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## Modeling and Simulation with Optimal Gear Ratio for a Forward-Looking, Velocity-Driven, Power-Split Hybrid Electric Vehicle

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MODELING AND SIMULATION WITH OPTIMAL GEAR RATIO FOR A FORWARD-  
LOOKING, VELOCITY-DRIVEN, POWER-SPLIT HYBRID ELECTRIC VEHICLE

by

Sonal Babasaheb Kanap

A thesis submitted to the Graduate College  
in partial fulfillment of the requirements  
for the degree of Master of Science in Engineering  
Mechanical Engineering  
Western Michigan University  
April 2019

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Finally, my deepest thanks to my parents for all the support, encouragement and love they have given me.

Sonal Babasaheb Kanap

# MODELING AND SIMULATION WITH OPTIMAL GEAR RATIO FOR A FORWARD-LOOKING, VELOCITY-DRIVEN, POWER-SPLIT HYBRID ELECTRIC VEHICLE

Sonal Babasaheb Kanap, M.S.E.

Western Michigan University, 2018

Increases in vehicle demand and fossil fuel consumption are major contributors to environmental problems, such as air pollution and climate change. This has led to research on alternative, energy-efficient vehicle technologies. Automobile users are now preferring comfortable vehicles with minimal fuel consumption and with more efficient engines. Hybrid cars are becoming common because of their advantage of running cleaner and with better gas mileage. A hybrid car runs on the power of both an electric motor and a gasoline engine. This mechanism helps cut fuel consumption and conserve energy. An additional advantage is a regenerative braking system that helps recharge the battery, which ultimately reduces load on the engine and hence produce lower emissions. In this thesis, components of hybrid electric vehicles are defined and a computational model of a typical hybrid system is developed. Modeling and simulation of these components is done in the Matlab/Simulink environment. This thesis underscores the HEV model that can be used in future and the importance of optimized powertrain components, especially the planetary gear ratio and its impact on vehicle performance and fuel economy. The planetary gear ratio ensures smooth transmission, propulsion capacity, acceleration, and fuel economy. This thesis investigates the effect of different gear ratios on fuel economy for the US06 and FTP75 drive cycles and proposes a strategy for optimizing gear ratio to maximize fuel economy without compromising vehicle drivability. The study presented in this thesis also emphasizes battery stability and optimal energy management at different gear ratios. The forward-looking, velocity-driven, power-split model developed in this study highlights the importance of gear ratio in engine operation and also ensures that generator does not overrun in the process.

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## CHAPTER 1

### INTRODUCTION

Due to increasing demand for reduced fuel consumption and emissions, researchers are developing new automotive technologies, such as hybrid electric vehicles (HEV), plug-in hybrid electric vehicles (PHEV), electric vehicles (EV), and autonomous vehicles (AV). Stringent emission standards were set due to the influence caused by automobiles to the environment. To protect the environment, US emission standards were enacted in 1970 by the Environmental Protection Agency (EPA) [1]. The EPA regulates the air quality in general and the engine emissions.

Hybrid electric vehicles offer an innovative solution to meet environmental standards. The technology was initially developed by Ferdinand Porsche in 1901[18]. Its modern roots were laid by Toyota motor corporation in 1997 under the first generation Prius, which features the Toyota Hybrid system (THS) and then by the Honda Insight in 1999.

HEV are intended to achieve good fuel economy, high efficiency, and reduced greenhouse gases emission. As the name suggests, the vehicle uses a combination of an internal combustion engine and an electric power source to deliver requested vehicle drive power. HEVs can be categorized as full hybrid or mild hybrid considering the degree of hybridization. Hybridization of an HEV depends upon the ratio of power of the propulsion motor to the power of engine. For example, Toyota Prius is the full hybrid its hybridization being 62.3% and Honda Civic is the mild hybrid its hybridization being 15.9%

HEV typically has limited features while the full HEV has numerous features such as power boost mode, mechanical mode, overdrive mode, electric mode, battery charge mode, etc. Full HEV can run on the combination of batteries and engine or only on batteries or on engine power. All-electric propulsion cannot be provided by turning off the engine with the mild hybrid electric vehicles while it can be provided with full hybrid electric vehicle.

There are three common types of configurations involved in hybrid electric vehicles: series, parallel, and series-parallel transmissions. A schematic of these configurations is shown in Figure 1, which is from [2].

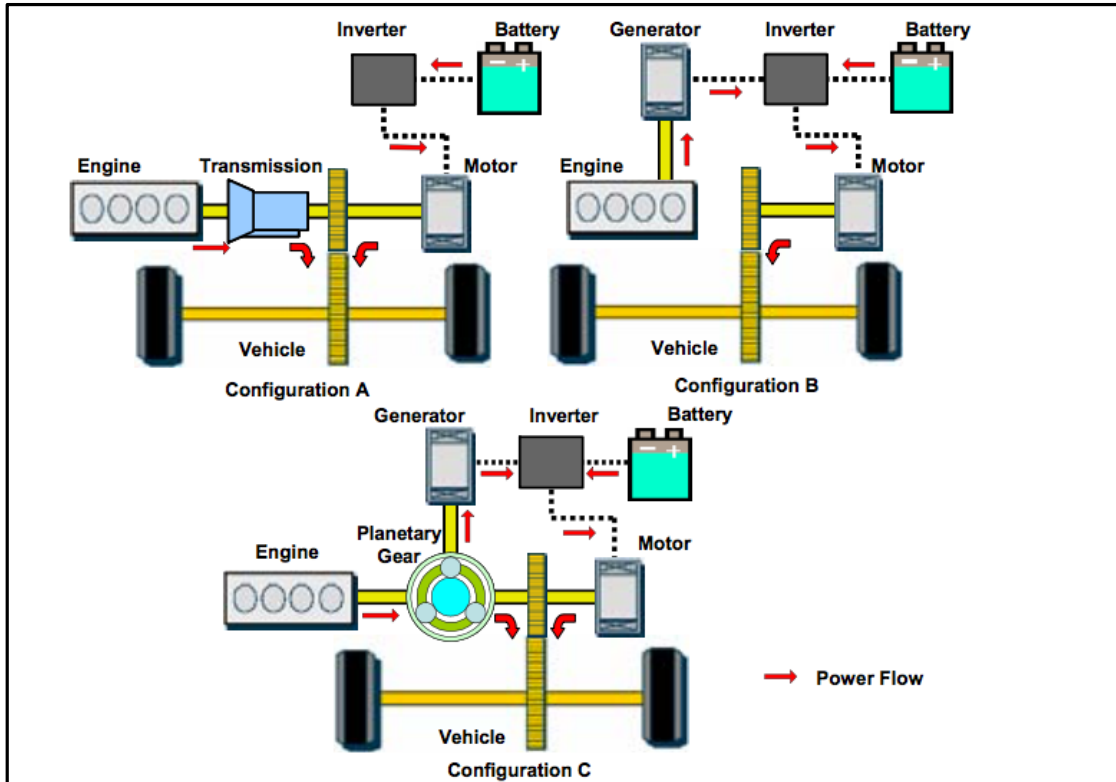


Figure 1. Hybrid Electric Vehicle (HEV) configuration: A. Parallel, B. Series, C. Power-Split [2]

In a series hybrid powertrain, electric power is connected in series with the mechanical input, which is comprised of the combustion engine and the generator. This mechanical configuration is not directly linked to the wheels. The generator does the job of transferring mechanical power to electric power. The electric motor does the job of converting electric power to mechanical power. This transmission always operates at optimal engine efficiency as the engine operation does not depend on the road conditions and vehicle speed.

In a parallel hybrid powertrain, there is an additional power source, a battery, that is placed parallel to the mechanical power source. This mechanical power source is connected to the wheels.

The main advantage of the parallel hybrid transmission is its high efficiency. The disadvantage with this transmission is its inability to operate at optimal engine efficiency invariably as speed of engine is directly connected to the speed of vehicle and road condition.

In a series-parallel hybrid powertrain, the power is drawn from two branches: a mechanical and an electric branch. The mechanical branch transfers power generated by combustion engine to the wheels. The electric branch transfers power to the wheels via the generator and motor. There is a planetary gear set that sets the gear ratio. Since the split-type hybrid transmission is a combination of series- and parallel-type hybrids, it can operate at optimal engine efficiency.

The powertrain configuration considered in this study is a split-type, series-parallel configuration. Power split hybrid electric vehicles are comprised of an internal combustion engine (ICE), planetary gear sets, motor, generator, battery, electronic control unit (ECU), mechanical brakes, and a transmission. In this thesis, the components are modeled and simulated in the MATLAB/Simulink environment. The overall goal of this effort is to develop a physics-based model for HEV fuel-efficiency research. This thesis focuses on reducing fuel consumption as well as increasing regeneration of energy due to braking.

## Literature Review

Series-parallel HEV can either be modeled based on velocity-driven, power-driven or torque-driven models. Computational modeling and simulation tools, as in this case MATLAB/Simulink, can be used to evaluate efficient results and to reduce the time required for modeling and simulation. A steady state dynamic model based on power is presented by Huei Peng, H. [2] Toyota Prius HEV model was developed and its overall behavior was controlled using rule based control strategy. The research on a parallel HEV model is presented Shen, T. et al. [3] strategy for improving the fuel economy is proposed. A torque-based dynamic model is presented by T. Purnout [4]. The importance for steady-state mathematical model for power split device is presented by Rizoulis et al. [5]. A dynamic forward looking, velocity driven model was developed and simulated based on work presented by Dr. Can [6]. This work provides foundation for investigating optimal gear ratio at different driving cycles. A backward looking velocity driven

model to investigate the importance of optimal gear ratio and its impact on fuel economy and vehicle performance was presented by Li, Yanhe [7].

Proper modelling of the planetary gear set is important since it is the heart of a power-split HEV [8]. The constants used in equations of planetary gear set remains consistent from [6] for 2.6 gear ratio and for all other gear ratio the constants used are based on [7] and [19]. The planetary gear set ratio ensures smooth transmission and better fuel economy. It affects engine and generator operation and hence careful consideration needs to be taken while selecting the optimal gear ratio for the planetary model. Duoba et al. [9] state the effect of gear ratio on vehicle operation and illustrate its importance to minimize fuel consumption under various modes of operation for HEV.

Components of the model developed in this thesis are based on several references. In a previous study related to this project [10], a novel ratio scheduling technique for a CVT transmission was developed using a backward-looking cycle-driven model. The work presents optimization techniques for generating an Optimum Operating Line (OOL), which is used in engine lookup tables in the model presented here [10]. The battery, which acts as secondary power source, ensures smooth acceleration in EV mode and stores energy during regenerative braking. Data presented in [11] is utilized for battery modelling and control. Relevant literature for engine operation without overrunning the generator is presented in [12].

Control strategies can be achieved with both global and local optimization techniques. Global optimization strategies can be developed with dynamic programming [13] or Pontryagin's Minimum Principle [14] and ECMS is a very popular local optimization technique [15]. Rule-based control strategy developed in this work is based on [6]. The control logic developed in this study such motor control logic to decide either to accelerate or decelerate, engine control logic to get engine torque and engine RPM and generator on/off is based on [20]. This forms the basis for optimal energy management and helps investigate optimal gear ratio.

### Assumptions

For this study operating conditions were assumed to be in steady-state. It was also assumed that engine operates inefficiently at low speeds and data for engine speed vs. vehicle speed remains

consistent [6] and engine acceleration and deceleration lookup tables remains consistent from [12]. Benchmark data for development of the HEV model was obtained with the use of study presented in [6]. Finding of optimal gear ratio was done with the use of benchmark data obtained in [7] and [19]. Control logic developed for operating engine, motor and generator is based on [20].

## CHAPTER 2

### FORWARD-LOOKING DYNAMIC MODEL OF HYBRID ELECTRIC VEHICLE

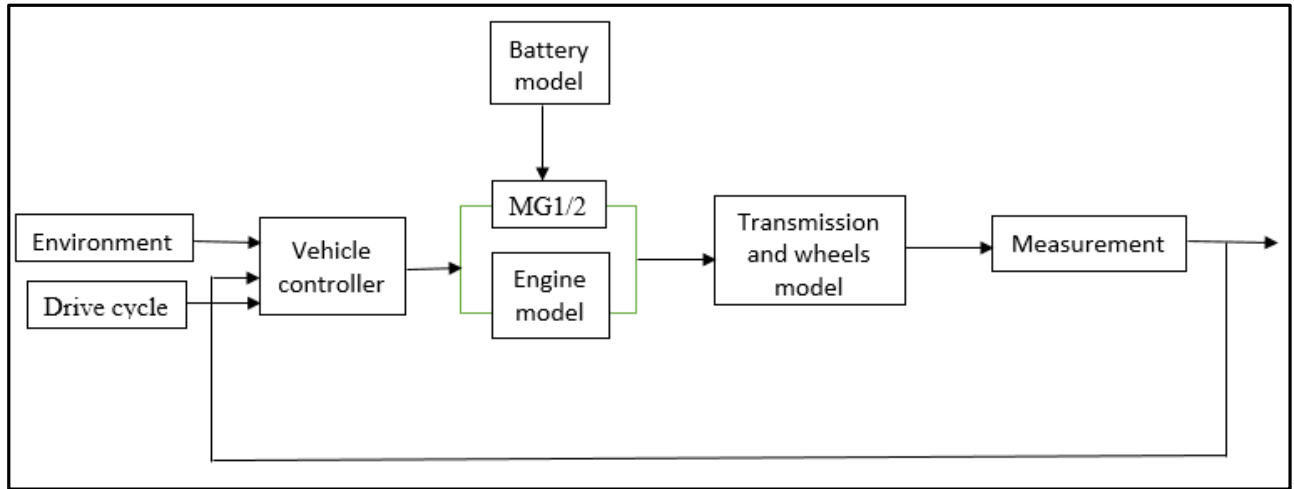


Figure 2. Overview of hybrid electric vehicle Simulink model

Vehicles models can be configured depending upon their direction of calculations as Backward looking models and Forward looking models. Actual speed of vehicle is controlled to match the target speed during simulation in forward looking model. When using forward looking models the component losses and constraints are considered by the controller and it makes decision quickly for the whole system. Control logic in backward looking model does not consider the systems constraints as feasible control options are allowed to the controller. An overview of the power-split hybrid electric vehicle Simulink model is shown in Figure 2. This forward-looking, velocity-driven HEV model has an input of a standard emission test cycle, such as the US06 and FTP-75 drive cycles, which specify a vehicle speed profile over a fixed time interval. The vehicle controller model is the brain of the hybrid system that controls the vehicle based on operating conditions. The battery model, motor and generator model, and engine model are used for generation of required or demanded power. The transmission and wheels model is used to find the

actual velocity of the vehicle. The measurement block is used for visualization of results after simulation.

### Planetary Gear

The power split device includes the planetary gear set, which is connected to the generator, drive shaft, and the engine model as can be seen in Figure 3 [17].

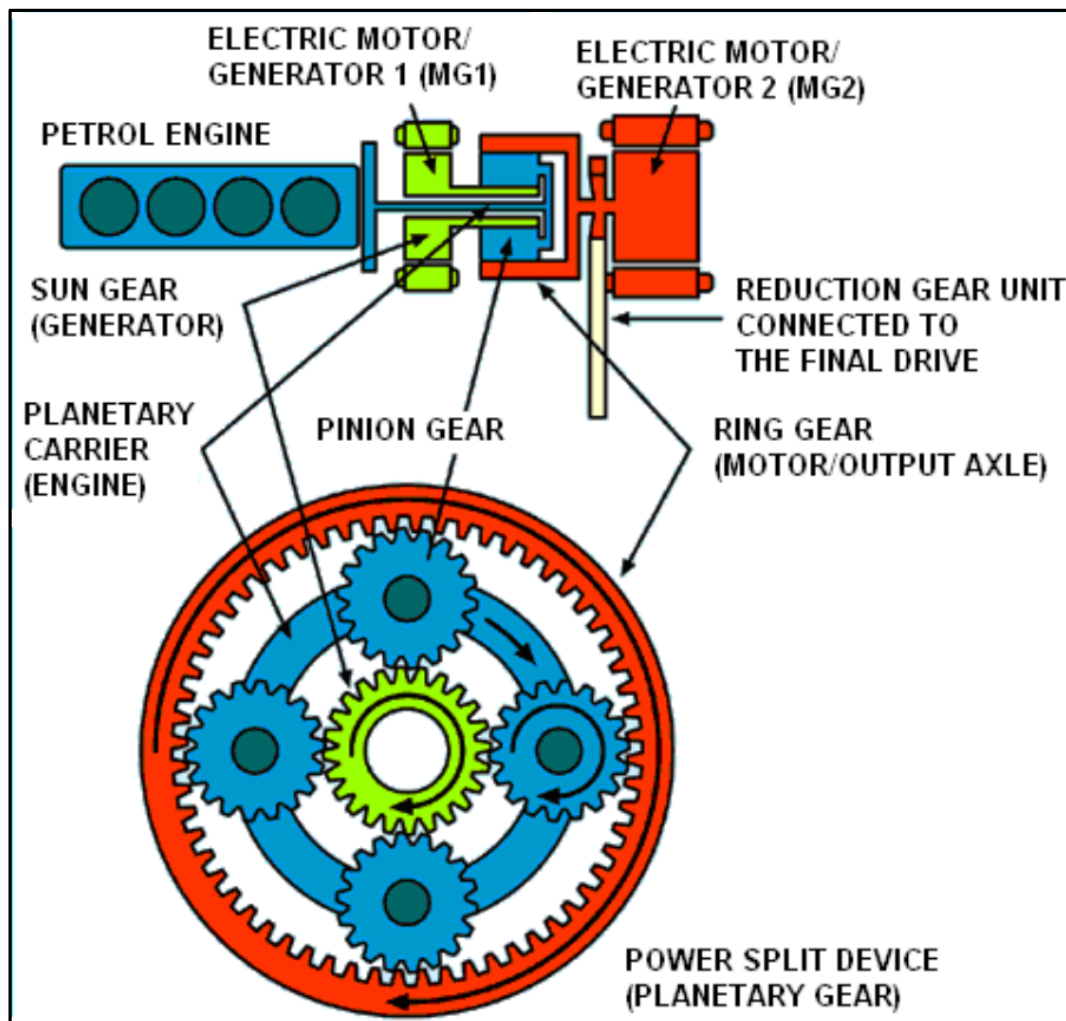


Figure 3: Planetary Gear set model of Toyota Prius with its connections [17]



There are three nodes of the planetary gear: the sun gear connected to generator, the carrier gear connected to the engine, and the ring gear connected to the drive shaft. The power coming from engine is divided by the power split device between the drive shaft and the generator.

