Maintenance of Cognitive Demand During Repeated Task Enactments Using a Teaching Practice That Builds on Student Thinking

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MAINTENANCE OF COGNITIVE DEMAND DURING REPEATED TASK ENACTMENTS USING A TEACHING PRACTICE THAT BUILDS ON STUDENT THINKING

by

Joshua M. Ruk

A dissertation submitted to the Graduate College in partial fulfillment of the requirements for the degree of Doctor of Philosophy Mathematics Western Michigan University December 2021

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Maintaining high levels of cognitive demand during task enactments can improve student learning in classrooms. Influences such as teachers' beliefs about their students’ abilities, and the curriculum that a teacher uses can affect the factors that contribute to the maintenance of cognitive demand. My study adds to what we know about such influences by looking at how a teaching practice that builds on student thinking affects the factors that contribute to the maintenance of cognitive demand, by asking:

1. What does the maintenance of cognitive demand look like when teachers are attempting to attend to student thinking during task enactment?
2. How do factors contributing to the maintenance of cognitive demand vary when the same cognitively demanding task is enacted by different teachers in multiple classes?

To answer these questions, I looked at 24 videotaped enactments of two tasks (12 enactments of each task). These enactments came from six teachers who were engaging in the teaching practice of building. My videotaped task enactments were analyzed using the Instructional Quality Assessment (IQA) and the Reorganized Factors that Undermine or Keep Cognitive Demand (RUK). In addition to my videotaped data, I also analyzed survey and interview data from the participating teachers to help answer my research questions. These data
were analyzed by comparing and contrasting the survey and interview responses to the results of applying the IQA and RUK to my videotaped data.

In regard to the first research question, I found that employing a teaching practice that attends to student thinking by building on it can improve the overall maintenance of cognitive demand by aiding some factors that maintain and minimizing other factors that lower cognitive demand during task enactments. Specifically, by implementing the building practice teachers offered a more appropriate amount of scaffolding than they otherwise would have. The building practice calls for teachers to draw out conceptual connections, and as a result there were more conceptual connections drawn out than has been found in previous work—an important part of maintaining cognitive demand. Finally, in order to make available the type of high leverage thinking that the building practice requires, teachers often focused their students on trying to understand misconceptions, and this helped to maintain the appropriateness of the task.

In regard to the second research question, I found that when the same tasks were enacted by different teachers or with different groups of students, the prominence of factors known to affect the maintenance of cognitive demand sometimes varied across enactments. Specifically, I found that offering the appropriate amount of time is highly task dependent, and that it can be affected by teachers focusing on other aspects of maintaining cognitive demand. I also found that allowing students enough time to make conceptual connections on their own is important for the maintenance of cognitive demand. Finally, I found that the number of solution strategies drawn out, as well as the order in which concrete and abstract strategies are presented, was associated with higher and lower levels of cognitive demand.

Overall, my findings showed that intentional use of a teaching practice can affect the maintenance of cognitive demand during task enactments. In light of this I provide some
pathways for future research, including looking at the teaching practice of building in different lights, as well as looking at other teaching practices.
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Joshua M. Ruk
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CHAPTER I
INTRODUCTION

Background

Cognitively demanding tasks are challenging problems, or sets of problems, that require students to use their existing knowledge, sometimes in new and unique ways, along solution pathways that are not immediately clear (Stein et al., 1996). The use of such problems has been shown to lead to student learning gains (Kane et al., 2013; Otten & Soria, 2014; Stein & Lane, 1996). As a result, the National Council of Teachers of Mathematics (NCTM) has called for the use of cognitively demanding tasks in mathematics classrooms (e.g., NCTM, 2014).

Stein et al. (1996) looked at the tasks that are given to students and identified four levels of cognitive demand in which all tasks can be grouped. Memorization—stating facts and formulas from memory, procedures without connections—students follow an algorithm to find a solution, procedures with connections—students use algorithms, but connections are made to important and underlying mathematical concepts, and Doing Math—framing problems, making conjectures, justifying, and explaining solution paths that are not immediately clear. The first two levels are considered low-level cognitive demand and the latter two are considered high-level cognitive demand.

Even tasks that begin with high-level cognitive demand, such as those written in some curricular materials, however, may not maintain that level of cognitive demand as they are enacted. The cognitive demand of the task as it is written can be lowered before it is even presented to students due to a wide variety of factors, including teachers’ opinions, values, or perceptions of students, or pressure from school districts or state officials. As the task is presented in class to the students the cognitive demand can change simply by the way that the
teacher chooses to phrase things, or any information that the teacher adds or withholds from the original task as it was written. Finally, the cognitive demand of the task can change when students are working on it, depending on what they do and what the teacher does during the task enactment.

As such, the level of cognitive demand of a task is anything but static, and the largest learning gains are seen in classrooms where the cognitive demand of a task remains high throughout the task enactment (Stein & Lane, 1996). To assist with maintaining high levels of cognitive demand, Stein et al. (1996) identified seven factors that help maintain the initial level of cognitive demand as the task is enacted and six factors that cause the initial level of cognitive demand to be lowered (see Figure 1).

Figure 1

Factors That Maintain and Lower Cognitive Demand

<table>
<thead>
<tr>
<th>Factors that Maintain Cognitive Demand</th>
<th>Factors that Lower Cognitive Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Appropriate task</td>
<td>1) Inappropriate task</td>
</tr>
<tr>
<td>2) Appropriate amount of time</td>
<td>2) Inappropriate amount of time</td>
</tr>
<tr>
<td>3) Modeled high-level performance</td>
<td>3) Focus shifts to finding a correct answer</td>
</tr>
<tr>
<td>4) Sustained pressure for meaning</td>
<td>4) Lack of accountability</td>
</tr>
<tr>
<td>5) Proper amount of scaffolding</td>
<td>5) Challenges become nonproblems</td>
</tr>
<tr>
<td>6) Students self-monitor</td>
<td>6) Classroom management</td>
</tr>
<tr>
<td>7) Drew conceptual connections</td>
<td></td>
</tr>
</tbody>
</table>

Note. Derived from Stein et al. (1996).

Aside from the 13 factors uncovered by Stein et al. (1996), there are other influences that can affect the maintenance of cognitive demand during task enactments. For example, influences such as the curriculum used (Stein & Kaufman, 2010), a teacher’s subject matter knowledge (Lunt, 2011), the type of lesson preparation that a teacher completes (Estrella et al., 2019), and even a teacher’s beliefs about their students’ abilities (Hong & Choi, 2018) can all have an effect
on the maintenance of cognitive demand during task enactments. Thus, getting the full value from cognitively demanding tasks requires better understanding the interplay of cognitively demanding tasks with other influences on classroom teaching.

Besides calls for the use of cognitively demanding tasks, the NCTM has also made calls for changes in teachers’ classroom practices. Teaching practices are intentional actions that have the goal of increasing student learning or understanding. In alignment with this call, as well as the call for the use of cognitively demanding tasks, Smith and Stein (2018) published *5 Practices for Orchestrating Productive Mathematics Discussions*. These five practices are based on what is known about the maintenance of cognitive demand, and they provide teachers with a framework for how to organize and execute class discussions that maintain high levels of cognitive demand.

There are a wide variety of practices available to teachers today (Charalambous & Delaney, 2019), and because of the wide scope of mathematics education in general, many of these practices have aims other than the maintenance of cognitive demand. For example, another NCTM (2014) call has been to incorporate student thinking into classroom instruction. When teachers incorporate student thinking into their instruction, there is a direct relationship to improved student learning (Fennema et al., 1996; Medrano, 2012). As such, there are newly developed teaching practices that help improve the way that teachers incorporate student thinking into their instruction (e.g., Van Zoest et al., 2016). We do not yet know, however, about the interaction of these teaching practices and maintenance of cognitive demand.

**Statement of the Problem**

Although mathematics education has made improvements over time, the current level of student learning still needs to be increased in order to adequately prepare students for college or their jobs after high school (NCTM, 2018). It has been shown that student learning can be
increased by employing teaching practices that use student thinking (Carpenter & Fennema, 1992) and by maintaining high levels of cognitive demand during task enactments (Stein & Lane, 1996). Such findings have led the NCTM to call for increased use of both student thinking and cognitively demanding problems in today's classrooms (NCTM, 2014). Such calls have been answered by numerous researchers, including those who look to create teaching practices that improve the way teachers use student thinking (e.g., Van Zoest et al., 2016) and those who look to improve the maintenance of cognitive demand (e.g., Smith & Stein, 2018). Even though the answers to both of these calls draw from what we already know about mathematics education, there has been little overlap in the study of teaching practices that improve the way teachers use student thinking and maintaining high levels of cognitive demand.

Previous work has shown that factors from outside the original scope of study can affect the maintenance of cognitive demand. For example, Boston and Smith (2009) showed that a teacher’s ability to maintain high levels of cognitive demand during task enactments increased due to participation in a Professional Development (PD) where increased cognitive demand was a goal. Since teacher’s participation in PD was outside of the original scope of research that led to what is known to affect the maintenance of cognitive demand, Boston and Smith (2009) provided us a better understanding of the intricacies that can affect the maintenance of cognitive demand. Olson et al. (2011) added even more to this understanding when they showed that even PD where increased cognitive demand was not a goal could affect the maintenance of cognitive demand during task enactments. Such work has enriched our understanding of the maintenance of cognitive demand, and in turn given us the opportunity to improve the quality of instruction in mathematics classrooms.
In a broad sense, the ultimate goal of PD research and cognitive demand research is to improve mathematics education. Each of these separate areas of study already accomplishes this goal in a distinct way, but when we look at the overlap of these areas we are able to realize new ways to meet this ultimate goal. Similarly, looking at the overlap of other areas of research with the maintenance of cognitive demand, such as teaching practices that use student thinking, could provide the same fruitful outcomes that were realized by Boston and Smith (2009) and Olson et al. (2011).

To capitalize on the potentially fruitful overlap between teaching practices that use student thinking and the maintenance of cognitive demand, my work looked at how cognitive demand was maintained during the enactment of a teaching practice that takes advantage of in-the-moment, high-leverage student thinking—the teaching practice of building on MOSTs (Van Zoest et al., 2016). I investigated enactments by the same teacher using different tasks and different teachers using the same task. My goal was to understand if there are factors contributing to the maintenance of cognitive demand that are more or less prominent in this particular teaching practice. Understanding any differences in the factors contributing to the maintenance of cognitive demand when enacting a specific teaching practice may lead to finding patterns in the maintenance of cognitive demand that could be used to improve the maintenance of cognitive demand overall, which in turn could aid in accomplishing the ultimate goal of helping improve mathematics education.

**Framework**

In their work to understand cognitive demand, Stein et al. (1996) uncovered a list of factors that help maintain the level of cognitive demand during task enactments, as well as a list of factors that lower the level of cognitive demand during tasks enactments (see Figure 1). These
factors have been used by numerous researchers to understand the maintenance of cognitive demand during task enactments (e.g. Estrella et al., 2019; Hong & Choi, 2018; Lunt 2011). Unfortunately, the studies that have used the factors to better understand the maintenance of cognitive demand have done so in different ways, and there has been no mechanism to uniformly measure the presence or absence of each factor across studies. Tools such as the Instructional Quality Assessment (IQA; Boston, 2012) look broadly at the maintenance of cognitive demand during task enactments, but again, do not look at each individual factor. To meet this need, I developed the Reorganized Factors that Undermine or Keep Cognitive Demand (RUK; Ruk, 2020). The RUK provides a streamlined way of assessing each individual factor that maintains or lowers cognitive demand. Thus, as a way to better understand these factors in the context of my work, the RUK served as both a tool for data collection and a framework that undergirded many of the decisions I made in my study (the RUK is described in detail in the Theoretical Framework section of this dissertation).

**Purpose**

Cognitive demand research is an important component of the current reform movement. What started as hierarchically categorized math problems has spawned research articles, books, ideals, pedagogies, and innovations that are now firmly embedded in the ongoing push to improve student learning in mathematics. However, as far reaching as cognitive demand research may be, it is still but one of many avenues being pursued to provide the students of today with the mathematics they will need tomorrow. Like cognitive demand research, research involving the use of student thinking is also prominent on the math education landscape. To meet our goal of improving student learning, it would be helpful to know how maintaining cognitive demand is affected by other influences, such as teachers attempting to use student thinking as part of their
instruction. To help us understand what interplay exists between these two areas of mathematics education research, I investigated what the maintenance of cognitive demand looks like during the enactment of a teaching practice that builds on in-the-moment, high-leverage student thinking. My goal was to understand if there are factors contributing to the maintenance or decline of cognitive demand that are more or less prominent in a teaching practice whose underlying goal is to use student thinking. Such prominences could signal that, like other influences—such as the curriculum used (Stein & Kaufman, 2010), a teacher’s subject matter knowledge (Lunt, 2011), the type of lesson preparation that a teacher completes (Estrella et al., 2019), or a teacher’s beliefs about their students’ abilities (Hong & Choi, 2018)—teaching practices can affect the maintenance of cognitive demand during task enactments.

**Research Questions**

1. What does the maintenance of cognitive demand look like when teachers are attempting to attend to student thinking during task enactment?

2. How do factors contributing to the maintenance of cognitive demand vary when the same cognitively demanding task is enacted by different teachers in multiple classes?

**Definition of Terms**

*Cognitive demand* is “the kind and level of thinking required of students in order to successfully engage with and solve the task” (Stein et al., 2009, p. 11).

*Cognitively demanding tasks* are problems or sets of problems “that engage students at a deeper level by demanding interpretation, flexibility, the shepherding of resources, and the construction of meaning” (Stein et al., 1996, p. 459).

*High-leverage student thinking* refers to instances of student thinking that “have considerable potential at a given moment to become the object of rich discussion about important
mathematical ideas” (Leatham et al., 2015, p. 90), and that can “significantly enhance student understanding of mathematics when made an object of discussion” (Leatham et al., 2015, p. 121).

Tasks are “a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea. An activity is not classified as a different or new task unless the underlying mathematical idea toward which the activity is oriented changes” (Stein et al., 1996, p. 460).

Procedures

This study looks at two tasks that were enacted two times each by six teachers for a total of 24 task enactments. These task enactments were coded using the IQA (Boston, 2012) and the RUK (Ruk, 2020). The results of this coding were used to provide initial answers to my research questions. The results of my coding were also compared to what is known about cognitive demand and were used to create hypotheses about the maintenance of cognitive demand during the task enactments. Based on these hypotheses and what is known about the maintenance of cognitive demand, questions for an online survey and teacher interviews were created. These questions were given to the five teachers who enacted the tasks and were able to participate in this part of the data collection. Their responses allowed me to verify or disprove hypotheses I derived from prior data analysis, and they helped to elaborate on, and provide detail and definition for my findings.

Significance

Many researchers have looked at cognitive demand and found that it plays a vital role in improving education today (e.g., Stein et al., 1996; Tekkumru Kisa & Stein, 2015). Likewise, researchers have shown that using student thinking plays a vital role in improving education
(e.g., Carpenter & Fennema, 1992; Sherin et al., 2011). My work extends this research by looking at the interplay between a teaching practice intended to utilize student thinking and the maintenance of cognitive demand as this teaching practice is enacted. Since both of these topics are at the forefront of mathematics education research today, understanding the interplay between them will benefit researchers from both areas. My work also furthers the fields’ understanding of maintaining cognitive demand and how to support teachers to improve their abilities to maintain cognitive demand while they are engaged in other teaching practices. Although my work alone cannot definitively show the interplay between teaching practices that utilize student thinking and the maintenance of cognitive demand, it does open a new line of inquiry that looks to bring two areas of mathematics education research together with the underlying goal of improving mathematics education as a whole.

**Limitations**

Although I look to shed light on the interplay between teaching practices and the maintenance of cognitive demand more generally, my work only looks at one specific teaching practice. As such my work alone cannot show how teaching practices in general affect the maintenance of cognitive demand; in fact, with the great number of teaching practices available today it seems very unlikely that they would all have the same effect on the maintenance of cognitive demand. However, by looking at this one teaching practice I can show the effect on the maintenance of cognitive demand that teaching practices can have, which would provide a model for looking at other teaching practices to see how they affect the maintenance of cognitive demand and would provide a baseline for comparison.

Besides looking at a single teaching practice, the teachers in my study are not a representative sample of teachers in general. Because my work utilizes data from a larger study,
the participating teachers were preselected, and were chosen because they were willing to actively change their teaching strategies to improve their use of student thinking in their instruction. Previous work has shown that teachers often say they are employing reform-based teaching ideals in their classrooms, but unless they are actively attempting to change their teaching strategies, they are likely just using traditional teaching methods with a veneer of reform-based terminology (Olson et al., 2011). Even though they were not representative, it is likely that this group of teachers provided more fruitful data because they were attempting to implement the reform-based teaching practice using high-leverage student thinking that was part of this study.

Finally, the data that were coded using the RUK were only coded by the author of this study. To mediate this potential limitation, a second coder checked some of the data, and the resulting codes were similar to the author's codes. Additionally, the IQA was applied to the data by trained outside coders, and since some of the IQA rubrics broadly overlap the RUK, I was able to see that in a general sense, the RUK codes aligned with the IQA codes. Still, because most of the RUK coding was done by a single coder, there is a higher likelihood of small errors than if more coders had looked at all of the data and reconciled their RUK coding as a group.

**Organization**

Now that I have introduced my dissertation work, I can begin discussing (in Chapter II) what research has uncovered about cognitive demand so far, and what horizons are still left to explore. To help with my personal exploration (in Chapter III) and introduce a tool that can be used to uniformly measure the factors that help maintain and lower cognitive demand during task enactments, I will then introduce the **Reorganized Factors that Undermine or Keep Cognitive Demand (RUK)**. I used the RUK to guide many of the decisions I made throughout my study,
and it undergirds (in Chapter IV) the architecture of my work and what I did to answer my research questions. After this I discuss (in Chapters V) what I found in my expansion of the cognitive demand horizon, and how (in Chapter VI) my findings affect the field.
CHAPTER II
LITERATURE REVIEW

In this literature review, I first describe cognitively demanding tasks, as they are the foundation of this study. I then use the Mathematical Task Framework (Stein et al., 1996), which outlines the different phases that a task goes through, as a roadmap to look at different facets of cognitive demand research. To situate my work, I next look at what is known about maintaining the cognitive demand of task enactments. After this I look briefly at measuring the maintenance of cognitive demand. Finally, I look at a small portion of what is known about student thinking, so that I can tie these results to my own work on cognitive demand.

Tasks

In general, a task has four main elements: (a) an outcome or solution; (b) the information given as part of the task, or the problem that students are asked to solve; (c) the knowledge or attributes that a student possesses that they can use for the task (Doyle & Carter, 1984), and (d) the level of accountability student will be held to for the task (Doyle, 1988). To speak more about level of accountability, students will do what a teacher expects of them, but usually little beyond that (Doyle, 1983). For example, unless there is a classroom norm for students to share their reasoning behind a solution, most students will generally give short one- or two-word answers. Using these four elements as a base, tasks come in many forms, including: opinion tasks—stating opinions, memory tasks—recognizing or reproducing previously learned information; routine tasks—applying algorithms; and comprehension tasks—using previously learned information in new ways (Doyle, 1983). Classifying a task often relies on looking at what sort of answers students are asked for and what solution strategies they must employ to find that answer (Doyle, 1983). For example, cognitively demanding tasks are generally
comprehension tasks with a high level of accountability. In other words, they require students to use a number of different strategies which are not immediately clear, and students must understand how to put these strategies together, and consequently, should be able to explain their process or reasoning.

Based on the elements and forms of tasks that we just discussed, one of the most widely known outcomes of cognitive demand research is Stein et al.’s (1996) separation of tasks by their level of cognitive demand (Figure 2). They developed four levels—memorization, procedures without connections, procedures with connections, and doing math—and explained that they were based on the level of cognitive demand that the tasks elicit from students. Memorization tasks involve simply stating facts and formulas from memory. Procedures without connections tasks are those where students follow a procedure, typically an algorithm, as a way to find a solution. Procedures with connections tasks rely on the use of procedures, but connections are made between these procedures and the mathematical ideas behind them, which allows students to gain a deeper understanding of the mathematical concepts with which they are working. Doing math tasks involve framing problems, making conjectures, justifying, and explaining as a way to find solutions to a problem, where the path to that solution is not clear. In cognitive demand research, there are often references to low and high cognitive demand tasks. When categorized in such a way, memorization tasks and procedures without connections tasks are considered to be low cognitive demand, while procedures with connections tasks and doing math tasks are considered to be of high cognitive demand. Next, I look more broadly at what is known about cognitive demand.
Figure 2

*The Four Levels of Cognitive Demand*

<table>
<thead>
<tr>
<th>Levels of Demands</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lower-level demands (memorization):</strong></td>
<td>Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.</td>
</tr>
<tr>
<td><strong>Lower-level demands (procedures without connections):</strong></td>
<td>Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task. Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. Have no connection to the concepts or meaning that underlie the procedure being used. Are focused on producing correct answers instead of on developing mathematical understanding. Require no explanations or explanations that focus solely on describing the procedure that was used.</td>
</tr>
<tr>
<td><strong>Higher-level demands (procedures with connections):</strong></td>
<td>Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.</td>
</tr>
<tr>
<td><strong>Higher-level demands (doing mathematics):</strong></td>
<td>Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example. Require students to explore and understand the nature of mathematical concepts, processes, or relationships. Demand self-monitoring or self-regulation of one’s own cognitive processes. Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.</td>
</tr>
</tbody>
</table>

Mathematical Task Framework

This section assumes the information in the prior section and delves deeper into what else is known about cognitive demand. To help position my own work, I will begin by giving a broad overview of cognitive demand research, and then I will narrow my focus to the aspects of cognitive demand that my work helps to better understand. For this broad overview, I begin by considering the Mathematical Task Framework (MTF; Stein et al., 1996; Figure 3). The MTF which illustrates the progression of a task throughout a lesson, and identifies phases of a mathematics lesson where the cognitive demands of a task have the potential to be altered and why those changes might occur. The relationships within the MTF are complex, and different research has helped to shed light on different aspects of the MTF. By looking at the different parts of the MTF separately, I provide details to help understand different aspects of the cognitive demand of a task; and then by considering the MTF as a whole, I give an overview of what is known about cognitive demand today.

The outcome of the MTF is Student Learning (represented by the triangle in Figure 3) and I start my discussion of the literature there to better understand why work on cognitive demand is important. Next, I discuss the literature on Mathematical Tasks as they appear in curricula (the leftmost rectangular box). I then move to what is known about Mathematical Tasks as set up by teachers, as well as the Factors Influencing Set Up (the middle rectangle and the leftmost circle). I then complete this framework by describing Mathematical Tasks as they are implemented and the Factors Influencing Implementation (the rightmost rectangle and the rightmost circle). Implementation and Factors Influencing Implementation are the aspects of the framework that have the most bearing on my dissertation work, so after my discussion of the
MTF as a whole, I look more in depth at what is known about this area, including the tools developed to explore it.

**Figure 3**

*The Mathematical Task Framework*


**Cognitive Demand and Student Learning**

To understand the importance of cognitively demanding tasks, we will begin with what is known about the last phase of the MTF: Student Learning (triangle in Figure 3). According to Otten et al. (2017), the benchmark for what is known today about student learning outcomes related to cognitive demand was set by Stein and Lane (1996). Stein and Lane (1996) collected data for 620 task enactments from six urban middle schools from across the U.S. A stratified
random sample of 144 of these task enactments from the four longest participating school sites was drawn from the original data set, so that the schools and teachers from these sites received equal representation in the data set. Because of this it was possible to use these data to compare the sites based on the cognitive demand during the set-up and implementation of the tasks. Student learning was measured using an empirically validated tool consisting of 36 open-ended questions that was administered during three consecutive years of data collection. The four sites were looked at and the results showed that the classroom site using the lowest proportion of cognitively demanding tasks also saw the lowest levels of cognitive demand during task enactments, and it saw the least amount of student learning gains. The classroom site that saw the greatest increase in student learning used predominantly doing math tasks, and even though the cognitive demand was lowered during the majority of the task enactments, this classroom site still maintained cognitive demand at a higher level than the other sites. Finally, the last two classroom sites had an even mixture of doing math, and procedures with connections tasks, but cognitive demand was lowered during enactment more often than the highest outcome site, and these two sites saw learning gains in between the highest and lowest sites. Overall this study showed that the consistent use of cognitively demanding tasks, where the level of cognitive demand is maintained through the lesson, will result in greater student learning gains.

Stein and Lane (1996) looked at urban schools in low-income districts when establishing a relationship between the cognitive demand of tasks, and student learning gains. However, Hiebert and Wearne (1993) found similar results when they looked at a rural school with low student turnover rates. In addition, Hiebert and Wearne found that the discourse in classrooms using high cognitive demand tasks differed from those using low cognitive demand tasks. In the high cognitive demand classrooms, students spoke more often and were asked to share their
thinking as opposed to simply reciting a formula or giving a solution. These characteristics of classroom discourse have been tied to increased opportunities for students to learn mathematics (e.g., Kazemi & Stipek, 2001; Malloy, 2010).

Similar to the findings of Stein and Lane (1996) and Hiebert and Wearne (1993), Boaler and Staples (2008) reported on a study of three high schools, two using “traditional” teaching methods and one using reform-based teaching methods. The reform-based teaching methods at the third school aligned with the use of cognitively demanding tasks. The results of this study showed that even though the reform-based school started with lower scores on content tests developed for this project, their scores caught up to the scores of the traditional schools after one year and surpassed them by the end of the second year.

The results of these studies show not only that using high cognitive demand tasks, and maintaining that cognitive demand, increases student learning gains, but also that such learning gains are not specific to a particular group of students (i.e., low-income urban students vs. middle class rural students and high school vs. middle school students). This is even more impressive when we consider that the learning gains discussed here were measured using tests that simply looked at students’ ability to apply algorithms. As previous work has shown, such tests do not assess students’ gains in conceptual understanding (Schoenfeld, 1992). Since cognitively demanding tasks have been shown to increase learning gains, and since it is likely that they also increase conceptual understanding (Stein & Lane, 1996), it is clear why cognitively demanding tasks have become a cornerstone of the NCTM standards-based mathematics reform movement.

The Measures of Effective Teaching (MET) project (Kane et al., 2013) used a nationally representative sample of 1,333 teachers, 7,491 videotaped lessons, and 44,500 students’ surveys,
state tests scores, and supplemental test scores. The work of this project echoed earlier findings that classroom environments aligned with the use of cognitively demanding tasks increased student learning gains on traditional tests more than for students in traditional classrooms. However, this project also used alternative tests that looked at students' conceptual understandings, and they found that conceptual learning gains for students in classroom environments aligned with the use of cognitively demanding tasks were also higher than the learning gains of students in traditional classrooms.

**Mathematical Tasks as Represented in Curriculum**

Now that I have established the benefits of cognitively demanding tasks, I move back to the beginning of the MTF (the leftmost rectangle in Figure 3) to explore where tasks start. Most of the time, the tasks that students see in classrooms come from textbooks, but they can also come from curriculum guides, descriptions of mathematical tasks, and instructional software (Remillard & Heck, 2014).

Since textbooks are the most common source for tasks we will first look at what sort of tasks they make available to teachers. Early work on tasks (Doyle, 1983) found that the majority of textbook tasks came from poorly written commercially produced textbooks that often explained things in convoluted ways, and at reading levels that were above or below the students for which they were written. More recent work (Jäder et al., 2020) has shown that for commonly used secondary school textbooks in the United States, 90% of algebra problems, and 81% of geometry problems found in textbooks could be solved by applying a recently learned algorithm, with no or little modification. The remaining 10% of algebra problems and 19% of geometry problems were more akin to high cognitive demand tasks, however these tasks are often not assigned by teachers, or if they are assigned, additional scaffolding may be provided that lowers
their cognitive demand. Similarly, Ruk (2019) looked at how textbooks address proportions and found that 82% of problems in commonly used textbooks either specifically ask for an algorithm when solving proportion problems (65%), or were simple memorization problems asking for known facts (17%). As opposed to the most commonly used textbooks, reform-based textbooks seem to do a better job of providing teachers with cognitively demanding tasks. Indeed, Stein and Kaufman (2010) found that when teachers used Everyday Mathematics or Investigations (two different reform-based curricula), 79% and 89% (respectively) of the tasks were of high cognitive demand.

Unfortunately, even when textbooks and adopted curriculum provides teachers with cognitively demanding tasks to use in their classrooms, teachers will find tasks from other sources, or design tasks themselves, that are of low cognitive demand and present those to students in place of the cognitively demanding tasks provided by the curricula (de Araujo, 2017). Even in a best-case scenario, where teachers were trying to present their students with the best possible tasks, Stein et al. (1996) showed that 30% of tasks came from standard textbooks (11%) or workbooks (19%), 39% were created by teachers, and only 30% came from reform-oriented textbooks. Thus, even before teachers begin presenting a task to their students, there are numerous reasons that the outlook for that task being of high cognitive demand is grim.

**Mathematical Tasks as Set Up by the Teacher**

I now explore how mathematical tasks are set up by the teacher and factors that influence that set up (the second rectangle and leftmost circle in Figure 3). Problems may originate in textbooks or curricular frameworks, but a lot can happen before they are presented to students. As Remillard and Heck (2014) describe, instructional tasks can be influenced by social, political, cultural, structural, and cognitive factors, such as what teachers, school districts, and state and
national organizations feel is important. In turn, these entities are influenced by factors ranging from the questions asked on consequential assessments, to empirical research, to popular opinion of the masses, to personal opinions, values, or knowledge. As an example, a textbook may present the problem, “The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price?” One teacher may take this problem and use it to help prepare their students for an upcoming consequential assessment by making it multiple choice and giving their students the possible answers: (a) Yes, because 50% is always the same, (b) No, because once the price of the necklace is increased you will be taking 50% of a different amount when you decrease the price, (c) Yes, but only if the original price of the necklace is $0, and (d) Both c and d are correct. Another teacher may take the same problem, and because they read a research article about the importance of students explaining their reasoning, the teacher may add “explain your reasoning” to the problem. Finally, another teacher may think this problem is too challenging for their students, so the teacher may change it to: “The original price of a necklace is $100, if the price is increased by 50%, to a new price of $150, will the necklace go back to the original price if the new price of $150 is decreased 50%? Overall, these factors and influences can lead to great differentiations in the tasks that are actually presented to students. If two students in different classrooms are being taught the same mathematical concept, what tasks, and how those tasks are presented to them can be vastly different due to the scores of influences that impacted how and why the concept was being taught.

Clearly, if teachers start with just a problem and no additional information about how it should be presented, the outcome is likely to vary. However, what about situations where teachers are given instructions about how a problem should be presented to students? According to Stein et al. (2007), reform-based curricula often provides instructions for teachers about how
content should be taught. However, when different teachers look at the same set of instructions, things like their individual mathematical knowledge, their understanding of reform ideals, and their personal preferences can lead them to interpret these instructions in very different ways. Because of these different interpretations, even well-meaning teachers who see the value in reform-based curricula and strive to adopt it in their classrooms can adapt tasks in ways that were not intended by, and even go against the original intent of curriculum developers and the reform-based ideals from which the curriculum evolved. Thus, even if we consider a single task from one standards-based reform curriculum, the task that is actually presented to two different classes by two different teachers can vary widely. As an example, Teacher A in one classroom could ask students, “Which is larger, $x$ or $x + x$? Explain your reasoning.” And Teacher B in another classroom could say, “Which is larger, $x$ or $x + x$? Explain your reasoning, and remember $x$ does not always have to be positive.” In such a scenario, Teacher B added a small caveat that totally changed the nature of this task. In Teacher A’s classroom, the students would likely have to cognitively struggle to come to the understanding that the value of $x$ plays an important role in this task. In Teacher B’s classroom, much of this cognitive struggle has been removed by the caveat, and students are left with simply finding a correct solution.

