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The Photonic Band Structure of Dielectric Systems with a Frequency Dependent Dielectric Function

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THE PHOTONIC BAND STRUCTURE OF DIELECTRIC SYSTEMS WITH A FREQUENCY DEPENDENT DIELECTRIC FUNCTION

by

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The concepts of band theory for electrons can be employed to describe the behavior of electromagnetic waves propagating in periodic dielectric structures with a frequency independent dielectric function. These periodic structures can produce photonic band gaps in which the propagation of electromagnetic waves is strictly prohibited. The introduction of a frequency dependent dielectric function in such systems gives rise to strong changes of the photonic bands in the neighborhood of the frequency resonance of the dielectric function.

In my thesis, the photonic band structure of a dielectric system with a frequency dependent dielectric function which undergoes a dielectric resonance will be studied. The transmittance and dispersion relation in the presence of periodic dielectric slabs will be derived theoretically and calculated numerically. Comparison with results for systems in the absence of dielectric resonance will be made.

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CHAPTER I

INTRODUCTION

It has been shown that when a quantum or classical wave propagates in a periodic structure in any number of spatial dimensions, frequency gaps at points of symmetry in the corresponding Brillouin Zones are produced in the dispersion relation of the wave. In some cases, an absolute frequency gap[1] occurs in which no wave, regardless of the wavevector value in the Brillouin Zone can propagate. Such a gap is called a band gap.

Photonic band structures are the band structures of electromagnetic waves propagating in periodic, dielectric media. They are thought to be of great technical interest. If the three-dimensional, periodic, dielectric structure is slightly disordered so that it remains periodic on average, it has been suggested[1], that it may be easier to observe the Anderson location of light whose frequency is close to an edge of an absolute band gap of the corresponding periodic structure than it would be in a disordered dielectric structure that is homogeneous on average. It has also been suggested[2,3] that since electromagnetic waves with frequencies in absolute band gaps are totally absent from the system, spontaneous emission for atoms placed in these structures is forbidden when the band gap overlaps the emission frequencies. This prohibition of spontaneous emission can improve the performance of many optical and electric devices which suffer energy losses due to spontaneous emission. The absence of electromagnetic modes in a certain frequency range can also modify the basic properties of many atomic, molecular and excitonic systems[4].

In the following introduction, we will give a brief review of the development of the theory and experiments on photonic band structures.

As early as 1990, S.Satpathy et al. reported their theoretical studies of the photonic band structures of three-dimensional, periodic, dielectric structures where the electromagnetic field is approximated as a scalar wave[2,5-8]. Soon after this, K.M.Leng and Y.F.Liu gave a different approach to the theoretical studies of the photonic band structures of three-dimensional, periodic, dielectric structures in which the electromagnetic field is computed as a full vector field[9,10]. However, the band structures obtained from these two methods are not in agreement with each other. The full vector treatment is needed to obtain accurate results.

The earliest investigations aimed at finding structures that possess absolute band gaps. Subsequently, many special cases were investigated. In 1991, R.D.Meade theoretically calculated the surface electromagnetic states that can exist at the planar surface of a semi-infinite three-dimensional, periodic, dielectric structure formed by cutting an infinite structure along some plane[13]. In recent years, E.Yablonovitch derived the spatially localized defect modes that can arise in the vicinity of a perturbation of a periodic, dielectric structure[14]. Finally, S.John and J.Wang associated quantum electromagnetic effects with the introduction of atoms and molecules into photonic band gap structures[15,16]. And M.Plihal, S.L.Mccall and others have begun to investigate both theoretically[19] and experimentally[22-25] the photonic band structures of two-dimensional, periodic, dielectric structures in order to find structures that possess absolute band gaps. The two dimensional, periodic structure plays an important role in the photonic band structure studies. Two-dimensional systems are often easier to fabricate than the three-dimensional structures[12,17,18]. Moreover, the localization of light in a disordered two-dimensional structure that is periodic on average, whose frequency is close to an edge of an absolute band gap of the corresponding periodic structure, may be easier to achieve than that of a disordered three-dimensional, dielectric structures.

The photonic band structure has been a subject of interest because of its many useful applications. It is known that when an electromagnetic plane wave is incident onto a periodic layered dielectric medium; it is evanescent and can not propagate in the medium if its frequency falls in the band gaps. Thus, the electromagnetic energy is totally reflected, and dielectric medium act as a high-reflectance reflector for the incident wave. By properly designing the periodic layered dielectric medium, extremely high reflectance for some selected spectral region can be achieved. The Fabry-Perot interferometer, the Gires-Tournois etalon, high-reflectance coating, antireflection coating, spectral filters, edge filters, Christiansen-Bragg filters, ellipsometry and so forth[27] are invented based on this simple one-dimensional band structure theory.

We already know from solid-state physics that some materials in some band gap regions have frequency-dependent dielectric constants. How this property affects photonic band structures is a challenging problem in the theoretical studies of the photonic band structures. In this thesis we will introduce a transfer-matrix method which has the advantage that transmission and reflection coefficients for incident electromagnetic waves of various frequencies can be obtained directly from calculations to study the photonic band structure of dielectric systems with a frequency dependent dielectric function . We are especially interested in the effects of a dielectric resonance on the photonic band structure. We will concentrate on one-dimensional layered optical system.

The order of our theoretical derivation and numerical calculation in Chapter II are as follows:

First, we study one slab, characterized by a frequency dependent dielectric function $\varepsilon(\omega)$, surrounded by vacuum . We compute the transmittance relations for the frequency dependent dielectric function ε (ω). We compare these results to those obtained for a frequency independent dielectric constant.

Second, we put slabs, characterized by a frequency dependent dielectric function $\varepsilon(\omega)$ into vacuum to form a one-dimensional, periodic, optical dielectric structure. We obtain the photonic dispersion relations of the periodic system and compare these to those obtained for a frequency independent dielectric constant.

Third, conclusions are presented in Chapter III.

The one-dimensional structure we study in this thesis illustrates all of the aspects of the two-dimensional and three-dimensional systems formed from frequency dependent dielectric media with a dielectric frequency resonance.

CHAPTER II

ONE-DIMENSIONAL PHOTONIC BAND STRUCTURES

In this chapter, we investigate the propagation of electromagnetic waves in the simplest layered structure, which is a single homogeneous and isotropic layer sandwiched between two semi-infinite media. The bounding media we selected are simply vacuum with dielectric constant $\varepsilon_0=1$. Specifically , we investigate transmittance functions in the presence of coupling between the lattice oscillations and electromagnetic wave. This coupling shows up as a frequency resonance in the dielectric function of the dielectric structure. We then generalized this special case into a more complicated one which is the one-dimensional, periodic, dielectric structure, formed by a periodic array of slabs. We are especially interested in the effects of the frequency dependent function $\varepsilon(\omega)$ on the dispersion relation near a resonance frequency of $\varepsilon(\omega)$.