An electronic control unit is used for controlling the power split device (PSD) and it acts as electronically controlled continuous variable transmission. It can allow the vehicle to use power from not only the engine but also from the motor and generator rotating at varying and dissimilar speeds. Generated power from the ICE can be carried to the vehicle via an electrical or mechanical path. Engine power transferred via the mechanical path is transferred from the carrier gear to the ring gear, which is attached to the vehicle's drive shaft. Power can also be carried to the sun gear and transferred to the generator, where the generated electrical power is used by the electric motor or is stored in the battery.

One of the most critical parameter of power split device is the gear ratio. The basic gear ratio equation of the planetary gear set is given by Eq. 1:

$$K = \frac{\text{Number of teeth of Ring Gear}}{\text{Number of Teeth of Sun Gear}} \quad [\text{Eq. 1}]$$

This ratio mentioned in Eq. 1 also sets the planet gears and carrier. Depending upon vehicle performance requirements, the planetary gear set is designed with the best gear ratio for various operating conditions, such as acceleration, maximum speed, and deceleration. Five gear ratios are considered further in this analysis based on literature review: 2.6, 2.75, 2.9, 3 and 3.4 [7]. For gear ratio 2.6, the number of teeth used by the ring gear is 78, planet gear is 23 and sun gear is 30. The number of teeth for the ring gear, planet gear, and sun gear remain consistent from the study presented in [4] based on the gear condition assembly.

The gear set can achieve an unlimited gear ratio by adjusting the speed of different components. To get better efficiency, the main aim of this thesis is to obtain an optimized gear ratio. The E-CVT overall planetary gear set model is as shown in Figure 4.

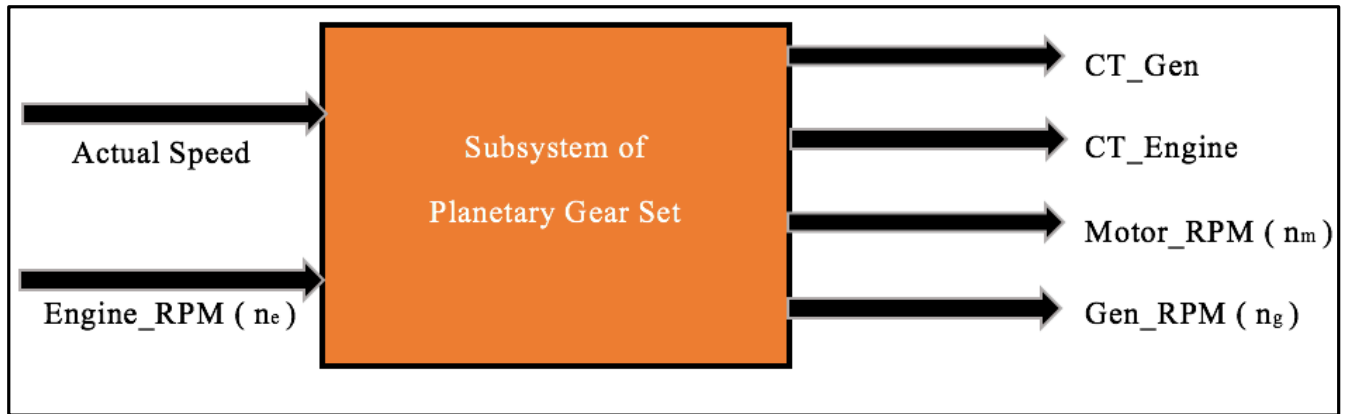


Figure 4. Overall planetary gear set model

Input to the subsystem in Figure 4 is the actual speed and engine RPM ( $n_e$ ). Motor RPM depends upon actual vehicle speed (Eq. 2) and generator RPM is evaluated from the planetary gear equation (Eq. 3).

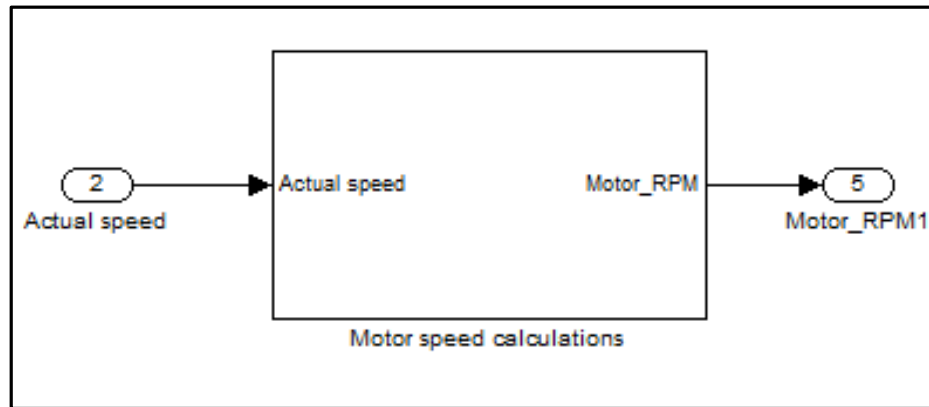


Figure 5. Simulink model of planetary gear set subsystem to find the Motor RPM

$$\text{Motor RPM} = R_{\text{wheel}} * \text{Actual speed} * \frac{1000}{2\pi*60} * \text{FDR} \quad [\text{Eq.2}]$$

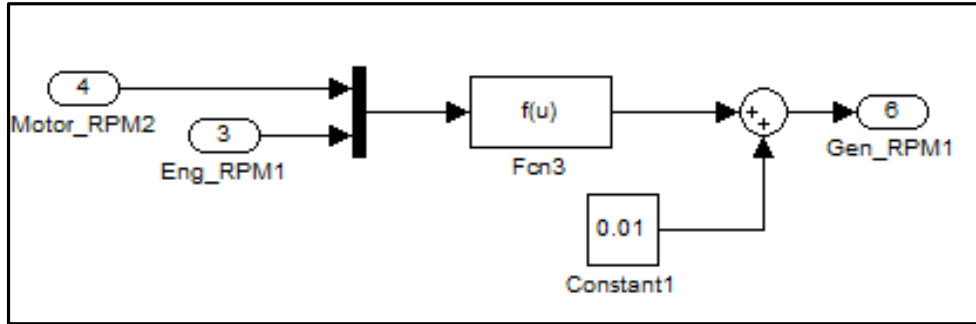


Figure 6. Simulink model of planetary gear set subsystem to find the Generator RPM

The equation used for planet gear is as follows:

$$\text{Eng\_RPM} = \frac{(\text{Gen\_RPM1} + (2.6 * \text{Motor\_RPM}))}{3.6} \quad [\text{Eq. 3}]$$

$$\text{Therefore, Fcn3} = 3.6 * \text{Eng\_RPM1} - 2.6 * \text{Motor\_RPM2} \quad [\text{Eq. 4}]$$

In Figure 6. 0.01 is used to avoid zero ratio coming out of Fcn3.

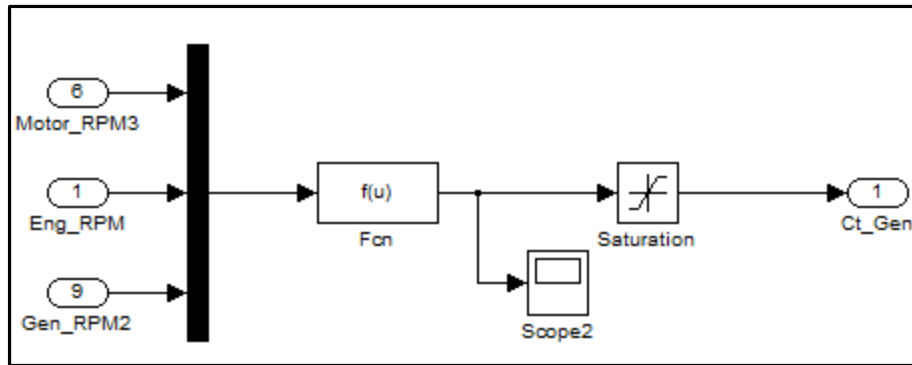


Figure 7. Simulink model of planetary gear set subsystem to find torque coefficient of generator

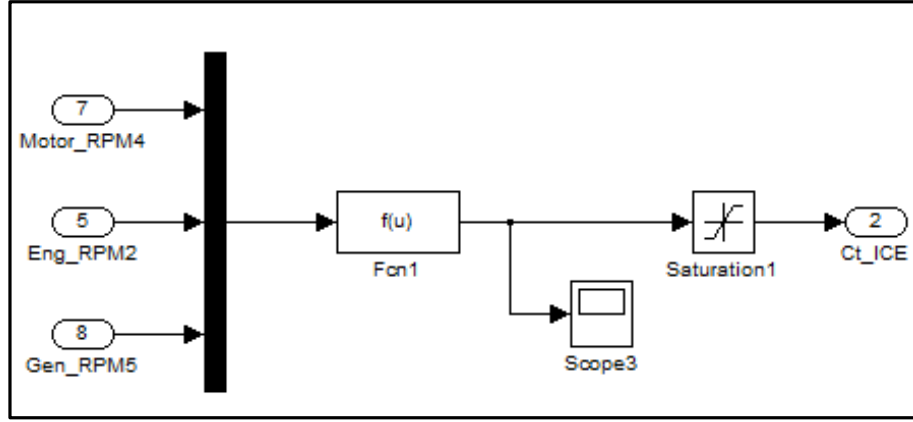


Figure 8. Simulink model of planetary gear set subsystem to find torque coefficient of ICE

Equations of CT (coefficient of torque) of generator and engine are as:

$$CT_{Gen} = \frac{(\text{speed of engine} \times 9.36) - (\text{speed of motor} \times 6.76)}{(\text{speed of engine} \times 3.6) - (\text{speed of motor})} \quad [\text{Eq. 5}]$$

$$CT_{ICE} = \frac{(\text{speed of engine} \times 3.6)}{0.722 + (\text{speed of generator} \times 2.6)} \quad [\text{Eq. 6}]$$

All numerical constants for gear ratio 2.6 in equation from 3-6 are adapted from Ref. [6]. The numerical constants for all other gear ratios were calculated using study presented in Ref. [19] and [7]. The constants in Eq. 3-6 are calculated as:

$$\frac{N_r}{N_r + N_s} = 0.722, \frac{N_s + N_r}{N_s} = 3.6, \frac{N_r}{N_s} = 2.6, \frac{(N_r + N_s) \times N_r}{N_s^2} = 9.36 \text{ and } \frac{N_r^2}{N_s^2} = 6.76$$

Where,  $N_r$  is the number of teeth of ring gear and  $N_s$  is the number of teeth of sun gear.

## Motor Model

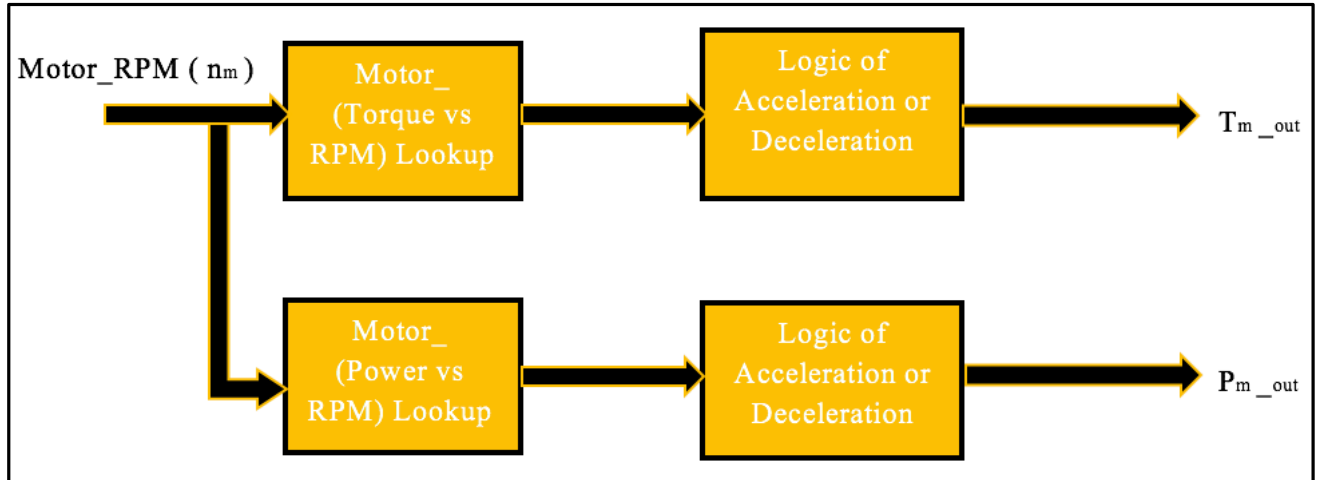


Figure 9. Overview of motor model

A brushless DC motor is modeled in Simulink. This electrical machine can be used for regenerative braking or as a driving electrical motor; it can accelerate the vehicle up to about 40km/h. Characteristics of torque vs. RPM and power vs. RPM based on the 35KW Toyota Prius motor [6] are determined by a lookup table. Motor speed, which is calculated in planetary gear set, is given as an input to the lookup tables. This motor speed is used to calculate the Torque and Power values. Control logic in the control subsystem block checks the condition of acceleration or deceleration i.e., regenerative braking depending upon pedal position of driver. The control logic is described in [20].

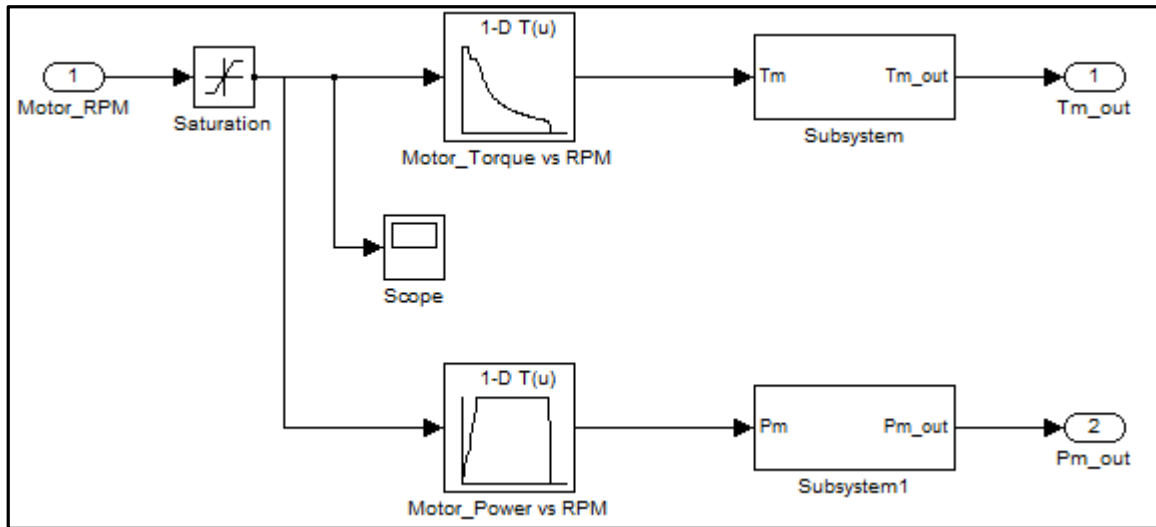


Figure 10. Simulink model of motor

### Generator Model

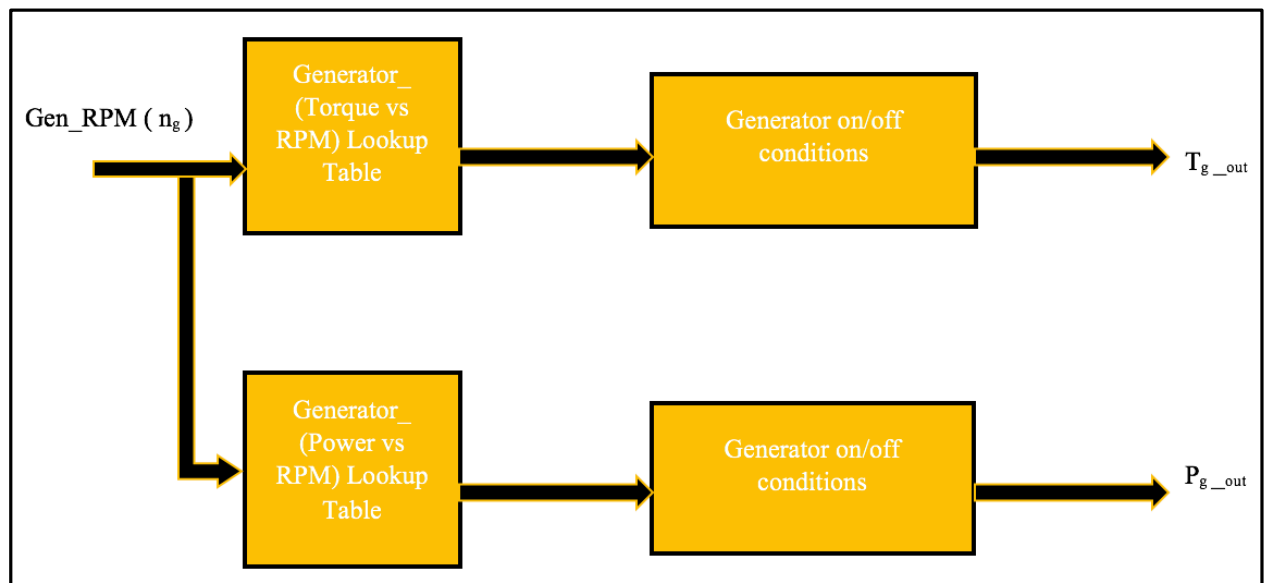


Figure 11. Overview of generator model

Figure 12 illustrates the Simulink model of the generator. It is used for gear ratio adjustment as described by Eq. 3. The generator is connected to the PSD and driven by the excess engine

power. During full throttle conditions, the generator runs with full power. Characteristics of torque vs. RPM and Power vs. RPM are used in Lookup table of generator model. These maps are based on data for the 15KW Toyota Prius system [6]. Generator speed is used as an input to these lookup tables and this calculation of generator speed is performed in planetary gear set model. The generator model is also used by the vehicle to generate the desired torque for starting up. The generator's on/off conditions is described in [20].

