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Control Law Synthesis for Lockheed Martin’s Innovative Control Effectors Aircraft Concept

Cameron James Segard

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CONTROL LAW SYNTHESIS FOR LOCKHEED MARTIN’S INNOVATIVE CONTROL EFFECTORS AIRCRAFT CONCEPT

by

Cameron James Segard

A thesis submitted to the Graduate College in partial fulfillment of the requirements for the degree of Master of Science
Aerospace Engineering
Western Michigan University
April 2019

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ACKNOWLEDGMENTS

I cannot express my gratitude to Western Michigan University and the College of Engineering and Applied Sciences Mechanical and Aerospace Engineering Department enough for all of their support. Without the guidance and education collectively provided during my time at WMU I would not have been able to achieve the level of education I have reached today. Enabling me to lead with awareness and strive for meaningful achievements that may one day benefit society.

I want to thank my advisor Dr. Kapseong Ro for guiding me throughout my academic carrier and thesis development with resolute knowledge and acceptance to intuitive adaptation. I have enjoyed the journey and the numerous lessons over the years, thank you!

I am thankful for the unwavering support from my Mother and Father for helping me keep my passion and focus steadfast. Particularly helping me stay in touch with the outside world as I became engrossed in my studies.

Finally to all of my friends throughout college; cheers to the late night study sessions and, the weekend breaks that were all too far and few between. My time as a Bronco would not have been the same without our camaraderie while tackling wave after wave of engineering assignments!

Cameron James Segard
CONTROL LAW SYNTHESIS FOR LOCKHEED MARTIN’S INNOVATIVE CONTROL EFFECTORS AIRCRAFT CONCEPT

Cameron James Segard, M.S.
Western Michigan University, 2019

This thesis documents a conventional and modern flight control system design process carried out on a tailless aircraft Simulink model with innovative control effectors provided by Lockheed Martin. To set scope and design requirements a performance analysis was carried out to categorize the aircraft. Evaluation of open-loop dynamics reveled modal instabilities as well as state and control coupling. Flight condition dependent pole migration mapping reveled large changes in the aircraft’s static stability. Leading to the development of a four channel proportional-integral-derivative (PID) stability and control augmentation system (SCAS) controlling pitch-rate, roll-rate, side-slip angle, and airspeed states. PID gains are scheduled via iterative constrained optimization throughout the linearized flight envelope generating a full flight envelope flight control system. The linear quadratic (LQ) servo design method provides optimal control allocation at every linearized flight condition, controlling angle of attack, roll angle, side-slip angle, and airspeed states. LQ controlled states were modified to be equivalent to the PID control system for real time handling qualities (HQ) evaluation. Both control systems required gain scaling to prevent state resonance and control saturation during nonlinear 6-degree-of-freedom (6DOF) real-time simulation. Either method achieved stable augmented control of the innovative control effectors (ICE) aircraft throughout the flight envelope. The HQ’s for the control methods are satisfactory for conventional aerobatics but became dissimilar for sustained super-maneuverability flight conditions.
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CHAPTER 1

INTRODUCTION

1.1 Overview

Modern and future high performance military aircraft configurations have considered tailless design concepts predominantly for inherent stealth and aerodynamic advantages. Flight control design for this configuration is challenging in many aspects such as multi-axes instabilities and unconventional control effectors. The main objective of this study were to develop full flight envelope flight control laws based on conventional and optimal control design methodologies, and to evaluate and compare the augmented aircraft performance and handling qualities via man-in-the-loop-simulation (MILS), real-time 6 degrees-of-freedom (DOF) flight simulation. Flight control system design and evaluation requires a high fidelity aircraft flight dynamics model constructed from aerodynamic derivatives. This study utilizes the nonlinear 6-DOF aircraft simulation model developed in MATLAB/Simulink environment provided by Lockheed Martin.

1.1.1 Literature Review

Lockheed Martin’s Innovative Control Effectors (ICE) Aircraft [1], the configuration was initially developed internally in 1991 and expanded to the Future Aircraft Technology Enhancements or FATE [2] in 1997. The design was evaluated for technology readiness to jump-start the Unmanned Combat Air Vehicle Advanced Technology Demonstrator or UCAV ATD in year 2000. The F/A-18E/F and F-22 were used as design benchmarks to set design requirements for the UCAV development.

Flight control design studies particularly for the ICE aircraft are very limited in the public domain literature due to limited availability of the aircraft model. Dynamic inversion and structured singular value methods were studied at Air Force Research Laboratory [3] by J. Buffington.
Evaluating various control allocation structures to minimize drag, surface deflection, etc... while minimizing coupled state responses. The potential for real time control surface reconfiguration was emphasized to accommodate for modeling error or physical system changes during operation. Handling qualities were constrained via command shaping to be comparable to be similar to the F-16 and F-18. Non-real time batch simulations evaluated the system, ultimately concluding Level 1 HQ were achieved for the majority of the presented flight envelope.

Dynamic gain-scheduled (DGS) eigenstructure assignment control method was proposed and applied to the ICE aircraft model [4] by C. D. D. Jones et al. Developed with reduced order system models the DGS allows for smooth transition of gain values between flight conditions. With the five aircraft dynamic modes assigned to satisfy Level 1 handling qualities requirements. Achieving the desired longitudinal and lateral directional state control with achievable control surface deflections.

The Eigenstructure Assignment technique is also studied for a linear model at a single point flight condition [5] by K.M. Sobel and F.J. Lallman. Designing a lateral directional control system decoupled from the longitudinal system then applying the lower order controller to the full system model. The control space is reduced via singular value decomposition allowing for specific dynamic modes to be regulated. Utilizing a pseudo control structure, individual control surface deflections are distributed from a higher level conventional control inputs. This approach improved the multivariable stability margins compared to earlier designs, utilizing control surfaces in proportion to their corresponding control power.

Since the recent release of the high fidelity simulation model to flight control community for research purpose [1], only a few number of control design study cases have been published. In reference [6] by A.R.J. Stolk, a mission-specific spline-based incremental control allocation technique was proposed to achieve a minimum drag profile via mission specific control allocation method. By locally linearizing the nonlinear input dynamics allowing spline-based nonlinear dynamic inversion (SNDI) to allocate surface deflection to control the desired aircraft states via gradient computation method control derivatives. Enabling improved tracking performance when compared to linear input dynamics methods in nonlinear flight conditions. Minimal drag profile was achieved by penalizing the highest drag control effector deflection or utilizes remaining control power to reduce deflection in high drag contributing effectors, both methods reduced drag by 6% and 6.5% from a
standard allocation method. It is stated that acceptable performance is not guaranteed through the entire flight envelope, but is possible with a high fidelity onboard aircraft model. Acceptable flight path tracking was observed with obtainable control surface deflection requirements.

Most recently, an incremental nonlinear control allocation (INCA) method is proposed for flight control design for ICE aircraft [7] by C.C. de Visser. Stating linear control allocation methods fail to provide satisfactory performance for highly nonlinear coupled aircraft such as with the ICE aircraft. Where as incremental nonlinear control allocation approach captures the nonlinearities coupled impact of each effector, while still being solvable with linear control allocation methods. Through continuous linearization of the the nonlinear control dynamics about the current surface position during real time simulation. Control is allocated based on a Jacobian model of the control effectiveness of the actuators, enabling the consideration of the nonlinear control moments. Driving the control surfaces to their optimal positions while considering actuator dynamics via the presented INCA method. Improving overall state tracking and control allocation performance and resulting maneuverability, exploiting the full potential of the ICE effector suite.

While there are abundant literature available in public domain for control studies as presented in [8] by T.A. Johansen and T.I. Fossen evaluating over-actuated mechanical system controlled with multi level control algorithms of various methods. Allowing for secondary objectives to be satisfied in addition to system control for either linear or nonlinear system models, accounting for complex constraints and objectives. Preventing control and deflection rate saturation with inherent effector fault tolerance. While an aircraft control focused survey in [9] by M.L. Steinberg evaluates several modern control method approaches to control multi-axis nonlinear high performance aircraft. Overall summarizing the capability of each by evaluating the absolute resulting error from each presented controller for a uniform maneuver sequence. Concluding with the advantages and challenge for each accounting for development time and computation power. It should be mentioned that only literature pertaining to the ICE aircraft and relevant to this thesis have been presented and is not an exhaustive literature review.
1.1.2 Thesis Organization

This thesis is organized as follows, Chapter 1 succinctly summarizes the general ICE aircraft performance characteristics obtained from the ICE simulation model provided by Lockheed Martin. Chapter 2 presents the traditional open-loop stability and control characteristics of the ICE aircraft based on linearized model. Chapter 3 provides a detailed description of control synthesis based on conventional design method and scheduling control gains for full-flight envelope. Chapter 4 provides the results from optimal control method in parallel to Chapter 3. Chapter 5 summarizes the design evaluation and comparison via nonlinear 6-DOF man-in-the-loop flight simulation.

1.2 Innovative Control Effectors Aircraft Familiarization

1.2.1 Aircraft Configuration

The ICE aircraft is a cambered tailless flying body configuration with a 65° leading edge sweep shown in Figure 1.1. It was designed to operate from approximately Mach 0.2 to 2.2 utilizing up to 45,000 Lbf of thrust at sea level. With an operational aircraft weight range from 25,989 lb to 37,084 lb both with internal stores and fuel [1], the difference is assumed to be usable fuel weight of 11,095 lb. As a result, the center of gravity (CG) only varies 4% from 0.34 to 0.38 in Mean Aerodynamic Chord (MAC, \(\bar{c}\)) a small variation considering a 43% change in ramp weight. Only the 37,084 lb aircraft configuration will be evaluated in this study[1]. The small variation in CG is likely due to fuel tanks centered about the CG, minimizing CG variation along the MAC as fuel is consumed.
1.2.2 Control Surface Configuration

Figure 1.1 shows all the control surfaces on the ICE aircraft, Table 1.1 has all of the symbols, labels and deflection limits, and Table 1.2 briefly describes the role of each control surface. The 14 control effectors of various designs incorporated into the airframe including a 3-D thrust vectoring nozzle to control the aircraft. Starting at the front, four leading edge flaps are positioned in pairs on the inboard and outboard half of the wing section which can all operate independently. The inboard pair can deflect up and down whereas the outboard pair can only deflect downward like traditional plane leading edge flaps or slats. At the wingtips are unique rotating wing tips not found on conventional aircraft which can only rotate trailing edge downward; inboard on the upper surface of the wing are traditional slotted spoiler deflectors. The trailing edge comprises of two primary surfaces made up of elevons located midspan and a symmetric pitch flap surrounding the exhaust nozzle of the engine. The exhaust nozzle can vector the available thrust laterally and longitudinally as needed to sustain various maneuvers or steady state flight conditions.

Figure 1.1: Control Surface Location From [1]
Table 1.1: Control Surface Key

<table>
<thead>
<tr>
<th>Number</th>
<th>Symbol</th>
<th>Label</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\delta_{LILEF}$</td>
<td>Left Inboard Leading Edge Flap</td>
<td>$-40^\circ \leq \delta \leq 0^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>$\delta_{LOLEF}$</td>
<td>Left Outboard Leading Edge Flap</td>
<td>$-40^\circ \leq \delta \leq 40^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>$\delta_{LAMT}$</td>
<td>Left All Moving Wingtip</td>
<td>$-60^\circ \leq \delta \leq 0^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>$\delta_{LELE}$</td>
<td>Left Elevon</td>
<td>$-30^\circ \leq \delta \leq 30^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>$\delta_{LSSD}$</td>
<td>Left Spoiler Slot Deflector</td>
<td>$0^\circ \leq \delta \leq 60^\circ$</td>
</tr>
<tr>
<td>6</td>
<td>$\delta_{PF}$</td>
<td>Pitch Flap</td>
<td>$-30^\circ \leq \delta \leq 30^\circ$</td>
</tr>
<tr>
<td>7</td>
<td>$\delta_{RIBLEF}$</td>
<td>Right Inboard Leading Edge Flap</td>
<td>$-40^\circ \leq \delta \leq 0^\circ$</td>
</tr>
<tr>
<td>8</td>
<td>$\delta_{ROBLEF}$</td>
<td>Right Outboard Leading Edge Flap</td>
<td>$-40^\circ \leq \delta \leq 40^\circ$</td>
</tr>
<tr>
<td>9</td>
<td>$\delta_{RAMT}$</td>
<td>Right All Moving Wingtip</td>
<td>$-60^\circ \leq \delta \leq 0^\circ$</td>
</tr>
<tr>
<td>10</td>
<td>$\delta_{RELE}$</td>
<td>Right Elevon</td>
<td>$-30^\circ \leq \delta \leq 30^\circ$</td>
</tr>
<tr>
<td>11</td>
<td>$\delta_{RSSD}$</td>
<td>Right Spoiler Slot Deflector</td>
<td>$0^\circ \leq \delta \leq 60^\circ$</td>
</tr>
<tr>
<td>12</td>
<td>$\delta_{PTV}$</td>
<td>Pitch Thrust Vectoring</td>
<td>$-15^\circ \leq \delta \leq 15^\circ$</td>
</tr>
<tr>
<td>13</td>
<td>$\delta_{LTV}$</td>
<td>Lateral Thrust Vectoring</td>
<td>$-15^\circ \leq \delta \leq 15^\circ$</td>
</tr>
<tr>
<td>14</td>
<td>$\delta_{T\text{hrust}}$</td>
<td>* Thrust x 1000</td>
<td>$0 \leq \delta \leq 45 Lbf$</td>
</tr>
</tbody>
</table>

*: Sea Level Performance  - : Control Surface Edge Down  + : Control Surface Edge Up
Table 1.2: Control Surface Descriptions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{LILEF}$</td>
<td>Minor effect on pitching and roll axis</td>
</tr>
<tr>
<td>$\delta_{LOLEF}$</td>
<td>Minor effect on pitching and roll axis</td>
</tr>
<tr>
<td>$\delta_{LAMT}$</td>
<td>Major effect on roll and yaw axis</td>
</tr>
<tr>
<td>$\delta_{LEL}$</td>
<td>Major effect on all axes</td>
</tr>
<tr>
<td>$\delta_{LSSD}$</td>
<td>Major effect on roll and pitch axis</td>
</tr>
<tr>
<td>$\delta_{PF}$</td>
<td>Major effect on pitch axis</td>
</tr>
<tr>
<td>$\delta_{RIBLEF}$</td>
<td>Minor effect on pitching and roll axis</td>
</tr>
<tr>
<td>$\delta_{ROBLEF}$</td>
<td>Minor effect on pitching and roll axis</td>
</tr>
<tr>
<td>$\delta_{RAMT}$</td>
<td>Major effect on roll and yaw axis</td>
</tr>
<tr>
<td>$\delta_{REL}$</td>
<td>Major effect on all axes</td>
</tr>
<tr>
<td>$\delta_{RSSD}$</td>
<td>Major effect on roll and pitch axis</td>
</tr>
<tr>
<td>$\delta_{PTV}$</td>
<td>Major effect on pitch axis - dependant on trim thrust</td>
</tr>
<tr>
<td>$\delta_{LTV}$</td>
<td>Major effect on yaw axis - dependant on trim thrust</td>
</tr>
<tr>
<td>$\delta_{T\text{hrust}}$</td>
<td>Compensates for aerodynamic drag</td>
</tr>
</tbody>
</table>

*: Critical Surface  Note: All surfaces have coupled response, primary effect described

1.3 Configuration Aerodynamics

1.3.1 Data Collection

Utilizing the aerodynamic model provided by Lockheed Martin [1], 36^3 trim points were evaluated throughout the flight envelope to collect the necessary forces and moments with the CG at .36 mean aerodynamic chord to represent the aerodynamic characteristics presented below. This method is more representative of a digital flight test in place of sourcing the data directly from the 6 degree-of-freedom (DOF) coefficient build up tables for the data being within the Simulink model and was the most direct method to extract the desired data.
1.3.2 Lift Characteristics

With a highly swept low aspect ratio delta wing planform the lift curve slope \((a, C_{L\alpha})\) of 0.032 / °. Figure 1.2 shows that even at various mach flight conditions between 2.5° and 12.5° Angle of Attack (\(\alpha\), AoA) they all follow the similar slope. Beyond 17° various nonlinear effects account for the variation in slope and stall characteristics possibly due to turbulent vortex lift.

![Coefficient of Lift vs Angle of Attack](image)

Figure 1.2: \(C_L\) vs \(\alpha\)

1.3.3 Drag Characteristics

Figure 1.3 shows drag polar at specified Mach flight conditions where the solid lines are selected linear regions. The slope of each line corresponds to the induced drag coefficient where as the Y-axis intercept is the parasitic drag coefficient at each Mach.

![Coefficient of Drag vs Coefficient of Lift Squared](image)

Figure 1.3: Coefficient of Drag \((C_D)\) vs Coefficient of Lift Squared \((C_L^2)\)
Figure 1.4 shows a decreasing trend for parasitic drag coefficient ($C_{Do}$) indicative of a high speed design operating at off design point flight conditions. Induced drag coefficient ($C_{Di}$) decreases as Mach increases, showing an increased Oswald efficiency factor ($e$) with all other factors being constant or canceling out as illustrated by eq. 1.1 up to mach 1.2; after which $K$ begins to increase in a linear fashion at mach above 1.2 due to a decreasing $e$.

$$C_{di} = \frac{C_{L}^2}{\pi e AR}$$ \hspace{1cm} (1.1)

Figure 1.5 represents the Mach effects on $C_D$, the decreasing subsonic trend can be supported by the corresponding decrease in $C_{Di}$ and $C_{Do}$. Transonic wave drag effects are delayed due to the highly swept configuration diverging from the linearly converged subsonic drag coefficient at Mach 0.87, increasing until Mach 1.2 then leveling out around 0.025 a low supersonic drag coefficient, as described on pp. 454 in [10]. Supporting the design objective of a purpose built supersonic aircraft, reduced wing sweep or lower wing loading would incur a higher wave drag penalty with earlier transonic drag divergence and a steeper slope. The penalty of a relatively large $C_{Di}$ is the rate at which kinetic energy is dissipated during high AoA maneuvers; requiring significant fuel consumption to prevent the energy state of the aircraft from being depleted to unfavourable levels.
Figure 1.6 represents the optimal AoA and Mach flight condition at each altitude to achieve best lift to drag performance. Aligning with the low aspect ratio of 1.74 and 65° leading edge swept wing the lift to drag ratio’s peak values are comparable to YF-16 drag polar data from [11] resulting at an approximate peak lift to drag ratio of 11.2 at Mach 0.9. The ICE aircraft’s peak lift to drag ratio ($L/D$) is 10.9 at 1000ft only decreasing to 10.1 by 30,000ft, indicating that the most efficient flight conditions will occur at lower altitudes; a trend not typically observed with modern aircraft designs.

Figure 1.7 confirms the observations from Figure 1.6 that optimal performance occurs below 20,000ft, where lower angles of attack in the denser air allow for more efficient flight due to the high $C_{Di}$. Not considering fuel consumption factor on aircraft weight the ICE aircraft achieves up to 3 hours of loiter time or 1,400 miles of cruise range while consuming the estimated available fuel weight of 11,095 Lb, and thus true range and endurance may have improved results.
1.3.4 Pitching Moment Characteristics

Figure 1.8 represents the change in pitching moment curve due to Mach number. The slope decreases as Mach increases. Above Mach 0.5 and $C_L$ greater than 0.1 the pitching moment coefficient ($C_M$) becomes purely negative indicating the positive static stability in longitudinal axis. This change may occur due to the aerodynamic center shifting rearward because of the strengthening standing oblique shock wave formation on the aircraft’s lifting surface.

Figure 1.8: $C_M$ vs $C_L$

Figure 1.9 shows the aerodynamic center shift rearward as Mach increases independent of AoA. The movement in aerodynamic center is displayed in % Static Margin (SM) calculated by dividing $C_M$ by the $C_L$, as shown in eq. 1.2. The SM moves from approximately 0% at Mach 0.3 to approximately 8% at Mach 1.6 increasing the static stability of the aircraft.
Static Margin = \frac{C_M}{C_L} \tag{1.2}

**Figure 1.9:** \( SM \) vs Mach

1.4 Aircraft Performance Characteristics

A Pratt and Whitney F135 style engine is considered to generate thrust to support comparable sustained turn rate performance of 11.6 deg/sec of a F/A-18E at 15,000 ft as specified in Navy Aviation [12].

1.4.1 Thrust Model

For this study, the sea level static thrust was selected to be 45,000 Lbf with an inversely proportional altitude vs. thrust relationship as specified in eq. 1.3. Above Mach 1 subtle ramjet thrust effects are factored linearly to an additional 10% thrust at Mach 2.0. These propulsion factors were adjusted to reach the aforementioned performance goals within engineering reasoning and should not be representative of a known engine thrust performance.
Figure 1.10: Thrust Available

\[ T_A = T(M) \left( \frac{\rho}{\rho_{ssl}} \right)^4 \]  \hspace{1cm} (1.3)
At 15,000 ft, a sustainable Turn Rate ($\omega$) of 9.8 deg/sec was achieved at Mach 0.45 with an instantaneous $\omega$ of 21 deg/sec at Mach 0.72 as shown in Figure 1.11. Higher initial $\omega$ may be achieved at the cost of excessive energy bleed rates degrading the follow on maneuver capability, testing the limits of how fast energy can be recovered to re-engage.

Figure 1.11: 15,000 ft E-M Diagram
1.4.2 1-G Specific Excess Power

Representing the feasible flight envelope of the ICE aircraft, Figure 1.12 demonstrates the variation in available excess power with maximum thrust applied. The contour lines represent rate of climb in ft/min which determine the available maneuver at various flight conditions ranging from 50,000 ft/min to 6,000 ft/min at higher altitude flight conditions. The bold and dashed red lines represent the maximum and minimum $\bar{q}$ allowed for this analysis. Selected to allow for flight conditions are up to Mach 1.05 or a max $\bar{q}$ of 1,655 Psf and a minimum $\bar{q}$ of 65 Psf or Mach 0.21 at sea level equivalent to a $C_L$ of 0.7 at the max evaluated AoA up to 25°.

Figure 1.12: Specific Excess Power
Figure 1.13 represents the maximum sustainable climb angle with maximum thrust applied, ranging from a peak of 90° down to 10° near the edge of the flight envelope at higher altitudes. Enabling effective short field takeoff procedures and superior climb angles when operating out of improvised or geographically challenging airports.