It has been established that tasks as set up by the teacher in the classroom can vary widely (Remillard & Heck, 2014). Even if students in different classrooms are presented with the same task, and even if that task has specific instructions for its implementation, the actual tasks that students end up engaging with can be quite different (Stein et al., 2007). However, this still leaves out when the same task is implemented the same way in different classrooms. Looking at the same cognitively demanding task that is implemented the same way in different classrooms may provide additional information about subtle differences of how cognitive demand is
maintained between these implementations that can be exploited and used to improve the maintenance of cognitive demand for all task enactments.

I have now explored the beginning and the end of the MTF, and drawn a relationship between the two. I saved the middle sections of the MTF for last because this is where my dissertation work lies. I will now focus on what is known about the middle section of the MTF and the enactment of cognitively demanding tasks.

**Instructional Tasks as Implemented**

To begin zeroing in on where my dissertation work is situated, I now move to instructional tasks as they are implemented (the rightmost rectangle in Figure 3). Analyzing a stratified sample of 520 videotaped task enactments from six school districts across the U.S., Stein et al. (1996) found that 74% of tasks in their data set started out as having high cognitive demand. However, of these tasks, only 30% maintained their cognitive demand when they were enacted (Figure 3). In a similar study, Hiebert et al. (2003), seeking explanatory evidence for the results of the 1999 Trends in International Mathematics and Science Study (TIMSS) study, looked at 83 task enactments from 83 different eighth grade classrooms across the U.S. Unlike the findings of Stein et al. (1996), the teachers in these classrooms were not necessarily attempting to use reform-based curricula. This does not mean that any of the teachers were specifically opposed to such curricula, but rather, this was a nationally representative sample as opposed to Stein et al. (1996) data which came from classrooms where teachers were actively attempting to use reform-based curricula. The results can be seen in Figure 4, and they show that only 17% of tasks in these classrooms started out as having high cognitive demand, and of these, less than 1% maintained their cognitive demand when they were enacted. In another study, Boston and Wilhelm (2017) looked at lessons presented by 114 teachers in four school districts.
The teachers were asked to select cognitively demanding tasks for this study, and 76% were able to do so, but unfortunately, only 34% were able to maintain that high level of cognitive demand when the task was enacted (Figure 4).

**Figure 4**

*Decline of Cognitive Demand During Task Enactment*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization</td>
<td>As Written: 1%, As Implemented: negligible</td>
<td>As Written: 13%, As Implemented: 100%</td>
<td>As Written: 0%, As Implemented: not applicable</td>
</tr>
<tr>
<td>Procedures without connections</td>
<td>18%</td>
<td>69%</td>
<td>25%</td>
</tr>
<tr>
<td>Procedures with connections</td>
<td>34%</td>
<td>17%</td>
<td>47%</td>
</tr>
<tr>
<td>Doing math*</td>
<td>40%</td>
<td>38%</td>
<td>29%</td>
</tr>
</tbody>
</table>

*Note.* “As Written” represents the total percentage of tasks from the study that fell in the corresponding category before the tasks were implemented. “As Implemented” represents the total percentage of tasks from the corresponding category that maintained that level of cognitive demand upon being implemented. * Due to differentiations in how the levels of cognitive demand were measured, it was not possible to attain results for doing math tasks and procedures with connections tasks separately, so they are both reported here simply as high cognitive demand tasks.

The results listed in Figure 4 clearly show that when tasks are implemented, especially tasks that begin as high cognitive demand tasks, the level of cognitive demand is frequently lowered upon implementation. Hiebert et al. (2003) showed that this is certainly true when teachers were not using reform-based curricula. Stein et al. (1996) showed this is still true when teachers are using reform-based curricula. And Boston and Wilhelm (2017) showed that this is still true even when teachers are specifically trying to use cognitively demanding tasks. Thus, it seems that cognitive demand being lowered during task enactments is a concern in all classrooms, regardless of the quality of the curricula or the specific tasks being used. However, we can also see that maintaining cognitive demand occurs more often in classrooms where teachers are using reform-based curricula and especially they are using cognitively demanding
tasks. So, if we seek to understand how this maintenance occurs, studying classrooms where teachers are attempting to improve their teaching and using cognitively demanding tasks, are likely to yield more fruitful results.

Factors Influencing the Maintenance of Cognitive Demand

I conclude my discussion of the MTF with the factors that influence implementation (the rightmost circle in Figure 1). To fully understand these factors, I look first at what causes cognitive demand to be lowered and then at what helps maintain cognitive demand. I then move on to work built on the maintenance and decline of cognitive demand. Finally, I consider specific actions teachers can take to maintain cognitive demand and ways in which others have built on earlier findings of cognitive demand research.

What Causes Cognitive Demand to be Lowered

As we have just seen, maintaining cognitive demand does not happen in a majority of lessons that begin with cognitively challenging tasks. In fact, according to Henningsen and Stein (1997), even under optimal circumstances, where a teacher starts with a cognitively demanding task, and also utilizes a widely accepted tool like Polya’s four-steps of problem solving to keep students on track, things can still go awry. Thus, we will look next at what causes cognitive demand to be lowered or maintained during task enactments.

Drawing from 520 task enactments, Stein et al. (1996) uncovered six predominant factors that cause the cognitive demand of tasks to be lowered during task enactments (see Figure 5). Even though Stein et al. (1996) were the first researchers who brought together these factors, they were based on prior work. As an example, *inappropriate amount of time* is a factor that reflects previous findings that if students do not have enough time to work on a task, the results will generally only lead to rote learning (Doyle, 1983). Doyle (1983) also showed that teachers
must have well-behaved students in their classrooms before they can successfully implement cognitively demanding tasks; this is related to Stein et al. (1996) finding that the factor of classroom management issues can lower cognitive demand. However, it should be noted that a well-behaved class in and of itself is not enough to successfully implement and maintain the cognitive demand of cognitively demanding tasks. As previous work has shown, teachers considered exemplary, who have excellent classroom management skills, can maintain discipline in their classrooms at the expense of maintaining cognitive demand (Doyle & Carter, 1984). Another thing that can lower the cognitive demand of a task is where the teacher sets the bar for student participation. If a teacher starts with a cognitively demanding task, and the students do not put forth enough effort to solve the task, and the teacher responds by allowing incorrect, vague, or strictly algorithmic thinking, students will come to accept this as the norm, and in the future, they will continue to not put forth the effort needed to maintain the cognitive demand of tasks (Doyle, 1983). This phenomenon is directly related to the lack of accountability factor uncovered by Stein et al. (1996).

It is not solely the fault of teachers for lowering the cognitive demand of a task, students can also act as catalysts that lead to lower cognitive demand during task enactments. For example, students can repeatedly ask teachers for help or what algorithm to use to solve a particular problem; a practice known as piloting (Doyle, 1983). This process can be split into two factors that lower cognitive demand: it can make challenges become nonproblems, or it can make the focus shift to finding a correct answer. Students can also delay a problem by simply moving slowly in their work, until a teacher simply gives them information to help speed up their work (Doyle, 1983).
Figure 5

Factors that Cause Cognitive Demand to be Lowered, and Frequency of Occurrence

<table>
<thead>
<tr>
<th>Factors that lower cognitive demand</th>
<th>Explanation</th>
<th>All tasks</th>
<th>Doing math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenges become nonproblems</td>
<td>“The problematic aspects of a task somehow became routinized, either through students’ pressing the teacher to reduce task ambiguity and complexity by specifying explicit procedures or steps to perform or by teachers’ taking over the challenging aspects of the task and either performing them for the students or telling them how to do them” (p. 479, Stein, Grover, &amp; Henningsen, 1996).</td>
<td>64%</td>
<td>39%</td>
</tr>
<tr>
<td>Inappropriate task</td>
<td>Either a task is too simple or too difficult, or students lack interest, motivation, or the prior knowledge necessary to complete a task.</td>
<td>61%</td>
<td>47%</td>
</tr>
<tr>
<td>Focus shifts to finding a correct answer</td>
<td>Rather than working towards an understanding of the mathematical concepts underlying a task, students become focused on only finding a correct solution</td>
<td>44%</td>
<td>25%</td>
</tr>
<tr>
<td>Inappropriate amount of time</td>
<td>If students have too little time, they will not be able to fully explore the underlying mathematics, and if students have too much time they will likely lose focus.</td>
<td>38%</td>
<td>47%</td>
</tr>
<tr>
<td>Lack of accountability</td>
<td>Students are not held accountable for understanding the underlying mathematics of the task because the teacher does not ask for, or sustain pressure for, explanations or reasoning.</td>
<td>21%</td>
<td>28%</td>
</tr>
<tr>
<td>Classroom management</td>
<td>Students have trouble working on tasks requiring a lot of cognitive activity, so they become disruptive and stop working.</td>
<td>18%</td>
<td>22%</td>
</tr>
</tbody>
</table>

Note. Derived from the work of Stein et al. (1996) and Henningsen and Stein (1997).

Besides determining what factors can lower cognitive demand, Stein et al. (1996) were also able to determine how often each of these factors occurred (see Figure 5). On average a task had 2.5 reasons for being lowered, thus the factors that cause the cognitive demand of tasks to be lowered are not exclusive. Going even further, Henningsen and Stein (1997) looked specifically at the 40% of tasks (58 tasks) that began as doing math, and determined what factors were responsible for causing the cognitive demand of these specific tasks to be lowered during set up and implementation (see Figure 5).

Looking at how often these factors play a role in lowering cognitive demand for all tasks as compared to doing math tasks, it appears there is often a difference in the prominence of individual factors. For example, when all tasks are considered, too much or too little time is only the fourth most common reason for the cognitive demand of tasks to be lowered, but if we look only at doing math tasks, this same factor is a tie for the most common reason. This shows that
these factors are not absolute, and clearly there are influences, such as the initial cognitive demand of a task in this case, that can cause factors to have different levels of prominence.

With all of the reasons that the cognitive demand of a task can be lowered, it is little wonder that findings have shown that math teachers do not always implement standards-based curricula as curriculum developers intended (Boston & Smith, 2009). These findings also shed light on why cognitively demanding tasks are not the norm in most classrooms; as evidenced by Weiss et al. (2003) who looked at 480 teachers/lessons in 120 schools in 40 districts throughout the United States and found that only 15% of lessons were intended to be cognitively demanding, and less than half of these were implemented as such. Even well-meaning teachers who want to see their students succeed can inadvertently lower the cognitive demand of tasks by helping students too much, or by the amount of time they give students to work on a task. However, such findings also offer hope: when research can uncover reasons that cognitive demand is lowered, these reasons can be explored further to provide pathways for teachers to improve the maintenance of cognitive demand during task enactments in their classrooms.

**What Helps Maintain Cognitive Demand**

A substantial amount of the work on cognitive demand has focused on what can be done to maintain the cognitive demand of tasks (e.g., Smith & Stein, 2018). Stein et al. (1996) uncovered seven factors that help maintain cognitive demand during task enactments (see Figure 6). As with the factors that lower cognitive demand (Figure 5), Stein et al. were able to determine the frequency of the factors that help maintain cognitive demand (see Figure 6). Tasks could employ more than one of these practices, so the percentages given are not exclusive. Also, Henningsen and Stein (1997) looked specifically at tasks that began as doing math, and
determined what factors were responsible for causing the cognitive demand of these specific tasks to be maintained during set up and implementation.

**Figure 6**

*Factors That Help Maintain Cognitive Demand and Frequency of Occurrence*

<table>
<thead>
<tr>
<th>Factors that help maintain cognitive demand</th>
<th>Explanation</th>
<th>All tasks</th>
<th>Doing math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriate task</td>
<td>Tasks are challenging but not overly difficult, and they utilize previously learned concepts.</td>
<td>82%</td>
<td>82%</td>
</tr>
<tr>
<td>Appropriate amount of time</td>
<td>Students have just enough time to explore and formulate ideas.</td>
<td>71%</td>
<td>77%</td>
</tr>
<tr>
<td>Modeled high-level performance</td>
<td>The teacher or a more capable peer shares their solution strategy for a task, thus assisting others to make connections necessary to understand the underlying mathematical concepts.</td>
<td>71%</td>
<td>73%</td>
</tr>
<tr>
<td>Sustained pressure meaning</td>
<td>The teacher continually asks for or expects students to students to provide evidence for their contributions or explanations for their reasoning.</td>
<td>64%</td>
<td>77%</td>
</tr>
<tr>
<td>Proper amount of scaffolding</td>
<td>The teach or other capable student provides assistance in solving a problem in such a way that this assistance did not make the task less complex or challenging.</td>
<td>58%</td>
<td>73%</td>
</tr>
<tr>
<td>Student self-monitoring</td>
<td>Students stay on track, and continue pressing themselves, evidence of this is when a teacher asks a student for an explanation, or to share their reasoning, the student is able to do so.</td>
<td>27%</td>
<td>36%</td>
</tr>
<tr>
<td>Drew conceptual connections</td>
<td>The teacher or a student explicitly makes connections between the task and the underlying mathematical concepts.</td>
<td>13%</td>
<td>14%</td>
</tr>
</tbody>
</table>

*Note.* Derived from the work of Stein et al. (1996) and Henningsen and Stein, 1997).

As can be seen here, the factors that help maintain cognitive demand are much more similar across cognitively demanding tasks and all tasks in general than were the factors that lower cognitive demand. No matter what types of tasks are being used, it is important that they are *appropriate tasks*, that students are given the *appropriate amount of time* to grapple with tasks, and that the teacher or more capable peer *modeled high-level performance*.

To get another perspective on the maintenance of cognitive demand, we again look to the work of Hiebert and Wearne (1993). The results of their work show that a smaller number of problems and more time spent on each problem was not related to student achievement, and as such, the amount of time a problem takes is not necessarily related to an increase in cognitive
demand. What really helps maintain the cognitive demand of a problem is how teachers and students interact with it. This study showed that the types of questions the teacher asked, such as asking students why a procedure works, asking students to consider the nature of a particular strategy, or asking students to come up with a story about a problem, had an effect on maintaining cognitive demand. Related directly to this, increased length of student utterances during whole class work also had a positive correlation with increased cognitive demand. Essentially, a teacher asking probing questions that require students to give explanations, rather than simple one-word answers, is an indicator of maintaining cognitive demand.

The findings of Hiebert and Wearne (1993) seem to at least partially align with those of Stein et al. (1996). As an example, the types of questions that maintain cognitive demand align with teachers’ sustained pressure meaning. Even more importantly however, the ideas of the Hiebert and Wearne and Stein et al. do not conflict. Instead, they bring together different dimensions of understanding what can be done to help students maintain cognitive demand.

Looking over what is known about the maintenance and decline of cognitive demand, it would seem that some of the main reasons that cognitive demand is maintained include appropriate task, appropriate amount of time, modeled high-level performance, sustained pressure meaning, and proper amount of scaffolding. Not surprisingly, the inverses of many of these factors are the reasons that the cognitive demand of tasks can be lowered. Because of this, teachers must be constantly vigilant in maintaining cognitive demand. A prime example of the fine line that teachers must walk when maintaining cognitive demand can be found in scaffolding: if a teacher gives struggling students an example that is too similar to the task that they are working on, the teacher is essentially giving the students a solution strategy, and thus lowering the cognitive demand. However, if a teacher gives these students a problem that is too
dissimilar, it will likely not help them, and they may continue to struggle unproductively, or worse still, get off task. With this in mind, it is clear that attempting to maintain cognitive demand is not a small undertaking because the slightest teacher action can have either a positive or negative effect on the maintenance of cognitive demand during task enactments. As such, it seems important for researchers to try to understand small differences in teacher actions, and how these affect the maintenance or decline of cognitive demand.

**Specific Actions Teachers Can Take to Maintain Cognitive Demand**

As noted earlier, a substantial amount of the work has focused on what can be done to maintain cognitive demand, and this extends far past the identification of factors that can maintain (or lower) cognitive demand. Utilizing all of these factors, a set of five practices was developed and has been the subject of papers (e.g., Stein et al., 2008) and books (e.g., Smith & Stein, 2018). These five practices were intended as a framework to help teachers facilitate discussion in their classrooms that maintains the demands of cognitively challenging tasks (Stein et al., 2008). The five practices—anticipating, monitoring, selecting, sequencing, and connecting—represent different stages of preparing for and enacting discussions surrounding cognitively demanding tasks.

According to Smith and Stein (2018), anticipating occurs before a task is enacted, and it consists of teachers discerning a variety of solution paths for a given task. Monitoring involves more than simply knowing where students are in the process of completing a task, which students are frustrated, or which students are off task. Monitoring involves the teacher asking specific types of questions as students work on their task, trying to understand the different solution paths that students have attempted, as well as what mathematics students have tried to apply to a given task. When the teacher is aware of these things, it will aid them in facilitating a
relatively smooth and, more importantly, productive discussion involving the different solution strategies that can be employed for a particular task.

Selecting when and what students will share their solution strategies with the class allows teachers to be in control of what mathematical ideas will be shared with the class. When a teacher does this, they can ensure that all of the mathematics that they want to be addressed will be, along with any misconceptions that the teacher hopes to make public so that they can be dispelled. For sequencing, the teacher must determine the order they want the student mathematics to appear. There are different ways to do this including: (a) first dispelling any common misconceptions that students have and then moving on to workable strategies, (b) having students share the most common strategies first, so that the largest number of students possible see that their thinking is validated, and (c) starting with the simpler strategies, which allows more students to get on board with the conversation, and gives everyone the stepping stones that they may need to be able to grapple with the more complex strategies that are presented towards the end. Finally, connecting is where the teacher helps students to make connections between the different strategies that have been presented and draws out the important mathematics.

It is clear that a great deal of work has already been done to help us understand the maintenance and decline of cognitive demand during task enactments, as well as to provide resources for teachers to help them maintain cognitive demand during task enactments. The understanding of the maintenance of cognitive demand during task enactments that we have so far is based on data that draws from a wide range of classroom environments. Some of them are traditional (e.g., Hiebert et al., 2003; Weiss et al., 2003) and some of them are reform-based (e.g., Boston & Wilhelm, 2017; Stein et al., 1996). These, however, are global descriptions of the
instruction. Less is known about the maintenance of cognitive demand of task enactments when teachers are using specific reform-based practices.

**Building on Known Factors That Affect the Maintenance of Cognitive Demand**

As described earlier, Stein et al. (1996) identified six factors that lower the level of cognitive demand during task enactments (Figure 5), and seven factors that help maintain cognitive demand (Figure 6). These factors have been applied to other research for an array of different reasons (see Figure 7). The commonality among these studies is that they all use Stein et al.’s (1996) factors that maintain or lower cognitive demand. However, beyond that commonality, Figure 7 shows how they vary in their purpose, context, and findings.

Based on the studies from Figure 7, we can see that there are a number of influences other than the factors described by Stein et al. (1996; Figures 5 and 6), that can have an effect on the maintenance of cognitive demand. For starters, the use of professional development can have an effect on the maintenance of cognitive demand during task enactments (Boston & Smith, 2009; Olson et al., 2011). This is sometimes the case even when the professional development was not specifically intended to affect the maintenance of cognitive demand (Bishop et al., 2012). Additionally, things that a teacher draws from for their lessons, including the curriculum that they use (Stein & Kaufman, 2010) and their own subject matter knowledge (Lunt, 2011), can have an effect on the maintenance of cognitive demand. Other influences include the type of preparation that a teacher uses for their lessons (Estrella et al., 2019) and even a teachers’ beliefs about their students’ abilities (Hong & Choi, 2018). However, it is not just teacher-related influences that can affect the maintenance of cognitive demand, the way students talk amongst each other can also be an influence (Cheng & Feldman, 2011). Considering all of the studies in Figure 7, it is clear that influences besides the factors uncovered by Stein et al. (1996) can affect
the maintenance of cognitive demand. This work is an important start, but there is still a lot to be done. Further understanding of influences that affect the maintenance of cognitive demand can be used to help teachers maintain the cognitive demand of task enactments in their classrooms.

When Stein et al. (1996) first uncovered the factors that can help maintain, or lower cognitive demand (Figures 5 and 6), they did so by drawing from a sample of classrooms using different reform-based curricula. Initially, no stratification was made between these factors and the specific curricula being used, but later work showed that the curricula itself could affect these factors (Stein & Kaufman, 2010). Similarly, influences such as teachers’ subject matter knowledge (Lunt, 2011), teachers’ beliefs about students’ abilities (Hong & Choi, 2018), and student discourse (Cheng & Feldman, 2011) can all affect these factors. Attempting to understand how influences such as these affect the factors that help maintain or lower cognitive demand is a recent endeavor, so looking at additional influences is fertile ground for research. For example, teaching practices that build on student thinking might affect the factors that help maintain or lower cognitive demand.

**Measuring Cognitive Demand**

Up until this point, I have explained what cognitive demand is, why it is important, the intricacies involved in maintaining cognitive demand during task enactments, and how this knowledge has been utilized by other researchers. In order to build on what is known and conduct a study involving the maintenance of cognitive demand, it is essential to have an effective measurement tool. Thus, I will now discuss the Instructional Quality Assessment (IQA), an important tool that has been developed to measure cognitive demand.
## Research That Expands Our Understanding of the Maintenance of Cognitive Demand

<table>
<thead>
<tr>
<th>Purpose of the Study</th>
<th>Context</th>
<th>Findings of the Study</th>
<th>Relationship to Maintaining Cognitive Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston and Smith (2009)</td>
<td>Do teachers’ selection and implementation of cognitively demanding tasks increase during their participation in a PD for that purpose</td>
<td>Do teachers integrate lesson study insights into practice, and what aspects of it promote reflection, collaboration, and change?</td>
<td>Teachers who participated in PD could select, implement, and maintain cognitive demand of at higher levels than a) before PD, and b) the control group.</td>
</tr>
<tr>
<td>Olson, White, and Sparrow (2011)</td>
<td>Teachers attended seven lesson study groups. The third group discussed maintenance of cognitive demand. No other group discussed any aspect of cognitive demand.</td>
<td>This study looked at student learning gains associated with PD.</td>
<td>Teachers at the schools that maintained the reform-based PDs were able to maintain cognitive demand of task enactments.</td>
</tr>
<tr>
<td>Bishop, Berryman, Peter, and Clapham (2012)</td>
<td>PD helped teachers understand Maori values and culture, including speaking in turn, and shared teacher/student responsibility to attain knowledge.</td>
<td>Maintenance of cognitive demand was measured in 511 lessons using Investigations (mostly doing math tasks), and Everyday Mathematics (mostly procedures with connections).</td>
<td>Teachers at the schools that maintained the reform-based PDs were able to maintain cognitive demand of task enactments.</td>
</tr>
<tr>
<td>Stein and Kaufman (2010)</td>
<td>To understand how cognitive demand is maintained by different small groups of students working on the same task.</td>
<td>A task was given to a classroom of students who worked in different groups, where the unit of analysis was the groups. Two groups were reported on in this study.</td>
<td>The curriculum that a teacher uses can have an effect on the maintenance of cognitive demand in their classrooms.</td>
</tr>
<tr>
<td>Cheng and Feldman (2011)</td>
<td>Understand how cognitive demand was maintained during a high school calculus class about limits.</td>
<td>In this class, the teacher felt as though they maintained the cognitive demand of tasks.</td>
<td>The different ways students talked about the problem lead to differences in maintenance of cognitive demand.</td>
</tr>
<tr>
<td>Hong and Choi (2018)</td>
<td>Understand how content knowledge affects the maintenance of cognitive demand during task enactments.</td>
<td>Three teachers were evaluated on student understanding, their ability to identify, explain, interpret, discriminate, apply, and generalize within the context of understanding addition and subtraction word problems.</td>
<td>The maintenance of cognitive demand was affected by teachers' beliefs in students abilities.</td>
</tr>
<tr>
<td>Lunt (2011)</td>
<td>Understand how lesson study affects the maintenance of cognitive demand.</td>
<td>Teachers participated in eight lesson study groups. Task enactments occurred after the fourth, and final groups. Both enactments were coded by looking for factors that maintain cognitive demand.</td>
<td>Maintenance of cognitive demand was affected by teachers own understanding of underlying concepts, and their ability to put this understanding into practice.</td>
</tr>
<tr>
<td>Estrella (2019)</td>
<td></td>
<td></td>
<td>Factors that maintain cognitive demand vary significantly, and lesson study sees cognitive demand maintained at higher levels.</td>
</tr>
</tbody>
</table>
The IQA consists of 10 rubrics that were originally developed and tested at the University of Pittsburgh’s Learning Research and Development Center (Boston & Wolf, 2006). These rubrics were later revised, refined, and tested again by Boston (2012). However, when I discuss the IQA, I am referring to an even later version from 2017 (M. Boston, personal communication, April, 10, 2019). The IQA was originally developed for use during live instruction, but the cognitive demand of task enactments of video recorded lessons can be ascertained using the IQA as well (see for instance, Boston & Candela, 2018). The ten rubrics of the IQA measure the Academic Rigor (AR) and Accountability Talk (AT) of enacted tasks (see Figure 8). AR measures students’ opportunities to work on challenging, cognitively demanding tasks and grapple with mathematical concepts, and AT looks at whole-group discussion to determine how effectively the discourse builds accountability to the learning community and accountability to knowledge and rigorous thinking.

**Figure 8**

*Rubrics of the Instructional Quality Assessment*

<table>
<thead>
<tr>
<th>Rubric Grouping</th>
<th>Rubric Name</th>
<th>What the Rubric Assesses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Academic Rigor</strong></td>
<td>Potential of the task</td>
<td>What level of cognitive demand the task is meant to depict</td>
</tr>
<tr>
<td></td>
<td>Implementation of the task</td>
<td>The level of cognitive demand that teachers guide students to engage with</td>
</tr>
<tr>
<td></td>
<td>Student discussion following the task</td>
<td>The extent to which students explain their thought processes</td>
</tr>
<tr>
<td></td>
<td>Rigor of teachers’ questioning</td>
<td>To what extent the teacher asks academically relevant questions</td>
</tr>
<tr>
<td></td>
<td>Mathematical residue</td>
<td>To what extent the important ideas of the task were uncovered during whole class discussion</td>
</tr>
<tr>
<td><strong>Accountability Talk</strong></td>
<td>Participation</td>
<td>How many students are part of whole the whole class discussion</td>
</tr>
<tr>
<td></td>
<td>Teacher’s linking contributions</td>
<td>To what extent the teacher supports student’s connecting big ideas during whole class discussion</td>
</tr>
<tr>
<td></td>
<td>Student’s linking</td>
<td>The extent to which students make connections between their and other students’ ideas</td>
</tr>
<tr>
<td></td>
<td>Asking (teacher press)</td>
<td>To whether the teacher continually ask students to explain their reasoning</td>
</tr>
<tr>
<td></td>
<td>Providing (students responses)</td>
<td>To what degree students provide reasoning for their statements</td>
</tr>
</tbody>
</table>

*Note. Derived from Boston (2012)*
According to Boston (2012), the 10 rubrics of the IQA are based on cognitive demand and discourse research, such as that of Stein et al. (1996), Hiebert and Wearne (1993), and O'Connor and Michaels (1996). The direct correlation of some IQA rubrics to aspects of cognitive demand that we have already discussed is almost immediately evident. For example, the Potential of the Task rubric directly correlates to Stein et al.’s (1996) levels of cognitive demand (Figure 2). As another example, the Asking (Teacher Press) rubric correlates with Hiebert and Wearne (1993) finding that the most cognitively demanding teacher questions involve asking students to share their reasoning behind a solution or statement. Similarly, the Teachers Linking Contributions rubric correlates with O'Connor and Michaels (1996) work around teacher discourse moves such as revoicing. Other rubrics drew more broadly from the cognitive demand research that we have already looked at. For instance, the Implementation of the Task rubric takes into account many of the factors that we have looked at related to the maintenance of cognitive demand.

The scores for all of the most recently updated rubrics (M. Boston, personal communication, April, 10, 2019) fall into six categories: N/A, zero, one, two, three, and four. N/A indicates that students did not engage with the task in any way. Zero indicates that either there was no mathematical activity, or no class discussion. One, two, three, and four are based on the four levels of cognitive demand theorized by Stein et al. (1996; Figure 2): memorization, procedures without connections, procedures with connections, and doing math, respectively. This scoring system is consistent through nine of the ten rubrics; the Mathematical Residue rubric does not have the N/A category.

When the IQA was originally piloted, Boston and Wolf (2006) utilized the rubrics of the IQA for 16 mathematics lessons from randomly sampled primary schools in two urban school
districts. Reliability tests between different raters who used the IQA found that the rubrics of the IQA were effective for evaluating school mathematics lessons. After the IQA was revised, it was again tested for interrater reliability by Boston (2012). After using the rubrics of the IQA on 26 lesson observations and 35 sets of assignments with student work samples collected from a middle school, again the results showed that the IQA can provide an assessment of the maintenance of cognitive demand of high-level tasks throughout the task enactment.

Since its initial pilot, the IQA has been used numerous times and for multiple purposes (e.g., Boston, 2012; Boston & Candela, 2018; Boston & Smith, 2011, 2019; Cassidy, 2009; Neergaard & Smith, 2012; Sullivan, 2019). Boston and Smith (2011) used the IQA as a professional development tool to help teachers implement cognitively demanding tasks. It was used by Boston (2012) to assess teachers’ ability to enact high-quality mathematics instruction. Cassidy (2009) used the IQA to compare the effectiveness of teachers who were trained through traditional programs versus teachers who received alternative certification. Neergaard and Smith (2012) used it to assess new teachers’ improvement in instructional quality over the first three years that they teach. Boston and Candela (2018) applied the IQA to videotaped lessons to assess how it could be used as a tool for professional learning and reflection. Sullivan (2019) used the IQA to measure change in the quality of classroom discourse over a three-year period as teachers attended 1000 hours of PD that was intended to improve the quality of classroom discourse. It has been found that the IQA is “a statistically sound instrument for collecting quantitative data on teachers’ selection and implementation of cognitively challenging tasks” (Boston & Smith, 2009, p. 147). Thus, the IQA has been shown to be a powerful research tool capable of assessing the maintenance of cognitive demand during task enactments.
The IQA is a tested and validated tool that can measure change over time, can be used in a variety of contexts (K-12 teacher, school, or district level measurement), and looks specifically at the quality of reform-oriented instruction that aligns with the implementation of cognitively demanding tasks (Boston et al., 2015). As versatile as it is, one thing that the IQA does not do is measure the level of each individual factor found by Stein et al. (1996) to either maintain or lower cognitive demand during a task enactment. As a tool, the IQA is more than sufficient in helping us understand if a task enactment was cognitively demanding, as well as some of the underlying causes that led to the maintenance of cognitive demand. However, to understand the role of, and interplay among, these individual factors known to either maintain or lower cognitive demand during a task enactment, I developed the Reorganized Factors that Undermine or Keep Cognitive Demand (RUK; Ruk, 2020), which I will discuss in greater depth in the Theoretical Framework section of this dissertation.