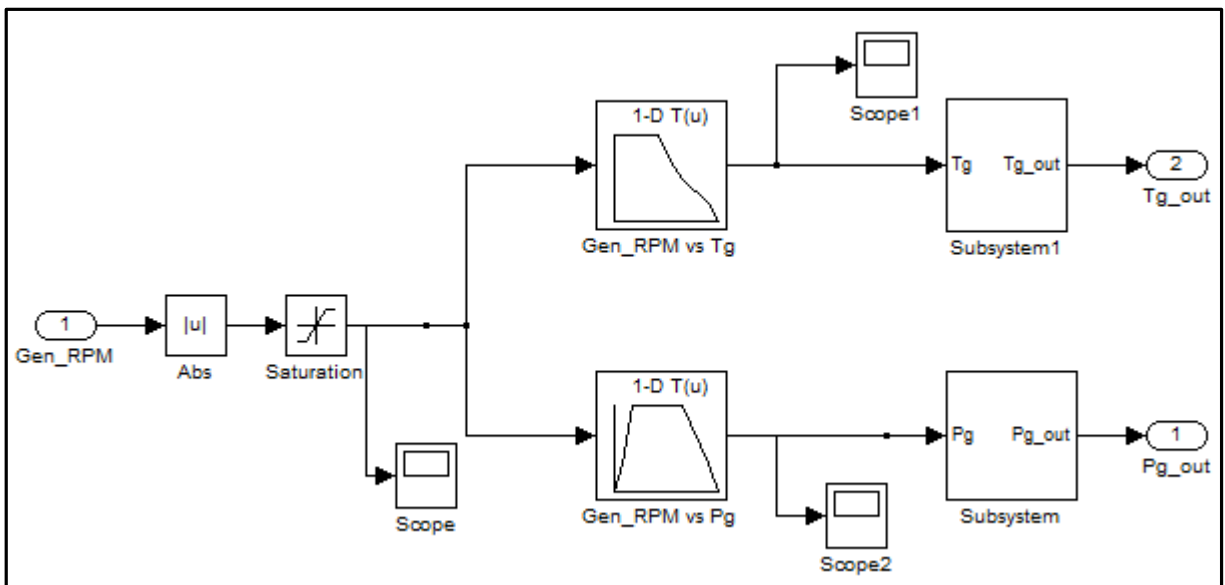


Figure 12. Simulink model of generator

## Engine Model

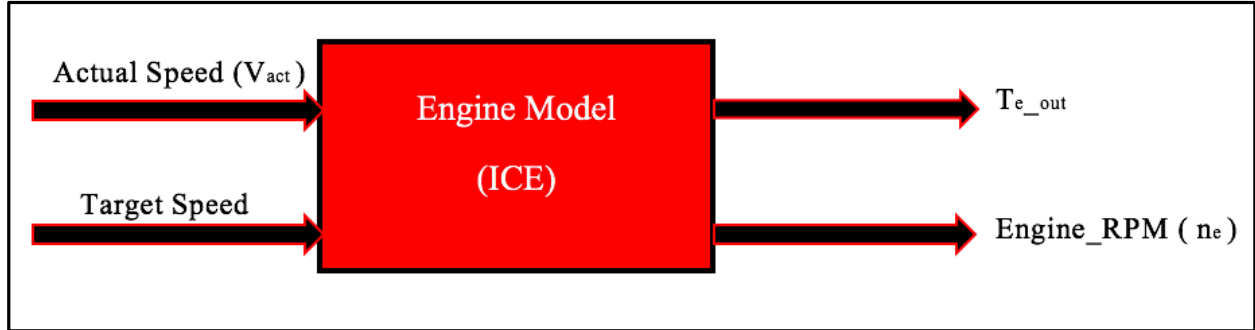


Figure 13. Overview of engine model

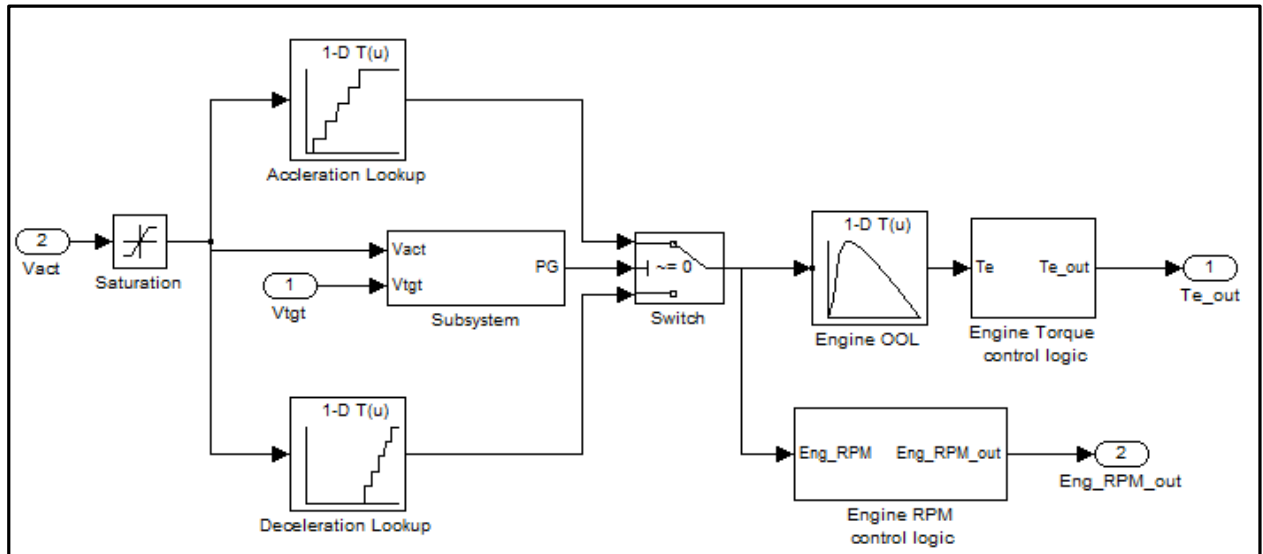


Figure 14. Simulink model of engine

Figure 14. shows the Simulink model of engine. Performance of the engine is modeled using lookup tables, with data from [12]. Input to the engine model is the actual speed and target speed of the vehicle. Depending upon the accelerating and decelerating conditions, the maximum and minimum speed values of engine are taken into consideration. The engine control logic [20] determines the operating condition and decides whether the engine is on or off. Figure 15 shows the OOL of engine from 1-D lookup table of ICE RPM vs ICE torque. OOL is the optimum



operating line that is indicated by red line as shown in figure 15. OOL is the line of minimum fuel consumption obtained by joining operating points for different power demand in engine's torque and speed plane.

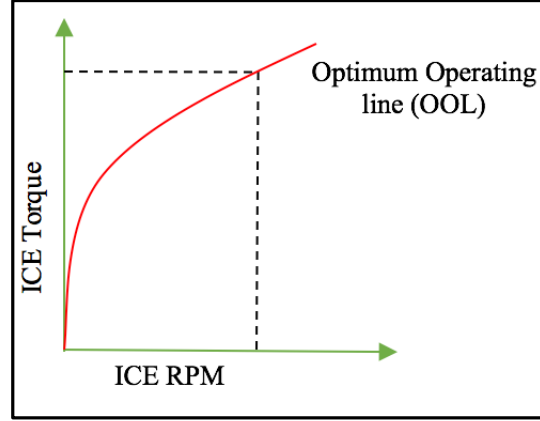


Figure 15: ICE RPM vs ICE Torque 1-D lookup table

### Transmission Model

Figure 16 show the transmission block modeled in Simulink. For calculating the vehicle's speed, all torques from the machines and all the loads are taken into transmission model. Torque of the engine, generator, and motor is multiplied by the final drive ratio and transmission efficiency. This torque value obtained is subtracted from the total load torque value acting on the wheels and then integrated. Using wheels speed it is possible to calculate actual speed of the vehicle.

$$F_{cn2} = \frac{\int \left( (T_{e\_out1} - T_{g\_out1} + T_{m\_out1}) * FDR * \text{Transmission efficiency} \right) - T_{w1} dt}{M_t * r_w^2} \quad [\text{Eq. 7}]$$

$$V_{act\_spd} = \int \left( \left( (T_{e\_out1} - T_{g\_out1} + T_{m\_out1}) * FDR * \text{Transmission effi} \right) - T_{w1} \right) dt * r_w * 3.6 \quad [\text{Eq. 8}]$$

Where,  $T_{e\_out1}$ ,  $T_{g\_out1}$ ,  $T_{m\_out1}$  and  $T_{w\_out1}$  are the torque of engine, generator, motor and wheel,  $M_t$  is the total weight of vehicle and  $r_w$  is the wheel radius, FDR is final drive ratio.

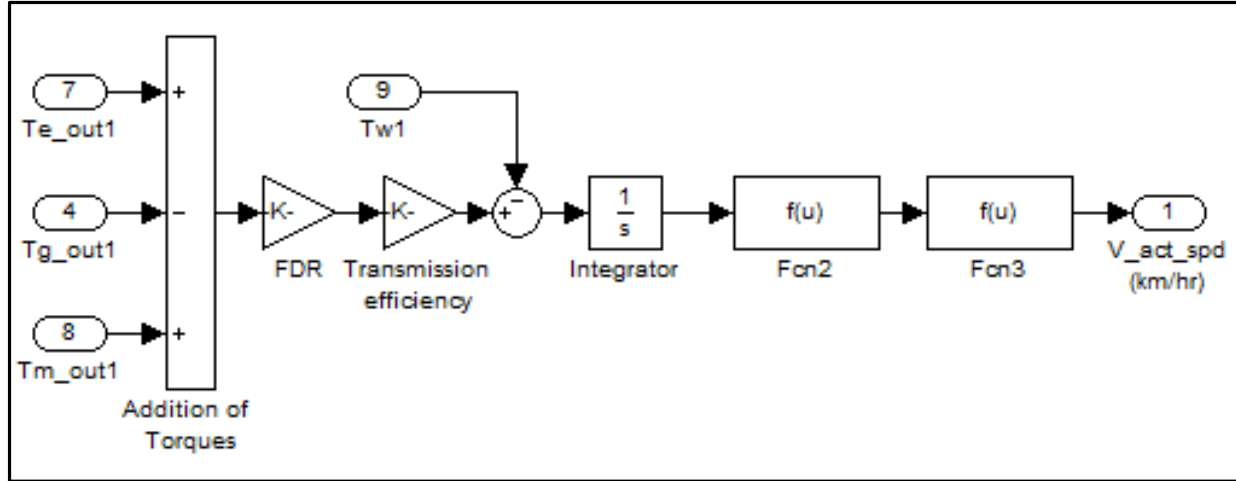


Figure 16. Simulink model of transmission and wheel

### Vehicle Dynamics

Forces acting on the vehicle's wheels are included in the vehicle dynamics model. The forces acting on vehicle are aerodynamic drag, rolling resistance, gradient resistance, tractive force, and the brake force. When the vehicle passes through air it experiences fluidic resistance, which is nothing but the aerodynamic drag. Rolling resistance acts on the vehicle due to the wheel rotation, and gradient resistance acts on the vehicle due to the slope of the road. Brake force is the required force for the vehicle to decelerate; traction force is the powertrain force used for driving the vehicle in the forward direction.

The Toyota Prius body is taken into consideration for modeling the dynamics of the vehicle. Equations 9-13 are used for modeling the vehicle in Simulink.

$$\text{Aerodynamic Resistance, } F_{\text{drag}} = 0.5\rho C_d A_f V^2 \quad [\text{Eq. 9}]$$

$$\text{Rolling Resistance, } F_{\text{roll}} = C_t mg \cos \theta \quad [\text{Eq. 10}]$$

$$\text{Gradient Resistance, } F_{\text{grad}} = mg \sin \theta \quad [\text{Eq. 11}]$$

$$\text{Accelerating force, } F_a = ma \quad [\text{Eq. 12}]$$

Vehicles linear dynamic equation can be represented as:

$$F_T = (ma) + (0.5\rho C_d A_f V^2) + (C_t mg \cos \theta) + (mg \sin \theta) \quad [\text{Eq. 13}]$$

where  $\rho$  is the air density,  $C_d$  is the coefficient of drag,  $A_f$  is the frontal area of the vehicle,  $V$  is the vehicle speed,  $C_t$  is the wheel rolling resistance,  $m$  is the vehicle's total mass,  $g$  is the gravity,  $\theta$  is the slope of road and  $a$  is the acceleration.

Table 1. Vehicle dynamics constants

Abbreviation	Definition	Value (Units)
$A$	Vehicle frontal area	1.746 m <sup>2</sup>
$g$	Gravitational constant	9.81 m/s <sup>2</sup>
$\rho$	Air density	1.17 kg/m <sup>3</sup>
$C_d$	Drag coefficient	0.3
$m$	Vehicle's total mass	1300 Kg
$V_{\text{act}}$	Actual vehicle's speed	m/s
$\theta$	Slope of the road	radians
$C_t$	Wheel rolling resistance	0.01

Using the vehicle's speed and road slope values as input, the above forces are calculated individually.

The total load torque and wheel revolution speed are then calculated. Required brake power is applied in the mechanical brake block [20].

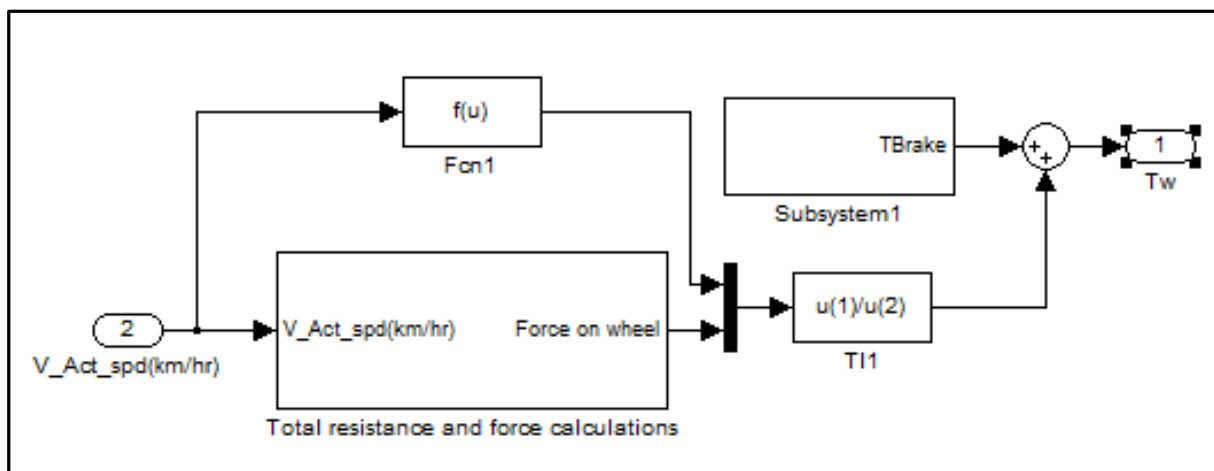


Figure 17. Simulink model of vehicle dynamics

### Battery Model

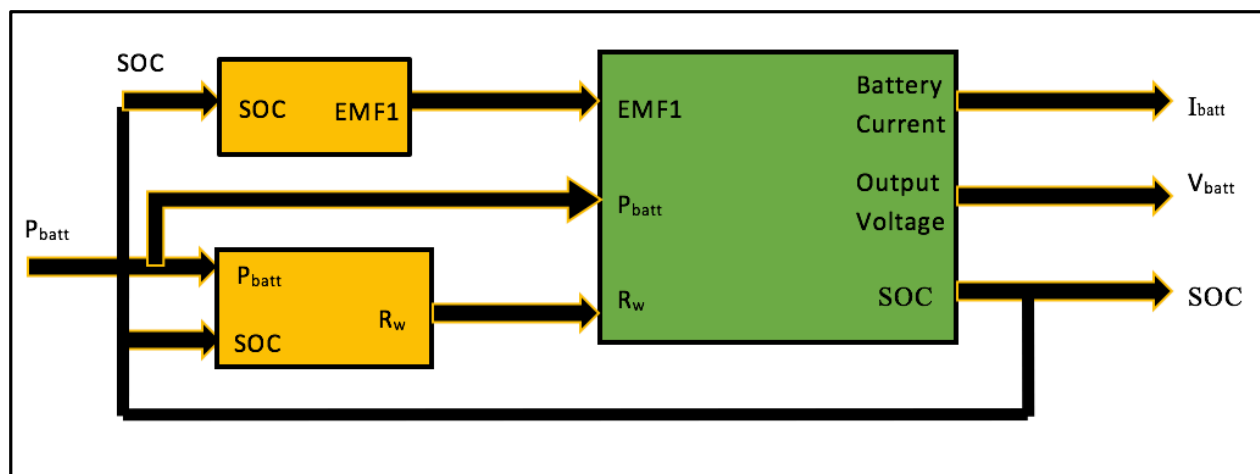


Figure 18. Block diagram of the battery model subsystem

The battery is used as the secondary power source in a Hybrid Electric Vehicle and thus its accurate modeling is very important. A 480V nickel metal hydride battery is modeled in Simulink as seen in Figure 18. The input to this model is the state of charge (SOC) of the battery and the power of the battery ( $P_{batt}$ ), which is calculated by subtracting the power of the generator from the power of the motor. A 1D-Lookup table of SOC vs. EMF is generated to find the EMF of the battery. The data in this table is based on [11]. Similarly, to determine the  $R_w$  that is the internal resistance of the battery, 1D-Lookup tables of SOC vs.  $R_{discharge}$  and SOC vs.  $R_{charge}$  are generated [11]. Output battery current and output voltage of the battery are then calculated using the above calculated values as:

$$\text{Output voltage, } U = \text{EMF1} - I R_w \quad [\text{Eq. 14}]$$

$$\text{Current, } I = \frac{(\text{EMF1} - \sqrt{(\text{EMF1}^2 - 4 P_{batt} R_w)})}{2R_w} \quad [\text{Eq. 15}]$$

Then finally the SOC of battery is determined as shown in Figure 19.

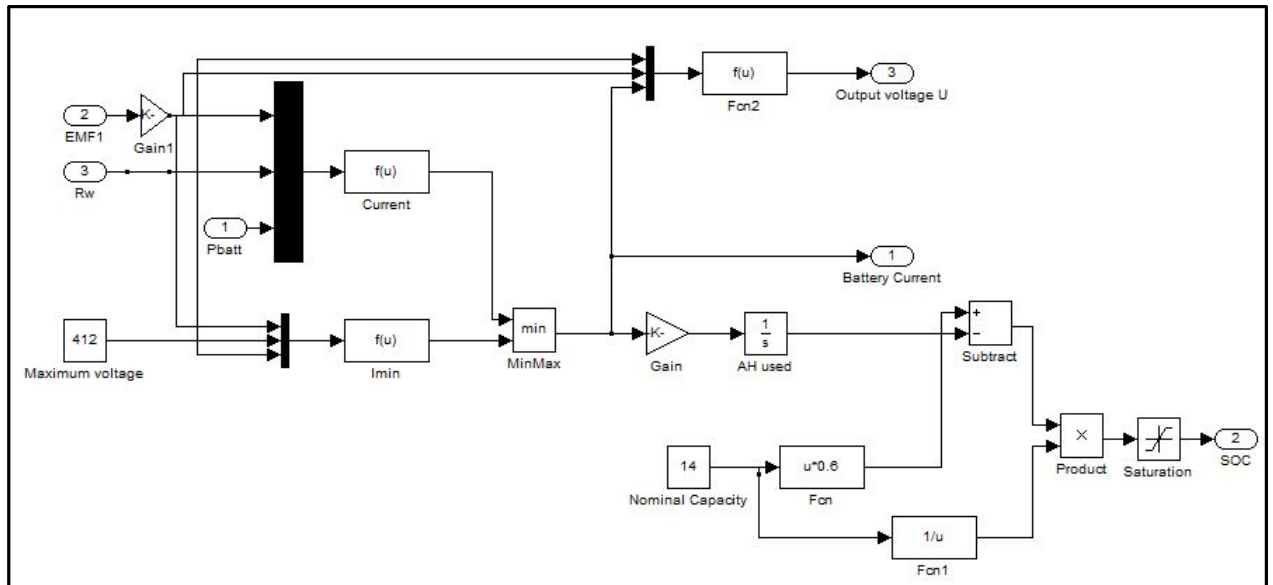


Figure 19: Simulink model of battery

## CHAPTER 3

### CONTROL STRATEGY

The hybrid controller is the brain of the hybrid electric vehicle that controls and manipulates requested data and also manages operating characteristics. The engine speed, generator speed, and electric motor speed are the inputs to the controller that are feedback from the planetary gear set. From the battery model, the battery power and SOC are provided as feedback to the controller.

