The Effects of Non-Uniform Blade Geometries on Cascade and Turbine Stage Performance

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THE EFFECTS OF NON-UNIFORM BLADE GEOMETRIES
ON CASCADE AND TURBINE STAGE PERFORMANCE

by

Hiroyuki Maeda

A Thesis
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Hiroyuki Maeda
THE EFFECTS OF NON-UNIFORM BLADE GEOMETRIES
ON CASCADE AND TURBINE STAGE PERFORMANCE

Hiroyuki Maeda, M.S.E.
Western Michigan University, 1994

It is known that in-service turbomachinery blades do not have uniform shapes. There are small differences due to blade erosion, manufacturing tolerances, or faulty installation. These differences can be in the size, shape, or relative angles of the blade. In an effort to predict losses resulting from changes in blade thickness and/or blade angles, a two-part investigation has been conducted. A two-dimensional unsteady Navier-Stokes flow analysis has been used to perform numerical experiments.

First, simulations were performed for an isolated cascade. One of the two stators was scaled and/or rotated. Results of the numerical simulations show that increasing the blade scaling causes higher total pressure losses compared to any rotation or reducing the scaling of the blades. The predicted numerical results show very good agreement with the available experimental data.

Second, turbine stage flow simulations were performed. One of the two rotors was scaled and/or rotated. Results of the turbine stage simulations show that the efficiency decreases almost 1.0% due to the scaling and rotation of the blade.

The significance of the results is that operational turbine blades must be well-maintained to minimize losses and maximize efficiency.
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<table>
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<th>English</th>
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<tbody>
<tr>
<td>$a$  Speed of sound</td>
<td>$\alpha$ Absolute inlet, exit flow angle</td>
</tr>
<tr>
<td>$e$  Specific energy</td>
<td>$\beta$ Relative inlet, exit flow angle</td>
</tr>
<tr>
<td>$e_t$ Total energy</td>
<td>$\gamma$ Ratio of specific heats</td>
</tr>
<tr>
<td>$J$  Jacobian of the coordinate transformation</td>
<td>$\lambda$ Second coefficient of viscosity</td>
</tr>
<tr>
<td>$M$  Mach number</td>
<td>$\mu$ First coefficient of viscosity</td>
</tr>
<tr>
<td>$P$  Thermodynamic pressure</td>
<td>$\rho$ Density</td>
</tr>
<tr>
<td>$Pr$ Prandtl number</td>
<td>$\tau$ Shear stress</td>
</tr>
<tr>
<td>$R$  Universal gas constant</td>
<td></td>
</tr>
<tr>
<td>$Re$ Reynolds number</td>
<td></td>
</tr>
<tr>
<td>$T$  Static temperature</td>
<td></td>
</tr>
<tr>
<td>$u$  Axial component of velocity</td>
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</tr>
<tr>
<td>$v$  Circumferential component of velocity</td>
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### Subscripts

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<thead>
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<tr>
<td>$i$</td>
<td>Inviscid</td>
</tr>
<tr>
<td>$v$</td>
<td>Viscous</td>
</tr>
<tr>
<td>$x, y$</td>
<td>First derivative with respect to $x$ or $y$</td>
</tr>
<tr>
<td>$xx, yy$</td>
<td>Second derivative with respect to $x$, $y$</td>
</tr>
<tr>
<td>$\xi, \eta$</td>
<td>First derivative with respect to $\xi$ or $\eta$</td>
</tr>
<tr>
<td>$\xi\xi, \eta\eta$</td>
<td>Second derivative with respect to $\xi$ or $\eta$</td>
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<th>Symbol</th>
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<tr>
<td>$*$</td>
<td>Dimensional quantity</td>
</tr>
<tr>
<td>$\sim$</td>
<td>Computational domain, relative reference frame quantity</td>
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CHAPTER I

INTRODUCTION

Overview

In actual turbomachinery stages, the airfoils in a given blade row do not have uniform shapes. There are small differences between adjacent blades. The discrepancies can be in the size, shape, or relative angles of the blades. These differences can occur because of tolerances in manufacturing processes, faulty blade installation, or due to blade erosion. Nonuniformities can affect the efficiency of the turbomachine. The roughness of the blade can also affect the efficiency. It has been shown that polishing rough blade surfaces can improve turbine efficiency by nearly two percent (Boyle et al., 1993). Since actual turbine blades are not uniform, it is beneficial to have the ability to predict the changes in turbine efficiency resulting from variations in blade surface roughness, blade thickness, blade angles, or leading edge radii. It is also very important to consider interaction effects between the stator and rotor blade rows.

Experimental Studies

The potential flow over a row of airfoils can cause unsteadiness in both the upstream and downstream rows if the axial gap between them is less than approximately the airfoil chord. The wake, on the other hand, is convected downstream. Thus, the need for treating the rotor and stator airfoils as a system when interaction effects are predominant is obvious. Considerable research has been performed
to determine the effect of unsteady interactions on turbine aerodynamics. Dring 
et al. (1982) performed large-scale, low speed turbine rig tests and investigated 
the effects of varying the spacing between the vane and blade rows. They detected 
considerable differences between the rotor exit conditions for the different vane­
blade spacings which were attributed to unsteady phenomena. Also, Rao et al. 
(1994) studied vane-blade interaction for two vane blade spacings and two vane 
setting angles in a short duration shock tunnel.

Computational Studies

An accurate analysis of the flows associated with rotor-stator configura­
tions is very important to optimize the performance of turbomachinery. However, 
such analyses tend to be computationally expensive and extremely complex. Nev­
ertheless, a clear understanding of the aerodynamic processes associated with tur­
bomachinery is necessary. Many calculations of cascade flow have been reported 
in the literature. These studies include two- and three-dimensional calculations 
using both the Euler and Navier-Stokes equations. Korakianitis (1993), Arnone 
et al. (1993), Zimmermann (1992), Nakanishi et al. (1989), and Lee et al. (1993) 
are examples of such efforts. Although analyses of flows through isolated rows 
can be used to study many of the fluid mechanical phenomena presented in tur­
bomachinery, they do not yield any information regarding the unsteady effects 
arising out of rotor-stator aerodynamic interaction. These interaction effects be­
come increasingly important as the distance between successive rows is decreased. 
The experimental results of Dring et al. (1982) show that the temporal pressure 
fluctuations near the leading edge of the rotor can be as much as 72\% of the 
exit dynamic pressure when the axial gap is reduced to 15\% of the chord length.
Therefore, it becomes imperative to use numerical methods that treat the rotor and stator airfoils as a system and provide information regarding the magnitude and the impact of unsteady effects. One approximation that was made in obtaining the results in Rai et al. (1989, 1990) and Dorney et al. (1992) was a rescaling of either the rotor or the stator geometry. The experimental turbine configuration of Dring et al. (1982) had 22 stator and 28 rotor airfoils. Therefore, an accurate calculation would require a minimum of 25 airfoils (11 in the stator row and 14 in the rotor row). In order to avoid the computational expense involved in such a simulation, either the rotor or the stator airfoils were rescaled and it was then assumed that the number of airfoils in both the stator and rotor rows were the same. This assumption made it possible to perform a calculation with only one rotor and one stator, thus reducing computation time by more than an order of magnitude. In other cases, a different scaling strategy is used. For example, in In Rao et al. (1994), no rescaling was used. In Madavan et al. (1993) and Rangwalla et al. (1992), the stator-to-rotor airfoil count ratio of (3:4) was used in the calculations. It is a close approximation to those in the actual turbine configurations, (22:28) (Madavan et al., 1993) and (38:52) (Rangwalla et al., 1992).

Roughness Studies

The manufacturing accuracy of turbine blades is judged by the difference between the designed and produced location of each point around the airfoils. As discussed by Boynton et al. (1993), turbine efficiency is higher for the smooth blades over a wide range of test conditions. Their results showed that the efficiency improvement for the polished coated (smooth) blades also improves turbine durability. In addition, the heat transfer is reduced with the smooth surface fin-
ish, improving the durability. Taylor (1990) measured the surface roughness of in-service turbine blades. His results showed that for typical Reynolds numbers the surfaces are not smooth. Since actual turbine blades are rough, heat transfer predictions assuming smooth surfaces can significantly underpredict external heat transfer.