1.4.3 Aircraft Specifications and Performance Summary

Table 1.3 is a summary of ICE aircraft properties, dimensions, performance, and limitations.
Table 1.3: Aircraft Specification Table

<table>
<thead>
<tr>
<th>Mass Properties</th>
<th>Aircraft Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weight</strong></td>
<td>37,084 Lbf</td>
</tr>
<tr>
<td><strong>$h_{CG}$</strong></td>
<td>36% $\bar{C}$</td>
</tr>
<tr>
<td><strong>$I_{XX}$</strong></td>
<td>42,576 slug – ft$^2$</td>
</tr>
<tr>
<td><strong>$I_{YY}$</strong></td>
<td>81,903 slugs – ft$^2$</td>
</tr>
<tr>
<td><strong>$I_{ZZ}$</strong></td>
<td>118,379 slug – ft$^2$</td>
</tr>
<tr>
<td><strong>$I_{XZ}$</strong></td>
<td>-525 slug – ft$^2$</td>
</tr>
<tr>
<td><strong>Empty Weight</strong></td>
<td>22,101 Lbf</td>
</tr>
<tr>
<td><strong>Fuel Weight</strong></td>
<td>11,095 Lbf</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$G_{\text{Instantaneous}}$</strong></td>
<td>9 $G$</td>
</tr>
<tr>
<td><strong>$G_{\text{Sustainable}}$</strong></td>
<td>6 $G$</td>
</tr>
<tr>
<td><strong>$\dot{\omega}_{\text{Instantaneous}}$</strong></td>
<td>26 $\frac{\circ}{s}$</td>
</tr>
<tr>
<td><strong>$\dot{\omega}_{\text{Sustainable}}$</strong></td>
<td>13 $\frac{\circ}{s}$</td>
</tr>
<tr>
<td><strong>Range$_{\text{Max}}$</strong></td>
<td>1,677 Miles</td>
</tr>
<tr>
<td><strong>Endurance$_{\text{Max}}$</strong></td>
<td>3 Hrs</td>
</tr>
<tr>
<td><strong>ClimbTime$_{30kFt}$</strong></td>
<td>38 Seconds</td>
</tr>
<tr>
<td><strong>ClimbTime$_{50kFt}$</strong></td>
<td>68 Seconds</td>
</tr>
</tbody>
</table>

* : SSL  ** : 10 kFt  $\otimes$ : Calculated  $\uparrow$ : Selected
1.5 Aerodynamic Data

Reference [1] states that data used to represent the tailless fighter aerodynamic model were extracted from five major wind tunnel test entries including: two low-speed tests at the Wright Laboratory Subsonic Aerodynamic Research Laboratory (SARL) 7x10 tunnel; a transonic test at the NASA LaRC 8-ft tunnel; a supersonic test at the NASA LaRC 4-ft unitary tunnel; and a rotary balance test at the Birhle Applied Research (BAR) vertical spin tunnel in Neuberg, Germany. The dynamic derivatives were collected for Mach $\geq 0.6$ from $0^\circ$ AoA to $90^\circ$, and aeroelastic flexibility is modeled from early F-16XL configuration with all moving wing tips. Similarly the computed hinge moment derivatives are modeled as F-16 and F-16XL computational predictions and pertinent wind tunnel data. Constructing an aerodynamic derivative database ranging from Mach .2 to 2.2, $\beta -30^\circ$ to $30^\circ$, and $\alpha -5^\circ$ to $90^\circ$

According to Etkin and Reid [13] lateral static stability is evaluated by the sign of the roll and yaw stiffness components. Negative $C_{l\beta}$ returns the aircraft to wings level by rolling in the direction of side slip. A positive $C_{n\beta}$ yaws the aircraft opposite the direction of side slip restoring a streamline aircraft orientation. The ICE aircraft exhibits unstable lateral stability characteristics according to Figure 1.14 and Figure 1.16, where $C_{l\beta}$ slope is negative for a most angles of attack except for for $-5$, 0 and a portion of $25^\circ$ where it is unstable. $C_{n\beta}$’s slope is exclusively negative, where any side slip angle will continue to increase without control input to oppose it. Included are the two control surfaces with control authority over the roll and yaw axis. The $\Delta C_{l\text{dELE}}$ in Figure 1.15 shows a positive rolling moment to the right trailing edge up deflection of the right side elevon as would be expected, the elevon’s control power is not effected by side slip angle. $\Delta C_{n\text{dSSD}}$ in Figure 1.17 displays a negative yawing moment yawing the aircraft to the right with some nonlinear variation due to side slip angle. For both axis the corresponding control power is enough to counter the the static lateral stability characteristics and return the aircraft to streamline flight. As long as the side slip angle is not allowed to increase beyond approximately $12^\circ$ the incorporation of the remaining control effectors will allow for additional control authority to regulate the undesirable static stability characteristics at larger side slip angles.
Figure 1.14: $C_l$ vs $\beta$

Figure 1.15: $\delta_{RELE} C_l$ vs $\beta$

Figure 1.16: $C_n$ vs $\beta$
1.5.1 Reference Flight Conditions

Table 1.4 reference flight conditions that span the feasible flight envelope for this study. These conditions allow for comparison between sub and supersonic flight conditions at low and high altitudes. Linearizing the general nonlinear 6 degrees of freedom equations of motion at a specific flight condition result in the linear open-loop dynamics, from which the local dynamic behaviour can be studied in terms of damping and frequency.

Table 1.4: Evaluation Flight Condition Table

<table>
<thead>
<tr>
<th>Evaluation Flight Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight Condition</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5*</td>
</tr>
</tbody>
</table>

*: Control System Design Point
Figure 1.18 shows the sustainable flight envelope limited by the maximum and minimum allowable $\bar{q}$ of 1,655 Psf and 65 Psf, allowing for supersonic flight from sea level and up to Mach 2.2 at 38,000 ft. Referencing the legend in Figure 1.18 will specify the location of each flight condition as specified in Table 1.4 with the yellow dots. Where as the red, green, blue, and purple dots all specify peak performance flight conditions for their specified condition. The initial design point flight condition was selected to be Mach 0.5 at 15,000 ft altitude as this flight condition will be regularly flown on any given mission.
1.6 Handling Qualities & Design Requirements

Presented in Table 1.5, 1.6 and Figure 1.19, the following Level 1 handling quality requirements were selected to meet the most stringent based on the MIL-8785C from [14] which applied to the ICE aircraft. As a result the following flight dynamics will be augmented and the presented requirements will be interpreted into relevant rise times to be applicable to a fly by wire flight control system which will be evaluated in Chapter 5.1.

Table 1.5: MIL 8785C - Long HQ Req

<table>
<thead>
<tr>
<th>Level</th>
<th>Phugoid $\zeta$</th>
<th>Flight Path Stab</th>
<th>Short Period $\zeta$ Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\zeta \geq .04$</td>
<td>.06 $\frac{\text{Deg}}{\text{Knot}}$</td>
<td>.35 1.3</td>
</tr>
<tr>
<td>2</td>
<td>$\zeta \geq 0$</td>
<td>.15 $\frac{\text{Deg}}{\text{Knot}}$</td>
<td>.25 2</td>
</tr>
<tr>
<td>3</td>
<td>$T_2 \geq 55$</td>
<td>.25 $\frac{\text{Deg}}{\text{Knot}}$</td>
<td>.15 -</td>
</tr>
</tbody>
</table>

Table 1.6: MIL 8785C - Lat./Dir. HQ Req

<table>
<thead>
<tr>
<th>Mode</th>
<th>Level 1</th>
<th>Level 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll $T_c$</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>Spiral</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>$T_2$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Hybrid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dutch Roll</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roll Sideslip</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Roll Performance - Phase CO

<table>
<thead>
<tr>
<th>Level 1</th>
<th>$0.2 \leq M \geq 0.3$</th>
<th>$0.3 \leq M \geq 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll $T$</td>
<td>1.4 [s]</td>
<td>1 [s]</td>
</tr>
</tbody>
</table>

Figure 1.19: Class 4 Short Period $\omega$ Req
CHAPTER 2

OPEN-LOOP FLIGHT DYNAMICS

2.1 Overview

The linearized model representing the aircraft operating at a particular trimmed flight condition is obtained via numerically linearizing the nonlinear 6-DOF Simulink model. The linearized model is also of a paramount importance to understand the dynamics of aircraft and the stability of the motion, which is an a priori assessment for successful control system design synthesis. This chapter summarizes the key flight dynamic characteristics from the analysis based on linearized models.

2.2 6-DOF State Space Realization

A linearized 6-DOF full order system model has been generated at 46,656 points within the limits of the achievable flight envelope. Altitude ranges from 0 to 60,000 ft, Mach ranging from 0.2 to 2.2 and AoA ranging from $-5^\circ$ to $25^\circ$ with 36 steps between each variables end point. At every flight condition the ‘linmod’ function in Matlab extracted a continuous time linear open loop state space model with the control effector inputs as shown in Table 2.1 and eq. 2.1. The ICE Simulink aerodynamic model initial conditions were set to zero body rates and fared control surface position at each linearization point. Thrust was the only input varied with ‘fmincon’ to minimize axial body acceleration within 0.1 ft/sec$^2$ of equilibrium. Trim vs. no trim control deflection did not impact resulting available linear model control power when compared. Five flight conditions as specified in Table 1.4 will be used to compare dynamics and the following control system performance in a linear analysis and design.
2.3 System Scaling

Before proceeding to analysis and design using the linear state-space model in Table 2.1, it is convenient to rearrange the order of the state variables into the longitudinal and the lateral-directional motion variables via coordinate transformation. Further, it is important to properly scale the model for accurate representation of the system dynamics. Scaling is performed on the system matrix for units among the state variables to be comparable, and on the input matrix for normalizing the input variable by the maximum deflection limit of each control effector. The scaling process can be expressed by the following eq. 2.1. Scaling the linear system does not change the eigenvalues but does change the eigenvectors, and the modal characteristics can be studied via the eigenvector diagram. (see Figure 2.2 and 2.3)

<table>
<thead>
<tr>
<th>Table 2.1: Full Order State Space Realization</th>
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</thead>
<tbody>
<tr>
<td>Vt</td>
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<tr>
<td>------------------</td>
</tr>
<tr>
<td>-0.0044179</td>
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<table>
<thead>
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<th>RAMT</th>
<th>REL</th>
<th>VecThrY</th>
<th>Thrust</th>
</tr>
</thead>
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<td>0.00042272</td>
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<td>-0.015428</td>
<td>-0.0096291</td>
<td>-0.0026697</td>
<td>-0.015428</td>
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<td>0</td>
</tr>
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<td>-0.00032992</td>
<td>-0.002129</td>
<td>-0.0012766</td>
<td>-0.00032992</td>
<td>-0.002129</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4.0149e-06</td>
<td>-3.0954e-05</td>
<td>0</td>
<td>4.0149e-06</td>
<td>3.0554e-05</td>
<td>0.00037466</td>
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<tr>
<td>0.00057034</td>
<td>0.0026097</td>
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<td>-8.4347e-06</td>
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<tr>
<td>-2.8833e-05</td>
<td>6.6112e-05</td>
<td>0</td>
<td>2.8833e-05</td>
<td>-6.6112e-05</td>
<td>-0.00060403</td>
<td>0</td>
</tr>
</tbody>
</table>
\[ \dot{x} = Ax + Bu \]

Where

\[ x = [X \ Y \ Z \ u \ v \ w \ \Phi \ \Theta \ \Psi \ p \ q \ r] \]

\[ u = [\delta_{\text{LILEF}} \ \delta_{\text{LOLEF}} \ \delta_{\text{LAMT}} \ \delta_{\text{LER}} \ \delta_{\text{PF}} \ \delta_{\text{ROLEF}} \ \delta_{\text{RMT}} \ \delta_{\text{REL}} \ \delta_{\text{SSD}} \ \delta_{\text{PTV}} \ \delta_{\text{LTV}} \ \delta_{\text{Th}}] \]

\[ \dot{x} = (S_x^{-1} A S_x) x + (S_x^{-1} B S_u) u = A_s x_s + B_s u \]

\[ S_x = \text{diag}[1 1 \ \frac{\pi}{180} \ \frac{\pi}{180} \ 1 1 \ \frac{\pi}{180} \ \frac{\pi}{180} \ \frac{\pi}{180} \ \frac{\pi}{180} \ 1], \ S_x \in R^{12 \times 12} \]

\[ S_u = \text{diag}[\frac{1}{40} \ \frac{1}{40} \ \frac{1}{60} \ \frac{1}{60} \ \frac{1}{30} \ \frac{1}{30} \ \frac{1}{40} \ \frac{1}{40} \ \frac{1}{60} \ \frac{1}{60} \ \frac{1}{15} \ \frac{1}{15}], \ S_u \in R^{14 \times 14} \]

\[ u : \text{SurgeVelocity} \]

\[ v : \text{SideVelocity} \]

\[ w : \text{HeaveVelocity} \]

\[ p : \text{RollRate} \]

\[ q : \text{PitchRate} \]

\[ r : \text{YawRate} \]

\[ \Phi : \text{BankAngle} \]

\[ \Theta : \text{PitchAngle} \]

\[ \Psi : \text{HeadingAngle} \]

\[ X : \text{NorthPosition} \]

\[ Y : \text{EastPosition} \]

\[ Z : \text{VerticalPosition} \]

(2.1)

2.4 Full vs Reduced-order Linear Models

Traditional flight dynamic analysis and control system design starts from the decoupled longitudinal and lateral-directional dynamics. For the ICE aircraft linear model, longitudinal vs. lateral-directional decoupling in the state matrix is apparent except for the roll and pitch coupling
but strong coupling does exist among all three axes through the control effectors see Table 2.1 and

\[ \dot{x}_{\text{Long}} = A_{\text{Long}} x_{\text{Long}} + Bu, \quad x_{\text{Long}} = [u \ w \ \Theta \ q]^T \]

\[ \dot{x}_{\text{LD}} = A_{\text{LD}} x_{\text{LD}} + Bu, \quad x_{\text{LD}} = [v \ p \ r \ \Phi \ \Psi]^T \] (2.2)

\[ \dot{x}_{\text{LLD}} = A_{\text{LLD}} x_{\text{LLD}} + Bu, \quad x_{\text{LLD}} = [u \ w \ \Theta \ q \ v \ p \ r \ \Phi \ \Psi]^T \]

Lower Order State Space Models

2.4.1 Design Flight Condition 5 - Modal Characteristics

The decoupled longitudinal system poles dependant on \([u \ w \ \Theta \ q]\) states in Figure 2.2 have negative real components resulting in damped oscillatory stable system response. The corresponding natural frequencies and damping ratios are not desirable for acceptable handling qualities as specified in Chapter 1. The damping values are too low and the natural frequencies do not allow the system to respond as desired. Faster longitudinal pair poles are categorized as short period where the slower pole pair are the phugoid, corresponding with the pitch angle response and the slower airspeed response.

\[ \text{Table 2.2: Longitudinal Mode Damping} \]

<table>
<thead>
<tr>
<th>Pole</th>
<th>Damping</th>
<th>Frequency</th>
<th>Time Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.51e-03 + 6.36e-02</td>
<td>7.07e-02</td>
<td>6.37e-02</td>
<td>2.22e+02</td>
</tr>
<tr>
<td>-4.51e-03 - 6.36e-02</td>
<td>7.07e-02</td>
<td>6.37e-02</td>
<td>2.22e+02</td>
</tr>
<tr>
<td>-4.97e-01 + 1.28e+01</td>
<td>3.62e-01</td>
<td>1.37e+00</td>
<td>2.01e+00</td>
</tr>
<tr>
<td>-4.97e-01 - 1.28e+01</td>
<td>3.62e-01</td>
<td>1.37e+00</td>
<td>2.01e+00</td>
</tr>
</tbody>
</table>
The lateral directional poles dependant on \([v \ p \ \Phi \ r]\) are real positive and negative component values in Figure 2.3 indicating a combination of over-damped stable and unstable system responses. Any unstable modal responses will result in divergence of the entire system from the given initial state and will need to be augmented to achieve the desired lateral directional response characteristics as specified in Chapter 1. Categorization of the poles in the lateral-directional dynamics is less intuitive without a vertical stabilizer to specify a traditional dutch roll mode pair. Roll and spiral mode are observed to be stable and correlate to the fastest pole and second fastest stable poles. As advised the two unstable poles are a version of a dutch roll mode indicating that \(P\) and \(R\) rate disturbances will diverge from this initial flight condition.

**Table 2.3: Lateral Directional Model Damping**

<table>
<thead>
<tr>
<th>Pole</th>
<th>Damping</th>
<th>Frequency</th>
<th>Time Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1.67e-01)</td>
<td>1.00e+00</td>
<td>1.67e-01</td>
<td>5.99e+00</td>
</tr>
<tr>
<td>(2.48e-01 + 1.12e-01i)</td>
<td>(-9.12e-01)</td>
<td>2.72e-01</td>
<td>-4.03e+00</td>
</tr>
<tr>
<td>(2.48e-01 - 1.12e-01i)</td>
<td>(-9.12e-01)</td>
<td>2.72e-01</td>
<td>-4.03e+00</td>
</tr>
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<td>(-1.05e+00)</td>
<td>1.00e+00</td>
<td>1.05e+00</td>
<td>9.51e-01</td>
</tr>
</tbody>
</table>

Evaluating the coupled system dependant on \([u \ w \ \Theta \ q \ v \ p \ \Phi \ r]\) in Table 2.4 the modal damping characteristics for longitudinal and lateral directional poles are intermixed within the system results, with only minor variation in pole locations from the decoupled results. Indicating how each mode is principally decoupled from the other, allowing for independent control and stabilization with appropriate control surface allocation. Observe Figure 2.1 to view all five evaluation flight conditions open loop pole locations for comparison.

**Table 2.4: Full Order Model Damping**

<table>
<thead>
<tr>
<th>Pole</th>
<th>Damping</th>
<th>Frequency</th>
<th>Time Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5.17e-03 + 6.34e-02i)</td>
<td>8.13e-02</td>
<td>6.36e-02</td>
<td>1.93e+02</td>
</tr>
<tr>
<td>(-5.17e-03 - 6.34e-02i)</td>
<td>8.13e-02</td>
<td>6.36e-02</td>
<td>1.93e+02</td>
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<tr>
<td>(-1.67e-01)</td>
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<td>(2.49e-01 + 1.13e-01i)</td>
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<tr>
<td>(2.49e-01 - 1.13e-01i)</td>
<td>(-9.11e-01)</td>
<td>2.73e-01</td>
<td>-4.02e+00</td>
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<tr>
<td>(-1.05e+00)</td>
<td>1.00e+00</td>
<td>1.05e+00</td>
<td>9.52e-01</td>
</tr>
<tr>
<td>(-4.97e-01 + 1.23e+001)</td>
<td>3.62e-01</td>
<td>1.37e+00</td>
<td>2.01e+00</td>
</tr>
<tr>
<td>(-4.97e-01 - 1.23e+001)</td>
<td>3.62e-01</td>
<td>1.37e+00</td>
<td>2.01e+00</td>
</tr>
</tbody>
</table>
2.4.2 Modal Characteristics

The stability of the linear systems depends only on the poles which are the eigenvalues of the system matrix. The ICE aircraft linear model eigenvalues at the reference flight condition 5 are listed in Figure 2.4. At this flight condition, the ICE aircraft has typical short-period characteristics but almost periodic unstable phugoid. Figure 2.2 shows the vector diagram (or Argand Diagram) depicting the primary aircraft motion variables (or state variables) associated with the longitudinal dynamics.
The lateral-directional dynamics has unstable oscillatory mode for lack of static stability properties as described in Chapter 1 in Figure 1.16, describing the instability due to the yawing moment stability derivative $C_{n\beta}$, and two over-damped stable poles. The unstable mode is composed of all coupled lateral-directional motion variables, as shown in Figure 2.3, while the two periodic stable modes are similar to a typical spiral and roll convergence mode. The spiral mode is slow dynamics involving $\Phi$ and $\Psi$, while the roll convergence mode is relatively fast with the primary motion variables involving $P$ and $\Phi$. The side-slip involvement in the roll mode is rather substantial than that of a conventional wing-body-tail configuration. Figure 2.4 shows also the modal response characteristics for lateral-directional dynamics.
Figure 2.3: Lateral Directional Mode Dependency

Figure 2.4: Lateral Directional Mode Convergence
The eigenvalues of the coupled longitudinal and lateral-directional dynamics exhibit noticeable differences on the stability of phugoid mode. Coupling effect between the longitudinal and lateral-directional dynamics can be examined through the modal decomposition of the state matrix as shown in the Figure 2.5 below. The phugoid mode shows noticeable coupling with the bank and heading angle.

![Figure 2.5: State To Mode Relationship](image)

2.5 Open Loop Effector Step State Response

The open loop step response of the longitudinal and lateral directional state responses in Figures 2.7 and 2.6, show the natural dynamic response to the system from the seven selected stepped control inputs. The longitudinal dynamics $Q$ state damps out faster as it has short period characteristics while the $U$ state continues to oscillate with minimal damping demonstrating phugoid characteristics. The lateral directional states $\beta$ and $P$ both diverge from equilibrium, following the unstable response of due to positive pole locations observed in Table 2.3.
Figure 2.6: Longitudinal State Step Response

Figure 2.7: Lateral Directional State Step Impulse Response
2.5.1 Modal Controllability

By transforming the state-space in the modal coordinates as represented in eq. 2.3, the controllability of a particular mode can be investigated.

\[ \dot{x}_m = \Lambda_m x_m + B_m u \]  

Figure 2.8 presents the longitudinal control effector contribution to the phugoid and short period modes as specified in Figure 2.2. The control effectors $\delta_{LELE}$, $\delta_{PF}$ and $\delta_{RELE}$ have predominant control over the short period and notably less control over phugoid mode, whereas effector $\delta_{Thrust}$ engine thrust has predominant control over phugoid mode and no control over short period. This supports intuition that body rate and orientation control is achieved via surface deflection and flight path angle is controlled via thrust.

Figure 2.8: Longitudinal Modal Controllability

Figure 2.9 represents the modal control contributions of each control effector on the lateral directional modes. The three identifiable modes according to Figure 2.3 are Heading, Spiral and Roll Convergence or $\lambda_1$, $\lambda_2$, and $\lambda_5$ in this figure. $\lambda_3$ and $\lambda_4$ are the unstable pair taking the place of
the conventional stable dutch roll mode, due to the lack of a vertical stabilizer to inherently counter side slip divergence. By observation the most controllable modes are $\lambda_2$, $\lambda_3$ and $\lambda_4$, as shown in Figure 2.9, allowing for the correlating contributing control effectors to augment their open loop dynamics.

![Modal Controllability: Lateral/Directional Modes](image)

Figure 2.9: Lateral Directional Controllability

With decoupled control understood, Figure 2.10 present the complicating facet of this aircraft configuration with the normalized absolute controllability contribution of each effector to each mode displayed below. Note the change in eigenvalue order, once coupled mode correlation to states is not intuitive; reference the key for identification in conjunction with Figure 2.8 and Figure 2.9. Several effectors have minimal contribution to any mode, where as other’s have large relative contributions to both longitudinal and lateral directional modes. This indicates that in addition of stabilizing a desired mode an effector also has to reject disturbances from other effectors as most have a coupled modal effect on the system. Note that the unique control effector to mode relationships change with flight condition, making eigenvector derived fixed control allocation via state pairing is limited to a local flight condition solution.
2.5.2 State Control

With the individual state and control effectors modal dependencies described, the resulting regulated state control is presented in Figure 2.11 and 2.12. Where the control matrix from the linearized system is normalized along each row to highlight the control inputs which have the largest control power over each state. Clearly representing the primary inputs required to successfully regulate the ICE aircraft’s longitudinal and lateral directional states.
Figure 2.11: Longitudinal State Control

Figure 2.12: Lateral Directional State Control
2.5.3 PID Control Input Selection

Summarizing the presented modal and state control data control input combinations may now be selected to control all necessary states. Where control effectors $\delta_{LELE}, \delta_{PF}, \delta_{RELE}$ and $\delta_{Thrust}$ clearly have the most longitudinal modal and state control authority; the selection of effectors $\delta_{LAMT}, \delta_{LELE}, \delta_{RAMT}, \delta_{RELE}$ and $\delta_{LTV}$ is less intuitive to control the lateral directional states. Particularly use of effectors $\delta_{LAMT}$ and $\delta_{RAMT}$ as they have less state control over yaw rate but more modal control over the unstable poles $\lambda_4$ and $\lambda_5$ than effectors $\delta_{LSSD}$ and $\delta_{RSSD}$. Therefore the addition of $\delta_{LAMT}$ and $\delta_{RAMT}$ will improve yaw rate suppression and have a larger effect on $\lambda_4$ and $\lambda_5$ than effectors $\delta_{LSSD}$ and $\delta_{RSSD}$ would. In addition effectors $\delta_{LAMT}$ and $\delta_{RAMT}$ decrease state coupling as they have relatively little effect on $\beta$ compared to $\delta_{LSSD}$ and $\delta_{RSSD}$, and the disturbance to roll can be easily compensated for by effectors $\delta_{LELE}$ and $\delta_{RELE}$. The remaining effectors are not utilized to minimize control distribution complexity. This combination provide an adequate foundation to develop a PID control system. Additionally, within the nonlinear model additional effectors are experimented with to improve deceleration and reduce control surface trim deflections at various flight conditions.

