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DEVELOPMENT OF A SIMULATION FOR THE PREDICTION OF PVDF SURFACE VELOCITY SENSOR RESPONSE FOR VIBRATING BEAMS AND RECTANGULAR PLATES

By

Brian C. Zellers

A Thesis
Submitted to the
Faculty of The Graduate College
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Brian C. Zellers
A simulation for the prediction of Polyvinylidene Flouride (or PVDF) sensor charge response to structural vibration displacements of beam and rectangular plate structures was developed. The simulation uses measured vibration displacements from an actual structure using point sensors (accelerometers) assembled in an array on the surface of the structure. The PVDF sensor charge output equation was written for a PVDF film of constant spatial sensitivity so that numerical analysis tools in Matlab® can be used to solve the equation. Preliminary experimental results showed that the bending strain component, due to transverse vibration displacements, is the dominant term for both the beam and the plate used in this work. As such, the simulation accounts for only the bending strain component experienced by the sensor. The simulation is verified analytically, using the PVDF sensor charge output equation, as well as experimentally. Comparisons show that the simulation provides good correlation to the analytical predictions and the experimental sensor results for the beam, however, deviations yet to be resolved are encountered for the plate. Several possible sources for these deviations were identified to be explored in the future. The simulation was also shown to serve as a useful tool in prediction of sensor fabrication errors (in shaping and placement).
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CHAPTER I

BACKGROUND

Introduction

The focus of this research is in part to that of a larger study aimed toward achieving quiet structures using the active noise control technique. Active noise control is the use of secondary acoustic actuators (i.e. loudspeakers) to cancel unwanted noise that is radiated from a vibrating structure. The most convenient measure of the noise that is radiated is to measure the surface volume displacement (i.e., total volume of air displaced by the vibrating surface) of the vibrating structure. This measured signal of total volume displacement is sent to an active controller, which then interprets this signal so as to excite the secondary acoustic actuators. The excitation signal sent to the secondary acoustic actuators drives them in a manner that the noise they radiate cancels that radiated by the structure through destructive interference. In an effort to cancel noise radiated by a structure over an entire frequency range of interest and over a wide area of interest, multiple sensors and actuators are necessary to describe and control sound radiation. As such, the larger the frequency range and area of interest the more complex the active controller becomes due to the growing number of inputs and outputs needed to accommodate this larger frequency range and area. One advantage of this type of active control system is that it does function as a single active controller responsible for the interface of all sensors and actuators. Despite this interfacing convenience, the need for a
multiple-input multiple-output controller (due to multiple sensors and multiple actuators) makes the system complex and costly.

It is possible to simplify the active control system, thereby reducing its complexity and cost, by sensing only local vibrating portions of a total vibrating surface and canceling the contribution of each portion. To achieve wide-area cancellation of sound, each cancellation loudspeaker will have to be located as close as possible to its corresponding vibrating portion of the structural surface. This strategy lends itself to allowing a single-input single-output controller replacing the more complex, multiple-input multiple-output controller described previously. However, in order to cancel radiated noise within an entire frequency range of interest as the more complex system can, multiple single-input single-output controllers for all localized areas would be necessary. In addition, the structural areas assigned to devices do not have to be equally divided. Depending on the structural radiation characteristics, it is possible to assign one device to a larger area. This further simplifies the overall active control system by using local controllers that address only the most significant structural areas of radiated noise.

Typically, point sensors are used to perform the structural sensing. However, these provide for a complex multiple-input multiple-output controller. Therefore, a single distributed sensor could be used to measure the volume displacement exactly thereby providing for a comparatively simpler single-input single-output controller. A prediction of this distributed sensor response to surface volume displacement would aid in the development of such distributed sensors. Such a prediction could lend itself to optimizing the shape, position, and number of localized sensors for the cancellation of
radiated noise within an entire frequency range of interest. In addition, such a prediction could show sensor response to fabrication and placement errors. Thus, the focus of this research is to develop a simulation for the prediction of sensor response based on shape and position using numerical techniques. A successful simulation would be validated both theoretically and experimentally for beam and rectangular plate structures.

Review of Relevant Literature

Active control of radiated noise is a technique used to eliminate unwanted radiated noise. This can be accomplished by either altering the dynamics of the radiating structure or acoustically canceling the radiated noise.

Active Control of Radiated Sound

Active noise control (ANC) is used to cancel an unwanted noise signal emitted from a source using a secondary acoustic actuator. This secondary noise is equal in magnitude and 180° out of phase with respect to the unwanted noise. The sum of these sounds results in destructive interference that ideally reduces the unwanted noise (Charette et. al., 1998). This noise control method is preferable over that utilizing secondary force actuators (which alter the dynamics of the structure) when the excitation force is very large in comparison to the secondary actuation force required (e.g. a transformer).

Active structural acoustic control (ASAC) is another noise control solution used to cancel unwanted noise radiated from a structure by altering the modal characteristics of the structure. These modal characteristics, and hence radiation efficiencies, are altered by using secondary force actuators on the structure with the goal of minimizing the sound
that is radiated (Charette et. al., 1998). Such secondary force actuators can be electromechanical shakers or piezoceramic patches. Structural sources are preferred over acoustic sources in that loudspeakers provide effective attenuation only over a limited frequency range. As a result, fewer structural secondary sources are needed in comparison to the number of acoustic secondary sources needed for noise control over a wide area and a wide frequency range (Charette et. al., 1998). Furthermore, by changing the modal characteristics, one has changed the noise source to a quieter one whereas in ANC, the source remains unchanged. Currently, the use of structural sensors in ANC settings is lagging far behind that of ASAC settings providing opportunity for research advancement.

Sound Radiation

As there exists a time delay between the propagation of sound from the radiation source to measuring microphones, structural sensing is the preferred method to radiation sensing (Preumont et. al., 1999). Radiation efficiency of vibrating structures is largest at low frequencies where the net volume displacement is the largest (Rex, 1991). Work by Rex has shown that at these low frequencies the net volume displacement of a vibrating surface is the most influential item in the determination of the sound power radiated by the surface. Rex showed that the total sound power radiated by a surface that is surrounded by a rigid baffle vibrating at a single frequency is given by

\[
W = \frac{\omega^4 \rho_0}{8\pi^2 c} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^S w(x, y) e^{jk\sin \theta (x \cos \theta + y \sin \theta)} dx dy \left| \sin \theta \right|^2 d\theta d\phi,
\]

(1)
where \( W \) is the total sound power, \( \rho_0 \) is the density of medium through which the sound is propagating, \( c \) is the speed of sound, \( \omega \) is the frequency of vibration, and \( w(x,y) \) is the transverse vibration displacement of the surface \( S \) at \( x \) and \( y \). The variables \( \theta \) and \( \phi \) indicate the direction of the position vector \( R \) from the vibrating surface to a far field point \( p \) (see Figure 1).

\[ W = \frac{\omega^2 \rho_0}{4\pi c} \int_0^s w(x,y)dx \, dy \tag{2} \]

The integral \( \int_0^s w(x,y)dx \, dy \) represents the net volume of air displaced by the surface \( S \) (called volume displacement). Equation (2) shows that the total sound power radiated is directly related to the square of the magnitude of the total volume displacement. As such, low order modes of vibration (less complex in vibration shape)
produce larger values of net volume displacement, and hence dictate the need for structural sensing at low frequencies.

Piezoelectric Materials

Traditionally, point sensors are used for volume displacement measurements; however, multiple point sensors are necessary in order to have sufficient resolution to describe the total volume displacement within a frequency range of interest. This necessitates the use of multiple input and output controllers which compounds the cost and complexity of the active controller system, as discussed. In turn, Polyvinylidene Fluoride (or PVDF) sensors can be used for the measurement of volume displacement of the vibrating structure. Using PVDF only one input and output controller is necessary thereby significantly simplifying the active controller system.

PVDF is a piezoelectric film that, by the nature of its piezoelectricity, will emit a charge when it experiences an external strain from an applied load (AMP 1994). The converse is also true where an applied voltage will result in internal strains and hence deflection in the PVDF film (AMP, 1994). The charge emitted is in direct relation to the amount (thickness and area) of PVDF used and the magnitude of the applied strains. Also, the charge emitted is in direct relation to the value of the anistropic electrical material coefficients \(e_{mn}\) of the PVDF used. The stress/charge coefficient \(e_{31}\) is the piezoelectric coefficient acting in the 3 plane in the 1 direction, and the stress/charge coefficient \(e_{32}\) is the piezoelectric coefficient acting in the 3 plane in the 2 direction. This is tensor notation representative of the material coordinates of Figure 2.
In addition, the sign of a voltage response (or the direction of a mechanical straining response) is related to the poling of the material where one side of the PVDF sheet is a negative pole and the opposite a positive pole. Once bonded to the structural surface, such a sensor would be subjected to strains equal to the surface strain of the structure. A sheet of uniform thickness is shaped by means of chemical etching or cutting of the film itself. As a vibrating structure deflects, the PVDF sensor emits a charge related to the surface strain of the structure.

One Dimensional Structures

Guigou et. al. (1994) successfully developed shaped PVDF sensors for measuring the total volume displacement of a simply supported beam. The charge equation of a shaped PVDF sensor for a one dimensional structure used was derived by Lee and Moon (1990), from a more general form of the shaped PVDF sensor charge output equation, as

$$q = -(h_b + h_s) e_{31} \int_0^l F(x) \frac{d^2 w(x)}{dx^2} dx$$  \hspace{1cm} (3)
where $h_b$ and $h_s$ are the beam and sensor thickness, $F(x)$ is the function describing the sensor shape, $e_{31}$ is the stress per charge output coefficient of the PVDF film, $L$ is the length of the beam, and $w(x)$ is the transverse vibration displacement of the beam. Figure 3 shows where the sensor equation would appear on a sensor adhered to a beam.

![Figure 3. Shaped PVDF Sensor on a Beam.](image)

It is important to note that the sensor is required to span the entire length of the beam so as to include the boundary conditions.

Zahui et al. (2001) successfully used shaped PVDF as a local volume displacement sensor for active control of noise radiated from a fixed-fixed vibrating beam. A PVDF sensor was cut from a roll of PVDF, cut to shape, and bonded to the beam surface such that it generated a charge output equal to the volume displacement of each half of the vibrating beam. The sensor measured the volume displacement of only an area of interest. For this work it was necessary that the shaped PVDF sensor span the entire length of the vibrating beam structure in an effort to include the boundary conditions. An active noise controller then processed the localized shaped PVDF signals for each half of the beam to determine a signal that was finally sent to two secondary acoustic sources to cancel the noise radiated from the beam. This work is an example of two single-input single-output controllers, one for each half of the beam.

Rozema et al. (2001) continued the sensor development aspect this work by developing an integrated sensor consisting of a shaped PVDF sensor in addition to two
point sensors placed at each end of the PVDF strip. Here, the shaped PVDF sensors spanned only the area of interest that radiated sound (i.e. halves, thirds, etc. were chosen) and did not traverse the entire length of the beam. Therefore, a localized sensor that spanned half the length of a fixed-fixed beam, using point sensors at each of its ends, would measure only the volume displacement of that beam half. The work by Rozema et. al. showed good correlation between the localized volume displacements measured using these integrated sensors and that measured using multiple accelerometers.