Present Work

The United Technologies Research Center (UTRC) Large Scale Rotating Rig (LSRR) (Dring et al., 1982) will be used for performing numerical studies of turbine blade nonuniformities. The LSRR is 5 ft (1.52m) in diameter and it can accept a wide variety of turbine and compressor models. Numerical simulations will be performed for two-dimensional viscous flow through a single blade row (the LSRR stator) and through the LSRR turbine stage. The blade scaling factor and blade stagger angle will be varied to study the effects of nonuniform blade shapes. In the stator cascade, there are two stator blades and one of them is scaled and/or rotated around the leading edge. In the turbine stage, there are two stator blades and two rotor blades. One of the rotor blades will be modified in scaling and rotation.
CHAPTER II

MATHEMATICAL AND PHYSICAL MODELS

Euler Equations

Cartesian Coordinates

For two-dimensional inviscid compressible flows, the Euler equations can be written in conservation form as

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) &= 0 \\
\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x} (\rho u^2) + \frac{\partial}{\partial y} (\rho uv) + \frac{\partial p}{\partial x} &= 0 \\
\frac{\partial (\rho v)}{\partial t} + \frac{\partial}{\partial x} (\rho uv) + \frac{\partial}{\partial y} (\rho v^2) + \frac{\partial p}{\partial y} &= 0 \\
\frac{\partial e_t}{\partial t} + \frac{\partial}{\partial x} (e_t + p) u + \frac{\partial}{\partial y} (e_t + p) v &= 0
\end{align*}
\]

These equations are, in sequence, the continuity equation, the \(x\)- and \(y\)-momentum equations, and the energy equation. For conciseness, it is useful to write these equations in vector form. Thus, returning to the conservative equations (Eqns. 1-4), the vectors \(Q\), \(E\), and \(F\) can be defined as

\[
Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e_t \end{pmatrix}, \quad E = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e_t + p)u \end{pmatrix}, \quad F = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e_t + p)v \end{pmatrix}
\]
Therefore, the two-dimensional Euler equations in conservative form and Cartesian coordinates \((x, y)\) can be written as

\[
\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0
\]  

(6)

Physically, \(\frac{\partial Q}{\partial t}\) is the time rate of change of the conserved flow variables, while \(\frac{\partial E}{\partial x}\) and \(\frac{\partial F}{\partial y}\) represent net fluxes.

**Body Fitted Coordinates Form**

For the analysis of arbitrary geometries, the formulations can be generalized by using body-fitted coordinates. The transformations to generalized coordinates is formally given by

\[
\begin{align*}
\xi &= \xi(x, y, t) \\
\eta &= \eta(x, y, t) \\
\tau &= t
\end{align*}
\]  

(7)

where the quantities \(\xi\) and \(\eta\) represent the body-fitted coordinates in the transformed space, and where the quantity \(\tau\) represents the transformed time. In the transformed (computational) domain, the two-dimensional Euler equations, in conservation form, can be written as

\[
\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{E}}{\partial \xi} + \frac{\partial \tilde{F}}{\partial \eta} = 0
\]  

(8)
The conserved variable and flux vectors can be written as

\[ \vec{E} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e_t \end{pmatrix} \xi_t J + \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e_t + p)u \end{pmatrix} \xi_x J + \begin{pmatrix} \rho v \\ \rho v^2 + p \\ \rho uv \\ (e_t + p)v \end{pmatrix} \xi_y J \] (9)

and

\[ \vec{F} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e_t \end{pmatrix} \eta_t J + \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e_t + p)v \end{pmatrix} \eta_x J + \begin{pmatrix} \rho u \\ \rho uv \\ \rho v^2 + p \\ (e_t + p)u \end{pmatrix} \eta_y J \] (10)

where

\[ \dot{Q} = JQ \]
\[ \vec{E} = J(Q\xi_t + E\xi_x + F\xi_y) \] (11)
\[ \vec{F} = J(Q\eta_t + E\eta_x + F\eta_y) \]

The Jacobian and metrics of the transformation are given by

\[ J = x_\xi y_\eta - x_\eta y_\xi \] (12)

\[ \xi_t = \frac{x_\eta y_\pi - y_\eta x_\pi}{Jt_\tau} \quad \xi_x = \frac{y_\eta}{J} \quad \xi_y = -\frac{x_\eta}{J} \]
\[ \eta_t = -\frac{x_\xi y_\pi - y_\xi x_\pi}{Jt_\tau} \quad \eta_x = -\frac{y_\eta}{J} \quad \eta_y = \frac{x_\xi}{J} \] (13)
Equations 9 and 10 can be re-written as

\[
\begin{bmatrix}
    \rho U J \\
    \rho u U J + p \xi_x J \\
    \rho v U J + p \xi_y J \\
    (e_t + p) U J - p \xi_t J
\end{bmatrix} \quad \begin{bmatrix}
    \rho V J \\
    \rho u V J + p \eta_x J \\
    \rho v V J + p \eta_y J \\
    (e_t + p) V J - p \eta_t J
\end{bmatrix}
\]

where

\[
U = \xi_t + u \xi_x + v \xi_y \\
V = \eta_t + u \eta_x + \eta_y
\]

are the contravariant velocities (i.e., velocity components normal to the \( \xi = \) constant and \( \eta = \) constant gridlines in the transformed plane).

Navier-Stokes Equations

Cartesian Coordinates

For two-dimensional viscous compressible flows, the Navier-Stokes equation can be written as

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0 \quad (15)
\]

\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} + \frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} = 0 \quad (16)
\]

\[
\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} = 0 \quad (17)
\]

\[
\frac{\partial e_t}{\partial t} + \frac{\partial}{\partial x} (e_t + p) u + \frac{\partial}{\partial y} (e_t + p) v - \frac{\partial \tau_{hx}}{\partial x} - \frac{\partial \tau_{hy}}{\partial y} = 0 \quad (18)
\]
where

\[
\begin{align*}
\tau_{xx} &= 2\mu \frac{\partial u}{\partial x} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\
\tau_{xy} &= \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
\tau_{yx} &= \tau_{xy} \\
\tau_{yy} &= 2\mu \frac{\partial v}{\partial y} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\
\tau_{hx} &= u\tau_{xx} + v\tau_{xy} + \gamma \mu Pr^{-1} \frac{\partial e}{\partial x} \\
\tau_{hy} &= u\tau_{yx} + v\tau_{yy} + \gamma \mu Pr^{-1} \frac{\partial e}{\partial y} \\
e_t &= \rho e + \frac{\rho (u^2 + v^2)}{2} \\
e &= \frac{p}{\rho (\gamma - 1)}
\end{align*}
\]

Equations 15 to 18 are, in sequence, the continuity equation, the \(x\)– and \(y\)–momentum equations, and the energy equation. For the present application, the second coefficient of viscosity, \(\lambda\), is calculated using Stoke’s hypothesis, \(\lambda = -2/3\mu\), where \(\mu\) is the dynamic viscosity. The equations of motion are completed by the perfect gas law. Returning to the conservative equations (Eqns. 15-18), the conserved variables and flux vectors can be defined as

\[
Q = \begin{pmatrix} 
\rho \\
\rho u \\
\rho v \\
e_t 
\end{pmatrix}
\]
The two-dimensional Navier-Stokes equation in conservative form and Cartesian coordinates \((x, y)\) can be written as

\[
\frac{\partial Q}{\partial t} + \frac{\partial E_i}{\partial x} + \frac{\partial E_v}{\partial y} + \frac{\partial F_i}{\partial x} + \frac{\partial F_v}{\partial y} = 0
\]  

(22)

Physically, \(\frac{\partial Q}{\partial t}\) is the time rate of change of the conserved flow variables, while \(\frac{\partial E_i}{\partial x}, \frac{\partial E_v}{\partial y}, \frac{\partial F_i}{\partial x}\) and \(\frac{\partial F_v}{\partial y}\) represent net fluxes.

