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Small Below-Endurance-Limit Cycle Damage Contributions in Variable Sequence Loading for 2024-T351 Aluminum Alloy

Christopher M. Ngiau
Western Michigan University

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SMALL BELOW-ENDURANCE-LIMIT CYCLE DAMAGE CONTRIBUTIONS
IN VARIABLE SEQUENCE LOADING FOR
2024-T351 ALUMINUM ALLOY

by

Christopher M. Ngiau

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
Degree of Master of Science in Engineering (Mechanical)
Department of Mechanical and Aeronautical Engineering

Western Michigan University
Kalamazoo, Michigan
December 2000
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2000
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SMALL BELOW-ENDURANCE-LIMIT CYCLE DAMAGE CONTRIBUTIONS IN VARIABLE SEQUENCE LOADING FOR 2024-T351 ALUMINUM ALLOY

Christopher M. Ngiau, M.S.E.
Western Michigan University, 2000

It is a common practice in industry today to totally discard the presence of small below-endurance/fatigue limit cycles while calculating damage accumulation using the Palmgren-Miner rule. However, it was experimentally found in a number of studies that small cycles with stress amplitudes below the fatigue/endurance limit do in fact contribute significantly to the damage accumulation in all types of materials. There was a need to investigate cases where small cycles would only be present for a certain percentage of the material’s life. An experimental study was done on 2024-T351 aluminum alloy. Loading was done in low and high cyclic fatigue regimes, and subjected to constant-amplitude fully reversed “regular” loading as well as variable amplitude block loading, sequenced either in block-to-constant or constant-to-block patterns. Additionally, the small cycles in those block loads were tested at stress amplitudes of 100% and 50% endurance limit. The results showed that there was significant damage caused by both amplitudes of small cycles, and this damage was influenced heavily by the sequencing pattern, regime, and the cutoff point. There were instances when the small cycles could be safely ignored, but in other cases the damage values from these supposedly harmless small cycles could be as high as 0.81.
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INTRODUCTION

A Historical Overview

Fatigue and Fracture

Fatigue is a subject that has been studied extensively for over 150 years. The definition of fatigue is: catastrophic full-separation failure caused by repetitive loads that have amplitudes that are lower than the monotonic ultimate strength of the material. It is a very important phenomenon because such failure, if not predicted and prevented, can result in damage to machines and danger to human life. The study of fatigue, hence, is to better understand the factors and conditions that influence the life of materials so that more reliable prediction methods can be modeled and, in turn, allows industries to produce safer, more reliable products. At this present time, even with the wealth of information on fatigue, this field of study is still not fully mature. The problems in fatigue are very often complex and influenced by a lot of variables. Fatigue studies usually deal with the number of cycles it takes to initiate a crack on specimens. Also, fatigue involves keeping track of how many cycles it takes to ultimately fail a specimen. Until lately, it has rarely involved the study of fatigue-crack propagation.

While fatigue is the study of crack initiation and the number of cycles to failure for specimens undergoing repetitive loading, fracture is the study of crack
growth, or propagation, for monotonic cases as well as fatigue loading conditions.

Specimen gage surfaces are usually studied using optical or electron microscopes, cellulose acetate replica methods, compliance methods, electric potential difference methods, and other means to chart the progress of cracks on the surface of a specimen. After that, theories or mathematical models are formulated based on empirical data to help predict the rate of crack growth under a variety of conditions.

Although fatigue and fracture have been treated as two separate disciplines for a very long time, it can be seen that they are not very different after all. Fatigue studies involve the beginning or the end stages of a repetitive loading history, while fracture focuses primarily on the intermediate stage. Within the last 40 years, with the efforts of visionary researchers such as Paris, Donald, Elber, Topper, and etc., the gap between fatigue and fracture continues to be bridged. They have seen the importance of studying the "whole story" and have incorporated many fracture concepts into their fatigue work. This has greatly helped efforts to better understand the nature of fatigue failures and the mechanisms that influence them.

From here on any mention of the term "fatigue" actually means a combination of both fatigue and fracture approaches. This reflects the experimental practice of research today, where fatigue and fracture have been integrated to help explain any repetitive loading phenomenon.
The Stress Amplitude vs. Fatigue Life (S-N) Curve

In the past, fatigue was studied using purely constant-amplitude fully reversed cycles; the maximum and minimum peaks of the loads were kept constant throughout the test and the load amplitudes were constant in magnitude and oscillated around a zero mean load. The load amplitude was varied and the number of cycles to failure, \( N_f \), was recorded for each case. From this fatigue data a stress vs. life curve was generated, commonly called a “S-N curve”. This curve was, and still is, a valuable prediction tool that helps determine the life of a material given the load amplitude it was subjected to. An illustration of a S-N curve is presented in Figure 1.

![S-N Curve for 2024-T351 Aluminum Alloy](image)

Figure 1. Stress (linear) vs. Life (log) Curve for 2024-T351 Aluminum Alloy.
One of the most valuable pieces of information provided by a S-N curve is the long-life “horizontal” line toward the right hand side of the curve that is known as the “fatigue limit” in the case of ferrous metals such as steel, or “endurance limit” in the case of aluminum alloys. This horizontal line denotes the minimum stress needed to cause fatigue failure in a material. Any stress below this level will, quite simply, not cause significant damage and thus, results in infinite life. This stress level is usually cross-referenced at $N_f = 10^6$ for materials such as steel. However, for some materials (aluminum alloys especially) this long-life horizontal line is never truly horizontal no matter how many cycles are involved; it will continue to taper off slowly as life increases. For these materials, it is called “endurance limit” instead, and the usual cross-referenced life is $N_f = 10^7$, $10^8$, or $10^9$ cycles. For theoretical purposes, fatigue and endurance limits are treated similarly i.e. any stress amplitude below this level is assumed to result in infinite life (or endurance life, in the case of endurance limit).

The Strain-Based Approach

Load control fatigue was a method used almost exclusively in the early days of fatigue studies. This is commonly called the “stress-based approach” where load is applied remotely to a gage section of a certain cross-section, and varying the load effectively varied the stress on the specimen. This method was used in all experiments in the early days, and although it is still used up till today, there is another loading method that was developed in the late 1950’s and early 1960’s called the “strain-based approach”. This approach was developed due to the high amount of
plastic deformation in most ductile metals especially at high load amplitudes. Depending on the nature of the material, this might cause a cyclically strain-hardening or softening effect. When a material cyclically hardens, more load is required to stretch the material and thus induces progressively more peak stress in the specimen as loading continues. Softening is the opposite of that, where the material softens as load is repetitively applied and thus, lesser and lesser stress amplitudes would be required. So, since stress does not stay at a constant level during the cycling of the specimen, this causes inconsistencies in very-short life tests. This brought about the strain-based approach or strain-controlled fatigue testing where instead of controlling the load amplitude, a strain gage is attached to the gage section of the specimen and a feedback loop keeps the strain-range constant instead. The use of the strain-based approach generates a curve quite similar to the regular S-N curve, but instead of relating stress amplitudes to number of cycles to failure, it is instead corresponds strain amplitudes with number of cycles to failure. This version of the S-N curve is called the “strain-life curve”.

The Problem With Fully Reversed Cycles

As mentioned above, constant-amplitude, fully reversed cycles were the focus of a lot of fatigue studies before the year 1960, and a lot of current equations and models are based on this simplistic approach to fatigue. However, as can be imagined, there are problems with this method of fatigue study, one of which is the obvious fact that real-life loading conditions are mostly never constant-amplitude fully reversed
cycles! Most actual loading has a mean stress that the stress amplitude oscillates between i.e. mean stress is very rarely zero. Usually, machine parts, components or structures have static tensile or compressive forces applied to them in addition to repetitive loads. Differences between these different kinds of loading conditions are illustrated in Figure 2.

Figure 2. Illustrations of Different Loading Types: (a) Fully Reversed Cycles; and (b) Positive (Non-Zero) Mean Stress Cycles.
The works of Goodman, Gerber, Morrow, Leis, and others [1] have allowed non-fully reversed cycles to be effectively translated into fully-reversed cycles using simple yet good approximation equations. This served to solve the problem of mean stresses being introduced into fatigue cycles.

Problems do not end here, however. Real-life problems, aside from rarely being fully reversed, are even more rarely constant-amplitude. In the real world, parts and components undergo what is called “spectrum loading”. Spectrum loading is essentially non-uniform repetitive loads that can vary in mean stress and stress amplitude every cycle or segment (half-cycle) all throughout the life of the specimen, part or component. This type of loading, to this day, still poses a considerable challenge to researchers. Being able to actually model real-world spectrum loads would in fact predict the actual behavior of realistic fatigue problems. This vision, unfortunately, is still years ahead of possibility.

Variable Amplitude Block Loading and the Palmgren-Miner Rule

A simpler form of spectrum loading is what is known as “variable amplitude block loading”, where mean stresses and amplitudes vary in a predictable block-like pattern that repeats itself over time, and is more often studied and easier to grasp compared to spectrum loads. An example of this is illustrated in Figure 3.

Variable amplitude loading can be a part of fabricated tests used to better understand load interaction effects and damage accumulation in specimens, or they
can be simplifications of actual spectrum loading cases using simplification techniques, which are rough approximations at best.

Figure 3. An Illustration of Variable Amplitude Block Loading.

This type of loading condition is much more manageable and has lead to some landmark findings [1] such as the works of Palmgren (1920) and Miner (1945). Their research resulted in the linear damage accumulation equation called the Palmgren-Miner Rule, which specifies that all the damage accumulation in any specimen, regardless of material or loading history, must be equal to unity. Their equation is as follows:
\[
\sum \frac{N_j}{N_{jf}} = 1
\]  
(Eq. 1)

where, \(N_j\) = the number of cycles at a certain fully reversed amplitude  
\(N_{jf}\) = the number of cycles to failure at constant amplitude

The ratio \(N_j / N_{jf}\) is commonly known as “damage”. Both \(N_j\) and \(N_{jf}\) are calculated assuming fully reversed conditions. If the cycles have a mean stress associated with them, they have to be converted to a fully reversed equivalent case using the equations provided by Goodman, Gerber, Morrow, Leis, etc. [1] The sum of all damage done to the specimen (\(\Sigma N_j / N_{jf}\)) is known as “damage accumulation” and according to the Palmgren and Miner, this should be equal to one, or unity.

Unfortunately, the Palmgren-Miner Rule does not hold in a lot of cases. Researchers like Bilir [2] proved that the Palmgren-Miner Rule does not consider load-interaction effects i.e. when there is a shift from high to low loads, or vice versa. In most cases, the damage accumulation in a specimen did not equal unity. Instead the resultant damage accumulation could be as low as 0.1, or as high as 10, according to Bilir’s experiments, depending on the load-interaction. A lot of studies have tried to better model variable amplitude damage accumulation and as a result there have been many modifications made to the Palmgren-Miner Rule by researchers to better model their own tests.

One of the biggest oversights of the Palmgren-Miner Rule is it disregards the contributions of cycles that have fully reversed stress amplitudes that are below the fatigue limit of a material. As can be seen from equation 1 above, all damage is
specified as \( \frac{N_j}{N_f} \). The denominator, \( N_f \), for small below fatigue limit cycles is considered infinity, which means the ratio \( \frac{N_j}{N_f} \) for small cycles should be effectively zero. Hence, all cycles that are below the fatigue/endurance limit are disregarded from any damage calculation, based on this approach. However, there have been many findings based on variable amplitude tests [3-10] that have shown that cycles below fatigue/endurance limit contributed to significantly reduced fatigue lives of materials. The study of overloads and underloads in constant amplitude cycles, an area of research done extensively by Topper and his students at the University of Waterloo, Canada, [11-14] also confirmed this weakness in the Palmgren-Miner Rule. From their work, they found that when a tensile or compressive load spike (at the cyclic yield strength of the material) was introduced periodically in otherwise constant amplitude below fatigue/endurance limit cycles, it actually caused the small cycles to contribute to the accumulated damage in the specimen. In short, cycles with amplitudes below fatigue/endurance limit cannot always be ignored as the Palmgren-Miner Rule suggests.

Fracture Mechanics Concepts in Fatigue Studies

As was mentioned earlier in this section, a lot of contemporary fatigue studies [3-14] involve some form of fracture mechanics analysis. And this studying of crack growth rates and the mechanisms that contribute to them have added an exciting new dimension to the study of fatigue. Paris produced the most ground-breaking work in fatigue crack growth analysis, with his efforts to incorporate fracture mechanics into
fatigue [15], as well as his method of studying the elastic crack tip stress intensity
factor range, \( \Delta K \), identifying it as a major driving force that influences crack growth
rate [16]. This was an important milestone in fatigue studies. And even though the
marriage between fatigue and fracture is relatively new (it started in the 1960’s), no
fatigue research today will ever be considered complete without considering crack
growth analysis.

An additional critical concept on fatigue crack growth came from Elber who
discovered “crack closure” in the late 1960’s [17-18]. Crack closure is defined as the
premature contact of the surfaces behind the crack tip which can be caused most
commonly by plasticity, surface roughness, or oxidation of the surfaces behind the tip
of the crack. These forms of closure are illustrated in Figure 4.

This premature closure was reported to affect crack growth, including
retardation effects that are prominent in certain loading conditions especially ones
containing overloads. Crack closure moved the focus of crack growth studies to the
area behind the crack tip, causing a lot of fatigue experts to ignore a lot of other
possibilities, namely what happens at or in front of the crack. It was a concept that
seemed to be versatile enough to explain most fatigue situations.

However, just lately, some researchers have had some doubts and brought
forward alternative theories regarding fatigue crack growth. The recent works of
Sadananda, Vasudevan, and Marci [19-21], among others, have pointed out the
overuse of the crack closure concept and the fact that this concept might actually be
over-exaggerated i.e. maybe crack closure exists but is not as significant as a lot of
advocates suggest it is. They argued that the primary driving forces for fatigue cracks
are not behind the crack, as the crack closure concept emphasizes, but they are instead
at or in-front of the crack tip. They stated that two parameters are required to advance
a fatigue crack: \( \Delta K \) and \( K_{\text{max}} \); \( \Delta K \) is the range of stress intensities (the difference between \( K_{\text{max}} \) and \( K_{\text{min}} \)) and \( K_{\text{max}} \) is the maximum stress intensity at the crack tip. They argued that these two independent factors play an essential role in crack growth even in special cases such as overloads, and although they do not completely deny the existence of crack closure effects, they deduced that it usually plays a very insignificant part in the fatigue process.

As expected, their findings caused quite a stir in the fatigue community because the supposedly universal crack closure concept was being challenged. In a lot of cases, the works of Sadananda and Vasudevan have been criticized as too extreme and uncompromising, seeing that there has been so much work done on crack closure to date [22]. Even then, just like how fatigue and fracture were considered different schools of thought back in the early days, this might be the same case between crack closure, and \( \Delta K \) and \( K_{\text{max}} \)-motivated crack growth. While crack closure emphasizes what happens behind the crack tip, the concept of \( \Delta K \) and \( K_{\text{max}} \) focuses on what happens ahead of it. So in fact, they might just be parts 1 and 2 of the same story, and it might only be a matter of time before both approaches are unified and utilized to better model fatigue cracks.

Studies Involving Small Below-Fatigue (or Endurance) Limit Cycles

Among all the available literature, the one subject that was the primary inspiration for this work was the contributions of small below fatigue/endurance limit cycles to fatigue crack growth and damage accumulation. Small cycles are usually a
part of service loading cases or variable amplitude loading conditions. There has been a lot of research done on overloads and crack retardation effects [23-26]. Recently, interesting work was done by the fatigue group at the University of Waterloo [11-14], which involved the interaction effects between overload or underload cycles with background cycles that had amplitudes slightly above or below the fatigue limit of their materials. Pompetzki et al [13], in their work on tensile overloads and compressive underloads on 2024-T351 aluminum alloy, employed the traditional crack closure model for long cracks to explain the damage interaction effects between the overload cycles and the small background cycles. Their primary equation was:

\[
\sigma_m - \sigma_o - \sigma_i > 0
\]  
(Eq. 2)

where, \(\sigma_m\) = maximum stress

\(\sigma_o\) = crack closure stress

\(\sigma_i\) = intrinsic material resistance to crack growth

This equation is defined as the fatigue crack driving force and basically translates to: if the maximum applied stress minus the sum of the crack closure stress (i.e. the stress at which the surfaces behind the crack tip achieve premature contact, and causes the crack growth to arrest) and the intrinsic ability of the material to resist cracks is larger than zero, then there will be damage to the specimen and the crack can advance. If not, no damage is caused by cyclic loading. Pompetzki and his colleagues used this concept to explain the effects of overloads and underloads. They suggested that when an overload/underload event occurs, it drastically reduces \(\sigma_o\) and
the crack driving force increases. And it takes awhile for $\sigma_o$ to reach steady state conditions again and until then, small cycles after the overload, even ones below endurance limit, may cause damage because of the enhanced crack driving force.