2.5.4 LQ Control Input Selection

Specifying control allocation for LQ control systems is not required as optimal control and state weights values handle control allocation to achieve desired state response times.

2.6 Flight Condition Pole Migration

As flight conditions change via Mach, Altitude, AoA, flight path angle, bank angle, center of gravity the dynamics of the aircraft change. To better understand how the primary flight condition variables (Mach, Altitude, AoA) impact the dynamics of 1 G flight as well as climbing and descending, the available linearized models have been evaluated within the end points of each variable to construct Altitude and Mach sweeps for the AoA ranges of $-20^\circ$ to $20^\circ$ to convey open loop system changes via pole locations change.
2.6.1 Altitude Sweep

Longitudinal

Figure 2.13 shows how the phugoid pole locations migrate due to altitude and flight path angle variation. Climbing flight decreases phugoid damping where descending flight increases phugoid damping of the open loop system. Overall the phugoid poles are not impacted by altitude as much as the short period poles are in Figure 2.14. Exhibiting little variation due to flight path angle and significant pole migration due to change in altitude, increasing the damping and natural frequency of the short period mode. Figure 2.15 is an alternative pole migration figure conveys the change for all longitudinal poles in a simple pole migration relationship due to altitude change. Of the four poles plotted, the short period poles can be seen to move the most when comparing the initial and final values to determine migration direction in the pole figures below.

![Longitudinal Phugoid Pole Migration - Altitude Sweep](image1)

**Figure 2.13: Phugoid Pole Migration - Altitude**

![Longitudinal Short Period Pole Migration - Altitude Sweep](image2)

**Figure 2.14: Short Period Pole Migration - Altitude**
Lateral Directional

Unlike the longitudinal poles the lateral-directional poles do not migrate significantly due to change in flight path angle as seen in Figure 2.16. Variation in altitude does have a stabilizing effect on the four lateral directional poles, likely due to the increased trim AoA as the air density decreases.

Figure 2.15: Alternative Longitudinal Pole Migration - Altitude

Figure 2.16: Lateral Directional Pole Migration - Altitude
2.6.2 Mach Sweep

Longitudinal

The Mach effects on the longitudinal phugoid poles are shown in Figures 2.18 which only vary slightly due to change in flight path angle. Figure 2.20 shows how the phugoid pair splits off of the real axis at Mach 1 where the poles become more sensitive to flight path angle as described in Figure 2.18. The short period poles like phugoid are not sensitive to change in flight path angle see Figure 2.19, but in contrast vary significantly as Mach increases from 0.2 to 1.2 then continue to move away from the imaginary axis at a lower rate as described in Figure 2.20.
Lateral-Directional

The lateral-directional poles offer a unique challenge as they don’t follow a uniform migration pattern nor consistently pair together as the system changes. So all four poles are plotted on the same Figure 2.21 which shows little movement due to change in flight path angle but large movement due to change in Mach. Figure 2.22 shows two poles almost symmetrically migrating from the imaginary axis in opposite directions destabilizing the system.
Figure 2.21: Lateral Directional Pole Migration - Mach

Figure 2.22: Alternative Lateral Directional Pole Migration - Mach
CHAPTER 3

PID CONTROL SYSTEM DESIGN AND ANALYSIS

3.1 Introduction

The linearized plant model representing the ICE aircraft has been obtained via linearizing a nonlinear Simulink based aerodynamic model. The full aerodynamic model has 14 separate control inputs with 21 state outputs. For the purpose of PID design study in this thesis only 7 inputs and 4 output states are being considered. Selection of the necessary inputs was done via control power evaluation at the selected design point flight condition. The feedback design results in four channel closed-loop system constructing a multi-input-multi-output (MIMO) PID control system around the full order linear ICE model. The body $Q$, $U$, $P$ and $\beta$ states were selected to be fed-back to their perspective controllers as feedback variables. Each controller commands a set of control effectors with minimal cross axis coupling and notable control authority in order to minimize any resulting coupled state disturbances, which are regulated by the corresponding axis controllers. The closed-loop control responses will be evaluated for acceptable handling qualities to determine performance of the gain scheduled PID control system design. The closed-loop system performance is evaluated at five flight conditions as specified in Table 1.4 by performing a linear model simulation with a uniform series of state command inputs.

3.2 PID Control Law Design

3.2.1 Control System Block Diagram

Tracking the control signal from input to output the state control command (CMD) passes through the handling quality (HQ) filter to remove unwanted high rate or abrupt system inputs. Then the state error is computed by calculating the difference between the current system output
state to the desired CMD state. The error is passed into the gain scheduled PID state controllers to compensate for the existing error in an effort to reduce it. Depending on the controller various combinations of proportional, integral, and derivative gains are relied on to regulate each state dependant on the corresponding airspeed, altitude, and angle of attack which specify the correct gain values to use. The resulting output is distributed to the necessary corresponding control effector inputs on the ICE Simulink model working in unison to control the system. Figure 3.1 depicts the flow of each system state and control input distribution.

![PID Block Diagram](image)

**Figure 3.1: Gain Scheduled PID Block Diagram**

3.2.2 Control Distribution Matrix

To specify the control distribution within the PID system a matrix was generated to specify the direction and distribution of each given state control. This facilitated the specific control distribution necessary to regulate each state as described in Chapter 2 reference Appendix A to see how control distribution is implemented. Note how $\beta$ control is only distributed to $\delta_{RAMT}$ and $\delta_{LTV}$ in the linear model but functions as a mirrored system in the nonlinear model. With the inability to limit control deflection in a linear model the $\delta_{RAMT}$ was left out to prevent symmetric $\delta_{LAMT}$ and $\delta_{RAMT}$ deflection which would have prevented the intended yaw control. Allowing for the correct gains to be obtained based on the control power available when deflecting the $\delta_{LAMT}$ independently from the $\delta_{RAMT}$ to dampen out side slip disturbances.
3.2.3 Controller Component Definition

Proportional Control

Proportional control responds to state error with a gain scheduled proportional response, accelerating or delaying the response of a state depending of the flight condition. The proportional gain controls the magnitude and direction of the stabilizing controller response. The proportional gain value can be scheduled with any given flight condition to compensate for plant variation in an effort to maintain consistent control response characteristics.

Integral Control

Integral control compensates for accumulated steady state error over time increasing the commanded response/deflection from the controller in a gain controlled proportion to the accumulated state error to eliminate steady state error. The integral gain value controls the magnitude and direction of the stabilizing integral control response and can be scheduled with flight condition to maintain consistent control responses.

Derivative Control

Derivative control compensates for the rate at which the state error is changing, opposing rapid state changes in proportion to the selected derivative gain value. Large derivative gain values can increase the damping of the system which can be scheduled with flight condition to maintain
consistent control responses.

Controller Component Summary

Between each components of the proportional integral derivative controller gains, the response of any causal or proper transfer function based controller can be successfully regulated. Utilizing the `pidTuner` function in Matlab allowed for a convenient initial tuning method to set the PID gain values. By experimenting with various desired controller bandwidth and phase margin values a satisfactory response was obtained for each control channel. Improper transfer function controllers can be constructed to compensate for non-causal systems but their performance is dependant on the pole location variances / modelling errors and is not recommended for robust controller designs.
3.2.4 Q Channel Control Law

The pitch rate control law is comprised of a second order proportional integral derivative controller, exhibiting no overshoot with a settling time of 0.67 seconds and no steady state error. The corresponding gains and additional response times are presented in Table 3.1 and Figure 3.3.

\[
\frac{Q_{CTRL}}{Q_{ERROR}} = -77.69s^2 - 849.2s - 2320
\]

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<th>Table 3.1: Q Step Info</th>
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<tr>
<td>Settling Time</td>
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<td>Overshoot</td>
</tr>
<tr>
<td>P Gain</td>
</tr>
<tr>
<td>I Gain</td>
</tr>
<tr>
<td>D Gain</td>
</tr>
</tbody>
</table>

Figure 3.3: Q Channel Step Response

3.2.5 U Channel Control Law

The airspeed control law is comprised of a second order proportional integral derivative controller, exhibiting no overshoot a settling time of 10.53 seconds with no steady state error. The corresponding gains and additional response times are presented in Table 3.2 and Figure 3.4.

\[
\frac{U_{CTRL}}{U_{ERROR}} = 1295s^2 + 660.5s + 6.475
\]

<table>
<thead>
<tr>
<th>Table 3.2: U Step Info</th>
</tr>
</thead>
<tbody>
<tr>
<td>U Step Info</td>
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<td>Settling Time</td>
</tr>
<tr>
<td>Overshoot</td>
</tr>
<tr>
<td>P Gain</td>
</tr>
<tr>
<td>I Gain</td>
</tr>
<tr>
<td>D Gain</td>
</tr>
</tbody>
</table>

Figure 3.4: U Channel Step Response
3.2.6 P Channel Control Law

Roll rate control law is comprised of a second order proportional integral derivative controller, exhibiting 10.6 % overshoot and a settling time of 2.18 seconds with no steady state error. The corresponding gains and additional response times are presented in Table 3.3 and Figure 3.5.

\[
P_{CTRL} = \frac{16.22s^2+45.42s+31.79}{s}
\]

Table 3.3: P Step Info

<table>
<thead>
<tr>
<th>P Step Info</th>
<th>Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.301</td>
</tr>
<tr>
<td>Settling Time</td>
<td>2.18</td>
</tr>
<tr>
<td>Overshoot</td>
<td>10.684</td>
</tr>
<tr>
<td>P Gain</td>
<td>31.791</td>
</tr>
<tr>
<td>I Gain</td>
<td>45.416</td>
</tr>
<tr>
<td>D Gain</td>
<td>16.22</td>
</tr>
</tbody>
</table>

Figure 3.5: P Channel Step Response

3.2.7 Beta Channel Control Law

The side slip angle control law is comprised of a first order proportional integral controller, exhibiting 21.2 % overshoot with a settling time of 3.39 seconds and no steady state error. The corresponding gains and additional response times are presented in Table 3.4 and Figure 3.6.

\[
\beta_{CTRL} = \frac{1.954s+7.6}{s}
\]

Table 3.4: \(\beta\) Step Info

<table>
<thead>
<tr>
<th>Beta Step Info</th>
<th>Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.846</td>
</tr>
<tr>
<td>Settling Time</td>
<td>3.399</td>
</tr>
<tr>
<td>Overshoot</td>
<td>21.227</td>
</tr>
<tr>
<td>P Gain</td>
<td>7.6003</td>
</tr>
<tr>
<td>I Gain</td>
<td>1.9543</td>
</tr>
<tr>
<td>D Gain</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.6: \(\beta\) Channel Step Response
3.2.8 Actuator and Engine Transfer Functions

The actuator transfer function is proper, with an over-damped response and a settling time of approximately 0.111 seconds. This allows for nearly uninhibited control actuation for all dependant state channels, response times are summarized in Table 3.5 and Figure 3.7.

\[
\frac{dS_{TEA}}{dSCMD} = \frac{4000}{s^2+140s+4000}
\]

Table 3.5: TEA Step Info

<table>
<thead>
<tr>
<th>TEA Step Info</th>
<th>Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.062</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.111</td>
</tr>
<tr>
<td>Overshoot</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.7: Actuator Step Response

The leading edge control surface actuator transfer function is proper, with an over-damped dynamic response and a settling time of 0.228 seconds. This allows for nearly uninhibited control actuation for leading edge flaps command inputs, response times are summarized in Table 3.6 and Figure 3.8.

\[
\frac{dS_{LEA}}{dSCMD} = \frac{180}{(s+18)(s+100)}
\]

Table 3.6: LEA Step Info

<table>
<thead>
<tr>
<th>LEA Step Info</th>
<th>Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.125</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.228</td>
</tr>
<tr>
<td>Overshoot</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.8: Actuator Step Response
The engine dynamics are represented as a proper system, with an over-damped dynamic response and a settling time of 9.78 seconds for a high performance low bypass turbofan. Representing the time needed for an engine to spool from idle to max thrust at sea level the response times are summarized in Table 3.7 and Figure 3.9, reference Figure 1.3 for thrust available performance.

\[
\frac{\text{Thrust}}{\text{Thrust}_{\text{CMD}}} = \frac{1}{s^{2.5}+1}
\]

Table 3.7: Engine Step Info

<table>
<thead>
<tr>
<th>Engine Step Info</th>
<th>Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>5.493</td>
</tr>
<tr>
<td>Settling Time</td>
<td>9.78</td>
</tr>
<tr>
<td>Overshoot</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.9: Engine Step Response

3.2.9 Control Input Filters

The ICE aircraft command inputs are filtered to achieve the desired handling qualities and prevent excitations of high frequency control system modes. The pitch and roll commanded rates use the same filter with a rise time of 0.38 seconds and a settling time of 0.677 seconds. The side slip command input goes through a slightly slower filter with a rise time of 0.591 seconds and a settling time of 1.05 seconds, see Table 3.8 and Figure 3.10 for comparison. Damping of the above inputs prevents controller saturation and excessive G onset. Airspeed command inputs are passed through a filter with a rise time of 5.9 seconds and a settling time of 10.52 seconds, see Table 3.9 and Figure 3.11 for comparison. Gradual variation of commanded airspeed prevents the engine from lagging behind the commanded state and regulates the longitudinal G onset, which can near 1 G in some low altitude and slow flight conditions.
Table 3.8: Control Step Info

<table>
<thead>
<tr>
<th>Control HQ</th>
<th>P/Q Time Sec</th>
<th>Beta Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.38</td>
<td>0.591</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.677</td>
<td>1.052</td>
</tr>
<tr>
<td>Overshoot</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.9: Airspeed Step Info

<table>
<thead>
<tr>
<th>Airspeed HQ</th>
<th>Time Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>5.908</td>
</tr>
<tr>
<td>Settling Time</td>
<td>10.523</td>
</tr>
<tr>
<td>Overshoot</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.10: Desired Control HQ

Figure 3.11: Engine Input CMD Filter

Applying the above state channel controllers and control input filters to their appropriate channels, results in acceptable performance found in Figure 3.12. No excessive control power is observed in $\delta_{PF}$ through $\delta_{LSSD}$ subplots to achieve the desired state responses in $Q$ through $P$ subplots in the desired response times as specified in Table 3.8 and Table 3.9.
Figure 3.12: Linear Model PID Step Response
3.3 Generating PID Gain Schedule

To generate the PID gain schedule, a series of three nested loops were used to interpolate among 180 points of flight condition evaluated using Matlab’s ‘fmincon’ function associated with equality and cost constraints see eq. 3.1. Then, by setting the fixed design point PID gains in Table 3.1 to Table 3.4 as the initial condition the optimization is carried out with the 12 PID gain values to obtain the desired response characteristics as seen in Table 3.10. This process was repeated twice to further refine the system response times. Mach ranged from 0.2 to 1.91, Altitude ranged from 0 ft to 51,429 ft both with an index step size of 6, and AoA ranged from 0.142° to 20.7° with an index step size of 5 to generate a course initial gain value schedule. For the next gain scheduling step, each of the 12 PID gain schedule matrices were interpolated from the original 180 flight condition points to 1,448 points across the altitude and Mach surface where in between the AoA layers to reduced the index step size by half. Next extrapolation around each of the available points to establish a 2x2 window of median gain values around each of the 144 points on each surface. Finally, linear interpolation between each of the discontinuous gain windows was performed to generate the final gain value surface. See Figure 3.13 for a generic example to visualize the transition from the 144 islands of data to a continuous surface. This gain scheduling method generates a continuous gain value surface with reasonable contours to be referenced at any given flight condition. For the flight conditions that fall between available AoA surface layers, linear interpolation fills the gaps of the map during linear or non-linear model simulation allowing for continuous gain value variation, improving accuracy and consistency of the system response.

\[
J_{RT} = (Q_{RT} - \text{Des}Q_{RT})^2 + (\beta_{RT} - \text{Des}\beta_{RT})^2 + (P_{RT} - \text{Des}P_{RT})^2 + (U_{RT} - \text{Des}U_{RT})^2
\]
\[
J_{ST} = (Q_{ST} - \text{Des}Q_{ST})^2 + (\beta_{ST} - \text{Des}\beta_{ST})^2 + (P_{ST} - \text{Des}P_{ST})^2 + (U_{ST} - \text{Des}U_{ST})^2
\]
\[
J_{OS} = (Q_{OS} - \text{Des}Q_{OS})^2 + (\beta_{OS} - \text{Des}\beta_{OS})^2 + (P_{OS} - \text{Des}P_{OS})^2
\]
Briefly describing the inequality constraint components in eq. 3.2, $U_{OS}$ represents airspeed overshoot (OS) %, $Q_{OS}$ represents $Q$ overshoot %, $Q_{ST}$ represents $Q$ settling-time, $Q_{RT}$ represents $Q$ rise-time. For inequality constraints to be satisfied within 'fmincon' each component must remain negative. So the upper and lower bounds are scripted opposite to enable the reverse constraint. The numerical values specified next to each component is the corresponding bound with units of seconds or % if an overshoot constraint. Refer to Appendix A to see entire constraint function.

\[
C(1) = U_{OS} - 5 \\
\ldots \\
C(25) = .125 - Q_{RT}
\]

### Table 3.10: State Response Requirements

<table>
<thead>
<tr>
<th>Response Req</th>
<th>Orientation Req Sec</th>
<th>Airspeed Req Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.55</td>
<td>6.5</td>
</tr>
<tr>
<td>Settling Time</td>
<td>1.1</td>
<td>13</td>
</tr>
<tr>
<td>Overshoot</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.13: Generic Gain Value Surface
3.3.1 Closed Loop PID Control System Pole Locations

Comparing the open loops pole locations of Figure 2.1 to the closed loop pole locations in Figure 3.14, all of the unstable poles have become stable moving to the left side of the pole zero plane. Enabling the desired response times to be obtained for each controlled state, stabilizing the initially unstable system.

![Closed Loop Pole Zero Map](image)

Figure 3.14: Pole Zero Location

3.3.2 PID Gain Schedule

Figure 3.15 represent the P, I and D gain variation throughout the flight envelope for an AoA equal to the design point flight condition. The other AoA gain surfaces compensate for the changes in system dynamics to achieve uniform response times throughout the flight envelope.
3.3.3 Linear Simulation State Response Comparison

Representing how uniform the response characteristics are for the gain scheduled closed loop PID control system is Table 3.11 and Table 3.12. Stating the rise time and overshoot for each state and the statistical variation of each state for the five selected flight conditions. Presenting
conclusive evidence of how well the iterative gain optimization process worked, with rise and settling time standard deviations only ranging from 0.027 to 0.569 seconds for the four controlled states. Overshoot standard deviation was even less comparatively, ranging from 0 to 2.45%.

<table>
<thead>
<tr>
<th>PID Flight Condition State Step Info</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Step Info</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>0.5M 1000ft 1deg</td>
</tr>
<tr>
<td>1M 1000ft 4deg</td>
</tr>
<tr>
<td>1.5M 50000ft 2deg</td>
</tr>
<tr>
<td>0.5M 35000ft 5deg</td>
</tr>
<tr>
<td>0.5M 15000ft 5deg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID State Step Response Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID Q Step Perf</td>
</tr>
<tr>
<td>Rise Time [Sec]</td>
</tr>
<tr>
<td>Settling Time [Sec]</td>
</tr>
<tr>
<td>Overshoot [%]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID P Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time [Sec]</td>
<td>5.845</td>
<td>5.895</td>
<td>5.916</td>
<td>0.027</td>
</tr>
<tr>
<td>Settling Time [Sec]</td>
<td>10.523</td>
<td>10.564</td>
<td>10.659</td>
<td>0.076</td>
</tr>
<tr>
<td>Overshoot [%]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3.3.4 Frequency Domain Evaluation

Figures 3.16 to 3.23 show how the open loop augmented PID system is impacted with increasing input frequencies. The open $Q$ loop bode plot shows consist amplification around 50 dB below 7.5 Rad/Sec with mild resonant frequency peaks. As input rates increase amplification decreases at approximately 7 dB per decade. The phase angle profiles trend together with decreasing phase angle lag then increase past their corresponding resonate frequency peaks, finally begin converging as input frequencies pass 10 Rad/Sec. The open $U$ loop bode plot shows uniform decreasing amplification as input rates increase crossing 0 at various frequencies, a single resonant peak is observed at the Mach 0.3 flight condition. Phase angle tracks uniformly for all flight conditions lagging beyond 180° around 0.38 Rad/Sec. The open $P$ loop bode plot shows uniform decreasing amplification as input rates increase with attenuation occurring beyond approximately
20 Rad/Sec. Phase angle has comparable trends for each flight condition with the below transonic flight conditions offset from the faster flight conditions by approximately 360° due to changes in system type. The open β loop bode plot shows clustered uniform decreasing input amplification with attenuation beyond around 15 Rad/Sec at an approximately linear 6.5 dB per decade. Phase angle loosely clusters around −90° beginning to converge beyond 9 Rad/Sec with increasing phase lag. All of the closed loop magnitude response have uniform trends with acceptable bandwidth, closed β loop is the only figure to have noticeable magnitude dropout most noticeable between 10 and 11 Rad/Sec.
Table 3.13 collects the stability margin parameters for each of the controlled states. All of the controlled states have acceptable bandwidth with a minimum of 1.93 Rad/Sec, more than adequate for all practical control input sequences, excluding U as it is not required to respond quickly with a minimum bandwidth of 0.34 Rad/Sec. MIL-F-872242 cited within AD-A279 [7] establishes the necessary gain margin to be $\pm 8$ dB and a phase margin to be $\pm 60^\circ$ for multi-variable control systems. $\beta$ is the only state to critically violate the established recommendation with a minimum gain margin of $-1.8$ dB and a minimum phase margin of $-9.24^\circ$ both occurring at the Mach 0.3 flight condition. All of the other flight conditions satisfy the gain margin recommendation whereas only the Mach 1 flight condition satisfies the phase margin requirement, the others come close but fall short of $\pm 60^\circ$ phase margin recommendation. The term critically was used as given some modeling error $\beta$ would be the first state to become unstable; whereas the $U$ state at Mach 0.85 has less than the recommended phase margin as well but is not a threat to the overall stability of the aircraft.
Evaluating each flight condition with a uniform series of state command inputs reference the blue boxes for the sequence of commands. The gain scheduled PID control system performs with uniform state tracking and supporting control surface deflections see Figure 3.24. Large surface deflections are observed under flight condition three at Mach 1.5 at 50,000 Ft supported by observations during nonlinear model flight simulation. Several points indicate the coupling effects of individual state commands, most notably in the $\beta$ state, displacing in reaction to $Q$ and $P$ state commands.