For efficient energy management of the battery and for better driving results, control logic is developed. The control strategy used is a rule-based strategy that makes decision based on the mode of operation of the vehicle. The vehicle driving modes include vehicle startup, vehicle sudden acceleration, low speed of the vehicle, normal working conditions, battery recharge at rest, and regenerative braking. To avoid under charge and over charge of the battery, its SOC is kept near 50%. The vehicle runs in silent mode that is on electric mode when its speed level is less than 40 km/h; the engine behaves inefficiently below this speed during heavy traffic and also while starting and stopping of the vehicle. Vehicle actual speed and SOC of battery are the two operating parameters for the control algorithm development.

Fig. 20 shows the SOC level, which is from Ref. [6]. The SOC of the battery for proper energy management is separated into 5 different levels as:

- When SOC of battery is less than 10%, all electric assist is canceled as the battery is undercharged. In this mode, the battery is charged until it increases the SOC level.
- When SOC is between 10% and 40%, it runs the vehicle only at its most efficient high-speed points, as ICE is inefficient at low speeds. It cancels the silent mode operation and tries to charge the battery by taking excess power from the generator.
- When SOC is between 40% and 60%, the SOC of the battery is in its adequate level. In this case, the engine runs at its optimum point and the vehicle runs silently when

speed level is below 40km/h. Extra generated power is stored in the battery to charge it and also to provide electric assist at full throttle conditions.

- When SOC is between 60% and 90%, energy in battery is only stored via regenerative braking, and the excess power from the engine is used by the motor with the help of generator.
- When SOC of the battery is between 90% and 100%, the vehicle runs as usual without any energy generation as the battery is overcharged.

The vehicle control logic that implements these energy management conditions is described in greater detail in [20].

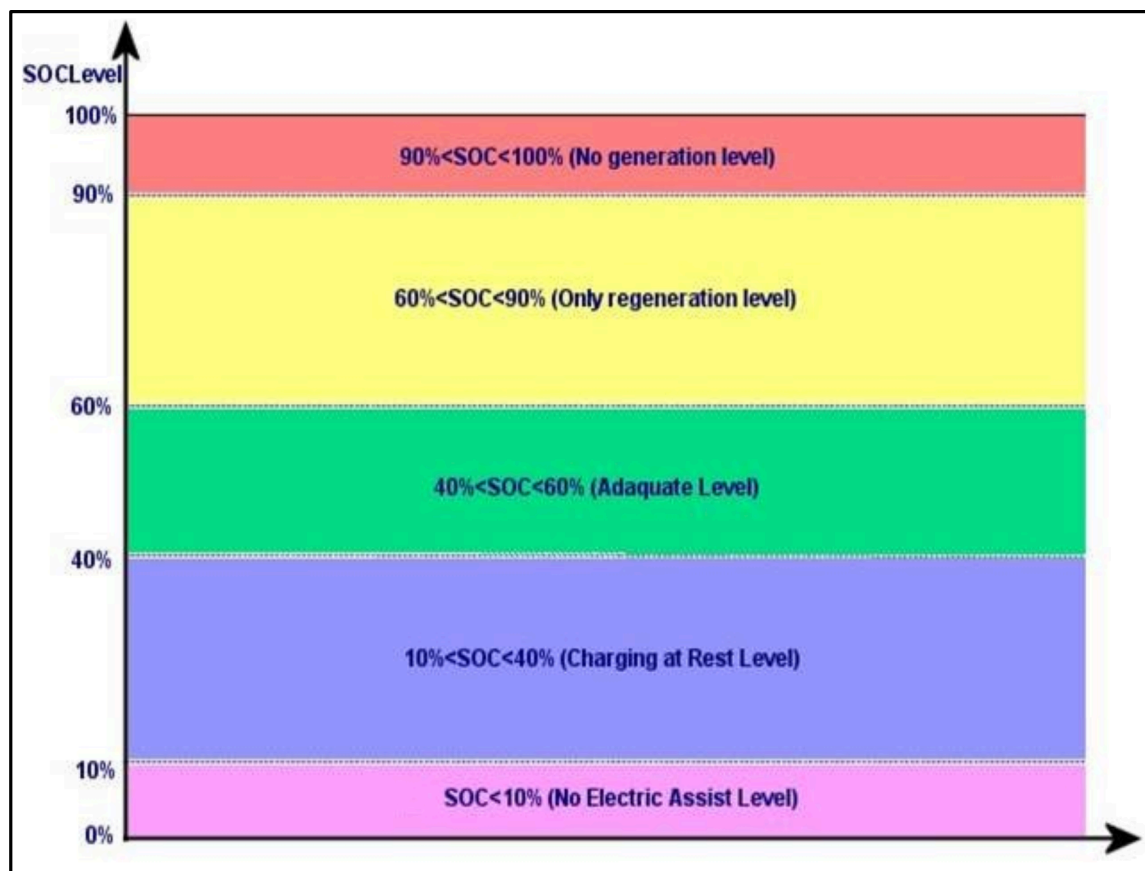


Figure 20. State of charge levels [6]

## CHAPTER 4

### SIMULATION

The power split Hybrid Electric Vehicle model was built on MATLAB R11 using Simulink. This model is run using US06 and FTP 75 driving cycles. Goals of these simulations are to test the model's performance at different gear ratios and determine which gear ratios provide the best overall efficiency for both drive cycles. Initially the SOC of battery is taken as 60% to maintain battery in its adequate level.

#### US06 Driving Cycle

US06 was developed as a representation of driving behavior during startup, rapid speed fluctuations, high speed, and high acceleration. The driving cycle travels a distance of about 12.6 km or 7.83 miles. Figure 21 represents the vehicle speed used for the US06 test [16]. US06 cycle is divided into three phases that is into bags for convenience, as shown in Table 2.

Table 2. Phases of US06 cycle

Bag#	Time in seconds
Bag 1	1 to 120 sec
Bag 2	120 to 370 sec
Bag 3	370 to 570 sec



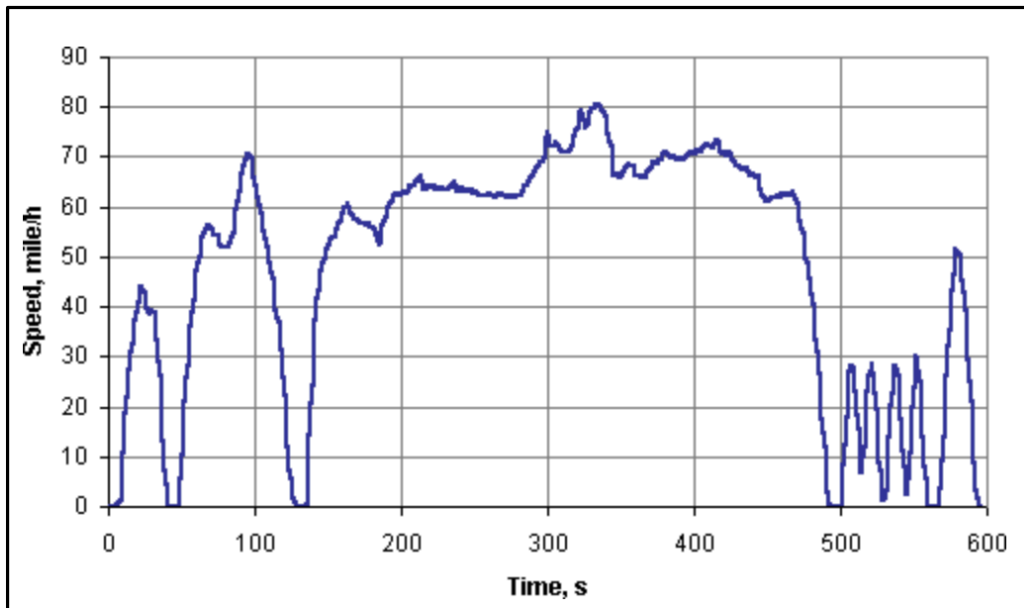


Figure 21. US06 drive cycle

Table 3: Fuel economy improvement compared with 2.6 gear ratio for US06 cycle

Gear Ratio	Fuel Economy improvement
2.75	- 4.57 %
2.9	- 4.74 %
3	- 6.045 %
3.4	- 3.73 %

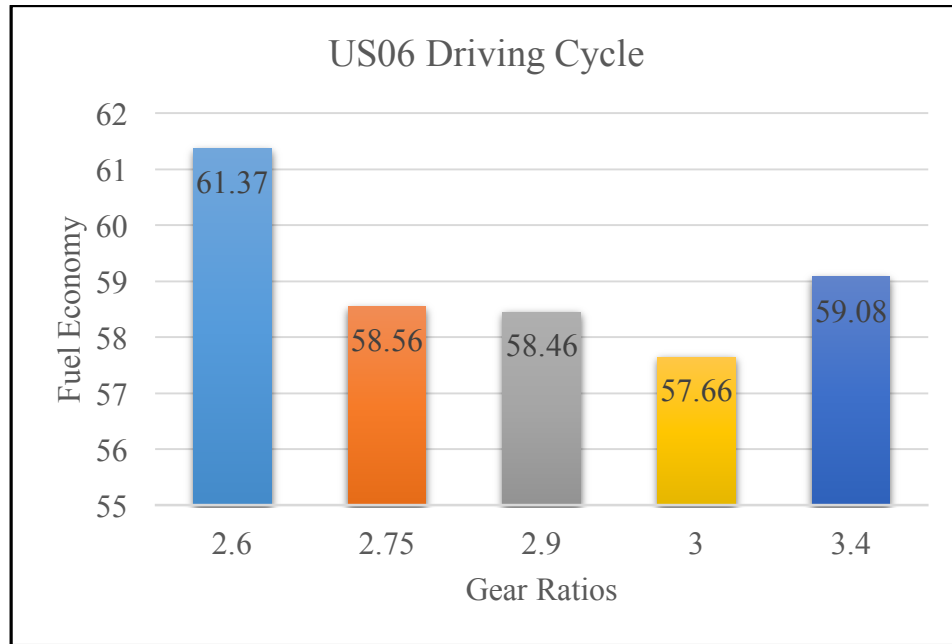


Figure 22: Fuel economy (mpg) for US06 driving cycle for 5 different PSD gear ratios

The US06 cycle was run with 5 different gear ratios: 2.6, 2.75, 2.9, 3, 3.4. The results of these simulations are shown in Table 3 and Figure 22. It is evident from Figure 22 that the best fuel economy is achieved for gear ratio 2.6. For gear ratio 3 the fuel economy obtained is less compared to all other ratios.

Velocity profile, torque results, and battery results were studied at different gear ratios and comparison was done to obtain the optimal gear ratio. Differences in engine torque results were highlighted for minimum and maximum gear ratio, 2.6 and 3.4, and these results demonstrate that the engine operates over more time steps at 3.4 gear ratio because of sudden acceleration and deceleration. Thus, it hampers fuel economy results. Figure 23 and 24 represents these engine torque results for a 200 second time slot for the Bag 3 cycle. The results for 2.6 gear ratio depict that energy recovered by the regenerative braking is 1.671 KW-hr which is greater than all other gear ratios. 1.22 KW-hr of mechanical energy consumed at 2.6 gear ratio is less compared to 2.75, 2.9, 3, 3.4 gear ratios. Thus, after analyzing the results obtained from the simulations, it is evaluated that 2.6 is the optimal gear ratio.

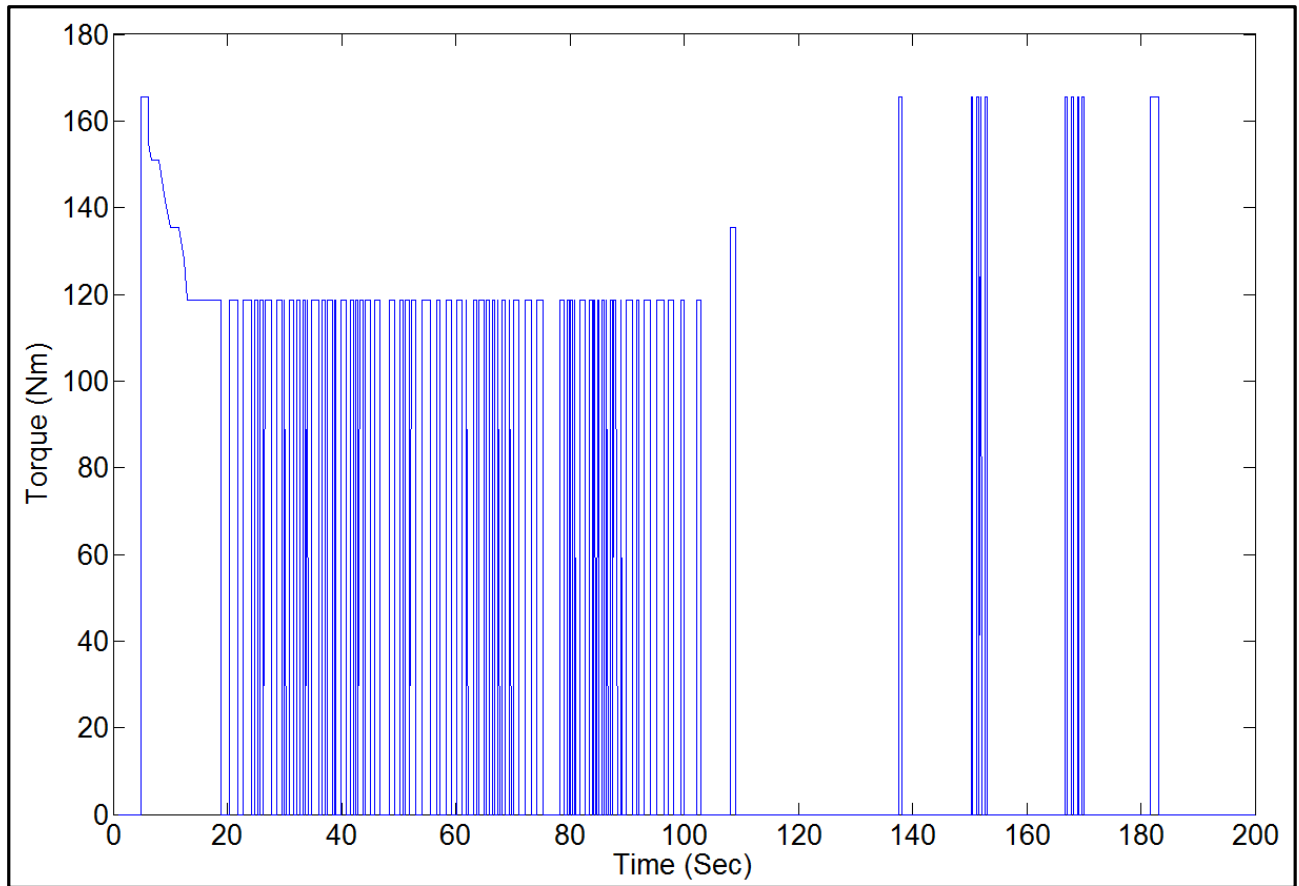


Figure 23: Engine torque results for gear ratio 2.6 for Bag 3 cycle

The engine torque results shown in Figure 23. seems to be unpleasant to the occupants but could be addressed in the future work for smoothing it. This can be done using ramp brake.