Reference [13] used underload cycles with a peak of $-414$ MPa and overloads with a peak of $359$ MPa, both corresponding to the cyclic yield stresses of 2024-T351 aluminum alloy. These overloads were introduced at the beginning of block loads that contained a certain number of small cycles, $\eta$, after the overload/underload. They found that when $\eta$ increases in each block, the number of blocks to failure decreases. This means that the more small cycles that are added, the more damage is done by them causing the life of the specimen to be shorter. However, it was observed that after an overload/underload event, damage would continue to accumulate for at least 1000 small cycles, but very little difference could be ascertained when more than 1000 small cycles were used. From their results they also deduced the following equations for interactive damage per block, $D_{ib}$:

Underload : $D_{ib} = D_b - D_{ss,b}$ \hspace{1cm} (Eq. 3a)

Overload : $D_{ib} = D_b - D_{ob} - D_{ss,b}$ \hspace{1cm} (Eq. 3b)

where, $D_b$ = damage per block (inverse of the number of blocks to failure)

$D_{ob}$ = overload damage per block ($N/N_f$ for just the overload cycle)

$D_{ss,b}$ = damage that would have been caused by the $\eta$ fully reversed cycles in a constant amplitude test. If the cycles are below fatigue limit, then this is zero.
Equations 3a and 3b are two separate equations that can be used to predict interactive damage done depending on whether the case has an overload or underload in it. They also show that even though below fatigue/edurance limit cycles will not contribute to damage while in steady state conditions ($D_{ss,b} = 0$, for those cycles), there would still be interactive damage because of the overload cycles. For this reason, the small cycles will cause damage for at least 1000 cycles following an overload/underload event.

Jurcevic et al [14], took this concept one step further by introducing reversed overloads instead of single overload or underload peaks as done by Pompetzki et al above. Reversed overloads involved a tensile overload followed directly by a compressive underload (termed “T-C”) or an underload followed by an overload (“C-T”). Aside from this difference, they used the same testing parameters as Pompetzki et al. Their results confirmed that 1000 small cycles following a reversed overload event caused the most significant damage and did not change much for small cycles exceeding this amount. They also found out that C-T overloads caused more damage than T-C cases. This finding was, however, contradictory to the works done by Mills et al [27] and Ohrloff [7] which found that a tension-compression overload was more detrimental than a compression-tension case, due to the fact that the compressive underload stage causes the asperities behind the crack to be crushed, and the crack tip to sharpen - this helps advance the crack. Other researchers also came up with identical conclusions [8-10] using other concepts such as residual stresses that build up in front of the crack tip to explain their findings. Based on the wealth of literature
it can be concluded that, in most cases, a T-C loading sequence does cause more damage than a C-T sequence.

Studies involving variable amplitude fatigue [4] have also confirmed that small cycles, even ones below fatigue/endurance limit, were damaging to the material. Reference [4], which used a strain-based approach to explain variable amplitude loading, also emphasized on the crack closure concept and explained that large loads would cause the closure stress, $\sigma_c$, to be drastically reduced so when loading shifts to small amplitude cycles, those small cycles will cause damage because the crack driving force is enhanced.

Problem Statement and Thesis Objective

This thesis work attempts to take an experimental approach to study the damage contributions of below-endurance-limit cycles in constant amplitude low-cyclic and high-cyclic fatigue loading. Many current variable amplitude calculation methods either do one of two things: (1) they ignore all small below-fatigue-limit cycles, which is not a very conservative choice as shown by many researchers; or (2) they calculate the effects of small cycles all throughout the life of the specimen. These are effectively two extremes to the problem of solving variable amplitude or spectrum loading conditions; one would involve a lot of overestimation in life because of the exclusion of small cycles, and the other would be too hard to calculate.

For the purpose of being practical, the following question is presented: “When can we ignore small cycles, and when can we not do so?” Hence, the author attempts
to study the contributions of small cycles as they are inserted in a variety of sequences, for LCF and HCF cases, to ascertain under what conditions, if any, can small cycle contributions be neglected completely. This endeavor, if successful, will allow engineers to simplify their computational and prediction methods significantly while not sacrificing any accuracy in their fatigue life predictions. It should be noted, however, that this study aims to introduce the concept of “selective negligence” by pointing out that, for more basic fatigue problems, there is a dependable way of neglecting non-contributing small cycles from calculations depending on load-sequencing, regime, small cycle amplitudes, and at what point the small cycles are introduced or removed. All results in this thesis are specific to 2024-T351 aluminum alloy subjected to fully reversed constant-amplitude load-controlled testing, and does not attempt to create a generic model for other materials or loading conditions. It will also not involve a detailed study of the crack propagation or acceleration/deceleration mechanisms such as crack closure, crack tip stress intensity analysis, or most other theories involving what is happening to the crack. The study was based on classical fatigue methods but backed by a cellulose acetate replica method to help quantify the effects of fracture mechanics and their effects on the fatigue results.
SPECIMENS

For this thesis, 2024-T351 aluminum alloy was chosen. This was based on the wealth of research involving this alloy [6, 13, 14, 23], and the fact that it is a material with excellent properties for industrial uses, specifically aircraft and automotive. Most of the references for this material came from the overload and underload research done by Pompetzki et al [13] and Jurcevic et al [14].

Specifications abide by ASTM E466-82 standards for hourglass-shaped LCF specimens and are as follows:

Specimen length, $L = 9$ in.

Continuous radius, $R = 6$ in.

Gage diameter, $d_{gage} = 0.5$ in.

Grip diameter (diameter of as-received bar-stock), $d_{grips} = 0.75$ in.

Although made as LCF specimens they worked just as well in HCF cycling; the thicker gage section diameter for LCF specimens served to prevent buckling in compression during high load applications. The details on the ASTM standards, specimen dimension calculations and machining details are provided in Appendix A. A fully machined and polished specimen can be seen in Figure 5.

The fully-prepared specimens were tested in load-controlled uniaxial constant-amplitude fully reversed cycling to generate a S-N curve. This curve was illustrated in
Figure 5. Picture of the Test Specimen.

Figure 1, in the introduction section. A total of 14 data points were generated from an equal number of tests to plot a S-N curve. Compared to references [13] and [14], there were slight variations with both in the form of stress magnitude as well as shape. This could be attributed to the different batches of aluminum alloy used in those references, that might have had slightly different manufacturing or chemical properties, and this could have caused the variance. This difference is illustrated in Figure 6.

This was an important matter, because using an incompatible S-N curve from references could result in erroneous results. Since there were significant differences between the references and the actual experimental results, the experimental S-N curve was used for all thesis procedures. All details on the S-N curve are available in Appendix B.
Figure 6. Comparison S-N Curves Between Experimental Data and the S-N Curves From References [13] and [14].

In addition to the S-N curve, fatigue tests using fully reversed constant-amplitude strain-controlled conditions were done at 5 different strain-amplitudes. The detailed calculation procedures, test results and tabulated data are provided in Appendix C. The cyclic stress-strain curve is shown in Figure 7.

From Figure 7, using a 0.2% offset to strain, the cyclic yield stresses were determined to be approximately 380 MPa in tension and –390 MPa in compression.
Figure 7. Cyclic Stress-Strain Curve for 2024-T351 Aluminum Alloy.
TEST PROCEDURES

Overview of Test Procedures

All testing in this thesis work was done on a MTS 810 servo-hydraulic test frame that was capable of a maximum uniaxial load of 20,000 lbf. The hydraulics were controlled by the Teststar IIIs control system and all programming for the tests was done using the MTS Multi-Purpose Testware software package (abbreviated MPT). Details on the programming procedures are presented in Appendix D.

Function generation and data acquisition were fully digital, and controlled exclusively by a Pentium-based personal computer.

An overview of the test procedure is as follows, but will be explained in more detail later in this section:

1. Testing was done at two different load regimes: LCF and HCF.

2. Two types of loading were used: (1) "regular" constant-amplitude loading; and (2) block loading. Block loads contained a certain number of regular cycles followed by 1000 small cycles.

3. There were two small cycle amplitudes tested: at 100% endurance limit and 50% endurance limit.

4. For both regimes, LCF and HCF, there were two types of sequencing: (1) constant amplitude to block loading; and (2) block loading to constant amplitude. The
transition between block to constant loads, or constant to block loads took place at
cutoff points, which were preset percentages of the expected life of the specimen.

5. Cellulose acetate replicas of the specimen gage section were taken at preset
intervals of the expected life to help map the progress of any fatigue cracks on the
surface.

Low and High Cyclic Fatigue (LCF and HCF)

By definition, LCF cycles involve very low lives usually below 10,000 cycles.
HCF cycles are the opposite; they can range from 100,000 cycles up to several
million cycles. This is illustrated in Figure 8.

![S-N Curve for 2024-T351 Aluminum Alloy](image)

Figure 8. Illustration of HCF and LCF Regimes for 2024-T351 Al. Alloy.
For this study, \( N_r = 5,000 \) cycles was used as the expected life for the LCF case, and \( N_r = 100,000 \) cycles was used for HCF. This was based on the shape of the S-N curve illustrated in Figure 8. The beginning and end sections of the S-N curve (corresponding to roughly \( N_r = 100 \) and \( N_r = 1,000,000 \)) were relatively horizontal. The biggest factor in these areas was the fact that even a small variation in stress amplitude, which is highly possible even in the case of very precise fatigue testing machines, would result in a large scatter in fatigue life. For example, when cycling a specimen around \( \sigma_a = 460 \) MPa, even a small deviation of \( \pm 2 \) MPa could cause the fatigue life to scatter anywhere from 100 cycles to 1000 cycles! Hence, for more accurate analysis, the LCF and HCF points were taken along the incline of the S-N curve, where the relationship between stress and life resulted in less scatter.

**Constant Amplitude Cycles and Block Loads**

Two types of cycling schemes were used in this thesis. They were:

1. Constant amplitude fully-reversed cycles at a stress amplitude of \( \sigma_a = 420 \) MPa (corresponding to \( N_r \approx 5,000 \) cycles) for LCF, or \( \sigma_a = 270 \) MPa (corresponding to \( N_r \approx 100,000 \) cycles) for HCF. These amplitudes were cross-referenced off the S-N curve similar to the one illustrated in Figure 8. Note: for convenience, this type of cycling will be referred to as “regular cycles” from here on.

2. Block loads – in each block there were 5 regular cycles followed by 1000 small cycles for LCF tests, or 100 regular cycles followed by 1000 small cycles for HCF tests.
Both of these schemes are illustrated in Figure 9:

**Figure 9.** Cycling Schemes: (a) Constant Amplitude Cycles (Regular Cycles); and (b) Block Loading.
This pattern of block loading was designed to assure that an equal number of blocks were applied to both LCF and HCF cases, thus introducing the same amount of small cycles to each case. To gain a better understanding of this concept, the actual philosophy behind it has to be emphasized. The expected life for LCF tests, as mentioned, was \( N_f \geq 5,000 \) cycles, and for HCF it was \( N_f \geq 100,000 \) cycles. Each block should, first of all, have the same amount of small cycles, and in the case of this study it was 1000 small cycles, based on the findings in references [13-14]. On the other hand, since LCF and HCF had different expected lives, not the same amount of regular cycles could be used in both cases. So to make blocks in LCF and HCF equivalent to each other, 5 regular cycles were inserted into the block loading scheme for LCF and 100 regular cycles inserted into the HCF case. Hence, one block in LCF would be equivalent to one block in HCF, primarily based on the fact that if you add up the damage (the sum of \( N_f/N_{ij} \), and for the small cycles \( N_f = 10^7 \) was assumed, but just for this instance) caused by a single block in HCF and compare it to the damage caused by one block in LCF, they would be numerically equivalent. In addition, one block in LCF would contain an equal amount of small cycles as one block in HCF.

Two Load Amplitudes for Small Cycles

Two levels of small cycle stress amplitude were studied in this thesis: (1) a stress amplitude of \( \sigma_a = 175 \) MPa, which is the endurance limit (or abbreviated, 100%EL) of 2024-T351 aluminum alloy; and (2) a stress amplitude of \( \sigma_a = 88 \) MPa, or 50% of the endurance limit (50%EL). It was predicted that although the 50%EL
cycles would cause significantly lesser damage to the specimen compared to the 100% EL cycles, it was interesting to study how much difference there really is, and whether or not cycles at this very low stress level should cause concerns for engineers. Bonnen and Topper stated in reference [11] that if the amplitude of the small cycles was at or below one third of the fatigue limit of a specimen, no amount of small cycles would cause damage. This applied to 1045 steel but it could also be the case for aluminum alloys. However, for the sake of brevity and to ensure that all tests provided some form of empirical results, only stress amplitudes above 1/3 EL were used and thus, their theory was not investigated further in this study.

**Load Sequencing**

As mentioned at the start of this section, small cycles, in the form of block loading, can be introduced either at the start or toward the end of otherwise regular cycles. This was done at preset intervals to allow the tests to identify the points when small cycles begin contributing to the damage accumulation in the specimens. In order to achieve this, the following testing sequences were used:

1. The specimen starts off with regular cycles till a certain “percent cutoff” (or %cutoff, for short), after which block loads would be introduced and applied repeatedly till failure occurs. This type of sequencing is called constant-to-block loading, or “C-B”, abbreviated. The %cutoff’s were scheduled at 20%, 40%, 60% and 80% of the specimen’s life. For LCF this meant the following lives: 1000, 2000, 3000 and 4000 cycles. For HCF it was: 20,000, 40,000, 60,000, and 80,000 cycles.
2. The specimens begin loading with block loads up till a certain %cutoff, after which regular cycles are introduced and run till failure. This is called block-to-constant loading, or "B-C". The %cutoff's would also be 20%, 40%, 60% and 80% of life for both LCF and HCF.

Load sequencing is illustrated in Figure 10:

Figure 10. Two Different Load Sequences: (a) Constant-to-Block Loading; and (b) Block-to-Constant Loading.
Cellulose Acetate Replicas

Cellulose acetate replicas were used very successfully to replicate the surface of the gage section to detect and help with the measurement of fatigue cracks. This was done using Fullam Inc. “thin replicating tape” and lab-grade acetone. Details on cellulose acetate tape placement, techniques, and replica results are presented in Appendix E.

The replicas were taken every 10% of expected life for both regimes i.e. every 500 cycles for LCF or every 10,000 cycles for HCF. However, since fatigue cracks appeared much earlier in LCF than in HCF, replicas were taken from the beginning for LCF, but in the case of HCF, replicas were not taken till 50% of expected life. This helped save time and resources but still accurately replicated fatigue cracks during the tests. It should be noted, however, that in both HCF and LCF cases, cracks tended to propagate very quickly especially close to failure and a good deal of experimental judgement was used to shorten the intervals between replica takes as the specimen approached failure. Shortening the intervals meant that more replicas were taken as the test approached failure for better resolutions on late-life crack growth, but this process was achieved with a lot of trial and error, and guess-work while running the tests.

Test Procedure Specifics

A total of 32 tests (not including any required re-tests) were run on the MTS 810 servo-hydraulic machine in uniaxial load-controlled fully reversed (R = minimum
load/maximum load = -1) conditions. To help identify each test, they were numbered 1 through 32, each having a combination of load sequencing, small cycle amplitude and %cutoff as show in Table 1.

Table 1

Test Numbers and Combinations Table

<table>
<thead>
<tr>
<th>Test #</th>
<th>Sequencing C-B</th>
<th>Sequencing B-C</th>
<th>Small cycle amplitude 100% EL</th>
<th>Small cycle amplitude 50% EL</th>
<th>%cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCF Tests (N_f ≥ 5,000)</td>
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<td></td>
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<tr>
<td>1</td>
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<td>16</td>
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<td>80</td>
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<tr>
<td>HCF Tests (N_f ≥ 5,000)</td>
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<td>17</td>
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<td>18</td>
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<td>24</td>
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<td>X</td>
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</tbody>
</table>
Since the tests were done in load-control, all stresses had to be converted to forces. LCF regular cycles had a stress amplitude of $\sigma_a = 420 \text{ MPa}$, and HCF had a stress amplitude of $\sigma_a = 270 \text{ MPa}$. Using the simple relationship: $\sigma = P / A$, and knowing that the area corresponds to a circular cross-section with a diameter 0.5 inches, the load amplitudes for LCF and HCF were calculated to be $P_a = 11,960 \text{ lbf}$ and $P_a = 7690 \text{ lbf}$, respectively. For the block loads that contained either 100%EL or 50%EL amplitude small cycles, their load amplitudes were calculated to be $P_a = 4980 \text{ lbf}$ and $P_a = 2490 \text{ lbf}$, respectively.