Transmittance Functions in the Presence of a Single Dielectric Slab

Referring to Figure 1, we consider the reflection and transmission of electromagnetic radiation at a thin dielectric layer between two semi-infinite media. We assume that the dielectric function of the thin dielectric layer is given by the frequency dependent $\varepsilon(\omega)$, the bounding media are just vacuum with $\varepsilon_0=1$, The whole structure for a slab of thickness a can be described by:

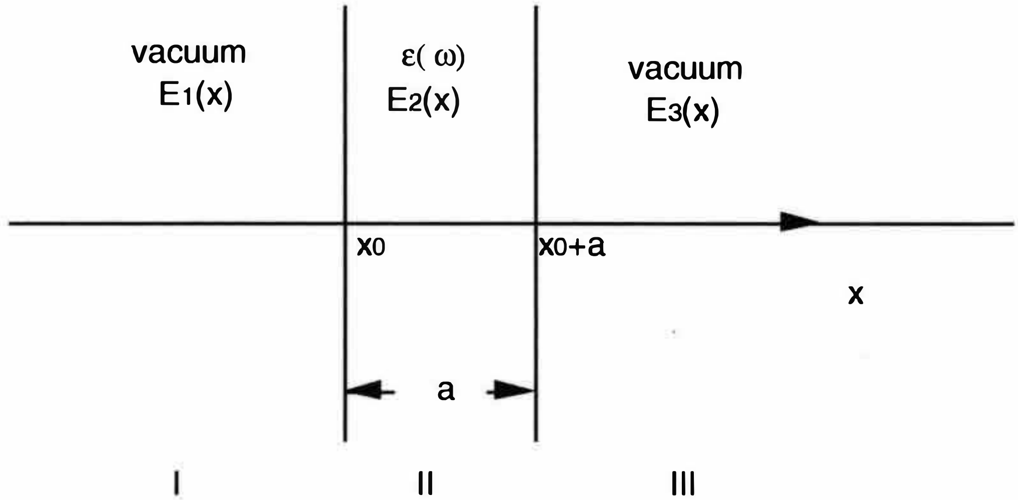

Figure 1. The Structure of a Single Dielectric Slab.

$$\varepsilon_0 = 1 \qquad \text{if } x \le x_0 \quad \text{or} \quad x \ge x_0+a$$
$$\varepsilon(\omega) \qquad \text{if } x_0 \le x \le x_0+a \tag{1.1}$$

For an electromagnetic wave propagating in the x direction, the electric field can be represented by:

$$E(x) = Ae^{i(kx-\omega t)} \tag{1.2}$$

where A is the amplitude of the wave, k is the wavevector in the x direction and ω is the frequency of the incident electric wave. For the motion of an electromagnetic wave through the system in Figure 1, a general solution of the wave E(x) propagating through a single isotropic layer, along the x-axis, can be written as:

$$E_1 = A_0 e^{i(k_0 x - \omega t)} + B_0 e^{i(-k_0 x - \omega t)} \qquad x \le x_0$$

$$E_2 = C e^{i(kx - \omega t)} + D e^{i(-kx - \omega t)} \qquad x_0 < x \le x_0 + a$$

$$E_3 = A e^{i(k_0 x - \omega t)} + B e^{i(-k_0 x - \omega t)} \qquad x \ge x_0 + a$$

(1.3)

where B=0 in our single slab transmission problem. The complex amplitudes A_0 , B_0, C, D, A and B are constants, k_0 and k are the x components of the wavevectors in the two different regions with dielectric constant $\varepsilon_0 = 1$ and $\varepsilon(\omega) \neq 1$

where:

$$K_0 = \frac{\omega}{c}\sqrt{\varepsilon_0} = \frac{\omega}{c}$$

$$k = \frac{\omega}{c}\sqrt{\varepsilon}$$

(1.4)

The constant A_0 is the amplitude of the incident wave. B_0 and A are amplitudes of the reflected and transmitted waves respectively.

According to the boundary conditions at the film surface, both the electric field and its normal derivative at $x=x_0$ and $x=x_0+a$ should be continuous. We find that at $x=x_0$ these conditions yield :

$$A_0 e^{i(k_0 x_0 - \omega t)} + B_0 e^{i(-k_0 x_0 - \omega t)} = C e^{i(k x_0 - \omega t)} + D e^{i(-k x_0 - \omega t)}$$

$$ik_0 A_0 e^{i(k_0 x_0 - \omega t)} - ik_0 B_0 e^{i(-k_0 x_0 - \omega t)} = ikC e^{i(k x_0 - \omega t)} - ikD e^{i(-k x_0 - \omega t)}$$

(1.5)

Solving for the relation between C, D and A_0, B_0, we have:

$$\begin{bmatrix} C \\ D \end{bmatrix} = -\frac{1}{2k}\begin{bmatrix} -(k+k_0)\exp[-i(k-k_0)x_0] & -(k-k_0)\exp[-i(k+k_0)x_0] \\ -(k-k_0)\exp[i(k+k_0)x_0] & -(k+k_0)\exp[i(k-k_0)x_0] \end{bmatrix}\begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

(1.6)

At $x=x_0+a$, our boundary conditions give:

$$Ae^{i[k_0(x_0+a)-\omega t]} + Be^{i[-k_0(x_0+a)-\omega t]} = Ce^{i[k(x_0+a)-\omega t]} + De^{i[-k(x_0+a)-\omega t]}$$

$$ik_0Ae^{i[k_0(x_0+a)-\omega t]} - ik_0Be^{i[-k_0(x_0+a)-\omega t]} = ikCe^{i[k(x_0+a)-\omega t]} - ikDe^{i[-k(x_0+a)-\omega t]}$$

(1.7)

Similarly, we have:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -(k+k_0)\exp[i(k-k_0)(x_0+a)] & (k-k_0)\exp[-i(k+k_0)(x_0+a)] \\ (k-k_0)\exp[i(k+k_0)(x_0+a)] & -(k+k_0)\exp[-i(k-k_0)(x_0+a)] \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix}$$

(1.8)

Combining equation (2.7), (2.9), we find the relation between A, B, and A_0, B_0:

$$\begin{bmatrix} A \\ B \end{bmatrix} =$$

$$\begin{bmatrix} [\cos(ka)+\frac{i}{2}(\frac{k}{k_0}+\frac{k_0}{k})\sin(ka)\exp(-ik_0a) & -\frac{i}{2}(\frac{k}{k_0}-\frac{k_0}{k})\sin(ka)\exp[-i(k_0a+2k_0x_0)] \\ -\frac{i}{2}(\frac{k}{k_0}-\frac{k_0}{k})\sin(ka)\exp[i(k_0a+2k_0x_0)] & [\cos(ka)-\frac{i}{2}(\frac{k}{k_0}+\frac{k_0}{k})\sin(ka)]\exp(ik_0a) \end{bmatrix}\begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

(1.9)

We can write this as:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} (\alpha_1+i\beta_1)\exp(-ik_0a) & i\beta_2\exp[-i(k_0a+2k_0x_0)] \\ -i\beta_2\exp[i(k_0a+2k_0x_0)] & (\alpha_1-i\beta_1)\exp(ik_0a) \end{bmatrix}\begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

(1.10)

Where:

$$\alpha_1 = \cos(ka)$$

$$\beta_1 = \frac{1}{2}\left(\frac{k}{k_0} + \frac{k_0}{k}\right)\sin(ka)$$

$$\beta_2 = \frac{1}{2}\left(\frac{k}{k_0} - \frac{k_0}{k}\right)\sin(ka)$$

(1.11)

This matrix can also be written as:

$$\begin{bmatrix} A \\ B \end{bmatrix} = P \begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

(1.12)

where:

$$P = \begin{bmatrix} (\alpha_1 + i\beta_1)\exp(-ik_0 a) & i\beta_2 \exp[-i(k_0 a + 2k_0 x_0)] \\ -i\beta_2 \exp[i(k_0 a + 2k_0 x_0)] & (\alpha_1 - i\beta_1)\exp(ik_0 a) \end{bmatrix}$$

(1.13)

Taking the origin of the coordinate at the left surface of the slab , $x_0=0$ we know there is no reflected is present. That means Bwavevector existing in area III, only the transmitted wave is present. That means B=0.