Non-Dimensional Form

The governing fluid dynamic equations are often into nondimensional forms to allow independent variation of the Mach and Reynolds numbers. The two-dimensional Navier-Stokes equations in conservative form and Cartesian coordinates \((x, y)\) (Eqn. 22) can be written in non-dimensional form as

\[
\frac{\partial Q}{\partial t} + \frac{\partial E_i}{\partial x} + \frac{\partial}{\partial x} \left( Re^{-1} E_v \right) + \frac{\partial F_i}{\partial x} + \frac{\partial}{\partial y} \left( Re^{-1} F_v \right) = 0
\]  

(23)
In Eqn. 23, the Reynolds number \((Re)\) is defined as

\[
Re = \frac{\rho_\infty V_\infty L}{\mu_\infty}
\]  

(24)

where \(\rho_\infty\) is the free stream density, \(V_\infty\) is the free stream velocity, \(L\) is the reference length, and \(\mu_\infty\) is the free stream dynamic viscosity.

Transformed Navier-Stokes Equations

Upon applying Eqn. 7, the two dimensional Navier-Stokes equations can be written as

\[
\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{E}_i}{\partial \xi} + \frac{\partial}{\partial \xi} \left( Re^{-1} \tilde{E}_v \right) + \frac{\partial \tilde{F}_i}{\partial \eta} + \frac{\partial}{\partial \eta} \left( Re^{-1} \tilde{F}_v \right) = 0
\]

(25)

where

\[
\begin{align*}
\tilde{Q} &= JQ \\
\tilde{E}_i &= J (Q \xi_t + E_i \xi_x + F_i \xi_y) \\
\tilde{E}_v &= J (E_v \xi_x + F_v \xi_y) \\
\tilde{F}_i &= J (Q \eta_t + E_i \eta_x + F_i \eta_y) \\
\tilde{F}_v &= J (E_v \eta_x + F_v \eta_y)
\end{align*}
\]

(26)

Thin-Layer Approximation

For high Reynolds number flow the viscous flux vectors can be simplified by incorporating the thin-layer approximation, which assumes that all viscous terms containing derivatives parallel to a solid surface are negligible compared to
terms containing derivatives normal to the surface (Baldwin and Lomax, 1978). Thus, for two-dimensional turbomachinery applications, the viscous terms in the direction normal to the blade surface ($\eta$ direction) are retained. In the case including the thin-layer approximation, the transformed Navier-Stokes equations (Eqn. 25) have the form

$$\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{E}_i}{\partial \xi} + \frac{\partial \tilde{F}_i}{\partial \eta} + \frac{\partial}{\partial \eta} \left( Re^{-1} \tilde{F}_v \right) = 0$$

where

$$\tilde{F}_v = -\begin{pmatrix} 0 \\ K_1 u_\eta + K_2 \eta_x \\ K_1 v_\eta + K_2 \eta_y \\ K_1 [Pr^{-1}(\gamma - 1)^{-1}(a^2)_\eta + (q^2/2)_\eta] + K_2 K_3 \end{pmatrix}$$

$$K_1 = \mu \left( \eta_x^2 + \eta_y^2 \right)$$

$$K_2 = \frac{\mu}{3} \left( \eta_x u_\eta + \eta_y v_\eta \right)$$

$$K_3 = u \eta_x + v \eta_y$$

$$q^2 = u^2 + v^2$$

In the limit as $Re \to \infty$, all viscous term become negligible and the inviscid (Euler) equations of motion (Eqn. 8) are obtained.
Solution Procedure

Approximate Factorization

In the approximate factorization (AF) technique (which is similar to the alternating-direction implicit (ADI) technique), the original multi-dimensional difference operators are replaced by a set of one-dimensional finite-difference operators, which can be represented as block-tridiagonal matrices. The purpose of the AF procedure is to simplify the solution process and permit more efficient code operation. The computational procedure in this investigation will use the AF method. Although the viscous (Navier-Stokes) equations will be solved, it is useful to illustrate the method for the inviscid (Euler) equations. Consider the AF scheme, which in semi-discretized form can be written as

\[
[I + \Delta t \left( \partial_x \tilde{A} \right)] [I + \Delta t \left( \partial_\eta \tilde{B} \right)] \Delta \tilde{Q} = -\Delta t \left( \partial_x \tilde{E} + \partial_\eta \tilde{F} \right)
\]  

(30)

where \( \tilde{A} \) and \( \tilde{B} \) are fluid dynamic Jacobian matrices. Equation 30 can be solved by the following procedure

1. Determine the residual as

\[
\Delta \tilde{Q}^* = -\Delta t \left( \partial_x \tilde{E} + \partial_\eta \tilde{F} \right)
\]  

(31)

2. Make a solution sweep in the \( \eta \) coordinate direction

\[
[I + \Delta t \left( \partial_\eta \tilde{B} \right)] \Delta \tilde{Q}^{**} = \Delta \tilde{Q}^*
\]  

(32)
3. Make a solution sweep in the $\xi$ coordinate direction

$$\left[ I + \Delta t \left( \partial_{\xi} \tilde{A} \right) \right] \Delta \tilde{Q} = \Delta \tilde{Q}^{**}$$  \hspace{1cm} (33)

**Boundary Conditions**

The unsteady, one dimensional Euler equations in characteristic form can be written as (Dorney, 1992; Giles, 1991)

$$\frac{\partial W}{\partial t} + \Lambda \frac{\partial W}{\partial x} = 0$$  \hspace{1cm} (34)

where

$$W = T^{-1} Q$$  \hspace{1cm} (35)

and

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$  \hspace{1cm} (36)

$$\lambda_1 = u$$

$$\lambda_2 = u + a$$  \hspace{1cm} (37)

$$\lambda_3 = u - a$$

where $\lambda_1$, $\lambda_2$, and $\lambda_3$ are eigenvalues. In Eqn. 34, $T^{-1}$ is a matrix of left eigenvectors. The vector $W$ contains the characteristic variables, specifically:

$$W = [w_1, w_2, w_3]^T$$  \hspace{1cm} (38)
If one assumes that for two-dimensional flow there are negligible variations in the $y$-direction, then there is a fourth characteristic given by Giles (1991)

$$w_4 = p au + P$$

Each of the characteristic variables is associated with a characteristic wave,

1. $w_1 = -\rho a^2 + P$ is a linear entropy wave
2. $w_2 = \rho au + P$ is a pressure wave
3. $w_3 = -\rho au + P$ is a pressure wave
4. $w_4 = \rho av$ is a vorticity wave

If the flow is subsonic ($u < a$), then the first, second, and fourth eigenvalues are positive, and the third eigenvalue is negative. At the inlet, three characteristic waves (the entropy wave, one pressure wave, and the vorticity wave) are specified, while a fourth characteristic wave (the second pressure wave) is calculated. At the exit the fourth characteristic wave is specified, while three characteristic wave are calculated. Since the characteristic variables are sometimes difficult to measure, they are often replaced by other quantities. For example, the entropy is often
replaced by the total pressure $P_T$, the specified pressure wave is often replaced by the total temperature $T_T$, and the vorticity wave is often replaced by the inlet flow angle $\beta$ (Dorney, 1992).

### Turbulence Modeling

The basic equations for the numerical solutions under consideration are the Navier-Stokes equations. The effects of turbulence are simulated in terms of an eddy viscosity coefficient $\mu_T$. Thus, the effective viscosity and effective thermal conductivity can be defined as

\[
\mu = \mu_L + \mu_T \\
P_r = \frac{\mu c_p}{\kappa} \\
\frac{\kappa}{c_p} = \frac{\mu_L}{P_r L} + \frac{\mu_T}{P_r T}
\]

The turbulent Prandtl number is assumed to be a constant, $Pr_T = 0.90$, and the turbulent viscosity, $\mu_T$, is calculated using the Baldwin-Lomax (1978) algebraic turbulence model. The Baldwin-Lomax (B-L) turbulence model is a two-layer model in which $\mu_T$ is given by

\[
\mu_T = \begin{cases} 
\mu_{T_{inner}} & s \leq s_{crossover} \\
\mu_{T_{outer}} & s > s_{crossover} 
\end{cases}
\]

where $s$ is the normal distance from the solid surface and $s_{crossover}$ is the smallest value of $s$ at which values from the inner and outer formulas are equal.
Grid Generation

An overlaid O-H grid arrangement is used. The H-grids are used upstream of the leading edge, downstream of the trailing edge, and in the inter-blade region. The O-type grid, which is body-fitted to the surface of the airfoil, is then overlaid upon the H-grid. The stability and accuracy of the flow solution is enhanced by increasing the amount of grid overlap.