<table>
<thead>
<tr>
<th>PID U State Margins</th>
<th>Gain Margin dB</th>
<th>Gain Crossover Freq Rad Sec</th>
<th>Phase Margin Deg</th>
<th>Phase Crossover Freq Rad Sec</th>
<th>CL Bandwidth Rad Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9M 1000Ps 1Deg</td>
<td>-39.35</td>
<td>10.07</td>
<td>-29.15</td>
<td>22.6</td>
<td>5.81</td>
</tr>
<tr>
<td>1.0M 50000Ps 2Deg</td>
<td>-20.88</td>
<td>7.91</td>
<td>-95.77</td>
<td>29.41</td>
<td>5.81</td>
</tr>
<tr>
<td>1.5M 50000Ps 2Deg</td>
<td>-32.40</td>
<td>6.61</td>
<td>-106.00</td>
<td>31.7</td>
<td>5.81</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>-26.69</td>
<td>6.56</td>
<td>-97.15</td>
<td>29.1</td>
<td>5.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID V State Margins</th>
<th>Gain Margin dB</th>
<th>Gain Crossover Freq Rad Sec</th>
<th>Phase Margin Deg</th>
<th>Phase Crossover Freq Rad Sec</th>
<th>CL Bandwidth Rad Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9M 1000Ps 1Deg</td>
<td>-45.52</td>
<td>0.4</td>
<td>-100.00</td>
<td>2.34</td>
<td>0.36</td>
</tr>
<tr>
<td>1.0M 1000Ps 1Deg</td>
<td>-71.46</td>
<td>0.42</td>
<td>-164.59</td>
<td>6.53</td>
<td>0.36</td>
</tr>
<tr>
<td>1.5M 50000Ps 2Deg</td>
<td>-65.45</td>
<td>0.38</td>
<td>-152.74</td>
<td>4.22</td>
<td>0.34</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>-12.94</td>
<td>0.38</td>
<td>-54.06</td>
<td>0.74</td>
<td>0.34</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>-33.25</td>
<td>0.38</td>
<td>-100.99</td>
<td>1.67</td>
<td>0.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID P State Margins</th>
<th>Gain Margin dB</th>
<th>Gain Crossover Freq Rad Sec</th>
<th>Phase Margin Deg</th>
<th>Phase Crossover Freq Rad Sec</th>
<th>CL Bandwidth Rad Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9M 1000Ps 1Deg</td>
<td>107.76</td>
<td>222.7</td>
<td>-129.52</td>
<td>15.28</td>
<td>3.94</td>
</tr>
<tr>
<td>1M 1000Ps 1Deg</td>
<td>-44.73</td>
<td>2.73</td>
<td>-159.82</td>
<td>27.63</td>
<td>6.81</td>
</tr>
<tr>
<td>1.5M 50000Ps 2Deg</td>
<td>2Inf</td>
<td>NaN</td>
<td>-135.6</td>
<td>21.13</td>
<td>5.25</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>-Inf</td>
<td>0</td>
<td>-152.14</td>
<td>23.84</td>
<td>5.96</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>162.43</td>
<td>704.23</td>
<td>18.97</td>
<td>4.14</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PID Beta State Margins</th>
<th>Gain Margin dB</th>
<th>Gain Crossover Freq Rad Sec</th>
<th>Phase Margin Deg</th>
<th>Phase Crossover Freq Rad Sec</th>
<th>CL Bandwidth Rad Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9M 1000Ps 1Deg</td>
<td>-1.29</td>
<td>9.67</td>
<td>-9.24</td>
<td>10.01</td>
<td>1.92</td>
</tr>
<tr>
<td>1M 1000Ps 1Deg</td>
<td>-12.93</td>
<td>12.22</td>
<td>-63.29</td>
<td>22.7</td>
<td>2.64</td>
</tr>
<tr>
<td>1.5M 50000Ps 2Deg</td>
<td>-6.28</td>
<td>11.27</td>
<td>-42.79</td>
<td>17.55</td>
<td>2.25</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>-11.22</td>
<td>11.47</td>
<td>-44.93</td>
<td>20.12</td>
<td>2.29</td>
</tr>
<tr>
<td>0.9M 150000Ps 3Deg</td>
<td>-8.22</td>
<td>11.55</td>
<td>-43.35</td>
<td>17.94</td>
<td>2.31</td>
</tr>
</tbody>
</table>
Figure 3.24: Off Design Flight Condition State Response
Representing the uniform response of the gain scheduled PID control system under 1 G flight conditions is Figure 3.25, presenting response time, settling time and overshoot percentage for $Q$, $P$ and $\beta$; the continuous surfaces were trimmed to match the allowable flight envelope.

![Figure 3.25: State Response Continuity Envelope](image)

Figure 3.25: State Response Continuity Envelope

A sample of the uniform control response times throughout the flight envelope are presented in Figure 3.26, with each surface trimmed to match the allowable flight envelope. Discontinuous surfaces do not indicate instability, only unconverged control response metrics, $\delta_{LTV}$ is a heavily utilized control effector and typically never reaches a steady state deflection.
The gain scheduled portion of the flight envelope is presented in Figure 3.27 representing the flight conditions in which there are no positive system poles. Overall the scheduled PID control system provides an acceptable operating envelope to maneuver the aircraft within that has demonstrated system stability. Flight conditions outside of the presented flight envelope will likely be controllable for transient periods of time as the unstable poles may require several minutes or longer to diverge near the edge of the known envelope. Large deviations from the presented boundaries will become unstable as the system will diverge within a matter of seconds.
Figure 3.27: Stable Flight Envelope with PID Control
CHAPTER 4

LQ OPTIMAL CONTROL SYSTEM DESIGN AND ANALYSIS

4.1 Design Method Introduction

The ICE aircraft flight dynamics were augmented to achieve the desired state response times to satisfy the required handling qualities for this aircraft. Utilizing the design point flight conditions in Table 1.4 the linear quadratic (LQ) optimal control structure was assembled around the linearized state space model. The resulting closed-loop system will ensure the system stability at the design flight condition with the minimum control energy. To achieve the command following or servo design requirements, the open loop system is augmented with integrator to transform control surface inputs to commanded inputs. This augmentation is achieved by solving for the DC gain matrix and converting the system input matrix \( B \) to form the new control input matrix \( B K_{dc} \). This process enables the decoupled control commands and system state responses by allowing the augmented control matrix to distribute control deflections to compensate for any tracking errors from inputs or system disturbances. Closing the loop is the last component of the design providing the tracking error state feedback. With the LQ control system assembled it is now feasible to evaluate various system state \( Q_s \), control input \( R_C \) and tracking error \( K_s \) weighting matrices. This also allows the system designer to use gradient based optimization tools like 'fmincon' in Matlab to balance the available state control power. The final weighting matrices that satisfies the desired response characteristics (handling qualities in section 1.6) and control constraints can be obtained through this optimization process. With the design flight condition the complete solution can now be scaled to cover the entire flight envelop via a series of nested loops to sequentially evaluate every flight condition. At each flight condition the necessary optimal gain matrix \( K \) is computed to augment the flight dynamics as previously constrained at any flight condition desired, allowing for either the linearized model or the non linear model to be gain scheduled to the nearest flight
condition and evaluated utilizing the same $K$ gain matrix solution. With 46,656 points available the transition between each solution is reasonably continuous ensuring uniform handling quality variations throughout the flight envelope. To close, the gain scheduled LQ optimal control system will be evaluated for performance and robustness throughout the available flight envelope.

The $K$ gain matrices is a result of solving the algebraic Riccati equation via the lqr Matlab function. The riccati equation follows Equation 4.1 and is solved via Equation 4.2 for $K$ provided $Q_S$, $R_C$, and $N$ is set to 0. Transforming $S$ the solution for $K$ is found by $K = R_C^{-1}(B^T S + N^T)$.

$$A^T S + S A - (S B + N) R_C^{-1} (B^T S + N^T) + Q_S = 0$$ (4.1)

$$\text{Minimize } J = \int_0^\infty (x^T Q_S x + u^T R_C u + 2 x^T N u) dt$$

$$\text{Subject To}$$

$$\dot{x} = A x + B u$$ (4.2)

A theoretical representation of the control system is shows in eq. 4.3 and 4.4 also summarized in block diagram form in Figure 4.1. Where $K$, $Kdc$, $K_s$ gain matrices will reside with their corresponding state space realization matrices to form a linear quadratic servo mechanism. These representations provide a fundamental approach building up from theory to application to enable the LQ control system design. For reference the original state space realization (SSR) dimensions are $A$ [8 x 8], $B$ [8 x 14], $C$ [8 x 8], and $D$ [8 x 14] once augmented to directly control system states and output control surface deflections the final SSR dimensions are $A$ [12 x 12], $B$ [12 x 4], $C$ [22 x 12], and $D$ [22 x 4]
\[ \dot{x} = A x + B u = A x + B(-K x + Kdc \xi) = (A - B K)x + B Kdc \xi \]

\[ \dot{\xi} = (r - Y)K_s = (r - C \chi)K_s \]

\[ Y = C \chi \]

\[ dS = K \chi \]

\[
\begin{bmatrix}
\dot{x} \\
\dot{\xi}
\end{bmatrix} =
\begin{bmatrix}
(A - B K) & B Kdc \\
-C K_s & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\xi
\end{bmatrix} +
\begin{bmatrix}
0 \\
K_s
\end{bmatrix} r
\]

\[ (4.3) \]

\[
\begin{bmatrix}
y \\
d
\end{bmatrix} =
\begin{bmatrix}
C & 0 \\
K & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\xi
\end{bmatrix}
\]

\[ (4.4) \]

4.2 System Design

4.2.1 LQ Design Flow Chart

Figure 4.2 is a graphical representation of the LQ servo control system and how the command inputs are processed into the desired state outputs. Starting with the commanded input the tracking error states are offset from equilibrium; causing the required control surface deflections to be generated and move the system states to the commanded values. The states converge to the
desired position at the specified rate depending on the corresponding augmented system poles. The output states allow for the system states to be monitored and control surface deflections to be observed. Following the arrows from input to output depicts the flow of the data in the LQ state feedback servo control systems. The colors of each block in Figure 4.1 are coordinated to match with Figure 4.2 to convey how each portion of the block diagram match up with the resulting matrix flow chart.

![Figure 4.2: LQ Design Flow Chart](image)

4.2.2 LQ Design and Potential Application

A Linear Quadratic Estimator (LQE) or Kalman filter’s role is to reconstruct states which are not easy to measure reliably or have high levels of disturbance and noise. By adjusting the weighting bias between measurement noise $R_N$ and process disturbance $Q_D$ weighting matrices, an accurate full state vector can be constructed under noisy measurement conditions. The LQE takes the available output states from the plant model along with the current control inputs to estimate the system states, while compensating for the systems measurement noise and process disturbances. Refer to the flow chart 4.3 to observe it’s structure, which replaces any preceding system’s state matrix A. If comparable states are required then the A matrix size expands to accommodate both the true state matrix A with the estimator state matrix $(A - LC)$ on it’s diagonal to keep the now dual A matrix square. Adjustment to the following system is necessary to support
the now larger A system matrix, but does not change the inherent system dynamics. Now with the true states and estimated states available, various weights for the states and the controls can be evaluated to achieve the desired state convergence settling times. The necessary settling times are dependant on the corresponding rise times for each state, allowing for some estimator poles to be slow where other must be fast to quickly converge on a high frequency changes.

Figure 4.3: LQE based Design Flow Chart

The LQ state feedback control structures role is to stabilize or augment the inherent system stability to behave as the user specifies. To achieve the desired behaviour the state weighting matrix $Q_S$ and the control weighting matrix $R_S$ values must be adjusted but not directly intuitive. Thus, iterative design is required with in depth knowledge of the system dynamics at hand. To optimize state response and control power allocation a constrained optimization algorithm can be applied. Which may decrease solution time while maximizing system performance to achieve the desired handling qualities with minimal cost and improved control feasibility. The LQ state feedback moves the eigenvalues of the system matrix $A$ by utilizing the available control power to $(A - BK)$. The $A$ matrix can also be expanded to incorporate servo design requirements, representing the difference between the desired states and the current state values. Unlike the open loop case now individual states are being controlled and the given B control matrix needs to be augmented to allow the designer to control individual states and not specific surface inputs. To achieve this adaptation for the linear system a $K_{dc}$ matrix is generated and applied to the $B$ matrix as follows $(B K_{dc})$. Now the augmented closed loop LQ state feedback is complete once a zero value matrix is added.
to the lower right hand corner of the final augmented closed loop system matrix. To complete the
LQ design a new control matrix is generated initially with identity values in the same row as the
feedback error states, allowing for individual states to be commanded. If the state disturbances are
also being modeled additional columns may be added with one value cells corresponding to each
state row to be disturbed. This feature allows for disturbance rejection testing or higher fidelity
system evaluation as no digital system is without background noise. The output $C$ matrix can
be used to handle any necessary unit conversions as well as decompose each state control input
into individual control surface deflections, allowing for the desired state units to be output and for
control surface activity to be monitored during system evaluation.

Linear Quadratic Gaussian (LQG) control structure is simply the placement of estimated
full states from LQE for the LQ state feedback as shown in Figure 4.4. The combination of the two
allow for system noise and disturbances to be filtered from the system states; and enable response
dynamics to be specified and tracked without excessive control effort with acceptable response
times for a well tuned system. Variation of the $Q_D$, $R_N$, $Q_S$, $R_C$, and $K_s$ values allow for the limits
of the given dynamic system to be obtained without excessive control surface effort, undesirable
overshoot, or slow response times. A LQG control structure is not evaluated within this thesis but
has been included to demonstrate the application and understanding of state estimation and follow
on state regulation to achieve the desired dynamic response of a given dynamic system [15].
4.2.3 Control Power Balancing

Control state selection was done by picking states with highest normalized control authority components of the DC gain matrix, to control airspeed, roll, pitch, and yaw. The DC gain matrix transforms the desired state change commands into control surface deflections, allowing for accurate comparison between control magnitudes. Therefore by selecting which state the control surfaces have the most control power over the desired controlled states will be well regulated. All of the control authority values have been normalized to 100 for easier comparison. Table 4.1 was generated by using 'fmincon' to minimize the difference in control power for each state as specified in eq.4.5, which is driving each control authority term to 100 or as near as each state can get given the 30 variables in the weight vector. The weight vector is comprised of 8 state weights ($Q_s$), 14 control weights ($R_C$), and 8 tracking error weights ($K_s$). Initializing the weighting vector with all values set to one, 'fmincon' was able to quickly find a solution as the objective was not obscure. Observing Table 4.1 $U, \alpha, \Theta, \beta, \Phi$, and $R$ all offer potential control states, the selected control states for this design will be $U, \alpha, \beta, and \Phi$. As the rate states $Q$ and $P$ do not offer sufficient available control power, pitch angle $\Theta$ state control augments the system more than desired for conventional
flight and yaw rate $R$ does not offer steady heading side slip capability like controlling side slip angle $\beta$ does.

Table 4.1: Balanced State Control Authority

<table>
<thead>
<tr>
<th>Control Authority</th>
<th>Ub</th>
<th>Alpha</th>
<th>Theta</th>
<th>Q</th>
<th>Beta</th>
<th>Phi</th>
<th>P</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Values</td>
<td>100</td>
<td>100</td>
<td>97.464</td>
<td>0</td>
<td>30.294</td>
<td>7.712</td>
<td>0.161</td>
<td>99.992</td>
</tr>
</tbody>
</table>

$$J = (100 - CtrAuth(1))^2 + (100 - CtrAuth(2))^2 + (100 - CtrAuth(3))^2 + (100 - CtrAuth(4))^2 + (100 - CtrAuth(5))^2 + (100 - CtrAuth(6))^2 + (100 - CtrAuth(7))^2 + (100 - CtrAuth(8))^2$$

(4.5)

4.2.4 State Response Tuning

The desired rise times were specified within Matlabs ‘fmincon’ algorithm as components of the quadratic cost function. Since the cost function is minimized the desired state rise times and settling times were used to offset to drive the measured rise and settling times to their desired values over several iterative solutions reference eq.4.6.

$$J = (10 \times (SpecQRT - RTQ))^2 + (SpecBetaRT - RT Beta)^2 + (SpecPRT - RTP)^2 + (SpecQST - STQ)^2 + (SpecBetaST - ST Beta)^2 + (SpecPST - STP)^2$$

(4.6)

The weighting vector constraints were kept generic only specifying that each values stay within the following range $0 < Weight < 3E6$, this ensured that the vector and resulting $Q_S$, $R_C$, and $K_S$ matrices remain positive definite and don’t use unreasonably large gain values.

The control surfaces travel limits were represented as inequality constraints along with overshoot and rise time limits to prevent excessive deflection and undesired state oscillation. (see eq. 4.7 for reference or Appendix B for the entire constraint) The first 13 terms constrained control effector deflection excluding thrust, 14 - 16 constrained state overshoot, while the remaining
components up to 22 constrain state rise time aiding in minimizing the cost function. This allowed for acceptable regions for each variable to reside within the set limits rather than rigid objectives as the cost function does, aiding the minimization of the provided cost function. A perfect solution was not obtained but adequately decoupled state response were obtained to control the response of the LQ design.

\[
C(1) = \text{MaxDfl}(1) - \text{dlefiLimitU} \times AD
\]

\[
\ldots
\]

\[
C(22) = PRT - 0.75
\]

4.2.5 State Weighting Vector Components - \( Q_S \)

The a balanced control power weighting vector was generated to initialize the optimization process ref eq. 4.8 for the initial state weighting components of the weight vector.

\[
X(1 : 8) = [30.63, 16.39, 0.47, 33.84, 24.27, 0.09, 0.0001, 0.0001]
\]  (4.8)

With 'fmincon' constrained by Equation 4.7 to minimize eq.4.6 the balanced control power weighting vector via eq.4.8 was refined to obtain the desired state response results. After several iterations of constraint and objective adjustments with some manual weight vector tuning to decrease rise time by scaling the entire vector uniformly. Increasing or decreasing the vector scaling factor has a nonlinear relationship to the desired CMD state rise times, which can also be extended to individual vector value bases for fine manual adjustment, resulting in the following optimal weight vector components see Equation 4.9.

\[
X(1 : 8) = [69.32, 43.91, 39.28, 35.99, 1070.7, 619.060, 18.02, 36.06]
\]  (4.9)
4.2.6 Control Weighting Vector Components - $R_C$

The balanced control power weighting vector generated to initialize the optimization process ref eq.4.8 for the initial control weighting components of the weight vector.

\[ X(9 : 22) = [9e-11, 13.98, 0.0034, 0.0780, 17.76, 3.84E-5, 0.0001, \\
2.35E-5, 0.0319, 0.0517, 0.2969, 1.8E-14, 1.1729, 1.9324] \]  

(4.10)

Unlike the state control weights these values were not independently adjusted to achieve a specified result beyond the overall applied scaling factor mentioned in Section 4.2.5. Observing Equation 4.11 3 control effectors the $\delta_{LELE}$, $\delta_{RELE}$, and $\delta_{LTV}$ have the lowest regulation cost, indicating they are the most free to be used to control the aircraft. Where as the remaining control weights are on the order of 5 to 40 times higher compared to the three primary control surfaces. Indicating they will have a proportional decrease in comparative control surface deflection.

\[ X(9 : 22) = [74.07, 80.21, 69.59, 1.76, 74.00, 71.65, 74.33, \\
74.93, 73.83, 1.78, 71.97, 23.85, 4.65, 74.71] \]  

(4.11)

4.2.7 State Error Weight Vector Components - $K_s$

Not part of the typical LQ optimal design process, scaling the state error was observed to be an important design variable. Used to adjust individual state response times to be within allowables by scaling the tracking error state feedback loop. Varying the state error as well as the commanded state to make them proportional; allows for independent amplification or attenuation of a specified state based on what was desired for each state. Each values of the error state weights corresponds to the 8 dynamic states of the system. The initial $K_s$ values from the balanced control power weight vector are in Equation 4.12.

\[ X(23 : 30) = [1, 1, 1, 1, 1, 1, 1, 1] \]  

(4.12)
Once again given the availability of automated constrained optimization algorithms all eight of these variables were allowed to be adjusted with some manual adjustments between iterations resulting in the final adjustment values seen in eq.4.13

\[ X(23:30) = [1.23E6, 5.34, 1.99, 1.99, 3.42, 2.01, 1.99, 1.99] \] (4.13)

4.2.8 Optimal Feedback Gain - \( K \)

The optimal feedback gain matrix in Figure 4.5 augments the 14 control input \( B \) matrix to the system from control effector deflection to 8 system state inputs. Notice how symmetric the control gains are between the left and right side of the aircraft, this was one of enabling reasons for the success of the LQ control system. Figure 4.2 system states block to see how the system poles are augmented by subtracting \((B\times K)\) from the open loop state matrix \( A \), stabilizing the system poles of the state matrix.

\[
\begin{array}{cccccccc}
-0.0104 & -0.0017 & 0.1022 & -0.0106 & 0.0086 & -0.0426 & 0.0026 & -0.0589 \\
0.0072 & 0.0029 & -1.6136 & -0.0388 & 0.0029 & -0.2762 & -0.0243 & -0.0565 \\
-0.0651 & -0.0335 & 3.9809 & -0.3435 & 0.1079 & 2.6387 & 0.5312 & -3.0940 \\
3.0194 & -4.1414 & -242.4603 & -113.6242 & 0.0204 & 567.6602 & 73.4298 & -155.7176 \\
-0.1643 & -0.0152 & 4.7458 & 0.3065 & 0.1104 & -3.1206 & -0.0263 & -3.7843 \\
-0.0355 & -0.0800 & 1.0365 & -1.5757 & -0.0005 & -0.0390 & -0.0611 & 0.0059 \\
-0.0103 & -0.0017 & 0.1014 & -0.0106 & -0.0085 & 0.0416 & 0.0627 & 0.0589 \\
0.0077 & 0.0042 & -1.7274 & -0.0380 & -0.0031 & 0.2952 & 0.0260 & 0.0584 \\
-0.0612 & -0.0317 & 3.7467 & -0.3206 & -0.1019 & -2.5071 & -0.5038 & 2.9210 \\
-0.1688 & -0.0156 & 4.0742 & 0.3170 & -0.1133 & 3.2166 & 0.0293 & 3.8977 \\
1.9573 & 0.5320 & -219.8309 & -6.8417 & -0.0007 & -0.0867 & -0.0107 & 0.0176 \\
-0.0119 & 0.0022 & 0.4832 & 0.0845 & 20.2564 & -130.5505 & 35.3285 & -603.1254 \\
0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 \\
\end{array}
\]

Figure 4.5: K Gain Matrix

4.2.9 Control Linear Model Augmentation DC Gain Matrix - \( K_{dc} \)

Feedback gain matrix was generated using the pseudo-inverse of the DC gain. But unique to this dynamic system application the result was modified to remove all airspeed control from the control surfaces and remove all state control from the engine thrust input. Therefore simplifying which control effectors are responsible for controlling their corresponding state pairing. Allowing control effects and engine thrust to compensate independently without directly interfering with the
manually decoupled control state. See figure 4.6 for how the $Kdc$ matrix is trimmed to decouple controls. With the generic matrix on the left and the resulting design point gain matrix on the right boxed in to specify each gain subsection.

**Figure 4.6: Kdc Gain Matrix**

### 4.2.10 Resulting State Optimization Control Power

With all of the above changes made to the LQ control system to achieve the desired state response times the normalized state control power vector has changed referencing Figure 4.7. It’s unintuitive magnitude distribution is difficult process to any meaningful takeaways, although interesting to note how several of the controlled states now have some of the lowest control powers in comparison to the unregulated states.

**Figure 4.7: State Control Authority**

### 4.2.11 Actuator and Engine Dynamics

Actuator and engine dynamics have not been incorporated into this design currently, follow on designs may incorporate actuator dynamics in series with all of the corresponding control effectors. Until then the following Table 4.2 has been provided to gauge if the resulting design...
can be supported by each sub system component. Sub system dynamics incorporation following this evaluation would increases the open loop state rank making the system more tedious to filter and control with the intended LQ design method. Depending on the resulting LQ design the sub system components may have no to minimal impact to the performance. Comparison of the rise time values in Table 4.2 to the required rise time values in Table 4.3 throughout the flight envelope will indicated the impact of incorporating actuators to the control system.