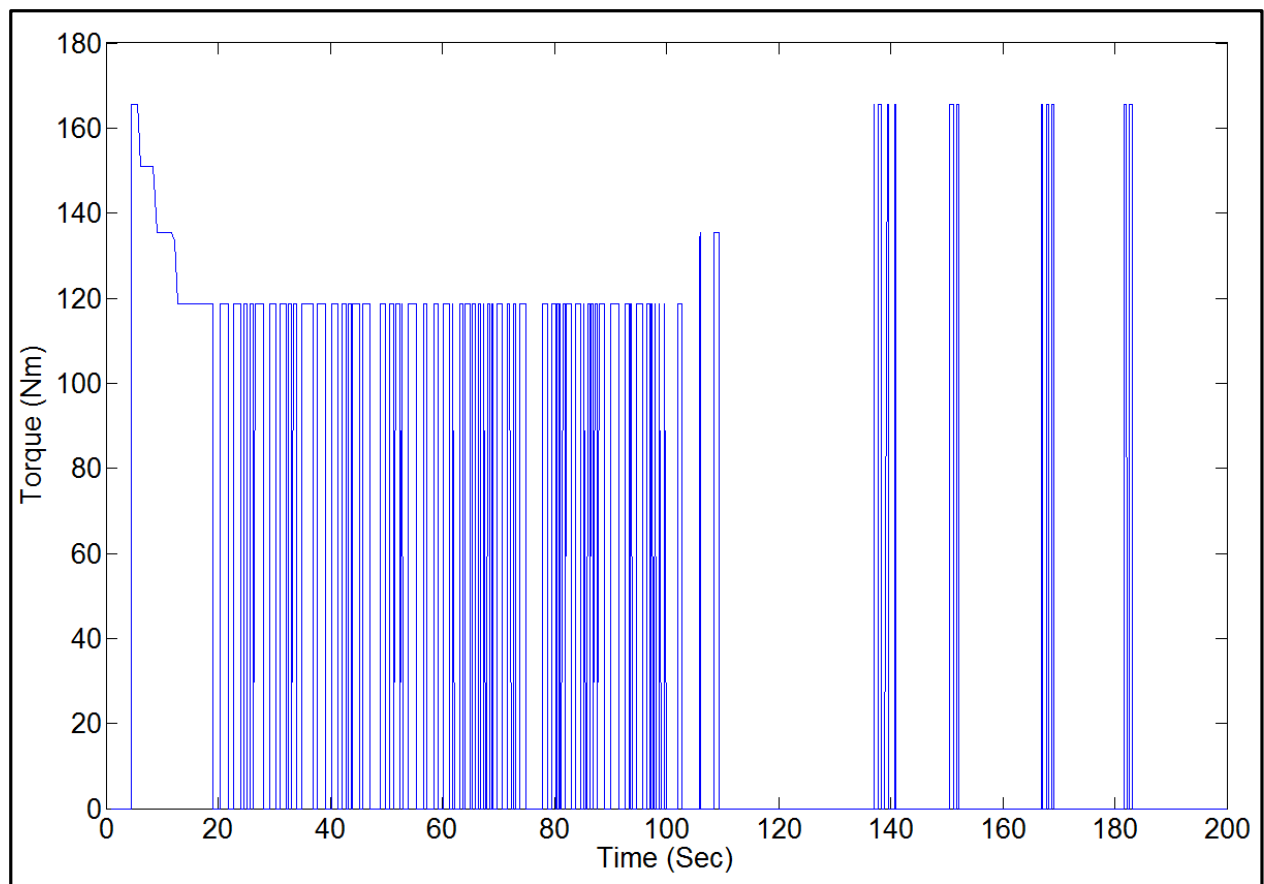


Figure 24: Engine torque results for gear ratio 3.4 for Bag 3 cycle

### US06 Cycle Results of Optimal Gear Ratio = 2.6

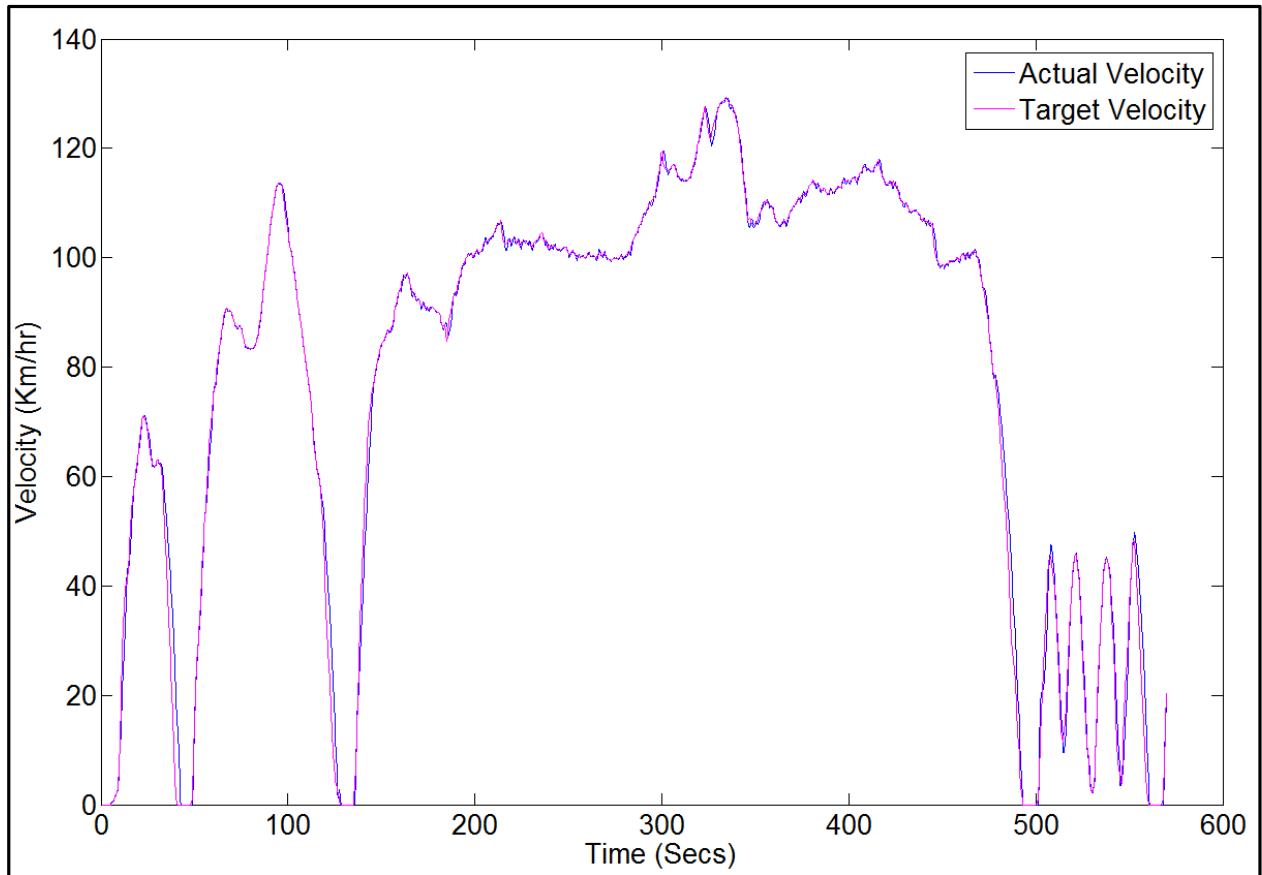


Figure 25: Velocity profile of full US06 driving cycle and for optimal gear ratio = 2.6

Figure 25 illustrates the velocity profile for 2.6 gear ratio of US06 cycle and the results represent that overall speed demanded by the vehicle (target speed) is satisfied by the actual vehicle speed. The root mean squared of the difference between target and actual speed over cycle is 3km/hr.

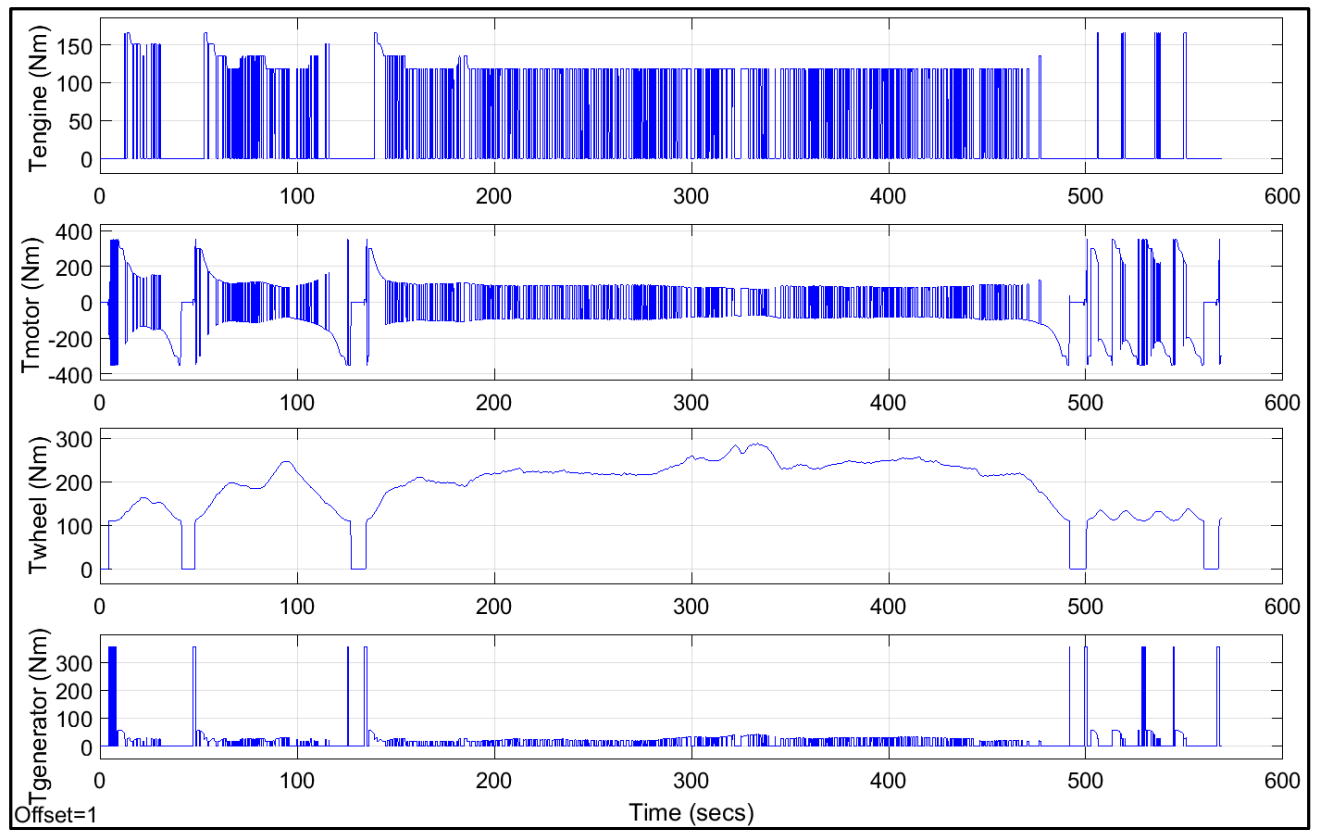


Figure 26: Torque results for engine, motor, wheels and generator for full US06 cycle

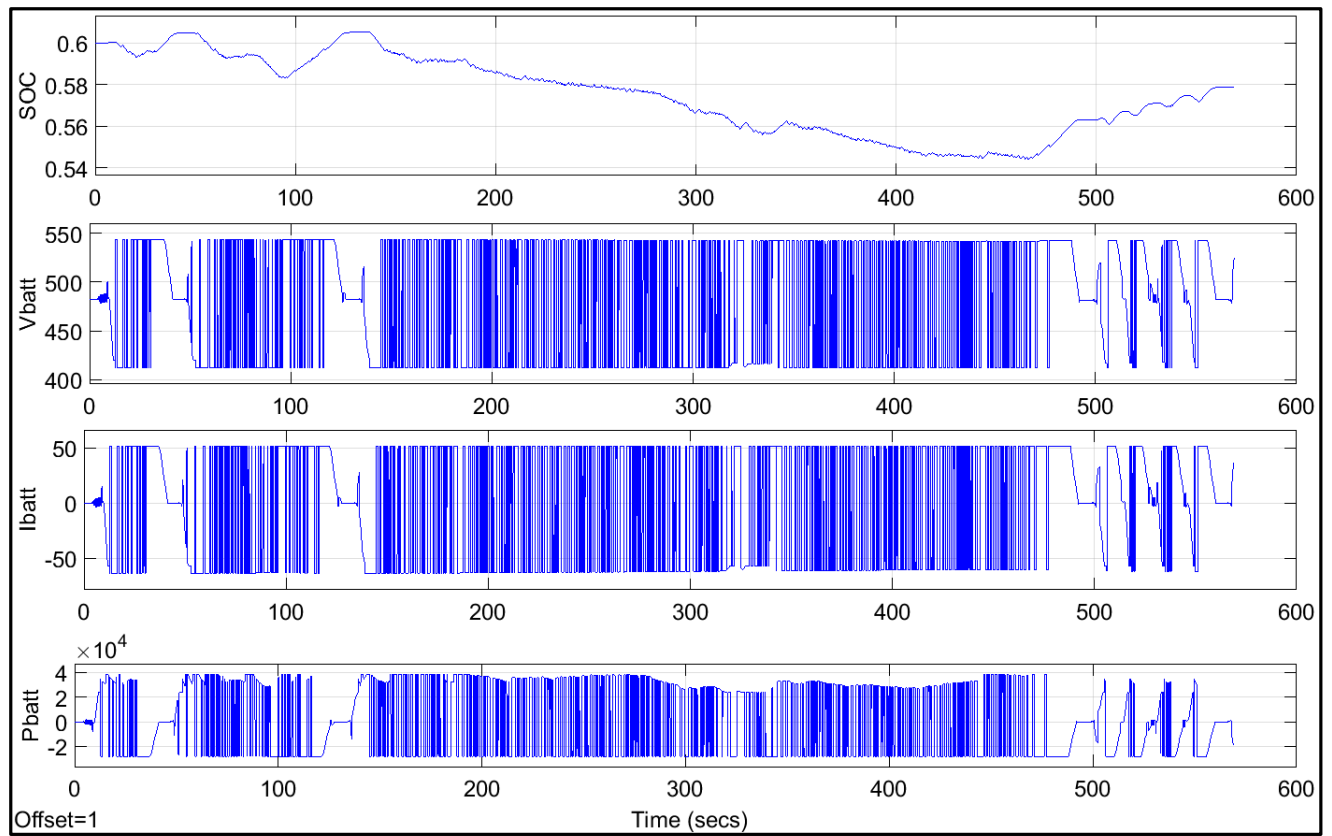


Figure 27: Results of battery for full US06 cycle and optimal gear ratio = 2.6

Figure 26-27 represents the torque results for engine, motor, wheels and generator and results of battery SOC, voltage, current and power for full US06 cycle and optimal gear ratio = 2.6