Preumont et. al. (1999) provided an alternative approach to the development of strain sensors that can measure the volume displacement of a vibrating beam through the use of PVDF sensors based on an adaptive linear combiner. Figure 4 shows rectangular PVDF patches on a beam where each patch is interfaced to the adaptive linear combiner.

Figure 4. Beam Volume Displacement Patches Based on Adaptive Linear Combiner. Preumont et. al. show that the volume displacement of a beam as measured by finite patches of PVDF is

$$V = \sum_{i=1}^{k} \alpha_i Q_i$$  \hspace{1cm} (4)
\[ \alpha_i = \sum_{j=1}^{m} \alpha_j V_j, \]  

where \( V \) is the beam total volume displacement, \( Q_i \) is the charge output of PVDF patch \( i \), and \( \alpha_i \) is the modal amplitude coefficient of patch \( i \) at mode \( j \). The adaptive linear combiner computes the modal amplitude coefficient based on the charge output of a PVDF patch and the actual beam total volume displacement, where the actual beam total volume displacement is measured by using a scanning laser vibrometer. Figure 5 shows a control scheme that computes the modal coefficients through the use of an error feedback control loop of the beam volume displacement.

![Diagram of Adaptive Linear Combiner Controller](image)

Figure 5. Adaptive Linear Combiner Controller.

The charge output of each patch is interfaced with the linear combiner where it receives a weighting \( \alpha_i \). Then the computed volume displacement (PVDF patch calculation) is compared against the actual volume displacement (determined using a laser vibrometer) to minimize the mean-square error. This feedback is then used to reevaluate the weighting values \( \alpha_i \).
However, the optimal size of a patch within the patch array is not addressed, although the influence of the size on the output is recognized. The adaptive linear combiner controller compensates the modal coefficients for patch size. Therefore, this work offers no design control or optimization over the patch size within the patch array.

Two Dimensional Structures

Johnson and Elliot (1995) developed a quadratically weighted sensor that covered the entire surface of a plate to measure the total volume displacement of the plate. Figure 6 shows the sensor used as it covers the plate.

![Quadratic Sensitivity to Measure Plate Total Volume Displacement.](image)

Figure 6. Quadratic Sensitivity to Measure Plate Total Volume Displacement.

The generalized form of the PVDF charge output equation used was developed by Lee (1990) as

\[
q = \int_{0}^{L_x} \int_{0}^{L_y} \left[ e_{31} \left( \frac{\partial u}{\partial x} - h \frac{\partial^2 w}{\partial x^2} \right) + e_{32} \left( \frac{\partial v}{\partial y} - h \frac{\partial^2 w}{\partial y^2} \right) \right. \\
+ e_{33} E_3 + e_{36} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - 2h \frac{\partial^2 w}{\partial x \partial y} \right) \left] \right] F(x, y) \, dx \, dy ,
\]

(6)
where \( u, v, \) and \( w \) are the vibration displacements of the plate in \( x, y, \) and \( z \) directions at \( (x,y) \), respectively, \( e_{mn} \) are the stress per charge output coefficients of the PVDF film, and 
\[
 h = \left( h_z^{pl} + h_z^s \right) / 2 .
\]
The symbols \( h_z^{pl} \) and \( h_z^s \) represent the plate thickness and sensor thickness, respectively. These constitute the bending moment arm of the plate and sensor. In addition, \( F(x,y) \) is the spatial sensitivity of the film, \( \varepsilon_{33} \) is the permittivity of the film, and \( E_3 \) is the electric field intensity across the \( z \) direction of the film. However, Johnson and Elliot made the assumption that the contribution of the in-plane strain (first order partial terms) were negligible in comparison to the bending strains and could be ignored. They also assumed no skew angle and thereby setting the \( e_{36} \) coefficient equal to zero. In addition, as they were measuring charge output of the sensor, they could eliminate the contribution of the \( E_3 \) component. The resulting equation was

\[
 q = \iint_{0}^{L_x} h F(x,y) \left[ e_{31} \frac{\partial^2 w(x,y)}{\partial x^2} + e_{32} \frac{\partial^2 w(x,y)}{\partial y^2} \right] dx \, dy .
\]  

Figure 7 shows the volume velocity results of Johnson and Elliot's quadratically weighted sensor. The dotted line is the volume velocity result from the PVDF and the
solid line is the volume velocity measured using a laser vibrometer at 49 discrete points. The plate volume velocity obtained by the PVDF is in good agreement with that found using the laser vibrometer.

Figure 7. Results of Volume Velocity from PVDF and Laser Vibrometer.

As the quadratically weighted sensor widths decrease, thereby increasing the number of sensor strips that can fit onto the plate, the accuracy of the PVDF strips in predicting higher order modes increases. However, this work offers no optimization relationship between sensor width and the largest vibration frequency of interest.

Gardonio et. al. (2001) added to previous work in the development of PVDF sensors for plates by considering the contribution of inplane strains, in addition to bending strains, to the total charge output. The shaped PVDF sensors that Gardonio et. al used are quadratically weighted and cover the entire surface of the plate as seen in Figure 5. The following equations arrive by separating the bending and in-plane terms of Eq. (6) developed by Lee (1990) as
\[ q_b(t) = -\frac{h_z^{pl} + h_z^{s}}{2} \int_0^L \int_0^L \left[ e_{31} \frac{\partial^2 w}{\partial x^2} + e_{32} \frac{\partial^2 w}{\partial y^2} + 2e_{36} \frac{\partial^2 w}{\partial x \partial y} \right] F(x, y) \, dx \, dy, \quad (8) \]
\[ q_i(t) = \int_0^L \int_0^L \left[ e_{31} \frac{\partial u}{\partial x} + e_{32} \frac{\partial v}{\partial y} + e_{36} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] F(x, y) \, dx \, dy, \quad (9) \]

where \( q = q_b(t) + q_i(t) \) and subscripts \( b \) and \( i \) refer to bending and in-plane components, respectively.

Figure 8 shows the results of the PVDF output when considering both in-plane and bending strains in comparison to a theoretical model. The bold line represents the theoretical model and the faint line represents the volume displacement results measured using the PVDF sensor. There exists good correlation in the results between the theoretical model and the experimental PVDF sensor output.

Figure 8. Comparison of Theoretical Model and Experimental PVDF Sensor Output.

It is important to note that the frequency at which in-plain contributions become significant is dependent on the material and geometry of the structure. The experimental plate used by Gardonio et. al. had a thickness of 1 mm. As plate thickness increases, the first in-plane natural frequency of a plate increases. However, this work reveals the possibility of PVDF sensitivity to in-plane strains and thereby shows a need to identify where in-plane resonances occur in relation to the frequency range of interest.
Charette et. al. (1998) incorporated PVDF sensors in part for use in active control of sound radiated from plates using Eq. (8). By assuming no skew angle in the sensor, and constant spatial sensitivity of the PVDF film, Eq. (8) can be broken into separate charge equations for the $x$ and the $y$ directions as

\begin{align}
q_x &= -\frac{h_x^2}{2} \int_0^{L_y} \int_{y_a}^{y_b} \left[ e_{31} \frac{\partial^2 w(x,y)}{\partial x^2} + e_{32} \frac{\partial^2 w(x,y)}{\partial y^2} \right] dy \, dx,
q_y &= -\frac{h_y^2}{2} \int_0^{L_x} \int_{x_a}^{x_b} \left[ e_{31} \frac{\partial^2 w(x,y)}{\partial x^2} + e_{32} \frac{\partial^2 w(x,y)}{\partial y^2} \right] dx \, dy,
\end{align}

where $q_b = q_x + q_y$ (charge output due to bending only). The maximum sensor width for the $x$ direction is $2\mu_x$ and $2\mu_y$ for the $y$ direction. This simplification allows for the construction of two separate sensors in both axial directions of the plate. Figure 9 shows these arbitrarily shaped PVDF sensors on a rectangular plate.

Figure 9. Arbitrarily Shaped PVDF Sensors on a Plate Structure in Two Directions.

The sensors developed by Charette et. al. (1998) spanned the entire length of the plate structure in both directions incorporating the boundaries. The two sensor shapes were developed by curve fitting experimental modal displacements of the plate. These authors were able to show that these two shaped PVDF sensors could successfully measure the
total volume displacement of the plate. Figure 10 shows the volume displacement results that were obtained using these shaped PVDF sensors in comparison to those developed using a laser vibrometer. In addition, this figure also shows the sensor shapes that were used to measure the total volume displacement of the plate.

Figure 10. Results of Shaped PVDF Sensors.

This work did not address the placement of these sensor strips in either direction through the use of analytical equations. Although the limits of integration used to determine the charge output for each strip are functions of the plate cartesian coordinates \((x_c, 0)\) and \((0, y_c)\), they offer no method to determine these coordinates in relation to the total volume displacement of the plate. In addition, there exists the opportunity to explore the optimization of sensor shape with respect to the location of the sensor strip positions, \((x_c', 0)\) and \((0, y_c')\).
As sensor shape and its position dictate the sensor charge output, the sensor must be shaped and placed precisely in order to generate the proper output. Errors in shape and placement allow for undesired modal contributions. As sensor placement is done by hand there exist opportunities for placement errors. Clark et. al. (1996) developed an exact analytical relation for predicting sensor charge output of modal sensors based on placement error for pinned beams only. Furthermore, Clark et. al. concluded that exact modal sensor shapes exist only for pinned plates, whereas exact modal sensors can be realized for beams from the entire class of boundary conditions, although these authors presented work for pinned beams only. As such, some deviation is expected from non-pinned plate modal sensors. In addition, their work on modal sensor placement error did not include the contributions of error due to shaping the sensor.

In conclusion, this points to further research opportunities for structural volume displacement sensors made of PVDF. Current structural sensor development requires sensors to span the entire length of a beam or plate in an effort to incorporate the boundary conditions. Those that do not incorporate the boundary conditions require the aid of point sensors, controllers, etc. to supplement the PVDF sensor signal. Furthermore, optimal sensor placement in relation to the sensor shape is yet to be determined. Precise shaping and placement of PVDF sensors is crucial as the sensor output is a function of shape and placement. However, exact error prediction exits only for shaped modal sensors on beams and pinned plates. Error prediction with regard to other boundary conditions for plates has yet to be realized. In addition to placement errors, no investigation of sensor shape error has been conducted.
Thesis Organization

This thesis is organized in the following manner. A discussion of the implementation of numerical techniques used to solve the PVDF charge output equation for the simulation is given in Chapter II. The derivation of the analytical relations for the PVDF charge output equation, for both beams and rectangular plates, is presented in Chapter III. These analytical relations are used to verify the numerical techniques used in the simulation. A discussion of the experimental setup used to gather structural data for the simulation is given in Chapter IV. It shows the experimental setup for both the beam and rectangular plate structures. Preliminary results that compare the analytical results to the simulated results for both the beam and plate are presented in Chapter V. Details of the experimental verification of the simulation are presented in Chapter VI by first discussing sensor fabrication and the experimental sensor measurement procedure. Next, the experimental sensor results for the beam and rectangular plate are compared with that of the simulation, and finally a discussion of errors is shared. Practical extensions of the simulation to predict fabrication and placement errors in experimental sensors are given in Chapter VII. Finally, a conclusion of this work and recommendations of future efforts to further the study are discussed in Chapter VIII.
CHAPTER II
NUMERICAL IMPLEMENTATION

