis. It should be noted that the underlined and bolded steps in Figure 4.4 only apply to the fused-image MEWMA.
Figure 4.4 Image-based MEWMA Simulation Design and Implementation Strategy

In the first stage, a random field generator is developed from two real-world images to create simulated images, for both historical data analysis and Phase II analysis. The original images are cropped to remove unnecessary information and converted to grayscale (RGB-to-grayscale). For each image, a median filter is applied to the grayscale image to remove noise. The filtered images are referred to as the nominal images, which are later subtracted from the original images to extract noise. The extracted noise is used to create the random field generator with the approach mentioned in Section 4.2.3. Specifically, the random field generator creates two cross-correlated random fields (i.e., one for each simulated image), which are added to their respective nominal images, creating two simulated images.

In the second stage, for creating single-image MEWMAs, two MPCA models are constructed from historical data, one for each image. The historical data is created using the random field generator, developed in the first stage, and consists of \( m \) simulated observations of both images, referred to as image-sets 1 and 2. To mimic process noise, each pair of simulated images (i.e., the
\( i \)th observations from images-sets 1 and 2) are randomly perturbed vertically and randomly perturbed horizontally by the same amount. This process is followed by resizing each image to a smaller size, reducing the computational load. For both image-sets, the \( m \) resized images are eigen-decomposed with MPCA to obtain eigentensors and eigenvalues. By selecting the \( k \) eigentensors with the largest eigenvalues from both mode-1 and -2 tensor spaces, a resized image can be projected onto the selected eigentensors to create a \( k \times k \) image. The projected images contain key features from the original images, which are used to develop two MEWMA charts by estimating the mean vector and covariance matrix from the projected images in image-sets 1 and 2.

In the second stage, for creating a fused-image MEWMA, the same process described in the previous paragraph is followed, except that the two image-sets are stacked together to create a 3\(^{rd}\)-order tensor space. Collecting \( 2m \) stacked images allows for eigen-decomposition to the 3\(^{rd}\)-order (i.e., up to mode-3) eigentensors and eigenvalues. By selecting 1 eigentensor on mode-3 and \( k \) eigentensors of the largest eigenvalues on mode-1 and -2 followed by a projection, image-sets 1 and 2 are fused into \( m \times k \times k \) images. These fused images are used to develop an MEWMA chart by estimating the mean vector and covariance matrix.

In the third stage, Phase II analysis is conducted to evaluate the control charts’ performances. Simulated images are preprocessed (i.e., image perturbation and resizing) and projected onto the eigentensors selected from the second stage. The projected images are monitored by their respective MEWMA chart, where their performances are assessed through the well-known ARL performance metric. In Phase II, the process shifts are created within images to evaluate the out-of-control ARLs for various shift areas, intensities, and locations.
Phase II analysis introduces process shifts for estimating out-of-control ARLs. Figure 4.5 illustrates how an image with a process shift is synthesized from a nominal image, a generated random field, and a process shift layer. The process shift layer is simulated by creating an irregular circle defined by a center location, a radius with variations, and a pixel intensity uniformly distributed in the irregular circle (mean shift). In this way, process shifts within the image can be specifically defined and quantified at various levels. It should be noted that each pair of generated images have the same process shift layer, but their nominal images and random fields are different.

4.3.2. Performance Comparisons at Different Shift Locations

Two images of a ceramic tile, shown in Figure 4.6, were captured with a digital camera. These two images (i.e., image 1 and image 2) were taken under different capturing parameters (shown in Table 4.1) and exhibit noticeable differences. The ceramic tile has patterns differentiated by black and white colors. Comparing image 1 with image 2, image 1 is darker and blurrier, and image 2 is brighter and sharper. To investigate the control charts’ sensitivity to process shifts at different locations, shift location A was set on the white area, and shift location B was set on the black area.