Solving eq.(1.13), we have:

$$\frac{A}{A0} = \frac{e^{-ik_0 a}}{\alpha 1 - i\beta 1}$$

Then transmittance in this case is :

$$T = \left|\frac{A}{A0}\right|^2 = \frac{1}{\alpha 1^2 + \beta 1^2}$$

(1.14)

Substituting eq.(1.11) and eq. (1.4) into eq. (1.14) we have:

$$T = \frac{1}{\cos^2\left(\frac{\omega a}{c}\sqrt{\varepsilon}\right) + \frac{1}{4}\left(\sqrt{\varepsilon} + \frac{1}{\sqrt{\varepsilon}}\right)^2 \sin^2\left(\frac{\omega a}{c}\sqrt{\varepsilon}\right)}$$

(1.15)

Now we discuss the special case - the frequency dependent dielectric function at dielectric resonance.

It is well known from solid state physics that the energy of a lattice vibration is quatized. The quantum of this energy is called a phonon in analogy with the photon of the electromagnetic wave. For each wavevector, there are three modes: one of longitudinal polarization and two of transverse polarization. Although longitudinal phonons do not couple to transverse photons in the bulk of crystal, the transverse optical phonons can interact with transverse electromagnetic waves. This interaction shows up as a frequency resonance in the dielectric constant of the media. Resonance means a condition in which the frequencies and wavevectors of two waves are approximately equal. At resonance this phonon-photon coupling entirely changes the character of the propagation and band gaps in photonic band structures. If there are N ion pairs of effective charge q and reduced mass M, then this dielectric function[26] is:

$$\varepsilon(\omega) = \varepsilon(\infty) + \frac{D}{\omega_r^2 - \omega^2}$$

(1.16)

Where:

$$D = \frac{4\pi N q^2}{M} \quad ,$$

ω is the frequency of incident wave. ω_T is the transverse optical phonon frequency in the absence of coupling with photons. $\varepsilon\,(\infty)$ is optical dielectric constant obtained as the square of the optical refractive index.

We put the dielectric function $\varepsilon(\omega)$ into eq. (1.15) and study the photonic transmittance near the dielectric resonance. It is a very complicated function. We can run a FORTRAN program to show this transmittance as a function of frequancy.

By my FORTRAN computer program, I get a set of solutions of eq. (1.15) for $\varepsilon(\omega)$ given by eq. (1.16). We plot the relation of transmittance versus $\omega a/c$ for an optical dielectric constant $\varepsilon\,(\infty) = 9.0$, $\omega_T a/c=1.5421$ where ω_T is taken to always be in the band gaps. The number of frequancy points in the graphes which we have taken is 200.

We plot the transmittance versus $\omega a/c$ relation at different values of M, where $M=Da^2/c^2$, for M=0.0, M=1.0, M=5.0, M=10.0 in Figure 2, Figure 3, Figure 4 and Figure 5 respectively. Referring to Figure 2, we find when M=0.0 (that is D=0.0) there is no resonance present, namely $\varepsilon(\omega)=\varepsilon_0$, transmittance is a continuous and periodic function of $\omega a/c$. When $M\neq0.0$, refering to Figure 3, Figure 4, Figure 5, there is a resonance present, the transmittance is no longer a continuous function of ω. Instead, band gaps are bound to exist between particular frequency ranges in which $\varepsilon(\omega)<0$. This means that the electromagnetic waves with frequencies in these ranges (band gaps) can not propagate though the dielectric structure being considered. At M=1.0, band gap is very narrow. Band gaps become gradually wider when M is increased. The band gaps are always around the transverse optical photon frequency in the presence of coupling with photons ω_T. It displays the effects of dielectric resonance on the transmittance of the single slab.

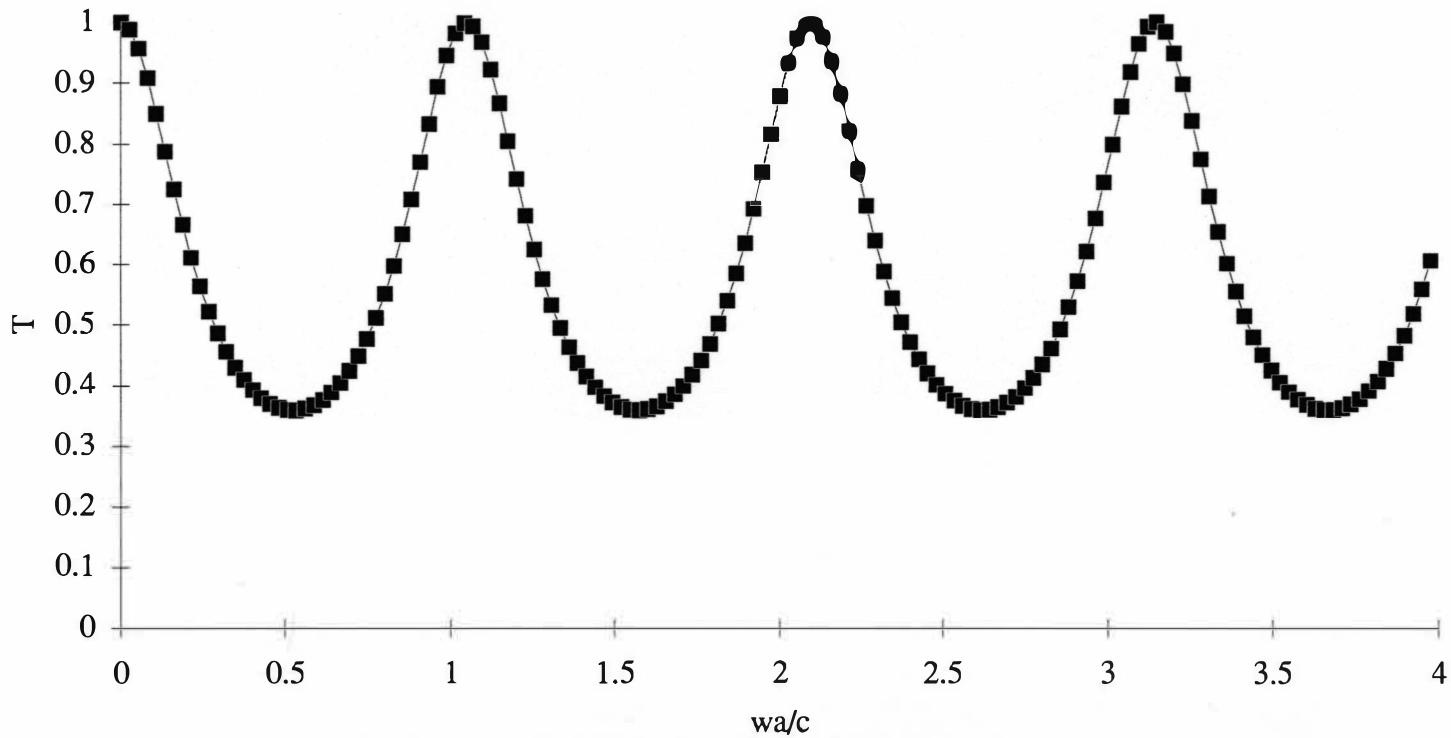

Figure 2. Transmittance T v.s. wa/c When M=0.0.