Quasi-One-Dimensional Analysis

A quasi-one dimensional analysis was also used to help validate the predicted numerical results. These equations are obtained by integrating the two-dimensional equations (Eqns. 1-4) across a flow passage to give average value of the various properties at each axial location. (An alternative procedure is to start from the equations of motion in generalized, $x, y$ coordinates and drop all derivatives and variations in the $y$ direction). Thus, rather than define a velocity, density and other related properties at every point in space, it is possible to define average values at each axial location. In this quasi-one-dimensional analysis, it is assumed that the density is constant. Thus, this analysis is strictly valid for incompressible flow, but will give accurate results in subsonic flow. Basically, the quasi-one-dimensional analysis in this investigation is solved using Mach number and/or flow angles. Portions of the results from the two-dimensional Navier-Stokes simulations are used as boundary values for the quasi-one-dimensional analysis.
Cartesian Coordinates

For the one-dimensional inviscid incompressible flows, the Euler equations can be written as

\[
\frac{\partial u}{\partial x} = 0
\]  
(46)

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial p}{\partial x} = 0
\]  
(47)

\[
\frac{\partial e_t}{\partial t} + \frac{\partial}{\partial x} (e_t + p) u = 0
\]  
(48)

These equations are, in sequence, the continuity equation, the \(x\)-momentum equation, and the energy equation. Returning to the conservative equations (Eqns. 46-48), the vectors \(Q\), \(E\), and \(F\) can be defined as

\[
Q = \begin{pmatrix} 0 \\ u \\ e_t \end{pmatrix} \quad E = \begin{pmatrix} u \\ u^2 + p \\ (e_t + p)u \end{pmatrix}
\]  
(49)

Therefore, the one-dimensional Euler equations in conservative form and Cartesian coordinates \((x, y)\) can be written as

\[
\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = 0
\]  
(50)

Physically, \(\frac{\partial Q}{\partial t}\) is the time rate of change of the conserved flow variables, while \(\frac{\partial E}{\partial x}\) represents net flux. Radius and streamtube contraction variations are included in the numerical implementation of equations of motion, resulting in a quasi-one-dimensional analysis. For this cases studied in this investigation, the radius and streamtube thickness were constant.
CHAPTER III
NUMERICAL EXPERIMENTS

Overview

Simulations have been performed for a stator cascade and for a turbine stage. The blade scaling and blade stagger angle were varied to study the effects of nonuniform blade shapes.

Test Cases

Experimental Geometry - Single Blade Row

The Large Scale Rotating Rig (LSRR) first-stage stator is used for the isolated cascade (Dring et al., 1982). The stator inlet Mach number is $M_1 = 0.07$. The inlet flow angle is $\alpha_1 = 0$. The Reynolds number is $1.0 \times 10^5$, based on the airfoil chord. The blade surface is assumed to be adiabatic.

Experimental Geometry - Turbine Stage

The geometry of the LSRR turbine model has an equal number of stator and rotor blades. In the real turbomachine, there are 22 stators and 28 rotors. Thus, in the numerical experiments it is assumed that there are 28 stator blades and the stator blades are scaled down by a factor of 22/28. The current model has the same gap-to-chord ratio as the experimental geometry. The turbine model has two rows of airfoils; the first-stage vane (stator) row and the first-stage rotor
row. Both airfoils have aspect ratios (span/axial chord) of approximately unity. The airfoils chords are approximately 5 times engine scale. In the experimental geometry, both the first blade (rotor) and the first vane (stator) have 22 pressure taps around their perimeters at midspan which are used to establish the steady-state, or time-averaged, pressure distributions (Dring et al., 1982). The axial spacing between the stator and rotor blades is 50 percent of the axial chord.

The test cases used to study blade nonuniformities in the stator cascade and the turbine stage are summarized in Tables 1 and 2, respectively. In the isolated stator cascade cases (Table 1), there are two stator blades and one of the blades is scaled and/or rotated around the leading edge. The blade passage is defined, from the bottom to top, as $B_1$ (Blade 1) and $B_2$ (Blade 2). A positive rotation is in the counterclockwise direction. In the case of the stator/rotor stage (Table 2), there are two stator blades and two rotor blades and only one rotor blade is modified. The predicted numerical results will be compared to the experimental data of Dring et al. (1982).

### Table 1

**Test Cases - Stator Blade Row**

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>Scaling($B_1/B_2$)</th>
<th>Rotation(Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>original</td>
<td>1.000/1.000</td>
<td>0.00/0.00</td>
</tr>
<tr>
<td>mod1</td>
<td>1.000/1.010</td>
<td>0.00/0.00</td>
</tr>
<tr>
<td>mod2</td>
<td>1.000/0.990</td>
<td>0.00/0.00</td>
</tr>
<tr>
<td>mod3</td>
<td>1.000/1.015</td>
<td>0.00/0.00</td>
</tr>
<tr>
<td>mod4</td>
<td>1.000/1.000</td>
<td>0.00/0.25</td>
</tr>
<tr>
<td>mod5</td>
<td>1.000/1.000</td>
<td>0.00/-0.25</td>
</tr>
<tr>
<td>mod6</td>
<td>1.000/0.985</td>
<td>0.00/0.00</td>
</tr>
<tr>
<td>mod7</td>
<td>1.000/1.000</td>
<td>0.25/-0.25</td>
</tr>
<tr>
<td>mod8</td>
<td>1.000/1.010</td>
<td>0.00/0.25</td>
</tr>
</tbody>
</table>
### Table 2

Test Cases - Stator/Blade Stage (S: Stator, R: Rotor)

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>Scaling(B1/B2)</th>
<th>Rotation(Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage1 (original)</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.000</td>
</tr>
<tr>
<td>stage2</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.010</td>
</tr>
<tr>
<td>stage3</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/0.990</td>
</tr>
<tr>
<td>stage4</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.000</td>
</tr>
<tr>
<td>stage5</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.000</td>
</tr>
<tr>
<td>stage6</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.000</td>
</tr>
<tr>
<td>stage7</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.010</td>
</tr>
<tr>
<td>stage8</td>
<td>S: 1.000/1.000</td>
<td>R: 1.000/1.020</td>
</tr>
</tbody>
</table>

### Numerical Results

Two-dimensional numerical simulations have been performed for a two-stator cascade and a two-stator/two-rotor turbine stage geometry.

For the two-dimensional two-stator cascade simulations, each of the stator grids was constructed with $121 \times 41$ (streamwise\texttimes tangential) grid points in the O-grid and $120 \times 45$ grid points in the H-grid (see Figure 1). The average value of $y^+$, the nondimensional distance of the first grid point above the blade surface, was 0.90. The cascade simulations were run for 18,000 iterations on SUN SPARC IPC workstations. Typical cascade calculations required 0.00364 seconds per grid point per iteration. A total of 20,722 grid points were used in the simulations.
Figure 1. Computational Grid for Stator Cascade Simulations.

and it approximately took 380 hrs for each test case.

For the two-dimensional stage simulations, each of the stator grids was constructed with $121 \times 41$ grid points in the O-grid and $109 \times 45$ grid points in the H-grid (see Figure 2). Each of the rotor grids was constructed with $121 \times 41$ grid points in the O-grid and $119 \times 45$ grid points in the H-grid. For this case, the values of $y^+$ for the stator and rotor blades were 0.930 and 0.850, respectively. The turbine stage simulations were run for twelve cycles at 3000 iterations per blade passing cycle on SUN SPARC IPC workstations to obtain time-periodic solutions. Typical calculations for this case required 0.00243 seconds per grid point per iteration. A total of 40,364 grid points were used in the simulations and it approximately took 980 hrs for each test case.

The results for the stator cascade cases and the stator/rotor stage cases are presented in the form of pressure coefficient distributions, momentum thickness distributions, skin friction coefficient distributions, Mach number contours, and
Figure 2. Computational Grid for Stator/Rotor Simulations.

entropy contours. The pressure coefficient distributions give information about the blade loading and can be compared with experimental data. The momentum thickness distributions give information about the development of the boundary layer. The skin friction coefficient distributions give an indication of any flow separation and can be used to determine the drag. The Mach number contours give information about development of the boundary layer and the convection of wakes. The entropy contours give information about the boundary layer development, wakes, and losses.