Table 4.1 Camera Capturing Parameters for Image 1 and 2

<table>
<thead>
<tr>
<th></th>
<th>Image 1</th>
<th>Image 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture</td>
<td>5.6</td>
<td>8</td>
</tr>
<tr>
<td>Shutter speed</td>
<td>1/60</td>
<td>1/250</td>
</tr>
<tr>
<td>ISO</td>
<td>800</td>
<td>3200</td>
</tr>
</tbody>
</table>
The two images were preprocessed (i.e., cropping and RGB-to-Grayscale), resulting in \(876 \times 936\) images. Their noise was extracted with a \(5 \times 5\) median filter and used to generate two cross-correlated random fields. The two random fields were considered to be cross-correlated along both horizontal x-axis and vertical y-axis. To account for spatial auto- and cross-correlation along two directions, an anisotropic variogram Gaussian model was used to estimate ranges and sills for random field generations, where the nugget was disregarded and set to zero. Overlaying both nominal images with their respective generated random fields creates a pair of simulated cross-correlated images (i.e., simulated images 1 and 2). This process was used to create images for historical data analysis and Phase II analysis. Once generated, each image was perturbed (vertically and horizontally) by uniformly distributed distances between +/- 3 pixel spaces, and then resized to \(88 \times 94\). For the historical data analysis, 500 pairs of images were generated to produce image-sets 1 and 2, which were eigen-decomposed with MPCA to obtain eigentensors and eigenvalues. By selecting \(k = 8\) eigentensors with the largest eigenvalues in mode-1 and -2 followed by a projection, both image-sets were projected to \(8 \times 8\) eigentensors that are directly monitored by two separate single-image MEWMAs. In addition, these image-sets were stacked.
together to build a 3rd-order eigen space. By selecting only 1 eigentensor in mode-3 and \( k = 8 \) eigentensors in mode-1 and -2, each pair of images were fused into an \( 8 \times 8 \) eigentensor that was monitored by a fused-image MEWMA. A summary of the MPCA models created, including the percent of variance retained, is shown in Table 4.2.

<table>
<thead>
<tr>
<th>MPCA Model</th>
<th>Mode</th>
<th>Number of Original Eigentensors</th>
<th>Number of Selected Eigentensors</th>
<th>Percent Retained Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image 1</td>
<td>1</td>
<td>88</td>
<td>8</td>
<td>0.4234</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>94</td>
<td>8</td>
<td>0.4240</td>
</tr>
<tr>
<td>Image 2</td>
<td>1</td>
<td>88</td>
<td>8</td>
<td>0.5190</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>94</td>
<td>8</td>
<td>0.5215</td>
</tr>
<tr>
<td>Fused Image</td>
<td>1</td>
<td>88</td>
<td>8</td>
<td>0.4403</td>
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<tr>
<td></td>
<td>2</td>
<td>94</td>
<td>8</td>
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<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0.9273</td>
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</tbody>
</table>

In this simulation, single-image MEWMA 1 monitors image 1 and single-image MEWMA 2 monitors image 2; the combined-image MEWMA monitored both image 1 and 2 together through two simultaneous single-image MEWMAs; and the fused-image MEWMA monitors fused images projected from image 1 and 2. It should be noted that all MEWMA chart’s tuning parameters were set to 0.2. A total of 10,000 simulations were conducted for all ARL estimates, and each control chart’s in-control ARL value was set to approximately 300. This means in-control ARLs estimates relied upon simulating 3,000,000 images for the single-image MEWMAs and 6,000,000 images for the combined- and fused-image MEWMAs, which required significant computational effort. To reduce the computational runtime, the simulations employed GPU processing, with a dedicated GPU (NVIDIA GTX Series), for its superb efficiency in matrix computations. Table 4.3 provides ARL comparisons of the two single-image MEWMAs.
Table 4.3 ARL Comparison of Single-image MEWMAs

<table>
<thead>
<tr>
<th>Shift Size</th>
<th>Shift Location A</th>
<th>Shift Location B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Image 1</td>
<td>Image 2</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radius</td>
<td></td>
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<tr>
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<td>10</td>
<td>3</td>
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<tr>
<td></td>
<td>5</td>
<td>27</td>
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<td></td>
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</tr>
<tr>
<td>0</td>
<td>0</td>
<td>300</td>
</tr>
</tbody>
</table>

In Table 4.3, the columns named “Image 1” and “Image 2” refer to single-image MEWMA 1 and single-image MEWMA 2, respectively. ARLs were estimated at various shifts (combinations of intensities and radii) for shift locations A and B. For every shift, smaller ARLs are given in bold font to emphasize better performance. The displayed ARL values are rounded to the nearest integer.

Based on Table 4.3, single-image MEWMA 1 uniformly outperforms single-image MEWMA 2 to detect shifts at location A, except at the largest shifts where both charts have indistinguishable
performances. At shift location B, single-image MEWMA 2 almost uniformly outperforms single-image MEWMA 1. For the shift with an intensity of 15 and a radius of 10 their performances are indistinguishable. In addition, for the smallest shift with a positive shift intensity, single-image MEWMA 1 outperforms single-image MEWMA 2. This could be due to simulation error, but what is most important is the trend that single-image MEWMA 2 is more sensitive to shift location B. As mentioned earlier, the difference between image 1 and image 2 was the capturing parameters. Therefore, Table 4.3 provides the evidence that capturing parameters affect a control chart’s ability to detect shifts within images (Objective 1, from Section 3). It also suggested that both images are needed to effectively detect both shifts at shift locations A and B, verifying the need of the proposed multi-image monitoring framework.