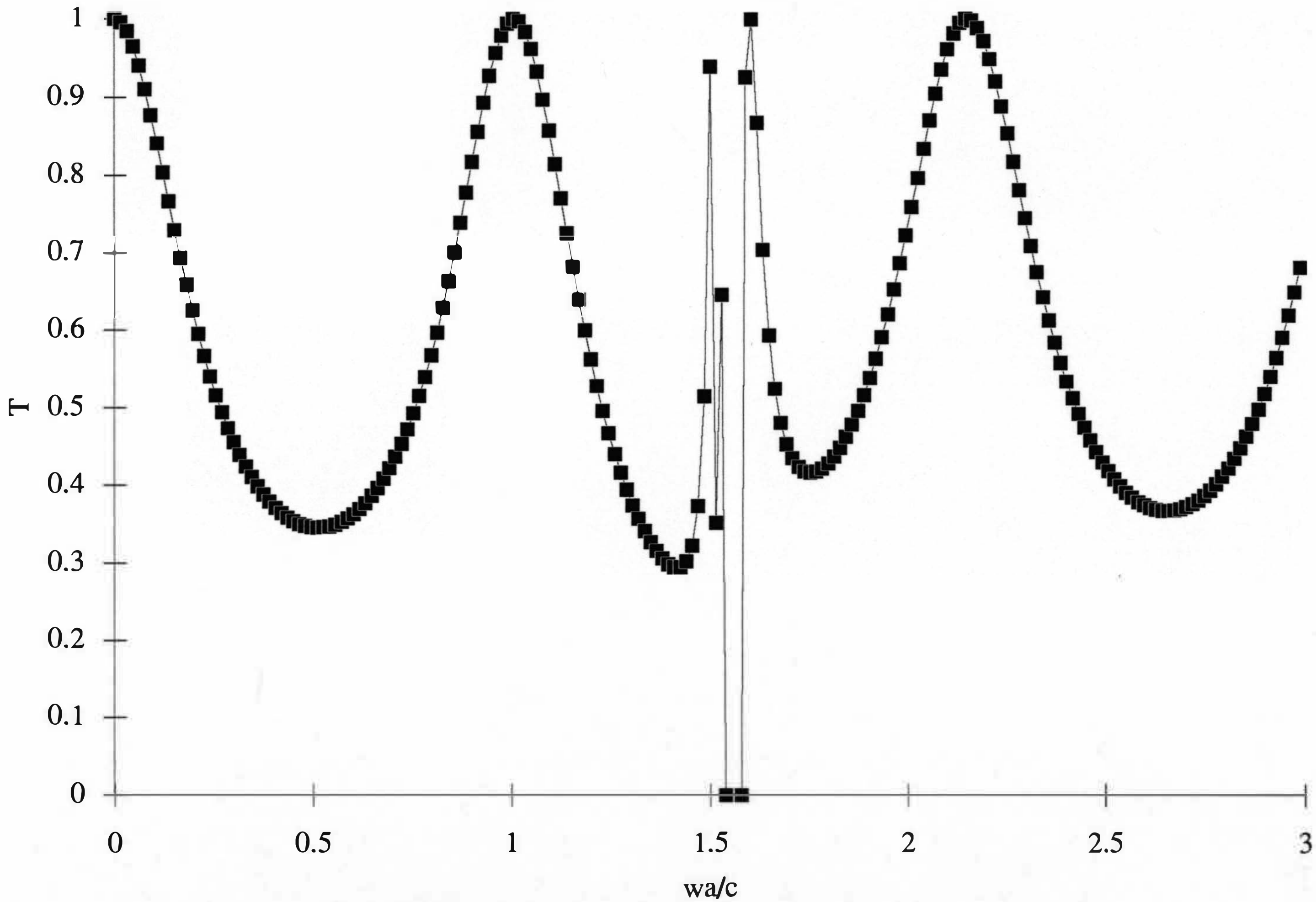

Figure 3. Transmittance T v.s. wa/c When M=1.0.

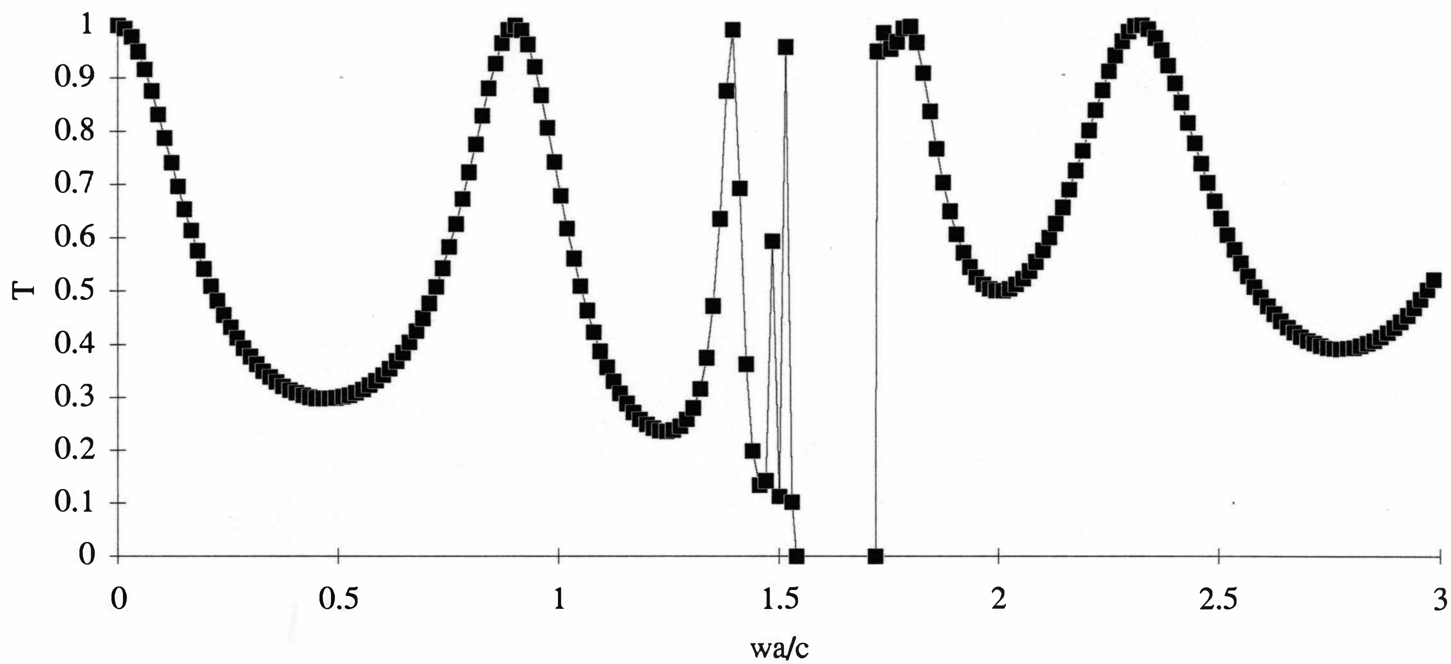

Figure 4. Transmittance T v.s. wa/c When M=5.0.