Results - Cascade Cases

The steady-state inlet and exit flow variables are presented for the stator cascade cases in Table 3. $M_1$ is the stator inlet Mach number, $M_2$ is the stator exit Mach number, $\alpha_1$ is the stator inlet flow angle, $\alpha_2$ is the stator exit flow angle, $P_1$ is the stator inlet static pressure, $P_2$ is the stator exit static pressure, and $\Delta P_T$
is the normalized total pressure loss. All flow angles are measured with respect to the axial direction. The normalized total pressure loss, \( \Delta P_T \), is defined as

\[
\Delta P_T = \frac{P_{T1} - P_{T2}}{P_{T1}}
\]

(51)

where \( P_{T1} \) is the inlet absolute total pressure and \( P_{T2} \) is the exit absolute total pressure.

Table 3

Flow Variables - Stator Blade Row

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( P_2/P_1 )</th>
<th>( \Delta P_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>original</td>
<td>0.0687</td>
<td>0.1855</td>
<td>0.00</td>
<td>-67.93</td>
<td>0.9787</td>
<td>0.000807</td>
</tr>
<tr>
<td>mod1</td>
<td>0.0684</td>
<td>0.1853</td>
<td>0.00</td>
<td>-67.93</td>
<td>0.9787</td>
<td>0.000857</td>
</tr>
<tr>
<td>mod2</td>
<td>0.0687</td>
<td>0.1854</td>
<td>0.00</td>
<td>-67.85</td>
<td>0.9787</td>
<td>0.000837</td>
</tr>
<tr>
<td>mod3</td>
<td>0.0681</td>
<td>0.1849</td>
<td>0.00</td>
<td>-67.93</td>
<td>0.9787</td>
<td>0.000966</td>
</tr>
<tr>
<td>mod4</td>
<td>0.0691</td>
<td>0.1854</td>
<td>0.00</td>
<td>-67.77</td>
<td>0.9788</td>
<td>0.000817</td>
</tr>
<tr>
<td>mod5</td>
<td>0.0680</td>
<td>0.1854</td>
<td>0.00</td>
<td>-67.99</td>
<td>0.9786</td>
<td>0.000837</td>
</tr>
<tr>
<td>mod6</td>
<td>0.0688</td>
<td>0.1857</td>
<td>0.00</td>
<td>-67.78</td>
<td>0.9787</td>
<td>0.000737</td>
</tr>
<tr>
<td>mod7</td>
<td>0.0719</td>
<td>0.1849</td>
<td>0.00</td>
<td>-66.82</td>
<td>0.9790</td>
<td>0.000957</td>
</tr>
<tr>
<td>mod8</td>
<td>0.0687</td>
<td>0.1849</td>
<td>0.00</td>
<td>-67.80</td>
<td>0.9787</td>
<td>0.000957</td>
</tr>
</tbody>
</table>

Figures 3-7 illustrate the Mach contours for the original, mod3, mod4, mod7, and mod8 test cases, respectively. In these figures, the wakes from the trailing edges of the stator blades can be recognized. It is observed that larger wakes cause higher total pressure losses. Comparing Figures 3-7, it is observed that increasing the scaling causes higher total pressure losses compared to any rotation or scaling down of the blades.

Figures 8-12 present the entropy contours for the original, mod3, mod4, mod7, and mod8 test cases, respectively. In these figures, the wakes from the
Figure 3. Mach Number Contours - Orig.

Figure 4. Mach Number Contours - Mod3.
Figure 5. Mach Number Contours - Mod4.

Figure 6. Mach Number Contours - Mod7.
trailing edges of the stator blades can also be recognized.

Figures 13-17 show the pressure coefficient distributions for the original, mod3, mod4, mod7, and mod8 test cases, respectively. Also shown in Figures 13-17 is the experimental data of Dring et al. (1982). In this investigation, the pressure coefficient is defined as

\[ C_P = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \]  

(52)

where \( P \) is the local pressure, \( P_\infty \) is the free stream pressure, \( \rho_\infty \) is the free stream density, and \( V_\infty \) is the free stream velocity. In Figure 13, the pressure distributions from both blades are identical, which is expected because the blade geometries are identical. The predicted numerical results show excellent agreement with the experimental data. Note, “P.S.” denotes the pressure surface of the airfoil and “S.S.” denotes the suction surface of the airfoil.
Figure 8. Entropy Contours - Orig.

Figure 9. Entropy Contours - Mod3.
Figure 10. Entropy Contours - Mod4.

Figure 11. Entropy Contours - Mod7.
Figure 12. Entropy Contours - Mod8.

Figure 13. Pressure Coefficient Distribution - Orig.
In Figures 14-15, the pressure distribution on the unmodified stator airfoil shows very good agreement with the experimental data. The pressure distribution along the suction surface of the modified stator blades exhibits differences from the unmodified blade. The pressure is slightly increased from the leading edge to the location of peak suction. The peak suction point is located on the suction surface of the blade and denotes the location where the velocity has a maximum value. In other words, the pressure at the peak suction location has the minimum value. This shows that increasing the scaling factor or giving the blade positive rotation reduces the blade loading. In Figure 16, a combination of the positive and negative rotation act to increase the loading on both stator blades. It is also observed that scaling down the blade or giving it negative rotation increases the loading. In Figure 17, the pressure from the leading edge to the location of peak suction is increased greater than that in Figures 14-15. In addition, the loading on the unmodified blade is also changed. This is because in the mod8 case, the scale factor was increased and the blade was given positive rotation. In all the test cases, the pressure distribution along the pressure surface of the modified blade is almost identical to that of the unmodified blade.

Figures 18-22 illustrate the skin friction coefficient distributions for the original, mod3, mod4, mod7, and mod8 test cases, respectively. In this investigation, the skin friction is defined as

\[ C_f = \frac{\frac{\tau_w}{\frac{1}{2} \rho_\infty V_\infty^2}} \]  

(53)
Figure 14. Pressure Coefficient Distribution - Mod3.

Figure 15. Pressure Coefficient Distribution - Mod4.
Figure 16. Pressure Coefficient Distribution - Mod7.

Figure 17. Pressure Coefficient Distribution - Mod8.
where \( \tau_w \) is the wall shear stress. The shear stress can also be defined as

\[
\tau_w = \mu \left( \frac{du}{dy} \right)_{y=0}
\]  

where \( \mu \) is the dynamic viscosity. If the wall shear stress, \( \tau_w \), is integrated with respect to the axial direction, the drag, \( D \), can be obtained. The drag, \( D \), can be defined as

\[
D = \int_0^c \tau_w \, dx
\]

where \( c \) is the chord length of the blade.

In Figures 19 and 20, the skin friction coefficients along the suction surface of the modified stator blades shows differences from the unmodified blade. The skin friction coefficient is slightly decreased from the leading edge to the location of peak suction. This shows that increasing the scale factor or giving the blade
positive rotation reduces the drag. Note that the presence of a modified blade also effects the skin friction distribution of the unmodified blade. In Figure 21, a combination of the positive and negative rotation act to increase the drag. It is also observed that scaling down the blade or giving it negative rotation increases the drag. In Figure 22, the skin friction coefficient from the leading edge to the location of peak suction decreased greater than that in Figures 19 and 20. This is because in the mod8 case, both the scale factor was increased and the blade was given rotation. In all the test cases, the skin friction coefficient distributions along the pressure surface of the modified blade are almost identical to that of the original case (no blade modifications). In Figures 19, 20, and 22, the modified blade has a slightly lower skin friction coefficient along the suction surface of the blades, which corresponds to the slightly larger pressure coefficients observed on the suction surface in Figures 14, 15, and 17. In Figure 21, the modified blade has a slightly higher skin friction coefficient distribution along the suction surface, which corresponds to the smaller pressure coefficients on the suction surface observed in Figure 16. In all the test cases investigated, the drag due to the blade modifications would be minimal.