To demonstrate the proposed multi-image monitoring framework, the fused-image MEWMA and combined-image MEWMA are used to monitor multiple images simultaneously in contrast to monitoring a single image. In this simulation, the fused-image MEWMA utilizes a single-image MEWMA chart created from fusing two image-sets. The combined-image MEWMA is comprised of two single-image MEWMAs whose individual control limits were chosen to give the combined-image MEWMA the desired in-control ARL ($\cong 300$). This was accomplished by ensuring individual single-image MEWMA had approximate the same in-control ARL when used alone, as suggested by Wang and Reynolds (2013).
Table 4.4 ARL Comparison for Single- and Multi-image MEWMAs

<table>
<thead>
<tr>
<th>Shift Size</th>
<th>Shift Location A</th>
<th>Shift Location B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intensity</td>
<td>Image 1</td>
</tr>
<tr>
<td>-20</td>
<td>10</td>
<td>2</td>
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<tr>
<td>5</td>
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<tr>
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<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

The estimated ARLs for the fused-image, combined-image, and single-image MEWMAs are given in tabular form in Table 4.4 and illustrated in Figure 4.7. In Table 4.4, the column named “Combined” refers to combined-image MEWMA, and the column name “Fused” represents the fused-image MEWMA. The smallest ARLs, for every shift, are presented by bold for shift locations A and B.
The ARL values presented in Figure 4.7 are sorted by ascending shift size (i.e., intensity $\times$ radius). Figure 4.7a shows the ARL comparison at shift location A. It can be observed that the multi-image control charts performed almost as well as the best single-image control chart (i.e., single-image MEWMA 1) and perform significantly better than the worst single-image control chart (i.e., single-image MEWMA 2).
A similar observation occurs at shift location B, as shown in Figure 4.7b. Here, the multi-image control charts still performed significantly better than the worst single-image MEWMA (i.e., single-image MEWMA 1) and the combined-image MEWMA performed almost as well as the best single-image MEWMA (i.e., single-image MEWMA 1). Furthermore, the fused-image MEWMA uniformly outperforms the best single-image MEWMA, especially for small shifts.

Altogether, the results in Figure 4.7 demonstrate the advantages of the proposed multi-image monitoring framework (Objective 2, from Section 3). Individually, neither single-image MEWMA 1 or 2 can effectively detect shifts at both shift locations A and B. However, when using a multi-image monitoring control chart, shifts at both shift locations A and B can be effectively detected without incurring a significant performance loss.

One final observation from Figure 4.7 is that the fused-image MEWMA consistently outperforms the combined-image MEWMA, which may result from the fact that the fused-image MEWMA considers cross-correlation. The combined-image MEWMA does not consider cross-correlation because the two images are handled by separate control charts. In contrast, the fused-image MEWMA was created using MPCA which does consider cross-correlation. These performance differences justify the need to further investigate the benefits of the fused-image over the combined-image MEWMA. This investigation is performed in the next subsection, which provides a performance comparison between the fused-image and the combined-image MEWMA at different levels of cross-correlation.

4.3.3. Process Shifts at Different Levels of Cross-correlation

This subsection investigates the hypothesis that the fused-image MEWMA will benefit, with respect to performance, from images with high cross-correlation. This is achieved by increasing the cross-correlation between the original two images, which had relatively low cross-correlation.
Cross-correlation between two random fields can be assessed by the cross-variogram’s range. The range represents the distance that two fields are spatially correlated. Since an anisotropic variogram model was used for generating random fields, cross-correlation ranges were evaluated for two spatial directions: horizontal (x-axis) and vertical (y-axis). The range of cross-correlation in both directions (i.e., x-axis and y-axis) can be changed by adjusting the range parameters during random field generations. The original images contained “low” cross-correlation with ranges of 1.1662 and 1.0948 in the x-axis and y-axis, respectively. By adjusting the range parameters, the simulated high cross-correlation images have ranges of 1.8915 and 1.7539 in the x-axis and y-axis, respectively. These ranges are summarized in Table 4.5. It should be noted that this approach does not change the original images’ sills nor their random field auto-correlations. In addition, the same nominal images were utilized for simulating images.