Governing Equations

In order to investigate the sensor shaping and placement errors for any arbitrarily-shaped sensor, the sensor charge output $q$ is determined numerically. This numerical prediction is arrived at by using measured surface vibrations of any planar structure that the sensor is to be bonded to within a frequency range of 0 Hz to 1600 Hz. Although a beam is considered a one dimensional structure, it does possess a finite width that does allow for variation/change in the $y$ direction requiring one to carry out the double-integration over its surface. However, in reality the contribution to the charge output from this width direction is negligible in comparison to the length direction. This offers the flexibility to solve for sensor charge output on either a beam (one dimensional) or rectangular plate (two dimensional) structure using the same numerical equations in the simulation. In order to numerically predict the charge output of the PVDF sensor, the limits of integration of Eq. (6) are changed to define a maximum rectangular boundary defined as $a_1 \leq x \leq a_2$ and $c_1 \leq y \leq c_2$ that encloses the arbitrary sensor shape. Equation (12) reflects the use of these new sensor integration boundaries seen as

$$
q = \int_{a_1}^{a_2} \int_{c_1}^{c_2} \left[ e_{31} \left( \frac{\partial u}{\partial x} - \frac{h_z^{pl} + h_z^s}{2} \frac{\partial^2 w}{\partial x^2} \right) + e_{32} \left( \frac{\partial v}{\partial y} - \frac{h_z^{pl} + h_z^s}{2} \frac{\partial^2 w}{\partial y^2} \right) \\
+ e_{36} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{h_z^{pl} + h_z^s}{2} \frac{\partial^2 w}{\partial x \partial y} \right) \right] F(x,y) \, dx \, dy.
$$

(12)
Also, as the solution is predicting charge output, $E_3$ is set to zero. Figure 11 shows how this arbitrary sensor would appear on a two dimensional vibrating surface enclosed by the maximum rectangular limits of integration. It should be noted that the structure in the figure could be seen as either a beam or a rectangular plate. If viewed as a beam, $c_1$ and $c_2$ would be the sides of the beam for an arbitrary sensor that spanned the entire width of the beam.

![Figure 11. Arbitrarily Shaped Sensor for Numerical Analysis.](image)

Equations (10) and (11) were based on the assumption of no skew angle in the sensor and by decomposing the integral into the separate axial directions where the limits of integration are governed by the sensor shape, as $F(x,y)$ is a constant value of one. The same assumption regarding the skew angle, method of integral decomposition, and integration limit substitution can be applied to Eq. (12) to yield

$$q_x = -e_{31} \int_{y_1 = F_1(x)}^{y_2 = F_2(x)} \int_{a_1}^{a_2} \left( \frac{h_{zl}^2 + h_z^2}{2} \frac{\partial^2 w}{\partial x^2} - \frac{\partial u}{\partial x} \right) \, dx \, dy, \tag{13}$$

$$q_y = -e_{32} \int_{x_1 = F_1(x)}^{x_2 = F_2(x)} \int_{a_1}^{a_2} \left( \frac{h_{zl}^2 + h_z^2}{2} \frac{\partial^2 w}{\partial y^2} - \frac{\partial v}{\partial y} \right) \, dy \, dx. \tag{14}$$
The limits of integration are now replaced by the equations that describe the upper and lower bounds of the sensor shape. The shape function is a function evaluated over the desired length the sensor will span \((a_1 \text{ to } a_2 \text{ or } c_1 \text{ to } c_2)\) in the \(x\) or \(y\) direction. It is a function that is evaluated for a vector of \(x\) or \(y\) points where these vectors are discretized to the same resolution as the structure. This allows every shape function point to lie on a nodal line of the structure (points cannot lie between nodes). It should be noted that \(q_{x'}\) and \(q_{y'}\) of Eqs. (13) and (14) refer to the two axial directions of the same sensor (indicated by the prime superscript), as opposed to two separate axial sensors as in Figure 9. For the numerical solution, it is possible that the equations of the sensor shape are more conveniently described as functions of \(y\) as \(x_1 = G_1(y)\) and \(x_2 = G_2(y)\). The implementation of these functions of \(y\) as limits of integration are seen as

\[
q_{x'} = -e_{31} \int_{x_1 = G_1(y)}^{x_2 = G_2(y)} \left[ \int_{x_1 = G_1(y)}^{x_2 = G_2(y)} \left( h^2_{x} + h^2_{z} \frac{\partial^2 w}{\partial x^2} - \frac{\partial u}{\partial x} \right) \right] \, dy, \tag{15}
\]

\[
q_{y'} = -e_{32} \int_{x_1 = G_1(y)}^{x_2 = G_2(y)} \left[ \int_{x_1 = G_1(y)}^{x_2 = G_2(y)} \left( h^2_{x} + h^2_{z} \frac{\partial^2 w}{\partial y^2} - \frac{\partial v}{\partial y} \right) \right] \, dx. \tag{16}
\]

Figures 12 and 13 show where the upper and lower equations (convenient functions of \(x\)) and the left and right hand equations (convenient functions of \(y\)) appear on the sensor relating to the limits of integration in Eqs. (13), (14), (15), and (16). The coordinate systems presented in Figures 11 and 12 are for presentation only (indicated by the prime superscript). The functions \(F_1(x)\), \(F_2(x)\), \(G_1(y)\), and \(G_2(y)\) are implemented relative to the coordinate system presented in Figure 11.
The method of sensor development for this work uses sensor equations that are functions of $x$. Therefore, the numerical solution uses Eq. (13) and Eq. (14) for the simulation.

Implementation for Simulation of Sensor Charge Output

The software package Matlab® provides a variety of numerical analysis functions that can be used to evaluate Eqs. (13) and (14). By assuming harmonic motion, the vibration displacements of the surface under study can be determined from measured surface acceleration using the relation

$$w(x, y, t) = \overline{W}(x, y)e^{i\omega t}, \quad (17)$$

where $w$ is the vibration displacement at position $(x, y)$ at time $t$, and the variable $\overline{W}$ is the magnitude of the vibration displacement. Knowing that the second derivative of displacement with respect to time $(\ddot{w})$ is acceleration, the vibration displacement is then found using
These experimental vibration displacements in $x$, $y$, and $z$ directions are collected using a single tri-axial accelerometer in a grid pattern covering the structural surface. For the beam, this grid pattern would be 1 row (in the $y$ direction) of $N$ equally spaced points (in the $x$ direction). For the rectangular plate, this grid pattern, or array, would be $N$ rows (in the $y$ direction) of $M$ equally spaced points (in the $x$ direction) on the surface. These arrays, which are functions of frequency, are interpolated to obtain a finer resolution.

The interpolation of the arrays is necessitated in order to minimize numerical error when integrating the first and second order partial differentials of the displacement arrays inside the integrals of Eqs. (13) and (14). For example, let $X$ be a vector seen as

$$X = \begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
7
\end{bmatrix}$$

Squaring this vector would yield

$$X^2 = \begin{bmatrix}
1 \\
4 \\
9 \\
16 \\
25 \\
36 \\
49
\end{bmatrix}.$$
The second derivative of this vector is known to be 2 from \( \frac{d^2(x^2)}{dx^2} = 2 \). However, when numerically evaluating this second derivative using a finite difference based method in Matlab\(^\circledR\) the solution of the second derivative becomes

\[
\begin{bmatrix}
1 \\
1.5 \\
2 \\
2 \\
2 \\
1.5 \\
1
\end{bmatrix}
\]

Note that the first and last two elements for this vector are not the actual second derivative value of 2, where only 3 of the 7 elements are 2. For a two dimensional array the entire first and last two rows and columns would have this numerical error. By interpolating the \( X^2 \) vector to be twice as resolute, 9 of the 13 elements would be the actual second derivative value of 2, however, the first and last two elements would still be numerical errors of 1 and 1.5. Yet, by interpolating the array, more actual second derivative values were present to dominate over those produced from numerical error. This improves the numerical accuracy by having a larger portion of correct values than incorrect values. Figure 14 shows a comparison plot of the second derivative values of the original vector \( X \) and an interpolation of vector \( X \). The original vector has only seven elements where the middle three are actually two. However, interpolating the vector \( X \) to identify points at increments of one-fifth the original resolution and then calculating the second derivative provides 27 elements whose value is actually the true second derivative.
of two. The plot also shows that despite the interpolation, both vectors contain four elements, the first and last two, whose value is not the actual second derivative value of two.

Figure 14. Example of Numerical Error in Derivatives

Sufficient interpolation is determined using a known function \( w(x,y) = x^2 + y^2 \). This equation is substituted into

\[
q = \iint_0^L \left[ \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} \right] \, dy \, dx,
\]

which is a form of the PVDF sensor charge output equation accounting for only bending. This equation exercises the derivating and integrating inherent in the simulation by taking second derivatives and performing double integration. As this function can be solved by hand, an exact solution can be developed in addition to solving it numerically. Therefore, by comparing the numerical solution to the exact solution, a percentage error can be
found between the numerical solution and the exact solution. The independent variables $x$ and $y$ were simply vectors meshed together to the size of the beam and plate arrays in the $x$ and $y$ directions, respectively. Therefore, by interpolating $w(x,y) = x^2 + y^2$ to finer resolutions, the percentage error between the exact solution and the numerical solution becomes less. As the independent variable sizes used are representative of either the beam or plate array dimensions, the percentage error also reflects the use of either the beam or plate array size. The first interpolation resolution to achieve a percentage error less than 5% was used as the final interpolation resolution. Although interpolation could continue to provide more accuracy, the resulting file size could become too large to manage or process.

After sufficient interpolation, the second partial derivatives of the transverse vibration displacements are computed numerically with respect to the $x$ and $y$ directions using the finite difference method. In addition, the first partial derivatives for the in-plane vibration displacements in the $u$ and $v$ directions are computed with respect to the $x$ and $y$ directions also using the finite difference method. Then, using the functions $F_1(x)$ and $F_2(x)$, the first and second partial derivate arrays are integrated over the sensor shape area using the Gaussian quadrature technique. Finally, multiplying by the stress per charge output coefficients of the PVDF material computes $q_x$ and $q_y$ as functions of frequency. The Matlab® code used to develop the simulation can be seen in Appendix A-G. Figure 14 shows a diagram graphically depicting the flow of the logic for the simulation highlighting the key numerical tools used for the computations.
As the sensor shape is a vector of points, which govern the limits of integration, discontinuities can be easily introduced into the shape vector and its location to simulate errors in a sensor. As the sensor shapes for the beam and plate are both described by functions of $x$, the following explanations serve to describe shaping and placement errors for both the beam and the rectangular plate.

**Shaping Errors**

The shape vectors in the numerical simulation describe the sensor shapes on the structure under study. These shape vectors are one dimensional arrays of discrete points (in the $y$ direction) generated by a function in the $x$ direction (the $F(x)$ shape function) of the structure. These shape vectors are used as the numerical limits of integration. By introducing deviations in the coordinates of the points that describe the sensor shape, one can simulate sensor shaping errors. Figure 16 shows a discritized first order beam sensor shape (triangle) shaped correctly.

---

Figure 15. Simulation Flow Diagram

Implementation for Simulation of Errors
Figure 16. Discritized Sensor Shape.

As there are eighty values, the shape array ($y$ array) is a 1 by 8 array. In order to simulate a shape error, a value from the $y$ array is chosen from the desired position (identified by the $x$ values) then it is changed by a small amount (either add or subtract this small amount from its original, correct value). For example, assume that at $x_3$, a shape error occurs that leaves 1 mm more electrode at the edge of the sensor. The $y_3$ value is identified from the shape array and, being consistent with the units, 1 mm is added to $y_3$ identifying it as $y'_3$. Figure 17 shows the sensor shape simulated with this error. This error mimics incorrect tracing of the sensor shape from the paper template, as will be described in Chapter VI.