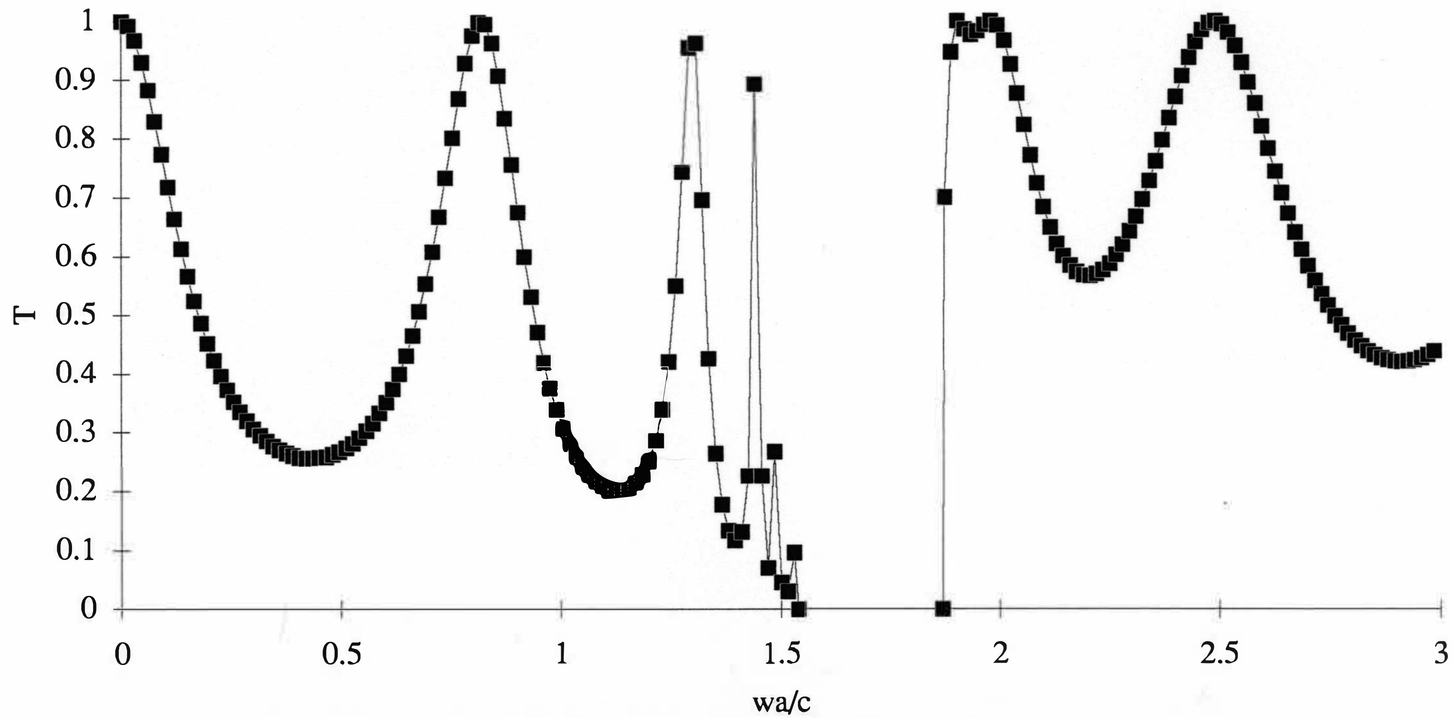

Figure 5. Transmittance T v.s. wa/c When M=10.0.

<u>Dispersion Relations in the Presence of One-dimensional Periodic Dielectric Slabs</u>

We now proceed to solve the problem in the presence of a one-dimensional, periodic, dielectric structure which is formed by the periodic array of slabs. Figure 6 shows a cut from this periodic array of slabs. The over-simplified shape treated here already exhibits the essential features of all such one-dimensional, periodic structures.

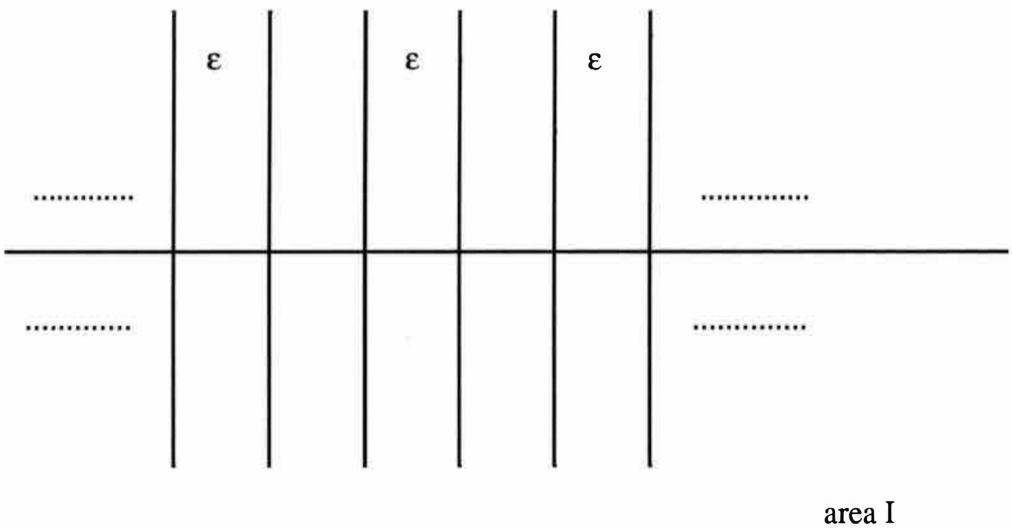

Figure 6. One-dimensional Periodic Structure Formed by One-dimensional Periodic Dielectric Slabs.

As a useful idealization we assume that slabs and vacuums follow each other in periodic succession indefinitely in x direction, although in reality the number of slabs is ,of course, finite, but large. Let d=a+b be the period of this structure in which a is the width of every dielectric slab and b is the width of each vacuum.

As in the single slab problem, a plane wave propagating along the x-axis is incident from the left on the periodic structure . Passing through the periodic dielectric structure, it finally arrives in area I. The wave function of the incident electric field can still be expressed as:

$$E(x) = Ae^{i(kx-\omega t)} \tag{2.1}$$

Suppose the coordinate of left surface of first slab is x=0, (see Fig.7), then the coordinate of the left and right surface of nth slab are x=nd and x=nd+a respectively. The electric field in the first vacuum and first slab are same as eq.(1.3). By the same method, we can derive the electric field in the nth vacuum and nth slab.

The nth electric field in the vacuum may be written in the form:

$$E_n(x) = A_n e^{i[k_0(x-nd)-\omega t]} + B_n e^{i[-k_0(x-nd)-\omega t]} \tag{2.2}$$

where $0 \le x\text{-}nd \le l$. Referring to Figure 7.

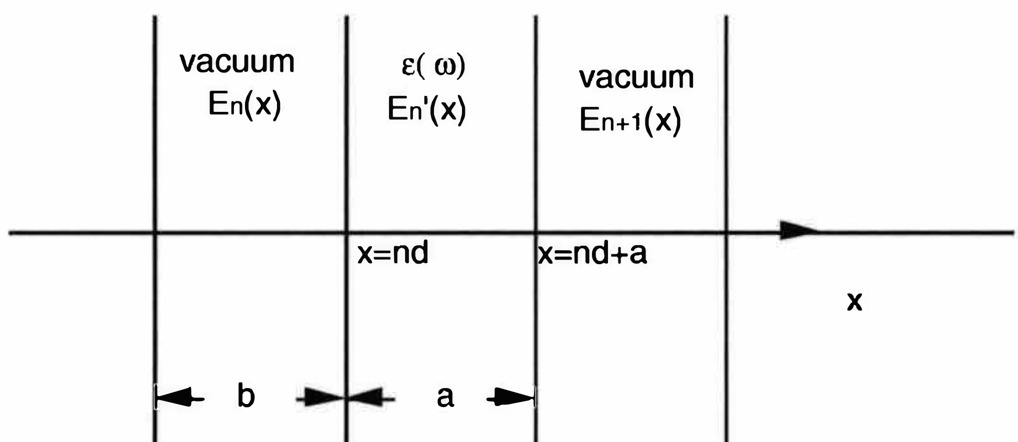

Figure 7. One Part of One-dimensional Periodic Structure Formed by Periodic Array of Slabs.