**Student Thinking**

So far, the theme of this dissertation has revolved around cognitively demanding tasks, the use of which has been called for by the NCTM (2014). However, with the overarching mission of advocating “for high-quality mathematics teaching and learning for each and every student” (NCTM, 2017), the NCTM has made additional calls to meet this goal. One such call is for the use of student mathematical thinking during instruction (NCTM, 2014). Initial calls of the NCTM in this area (e.g., NCTM 1991) were based on findings such as those of Carpenter and Fennema (1992) who showed an increase in student achievement when teachers used knowledge about students' thinking to help guide their instruction during lesson enactments. Since these initial calls, additional work has been done to understand what, when, and how using student thinking can be beneficial to students (e.g. Leatham et al., 2015).
One of the first steps in using student thinking is noticing that thinking. According to Sherin et al. (2011), teacher noticing is not just teachers passively being aware that student thinking has occurred. Instead, teacher noticing of student thinking involves teachers taking the constant barrage of thinking available in classrooms and choosing what thinking to pay attention to and what thinking to disregard. When teachers pay attention to thinking, they must also determine how that thinking relates to their mathematical goals and instruction. Because teachers cannot know for certain exactly what thinking will be available during a class or how and when it will occur, teachers who use student thinking in their instruction cannot create rigid lesson plans of exactly how material will be covered. Instead, they must plan adaptive and dynamic lessons that are fueled by the ideas that students share as the lesson is being enacted.

To help guide the dynamic nature of teaching that uses student thinking, researchers have tried to understand the characteristics and teacher actions that lead to successfully using student thinking. Cengiz et al. (2011) discussed three main types of teacher actions: eliciting—providing students opportunities to express their thinking, extending—providing students opportunities to move beyond their initial understandings, and supporting—teacher telling to help facilitate continued discussion. Of these three actions, they found that extending was the most important, but a combination of all three were necessary to create extending opportunities. However, they also found that facilitating discussions that utilize all of these actions can be a difficult undertaking for teachers.

In similar work, the questions that a teacher asks can also have an effect on the successful use of student thinking. Franke et al. (2009) found that teachers frequently asked questions that elicited student thinking; specifically, teachers were able to get students to explain their reasoning and thought processes. However, after this, teachers were less likely to ask follow-up
questions that would extend the student thinking. Essentially, teachers often asked eliciting questions to elicit students’ thinking, but they had trouble following up on that thinking.

Since following up on, and facilitating discussions about, student thinking that has been elicited has proven to be troublesome for teachers, other researchers have looked to better understand this phenomenon. Frameworks have been developed to better understand the nuances of teacher actions (e.g. Lineback, 2015), and it has been shown that with guidance, educators can learn to better utilize student thinking (Barnhart & van Es, 2015).

Others still have taken what is known about using student thinking and are attempting to develop practices for teachers to help them use, and make the most out of the student thinking that occurs in their classrooms. For example, Leatham et al. (2015) characterized Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs). Essentially, a MOST is a teachable moment—an instance of a student thinking that if taken advantage of in the moment it occurs will likely lead to the students in the class having a better understanding of the mathematics behind the student-thinking. Conversely, not taking advantage of a MOST would be a missed opportunity for student learning. In order to take advantage of MOSTs, the MOST Research Group (e.g., Van Zoest et al., 2016; Leatham et al., in press) theorized the teaching practice of building on MOSTs. Building has four elements: (a) establish—taking the instance of student thinking and making clear the object that the class will be considering; (b) grapple toss—turning the object of consideration to the class, for them to make sense of it; (c) conduct—facilitating a whole-class discussion where students work together to make sense of the object of consideration; and (d) make explicit—drawing out and verbalizing the underlying mathematics of the object that was considered. Overall, the theorized teaching practice of building is intended to give teachers a set of tools to help them use the student thinking that occurs in their classrooms.
I have discussed some of the research being done in response to the NCTMs call for increased use of student thinking in classrooms and the call for more cognitively demanding problems in today's classrooms (NCTM, 2014). Although both of these calls ultimately have the same goal of improving math education, there is little overlap between the research done to answer each individual call. To help fill this gap, my work looks to better understand what can be learned from making connections between the work that has been done to answer these two separate calls. Specifically, my work looks at the interplay between the maintenance of cognitive demand, and using student thinking through the teaching practice of building on MOSTs.

Research Questions

In an effort to better understand the maintenance of cognitive demand during the enactment of the specific teaching practice, building on MOSTs, my study seeks to answer the following research questions:

1. What does the maintenance of cognitive demand look like when teachers are attempting to attend to student thinking during task enactment?
2. How do factors contributing to the maintenance of cognitive demand vary when the same cognitively demanding task is enacted by different teachers in multiple classes?

In the methodology chapter of my dissertation, I will explain what I did to answer these two research questions. However, I will first introduce the theoretical framework I used for my study.

Theoretical Framework

In order to understand the cognitive demand of tasks that are being enacted, Stein et al.'s (1996) factors that maintain and lower cognitive demand, captured in Figures 4 and 5, have been utilized by numerous studies (e.g., Cheng & Feldman, 2011; Estrella et al., 2019; Hong & Choi, 2018; Lunt, 2011). Since my work looks to understand the maintenance of cognitive demand
during task enactments, I will likewise utilize these same factors as a framework for my study. In order to understand their interaction, and the role that each of these factors play during a task enactment, it is essential to have a way to consistently measure each factor during a task enactment. As I considered each factor, I noticed that there is some overlap between them. Stein et al. (1996) identified seven factors that lower the cognitive demand of a task enactment and six factors that help maintain it. This means that in order to measure every factor, one would have to look at a total of 13 factors for every task enactment. However, by taking advantage of the overlap that I saw, I was able to create a tool with only eight categories that can measure each of the 13 factors during a task enactment. The resulting instrument is the Reorganized Factors that Undermine or Keep Cognitive Demand (RUK; Figure 9; Ruk, 2020). As a way to explain the RUK, I will now discuss how I used the overlap between the factors known to maintain and lower cognitive demand to create a succinct tool to consistently measure these factors.

**Figure 9**

*Categories of the Reorganized Factors that Undermine or Keep Cognitive Demand*

<table>
<thead>
<tr>
<th>Reorganized Factors that Undermine or Keep Cognitive Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Appropriateness of Task</strong></td>
</tr>
<tr>
<td><strong>Appropriateness of Time</strong></td>
</tr>
<tr>
<td><strong>Solution Strategies Discussed</strong></td>
</tr>
<tr>
<td><strong>Held Accountable</strong></td>
</tr>
<tr>
<td><strong>Amount of Scaffolding</strong></td>
</tr>
<tr>
<td><strong>Classroom Management</strong></td>
</tr>
<tr>
<td><strong>Explain Thinking</strong></td>
</tr>
<tr>
<td><strong>Conceptual Connections</strong></td>
</tr>
</tbody>
</table>

*Note.* Derived from the work of Stein et al. (1996). An expanded version of the RUK, that includes rating criteria, is available in Appendix D.
Some of the factors known to affect the maintenance or decline of cognitive demand (Figures 4 and 5), are actually two sides of the same coin. For example, if students are given an appropriate task based on their level of previous knowledge, interest, and motivation, then the task is more likely to maintain its cognitive demand. However, if students are given an inappropriate task, where they are not ready to grapple with a specific concept, the task is too easy, or they are uninterested in it, then the cognitive demand will likely be lowered. If we put these together, they simply look at the extent to which the task was appropriate for students. Similarly, if students are given an appropriate amount of time to work on a task, cognitive demand is more likely to be maintained, but if students are given an appropriate amount of time (too much or too little) cognitive demand is likely to be lowered. Combined, these simply look at the extent to which the amount of work time was appropriate for students. Also, if a teacher or more capable student modeled high level-performance by sharing different solution strategies and discussing mathematical ideas needed to engage with the task, cognitive demand is more likely to be maintained. However, if the tasks’ focus shifts to finding a correct answer and solution strategies and ideas needed to engage with the task are not attended to, cognitive demand is likely to be lowered. If we consider these two factors together, they can be measured by looking at the extent to which solution strategies were discussed. Next, if a teacher continues to pressure students for explanations and meaning through sustained pressure for meaning, cognitive demand is more likely to be maintained. However, if the teacher does not hold students accountable for such things and there is a lack of accountability, cognitive demand is likely to be lowered. Combined, these simply look at the extent to which students are held accountable for explaining their reasoning. Finally, if the teacher or more capable peer provides the proper amount of scaffolding for a task, where they help students, but do not remove any of the
cognitive complexity of the problem, cognitive demand is more likely to be maintained. However, if too much information is given away, and the complexity of the problem is lost, to a point where *challenges me nonproblems*, cognitive demand is likely to be lowered. Together, these simply look at the extent to which solution strategies were given away.

As shown in the previous paragraph, many of the factors that maintain, and lower cognitive demand are two sides of the same coin. However, this is not true of all the factors. As an example, if *students self-monitor* themselves, they have been doing their work and are likely able to provide evidence for their claims or explain their thinking, which maintains cognitive demand, but not doing this will not necessarily lower cognitive demand. Certainly, if students are not self-monitoring, it is more likely that *classroom management* issues would arise—a factor that lowers cognitive demand. However, since not all classroom management issues are caused by students not self-monitoring themselves, we cannot say that there is a direct relationship here. Also, since we generally do not know what students are thinking until they speak, a reasonable way to understand self monitoring is to ask, to what extent can students explain their thinking. Because certainly if a student was self-monitoring, they would be able to explain their reasoning. Additionally, for the *classroom management* factor, we can look at the extent to which classroom management issues occur. Lastly, if the teacher *drew conceptual connections* out, this will certainly help to maintain the cognitive demand of a task, but just because these connections are not explicitly drawn out does not mean that students did not make them on their own, and so we cannot infer that cognitive demand was necessarily lowered if such connections are not explicitly made. So, all we can look at is the extent to which conceptual connections were drawn.

Thus, as we can see in many, but not all cases, the factors that maintain or lower cognitive demand are simply two sides of the same coin, and the RUK provides us with an
alternative streamlined way of viewing these factors that undergirds the decisions I make in my study. This alternative view becomes particularly useful as I explain my methodology in the next chapter.
CHAPTER III

METHODOLOGY

I first describe the teachers who were participants in my study. I then describe the data for the study—videotaped task enactments that were already collected from these teachers as part of a broader study, as well as the survey and interview data that I collected as part of this study. Finally, I describe how I analyzed these data to answer my research questions.

Participants

The six participants in this study were drawn from secondary school mathematics teachers who served as Teacher Researchers (TRs) for the MOST Project (BuildingonMOSTs.com). These teachers were chosen to be TRs because of their expressed desire to improve their teaching through the utilization of reform-based teaching practices and demonstrated ability to engage in high-level discussions of teaching. The TRs were diverse in both their geographic locations and years of teaching experience. They were located in school districts in Colorado, Michigan, New Mexico, Oregon, and Utah, and had between 3 and 25 years of experience. Some of the more experienced TRs had been professional development leaders, district leaders, prestigious teaching award recipients, book authors, and National Council of Teachers of Mathematics board members.

The MOST Project was testing and refining a theorized teaching practice, called building (Van Zoest et al., 2016), and the TRs were active participants in this process. The building practice was theorized before the TRs joined the project, but when they were onboarded through a two-day summer retreat in 2018, they became fully immersed in the practice. Through each of four separate cycles across the following year, the TRs enacted the building practice in their classrooms and engaged in discussions with the MOST Research Team to help refine the
practice. These four cycles consisted of enacting two separate cognitively demanding tasks (one during each cycle), included in the MOST Project’s MOST Eliciting Prompts (MEPs; See Appendix B). MEPs are designed to elicit predictable MOSTs. For the purposes of my study, I focused on the two tasks that were enacted twice by each TR. Because the TRs in my study were committed to using reform-based ideals to build on student thinking, they provided a best-case scenario for looking at the interaction of the building practice and maintenance of cognitive demand.

Data Collection and Analysis

The data column in Figure 10 illustrates the three data sources for this study: (a) videotaped classroom task enactments from all four MEP enactment cycles, (b) online TR surveys, and (c) recorded TR interviews in online meeting rooms. The videotaped classroom task enactment data were collected first, followed by the survey and interview data. The survey data were based on analysis of the videotaped classroom task enactments, and the interview data were based on analysis of both the videotaped classroom task enactments and the survey data. Thus my data required multiple levels of analysis and to fully understand all of my data collection and analysis, it is helpful to employ a framework to aid my description.

To help organize my data analysis, I draw on the work of Simon (2019), who uncovered three levels of qualitative data analysis. During the first level of analysis, the raw data are interpreted and coded to provide the basis for further levels of analysis. At this level the analysis stays close to the raw data and does not yet provide answers for the research questions of the study. The second level of analysis is not simply another pass over the raw data, but rather it builds on the first level of analysis. Here, the analysis is extended past the first level and “we use the results of the first level as the data for the second level” (Simon, 2019, p.118). This second
level of analysis may require numerous passes that, besides just drawing on the first level of
analysis, may also draw on raw data as well as previous passes of the second level of analysis.
Finally, the third level of data analysis offers explanations for the previous levels of analysis. In
contrast to the first two levels of analysis, which were based primarily on the data collected, this
level takes what was found and moves beyond that to answer the research questions that have
been posed.

Figure 10 illustrates how I used the ideas of Simon (2019) to organize my data. The First
Column (Previous Work) was not an explicit part of Simon’s framework, but it helps explain my
data collection and analysis. I have already discussed Box 1 of this Column (Cognitive Demand
Research), Box 2 (MOST Research), and Box 3 (the IQA) in my literature review. I showed how
the IQA (Boston, 2012) used what was known from research on cognitive demand and classroom
discourse to create a tool to evaluate the cognitive demand of task enactments. I discussed Box 4
(the RUK) in my theoretical framework. There I showed how I drew from cognitive demand
research, specifically the factors known to maintain or lower cognitive demand (Stein et. al.,
1996), in order to create a tool that could measure these factors during task enactments. Thus, in
the following I begin by discussing my data (Column 2). I start with the Task enactment data
(Column 2, Box 1), and how I needed to conduct first and second level analysis before I could
collect my survey response data (Column 2, Box 2). Then I discuss how I needed to conduct
additional first and second level analysis before I could collect my survey Interview data
(Column 2, Box 3). In conjunction with this I was also conducting other second level analysis
that lead directly to third level analysis, as opposed to being needed for data collection. After all
of my data were collected, I conducted more first level analysis, which in conjunction with
second level analysis I had already completed led to third level analysis, as well as another round
of second level analysis. Finally, after this I was able to conduct my final rounds of third level analysis, thus completing my research.

Figure 10

Data and Analysis Organizational Chart

![Data and Analysis Organizational Chart](image)

Task Enactment Data Collection and Analysis

The task enactment data (Figure 10, Column 2, Box 1) came from the MOST Project’s efforts to refine a conceptualized framework for building on MOSTs by studying the TRs’ enactments of MEPs. The complete MEP enactments, from the launch of the task to its completion, were videotaped by the MOST Project. These enactments include extended whole-class discussion during the implementation of the conceptualized building practice, and thus lend themselves very well to cognitive demand analysis. The maintenance or decline of cognitive demand during task enactment depends largely on what actions students and teachers take during the enactment, and the MOST Project’s MEP video recordings capture these actions. Thus, these data support answering Research Question 1: What does the maintenance of cognitive demand
look like when teachers are attempting to attend to student thinking during task enactment?

Since each of two MEPs were enacted twice by each TR (24 enactments total), this data set allowed me to look at multiple enactments of the same cognitively demanding task so that I could answer Research Question 2: How do factors contributing to the maintenance of cognitive demand vary when the same cognitively demanding task is enacted by different teachers in multiple classes? Each of the 24 videotaped task enactments were coded from beginning to end using the Angles (Fulcrum Technologies, 2017) video analysis software. There were two passes of coding: the IQA (Boston, 2012) and the RUK (Ruk, 2020), both of which can be found in Appendix D. Figure 10, Column 3 Boxes 1 and 2, shows that applying the IQA and the RUK to the MEP data was part of my first level of analysis.

**IQA Analysis**

The IQA is designed to be applied to task enactments in the classroom as the task is being enacted or during a first pass through a video recorded enactment. I was already familiar with the task enactments that are part of my dissertation work, due to my involvement in the larger ongoing study from which these data were drawn. Because of this the IQA analysis was completed by trained raters who were not otherwise affiliated with my dissertation work (or the larger project) in order to avoid bias.

As described in the literature review, the IQA utilizes a four-point rating system that was developed based on the four levels of cognitive demand developed by Stein et al. (1996)—Memorization, Procedures Without Connections, Procedures With Connections, and Doing Math. The second highest level for many of the AT rubrics of the IQA is often attained by doing something a minimum number of times (ex: At least twice during the lesson, the teacher explicitly connects speakers’ contributions to each other), while the highest level is attained by
doing something *consistently* (ex: The teacher consistently (at least 3 times) explicitly connects speakers’ contributions to each other). During an initial analysis of a small subset of task enactments for work related to this dissertation, it was found that two different teachers could both get the highest rating on the IQA, while still being quite differentiable from each other (ex: one teacher would make connections 13 times during a task enactment, while another made only 6). Because of this, each occurrence of something measured by the IQA was also documented for the 24 task enactments that are part of my dissertation data. Thus, even if the IQA shows that nearly every task enactment is cognitively demanding, this additional data allowed me to identify more nuanced differences that helped with my second level of analysis (Column 4, Box 2).

**RUK Analysis**

The task enactments were also analyzed using the RUK (Figure 10, Column 3, Box 2). A version of the RUK that includes the criteria for measuring each factor can be found in Appendix D. The RUK combines the factors known to lower and maintain cognitive demand (see Figures 4 & 5) and, like the IQA, puts them on a four-point continuum meant to align with the four levels of cognitive demand uncovered by Stein et al. (1996). Additionally, some aspects of the RUK are meant to look at, with more differentiation, the same aspects looked at by the IQA. Considering the categories on a continuum supports quantifying the existence of the factors known to lower and maintain cognitive demand. The results of this coding helped me to look for differences among teachers and task enactments, which helped answer my second research question. Additionally, comparing my findings to those of the earlier researchers who looked at the maintenance of cognitive demand (e.g. Stein et al., 1996) allowed me to uncover variations that could be specific to the teaching practice of building, which helped with my second level of analysis (Figure 10, Column 4, Box 3).
Further Analysis of the IQA and RUK

Overall, the combined findings of applying the IQA and the RUK were used in my second level of analysis (Figure 10, Column 4, Box 2), which ultimately helped to answer my first research question during the third level of analysis (Figure 10, Column 5, Boxes 1 & 2). By reporting on the results of these tools, I am able to provide a rich description of what it looks like to maintain cognitive demand during the enactment of a teaching practice that builds on in-the-moment, high-leverage, student thinking. Also, by rating each task using both the IQA, and the RUK, I received a wealth of data for each individual task enactment that allowed me to make many different comparisons across task enactments during the second level of analysis (Figure 10, Column 4, Box 2). When making these comparisons, I was able to identify subtle differences between these various task enactments, which helped answer my second research question during the third level of analysis (Figure 10, Column 5, Boxes 1 & 2).

Survey Data Collection

In order to understand the survey, let’s again turn to Figure 10, in particular, Column 4, Box 1. Here you can see that my survey was a form of second level analysis based on a) Cognitive demand research, b) the IQA, and c) the results of applying the RUK. I begin by explaining how each of these three factors went into creating the survey, and as I do so, the need for the survey will become apparent. After that, I look at the surveys overall and discuss how the results were analyzed. A copy of the survey is provided in Appendix C.

Cognitive Demand Research

As I have already discussed, the factors that help maintain or lower cognitive demand, originally uncovered by Stein et al. (1996) played an important role in my dissertation work. Indeed, they were the basis for creating the RUK, which underpins much of my data collection.
and analysis. Unfortunately, there are some factors that the RUK cannot fully capture through videotaped task enactments. Thus, when I created my survey, I based it partially on aspects of maintaining cognitive demand that are not observable and thus beyond the scope of the RUK. For example, the RUK looks at the extent to which students were prepared to engage with a task. Certainly, there are cues that one can ascertain about a student’s preparedness to deal with a specific task when watching an enactment, but there is no way to know for sure if students have the background knowledge to grapple with a task unless you know what mathematics they have previously learned. As such I created a question on my survey to ascertain this information (see Appendix C, Question 12). Similarly, if students are given the right amount of time to work on a task, cognitive demand is more likely to be maintained, but if students are given too much or too little time, cognitive demand is likely to be lowered. Again, there are cues such as students fidgeting or talking to one another that could indicate they are being given too much time to work, but since a video recording cannot capture exactly what every student is doing, it is impossible to know for sure what they are doing or thinking. As such, I created a survey question to help me understand how teachers decide to bring a class together for whole group discussion (see Appendix C, Question 9). This question, along with visual cues, helped me to better understand if students had the appropriate amount of work time or not. Thus, as far as the cognitive demand research is concerned, the survey was needed to provide richer data than the results of the RUK alone provide.

**IQA**

Like cognitive demand research, my survey is also partially based on the IQA. The IQA part of the survey consists of many of the same items that are on the IQA, but the wording of the questions was changed from third person to second person (i.e. the questions now read, “do
you,” as opposed to, “does the teacher”), and some longer questions were paraphrased and shortened to make them more palatable for the online survey format. Additionally, the IQA is designed to look at a specific task enactment, whereas TRs were asked about general classroom practices, so the wording of the questions was changed from asking about a specific task enactment, to how teachers do something during a typical task enactment. The reworded IQA items can be found in Appendix C (questions 1-9). By collecting this data as part of my survey, I was able to compare and contrast teachers’ self-interpretations against the IQA results during my second level of analysis (Column 4, Box 2), which gave me another dimension to consider during subsequent rounds of analysis.

RUK

Finally, my survey is also partially based on the results of the RUK. Since the results of the RUK are part of my first level of analysis (Figure 10, Column 3, Box 2), this is what makes the survey itself part of my second level of analysis (Figure 11, Column 4, Box 1). As an example, when I analyzed the results of the RUK, it seemed that some teachers naturally geared their whole-class discussions towards understanding underlying mathematical concepts. In order to understand if this was the case, I created a survey question to help me understand the moves a teacher makes after an underlying mathematical concept had been uncovered (see Appendix C, question 8). Similarly, when I analyzed the results of the RUK, it seemed like some teachers were more comfortable offering their students the appropriate amount of scaffolding when needed, and other teachers seemed to offer too much or too little. In order to better understand this, I created a survey question to better understand the amount of scaffolding that teachers generally offer (see Appendix C, question 10). It is worth noting that the focus of my research is not to provide a detailed account of the differences between TRs’ general classroom practices
and what occurred in the videotaped task enactments. Rather, my work will contribute to the field’s understanding of cognitive demand by providing an account of the maintenance of cognitive demand in relation to a specific teaching practice. As such, if the survey results showed that a teacher offered a different amount of scaffolding than usual, I needed to ask follow-up interview questions to understand how this related to the teaching practice of building on MOSTs. Thus, the survey was also needed to shed additional light on the results of the RUK, which in turn allowed me to provide richer descriptions as I answered my research questions.

Survey Analysis

Now that I have explained the creation of my survey, I can use the framework (Figure 10) to explain how this creation led to the analysis of the survey results. Figure 10 illustrates that the creation of the survey questions was part of my second level analysis (Column 4, Box 1), and that the responses to the survey questions that the teachers gave are additional data (Column 2 Box 2). The analysis of these data occurred in four ways, a) comparing the results of applying the IQA to the survey responses (Column 3, Box 3), b) comparing the results of applying the RUK to the survey responses (Column 3, Box 4), c) serving as the foundation for interview questions (Column 4, Box 4), and d) helping to explain connections between hypothesis made during second level analysis (Column 5, Box 3). Each of these four analyses will be discussed in the following sections.

Comparing the IQA to the Survey Responses

The purpose of comparing the results of the IQA to the survey responses (Figure 10, Column 3, Box 3) was threefold: (a) to find differences and similarities between the results of this instrument and teachers self-assessments that lead to hypotheses that could help explain the data (Column 4, Box 3); (b) to help inform the questions that were asked during the teacher
interviews (Column 4, Box 4), and (c) to help explain connections between hypothesis made during second level analysis (Column 5, Box 3).

**Comparing the RUK Results to the Survey Responses**

As I noted earlier, the survey responses helped enrich the data that the RUK provided. For example, I gained insight into students' prior knowledge as well as the amount of work time students were given. This in turn helped me provide richer explanations when I explained connections between hypothesis made during second level analysis (Column 5, Box 3). Additionally, like the comparison of the IQA to the survey responses, the comparison of the RUK results to the survey responses (Column 3, Box 4) helped inform the teacher interview questions (Column 4, Box 4). For example, when the results of the RUK showed that a task enactment had the proper amount of scaffolding, but the survey indicated that the teacher generally offers more scaffolding, I was able to ask a follow-up question to understand this discrepancy.

**Using Survey Responses to Create Interview Questions**

In addition to using differences and similarities between the results of the IQA and the survey responses, and the results of the RUK and the survey responses to help inform my interview questions, the survey results themselves also directly informed my interview questions (Figure 10, Column 4, Box 4). For example, one of my survey questions asked how teachers determine that students have had enough work time before they began whole-class discussion (see Appendix C, question 9). If a teacher was vague in their survey response, or if I could not see any of the cues that the teacher gave in their response in the videotaped enactment, I asked follow-up interview questions to better understand how the teacher determined when to begin whole-class discussion.
Using Survey Responses to Explain Connections

Finally, just as I used differences and similarities between the results of the IQA and the survey responses, and the results of the RUK and the survey responses to help explain connections among hypotheses made during second level analysis, I did this with the survey responses themselves as well (Figure 10, Column 5, Box 3). For example, for some of the teachers in my study, the IQA and RUK results showed that students shared the reasoning behind their ideas more often than they were asked to do so. This would seem to indicate that there was a norm in some classrooms where students are expected to share their reasoning. My third survey question enabled me to determine that this was indeed a norm in some teachers’ classrooms.

Interview Data Collection

Similar to the survey questions that I created, the interview questions that I created were part of my second level of analysis (Figure 10, Column 4, Box 4). As I explained previously, some interview questions were based directly on survey responses, and some were based on the comparison of the results of the IQA and the RUK to the survey responses. However, there were additional sources that I drew from to create my interview questions. This included the results of the IQA and the RUK themselves (Column 3, Boxes 1 and 2), as well as hypotheses I formed based on the results of the IQA and the RUK (Column 4, Box 2), and hypotheses I formed based on the comparison of the results of the IQA and the RUK (Column 4, Box 3). All of these are very similar, however there are subtle differences. For example, if the results of the RUK showed that a student was off task for the entire class period, I could ask the teacher an interview question about this particular student. This would be based strictly on the results of the RUK because I would not be able to hypothesize (with any degree of accuracy) why this student was always off task. Comparatively, if the results of the RUK showed that a student was off task after
working on a problem for a few minutes, I could hypothesize that the student solved the problem quickly, and then had nothing else to do. In this case I could ask the teacher a more pointed question during the interview. Finally, if the results of the RUK showed that a number of students were off task, and on the survey the teacher said that students were rarely off task, I could hypothesize that this had something to do with the enactment of this specific task, and I could ask the teacher an interview question to understand if that were the case or not.

Interview Analysis

Just like the surveys, the interview questions that I developed were part of my second level of analysis (Figure 10, Column 4, Box 4), while the responses that the teachers gave fall in the data column (Column 2, Box 3). During, or shortly after, each interview, I wrote down the responses that the teachers gave to my questions, as well as my impressions of these responses. These interview notes were then part of my first level of analysis (Column 3, Box 5). Then, using these notes, I came up with additional hypotheses during my second level of analysis (Column 4, Box 5). I also used these notes to confirm or disconfirm hypotheses during my third level of analysis (Column 5, Box 2). For example, if I had hypothesized that a task was too easy for students, I could use the explanation that the teacher(s) gave me during their interview(s) to explain why this was (or was not) the case. Finally, I used my interview notes, in conjunction with other data sources, during my third level analysis to help explain connections between hypotheses made during second level analysis (Column 5, Box 3).

Summary

As I have shown, the data collection and analysis for my dissertation was quite complex. This is due largely to the fact that although the survey results are data themselves, the survey was created from previously collected data (task enactments) and analysis of this data (RUK Results),
as well as previous work in the field (cognitive demand research and the IQA). Similarly, although the interview question answers were data, the interview questions themselves were created from previously collected data (survey results), first level analysis (RUK and IQA Results and comparisons of these to survey results), as well second level analysis (first and second round hypothesis). Then all of these data were put through the cycle again: first level analysis where raw data were processed, second level analysis where hypotheses were formed during numerous passes through raw data (again), first level analysis, and previous passes of second level analysis, and finally third level analysis where all of the previous levels of analysis, raw data, and even previous work from the field were drawn from to provide answers to the research questions of the study. In the next chapter, I use the results that slowly emerged from these complex cycles of data collection and analysis to answer my research questions.
CHAPTER IV
MAINTENANCE OF COGNITIVE DEMAND WHEN ATTENDING TO THINKING

In this chapter, I discuss the findings of my work as they relate to my first research question: *What does the maintenance of cognitive demand look like when teachers are attempting to attend to student thinking during task enactment?* Overall, my findings show that the maintenance of cognitive demand looks better than expected when teachers are attempting to attend to student thinking, as cognitive demand was maintained at higher levels in my study than has been found in previous studies. In the following I will first show how my data, and analysis of these data, led to the conclusion that the maintenance of cognitive demand was supported by teachers engaging in the building practice. Then I will look more deeply into what my data tell us about aspects of the maintenance of cognitive demand that seem to be enhanced when student thinking is attended to through the practice of building.

**The Maintenance of Cognitive Demand is Supported by Building**

I begin answering my first research question by looking broadly at how the level of cognitive demand for the task enactments that were part of my study compares to what has been uncovered by previous work. Two of the IQA rubrics, the Potential of the Task and Implementation of the Task, are ideally suited to assist with this comparison. The Potential of the Task rubric measures a task's potential to be cognitively demanding, with an emphasis on whether or not the task (as written) explicitly asks students to explain their reasoning. This rubric allowed me to determine which tasks from the previous studies are the most comparable to the tasks in my study. When applied to the two tasks that were part of this study, this rubric shows that both of them had the potential, as they were written, to be *Doing Math* tasks.
The IQA Implementation of the Task rubric measures the level of cognitive demand at which a task was enacted, with an emphasis on explicit examples of students explaining their reasoning. Drawing from Figure 6 (in the Literature Review chapter of this study), previous work has isolated Doing Math tasks, and looked at how cognitive demand was maintained during task enactment. By comparing this to the IQA Implementation of the Task rubric results for my dissertation study, I found that a higher percentage of the tasks in my study were implemented at the Doing Math level of cognitive demand (Figure 11). Eleven of the 24 task enactments in my study (46%) were implemented at the Doing Math level. When compared to previous work, where at best 38% of Doing Math tasks maintained their level of cognitive demand during enactment, it would seem that attending to student thinking by attempting to build on it supports maintaining cognitive demand.