Every cycle in this thesis started at zero, moved in the tensile direction to the maximum load level, then moved in the compressive direction to the minimum load level, and then ended back at zero. This created a tensile-compressive load sequence to ensure that the most damaging case was utilized, as suggested by references [7-10, 27].
Based on trial and error empirical fully-reversed tests done to generate the S-N curve (as explained in the specimens section of this manuscript), it was found that the optimum frequencies for the different load amplitudes are as listed in Table 2.

Table 2
Optimum Cycling Frequencies Depending on Load Amplitude

<table>
<thead>
<tr>
<th>Load Amplitude</th>
<th>Optimum Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCF cycles : 11,960 lbf</td>
<td>5 Hz</td>
</tr>
<tr>
<td>HCF cycles : 7690 lbf</td>
<td>10 Hz</td>
</tr>
<tr>
<td>100%EL small cycles : 4980 lbf</td>
<td>20 Hz</td>
</tr>
<tr>
<td>50%EL small cycles : 2490 lbf</td>
<td>20 Hz</td>
</tr>
</tbody>
</table>

In general, the frequencies were not allowed to exceed 20 Hz to prevent excessive machine noise. The above frequencies were called “optimum frequencies” because they were a realistic compromise between time-saving, noise generation, and the ability to keep signal integrity by lowering frequencies to accommodate higher load amplitudes. An interesting occurrence was observed with HCF cycles ($P_a = 7690$ lbf) that were tested at 20 Hz instead (not 10 Hz as stated above) - failure would occur at the grips instead of at the gage section! The grips on the MTS machine had teeth that worked to prevent the specimen from slipping. Normally, even with stress raisers induced by the teeth marks on the specimen’s grip sections, no failures would occur here since the highest stress would exist at the relatively narrower gage section. However, interestingly enough, if the frequency of loading was too high, as in the case of the aforementioned HCF cycles, a fatigue crack started to grow at the grip section of the specimen long before any cracks formed on the gage section. One
theory to explain this undesirable phenomenon is high frequencies didn’t allow
plasticity around teeth marks, and this caused the material around this notch to act in
a brittle manner, and since brittle-behavior materials are very notch-sensitive, it
caused failure at the grips early in the specimen’s life. To prevent this from
happening, the correct frequency for certain loading conditions had to be carefully
picked.
RESULTS AND ANALYSIS

This section will describe the results from the 32 tests that were run. It will discuss the results and augment them with any observations, trends, peculiarities etc. for each case.

Before full-scale testing was conducted, two initial tests were run to attain reference values for $N_f$ in LCF and HCF. These tests were simple fully reversed constant amplitude load-controlled tests done on the MTS 810 testing machine at load amplitudes $P_a = 11,960$ lbf for LCF, and $P_a = 7690$ lbf for HCF. The results for these reference tests are shown in Table 3.

Table 3

<table>
<thead>
<tr>
<th>Reference Test</th>
<th>Fatigue Life, $N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCF : $P_a = 11,960$ lbf</td>
<td>4870 cycles</td>
</tr>
<tr>
<td>HCF : $P_a = 7690$ lbf</td>
<td>124,000 cycles.</td>
</tr>
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</table>

These lives are single-test values that corresponded very closely to the expected lives of $N_f = 5,000$ cycles for LCF and $N_f = 100,000$ cycles for HCF, which confirmed that the proper load amplitudes were used and also served as reference lives for the main tests.

A summary the test main results are listed in Table 4.
Table 4
Results of the 32 Main Tests

### Low Cyclic Fatigue Tests (LCF)

<table>
<thead>
<tr>
<th>Test #</th>
<th>C-B</th>
<th>B-C</th>
<th>100% EL</th>
<th>50% EL</th>
<th>%cutoff</th>
<th>cutoff at</th>
<th>Ref N_f = 4870 cycles</th>
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### High Cyclic Fatigue Tests (HCF)

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<th>%cutoff</th>
<th>cutoff at</th>
<th>Ref N_f = 124000 cycles</th>
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<th>50% EL</th>
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<th>cutoff at</th>
<th>N_r</th>
<th>%drop</th>
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All analysis for these main tests will be presented later in this section according to the load sequencing (C-B or B-C) and loading regime (LCF or HCF) combinations, and will involve the study of: (a) fatigue life versus %cutoff; (b) the number of blocks used in the tests; and (c) crack growth analysis obtained from the cellulose acetate replicas.

Detailed information on replica data points, re-tests, and other miscellaneous material concerning this section are made available in Appendix E. Any test that was repeated once or more are presented in diagrams as square shaped “off-trend” points, which show up in a few figures in this section.

Note that the general practice in this thesis was to assume that any fatigue life, N_r, result that fell within ± 10% of the reference lives for either LCF or HCF cases, which were 4870 cycles and 124,000 cycles respectively, would be considered data scatter and were deemed to have “no effect” on fatigue life. Also, the term “percent drop” (or %drop, for short) used in this thesis pertains to the difference, in percentage, between the reference life for LCF or HCF, and the fatigue life of the test.
LCF Block-to-Constant (B-C) Loading

Small Cycles With Amplitudes of 100%EL

The first case that was investigated was LCF B-C loading containing 100%EL amplitude small cycles. The findings for this series of tests were very interesting. The very first test (Test #3) involved loading the specimen with block loads containing 100%EL small cycles until 20% of the expected life, and then switching to regular constant amplitude cycles. Surprisingly, there was failure after only 186 block loads were applied, even before the transition to regular cycles could occur. This showed that 100%EL small cycles were very damaging to the specimen in LCF cases, even when they were applied at the beginning of life when no cracks were present on the specimen. The life of 186 blocks, or 930 cycles (5 regular cycles per block for LCF cases multiplied by 186), translated to a %drop in expected life of 80.9%, confirming that a large amount of damage was done by the 100%EL small cycles.

This finding was very significant because in other tests involving LCF cycles (which will be discussed later in this section), the number of blocks with 100%EL small cycles never exceeded 186 blocks, meaning that this was the most damaging amount of blocks for LCF cases. Since each block contained 1000 small cycles, it could be deduced that for LCF cases, the critical number of 100%EL small cycles was approximately 186,000 cycles, assuming that these cycles were introduced intermittently in amounts no less than 1000 between larger cycles.
Because of the unique nature of this result, Tests #7, #11 and #15 were not run because they involved B-C loading with 40%, 60% and 80% cutoffs, which would have given the same result as Test #3.

The crack growth profile for Test #3 is shown in Figure 11 (Note: in the crack profile figures, \( a = \) crack length, \( N = \) number of cycles):

![Figure 11. Crack Growth Profile for Test #3: LCF, B-C, 20\%Cutoff With 100\%EL Small Cycles.](image)

This profile was recorded using cellulose acetate replicas and a crack as big as 0.5 mm (which is considered the minimum crack length needed to declare crack initiation in this thesis) was detected as early as 450 cycles and required around 480 cycles of propagation from this 0.5 mm length until failure occurred (this will be referred to as “propagation range” from here on, measuring how fast a crack
propagated; the lower the number the faster). This crack growth profile was compared against a constant amplitude LCF loading case in Figure 12.

![Crack Comparison Chart for LCF, B-C, 100%EL Small Cycles.](image)

Figure 12. Crack Comparison Chart for LCF, B-C, 100%EL Small Cycles.

The results showed that the small cycles used in Test #3 caused the crack to initiate very early (compared to 3900 cycles for the constant amplitude case, nearly 9 times earlier) and to propagate extremely fast (compared to the constant amplitude case that needed 900 cycles, 1.9 times faster). Since this entire profile was influenced by small cycles it can be assumed that this would be the general behavior of cracks under pure LCF 100%EL block loading conditions.

Hence, for LCF block-to-constant cycles with 100%EL intermittent small cycle cases, all small cycles must be considered since they are so damaging and affect fatigue life even when introduced at the very beginning of cycling.
Small Cycles With Amplitudes of 50%EL

The 50%EL cases (Tests #4, #8, #12 and #16) provided more results to work with than the 100%EL case. A plot of fatigue life, $N_f$, versus %cutoff is provided in Figure 13.

![Figure 13. Fatigue Life vs. %Cutoff Plot for LCF, B-C, 50%EL Small Cycles.](image)

It revealed that as %cutoff was increased, the fatigue life of the specimen decreased. This meant that when more small cycles at 50%EL were added to the beginning of cycling, it caused earlier crack initiation and possibly also enhanced propagation causing earlier failure.

In terms of %drop versus %cutoff, Figure 13 would be presented in the form of Figure 14.
Figure 14. %Drop vs. %Cutoff Plot for LCF, B-C, 50%EL Small Cycles.

The first test (#4) which only involved a 20% cutoff between the block loads and the subsequent regular cycles resulted in a fatigue life of 4849 cycles (%drop = 0.43%). This signified that there was no effect even though small cycles were introduced, i.e. 200 blocks of small cycles at 50%EL introduced at the beginning of loading were not enough to cause any significant damage to the specimen and thus, did not alter the fatigue life.

A comparison between the crack growth diagrams for these tests were made against a constant amplitude LCF case and is shown in Figure 15. This figure confirmed that the fatigue life for Test #4 was not altered since it very closely resembled the constant amplitude LCF curve, displaying roughly similar initiation
Figure 15. Crack Comparison Chart for LCF, B-C, 50%EL Small Cycles.

points and propagation range. A more detailed version of the crack growth diagram for Test #4 is available as Figure 16.

Test #4 displayed a 0.5 mm crack at approximately 3700 cycles and a propagation range of roughly 1000 cycles. This compared very closely to the constant amplitude LCF case that had initiation at 3900 cycles and a propagation range of 900 cycles.

Test #8, had a 40% cutoff between blocks and regular constant amplitude cycles, and resulted in $N_f = 3190$ cycles (%drop = 34.5%), meaning that, in this case, small cycles did in fact influence fatigue life. This was a substantially high %drop compared to 20% cutoff (Test #4) meaning that the loading process had to overcome some sort of threshold before the small cycles could incur damage.
Figure 16. Crack Growth Profile for Test #4: LCF, B-C, 20% Cutoff With 50% EL Small Cycles.

From the crack growth diagram in Figure 17, a 0.5 mm crack was detected at around 1650 cycles and involved a propagation period of approximately 1600 cycles. This showed that the crack initiated early but took a little longer to fail than Test #4.

Test #12, a 60% cutoff case, resulted in \( N_f = 532 \) blocks or 2660 cycles (%drop = 45.4%); there was no transition between block loads and regular cycles because failure occurred before the planned 600 blocks. This case represented a full block loading till failure case, very much like Test #3, but this time for 50% EL small cycles. For this case, as illustrated by Figure 18, a 0.5 mm crack was detected at around 1700 cycles, which was very similar to what was found for Test #8, meaning
Figure 17. Crack Growth Profile for Test #8: LCF, B-C, 40% Cutoff With 50%EL Small Cycles.

that the initiation happened at the same instance for both sequences. The propagation range was measured to be approximately 1000 cycles, very similar to Test #4 in this respect. This meant that the small cycles with amplitudes at 50%EL only influenced the initiation of the crack but hardly modified the propagation range.

Test #16 was not run because of the nature of the results for Test #12. Test #16 would not reach the 80% cutoff since, as been shown by Test #12, failure would happen much earlier, around 532 blocks, hence making Test #16 essentially similar in result as Test #12 if it was run. This also pointed out that approximately 532 blocks,
or 532,000 cycles, was the maximum amount of intermittent 50%EL small cycles that could be introduced into a LCF case.

For LCF block-to-constant 50%EL small cycle cases, the small cycles can be ignored only if they are introduced up to 20% fatigue life or earlier, based on the above empirical findings. If block loads are present past 20% life, then the small cycles would cause damage in increasing quantity. It should be noted that a peculiar "threshold" effect was observed for the first time, and this occurred just after 20% cutoff.
LCF Constant-to-Block (C-B) Loading

Small Cycles With Amplitudes of 100%EL

Constant-to-block loading was studied a bit differently compared to B-C loading. Aside from using plots that involved %drop versus %cutoff, another type of curve was introduced, which plotted the number of blocks used in the test versus the %cutoff. The logic behind this is that fatigue life will, in general, continue to increase even as %cutoff increases; the regular cycles, which should theoretically cause less damage than block loads, will amount for most of the life, and the fatigue life of the specimens will see an steady increase despite of any increase in %cutoff. Any possible enhanced damage could only be caused by the small cycles after the switch from regular cycles to block loads toward the end. However, as the cutoff increased and more constant amplitude cycles were applied before block loads were introduced, it caused the regular cycle loading to start damaging the specimen. Thus, as more and more regular cycles were introduced before the cutoff, fewer and fewer blocks would be required to cause failure. Hence, keeping track of the number of block loads would make prefect sense for C-B cases.

The tests that were involved for this load sequencing were: Test #1, #5, #9 and #13, for 20%, 40%, 60% and 80% C-B cutoffs, respectively. The number of blocks vs. %cutoff graph is illustrated in Figure 19 and the %drop vs. %cutoff graph is presented in Figure 20.
Figure 19. Number of Blocks vs. %Cutoff Plot for LCF, C-B, 100%EL Small Cycles.

Figure 20. %Drop vs. %Cutoff Plot for LCF, C-B, 100%EL Small Cycles.
From Figure 19, it was found that as \%\text{cutoff} increased, the number of blocks needed till failure decreased nearly linearly. This pointed out that the regular LCF cycles generally caused damage to the specimen and as more of these regular cycles were introduced before block loads, the fewer block loads were needed to cause final failure. As could be expected, the small cycles were very damaging to the specimens and resulted in relatively few blocks to failure all throughout the plot.

It was interesting to note that it required 170 blocks to fail in Test #1 after regular cycles were introduced. This number of blocks was very similar to the results in Test #3, which was representative of a pure block loading case. This meant that the 1000 regular cycles (which pertained to the 20\%\text{cutoff}) that were introduced in the beginning did little or no damage to the specimen, and all damage was done almost exclusively by the block loads, in the case of Test #1.

A comparison plot for the different crack growth profiles compared to the constant amplitude LCF case is shown in Figure 21. Also, Figure 22 shows the crack growth profile for Test #1.

Looking at the crack growth diagram for Test #1 revealed that a 0.5 mm crack formed at around 1200 cycles, and required a propagation range of approximately 680 cycles till failure, which could be a bit erroneous given the coarseness of the plot. The crack growth profile plots for Tests #5 and #9 are presented in Figures 23 and 24 respectively. Test #5, at 40\%\text{cutoff}, required 95 blocks after the regular cycles to cause failure, a little over half of what was needed for the 20\%\text{cutoff} case.
Figure 21. Crack Comparison Chart for LCF, C-B, 100%EL Small Cycles.

Figure 22. Crack Growth Profile for Test #1: LCF, C-B, 20%Cutoff With 100%EL Small Cycles.
Initiation was detected at approximately 2100 cycles and the propagation range was around 300 cycles. Test #9, 60%cutoff, needed 62 blocks till failure and had crack initiation happen at 3050 cycles and a propagation range of around 300 cycles. Test #13, 80%cutoff, only needed 32 blocks, with a crack initiation at 3300 cycles and a propagation range of about 900 cycles.

Test #5 and #9 shared a similar propagation range of 300 cycles because propagation happened under the influence of block loads. Test #1 was over double this value, at 680 cycles for propagation range, but this variance might have been caused by the coarseness of the crack growth diagram in this case. Test #13, as shown in Figure 25, had a relatively long range of 900 cycles based on the fact that almost all of its propagation happened during regular cycles, which were less damaging that block loads. This observation was confirmed by the fact that the constant amplitude LCF case had a propagation range of 900 cycles as well.

Initiation happened much earlier for Test #9 than for Test #13 based on the fact that, in Test #9, block loads were introduced just after a micro-crack was formed (lengths below 0.5 mm) but before the crack reached 0.5 mm. Block loading helped accelerate crack growth causing crack initiation to happen much earlier. Test #13 experienced only constant amplitude cycles during the initiation stage and thus had a later initiation point due to the less damaging regular cycle conditions.

Hence, for LCF constant-to-block loading containing 100%EL small cycles, all of the small cycles cannot be ignored since they are too damaging and have the ability to cause very early crack initiation and very fast propagation.
Figure 23. Crack Growth Profile for Test #5: LCF, C-B, 40% Cutoff With 100% EL Small Cycles.

Figure 24. Crack Growth Profile for Test #9: LCF, C-B, 60% Cutoff With 100% EL Small Cycles.
Figure 25. Crack Growth Profile for Test #13: LCF, C-B, 60% Cutoff With 100%EL Small Cycles.