The electric field in the (n+1)th slab is:

$$E_{n+1}'(x) = C_{n+1}e^{ik[x-(n+1)d]-i\omega t} + D_{n+1}e^{-ik[x-(n+1)d]-i\omega t} \tag{2.3}$$

And the electric field in (n+1)th vacuum is:

$$E_{n+1}(x) = A_{n+1}e^{ik_0[x-(n+1)d]-i\omega t} + B_{n+1}e^{-ik_0[x-(n+1)d]-i\omega t}$$

(2.4)

According to the boundary conditions at the film surface, both the electric field and its normal derivative at x=nd and x=nd+a should be continuous. That is:

at x = nd,

$$A_ne^{-i\omega t} + B_ne^{-i\omega t} = C_{n+1}e^{-i(kd+\omega t)} + D_{n+1}e^{i(kd-\omega t)}$$

$$ik_0A_ne^{-i\omega t} - ik_0B_ne^{-i\omega t} = ikC_{n+1}e^{-i(kd+\omega t)} - ikD_{n+1}e^{i(kd-\omega t)}$$

(2.5)

This equation can be written as a matrix equation:

$$\begin{bmatrix} C_{n+1} \\ D_{n+1} \end{bmatrix} = \frac{1}{2k}\begin{bmatrix} (k+k_0)\exp(ikd) & (k-k_0)\exp(ikd) \\ (k-k_0)\exp(-ikd) & (k+k_0)\exp(-ikd) \end{bmatrix}\begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

(2.6)

at x = nd+a,

$$A_{n+1}e^{-i(k_0b+\omega t)} + B_{n+1}e^{i(k_0b-\omega t)} = C_{n+1}e^{-i(kb+\omega t)} + D_{n+1}e^{i(kb-\omega t)}$$

$$ik_0A_{n+1}e^{-i(k_0b+\omega t)} - ik_0B_{n+1}e^{i(k_0b-\omega t)} = ikC_{n+1}e^{-i(kb+\omega t)} - ikD_{n+1}e^{i(kb-\omega t)}$$

(2.7)

The matrix form of this equation is:

$$\begin{bmatrix} A_{n+1}\exp(-ik_0b) \\ B_{n+1}\exp(ik_0b) \end{bmatrix} = \frac{1}{2k_0}\begin{bmatrix} (k_0+k)\exp(-ikb) & (k_0-k)\exp(ikb) \\ (k_0-k)\exp(-ikb) & (k_0+k)\exp(ikb) \end{bmatrix}\begin{bmatrix} C_{n+1} \\ D_{n+1} \end{bmatrix}$$

(2.8)

Substituting (1.12) into (2.8) we find:

$$\begin{bmatrix} A_{n+1}\exp(-ik_0b) \\ B_{n+1}\exp(ik_0b) \end{bmatrix} = \frac{1}{4kk_0}\begin{bmatrix} \alpha_1+\beta_1 & i\beta_2 \\ -i\beta_2 & \alpha_1-i\beta_1 \end{bmatrix}\begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

(2.9)

where we have defined α_1, β_1 and β_2 in section 2.1.

This may also be written as:

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} = \frac{1}{4kk_0}\begin{bmatrix} (\alpha_1+i\beta_1)\exp(ik_0b) & i\beta_2\exp(ik_0b) \\ -i\beta_2\exp(-ik_0b) & (\alpha_1-i\beta_1)\exp(-ik_0b) \end{bmatrix}\begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

(2.10)

Defining:

$$P = \begin{bmatrix} (\alpha_1+i\beta_1)\exp(ik_0b) & i\beta_2\exp(ik_0b) \\ -i\beta_2\exp(-ik_0b) & (\alpha_1-i\beta_1)\exp(-ik_0b) \end{bmatrix}$$

(2.11)

Then we find by iteration:

$$\begin{bmatrix} A_n \\ B_n \end{bmatrix} = P^n\begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

(2.12)

Applying these considerations to an infinite periodic lattice, we must clearly demand that for $n \to +\infty$ the limit P^n should be finite for a propagating wave to exist. That is most conveniently discussed in terms of the eigenvalue problem of the matrix P.

We now discuss the eigenvalue of P .The eigenvalues of P are roots of the characteristic equation.

$$\begin{vmatrix} (\alpha_1 + i\beta_1)\exp(ik_0b) - \lambda & i\beta_2\exp(ik_0b) \\ -i\beta_2\exp(-ik_0b) & (\alpha_1 - i\beta_1)\exp(-ik_0b) - \lambda \end{vmatrix} = 0$$

(2.13)

That is:

$$\lambda^2 - 2\lambda[a_1\cos(k_0b) - b_1\sin(k_0b)] + a_1^2 + b_1^2 + b_2^2 = 0$$

The roots λ_+ and λ_- are given by:

$$\lambda_+ = \alpha_1\cos(k_0b) - \beta_1\sin(k_0b) + \sqrt{\Delta}$$
$$\lambda_- = \alpha_1\cos(k_0b) - \beta_1\sin(k_0b) - \sqrt{\Delta}$$

(2.14)

where:

$$\Delta = [a_1\cos(k_0b)-b_1\sin(k_0b)]^2 - (a_1^2 + b_1^2 + b_2^2)$$

(2.15)

If $\Delta > 0$, λ_+ and λ_- are real and $\lim_{n\to\infty}|P^n| \to \infty$, the solution of the transfer matrix blows up. Such solution is in conflict with the requirement that electric field must remain finite. Hence, an acceptable solution is obtained and a particular eigenvalue allowed only if $\Delta \le 0$. If this condition holds we may write:

$$\lambda_+ = e^{iKd}$$

$$\lambda_- = e^{-iKd}$$

(2.16)

Then:

$$\lambda_+ + \lambda_- = e^{iKd} + e^{-iKd}$$

$$= 2\,[a_1\cos(k_0b) - b_1\sin(k_0b)]$$

So, we obtain the dispersion relation in this case:

$$\cos(Kd) = \cos(ka)\cos(k_0b) - \frac{1}{2}\left(\frac{k}{k_0} + \frac{k_0}{k}\right)\sin(ka)\sin(k_0b)$$

(2.17)

Substituting (1.4) into (2.17):

$$\cos(Kd) = \cos\left(\frac{\omega a}{c}\sqrt{\varepsilon}\right)\cos\left(\frac{\omega b}{c}\right) - \frac{1}{2}\left(\sqrt{\varepsilon} + \frac{1}{\sqrt{\varepsilon}}\right)\sin\left(\frac{\omega a}{c}\sqrt{\varepsilon}\right)\sin\left(\frac{\omega b}{c}\right)$$

(2.18)

For the wave media, K is the function of w.