<table>
<thead>
<tr>
<th>Cross-correlation Range</th>
<th>x-axis</th>
<th>y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (original)</td>
<td>1.1662</td>
<td>1.0948</td>
</tr>
<tr>
<td>High (new)</td>
<td>1.8915</td>
<td>1.7539</td>
</tr>
</tbody>
</table>

Subsequently, a new set of ARL simulations, using the same parameters (e.g., same MEWMA weight, number of simulations, in-control ARL) were performed with high cross-correlation. The results of this new simulation and the original simulation for the combined-image and fused-image MEMWAs are given in tabular form in Table 4.6 and illustrated in Figure 4.8. In Table 4.6, within each level of correlation, the smallest ARL values for each shift are displayed as bold text. The overall smallest ARL (considering both correlation levels) for each shift are underlined. For all considered shift levels, the fused-image MEWMA outperforms the combined-image MEWMA, at both low high cross-correlation levels.
Table 4.6 ARL in Different Image Cross-correlation Levels

<table>
<thead>
<tr>
<th>Shift Size</th>
<th>Low Cross-correlation</th>
<th>High Cross-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity</td>
<td>Radius</td>
<td>Combined</td>
</tr>
<tr>
<td>-20</td>
<td>10</td>
<td>2</td>
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</tbody>
</table>

Similar to Figure 4.7, ARL values presented in Figure 4.8 are sorted by ascending shift size. Figure 4.8a displayed the ARL values for the combined-image MEWMA. As can be observed, the performance at high cross-correlation are only slightly better than low-correlation levels are for the ARLs at high cross-correlation levels. Overall, the changes in cross-correlation has almost no effect on the performance of combined-image MEWMA. On the other hand, Figure 4.8b illustrated the ARL values of the fused-image MEWMA. Unlike its counterpart, fused-image MEWMA gained noticeable ARL improvements from high cross-correlation levels. The ARL values were
reduced significantly when the shift sizes were small (-135 to +135), and the improvement became negligible for larger shift sizes.

The simulation results from different levels of cross-correlation supported the initial hypothesis that fused-image MEWMA’s performance can be improved by images with higher cross-correlation. Specifically, the images with higher cross-correlation increased fused-image MEWMA’s sensitivity to detect small shifts. On the other hand, the changes in image cross-correlation had no significant effect on the combined-image MEWMA chart. The results also show that the fused-image MEWMA outperforms the combined-image MEWMA across every shift size.
regardless of the level of cross-correlation (Objective 3, from Section 3). The fused-image MEWMA had shown significant performance advantages over the combined-image MEWMA in the multi-image monitoring framework. This performance gain can be attributed to the effectiveness of fusing images with MPCA. The fused images extract critical features from multiple images. This process not only reduced data redundancy in the original images but also increased the efficiency to detect shifts. Moreover, the fused images contain cross-correlation information from multiple images. Multiple cross-correlated images of the same part would increase control chart’s ability to detect process shifts within a part.

4.4. Experimental Laboratory Study

An experimental case-study was designed to verify the effectiveness of the proposed multi-image monitoring framework with a real-world dataset. A DSLR camera (Canon EOS Rebel T2i) was mounted on a stand facing downward to capture images of a bowl, as shown in Figure 4.9. In a manufacturing environment, images of a part can be captured automatically and continuously. In this case-study, the camera was connected to a controller (Raspberry Pi) which was programmed to automatically adjust the camera’s settings and capture images.

![Figure 4.9 Experimental Setup](image)

Blurriness in images is a source of noise that increases the difficulty to detect shifts. Blurriness in an image can occur when capturing an object in motion or when the depth of field is too shallow.
In this case-study, blurriness occurs due the camera’s shallow depth of field with respect to the bowl’s concave geometry. This was taken advantage of in this case-study to create two sets of images that exhibited different blurriness. The resulting image-sets (i.e., image-set 1 and 2) are shown in Figure 4.10. In image-set 1, the camera focuses on the bowl’s edge, and the bowl’s center is blurred. This can be seen in Figure 4.10 (a)(c)(e). Image-set 2 are captured by focusing the camera on the bowl’s center, and the bowl’s edge is blurred. This can be seen in Figure 4.10 (b)(d)(f).

Image-set 1 and 2 have opposite blurry and non-blurry regions, which provides an excellent scenario where the ability to detect shifts at different locations differ between the two images. The hypothesis is that shifts occurring on blurry region will be harder detected. To test this hypothesis, an experimental study was designed where shifts appear on both blurry and non-blurry regions. Shift A (a stain) was created at the bowl’s edge highlighted by a circle as shown in Figure 4.10 (c)(d), and shift B (a stain) appears at the bowl’s center as shown in Figure 4.10 (e)(f). This design provides two opposite cases: shift A is in image-set 1’s non-blurry region and image-set 2’s blurry region. Shift B is in image-set 1’s blurry region and image-set 2’s non-blurry region. Monitoring images 1 and 2 together may help detect shift A and B despite image blurriness.
Figure 4.10 Example Images Captured from Case-Study
In this study, for image sets 1 and 2, the camera continuously captured 200 images in which 100 images have no shifts, 50 images have shift A, and the other 50 images have shift B. The details are provided in Table 4.7.