<table>
<thead>
<tr>
<th>Subsystem Step Info</th>
<th>LES Actuator</th>
<th>TES Actuator</th>
<th>Generic Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time [Sec]</td>
<td>0.062</td>
<td>0.125</td>
<td>5.493</td>
</tr>
<tr>
<td>Settling Time [Sec]</td>
<td>0.111</td>
<td>0.228</td>
<td>9.78</td>
</tr>
<tr>
<td>Overshoot [%]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4.3 LQ Control System Performance

4.3.1 Subsystem Performance

Evaluation of the control surface response requirements in Figure 4.3 to the rise time and settling times in Figure 4.2. Providing an indicator for which actuator will most likely saturate, starting with the Leading Edge Surface (LES) Actuator the rise time only has a 0.033 second or 50% margin with an acceptable settling time margin by observation over the capability of the LES actuator in Table 4.2. The Trailing Edge Surface (TES) actuator has a rise time margin of 0.153 seconds or 122%, with a settling time margin of 0.175 seconds or 76%. The LES and TES response requirements are within the capabilities of the provided actuator dynamics for the design flight condition. Transitioning these results to the non linear environment may face scaling challenges as well as nonlinear effects making equivalent real time performance infeasible without additional system design.

<table>
<thead>
<tr>
<th>Actuator Response Req</th>
<th>LES Actuator</th>
<th>TES Actuator</th>
<th>Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time [Sec]</td>
<td>0.095</td>
<td>0.278</td>
<td>3.292</td>
</tr>
<tr>
<td>Settling Time [Sec]</td>
<td>10.29</td>
<td>0.403</td>
<td>22.548</td>
</tr>
<tr>
<td>Overshoot [%]</td>
<td>3.297</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
4.3.2 LQ Closed Loop vs Open Loop System Comparison

Comparing the closed-loop pole locations to the open loop system in Figure 4.8 it is first critical to see no unstable poles in the augmented system. With no poles on the right hand side of the imaginary axis the aircraft flight dynamics have been successfully stabilized, next the response time can be broadly compared between the two systems. With the fastest pole located just past -1 the open loop system is much slower than the closed loop system with it’s fastest poles near -13, indicating that some of the augmented states will converge significantly faster than the open loop system states would have if it were to have started as a statically stable system.

Figure 4.8: Open Loop and Resulting LQR System Poles At Design Flight Condition
4.4 Flight Condition Control Law Evaluation

Figure 4.9: ICE Aircraft Concept Rendering [16]
4.4.1 Pole Locations

The poles of the system migrate uniformly as the flight conditions change in Figure 4.10, indicating uniform response of the LQ feedback system controlled at the specified flight conditions.

![Figure 4.10: LQ Control System Pole Zero Map](image)

4.4.2 Frequency Domain Characteristics

Representing each controlled state from the frequency domain is Table 4.4 and Figures 4.11 to 4.18 to represent how increasing control frequencies impact the system. For a linear quadratic derived control system stability margins are guaranteed as we see in the following bode plots. Figure 4.11 shows uniform attenuation trends for the subsonic flight conditions with additional attenuation for supersonic flight conditions. By observation the higher dynamic pressure flight conditions attenuated more than the lower dynamic pressure flight conditions. Phase angle never crosses $-180^\circ$ ensuring stability with overall uniform variation as input frequencies increase. Figure 4.13 shows how state $U$ state has a linearly increasing attenuation as input frequencies increase as well as converging constant phase angle, resulting in $\infty$ gain and phase margins (GM, PM). Figure 4.15 shows how state $U$ has no attenuation below 4 Rad/Sec then attenuation increases by approximately 5 dB per decade. Phase angle for all flight conditions has a uniform trend with little
variation approaching $-180^\circ$ in an asymptotic fashion. Figure 4.17 shows no input amplification or attenuation below 7 Rad/Sec then begins to attenuate at 4 dB per decade. Phase angle shows acceptable trends reaching a peak lag of only $-135^\circ$ the system remains stable. Without any of the phase angles ever crossing $-180^\circ$ all of the controlled aircraft states have ensured stability and resulting control at all input frequencies. The closed loop bode plots have acceptable trends and reduced attenuation due to as expected. MIL-F-872242 cited within AD-A279 [17] establishes the necessary GM to be $\pm 8$ dB and a PM to be $\pm 60$ deg for multi-channel control systems. With a minimum GM of $\infty$ where as the lowest PM is $177.4^\circ$ the recommended stability margins are satisfied. The available bandwidth of each loop is acceptable overall, for state $U$ the low bandwidth does not pose a problem as airspeed is not required to respond quickly to command changes.

Table 4.4: Feedback Loop Stability Margins

<table>
<thead>
<tr>
<th>UQR Alpha State Margins</th>
<th>Gain Margin dB</th>
<th>Gain Crossover Freq Rad Sec</th>
<th>Phase Margin Deg</th>
<th>Phase Crossover Freq Rad Sec</th>
<th>CL Bandwidth Rad Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3M 10000ft 10deg</td>
<td>Inf</td>
<td>NaN</td>
<td>177.06</td>
<td>0.04</td>
<td>16.2</td>
</tr>
<tr>
<td>1.5M 50000ft 20deg</td>
<td>Inf</td>
<td>NaN</td>
<td>179.99</td>
<td>0</td>
<td>6.71</td>
</tr>
<tr>
<td>0.5M 35000ft 50deg</td>
<td>Inf</td>
<td>NaN</td>
<td>177.39</td>
<td>0.04</td>
<td>10.73</td>
</tr>
<tr>
<td>0.5M 15000ft 50deg</td>
<td>Inf</td>
<td>NaN</td>
<td>177.30</td>
<td>0.05</td>
<td>12.64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UQR Beta State Margins</th>
<th>Gain Margin dB</th>
<th>Gain Crossover Freq Rad Sec</th>
<th>Phase Margin Deg</th>
<th>Phase Crossover Freq Rad Sec</th>
<th>CL Bandwidth Rad Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3M 10000ft 10deg</td>
<td>Inf</td>
<td>NaN</td>
<td>179.74</td>
<td>0.07</td>
<td>12.57</td>
</tr>
<tr>
<td>1M 10000ft 40deg</td>
<td>Inf</td>
<td>NaN</td>
<td>179.73</td>
<td>0.14</td>
<td>9.15</td>
</tr>
<tr>
<td>1.5M 50000ft 20deg</td>
<td>Inf</td>
<td>NaN</td>
<td>179.93</td>
<td>0.04</td>
<td>9.60</td>
</tr>
<tr>
<td>0.5M 35000ft 50deg</td>
<td>Inf</td>
<td>NaN</td>
<td>179.88</td>
<td>0.05</td>
<td>10.49</td>
</tr>
<tr>
<td>0.5M 15000ft 50deg</td>
<td>Inf</td>
<td>NaN</td>
<td>179.30</td>
<td>0.06</td>
<td>12.21</td>
</tr>
</tbody>
</table>
Figure 4.11: Open Alpha Loop Bode Plot

Figure 4.12: Closed Alpha Loop Bode Plot

Figure 4.13: Open U Loop Bode Plot

Figure 4.14: Closed U Loop Bode Plot
4.4.3 Time Domain Response

Figure 4.19 presents each states rise time and overshoot or peak value depending on the state and what is most applicable. Overall it can be observed that at higher trim thrust flight conditions the LQ control system takes longer to respond, where as at lower trim thrust flight conditions the system is more responsive with lower rise times and less overshoot. With more thrust available as the lower Mach flight conditions this variation in system response is expected, Mach 0.3 is an outlier to this reasoning because it is operating on the backside of the power required curve. Where there is less additional available thrust than other subsonic flight conditions, and takes longer to respond as a result.
Figure 4.19: Flight Condition State Step Information

Between Table 4.5 and 4.6 the system rise time, settling time, and overshoot % statistics are presented to convey how little variation occurs in state and control response times over the 5 flight conditions evaluated. The rate based states Alpha, β, and Phi have rise time standard deviations between 0.047 and 0.288 seconds undetectable from a pilots perspective. The corresponding control surface statistics are consistent with all three having the same standard deviation of approximately 2.56 seconds. Demonstrating how well the LQ control method regulates control surface deflection throughout the flight envelope, saturation of engine response has a non detrimental impact.

Table 4.5: State Step Response Statistics

<table>
<thead>
<tr>
<th>State Step Info</th>
<th>Airspeed Nt [Sec]</th>
<th>Peak Thrust</th>
<th>Alpha Nt [Sec]</th>
<th>Alpha OS</th>
<th>Data Nt [Sec]</th>
<th>Data OS</th>
<th>Phi Nt [Sec]</th>
<th>Phi OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8M 1000ft 10Deg</td>
<td>94.818</td>
<td>1.4701e+05</td>
<td>0.150</td>
<td>9.32</td>
<td>0.179</td>
<td>3.081</td>
<td>0.293</td>
<td>13.675</td>
</tr>
<tr>
<td>1.5M 1000ft 4Deg</td>
<td>374.12</td>
<td>61043</td>
<td>0.047</td>
<td>20.701</td>
<td>0.239</td>
<td>0</td>
<td>0.314</td>
<td>0</td>
</tr>
<tr>
<td>1.5M 5000ft 2Deg</td>
<td>16.36</td>
<td>24635</td>
<td>0.329</td>
<td>1.235</td>
<td>0.228</td>
<td>0</td>
<td>0.286</td>
<td>5.038</td>
</tr>
<tr>
<td>0.8M 3500ft 5Deg</td>
<td>7.877</td>
<td>34642</td>
<td>0.223</td>
<td>0.75</td>
<td>0.214</td>
<td>0.005</td>
<td>0.293</td>
<td>2.261</td>
</tr>
<tr>
<td>0.8M 15000ft 5Deg</td>
<td>4.273</td>
<td>48566</td>
<td>0.199</td>
<td>1.218</td>
<td>0.181</td>
<td>1.094</td>
<td>0.269</td>
<td>4.858</td>
</tr>
</tbody>
</table>

Table 4.6: Control Step Response Statistics

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<tr>
<th>LQR U Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
<th>LQR Beta Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
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<tbody>
<tr>
<td>Rise Time [Sec]</td>
<td>4.262</td>
<td>99.272</td>
<td>373.53</td>
<td>157.72</td>
<td>Rise Time [Sec]</td>
<td>0.179</td>
<td>0.217</td>
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<tr>
<td>Settling Time [Sec]</td>
<td>25.698</td>
<td>184.68</td>
<td>663.66</td>
<td>273.95</td>
<td>Settling Time [Sec]</td>
<td>0.291</td>
<td>0.412</td>
<td>0.46</td>
<td>0.074</td>
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<tr>
<td>Overshoot [%]</td>
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<td>3.748</td>
<td>15.586</td>
<td>6.757</td>
<td>Overshoot [%]</td>
<td>0</td>
<td>0.997</td>
<td>3.687</td>
<td>1.683</td>
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</tbody>
</table>

<table>
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<tr>
<th>LQR Alpha Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
<th>LQR Phi Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
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<tr>
<td>Rise Time [Sec]</td>
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<td>0.351</td>
<td>0.553</td>
<td>0.205</td>
<td>Rise Time [Sec]</td>
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<td>Overshoot [%]</td>
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<td>20.673</td>
<td>8.624</td>
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<td>5.172</td>
<td>13.672</td>
<td>5.182</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>LQR Engine Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
<th>LQR LTV Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
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</thead>
<tbody>
<tr>
<td>Rise Time [Sec]</td>
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<td>98.403</td>
<td>369.64</td>
<td>156.97</td>
<td>Rise Time [Sec]</td>
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<td>1.74</td>
<td>6.273</td>
<td>2.595</td>
</tr>
<tr>
<td>Settling Time [Sec]</td>
<td>22.548</td>
<td>170.45</td>
<td>642.65</td>
<td>266.04</td>
<td>Settling Time [Sec]</td>
<td>0.311</td>
<td>2.147</td>
<td>7.685</td>
<td>3.172</td>
</tr>
<tr>
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<td>4.457</td>
<td>Overshoot [%]</td>
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<td>0.309</td>
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<td>0.457</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>LQR FF Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
<th>LQR Elevon Step Perf</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time [Sec]</td>
<td>0.095</td>
<td>1.091</td>
<td>6.059</td>
<td>2.558</td>
<td>Rise Time [Sec]</td>
<td>0.276</td>
<td>1.757</td>
<td>6.273</td>
<td>2.595</td>
</tr>
<tr>
<td>Overshoot [%]</td>
<td>3.537</td>
<td>39.8</td>
<td>92.088</td>
<td>33.591</td>
<td>Overshoot [%]</td>
<td>0</td>
<td>1.025</td>
<td>3.616</td>
<td>1.493</td>
</tr>
</tbody>
</table>
Figure 4.20 represents full state response to commanded state step inputs to the LQ control system. Highlighted in blue boxes are the commanded states individual step responses, state coupling can be observed by comparing the non highlighted graphs within individual columns. Note the scale magnitude for each Y axis as they change for each subplot.

Figure 4.20: Flight Condition State Response
Figure 4.21 represents the right side control response to the same commanded state step inputs to the LQ control system, left side surface deflections are nearly equivalent to the right side surface deflections. Each state utilizes different control distributions to achieve a given command input. Compare $\alpha$ to $\phi$ control deflection responses increase by one to two magnitudes when comparing individual effectors.
4.5 Flight Envelope System Response

Figure 4.22 evaluates the LQ control system at the specified flight conditions with the same command time series. Allowing for side by side tracking performance comparisons along with a
sample of the required control effector activity. This figure is formatted the same as Figure 3.24 for a brief comparison between PID and LQ control performance before Chapter 5 expands on the topic in depth.

A cumulative representation of the uniform response of the LQ control system are the state continuity envelopes in Figure 4.23. Note the heat bar below each figure to compare the magnitude of variation for each proposed metric. Some of the flight conditions have gaps in the surface, but that doesn’t indicate the system is unstable in those regions.

Representing the three most critical control surfaces are the continuity envelopes in Figure 4.24. Referencing the heat bar below each figure to compare the magnitude of each metrics variation. Overall acceptable variation has allowed for consistent control deflection requirements with no non continuous outliers mid surface.
Figure 4.22: Evaluation Flight Condition State Responses
Figure 4.23: 1G State Response Continuity Envelope
Figure 4.24: 1G Control Continuity Envelope
Representing the stable and sustainable flight condition envelope is Figure 4.25. Each layer of the envelope represents the stable poles at each flight condition within the allowable $\bar{q}$ range specified in Table 1.3 and the thrust available limits as represented by Figure 1.10. When limiting max AoA the peak turn rate performance is decreased as seen in Figure 1.11 of the aircraft, with the benefit of being a sustainable maneuver. Operating above the AoA limit the aircraft is unable to sustain turn performance and will bleed kinetic energy throughout the maneuver. At higher altitudes this will prove to be detrimental as thrust available is reduced and will take longer to recover the energy required to carry out follow up maneuvers.

Figure 4.25: Stable Flight Envelope with LQ Optimal Control
CHAPTER 5

REAL-TIME FLIGHT SIMULATION AND HANDLING QUALITIES EVALUATION

5.1 Flight Simulator Development

With the support from the Office of Sustainability and the Department of Mechanical and Aerospace Engineering at Western Michigan University, an engineering flight simulator was built as a part of this project. MAEFS (Mechanical and Aerospace Engineering Flight Simulator), shown in Figure 5.1, will be utilized within the current aerospace engineering curriculum, exposing undergraduate students to the flight dynamics and nuances of flight in early introductory level classes. As students progress through the aerospace engineering curriculum, the simulator will provide opportunity to flight test their own design. It also will serve as a research tool for graduate students who are interested flight dynamics and control discipline.
5.1.1 Simulator Configuration

The physical design and construction was a collaborative effort to establish the appropriate support structure, screen angles, and ergonomic positioning of the pilot and adjustable seating position to accommodate all size pilots. All design and fabrication was jointly carried out with a senior capstone design project team with Scott Miller and Heather Irish and a research assistant Nick Goberville according to the schedule shown in Figure 5.2.
5.1.2 Real-time Simulation Software

Matlab/Simulink software by MathWorks is the primary development and operating environment for MAEFS. All the necessary tools are already available to represent aircraft flight dynamics and also design various flight control architecture accurately and rapidly. It is only limited by the fidelity and range of the associated aerodynamic data. The user has the opportunity to build a wide variety of systems to evaluate and design control laws for a given aircraft at hand. The aerospace toolbox and the aerospace block set are the pivotal add-on components for seamless progress on any aerodynamic dependant model being simulated, providing input for external controls from any USB based or emulated user controls.

5.1.3 Out-the-window Image Generation

MAEFS currently can use either X-Plane 11 or an open source flight simulation software FlightGear for the out-the-window (OTW) Image Generation. Currently, the OTW Image Screen from X-Plane 11 is used for an introductory aerospace engineering class familiarizing the students with the aircraft and the flight in general. Externally driven flight dynamics are represented to the
user through FlightGear and an aircraft file modified to use a network based flight dynamic model. This allows for Simulink to connect with FlightGear and represents how the aircraft translates through its environment as the pilot maneuvers the aircraft. A block-set in Simulink already existed connecting to FlightGear making the connection between the two software packages seamlessly. The OTW images are channelized on three 55” LED TV’s providing an immerse 160° lateral field of view through high-performance liquid cooled graphics hardware mounted on a Microsoft Windows based work station platform on which MATLAB/Simulink is running.

5.1.4 Control Loading System

The Brunner Elektronik programmable control loading system (CLS-P) was procured from Switzerland where Brunner manufactures flight simulation equipment, servicing the pilot training industry. The purchased CLS-P system included a right hand joystick with an F-18 style grip and toe brake incorporated rudder pedals, see Figure 5.3. The joystick is able to generate up to 16.22 Lb-ft of torque to resist pilot inputs, to replicate the desired stick force gradient. The rudder pedals can generate up to 202 lb-ft of opposing moment or approximately 171 lb of opposing force. The CLS-P user interface allows for a multitude of control features to be replicate the desired stick force gradient \( \frac{dS}{\text{Control Variable}} \), dead band, hysteresis, stick shaker vibration, ground vibration, thrust effects on manual control systems, and input travel limits if applicable. The loading system can also be schedule with Airspeed to achieve the desired \( \frac{dS}{\varphi} \), \( \frac{dS}{\Omega} \), and \( \frac{dS}{\beta} \) at each flight condition. Brunner’s interface is not directly compatible with Simulink and would require an add-in to be developed to send the airspeed from Simulink to Brunner’s CLS-P software. The CLS-P system is suitable for a FAA certifiable flight simulator and handling qualities evaluation for MAEFS.

5.1.5 Cockpit Display System

Two 17” LCD touch screens are used to display electronic flight instruments which are again being capable for a new design. Co-pilot side screen can be used for navigation, operator interface, or engineering analysis display like in this study in the following sections. Figure 5.3 also shows the cockpit display system.
5.2 Real-time Simulator based HQ Evaluation

Figure 5.4 presents the airspeed, orientation, and acceleration data to the user through a cluster of classic and non-traditional panel gauges. This layout is duplicated for both the PID and LQ optimal control systems with the addition or modification of nuance features for both. Located on the left side panel monitor it is convenient to reference while transitioning flight conditions and trying to monitor or maintain specific test parameters. Figure 5.5 presents control surface activity and aircraft states for the PID control system, allowing the user to monitor control surface saturation and any occurrences of resonance or coupled state control coupling. This is an essential diagnostic tool when developing any gain scheduled control system for ICE aircraft model.

With the flight dynamics of the ICE aircraft augmented, the MIL-8785C requirements have been interpreted and applied to both control system to be as constricting as possible while being applicable to available controlled states. Due to the traditional control axis pole pairs being intermixed with state error poles, it is not representative of system performance to use individual pole locations to evaluate the overall system response.
Figure 5.4: Gauge Cluster

Figure 5.5: Data Scopes
5.2.1 HQ Requirements Interpretation

Initially presented in Figure 4.23 the time response per axis is now presented in the scope of allowable time response ranges based on handling qualities metrics presented in Section 1.6. For the necessary requirements the corresponding natural frequency and damping ratio ranges have been converted into equivalent rise time requirements. By finding the extreme limits of the Level 1 handling quality range, a reasonable comparison can be made based on the available rise time performance for each axis of control. Going forward it is plausible to constrain the presented optimization methods to achieve all desired state rise times to be within the bounds of the desired handling quality range, through scheduling the constraints and cost function based on flight condition to the median value of the allowable handling quality range.

5.2.2 PID Control HQ Results

Pitch Handling Qualities

Figure 5.6 presents the consistency of the pitch rate rise time with PID control of the linearized model at various altitudes compared to relative AoA under level flight cruise conditions. The lower bound of the Level 1 range is predominantly linear increasing from 0.1 seconds at 1° to 0.45 seconds by 25° where as the resulting pitch rate response rise time is constant at 0.4 seconds throughout the evaluated AoA range. The upper bound has natural log profile characteristics increasing at a decreasing rate as AoA increases, which is not a constraining factor for the PID control system. The high AoA Level 1 region departure at 22° is minor and does not noticeably increase the required workload of the maneuver at hand, allowing for other mission relevant tasks to take priority.
Figure 5.6: 1-G Pitch Rate Rise Time Vs Alpha

Figure 5.7 presents the consistency of pitch rate rise time with PID control of the linearized model under an instantaneous 6-G maneuver at various specified altitudes. The upper and lower bounds have the same trends as described in the 1-G level flight figure with overall lower allowable rise time constraints, increasing the performance requirements on the PID control system. The resulting pitch rate rise time performance resides completely within the level 1 handling qualities region during an instantaneous 6-G pull up maneuver. Completely satisfying the highest class of fixed-wing handling quality requirements as specified by MIL-8785C. [14]

Figure 5.7: 6-G Pitch Rate Rise Time Vs Alpha

Roll Handling Qualities

Figure 5.8 presents the resulting roll rise time response under PID control of the linearized model throughout the flight envelope in comparison to the maximum allowable roll rise time based on the minimum roll time period in MIL-8785C. This interpretation may have higher expectations than necessary but pushes what is expected from the control system to be capable of any future tasks. The roll channel rise times with PID control system are consistently between 0.2 and 0.4 seconds with the maximum rise time requirement of 1 second, the roll handling quality require-
ments are satisfied throughout the entire flight envelope.

![Figure 5.8: Roll Rate Rise Time Vs Mach](image)

Figure 5.8: Roll Rate Rise Time Vs Mach

**Yaw Handling Qualities**

Figure 5.9 presents the variation of side slip angle rise time with PID control of the linearized model throughout the flight envelope, in comparison to the interpreted Level 1 yaw rise time requirement. With augmented side slip angle dynamics typical time-to-double spiral divergence time periods do not apply, with no resulting heading angle drift overtime. So a response time double that of the roll rise time was selected, given that this is now a commanded state it is reasonable that yaw angle can response slower than roll but not excessively so. Again this requirement may be overly constrictive of the side slip angle rise time but by doing so allows the system to adequately respond to the majority of the commands and coupled disturbances. The overall decreasing trend of the side slip angle rise times is correlated to the increasing $\ddot{q}$ and corresponding thrust required, increasing the available lateral directional control power of the ICE aircraft.