Figure 17. Sensor Shaped Simulated with an Error at $x_3$. 
The value of \( y'_3 \) is not a value found using \( F(x_3) \) alone, but rather \( F(x_3) + 1 \) mm. Figure 18 shows the gain in area of the original sensor area from the shape error.

![Error Area](image)

**Figure 18. Added Sensor Area from Simulated Error.**

This new sensor array is saved and used as the numerical limits of integration for the simulated sensor with a shaping error. The shaping errors could be introduced in the form of any number of deviations from the original sensor shape at any number of points. Furthermore, the deviations could be represented by a new sensor shape equation. For example, \( F_1(x) \) could be a linear equation that extends from \( x_1 \) to \( x_4 \), \( F_2(x) \) could be a quadratic equation that extends from \( x_4 \) to \( x_9 \), and \( F_3(x) \) could be a constant that extends from \( x_9 \) to \( x_{15} \) such that the shape array is a 1 by 15 array made up of three different equations. This array now makes up the limits of integration for the sensor simulating an error.

In addition to manipulating the sensor shape at its boundaries, it is also possible to manipulate the sensor area within the boundaries of the sensor shape to simulate shaping errors. Figure 19 shows the same first order (triangle) sensor as in Figure 16, however, a square of material has been removed from the area within the sensor shape boundaries indicated by the blue outlined square. This error mimics the possibility of etchant splashing onto the sensor surface during the etching of the shape.
For this shaping error simulation, the shape that defines the boundaries of the sensor is not altered. Rather, the simulated sensor charge output from the area defined by the boundaries of $E(x)$ and $-E(x)$ is subtracted from the total charge output of the original sensor without an inner area error. Written another way, $q_{\text{sensor}} - q_{\text{error}} = q_{\text{simulated error}}$ where $q_{\text{sensor}}$ is the charge output for the original sensor, $q_{\text{error}}$ is the charge output for the area covered by the error functions, and $q_{\text{simulated error}}$ is difference of the two representing the charge output of the sensor with an error, which represents the principle of superposition for sensor charge output.

**Placement Errors**

Placement errors do not alter the shape of the sensor, rather they position the sensor at some value $x_n \pm \varepsilon_a$ and $y_n \pm \varepsilon_b$. The variable $x_n$ is the $n^{th}$ $x$ position at which to evaluate $F(x_n)$ and $\varepsilon_a$ is the placement error value along the $x$ axis, and $\varepsilon_b$ is the placement error along the $y$ axis. Figure 20 shows how the placement error simulation manipulates a sensor shape along the $x$ axis.
Figure 20. Discritized Sensor Shape with Placement Error in x Direction.

Figure 20 shows that the sensor shape simply translates linearly down the x axis of the structure when a placement error of $\varepsilon_{a}$ is added to $x$. This is equivalent to having another linear equation with the sample slope as $F(x)$ yet the $y$ axis intercept has changed because the line is evaluated at another range of $x$ values. Figure 21 shows how the placement error simulation manipulates a sensor shape along the $y$ axis.

Figure 21. Discritized Sensor Shape with Placement Error in y Direction.

This placement error shows a translation of the sensor shape in the $y$ direction due to a placement error of $\varepsilon_{b}$. This value of $\varepsilon_{b}$ is simply added to the resulting sensor shape array. However, for a beam with a finite width $b$ and a beam sensor whose shape functions have a least one element in the sensor shape array equal to $b$ (the sensor shape goes to the edge), no $y$ direction placement errors can be simulated. Placement errors in the $y$ direction are for sensors on a rectangular plate only.
CHAPTER III
ANALYTICAL VERIFICATION

In order to verify that the numerical solution does in fact produce accurate results, analytical calculations using the PVDF charge output equations are used for comparison. Using preliminary experimentation, it was found that within the frequency range of interest, from 0 Hz to 1600 Hz, sensor charge response is dominated by bending strains (compared to in-plane strains). This allows for the analytical verification to be derived simply as a function of bending strains only. As such, Eq. (3) is analytically solved for the verification of sensor shape simulation on a beam, and Eq. (7) is analytically solved for the verification of sensor shape simulation on a rectangular plate.

Bending Strains versus In-plane Strains

Preliminary experiments, using techniques that will be addressed in Chapter VI, show that the sensor response due to bending strains dominates over the sensor response due to in-plane strains. These bending and in-plane strains can be separated by simply adding or subtracting sensor charge output from two separate sensors; one located on the top of the structure and one located on the bottom of the structure in the same position. Figure 22 shows, for an arbitrary structure, the charge response due to bending strains \(q_b\) and in-plane strains \(q_i\) for a sensor on top and on bottom of the structure.
Figure 22 shows a structure undergoing both bending and in-plane strains. As such, a sensor located on the top surface of the structure would experience a tension loading due to bending as well as an in-plane loading. Conversely, a sensor on the bottom surface would undergo a compressive loading due to bending as well as the same in-plane loading experienced by the sensor on the top surface of the structure. Therefore, a sensor located on the top surface of a structure would experience strain due to bending that would be 180° out of phase with respect to a sensor experiencing strain due to bending on the bottom surface of the structure. However, a sensor on the top surface of the structure would experience strain due to in-plane loading that would be in phase with a sensor on the bottom surface of the structure experiencing the same strain due to in-plane loading.

In order for these signals to add or subtract such that one can isolate $q_b$ and $q_i$ experimentally, the two sensors must be in the same in position on the structure and possess the same shape. Furthermore, the sensor polarity must be the same between these two sensors. In this way, by adding the sensor charge output of both sensors, the bending strains would cancel leaving only $2q_i$, where as subtracting the charge output of both sensors would cancel the in-plane strains leaving only $2q_b$. Figure 23 and Figure 24 show the sensor charge output due to bending strains in comparison to sensor charge output due to in-plane strains on the beam and the rectangular plate, respectively. The $y$-axis label
Charge Output (dB) of these experimental sensor charge output plots is defined as

\[ 20 \cdot \log_{10} \] (with re.: 1 N).

![Experimental Charge Outputs](image)

Figure 23. Bending Strain Versus In-plane Strain Response for the Beam

It can be seen from the plot for the beam that the charge output of the sensor from in-plane strains is approximately 10 dB below the charge output of the sensor due to bending strains across the frequency range. However, this difference is smaller at approximately 60 Hz and 1400 Hz. This 10 dB difference is large enough to conclude that the charge output due to bending strains dominates over the charge output due to in-plane strains for the beam and can be neglected.
Figure 24. Bending Strain Versus In-plane Strain Response for the Rectangular plate

The influence of in-plane strains in comparison to bending strains on sensors for the rectangular plate seems to follow the same 10 dB difference trend as that experienced by the beam. However, this influence of in-plane strains is more significant at approximately 400 Hz and between 900 Hz to 1100 Hz. As with the beam, this in-plane influence for plates can be neglected.

Beam Derivations

Equation (3) can be used to analytically solve for the sensor charge output on a beam for a limited number of sensor shapes. The analytical calculation requires experimental data only for select accelerometer points. The derivation below reveals which experimental accelerometer points are needed depending on the sensor shape.

Equation (20) shows that the sensor charge output on a beam is
\[ \hat{q} = -(h_b + h_s)e_{31} \int_a^c F(x) \frac{d^2 w}{dx^2} \, dx. \]  

(20)

It is important to note that Eq. (20) is equal to Eq. (3), however, the limits of integration are for any arbitrary sensor length, \(a\) to \(c\), on a beam length. This is done for the analytical convenience of deriving one equation suitable for a variety of sensor positions along the beam length. As the charge output of Eq. (20) is not representative of the entire beam length it is referred to as the local charge output and is designated by the carrot symbol above the charge variable \(q\). Figure 25 shows where this sensor length segment would appear on a beam.

![Figure 25. Arbitrary Sensor Shape on a Beam.](image)

Integration of Eq. (20) by parts once yields

\[
\hat{q} = -(h_b + h_s)e_{31} \left\{ F(x) \frac{dw(x)}{dx} \Bigg|_a^c - \int_a^c \frac{dw(x)}{dx} \frac{dF(x)}{dx} \, dx \right\}. 
\]

(21)

Integration by parts once more yields

\[
\hat{q} = -(h_b + h_s)e_{31} \left\{ F(x) \frac{dw(x)}{dx} \Bigg|_a^c - \frac{dF(x)}{dx} w(x) \Bigg|_a^c + \int_a^c w(x) \frac{d^2 F(x)}{dx^2} \, dx \right\}. 
\]

(22)

From this point on, function derivatives will be designated by prime indicators as they are functions of only one variable, \(x\). Substituting the integration limits into Eq. (22) and rearranging and factoring the terms gives
\[
\dot{q} = -(h_b + h_s) e_3 \left\{ \left[ F(c) w'(c) - F'(c) w(c) \right] - \left[ F(a) w'(a) - F'(a) w(a) \right] + \int_a^c w(x) F''(x) dx \right\}.
\] (23)

Equation (23) shows a single integral of the product of \(w(x)\) and \(F''(x)\) remaining. By allowing a second order equation as the largest order equation for \(F(x)\), \(F''(x)\) will become a constant and can be brought outside the integral leaving only the integral of \(\int_a^c w(x) dx\).

The integral of \(w(x)\) is the local volume displacement divided by the beam width \(b\) and can be calculated analytically using the measured beam vibrations and utilizing the trapezoidal rule (Rozema et al. 2001).

One might wonder why the author did not use this method for prediction of sensor charge output of an arbitrary sensor. However, for zero and first order equations of \(F(x)\), \(F''(x)\) would be equal to zero thereby making the integral term zero as well. If \(F(x)\) was a third order equation or higher, the remaining integral would be of a product of \(w(x)\) and some function of \(x\) which cannot be analytically reduced any further. This shows the limitation of the analytical method of solving for sensor charge output by requiring sensor shapes to be described by equations whose order is no greater than two. However, despite this analytical limitation of sensor shape equation order, a sufficient variety of zero, first, and second order shapes can be used for the verification of the simulation.

The variables \(w(x)\) and \(w'(x)\) refer to the experimentally measured transverse vibration displacements and the first derivative (with respect to length) of the experimentally measured transverse vibration displacements, respectively, at \(x\).

Four arbitrary shapes have been chosen to analytically solve for the local sensor charge output. A rectangle is used as a representative for a zero order equation, a triangle
and diamond are used to represent first order equations, and a quadratic equation is used to represent second order shapes.

**Zero Order Sensors**

Figure 26 shows a rectangular zero order sensor on an arbitrary length of a beam, where \( b \) is the width of the beam. Also, the sensor shape equations are identified in Figure 26.

![Figure 26. Rectangular Zero Order Sensor on an Arbitrary Length of a Beam.](image)

Equation (23) is based on sensor equation values at the boundaries of the sensor shape. The sensor equation and its values at the boundaries from Figure 26 are

\[
\begin{align*}
F(x) &= \frac{b}{2}, \\
F(c) &= \frac{b}{2} \quad F'(c) = 0 \quad F''(c) = 0 \\
F(a) &= \frac{b}{2} \quad F'(a) = 0 \quad F''(a) = 0.
\end{align*}
\]

Substituting these values into Eq. (23) and factoring yields

\[
\dot{q} = -(h_b + h_c)e_{31} \frac{b}{2}[w'(c) - w'(a)].
\]

**First Order Sensors**

Figure 27 shows a triangular (first order) sensor on an arbitrary length of a beam. Figure 27 also shows where the sensor shape equations appear.
Figure 27. Triangular First Order Sensor on an Arbitrary Length of a Beam.