### US06 Cycle Battery Results for Gear Ratios 2.75,2.9,3,3.4

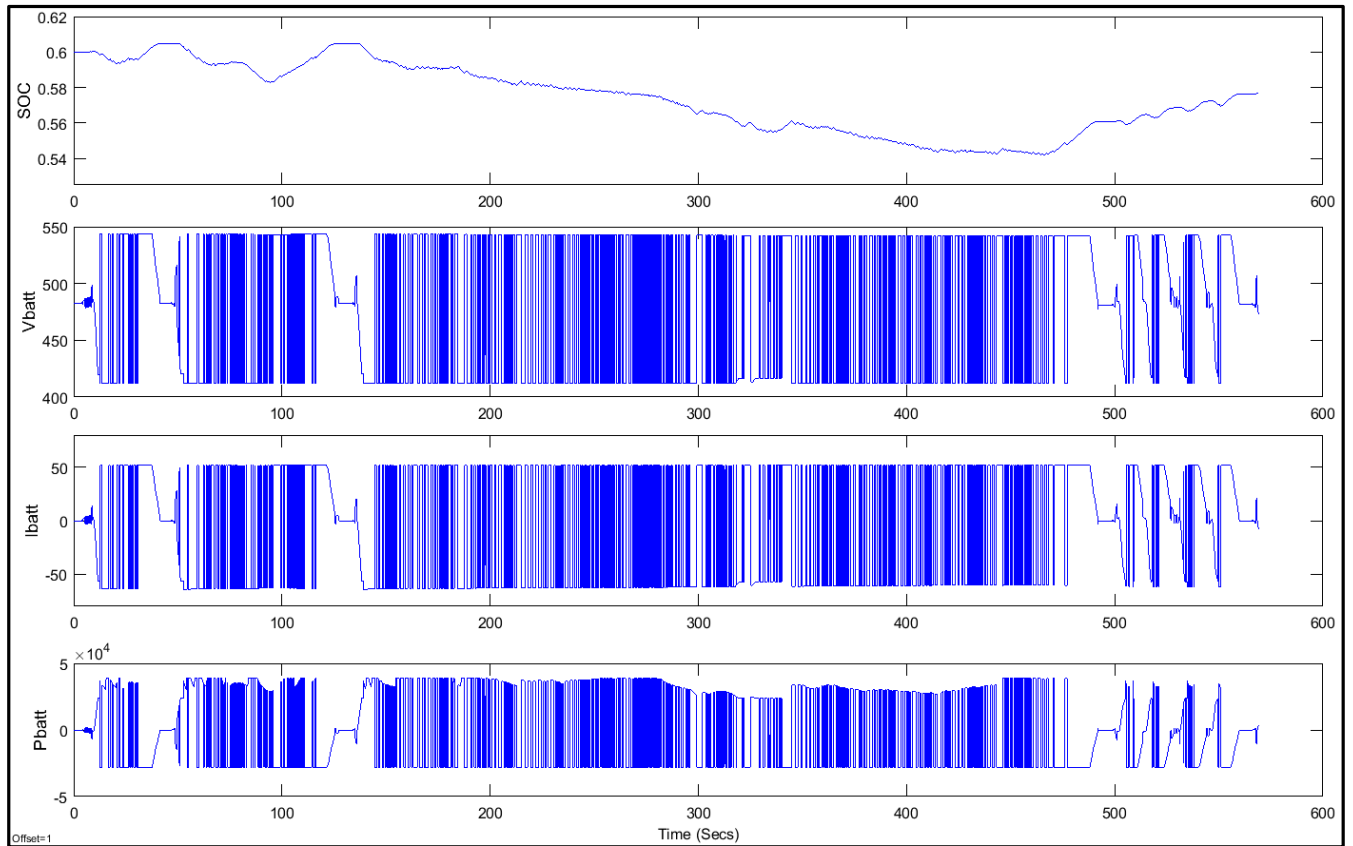


Figure 28: Results of battery for full US06 cycle and optimal gear ratio = 2.75



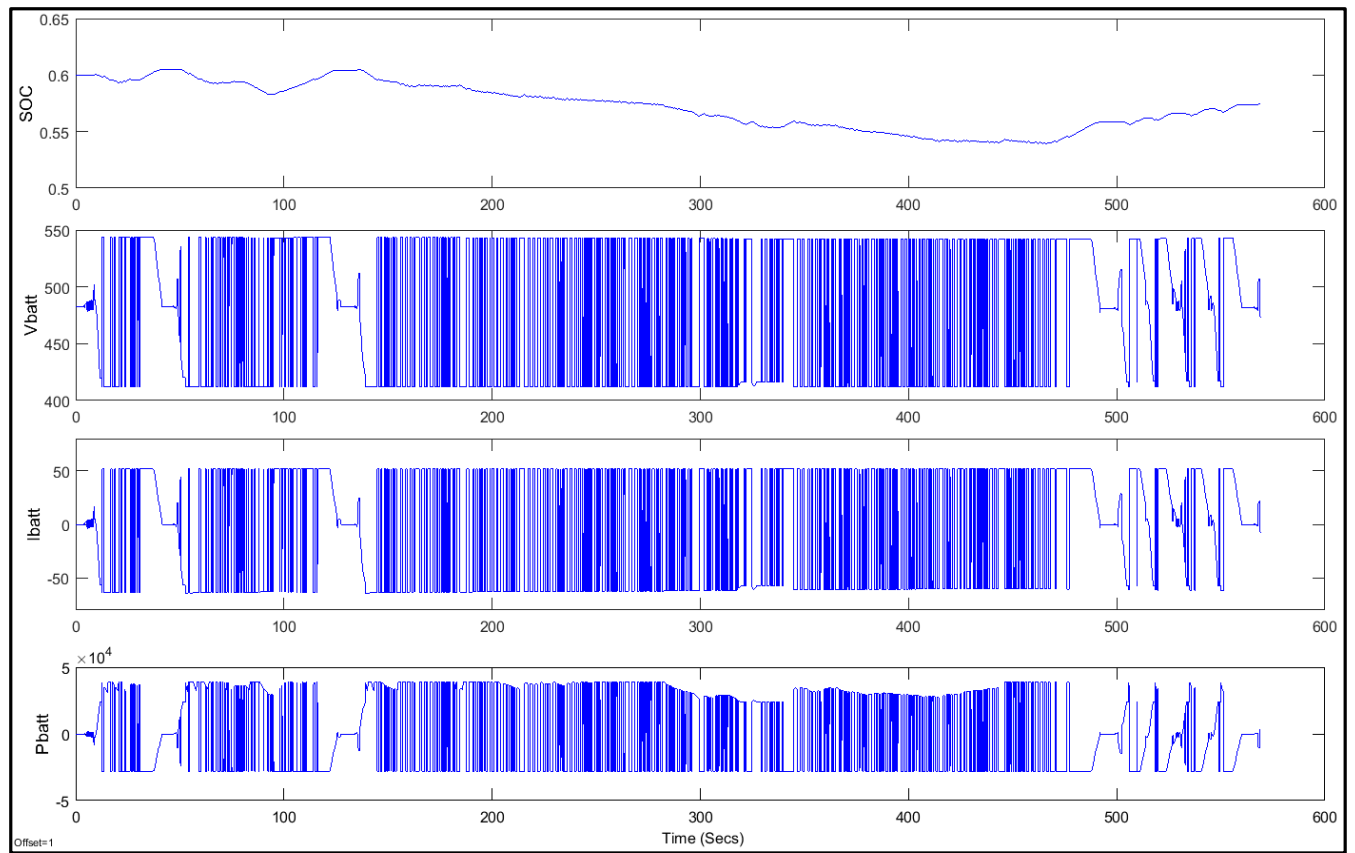


Figure 29: Results of battery for full US06 cycle and optimal gear ratio = 2.9

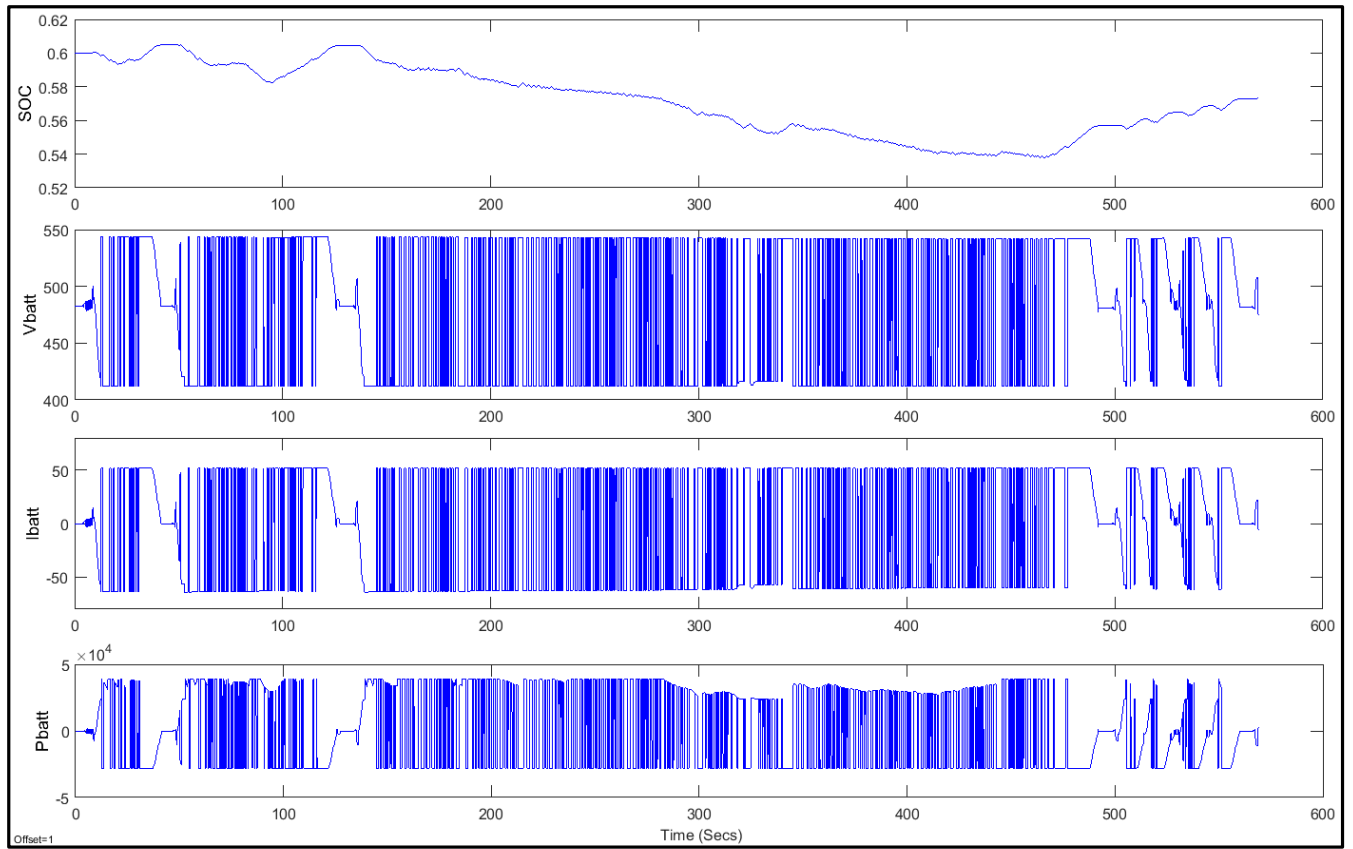


Figure 30: Results of battery for full US06 cycle and optimal gear ratio = 3

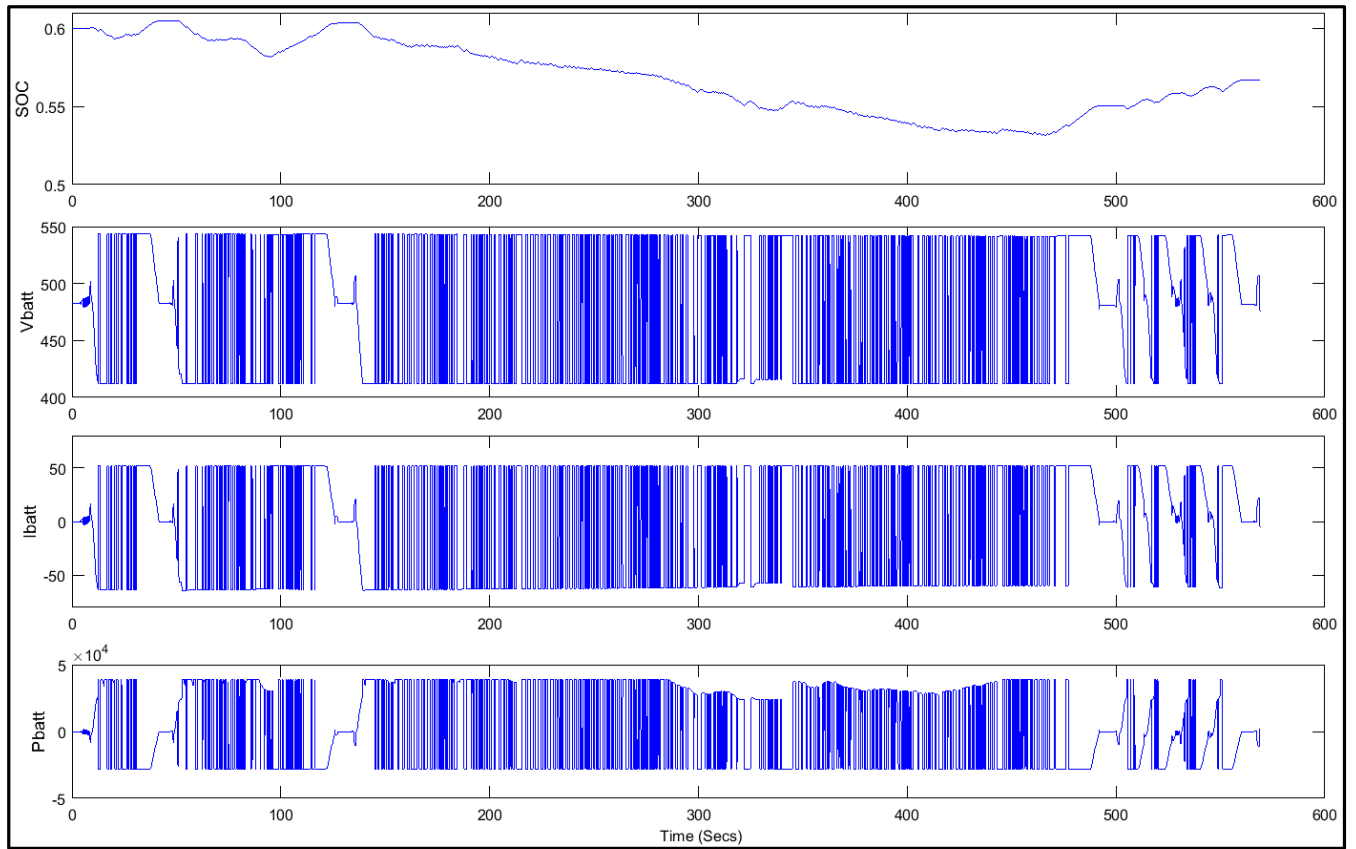


Figure 31: Results of battery for full US06 cycle and optimal gear ratio = 3.4

Figure 28–31 represents the results of battery SOC, voltage, current and power for full US06 cycle with gear ratios 2.75, 2.9, 3 and 3.4.

## FTP 75 Driving Cycle

FTP-75 (Federal Test Procedures) cycle is used for testing fuel economy in the United States for light duty vehicles [16]. FTP-75 cycle has three phases: cold start phase, stabilized phase, and hot start phase. The driving cycle travels a distance of about 6.11 km. FTP-75 cycle is simulated for 570 secs and is divided into three bags for convenience:

Table 4: Bags of FTP-75 cycle

Bag#	Time in seconds
Bag 1	1 to 220 sec
Bag 2	220 to 370 sec
Bag 3	370 to 570 sec

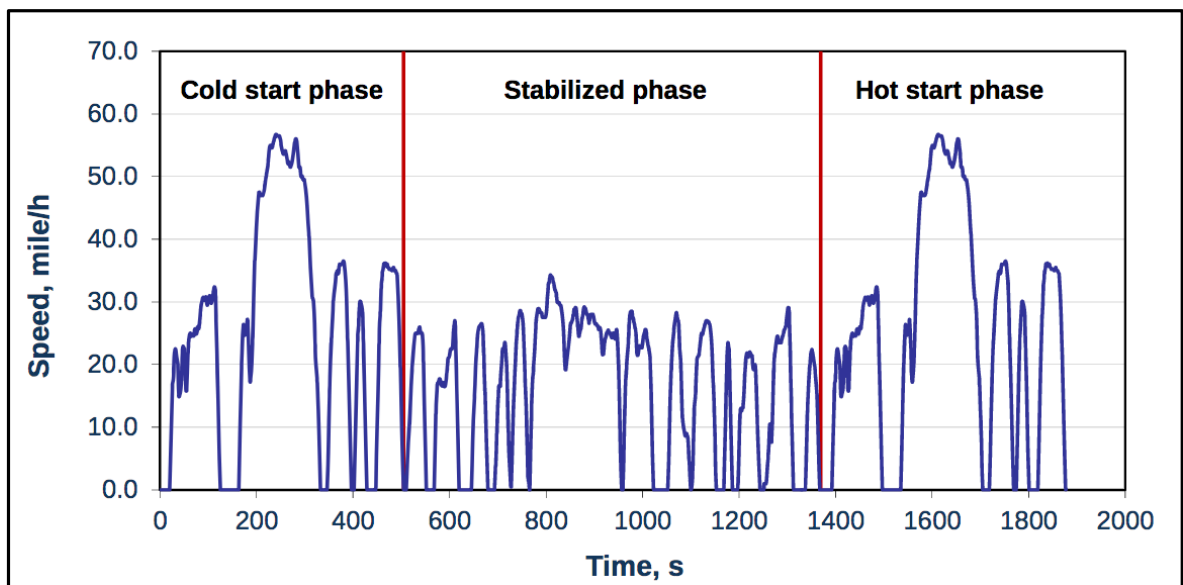


Figure 32. FTP 75 Driving cycle

Table 5: Fuel economy improvement compared with 2.6 gear ratio for FTP-75 cycle

Gear Ratio	Fuel Economy improvement
2.75	1.91 %
2.9	2.15 %
3	- 0.98 %
3.4	- 1.81 %

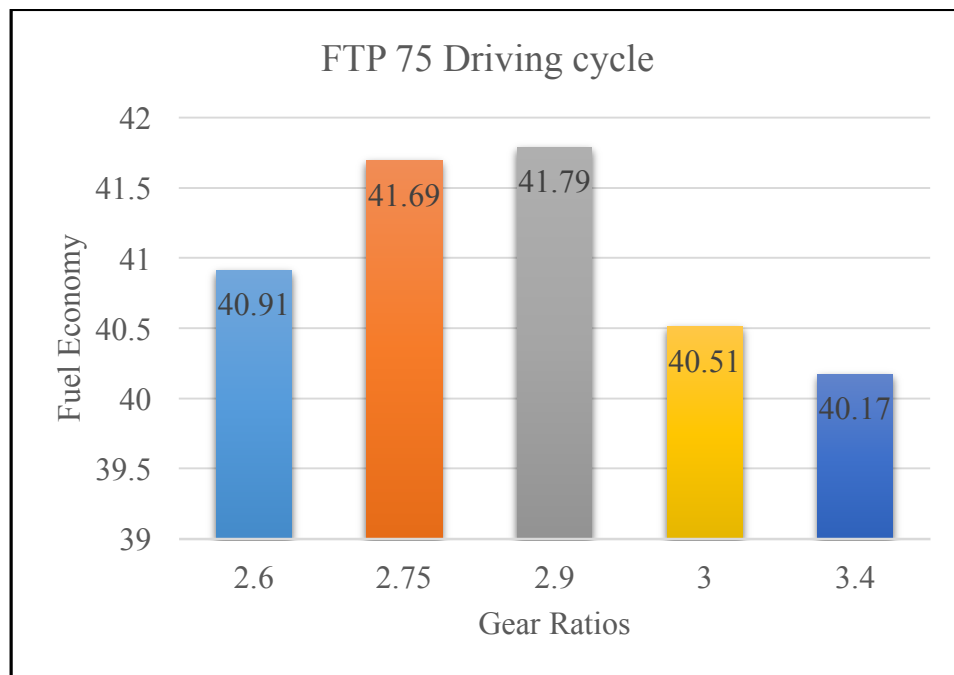


Figure 33: Fuel economy (mpg) for FTP 75 driving cycle for 5 different PSD gear ratios

FTP-75 cycle was run with 5 different gear ratios 2.6, 2.75, 2.9, 3, 3.4. Figure 33 shows that there are no major changes in fuel economy for FTP-75 cycle. The best fuel economy is achieved for gear ratio 2.9. For gear ratio 3.4, the fuel economy obtained is less compared to all

other gear ratios. There is no drastic improvement in fuel economy for 2.9 gear ratio compared to gear ratio 2.6 as can be seen from table 5.

Velocity profile, torque results, and battery results were studied at different gear ratios and comparison was done to obtain the optimal gear ratio. Differences in engine torque results were highlighted for minimum and maximum gear ratio, 2.6 and 3.4, and these results demonstrate that engine operation is almost same for 2.6 gear ratio. This is because during sudden acceleration and deceleration, the engine has a lower capacity of propulsion than at a lower gear ratio. This is evident when the vehicle cycle changes from the cold start phase to the stabilized phase, as observed in velocity profiles for gear ratio=2.6 in Figure 34 and gear ratio=3.4 in Figure 35.

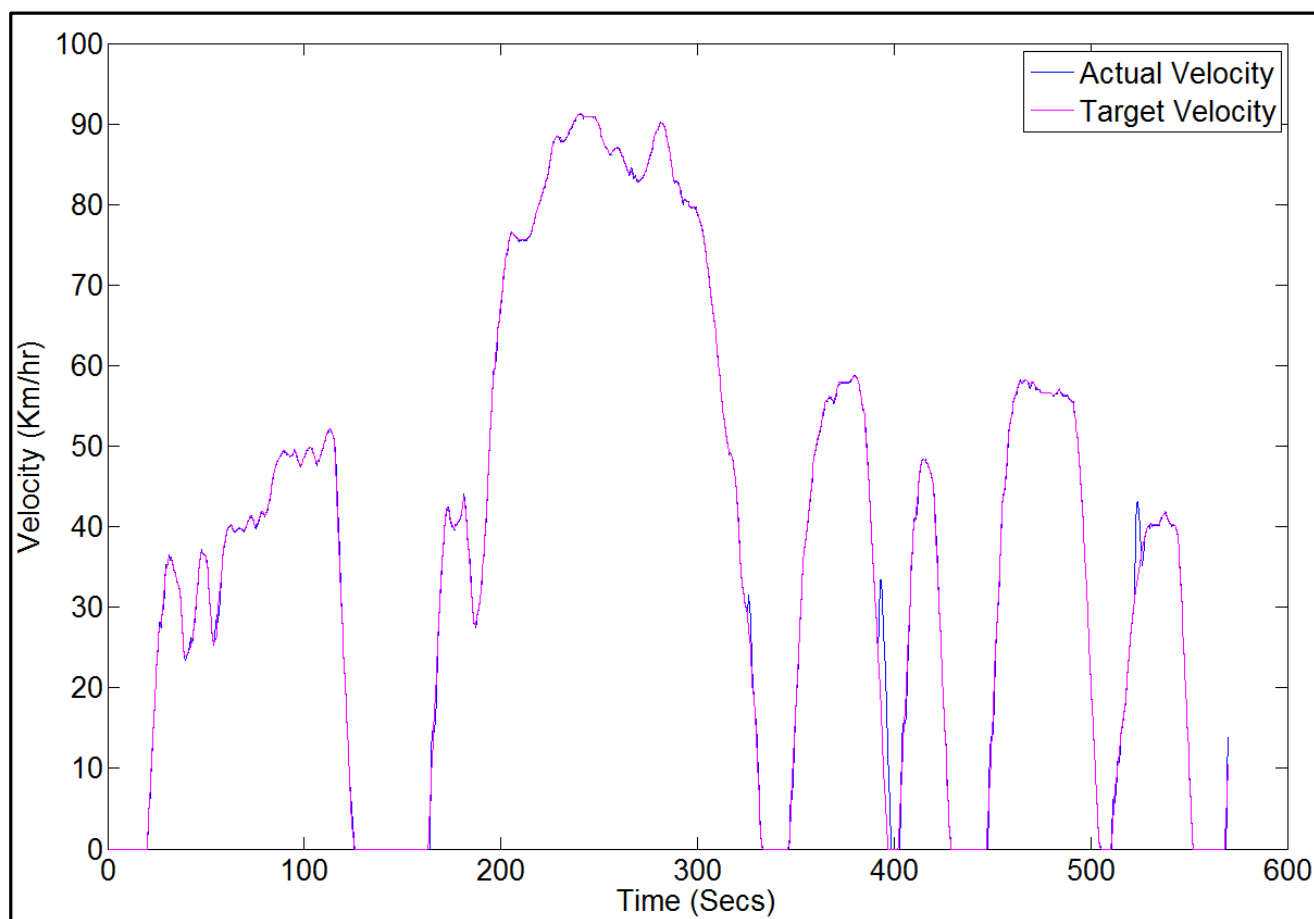


Figure 34: Velocity profile of FTP 75 driving cycle for 570 secs and for 2.6 gear ratio

The root mean squared of the difference between target and actual speed over cycle is 2.48km/hr.