Figures 23-27 present the momentum thickness distributions for the original, mod3, mod4, mod7, and mod8 test cases, respectively. The momentum thickness is relatively thin on the pressure surface, but steadily increases along the suction surface. In Figures 24 and 25, the momentum thickness along the suction surface of the modified stator blades shows differences from the unmodified blade. The momentum thickness is slightly decreased along the suction surface. The momentum thickness along the pressure surface is slightly increased. This shows that increasing the scale factor or giving the blade positive rotation reduces
Figure 19. Skin Friction Coefficient Distribution - Mod3.

Figure 20. Skin Friction Coefficient Distribution - Mod4.
Figure 21. Skin Friction Coefficient Distribution - Mod7.

Figure 22. Skin Friction Coefficient Distribution - Mod8.
the momentum thickness. In Figure 26, a combination of the positive and negative rotation act to increase the momentum thickness. It is also observed that scaling down the blade or giving it negative rotation increases the momentum thickness. In Figure 27, the momentum thickness along the suction surface decreased greater than that in Figures 24 and 25. This is because in the mod8 case, both the scale factor was increased and the blade was given rotation. As before, the distributions on the unmodified blades are changed due to the presence of modified blades.

Figures 28-37 illustrate $M_1$, $M_2$, $\alpha$, $P_2/P_1$, $\Delta P_T$ versus rotation and scaling, respectively. In Figure 28, the stator inlet Mach number is higher for positive rotation and lower for the negative rotation. From the previous results, the positive rotation causes a smaller blade loading and the negative rotation causes a larger blade loading. In Figure 29, the stator inlet Mach number is smaller for larger scaling factors and larger for smaller scaling factors. From the previous
Figure 24. Momentum Thickness Distribution - Mod3.

Figure 25. Momentum Thickness Distribution - Mod4.
Figure 26. Momentum Thickness Distribution - Mod7.

Figure 27. Momentum Thickness Distribution - Mod8.
results, increasing the scaling of the blade causes a smaller blade loading and decreasing the scaling of the blade causes a larger blade loading. In Figure 30, the stator exit Mach number goes down as the blade is rotated in any direction. In Figure 31, scaling down the blade, in general, gives larger stator exit Mach numbers and increasing the scaling of the blade gives lower stator exit Mach numbers. In Figure 32, the exit flow angle increases with the positive rotation. In Figure 33, the exit flow angle decreases with increasing scaling factor. In Figure 34, the ratio of the exit-to-inlet static pressure is greater for the positive rotation than for negative rotation. In other words, the negative rotation causes a larger static pressure drop, which induces higher velocities. A positive rotation causes a smaller static pressure drop which results in lower velocities. In Figure 35, the ratio of the exit-to-inlet static pressure is the same with any scaling. In other words, both increasing and decreasing the scale factor causes the same static pressure drop. As seen in the plots of the Mach contours and entropy contours, the larger wakes induced by any rotation and scaling, in general, cause higher total pressure losses (see Figures 36 and 37).

Results - Turbine Stage Cases

The time-averaged inlet, inter-blade, and exit flow variables for the stator/rotor cases are presented in Table 4 and Table 5, respectively. $M_1$ is the inlet Mach number, $M_2$ is the stator exit Mach number, $M_{2\text{rel}}$ is the rotor inlet relative reference frame Mach number, $M_3$ is the rotor exit Mach number, $M_{3\text{rel}}$ is the rotor exit relative reference frame Mach number, $\eta$ is the turbine efficiency, $\Delta P_T$ is the normalized total pressure loss, $\alpha_1$ is the stator inlet absolute flow angle, $\alpha_2$ is the stator exit absolute flow angle, $\beta_2$ is the rotor inlet relative flow angle, $\alpha_3$
Figure 28. $M_1$ vs Rotation - Cascade Case.

Figure 29. $M_1$ vs Scaling - Cascade Case.
Figure 30. $M_2$ vs Rotation - Cascade Case.

Figure 31. $M_2$ vs Scaling - Cascade Case.
Figure 32. $\alpha_2$ vs Rotation - Cascade Case.

Figure 33. $\alpha_2$ vs Scaling - Cascade Case.
Figure 34. $P_2/P_1$ vs Rotation - Cascade Case.

Figure 35. $P_2/P_1$ vs Scaling - Cascade Case.
Figure 36. Total Pressure Loss vs Rotation - Cascade Case.

Figure 37. Total Pressure Loss vs Scaling - Cascade Case.
is the rotor exit absolute flow angle, and $\beta_3$ is the rotor exit relative flow angle. The efficiency, $\eta$, is defined as

$$\eta = \frac{T_{T_1} - T_{T_3}}{T_{T_1} \left[ 1 - \left( \frac{P_{T_3}}{P_{T_1}} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$  \hspace{1cm} (56)$$

where $T_{T_1}$ is the absolute total temperature at the stator inlet, $T_{T_3}$ is the absolute total temperature at the rotor exit, $P_{T_1}$ is the stator inlet absolute total pressure, and $P_{T_3}$ is the rotor exit absolute total pressure. The normalized total pressure loss, $\Delta P_T$, is defined as

$$\Delta P_T = \frac{P_{T_1} - P_{T_3}}{P_{T_1}}$$  \hspace{1cm} (57)$$

where $P_{T_1}$ is the stator inlet absolute total pressure and $P_{T_3}$ is the rotor exit absolute total pressure.

Table 4

Flow Variables - Stator/Rotor Stage

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_{2rel}$</th>
<th>$M_3$</th>
<th>$M_{3rel}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage1</td>
<td>0.0684</td>
<td>0.1852</td>
<td>0.1077</td>
<td>0.0869</td>
<td>0.1576</td>
<td>0.938</td>
</tr>
<tr>
<td>stage2</td>
<td>0.0680</td>
<td>0.1842</td>
<td>0.1067</td>
<td>0.0866</td>
<td>0.1573</td>
<td>0.930</td>
</tr>
<tr>
<td>stage3</td>
<td>0.0682</td>
<td>0.1846</td>
<td>0.1071</td>
<td>0.0862</td>
<td>0.1567</td>
<td>0.928</td>
</tr>
<tr>
<td>stage4</td>
<td>0.0679</td>
<td>0.1840</td>
<td>0.1065</td>
<td>0.0864</td>
<td>0.1572</td>
<td>0.929</td>
</tr>
<tr>
<td>stage5</td>
<td>0.0684</td>
<td>0.1852</td>
<td>0.1077</td>
<td>0.0866</td>
<td>0.1570</td>
<td>0.931</td>
</tr>
<tr>
<td>stage6</td>
<td>0.0687</td>
<td>0.1859</td>
<td>0.1082</td>
<td>0.0867</td>
<td>0.1571</td>
<td>0.929</td>
</tr>
<tr>
<td>stage7</td>
<td>0.0682</td>
<td>0.1845</td>
<td>0.1069</td>
<td>0.0870</td>
<td>0.1580</td>
<td>0.930</td>
</tr>
<tr>
<td>stage8</td>
<td>0.0681</td>
<td>0.1843</td>
<td>0.1068</td>
<td>0.0869</td>
<td>0.1577</td>
<td>0.932</td>
</tr>
</tbody>
</table>

Figure 38 shows the pressure history for one blade passing cycle at 10% of the pressure surface on the rotor for the turbine stage case in which the rotor blades
Table 5

Flow Variables - Stator/Rotor Stage

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_3$</th>
<th>$\Delta P_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage1</td>
<td>0.00</td>
<td>-67.74</td>
<td>-49.23</td>
<td>35.86</td>
<td>63.47</td>
<td>0.0294</td>
</tr>
<tr>
<td>stage2</td>
<td>0.00</td>
<td>-67.71</td>
<td>-49.00</td>
<td>35.93</td>
<td>63.56</td>
<td>0.0294</td>
</tr>
<tr>
<td>stage3</td>
<td>0.00</td>
<td>-67.71</td>
<td>-49.06</td>
<td>35.50</td>
<td>63.42</td>
<td>0.0294</td>
</tr>
<tr>
<td>stage4</td>
<td>0.00</td>
<td>-67.72</td>
<td>-48.96</td>
<td>35.98</td>
<td>63.61</td>
<td>0.0303</td>
</tr>
<tr>
<td>stage5</td>
<td>0.00</td>
<td>-67.73</td>
<td>-49.18</td>
<td>35.51</td>
<td>63.36</td>
<td>0.0294</td>
</tr>
<tr>
<td>stage6</td>
<td>0.00</td>
<td>-67.74</td>
<td>-49.31</td>
<td>35.44</td>
<td>63.30</td>
<td>0.0294</td>
</tr>
<tr>
<td>stage7</td>
<td>0.00</td>
<td>-67.74</td>
<td>-49.10</td>
<td>36.30</td>
<td>63.65</td>
<td>0.0293</td>
</tr>
<tr>
<td>stage8</td>
<td>0.00</td>
<td>-67.72</td>
<td>-49.03</td>
<td>36.14</td>
<td>63.61</td>
<td>0.0294</td>
</tr>
</tbody>
</table>

are identical. This figure shows that the solution has become time-periodic. The two peaks in the pressure history are related to the passing wakes of the upstream stator blades.