Small Cycles With Amplitudes of 50%EL

The tests involved with this case were Test #2 (20% cutoff), #6 (40% cutoff), #10 (60% cutoff) and #14 (80% cutoff). The number of blocks versus % cutoff graph for this loading condition is illustrated in Figure 26.

This graph showed that up to 40% cutoff (or 2000 regular cycles) it would require between 272 and 311 blocks after regular cycles to cause failure. As more regular cycles were introduced before block loads, fewer and fewer blocks were needed to cause failure (202 and 165 blocks, results from Tests #10 and #14 respectively). Looking at the trend, the disparity between 272 blocks and 311 blocks could be attributed primarily to scatter and hence, were considered similar cases. The
average of both these values was 292 blocks. Hence, at or before 40% cutoff, the amount of damage caused by the regular cycles was deemed to be uniform and thus needed an average of 292 blocks after those cycles to cause failure. After that, if more regular cycles were added past 40%, there would be considerably more damage caused by these constant amplitude cycles and thus required less blocks to cause failure afterwards. However, comparing this average of 292 blocks to the result from Test #12, a representative case of pure LCF block loading, which was 532 blocks to failure, pointed out that some amount of damage might have been done by the regular cycles before block loading took place in Tests #2 and #6. This damage, by observation, should be uniform between 20% and 40% cutoff.
The results in fatigue life for the tests were \( N_f = 2360 \) cycles (%drop = 51.5%) for Test #2, \( N_f = 3560 \) cycles (%drop = 26.9%) for Test #6, and \( N_f = 4010 \) cycles (%drop = 17.7%) for Test #2. More importantly, Test #14 resulted in a fatigue life of 4825 cycles (%drop = 0.92%) meaning that block loads had no effect in this case. The %drop vs. %cutoff graph is presented as Figure 27.

Also, Figure 28 illustrates the comparisons between crack growth profiles of all these tests plotted against a constant amplitude LCF case.
Figure 28. Crack Comparison Chart for LCF, C-B, 50%EL Small Cycles.

Test #2, at 20%cutoff, displayed crack initiation at about 1800 cycles and a propagation range of 500 cycles. Test #6, 40% cutoff, had a crack initiate at 3000 cycles and a propagation range of 510 cycles. Test #10, 60% cutoff, initiated a crack at 3250 cycles and had a propagation range of 750 cycles. And finally Test #14, with a cutoff at 80%, had initiation happen at 4000 cycles and a propagation range of 800 cycles. Figures 29, 30, 31 and 32 illustrate the crack growth profiles for Tests #2, #6, #10 and #14 respectively.

These results helped shed some light on what was going on with Tests #2 and #6 which incidentally shared similar initiation behavior. Both these tests had initiations that happened during a pure block loading.
Figure 29. Crack Growth Profile for Test #2: LCF, C-B, 20% Cutoff With 50% EL Small Cycles.

Figure 30. Crack Growth Profile for Test #6: LCF, C-B, 40% Cutoff With 50% EL Small Cycles.
Figure 31. Crack Growth Profile for Test #10: LCF, C-B, 60% Cutoff With 50% EL Small Cycles.

Figure 32. Crack Growth Profile for Test #14: LCF, C-B, 80% Cutoff With 50% EL Small Cycles.
environment and both had the same propagation range, meaning that both of them were essentially identical, thus confirming the observations mentioned earlier. However, no evidence pointed to any cracks developing in the regular cycle section of these tests and this probably meant that even though there was some form of weakening caused by the constant amplitude cycles prior to block loading, it was not caused by crack initiation.

Test #10 had an earlier initiation point due mainly to the effects of the more damaging block loads as its crack entered the middle stages of propagation, and its propagation was influenced by both block loads and regular cycles, hence it seemed to have a propagation range (750 cycles) that was in between the 480 cycles for pure block loads and the 900 cycles for the constant amplitude LCF case.

Test #14, which had no effect on fatigue life, had a very similar profile with the constant amplitude LCF case confirming that the 50%EL small cycles were introduced very late and thus, featured a propagation range (of 800 cycles) closer to HCF constant amplitude loading compared to the less influencing block loads which were only introduced towards the end.

Based on the information presented above, hence, for LCF constant-to-block cases with 50%EL intermittent small cycles, the small cycle effects can only be ignored if they are introduced very late in life, at 80% of later. Before that, the small cycles would have an effect, but this effect would become less and less severe as percent cutoff increases in value.
HCF Block-to-Constant (B-C) Loading

Small Cycles With Amplitudes of 100%EL

The fatigue life vs. %cutoff, and %drop vs. %cutoff graphs are presented in Figures 33 and 34 respectively.

![Graph showing fatigue life vs. %cutoff](image)

Figure 33. Fatigue Life vs. %Cutoff Plot for HCF, B-C, 100%EL Small Cycles.

In the fatigue life versus %cutoff plot for Tests #19 (20%cutoff), #23 (40%cutoff), #27 (60%cutoff) and #31 (80%cutoff), it was found that Tests #19 and #23, with results $N_f = 120,300$ cycles (%drop = 2.98%) and $N_f = 131,000$ cycles (%drop = -5.65%), respectively, had no effect on the expected life while at later cutoffs, 60% and 80%, the block loads started to influence life. Test #27 resulted in
\[ N_f = 72,650 \text{ cycles (}\%\text{drop} = 41.4\%) \] and \#31 had \( N_f = 65,200 \text{ cycles (}\%\text{drop} = 47.4\%) \).

This trend pointed out that although 100\%EL small cycles were not influential when introduced into cycles before 40\% life, it had a very drastic effect if more blocks were added past that point, causing up to a 40\% drop in life in less than 200 additional blocks (i.e. in between the 40\% and 60\% cutoff points).

![Figure 34. %Drop vs. %Cutoff Plot for HCF, B-C, 100\%EL Small Cycles.](image)

Crack growth profile comparisons were made and illustrated in Figure 35. Tests \#19 and \#23 did not have replicas available for analysis, but since they had no effect on the fatigue life of the specimens, their crack growth diagrams would look like the constant amplitude HCF case, which had crack initiation at 100,000 cycles.
Figure 35. Crack Comparison Chart for HCF, B-C, 100%EL Small Cycles.

and a propagation range of 20,000 cycles. It was safe to predict that no cracks initiated in both Tests #19 and #23 prior to 105,000 cycles since their cutoffs were 20% = 20,000 cycles, and 40% = 40,000 cycles, respectively, which was very early even in the case of block-to-constant loading where earlier initiation was expected.

Test #27 had a crack growth profile as shown in Figure 36. Test #27 was for HCF B-C sequencing with 100%EL intermittent small cycles. The crack growth diagram for this particular test displayed a fatigue initiation crack (0.5 mm) after approximately 54,000 cycles, showing that the block loading prior to the 60% cutoff (which amounted to 600 blocks) worked to initiate the crack. Its propagation range
Test #27 also displayed a fatigue crack initiation at around 54,000 cycles. This result worked to confirm that the blocks loads affected initiation in this test just like it did in Test #27 above (which had an initiation point at 54,000 cycles as well). The propagation range for Test #31 was shortened to 10,000 cycles, caused primarily by the damaging block loads all throughout crack initiation and propagation. This crack growth profile is shown in Figure 37.

Figure 36. Crack Growth Profile for Test #27: HCF, B-C, 60% Cutoff With 100% EL Small Cycles.

was 20,000 cycles, which was similar to the constant amplitude HCF case since most of its propagation happened in the regular cycle constant amplitude loading stage.
Figure 37. Crack Growth Profile for Test #31: HCF, B-C, 80% Cutoff With 100%EL Small Cycles.

There was no transition from blocks to regular cycle loading because failure happened after 652 blocks (or 65,200 cycles) which was short of the planned 80% cutoff (800 blocks). Test #31 was representative of pure HCF block loading conditions with 100%EL small cycles.

As a summary, in the case of high cyclic fatigue block-to-constant cases with 100%EL intermittent small cycles, as long as the small cycles are introduced for no more than 40% from the beginning of the expected life, they can be ignored. After that, the small cycles have the ability to do a lot of damage very quickly. This set of tests also displayed that a certain threshold had to be overcome after 40% cutoff in order for the small cycles to cause significant damage.
Small Cycles With Amplitudes of 50%EL

Based on the fact that Tests #19 and #23 had no effect on fatigue life, even with 100%EL small cycles, Tests #20 and #24 were skipped since 50%EL small cycles would very obviously cause lesser damage and would thus, not have any effect either.

Tests #28 and #32 were run as normal and yielded fatigue life results of $N_f = 112,800$ cycles (%drop = 9.03%) and $N_f = 91,500$ cycles (%drop = 26.2%). On the fatigue life versus %cutoff plot, it can be seen that there was no significant change in fatigue life even up to 60% cutoff between block loads and regular cycles, meaning that the number of small cycles at 50%EL was not enough to cause damage. The only obvious drop in life came at the 80%cutoff point, after 800 blocks, meaning that the intermittent small cycles worked to weaken the specimen significantly between 600 and 800 blocks.

Figures 38 and 39, illustrate the fatigue life vs. %cutoff, and the %drop vs. %cutoff relationships, respectively. The comparisons between crack growth profiles are presented in Figure 40.

Test #28, which was done in HCF B-C with 50%EL small cycles at 60%cutoff, illustrated in Figure 41, showed initiation at 100,000 cycles, which was similar to the constant amplitude HCF case, but failed a little early after a propagation range of 12,000 cycles (compared to 20,000 cycles for constant amplitude HCF).
Figure 38. Fatigue Life vs. %Cutoff Plot for HCF, B-C, 50%EL Small Cycles.

Figure 39. %Drop vs. %Cutoff Plot for HCF, B-C, 50%EL Small Cycles.
Figure 40. Crack Comparison Chart for HCF, B-C, 50%EL Small Cycles.

Test #20 (20%cutoff) and Test #24 (40%cutoff) had no effect and would look like the const amp HCF curve.

Figure 41. Crack Growth Profile for Test #28: HCF, B-C, 60%Cutoff With 50%EL Small Cycles.
This suggested that there might have been a small amount of damage caused by the small cycles and this had some influence on the propagation rate of the fatigue crack, although it was not linked to initiation since Test #28 shared the same crack initiation value with the constant amplitude HCF case.

Test #32 displayed crack initiation at 81,000 cycles, showing that there was damage done by the small cycles causing early crack initiation. The propagation range was a low 10,000 cycles; the block loads must have damaged the specimen in such a way that the propagation happened quicker even though most of it occurred in the regular constant amplitude range. This was shown in Figure 42.

This finding did not agree with what was observed with 100%EL small cycles, reported above. For the 100%EL small cycles case, which involved obviously more damaging block loads, cracks that propagated under the influence of constant amplitude regular cycles (Test #27) had a longer propagation range compared to the surprisingly shorter ones reported for the 50%EL small cycles cases (Tests #28 and #32), which should have been less damaging.

It was expected that the 50%EL cases would allow cracks propagating under the influence of less damaging regular cycles to span over a longer range than the 100%EL cases. These results, however, reveal the contrary. Unfortunately no explanation can be provided toward the analysis of this behavior. Nonetheless this phenomenon might be a topic of interest in any future work based on the findings of this thesis study.
Figure 42. Crack Growth Profile for Test #32: HCF, B-C, 80% Cutoff With 50% EL Small Cycles.

In partial conclusion for this series of studies which were based on high cyclic fatigue loaded in block-to-constant pattern cases involving 50% EL small cycles, it is safe to ignore small cycles that are introduced from the beginning of cycling to, at most, 40% fatigue life, but caution should be used as 60% cutoff is approached because there is some evidence that the small cycles begin to marginally influence the material at that cutoff point. After 60% cutoff, small cycles at 50% EL must be accounted for because the damage is substantial. Again, there was a threshold that needed to be overcome up until 60% cutoff. Only after 60% cutoff did the small cycles act toward causing significant damage.
HCF Constant-to-Block (C-B) Loading

Small Cycles With Amplitudes of 100%EL

This category involved Tests #17, #21, #25 and #29 corresponding respectively to 20%, 40%, 60% and 80% cutoffs under HCF C-B loading with 100%EL small cycles. The blocks versus %cutoff plot is illustrated in Figure 43.

Figure 43. Number of Blocks vs. %Cutoff Plot for HCF, C-B, 100%EL Small Cycles.

This graph revealed that 650 blocks were initially needed to fail a specimen after regular cycles turned to block loading at the 20% cutoff point. This seemed to increase a bit at 40% cutoff, requiring 778 blocks after constant amplitudes to cause
failure. Even though these results show some difference, it is suspected that it was a product of scatter and that Tests #17 and #21 should require an equal number of blocks to cause failure under ideal conditions. Hence, the average number of blocks – 714 blocks – could be how many blocks would be required to cause failure if regular cycles were only applied up to 40%cutoff. An addition to this observation was that the regular cycles at or below 40%cutoff (or 40,000 cycles) did not significantly cause damage to the specimen; all the damage was done exclusively by the blocks loads proceeding them. However as the cutoff moved up to 60%, a drastic drop (to 202 blocks) was observed meaning that the regular cycles between 40% and 60% cutoffs worked to damage the specimen, thus requiring fewer blocks after that to cause failure. At 80% it dropped further to 110 blocks, suggesting that a majority of the damage to the specimen was done by the large amount of regular cycles before the block loads.

As far as fatigue life results are concerned, Test #17 had a fatigue life of Nf = 85,000 (%drop = 31.5%), Test #21 – Nf = 117,800 (%drop = 5.0%, no effect), Test #25 – Nf = 80,200 (%drop = 35.3%), and Test #29 – Nf = 91,100 (%drop = 26.53%). Test #17 displayed a visible drop in fatigue life because the regular cycles were stopped early enough (at 20%cutoff) to allow a larger amount of block loads after that to cause damage and result in earlier failure. On the other hand, the “no effect” case observed from Test #21 could be attributed to the fact that the regular cycles up to 40% cutoff did little to cause damage, and the 778 blocks after that did not cause significant damage early enough to affect the point of final failure, and the specimen
was able to cycle to its full expected fatigue life. The %drop vs. %cutoff behavior for this series of tests is illustrated in Figure 44.

![Figure 44. %Drop vs. %Cutoff Plot for HCF, C-B,100%EL Small Cycles.](image)

Figure 44 had an unusual trend that pointed a decreasing effect of small cycle damage up to 40%cutoff, and then showed a rapid increase in small cycle effects as the cutoff approached 60%. This trend illustrated that at 20% and 40% cutoff, the regular cycles that were applied before the block loads did not help damage the specimen, and thus, helped reduce the %drop value as more and more regular cycles were added in between 20% and 40% cutoffs. However, after 40%, the regular cycles
started causing significant damage and helped the small cycles to affect crack growth which lead to premature failures all the way up to 80%cutoff.

The fatigue crack profile comparisons are presented in Figure 45.

Figure 45. Crack Comparison Chart for HCF, C-B, 100%EL Small Cycles.

The crack growth profiles for Tests #17, #25 and #29 are presented in Figures 46, 47 and 48 respectively. Test #17 displayed a 0.5 mm crack initiation length at 71,000 cycles and a crack propagation range of about 12,000 cycles. Test #21 did not have any replicas for study but since it caused no effect on the specimen’s life, its crack growth profile could be assumed to be very similar to the constant amplitude HCF case.
Figure 46. Crack Growth Profile for Test #17: HCF, C-B, 20% Cutoff With 100% EL Small Cycles.

Figure 47. Crack Growth Profile for Test #25: HCF, C-B, 60% Cutoff With 100% EL Small Cycles.
Test #25 had crack initiation at 74,000 cycles and a propagation range of around 7,000 cycles, while Test #29 experienced crack initiation at 82,000 cycles with a propagation range of approximately 8,000 cycles.

Faster propagation seemed to happen in Tests #25 and #29, while it was much slower in Test #17 that had an earlier cutoff than the former two cases. This suggested that the larger amount of regular cycles before the block loads in Tests #25 and #29 (60,000 and 80,000 cycles respectively), as compared to the 20,000 cycles for Test #17, might have helped accelerate crack growth rate.

In summary, for HCF C-B cases involving 100%EL small cycles, the only instance where these small cycles can be ignored is at 40% cutoff, and even so, very tentatively. At all other cutoff levels, small cycle effects must be considered.
Small Cycles With Amplitudes of 50%EL

Tests #18 (20%cutoff), #22 (40%cutoff), #26 (60%cutoff) and #30 (80%cutoff) were used to study this category. These tests displayed a very well-behaved and interesting trend on the number of blocks versus %cutoff graph, as shown in Figure 49.