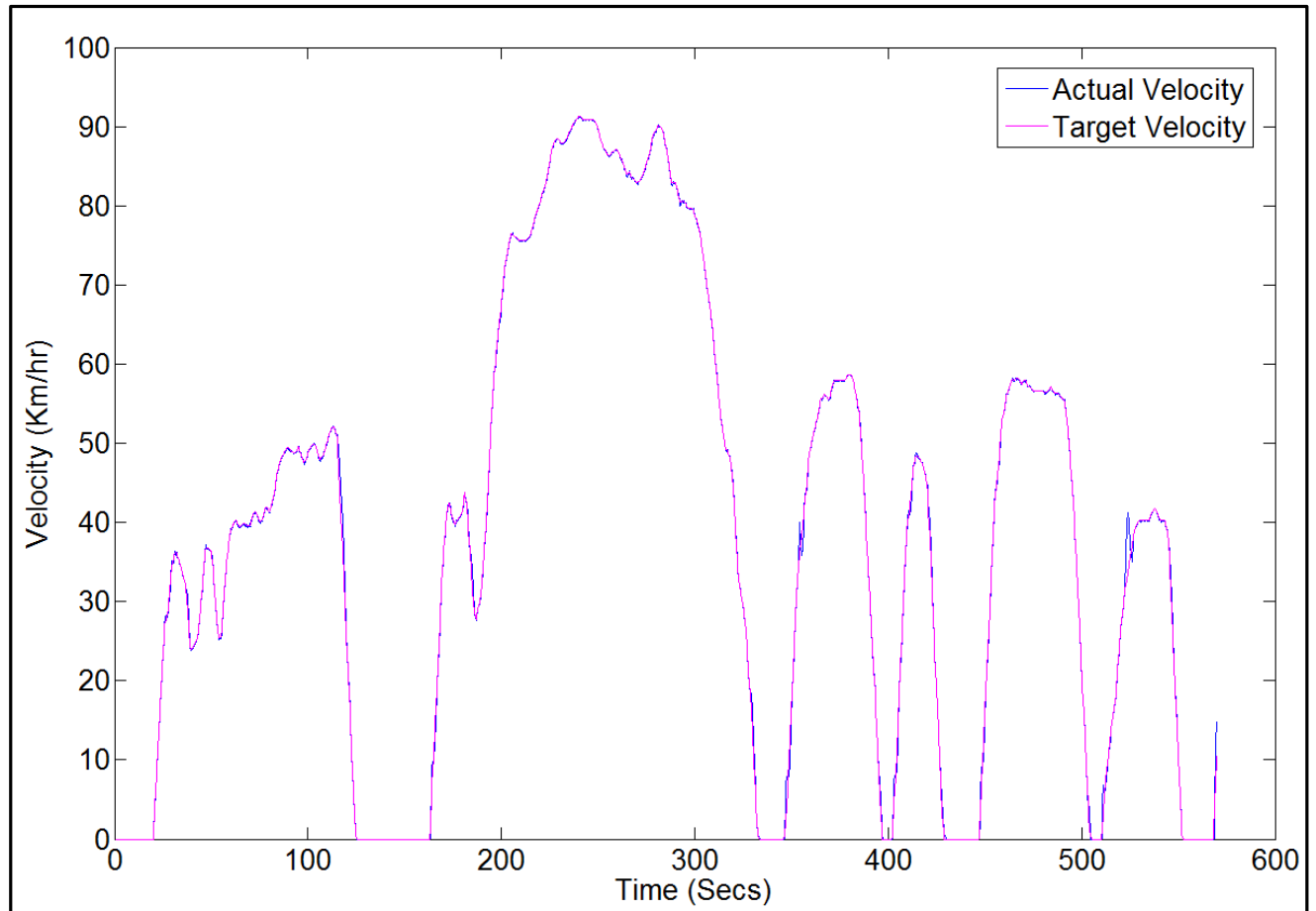


Figure 35: Velocity profile of FTP 75 driving cycle for 570 secs and for 3.4 gear ratio

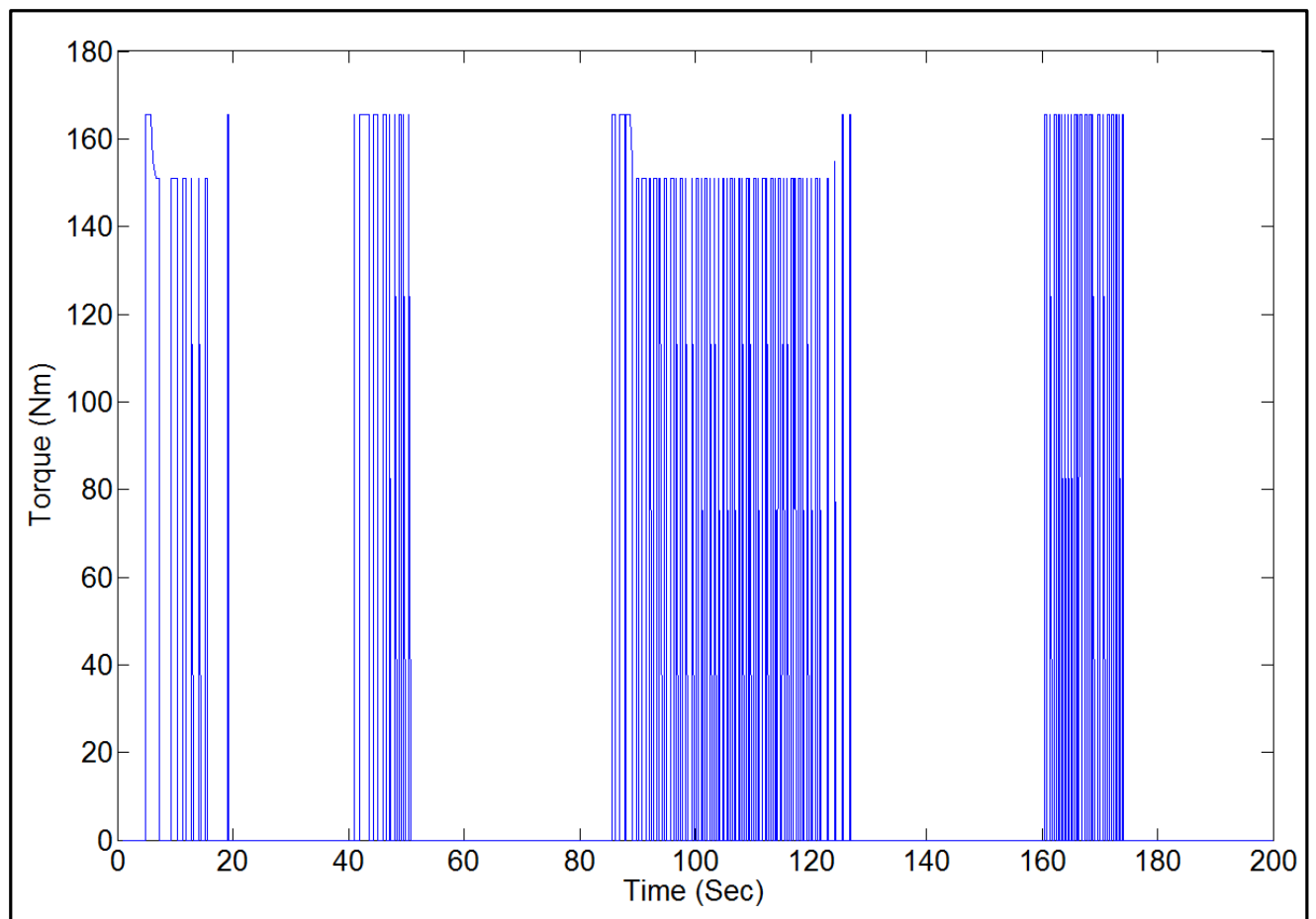


Figure 36: Engine torque results for gear ratio 2.6 and Bag 3



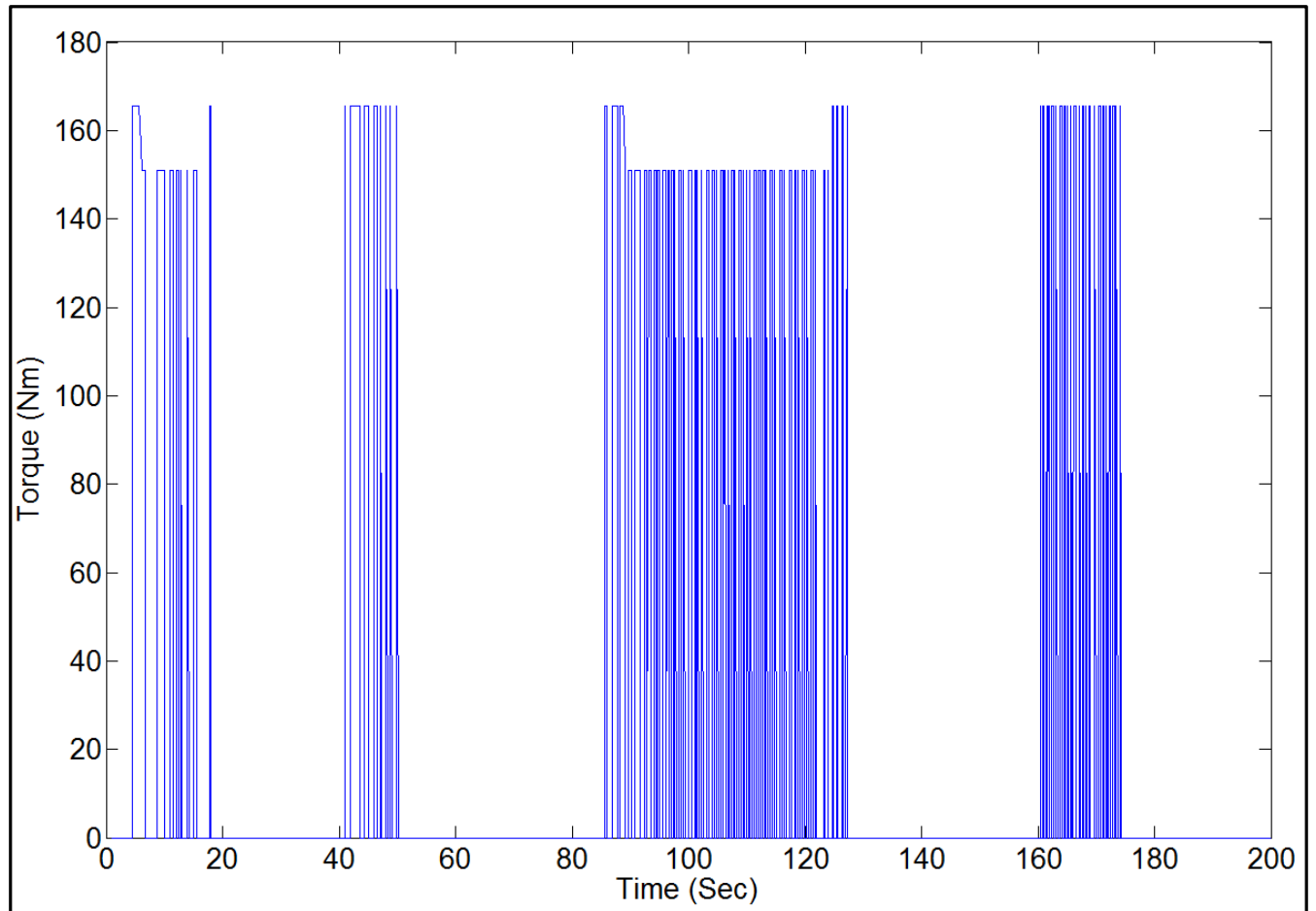


Figure 37: Engine torque results for gear ratio 3.4 and Bag 3

Figure 36 and 37 represents the engine torque results for a 200 second time slot for the Bag 3 cycle. It can be seen from these figures that there are chatters in engine torque with short on-off time periods and this could be addressed in the future work for smoothing it. This can be done by using ramp brake. Results for 2.6 gear ratio show that energy recovered by the regenerative braking is 1.2 KW-hr and for 3.4 is 1.43 KW-hr. This is because at higher gear ratio, there is more capacity of propulsion and thus more ability to regenerate energy. Thus, after analyzing the results obtained from simulation, it is evaluated that there not much difference in the results for gear ratio 2.6 and 3.4.

Results show that for the FTP-75 cycle, there is no drastic improvement in fuel economy over the optimal 2.6 gear ratio selected for the US06 cycle. Therefore, it is reasonable to use 2.6 as the optimal gear ratio for all driving cycles. Torque results for engine, motor, generator, and battery results for gear ratio=2.6 are shown in Figures 38-39.

FTP 75 Cycle Results for Optimal Gear Ratio =2.6

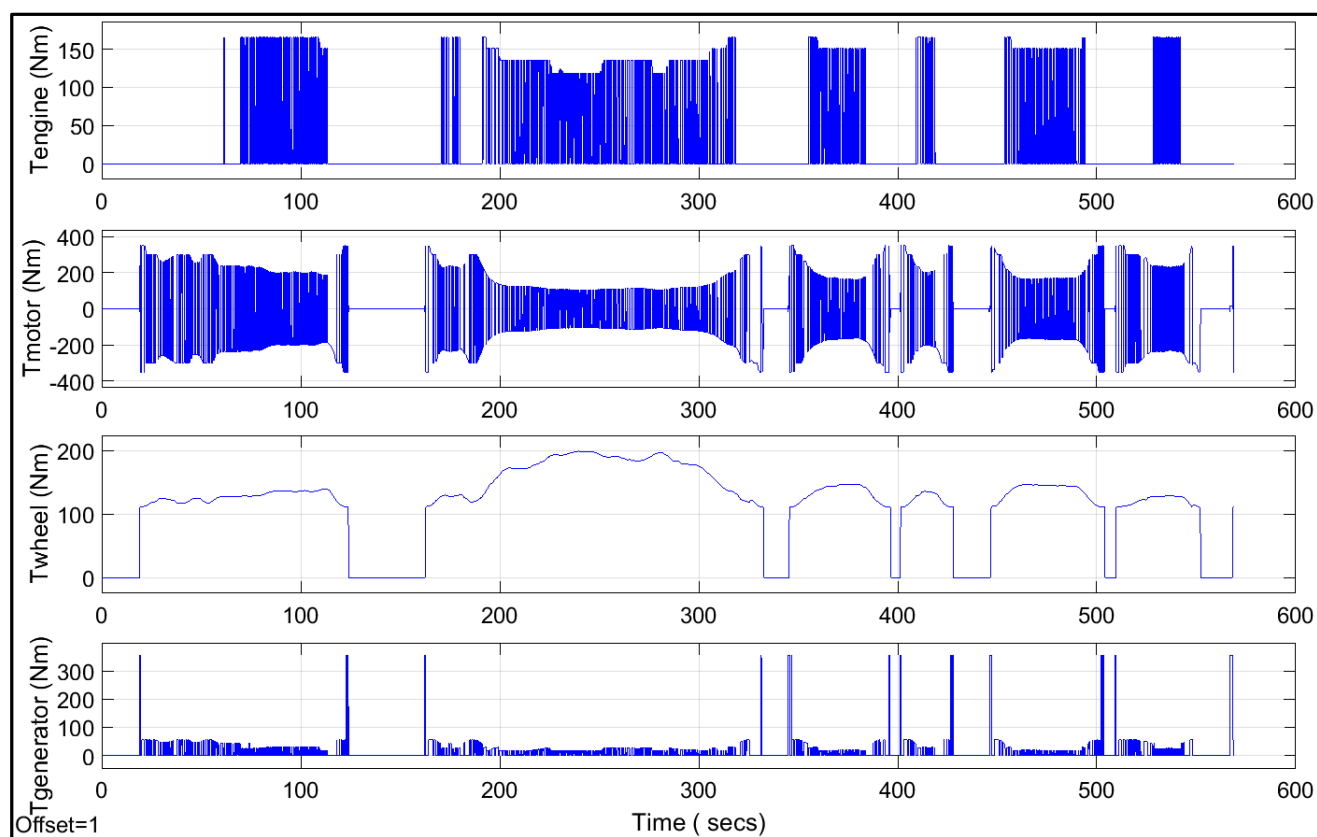


Figure 38: Torque results for engine, motor, wheels and generator for FTP 75 driving cycle

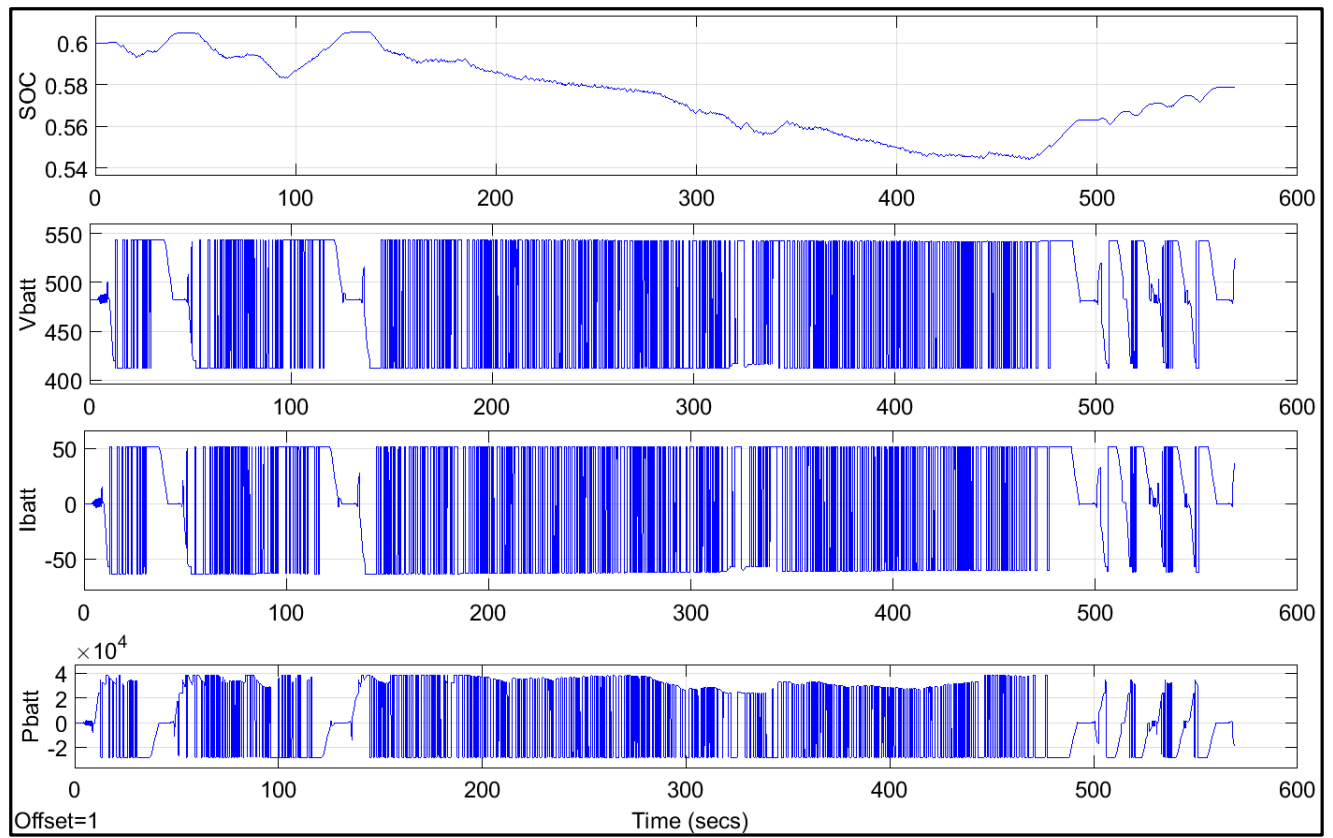


Figure 39: Results of battery for FTP 75 driving cycle and optimal gear ratio = 2.6