**Figure 11**

*Level of Cognitive Demand During Doing Math Task Enactments*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tasks</td>
<td>As Written</td>
<td>As Implemented</td>
<td>As Written</td>
<td>As Implemented</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>58</td>
<td>22</td>
<td>34</td>
</tr>
<tr>
<td>% of Tasks</td>
<td>17%</td>
<td>0%</td>
<td>40%</td>
<td>38%</td>
</tr>
</tbody>
</table>

Of course, there are other factors besides attempting to build on student thinking that could have affected the level of cognitive demand during task enactment. For example, I previously described the teachers in my study as having a desire to improve their teaching through the utilization of reform-based teaching practices. Since the teachers that Hiebert et. al. (2003) looked at were a random sample, the simple fact that the teachers in my study were trying to improve their teaching through the use of reform-based teaching practices may account for the difference in the level of cognitive demand of the task enactments in these two studies. However,
Stein et al.’s (1996) participants were “working with each other and with ‘resource partners’–typically mathematics educators from a local university–to enhance the school’s mathematics instructional program with an emphasis on mathematical understanding, thinking, reasoning, and problem solving” (Silver & Stein, 1996, p. 481), and Boston and Wilhelm’s (2017) participants shared “the vision of ambitious mathematics instruction” and their work was designed “to improve student achievement in middle school mathematics by supporting teachers’ development of ambitious instructional practices” (p. 839). As such, the results of these two studies are quite comparable to my work. The main difference between the earlier studies as opposed to mine are that Stein et al. (1996), and Boston and Wilhelm (2017) looked at a broader spectrum of teaching practices that fall under the umbrella of reform-based teaching, whereas I looked specifically at the single teaching practice of attempting to build on student thinking. Thus, as can be seen in Figure 11, teachers attempting to build on student thinking seems to correlate with higher levels of cognitive demand during task enactments.

It is also worth noting that the other studies in Figure 11 only looked at a single enactment of each task from each teacher. Because my study looked at two enactments of the same task for every teacher, it provides us the opportunity to see what happens when we contrast the most comparable enactments from my study to previous studies. For example, Stein et. al. (1996) collected data for 620 task enactments, but used a stratified random sample of 144 task enactments for their work. With only six teachers in my study, I cannot replicate such a random sampling procedure, but I can stratify my data in other ways. For example, if I look only at the first enactment of a task, as can be seen in Figure 12, my results show that 58% of such task enactments maintained cognitive demand at the Doing Math level. By stratifying my data in this
way, it still seems that attempting to enact the teaching practice of building on student thinking helps to maintain cognitive demand at higher levels than has been previously found.

**Figure 12**

*Level of Cognitive Demand During Enactments of Doing Math Tasks With Different Stratifications of My Data*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Enactments</td>
<td>First Enactment</td>
<td>Best Enactment</td>
</tr>
<tr>
<td>Number of Tasks</td>
<td>As Written</td>
<td>As Implemented</td>
<td>As Written</td>
</tr>
<tr>
<td></td>
<td>58</td>
<td>22</td>
<td>34</td>
</tr>
<tr>
<td>% of Tasks</td>
<td>40%</td>
<td>38%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Choosing a stratification of the first enactment of a task is quite different from the stratified random sample method used by Stein et al. (1996). As such, it seems that I can attain the most accurate comparison of my results when I look at a subset of them next to the findings of Boston and Wilhelm (2017), who collected data for two days in each of the 118 classrooms they looked at. However, if these data resulted in more than one task from an individual teacher, the best–most aligned with ambitious teaching–task enactment was selected to be part of their work, because they were looking for best case scenarios. Thus, they started with the 118 best task enactments that each teacher provided, and we have been considering the 34 that started out as *Doing Math* tasks. For my work, as can be seen in Figure 12, five of the six teachers who were part of my study had at least one out of four task enactments where they maintained the *Doing Math* level of cognitive demand during the task enactment. This means that if I consider only the best-case scenario for the teachers in my study, as Boston and Wilhelm (2017) did in theirs, my findings show that for 83% of task enactments, the teachers in my study who were attempting to build on student thinking, were able to maintain the *Doing Math* level of cognitive demand during task enactments. Thus, no matter how I compare it to previous findings, attempting to
build on student thinking seems to correlate with higher levels of cognitive demand during task enactments. I next look at some potential reasons for this finding.

Specific Factors Supported by Building

As discussed in my literature review, previous work has shown that there are seven main factors that help maintain cognitive demand during task enactments, and six factors that lower cognitive demand (Stein et al., 1996). However, the RUK condensed these down into eight categories on continuums that combine these original 13 factors. Because of this, I use the categories of RUK to describe what the maintenance of cognitive demand looks like when teachers attempt to attend to student thinking during task enactment. The results of the RUK can be seen in Figure 13. This table is organized by task, enactment, and RUK category. The numerical results in each cell are organized by teacher. For example, the first number in each cell represents the RUK score for a particular teacher, the second number represents a different teacher, and so on. By organizing my data in this way, we can see each individual teacher’s RUK score for each task and enactment, but attention is drawn more to the overall scoring. By considering the reorganized factors in this way we can identify which were highest and lowest, and most and least stable across tasks and enactments. Consequently, this helped to guide which reorganized factors seemed the most fruitful for further exploration. For example, the RUK Amount of Scaffolding category was extremely stable across all tasks and enactments, and trying to understand this stability is how I delve into the specific reorganized factors that are supported by building. On the other hand, the reorganized factor of Appropriate Task seemed to be extremely variable, with much higher ratings for the first task than the second. Thus, I continue answering my first research question by trying to understand this variability. Finally, the Conceptual Connections category is also highly variable, while simultaneously having the lowest
overall ratings of all the reorganized factors. As such, I end the discussion of my first research question by looking at this variability in conjunction with overall low ratings.

Figure 13

Results of the RUK

<table>
<thead>
<tr>
<th>Task</th>
<th>Enactment</th>
<th>Appropriateness of Task</th>
<th>Appropriateness of Tim</th>
<th>Solution Strategies Discussed</th>
<th>Held</th>
<th>Accountable</th>
<th>Amount of Scaffolding</th>
<th>Classroom Management</th>
<th>Provide</th>
<th>Evidence</th>
<th>Conceptual Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td></td>
<td>2</td>
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</table>

Note. The numerical results in each cell are organized by teacher. For example, the first number in each cell represents the RUK score for a particular teacher, the second number represents a different teacher, and so on. The shading of each cell is derived from the mean of the scores in that cell, a mean of 3.6-4 is maroon, 3-3.5 is red, 2.5-2.9 is orange, and below 2.5 is yellow.

Amount of Scaffolding

The RUK Amount of Scaffolding category looks at how much scaffolding the teacher or more capable peers offered the class. Task enactments with scaffolding that supports students without taking away necessary struggle receive a high score, while enactments where so much scaffolding is offered that solution strategies are given away receive the lowest score. In my study, 23 out of the 24 task enactments (96%) had appropriate scaffolding (a rating of 4 on the RUK; see Figure 13), and the remaining enactment had appropriate scaffolding at the beginning of the enactment (a rating of 3). Contrast this with Henningsen and Stein’s (1997) study of Doing Math tasks where they found appropriate scaffolding in only 73% of the enactments. Since the teachers in my study were particularly successful in offering the appropriate amount of
scaffolding, to give us further insight into this important aspect of maintaining cognitive demand, I focus in this section on the nature of the scaffolding they offered.

The survey question that dealt specifically with scaffolding (Appendix C, Question 10) asked: “To what extent did doing the MEP enactments change the amount of scaffolding that you normally offer (as opposed to other problems that you give your students).” Two teachers responded that they did the same amount of scaffolding (one of them the teacher who received the rating of 3), two said that they did a little less scaffolding, and one said that they did a lot less scaffolding. To better understand the scaffolding, they provided and how it may have affected the maintenance of cognitive demand, I draw on my interview data where I asked follow-up questions regarding the teachers’ responses to the survey question about scaffolding. I begin by looking at the three teachers who said they provided less scaffolding than usual, and then discuss the two who said they provided the same amount.

During her interview, one of the two teachers who said that they offered a little less scaffolding gave an example of the scaffolding she would have offered for the Variables task by saying that she would “have started a little bit with a conversation about what is a variable, and what different ways a variable can be used.” However, this teacher recognized that offering this scaffolding “takes away from the mathematics that [the students] experienced as they went through [the task]” and, because she believed that experiencing this mathematics was needed for the building practice, she did not offer this additional scaffolding. If the teacher had offered this scaffolding before students began grappling with this task, cognitive demand would likely have been lowered significantly. Students would have, as part of the scaffolding, discussed and likely resolved ideas related to the underlying mathematics of this task before they had the opportunity to engage with these ideas themselves through exploration of the task. Thus, it seems that this
teacher offered an appropriate amount of scaffolding because she believed that it was a necessary part of the building practice.

The other teacher who said that she offered a little less scaffolding, revealed during her interview that for the Variables task, if it had not been for enacting a teaching practice of building on student thinking, she “might have given them: try it with two positive numbers, try it with two negative numbers, you know, can you generalize what happened.” This scaffolding would have lowered the cognitive demand of this task because students would have lost the opportunity to explore for themselves and discover that positive and negative numbers (as well as zero) lead to different outcomes for this task, which in turn leads to uncovering the mathematics underlying this task—all possible values within a domain must be considered to determine relative values of variable expressions. This teacher also said that before understanding the practice of building on student thinking:

I would have [scaffolded] right away. I would have probably had it all in there and scaffolded, but now I would probably give it to them, but if you do if I walked around the room and nobody was going in the right direction. You know what I mean, like if I didn't see that there was going to be a good discussion than I might say, okay why don't you guys try it with different types and numbers and see what happens.

However, because of coming to understand the building practice the teacher now concludes that:

Well I think realizing that it's okay for the wrong answer to come out, and we could build on it and still build towards the right thinking. Because again, I think I used to be so afraid that if we spent too much time to talk to you about the wrong thing that they might internalize that, as like, oh that's what I'm supposed to do,
you know? They might walk away thinking too negatives equals a positive is always right, but now I realize after doing this, that if you know like, you should kind of like, facilitating those discussions, there's a way to go from the student thinking and have the students keep building until they get to the right conclusion. And they actually will make more sense to them because they came up with those ideas, it wasn't me telling them, oh you were wrong see here’s your counter example. You know, that they can kind of, like, work through that muddiness themselves, and come out hopefully with a more clear picture.

As part of learning about the practice of building this teacher came to see the value in allowing students to productively struggle in her classroom, not just in service of the building practice, but in general as well. This suggests that learning to enact a practice that builds on student thinking can help teachers not only with offering an appropriate amount of scaffolding in their classrooms, but to see the value in doing this as well.

The teacher who said they offered a lot less scaffolding, revealed in her interview that if it had not been for enacting the teaching practice of building on student thinking, and wanting “all of those different misconceptions to come out,” she would have started the Variables task by asking her students to think “about different numbers. Like, make sure you think about all the numbers, or something like that.” Although this is not as explicit as telling students to consider positive and negative numbers, it would have likely had a similar effect of lowering the cognitive demand of this task because students would have been given clues about the underlying mathematics before they started grappling with the task. Thus, it also seems that offering an appropriate amount of scaffolding when enacting the building practice can stem from a desire to draw out specific student thinking.
Out of the two teachers who said they would have offered the same amount of scaffolding, one did not offer an appropriate amount of scaffolding. The teacher who did offer an appropriate amount of scaffolding was the teacher in my study who had the most experience with cognitive demand research. This teacher said that Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development (Stein et al., 2009), a book describing the Mathematical Task Framework and cognitive demand research, “made a huge impact on me, and my practice” and that “I think it's extremely important that students are engaged and doing significant math and working at a high level.” This teacher was entrenched in applying cognitive demand research in his classroom and thus was focused on continuously attempting to provide an appropriate amount of scaffolding.

The teacher who offered a little too much scaffolding did so by working through a specific example of the Percent Discount task. The class had been discussing the task, and a student had already shared the idea that if you started with a price of $30, and then added 50% you would get $45, but then 50% decrease would not go back to $30—an idea that could be used to help reach the generalized underlying mathematical concept that \( n\% \text{ of } A \) is a quantity that changes as the value of \( B \) changes—however, rather than build on this thinking, the teacher collected other student ideas, which led to writing “\( $10 + 50\% - 50\% = $10 \)” on the board. She then engaged her students in the exchange seen in Figure 14.

By looking at this exchange, we can see that the teacher broke the problem down into smaller steps and then asked students for only small bits of information, such as simplification of expressions, or what specific numbers represented—a type of questioning pattern that Wood (1998) identified as funneling. Also, rather than letting students work through incorrect ideas by asking follow-up questions, the teacher said “no,” and waited for correct thinking to emerge.
These actions run contrary to the practice of building on student thinking because, although the interaction began with the student’s thinking, the attention quickly shifted to the teacher’s way of thinking. These teacher actions removed the challenge from this task by breaking it down into smaller parts, controlling the conversation, and only moving forward when students shared the correct thinking that the teacher was looking for. Fortunately, the teacher did this towards the end of the enactment, so the students had time to grapple with the task before the cognitive demand was lowered. Had the teacher provided this scaffolding earlier in the enactment, cognitive demand likely would have been lowered even more. To gain an understanding of why cognitive demand was lowered in this way, I now turn to my interview data.

During this teacher’s interview, I showed her the videotaped exchange represented in Figure 14, and she said that she felt her students were really struggling with this problem, and she offered this scaffolding to help them understand it. However, after watching the video, she also realized that this ran contrary to the building practice—that she had gone too far and had given too much information away to her students. Even with this realization, she also noted that if she had not been enacting the practice of building on students thinking, that she likely would have worked through an example like this much earlier in the class discussion but held off because she believed that properly enacting the building practice required more time for students to work through this task on their own. This implies that even though the teacher was able to recognize that she offered too much scaffolding, she still would have offered it had she not been enacting the building practice. Furthermore, if it hadn't been for enacting the building practice, she likely would have lowered the cognitive demand even further than she did by offering this scaffolding earlier and giving her students even less time to productively struggle with this task. So, it seems that even though she did not adhere to the guidelines of the practice, this teacher still
maintained cognitive demand at a higher level than if she had not been attempting to enact the building practice.

**Figure 14**

**Too Much Scaffolding Exchange**

| Teacher: Gabrielle, what are you thinking? |
| Lucas: 10 times .5 |
| Teacher: 10 times the .5. Okay, |
| Lucas: And then subtract that from 10, oh wait no. |
| Teacher: You were thinking do 10 times the .5. So 50 percent of 10 would give you the $5 [writes 10 (.5) = 5 on the bord] okay. So now Mitch you started with your $10 right? |
| Liam: Yeah |
| Teacher: You’re gonna add $5 to that, now you have |
| Loiam: 15 |
| Teacher: $15 okay. Now your necklace cost how much now? |
| Student: Um 20 bucks. |
| Teacher: No |
| Student: 15. |
| Teacher: 15. Your necklace is $15 now. Now if you wanna take 50 percent off. How.. |
| Student: Subtract 5. |
| Teacher: What are you gonna take 50 percent of? Are you gonna take the 50 percent of the $10 or the $15? |
| Student: The $10 [long pause] $15. I don’t know I just guessed. |
| Teacher: I know, I know. What is the price of your necklace now? |
| Student: 15 |
| Teacher: It’s $15. So if you wanted to take 50 percent off of your the price of your necklace, what price do you think you’d wanna use? |
| Student: 15 |
| Teacher: The 15 yeah. Now you have a new starting amount and if you wanna take 50 percent off of there, you’re gonna have to figure out what’s 50 percent of $15. |
| Student: 7.5 |
| Teacher: 7.5 |
| Student: Yeah |
| Teacher: Cause what is it about percents. I mean is a 50 percent add and a 50 percent subtract, are they the same thing? |
| Robyn: No |
| Teacher: Robyn says no, what do you think Andrew? |
| Al: It’s our static number but more to like get the number that you’re adding or dividing |
| Teacher: Okay percents are the static number, Andrew said. They morph depending on the other numbers that you’re using them with right. They do, they change, they're not just a constant okay. |

Interestingly this teacher enacted the Percent Discount task with two classes, but only offered this amount of scaffolding during one of the enactments. During her interview, the teacher shed light on this discrepancy by saying that she felt the students in her other class were
more advanced and that they were able to uncover the underlying mathematics by themselves, whereas the students in this class were weaker mathematically and thus she provided this scaffolding because she felt that the students would not be able to uncover the underlying mathematics on their own. This seems to support the findings of Hong and Choi (2018) who found that teachers' perceptions of students' abilities affect the maintenance of cognitive demand.

If we now consider all of the cases that I have described, the results of applying the RUK, and my survey and interview data, it seems that the teachers in my study are able to recognize what a proper amount of scaffolding would be. However, even though they can recognize this, they may still offer too much scaffolding, and thus lower the cognitive demand of the tasks that they are enacting. However, attempting to build on student thinking seemed to mitigate this and support teachers to provide appropriate amounts of scaffolding because they believed the building practice required it, because they saw the value in it, and because they felt doing so provided student thinking for the building practice to utilize.

**Appropriate Task**

There are a number of scoring criteria that help determine where a task falls on the RUK Appropriate Task continuum; criteria such as the difficulty level of the task, prior knowledge, and student interest and engagement in the task. These criteria can be situationally dependent, and I found that this was the case for the difficulty level of the Percent Discount task in my study where in seven of the twelve classes (58%) over 90% of the students were quickly able to get the correct answer. However, I also found that teachers can maintain high cognitive demand if they ask a slightly altered question from the original task that pushes students to understand a misconception. In this section I first discuss appropriateness more generally and the classes in
which the Percent Discount task was appropriate. I then look at what we can learn from the teachers’ actions in classrooms for which the Percent Discount task was not appropriate.

The appropriateness of the task is relative to the students engaging in a task enactment. For example, asking kindergarteners to add two and three would likely be a high cognitive demand task for them. However, asking a group of fourth graders the same question, would likely be of very low cognitive demand. This example is straightforward, but appropriateness can be much more complex. Clearly, kindergarteners and fourth graders possess very different mathematical knowledge, but what about two students in the same grade? The mathematical knowledge that students possess is based on the mathematics that they are exposed to, how they are exposed to it, and what they retain. This can vary widely from one student to the next, especially when we consider students in different classrooms, different school districts, and even different states. Because of this, the mathematical knowledge that one classroom of middle school students possess may lead to the difficulty level of a specific task being appropriate for them, whereas the difficulty level of the same task may be inappropriate for another classroom of middle school students possessing different mathematical knowledge. This was true for the Percent Discount task in my study.

The building practice begins by drawing out a well-formed instance of high-leverage student thinking and making it the object of discussion. For the Percent Discount task, high-leverage thinking is found in the incorrect assertion that the final and original price are the same. If students simply pick a value, calculate a 50% increase to produce a new value, calculate a 50% decrease to produce a final value, then compare the original and final value, they could easily perform these calculations and never think more deeply about percentages. However, trying to understand an incorrect solution strategy focused students’ attention on the underlying
mathematics, and presented them with a question that they needed to think more deeply about, as opposed to just applying a percent to a value.

In the five enactments of the Percent Discount task where the difficulty level was appropriate, the majority of the students came up with only incorrect solution strategies, concluding that the final and original price are the same. Thus, the stage was set for the students to grapple with the high-leverage thinking that the building practice called for. For four of these enactments, the focus of the class discussion stayed on working to understand these incorrect strategies and uncover the underlying misconception. For the remaining enactment, as the teacher explained during her interview, even though an incorrect solution strategy was initially shared, one student in the class quickly presented a correct solution strategy, and because the other students in the class recognized that this student was generally correct, they got on board with her strategy even if they did not necessarily understand it. If we look at the IQA Implementation of the Task rubric results for these enactments, they show that the four enactments in which the focus remained on understanding the incorrect thinking were implemented at a high level of cognitive demand, and the one enactment in which the incorrect thinking was quickly abandon was implemented at a low level of cognitive demand.

In the seven enactments of the Percent Discount task where the difficulty level was not appropriate, the Percent Discount task was correctly solved by nearly all the students before class discussion about it began. This is an indication, corroborated by interview statements made by the teachers from these classes (Figure 15, Column 1), that the difficulty level of the question that this task posed was inappropriate for the students in these seven classrooms. The IQA Implementation of the Task rubric showed four of these seven enactments were implemented at the Doing Math level, one was implemented at the Math With Connections level, and two were
implemented at the *Math Without Connections* level. This means that the task not being of the appropriate level of difficulty for the students in the class was likely a factor that lowered the cognitive demand for three of the total enactments (25%). Had cognitive demand instead been lowered for all seven of these enactments (58%), the results would have aligned with Stein et al. (1996) finding that inappropriateness contributed to lowered cognitive demand during 61% of task enactments.

**Figure 15**

*Teacher Interview Responses Related to Task Appropriateness*

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Was the Percent Discount Task too easy for your students?</th>
<th>What do you think of the teaching strategy of trying to understand incorrect thinking?</th>
<th>Would you have used this strategy if you had not been enacting the building practice?</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL:</td>
<td>So this class of students is strong mathematically and there wasn't anybody as I monitored the room and watched what they were doing, there wasn't anybody out there that had, that I could see, that had incorrect thinking.</td>
<td>I think it helps them clarify their own thinking and helps them figure up their understanding. In a lot of cases because they have to think through, and you know articulate an argument as to why there's a mistake, and how it needs to be corrected, and why. So, I think it's a good thing to do to help students strengthen their understanding.</td>
<td>So it would, if I were giving this problem, and it was my goal for the day to get at finding percentages and comparing percent increase and decrease and those things I would. If I was doing it just for the sake of review and that kind of thing and time was an issue and I needed to get to other things, then I probably wouldn't.</td>
</tr>
<tr>
<td>RV:</td>
<td>It might have been a little easy, but I think it was a really good problem for them to think about.</td>
<td>I really like that strategy. That's actually a lot of what you see on the state tests. Students have to be able to go and see why other people's work is correct or not correct.</td>
<td>I'm not sure, but I did want to make sure that they look at what other people are thinking in the class, and they can understand, you know, where those misconceptions come from.</td>
</tr>
<tr>
<td>JAP:</td>
<td>I think this was a easy one for my students</td>
<td>I always think doing error analysis is really good stuff. Figuring out what someone might have done wrong, I think that that helps deepen your own thinking if you're coming at it from different perspectives.</td>
<td>Probably not. Because given my perception was that the vast majority of the kids had this. So I think that error analysis comes in, is more important when there's a propensity of students to make these errors ... A typical wrong answer to a problem, so that would be something that I would, like, want to talk about. So yeah, it does depend on what students, where students are with their mastery.</td>
</tr>
<tr>
<td>JP:</td>
<td>I can't remember but I feel like barely anybody had the wrong answer.</td>
<td>It takes a lot of time. I usually don't introduce misconceptions as much because I need to get on to other other things.</td>
<td>No, I probably wouldn't. It would depend on how much time I have and what else I needed to get to.</td>
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</table>

Since my work found that the cognitive demand of task enactments was lowered less often than has previously been found, I took a deeper look into my data. In doing so, I found a pattern. In each of these enactments, the originally asked question was altered by the teachers, and this altered question was grappled with by students. As written, the task stated that “The price of a necklace was first increased 50% and later decreased 50%,” and students were asked
“Is the final price the same as the original price? Why or why not?” However, instead of this statement and question, these five task enactments saw teachers give or draw out examples of incorrect thinking where the final price and the original price are the same (e.g. a necklace costs $20, the price is increased by 50% to $30, and then decreased by 50% so it goes back to $20), and students were asked to explain why they either agreed or disagreed with this thinking.

Drawing out the misconception in this way was part of the instructions for enacting the building practice. No matter the reason, we can see that the task and the underlying mathematics have not changed; students are still looking at a percent increase and decrease as a way to understand that a percent is not a static value. However, the exact question the task is asking has been altered. As written, students were asked if the price would change, but the altered question asks why the price did not change in a given example. Making this alteration to the question that the task originally asked seemed to help maintain cognitive demand of the task enactment. I next look more deeply at the relationship between this alteration and enacting the building practice.

To understand this relationship, I turn to my interview data. As can be seen in Column 2 of Figure 15, the four teachers who asked this altered question during the seven enactments where the original question was not appropriately difficult (three during both of their enactments) were all familiar with the teaching strategy of trying to understand incorrect thinking. The teachers had different names for this strategy, such as error analysis, or working with misconceptions, but all of the teachers said that they used this strategy, to varying degrees, prior to learning about and enacting the building practice. However, only one teacher said that they would have used this strategy for the Percent Discount task had it not been for attempting to build on student thinking (Figure 15, Column 3). This teacher noted the value in explorations like this and said that “I think it's a good thing to do to help students strengthen their
understanding.” The other teachers either felt that this strategy had more limited value, such as for test preparation because, “That's actually a lot of what you see on the state tests"; they saw the strategy as most effective as a way to correct misconceptions when the majority of the class has “a typical wrong answer to [a] problem,” or they used the strategy sparingly because they felt that it took a lot of time, and was not compatible with the amount of material they needed to cover in a given lesson. Interestingly, the teacher who saw the most value in this strategy, who said they would have used it regardless of attempting to build on student thinking, is the same teacher who I discussed in the previous section, whose beliefs about teaching and practices are thoroughly entrenched in cognitive demand research. Overall, however, it would appear that if the teachers in this study had not been enacting the building practice, they would not have used this altered task. Thus, it appears that drawing out and focusing on high-leverage student thinking as part of the building practice helped to maintain the appropriate level of difficulty and thus the cognitive demand of the task enactments in my study.

To further explore the phenomenon of slightly altered questions helping to maintain the cognitive demand, I now discuss the two out of seven task enactments where the difficulty level of the task was inappropriate for students, and cognitive demand was not maintained. Both of these enactments were implemented at the Math without Connections level, and both were enacted by the same teacher. Additionally, like the five task enactments where cognitive demand was maintained during implementation, the teacher of these two enactments also made slight alterations to the question that the task was asking by focusing on an incorrect solution strategy. However, during both of her enactments, students quickly brought up a correct solution strategy, and the whole-class discussion switched to this strategy without continuing to discuss or make any connections to the incorrect strategy. Interestingly this teacher was the one who felt that
trying to understand incorrect solution strategies was too time consuming (Figure 15, Row 4). Thus, it is likely that even though this teacher began by altering the question that the task was asking in order to draw out high-leverage student thinking, her personal beliefs about the time commitment that would be needed for students to engage in making sense of this incorrect solution strategy, caused her to revert to focusing on the lower leverage thinking associated with a question that was at an inappropriate level of difficulty. In turn this implies that simply asking students a slightly altered question is not enough to maintain cognitive demand. It seems that teachers must persist in their support of students putting forth the cognitive energy needed to follow through and answer the slightly altered question that teachers posed. Indeed, this is what occurred for the five task enactments where cognitive demand was maintained when teachers asked a slightly altered question. When the teacher followed up on the correct thinking, and gave up on pursuing the incorrect solution strategy, she also veered away from enacting the building practice. Thus, it would seem that enacting the building practice helps maintain cognitive demand, and conversely, if a teacher stops enacting the building practice, cognitive demand could be lowered.

To better understand how slightly altered questions can help maintain the appropriate level of task difficulty, I now look more deeply at the enactment data for the five task enactments where this successfully occurred. I begin by looking at a representative sample of teacher utterances that occurred right after the slightly altered question was posed. One teacher said:

So, let’s take a moment to think about what this claim says here on the board. What do you all think about this claim here [motions hand towards board]? I’m not asking for your answers, remember, we’re asking to see what this claim is,
look at it, and what you guys feel or think, what are your thoughts about this claim?

Another teacher said:

What I would like for you to do on your paper that you have, underneath so like where you stopped, draw a line so that we know like this is, if you don’t have room on the front turn it over and you can do it on the back, ‘kay? So, what I would like for you to do is take just a minute and write your response about [turns and gestures to board] what you think in connection with this. Do you agree, disagree? And why or why not, ‘kay? So, make sure it’s, like, you got a line so it’s clear that this is a response to this, thinking instead of the original, ‘kay? Just a minute. Go.

And another teacher said:

All right, so what do you all think about this claim? Remember to focus on this and not your answer. Focus on this claim here [referring to the smart board].

As we can see from these three examples, the teachers all explicitly stated, multiple times, that students needed to remain focused on an incorrect strategy that had just been presented. They also all explicitly stated that students should not focus on their own answers (from when they were answering the original question, if the price would change). In contrast, one of the enactments where an altered question was posed, but the appropriateness of the task was not maintained, the teacher utterance that occurred right after the slightly altered question was:
Ok, so, this is Leanna’s claim [points to the board]: the price of the necklace is going to increase and decrease by the same amount. It’s going to go up, it’s going to go down, you get the same thing you started with. And I saw that on a few other papers as well. So, what I would like to know is what do other people think of this claim?

As we can see from this representative example, the teacher recapped the strategy that had just been shared, and then at the end, students were asked what they thought about it. This seemed more like an afterthought, and it was not explicitly stated that students needed to focus on the altered question, nor was it explicitly stated that students could not bring up their own answers. Indeed, thirty seconds later, a student presented a correct solution strategy, and the class focus shifted to that, and the discussion of this altered question ended.

In contrast, for the five enactments where the cognitive demand was maintained, the discussion about the exact incorrect strategy that was shared in conjunction with the altered task was the sole focus of the class discussion for three to nine minutes. Sometimes teachers needed to redirect students back to the slightly altered task by saying things like “You disagree with this claim? Remember to focus on the claim, okay? So, what do you think here?”, but they maintained focus on the incorrect strategy at least until one student explicitly stated what was wrong with the strategy and shared the correct thinking. For example, an incorrect strategy that was shared was that $4 plus 50% is $6, and then a 50% decrease brings the price back to $4. The class did not stop discussing this example until four minutes later, when after reasoning had been shared, a student concluded that “He said 50 percent of the--not the original price but the new price and that would be 4 dollars instead of 3 dollars.”
The five high, and two low, cognitive demand enactments that I have discussed present a picture of what the teachers in my study did when they asked a slightly altered question, and how this relates to the maintenance of cognitive demand. It would seem that maintaining high levels of cognitive demand requires, first and foremost, that teachers pose the altered question, and then get students to remain focused on it for the sake of adhering to the building practice and, coincidentally, the maintenance of cognitive demand. The teachers in my study did this by explicitly stating that the focus must remain on the incorrect solution strategy that was shared with altered questions, and that students could not share their own thinking. This sometimes required teachers to redirect students, and the whole-class discussion about the incorrect strategy continued until the correct 50% value, corresponding to the incorrect solution strategy that had been shared, had been revealed.

Overall, the results of my study seem to indicate that working to understand a misconception found in a publicly shared task solution is a teaching strategy that can not only be used as part of the building practice, but can also be used to help maintain high levels of cognitive demand. This is applicable if either a student, or the teacher themselves introduced the task solution containing the misconception. However, this strategy only seems to work if teachers ensure that the focus of the whole-class discussion remains on trying to understand the misconception. To do this, it seems as though teachers must explicitly state that students need to remain focused on the misconception, and this may require redirection during the task enactment. However, if teachers can adhere to the building practice, and keep students focused on the misconception until a student provides an utterance of the underlying mathematics as it applies specifically to the error in the publicly shared task solution, it appears that cognitive demand will be maintained.
**Conceptual Connections**

Finally, the RUK Conceptual Connections category showed lower ratings than the other categories of the RUK (Figure 13) but was still rated more highly in my study than in previous work. Henningsen and Stein (1997) found that conceptual connections were drawn out during 14% of the *Doing Math* tasks in their study. In my study, 54% of task enactments saw a RUK score of 3 or 4. The Mathematical Residue rubric of the IQA is very similar to the RUK Conceptual Connections category, and the scores found when applying this rubric largely mirrored the results of applying the RUK and showed that 58% of the task enactments in my study were rated at a high level of cognitive demand. The Mathematical Residue rubric is the newest addition to the IQA toolset, and as such there is a very limited amount of data available to compare my IQA scores to other studies. However, Sullivan (2019) found that a group of 16 middle school teachers, who over the course of three years underwent 300 hours of professional development to support high quality classroom discourse, were able to increase their average Mathematical Residue rubric score from 1.85 to 2.07; the teachers in my study had an average score of 2.83 for this rubric.