![Figure 49. Number of Blocks vs. %Cutoff Plot for HCF, C-B, 50%EL Small Cycles.](image)

It was found that for the first three cutoff stages, 20%, 40% and 60%, almost the same number of blocks were needed in each case to cause failure (results were 618, 636 and 609 blocks respectively, for an average of 621 blocks) after the regular cycles changed to block loading. The only change observed in the blocks versus %cutoff graph was the dip seen at 80%cutoff. This can be attributed to damage
accumulation from the regular cycles finally adding up at 80% fatigue life and thus, began to affect the number of blocks needed for failure. It was deduced that the majority of the regular cycle damage occurred significantly between 60,000 and 80,000 regular cycles.

A comparison was made between these results and those attained from Tests #17 and #21, and it was observed that fewer blocks were needed in this instance to cause failure (average of 621 blocks) compared to the 100%EL cases (average of 714 blocks). This was an unexpected result since 100%EL small cycles were always believed to be more damaging and hence would require fewer blocks to failure; these turn of events were contradictory to this logic. As concession for this development, it was assumed that Tests #17, #18, #21, and #22 were, in fact, identical cases, i.e. for all these cases which involved 20% and 40% cutoffs, the regular cycles did not damage the specimens, and somehow, 100%EL and 50%EL small cycles caused equal amounts of damage! This might not be a bad approximation since the average 714 blocks in the 100%EL case was only around 1.2 times more than the 621 blocks found in 50%EL results, showing not much of a difference between them.

A report on fatigue lives for each test are as follows: Test #18 had \( N_f = 81800 \) cycles (%drop = 34.0%), Test #22 had \( N_f = 103,600 \) cycles (%drop = 16.45%), Test #26 had \( N_f = 120,900 \) cycles (%drop = 2.50%, no effect), and, finally, Test #30 had \( N_f = 105,400 \) (%drop = 15%). The %drop vs. %cutoff graph is illustrated fully in Figure 50.
For all cases, except at 60% cutoff, the small 50%EL small cycles contributed to damage and caused premature failure. An observation of this trend suggested that the 50%EL small cycles caused larger % drops when block loads were introduced early (as was the case in Tests #18 and #22) because that gave them enough repetitions to build up damage and eventually cause premature failure. However, when 60% cutoff was approached (in Test #26), the number of blocks needed to cause significant damage would be reached only at the end of the specimen’s life, hence the final fatigue life was not affected. In the case of Test #30, as regular cycles were applied up to 80% cutoff, damage began to build up significantly in the regular cycles.
between 60% and 80% cutoffs and, after that, adding block loads till failure, helped build up more damage resulting in fewer number of blocks needed to cause failure.

The crack growth profile comparisons for this series of tests is presented in Figure 51:

Figure 51. Crack Comparison Chart for HCF, C-B, 50%EL Small Cycles.

Crack growth profiles are featured in Figures 52 for Test #18, 53 for Test #22 and 54 for Test #30.

Test #18, which was at 20% cutoff showed crack initiation at 60,000 cycles and had a propagation range of 22,000 cycles. Test #22 (at 40% cutoff) had a crack initiate at 85,000 cycles and a propagation range of 20,000 cycles. Test #26 (which was at 80% cutoff), unfortunately, did not have any replicas, however since it had no
Figure 52. Crack Growth Profile for Test #18: HCF, C-B, 20% Cutoff With 50% EL Small Cycles.

Figure 53. Crack Growth Profile for Test #22: HCF, C-B, 40% Cutoff With 50% EL Small Cycles.
effect, it could be assumed that its crack growth profile would mimic the one for constant amplitude HCF. Finally, Test #30 experienced crack initiation at 85,000 cycles and a propagation range of 20,000. The striking similarity in all these tests was the propagation range, which did not very much differ between the tests.

Furthermore, the propagation range for constant amplitude HCF was approximately 20,000 cycles. This revealed that, HCF block loads with 50%EL small cycles helped to initiate fatigue cracks earlier but did not cause acceleration in crack growth rate. This finding did not, however, agree with what was found in HCF block-to-constant loading with 50%EL small cycles. That B-C category of tests, as described earlier in this section, showed that there might have been crack growth acceleration caused by
the 50%EL small cycles, something that was not evident in this case. So this phenomenon might be specific only to constant-to-block loading conditions.

In summary, for HCF C-B loading with 50%EL small cycles, most cases cannot be ignored, but if small cycles were to be introduced less severely (i.e. lower stress amplitudes and/or with frequencies of less than 1000 intermittent cycles) they can be ignored if introduced no sooner or later than 60% of fatigue life.

Pure Block Loading Tests

In addition to the main tests, two other experiments were run to quantify what happens in pure block loading conditions. There were four possible pure block loading conditions: LCF with 100%EL small cycles, LCF with 50%EL small cycles, HCF with 100%EL small cycles, and HCF with 50%EL small cycles. Of these four tests, data for two were already available in the form of Test #3 and Test #12, for LCF 100%EL and LCF 50%EL respectively. The last two tests were additional tests that were run and produced the following fatigue life results:

Table 5

<table>
<thead>
<tr>
<th>Regime</th>
<th>Small Cycle Amplitude</th>
<th>Fatigue Life, Nf</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCF</td>
<td>100%EL</td>
<td>930 cycles (from Test #3)</td>
</tr>
<tr>
<td>LCF</td>
<td>50%EL</td>
<td>2660 cycles (from Test #12)</td>
</tr>
<tr>
<td>HCF</td>
<td>100%EL</td>
<td>75,900 cycles (759 blocks)</td>
</tr>
<tr>
<td>HCF</td>
<td>50%EL</td>
<td>141,500 cycles (1415 blocks)</td>
</tr>
</tbody>
</table>
The LCF block loading cases were already analyzed earlier in this section and provided results that were well-behaved. However, for the HCF cases, pure block loading produced very interesting results! Before proper analysis begins, it should be noted that the block loading, as prescribed by the author for HCF block loads, contained 100 regular cycles followed by 1000 small cycles, which is not the most damaging case. As mentioned before, this loading pattern was designed to ensure that LCF and HCF sequences experienced an equal number of small cycles throughout the their respective expected lives, and also to save time on tests. The most damaging case for the same regular and small cycle amplitudes would be 1 regular cycle followed by 1000 or more small cycles per block, which pertains essentially to a fully-reversed overload case as done by Reference [14].

Under pure block loads containing 100 regular cycles followed by 1000 small cycles, it was observed that at a small cycle amplitude of 100%EL, the small cycles contributed as expected and caused premature failure after 759 blocks, or 75,900 cycles. This agreed with the results for Tests #17, #18, #21, #22, #26 and #31, and confirmed once again that 100%EL small cycles had a profound effect on HCF loading. The surprise, however, came from the pure block loading case that involved 50%EL small cycles. It was found that only after 1415 blocks (or 141,500 cycles) was there full separation failure, meaning that the small cycles did not effect fatigue life at all! This case was very possible because, as mentioned in the paragraph above, the block loading conditions used were not the most damaging case, and if a reversed
overload case was used instead, premature failure due to the small cycles would have been expected.

On the other hand, this finding was still very unexpected because in tests involving B-C or C-B HCF loading with 50%EL small cycles (refer to Figures 38, 39 and 48), the trends gave no hint of this kind of behavior. In the HCF B-C with 50%EL small cycles case, for instance, it can be seen that in Figure 39 the %drop seemed to sharply increase all the way up to 80% cutoff and showed no sign of dropping any lower. But with the data provided by the pure block loading test, Figure 39 would, in fact, look like Figure 55.

Figure 55. %Drop vs. %Cutoff Plot for HCF, B-C, 50%EL Small Cycles Including Pure Block Loads at 100%Cutoff.
This trend suggested that the block loads applied up to any cutoff level still required regular cycles to advance any damage done by the small cycles before them. A possible explanation for this behavior would be that the small cycles caused some micro-cracks on the surface of the specimen but the limited number of regular cycles (100 cycles) in each block were not enough to advance these cracks. Only regular cycles applied continuously had the ability to advance the micro-cracks past their dormant state, create initiation cracks and finally propagation till failure. This observation was further strengthened by Figure 40, shown earlier in this section, that confirmed the fact that no initiation cracks showed up during block loading, and the cracks only started to advance in the presence of regular constant amplitude cycles. Without the presence of regular cycles toward the end of the loading, there would not be enough damage to cause premature failure.

This same analysis was done to Figure 50, which was for the HCF C-B 50%EL small cycle case. That figure, with the data point from pure block loading would resemble Figure 56.

Note that the pure block loading point was placed at 0% cutoff this time, because at that cutoff, there would be no regular constant amplitude cycles, only block loads. In the case of these tests the regular cycles, which came before the block loads, lightly damaged the specimens, and this helped to form the few micro-cracks necessary for the block loads to act upon and cause early failure of the specimens. Between 20% and 60% cutoffs, the regular cycles caused the same amount of damage
regardless of cutoff, seemingly unable to get past a certain threshold. However, after 60% cutoff this threshold was overcome by the regular cycles and more damage started to build up with more regular cycle inclusions. The %drop decreased as cutoff increased because the regular cycles helped to delay the effects of the small cycle damage, which required a certain number of blocks before they could have any effect (as shown in Figure 51, above). At 80% cutoff, the regular cycles began to damage the specimen more and with the addition of block loads toward the end, the crack experienced quick initiation during block loading and caused failure to happen.
prematurely. It should be noted that at 100% cutoff, which is essentially normal constant-amplitude fully reversed cycling from beginning to end, the %drop would be zero.
DISCUSSION

Damage Calculations

The Palmgren-Miner Rule is still widely used in industry to approximate the fatigue lives of parts and components although, as mentioned in the introduction section of this thesis, the indiscriminate omissions of small cycle effects from spectrum or variable amplitude loading calculations is not a satisfactory practice. Hence, based on the data provided by the thesis testing procedure, a practical damage prediction method can be introduced based on when the small cycles are introduced or removed, and this model will work to correct any under-predictions made by the Palmgren-Miner Rule.

Recollecting what was introduced earlier in this thesis, the Palmgren-Miner Rule, described by Equation 1, is reproduced here:

\[
\sum \frac{N_j}{N_f} = 1
\]

(Eq. 4)

where, \(N_j\) = the number of cycles at a certain fully reversed amplitude

\(N_f\) = the number of cycles to failure at constant amplitude

Small below-endurance limit cycles are usually omitted completely from this equation because their fatigue lives are supposedly infinite. However, as can be seen from the results attained in this thesis, this equation resulted in a value of less than 1
on almost all occasions. Rearranging the Palmgren-Miner Rule, the damage that came from the small cycles can be calculated to be:

\[
D_{\text{small}} = 1 - \frac{N_{\text{regular}}}{N_{f,\text{regular}}} \tag{Eq. 5}
\]

where, \(D_{\text{small}}\) = damage caused by the small intermittent below-endurance limit cycles.

\(N_{\text{regular}}\) = the number of regular cycles

\(N_{f,\text{regular}}\) = the expected fatigue life of the regular cycles.

The value for \(D_{\text{small}}\), once calculated, can be applied back into the Palmgren-Miner Rule and will thus allow it to better approximate the contributions of intermittent small cycles. This equation was used to find the \(D_{\text{small}}\) for all the tests in this thesis and are tabulated in Table 6 below:

### Table 6

<table>
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<tr>
<th>Test #</th>
<th>Regime</th>
<th>Sequence</th>
<th>100% or 50% EL</th>
<th>Nf</th>
<th>Number of blocks</th>
<th>(D_{\text{regular}})</th>
<th>(D_{\text{small}})</th>
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<td>11500</td>
<td>800</td>
<td>0.74</td>
<td>0.26</td>
</tr>
</tbody>
</table>

In Table 6, the number of regular cycles were calculated by adding up the regular cycles in the constant amplitude loading history and adding to them the number of regular cycles from the number of blocks used in the test. As an example, in Test #5, which was a LCF B-C 100%EL test with a cutoff at 40%, the 2000 regular cycles from before the cutoff were added to the 405 cycles that came from the 81 block loads proceeding them (5 regular cycles per block multiplied by 81 blocks = 405 cycles) to give a total of 2405 regular cycles.
The damage from the regular cycles, $D_{\text{regular}}$, was calculated by dividing the total number of regular cycles with the expected fatigue life based on loading regime i.e. 4870 cycles for LCF, or 124,000 cycles for HCF. Then the small cycle damage was calculated for each test using equation 5 above. After all calculations were done, the tests were grouped into their regime (LCF or HCF) and sequencing (C-B or B-C) combinations and plotted using Microsoft Excel. With the data points, a trend-line was added for each case (only if possible) using polynomial or linear curve fits. These plots are made available in Figures 57 through 63. This provided very useful equations or (cross-reference lines) that can be used to predict damage from below-fatigue limit cycles at any cutoff between 0% and 100%, given the amplitude (100%EL or 50%EL), regime and sequencing.

Figure 57. 100%EL Small Cycle Damage vs. %Cutoff for LCF C-B Loading.
Figure 58.  50%EL Small Cycle Damage vs. %Cutoff for LCF C-B Loading.

y = -2.954E-08x^4 + 7.816E-06x^3 - 6.509E-04x^2 + 1.185E-02x + 4.583E-01

Figure 59.  50%EL Small Cycle Damage vs. %Cutoff for LCF B-C Loading.

y = -1.837E-09x^5 + 5.474E-07x^4 - 5.915E-05x^3 + 2.657E-03x^2 - 3.336E-02x - 1.924E-11
Figure 60. 100%EL Small Cycle Damage vs. %Cutoff for HCF C-B Loading.

Figure 61. 50%EL Small Cycle Damage vs. %Cutoff for HCF C-B Loading.
Figure 62. 100%EL Small Cycle Damage vs. %Cutoff for HCF B-C Loading.

Figure 63. 50%EL Small Cycle Damage vs. %Cutoff for HCF B-C Loading.
For convenience, any equation in the figures above are reproduced in Table 7 below based on regime, sequencing, and small cycle amplitudes. It should be noted that some of the equations allow for damage results to go below zero. Any negative damage values that might result from these equations should be treated as zero.

### Table 7

List of Small Cycle Damage Equations

<table>
<thead>
<tr>
<th>Regime</th>
<th>Sequencing</th>
<th>Small Cycle Amplitude</th>
<th>Small Cycle Damage Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCF</td>
<td>C-B</td>
<td>100%EL</td>
<td>$(-8.10E-03 \times %\text{cutoff}) + (8.03E-01)$</td>
</tr>
<tr>
<td>LCF</td>
<td>C-B</td>
<td>50%EL</td>
<td>$(-2.954E-08 \times %\text{cutoff}^4) + (7.816E-06 \times %\text{cutoff}^3) - (6.509E-04 \times %\text{cutoff}^2) + (1.185E-02 \times %\text{cutoff}) + (4.583E-01)$</td>
</tr>
<tr>
<td>LCF</td>
<td>B-C</td>
<td>50%EL</td>
<td>$(-1.837E-09 \times %\text{cutoff}^5) + (5.474E-07 \times %\text{cutoff}^4) - (5.915E-05 \times %\text{cutoff}^3) + (2.657E-03 \times %\text{cutoff}^2) - (3.336E-02 \times %\text{cutoff}) - (1.924E-11)$</td>
</tr>
<tr>
<td>HCF</td>
<td>C-B</td>
<td>100%EL</td>
<td>No equations. Damage cross-referenced from Figure 60.</td>
</tr>
<tr>
<td>HCF</td>
<td>C-B</td>
<td>50%EL</td>
<td>No equations. Damage cross-referenced from Figure 61.</td>
</tr>
<tr>
<td>HCF</td>
<td>B-C</td>
<td>100%EL</td>
<td>$(-2.688E-08 \times %\text{cutoff}^4) + (1.862E-06 \times %\text{cutoff}^3) + (1.883E-04 \times %\text{cutoff}^2) - (6.866E-03 \times %\text{cutoff}) + (8.564E-03)$</td>
</tr>
<tr>
<td>HCF</td>
<td>B-C</td>
<td>50%EL</td>
<td>No equations. Damage cross-referenced from Figure 63.</td>
</tr>
</tbody>
</table>

It should be apparent that the LCF B-C 100%EL case was not formulated since it involved pure block loads and hence, has a uniform small cycle damage of 0.81 (shown for Test #3 in Table 6 above) for all cutoffs except 0% where damage is zero. Between 0% cutoff and 20% cutoff, the relationship between damage and cutoff can be assumed to be linear. Also, Figures 60, 61 and 62 could not be curve-fitted.
since their trends were less predictable. If any small cycle damage value was needed, it can be found by directly cross-referencing the appropriate figures.