At the dielectric resonance which we have discussed in section 2.1, ε is a function of ω , given by eq. (1.16) for $\omega \cong \omega_T$. Similarly we substitute eq.(1.16) into eq. (2.18) to get the dispersion relation for one-dimensional, periodic, dielectric slabs and to study the effects of the resonance on the band structure. We also run a FORTRAN program to show the dispersion relation and to find band gaps for this case.

We numerically calculated eq.(2.18) using a FORTRAN computer program and got a set of solutions of Kd in different $\omega d/c$ at condition of eq.(1.16) . Then we plotted this dispersion relation Kd versus $\omega d/c$. The width of the slabs a=0.5d, the width of the vacuum b=0.5d, where d is the period of the structure formed by a periodic array of the slabs d=a+b. $\omega_T d/c=1.5421$, optical dielectric constant $\varepsilon(\infty)=9.0$, and the number of frequancy points in the graphs which we selected is 200. We plotted dispersion relations at different $M=Dd^2/c^2$, i.e. M=0.0 (that is D=0.0), M=1.0, M=5.0, and M=10.0, where M=0.0 is in no dielectric resonance case and $M\neq0.0$ is in dielectric resonance case. In Figure 8 We present the dispersion relation when no dielectric resonance presented. From Figure 9, Figure 10, Figure 11, we see that dispersion relations altered near the resonance frequency, ω_T of the dielectric function. A new set of bands centered around a gaps at ω_T is observed. The width of the gap increase with increasing M.

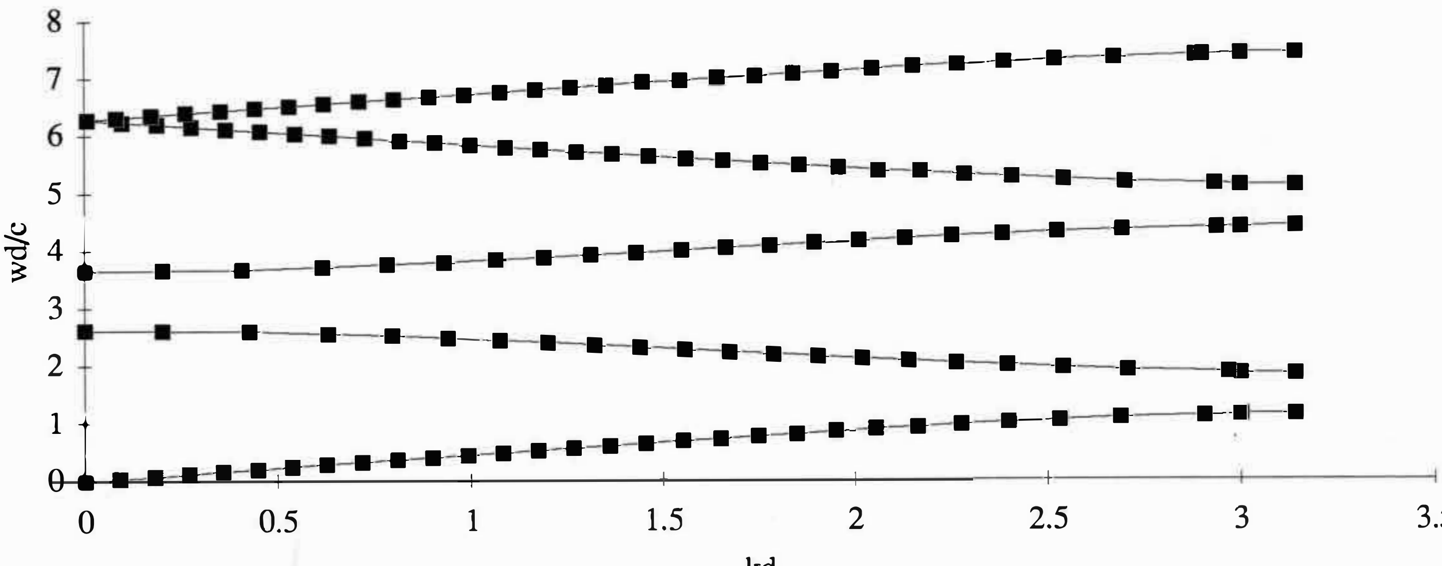

Figure 8. Dispersion Relation When M=0.0.

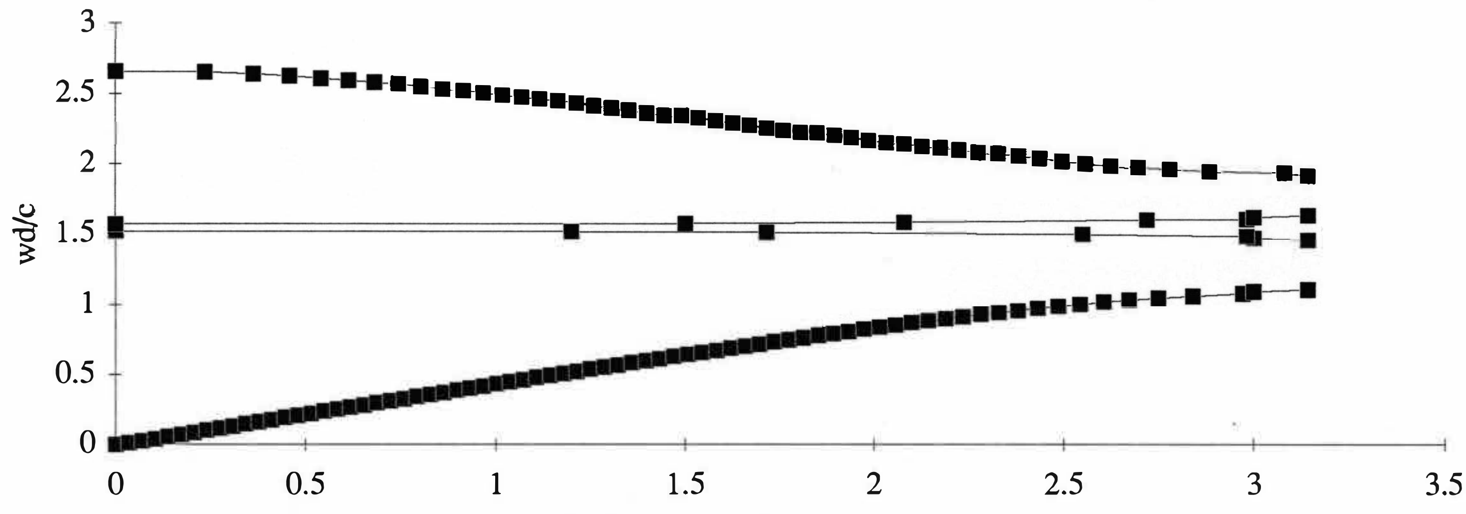

Figure 9. Dispersion Relation When M=1.0.