<table>
<thead>
<tr>
<th>Focus</th>
<th>Shift Location</th>
<th># Samples</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image-set 1</td>
<td>Edge</td>
<td>none</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>A (edge)</td>
<td>50</td>
<td>Out-of-control</td>
</tr>
<tr>
<td></td>
<td>B (center)</td>
<td>50</td>
<td>Out-of-control</td>
</tr>
<tr>
<td>Image-set 2</td>
<td>Center</td>
<td>None</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>A (edge)</td>
<td>50</td>
<td>Out-of-control</td>
</tr>
<tr>
<td></td>
<td>B (center)</td>
<td>50</td>
<td>Out-of-control</td>
</tr>
</tbody>
</table>

Within the 100 images without shifts, the first 90 of those are used for historical data analysis, and the remaining 10 images are used as non-historical in-control datasets. In historical data analysis, MPCA was used for feature extraction and reducing the image size to $5 \times 5$ down from $340 \times 356$, which is a significant dimension reduction. The determination of the size $5 \times 5$ was based on the fact that the number of monitored variables should be much less than the historical samples size (i.e., 90 images).

In phase II, the projected images are monitored by image-based control charts. Table 4.8 elaborates on how the image-sets are organized for monitoring. “Image 1” refers to single-image MEWMA 1 that monitors image-set 1. Similarly, “Image 2” refers to single-image MEWMA 2 that monitors image-set 2. To implement a multi-image control chart, image-sets 1 and 2 are fused together with MPCA and monitored by a fused-image MEWMA chart, referred by “Fusion 12”. Every control chart monitored 150 images from the corresponding image-set. Within the 150 images, the first 100 are in-control images and the remaining 50 are out-of-control images with either shift A or B.
Table 4.8 Control Chart Monitoring Scheme

<table>
<thead>
<tr>
<th></th>
<th>No shift</th>
<th>Shift A</th>
<th>Shift B</th>
<th>Total</th>
<th>Control chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image-set 1</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>150</td>
<td>Image 1A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Image 1B</td>
</tr>
<tr>
<td>Image-set 2</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>150</td>
<td>Image 2A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Image 2B</td>
</tr>
<tr>
<td>Fused image-set</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>150</td>
<td>Fusion 12A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Fusion 12B</td>
</tr>
</tbody>
</table>

The comparisons between single- and fused-image MEWMA charts are shown in Figure 4.11. Figure 4.11a has the three control charts displayed together in detecting shift A. It can be observed that all three control charts successfully detect shift A after the first 100 images. The fused-image MEWMA shows more sensitivity to shift A with clearly higher statistical values. The similar results are displayed in Figure 4.11b for detecting shift B. Both single- and fused-image MEWMA charts successfully detect shift B, while the fused-image MEWMA outperform both single-image MEWMAs with much greater statistical values after the first 100 images. It should be noted that both single and fused images monitored have the same size of $5 \times 5$ after eigentensor projection, which means both single- and fused-image MEWMAs monitor the same amount of variables per sample. This indicates two advantages for the fused-image MEWMA chart. 1) It is more sensitive to multiple shifts within the image. 2) It is as efficient as the single-image MEWMA because the fused-images do not increase computational burden to the control chart for process monitoring.
Figure 4.11 Single-image MEWMA vs. Fused-image MEWMA

Figure 4.11 shows a good visual comparison of control charts. To quantify their actual performance, the second measurement, signal noise ratio (SNR), was calculated to measure control chart’s strength to detect shifts adjusted by noise. The SNR equation is $\frac{\bar{x}_{out} - \bar{x}_{in}}{s_{in}}$, where $\bar{x}_{out}$ is the average statistical value for samples that are out of control limit, and $\bar{x}_{in}$ is the average statistical value for samples that are in control limit. $s_{in}$ is the standard deviation of statistical values for samples that are in control.
Table 4.9 Signal Noise Ratio of T^2 Statistics

<table>
<thead>
<tr>
<th></th>
<th>Shift A</th>
<th>Shift B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fused image</td>
<td>23.90</td>
<td>13.94</td>
</tr>
<tr>
<td>Image 1</td>
<td>13.50</td>
<td>7.20</td>
</tr>
<tr>
<td>Image 2</td>
<td>11.36</td>
<td>7.83</td>
</tr>
</tbody>
</table>

The signal noise ratio is calculated for the three image-based T^2 charts on detecting shift A and B and recorded in Table 4.9. Unlike MEWMA chart, T^2 chart’s statistics for every image is independent from the previous images, which is critically important for understanding the true signal and noise. The larger SNR is in bold text between image 1 and 2, and the largest SNR is in bold text and underscored. Comparing image 1 and 2 T^2 chart, it can be observed that image 1 T^2 chart is more sensitive to shift A with larger SNR, and image 2 T^2 chart is more sensitive to shift B with larger SNR. Both single-image T^2 charts have shown better SNR on detecting shifts on image’s non-blurry zone. This is in line with initial hypothesis that shifts on non-blurry zone in an image are easier to be detected than shifts on blurry zone in single-image control chart.