The higher ratings for conceptual connections being drawn out in my study are likely due to the final stage of the building practice specifically calling for teachers to draw out explicit utterances of underlying mathematics. Such a specific call likely did more to see conceptual connections drawn out than simply utilizing reform-based teaching practices, as with Henningsen and Stein (1997), or attempting to provide high quality classroom discourse, as with Sullivan (2019). Even though the building practice specifically called for teacher actions that drew out conceptual connections, understanding exactly what the teachers in my study did to draw them
out is a valuable finding that could lead to conceptual connections being made more often during task enactments.

In Figure 16, I present the teacher utterances from all of the task enactments, rated at 3 or 4 by the RUK, where the teachers first try to draw out explicit student utterances of the underlying math. After all of these utterances, students uncovered the underlying mathematics and connected it in some way to previous thinking that had occurred during the task enactment. For enactments rated 4 this was done more thoroughly and included explicit references to previous thinking, for enactments rated 3 the references were vaguer, and not necessarily understood by all students. In many of the utterances (Figure 16) we can see that the teachers are trying to direct students back to the original question that the problem posed. Since this question included an explanation of reasoning, this seems like a good initial step towards getting students to uncover the underlying mathematics and connect it to what they have been doing in class. However, for many of the utterances coming from enactments rated 4 (and some rated 3), we can see that the teachers make additional statements that specifically request students consider thinking that has been shared during the task enactment. Since this did not occur as part of all enactments rated 4, this does not seem like a necessity for making conceptual connections, but because many of the enactments rated 4 had this, it does seem like it helps draw out conceptual connections. Overall, however, from looking at the utterances in Figure 16, it seems like one of the most important things for teachers to do in order to make conceptual connections is to ask for them. All of these utterances do that in some way, some directly ask for underlying mathematics by saying things like “What’s the main kinda thing to think about when you’re considering a question like this?”, while others simply direct students back to the original question and look for an explanation of reasoning that brings about the underlying mathematics.
### Variables Task

<table>
<thead>
<tr>
<th>RUK Score</th>
<th>Teacher Utterances That Drew Out Conceptual Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>So, the original question is which is larger, x or x plus x? Which one’s larger?</td>
</tr>
<tr>
<td>4</td>
<td>Okay, because some of you initially just said that x+x is bigger cause it’s double the size but now you guys are saying it really depends on the value of the variable. So, if you go back to the original statement what did we figure out? Can somebody summarize for us what we just learned?</td>
</tr>
<tr>
<td>4</td>
<td>It depends on the number that x is? Can somebody restate what we just learned in this one problem combining everyone’s thinking? What should we consider when we are comparing two expressions?</td>
</tr>
<tr>
<td>3</td>
<td>So in general, we should be thinking of what when we’re comparing two different expressions?</td>
</tr>
<tr>
<td>3</td>
<td>Good. Perfect. So, then I have one last question for you guys. So Brandon just restated that whole thing. He said if x is zero it’s equal, if x is positive x plus x is bigger, and if x is negative then x is bigger, right? So my one last question for you guys is what do you think this means, what do we have to do, when we are comparing two different variable expressions? So a variable expression is like x plus x and x. What do you think we have to do when we’re comparing those? What do we have to consider?</td>
</tr>
<tr>
<td>3</td>
<td>There you go, are you alright there? Ok, so now we’re coming back. We figured out that it’s ok for it to be a negative, what do you think our answer is here?</td>
</tr>
<tr>
<td>3</td>
<td>Ok, alright, so, would you please on the, on the back of your paper, would you summarize what we’ve just talked about? What’s the main kinda thing to think about when you’re considering a question like this?</td>
</tr>
<tr>
<td>3</td>
<td>Alright. So when you’re given two expressions like this [points to “x or x+x” in the original question on the board “Which is larger x or x+x?”] that are represented with letters, like, you know, x, right? What do you need to do to decide which one’s gonna be bigger?</td>
</tr>
</tbody>
</table>

### Percent Discount Task

<table>
<thead>
<tr>
<th>RUK Score</th>
<th>Teacher Utterances That Drew Out Conceptual Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>So why do we end up with differences? Why does that price end up being the final price end up being different than the original price? [gesturing towards board]</td>
</tr>
<tr>
<td>4</td>
<td>With this, okay okay. All right so I’m wondering then, when we do these increase up and increase down that same amount, what do we need to make sure we’re careful with here? What’s the thing to be really careful about so you don’t get twisted up? Because wouldn’t it be easy to say, oh 50 percent plus 50 percent minus [points to different parts of Silas’s equation on the board], those just knock each other out, we’ll go back to the same spot.</td>
</tr>
<tr>
<td>4</td>
<td>The second price after the 50 percent decrease. So how does this relate to what the original claim said over here? So this one right here said 4.50 plus 9 dollars so it was increased. This one over here said $1 plus 50 cents which is a $1.50 it increased. How do those relate? What is it that we need to remember?</td>
</tr>
<tr>
<td>3</td>
<td>Okay, what do we think the important part is of this when we’re dealing with percents? What’s important with percents?</td>
</tr>
<tr>
<td>4</td>
<td>Like what’s the main thing to pay attention to. When you have a problem like this where there’s a percentage increase and a percentage, oh let’s go to the next page right here. Okay jot down like what you need to be careful with or what you need to pay attention to when we have a percent increase and then a percent decrease by the same amount.</td>
</tr>
</tbody>
</table>
Ultimately however, the teachers in my study drew out explicit utterances of underlying mathematics and made conceptual connections because this is exactly what the building practice called for. The underlying goal of the building practice is for students to gain an understanding of important mathematical ideas when opportunities to understand these ideas come up through student thinking shared during whole-class discussion. Explicit utterances of these ideas are likely to, and did, occur under such circumstances. I delve more deeply into making conceptual connections when I answer my second research question and I look more deeply at differences that occur during different enactments of the same tasks.

The findings reported in this chapter present rich examples of student-teacher interactions that seem to support the maintenance of cognitive demand during task enactments where teachers are attempting to build on student thinking. Many of these examples focus on specific aspects of the building practice that support teachers offering appropriate amounts of scaffolding, teachers ensuring that tasks ask the right questions, and teachers drawing out conceptual connections. In turn this provides insights into the importance teachers place on scaffolding, attempting to understand misconceptions to maintain appropriateness, and improved conceptual connections being an inherent byproduct of the building practice. Thus, I now move to the next chapter where I answer my second research question and consequently expand further on the insights my work brings to teachers’ drawing out of conceptual connections.
CHAPTER V

PROMINENCE OF FACTORS DURING DIFFERENT ENACTMENTS
OF THE SAME TASKS

In this chapter, I discuss the findings for my second research question: How do factors contributing to the maintenance of cognitive demand vary when the same cognitively demanding task is enacted by different teachers in multiple classes? My dataset gave me the opportunity to look at multiple enactments of the same task in different settings. Because of this vantage point I was able to compare task enactments and look at subtle differences in the maintenance of cognitive demand. I now report on some of these subtle differences. First, I look at the appropriateness of the amount of time students are given to work during task enactments and I discuss how different task features can affect the appropriateness of time in different ways. Next, I discuss in greater depth a finding that was introduced in the last chapter, teachers drawing out Conceptual Connections. Specifically, I compare high- and low-rated task enactments for this category to understand how teacher actions affect the maintenance of cognitive demand. Finally, I compare high- and low-rated task enactments for the Solution Strategies category to understand what aspects of this category help and hinder the maintenance of cognitive demand.

Appropriateness of Time

The RUK category of Appropriateness of Time looks at how much time students are given to engage with a task. For high-rated enactments, students are given enough time to explore a task and come up with solution strategies, but not so much time that they get off task. A low rating for enactments can either be due to students not being given enough time to fully explore a task and develop solution strategies, or it can be due to students being given so much time that they get off task. The RUK score for all enactments of both tasks was relatively stable, with 19 of 24 enactments receiving a score of 3. Although a score of three still means that
cognitive demand was maintained at a high level, there were very few that were at the highest possible level of 4. My findings seem to indicate that the specific reasons this score was a 3 and not a 4 were quite task dependent.

Many enactments of the Variables task were scored at a three because when the task was given, many students seemed to finish their work, but teachers generally seemed to wait for all or nearly all students to finish before they began whole-class discussion of the task. In general, students sat quietly and did not get off task, but initially it seemed that this additional time could have been better spent engaging the class in a discussion or extending the task for those who finished early. After further investigation, I found that even if students finished their exploration, and were not actively working on a task, this did not automatically lead to an overall lowering of cognitive demand. Instead, teachers were using this time to set the foundation for maintaining high levels of cognitive demand during the remainder of the task enactment.

For enactments of the Percent Discount task, whole-class discussion typically began immediately after most (but not all) of students had finished working on the task; that is, more quickly than enactments of the Variables task. However, it was much more common during enactments of the Percent Discount task for students to not have evidence or reasoning to support thinking that they shared during the whole-class discussion or to stumble when describing it. However, these shortcomings were related to the revised question that students were asked, not the original question of the task. Since this was such a common issue in many classes, it seemed as though students had not been given enough time to sufficiently develop their ideas about the revised question to participate in whole-class discussion. Thus, as tasks evolve, additional exploration time may be needed to maintain the cognitive demand.
In order to understand the additional time given to students during the Variables task, I showed many of the teachers’ video clips of this elapsed time and asked them to describe what signaled them that it was time to begin whole-class discussion. I had initially anticipated that teachers would notice that many students had finished working and they would say that they actually should have started the whole-class discussion earlier, but that was not the case. Instead, teachers gave similar explanations for this lapse in time, as captured in this teacher’s detailed explanation:

So yeah, so the cues, the evidence of me gathering those cues are hard to see in the video. But at the beginning of the video, you noticed I was writing several things, so as I go around and monitor the work of students, I'm looking for specific things to help as we come to have our discussion. I'm looking for the student thinking that can be used to build towards the goals that we’re trying to achieve, right? And so, I'm making note of those things, you can see I move up there at the front, I get to the front, I pause and write several things down based on what I've seen. And then I make another lap around, you see right before we go to the discussion, and that’s to check and see, do we have access, right? So usually when I launch a task, and get students working on it. I make laps around the room. Initially some of those laps are just to make sure everyone is engaged and working and on task. Once I know the kids are on task and working then I start monitoring, looking for those things that I am hoping to find to use, to build towards the understanding that we're going for. And making note of those, and I try and identify those as quickly as I can and even if students haven’t completed all tasks, if I can tell they're headed down a path that's going to be helpful, I make
note of that so that I can go back and check and see if they actually went there. And then after I have the students that I think I'm going to need, then my next round is to see how far the classes is, as a whole. So once I'm comfortable with the fact that there are things out there that I can use to bring together for the whole-class conversation, then I start looking for, you know, how much is completed on the task by all students, and are there any novel things outside of those core things that I could also connects and help. So, kind of those bonus ideas that we might go to, what’s out there. But I make sure those core kinds of needed bits of student work and student thinking are taken care of first. I make note of those. Sometimes I revise those notes. I might find a student who has the same line of reasoning but has done a better job at demonstrating it on their paper, or that I have asked to articulate to me and I can articulate it really well. And then I start to check on; okay, so how many students, how far along are they, are we going to be okay.

The statements made by this teacher clearly show that he was not just walking around waiting for everyone to finish. He was instead doing valuable work that would likely help maintain cognitive demand of this task enactment after whole-class discussion of this task began. Although other teachers were not this thorough in their explanations, the ideas were still the same. They were not simply waiting for everyone to finish; instead, they were walking around looking at the student thinking that was available to them, and deciding how they would use this to meet the goals of the task, and ultimately draw out its underlying mathematics. As such, it became clear that even though there was additional time at the beginning of this task enactment
that could have been spent engaging the students in whole-class discussion, this time was instead spent preparing to make the eventual discussion as fruitful as possible.

Now I turn to the Percent Discount task, where students often had trouble sharing the reasoning behind thinking that they shared as part of the whole-class discussion. As I discussed when answering my first research question, many of the enactments of this task saw teachers asking a slightly altered question, that ultimately helped maintain the appropriateness of the task. In nearly all of the enactments where this occurred, this slightly altered question was posed after students had been given time at the beginning of the enactment to come up with solution strategies, either individually, or in small groups. Then when the altered question was posed, teachers would say things like:

What do you all think about this claim here [motions hand towards board]? I’m not asking for your answers; remember, we’re asking to see what this claim is, look at it, and what you guys feel or think, what are your thoughts about this claim?

or

What I would like for you to do on your paper that you have, underneath so like where you stopped, draw a line so that we know like this is, if you don’t have room on the front turn it over and you can do it on the back, ‘kay? So what I would like for you to do is take just a minute and write your response about [turns and gestures to board] what you think in connection with this. Do you agree, disagree? And why or why not, ‘kay? So make sure it’s, like, you got a line so it’s clear that this is a response to this, thinking instead of the original, ‘kay? Just a minute. Go.
All right, so what do you all think about this claim? Remember to focus on this and not your answer. Focus on this claim here [referring to the smart board].

In these statements, the teachers are clearly asking students to think about something new, and not to simply discuss the solution strategies that they had come up with during their original work time. Some teachers did give students a couple of minutes to contemplate this new thinking, either individually or in their small groups. However, even in enactments where this occurred, students were still not given enough time to work through this new thinking. Because of this, it seemed that students were not completely prepared to discuss this new thinking when whole-class discussion began. This means that, although students were given enough time initially to grapple with the original question, they were not given enough time to grapple with the new slightly altered question. This in turn may be what led to students stumbling more often, or not being able to share reasoning for statements that they made in regard to this new question.

Overall, the reasons that task enactments did not receive the highest possible score on the RUK for Appropriateness of Time was quite dependent on the task itself. For the Variables task, students were given more time than they needed at the beginning of the enactment to grapple with the task, and complete their solution strategies, but this was not an oversight by teachers. Instead, they were using this time to prepare for the whole-class discussion. For the Percent Discount task, the fact that students did not seem completely prepared to share the reasoning behind their thinking, was not due to being given too little time to grapple with the question that the task posed, but rather they weren’t given enough time to grapple with the slightly altered question that the teachers posed.
Conceptual Connections

In answering my first research question, I found that when teachers attempted to build on student thinking, drawing conceptual connections was a factor that helped maintain cognitive demand more often than has been seen in previous work. In answering my second research question, I look more closely at the differences between task enactments with high (six) and low ratings (eight) for the Conceptual Connections category of the RUK. Engaging in the building practice seemed to lead to higher ratings, but regardless of the reason, understanding the subtle differences in teacher actions between the high- and low-rated enactments can be used to help teachers draw out conceptual connections at higher levels during all task enactments. I emphasize teacher actions because there was no indication that the students in higher-rated enactments were significantly different than the students in the lower-rated enactments. For example, enactments that were rated high and low for conceptual connections had the same average score (3) on the RUK category of Prepared to Engage, which looks at students' background knowledge in regard to their preparedness for a given task.

When teachers attempt to draw out conceptual connections, the end goal is to have connections made between the work being done in a class and the mathematical concepts underlying this work. In rating the conceptual connections made during a task enactment, the RUK looks not only at this end goal, but also at how well the connections made seemed to be understood by students. Thus, as I describe the highest- and lowest-rated enactments, I discuss not only what teachers do and say to make and help draw out conceptual connections, but also what students do and say as they make these connections. Student discourse beyond one- or two-word answers has been shown to improve retention of the mathematical ideas being discussed.
(Hiebert & Wearne, 1993). As such, I first look at the student discourse involving utterances that began to move the class toward making conceptual connections.

Interestingly, all 14 task enactments that I analyzed had student utterances that helped move the class toward making, but did not yet actually make, connections to underlying mathematical concepts of the task (see Figure 17 for examples of these student utterances). Because this happened in both high- and low-rated task enactments, it seems that a student utterance at some point during the whole-class discussion that moves students toward, but does not yet actually make, a connection to underlying mathematical concepts is not enough, in and of itself, to see cognitive demand maintained. Thus, I now look more closely at the exact content of student utterances as well as at what point during the discussion these utterances occurred.

**Figure 17**

*Tasks, Underlying Mathematical Concepts, and the Utterances That Revealed These Concepts*

<table>
<thead>
<tr>
<th>Task</th>
<th>Underlying mathematical concept</th>
<th>Examples of utterances revealing the underlying mathematical concept</th>
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<tbody>
<tr>
<td>Task 1 (Variables): Which is larger, x or x+1? Explain your reasoning.</td>
<td>All possible values within a domain must be considered to determine relative values of variable expressions.</td>
<td>“If the x is a positive then x plus x is bigger. If x is a negative then x is bigger and then if x is zero then they’re equal.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>“It depends on what the number itself is. Like, if it’s a negative then x would be greater.”</td>
</tr>
<tr>
<td>Task 2 (Percent Discount): The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?</td>
<td>A percentage is not a fixed value, it is dependent upon the value that the percent is being applied to.</td>
<td>“Well the 50 percent, like once you add the 50 percent, you have like a different number then.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>“The 50 percent of different prices is not gonna be the same when you decrease as when you increase.”</td>
</tr>
</tbody>
</table>

As I compared the task enactments that the RUK rated high and low for conceptual connections, patterns began to emerge. Specifically, there were differences between high- and low-rated enactments in regard to a) who ultimately uttered the underlying mathematics, b) the amount of time students were given to engage with the underlying mathematics after it was uttered, and c) the outcome of exploring previously learned abstractions. First, it appears that whole-class discussion during lower-rated task enactments often saw teachers as the ones who
made, or who heavily steered students toward making utterances that moved the class toward the underlying mathematics of the task. It was also often teachers who ultimately uttered the underlying mathematics of the task. Second, students in lower-rated enactments were given little time, after the underlying mathematics was uttered, to continue exploring the underlying mathematics of the task. Conversely, during higher-rated task enactments, students were the ones who made utterances that moved the class toward, and also ultimately made utterances of, the underlying mathematics. Additionally, for higher-rated enactments, after utterances of the underlying mathematics were made, students were generally given additional time to develop and explore the conceptual connections made by such utterances. Third, I also found that the exploration of previously understood abstractions seemed to be a necessary prerequisite for understanding the underlying mathematics of a task. For task enactments where conceptual connections were rated highly, this exploration was used predominantly to move towards understanding the new abstract mathematical concepts underlying the task. Whereas for lower-rated task enactments, this exploration was used partially for helping move the class toward new abstractions of underlying mathematical concepts, but was used mostly for solidifying the abstract concepts needed to engage with the task in the first place. To help show the emergence of these patterns, I first look at low-rated enactments of the Variables task, then high-rated enactments of that task, followed by low- and high-rated enactments of the Percent Discount task. I end by summarizing the overall patterns across both tasks.

Variables Task

The underlying mathematics of the Variables task (Figure 17) centers on the domain of variables. In the context of this task, students must consider positive and negative variables, as well as the possibility of the variable being zero. All task enactments, whether high- or low-rated
for the Conceptual Connections category of the RUK, saw students having little trouble understanding and discussing positive variable values, but negative and especially zero variables proved to be more challenging for many students, especially students in low-rated enactments. Thus, to understand the subtle differences between high- and low-rated enactments, I focus on utterances related to these two concepts as I look at student utterances that led to uncovering the underlying mathematics of the task.

**Uttering the Underlying Math**

The two higher-rated task enactments of the Variables task showed that when students first made an utterance relating to negative numbers, they uncovered the concept that if $x$ is negative, then $x + x < x$. For example, students said things such as: “because if it was negative then $x$ would be bigger,” or “in my opinion I believe that $x + x$ is larger … unless it is a negative.” However, for the three lower-rated enactments of the Variables task, students first utterance relating to negative numbers, was either a misconception, such as “but if it’s like, a negative number, then two negatives make a positive,” or it was a question, such as “um, can $x$ be a negative number? Like, can $x$ be a negative number?” This is not to say that for lower-rated task enactments students never uncovered the concept that if $x$ is negative, then $x + x < x$; indeed, after initial misconceptions were dealt with or questions were answered, students went on to say things such as, “but if $x$ is a negative number, um, $x$ will be larger,” or “if it’s negative it will be negative … and then $x$ will be bigger.” However, statements such as these appeared later in whole-class discussion for lower-rated enactments, and were the subject of whole-class discussion for shorter periods of time as compared to higher-rated enactments. Additionally, there was often not a clear shift from the initial discussion of the misconceptions that occurred in low-rated enactments to later discourse surrounding the concept that if $x$ is negative, then $x + $
Indeed, utterances of the initial misconceptions and correct concepts often overlapped in lower-rated enactments and in one classroom, a misconception persisted until shortly before the end of the task enactment. Overall, however, it seemed that the great majority of students in all enactments eventually came to see the importance of considering negative numbers in the context of this problem. The same was not true, however, for the case of $x = 0$.

For the three low-rated enactments of the variables task, the case of $x = 0$ seemed to be largely overlooked and only discussed briefly. As an example, in one low-rated enactment, the students had not come up with $x + x = x$ if $x = 0$, so the teacher asked, “Is there anything else we should look at? Are there any other numbers out there besides positives and negatives?” Clearly the teacher offered quite a bit of scaffolding, and essentially told students to consider zero without explicitly saying it. After a student said zero, the teacher offered additional scaffolding, which eventually led to a student saying “[if $x$ is zero] None of them are any bigger than the other,” and then the teacher extrapolated that this meant $x$ and $x + x$ are equal.

Following this, the teacher tried to draw out the underlying mathematics of the task by asking students to recite and consider all of the cases that they looked at ($x$ is positive, negative, and zero) in relation to the original problem. Students again had trouble considering zero, and rather than saying anything clearly related to domain, the students made vaguely related utterances such as “they’re both right and they’re both wrong,” “try with both negative and positive numbers if it’s a variable,” and to solve similar problems, one needs to “think about the problem.” Ultimately, the students gave little consideration to the case of $x = 0$, and they never uttered an encompassing statement that revealed the underlying mathematics. The teacher was trying to draw out the underlying mathematical concept, but students got a little off track and the teacher did not redirect them.
After a few minutes the teacher uttered her own version of the underlying math by saying, “or be more general, just think [of] different numbers for x. You might have to try other values. Not just positives and negatives.” Very little time elapsed between the teacher asking students to consider all the cases, and her utterance of the underlying mathematics. So, even though the teacher did not redirect the students, it is possible they could have moved the conversation towards the underlying math themselves had they been given time. However, the teacher precluded this by uttering the underlying math herself before the conversation could come back around to it.

The other two low-rated task enactments also showed the teacher struggling to draw out the underlying mathematical concept, and in one of these enactments, like the one discussed in the previous paragraph, the teacher was ultimately the one to utter the mathematics underlying this task. Also like the previously described enactment, it seemed like the students could have uncovered the underlying mathematics themselves had they been allowed more time to discuss the task. In the other low-rated enactment, a student did say, “it depends on the value of x.” However, the teacher did not press the student further, or ask other students to consider this idea. As it stood, the students’ statement was rather ambiguous and it did not include the context from this task where positive, negative, and zero needed to be considered. Had the teacher continued discussing this student’s idea, it seems likely that connecting this idea to the context of this problem would have occurred, since the class had discussed positive, negative, and zero earlier.

**Engaging with the Underlying Math**

Directly after the teacher’s utterance of the underlying mathematics that I previously discussed, “or be more general, just think [of] different numbers for x. You might have to try other values. Not just positives and negatives,” without a pause, the teacher concluded the task
by saying “Ok, make sure your name is on your paper.” Students were not given time to discuss or even process the teacher's utterance of the underlying mathematics of this task. Instead they were left to internalize the underlying mathematics on their own. Additionally, even though the teacher made an utterance of the underlying mathematics, she did not include the case of $x = 0$. This case was only briefly discussed earlier in the enactment, and it did not seem to be well understood by most of the class. As a result, the students were not only left to internalize the underlying mathematics on their own, but they also had to make sense of an important case in the context of this problem that was only briefly discussed by the class, and understand how this case related to the underlying math. Overall, the students simply were not given any time to discuss the underlying mathematics of the task for this low-rated enactment. Similar patterns were present in the other low-rated enactments of this task as well.

The other two low-rated task enactments also ended shortly after the underlying mathematics was uncovered. One enactment was very similar to the one described in the previous paragraph, in that the teacher uttered the underlying mathematics herself and ended the task enactment, without pause, directly after that utterance. In the third low-rated enactment a student made an utterance of the underlying mathematics, and although there was a brief exchange afterwards, the task enactment still ended less than a minute later. For both of these enactments, the students were not given additional time to discuss the mathematics underlying the task. Also, in both of these enactments, the contextual cases of negative and zero were not discussed in conjunction with the underlying mathematics, so in addition to internalizing the underlying ideas themselves, students again had to make the connections between the contextual examples from this problem and the underlying math themselves. Finally, during all three low-rated enactments of this task, $x = 0$ was brought up infrequently, and often only when prompted
by the teacher. Because of this, students were not even given the tools to consider the full scope of domain in the context of this problem, which implies only a superficial understanding of the mathematics underlying this task.

Overall, for the low-rated Variables task enactments, the underlying mathematics was not discussed in a way that would help students make conceptual connections. To contrast this, I now turn to the two high-rated Variables task enactments. In one such enactment, during the whole-class discussion a student said:

For this problem, it will be like, uh, it will depend on x. If it is a positive, x plus x will be greater, but if it is a negative x, x it will be greater. Or if it was a 0, they’d be equal, too.

Similarly, in the other high-rated enactment, a student said, “I wrote, [it] depends on the x-value, if it’s positive, negative, or zero.” In both of these utterances, the students discussed positive, negative and zero—the three components needed to fully consider the domain in the context of this task. Additionally, these utterances were made earlier during whole-class discussion, than any utterances considering zero that occurred during lower-rated enactments. After these initial utterances considering negative values and zero, the ideas continued being discussed, and several other students in both of the high-rated enactments made similar utterances, so it was ultimately multiple students in both enactments who uttered these complete examples of how the underlying concept of domain needs to be considered in the context of this task. Again, this is in comparison to low-rated enactments of this task where a single student, or only the teacher uttered such complete examples, and the whole-class discussion ended after the utterance. Ultimately, the teachers in the enactments rated higher for Conceptual Connections provided the opportunity for students to provide robust utterances regarding the underlying
mathematics of the task, as opposed to the low-rated enactments where teachers allowed only
enough time for more superficial utterances of the underlying math.

Previously Learned Abstractions

For all 12 of the enactments of the Variables task, at some point during the enactment,
students gave a numerical example of a negative value to show that if $x$ is negative, then $x +
x < x$. For example, students said things like: “Negative 9 plus negative 9 equals to negative 18,
and negative 18 would be less than negative 9.” However, for high-rated enactments, such
examples always emerged after a more abstract utterance that if $x$ is negative, than $x + x < x$,
and for low-rated enactments at least one concrete example came before the abstract utterance.
This shows a different pattern for the whole-class discussions when comparing high and low-
rated enactments. Overall, the low-rated enactments started out with misconceptions, moved to
concrete examples, then to an abstraction of those concrete examples, and then concluded the
task by giving a brief statement related to the underlying mathematics that was not discussed
further by the class. For these enactments, the concrete examples seemed to be used, at least
partially, to understand the mathematical concepts needed to engage with the task. For example,
if a student said, “Isn’t $(-3) + (-3) = (-6)$,” they were not saying this strictly as an example
of the more abstract concept that if $x$ is negative, then $x + x < x$, but rather they were saying it
as a way to verify their understanding of adding negative numbers. Conversely, the higher-rated
task enactments started out with an abstraction of the mathematics needed to find a solution of
the task, moved to concrete examples of this abstraction, and then the discussion moved to a new
abstraction representing the underlying mathematics of the task that was discussed by, and
seemed to be understood by, the majority of students in the class. For these enactments, the
concrete examples seemed to be in service of understanding the mathematical concepts underlying the task, as opposed to unnecessary focus on understanding prerequisite concepts.

These observations suggest that exploration of relevant previously-understood abstractions may be a necessary prerequisite to understanding the underlying mathematics of a task. However, for high-rated task enactments, this exploration was predominantly used to move towards understanding the new abstract mathematical concepts underlying the task—the underlying mathematics of the task. Whereas for lower-rated enactments, this exploration was primarily focused on solidifying the abstract concepts needed to engage with the task—if $x$ is negative, positive, or zero, then $x + x < x$, $x + x > x$, and $x + x = x$, respectively. To further explore the differences and similarities between high- and low-rated enactments in regard to conceptual connections, I now turn to the Percent Discount task.

**Percent Discount**

Just as with my discussion of the Variables task, I again compare enactments rated high and low on the Conceptual Connections category of the RUK. Although they surface in slightly different ways, many of the patterns that I observed for the Variables task are evident for the Percent Discount task as well. As such, I follow the same format as I did when discussing the Variables task. First, I look at who ultimately uttered the underlying mathematics, then at the opportunities students are given to engage with the underlying mathematics after it has been uttered, and finally at the way in which previously learned abstractions were explored.