The sensor equation and its boundary values for this sensor shape are

\[ F(x) = \frac{b}{2(c-a)}(x-a), \]  

(27)

\[ F(c) = \frac{b}{2}, \quad F'(c) = \frac{b}{2(c-a)}, \quad F''(c) = 0 \]  

(28)

\[ F(a) = 0, \quad F'(a) = \frac{b}{2(c-a)}, \quad F''(a) = 0. \]

Substituting these values into Eq. (23) and factoring shows the analytical formulation of the sensor charge output as

\[ \hat{q} = -(h_b + h_r) b e_{31} \left\{ w'(c) + \frac{1}{c-a} [w(a) + w(c)] \right\}. \]  

(29)

Figure 28 shows a diamond-shaped first order sensor on an arbitrary length of a beam. This sensor is analytically evaluated as the sum of two separate triangular sensors, as in Figure 27, as \( \hat{q} = \hat{q}_l + \hat{q}_r \). The variable \( \hat{q}_l \) is charge output of the left hand triangular portion of the sensor and \( \hat{q}_r \) is the charge output of the right hand triangular portion of the sensor. Figure 28 shows where the two separate sensor shape equations appear.
Figure 28. Diamond-Shaped First Order Sensor on an Arbitrary Length of a Beam.

The function $F_1(x)$ represents the left hand side of the first order diamond-shaped sensor and its position is identified in Figure 29. This sensor equation and its boundary values are

$$F_1(x) = \frac{b}{(c-a)}(x-a)$$

(30)

$$F_1\left(\frac{c+a}{2}\right) = \frac{b}{2} \quad F_1'\left(\frac{c+a}{2}\right) = \frac{b}{2(c-a)} \quad F_1''\left(\frac{c+a}{2}\right) = 0$$

(31)

$$F_1(a) = 0 \quad F_1'(a) = \frac{b}{2(c-a)} \quad F_1''(a) = 0.$$

Figure 29. Left Hand Portion of Diamond-Shaped Sensor.

Substituting these boundary values into Eq. (23) and factoring yields

$$\hat{q}_l = -(h_b + h_s)e_{31}\left\{\frac{b}{2}w'\left(\frac{c+a}{2}\right) + \frac{b}{c-a}\left[w(a) - w\left(\frac{c+a}{2}\right)\right]\right\},$$

(32)

which represents the charge output of the left hand side of the diamond-shaped sensor in Figure 28. The calculation of $\hat{q}_r$ follows in the same manner and represents the charge output of the right hand side of the diamond-shaped sensor. Figure 30 shows where the
right hand side of the diamond-shaped sensor equation appears. The sensor equation and boundary values for the right hand side are

\[ F_2(x) = \frac{-b}{(c-a)}(x-c), \]  

\[ F_2(c) = 0 \quad F_2'(c) = \frac{-b}{(c-a)} \quad F_2''(c) = 0 \]  

\[ F_2\left(\frac{c+a}{2}\right) = \frac{b}{2} \quad F_2'\left(\frac{c+a}{2}\right) = \frac{-b}{(c-a)} \quad F_2''\left(\frac{c+a}{2}\right) = 0. \]  

Substituting these values into Eq. (23) and factoring yields

\[ \hat{q}_r = -\left(h_b + h_s\right)e_{31} \left\{ -\frac{b}{2} w\left(\frac{c+a}{2}\right) + \frac{b}{c-a} \left[w(c) - w\left(\frac{c+a}{2}\right)\right] \right\}. \]  

\[ \hat{q} = -\left(h_b + h_s\right)e_{31} \frac{b}{c-a} \left[w(c) + w(a) - 2w\left(\frac{c+a}{2}\right)\right]. \]

Second Order Sensors

Figure 31 shows a second order sensor shape on an arbitrary length of a beam.
The quadratic sensor equation has the form

$$F(x) = Ax^2 + Bx + C,$$  \hspace{1cm} (37)

where the coefficients $A$, $B$, and $C$ are unknowns. The coefficients can be determined from the boundary values given by

$$F(c) = 0$$
$$F(a) = 0$$
$$F'(a) = -F'(c)$$
$$F\left(\frac{c + a}{2}\right) = \frac{b}{2}.$$  \hspace{1cm} (38)

Equating the opposites of the first derivatives of the sensor shape evaluated at $a$ and $c$, the coefficient $B$ can be found as a function of the coefficient $A$ as

$$B = -A(a + c).$$  \hspace{1cm} (39)

Substituting Eq. (39) into Eq. (37) yields

$$F(x) = Ax^2 - A(a + c)x + C.$$  \hspace{1cm} (40)

Evaluating Eq. (40) at $x = a$ or $c$ determines the coefficient $C$ as a function of the coefficient $A$ as

$$C = Aac.$$  \hspace{1cm} (41)

Substituting Eq. (41) into Eq. (40) yields

$$F(x) = Ax^2 - A(a + c)x + Aac.$$  \hspace{1cm} (42)
Evaluating Eq. (42) at \( x = (c+a)/2 \) determines the coefficient \( A \) as

\[
A = \frac{-2b}{(a-c)^2}.
\]  

(43)

Substituting Eq. (43) into Eq. (42) and factoring determines the quadratic sensor shape as

\[
F(x) = \frac{-2b}{(a-c)^2} \left[ x^2 - (a+c)x + ac \right].
\]  

(44)

Knowing the sensor shape equation allows for the determination of the sensor boundary values required by Eq. (23) given by

\[
F(c) = 0 \quad F'(c) = \frac{2b}{(a-c)} \quad F''(c) = \frac{-4b}{(a-c)^2}
\]

\[
F(a) = 0 \quad F'(a) = \frac{-2b}{(a-c)} \quad F''(a) = \frac{-4b}{(a-c)^2}.
\]

(45)

Substituting these values into Eq. (23) yields the sensor charge output for a quadratic sensor as

\[
\hat{q} = -e_{31} \left( h_b + h_s \right) \left\{ \frac{-2b}{(a-c)} \left[ w(a) + w(c) \right] - \frac{4b}{(a-c)^2} \int_a^c w(x) dx \right\}.
\]

(46)

As mentioned previously, the remaining integral of the transverse vibration displacement of the beam is simply the volume displacement of the beam per unit width. This volume displacement per unit width can be calculated analytically using the trapezoidal rule. Multiplying this trapezoidal calculation by the width of the beam determines the volume displacement of the local segment of the beam from \( x = a \) to \( x = c \). The local volume displacement of the beam is

\[
\int_a^c w(x) dx = \frac{-(c-a)b}{N} \left\{ \frac{5}{2} \left[ w(a) + w(c) \right] + \frac{N-1}{2} \left[ \sum_{i=2}^{N-1} w(x_i) \right] \right\},
\]

(47)
where \( N \) is the number of equally spaced vibration displacement measurement points \( w(x_i) \) along the length of the beam segment \( (a \leq x \leq c) \).

Rectangular Plate Derivation

The analytical derivation of the charge output of an arbitrarily shaped sensor on a rectangular plate structure attempts to follow that of the beam derivation. In the same efforts as for the beam, the local charge output of a sensor from bending strain in two dimensions is given by

\[
\hat{q} = -\frac{h_{\text{pl}}^2 + h_{\text{p}}^2}{2} \int_{c_1}^{c_2} \int_{a_1}^{a_2} \left[ \varepsilon_{31} \frac{\partial^2 w(x,y)}{\partial x^2} + \varepsilon_{32} \frac{\partial^2 w(x,y)}{\partial y^2} \right] F(x,y) \, dx \, dy. \tag{48}
\]

Figure 32 shows where the sensor would appear on a rectangular plate surface as well as identifying the limits of integration in Eq. (48). The limits of integration identify a rectangular region around the sensor whose dimensions are defined by the minimum and maximum limits of the sensor itself.

![Figure 32. Arbitrary Sensor Shape on a Rectangular Plate.](image-url)
As with the beam, by distributing the double integral to each second order partial
differential term, Eq. (48) can be written as the sum of two charge output terms \( \hat{q}_x \) and\( \hat{q}_y \) seen as

\[
\hat{q}_x = -\frac{h_x^p + h_y^s}{2} e_3 \int \frac{\partial^2 w(x, y)}{\partial x^2} F(x, y) dx \, dy,
\]

(49)

\[
\hat{q}_y = -\frac{h_x^p + h_y^s}{2} e_3 \int \frac{\partial^2 w(x, y)}{\partial y^2} F(x, y) dy \, dx,
\]

(50)

where \( \hat{q} = \hat{q}_x + \hat{q}_y \).

Integration of Eq. (49) and Eq. (50) by parts once yields

\[
\hat{q}_x = -\frac{h_x^p + h_y^s}{2} e_3 \int \left[ \left( \frac{\partial w(x, y)}{\partial x} \right)^2 - w(x, y) \frac{\partial^2 F(x, y)}{\partial x^2} \right] dx \, dy,
\]

(51)

\[
\hat{q}_y = -\frac{h_x^p + h_y^s}{2} e_3 \int \left[ \left( \frac{\partial w(x, y)}{\partial y} \right)^2 - w(x, y) \frac{\partial^2 F(x, y)}{\partial y^2} \right] dy \, dx,
\]

(52)

Integration by parts once more yields

\[
\hat{q}_x = -\frac{h_x^p + h_y^s}{2} e_3 \int \left[ \left( \frac{\partial w(x, y)}{\partial x} \right)^2 - w(x, y) \frac{\partial^2 F(x, y)}{\partial x^2} \right] dx \, dy + \int_{a_1}^{a_2} \int_{a_1}^{a_2} \left( \frac{\partial^2 w(x, y)}{\partial x^2} \frac{\partial^2 F(x, y)}{\partial x^2} \right) dx \, dy,
\]

(53)

\[
\hat{q}_y = -\frac{h_x^p + h_y^s}{2} e_3 \int \left[ \left( \frac{\partial w(x, y)}{\partial y} \right)^2 - w(x, y) \frac{\partial^2 F(x, y)}{\partial y^2} \right] dy \, dx + \int_{a_1}^{a_2} \int_{a_1}^{a_2} \left( \frac{\partial^2 w(x, y)}{\partial y^2} \frac{\partial^2 F(x, y)}{\partial y^2} \right) dx \, dy.
\]

(54)
At this point for the beam derivation, an analytical solution was found, yet that is not the case for the rectangular plate derivation. Looking closer, the function \( F(x,y) \) acts as a scalar weighting function for the sensor film area contained with the limits of integration. However, because the film used in this research has a constant electrode thickness, the polarization profile function \( F(x,y) \) has a constant value of one anywhere at or within the sensor area, and a constant value of zero anywhere outside of the sensor area and within the limits of integration. Figure 33 shows where these polarization profile values are located with regard to the shape of the sensor and the limits of integration.