However, Fused-image T^2 chart’s signal noise ratio is significantly higher than both single-image T^2 charts on detecting shift A and B. It shows the evidence that even in real-world applications the fused-image control chart outperforms single-image control chart by being more sensitive to multiple shifts. Regardless of blurry and non-blurry zones within an image, fused image with MPCA has the ability to extract critical features across multiple images and consider the spatial correlation between multiple images. These factors together significantly enhance fused-image control chart’s efficiency to detect shifts with better SNR than either single-image control charts.

4.5. Conclusion

We developed a multi-image monitoring control charting framework that can be used for monitoring multiple different images of the same thing simultaneously. The MEWMA chart was
recommended and used in this framework for its good performance on detecting small shifts. Through rigorous simulations and experiments, it was discovered that the proposed combined- and fused-image MEWMA charts had significant performance gain on detecting multiple process shifts within the image compared to single-image MEWMA chart. Especially, the fused-image MEWMA chart, that considers spatial cross-correlations, had shown outstanding efficiency on detecting multiple shifts within the image. It was proved that high cross-correlation between images further enhanced fused-image MEWMA chart’s sensitivity to detect small shifts. Moreover, we provided the details of implementing the proposed framework that handles images from capturing to monitoring on control charts, which can help practitioners apply this framework to more applications.

Some research opportunities can be derived from the proposed multi-image monitoring framework. This paper only examined the scenario where two images captured with different camera parameters of the same thing were monitored together. The proper combinations of camera parameters for capturing images could be further investigated. The proposed framework can be easily extended to handle applications that require monitoring three or more images at the same time. Additionally, this framework can monitor multiple images beyond being captured by different camera parameters. Images can be monitored in different angles, frequencies, and types including X-ray, infrared, and ultrasound images often found in medical applications. Furthermore, it is worth investigating further the use of MPCA in multi-image monitoring applications. The performance of our method is highly dependent on the number of eigentensors being selected and number of historical samples used for historical data analysis. The number of eigentensors being selected with respect to the different scenarios (i.e., image sizes, contents, types, and volume) being monitored for multi-image control charts could be investigated in future studies.
4.6. References


Cressie, N., 1993, Statistics for spatial data, Wiley Interscience


5. CONTRIBUTIONS AND FUTURE WORK

This dissertation is dedicated to improving statistical process control with the use of image data, which is a growing area of research within the quality control research. The research status quo in the area is reviewed. As a result, state-of-art techniques that handle images for the purpose of process monitoring were investigated. This investigation revealed a research gap, which was addressed in this dissertation by proposing a new framework that enables control charts to monitor multiple images simultaneously. Two approaches towards implementing this multi-image monitoring framework were introduced. The performance gains from the proposed approaches are evaluated through both extensive simulations and real-world applications. In addition, a computationally efficient and accurate image simulation approach was developed to generate cross-correlated images that can mimic the changes in system to evaluate performance metric of image-based control charts. Based off the accomplished works, this chapter summarizes contributions made in this dissertation and identifies future work in the use of image data for SPC.

5.1. Contributions

The control charting tools created in this dissertation provide the foundation for expanding the applicability of using high-dimensional image data for control charting. The developed tools can transform high-dimensional data into content-rich information. Its contributions and anticipated applications are discussed in the following two subsections to their respective research areas.

5.1.1. A Simulation Approach for Evaluating Performance Metric of Control Charts

The main contribution of this area is the proposed simulation approach that provides the ability to evaluate the performance control charts with high-dimensional image data. Control charting performance metrics are a crucial aspect in SPC research. A reliable performance metric provides a fair comparison for different control charting methods. It is customary that performance metrics for SPC are evaluated through computer simulations where data is replicated to approximate real-
world system behaviors. As described in Chapter 2, most of the literatures on simulating images for evaluating control chart performance metric focus on generating either Gaussian or Poisson noise. Both methods are utilized to generate random fields that are independent and identically distributed (i.i.d.). However, for multi-image or high-dimensional image monitoring, random fields are spatially auto- and cross-correlated which influence a control chart’s performance. This spatial correlation behavior cannot be simulated in i.i.d. based methods. In chapter 3, the proposed simulation approach addresses such spatial correlation by generating multiple cross-correlated random fields. This approach allows different levels of cross-correlations to be generated in images in order to evaluate the corresponding performance metric of multi-image control chart. In addition, the computational efficiency in the proposed approach enables fast and accurate generation of cross-correlated random fields. This allows us to estimate the well-known ARL metric, an effective but computationally intensive performance metric commonly used for SPC applications, to evaluate multi-image monitoring control charts. This simulation approach was never done in SPC literature and it is very crucial component for SPC applications dealing with high-dimensional data.