**Uttering the Underlying Math**

The underlying mathematical concept of the Percent Discount task is that a percentage is not a fixed value (Figure 17), and in Figure 18 we can see the first time in each high- (three) and low-rated (five) enactment when a student utterance moves towards understanding this concept.
by revealing that the final and original price of the necklace in the Percent Discount task are not the same. I have ordered these utterances so that there are matches between them from the high and low-rated enactments. For example, most of the initial utterances see students using 50% as opposed to making a statement about percentages in general. However, there was one initial utterance each from a high and low-rated enactment that considered percentages in general. These utterances can be seen in Row 1 of Figure 18. If we compare these two utterances, the higher-rated enactment includes reasoning as opposed to just the generalized statement that a percent increase and decrease are not the same. This additional information included in the high-rated utterance provides all students in the class a better opportunity to comprehend the correct thinking that is being shared. Similarly, if we look at Row 2, both of these utterances do specifically use 50%, but they are also unique (and matched together) because they are the only two initial utterances that specifically use 50%, but do not apply it to a concrete value. If we compare these two utterances, the utterance from the higher-rated task enactment uses the clearer, more precise language of “original price” and “second price” to describe the generalized values that they are referring to. Whereas the utterance from the lower-rated enactment uses the language “lower number” and “higher number,” which is more ambiguous and likely more difficult for some students in the class to understand.
Figure 18

First Student Utterance Addressing Percent Increase Not Being the Same as Percent Decrease

<table>
<thead>
<tr>
<th>Rated four for conceptual connections</th>
<th>Rated one for conceptual connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treating two percents as the same value is incorrect since percents can change depending on the value of the original number which is getting the percentage on</td>
<td>if you increase it, it’s gonna change the number so you decrease it, it's gonna be a different number.</td>
</tr>
<tr>
<td>We have to divide it, the original, not the, you would have to decrease the second price by 50% of itself, not by the original.</td>
<td>50% of, like, the lower number, isn’t gonna be 50% of the higher number</td>
</tr>
<tr>
<td>[referring to publicly shared student thinking: “I said that it’s the same price as the original because I used an example that if you add 5 to 9 which gives you 14, and it will give you 14, and then if you subtract 5 from 14 it will give you 9 once again, which was what’s the original”) So um 50% of 9 is not um 5, it’s 4 dollars and 50 cents, and also ... I, I don’t think she should’ve decreased it by 4.5 ... cause uh 50 percent of 14 is 7</td>
<td>You take 10 plus add 5 is 15 ... then half of 10 is 5 so ... 50% increase ... and then you divide by 2 plus half of 15. if a necklace is 100 dollars, and it increases by 50%, then it goes up to 150 dollars. And if it decreases by 50%, it goes down to 75 dollars.</td>
</tr>
<tr>
<td>if you take half of 10, which is 5, and make that 15, then 15 divided by 2, which will be the equivalent of 50% of 15, would be 7.5 which is not the original.</td>
<td></td>
</tr>
</tbody>
</table>

Finally, if we look at the rest of the utterances in Figure 18, they are all similar because they use 50% as opposed to a percent in general, and they apply this 50% to a concrete example. Prior to the utterances in Figure 18, all of the enactments, whether high or low-rated, started out with a concrete example of an incorrect solution strategy, and students were asked whether they agreed or disagreed with that example. The utterances in Figure 18 are the first time that disagreement with the incorrect example was publically shared. Interestingly, the utterance from the high-rated task enactment is the only one that used the exact numbers from the incorrect example and showed exactly why this example was incorrect. In the three low-rated enactments, the utterances, although they provided correct reasoning, did not use the numbers from the incorrect example that students were asked to consider. This means that the utterance from the high-rated enactment is the only one in this category that actually made a direct connection between correct reasoning and the exact misconception that the class was asked to consider. By making this explicit connection, the other students in the class were given a better opportunity to comprehend the correct thinking that was being shared.
Overall, for highly-rated enactments of the percent discount task, teachers seem to offer students greater opportunity to reach the underlying mathematics, which seems to allow students to make utterances that are more understandable, and provide more information that can be used by the class to understand these utterances and how they relate to the underlying mathematics of the task. Understanding this relationship in turn can lead to a stronger foundation for actually making conceptual connections. These differences may seem like they were due mostly to student actions; however, as we will see, the teachers’ actions were actually what drove these differences. To understand these subtle differences, and how they can lead to the class actually making conceptual connections, I now move past the initial utterances that helped move the class towards the underlying mathematics, and look at when the underlying mathematics itself was actually uttered.

The five low-rated enactments of the Percent Discount task all had similar patterns. As an example, in one enactment a student shared a concrete case of how a 50% increase and a 50% decrease are not the same. For the next six minutes of the task enactment, the class went through three additional concrete cases (all using different starting values) to show that the final and original price were not the same. However, none of these cases used the same numbers as the original incorrect thinking that the class was asked to consider. After these cases had been shared the teacher said, “I feel like you guys are at a point that we could probably generalize this. You picked two specific prices for the necklace. Did they tell us a price for the necklace in this problem?” Clearly the teacher was trying to move the class toward the underlying mathematics. If the students were to generalize what they had been doing, they would likely come to a statement that represents the underlying mathematics of this task. However, instead of taking up this prompt in a way that would move toward the underlying math, the next five minutes of
whole-class discussion was spent coming to the conclusion that a 50% increase followed by a
50% decrease is the same as 75% of the original price. Immediately after reaching this
conclusion, the teacher asked, “Okay so what’s the important thing to remember when you’re
doing percent’s?” and a student responded “multiply.” The teacher then ended the task enactment
by saying, “One is to multiply, you’re not just adding and subtracting and then it matters what
you’re taking the percent of, right?” No student utterances had touched on the idea that it matters
what you’re taking the percent of, so the teacher’s utterance was the only time this idea was
publicly shared. Overall, it seemed like the teacher tried to move the class in the right direction
earlier, but followed up on another idea, and never redirected the class back towards the
underlying mathematics.

As I noted, the remaining four lower-rated enactments of the Percent Discount task all
followed a pattern similar to what I just described. The classes went through two or more
concrete examples to show that the final price and the original price are not the same. In some
enactments, students used more complex concrete examples, such as where the final price turns
out to be a decimal. In one enactment, they explored a 25% increase and decrease and
determined (after using a concrete example) that the final and original price would not be the
same with that percent either. However, none of the low-rated enactments ended with a student
uttering the underlying mathematics and connecting it to work that had been done in class—it
was always the teacher who made such utterances. As can be seen in Figure 19, Enactment A,
the teacher was trying to draw out the underlying mathematics, and although the students seemed
to be skirting around it, there was no student utterance of the underlying mathematics. Just as
with the example in the previous paragraph, the teacher ultimately ended up sharing the
underlying math herself. At the end of another enactment (Figure 19, Enactment B), after the
teacher pointed to two concrete examples on the board, a student touched on the underlying mathematics of the task. Then the teacher elaborated and shared the underlying mathematics herself. This exchange is the closest that any of the low-rated enactments of this task got to having a student be the one to share the underlying mathematics of the task. The remaining two low-rated enactments ended by the teachers saying, “that thing we’re taking away 50% of is now different,” and “percents are the static number. They morph depending on the other numbers that you’re using.” Here, the teachers ended the enactments without ever fully expressing the underlying mathematics of the task, and students were left to make sense (or not) of the mathematics of the task on their own.

**Figure 19**

*Ending Utterances of Low-Rated (for Conceptual Connections) Enactments of the Percent Discount Task*

<table>
<thead>
<tr>
<th>Utterances that ended low-rated enactments of the Percent Discount task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Enactment A</strong></td>
</tr>
<tr>
<td>Teacher: So now what was the important piece that we had to change there? [points ambiguously to board]</td>
</tr>
<tr>
<td>What was the important piece right there? Somebody other than people that have said something already.</td>
</tr>
<tr>
<td>What’s the important piece that we had to change right there?</td>
</tr>
<tr>
<td>Student 1: Um specifying what you’re doing.</td>
</tr>
<tr>
<td>Teacher: Okay you’re very very close, specifying what you’re doing, what does he mean by that?</td>
</tr>
<tr>
<td>Specifying what? Go ahead</td>
</tr>
<tr>
<td>Student 2: It’s like you have to say what, or you have to say it more clearly.</td>
</tr>
<tr>
<td>Teacher: Okay so we had just specified what in this case? We had to specify… [points to revised claim]</td>
</tr>
<tr>
<td>Student 3: What you’re taking away and what you’re.</td>
</tr>
<tr>
<td>Teacher: [points to parts of the board] Kay. What we were taking a what of?</td>
</tr>
<tr>
<td>Student 2: The 50 percent of.</td>
</tr>
<tr>
<td>Teacher: The 50 percent of right, do you think that’s important of percents? That we have to be careful, we have to know exactly what we’re taking the percent of and that changes things. Yeah okay. Did we, how many of us wanted to change our answer now? Is that okay? That we wanted to change our answer. Right, that's always okay, remember that. Okay anything else we wanna add?</td>
</tr>
<tr>
<td><strong>Enactment B</strong></td>
</tr>
<tr>
<td>Teacher: OK these are both cheaper than the original price. What's the reason for this?</td>
</tr>
<tr>
<td>Student: the price is changing like so you’re taking 50 percent of two different numbers</td>
</tr>
<tr>
<td>Teacher: Yeah, the price is changing so you’re taking 50 percent of two different numbers, so when you’re doing percent, it matters what you’re taking the percent of.</td>
</tr>
</tbody>
</table>
Now, to contrast these low-rated enactments, I look at the utterances of high-rated task enactments that occurred after the initial utterances captured in Figure 19. All three of the high-rated enactments showed similar patterns that I explain by using an example from one representative enactment. In this enactment, students shared multiple concrete examples to show that the final price will not be the same as the original, and following this a student said:

If a necklace was first increased 50 percent and later decreased 50 percent; no, the final price is not the same as the original. For example, if the original price of the necklace was $20, then it increased to $30. $30 minus the 50 percent is 15 which is not the same as 20 [dollars]. Therefore, the final price is not the same as the original.

Here we can see the student (and not the teacher) was the one who made an utterance of the underlying mathematics. Additionally, the original misconception that the class was working with had a starting amount of $20, so this student utterance also saw the underlying mathematics connected directly to what students in the class were doing. The other two high-rated enactments were similar to this, in that students were the ones who both shared and connected the underlying mathematics to what had been done in class.

**Engaging with the Underlying Math**

Earlier, I described an enactment that was low rated on the RUK Conceptual Connections category, where the teacher tried to move students toward the underlying math by saying, “I feel like you guys are at a point that we could probably generalize this. You picked two specific prices for the necklace. Did they tell us a price for the necklace in this problem?” The students got off track and came to the conclusion that a 50% increase followed by a 50% decrease is the same as 75% of the original price. Following this, the teacher ended the enactment by saying,
“One is to multiply, you’re not just adding and subtracting and then it matters what you’re taking the percent of right?” Although no student utterances had said anything about what the percent is being taken of mattering, the teacher simply ended the task enactment here without giving students any time to consider the underlying mathematics.

Similarly, for the other two low-rated task enactments, the teachers' ending statements in Figure 19, Enactments A and B, saw the teacher as the one who discussed the underlying mathematics, and then ended the task enactment right afterwards. As such, the students were not given time to discuss the underlying mathematics, let alone make explicit connections between it and the work that had been done in class.

In contrast to this, the high-rated enactments saw students given time to not only discuss the underlying mathematics, but also show how it was connected to the work that they had done in class. As an example, in one class the teacher said:

The second price after the 50 percent decrease. So how does this relate to what the original claim said over here? So, this one right here said 4.50 plus 9 dollars so it was increased. This one over here said $1 plus 50 cents which is a $1.50 it increased. How do those relate? How does this original claim [$9 increased by 50% is $14, and a 50% decrease if that goes back to $9] relate to what [other students were saying: 50 percent of $9 is not $5, it’s $4.50]. How can we relate those two? Is there any relationship between those two? Esteban, is there a relationship? How can we relate what they are saying to the original claim that was said?

Here we can see that even though the teacher is still only considering concrete examples of 50% increases and decreases, she is asking the class to consider two of them at the same time,
as well as how they relate to the original (incorrect) claim that was being considered. By doing this, the teacher is trying to draw out the underlying concept and connect it to what the class has been doing. There was more whole-class discussion after this, and the teacher kept the class on track moving towards making a conceptual connection by saying things like, “They’re both trying to increase and then decrease but what is important in this?,” “50 percent is the same as 50 percent right, but in this problem, what was the problem that we had originally in the original work?,” and “if you want, you can use an example. Somebody give me what they wrote.” Shortly after this a student said:

If a necklace was first increased 50 percent and later decreased 50 percent; no, the final price is not the same as the original. For example, if the original price of the necklace was $20, then it increased to $30. $30 minus the 50 percent is 15 which is not the same as 20 [dollars]. Therefore, the final price is not the same as the original.

Here we can see that the teacher's first utterance pushing students to find the underlying math is similar to the low-rated enactments. The main difference, however, is that after this initial utterance, the teacher continued pushing students to make a conceptual connection. Even after a student said, “The 50 percent of different prices is not gonna be the same,” the teacher continued pushing the class to connect this to their work, which a student eventually did.

The other two high-rated enactments are similar to this, in that the teacher kept students on track to uncovering the underlying mathematics, and even after they stated the underlying mathematics, the teacher continued pushing students to make sure they connected this to what had been done in class. Overall, the students in these enactments were given the opportunity to explore and solidify their understanding of the underlying mathematics during the discussion.
Previously Learned Abstractions

In general, the high-rated task enactments saw fewer concrete examples shared than the low-rated enactments. All three high-rated enactments saw students share only one concrete example before discussing in more abstract terms that a percent increase and decrease are not the same. However, four of the five low-rated task enactments saw two concrete examples, and one enactment saw three concrete examples. Generally, these additional concrete examples were used to overcome additional misconceptions found in low-rated task enactments. More importantly however, spending time working through these additional concrete examples seemed to take up additional class time that teachers in high-rated enactments used to better understand the underlying mathematics, as opposed to simply working through concrete examples. Obviously, the misconception that all task enactments had to deal with was that a percent increase and decrease are the same amount. However, for the low-rated enactments, there were also misconceptions about calculating a percent that emerged during the discussion. In two enactments, there were students who believed that a 50% increase of $20 would be $20.50 (Interestingly both of these enactments saw students use a starting value of $20). Another low rated enactment had a student express that a 50% increase of $10 would be $60. These misconceptions often came up after a percent increase and decrease being the same had already been brought up, and the teacher asked if students had any other methods for finding a final value.

To overcome these misconceptions, the classes all worked through the example that was presented to show that a 50% increase of $20 would not be $20.50, but rather $30, and a 50% increase of $10 would not be $60, but rather $15. Following this, the students went on to show that a 50% decrease for each of these would be $15 and $7.50 respectively. In doing so, the
students in these low-rated enactments were clearly using what they knew about percent’s to better understand how to calculate percentages, which is prerequisite knowledge that the students need to engage with this problem. Conversely, the high-rated enactments saw no misconceptions publically shared, so even though a student may have had a misconception on their own, since it was never publicly shared the class as a whole did not need to spend time reinforcing the prerequisite knowledge needed to engage with this task.

    Also, as I mentioned earlier, during the high-rated enactments, students spent more time discussing the underlying mathematics after it had been uncovered. In one such enactment the class had already established that the final and original price are not the same, and a student said:

        So, first number, that’s its own 100 percent there. When you add 50 percent, you’re just taking half of that and putting it into the same thing. But that new number which you now get from that, is its own 100 percent so then when you minus 50 percent from that, that would give you a different number than the first one cause it’s a different relevance of which percentage we’re using.

        In this enactment, the student is trying to make sense of a percent remaining the same, but that the value of the percentage changes as the value that the percent is being applied to changes. Even though the student was searching for the precise wording, it is clear that they were trying to understand this task past just reinforcing the mathematical concepts needed to engage with it.

        This discussion of the underlying mathematics was typical of the high-rated enactments in that students really seemed to be trying to understand why a percent increase and decrease were not the same. Also, as I mentioned previously, the low-rated enactments ended shortly after the underlying mathematics was shared, and as a consequence, students in these enactments were not afforded the opportunity to try to understand the “why” of the underlying mathematics.
Overall, the students in high-rated enactments did not spend time reinforcing their understanding of calculating percentages, but instead spent their whole-class discussion time using what they did know about percentages to try to understand why a percent increase and decrease are not the same. Conversely, in the low-rated enactments students often spent time reinforcing their understanding of calculating percentages because a single student had shared a misconception, and they spent little to no time using what they did know to move toward an understanding of the underlying math. When we consider these things together, we can see the pattern emerge that although all the Percent Discount task enactments saw students using abstractions of the mathematics needed to engage with this task—how to calculate a percent—students in low rated enactments often used this to understand this idea itself, while students in high-rated enactments used this to understand abstractions of the underlying math.

Summary of Conceptual Conceptions across Tasks

Overall, the patterns that emerged in regard to the Conceptual Connections category of the RUK during enactments for the Variables task and the Percent Discount task were similar. The students in the lower-rated enactments spent time working through concrete examples, predominantly to reinforce an understanding of the mathematics needed to grapple with the task—if $x$ is negative, positive, or zero, then $x + x < x$, $x + x > x$, and $x + x = x$; and how to calculate percentages, respectively. Also, low-rated task enactments ended shortly after a single student or the teacher made an utterance uncovering the mathematics underlying the task. Additionally, students in these enactments were not given the opportunity to explore and gain a deeper understanding of this underlying mathematics. Conversely, for higher-rated enactments, the majority of time students spent working through concrete examples was in service of moving towards the underlying mathematics of the task, students were generally the ones who made the
utterances uncovering the mathematics underlying the task, and students were given the opportunity after the underlying mathematics was uncovered to explore and gain a deeper understanding of it. With such differences between the high and low-rated tasks, it may seem that I am describing two groups of students with differing mathematics abilities. However, as I noted earlier, that was not the case.

What is interesting about the differences between the high and low-rated enactments that I saw was that there was no indication during these task enactments or by the RUK results that the students in higher-rated enactments were significantly different than or any better prepared to deal with these tasks than the students in the lower-rated enactments. Indeed, the RUK category of Prepared to Engage looks to understand if students possess the appropriate background knowledge to deal with a task, and both the high- and low-rated enactments (for Conceptual Connections) had the same average score of three in regard to the Prepared to Engage category. Similarly, I did not see anything in the task enactment videos that led me to believe that one group of students was significantly different than another. Certainly, more misconceptions for the Variables task were uttered in the lower-rated enactments, as well as less robust utterances of the underlying mathematics for both tasks, but aside from this it seemed as though the students in the low- and high-rated task enactments were on very similar paths moving toward the underlying mathematics of the task. Indeed, the higher-rated task enactments seemed to have the same less robust utterances early in the task enactments, and there was no indication that after additional discussion, the students in lower-rated enactments would not have eventually moved towards the same robust utterances of the underlying mathematics that were found at the end of the high-rated task enactments. Simply stated, if students in lower-rated enactments had been given the opportunity to explore and to continue discussing underlying concepts, as was done
with students in the higher-rated enactments, it seems likely that they too would ultimately have made more robust utterances. As such, the differences in these enactments seem to be a product of actions the teachers took that cut discussions short in the lower-rated enactments and allowed more open discussions in higher-rated enactments.

Next, I share additional patterns that emerged as students discussed various solution strategies during their task enactments.

**Solution Strategies Discussed**

The RUK Solution Strategies Discussed category looks at how many solution strategies and mathematical ideas needed to engage with the task are discussed. Task enactments where many solution strategies and the mathematical ideas needed to engage with a task are discussed receive a high score, while enactments where most of these strategies and ideas are not attended to receive a low score. The results of applying the RUK showed that (especially for the Percent Discount task) the scores for this category were predominately rated at a 2 or a 4 (Figure 13). This means that either many or very few solution strategies and ideas needed to engage with the task were discussed, with very little middle ground. Since the enactments in my study were so split for this particular category, I contrast high-rated enactments against low-rated enactments to understand what teacher and student actions helped (or hindered) the maintenance of cognitive demand for this category. The overall pattern that was most apparent when comparing high- and low-rated task enactments is that for the high-rated enactments, abstract strategies seemed to precede concrete strategies. Also, the questions that teachers asked during high-rated task enactments seemed to be a little more pointed and kept students on track to uncover the mathematics underlying the task. In comparison, the low-rated enactments saw students focusing more, and sometimes solely, on concrete examples. Additionally, teachers tended to ask more
vague questions during low-rated enactments, which led to strategies, sometimes important ones, not being uncovered.

**Variables Task**

To begin looking at the contrast between high and low-rated tasks, I first look at the Variables task. There were six enactments of this task rated at a 4, and two enactments rated at a 2. Figure 20 shows all the different solution strategies and ideas that I encountered when coding my data. I found that none of the task enactments I looked at had all of these strategies, and that lower-rated tasks had fewer strategies than the higher-rated tasks. I also found that it was more common for abstract strategies (Figure 20, Lines 4, 5, & 7) to be presented before concrete strategies (Lines 2, 3, & 6) during high-rated task enactments, while the reverse was true for low-rated enactments. Additionally, the most encompassing strategies (Lines 8 & 9)—strategies that seem the most fruitful for students to draw from to help them understand the mathematics underlying the task—were discussed for much shorter lengths of time (if at all) for low-rated task enactments. Finally, I found that during high-rated enactments, teachers asked probing questions to draw out these strategies, but during lower-rated enactments teachers usually asked much more vague questions, and, on occasion, questions that gave away too much information. To understand these patterns, I describe some of the high and then all of the low-rated enactments of the variables task.
Figure 20

Strategies Uncovered During the Whole-Class Discussion Phase of Variables Task Enactments.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Combining Variables</td>
<td>$x+x=2x$ and $2x&gt;x$</td>
<td>$x+x&gt;x$</td>
</tr>
<tr>
<td>2 Select Positive Values</td>
<td>If $x=3$ then $x+x=6$, and since $6&gt;3$</td>
<td>$x+x&gt;x$</td>
</tr>
<tr>
<td>3 Select Negative Values</td>
<td>If $x=(-7)$ then $x+x=(-14)$, and since $(-14)&lt;(-7)$</td>
<td>$x+x&lt;x$</td>
</tr>
<tr>
<td>4 Abstract Positives</td>
<td>Two positive $x$'s are greater than one $x$</td>
<td>$x+x&gt;x$</td>
</tr>
<tr>
<td>5 Abstract Negatives</td>
<td>Two negative $x$'s are less than one $x$</td>
<td>$x+x&lt;x$</td>
</tr>
<tr>
<td>6 Selecting Zero</td>
<td>If $x=0$ then $x+x=0$, and since $0=0$</td>
<td>$x+x=x$</td>
</tr>
<tr>
<td>7 Abstract Zero</td>
<td>Two zeros are equal to one zero</td>
<td>$x+x=x$</td>
</tr>
<tr>
<td>8 Non-zero Numbers</td>
<td>$x$ can be positive or negative (Using 2-5)</td>
<td>$x+x&gt;x$ &amp; $x+x&lt;x$</td>
</tr>
<tr>
<td>9 Real Numbers</td>
<td>$x$ can be positive, negative, or zero (using 2-7)</td>
<td>$x+x&gt;x$, $x+x&lt;x$, &amp; $x+x=x$</td>
</tr>
</tbody>
</table>

Enactments of the Variables task that were high rated for the RUK Solution Strategies Discussed category had numerous similarities. Figures 21 and 22 show the teacher and student utterances surrounding the first-time solution strategies (Figure 20) are discussed during two representative high-rated task enactments. We can see that both of these task enactments began with Strategy 1 (Figure 20). Next, we can see that in the enactment represented by Figure 21, a student made the abstraction that $x + x > x$ when $x$ is positive (Strategy 4, Figure 21). The teacher followed this up by asking, “So you think that $x$ plus $x$ is larger than $x$ if it’s a positive only?” By saying this the teacher noted that positive values are important in this problem, but also pushed students to consider how they are important. Following this, a student brought up the strategy that $x = 0$ is also important in this problem (Strategy 7, Figure 21). Next, both of these more abstract strategies were made concrete by students sharing that if $x = 0$ then $0 + 0 = 0$, and if $x = 5$ then $5 + 5 > 5$ (Strategies 6 & 2, Figure 21). This same pattern of abstraction followed by concrete examples was continued when a student brought up the strategy that if $x < 0$ then $x + x < x$, followed by another student who says that if $x = (-9)$ then $(-9) + (-9) < (-9)$ (Strategies 5 & 3, Figure 21). Finally, the teacher asked students to reconsider the original
question that the task asked in the context of all the strategies that had been shared, and ultimately a student uttered Strategy 9 (Figure 21) and the class continued discussing this strategy for the next five minutes. Nearly all of the strategies from Figure 20 were discussed during this enactment, and the teacher asked some probing questions that provided just enough scaffolding to push students to utter these strategies. Also, although not all of the abstract strategies were brought up before all of the concrete examples, the abstract strategies corresponding to concrete strategies were brought up before those particular concrete strategies.

Turning to the other representative example of a high-rated enactment (Figure 22), we can see that when the teacher asked for reasoning by saying, “You’re just saying he’s flat out wrong? Why?” and “You think they’re both wrong? Okay, why do you think they’re both wrong?” students uncovered, abstractly, that for \( x > 0 \), \( x + x > x \), for \( x < 0 \), \( x + x < x \), and for \( x = 0 \), \( x + x = x \) (Strategies 1, 5, & 7, Figure 22). Following this, the teacher asked “So, how are we going to make sense of these three competing answers?” and the students uncovered Strategy 9 (Figure 22). The class continued discussing and reinforcing this strategy when the teacher drew out the concrete examples: if \( x = 4 \), then \( x \) then \( 4 + 4 > 4 \), if \( x = (-3) \) then \( (-3) + (-3) < (-3) \), and if \( x = 0 \) then \( 0 + 0 = 0 \) (Strategies 2, 3, & 6, Figure 22), by saying things like “Okay. So, can somebody give me an example of when \( x + x \) would be bigger?” Overall, many of the strategies from Figure 20 were discussed during this enactment, and again the teacher asked probing questions that pushed students to utter these strategies.
Figure 21

Teacher (RV) and Student Utterances of Solution Strategies for a High-Rated (for Solution Strategies Discussed) Task Enactment
Table 22

Teacher (JP) and Student Utterances of Solution Strategies for a High-Rated (for Solution Strategies Discussed) Task Enactment

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Dialogue that drew the strategy out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>04:21 JP: Okay, so as I look around the room I saw quite a few people that said they think x plus x is bigger. I don’t want you to say anything else right now, but raise your hand if you answered that x plus x is bigger. [some students raise their hand] Okay, almost half. So, one of you that just had your hand up, does somebody want explain why they thought x plus x is bigger? Broden? 04:56 Broden: Because x plus x will be double the size of x.</td>
</tr>
<tr>
<td>5</td>
<td>05:31 JP: Okay, so Broden’s answer is x plus x is bigger because it is double the size of x. What do the rest of you guys think of Broden’s answer? Liana? 05:47 Liana: He’s wrong. 05:49 JP: You’re just saying he’s flat out wrong? Why? 05:53 Liana: Because if it was negative then x would be bigger. If it was like negative one plus.</td>
</tr>
<tr>
<td>7</td>
<td>06:58 JP: Brooklyn, what do you think about what Liana and Broden said? 07:02 Brooklyn: They’re both wrong! 07:03 JP: You think they’re both wrong? Okay, why do you think they’re both wrong? 07:08 S: They’re both right. 07:09 Brooklyn: Because you don’t know what x is and if x was zero, they’d both be equal.</td>
</tr>
<tr>
<td>9</td>
<td>07:15 JP: Okay, so she’s saying you don’t know what x is. [writes “don’t know what x is”] And if x is zero they would both be equal. [writes “if x is zero, they would both be equal!”] So what is your answer to the original question then, Brooklyn? Did you say - which one did you say is bigger? 07:51 Brooklyn: Uh, it depends. 07:52 JP: Okay, so your answer was it depends. So we have three answers up here right now, competing. Broden says x plus x is bigger. Liana says x is bigger. 08:05 Liana: No, I said it depends too. 08:07 JP: Oh, you’re saying it depends and this is your why it depends? And Brooklyn is saying it depends. So, how are we going to make sense of these three competing answers? Q1: what do you think? 8:21 Q1: I wrote, depends on the x-value, if it’s positive, negative, or zero.</td>
</tr>
<tr>
<td>2</td>
<td>10:10 JP: Okay. So, can somebody give me an example of when x+x would be bigger? Gunter. 10:18 Gunter (?): When x equals 4. 10:20 JP: Okay, so Gunter says, when x equals four, that x plus x is bigger. [writing] Ugh… I can’t write today! Do you guys agree with that? 10:38 Multiple students: Yeah. 10:39 JP: Okay. Why? 10:41 S: Because 4 plus 4 equals 8</td>
</tr>
<tr>
<td>3</td>
<td>10:55 JP: Okay, and you’re saying x is just 4 (10:57 S: Yeah.) and 8 is bigger than 4. Okay. So, when is x going to be bigger? Zoe. 11:05 Zoe: If it equals negative three. 11:07 JP: Okay, so you’re example is when x equals negative 3, x is bigger? 11:18 Zoe: Yeah. 11:20 JP: Let’s try it. What’s negative 3 plus negative 3? 11:23 Multiple students: Negative 6.</td>
</tr>
</tbody>
</table>
I now contrast the two task enactments rated highly by the RUK Solution Strategies Discussed category (Figures N and O, discussed in the previous paragraphs) against the two low-rated enchantments. Figures 23 and 24 show the teacher and student utterances of solution strategies for these two low-rated task enactments. We can see that like the high-rated task enactments, both of these enactments began with Strategy 1 (Figure 20). Following this, both of these enactments were quite similar in the order in which solution strategies were presented. Interestingly both enactments first saw students present the same concrete example of \( x = 5 \) (Strategy 2, Figures 23 & 24) by saying, “Cause like, if \( x \) equals 5, then like, it’s just \( x \) plus 5, when you get like, \( x \) plus \( x \) equals 10” in one enactment and “\( X \), if \( x \) plus \( x \) could be bigger it could be, let’s say, if it was 5 plus 5, and then just \( x \) was 5” in the other enactment. This was followed by the same concrete example of \( x = (−5) \) (Strategy 3, Figures 23 & 24). In one enactment a student succinctly said, “I also put negative 5, and then negative 5 plus negative 5 because that equals negative 10, and then 5. Or, negative 5,” while in the other enactment the class struggled with adding two negative numbers, and it took a few minute-long discussions between the teacher and five students to determine that \( (−5) + (−5) = (−10) \).

The final strategies brought up in both low-rated enactments were the more abstract strategies of \( x < 0 \) and \( x = 0 \). In one enactment, the teacher was having trouble getting students to consider the \( x = 0 \) strategy, so she asked, “Is there anything else we should look at? Are there any other numbers out there besides positives and negatives?” Although the teacher asked the question to prompt the students to think about whether all the numbers in the domain had been considered, it was vague enough to run the risk of students going off in other directions, such as when a student responded, “What about fractions?” Many of the other teacher questions were even more vague; questions such as, “But what about this other example [two negatives make a
positive] that Kianna started us off with?” Here the teacher was trying to get students to think of negative numbers, but the example that the teacher was referring to was not a strategy for this problem, but rather the incorrect thinking that the sum of two negative numbers is positive.