![Figure 33. Polarization Profile Values.](image)

Therefore, any order partial derivates of the polarization profile \( F(x,y) \) with respect to \( x \) or \( y \) directions are zero. This reduces Eq. (53) and Eq. (54) back to the original form of Eq. (49) and Eq. (50). As a result of the polarization profile being a constant value of one anywhere on the sensor area, the limits of integration can be defined by a function representing the sensor shape itself. This removes the need for a polarization profile because the limits of integration contain only the sensor area where the profile is a constant value of one. This allows Eq. (49) and Eq. (50) to be written as

\[
\dot{q}_x = -\frac{h_z^{pl} + h_z^p}{2} e_{31} \int_{y_1 = F_1(x)}^{y_2 = F_2(x)} \frac{\partial^2 w(x,y)}{\partial x^2} \, dx \, dy,
\]

\[
\dot{q}_y = -\frac{h_z^{pl} + h_z^p}{2} e_{32} \int_{y_1 = F_1(x)}^{y_2 = F_2(x)} \frac{\partial^2 w(x,y)}{\partial y^2} \, dx \, dy.
\]
Figure 34 shows where the arbitrary sensor shape equations would appear on a rectangular plate structure.

Figure 34. Arbitrary Sensor Shape Equations on a Rectangular Plate.

Equation (55) and Eq. (56) are simply the numerical formulation used in the numerical simulation of the sensor. This appears to have brought the rectangular plate derivation full circle. However, in an effort to have a separate and uniquely distinguishable check for the rectangular plate simulation, as for the beam, the beam verification equations can be used. By assuming a sensor on the rectangular plate that has a width that is small in comparison to the smallest vibration half-sine wavelength from the largest frequency within the frequency range of interest, the sensor can be assumed to experience no vibration straining in the width direction. Figure 26 shows this idealized sensor width on the smallest half-sine wavelength.
Figure 35. Idealized Sensor.

According to the assumptions and coordinate system of Figure 35, Eq. (56) can be set equal to zero leaving only Eq. (55), which can be integrated once to yield Eq. (20). Therefore, any of the analytical charge output equations for the beam can be used by simply substituting $w(x,y)$ data from the plate for $w(x)$ data from the beam. It should be noted that the horizontal axis of the coordinate system of Figure 35 could also be of the $x$ direction. This requires the substitution of $F(y)$ for $F(x)$ into Eq. (23).
CHAPTER IV
EXPERIMENTAL SETUP

This chapter provides a description of the equipment and experimental setup used to gather the experimental data required to check the validity of the beam and the rectangular plate sensor simulations.

Tri-axial accelerometer measurements measured not only the transverse \( z \) direction) displacements but also the in-plane \( x \) and \( y \) directions) displacements. Figure 36 shows the tri-axial accelerometer (each axis powered by a signal conditioner).

![Tri-Axial Accelerometer](image)

Figure 36. PCB 356A21 Tri-Axial Accelerometer.

In order to determine the point measurement (accelerometer) array size necessary to describe the displacement surface of a vibrating structure within a frequency range of interest, the structural modes within this frequency range of interest must be known. This is necessary so as to properly sample the structure’s surface in an effort to discern all natural mode shapes present within the frequency range of interest. Each mode shape can be approximated as a series of sinusoidal waves in the axial directions of a structure.
Figure 37 shows a sinusoidal waveform being sampled at various points along its wavelength in an effort to describe the wave (Orfanidis 1996).

![Sampling Frequencies](image)

$f_s = 8f$

$f_s = 4f$

$f_s = 2f$

Figure 37. Three Different Sampling Frequencies for a Sinusoidal Wave.

It is important to note that the three different sampling plots in Figure 37 show a decreasing sampling rate, as a function of frequency, from left to right. The right most sampling plot shows that a sinusoidal waveform can accurately be described by measuring only two points per period. However, if these two points were to fall on the nodes of this waveform, then the wave would be aliased as a straight line and would appear not to exist (Orfanidis 1996). Therefore, a point measurement array of insufficient accuracy (in placement and size) may not provide useful information within the frequency range of interest. As broadband structural vibration is the summation of structural vibration modes at all frequencies, the measurement array must be sensitive at least to the most complex natural mode shape experienced. This would provide sufficient discritization of vibration response to observe all other less complex mode shapes. As this work is concerned with sound radiation between a known frequency range of 0 Hz and 1600 Hz, a simple single accelerometer sampling of the structural surface at various points could be used to predict the number of natural structural modes within this frequency range. As structural mode shapes are not unique, simply knowing the total number of naturally occurring mode shapes (if beginning from 0 Hz) is sufficient to
identify the most complex structural mode shape present within the frequency range of interest. This is due to the fact that structural modes follow a sequential progression across a frequency range of interest and can be identified by their order of occurrence. Once this is accomplished, the most advantageous array size and position can be determined based on the most complex structural mode shape present.

Experimental Beam Setup

A T-6061 aluminum .52 meter long beam that is .0381 meters wide and .0127 meters thick with fixed end conditions was used to gather the structural tri-axial accelerometer data for one dimensional simulations. Figure 38 shows this experimental fixed-fixed beam.

![Experimental Fixed-Fixed Beam](image)

Figure 38. Experimental Fixed-Fixed Beam.

Table 1 shows the bending modes present within the frequency range of 0 Hz to 1600 Hz for the beam.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Beam Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>216</td>
<td>1</td>
</tr>
<tr>
<td>568</td>
<td>2</td>
</tr>
<tr>
<td>1095</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 1. Beam Bending Modes
For this fixed-fixed beam, up to the third bending mode can be found within the frequency range of 0 Hz to 1600 Hz. This would necessitate three accelerometer points along the middle of the beams width positioned at the nodes of the third mode. However, due to modal spillover it is advantageous, from a control standpoint, to discritize the beams surface to confidently describe up to the seventh bending mode. Modal spillover is the result of modal contributions within a specified frequency range from modes that do not occur within that frequency range. Observing three to four modes beyond the frequency range of interest is sufficient for spillover considerations for structures with low structural damping. Based on these considerations, a 1 by 31 measurement array was utilized.

The beam was excited with a broadband noise signal supplied from a model 29 Wavtek signal generator. The broadband noise was amplified, using a Techron 5507 power amplifier, before arriving at an LDS 200 series electrodynamic shaker. The shaker stinger was threaded into the beam and held in place with a jam nut as seen in Figure 39.

Figure 39. Electromechanical Shaker for Beam.
The force input to the beam was measured using a PCB 208B02 force gage. All data were recorded on a dat tape recorder TEAC RD135T. Table 2 shows the recorded signal TEAC channel assignment.

<table>
<thead>
<tr>
<th>TEAC Channels</th>
<th>Recorded Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Excitation Source</td>
</tr>
<tr>
<td>2</td>
<td>Input Force</td>
</tr>
<tr>
<td>3</td>
<td>X-Axis Acceleration</td>
</tr>
<tr>
<td>4</td>
<td>Y-Axis Acceleration</td>
</tr>
<tr>
<td>5</td>
<td>Z-Axis Acceleration</td>
</tr>
</tbody>
</table>

Table 2. Recorded Signal TEAC Channel Assignment

Figure 40 shows the position transition of the accelerometer point measurements across the length of the beam and the corresponding beam coordinate system.

Figure 40. Accelerometer Point Measurement Progression on the Beam Length.

After each point on the beam was recorded, the tape was played back and frequency response function (FRF) data (acceleration divided by force input) was analyzed using an HP 35670A dynamic signal analyzer (DSA). The DSA analyzed the data from 0 Hz to 10000 Hz which is the limiting sampling frequency range for the TEAC when using five or more channels of acquisition. In addition, the frequency range of 0 Hz to 10000 Hz is large enough to discover in-plane vibration responses as well. Figure 41 shows this signal flow diagram for the beam.
The analyzed data was saved to disk and loaded into the simulation for post processing.

Figure 42 shows an example of an FRF plot (z direction) taken for the beam. The sample point shown in this plot is approximately at a quarter of the way along the beam's length so it shows the contribution of all three bending modes fairly well. The x-axis of the plot is frequency (0 Hz to 1600 Hz) and the y-axis is acceleration calculated as $20 \cdot \log_{10}$ (with re.: 1 N).
Experimental Rectangular Plate Setup

The rectangular plate is made of 6.35 millimeter thick aluminum and its dimensions are .610 meters in the $x$ direction and .350 meters in the $y$ direction. It is bolted in a fixed manner on all four of its sides to an otherwise rigid steel box made of 2.54 cm thick plate steel. The steel box is acoustically sealed and is lined with a sound absorbing material in an effort to prevent the reflection of sound from the bottom of the box to the underside of the plate thereby disturbing the measured vibration response of the plate. Figure 43 shows the experimental rectangular plate.
Table 3 shows the bending modes present within the frequency range of 0 Hz to 1600 Hz for the plate.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Plate Mode (mₓ,nᵧ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>278</td>
<td>(1,1)</td>
</tr>
<tr>
<td>396</td>
<td>(2,1)</td>
</tr>
<tr>
<td>600</td>
<td>(3,1)</td>
</tr>
<tr>
<td>684</td>
<td>(1,2)</td>
</tr>
<tr>
<td>805</td>
<td>(2,2)</td>
</tr>
<tr>
<td>893</td>
<td>(4,1)</td>
</tr>
<tr>
<td>1000</td>
<td>(3,2)</td>
</tr>
<tr>
<td>1280</td>
<td>(4,2)</td>
</tr>
</tbody>
</table>

Table 3. Plate Bending Modes

The accelerometer array size for the rectangular plate was determined in the same fashion as for the beam. The most complex mode shape for the rectangular plate within the 0 Hz to 1600 Hz frequency range in the x direction alone is the 4 by nᵧ, and the most complex mode shape in the y direction alone is the mₓ by 2, where mₓ and nᵧ are the corresponding mode numbers in the x and y directions, respectively. This would require an array size of 4 by 2, however, when considering modal spillover, a 13 by 7 was implemented.

The rectangular plate was excited using a Labworks ET-132 electrodynamic shaker. Figure 44 shows the shaker within the experimental rectangular plate structure.
As the box is completely sealed, a front interfacing panel for cables holds BNC couplers to supply and receive force gage and shaker signals from within the box.

Figure 44. Shaker in Experimental Rectangular Plate Structure.

The data from the source signal, the force input, and the three axial signals of the tri-axial accelerometer were recorded in the same manner as with the beam on the TEAC. Figure 45 shows the signal flow diagram for the rectangular plate.

Figure 45. Tri-Axial Accelerometer Experimental Setup for the Rectangular Plate.
In addition, Figure 46 shows the point measurement transition from point to point on the surface of the rectangular plate.

![Diagram of point measurement progression on a rectangular plate.]

**Figure 46. Accelerometer Point Measurement Progression on the Plate Surface.**

Figure 47 shows a sample FRF plot taken from the plate. The axis labels are the same as for the beam.

![Sample FRF plot with phase angle and acceleration vs. frequency.]

**Figure 47. Sample FRF Plot From Plate.**
CHAPTER V

PRELIMINARY RESULTS

The preliminary results of the sensor charge output simulation verification are presented for the beam and the rectangular plate. The simulated results are compared against the analytical results for the same zero, first, and second order shape sensors. Analytical equations for the sensor shapes were derived in Chapter III. Equation (26) is the analytical equation derived for the zero order sensors, Eq. (29) is the analytical equation derived for the triangular, first order sensor, Eq. (36) is the analytical equation derived for the diamond shaped, first order sensor, and Eq. (46) is the analytical equation derived for the second order sensor. From this point on, the y-axis label Charge Output (dB) on all simulated and experimental sensor charge output plots is defined as $20 \cdot \log_{10}$ (with re.: 1 N).