5.1.2. A Framework for Monitoring High-Dimensional Images In SPC

The main contribution of this area is the proposed framework that allows a single control chart to monitor multiple images at the same time. The use of image data in SPC play an important role in mass production operations. Typically, an image of a part captured by imaging device is monitored by control charts for fault detection and diagnosis. More specifically, control charting systems process one image of a part at a time to provide a statistical estimate. Considering the fact that systems may need to monitor multiple images of a part or several parts, the same number of control charts as images would be required. Such a monitoring system would be exhausted with too many sets of images. In this case, the proposed multi-image monitoring framework shows absolute
advantage to monitor multiple images at the same time by a single control chart. Furthermore, the study indicates the proposed framework not only reduce the number of control charts being used for process monitoring but also increase the shift detection capability compared to individual control chart, especially when those images are cross-correlated. The framework’s ability to monitor multiple images at the same time with enhanced control charting performance could have a greater impact in many industries.

5.2. Future Work

This dissertation provides a methodology that allows control charts to monitor high-dimensional image data for fault detection and diagnosis. The following two subsections discusses the possible research gaps in the area that are related to this dissertation. They are 1) the discussion of Multi-image monitoring applications and 2) the removal of lighting variation in Image-based SPC

5.2.1. The Discussion of Multi-Image Monitoring Applications

The multi-image monitoring framework proposed in this dissertation investigates the scenario that two images, obtained with different capturing parameters, of the same part are monitored simultaneously. This framework has the capability to be applied in more complex monitoring scenarios and is anticipated to research in the following areas.

1) Investigate the utilization of multi-image monitoring framework for monitoring three or more images at the same time. This can be achieved by stacking multiple images together to construct a high-dimensional stacked image that is further processed by MPCA for feature extraction. Monitoring three or more images through the proposed framework is viable, but the benefit of doing that needs to be investigated.

2) Monitor multiple images that are captured from different angles or types of the same thing. Most parts being manufactured have a three-dimensional geometric shape. That requires views
from different angles towards the part in order to take a full image of it. In Image-based SPC, the imaging system can be naturally adapted to capture the images of the part from different angles in 360-degrees, and the multi-image monitoring framework is worth to be considered to monitor those images simultaneously. A similar case to consider is to monitor images of different types including X-ray, infrared, and ultrasound images often found in medical applications. It is beneficial to understand the performance gain of using multi-image monitoring framework in those scenarios.

3) Analyze the use of MPCA in multi-image monitoring applications. MPCA is a powerful tool to extract features from high dimensional images without breaking its spatial correlation. However, control chart’s behavior is dependent on the number of eigentensors being selected in MPCA. Selecting too many eigentensors causes extremely large test statistics, while choosing too small eigentensors underrepresents the dataset. Selecting the number of eigentensors for feature extraction is highly problem dependent and requires manual adjustment to determine the most appropriate eigentensors. It is worth investigating the number of eigentensors being selected with respect to the different scenarios (i.e., image sizes, contents, types, and volume) being monitored for multi-image control charts in future studies.

5.2.2. The Removal of Lighting Variation in Image-based SPC

In the field of Image-based SPC, the implementation of Image-based control charts in manufacturing applications is rather limited. The major reason is that image’s brightness is susceptible to the changes of environmental lighting conditions. A slight change in lighting conditions may cause control charts to signal false alarms. Maintaining stable lighting conditions is an important requirement for these systems. In manufacturing environments, minimizing
variations in lighting conditions is often challenging and not economical. This is one of the problems that needs to be addressed in future studies.

To understand how changing light affecting control chart’s accuracy for process monitoring, a new study was conducted for such investigation. In the study, a total of 100 real images of the same ceramic tile were captured intentionally with different lighting conditions and shifts to examine control chart’s corresponding response. The description of captured images is shown in Table 5.1. Based on the description, the first 30 images have no sustained shifts and is used as in-control historical dataset. From image 31 to 46, an ambient light source is turned on to provide extra lights. From image 47 to 50, the light intensity level is increased to provide even stronger lights. From image 51 to 79, this light source is removed to recover the original lighting condition. Meanwhile, a surface shift A is added on tile’s white pattern for the sustained shifts. Finally, from image 80 to 100, an extra surface shift B is added on tile’s black pattern to provide additional sustained shifts. It should be noted that from image 1 to 50 there is no process shifts observed.