Finally, there was a brief period (less than a minute) where students discussed strategies 8 and 9 (Figure 23) in one of these two low-rated enactments, which concluded in a student saying, “It depends on what x equals, like if it, x equals a negative, or zero.” The other low-rated enactment saw no instances of strategies 8 or 9 at all. Overall, during the low-rated enactments students brought up fewer strategies than during the high-rated enactments. Additionally, the lower-rated enactments saw concrete strategies come to light before the more generalized abstract strategies, whereas the opposite was true of the higher-rated enactments. Lastly, the questions that the teachers asked to help draw out these strategies were quite different for low-rated task enactments than they were for the higher-rated enactments. During the high-rated enactments, teachers asked probing questions that put students on the right track to uncover new strategies, but they generally did not give away too much information, so students still had to uncover the strategies for themselves. In comparison, during the low-rated task enactments, the teachers generally asked much more vague questions that did not push students down any certain path, or (less frequently) they asked questions that gave away far too much information, and basically gave away the strategies that they were pushing students to uncover.
Figure 23

*Teacher (SJ) and Student Utterances of Solution Strategies for a Low-Rated (for Solution Strategies Discussed) Task Enactment*

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Dialogue that drew the strategy out</th>
</tr>
</thead>
</table>
| 1        | 6:46 SJ: Ok, that’s ok, that’s ok. So if you think x plus x is bigger. Alright Carly, can you tell me why?  
          | 6:52 Carly: Because they have like, the same shirt. So if we add it together, so then x plus x will be 2x, and then just x will, is the same as 1x. [SJ writes “x+x is larger” while the student is talking] |
| 2 & 3    | 7:56 SJ: Can somebody add on to what Forest is saying about x can be anything? Chance.  
          | 8:01 Chance: Ok, so like, I did it both ways, how like it, how it could be, how could x, x plus x could be bigger or x could be bigger.  
          | 8:07 SJ: Ok.  
          | 8:08 Chance: X, if x plus x could be bigger it could be, let’s say, if it was 5 plus 5, and then just x was 5.  
          | 8:13 SJ: Ok.  
          | 8:14 Chance: Like, obviously 5 plus 5 is bigger than just 5.  
          | 8:18 SJ: Ok.  
          | 8:19 Chance: But then I also put negative 5, and then negative 5 plus negative 5 because that equals negative 10, and then 5. Or, negative 5. |
| 8        | 8:24 SJ: Ok, so, so with that what you just said there, what an-, what answer do you have to the problem? Which is larger, x or x plus x?  
          | 8:31 S: I put x. |
| 9        | 8:32 Chance: I put, well technically both.  
          | 8:33 SJ: Ok, so you said x plus x is larger and you said…  
          | 8:37 Chance: X is larger.  
          | 8:38 SJ: Ok, x is larger [SJ writes on the board].  
          | 8:40 Amber: It depends on what x equals, like if it, x equals a negative, or zero. |
| 7        | 8:44 SJ: Ok, ok, so we got some other ideas on here. Can we go piggy bank off what Amber just said? It depends?  
          | 8:50 Amber: Like if you have zero, then it’s just x equals zero, but if it’s zero plus zero it’s still zero, so they’re equal.  
          | 10:35 SJ: Ok, so Amber is saying [SJ writes on the board] it depends on what the number is.  
          | 10:43 Amber: Like, can’t I point it out, if it’s a negative then x would be greater.  
          | 10:46 SJ: Ok.  
          | 10:46 Destiny: If it was a negative wouldn’t there be a negative sign in front of the x’s?  
          | 10:50 SJ: Ok, hang on, hold that thought. So Amber just said, what did you say about x is larger here, Amber? [SJ writes Amber’s statement on the board]  
          | 10:55 Amber: If there’s a negative, if the x, x equals a negative, then x is larger. |
Figure 24

*Teacher (JP) and Student Utterances of Solution Strategies for a Low-Rated (for Solution Strategies Discussed) Task Enactment*

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Dialogue that drew the strategy out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4:04 JP: Ok, I noticed that most of you said that x plus x is larger. Raise your hand if you said that x plus x is larger. [most students raise their hand.] Ok, you can put your hands down. Does anybody want to tell me why they think x plus x is larger? Brett? 4:30 Brett: x plus x equals 2x, and x plus x equals 2x.</td>
</tr>
<tr>
<td>2</td>
<td>5:30 Ok, quite a few of you. Did anybody pick that answer, [Pointing to the public record] but for a different reason, because they had maybe a different explanation? Kianna, what did you say? I feel like when I looked at your paper you had something a little bit different. 5:49 Kianna: Cause like, if x equals 5, then like, it’s just x plus 5, when you get like, x plus x equals 10.</td>
</tr>
<tr>
<td>3</td>
<td>5:58 JP: Ok, did you get...oh, sorry. 6:00 Kianna: But if it’s like, a negative number, then two negatives make a positive. 6:03 JP: Did you guys hear what she just said? So she actually tried a number[begins writing on document camera]. She said if x equals 5, then x plus x equals 5, which equals 10 [concludes writing &quot;if x=5 then x+x=5+5=10 on document camera]. But then what was the second part of what you said? 6:28 Kianna: And if it’s like, negative 5, then two negatives make a positive. 6:34 JP: [continues writing &quot;if x=−5 then 2 negatives equal a positive] And if x equals negative 5, then two negatives equal a positive. What do you guys think about what Kianna said? 6:55 S: I think it could be true.</td>
</tr>
<tr>
<td>4</td>
<td>7:01 JP: It could be true. What about the first part of what she said? How do you feel about that? She said this [pointing to the statement she wrote about ‘if x=5’] is an example that is showing that x plus x is larger. That if x is 5, then you have 5 plus 5 which is 10, and 10 is bigger than 5. Do you guys agree with that? Now think about the second part of what she said [points to the ‘x=−5’ portion of the public record]. If x equals negative 5, then two negatives equal a positive. Do you agree with that? I see a few of you shaking your head, and then some of you are giving me blank stares. What happens if we actually try x equals negative 5? 7:49 Luke: 10 7:56 JP: Brett? Orren? 7:58 Orren: Negative 5.</td>
</tr>
<tr>
<td>5</td>
<td>8:00 JP: what do you mean negative 5? What she is saying, if x equals 5 then two negatives equal a positive [pointing to the “if x=−5” statement on the board], so what is she really saying? That x plus x [pointing to the ‘x+x=−2x’ statement], what if x is negative 5? What would that look like? What addition problem is she talking about? Bruce? 8:23 Bruce: Negative 5 plus negative 5. 8:26 JP: So, she is saying negative 5 plus negative 5 [writes -5+-5=] would equal a positive. Do you agree with that statement? 8:34 JP: Madison. 8:34 Madison: No, it’s equal to negative 10. 12:24 JP: Say that one more time, if x is negative, then what? 12:26 Orren: It will be negative. 12:27 S: And then x will be bigger.</td>
</tr>
</tbody>
</table>
Percent Discount

Just as with the Variables task, I will begin by presenting the different solution strategies I encountered when coding my data for the Percent Discount task (Figure 25). Some of these strategies are not necessary to engage with, or uncover the underlying mathematics of, this task. However, they occurred frequently enough during the task enactments in my dataset that they warranted noting here. For example, Strategy 1—where students recognize that 50% is not the same as the number 50, is knowledge that most students have prior to beginning this task, and as such it is not necessary to bring this up during the discussion. Also, Strategy 5—where students combine a 50% increase followed by a 50% decrease into a single operation (a 25% decrease)—provides a way for students to explore and further their understanding of percent’s, but it is not needed to grapple with, or understand the underlying mathematics of the Percent Discount task. It is interesting to note that one of these additional strategies (either strategy 1 or 5) occurred during every high-rated enactment of the Percent Discount task, but did not occur during any of the low-rated enactments. Generally, these strategies came up because the teacher asked a question trying to move the class toward the underlying mathematics, but the students moved the conversation in a different direction. Additionally, like the Variables task, most of the seven high-rated enactments of the Percent Discount task saw abstract thinking (Strategy 3) brought out before concrete thinking (Strategy 2). For the three low-rated task enactments, abstract thinking was often left out altogether, as students only brought up concrete strategies as they engaged with the task. I now describe some representative task enactments in my study to illustrate these patterns.
Just as with the Variables task, I will begin by looking at the seven high-rated enactments of this task, and then make a comparison to the three low-rated enactments. In nearly all of the high-rated enactments, the abstract strategy (Strategy 3, Figure 25) was brought up before the concrete strategy (Strategy 2). In these enactments, the teacher, or a student began the overall enactment by presenting a concrete example of the misconception that the final and original price would be the same. Then the teacher asked students to consider this misconception by saying things like, “Okay. Alright, if you disagree, raise your hand. [students raise their hand] Who wants to share why they disagree? Shae, go for it.” or “I’m not asking for your answers, remember, we’re asking to see what this claim is, look at it, and what you guys feel or think, what are your thoughts about this claim?” Students responded to these teacher questions by saying things such as, “because 50 percent of, like, the lower number, isn’t gonna be 50 percent of the higher number.” “We have to divide it, the original, not the, you would have to decrease the second price by 50 percent of itself, not by the original,” or:

- they took, they increased it by 50 percent [gestures hand towards board], which was good, and then instead of taking 50 percent [gestures hand towards board] of the new price, which is what the question was asking, they took 50 percent of the

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Basic Understanding</td>
<td>50% is not the same as the number 50</td>
</tr>
<tr>
<td>2 50% is Not Static Concrete</td>
<td>An 50% of an original price of $8 yields $12, and a 50% decrease of this yields $6, which is not the same as $8</td>
</tr>
<tr>
<td>3 50% is Not Static Abstract</td>
<td>A 50% increase followed by a 50% decrease does not yield the original value</td>
</tr>
<tr>
<td>4 A Percent is Not Static</td>
<td>An x% increase followed by a x% decrease does not yield the original value</td>
</tr>
<tr>
<td>5 Simplification</td>
<td>A 50% increase followed by a 50% decrease is equivalent to a 25% decrease</td>
</tr>
</tbody>
</table>
original price and they umm decreased it by that much, rather than the 50 percent
of the new price.

Then, shortly after this abstract concept came up, students quickly (either directly after or
within two minutes), provided a concrete example. The teacher had to do very little to draw out
these concrete examples, and generally only had to say things like, “It won’t increase and
decrease by the same amount? Why not?”, “What do you all think? What does [a 50% increase
and decrease not being the same amount] mean here with [this (incorrect) example], what does
that have to do with this [same example]?” or “Could you hear what he said? ‘Kay, say that one
more time. Yeah yeah. What did he just say, what is he talking about?” Students then brought up
concrete strategies by saying things like, “Cause, if a necklace is 100 dollars, and it increases by
50%, then it goes up to 150 dollars. And if it decreases by 50%, it goes down to 75 dollars,” or
“He said 50 percent of the--not the original price but the new price and that would be 4 dollars
instead of 3 dollars,” or:

   I agree with them and I had an example in my work and I used that the price was
   50 dollars and 50 percent of 50 is 25 dollars, so 50 plus 25 is 70 and--75, so if you
   took away 50 percent of 75, it--the result would be 37 dollars and 50 cents, which
   is different than the original price of 50.

Aside from the student and teacher utterances around the strategies that they shared and
the order that these strategies were shared in, the high-rated task enactments also shared similar
timing. Students began sharing the abstract (and concrete) strategies described here, between
seven and a half and nine minutes after the task enactment first began. Additionally, the total
time it took to share this initial abstract and concrete thinking took between one and six minutes
for all high-rated task enactments.
The timing and manner that the abstract and concrete strategies (strategies 3 & 2, Figure 25) were brought up for high-rated enactments of the Percent Discount task were quite uniform, but the additional strategies (strategies 1, 4, & 5, Figure 25) that were brought up were less so. For the high-rated enactments the most common additional strategy was strategy 5; three enactments saw only this strategy presented—one, three, and six minutes after the abstract and concrete strategies. One high-rated enactment saw only Strategy 4—one minute after the abstract and concrete strategies, one enactment saw only Strategy 1 presented—directly before the abstract and concrete strategies. One high-rated enactment saw strategies 5 and 4—10 and 12 minutes after the abstract and concrete strategies, respectively, and one enactment saw no additional strategies (besides strategies 2 & 3). The reason that these strategies came up varied from enactment to enactment, but a common reason was that the teacher asked a question that attempted to move the discussion towards the underlying mathematics of the task, but then students took it in a different direction (see Figure 26). For other enactments, the teacher brought up the additional strategy, likely as a way to give the class a stronger understanding of the underlying math. For example, in one enactment a teacher said:

You think it would work the same? So, there’s nothing magical about the 50 percent? Should we try it with a different percent? Let’s take that 10-dollar necklace again, just for giggles. The thing is… let’s do 25 percent. Can you guys figure out what 25 percent of 10 would be?

Finally, for some enactments, the additional strategy was used to correct an error. For example, in one task enactment a student said, “This means 50 - 50 is zero, or in this case, back to the original price,” and another student tried to correct this by saying “instead of putting the same number he should have put the same percent.”
Overall, the high-rated enactments of the Percent Discount task show an interesting pattern in that the abstract and concrete strategies are brought up in a fairly uniform manner in all task enactments. The teachers say a few things to move the enactments forward, but the statements that they make are very general prompts. Aside from these two strategies however, the remaining strategies do not seem to show any particular pattern. To enhance my description of patterns that occur during enactment of the Percent Discount task, I now turn to the low-rated enactments of this task.

Three enactments of the Percent Discount task received a rating of 2 for Solution Strategies Discussed on the RUK. These enactments were quite different from the high-rated enactments in that none of them contained important strategies, such as Strategies 1, 4, or 5 (Figure 20), and only one of them contained Strategy 3. As such, the concrete strategy (Strategy 2), which occurred in every enactment, was by far the most prominent strategy for these enactments. Overall, teachers were able to elicit this strategy by saying things like:

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<table>
<thead>
<tr>
<th>Teacher: Why did the final price end up being different than the original price, not the same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1: Because, when you increase by 50 percent of the original price, you get a new value and then, if you do—if you times that by 50 percent, it’s going to be less than the original value.</td>
</tr>
<tr>
<td>Teacher: What do you think about what she said there? Can you add to that?</td>
</tr>
<tr>
<td>Student 2: On mine, I used the variable x to represent the original price.</td>
</tr>
<tr>
<td>Teacher: Okay, so wait. Are you talking about a new strategy or what…?</td>
</tr>
<tr>
<td>Student 2: (simultaneous with JaP) [Inaudible]... it’s about that.</td>
</tr>
<tr>
<td>Teacher: Oh, okay. Go ahead.</td>
</tr>
<tr>
<td>Student 2: So, you would multiply it by 1.5, to represent adding 50 percent of it. And then, if you—1.5 x over 2 can be simplified to 3 fourths x, getting that it’s less.</td>
</tr>
<tr>
<td>Teacher: Oh, so we’re taking 3 fourths of that original price?</td>
</tr>
</tbody>
</table>

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Ok, so, um, that was actually, that was a very common answer for that one. The 9, that if you add 50%, um, and you take away 50%, then people, a lot of people said that they were at the original price. Ok, now, um, what do you think about this idea that if you’re adding 50% and you’re taking 50% away that you would be at the same price that you started with? Um, Bryce. What do you think about it?

and:

So, let’s just clear that up first. Okay, so, still thinking about this [points to solution on the board showing that the final and original price are the same], what do we think about the price, the original price and the final price after that increase and decrease? Do you agree that it would stay the same or do you think that it would be different? What do you think Brandon?

Interestingly, these teacher prompts are very similar to the prompts for the high-rated task enactments. The main difference between these and the high-rated task enactments however is that after the teachers in these low-rated enactments elicited a concrete solution strategy they asked follow-up questions about the concrete strategy itself. In the high-rated enactments the teachers seemed to ask more comparative questions where they asked students to think about both the correct and incorrect thinking about this problem.

Overall, the high-rated task enchantments for the percent discount task saw a fairly uniform pattern of an abstract strategy (Strategy 3, Figure 20) being shared followed by a concrete strategy (Strategy 2). These enactments almost always also saw an additional strategy (either Strategy 1, 4, or 5) being shared. This additional strategy did not follow the same uniformity as the others, but was generally the result of teachers pushing students to uncover the underlying mathematics of the task. In contrast, the low-rated enactments generally only saw the
concrete strategy (Strategy 2) being shared, and also generally saw teachers not pushing students towards the underlying math as was seen in the high-rated enactments.

**Summary of Solution Strategies Discussed Across Tasks**

There were a number of similarities between the Variables and Percent Discount task enactments in regard to the Solution Strategies Discussed category of the RUK. For high-rated task enactments, the unifying theme seemed to be that abstract strategies were shared before concrete strategies. For the Percent discount task, these strategies were presented in an extremely uniform manner, but even with the Variables task, there was not much variation from one highly-rated task enactment to the next. When eliciting strategies past these initial abstract and concrete ones, teachers tended to ask more pointed questions that helped move students toward additional strategies, and ultimately towards the underlying mathematics of the task. This elicitation drew out additional strategies, and in the case of the Variables task, this seemed to be what drew out the most important strategies. In the case of the Percent Discount task, this elicitation often took the form of the teacher asking students to compare correct and incorrect thinking. In general, high-rated enactments of tasks saw more strategies drawn out than low-rated enactments.

In comparison, the low-rated task enactments saw a much heavier tendency towards concrete strategies. For the Variables task, these strategies were presented before the abstract strategies, and for the Percent Discount task, these were often the only strategies that students presented at all during the enactment. After these strategies were presented, the teachers tended to ask vague questions. It seemed like they were attempting to draw out additional strategies or move the class toward the underlying mathematics of the task, but they generally did not draw out any additional strategies. For the variables task this equated to the most important strategies either not being drawn out at all, or only discussed briefly. For the Percent discount task this
generally equated to no other strategies being drawn out at all. Overall, the low-rated enactments of both tasks saw not only fewer strategies drawn out in total, but also few or no important strategies drawn out at all.
CHAPTER VI
DISCUSSION

In this final chapter, I conclude my dissertation by summarizing my work and discussing the implications of this work, including ways future research could continue to build on what I have done. I then identify the limitations of my work and provide suggestions for additional research.

Summary

My study investigated how engaging in a teaching practice that builds on student thinking affects the factors which contribute to the maintenance of cognitive demand. To do this I looked at 12 enactments each of two tasks (24 total). These enactments came from six teachers who were engaging in the teaching practice of building. I looked to understand how the factors that affect the maintenance of cognitive demand varied among teachers and enactments, and I ultimately answered these research questions:

1. What does the maintenance of cognitive demand look like when teachers are attempting to attend to student thinking during task enactment?

2. How do factors contributing to the maintenance of cognitive demand vary when the same cognitively demanding task is enacted by different teachers in multiple classes?

In response to my first research question, my study found that employing a teaching practice that attends to student thinking by building on it can improve the overall maintenance of cognitive demand by aiding some factors that maintain and minimizing other factors that lower cognitive demand during task enactments. Specifically, my study found that engaging in the building practice seems to help teachers (a) offer a more appropriate Amount of Scaffolding than they otherwise would have, (b) ensure that the appropriateness of a task is maintained by
altering the difficulty level of the tasks’ questions, and (c) draw out conceptual connections at higher levels than has been found in previous work.

In response to my second research question, my study found that when the same tasks are enacted by different teachers or by the same teachers with different groups of students, the prominence of factors known to affect the maintenance of cognitive demand can vary between enactments. Specifically, I described how different aspects of task enactments affected the maintenance of cognitive demand in regard to (a) the amount of time students are given to engage with a task, (b) making conceptual connections, and (c) solution strategies.

**Implications**

I now look at the answers that the data provided for my research questions and discuss the importance of these findings and what they mean for our understanding of the maintenance of cognitive demand overall, as well as opportunities they present for future research. I begin by looking at the answer to my first research question, and then move to my second research question.

**Maintenance of Cognitive Demand when Attending to Thinking**

In answering my first research question, I found that when teachers engaged in the practice of building on student thinking, aspects of this practice directly influenced the maintenance of cognitive demand. The building practice was the vehicle that drove these improvements in the maintenance of cognitive demand; however, implementing the building practice may not necessarily be the only way to see these results. My study adds to what we know about the influences that affect the maintenance of cognitive demand, and in this broader sense, the results of my work are applicable beyond task enactments where the building practice
is being implemented. In the following, I look at the implications related to each of my major findings for this question.

**Amount of Scaffolding**

My findings showed that the teachers in my study were able to recognize what a *proper amount of scaffolding* looks like. Indeed, almost unanimously they provided an appropriate amount of scaffolding for every task they enacted as part of this study. However, even though they could all recognize, and successfully provide an appropriate amount of scaffolding, the majority of them would have offered more scaffolding had they not been enacting the building practice as part of the larger study from which my data were drawn. Furthermore, if they were all to provide the scaffolding that they described in their interviews, they almost assuredly would have lowered cognitive demand for their task enactments. However, it seems that if teachers have a specific reason (e.g. enacting a MEP, or a belief in the importance of maintaining cognitive demand), they are able to offer an amount of scaffolding that supports student learning.

Since teachers can recognize what an appropriate amount of scaffolding is and they can offer it if they try, it seems all they need is a reason to do so. Interestingly, what is already known about maintaining cognitive demand may be able to assist with this. The Five Practices (Smith & Stein, 2018) are already an important tool that can help teachers maintain cognitive demand in their classrooms, but one of these practices seems like it could lend itself very well to offering the proper amount of scaffolding too; specifically, the practice of anticipating. In the five practices, anticipating means that as part of planning their lesson teachers think of the possible solution strategies for a given task that students may come up with. By doing this, the teachers are prepared to act when students actually come up with these same strategies in the classroom. Similarly, as they are planning their lessons, teachers could plan ahead for how much
scaffolding to offer, and then offer that amount of scaffolding when they enact the task. Indeed, since teachers can recognize what an appropriate amount of scaffolding is and they can offer it if they try, simply planning ahead may offer a solution. As such, a possible topic for future investigation could be to compare one group of teachers who simply enact a task against another group of teachers who considered the amount of scaffolding they should offer prior to enacting the task. By doing so, we could gain a better understanding of the relationship between thinking about such scaffolding and the maintenance of cognitive demand during task enactments.

**Appropriateness of Task**

The results of my study showed that the teaching strategy of *working to understand a misconception found in a publicly shared (incorrect) task solution* can be used to help maintain appropriate levels of difficulty, and consequently high levels of cognitive demand for task enactments. This applies regardless of whether the teacher, or a student is the one who shared the misconception. However, this strategy only works if teachers ensure that the focus of the whole-class discussion remains on trying to understand the misconception. This can be accomplished by teachers explicitly stating that students must remain focused on the misconception, and then redirecting them as needed. If teachers can keep students focused on the misconception until a student explicitly utters the error in the publicly shared (incorrect) task solution and the task is resolved, cognitive demand can be maintained.

The results of my study showed that when students work through misconceptions, high levels of cognitive demand can be maintained whether the misconception was held by the whole class, or if no students held it and it was introduced by the teacher. The latter of these options seems to be the most useful finding. If students are given a cognitively demanding task and they make errors, those errors can be used to engage them in uncovering the important mathematics
involved, and cognitive demand is maintained; one could simply say that the task enactment was successful. However, if students are given a cognitively demanding task that turns out to be too easy for them and the whole class gets the correct answer, the cognitive demand will be lowered unless the teacher does something to maintain it. The results of my work suggest that when a teacher asks students to understand a misconception as opposed to simply answering the question that the task originally posed, they were able to maintain high levels of cognitive demand. This is important because posing a misconception is not dependent on the building practice; asking students to consider a misconception can be done even if a teacher does not know about the building practice. In other words, teachers enacting any cognitively demanding task that turns out to be too easy for students could maintain high levels of cognitive demand for the task enactment by simply asking students to understand a misconception as opposed to answering the question that the task originally posed. As such it may be a worthy endeavor for future research to look more deeply at the relationship between working to understand a misconception, and the maintenance of cognitive demand during task enactments.

**Conceptual Connections**

Finally, the RUK Conceptual Connections category showed lower ratings than the other categories of the RUK when it was applied to the task enactments in my study, but the ratings in my study were still higher than has been found in previous work. Higher ratings for conceptual connections being drawn out in my study are likely due to the last element of the building practice, Make Explicit, which calls for teachers to draw out explicit utterances of the underlying mathematics. When looking over the teacher statements that drew out conceptual connections, it seems like the most important thing was for teachers to ask for the underlying idea of the
problem and how it related to what had been done in class. The results of my study showed that more explicit teacher questions translated into maintaining higher levels of cognitive demand.

So, it seems that the best way for teachers to draw out conceptual connections is simply to ask for them explicitly. When students make conceptual connections, they can gain a deeper understanding of the mathematics underlying a task. In turn, this can lead to students having more mathematical understandings at their disposal to draw from when trying to navigate future math problems. As such, it seems like supporting students to draw conceptual connections should be an important part of maintaining cognitive demand. Although teaching practices meant to help maintain cognitive demand, such as the five practices, seem to focus on connecting solution strategies to each other, they do not emphasize connecting strategies to the underlying mathematics. Since the teaching practice of building addresses this directly, it seems like this part of the building practice could be utilized for the improvement of the maintenance of cognitive demand overall. If teachers simply take the time at the end of a task to discuss the underlying mathematics, and how it relates to what has been done, the maintenance of cognitive demand can be improved. As such, a possibility for future research would be to look more deeply at task enactments where teachers discuss and make connections to the underlying mathematics of a task and what has been done in class, and see if there are things besides directly asking for conceptual connections to be made that teachers can do to improve the maintenance of cognitive demand in this area.

Prominence of Factors During Different Enactments of the Same Tasks

In the previous sections, I discussed how the practice of building affected the maintenance of cognitive demand during task enactments, and the implications of this. I now compare the task enactments in my study against each other to better understand the implications
of what teachers and students do that affects maintenance of cognitive demand during task enactments.

**Appropriateness of Time**

The RUK Appropriateness of Time scores were consistent across most of the task enactments in my study. Even though they were often highly rated, at a 3 by the RUK, enactments generally did not reach the highest rating for this category (i.e., 4). The results of my work also showed that the reason tasks did not reach this highest level was task specific. For the Variables task, many students appeared to be given too much time to engage with the task at the beginning of the enactment. Students were often given so much time that they finished working in their groups, and simply waited for the teacher to begin the whole-class discussion. However, the students did not usually get off-task during this extra time, instead they just sat quietly, and as a result the RUK score for this category generally did not go below a 3. Also, as the results of my study showed this extra time that students were given was not a waste. Quite to the contrary, teachers used this time to actively understand what student work was available to them, which is knowledge that could help them apply the five practices, and in turn maintain the overall level of cognitive demand during these task enactments.

For the Percent Discount task, enactments often did not reach the highest level because students were not given enough time to engage with the slightly altered task that many of the classes were given. They were given enough time at the beginning of the task enactment to grapple with the original question that the task asked, but after teachers posed slightly altered questions they did not give students enough time to grapple with this newly posed question.

Thus, the reason that the Percent Discount task did not achieve higher scores for the appropriateness of time was quite different from the reasons that the Variables task did not do
this. In turn this simply means that there is no single answer to help teachers offer an appropriate amount of time during task enactments. Even if teachers give students an appropriate amount of time at the beginning of a task enactment, they must still be cognizant of what they ask students to do during the enactment, and if this requires more think time, they should offer it.

Thus, the results of my work indicate that sometimes maximizing one factor that helps maintain the cognitive demand of a task, is done at the expense of others. Indeed, if teachers had ended the variables task as soon as the majority of students were ready to discuss it, they likely would not have had enough time to make sure that they understood all of the student work that was available to them, which in turn could have stunted the whole-class discussion and lowered other factors that help maintain cognitive demand. Overall it would seem that attending to the Appropriateness of Time category requires a multifaceted approach that requires teachers to consider appropriate time throughout the task enactment, as well as to make trade offs to help maintain cognitive demand in other ways. As such, future research could look more closely at the enactments of other tasks to understand if the patterns I uncovered here are common for other tasks as well, or if these patterns are specific in some way to the task enactments in my study.

**Conceptual Connections**

As I noted earlier, engaging in the building practice made teachers quite adept at drawing out conceptual connections. However, regardless of the reason, having a deeper understanding of what classroom interactions occurred to accomplish this is something that could be drawn upon to help maintain higher levels of cognitive demand for all task enactments. Overall, the results of my work showed that even though the students in high- and low-rated enactments seemed to have no notable differences in their prior knowledge or ability to engage with this task, conceptual connections were drawn out at higher levels of cognitive demand when multiple
students made utterances that collectively contained all of the underlying mathematics relevant to the task. These higher-rated enactments also saw students given time to develop and explore these ideas. Conversely, lower levels of cognitive demand were found in task enactments where a single student (or the teacher) was one of a few, or the only person to make an utterance containing some (but likely not all) of the underlying mathematics relevant to the context of the task shortly before the conclusion of the task enactment. This does not imply that students in lower-rated task enactments were less ready or able to engage with the task. It does mean, however, that the content of whole-class discussions, along with at what point in the discussion the underlying mathematics was first uttered, was different for high and low-rated enactments of the same task. Specifically, I found that the exploration of previously understood abstractions is a necessary prerequisite to understanding the underlying mathematics of a task. However, for task enactments where drawing conceptual connections was rated highly, this exploration was used predominantly to move towards understanding the new abstract mathematical concepts underlying the task. Whereas for lower-rated task enactments, the focus of this exploration was on solidifying the abstract concepts needed to engage with the task.

Overall, it would seem that in order to draw out more conceptual connections, teachers need to not only explicitly ask for them, as I discussed when answering my first research question, but they also need to allow students enough time to bring up and discuss the underlying mathematics, so that the underlying mathematics is uttered by numerous students, and to make sure that students are taking what they learn from engaging with the task and using it to move towards new abstractions, rather than simply rehashing previously learned abstractions. Most importantly, however, the results of my work show that the level at which conceptual connections are drawn out is more dependent on what the teacher does than what students do.
Students in both high- and low-rated task enactments often have the same abilities and even insights when exploring a task. If they are allowed time to explore these insights, and if they are guided by the teacher to uncover the underlying math, as opposed to the teacher directly telling them what the underlying math is, it would appear that all students have the ability to maintain high levels of cognitive demand in regard to making conceptual connections. To further understand this, another possible direction for future research would be to look more closely at task enactments where teachers steer whole-class discussion toward making conceptual connections to better understand effective ways to do this.

**Solution Strategies Discussed**

When comparing high- and low-rated task enactments, my work found that for the high-rated enactments, abstract strategies seemed to precede concrete strategies. Also, the questions that teachers asked during high-rated task enactments seemed to be more pointed and kept students on track to uncover the mathematics underlying the task. In comparison, the low-rated enactments saw students focusing more, and sometimes solely, on concrete examples. Additionally, teachers tended to ask questions more vaguely during low-rated enactments, which often led to important strategies not being uncovered.

One of the most unexpected findings here is the importance of focusing on abstract strategies as opposed to concrete strategies. This seems to imply that although working through concrete examples can be important for maintaining cognitive demand, it is even more important that students understand the mathematics behind these concrete strategies, so that they can make more abstract generalizations. What’s unclear however, is the importance of the timing for discussion these abstract strategies. The results of my study showed that for high-rated enactments, the abstract strategies predominantly were discussed before the concrete strategies,
but there is no evidence that this had to be the case. It is possible that this was just a coincidence, but it is also possible that discussing abstract strategies first has a causational link to the maintenance of cognitive demand. As such this is an area that could be explored in the future to understand if such a causational relationship exists.

**Limitations**

For my study I had a sample size of only six teachers. As I discussed when I began answering my first research question, the teachers in my study were comparable in many ways to participants in previously conducted cognitive demand research with much larger sample sizes. While this is certainly encouraging, my small sample size limits the generalizability of my results.

Adding to the limitation of my small sample size is that only five of the six teachers in my study participated in the survey and interview portions of my data collection. While the teachers in my study sometimes said things that were very similar, there were also times where explanations for certain things they did varied widely from one teacher to the next. An additional perspective from this sixth teacher might have enhanced some of my findings. For example, this teacher’s enactments of the Variables task had the largest difference of average RUK scores of all other comparisons of one enactment of a task to the next in my study. Thus this teacher may have been able to provide insight on this difference that could have helped understand the difference in scores from one enactment of a task to the next in general.

Another limitation of my study is that the building practice that the teachers were engaging in was not fully understood when my data were collected. In fact, the data from my study was actually taken from a larger study in which the teachers in my study were working with researchers to explore and articulate the building practice. Thus, another possible area for
future research would be to look at task enactments utilizing the building practice after this practice has been fully developed.

**Recommendations for Future Research**

In the previous section I discussed some possibilities for future research, including looking more deeply at: a) what teachers can do to offer the proper amount of scaffolding; b) the relationship between working to understand a misconception, and the maintenance of cognitive demand; c) what teachers can do, in addition to asking directly, to help students make conceptual connections; d) the relationship between when abstract and concrete strategies are discussed, and the maintenance of cognitive demand; e) what teachers can do to steer whole-class discussion toward making conceptual connections; and f) multiple enactments of other tasks to understand if the patterns I uncovered here are common for other tasks as well.