Beam Sensors

As mentioned before, the simulation is suitable for simulating sensor charge output for either one or two dimensional structures. This is accomplished by programming the simulation for two dimensional structures explicitly. The beam can be simulated as two dimensional surface by simply copying the original 1 by 31 point measurement array into rows two and three of a 3 by 31 array. Although the beam is now viewed as two dimensional surface, the change in displacements in the y direction is zero so the simulation will not predict any sensor charge output due to changes in the y direction. This yields a displacement array of 3 by 31 where rows one, two, and three are
equal to each other. For each sensor shape described above, four positions are used for
the simulated and analytical comparison at the lengths of \((0, L), \left(0, \frac{1}{2} L\right), \left(0, \frac{1}{3} L\right),\) and
\(\left(\frac{1}{3} L, \frac{2}{3} L\right)\) of the beam length. Figure 48 shows specifically where these sensor lengths
are on the beam itself. Each beam length is color coded to show the beam length of
interest. The blue shows the beam length of \((0, L)\), the green shows the beam length of
\(\left(\frac{1}{3} L, \frac{2}{3} L\right)\), the purple shows the beam length of \(\left(0, \frac{1}{2} L\right)\), and the yellow shows the
beam length of \(\left(0, \frac{1}{3} L\right)\).

Figure 48. Beam Lengths of Interest for Simulation.
These positions \((x_1, x_2)\) are substituted into the positions \((a, c)\) of Eq. (26), Eq. (29), Eq. (36), and Eq. (46). At each of the position values, the experimental terms in the analytical relations are interpolated from the displacement arrays in order to find the corresponding value at those positions.

**Zero Order Sensor**

Figure 49 through Figure 52 show the comparison results for the rectangular (zero order) sensor. The preliminary results of the comparison of the sensor output predicted from analytical calculations and those predicted by the simulation for the beam show excellent agreement.

![Graph showing comparison of calculated vs simulated charge output](image1)

**Figure 49. Eq. (26) Versus Simulated Results for Rectangular Sensor at \((0, L)\).**

The simulation tends to follow the general trend of the analytical solution, however, there exists noise in the simulated results of Figure 49. This is most likely due to the numerical errors discussed earlier that are encountered in the first and last two rows and columns of...
an array when calculating the first and second order partial differentials. This assumption
is legitimate in that according to Eq. (26), \(w'(c)\) and \(w'(a)\) for the coordinate positions of
\(a = 0\) and \(c = L\), respectively, occur at the first and last elemental positions within the
second order partial differential array, where numerical errors are anticipated. As
expected, this problem vanishes for the other cases where the sensor does not end at both
beam boundaries (Figure 50 through Figure 52).

![Calculated vs Simulated Charge Output - Half](image)

Figure 50. Eq. (26) Versus Simulated Results for Rectangular Sensor at \((0, \frac{1}{2}L)\).
Figure 51. Eq. (26) Versus Simulated Results for Rectangular Sensor at \( \left( 0, \frac{1}{3} L \right) \).

Figure 52. Eq. (26) Versus Simulated Results for Rectangular Sensor at \( \left( \frac{1}{3} L, \frac{2}{3} L \right) \).
First Order Sensor

Figure 53 through Figure 56 show the comparison results for the triangular sensor and Figure 57 through Figure 60 show the comparison results for the diamond shaped sensor. The comparison results between the analytical and simulated predictions for the first order sensors (triangle and diamond) show excellent agreement.

![Graph](image)

Figure 53. Eq. (29) Versus Simulated Results for Triangular Sensor at $(0, L)$. 
Figure 54. Eq. (29) Versus Simulated Results for Triangular Sensor at $\left(0, \frac{1}{2}L\right)$.

Figure 55. Eq. (29) Versus Simulated Results for Triangular Sensor at $\left(0, \frac{1}{3}L\right)$.  

65
Figure 56. Eq. (29) Versus Simulated Results for Triangular Sensor at \( \left( \frac{1}{3} L, \frac{2}{3} L \right) \).

Figure 57. Eq. (36) Versus Simulated Results for Diamond Sensor at \( (0, L) \).
Figure 58. Eq. (36) Versus Simulated Results for Diamond Sensor at \( \left( 0, \frac{1}{2} L \right) \).

Figure 59. Eq. (36) Versus Simulated Results for Diamond Sensor at \( \left( 0, \frac{1}{3} L \right) \).
Second Order Sensor

Equation (46), used to analytically represent the sensor charge output for a quadratic sensor, contains an integral representing the volume displacement per unit width from $a$ to $c \left( \int_a^c w(x) \, dx \right)$. This integral can be calculated through a weighted summation of point sensors using the trapezoidal rule as detailed in Chapter III, or it can be calculated by using the Gaussian quadrature technique. This numerical technique offers a convenient alternative to the trapezoidal rule in that it requires less written code. Figure 61 shows the comparison of the solution for Eq. (46) when computing the integral numerically and when computing the integral using the trapezoidal rule for a quadratic sensor at $(0, L)$. 
Figure 61. Comparison of Integration Techniques.

The two separate techniques of integration yield very similar results and as such, the Gaussian quadrature technique is used for all further calculations of the integral of Eq. (46) in an effort to simplify written code.

Because of the shape of the quadratic sensor, a numerical problem was encountered within the simulations numerical integration routine. In order to overcome this numerical problem the shape was extended to accommodate an extra measurement point before and after points \(a\) and \(c\), respectively, of Eq. (46). Figure 62 shows a quadratic sensor that, based on its shape, posses a potential numerical integration error.

Figure 62. Quadratic Sensor Possessing a Numerical Integration Error.
However, Figure 63 shows a correction to this shape that allows for the numerical integration to take place. It should be noted that although the shape functions $F(x)$ and $-F_1(x)$ overlap, the simulated sensor charge output is considered only within the union of these two shapes and is therefore integrated between $a$ and $c$ as required by Eq. (46).

Figure 63. Quadratic Sensor Possessing No Numerical Integration Error.

Excellent agreement is found between the analytical and simulated prediction of quadratic sensor charge output. Figure 64 through Figure 67 show the comparison results for the quadratic sensor.

Figure 64. Eq. (46) Versus Simulated Results for Quadratic Sensor at $(0, L)$. 
Figure 65. Eq. (46) Versus Simulated Results for Quadratic Sensor at \( \left( 0, \frac{1}{2} L \right) \).

Figure 66. Eq. (46) Versus Simulated Results for Quadratic Sensor at \( \left( 0, \frac{1}{3} L \right) \).
Rectangular Plate Sensors

As detailed in Chapter III, the analytical equations derived to verify the simulated charge output of sensors on the beam can be used to verify the simulated charge output of sensors on the plate. However, an assumption was made with regard to the width of a sensor on the rectangular plate in order for this to hold true. It was assumed that the sensor width is small in comparison to the smallest structural wavelength encountered. The relationship between frequency and wavelength in an infinite structure is

\[ \lambda = \frac{1}{f} \sqrt{\frac{E}{\rho}}, \] (57)

where \( E \) is the material modulus, \( \rho \) is the mass density of the material, \( f \) is the frequency of interest, and \( \lambda \) is the structural wavelength. Equation (57) does not give the structural
wavelength as a function of the boundary conditions. However, this equation shows that as the frequency increases, the structural wavelength decreases. Therefore, for a sensor of constant width, as the frequency increases the influence of the sensor width on the charge output in the direction of the changing wave increases. As such, using a one dimensional analytical charge output equation would show increasing deviation as the frequency range increases. As the sensor width increases, the point of considerable deviation begins to appear at lower frequencies.

This presents one with the question of how wide a plate sensor can be such that considerable deviation, in comparison to the analytical solution, does not appear within the frequency range of interest. In an effort to validate a maximum sensor width, a simulation was performed for a rectangular sensor having several widths at two arbitrary locations on the rectangular plate. The results of these simulations were compared against the one dimensional analytical prediction to observe where deviations occur.

![Figure 68. Sensor in Position One for Width Assumption Validation.](image)

Figure 68 shows the first location of the sensor, with width $d$, that was simulated.
The rectangular sensor (zero order sensor) was chosen over the other first and second order sensors for the width validation study in that it covers the greatest amount of surface area. This would provide for the greatest coverage of the displaced surface and thereby experience the most straining in the $y$ direction over the area of coverage. This serves as a “worst case” for the sensor shapes used in the validation. The sensor width $d$ for the simulation of the first location was varied from 1.5", to 2", to 5", to 7". It should be noted that 1.5" is the width of the beam. The simulated results are compared to the analytical prediction using Eq. (26) where $a = \frac{1}{3} L_{pl}^r$ and $c = \frac{2}{3} L_{pl}^r$. Figure 69 shows the results of the comparison of the simulated and analytical prediction of the sensor in location one with a width of 1.5" over the frequency range of 0 Hz to 10000 Hz.

Figure 69. Width Validation at Location One for $d = 1.5''$ From 0 Hz to 10000 Hz.
This frequency range offers the ability to observe any deviation in the one dimensional analytically predicted sensor charge output due to decreasing structural wavelength. Although the one dimensional analytical prediction is anticipated to deviate at higher frequencies depending on the sensor width, it is important to consider specifically the deviations in the frequency range of interest (0 Hz to 1600 Hz). Figure 70 shows the results of the comparison of the simulated and analytical prediction of the sensor in location one with a width of 1.5” over the frequency range of 0 Hz to 1600 Hz.

Figure 70. Width Validation at Location One for $d = 1.5"$ From 0 Hz to 1600 Hz.

Figure 69 shows excellent correlation at the higher frequencies as does Figure 70. However, in an effort to discern whether the one dimensional analytical prediction will deviate from the simulation as expected, sensor width of 2”, 5”, and 7” were simulated as well. For each width, plots from 0 Hz to 10000 Hz are provided to observe the anticipated high frequency deviation. Also, plots from 0 Hz to 1600 Hz are provided to
observe the effect of the higher frequency deviations on the frequency range of interest of 0 Hz to 1600 Hz. Figure 71 and 72 show the results for a sensor width of 2", Figure 73 and 74 show the results for a sensor width of 5", and Figure 75 and 76 show the results for a sensor width of 7".

Figure 71. Width Validation at Location One for $d = 2$" From 0 Hz to 10000 Hz.
Figure 72. Width Validation at Location One for $d = 2''$ From 0 Hz to 1600 Hz.

Figure 73. Width Validation at Location One for $d = 5''$ From 0 Hz to 10000 Hz.
Figure 74. Width Validation at Location One for $d = 5''$ From 0 Hz to 1600 Hz.

Figure 75. Width Validation at Location One for $d = 7''$ From 0 Hz to 10000 Hz.
Figure 76. Width Validation at Location One for \( d = 7'' \) From 0 Hz to 1600 Hz.

These results show that, the one dimensional analytical calculations do in fact deviate as the frequency increases due to an increase in sensor width. From these plots, the most precise validation of the simulation using the one dimensional analytical calculations occurs when the sensor width does not exceed 1.5''.

The second sensor position used for the width validation study can be seen in Figure 77. This second position for width validation lends itself to supporting the conclusions drawn from the first position, and it removes any possibility of simulation conveniences by the sensor not being located symmetrically on the rectangular plate. This sensor length is the same as the sensor length used in the first position of Figure 68. In addition, the same progression of sensor widths of 1.5'', 2'', 5'', and 7'' is used as for position one.
Figure 77. Sensor in Position Two for Width Assumption Validation.