Figures 39-41 show instantaneous Mach contours for the stage1, stage6, and stage8 test cases, respectively. The wakes can be recognized at the trailing edges and downstream of the stator and rotor blades. The Mach contours around the rotor blades have complicated shapes compared to the stator blades. This is because the rotor blades are moving and the flow is disturbed by the wakes of the upstream stator blades. Also, the Mach contours from the location of peak suction to the trailing edge of the stator blades do not have uniform shapes. This is due to potential interactions between the stator and rotor blades.

Figures 42-44 present instantaneous entropy contours for the stage1, stage6, and stage8 test cases, respectively. The wakes can be recognized at the trailing edges and downstream of the stator and rotor blades. The wakes are chopped by the suction surface of the rotor blades and convected towards the trailing edge.
Figure 38. Pressure History - Rotor - 10% Chord - Stage1.

Figure 39. Instantaneous Mach Number Contours - Stage1.
Figure 40. Instantaneous Mach Number Contours - Stage6.

Figure 41. Instantaneous Mach Number Contours - Stage8.
Figure 42. Instantaneous Entropy Contours - Stage1.

of the rotor. The entropy contours show that losses are being generated in the surface boundary layers.

Figures 45-47 illustrate the pressure coefficient envelopes for the stage1, stage6, and stage8 test cases, respectively. Also shown in Figures 45-47 is the experimental data of Dring et al. (1982). The predicted numerical results for stator blades show very good agreement with the experimental data. The predicted numerical results for rotor blades show good agreement, although the blade loading is slightly less than the experimental data indicates. The pressure coefficient envelopes for all test cases are similar.

Figures 48-53 show the skin friction coefficient envelopes for the stage1, stage6, and stage8 test cases, respectively. In Figures 50 and 51, the skin friction coefficient envelopes for the stator and rotor blades are similar to the unmodified case (see Figures 48 and 49). This means that the effects of giving the negative blade rotation on the skin friction coefficient are quite small. In Figure 52, the
Figure 43. Instantaneous Entropy Contours - Stage6.

Figure 44. Instantaneous Entropy Contours - Stage8.
Figure 45. Pressure Coefficient Envelope - Stage1.

Figure 46. Pressure Coefficient Envelope - Stage6.
skin friction coefficient envelope for the stator blade is similar to the unmodified case (see Figure 48). In Figure 53, the skin friction envelope along the suction surface of the rotor exhibits differences from the unmodified blade. The skin friction coefficient is slightly decreased along the suction surface. This means that increased scaling decreases the skin friction coefficient. This result agrees with the trend observed in the cascade simulations. Along the pressure surface of rotor blades, the value of the skin friction coefficient is nearly zero, but the flow does not separate. A negative value of the skin friction coefficient is the critical condition of flow separation. Along the suction surface of rotor blades, the value of the skin friction coefficient varies periodically. These periodic changes are caused by the passing wakes from stator blades. Along the stator, small changes in the skin friction are evident from the location of peak suction to the trailing edge. The variations are caused by potential interactions between the rotor and stator.
blades.

Figures 54-59 show the momentum thickness envelopes for the stage1, stage6, and stage8 test cases, respectively. In Figure 56, the momentum thickness envelope for the stator is very similar to the unmodified case (see Figure 54). In Figure 57, the momentum thickness along the suction surface of the rotor exhibits differences from the unmodified blade. The momentum thickness is slightly increased along the suction surface. This shows that giving the negative rotation increases the momentum thickness. These results agree with those of the isolated cascade simulations. In Figures 58 and 59, the momentum thickness envelopes for the stator and rotor blades are very similar to the unmodified case (see Figures 54 and 55). This means that the effects of increased scaling on the momentum thickness are quite small. Along the stator surface, the momentum thickness varies periodically due to potential interactions with the rotor. Along the surface of the
Figure 49. Skin Friction Coefficient Envelope - Rotor - Stage1.

Figure 50. Skin Friction Coefficient Envelope - Stator - Stage6.
Figure 51. Skin Friction Coefficient Envelope - Rotor - Stage6.

Figure 52. Skin Friction Coefficient Envelope - Stator - Stage8.
rotor, the momentum thickness on the suction surface changes periodically, while it stays relatively constant along the pressure surface. As seen in the entropy contours, this is because the suction surface of the rotor intersects the wakes from the stator blades.

Figures 60-81 illustrate $M_1$, $M_2$, $M_{2rel}$, $M_3$, $M_{3rel}$, $\alpha_2$, $\beta_2$, $\alpha_3$, $\beta_3$, $\Delta P_T$, and $\eta$ versus rotation and scaling for the turbine stage test cases, respectively. In Figure 60, the stator inlet Mach number decreases with positive rotation. In Figure 61, the stator inlet Mach number decreases with any scaling. In Figure 62, the stator exit Mach number decreases with positive rotation. In Figure 63, the stator exit Mach number goes down with any scaling. In Figure 64, the rotor inlet relative Mach number decreases with positive rotation. In Figure 65, the rotor inlet relative Mach number goes down with any scaling. However, scaling down causes larger Mach numbers at the stator inlet, stator exit, and rotor inlet (rel-
Figure 54. Momentum Thickness Envelope - Stator - Stage1.

Figure 55. Momentum Thickness Envelope - Rotor - Stage1.
Figure 56. Momentum Thickness Envelope - Stator - Stage6.

Figure 57. Momentum Thickness Envelope - Rotor - Stage6.
Figure 58. Momentum Thickness Envelope - Stator - Stage8.

Figure 59. Momentum Thickness Envelope - Rotor - Stage8.
ative) compared to increased scaling. In Figure 66, the rotor exit absolute Mach number decreases with any rotation. In Figure 67, the rotor exit absolute Mach number decreases, in general, with any scaling. In Figure 68, the rotor exit relative Mach number decreases with any rotation. In Figure 69, the rotor exit relative Mach number decreases, in general, with any scaling. However, it is noticeable that scaling down causes lower rotor exit absolute and relative Mach numbers relative to increased scaling. Also, giving a positive rotation causes lower rotor exit absolute and relative Mach numbers compared to negative rotation. In Figure 70, the stator exit absolute flow angle increases with positive rotation. Compared to Figure 62, the stator exit Mach number is high when the stator exit absolute flow angle is low. In Figure 71, the stator exit absolute flow angle increases with any scaling. In Figure 72, the rotor inlet relative flow angle increases with positive rotation. Comparing Figures 64 and 72, the rotor inlet relative Mach number is high when the rotor inlet relative flow angle is low. In Figure 73, the rotor inlet relative flow angle increases with any scaling. In Figure 74, the rotor exit absolute flow angle increases with positive rotation. In Figure 75, the rotor exit absolute flow angle increases with increasing scale factor. In Figure 76, the rotor exit relative flow angle increases with positive rotation. In Figure 77, the rotor exit relative flow angle increases with increasing scale factor. The explanation for this phenomenon is that the thickness of the airfoil increases with increasing scale factor, causing more flow turning. In Figure 78, the total pressure loss is constant except for positive rotation. If correct, this means that negative rotation results in more efficient turbine operation compared to positive rotation. In Figure 79, the total pressure loss is constant with any scaling. In Figures 80 and 81, the efficiency decreases with any rotation or scaling. The results indicate efficiency
decreases of almost 1.0%. This result is significant because it indicates that blade wear, faulty installation, or exceeding manufacturing tolerances will reduce turbine efficiency. For jet engines, an efficiency reduction of 1.0% can cost millions of dollars due to increased fuel consumption.