![Figure 5.9: Side Slip Rise Time Vs Mach](image)

Figure 5.9: Side Slip Rise Time Vs Mach
5.2.3 PID Simulink Model

Tracing the flow of data in Figure 5.10 from the user inputs to the resulting model depiction in Flight Gear each sections purpose and construction will be summarized. The control input section passes the analog signals from the control loading system to the system logic section which scales, limits, and filters necessary command inputs. Those are then sent to the control distribution and actuator dynamics section, where the chosen state commands are appropriately compensated by the gain scheduling section and passed through corresponding actuator or engine dynamics in the engine model flight dynamics section. This section controls how the available control effectors are utilized in a fixed structure control allocation approach, where the leading edge flaps have been scheduled with $\alpha$ and the slotted spoiler deflectors can be manually deployed to act as speed brakes. The addition of these control inputs improves the sustained turn rate performance and deceleration rates of the ICE aircraft. Landing gear position has also been incorporated as a commanded state, allowing for landing sequences to be evaluated and as an additional test of the existing PID control system. The gain scheduling section references 12 different 3 dimensional tables of corresponding P, I, and D gain values for each of the $Q, R, \beta$, and $U$ channels. Each table requires interpolation between available points to ensure smooth transition between flight conditions, experimentally developed integrator reset rates were used to avoid integrator wind up. While $\ddot{q}$ gain scaling rates were also found to prevent coupled state resonance throughout the flight envelope, trading state error for less susceptible state resonance. Particularly at high $\dot{q}$ flight conditions as the relative control power increases for each controlled state, the system allows for minimal state errors to attenuate the system unnecessarily. Now with all of the scaled and scheduled control commands available they are fed into the aerodynamic model developed by Lockheed Martin to control the desired states to the system. Some of the outputs from the nonlinear aerodynamic model are converted and sent to the gauge cluster to appropriately inform the operator of the current state of the aircraft. Engine state parameters are also sent to the gauge cluster to monitor cruise thrust, fuel flow, and projected endurance with a full fuel on-board. Finally the translation and orientation states are converted and sent to the final Simulink block to orient and move the connected aircraft in Flight Gear around in the desired geographical environment.
Figure 5.10: PID Simulink Block Diagram
5.2.4 LQ Optimal Control HQ Results

Pitch Handling Qualities

Figure 5.11 represents the variation in the LQ optimal control of the linearized model’s $\alpha$ rise time in comparison to the established handling qualities Level 1 rise time region. Using the same metric to evaluate different pitch states are reasonable as they both are dependant on the short period oscillatory mode, therefore can be evaluated in the same way. With a decreasing trend the LQ optimal control system rise time exits the Level 1 region beyond approximately $10^\circ$ and stays nearly constant at 0.2 seconds out to $25^\circ$. Limited by what the linear model could control the $\alpha$ state requires significant pilot effort to maintain a given flight condition. Within the nonlinear Simulink model the commanded $\alpha$ state was successfully replaced with pitch rate command to make flight simulation experiences equivalent between the PID and LQ optimal control systems.

![Figure 5.11: 1G $\alpha$ Rise Time vs $\alpha$](image)

Figure 5.11 represents the variation in $\alpha$ state rise time of the linearized model during a 6 G pull up maneuver, paralleling the upper rise time bound up then sharply departing and quickly decreasing toward the lower bound. Overall it not a consistent response at different altitudes requiring constant attention to maintain a given flight condition, sufficient to classify for Level 1 handling qualities for a portion of the flight envelope while degrading to Level 2 for boarder line or departing rise time cases. The pitch rate control state modification within the nonlinear Simulink model bypassed this unique challenge that was initially experienced, but not reproducible at this time with the linear model.
Figure 5.12: 6G $\alpha$ Rise Time vs $\alpha$

Roll Handling Qualities

Figure 5.13 presents the roll angle rise time with LQ control of the linearized model in comparison to the established requirement of 1 second roll angle rise time. Sufficiently fast and consistent at all altitudes the system satisfies the roll handling quality requirements. As roll angle control makes for a highly augmented flying experience and limits the maneuver potential of the aircraft, the control state was replaced with roll rate equivalent to the PID control system. Like pitch the possible control states in the linear model prevented equivalent control states between PID and LQ optimal control from being selected. But after modification the nonlinear models allow for equivalent flying sequences to be evaluated and establish equitable handling qualities.

Figure 5.13: $\Phi$ Rise Time vs Mach

Yaw Handling Qualities

Figure 5.14 presents a consistent side slip angle rise time with LQ control of the linearized model of 0.2 seconds within the flight envelope in comparison to the established rise time requirement of 2 seconds. Indicating that regulation and control of the side slip angle will achieve and

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maintain the commanded value throughout the flight envelope according to the linear model well within Level 1 requirements.

![Graph of β Rise Time vs Mach](image)

**Figure 5.14: β Rise Time vs Mach**

LQ Optimal Control Simulink Model

Like the PID Simulink model the LQ optimal control system model is constructed to convert the pilots control inputs into the correct control effector deflections to achieve the commanded state of the system. Without being redundant only the differences between the the LQ and PID models will be expanded described. Particularly the system logic, gain scheduling, and control distribution sections will be described at length. Following the signal input as before the system logic filters out excessively high input rate commands and scales them as appropriate for each controlled state. Each state control block is not summarized into one large sub system to zero out control inputs with hands free to prevent state drifting from unintended near zero command values. Gain scheduling is less complicated as only a single matrix needs to be recalled that is nearest to the current flight condition. A byproduct of being developed in a linear environment was the symmetric deflection of elevons to control $\beta$, to correct for this only TEU deflection was allowed and regulate the $\beta$ error state. The control distribution and deflection limit block distributes the vector command to individual surface deflections operating within the corresponding limits as stated in Table 1.1. The resulting control input utilized all 14 surfaces to mitigate coupling and dampening out disturbances. What follows mirrors the PID system as the control inputs feed into the aerodynamic model are ultimately output to FlightGear as displayed to the pilot on MAEFS.
Figure 5.15: LQR Simulink Block Diagram
5.3 PID and LQ Optimal Control Handling Qualities Comparison

Referring back to the 5 specified evaluation flight conditions Table 5.1 compares the corresponding roll, pitch, and yaw axis state rise times between PID and LQ control. Within the magnitude of each perspective state there are no noticeable outliers, with each column being very consistent overall the average standard deviation is 0.04 seconds. Completely satisfying all Roll and Yaw requirements, while pitch is near the lower limit of Level 1 HQ depending on the corresponding trim angle of attack.

Table 5.1: PID and LQR Linear Model HQ Metrics Comparison

<table>
<thead>
<tr>
<th>Rise Time Table</th>
<th>PID Roll</th>
<th>LQR Roll</th>
<th>PID Pitch</th>
<th>LQR Pitch</th>
<th>PID Yaw</th>
<th>LQR Yaw</th>
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<tbody>
<tr>
<td>0.3M 1000Ft 10Deg</td>
<td>0.33</td>
<td>0.31</td>
<td>0.37</td>
<td>0.17</td>
<td>0.53</td>
<td>0.19</td>
</tr>
<tr>
<td>1M 1000Ft 4Deg</td>
<td>0.28</td>
<td>0.29</td>
<td>0.38</td>
<td>0.27</td>
<td>0.74</td>
<td>0.23</td>
</tr>
<tr>
<td>1.5M 5000Ft 2Deg</td>
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<td>0.29</td>
<td>0.4</td>
<td>0.36</td>
<td>0.51</td>
<td>0.25</td>
</tr>
<tr>
<td>0.85M 35000Ft 5Deg</td>
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<td>0.29</td>
<td>0.38</td>
<td>0.27</td>
<td>0.55</td>
<td>0.23</td>
</tr>
<tr>
<td>0.5M 15000Ft 5Deg</td>
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<td>0.3</td>
<td>NaN</td>
<td>0.24</td>
<td>0.51</td>
<td>0.2</td>
</tr>
<tr>
<td>HQ Required</td>
<td>1</td>
<td>1</td>
<td>NaN</td>
<td>NaN</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

5.3.1 Observation based on Real-time Flight Simulation

Transitioning to a nonlinear real time flight simulation, both the PID and LQ gain schedules had to be scaled to make them perform as intended. Each channel of the PID control system is scheduled to decrease control authority at various rates as $\ddot{q}$ increases, preventing unwanted state resonance, poor or loss of control. It was observed that for LQ optimal control system high error state flight conditions that less control deflection improved the likelihood of recovery, as excessive control which saturated surface deflections increased state error. Therefore the entire LQ gain matrix excluding the U column was scheduled to decrease as integrated roll state error accumulated to decrease the control deflection commands, preventing deflection saturation. Roll state error was selected as it was the least damped and was the first to indication that excessive control power would be requested to stabilize the aircraft, in an effect this is an active damper improving state settling time. The integrated roll state error unwinds at a predetermined fixed rate to increase available control power when the current state error returns to normal magnitudes.
PID Control System

The PID flight control system exhibited excellent command tracking performance during all phases of flight from low speed flight up to extreme aerobatic maneuvers where it diverges laterally beyond $\alpha$ of 45°. At low speed the PID system is able to manage sustained $\alpha$ of 25° pattern work with the landing gear extended or retracted, exhibiting very controllable decent rates for landing. Roll rate tracking is excellent and uninhibited at all air speeds, even during sustained 4 to 6-G pull ups the desired roll rate is achieved. Resulting in a coupled rolling and pitching maneuver at 15 to 30° $\alpha$, which from an external perspective appears as a coning roll about the velocity vector see Figure 5.16. Sustained roll rate performance up to 180°/sec was demonstrated throughout the flight envelope, allowing for rapid change in direction at the will of the pilot. When exposed to high $\ddot{q}$ in excess of approximately 1300 Lbf/Ft° usually obtained at low altitudes the roll axis resonates and becomes unstable, limiting the maximum obtainable airspeed. Mach limiters were put in place to prevent departure but this is not an ideal solution and leaves the PID system 0.15 Mach short of the $\ddot{q}_{Max}$ limit at sea level. Cruise flight stabilizes quickly with acceptable airspeed tracking, climb performance offers no challenges, and with the incorporation spoilers the deceleration performance increased the sustainable decent rate at a constant speed. No lateral directional challenges are experienced with trust reduced, as the control surfaces are able to maintain extended decent periods without the use of thrust to stabilize the system. Pitch rate commands were limited to 30°/sec with peak G forces typically between 9 to 10 more than sufficient for traditional aerobatic maneuvers. Excessive sustained pitch rate maneuvers do cause for unrecoverable energy loss and high $\alpha$ flight conditions resulting at best sluggish recovery of energy and body rate response or a divergent unrecoverable flight condition. With proper management a sequence of aggressive maneuvers may be performed at the cost of high pilot workload. Evaluating the PID flight control system was done by flying the ICE aircraft through the Grand Canyon between surface level and at most 500 feet while tracking the river below. This exposed the PID control system to a long sequence of rapid unique commands to track the river at speeds ranging from Mach 0.5 to 0.9. Overall the PID control system has excellent performance for all standard phases of flight only challenged by $\alpha$ beyond 45° at Mach $\leq$ 0.25 an easily avoidable flight condition.
The LQ optimal control system exhibited acceptable command tracking during standard operation ranging to extreme aerobatic maneuvers including varying departure recovery. During simulated take off from low speed between Mach 0.25 to 0.5, aircraft orientation is maintained as airspeed increases and $\alpha$ decreases to maintain a neutral pitch rate. Climb out is effortless with superior thrust availability maintaining the commanded airspeed, transitioning to cruise the throttle reduces and control activity begins to decreases as the system nears equilibrium as time progresses. Desecrating with the LQ control system presents lateral control challenges, as once thrust is reduced the use of lateral thrust vectoring is diminished and regulation is solely dependant on the distributed control surface compensation. Rapid roll rate commands can disrupt side slip angle to the point of divergence while decelerating, a compensating approach is to use a moderate wide up turn to bleed of excess airspeed as the cost of high work load by the pilot to not overshoot. During mild to moderate maneuvering at constant airspeed, noticeable roll rate response oscillations are observed particularly during cyclical opposing roll rate command. With the active damping $K$ matrix scaling, the disturbances dampen out within a few seconds allowing for uninhibited operation. During maximum opposing roll rate inputs the roll rate overshoot oscillations can overwhelm the commanded state response, but do dampen out once commands are neutralized. To prevent control saturation the allowable commanded roll rate is scaled in proportional to the current flight conditions $\bar{q}$ divided by max allowable $\bar{q}$ of 1655 Lbf/ft$^2$. As the roll axis is the most stable in comparison to pitch or yaw it requires the most control power to archive a given input at low $\bar{q}$. The
required control deflections for a commanded roll rate decrease with the increase of $\bar{q}$ allowing for higher sustained roll rates to be commanded. This prevents loss of control by allowing the other state error inputs to be compensated for with the remaining control power. Pitch rate commands up to $60^{\circ}$/sec allow for extreme maneuvers in excess of 12 G’s and controlled $\alpha$ maneuvers up to $90^{\circ}$ particularly at low $\bar{q}$ flight conditions between 50 and 200 Lbf/ft$^2$. Excessive sustained high pitch rate commands result in unrecoverable energy loss and side slip angle departure as Mach drops below 0.25. Proper energy management allows for repeatable maneuver performance at the cost of maximum pilot effort to obtain repeatable results. A common evaluation of the LQR control system was done by maneuvering through the Grand Canyon between 100 and 500 feet above ground level (AGL) and Mach 0.5 to 1.05 which did not require unreasonable pilot effort to maintain the desired flight path through the terrain. Opposite to roll the allowable commanded pitch rate decreases as $\bar{q}$ increases to limit excessive G or $\alpha$ maneuvers at high $\bar{q}$ flight conditions. When a side slip angle departure is experienced given enough time and altitude available the resulting $\pm 25^{\circ}$ $\beta$ excursion or flat spins have recovered given the right conditions. Tumbling flight conditions proved to be rarely recovered and were highly disorienting, which would require the pilot to punch out and loss of the aircraft. Overall the LQ control system has acceptable handling qualities while utilizing all of the control surfaces to regulate and sustain a wide range of command inputs from the most demanding of users when specific energy was properly managed.

Comparison

Figure 5.17 describes the F-35 aerial demonstration of which the ICE LQ and PID control systems can perform all but maneuver # 4 which requires a sustained yaw rate with approximately an $\alpha$ of $90^{\circ}$ as the aircraft descends vertically. Categorizing both flight control systems comfortably as an 4th generation fighter aircraft control system, noting the absence of a vertical stabilizer yet equivalent achievable performance for the majority of the aerial demonstration. Closer examination of F-35 pedal turn footage shows the descent is more of a helix than a straight line improving the feasibility of such a maneuver by the ICE aircraft. To start the maneuver a high turn rate over the top loop is performed and as peak $\alpha$ is reached near the bottom of the loop the helical rotating
decent is initiated. Resulting in a high $\alpha$ (Estimated to be 70 to 80°) slow roll about the velocity vector compared to the proposed flat spin as depicted in Figure 5.17. Continued attempts will be made to perform the pedal turn to evaluate the capabilities of the LQ control system under extreme $\alpha$ flight conditions.

A distinct advantage of the LQ system is its ability to operate a larger $\alpha$ than the current PID system. A signature maneuver of high $\alpha$ control is the Cobra Maneuver, “characterized by flight in the longitudinal axis of the aircraft with a small loss of altitude and a pitching up of the aircraft about its pitch axis greater than 90° while maintaining approximately the same altitude and its velocity vector” [18]. The LQ system has repeatedly demonstrated sustained 80° to 90° $\alpha$ pitching maneuvers without diverging.
Summarizing the capabilities of both the PID and LQ control system is Table 5.2, were a variety of selected maneuvers are listed with an X listed next to each maneuver that has been successfully demonstrated. Where as a O marks an unsuccessful attempt, where the PID falls behind the LQ by one mark the differing areas of peak performance offer advantages that the other lacks. Where the PID system leads in fast roll rates and controlled descents the LQ control offers extreme $\alpha$ control, spin recovery, and unrestricted Mach capabilities, combined these control systems would pose quite an impressive array of capabilities.

Table 5.2: PID vs LQ System Maneuver Comparison

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>PID</th>
<th>LQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Acceleration</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Vertical Climbing Roll</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Square Loop</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>25° AoA Level Flight</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Vert Climb From Mach 0.25</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>180 Deg Roll Rate</td>
<td>X</td>
<td>O</td>
</tr>
<tr>
<td>Minimum Radius Turn</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Cobra Maneuver</td>
<td>O</td>
<td>X</td>
</tr>
<tr>
<td>Idle Throttle Decent</td>
<td>X</td>
<td>O</td>
</tr>
<tr>
<td>Steady Heading Side Slip</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Spin Recovery</td>
<td>O</td>
<td>X</td>
</tr>
<tr>
<td>Pedal Turn</td>
<td>O</td>
<td>X</td>
</tr>
<tr>
<td>Unrestricted Airspeed</td>
<td>O</td>
<td>X</td>
</tr>
</tbody>
</table>
5.3.2 Future Work

With both the PID and LQ flight control systems performing continuously within their own respective operating envelopes, the next logical step would be to create a hybrid PID and LQ scheduled flight control system. Where the advantages of the PID system are used for \( \alpha \) less than 25° and below \( \bar{q} \) approximately 1300 Lbf/ft\(^2\) and the LQR system can be engaged to control flight conditions up to 90° \( \alpha \) and max \( \bar{q} \). In addition the gain scheduled envelop can be increased from 25° up to 90° \( \alpha \) with potentially the same number of steps taken between each flight condition limit; or additional steps can be taken to increase resolution as the gain schedule window increases. State estimation incorporating an LQE may be practical once sensor models and noise are incorporated, in addition to aero model uncertainty to evaluate how robust either control system is to modeling errors. Installation of actuator dynamics into the LQ and following LQG system will constrain the obtainable deflection rates increasing the realism of future control system models. Along with various state control methods, where G control may be more appealing than Q, or potentially \( \Theta \) for a highly augmented flying experience. The limitations for what can be controlled are only bounded by available development time and the creativity and knowledge of the designer to complete the objective at hand.

Brief evaluation of mass property sensitivity the presented flight control systems were evaluated at the lightweight 25,989 Lb configuration presented in [1]. Using the existing gain schedule for the 37,084 Lb configuration the nonlinear model was flown to evaluate the lightweight configuration’s resulting stability and control in real time pilot in the loop simulation on MAEFS. LQ results were promising with noticeably less time required to recover from a given spin, while maintaining the same \( Q, P \) and \( \beta \) control response times. As Mach increased to 2.2 pitch rate had greater steady state error likely due to a 4% decrease in static margin. Overall stability and control were maintained, with a noticeable improvement in pitch rate control allowing for repeatable Cobra maneuvers and body axis back flips that have been demonstrated at this time. The PID control system had a noticeable decrease in pitch damping as \( Q \) would begin to resonate a lower \( \bar{q} \) compared to the heavyweight configuration. A reasonable system response considering the 4% decrease in static margin decreasing longitudinal stability. No change in \( P \) or \( \beta \) state control was observed while the same limitations to the system persisted with overall state divergence beyond approximately
45° $\alpha$. In summary, neither the PID nor the LQ control system were detrimentally impacted by the change in mass properties to the ICE aircraft. While the sustainable turn performance and rate of climb greatly benefited as the thrust to weight ratio increased to 1.73 at sea level flight conditions.

5.4 Conclusion

Flight control system synthesis for the Lockheed Martin Innovative Control Effectors aircraft was successful. Demonstrating how to harness unstable high performance aircraft flight dynamics, due to properly understanding of the flight dynamics throughout the flight envelope and control by correctly applying PID and LQ control systems to the ICE aircraft. Providing an example of how to control unstable flight dynamics with innovative control effectors on a modern aircraft design. The overall design and evaluation process was enabled by MATLAB’s constrained optimization function ‘fmincon’ and nested loop code logic, without it the overall scale of the thesis would not have been practical in the time permitted.
REFERENCES


APPENDIX A

Sample PID Gain Optimization Script
%% Fmincon Based PID Gain Schedule Generation
clc
clear all
close all
format compact
savea = true;

D2R = 3.14/180;
R2D = 180/3.14;

% Linear Model Files - Variables, Mach, Alpha, Altitude
clear TrimMap ICEMDL
load('Linear_ICE_Model_Supersonic_2M_22M_0Alt_60Alt_-5Alt_25Alt_37084Weight_01-02-2019-00-31'
')
load('ICE_Trim_Map_36Stp_2M_22M_0Alt_60Alt_-5Alt_25Alt_37084Weight_12-16-2018-12-03')
load PID_GainSchedule_ReTuneRef_12819

AltitudeV = ICEMDL.Altitude;
MachV = ICEMDL.Mach;
AlphaV = ICEMDL.Alf;
Steps = ICEMDL.Steps;

%% Begin Loop Here

Gain_Iteration = 1;
Iteration_time = 120;
Time_Sec = 0;
MaxThrust = 45000;
warning('off','all')

VectorStep = 3;

% Arbitrary Initial Condition Manually Developed - Gain Order Q U P Beta

% xstart= [...
% 8.4920e+02 % Q Initial PID Control Gains
% 2.3204e+03
% 7.7694e+01
% 4.5416e+01 % P Initial PID Control Gains
% 3.1791e+01
% 1.6220e+01
% 1.9543e+00 % Beta Initial PID Control Gains
% 7.6003e+00
% 0
% 660.5 % U Initial PID Control Gains
% 6.48
% 1295]';

iteration_limit = max(size(REF_PID_GainSchedule));

try
load('PID_GainSchedule_12919')
load('Gain_Scheduled_PID_SYS_Envelope_12919')

catch

% 5:3:length(MachV)
% for Machi = 1:VectorStep:length(MachV)
for Machi = 1:VectorStep:iteration_limit
    Mach_Des = MachV(Machi);

119
% for Altitudei = 1:VectorStep:length(AltitudeV)
Altitude_Des = AltitudeV(Altitudei);
% for Alphai = 7:VectorStep:length(AlphaV)
Alpha_Des = AlphaV(Alphai);

format short
tic
clc

Mach_Des
Altitude_Des
Alpha_Des

i = interp1(MachV, linspace(1, Steps, Steps), Mach_Des, 'nearest');
p = i;

j = interp1(AltitudeV, linspace(1, Steps, Steps), Altitude_Des, 'nearest');
o = j;

k = interp1(AlphaV, linspace(1, Steps, Steps), Alpha_Des, 'nearest');
q = k;

Iteration = ICEDL.ABCD_Location(i, j, k, :);

% 1−13 : U V W Phi Theta Psi P Q R X Y Z Thrust
% 14−27 : [Initial_Thrust, LIBLEF, RIBLEF, LOPLEF, ROPLEF, LEL, REL, PF, LAMT, RAMT, LSSD, RSSD, VecThrZ ∗D2R, VecThrY ∗D2R]
% 28 : Trim Solution Cost

Trim_Thrust = ICEDL.Forces.Drag(i, j, k)
U = TrimMap(i, j, k, 1);
V = TrimMap(i, j, k, 2);
W = TrimMap(i, j, k, 3);
Alpha = atand(W/U)
Beta = atand(V/U)
V_FPS = sqrt(U^2 + V^2 + W^2)

Gain_Iteration
Iteration_time =Iteration_time;
Time_Sec = Time_Sec + Iteration_time;
Minutes_Elapsed = Time_Sec/60
Minutes_Required = Minutes_Elapsed/((Gain_Iteration+1)/((Steps/VectorStep)^3))
Minutes_Remaining = (Minutes_Elapsed/((Gain_Iteration+1)/((Steps/VectorStep)^3)))-Minutes_Elapsed
Hours_Required = Minutes_Required/60
Days_Required = Hours_Required/24

Percent_Complete = (Gain_Iteration/((Steps/VectorStep)^3))∗100
Iterations_Remaining = ((Steps/VectorStep)^3)−Gain_Iteration

%% Full Order Model
clear i
Asize = ICEMDL.Asiz(e Iteration,:);

A = reshape(ICEMLD.A(Iteration,:),Asize(1),Asize(2));
A = A(1:8,1:8); % Removing the Altitude State

% Notes – B Matrix is already scaled to represent surface deflection range

Bsize = ICEMLD.Bsize(Iteration,:);
Bint = reshape(ICEMLD.B(Iteration,:),Bsize(1),Bsize(2));
B = Bint(1:8,:); % Removing the Altitude State