Table 5.1 Description of Captured Images

<table>
<thead>
<tr>
<th>Examples</th>
<th>0–30</th>
<th>31–46</th>
<th>47–50</th>
<th>51–79</th>
<th>80–100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image-sets</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>Shift type</td>
<td>None</td>
<td>None</td>
<td>None</td>
<td>Shift A</td>
<td>Shift A and B</td>
</tr>
<tr>
<td>Lighting condition</td>
<td>Level 0</td>
<td>Level 1</td>
<td>Level 2</td>
<td>Level 0</td>
<td>Level 0</td>
</tr>
</tbody>
</table>

The selected images from these five different image-sets are displayed in Figure 5.1 from (a) to (e). Figure 5.1a shows no shifts and no extra light. Figure 5.1b has no shifts with an extra light at low level, and Figure 5.1c has no shifts with this light at high level. Figure 5.1d has sustained shift A, and Figure 5.1e has sustained shift A and B. The surface shifts are emphasized by the red circles within the image (d) and (e). It should be noted that image (d) and (e) have the same lighting condition as image (a) without extra light source.
Figure 5.1 Captured Images in Different Shift Types
These 100 observations or images are subsequently monitored by a single-image MEWMA chart. The selected five images (i.e., Figure 5.1 (a) to (e)) are explicitly marked within Figure 5.2 indicating the beginning of each images-set. It can be observed that the second image-set (light at low level) increases MEWMA value significantly starting from image (b). The third image-set (light at high level) raises the value even higher since image (c). When the fourth image-set (shift A) arrives at image (d), the MEWMA value plummets due to the removal of lights but is still higher than initial values where neither shifts nor extra light exhibit. After about 30 images, the MEWMA value gradually becomes stable until the fifth image-set (shift B) are introduced starting at image (e). The sustained shift B cause a sudden reduction in MEWMA values.

![Figure 5.2 MEWMA Chart Monitoring for Different Shift Types](image.png)

In summary, both sustained shifts A and B intentionally created in the study are successfully detected by the single-image MEWMA chart as expected. However, the change of light in the second and third image-sets are also detected by control chart to signal process shifts, even though they have no physical shifts. In fact, the control chart is more sensitive to the change of light than the actual process shifts. As a result of that, the change in light have caused a great amount of false
alarms in the system. Therefore, maintaining a stable lighting condition in Image-based SPC is crucially important.

Lighting variations in Image-based SPC are strictly controlled, because it can dramatically increase a control chart’s false alarm rate. Zuech (2000) highlighted that current machine vision systems are highly susceptible to variations in lighting conditions, reflections, and other environmental noise as compared to human capabilities. Liu et al. (2005) attempted to remove lighting variations with the uses of a Discrete Wavelet Transform (DWT). They omitted the last sub-image from a set of decomposed images to make the analysis robust to lighting variations. Megahed et al. (2012) suggested to perform a lighting survey by measuring variability in illumination at various locations with different positioning of lighting sources. Then select a location with minimum lighting variations. He et al. (2016) fixed the relative position between cameras and parts during process monitoring and controlled the light source to maintain a consistent lighting condition. Even though researcher have mentioned the ways to deal with lighting variations, a systematic methodology to address the lighting variation is still lack of discussion. The answer could be found in advanced machine vision technologies.

In recent years, many machine vision techniques have emerged in the research field of SPC. Wang et al. (2018) proposed a convolutional neural network-based defect detection model for quality control. Their model achieved 99.8% accurate detection rate from a benchmark dataset. Villalba-Diez et al. (2019) utilized deep neural network to perform automatic quality control in the printing industry. Lee et al. (2019) employed convolutional neural networks (CNNs) to monitor the localized defect regions within images for steel defect diagnostics. Similar to SPC, CNN models first need to be trained with training dataset and then tested with a testing dataset for evaluating a model’s performance. Unlike SPC whose historical data analysis only contain in-
Control image data, a CNN’s training dataset includes both in-control and out-of-control image data that are preferably balanced. Another major difference between SPC and CNN is that SPC’s phase II analysis focuses on monitoring new datasets for process shifts, and CNN’s testing stage concentrates on predicting outcomes from a test dataset.

Combining CNN with SPC probably improve process monitoring efficiency by removing unnecessary interference (i.e., lighting variations). In theory, CNN is a combination of image convolution and deep neural network. Image convolution extracts image features, and deep neural network analyzes features to predict outcomes. Considering the structure of CNN and SPC, it is possible to combine CNN models with control charting techniques to monitor process shifts while filtering out lighting variations. Specifically, lighting interference can be filtered out in a CNN’s image convolution stage where quality features are extracted. The extracted features would then be monitored by control charts to detect process shifts. Using image convolution trained from CNN alongside statistical process control would allow control chart less sensitive to the change of light and more sensitive to the actual process shifts. There are more future works can be accomplished in this direction and more applications can be investigated in this area.
REFERENCES


Cressie, N., 1993, Statistics for spatial data, Wiley Interscience