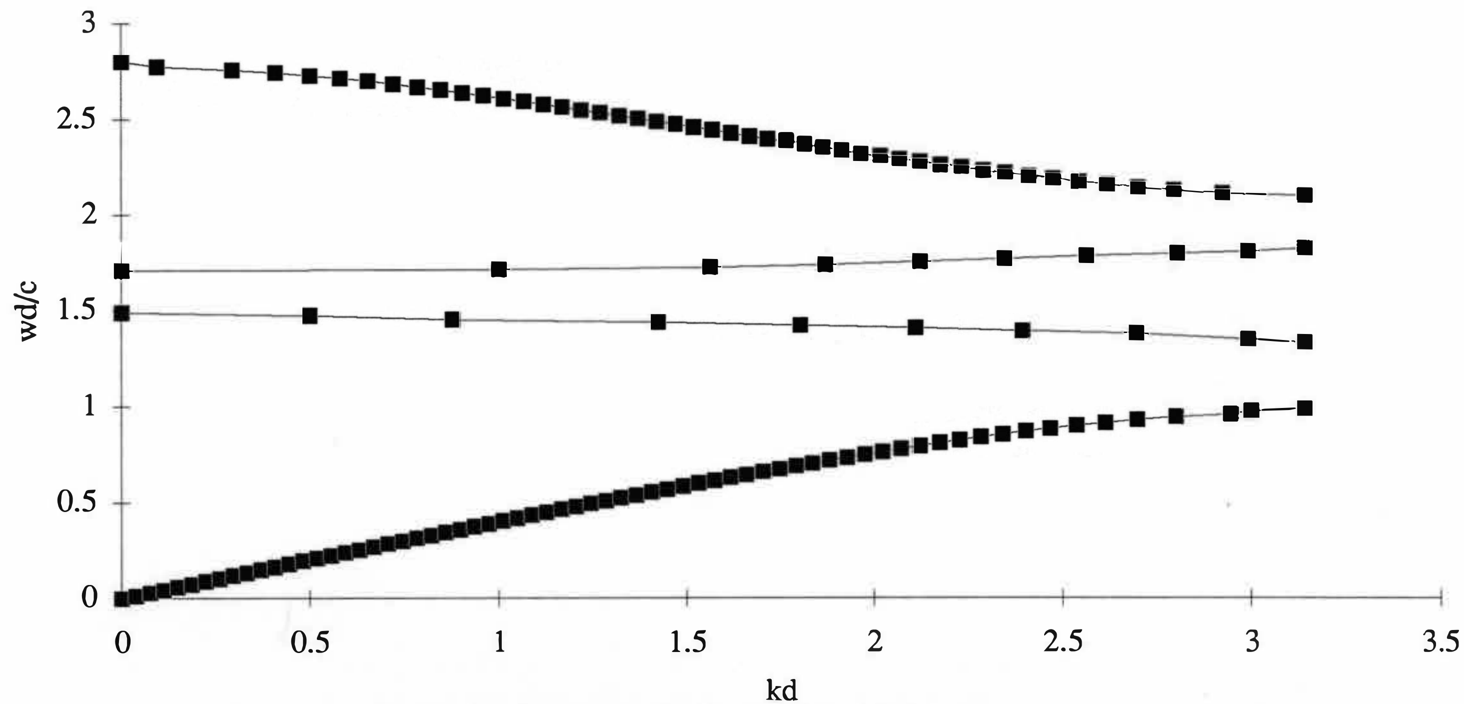

Figure 10. Dispersion Relation When M=5.0.

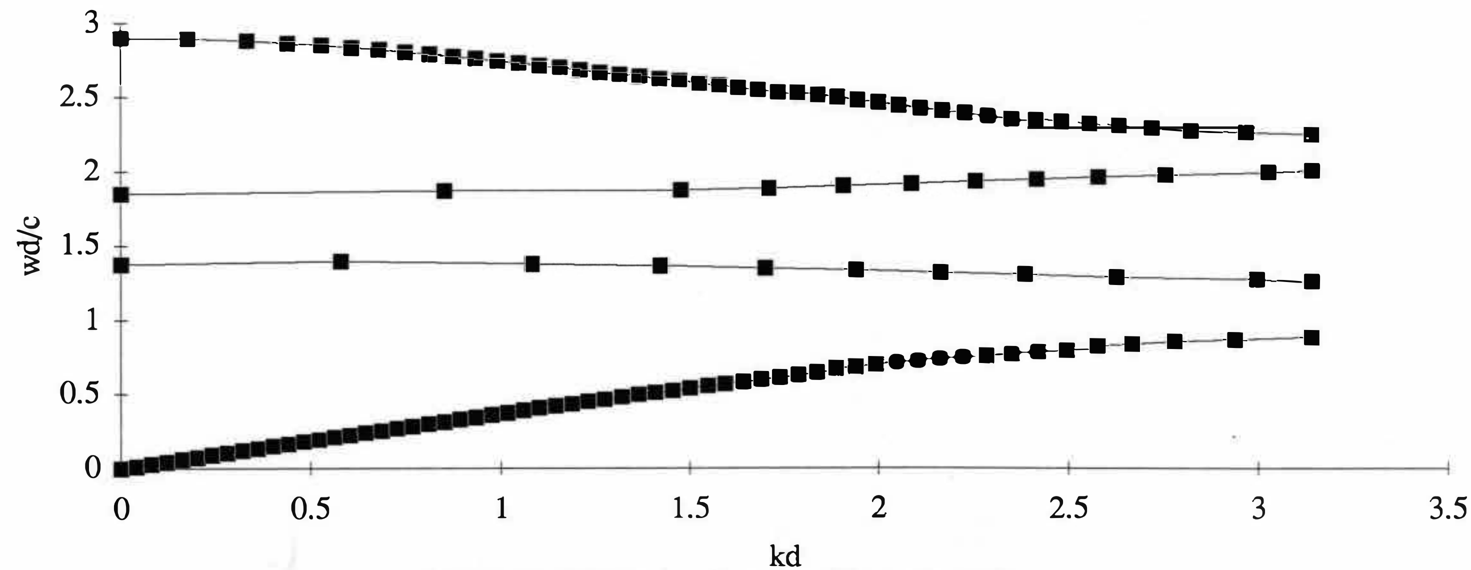

Figure 11. Dispersion Relation When M=10.0.

CHAPTER III

CONCLUSIONS

According to the band theory in solid state physics, electrons in periodic crystals are arranged in energy bands separated by band gaps. Similarly, this band theory can also be employed to explain the propagation of electromagnetic waves in periodic structures in any number of spatial dimensions. Electromagnetic waves with frequencies falling in the photonic band gaps are totally absent from the system. When electromagnetic waves propagate in the dielectric systems with a frequency dependent dielectric function , the band gaps can be changed especially near a dielectric resonance.

In chapter II, we have discussed first the propagation of electromagnetic waves through a single dielectric slab. The transmittance function versus the frequencies of electromagnetic waves was derived theoretically for the case of one single dielectric slab. eq.(1.15) shows the transmittance function. It is calculated by a FORTRAN program. From Figure 3, Figure 4 and Figure 5, we see that, the transmittance is no longer a continuous function of frequency, but instead, there are band gaps in the transmittance curves. The band gaps occur around ω_T the transverse optical phonon frequency in the absence of coupling with photons. The band gaps are presented in different frequency ranges with different M selected.

Then we discussed the propagation of electric waves through a one-dimensional, periodic structure consisting of an array of parallel dielectric slabs with a frequency dependent dielectric function. The dispersion relation was derived theoretically and calculated by a FORTRAN program. eq.(2.18) shows the dispersion relation. From the figures of the dispersion relation (Figure 9, Figure 10 and Figure 11), we see that the band gaps were changed in the dielectric system with a frequency dependent dielectric function from those of the frequency independent system. The changes of band gaps are most pronounced around the transverse optical phonon resonance frequency.

All the photonic band structures of dielectric systems with a frequency dependent dielectric function studied in my theses are one-dimensional system. But most systems in our natural world are two-dimensional or three-dimensional systems. So, the future work in this area can be done in the area of electromagnetic waves propagating in two-dimensional or three-dimensional dielectric structures with a frequency dependent dielectric function.

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