Scale_B = zeros(14,14);
Scale_B(1,1) = 1;
Scale_B(2,2) = 1;
Scale_B(3,3) = 1;
Scale_B(4,4) = 1;
Scale_B(5,5) = 1;
Scale_B(6,6) = 1;
Scale_B(7,7) = 1;
Scale_B(8,8) = 1;
Scale_B(9,9) = 1;
Scale_B(10,10) = 1;
Scale_B(11,11) = 1;
Scale_B(12,12) = 1;
Scale_B(13,13) = 1;
Scale_B(14,14) = MaxThrust*Trim_Thrust; % Approximate Max Sea Level Installed Thrust Then Scaled By Trim Thrust

% Scale B To Represent Deflection Range
B = (Scale_B*B')';

Csize = ICEMLD.Csize(Iteration,:);
C = reshape(ICEMLD.C(Iteration,:),Csize(1),Csize(2));
C = C(,:,1:8); % Removing the Altitude State

CSS = C;

Dsize = ICEMLD.Dsize(Iteration,:);
D = reshape(ICEMLD.D(Iteration,:),Dsize(1),Dsize(2));

C_Feedback = [CSS(14,:);CSS(4,:);CSS(5,:);CSS(13,:)];

% Convert Output From Radians To Degrees
SC = zeros(4);
SC(1,1) = R2D; % Pitch Rate rad/sec to deg / sec conversion
SC(2,2) = 1; % Airspeed ft/s – no conversion needed
SC(3,3) = R2D; % Sideslip ft/s – no conversion needed
SC(4,4) = R2D; % Roll Rate rad/s to deg /sec conversion

C_Degrees = SC*C_Feedback;
D_Feedback = [D(14,:);D(4,:);D(5,:);D(13,:)];

SYS OL = ss(A,B,C_Degrees,D_Feedback);

%% Open Loop Full Order Reduced Input & Output Model

clear G C Ahat Bhat i

% Desired Inputs
% Input1 = 6; % Pitch Flap Input
% Input21 = 4; % Left Elevon
% Input22 = 10; % Right Elevon
% Input3 = 13; % Yaw Thrust Vectoring
% Input4 = 14; % Engine Thrust

Full_SYS_Ol = SYS_Ol(:,[4,6,10,13,14,3,9]);

%% Q5 Tuned Full Order Closed Loop System

clear G CL Sum1 Sum2 Sum3 Sum4 Pass1 Pass2 Pass3 Pass4 Pass5 y

CTRLimits = [30 0 0 0 0 0
0 30 0 0 0 0
0 0 30 0 0 30
0 0 0 15 0 0 30
0 0 0 0 Trim_Thrust/MaxThrust 0 0
0 0 0 0 60 0 0
0 0 0 0 0 60
0 0 0 0 0 0 60];

RescaleB = Full_SYS_Ol.B*CTRLimits;

Scaled_Full_SYS_Ol = ss(Full_SYS_Ol.A,RescaleB,Full_SYS_Ol.C,Full_SYS_Ol.D);

%% Optimal Weight FMinCon Solution

tol = 1E-6;

options = optimset ('Display', 'iter', 'TolFun', tol, 'TolX', tol, 'MaxIter', 50, 'Diagnostics', 'off', 'Algorithm', 'interior-point', 'UseParallel', false);

options.TolCon = tol;

Ub = 10000*ones(1,12);
Lb = -.000001*ones(1,12);

format long

xstart = REF_PID_Gain_Schedule(p,o,q,:);

[GainVector, fval, exitflag]=fmincon(@(var) State_Cost(var,Scaled_Full_SYS_Ol,MaxThrust), xstart,[],[],[],[],[],[],[],LB,Ub, @(var) Constraints(var,Scaled_Full_SYS_Ol,MaxThrust),options);

% GainVector';

% format short

% Did_IT_Change = norm(xstart)-norm(GainVector)

GainSchedule(p,o,q,:) = GainVector;

[DES_HQ] = PIDFnx(GainVector,Scaled_Full_SYS_Ol,MaxThrust);

%% Data Collection

SQ = stepinfo(DES_HQ(1,1)); % Q CMD to Q Output
SP = stepinfo DES_HQ(3,3); % P CMD to P Output
SB = stepinfo DES_HQ(4,4); % Beta CMD to Beta Output
SPF = stepinfo DES_HQ(5,1); % Q CMD to Pitch Flap Deflection
SEL = stepinfo DES_HQ(7,3); % P CMD to Left Elevon
SLTV = stepinfo DES_HQ(9,4); % Beta CMD to Yaw Thrust Vectoring

% SQ.RiseTime
% SQ.SettlingTime
% SQ.Overshoot
%
% SP.RiseTime
% SP.SettlingTime
% SP.Overshoot
%
% SB.RiseTime
% SB.SettlingTime
% SB.Overshoot

SYSEVE.Q.RiseTimeMatrix(p,o,q) = SQ.RiseTime;
SYSEVE.Q.SettlingTimeMatrix(p,o,q) = SQ.SettlingTime;
SYSEVE.Q.OvershootMatrix(p,o,q) = SQ.Overshoot;

SYSEVE.P.RiseTimeMatrix(p,o,q) = SP.RiseTime;
SYSEVE.P.SettlingTimeMatrix(p,o,q) = SP.SettlingTime;
SYSEVE.P.OvershootMatrix(p,o,q) = SP.Overshoot;

SYSEVE.B.RiseTimeMatrix(p,o,q) = SB.RiseTime;
SYSEVE.B.SettlingTimeMatrix(p,o,q) = SB.SettlingTime;
SYSEVE.B.OvershootMatrix(p,o,q) = SB.Overshoot;

SYSEVE.PF.RiseTimeMatrix(p,o,q) = SPF.RiseTime;
SYSEVE.PF.SettlingTimeMatrix(p,o,q) = SPF.SettlingTime;
SYSEVE.PF.OvershootMatrix(p,o,q) = SPF.Overshoot;

SYSEVE.EL.RiseTimeMatrix(p,o,q) = SEL.RiseTime;
SYSEVE.EL.SettlingTimeMatrix(p,o,q) = SEL.SettlingTime;
SYSEVE.EL.OvershootMatrix(p,o,q) = SEL.Overshoot;

SYSEVE.LTV.RiseTimeMatrix(p,o,q) = SLTV.RiseTime;
SYSEVE.LTV.SettlingTimeMatrix(p,o,q) = SLTV.SettlingTime;
SYSEVE.LTV.OvershootMatrix(p,o,q) = SLTV.Overshoot;

SYSEVE.info.Trim_Thrust(p,o,q) = Trim_Thrust;
SYSEVE.info.Alpha(p,o,q) = Alpha;

try
SYSEVE.P = pole DES_HQ;
SYSEVE.PositiveDamping(p,o,q) = sum(SYS_P(SYS_P>0));
catch
disp('SVD Error Encountered')
SYSEVE.PositiveDamping(p,o,q) = .123;
end

%% Time Remaining Calculator
Gain_Iteration = Gain_Iteration + 1;
Iteration_time = toc;
end
end
end
save('Gain_Scheduled_PID_SYS_Envelope_12919','SYSEVE')

save('PID_Gain_Schedule_12919','GainSchedule')

done

warning('on','all')

%% Functions

function [C, Ceq] = Constraints(G, Scaled_Full_SYS_OL, MaxThrust)

[DES_HQ] = PIDFnx(G, Scaled_Full_SYS_OL, MaxThrust);

%% State Constraint

try

% opt2 = stepDataOptions('InputOffset',0,'StepAmplitude',1);
[Y2, T2, ~] = step(DES_HQ([1, 3, 4],[1, 3, 4]));
StateStepInfo = stepinfo(Y2, T2, 1);

% State Overshoot
QOS = StateStepInfo(1,1).Overshoot;
PST = StateStepInfo(2,2).Overshoot;
BetaOS = StateStepInfo(3,3).Overshoot;

if is nan(QOS) == 1
QOS = 120;
end

if is nan(PST) == 1
PST = 120;
end

if is nan(BetaOS) == 1
BetaOS = 120;
end

% State Overshoot
QST = StateStepInfo(1,1).SettlingTime;
PST = StateStepInfo(2,2).SettlingTime;
BetaST = StateStepInfo(3,3).SettlingTime;

if is nan(QST) == 1
QST = 120;
end

if is nan(PST) == 1
PST = 120;
end

if is nan(BetaST) == 1
BetaST = 120;
end

% State Rise Time
QRt = StateStepInfo(1,1).RiseTime;
PRT = StateStepInfo(2,2).RiseTime;
BetaRt = StateStepInfo(3,3).RiseTime;

if is nan(QRt) == 1
QRt = 120;
end

if is nan(PRT) == 1
PRT = 120;
end
if isnan(BetaRt) == 1
    BetaRt = 120;
end

% Airspeed Step Info
opt = stepDataOptions('InputOffset',0,'StepAmplitude',150);
[Y1,T1,~] = step(DESHQ(2,2),opt);
EngineStepInfo = stepinfo(Y1,T1,150);

% Airspeed and Thrust States
URt = EngineStepInfo(1,1).RiseTime;  % Rise Time
UOS = EngineStepInfo(1,1).Overshoot; % Over Shoot

if isnan(URt) == 1
    URt = 120;
end

if isnan(UOS) == 1
    UOS = 120;
end

catch
    QRt = 120;
    PRt = 120;
    BetaRt = 120;
    QST = 120;
    PST = 120;
    BetaST = 120;
    QOS = 120;
    POS = 120;
    BetaOS = 120;
    URt = 120;
    UOS = 120;
end

% Available Deflection %
% AD = .5; % 50% Safety Factor On Control Surface Deflection

% C(1) = MaxDfl(1) - dilefi_limit_u*AD;
% C(2) = MaxDfl(2) - drlefi_limit_u*AD;
% C(3) = MaxDfl(3) - dilefo_limit_u*AD;
% C(4) = MaxDfl(4) - drlefo_limit_u*AD;
% C(5) = MaxDfl(5) - dlamt_limit_u*AD;
% C(6) = MaxDfl(6) - dramt_limit_u*AD;
% C(7) = MaxDfl(7) - dilele_limit_u*AD;
% C(8) = MaxDfl(8) - drele_limit_u*AD;
% C(9) = MaxDfl(9) - dissd_limit_u*AD;
% C(10) = MaxDfl(10) − drssd_limit_u*AD;
% C(11) = MaxDfl(11) − dpf_limit_u*AD;
% C(12) = MaxDfl(12) − dptv_limit_u*AD;
% C(13) = MaxDfl(13) − dytv_limit_u*AD;
% C(14) = MaxDfl(14) − 45000;

C(9) = UOS - 5; % Set Allowable Airspeed Overshoot
C(10) = URt - 15; % Set Upper Limit For Range For Airspeed Rise Time
C(11) = 5 - URt; % Set Lower Limit For Range For Airspeed Rise Time

C(12) = QOS - 0; % Set Allowable Alpha Overshoot
C(13) = BetaOS - 0; % Set Allowable Beta Overshoot
C(14) = POS - 0; % Set Allowable Phi Overshoot

C(15) = QST - 1.5; % Set Allowable Alpha Settling Time
C(16) = BetaST - 3; % Set Allowable Beta Settling Time
C(17) = PST - 1.5; % Set Allowable Phi Settling Time
C(20) = .125 - QRt; % Enforces A Minimum Rise Time Constraint
C(21) = .125 - BetaRt; % Enforces A Minimum Rise Time Constraint
C(22) = .125 - PRt; % Enforces A Minimum Rise Time Constraint
C(23) = QRt - 1; % Enforces A Maximum Rise Time Constraint
C(24) = BetaRt - 1.25; % Enforces A Maximum Rise Time Constraint
C(25) = PRt - 1; % Enforces A Maximum Rise Time Constraint

Ceq = [];
end

function [J] = State_Cost(G, Scaled_Full_SYS_DL, MaxThrust)

[DES_HQ] = PIDFnx(G, Scaled_Full_SYS_DL, MaxThrust);

%%%% State Constraint
try
    [Y2, T2, ~] = step(DES_HQ([1, 3, 4],[1, 3, 4]));
    StateStepInfo = stepinfo(Y2, T2, 1);
end

% State Overshoot
QST = StateStepInfo(1,1).SettlingTime;
PST = StateStepInfo(2,2).SettlingTime;
BetaST = StateStepInfo(3,3).SettlingTime;

% State Rise Time
QRt = StateStepInfo(1,1).RiseTime;
PRt = StateStepInfo(2,2).RiseTime;
BetaRt = StateStepInfo(3,3).RiseTime;

if isnan(QST) == 1
    QST = 120;
end

if isnan(PST) == 1
    PST = 120;
end

if isnan(BetaST) == 1
    BetaST = 120;
end

if isnan(QRt) == 1
    QRt = 120;
end

if isnan(PRt) == 1
    PRt = 120;
end

if isnan(BetaRt) == 1
    BetaRt = 120;
end

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\begin{verbatim}
BetaRt = 120;
end

% State Overshoot
QOS = StateStepInfo(1,1).Overshoot;
POS = StateStepInfo(2,2).Overshoot;
BetaOS = StateStepInfo(3,3).Overshoot;

if isnan(QOS) == 1
    QOS = 120;
end

if isnan(POS) == 1
    POS = 120;
end

if isnan(BetaOS) == 1
    BetaOS = 120;
end

% Airspeed Step Info
opt = stepDataOptions('InputOffset',0,'StepAmplitude',150);
[Y1,T1,~] = step(DES_HQ(2,2),opt);
EngineStepInfo = stepinfo(Y1,T1,150);

% Airspeed and Thrust States
URt = EngineStepInfo(1,1).RiseTime;  \% Rise Time
USt = EngineStepInfo(1,1).SettlingTime;  \% Settling Time

if isnan(URt) == 1
    URt = 120;
end

if isnan(USt) == 1
    USt = 120;
end

catch  \% --- --- --- --- ---

    QST = 120;
PST = 120;
BetaST = 120;
USt = 120;

    QRt = 120;
PRt = 120;
BetaRt = 120;
URt = 120;

    QOS = 120;
POS = 120;
BetaOS = 120;

end

%% Desired State Response Times

RtQ  = .55;  \% [Sec]
RtBeta  = .55;  \% [Sec]
RtP  = .55;  \% [Sec]
RtU  = 6.5;  \% [Sec]

StQ  = RtQ*2;  \% [Sec]
StBeta  = RtBeta*2;  \% [Sec]
StP  = RtP*2;  \% [Sec]
StU  = RtU*2;  \% [Sec]

OSQ  = 0;  \% [Sec]
\end{verbatim}
OSBeta = 0; % [Sec]
OSP = 0; % [Sec]

% Response_Req = {'Rise Time','Settling Time','Overshoot'};
% BOutput_Table = cell2table(Response_Req);
% B_Matrix = array2table([RtQ,StQ,OSQ;RtU,StU,OSQ],...'
% Orientation_Req_Sec',
% Airspeed_Req_Sec');
% Response_Req = [BOutput_Table,B_Matrix]

% Constraint Cost Components
% Rise Time Constraints
% Deflection Cost
% Kalman Filter Accuracy Cost
J = (QRt-RtQ).^2+(BetaRt-BtBeta).^2+((URt-RtU)/1000).^2 + ...
% Rise
(QST-StQ).^2+(BetaST-StBeta).^2+((UST-StU)/1000).^2 + ... % Settling
(OQS-OSQ).^2+(BetaOS-OSBeta).^2+(POS-OSP).^2 ; % Over Shoot

function [DES_HQ] = PIDFnx(Gain,Scaled_Full_SYS OL,MaxThrust)

C_Q = tf(-1*pid(Gain(1),Gain(2),Gain(3))); % Q Channel Controller
C_P = tf(pid(Gain(4),Gain(5),Gain(6))); % P Channel Controller
C_Beta = tf(pid(Gain(7),Gain(8),Gain(9))); % Beta Channel Controller
C_U = tf(pid(Gain(10),Gain(11),Gain(12))); % U Channel Controller

Law = [C_Q 0 0 0 0 0 0; % Q Control Law
0 C_U 0 0 0 0 0; % U Control Law
C_Q 0 0 0 0 0 0; % Left Elevon Control Law
0 0 0 0 -C_Beta 0 0; % Right Elevon Control Law
0 0 0 0 -C_Beta 0 0; % Lateral Thrust Vectoring
0 0 0 0 0 -C_Beta 0 0; % Right AMT Beta Control
0 0 0 0 0 0 0]; % Left AMT Beta Control Law

% Re-scales control power from normalized control power matrix
Control_Power_Scaling = [1/30 0 0 0 0 0 0
0 1/30 0 0 0 0 0
0 0 1/30 0 0 0 0
0 0 0 1/30 0 0 2/30
0 0 0 0 1/15 0 0
0 0 0 0 0 0 4/60
0 0 0 0 0 0 4/60];

CL = Law*Control_Power_Scaling;

% Feedback Loop
Sum1 = sumblk('e(1) = r(1) - y(1)'); % Q Error - Q CMD - Q Actual
Sum2 = sumblk('e(2) = r(2) - y(2)'); % U Error - U CMD - U Actual
Sum3 = sumblk('e(3) = r(3) - y(3)'); % P Error - P CMD - P Actual
Sum4 = sumblk('e(4) = r(4) - y(4)'); % Beta Error - Beta CMD 0 Beta Actual

% Controller Input & Output Relationship
% Pitch Rate
CL(1,1).InputName = 'e(1)'; % Q Error
CL(1,1).OutputName = 'u(2)'; % Pitch Flap
CL(3,1).InputName = 'e(1)'; % Q Error
CL(3,1).OutputName = 'u(1)'; % Left Elevon

CL(4,1).InputName = 'e(1)'; % Q Error
CL(4,1).OutputName = 'u(3)'; % Right Elevon

% ------------------  Airspeed  ------------------
CL(2,2).InputName = 'e(2)'; % U Error
CL(2,2).OutputName = 'u(5)'; % Engine Thrust

% ------------------  Roll Rate  ------------------
CL(3,3).InputName = 'e(3)'; % P Error
CL(3,3).OutputName = 'u(1)'; % Left Elevon

CL(4,4).InputName = 'e(3)'; % P Error
CL(4,4).OutputName = 'u(3)'; % Right Elevon

% ------------------  Side Slip Angle  ------------------
CL(5,5).InputName = 'e(4)'; % Beta Error
CL(5,5).OutputName = 'u(4)'; % Lateral Thrust Vectoring

CL(6,6).InputName = 'e(4)'; % Beta Error
CL(6,6).OutputName = 'u(6)'; % Right AMT

CL(7,7).InputName = 'e(4)'; % Beta Error
CL(7,7).OutputName = 'u(7)'; % Left AMT

CL(3,6).InputName = 'e(4)'; % Beta Error
CL(3,6).OutputName = 'u(3)'; % Right Elevon

CL(3,7).InputName = 'e(4)'; % Beta Error
CL(3,7).OutputName = 'u(1)'; % Left Elevon

% Longitudinal Effector Deflection Variables
Pass1 = sumblk ('y(5) = u(2)'); % Pitch Flap Deflection

Pass2 = sumblk ('y(6) = u(5)'); % Thrust Required

% Lateral - Directional Effector Deflection Variables
Pass3 = sumblk ('y(7) = u(1)'); % Left Elevon Deflection

Pass4 = sumblk ('y(8) = u(3)'); % Right Elevon Deflection

Pass5 = sumblk ('y(9) = u(4)'); % Yaw Thrust Vectoring

Pass6 = sumblk ('y(10) = u(6)'); % Right SSD

Pass7 = sumblk ('y(11) = u(7)'); % Left SSD

% Actuator TF's
s = tf('s');
GC(1) = (40)*(100)/((s+40)*(s+100)); % Control Surface Actuators Not Including Leading Edge Surfaces
GC(2) = (1/(s+2.5+1)); % Engine Time Delay

Actuator_Dynamics = [GC(1) 0 0 0 0 0
0 GC(1) 0 0 0 0
0 0 GC(1) 0 0 0
0 0 0 GC(1) 0 0
0 0 0 0 GC(2) 0
0 0 0 0 0 GC(1) 0
0 0 0 0 0 0 GC(1)];

G = series(Actuator_Dynamics,Scaled_Full_SYS_OL);
G.InputName = 'u';
G.OutputName = 'y';

Full_PID_CL = connect(G,CL,Sum1,Sum2,Sum3,Sum4,Pass1,Pass2,Pass3,Pass4,Pass5,Pass6,Pass7,'r','y','u');

Full_CL_SYS = ss(Full_PID_CL);

F1 = 1/((.09*s)+1)^3;  % Pitch Input Filter
F2 = 1/((1.4*s)+1)^3;  % U Input Filter
F3 = 1/((.09*s)+1)^3;  % P Input Filter
F4 = 1/((.1*s)+1)^3;  % Beta Input Filter

Input_Filter = [F1 0 0 0
                0 F2 0 0
                0 0 F3 0
                0 0 0 F4];

DES_HQ = series(Input_Filter,Full_CL_SYS);

end
APPENDIX B

Sample LQ Gain Optimization Script
%% Fmincon Based LQR Weight Generation
clc
clear all
close all
format short
savea = true;

D2R = 3.14/180;
R2D = 180/3.14;

%% Specify Desired Flight Condition of Interest
Altitude_Des = 15000; % [Ft]
Mach_Des = .5;
Alpha_Des = 5; % [Deg]

% Linear Model Files - Variables, Mach, Alpha, Altitude
clear TrimMap ICEMDL
load(' Linear_ICE_Model_Supersonic_2M_22M_0Alt_60Alt_-5Alt_25Alt_37084Weight_01-02-2019_00-31')
load(' ICE_Trim_Map_36Stp_2M_22M_0Alt_60Alt_-5Alf_25Alf_37084Weight_12-16-2018_12-03')
Altitude = ICEMDL.Altitude;
Mach = ICEMDL.Mach;
Alpha = ICEMDL.Alf;
Steps = ICEMDL.Steps;
i = interp1(Mach, linspace(1,Steps,Steps),Mach_Des,'nearest'); % Mach is only 12 steps per file, Linear Model Solution Crashed half way through
j = interp1(Altitude, linspace(1,Steps,Steps),Altitude_Des,'nearest');
k = interp1(Alpha, linspace(1,Steps,Steps),Alpha_Des,'nearest');
Iteration = ICEMDL.ABCD_Location(i,j,k,:);

% 1 2 3 4 5 6 7 8 9 10 11 12 13
% 1-13 : U V W Phi Theta Psi P Q R X Y Z Thrust
% 14 15 16 17 18 19 20 21 22 23 24 25 26
% 14-27 : [Initial_Thrust,LIBLEF,RIBLEF,LOBLEF,ROBLEF,LEL,REL,PF,LAMT,RAMT,LSSD,RSSD,VecThrZ *D2R,VecThrY*D2R]
% 28
% 28 : Trim Solution Cost

%% State Space Relization
clear i
Asize = ICEMDL.Asizel(Iteration,:);
A = reshape(ICEMDL.A(Iteration,:),Asize(1),Asize(2));
A = A(1:8,1:8); % Removing the Altitude State

% Notes - B Matrix is already scaled to represent surface deflection range
Bsize = ICEMDL.Bsize(Iteration,:);
Bint = reshape(ICEMDL.B(Iteration,:),Bsize(1),Bsize(2));
B = Bint(1:8,:); % Removing the Altitude State

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Csize = ICEMDL.Csize(Iteration,:);
C = reshape(ICEMDL.C(Iteration,:),Csize(1),Csize(2));

% Specify Output States
C = C([4,6,11,14,5,10,13,15],1:8); % Removing the Altitude State

Dsize = ICEMDL.Dsize(Iteration,:);
D = reshape(ICEMDL.D(Iteration,:),Dsize(1),Dsize(2));

% Specify Feedthrough States
D = D([4,6,11,14,5,10,13,15],:);

sys = ss(A,B,C,D);