Looking at Figures 57 through 63 and comparing them to the findings in the results and analysis section, it can be seen that there is an agreement on when small cycles can be ignored in calculations. Essentially, any small cycle damage value that is at or below 0.1 can be safely ignored since their effects are negligible. Hence the very first check should be to find out if the small cycles are negligible. If they are, then no calculations would be needed for them. Figures 57 through 63 provide a very practical and quick way of cross-referencing the case in question to find out if there are small cycle contributions or not.

If the small cycles cause more than 0.1 damage, then damage calculations can be done using the equations in Table 7 above, for any %cutoff. These small cycle damage equations can only be used if the following assumptions hold true:

1. The material is 2024-T351 aluminum alloy.
2. Loading is fully reversed.
3. There is only two levels of loading: constant amplitude regular cycles (above the endurance limit) and small cycles (below-endurance-limit) and both have uniform amplitudes all throughout their respective loading schemes.
4. Regular LCF cycles have a fully reversed constant-amplitude fatigue life of 5000 cycles ±20%.
5. Regular HCF cycles have a fully reversed constant-amplitude fatigue life of 100,000 cycles ±20%.
6. Small cycles have stress amplitudes of either 100%EL or 50%EL, and not both at once.

7. Small cycles introduced intermittently into the regular cycles do not exceed the empirical number of blocks multiplied by 1000 for a specific case, for example for LCF C-B 100%EL at 40%cutoff (which would be akin to Test #5), there cannot be more than 81,000 small cycles introduced (81 blocks multiplied by 1000).

Among all the above assumptions, the only ones that are completely inflexible are assumptions number 1, 2 and 3. The first assumption cannot be modified since this study was done exclusively for 2024-T351 and was never meant to represent any other material. For assumption number 2, no tests were done for anything but fully-reversed cycles and hence, the damage equations should not be used for other non-zero mean cases. Assumption number 3 is also inflexible since no interaction studies were done between any case other than a two stage loading used in this thesis. If more complex variable amplitude loading conditions are used, it might introduce more variables into the problem. Other assumptions, however, can be relaxed to a good extent.

Assumptions number 4 and 5 specify that only very specific LCF and HCF cases can be predicted by the equation set. However, this restriction can relaxed by assuming that cases between 5000 and 100,000 cycles at a specific cutoff can be linearly interpolated using a simple equation such as Equation 6a.

\[ D_{small \ (interpolated)} = x \ast D_{small \ (HCF)} + (1 - x) \ast D_{small \ (LCF)} \]  

(Eq. 6a)

where, \( x = \frac{(N_i \ast of \ interpolated \ cycles) - 5000}{95000} \)
The number 5000 in equation 6a above is simply the life for LCF cycles, and 95,000 is the total of 100,000 minus 5000 cycles. Hence, if \( N_f \) is 5000 cycles then \( x = 0 \), and if \( N_f = 100,000 \) then \( x = 100,000 \) cycles.

As an example of the use of equation 6a, at 48% cutoff in C-B sequencing involving regular cycles that has a fully reversed constant amplitude fatigue life of \( N_f = 23,000 \) cycles and small cycles with 100%EL intermittent small cycles, the value for \( x \) would be \( x = 0.19 \). The \( D_{sma} \) values for LCF and HCF can be calculated, using the small cycle damage equations in Table 7, to be 0.414 and 0.153 respectively. Applying these numbers into Equation 6a above yields a result of 0.36 small cycle damage.

Also, assumption number 6 can be relaxed using an equation similar to the one above allowing for linear interpolations between 50%EL and 100%EL small cycles. This equation would be modified into:

\[
D_{sma} \text{ (interpolated)} = y \times D_{sma} \text{ (100\%EL)} + (1 - y) \times D_{sma} \text{ (50\%EL)}
\]

where, \( y = (\text{small cycle amplitude in \%EL} - 50) / 50 \). \hspace{1cm} (Eq. 6b)

For cases where both the \( N_f \) and small cycle amplitude have to be interpolated, equation 6a can be used to first interpolate the cases for \( N_f \) that is in between 5000 cycles and 100,000 cycles, for both 100%EL and 50%EL conditions. After that, with these two new sets of damage values, equation 6b would be used to interpolate the cases between 100%EL and 50%EL small cycles. An example follows: loading is done with C-B sequencing containing 80%EL intermittent small cycles up to
67% cutoff, and the constant amplitude fully reversed fatigue life of the regular cycles is \( N_t = 40,000 \) cycles. First, interpolation is done between 5000 and 100,000 cycles for 100% EL small cycles, yielding a damage value of 0.294. After that, interpolation is done between 5000 and 100,000 cycles for 50% EL small cycles, resulting in a damage value of zero. This was interesting since the damage at 67% cutoff for HCF C-B 50% EL was below 0.1 and was thus ignored. The damage at 67% cutoff for LCF C-B 50% EL was 0.086 which could also be ignored. This ultimately resulted in no damage for 50% EL small cycles. Finally, the damages of 0.294 for 100% EL, and 0 for 50% EL were evaluated over \( y = (80 - 50)/50 = 0.6 \) resulting in a final damage value of 0.176.

Assumption number 7 above is usually a safe one since it represents extreme cases of intermittent small cycles being present within otherwise regular cycle loading conditions. In LCF tests done in this thesis, there were anywhere from 17.3 to 288 times more small cycles compared to regular cycles, and in HCF cases there were 59 to 432 times more small cycles. And since this study was focusing on a background loading condition that consists of primarily regular cycles intermittently interrupted by a smaller quantity of small cycles, it can be easily seen that this assumption should pose no problems. Additionally, if the intermittent small cycles are introduced in amounts lesser than 1000 cycles in between larger loads, the effects would be less severe, hence the equations would give a conservative prediction. If it exceeds 1000 intermittent cycles by any amount, there should be no real discernable
effect compared to an exact 1000 intermittent cycles case, and thus the equations would still be valid. Hence in a lot of cases, assumption 7 should hold true.

The Significance of Damage Variation Based on %Cutoff

There are already a lot of damage models available commercially, such as the one explained as additional information in Appendix F, that were shown to be inaccurate by the empirical results from this thesis. And also, a lot of researchers have attempted to simplify below-fatigue/endurance limit cycle contributions into single equations that attempt to model small cycle damage similarly for all conditions regardless of when (decided by %cutoff) and how (sequencing: B-C or C-B) those cycles were introduced.

One of the questions that might arise at this point would be, “Is it sensible to introduce a variable damage model based on sequencing?” Or, “Wouldn’t one general damage model for interaction effects be sufficient to model small cycle damage in any instance? Why make things more complicated?”.

The best way to answer these questions would be to analyze the empirical data. In order to do that, the data was processed again and this time, a value called “damage per block”, or $D_{\text{per\_block}}$, was calculated. Damage per block is essentially the damage done by each block in a loading sequence and is represented by equation 7 below:

\[
D_{\text{per\_block}} = \left( \frac{N_{\text{regular}}}{N_{f,\text{regular}}} + D_{\text{small}} \right) / N_b
\]  

(Eq. 7)

where, $N_{\text{regular}} = \#$ of regular cycles in each block (5 for LCF, 100 for HCF)
\( N_{t, \text{regular}} = \) fatigue life for regular cycles (4870 for LCF, 124,000 for HCF)

\( D_{\text{small}} = \) small cycle damage calculated using equations in Table 7

\( N_b = \) the number of blocks used in the test

These values were tabulated and plotted as \( D_{\text{per block}} \) vs. \( \%\text{cutoff} \) for each regime/sequencing combination and is explained in detail in Appendix G, since this represents supplementary information.

From the information provided in Appendix G, an interesting discovery was observed. From the figures it was deduced that damage per block varied very drastically with \( \%\text{cutoff} \) and sequencing! Hence, there cannot be just one single generic damage equation for interaction/small cycle effects that can be used to accommodate for any insertion of small below-endurance-limit cycles.

Hence, this validates the need for the small cycle damage equations introduced in the previous sub-section.
CONCLUSIONS

Low Cyclic Fatigue vs. High Cyclic Fatigue

In general, it was found that LCF regular cycles, which are known to be very damaging themselves and have fatigue lives that are governed primarily by the initiation stage, were very sensitive to small below-endurance-limit cycles. Small cycles with a stress amplitude of 100%EL are very damaging especially in B-C loading, allowing no more than 186 block loads to build up under most circumstances. Hence, in almost all except two rare cases (between 0% and 2% cutoff for B-C loading, and 88% and 100% for C-B), 100%EL small cycles intermittently included in LCF cases must always be considered in damage calculations.

For small cycles with a stress amplitude of 50%EL, the effects are less profound, and can even be ignored under certain circumstances. Empirical evidence points out that for C-B sequencing, if constant amplitudes are allowed to run up to approximately 65% cutoff (according to Figure 58) before introducing block loads containing the small cycles, then the small cycle effects are negligible. Before 65% cutoff, all small cycles even at 50%EL must be calculated.

For B-C sequencing, the case is almost reversed. Based on the trend attained from Figure 59, if block loads are present from the beginning up to no more than approximately 25% cutoff, there would be no significant damage contributed by the small cycles. After that cutoff point, all small cycles at 50%EL must be considered.
In HCF cases, which are much less damaging in nature compared to LCF cycles and have fatigue lives governed by the propagation stage i.e. very late in life, it generally took a larger number of block loads to cause significant small cycle damage. The maximum number of blocks at any instance in HCF loading did not exceed 1415 blocks in pure block loading, and this was almost 7.6 times higher than the maximum number of blocks for the LCF cases. This signified, by itself, that HCF cycles were not affected as drastically by the small cycle damage as with LCF cycles.

From Figure 60 it can be seen that for a HCF C-B 100%EL small cycle case, if the small cycles are introduced too early, at or before 20% cutoff, there would be a good amount of damage that comes from the small cycles. However, as the cutoff point moves to 30%, the small cycle damage becomes negligible. From 30% cutoff to 45% cutoff, there is no sign of significant damage pointing out that the block loads were introduced too late and took too many blocks to affect the final life of the material. From 45% onwards damage contributions from small cycles increases to very high levels and should not be ignored. A similar trend was featured in the HCF B-C 100%EL case (Figure 62) but this time small cycles did not start contributing until the approximately 42% cutoff which means that the block loads, even with a severe small cycle amplitude of 100%EL, had no significant effect on regular cycles if introduced from the beginning of life until 42% expected life.

When 50%EL small cycles were used, there was generally less damage caused compared to the 100%EL small cycle cases which was expected. However, the trends from the small cycle damage diagrams in the discussion section were very
unpredictable and did not point toward simple trend shapes of any kind. For the HCF C-B 50%EL small cycle case, damage was insignificant under several circumstances: between 0% and 3% cutoffs, between 45% and 75% cutoffs, and between 96% and 100% cutoffs, as reported in Figure 61. In this case, it was observed that the constant amplitude cycles, as they were cutoff later and later in the life of the specimens, were virtually "pushing" the needed number of block loads further and further out. Also since, according to Figure 49, a constant amount of blocks were needed to cause failure all the way up to 60% cutoff, this meant that the regular cycles implemented before these block loads (for 20% to 60% cutoff cases) caused a small but constant amount of damage, and thus required the same amount of block loads to cause failure.

In the B-C sequencing case as illustrated by the trend in Figure 63, no significant damage was noticed until about 61% cutoff, meaning that before that point, introducing 50%EL small cycles into regular cycling had no effect. After that point damage contributions from the small cycles escalated to much higher values, but dropped to negligible levels again between 97% and 100% cutoffs as non-damaging pure block loads (explained in detail in the results and analysis section above) began to dominate.

In conclusion, LCF cycles are generally more sensitive to small cycle damage at both levels (100%EL and 50%EL) of small cycle amplitude. Since the damage done by LCF regular cycles are extremely damaging, any introduction of small cycles had profound effects. HCF cases were not as sensitive, with the regular cycles causing close to no damage for a large portion of early loading, only showing crack
growth very late in life. However, the regular cycles had an interesting effect that helped 50%EL small cycles produce more damage compared to pure block loading conditions that showed no damage from the 50%EL small cycles whatsoever.

100%EL Small Cycles vs. 50%EL Small Cycles

The best criteria for evaluating the differences between 100% and 50%EL small cycles would be the small cycle damage provided in Table 6 and Figures 57 through 63. A direct comparison between the maximum small cycle damage values in LCF loading cases showed that for the 100%EL small cycles the maximum damage caused was 0.81 while for the 50%EL small cycles the maximum damage was only 0.52. This signified a drastic difference in effect between the two amplitudes. The same observation was made for HCF cases where the maximum damage from 100%EL small cycles was 0.47. For 50%EL small cycles the damage never went past 0.34.

However, as stated in the sub-section above, although there was an expected difference between the 100%EL small cycles and 50%EL small cycles (which was shown by the numbers reported above), how and when they affected regular cycles were more important. The small cycle damage diagram trends, blocks vs. %cutoff plots, %drop vs. %cutoff graphs and additional information provided in Appendix G helped to map out this phenomenon. Among all other findings, this one was the most interesting since it proved, for a fact, that small cycles damaged the material very differently, and at varying severity depending on when and how the small cycles were
introduced! Hence, a single generic damage model for interaction or small cycle
damage cannot be introduced without taking into account sequencing (C-B or B-C
effects) and %cutoff. Depending on when and how the small cycles are introduced
into the loading spectrum, the damage model for the small cycles would also vary
which was a phenomenon that was addressed by the damage model introduced in the
previous section of this thesis.

Crack Growth Behavior

The study of crack growth in this thesis was done to augment the classical
fatigue approach with additional information which helped to shed some light on
what influenced fatigue life results. From the results and analysis, there were a few
trends that were prevalent.

In LCF cases, the small cycle inclusions generally helped initiate the crack
earlier in life for both 100%EL and 50%EL conditions. Also, it was found that
100%EL small cycles caused much higher propagation rates in LCF cycles, compared
to 50%EL small cycles that showed little or no change in crack growth rate. Hence, in
summary for LCF cases, 100%EL small cycles heavily influenced initiation points
(initiations happened much earlier in life than usual) and also accelerated crack
propagation. On the other hand, while 50%EL small cycles also influenced initiation
(although less severely), they did almost nothing to enhance propagation.

HCF cases were less predictable since the regular cycles themselves generally
caused no damage until very late in life, and for many cutoff levels the small cycle
inclusions were the sole contributors to damage. The differences, in terms of severity, between 100%EL and 50%EL small cycles were well-behaved: 100%EL small cycles generally caused initiations to happen much earlier in life, while the 50%EL small cycles had close to no effect on initiation except in a few severe block-loading instances such as for constant-to-block loading with a 20% cutoff which introduced the small cycles very early allowing them to accumulate damage over a larger number of blocks. The propagation effects of small cycles were more eccentric in HCF cases although generally, it was observed that 100%EL small cycles caused quicker propagation more often than 50%EL cases. The data does suggest that there was a very strong dependence between propagation range and sequencing; evidence of this comes primarily from comparing the B-C and C-B tests involving 50%EL small cycles – the B-C sequencing appeared to have caused faster propagation rates as %cutoff increased, while in the C-B case there was almost no difference whatsoever in the propagation range regardless of %cutoff. This meant that the inclusion of regular cycles, which are more damaging since they are continuously repeating above-endurance-limit cycles, at the end of the tests helped accelerate crack growth rates. When the regular cycles were introduced before block loads, they only helped the block loads to form fatigue cracks earlier. However, the 50%EL small cycles themselves were not severe enough, when applied toward the end of cycling, to help accelerate crack growth.

In LCF cases, shortened lives in specimens could be directly traced to the fact that initiation happened, in all instances, under the influence of block loads. Because
of the damage done by the small cycles during block loading stages, in addition to any damage that might have been done by the regular cycles up to that point, cracks tended to initiate during this stage and this worked to cause failure prematurely in the specimens.

The same observations were made in all but a few HCF cases. Block loads regardless of stress amplitude caused premature initiation which lead to shortened specimen life. The only cases when this was not evident were in the B-C sequencing with 50%EL small cycle tests where the less damaging small cycles failed to cause significant damage when introduced from the beginning to 60% cutoff. In those tests, initiation happened only under the influence of regular cycles. This helped to reinforce the notion that 50%EL small cycles were generally not severe and were very dependent on regular cycles as damage catalysts.

Final Conclusion

As a conclusion for this thesis, it was shown that for 2024-T351 aluminum alloy there were many cases, depending on loading regime, sequencing, small cycle amplitude and %cutoff, where the effects of small cycles in an otherwise above-endurance-limit cycle pattern could be completely ignored. It was also found that small cycle damage was decisively dependent not only on sequencing, small cycle amplitude and regime, but also %cutoff. In instances when small cycles do cause damage to the material, the small cycle damage equations in Table 7 provided the solution. These equations would more accurately predict the effects of small cycle
inclusions since they took into account cutoff effects. The equations would be conservative because the testing procedure used in this thesis modeled unusually high densities of small cycles compared to regular cycles. Most cases would involve loading patterns that would contain predominantly above-endurance-limit cycles with notably fewer small cycles introduced intermittently between them, so the damage prediction introduced in this thesis would provide a conservative damage estimate in a majority of circumstances.
RECOMMENDATIONS ON FUTURE WORK

This work, even after intense effort and research, could still be improved upon and this section provides future researchers with some suggestions on where to go from here.