The last of these suggestions for future research is particularly interesting because comparing the results of another study that looks at multiple enactments of a task could provide both validation for my work, as well as the uncovering of additional nuances that may not have been as prominent in my data as the nuances that I have discussed here. Another study that looks at the maintenance of cognitive demand during multiple enactments of the same task could be in a completely different context than my work, yet the results could still be compared to mine. For example, giving teachers a specific task to enact without any additional directives, such as those related to enacting the building practice, could provide additional insights into the maintenance of cognitive demand that compliment the work that I have done here.

In setting up my study, I noted that there were known influences such as teachers' beliefs about their students’ abilities, and the curriculum that a teacher uses, that can affect the factors that contribute to the maintenance of cognitive demand. As my work has shown, the teaching
practice of building on student thinking can also be an influence on the maintenance of cognitive demand. However, building on student thinking is only one teaching practice, and the foundation that my work has laid can be used to look at other teaching practices to understand how they may influence the maintenance of cognitive demand during task enactments. Some of my findings, such as conceptual connections being rated highly because teachers directly asked for such connections as part of the building practice, were clearly related to the specific teaching practice of building. However other findings, such as the timing of when abstract and concrete strategies are brought up during whole-class discussion, are not necessarily related to the teaching practice of building. As such, looking at the maintenance of cognitive demand during the implementation of other teaching practices could yield interesting results that could be compared and contrasted against my own to further increase our overall understanding of the influence that teaching practices can have on the maintenance of cognitive demand.

Finally, my study looked at a small sample of teachers. As such, a worthwhile future exploration would be to have a larger group of teachers enact the building practice, possibly for repeated enactments of a different task than those that I have already looked at. A study that looks at a larger sample size and a different task being enacted could be used to provide additional support, as well as generalizability, for the results that I have discussed here.

**Conclusion**

The work that I have presented here shows that a specific teaching practice can be an influence on the maintenance of cognitive demand during task enactments. Such an influence can be directly related to aspects of a teaching practice, or it can be from a more indirect relationship. Overall, however, it seems that a teacher’s actions—both what they say and how they direct students—can create subtle nuances that affect the maintenance of cognitive demand during task
enactments. My work has provided a better understanding of some of these subtle nuances. This understanding in turn can be leveraged to improve the maintenance of cognitive demand for all task enactments.
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doi:10.1080/10508406.2014.930707


Appendix A

Percent Discount MEP
Percent Discount MEP

Percent Discount Mini-Task
The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Teacher Orientation to the Task
This prompt is designed to elicit some common misconceptions related to the meaning of percent. These misconceptions include thinking that n% represents a quantity n and thinking that the value n% of an amount B remains the same as the quantity B changes. We envision a good discussion would arise from allowing at least one student who has such a misconception to share their thinking and then inviting the class to argue why they agree or disagree. This activity will likely not be successful unless these misconceptions are allowed to surface during the discussion, so it would be best to avoid doing things that might prevent this surfacing (like explaining what is wrong while monitoring or asking someone with a correct answer to share their thinking first).

Posing the Task
Distribute the task handout to students and give them 3-5 minutes to work on the task individually.

Anticipated Student Approaches

Incorrect
Reflects Misconception 1: the value n% of an amount B remains the same as the quantity B changes
A: Yes. Increasing a price by 50% then decreasing by 50% cancels the change
B: Yes. It is the same because it was increased and decreased by the same amount [would reflect this misconception if the student is interpreting the increase and decrease is taking 50% of the same value]
C: Yes. 

\[(1.5-0.5)\times 1\times\]

Reflects Misconception 2: n% represents a quantity n.
D: Yes. [chooses a number to (incorrectly) illustrate] 100+50%=150; 150-50%=100.
E: Yes. It is the same because it was increased and decreased by the same amount [would reflect this misconception if the student is interpreting the increase and decrease is taking 50% of the quantity 50]
F: Yes. 

x+50=50=x

Other Misconception
G: Yes, multiplying or increasing the original price by (.5, .3, or 50%) and dividing or decreasing the resulting price by (.5, .3, or 50%) cancels the change.

Correct
H: No, because if you increase the price by 50%, the new price will be 150% of the original price. If you decrease the new price by 50%, it will be 75% of the original price, which is not what we started with. [or 0.8*1.5x=0.75x, which is not x]
I: No. It will cost less than it originally did because [uses a counterexample such as 10*0.5=5; 10+5=15 (new price)]; 15*0.5=7.5; 16-7.5=7.5, which does not equal the 10 that we started with.
J: No, because if you increase a price by 50%, the new price will be 150% of the original price. If you decrease the new price by 50%, it will be 75% of the original price. [or \((x+1.5)\times.5=0.75x\)]
K: No. It is different, because the amounts on which the 50% increase and 50% decrease are applied are different.

Monitoring Students at Work
As students are responding to the prompt, monitor their progress and, if necessary, encourage them to provide a justification. Also make note of which students have various solutions that might be productive to discuss, including those with common responses (see eliciting notes below). Avoid explaining or correcting while monitoring as that may undermine the discussion.

Initial Moves to Elicit Student Solutions
Teacher Move 1: I noticed that many of you think the final price is the same as the original price. Choose a preselected student who has incorrectly assumed that the value n% of an amount B remains the same as the quantity B changes like students A and B above.
Teacher Move 2: [Student name] had this solution. Will you please share your thinking?

Mathematical Understanding
Continue to engage students in a discussion of the MOST, tying new student thinking back to the object of discussion as a means of developing the following mathematical understanding: "n% of an amount B is a quantity that changes as the value of B changes."
Appendix B

Comparing Variables MEP
Comparing Variables MEP

Comparing Variable Expressions Mini-Task
Which is larger, \( x \) or \( x + x \)? Explain your reasoning.

Teacher Orientation to the Task
This task is designed to elicit some common misconceptions related to values assigned to variables and to constructing valid arguments. These misconceptions include thinking that the sign on a variable expression (or operations used therein) indicates the sign of the values that can be assigned to the variables, thinking that a single example is sufficient to conclude that something is true, and thinking that addition or multiplication always make bigger. We envisage a good discussion would arise from allowing at least one student who has such a misconception to share their thinking and then inviting the class to argue why they agree or disagree. This activity will likely not be successful unless these misconceptions are allowed to surface during the discussion, so it would be best to avoid doing things that might prevent this (like explaining what is wrong while monitoring or asking someone with a correct answer to share their thinking first).

Posing the Task
Distribute the task handout to students and give them a few minutes (less than 5) to work on the task individually.

Anticipated Student Approaches
Only considers positive domain
A: \( x + x \) is larger because \( 2x \) is larger than \( 1x \).
B: \( x + x \) is larger because it is addition.
C: \( x + x \) is larger. [uses an example with positive numbers to illustrate] If \( x = 4, 4 + 4 = 8, 8 > 4 \).

Only considers + and domain
D: It depends on whether \( x \) is positive or negative [ignores the case of zero].

Considers +, 0, domain
E: It depends. When \( x \) is positive, \( x + x \) is larger. When \( x \) is negative, \( x \) is larger. When \( x \) is zero, the two expressions are equal.
F: (with examples of +, 0): It depends. [uses an example with positive numbers to illustrate] If \( x = 4, 4 + 4 = 8, 8 > 4 \). [uses an example with negative numbers to illustrate] If \( x = -2, -2 + -2 = -4, -2 > -4 \). [uses an example with 0 to illustrate] If \( x = 0, 0 + 0 = 0, 0 = 0 \).

Answers that Arise from Other Misconceptions
G: Cannot determine. A variable can represent any value. [uses an example with \( x \) taking on at least two different values simultaneously] If \( x = 16, x + x = 2 + 2, 16 > 4 \). If \( x = -2, -2 + -2 = -4, -2 > -4 \).

H: is larger because is only addition, while \( x \) can be multiplied by any number. [The student seems to see \( x \) as an object that can be operated on but \( x + x \) is seen as a fixed entity.]

Monitoring Students at Work
As students are working on the task, monitor their progress and, if necessary, prompt them to provide a justification. Also make note of which students have various solutions that might be productive to discuss, including those with common responses [see Initial Moves notes below]. From previous experience with this task we have found that responses like those of students A, D, and E are the most common.

Initial Moves to Elicit Student Solutions
Teacher Move 1: “I noticed that [Pick One: many, a few, a number] of you think \( x + x \) is larger. Please raise your hand if you think \( x + x \) is larger.”
Choose a presellected student who incorrectly uses a general argument that multiplication or addition always makes larger [like student A or B above].
Teacher Move 2: [Student name], will you please share your thinking.

Orchestrating the Discussion
The initial moves above are designed to initiate a discussion aimed at the following mathematical understanding:

All possible values within a domain must be considered to determine relative values of variable expressions.

Continuing to engage students in a discussion has the potential to bring them to one or more of the following additional mathematical understandings:

• Concluding that something is true requires considering ALL possible cases; concluding that something is false requires only a single counter example.
• The input(s) at which variable expressions are equal are locations of potential change in the relative value of the expressions.
• Adding a nonzero number to itself results in a number that is further away from zero in the direction of the original number.
• All uses of the same variable in a given situation represent the same value.
Appendix C

The TR Survey
## The TR Survey

<table>
<thead>
<tr>
<th>Question</th>
<th>Purpose</th>
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<tbody>
<tr>
<td>1) If you facilitate a discussion in class, what percentage of students participate in the discussion for any given problem?</td>
<td>IQA (AC #1)</td>
</tr>
<tr>
<td>a) Over 75%</td>
<td></td>
</tr>
<tr>
<td>b) 50-75%</td>
<td></td>
</tr>
<tr>
<td>c) 25-49%</td>
<td></td>
</tr>
<tr>
<td>d) Less than 25%</td>
<td></td>
</tr>
<tr>
<td>e) I don’t often use problems that involve a whole-class discussion</td>
<td></td>
</tr>
</tbody>
</table>

| 2) When you are having a whole-class discussion about a specific problem, how often do you make explicit connections, or ask students to make explicit connections between ideas that other students have shared? | IQA (AC #2) |
| a) Three or more times per discussion | |
| b) Twice per discussion | |
| c) Once per discussion | |
| d) I don’t usually make, or ask students to make, these connections | |
| e) I don’t often use problems that involve a whole-class discussion | |

| 3) When you are having a whole-class discussion about a specific problem, how often do students, whether they were asked to or not, make explicit connections between their ideas and ideas that other students have shared? | IQA (AC #3) |
| a) Three or more times per discussion | |
| b) Twice per discussion | |
| c) Once per discussion | |
| d) Students don’t usually make these connections | |
| e) I don’t often use problems that involve a whole-class discussion | |

| 4) When you are having a whole-class discussion about a specific problem, how often do you ask students to provide evidence for their contributions, or explain their reasoning? | IQA (AC #4) |
| a) Three or more times per discussion | |
| b) Once or twice per discussion | |
| c) I usually only ask students to explain a procedure or how they did their calculations | |
| d) I don’t usually ask students to explain their reasoning | |
| e) I don’t often use problems that involve a whole-class discussion | |

| 5) When you are having a whole-class discussion about a specific problem, how often do students, whether they were asked to or not, provide evidence for their contributions, or explain their reasoning? | IQA (AC #5) |
| a) Three or more times per discussion | |
| b) Once or twice per discussion | |
| c) My students do this, but they are often vague, or they explain a procedure, or how they did their calculations | |
| d) My students don’t usually explain their reasoning | |
| e) I don’t often use problems that involve a whole-class discussion | |

| 6) Which of these options best describes your students when they are in the midst of solving a problem? | IQA (AR #2) |
| a) My students generally explore problems that lead to them having a deep understanding of the underlying mathematical concepts. | |
| b) My students generally do some complex thinking about mathematical concepts, but I also give them procedures to work with. | |
| c) My students generally work on applying procedure that I just explained to them or other procedure that they were well familiar with. | |
| d) My students generally work on memorizing facts, rules, formulas, or definitions. | |
7) Which of these options best describes the discussions you generally have in your classroom after working on a problem?

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>I try to have my students explain their strategies or thinking, and why it works for that particular problem, and they are generally able to do this. My students are able to fully express their thinking, and why it works, and they can make connections to other students’ strategies as well as underlying mathematical ideas.</td>
</tr>
<tr>
<td>b)</td>
<td>I try to have my students explain their strategies or thinking, and why it works for that particular problem, but they often have trouble thinking more in depth about what they did, so their ideas are often incomplete, and it is difficult for them to make connections between different strategies that others used.</td>
</tr>
<tr>
<td>c)</td>
<td>I have my students explain the steps that they used or present the work that they did to the rest of the class. Most of what they share is just a repeat of earlier thinking that they did to solve the problem.</td>
</tr>
<tr>
<td>d)</td>
<td>I generally lead the discussion and have my students answer a few questions such as the answer to the problem, or other small bits of information about the problem. When they give longer responses, my students tend to be vague, and do not help the discussion.</td>
</tr>
<tr>
<td>e)</td>
<td>We do not usually have discussions following the problems that we do.</td>
</tr>
</tbody>
</table>

8) Which of these options best describes what you do in class while your students are working on a problem?

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>I continually ask my students questions that allow them to explain their mathematical work and thinking, identify important mathematical ideas of the lessons, or make connections between different ideas representations or strategies.</td>
</tr>
<tr>
<td>b)</td>
<td>At least three times during the course of a problem I ask my students questions that allow them to explain their mathematical work and thinking, identify important mathematical ideas of the lessons, or make connections between different ideas representations or strategies.</td>
</tr>
<tr>
<td>c)</td>
<td>Once or twice during the course of a problem I ask my students to explain their reasoning, identify important mathematical ideas of the lessons, or make connections between different ideas representations or strategies.</td>
</tr>
<tr>
<td>d)</td>
<td>I ask my students facts about a problem, or I ask them to share mathematical facts or procedures related to the problem, but I generally do most of the work.</td>
</tr>
<tr>
<td>e)</td>
<td>I usually show my students what needs to be done.</td>
</tr>
</tbody>
</table>

9) Which of these options best describes the discussions you generally have in your classroom after working on a problem?

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>Through my student’s contributions, important mathematical ideas and concepts from the problem surface and the discussion helps to solidify their understanding of the main mathematical goals of the lesson.</td>
</tr>
<tr>
<td>b)</td>
<td>Through my student’s contributions, important mathematical ideas and concepts from the problem begin to surface but the discussion does not completely solidify their understanding of the main mathematical goals of the lesson; this could be due to factors such as time, or student readiness.</td>
</tr>
<tr>
<td>c)</td>
<td>My student’s contributions do not usually uncover, important mathematical ideas and concepts from the problem. Usually, I must show students the connections that need to be made to understand these ideas and concepts. Without my guidance, the conversation would be mathematical, but would not lead to my students uncovering the main mathematical goals of the lesson.</td>
</tr>
<tr>
<td>d)</td>
<td>Our conversations are mathematical, but they do not uncover, important mathematical ideas and concepts from the problem, and my students often don’t understand the main mathematical goals of the lesson.</td>
</tr>
<tr>
<td>e)</td>
<td>We do not usually have discussions following the problems that we do.</td>
</tr>
</tbody>
</table>

10) In general, after you present an example or other information to your classes, how is what you presented related to the problem(s) that you assign your students afterwards?

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>Students can draw from an example I present to them but replicating it will not lead them to a correct solution, they must draw from other previous knowledge to find a solution.</td>
</tr>
<tr>
<td>b)</td>
<td>Replicating the example, I present makes their problem a little easier, but students still need to apply other strategies to find a solution.</td>
</tr>
<tr>
<td>c)</td>
<td>Replicating the example, I present makes their problem simpler, but there is still usually a small step that was not part of the example.</td>
</tr>
<tr>
<td>d)</td>
<td>Except for changes in numbers, and possibly mathematical operations, students can closely replicate my steps to find a solution.</td>
</tr>
</tbody>
</table>
11) In general, after you present an example or other information to your classes, how is what you presented related to the problem(s) that you assign your students afterwards?

a) Students can draw from what I present to them but replicating it will not lead them to a correct solution, they must draw from other previous knowledge to find a solution.
b) Replicating what I present makes their problem a little easier, but students still need to apply other strategies to find a solution.
c) Replicating what I present makes their problem simpler, but there is still usually a small step that was not part of the example.
d) Except for changes in numbers, and possibly mathematical operations, students can closely replicate my steps to find a solution.

12) When they are working on a complex/difficult problem, how often (1-5; never- always):

a) does the problem build on students’ prior knowledge/skills? ____
b) do students (individual, small group, or whole class) press you for additional information that makes the problem easier to solve? ____
c) do you provide additional information that makes the problem easier to solve, based on students’ requests for such information? ____
d) do you (based on student requests or not) find that you provide enough additional information that the problem loses its complexity or is broken down into smaller less complex parts? ____
e) do students remain motivated to continue working on the problem until it has been completed? ____

13) During an average class, how often (1-5; never- always):

a) do students speak out of turn in an unproductive way (not including activities where students are allowed to speak freely, and the conversation remains on task)? ____
b) do students ask you to re-explain a problem, it’s directions, or other information that you previously shared with the class? ____
c) do students stay on task? ____
d) do at least half the students not complete problems or other work due to time constraints? ____
e) do at least half the student’s complete problems or other work before their peers and have no other assigned work to do? ____

14) Which of the following were your students exposed to before they participated in the first variables MEP enactment in your class? If this topic was covered previously, approximately how many class periods were devoted to it? (If you know that it was covered, but not how many class periods, just type an x.)

a) Variables in real life application problems
   _____In a previous course
   _____In the current course
b) Variables representing negative values
   _____In a previous class
   _____In my class (how many class periods were devoted to this? _____)
c) The underlying concept that variables can represent any value
   _____In a previous class
   _____In my class (how many class periods were devoted to this? _____)
d) Percent’s
   _____In a previous class
   _____In my class (how many class periods were devoted to this? _____)
e) Application problems where a numerical value must be increased or decreased by a percent
   _____In a previous class
   _____In my class (how many class periods were devoted to this? _____)

The IQA questions are derived from the work of Boston (2012), the remaining questions are derived from the work of Stein et al. (1996).
Appendix D

Expanded RUK
Expanded Reorganized Factors that Undermine or Keep Cognitive Demand (RUK)

<table>
<thead>
<tr>
<th>RUK Category</th>
<th>Rating</th>
<th>Description of Assessment Corresponding to Each Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriateness of Task</td>
<td>4</td>
<td>Students have a strong interest in this task, they understand the underlying mathematics needed to approach it, and it requires considerable cognitive effort.</td>
</tr>
<tr>
<td>To what extent was the task</td>
<td>3</td>
<td>Students are interested in this task, but a complete grasp of the underlying math needed to approach it is lacking, or it does not require considerable cognitive effort for some.</td>
</tr>
<tr>
<td>appropriate?</td>
<td>2</td>
<td>Many students are not interested in this task, they struggle with the underlying mathematics needed to approach it, or most are able to solve it with minimal cognitive effort.</td>
</tr>
<tr>
<td>1</td>
<td>Nearly all students lack the interest or underlying mathematics needed to approach this task, or they can solve it with almost no cognitive effort.</td>
<td></td>
</tr>
<tr>
<td>Appropriateness of Time</td>
<td>4</td>
<td>Students had enough time to explore this task and come up with solution strategies, but not so much time that’s students started getting off task.</td>
</tr>
<tr>
<td>To what extent was the amount of</td>
<td>3</td>
<td>Students had time to explore, but this time may have been cut short for a few students, or a few students may have started getting off task.</td>
</tr>
<tr>
<td>time appropriate?</td>
<td>2</td>
<td>Some students had enough time to explore and share solution strategies, but some students did not, or some students may have been off task.</td>
</tr>
<tr>
<td>1</td>
<td>The teacher allowed very little time for exploration, or so much time that many students started getting off task.</td>
<td></td>
</tr>
<tr>
<td>Solution Strategies Discussed</td>
<td>4</td>
<td>Numerous solution strategies, and all of the mathematical concepts needed to engage with this task were discussed.</td>
</tr>
<tr>
<td>To what extent were solution</td>
<td>3</td>
<td>Some solution strategies are, and most of the mathematical concepts needed to engage with this task were discussed.</td>
</tr>
<tr>
<td>strategies discussed?</td>
<td>2</td>
<td>At least one solution strategy, and some of the mathematical concepts needed to engage with this task were discussed.</td>
</tr>
<tr>
<td>1</td>
<td>No solution strategies, or none of the mathematical concepts needed to engage with this task were discussed.</td>
<td></td>
</tr>
<tr>
<td>Held Accountable</td>
<td>4</td>
<td>The teacher consistently asks students to share their reasoning, or provide evidence for their conclusions.</td>
</tr>
<tr>
<td>To what extent were students</td>
<td>3</td>
<td>The teacher occasionally asks students to share their reasoning, or provide evidence for their conclusions.</td>
</tr>
<tr>
<td>held accountable for their thinking?</td>
<td>2</td>
<td>The teacher asks students to share procedures they used, or other memorized knowledge.</td>
</tr>
<tr>
<td>1</td>
<td>The teacher does not ask students to share their reasoning, nor provide evidence for their conclusions.</td>
<td></td>
</tr>
<tr>
<td>Amount of Scaffolding</td>
<td>4</td>
<td>The teacher or more capable peer offers assistance to others without reducing the cognitive complexity of the task.</td>
</tr>
<tr>
<td>To what extent were solution</td>
<td>3</td>
<td>The teacher or more capable peer gives away some or parts of solution strategies, which somewhat reduce the cognitive complexity of the task.</td>
</tr>
<tr>
<td>strategies given away?</td>
<td>2</td>
<td>The teacher or more capable peer gives away many or large parts of solution strategies, which considerably reduces the cognitive complexity of the task.</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>The teacher or more capable peer gives away all or entire solution strategies, thus reducing the task to simply applying known procedures.</td>
</tr>
<tr>
<td>Classroom Management</td>
<td>4</td>
<td>Students were on task for the entire task enactment.</td>
</tr>
<tr>
<td>To what extent did classroom</td>
<td>3</td>
<td>Some students may have gotten off task for a short time, but for the most part the class was on task.</td>
</tr>
<tr>
<td>management issues occur?</td>
<td>2</td>
<td>Many students were off task or a short time, or a few students were off task for longer periods of time.</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Many students were off task for long periods of time during the task enactment.</td>
</tr>
<tr>
<td>Explanin Thinking</td>
<td>4</td>
<td>When prompted, students can consistently share their reasoning, or provide evidence for their conclusions.</td>
</tr>
<tr>
<td>To what extent could</td>
<td>3</td>
<td>When prompted, students can occasionally share their reasoning, or provide evidence for their conclusions.</td>
</tr>
<tr>
<td>students explain their thinking?</td>
<td>2</td>
<td>When prompted, students can share procedures they used, or other memorized knowledge.</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>When prompted, students can share (at most) their answers, but little else.</td>
</tr>
<tr>
<td>Conceptual Connections</td>
<td>4</td>
<td>Explicit connections to other students strategies or to underlaying mathematical concepts were consistently made, and understood by most students.</td>
</tr>
<tr>
<td>To what extent were conceptual</td>
<td>3</td>
<td>More that once, explicit connections to other students strategies or to underlaying mathematical concepts were made, and understood by some students.</td>
</tr>
<tr>
<td>connections made?</td>
<td>2</td>
<td>A few, or weak attempts at, explicit connections to other students strategies or to underlaying mathematical concepts were made, and understood by a few students.</td>
</tr>
<tr>
<td>1</td>
<td>No explicit connections to other students strategies or to underlaying mathematical principles were made, or if they were made, they were not understood by students.</td>
<td></td>
</tr>
</tbody>
</table>

Note. Derived from the work of Boston (2012), and Stein et al. (1996).
Appendix E

HSIRB Documents
Western Michigan University  
Department of Mathematics

Student Investigator: Joshua M. Ruk (working under Dr. Laura R. Van Zoest)  
Principal Investigator: Dr. Laura R. Van Zoest  
Title of Study: Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking

Due to your participation in the Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking study, you are being invited to participate in my dissertation study. This consent document will explain the purpose of my dissertation study and will go over all of the time commitments, the procedures used in the study, and the risks and benefits of participating in this study. Please read this consent form carefully and completely and please ask any questions if you need more clarification.

What am I we trying to find out in this study?  
I am trying to understand the interplay between general teaching practices and the enactment of the MOST Eliciting Prompts (MEPs).

Who can participate in this study?  
Teacher Researchers who are already participating in the Building on MOSTs research project.

Where will this study take place?  
A videoconference room, that you can participate in from a location of your choosing.

What is the time commitment for participating in this study?  
Approximately thirty minutes to complete a survey and an hour-long interview.

What will you be asked to do if you choose to participate in this study?  
1. Complete a pre-interview survey; this is expected to take no more than 30 minutes.  
2. Participate in a single videoconference interview. This interview is anticipated to last about one hour, but could last up to two hours.  
3. Grant permission for the data collected in this interview to be used for research purposes.

What information is being measured during the study?  
General teaching practices, and how these practices relate to the implementation of the MEPs.

What are the risks of participating in this study and how will these risks be minimized?  
The only potential known risk is the loss of time spent participating in this study.

What are the benefits of participating in this study?  
Your participation in this study has the potential to benefit the mathematics education community through a better understanding of how general teaching practices are related to the

Building on MOSTs: Teacher-Researchers
implementation of MEPs. Your participation in this interview will also help the student investigator, who will be interviewing you, attain his PhD.

Are there any costs associated with participating in this study?  
There are no costs associated with participating.

Is there any compensation for participating in this study?  
There is no compensation associated with participating.

Who will have access to the information collected during this study?  
Only the MOST research team will have access to the raw data. Each participant will be assigned a pseudonym that will be used in my dissertation as well as any subsequent publications.

Do you have to participate in this interview as part of the Building on MOSTs project?  
You can choose to not participate in this survey and interview for any reason, and it will not affect your participation in the Building on MOSTs project. You can also choose to stop participating at any time during the survey or interview process. You will not suffer any prejudice or penalty by your decision to not participate. You will experience NO consequences either academically, personally, or in regard to your participation in the larger Building on MOSTs project if you choose not participate in this survey and interview.

Should you have any questions prior to the survey or interview, you can contact the student investigator, Joshua M. Ruk at 712-204-1418 or joshua.m.ruk@wmich.edu, or Dr. Laura R. Van Zoest at 269-387-4527 or laura.vanzoest@wmich.edu. You may also contact the Chair, Human Subjects Institutional Review Board at 269-387-8293 or the Vice President for Research at 269-387-8298 if questions arise during the course of the study.

This consent document has been approved for use for one year by the Human Subjects Institutional Review Board (HSIRB) as indicated by the stamped date and signature of the board chair in the upper right corner. Do not participate in this study if the stamped date is older than one year.

I have read this informed consent document. The risks and benefits have been explained to me. I agree to take part in this study.

Please Print Your Name

Participant’s signature  Date

Building on MOSTs: Teacher-Researchers
Western Michigan University
Department of Mathematics

Principal Investigator: Dr. Laura R. Van Zoest
Title of Study: Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking

You have been invited to participate in a research project titled Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking. This consent document will explain the purpose of this research project and will go over all of the time commitments, the procedures used in the study, and the risks and benefits of participating in this research project. Please read this consent form carefully and completely and please ask any questions if you need more clarification.

What are we trying to find out in this study?
We are trying to understand the nature of student mathematical thinking that is made public during mathematics instruction and how teachers interact with that thinking.

Who can participate in this study?
Secondary school mathematics teachers interested in learning how to productively use student thinking.

Where will this study take place?
At your school and a convenient location that you determine.

What is the time commitment for participating in this study?
Approximately 68 hours spread over two and a half years.

What will you be asked to do if you choose to participate in this study?
1. Participate in up to 4 days of professional development at a Central US location to prepare to implement a series of mathematics tasks in your classroom.
2. Distribute and collect student/parent permission forms prior to the teaching/recording of a lesson.
3. Teach and video record up to six enactments of provided mathematics tasks.
4. Complete post-lesson surveys following each task enactment; these are expected to take no more than 30 minutes.
5. Participate in two online meetings following each task enactment. Each meeting is anticipated to last approximately two hours, 4 hours total. This may result in up to 24 hours of online meetings over three different academic semesters.

Building on MOSTs: Teacher-Researchers
6. Grant permission for the data collected in your classroom to be used for research purposes and in teacher development materials that may be developed to support teacher learning.

**What information is being measured during the study?**
The student thinking that is made public in the classroom in response to provided mathematics task(s) and responses to that thinking.

**What are the risks of participating in this study and how will these risks be minimized?**
The only potential known risk beyond those incurred in your daily work as a classroom teacher is the loss of time spent participating in project activities.

**What are the benefits of participating in this study?**
The project activities have the potential to help you think about your teaching practice in new ways. Your participation in the project has the potential to benefit the mathematics education community through the development of theory and teacher education materials that are anticipated to help improve mathematics teaching and learning.

**Are there any costs associated with participating in this study?**
There are no costs associated with participating.

**Is there any compensation for participating in this study?**
You will be given stipends to compensate you for time spent participating in this study, including $200 for each professional development day (up to $800), $625 for participating in the first four task enactments in 2018-19, and $300 for participating in the last two task enactments in 2020-21. These stipends will be processed by Michigan Technological University. To be compensated for participation in this study, you will be asked to complete and sign a Receipt of Compensation Form along with a W-9; you may receive a 1099-MISC at the end of the year and you may need to report this compensation as income when filing your tax return with the IRS.

**Who will have access to the information collected during this study?**
Only the research team will have access to the raw data. Each participant will be assigned a pseudonym that will be used in all written reports resulting from this work.

**What if you want to stop participating in this study?**
You can choose to stop participating in the study at anytime for any reason. You will not suffer any prejudice or penalty by your decision to stop your participation. You will experience NO consequences either academically or personally if you choose to withdraw from this study. The investigator can also decide to stop your participation in the study without your consent.
Should you have any questions prior to or during the study, you can contact the primary investigator, Dr. Laura R. Van Zoest at 269-387-4527 or laura.vanzoest@wmich.edu. You may also contact the Chair, Human Subjects Institutional Review Board at 269-387-8293 or the Vice President for Research at 269-387-8298 if questions arise during the course of the study.

This consent document has been approved for use for one year by the Human Subjects Institutional Review Board (HSIRB) as indicated by the stamped date and signature of the board chair in the upper right corner. Do not participate in this study if the stamped date is older than one year.

I have read this informed consent document. The risks and benefits have been explained to me. I agree to take part in this study.

Please Print Your Name

Participant’s signature __________________________ Date ___________
Date: December 12, 2019

To: Laura Van Zoest, Principal Investigator
Mary Ochieng, Co-Principal Investigator
Carlee Hollenbeck, Student Investigator
Joshua Ruk, Student Investigator
Amanda Seiwell, Student Investigator

From: Amy Naugle, Ph.D., Chair

Re: IRB Project Number 16-12-01

This letter will serve as confirmation that the changes to your research project titled “Collaborative Research: Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking” requested in your memo received December 11, 2019 (to add survey an interview with subset of original participants with consent documents) have been approved by the Institutional Review Board.

The conditions and the duration of this approval are specified in the Policies of Western Michigan University.

Please note that you may only conduct this research exactly in the form it was approved. You must seek specific board approval for any changes in this project. You must also seek reapproval if the project extends beyond the termination date noted below. In addition, if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the IRB for consultation.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: March 29, 2020