%% Optimal Weight FMinCon Solution

tol = 1E-6;

options = optimset('Display','iter','TolFun',tol,'TolX',tol,'MaxIter',1000,'Diagnostics','off','Algorithm','interior-point','UseParallel',true);

options.TolCon = tol;

Ub = 1e3*ones(1,30);

Lb = [zeros(1,22),-1e3*ones(1,8)];

% Arbitrary Initial Condition Manually Developed

% State Weights 1 - 8
% Control Weight 9 - 22 (reduced to length 1 simplify solution)
% State Scaling 23 - 30

xstart= [ ... 
% % State Weighting
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% % Control Surface Weights
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% % State Scaling
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1
% 1

133
% 2-24-19
xstart = 1.0e+06 * [...
0.000069322794383
0.000043906657691
0.000039284018617
0.000035990243550
0.000061181696231*17.5
0.00009528120771*200
0.000018022516677
0.00003604340110
0.000037035444724
0.000040104145560
0.000034796083636
0.000000882022075
0.000037001423191
0.000035827332209
0.000037167137483
0.000037468165877
0.000036917801593
0.000000892223232
0.000035989583602
0.000011927296292
0.00002329384251
0.000037356064814
1.238307752326274
0.000005347580446
0.000001999334923
0.000001999334923
0.000003420274723
0.000002016055317
0.000001999334923
0.000001999334923]*2’;

Xmin = xstart;

%% Constrained Optimization

% Run First To Blance State Weight Vector

{Xmin,fval,exitflag} = fmincon(@(var) Control_Cost(var,sys,A,B,C,D),xstart,[],[],[],[],[],[],options);

% xstart = Xmin

% Run Second To Begin Optimizing State Weights To Achieve State Response

% Times

{Xmin,fval,exitflag} = fmincon(@(var) State_Cost(var,sys,A,B,C,D),xstart,[],[],[],[],[],options);

% Solution_Cost = fval
% Did_It_Change = norm(xstart)-norm(Xmin)
%
% format long
% % Xmin’
%
%%

format shorte

[Augmented_CL_SYS,OL_SYS,K,Kdc,Adj_Matrix,CtrAuth] = LQR(Xmin,sys,A,B,C,D);
K_Degrees = K*D2R;
K_Degrees(1:14,1) = 0;
K_Degrees(14,1:8) = 0;
K_Degrees(:,[3,4,7,8]) = 0

stepi = .001;
T3 = 0:stepi:30;
CMD = [ones(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),ones(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3))];
Time = 0:stepi:(max(T3)*5)+.004;

figure(3)
lsim(Augmented_CL_SYS(1:8,1:4),[CMD]',Time), grid on
title('LQR Step Response')

opt1 = stepDataOptions('InputOffset',0,'StepAmplitude',150);
[Y1,T1,~] = step(Augmented_CL_SYS([1,22],1),opt1);
EngineStepInfo = stepinfo(Y1,T1,150);
EngInfo = struct2cell(EngineStepInfo);

AirRt = cell2mat(EngInfo(1,1)); % Rise Time
Peak_Thrust = cell2mat(EngInfo(7,2)); % Peak Thrust

opt = stepDataOptions('InputOffset',0,'StepAmplitude',1);
[YS,Ts,~] = step(Augmented_CL_SYS([9,22],2:4),opt);

ControlStepInfo = stepinfo(YS,Ts,1);
CtrInfo = struct2cell(ControlStepInfo);

Peak_Deflection = [cell2mat(CtrInfo(7,:,1));cell2mat(CtrInfo(7,:,2));cell2mat(CtrInfo(7,:,3))];

MaxDfl = [max(Peak_Deflection),Peak_Thrust];

% Constraint States
MaxDfl = [max(Peak_Deflection),Peak_Thrust];

% State Step Info
opt2 = stepDataOptions('InputOffset',0,'StepAmplitude',1);
[Y2,T2,~] = step(Augmented_CL_SYS(1:8,2:4),opt2);
StateStepInfo = stepinfo(Y2,T2,1);

Peak,Thrust
disp(' ')

% State Rise Time
AirRt = StateStepInfo(2,1).RiseTime
BetaRt = StateStepInfo(5,2).RiseTime
PhiRt = StateStepInfo(6,3).RiseTime
disp(' ')
% State Overshoot
AlphaOS = StateStepInfo(2,1).Overshoot
BetaOS = StateStepInfo(5,2).Overshoot
PhiOS = StateStepInfo(6,3).Overshoot

figure(6)
subplot(121)
step(Augmented_CL_SYS(2:8,1:4),5)
subplot(122)
step(Augmented_CL_SYS(9:21,1:4),5)

%% Linear Model Simulation

clear CMD
format short
T3 = 0:.001:10;
CMD = [zeros(1,length(T3)),50*ones(1,length(T3)),zeros(1,length(T3)),
       zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),
       10*ones(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),
       zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),
       zeros(1,length(T3)),zeros(1,length(T3)),zeros(1,length(T3)),
       -10*sin(T3.^1.2),10*ones(1,length(T3)),10*ones(1,length(T3))]

Time = 0:.001:max(T3)*9+.008;

[Y,T,X] = lsim(Augmented_CL_SYS(9:22,:),CMD,Time);

MaxY = max(abs(Y));
Norm_Deflection = norm(MaxY);

% Closed Loop System Evaluation Tables & Figures

Peak_Deflection = [{'Normalized Values'}];
AOutput_Table = cell2table(Peak_Deflection);
A_Matrix = array2table(round(MaxY,2),{'LIBLEF_1','LOBLEF_2','LAMT_3','LEL_4','LSSD_5','PF_6','RIBLEF_7','ROBLEF_8','RAMT_9','REL_10','RSSD_11','VecThrX_12','VecThrY_13','Thrust_14'});
Peak_Deflection = [AOutput_Table,A_Matrix]

%% Functions

function [C,Ceq] = Constraints(var,sys,A,B,C,D)

[Augmented_CL_SYS,~,~,~] = LQR(var,sys,A,B,C,D);

% Airspeed Step Info
opt1 = stepDataOptions('InputOffset',0,'StepAmplitude',150);
[Y1,T1,X1] = step(Augmented_CL_SYS([1,22],1),opt1);
EngineStepInfo = stepinfo(Y1,T1,150);
EngInfo = struct2cell(EngineStepInfo);

% Airspeed and Thrust States
AirRt = cell2mat(EngInfo(1,1)); % Rise Time
Peak_Thrust = cell2mat(EngInfo(7,2)); % Peak Thrust

if isnan(AirRt) == 1
    AirRt = 120;
end

if isnan(Peak_Thrust) == 1
    Peak_Thrust = 120000;
end

%% Control Surface Step Info
opt = stepDataOptions('InputOffset',0,'StepAmplitude',10);
[Y,T,X] = step(Augmented_CL_SYS(9:21,2:4),opt);

ControlStepInfo = stepinfo(Y,T,10);
CtrInfo = struct2cell(ControlStepInfo);

Peak_Deflection = [cell2mat(CtrInfo(7,:,[1])); cell2mat(CtrInfo(7,:,[2])); cell2mat(CtrInfo(7,:,[3]));];

% Constraint States
MaxDfl = [max(Peak_Deflection), Peak_Thrust];

for surf_i = 1:14
    if isnan(MaxDfl(surf_i)) == 1
        MaxDfl(surf_i) = 12;
    end
end

%% State Constraint
opt2 = stepDataOptions('InputOffset',0,'StepAmplitude',10);
[Y2,T2,~] = step(Augmented_CL_SYS(1:8,2:4),opt2);
StateStepInfo = stepinfo(Y2,T2,10);

% State Overshoot
QOS = StateStepInfo(2,1).Overshoot;
BetaOS = StateStepInfo(5,2).Overshoot;
POS = StateStepInfo(6,3).Overshoot;

if isnan(QOS) == 1
    QOS = 12;
end

if isnan(BetaOS) == 1
    BetaOS = 12;
end

if isnan(POS) == 1
    POS = 12;
end

% State Rise Time
QRt = StateStepInfo(3,1).RiseTime;
BetaRt = StateStepInfo(5,2).RiseTime;
PRt = StateStepInfo(6,3).RiseTime;

if isnan(QRt) == 1
    QRt = 12;
end

if isnan(BetaRt) == 1
    BetaRt = 12;
end
end

if isnan(PRt) == 1
    PRt = 12;
end

dllefi_limit_u = 40;
drlefi_limit_u = 40;
dllefo_limit_u = 40;
drlefo_limit_u = 40;
dlamt_limit_u = 60;
dramt_limit_u = 60;
dele_limit_u = 30;
drele_limit_u = 30;
dlssd_limit_u = 60;
drssd_limit_u = 60;
dpf_limit_u = 30;
dptv_limit_u = 15;
dytv_limit_u = 15;

% % Available Deflection %
AD = 1;  % 0% Safety Factor On Control Surface Deflection
C(1) = MaxDfl(1) - dllefi_limit_u*AD;
C(2) = MaxDfl(2) - drlefi_limit_u*AD;
C(3) = MaxDfl(3) - dllefo_limit_u*AD;
C(4) = MaxDfl(4) - drlefo_limit_u*AD;
C(5) = MaxDfl(5) - dlamt_limit_u*AD;
C(6) = MaxDfl(6) - dramt_limit_u*AD;
C(7) = MaxDfl(7) - dele_limit_u*AD;
C(8) = MaxDfl(8) - drele_limit_u*AD;
C(9) = MaxDfl(9) - dlssd_limit_u*AD;
C(10) = MaxDfl(10) - drssd_limit_u*AD;
C(11) = MaxDfl(11) - dpf_limit_u*AD;
C(12) = MaxDfl(12) - dptv_limit_u*AD;
C(13) = MaxDfl(13) - dytv_limit_u*AD;
C(14) = QOS - 15;  % Set Allowable Alpha Overshoot
C(15) = BetaOS - 15;  % Set Allowable Beta Overshoot
C(16) = POS - 15;  % Set Allowable Phi Overshoot
C(17) = .25 - QRt;  % Enforces A Minimum Rise Time Constraint
C(18) = .25 - BetaRt;  % Enforces A Minimum Rise Time Constraint
C(19) = .25 - PRt;  % Enforces A Minimum Rise Time Constraint
C(20) = QRt - .75;  % Enforces A Max Rise Time Constraint
C(21) = BetaRt - .75;  % Enforces A Max Rise Time Constraint
C(22) = PRt - .75;  % Enforces A Max Rise Time Constraint

Ceq = [];

function [J] = State_Cost(var,sys,A,B,C,D)
[Augmented_CL_SYS,~,~,~,~] = LQR(var,sys,A,B,C,D);
end
% Airspeed Step Info
opt1 = stepDataOptions ('InputOffset',0,'StepAmplitude',150);
[Y1,T1,~] = step(Augmented_CL_SYS([1,22],1),opt1);
EngineStepInfo = stepinfo(Y1,T1,150);
EngInfo = struct2cell(EngineStepInfo);

% Airspeed and Thrust States
AirRt = cell2mat(EngInfo(1,1)); % Rise Time
Peak_Thrust = cell2mat(EngInfo(7,2)); % Peak Value Thrust
if isnan(AirRt) == 1
    AirRt = 120;
end
if isnan(Peak_Thrust) == 1
    Peak_Thrust = 12000;
end

% Control Surface Step Info
opt = stepDataOptions ('InputOffset',0,'StepAmplitude',10);
[Y,T,~] = step(Augmented_CL_SYS(9:21,2:4),opt);
ControlStepInfo = stepinfo(Y,T,10);
CtrInfo = struct2cell(ControlStepInfo);
Peak_Deflection = [cell2mat(CtrInfo(7,:,[1]); cell2mat(CtrInfo(7,:,[2]); cell2mat(CtrInfo(7,:,[3]);

% Constraint States
MaxDfl = [max(Peak_Deflection),Peak_Thrust];
for surf_i = 1:14
    if isnan(MaxDfl(surf_i)) == 1
        MaxDfl(surf_i) = 12;
    end
end

% State Step Info
[Y3,T3,~] = step(Augmented_CL_SYS(1:8,2:4),opt);
StateStepInfo = stepinfo(Y3,T3,1);
% State Rise Time
QRt = StateStepInfo(2,1).RiseTime;
BetaRt = StateStepInfo(5,2).RiseTime;
PRt = StateStepInfo(6,3).RiseTime;
if isnan(QRt) == 1
    QRt = 12;
end
if isnan(BetaRt) == 1
    BetaRt = 12;
end
if isnan(PRt) == 1
    PRt = 12;
end
% State Overshoot
QST = StateStepInfo(2,1).SettlingTime;
BetaST = StateStepInfo(5,2).SettlingTime;
PST = StateStepInfo(6,3).SettlingTime;

if isnan(QST) == 1
    QST = 12;
end

if isnan(BetaST) == 1
    BetaST = 12;
end

if isnan(PST) == 1
    PST = 12;
end

%% Desired State Response Times

% RtAir = 15;
RtQ = .5; % [Sec]
RtBeta = .5; % [Sec]
RtP = .5; % [Sec]

STQ = RtQ*3; % [Sec]
STBeta = RtBeta*3; % [Sec]
STP = RtP*3; % [Sec]

% Surf_Power = 2;

% Constraint Cost Components
% Rise Time Constraints
% Deflection Cost
% Kalman Filter Accuracy Cost
J = (10*(QRt - RtQ)).^2+(BetaRt - RtBeta).^2+(PRt - RtP).^2 + ... 
(QST-STQ).^2+(BetaST-STBeta).^2+(PST-STP).^2; %+ ... 
% MaxDfl(12).^Surf_Power+ MaxDfl(13).^Surf_Power;
end

function [J] = Control_Cost(var,sys,A,B,C,D)
    [~,~,~,~,~,CtrAuth] = LQR(var,sys,A,B,C,D);

    %% Desired State Response Times
    SatePower = 100;
    %Surf_Power = 0;
    J = (((SatePower - CtrAuth(1))).^2+((SatePower - CtrAuth(2))).^2 + ... 
        ((SatePower - CtrAuth(3))).^2+((SatePower - CtrAuth(4))).^2+ ... 
        ((SatePower - CtrAuth(5))).^2+((SatePower - CtrAuth(6))).^2+ ... 
        ((SatePower - CtrAuth(7))).^2+((SatePower - CtrAuth(8))).^2;
end

D2R = 3.14/180;
R2D = 180/3.14;

[Ay,Ax] = size(A);
[By,Bx] = size(B);
[Cy,Cx] = size(C);

% Converting Control Surface Inputs From Degrees To Radians
Convert_B(1,1) = R2D;
Convert_B(2,2) = R2D;
Convert_B(3,3) = R2D;
Convert_B(4,4) = R2D;
Convert_B(5,5) = R2D;
Convert_B(6,6) = R2D;
Convert_B(7,7) = R2D;
Convert_B(8,8) = R2D;
Convert_B(9,9) = R2D;
Convert_B(10,10) = R2D;
Convert_B(11,11) = R2D;
Convert_B(12,12) = R2D;
Convert_B(13,13) = R2D;
Convert_B(14,14) = 1;

RescaleB = sys.B*Convert_B;

% OL_SYS = [A,RescaleB;C,D];
SYS = ss(A,RescaleB,C,D);

%%
[ax,ay] = size(SYS.A);
[bx,by] = size(SYS.B);

% LQR State Weighting
Q = eye(ax,ay);
Q(1,1) = X(1); % 1 Airspeed
Q(2,2) = X(2); % 2 Alpha
Q(3,3) = X(3); % 3 Theta
Q(4,4) = X(4); % 4 Q
Q(5,5) = X(5); % 5 Beta
Q(6,6) = X(6); % 6 Roll Angle
Q(7,7) = X(7); % 7 Roll Rate
Q(8,8) = X(8); % 8 Yaw Rate

% LQR Control Weighting
R = eye(by,by);
R(1,1) = X(9);
R(2,2) = X(10);
R(3,3) = X(11);
R(4,4) = X(12);
R(5,5) = X(13);
R(6,6) = X(14);
R(7,7) = X(15);
R(8,8) = X(16);
R(9,9) = X(17);
R(10,10) = X(18);
R(11,11) = X(19);
R(12,12) = X(20);
R(13,13) = X(21);
R(14,14) = X(22);

% LQR Optimal Gain Matrix
[K,~,~] = lqr(SYS,Q,R);

% Rescaling B Matrix Control Deflection
% SYS.B;
% ReScale_B1 = eye(by,by);
ReScale_B1(1,1) = D2R;
ReScale_B1(2,2) = D2R;
ReScale_B1(3,3) = D2R;
ReScale_B1(4,4) = D2R;
ReScale_B1(5,5) = D2R;
ReScale_B1(6,6) = D2R;
ReScale_B1(7,7) = D2R;
ReScale_B1(8,8) = D2R;
ReScale_B1(9,9) = D2R;
ReScale_B1(10,10) = D2R;
ReScale_B1(11,11) = D2R;
ReScale_B1(12,12) = D2R;
ReScale_B1(13,13) = D2R;
ReScale_B1(14,14) = 1;
Temp = SYS.B * ReScale_B1;
% ReScale_B2 = eye(by,by);
ReScale_B2(1,1) = 40;
ReScale_B2(2,2) = 40;
ReScale_B2(3,3) = 60;
ReScale_B2(4,4) = 30;
ReScale_B2(5,5) = 60;
ReScale_B2(6,6) = 30;
ReScale_B2(7,7) = 40;
ReScale_B2(8,8) = 40;
ReScale_B2(9,9) = 60;
ReScale_B2(10,10) = 30;
ReScale_B2(11,11) = 60;
ReScale_B2(12,12) = 15;
ReScale_B2(13,13) = 15;
ReScale_B2(14,14) = 45000;
Temp2 = Temp * ReScale_B2;
% ReScale_B3 = eye(by,by);
ReScale_B3(1,1) = R2D;
ReScale_B3(2,2) = R2D;
ReScale_B3(3,3) = R2D;
ReScale_B3(4,4) = R2D;
ReScale_B3(5,5) = R2D;
ReScale_B3(6,6) = R2D;
ReScale_B3(7,7) = R2D;
ReScale_B3(8,8) = R2D;
ReScale_B3(9,9) = R2D;
ReScale_B3(10,10) = R2D;
ReScale_B3(11,11) = R2D;
ReScale_B3(12,12) = R2D;
ReScale_B3(13,13) = R2D;
ReScale_B3(14,14) = 1;
Scaled_B = Temp2 * ReScale_B3;

% Open Loop LQR System
Stab_OL_Matrix = [SYS.A-SYS.B*K,Scaled_B;SYS.C-SYS.D*K,SYS.D];
Stable_OL_SYS = ss(SYS.A-SYS.B*K,Scaled_B,SYS.C-SYS.D*K,SYS.D);
Ktest = pinv(dcgain(Stable_OL_SYS));
Control_Auth1 = [norm(Ktest(1:14,1)),norm(Ktest(1:14,2)),norm(Ktest(1:14,3)),norm(Ktest(1:14,4)),norm(Ktest(1:14,5)),norm(Ktest(1:14,6)),norm(Ktest(1:14,7)),norm(Ktest(1:14,8))];
Norm_Control = 100*(Control_Auth1/max(Control_Auth1));
% format short

% Control_Authority = [{‘Normalized Values’}];
% AOutput_Table = cell2table(Control_Authority);
% A_Matrix = array2table(round(Norm_Control,4),’VariableNames’,{’Ub’,’Alpha’,’Theta’,’Q’,’Beta’,’Phi’,’P’,’R’});
% Control_Authority = [AOutput_Table,A_Matrix]

% Control / Feedback State Control Selection
UCI = 1; %1 Airspeed
ACI = 1; %2 Alpha
TCI = 0; %3 Theta
QCI = 0; %4 Q
BCI = 1; %5 Beta
PhCI = 1; %6 Roll Angle
PCI = 0; %7 Roll Rate
RCI = 0; %8 Yaw Rate

Convert_Inputs = eye(8,8)*D2R;
Convert_Inputs(1,1) = 1; % Prevents Airspeed From Being Incorrectly Converted

CSi = [UCI 0 0 0 0 0 0 0
       0 ACI 0 0 0 0 0 0
       0 0 TCI 0 0 0 0 0
       0 0 0 QCI 0 0 0 0
       0 0 0 0 BCI 0 0 0
       0 0 0 0 0 PhCI 0 0
       0 0 0 0 0 0 PCI 0
       0 0 0 0 0 0 0 RCI]*Convert_Inputs;

[~,~,NCS] = find(CSi);
NumControlStates = length(NCS);

[rows,~,~] = find(CSi);
CVar = rows';

% DC Gain
Kdc = pinv(dcgain(Stable_OL_SYS(CVar(:,1:NumControlStates),:)));
Kdc(1:13,1) = 0; % Only Airspeed Error State Controlls Thrust
Kdc(14,2:4) = 0; % Nothing But Airspeed Impacts Thrust

AugB = Stable_OL_SYS.B(:,1:14)*Kdc(1:14,:);
AugD = Stable_OL_SYS.D(:,1:14)*Kdc(1:14,:);

% Augmented Open Loop LQR System
Aug_OL_Matrix = [Stable_OL_SYS.A,AugB;Stable_OL_SYS.C,AugD];
Augmented_OL_SYS = ss(Stable_OL_SYS.A,AugB,Stable_OL_SYS.C,AugD);

% State Time Response Adjustment Matrix
[~,Y] = size(Augmented_OL_SYS.B);
Adj_Matrix = eye(8,8);
Adj_Matrix(1,1) = X(23); % Airspeed
Adj_Matrix(2,2) = X(24); % Alpha
Adj_Matrix(3,3) = X(25); % Theta
Adj_Matrix(4,4) = X(26); % Pitch Rate
Adj_Matrix(5,5) = X(27); % Beta
Adj_Matrix(6,6) = X(28);  % Roll Angle
Adj_Matrix(7,7) = X(29);  % Roll Rate
Adj_Matrix(8,8) = X(30);  % Yaw Rate

ACM = -Adj_Matrix(1:8,:);

CS_Scaled = CSi * ACM;

BControl = CS_Scaled(CVar,CVar);

ACM = ACM(CVar,:);

[CtrY,~] = size(ACM);

% Converting Kdc Surface Deflections From Radians to Degrees

dEConvert = eye(by,by);
dEConvert(1,1) = R2D;
dEConvert(2,2) = R2D;
dEConvert(3,3) = R2D;
dEConvert(4,4) = R2D;
dEConvert(5,5) = R2D;
dEConvert(6,6) = R2D;
dEConvert(7,7) = R2D;
dEConvert(8,8) = R2D;
dEConvert(9,9) = R2D;
dEConvert(10,10) = R2D;
dEConvert(11,11) = R2D;
dEConvert(12,12) = R2D;
dEConvert(13,13) = R2D;
dEConvert(14,14) = 1;

dEffectors = dEConvert * Kdc;

% Converting State Outputs From Radians to Degrees

StateConvert(1,1) = 1;
StateConvert(2,2) = R2D;
StateConvert(3,3) = R2D;
StateConvert(4,4) = R2D;
StateConvert(5,5) = R2D;
StateConvert(6,6) = R2D;
StateConvert(7,7) = R2D;
StateConvert(8,8) = R2D;

Out_States = eye(8,8) * StateConvert;

% Closed Loop LQR System

AM = [Augmented OL_SYS.A,Augmented OL_SYS.B;
      ACM,
      zeros(CtrY,Y)];
SizeAM = size(AM);

BM = [zeros(8,4);BControl];
SizeBM = size(BM);

CM = [Out_States,zeros(8,Y);
      zeros(by,8),dEffectors];
SizeCM = size(CM);

DM = zeros(ay+by,4);

format short

Aug_CL_Matrix = [AM,BM,CM,DM];
Augmented_CL_SYS = ss(AM,BM,CM,DM);

end