It should be noted that as confident as the author is of the damage prediction method that was introduced in this thesis, there was no intention of generalizing the use of this method for any other material besides 2024-T351 aluminum alloy. Only future work using the outlined test procedure on other materials such as steels, ceramics, polymers etc. can prove that these findings are not material specific.

The number of tests done in this thesis was constrained by time and resources thus, only a minimal number of runs were performed. This was compensated by making sure all test results fit within plausible trends, thus validating all findings even with the absence of statistical evidence. However, with more time and resources, at least 6 repeats of each test should be conducted to present the researcher with a statistical spread that could be used to strengthen and improve these trends.

Additionally, since there was very eccentric damage trend behavior especially in HCF cases, which might or might not be a source of error, these series of tests deserve more testing efforts to investigate the validity of the trends. There is a reason to suspect that a lot of the wavy damage trends seen in Figures 60 through 63 might be more predictable if more data points were available to better map the trend but that is something only more time and resources will allow.
In the discussion section above, it was noted that there were seven assumptions that had to hold true in order for the small cycle damage equations to be valid. This was a product of the limited number of tests. It would be desirable to break away from most, if not all, of these assumptions but to do that, more tests would have to be run between and beyond $N_f = 5000$ and $N_f = 100,000$ cycles, and also between 100\%EL and 50\%EL small cycle amplitudes. Right now, only simplistic linear interpolation equations are available to estimate cases between these loading conditions but it should be stressed that these would be rough approximations at best. The only way that any in-between trends can be correctly predicted would be to run experimental tests.

One area that might be interesting to work on more would be the crack growth behavior displayed during the tests. Intensive crack growth studies into why, in some instances, would small cycle damage affect initiation and propagation, or sometimes, not affect them at all, could produce evidence that crack growth might not be influenced by factors as simple as crack-closure or $\Delta K$ and $K_{\text{max}}$-motivated advancement.
REFERENCES


Appendix A

Details on Specimen Standards, Calculations and Machining Methods
The specimens used in this thesis adhered to the standards prescribed by ASTM E466-82 “Standard Practice for Conducting Constant Amplitude Axial Fatigue Tests of Metallic Materials”. The acceptable dimension ranges for specimens abiding by these standards are listed as follows:

- \( d_{gage} \) = the diameter of the gage section: \( 0.200 \text{ in} \leq d_{gage} \leq 1.000 \text{ in} \)
- \( d_{grips} \) = diameter of the grip sections: \( 1.5 d_{gage} \leq d_{grips} \leq 4 d_{gage} \)
- \( R \) = radius of hourglass gage section \( \geq 8 d_{gage} \)
- \( L_{gage} \) = length of gage section \( \geq 3 d_{gage} \)

Within these restrictions, the dimensions of the specimen were calculated and followed the standards strictly. These dimensions are listed below once again for emphasis: \( d_{gage} = 0.5 \text{ in} \), \( d_{grips} = 0.75 \text{ in} \), \( R = 6 \text{ in} \), \( L_{gage} = 2.44 \text{ in} \), and \( L = \) total specimen length = 9 in.

The specimens were ordered as \( \frac{3}{4} \) inch round bars from Schupan Aluminum Sales, Kalamazoo, Michigan. The aluminum was purchased as 12 foot long bar-stocks and were cut into 9 inch rods using a band-saw. These rods were then faced-off and center-drilled on a standard horizontal lathe with a collared grip to assure almost perfect rotational alignment. Facing-off was done using a hard steel bit fed into the specimen axially, removing around 2 mm of material from the center of the rod out to its circumference. This was done to insure a smooth, right angled surface before drilling commenced. Center-drilling was done using a tapered bit so as to form a small cone-shaped hole roughly \( \frac{1}{4} \) inch in diameter, \( \frac{1}{4} \) inch deep at the center of the
cross-section. This hole would later serve as the nesting point for the engine lathe centering wheel.

All of the rods were machined to exact specimen specifications using an engine lathe with a servo-hydraulic tracer. An illustration of the lathe that was used can be seen in Figure 64 below:

![Illustration of the Lathe](image)

Figure 64. Illustration of the Lathe.

The lathe works as follows: it moves the tool-bit in or out following the shape of a template as the feed is advanced, virtually replicating the shape of the template.
onto the specimen which is turning on the lathe. This template was a flat metal plate measuring \( \frac{1}{4} \) by 3 by 6 inches and had a shallow 6 inch radius curve cut into one of its long edges. Since the rod had to be cut as deep as 0.125 inches per side (equaling 0.250 inches reduction in diameter) the template would have to be a deeper than 0.125 inches at the lowest point of the curve. The template that was used was about 0.2 inches deep. Figure 65 below shows a schematic of the template.

```
R = 6 in
```

Figure 65. Schematic of Lathe Template.

The template was secured onto the tracing holder of the lathe and aligned. The center of the template’s curve was marked with a marker-pen. The tracer was then activated and it’s “feedback arm” allowed to rest on the template. The feedback arm virtually tells the machine when to hydraulically move the tool-bit in or out as it runs along the shape of the template, hence cutting the template’s shape onto the turning specimen.
The tracer was moved so that its feedback arm would sit squarely on top of the center that was marked on the template. This was done to center the tool-bit. Next, the middle of the 9 inch length of the specimen rod was marked using a dark marker-pen. Then, the rod was clamped lightly down on the lathe’s 8-jaw chuck, specially designed to assure consistent rotational alignment. The other end of the rod, the end with the center-drilled hole, was nested snugly on the centering wheel. This specimen was then adjusted until the center of it’s length was in line with the now-centered tool-bit. Then the chuck was tightened down and the centering wheel securely fitted inside the center-drilled hole. Then the centered tool-bit was advanced inward till it almost touched the center of the specimen. The datum was then taken from the feed dial and the tracer would be moved all the way back to its starting position.

Once everything was ready, the lathe was turned on and allowed to spin up to 800 rpm. The depth-dial was adjusted to move inward 0.03 inches for the first cut. The specimen was lubricated with a lubricant spray and the automatic feed was activated, moving the tracer from right to left along the length of the turning specimen. The tool-bit then cut into the material and replicated the continuous 6 inch radius into the test section of the specimen as deep as 0.03 inches. And once it was done with the first pass, the tracer was returned to its starting position for the next pass. 0.03 inches of material was removed on the first three passes made by the tool-bit, but was reduced to 0.02 inches for the forth pass, and down further to 0.015 inches on the final pass to improve surface finish. The tracer feed speed was set at 0.04 inches/revolution for all but the last pass which was set at a slow 0.02
inches/revolution, and also, the rotational speed of the lathe was ramped up to 1200 rpm for the final cut.

Specimens had an average surface roughness of 60 microns after machining. Polishing involved using the following grades of emery paper starting from the coarsest to the finest (by grit): 280, 400, 600, 800, and 1200. All polishing was done on a horizontal lathe in the rolling direction. The papers were used in dry condition because when wetted, the aluminum particles tended to mud up with the water and interfere with the polishing process. Each stage of polishing involved around 20 passes or until that grit of emery paper didn’t visibly effect the specimen anymore. This polishing scheme managed to give the specimens an average roughness of 15 microns, which was more than enough for cellulose acetate replica application. The direction of the surface scratches after polishing was directly perpendicular to the direction of force application during testing.
Appendix B

Data for the S-N Curve for 2024-T351 Aluminum Alloy
Testing was done on a MTS 810 servo-hydraulic machine using load-controlled constant-amplitude fully reversed cycles. The stress amplitudes, $\sigma_a$, were arbitrarily chosen using the S-N curves from References [13] and [14] as guides. From these stresses, the corresponding load amplitudes $P_a$ were calculated using the simple stress equation: $\sigma = P / A$, where the stress, $\sigma$, was converted from MPa to psi using the conversion factor of 1 MPa = 1.4504E-4 psi, and $A$ was the gage area $\pi / 4 * d_{gage}^2$ using a constant $d_{gage}$ value of 0.5 inches. The fatigue life from these tests were recorded. The results from the 14 S-N tests that were done are tabulated in Table 8 below:

Table 8
Data Points for the S-N Curve

<table>
<thead>
<tr>
<th>Stress Amplitude, $\sigma_a$ (MPa)</th>
<th>Load Amplitude, $P_a$ (lb)</th>
<th>Resulting Fatigue Life, $N_f$ (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>475</td>
<td>13529</td>
<td>100</td>
</tr>
<tr>
<td>455</td>
<td>12959</td>
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<td>6266</td>
<td>420000</td>
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<td>5696</td>
<td>2290000</td>
</tr>
<tr>
<td>178</td>
<td>5070</td>
<td>6030000</td>
</tr>
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</table>
Appendix C

Cyclic Stress-Strain Data and Calculations
The cyclic stress-strain curve was generated using the MTS 810 servo-hydraulic machine using a strain-controlled constant-amplitude fully reversed cycles. It should be noted that different types of specimens were used for these tests since it involved the use of a strain gage and hence could not be in the hourglass configuration used in the main tests. The 2024-T351 aluminum was machined into uniform cross section gage area specimens as prescribed by ASTM E606-92 standards “Standard Practice for Strain-Controlled Fatigue Testing” and had the dimensions as follows:

\[
\begin{align*}
D_{gage} &= \text{gage section diameter} = 0.5 \text{ in.} \\
D_{grips} &= \text{grip section diameter} = 0.75 \text{ in.} \\
R &= \text{fillet radius} = 4 \text{ in.} \\
L_{gage} &= \text{gage section length} = 1 \text{ in.} \\
L_{total} &= \text{total length of specimen} = 8 \text{ in.}
\end{align*}
\]

The strain amplitudes were calculated using the Morrow equation:

\[
\varepsilon_a = \frac{\sigma_f'}{E} \left(2N_f\right)^b + \varepsilon_f' \left(2N_f\right)^c
\]

(Eq. 8)

For 2024-T351 aluminum alloy, the material constants were referenced as: 
\(E=73,100 \text{ MPa}, \sigma_f' = 927 \text{ MPa}, b = -0.113, \varepsilon_f' = 0.409\), and \(c = -0.713\). These values were attained from reference [1]. Five arbitrary fatigue lives, \(N_f\), values were chosen. These five values were: \(10^2\), \(10^3\), \(10^4\), \(10^5\) and \(10^6\) cycles. Using these 5 values, the
strain amplitudes for each point were calculated to be: 0.01632, 0.0078, 0.0049, 0.00326 and 0.00247.

The tests for each strain amplitude were run and the loads, $P_a$, and the corresponding strain amplitudes, $\varepsilon_a$, were recorded by the computer at half-life (i.e. 50 cycles for $N_t = 10^2$, 500 cycles for $N_t = 10^3$, and etc.). The load amplitudes $P_a$ were translated into stress amplitudes, $\sigma_a$, using the equation: $\sigma = P / A$, where $A$ was the gage area $\pi / 4 * d_{gage}^2$ with a constant $d_{gage}$ value of 0.5 inches, and the stress, $\sigma$, was converted from psi to MPa using the conversion factor of 1 MPa = 1.4504E-4 psi.

The results from the tests were tabulated in Table 9 below:

<table>
<thead>
<tr>
<th>Strain, $\varepsilon$</th>
<th>Recorded Stress, $\sigma$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01640</td>
<td>422.68</td>
</tr>
<tr>
<td>0.01100</td>
<td>417.76</td>
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<tr>
<td>0.00720</td>
<td>371.88</td>
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<tr>
<td>0.00451</td>
<td>350.43</td>
</tr>
<tr>
<td>0.00252</td>
<td>177.29</td>
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<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.00250</td>
<td>-188.20</td>
</tr>
<tr>
<td>-0.00453</td>
<td>-304.37</td>
</tr>
<tr>
<td>-0.00720</td>
<td>-377.11</td>
</tr>
<tr>
<td>-0.01100</td>
<td>-428.30</td>
</tr>
<tr>
<td>-0.01630</td>
<td>-440.94</td>
</tr>
</tbody>
</table>

These data points were used to generate the cyclic stress-strain curve seen in Figure 7 in the specimens section.
Appendix D

MTS Multi-Purpose Testware Programming Details
The MTS 810 servo-hydraulic test frame used at Western Michigan University is a uniaxial machine capable of 20,000 lb of load. The system is managed by the Teststar IIIs control system and the front end interface can be either the Basic Testware or Multi-Purpose Testware software packages.

For easier constant-amplitude fully-reversed tests, Basic Testware was sufficient. For the complex loading patterns used in this thesis, however, Multi-Purpose Testware (MPT), which allowed for flexible and customizable programming, was used instead. This appendix briefly covers the details on the MPT programming used in this thesis's test procedures, but requires that the reader be fairly knowledgeable with MTS test machines in general, and the MPT software specifically.

The Station Builder was configured to only run one axial channel, which was sufficient for all testing purposes, and had to be activated before the Station Manager, the primary program, could be run. In the “Adjust Input Signals” menu for the Station Manager, the “Signal Definition” was set to a Current Range = 20,000 lb. The “Signal Offset” section adjusts for the weight of the upper-clamps and could be automatically compensated by hitting the “Auto Offset” button. The “Signal Limits” should be set to “Disable”.

Next, the “Detectors” menu was set to only “Error Detectors”, and the Outer and Inner Actions for “Axial Force Absolute Errors” were set to “Interlock”. All other settings were set to “Disable”. For the Outer and Inner Axial Force absolute Errors, the value was set to 9000 lb for LCF tests, and 6000 lb for HCF tests, and this
allowed the machine to shut itself down when it detected that magnitude of error between the input signal from the Teststar IIs control system and the load cell readings from the MTS 810.

The “Control Panel” menu controlled the pump power and was used to tell the machine what load/displacement/strain to apply to a specimen. The pump (HPS) and machine (HSM) settings were always set to high power before a test was started. When specimens were loaded into the MTS 810 machine, it would first be gripped by the upper-clamps and, using Manual Axial Displacement control on the Control Panel, the lower piston was raised to the clamping level for the lower-clamps. To ensure that there would be zero load as the lower-clamps were activated, the Manual Axial Displacement was changed to Manual Axial Force and a value of zero was entered for the force, after which the lower-clamps could be closed. Caution had to be exercised to ensure that the “Input Signals” window read zero load upon zeroing the Manual Axial Force setting. If this was not the case, then the “Input Signal” for Axial Force could be manually zeroed to calibrate it with the Control Panel. As a note, the clamping pressure used for all tests was 1500 MPa.

The MPT program allowed for customizable programming of loading patterns and precise control of the MTS 810 machine. Regular cycles were programmed using the “Cyclic Command” Process. For all regular cycles, the Segment Type was set to “True Sine”, Adaptive Compensators = “None”, Relative End-Level = off, Cannel = “Axial”, and Control Mode = “Force”. The other settings for the process depended on LCF or HCF loading: (a) for LCF the Frequency was set to “5 Hz”, Absolute End
Level 1 = “11960 lbf” and Absolute End Level 2 = “-11960 lbf”; and (b) for HCF – Frequency = “10 Hz”, Absolute End Level 1 = “7690 lbf” and Absolute End Level 2 = “-7690 lbf”. These absolute end levels allowed for the desired tension-compression effect in each cycle. The Count could be set to a specific number of “cycles”, for instance – in the case of HCF C-B 20% cutoff, the number of regular cycles would be set to 20,000.

For block loads, the “Group” Process was used, which allowed for several Processes to be grouped together into one single repeatable Process. Within this “Group” Process, there were two “Cycle Command” Processes – one for the regular cycles within the block load, and the other for the 1000 small cycles. The regular cycles “Cycle Command” Process was set up in the same way as described above and, depending on whether the case was LCF or HCF, the Count would be “5 cycles” or “100 cycles” respectively. For the small cycles “Cycle Command” Process, the following parameters were set: Segment Type = “True Sine”, Frequency = “20 Hz”, Adaptive Compensators = “None”, Relative End-Level = off, Cannel = “Axial”, and Control Mode = “Force”. Additionally depending on small cycle amplitude, the other parameters would be entered as follows: (a) for 100%EL small cycle amplitude – Absolute End Level 1 = “4980 lbf” and Absolute End Level 2 = “-4980 lbf”; and (b) for 50%EL small cycle amplitude – Absolute End Level 1 = “2490 lbf” and Absolute End Level 2 = “-2490 lbf”. The number of blocks could be specified by double-clicking on the block load main “Group” Process, and in the in the Execute Process entry, the number of “times” the process would be repeated could be entered. For
example, to run 200 blocks, “200 times” would be entered in the Execute Process box.