### FTP 75 Cycle Battery Results for Gear Ratios 2.75,2.9,3,3.4

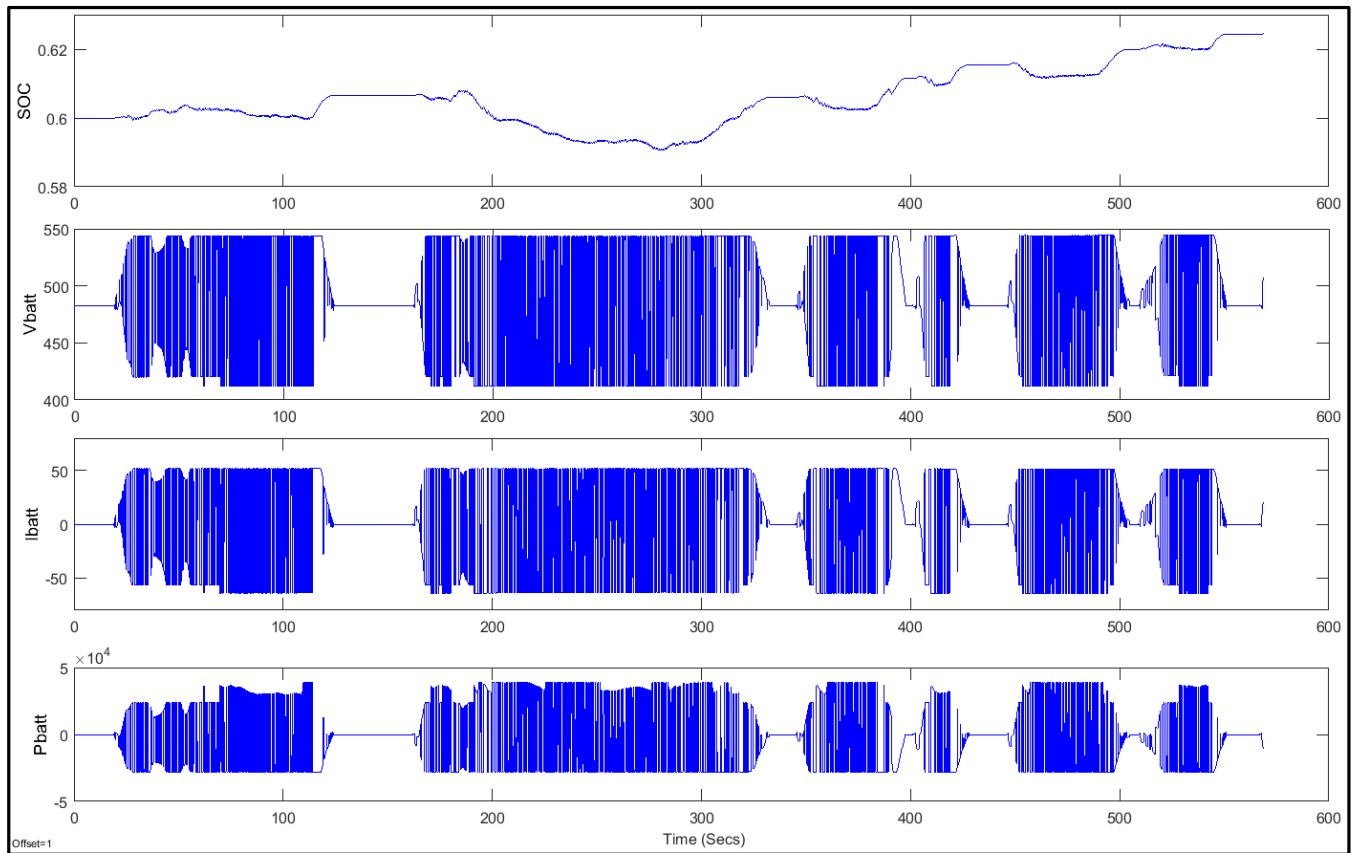


Figure 40: Results of battery for FTP 75 driving cycle and optimal gear ratio = 2.75

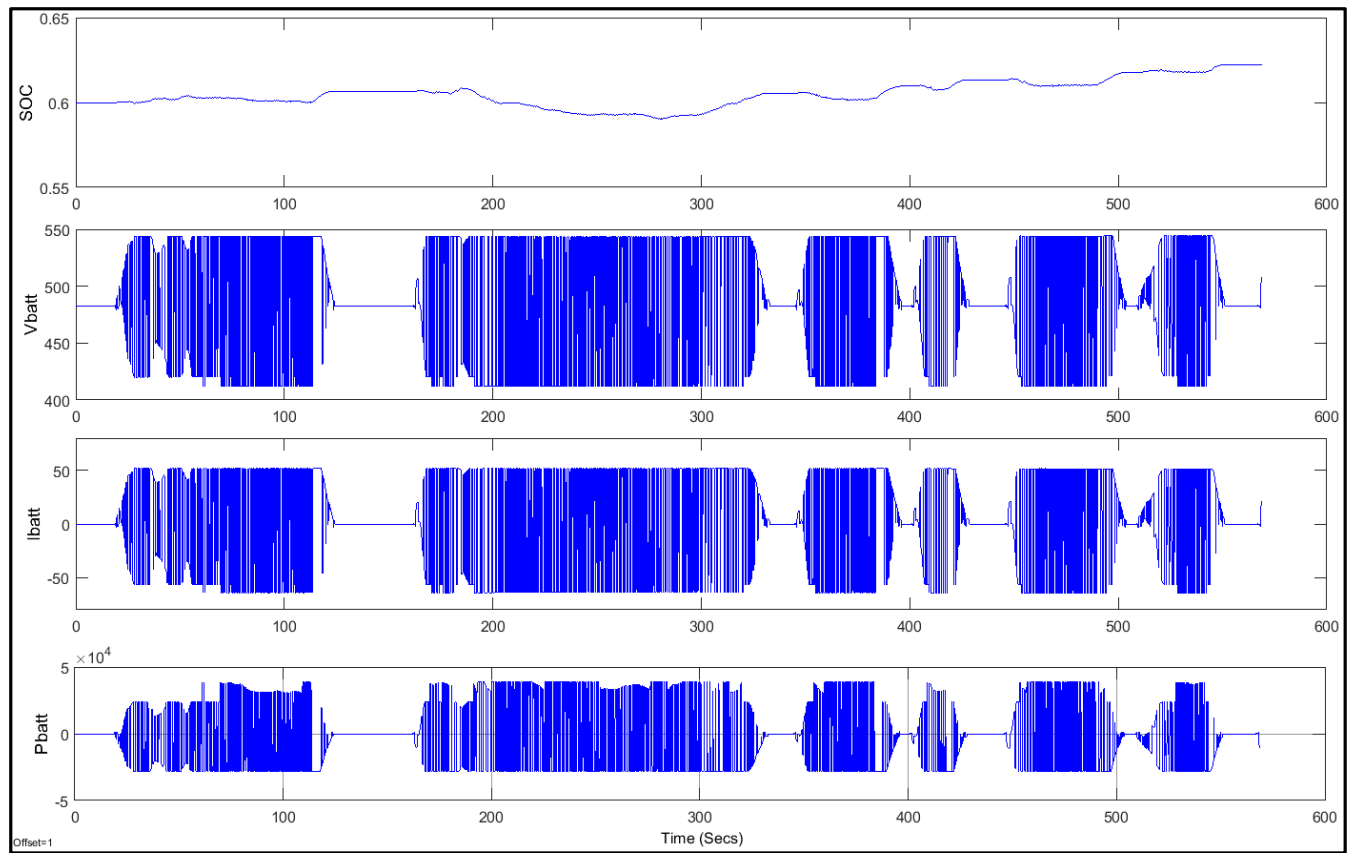


Figure 41: Results of battery for FTP 75 driving cycle and optimal gear ratio = 2.9

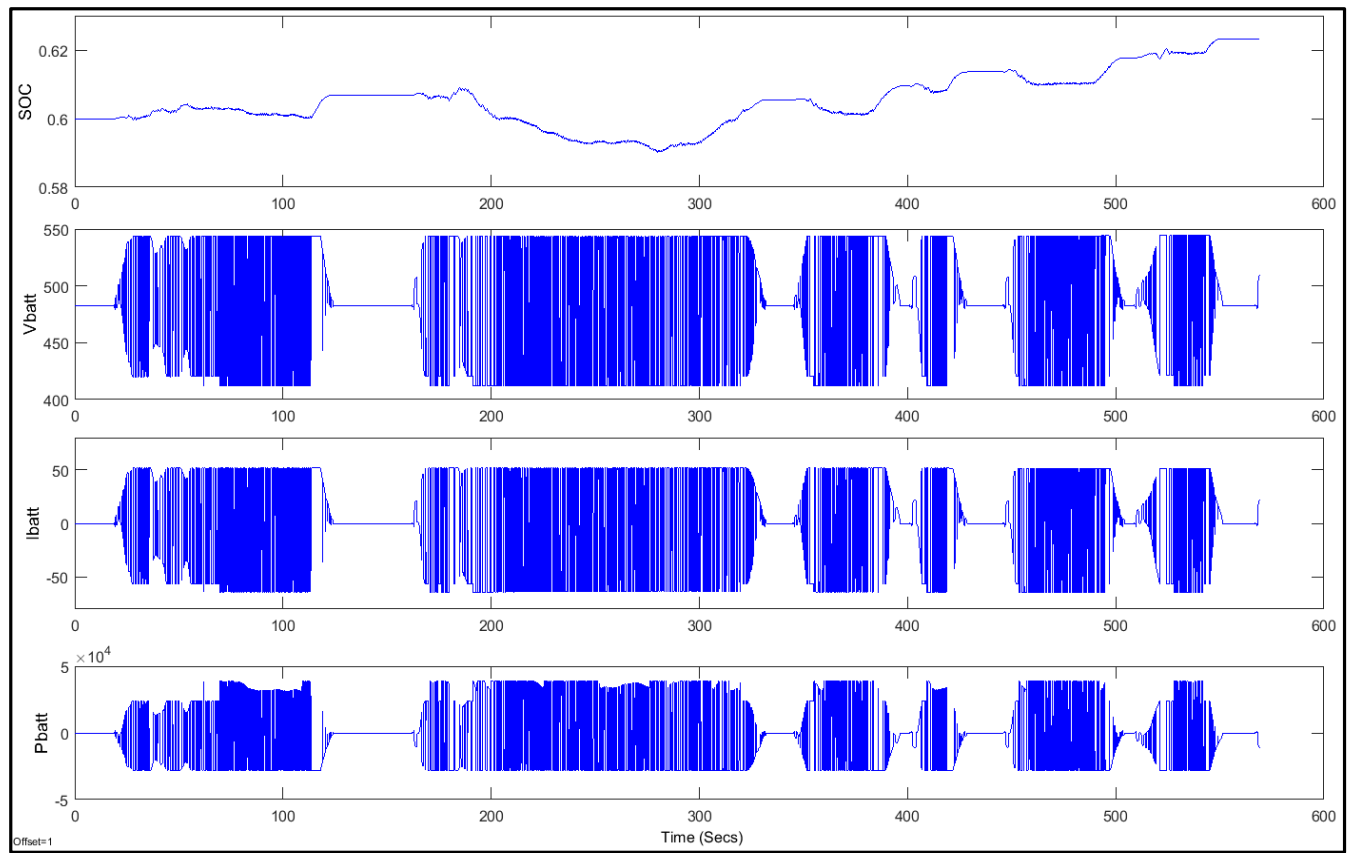


Figure 42: Results of battery for FTP 75 driving cycle and optimal gear ratio = 3

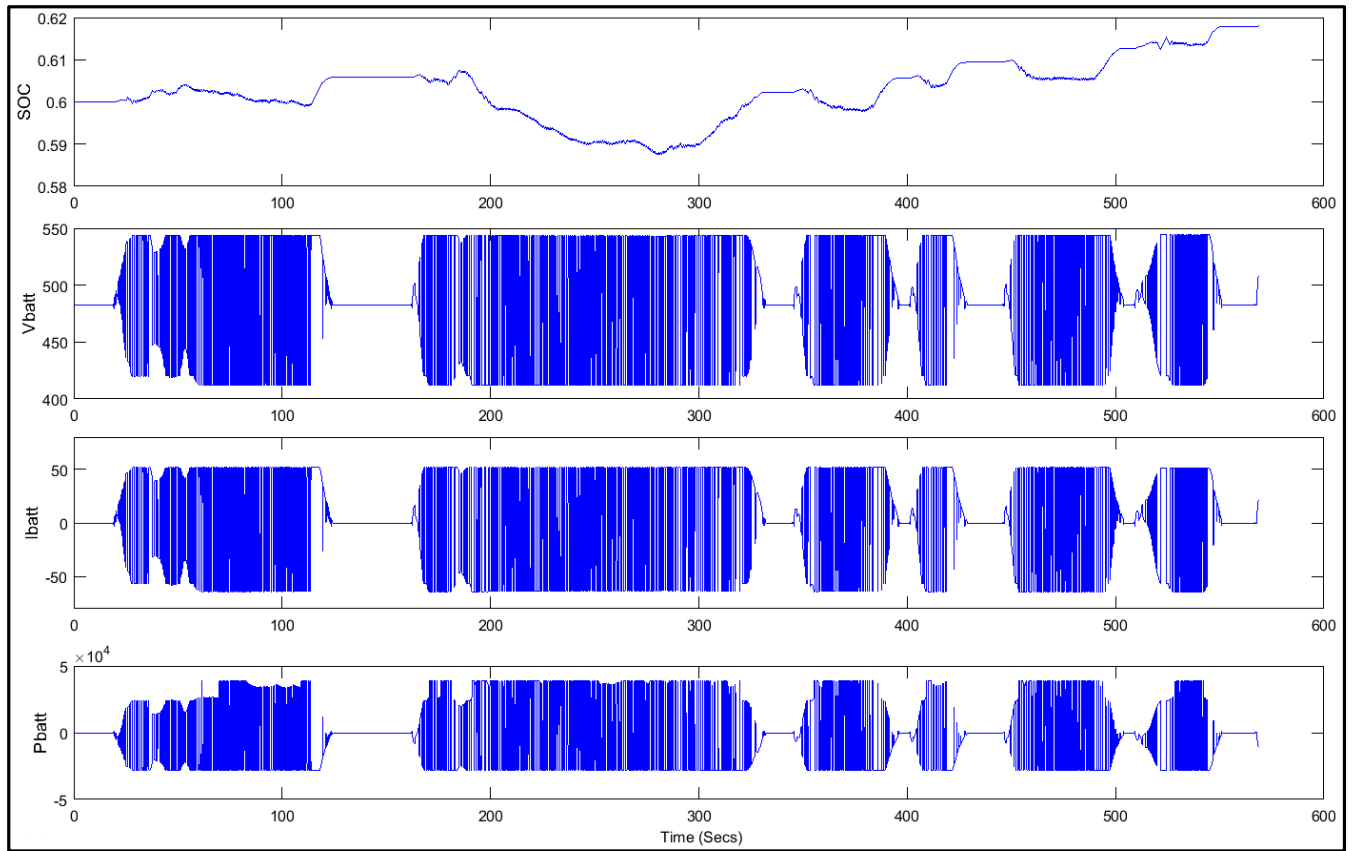


Figure 43: Results of battery for FTP 75 driving cycle and optimal gear ratio = 3.4

Figure 40 – 43 represents the results of battery SOC, voltage, current and power for FTP 75 driving cycle and optimal gear ratio 2.75, 2.9, 3, 3.4

## CHAPTER 5

### CONCLUSION

In this thesis, a forward-looking, velocity-driven, power-split, hybrid electric vehicle model is developed and simulation is run in the MATLAB R11/Simulink environment. This HEV model includes a planetary gear set model for obtaining optimal gear ratio to achieve the best fuel economy. The power-split HEV model is run for two driving cycles, US06 and FTP-75, with five different gear ratios. Velocity profile, torque results, battery results, energy recovered due to regenerative braking, and mechanical energy consumed were studied.

Fuel economy results were in good agreement at optimal gear ratio 2.6 for US06 and FTP-75 drive cycle. Although fuel economy was slightly better for 2.9 gear ratio for FTP-75 cycle, it is due to the greater propulsion capacity of the vehicle at higher gear ratios, which helps in regenerating more energy. But as there is not much difference in fuel economy between gear ratios for FTP-75 cycle, 2.6 is considered as optimal overall gear ratio. Velocity profile and engine torque results obtained after simulation of US06 cycle have also proven that 2.6 is the optimal gear ratio, as engine operates for less time steps for gear ratio=2.6. Also more energy is recovered by regenerative braking and less mechanical energy is consumed compared to all other gear ratios for gear ratio=2.6 in US06 drive cycle.

This thesis underscores the HEV model that can be used in future and the importance of optimized powertrain components, especially the planetary gear ratio and its impact on vehicle performance and fuel economy. The planetary gear ratio ensures smooth transmission, propulsion capacity, acceleration, and fuel economy. This thesis investigates the effect of different gear ratios on fuel economy for the US06 and FTP75 drive cycles and proposes a strategy for optimizing gear ratio to maximize fuel economy without compromising vehicle drivability. The forward-looking, velocity-driven, power-split model developed in this study highlights the importance of gear ratio in engine operation and also ensures that generator does not overrun in the process.



This study underscores the dynamic model of hybrid electric vehicle that can be used in future studies for developing test designs, control systems, dynamic analysis and prototype simulation. This study also underscores the benefit of using optimal gear ratio for modeling of planetary gear set model of HEV for achieving better fuel economy.

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