As a final check on the accuracy of the predicted two-dimensional numerical results, a quasi-one-dimensional analysis was performed. The predicted two-dimensional results were used as boundary input for the quasi-one-dimensional analysis. The results of a quasi-one-dimensional analysis for the stator/rotor cases are presented in Tables 6 and 7, respectively, where $W$ is the work done by the rotor blades. The stator and the rotor passages were solved separately. The stator exit Mach number, $M_2$, was calculated based on the absolute inlet flow angle, $\alpha_1$, and the absolute exit flow angle, $\alpha_2$ (see Table 3). The rotor inlet relative Mach number, $M_{2_{rel}}$, the rotor exit relative Mach number, $M_{3_{rel}}$, and the work, $W$, were calculated based on the rotor inlet relative flow angle, $\beta_2$, and the rotor exit relative flow angle, $\beta_3$ (see Table 3). For pure rotation test cases, it was inherently assumed that both rotor blades are modified. In Table 7, the subscript “mod” refers to the fact that the quasi-one-dimensional analysis for the rotation cases was also performed independently of the two-dimensional results. The results of the quasi-one-dimensional analysis compare favorably with the results of the two-dimensional Navier-Stokes simulations. This gives confidence that the two-dimensional numerical analysis is accurately predicting the turbine stage aerodynamic and performance quantities. In addition, the quasi-one-dimensional results indicate that the work is larger for positive rotation compared to negative rotation. Also, scaling up causes the larger work compared to scaling down. From the previous results, increasing scale factor causes more flow turning. Thus, more
Figure 60. $M_1$ vs Rotation - Stage Case.

Figure 61. $M_1$ vs Scaling - Stage Case.
Figure 62. $M_2$ vs Rotation - Stage Case.

Figure 63. $M_2$ vs Scaling - Stage Case.
Figure 64. $M_{2\text{rel}}$ vs Rotation - Stage Case.

Figure 65. $M_{2\text{rel}}$ vs Scaling - Stage Case.
Figure 66. $M_3$ vs Rotation - Stage Case.

Figure 67. $M_3$ vs Scaling - Stage Case.
Figure 68. $M_{3_{rel}}$ vs Rotation - Stage Case.

Figure 69. $M_{3_{rel}}$ vs Scaling - Stage Case.
Figure 70. $\alpha_2$ vs Rotation - Stage Case.

Figure 71. $\alpha_2$ vs Scaling - Stage Case.
Figure 72. $\beta_2$ vs Rotation - Stage Case.

Figure 73. $\beta_2$ vs Scaling - Stage Case.
Figure 74. $\alpha_3$ vs Rotation - Stage Case.

Figure 75. $\alpha_3$ vs Scaling - Stage Case.
Figure 76. $\beta_3$ vs Rotation - Stage Case.

Figure 77. $\beta_3$ vs Scaling - Stage Case.
Figure 78. Total Pressure Loss vs Rotation - Stage Case.

Figure 79. Total Pressure Loss vs Scaling - Stage Case.
Figure 80. Efficiency vs Rotation - Stage Case.

Figure 81. Efficiency vs Scaling - Stage Case.
flow turning produces more work.

Table 6
Quasi-One-Dimensional Analysis

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>$M_2$</th>
<th>$M_{2\text{rel}}$</th>
<th>$M_{3\text{rel}}$</th>
<th>$W \times 10^5$</th>
<th>$\frac{ft^2}{sec^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage1</td>
<td>0.184</td>
<td>0.106</td>
<td>0.155</td>
<td>0.250</td>
<td></td>
</tr>
<tr>
<td>stage2</td>
<td>0.184</td>
<td>0.106</td>
<td>0.157</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>stage3</td>
<td>0.184</td>
<td>0.106</td>
<td>0.156</td>
<td>0.250</td>
<td></td>
</tr>
<tr>
<td>stage4</td>
<td>0.184</td>
<td>0.106</td>
<td>0.157</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>stage5</td>
<td>0.184</td>
<td>0.106</td>
<td>0.157</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>stage6</td>
<td>0.184</td>
<td>0.106</td>
<td>0.154</td>
<td>0.249</td>
<td></td>
</tr>
<tr>
<td>stage7</td>
<td>0.184</td>
<td>0.106</td>
<td>0.157</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td>stage8</td>
<td>0.184</td>
<td>0.106</td>
<td>0.157</td>
<td>0.251</td>
<td></td>
</tr>
</tbody>
</table>

Table 7
Quasi-One-Dimensional Analysis

<table>
<thead>
<tr>
<th>Name of Test Cases</th>
<th>$M_{2\text{rel}}$</th>
<th>$M_{3\text{rel}}$</th>
<th>$W \times 10^5$</th>
<th>$\frac{ft^2}{sec^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage4$_{mod}$</td>
<td>0.106</td>
<td>0.157</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td>stage5$_{mod}$</td>
<td>0.106</td>
<td>0.154</td>
<td>0.248</td>
<td></td>
</tr>
<tr>
<td>stage6$_{mod}$</td>
<td>0.106</td>
<td>0.153</td>
<td>0.247</td>
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</tr>
</tbody>
</table>
A series of two-dimensional Navier-Stokes simulations have been performed for an isolated turbine cascade and a turbine stage. For each case, the effects of rotation and/or scaling have been investigated. The predicted results showed very good agreement with the available experimental data. Certain trends became evident upon interrogation of the predicted results.

For the isolated cascade, giving the blade positive rotation resulted in: (a) generation of larger wakes, (b) decreased blade loading, (c) decreased drag, (d) decreased momentum thickness, and (e) increased total pressure losses.

For the isolated cascade, giving the blade negative rotation resulted in: (a) generation of larger wakes, (b) increased blade loading, (c) increased drag, (d) increased momentum thickness, and (e) increased total pressure losses.

For the isolated cascade, increasing the size of a blade resulted in: (a) generation of larger wakes, (b) decreased blade loading, (c) decreased drag, (d) decreased momentum thickness, and (e) increased total pressure losses.

For the isolated cascade, scaling down the blade resulted in: (a) generation of smaller wakes, (b) increased blade loading, (c) increased drag, (d) increased momentum thickness, and (e) decreased total pressure losses.

For the isolated cascade, giving the both blade negative and positive rotation resulted in: (a) generation of larger wakes, (b) increased blade loading, (c) increased drag, (d) increased momentum thickness, and (e) increased total pressure losses.
pressure losses.

For the isolated cascade, scaling and rotating the blade resulted in: (a) generation of larger wakes, (b) decreased blade loading, (c) decreased drag, (d) decreased momentum thickness, and (e) increased total pressure losses.

For the turbine stage, giving the blade positive rotation resulted in: (a) decreased loading for the rotor blades, and (b) decreased efficiency.

For the turbine stage, giving the blade negative rotation resulted in: (a) decreased loading for the rotor blades, (b) increased momentum thickness, and (c) decreased efficiency.

For the turbine stage, increasing the size of the blade resulted in: (a) decreased loading for the rotor blades, (b) decreased drag, and (c) decreased efficiency.

For the turbine stage, scaling down the blade resulted in: (a) decreased loading for the rotor blades, and (b) decreased efficiency.

The isolated cascade results show that scaling up causes higher total pressure losses compared to giving the blade any rotation and scaling down the blade. The turbine stage results indicate that any rotation and/or scaling causes a decrease in the efficiency. At the maximum, the efficiency decreased almost 1.0%. It is observed that the turbine stage case has more complicated flow features compared to the isolated cascade case due to interactions between the rotor and stator blade. It is important to mention that the results of the turbine stage simulations agree with the trends observed in the turbine cascade simulations. This means that it is possible to predict the flow field of the turbine stage using the turbine cascade simulations.

The results of this investigation are significant because they indicate that
blade wear, faulty installation, or exceeding manufacturing tolerances will reduce turbine efficiency. For jet engines, an efficiency reduction of 1.0% can cost millions of dollars due to increased fuel consumption.

Recommendations for Future Work

The two-dimensional unsteady Navier-Stokes flow analysis in this study should be applied to different turbine test cases in an effort to improve the understanding of unsteady flows in turbomachines. In particular, the following studies are proposed:

1. Investigation of the effects of surface roughness on turbine performance.

2. Investigation of modifying stator and rotor blades on turbine performance.