One other “Group” Process was used to facilitate scheduled pauses in testing for replica taking. This involved grouping three processes together: (1) a “Segment command” Process, followed by; (2) a “Program Control” Process; and ending with (3) another “Segment Command” Process. The first “Segment Command” Process involved applying 75% of the regular cycle maximum load in tension to the specimen to allow for any surface cracks to gap open during replica taking. This was done by these following settings: Segment Shape = “Ramp”, Time = “1 second”, Adaptive Compensators = “None”, Relative End Level = off, Cannel = “Axial”, Control Mode = “Force”, and depending on whether it was a LCF of HCF case, the Absolute End Level would be “8970 lbf” or “5770 lbf” respectively. For the next process, which was the “Program Control” Process, the settings were: Action = “Program Hold” and Log Message as = “Information”. This simply paused the test at the 75% maximum tensile load level and allowed replicas to be taken. The program could then be manually resumed by hitting the “Run” button on the MPT Control Panel. Finally, the second “Segment Command” Process, which had identical settings as the first one except this time the Absolute End level was set to “0 lbf”, would return the load back to zero and end the replica taking process.

These regular cycle, block load and replica taking processes could be modularly arranged and modified to run any manner of tests outlined in the test procedure.
Control issues involved using the MPT Control Panel. Before any program could be run, the “Reset” button had to be depressed. This would lock the system in “Execute” mode and only by hitting the “Execute/Edit” button would allow editing again. A specimen name would have to be entered. Any name would suffice but one had to be entered. Hitting the “Run” button started the test and it would run until replica intervals are reached or to failure, when the detectors would trip and shut off the hydraulics. “Stop” would cut the program short and return control back to the user, while “Hold” would merely pause the program allowing for the user to resume it manually.
Appendix E

Cellulose Acetate Replica Techniques, Data-Points, Retest Data and Additional Notes on Testing

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Replicas were taken using Fullam Inc. "thin replicating tape" and lab-grade acetone. The replicating tape was ordered directly from the supplier and the acetone was ordered from Thomas Scientific Inc. The tape came in long 1 inch wide rolls which were cut into 1 by 1 inch squares. This square piece of tape was then further divided into 4 equal ½ by ½ inch patches. Acetone was applied onto the specimen gage section (i.e. the middle, narrowest section of the hourglass shape) using a medical syringe.

The best results were achieved by applying constant pressure to the syringe piston, creating a gentle but continuous flow of acetone down the specimen surface. As the surface of the gage section was wetted with acetone, cellulose acetate patches were carefully applied with tweezers. In general, replicas were quality controlled to ensure that they were free of creases, waviness, air bubbles (especially) or tears.

There was a system to placing the patches: the first patch would be placed at the front of the specimen (relative to the facing of the MTS 810 machine), and another at the back of it. This was done for two reasons: (1) to prevent overlapping the patches; and (2) to save time – placing two at once was definitely faster than placing one patch at a time and waiting for it to dry. After the front and back patches were dry (this usually took around 1 minute), they were carefully removed with tweezers and taped onto a microscope micro-slide. After that, the left and right side patches were applied, dried, removed and taped onto the micro-slide as well. All four patches worked well to cover the whole circumference of the gage section and was able to capture almost all cracks that showed up in their wide areas of coverage.
As an additional note, the right end of each patch was placed in such a way that it replicated about \( \frac{1}{4} \) of the left hand section of the next patch. This redundant technique was done to help capture any cracks that were around the edges of the patches because these areas were very prone to damage while the replicas were removed. For example, the right edge of the front patch would replicate the area that should also be captured by the left end of the right patch.

The replica results for the main tests are presented in Table 10 below:

### Table 10

**Cellulose Acetate Replica Results for Main Tests**

<table>
<thead>
<tr>
<th>Test #1</th>
<th>Test #2</th>
<th>Test #3</th>
<th>Test #4</th>
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</thead>
<tbody>
<tr>
<td>N</td>
<td>a (mm)</td>
<td>N</td>
<td>a (mm)</td>
</tr>
<tr>
<td>1850</td>
<td>7</td>
<td>2360</td>
<td>9.27</td>
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<tr>
<td>1500</td>
<td>1.4</td>
<td>2250</td>
<td>7.32</td>
</tr>
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<td>2000</td>
<td>1.63</td>
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<td></td>
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</table>

<table>
<thead>
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<th>Test #5</th>
<th>Test #6</th>
<th>Test #7</th>
<th>Test #8</th>
<th>Test #9</th>
</tr>
</thead>
<tbody>
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<td>a (mm)</td>
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<table>
<thead>
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<td>N</td>
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<td>a (mm)</td>
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<td>12.27</td>
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</tbody>
</table>
For several of the main tests, there was a need for re-tests to satisfy one or both of the following: (1) to better fit the data points to the trend; and/or (2) to verify the results when an unexpected phenomenon occurred. Only the best results were used as part of the thesis analysis. However, the other re-tests and their results are tabulated in Table 11 below for record-keeping purposes:

Table 11

Results From Re-Tests (Not Used in Analysis)

<table>
<thead>
<tr>
<th>Test #</th>
<th>Regime</th>
<th>Sequencing</th>
<th>Small Cycle Amplitude</th>
<th>%cutoff</th>
<th>Nf</th>
<th>%drop</th>
<th>Number of blocks</th>
</tr>
</thead>
<tbody>
<tr>
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<td>LCF</td>
<td>C-B</td>
<td>50%EL</td>
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<td>44.86%</td>
<td>250</td>
</tr>
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<td>3</td>
<td>LCF</td>
<td>B-C</td>
<td>100%EL</td>
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<td>820</td>
<td>69.35%</td>
<td>164</td>
</tr>
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<td>5</td>
<td>LCF</td>
<td>C-B</td>
<td>100%EL</td>
<td>40</td>
<td>2475</td>
<td>41.01%</td>
<td>95</td>
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<tr>
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<td>LCF</td>
<td>C-B</td>
<td>100%EL</td>
<td>60</td>
<td>3310</td>
<td>26.71%</td>
<td>62</td>
</tr>
<tr>
<td>13</td>
<td>LCF</td>
<td>C-B</td>
<td>100%EL</td>
<td>80</td>
<td>3620</td>
<td>21.40%</td>
<td>0</td>
</tr>
<tr>
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<td>C-B</td>
<td>100%EL</td>
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<td>63200</td>
<td>49.03%</td>
<td>232</td>
</tr>
<tr>
<td>25</td>
<td>HCF</td>
<td>C-B</td>
<td>100%EL</td>
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<td>34.68%</td>
<td>210</td>
</tr>
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<td>26</td>
<td>HCF</td>
<td>C-B</td>
<td>50%EL</td>
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<td>-33.87%</td>
<td>1060</td>
</tr>
<tr>
<td>28</td>
<td>HCF</td>
<td>B-C</td>
<td>50%EL</td>
<td>60</td>
<td>106500</td>
<td>14.11%</td>
<td>600</td>
</tr>
<tr>
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<td>B-C</td>
<td>50%EL</td>
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<td>40.89%</td>
<td>600</td>
</tr>
<tr>
<td>30</td>
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<td>C-B</td>
<td>50%EL</td>
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<td>86500</td>
<td>30.24%</td>
<td>65</td>
</tr>
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<td>31</td>
<td>HCF</td>
<td>B-C</td>
<td>100%EL</td>
<td>80</td>
<td>57300</td>
<td>53.79%</td>
<td>573</td>
</tr>
</tbody>
</table>
The variance between the acceptable results used in the thesis analysis compared to the results in some of the re-tests could have been caused by a multitude of factors: (a) inclusions in the material; (b) machining defects such as scratches or inaccurately machined gage sections; (c) imperfections caused by manufacturing processes; or etc. Evidence of inclusions were found when examining some of the unacceptable test specimen fracture surfaces under an optical microscope. The evidence of these inclusions were in the form of identical spherical indentations on the mating fracture surfaces of those specimens, which was a sign that some kind of harder material was present on the surface of the material and helped to cause failure to happen too early. Three of the faulty tests were linked to inclusions. In only a select few cases was there proof that machining faults were the cause of problems mainly because small scratches were hard to distinguish from the roughness of the gage section. Tests that did show machining mistakes that could be picked up by the cellulose acetate replicas constituted 4 out of the 12 faulty re-test cases. Imperfections from manufacturing processes were impossible to detect and was treated as the default problem if cases (a) and (b) did not hold true.
Appendix F

Analysis of a Traditional Damage Compensation Method
One of the most popular methods used in industry to compensate for the absence of small below-fatigue/endurance-limit cycles in the Palmgren-Miner Rule involves manipulating the S-N curve and artificially attaining the Nf value for the small cycles. An example of this method was exercised on the S-N curve for 2024-T351 aluminum alloy. Figure 66 below illustrates this process for small cycles that have stress amplitudes at 100%EL and 50%EL:

![S-N Curve for 2024-T351 Aluminum Alloy](image)

Figure 66. S-N Curve for 2024-T351 Aluminum Alloy With Compensation Line.

The "compensation line" applied to the S-N curve in Figure 65 above was virtually a line that was extrapolated from the linear slope section and, based on that tangential line, the fatigue life was cross-referenced from the stress amplitudes that were being studied (in this case it was 175 MPa and 88 MPa pertaining to 100%EL
and 50%EL small cycle amplitudes). From this illustration, it was determined that the predicted fatigue lives of these stress amplitudes were: \( N_{f,100\text{EL}} = 500,000 \) cycles (for 100%EL small cycles) and \( N_{f,50\text{EL}} = 2,700,000 \) cycles (for 50%EL small cycles).

The values \( N_{f,100\text{EL}} \) and \( N_{f,50\text{EL}} \) found above were used in the damage accumulation calculations for the test and this was supposed to get a sum of, or close to, one (unity) as specified in the Palmgren-Miner Rule.

This was tested using the data generated during this thesis and the results are presented in Table 12 below:

**Table 12**

Small Cycle Compensation Using Thesis Data

<table>
<thead>
<tr>
<th>Test #</th>
<th>Fatigue life, ( N_f )</th>
<th>Regular Cycles</th>
<th>Number of Blocks</th>
<th>Damage Accumulation without Compensation</th>
<th>Damage Accumulation with Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1850</td>
<td>1000</td>
<td>170</td>
<td>0.38</td>
<td>0.7198768</td>
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<tr>
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<td>2360</td>
<td>1000</td>
<td>272</td>
<td>0.48</td>
<td>1.0285996</td>
</tr>
<tr>
<td>3</td>
<td>930</td>
<td>0</td>
<td>186</td>
<td>0.19</td>
<td>0.5629651</td>
</tr>
<tr>
<td>4</td>
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<td>3849</td>
<td>200</td>
<td>1.00</td>
<td>1.3956879</td>
</tr>
<tr>
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<td>2000</td>
<td>311</td>
<td>0.73</td>
<td>1.3519795</td>
</tr>
<tr>
<td>7</td>
<td>8 3190</td>
<td>1190</td>
<td>400</td>
<td>0.66</td>
<td>1.4550308</td>
</tr>
<tr>
<td>9</td>
<td>3390</td>
<td>3000</td>
<td>78</td>
<td>0.70</td>
<td>0.8520986</td>
</tr>
<tr>
<td>10</td>
<td>4010</td>
<td>3000</td>
<td>202</td>
<td>0.82</td>
<td>1.2274086</td>
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<td></td>
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<tr>
<td>12</td>
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<td>0</td>
<td>532</td>
<td>0.55</td>
<td>1.6102012</td>
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<td></td>
</tr>
<tr>
<td>17</td>
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<td>20000</td>
<td>650</td>
<td>0.69</td>
<td>0.9262246</td>
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</tbody>
</table>
Table 12 – Continued

<table>
<thead>
<tr>
<th>Test #</th>
<th>Fatigue life, (N_f)</th>
<th>Regular Cycles</th>
<th>Number of Blocks</th>
<th>Damage Accumulation without Compensation</th>
<th>Damage Accumulation with Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
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<td>20000</td>
<td>618</td>
<td>0.66</td>
<td>0.8885663</td>
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<tr>
<td>19</td>
<td>120300</td>
<td>100300</td>
<td>200</td>
<td>0.97</td>
<td>1.0442354</td>
</tr>
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<td>20</td>
<td>0</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
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<td>91000</td>
<td>400</td>
<td>1.06</td>
<td>1.2045998</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>80200</td>
<td>60000</td>
<td>202</td>
<td>0.65</td>
<td>0.721589</td>
</tr>
<tr>
<td>26</td>
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<td>60000</td>
<td>609</td>
<td>0.98</td>
<td>1.2005556</td>
</tr>
<tr>
<td>27</td>
<td>72650</td>
<td>12650</td>
<td>600</td>
<td>0.59</td>
<td>0.8081093</td>
</tr>
<tr>
<td>28</td>
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<td>52800</td>
<td>600</td>
<td>0.91</td>
<td>1.1318996</td>
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<tr>
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<td>0.73</td>
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<tr>
<td>31</td>
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<td>0.53</td>
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<td>11500</td>
<td>800</td>
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</table>

From these results, Figures 67, 68, 69 and 70 were generated to illustrate the effectiveness of this compensation method.
Figure 67. Damage Accumulation for LCF Cycles Without Compensation.

Figure 68. Damage Accumulation for LCF Cycles With Compensation.
Figure 69.  Damage Accumulation for HCF Cycles Without Compensation.

Figure 70.  Damage Accumulation for HCF Cycles With Compensation.
From the above figures, it can be observed that without compensation there was a large amount of over-prediction by the damage accumulation calculations which used the Palmgren-Miner Rule as its equation. Although it might have been expected that the compensation method would alleviate this problem to acceptable levels, this was shown not to be the case. For the LCF tests, the values did not narrow down to the expected unity value. In fact, the calculated values ranged from 0.56 to 1.46, which were very large variations and were definitely not reliable damage estimates. Results for the HCF case where the damage accumulation calculations narrowed down to a range of 0.72 to 1.24, which was an improvement, but still showed too large a variation to be considered reliable.

This analysis points out the weaknesses of current damage estimation methods, suggesting that the fatigue problem cannot be so easily simplified.
Appendix G

Damage per Block vs. %Cutoff Calculations and Figures
The damage per block calculations for each of the main tests were done using the following equation (similar to equation 7):

\[ D_{\text{per\_block}} = \frac{D_{\text{regular}} + D_{\text{small}}}{N_b} \]  
(Eq. 9)

where, \( D_{\text{regular}} \) = regular cycle damage = \( \frac{N_{\text{regular}}}{N_{f\_\text{regular}}} \)

\( N_{\text{regular}} \) = # of regular cycles in each block (5 for LCF, 100 for HCF)

\( N_{f\_\text{regular}} \) = fatigue life for regular cycles (4870 for LCF, 124,000 for HCF)

\( D_{\text{small}} \) = small cycle damage as shown in Table 7

\( N_b \) = the number of blocks used in the test

The results for these calculations are provided in Table 13 below:

Table 13

<table>
<thead>
<tr>
<th>Test #</th>
<th>Fatigue Life, ( N_f )</th>
<th>Regular Cycles</th>
<th>Number of Blocks</th>
<th>Regular Cycle Damage, ( D_{\text{regular}} )</th>
<th>Small Cycle Damage, ( D_{\text{small}} )</th>
<th>Damage per Block, ( D_{\text{per_block}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1850</td>
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<td>170</td>
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Table 13 – Continued

<table>
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<tr>
<th>Test #</th>
<th>Fatigue Life, (N_f)</th>
<th>Regular Cycles</th>
<th>Number of Blocks</th>
<th>Regular Cycle Damage, (D_{\text{regular}})</th>
<th>Small Cycle Damage, (D_{\text{small}})</th>
<th>Damage per Block, (D_{\text{per_block}})</th>
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</table>

From these results, Figures 71 through 77 were generated for each case based on regime, sequencing, and small cycle amplitude.
Figure 71. Damage per Block vs. %Cutoff for LCF C-B 100%EL Small Cycles.

Figure 72. Damage per Block vs. %Cutoff for LCF C-B 50%EL Small Cycles.
Figure 73. Damage per Block vs. %Cutoff for LCF B-C 50%EL Small Cycles.

Figure 74. Damage per Block vs. %Cutoff for HCF C-B 100%EL Small Cycles.
Figure 75.  Damage per Block vs. %Cutoff for HCF C-B 50%EL Small Cycles.

Figure 76.  Damage per Block vs. %Cutoff for HCF B-C 100%EL Small Cycles.
Figure 77. Damage per Block vs. %Cutoff for HCF B-C 100%EL Small Cycles.