Figure 78 and 79 show the results for a sensor width of 1.5", Figure 80 and 81 show the results for a sensor width of 2", Figure 82 and 83 show the results for a sensor width of 5", and Figure 84 and 85 show the results for a sensor width of 7".

Figure 78. Width Validation at Location Two for \( d = 1.5" \) From 0 Hz to 10000 Hz.
Figure 79. Width Validation at Location Two for $d = 1.5''$ From 0 Hz to 1600 Hz.

Figure 80. Width Validation at Location Two for $d = 2''$ From 0 Hz to 10000 Hz.
Figure 81. Width Validation at Location Two for $d = 2''$ From 0 Hz to 1600 Hz.

Figure 82. Width Validation at Location Two for $d = 5''$ From 0 Hz to 10000 Hz.
Figure 83. Width Validation at Location Two for \( d = 5'' \) From 0 Hz to 1600 Hz.

Figure 84. Width Validation at Location Two for \( d = 7'' \) From 0 Hz to 10000 Hz.
Figure 85. Width Validation at Location Two for $d = 7''$ From 0 Hz to 1600 Hz.

Given these results, the same conclusions can be drawn, regarding the influence of width on the simulation validation, as those made from the results of position one. The deviation of the one dimensional analytical calculation from the simulation does occur at the higher frequencies. The results of position two also support that the most precise results for the simulation validation occur for a sensor with a maximum width of 1.5".

Having support for the use of a sensor width of 1.5" for the simulation validation using the one dimensional analytical calculation on the rectangular plate, the validation sensors of zero, first, and second order can be implemented. Figure 86 through Figure 88 show the three validation sensor lengths and their locations. These positions were chosen as they are arbitrary and do not possess any location symmetry with respect to either axis of the plate. These figures specifically show the length and position of the rectangular (zero order) sensor on the rectangular plate, however, these positions are the same for the
first and second order sensors. The first and second order sensors are positioned at the same locations on the x axis as seen in Figure 86 through Figure 88 using the same orientation as for the beam. The maximum $y$ values of these sensors is equal to the width $d$, where half of the sensor width value is located at the $y$ axis position as indicated in Figure 86 through Figure 88.

![Figure 86. Medium Sensor Length and its Position on the Rectangular Plate.](image)

![Figure 87. Long Sensor Length and its Position on the Rectangular Plate.](image)
Figure 88. Short Sensor Length and its Position on the Rectangular Plate.

Zero Order Sensor

Preliminary results for the comparison between the analytical and simulated predictions for the zero order sensors show excellent agreement. Figure 89 through Figure 91 show these comparative results.

Figure 89. Analytical Versus Simulated Results for the Short Rectangular Sensor.
Figure 90. Analytical Versus Simulated Results for the Medium Rectangular Sensor.

Figure 91. Analytical Versus Simulated Results for the Long Rectangular Sensor.
First Order Sensors

The preliminary results using the first order sensors for the comparison of the analytical and simulated predictions shows excellent agreement. Figure 92 through Figure 94 show the comparison results for the triangular sensor and Figure 95 through Figure 97 show the comparison results for the diamond shaped sensor.

Figure 92. Analytical Versus Simulated Results for the Short Triangular Sensor.
Figure 93. Analytical Versus Simulated Results for the Medium Triangular Sensor.

Figure 94. Analytical Versus Simulated Results for the Long Triangular Sensor.
Figure 95. Analytical Versus Simulated Results for the Short Diamond Sensor.

Figure 96. Analytical Versus Simulated Results for the Medium Diamond Sensor.
Figure 97. Analytical Versus Simulated Results for the Long Diamond Sensor.

**Second Order Sensors**

The preliminary results for the second order sensor comparison of the analytical and simulated predictions show excellent agreement. Figure 98 through Figure 100 show these comparative results.
Figure 98. Analytical Versus Simulated Results for the Short Quadratic Sensor.

Figure 99. Analytical Versus Simulated Results for the Medium Quadratic Sensor.
Figure 100. Analytical Versus Simulated Results for the Long Quadratic Sensor.
CHAPTER VI
EXPERIMENTAL VERIFICATION

Once the simulation had been verified by the analytical calculations, the actual sensors simulated and analytically modeled needed to be fabricated and experimentally evaluated.

Sensor Fabrication

The fabrication and placement of a sensor on a structure follows a systematic approach and is consistent for both the beam and the rectangular plate. The first step in the fabrication process is to plot the sensor shape and to scale it onto paper. Some plotted shapes might fit onto 8½ by 11 inch paper, however some are large enough to necessitate the use of a plotter from an AutoCAD® file. Once the shape has been plotted it can be cut out to form a paper template.

The second step in the fabrication process is to apply double sided tape (like carpet tape) onto the structure where the sensor is to be placed over a length and width slightly larger than the sensor itself. This provides for extra working room when performing the sensor shape tracing and etching steps (yet to be discussed). Figure 101 shows a double sided taping layout for a sensor applied to a rectangular plate.
Once the double sided tape has been adhered to the structure (the wax paper protecting the outer side of the tape should be left on at this point) the paper template should be laid onto the tape. The tape adhered to the structure should be observed in relation to the size and location of the sensor and, if necessary, should be trimmed within close proximity to the location and shape of the sensor. This allows room for electrical leads to extend from the sensor. Figure 102 shows the sensor template on the taping layout in an effort to size the taping layout accordingly.

The third step is to adhere the PVDF material to the structure by laying it on the exposed taping layout. As PVDF is an anisotropic material, it is important to align the appropriate directions of the film with the structure’s coordinate system. This is imperative so as to properly assign the stress/charge coefficients to the appropriate
coordinate system used for simulating and analytically modeling the sensor. Figure 103 shows the “grain-like” appearance of the roll axis of the material, which indicates the direction of the $e_{31}$ coefficient. The $e_{32}$ coefficient is orthogonal to this roll axis.

![Figure 103. Orientation of PVDF stress/charge Coefficients.](image)

The PVDF material is cut from a bulk roll and should be simply a rectangle whose length and width are slightly larger than the largest dimensions of the taping layout. This ensures that PVDF material is covering all the necessary area as required by the sensor template. Figure 104 shows a PVDF rectangle being applied to a taping layout and Figure 105 shows a PVDF rectangle completely adhered to a rectangular plate structure.

![Figure 104. A PVDF Rectangle Being Adhered to a Taping Layout.](image)
The fourth step in the fabrication process is to place the sensor template onto the PVDF material and trace the sensor shape from the template onto the PVDF material using a permanent marker as those produced by Sharpie®. Figure 106 shows a completed sensor shape traced onto the PVDF material.

Once the shape has been traced it should be filled in completely with the marker. Figure 107 shows a sensor shape being filled in with the marker, and Figure 108 shows the sensor completely filled in with marker.
The fifth step in the fabrication process is to make electrical leads that allow measurement equipment to take signal from the sensor. The leads are drawn, using the marker, onto the excess PVDF material that is not adhered to the structure. Figure 109 shows electrical leads being created for a sensor.
It is important to note that the lead is narrow at its neck (point of connection to the sensor shape) because not all of the taping layout would have been trimmed exactly to the outer bound of the sensor shape. As such this small portion of the electrical lead will be adhered to the structure and therefore contribute to the charge output of the sensor when excited (representing a deviation in sensor shape). By making it small, it is anticipated that its contribution to the overall output of the sensor is negligible. This step enforces the instructions of step three in the need to have the PVDF material slightly larger than the taping layout in an effort to have available material to create the leads. In addition, the need to trim the taping layout as close to the sensor as possible becomes obvious, otherwise the electrical lead material could be taped down to the structure rendering them inaccessible to measurement equipment.

The sixth step in the fabrication process is to etch the metalized surface electrode of the PVDF material, thereby creating the sensor. It is necessary to etch only one side of the PVDF material with the sensor shape to yield an effectively shaped sensor. Therefore, the sensor is etched using ferrous chloride (FeCl₃), which is brushed onto the metalized surface electrode around the sensor shape colored with the marker. The marker inhibits the ferrous chloride solution from etching the metalized electrode that defines the sensor. Any portion of the PVDF material covered by the marker is protected from the etching solution and will remain after the etching process is complete. Figure 110 shows this etching process.
Figure 110. Etching of the Metalized Surface Electrode on the PVDF Material.

This process is continued around the entire sensor including the electrical leads. Once the ferrous chloride solution has etched the sensor from the PVDF material, the excess solution is wiped away. The remaining material takes a copper tone color. At this point, the sensor is electrically isolated from the remaining portions of the PVDF material.

Figure 111 shows the excess ferrous chloride solution being removed with a paper towel.

Figure 111. Wiping Away of Excess Ferrous Chloride Etching Solution.

The seventh step in the fabrication process is to remove the excess PVDF material that has been etched from the sensor and clean off the electrical leads of the sensor. A razor blade is moved through the etched portion of the PVDF material around the sensor and its electrical leads cutting away the excess material. Figure 112 shows a razor blade cutting this excess material away.
Finally, the electrical leads of the sensor are cleaned of the marker colored on them by using isopropyl alcohol. This ensures good conductivity between the sensor leads and the measurement leads. Figure 113 shows the electrical leads being cleaned.

Figure 113. Cleaning of Sensor Electrical Leads.

Figure 114 shows a completed sensor with electrical leads for measurement equipment. At this point the sensor is ready to measure surface vibrations and can transmit this signal to a measurement device.

Figure 114. Completed Sensor.
In order to get the electrical signal from the sensor to the instrumentation, 28 gage wire is “electrically taped” to the sensor leads as seen in Figure 115. One end of the 28 gage wire is soldered onto a piece of 3M® copper tape whose adhesive backside can conduct electrical current, where one wire is adhered to the top side of the sensor lead and another to the bottom side of the sensor lead.

Figure 115. Copper Taped Leads.

The opposite end of this wire is fixed to a BNC adapter that allows the instrumentations co-axial cabling to receive the sensor signal, as seen in Figure 116.

Figure 116. BNC Adaptor for Electrical Wire Leads From Sensor.
Measurement Procedure

With an understanding of how to fabricate and place sensors onto structures and receive their signal, one is now ready to actually measure the charge output of a PVDF sensor due to vibrations of the structure.

The procedure for taking a measurement from a sensor requires equipment and sensor connections that are isolated from electrical interference. PVDF electrically exposed to the surrounding environment can behave as an “antenna” in that it can receive and conduct electromagnetic interference, or EMI. Such EMI can come from overhead lighting or electrical outlets. In an effort to eliminate this EMI from a sensor signal, a shield is constructed to conceal the sensor from EMI. Figure 117 shows a shielded sensor on a beam.

Figure 117. Shielded Sensor.

The shield is simply a rectangular piece of PVDF whose length and width are slightly larger than the length and width of the sensor. These shield dimensions ensure that the sensor is completely covered by the shield. The side of the shield that is in contact with the sensor surface is electrically isolated from the sensor by etching off the entire electrode on that side of the shield. The opposite side (the exposed side) is left with its entire electrode intact. Figure 118 shows the two sides of the same shield indicating the
etched and unetched side. The blue edges on the shield are permanent marker that protects the electrode of the unetched side from the FeCl₃ used to etch the side that comes in contact with the sensor surface.

Figure 118. Two Sides of a Shield.

The copper color of the bottom of the shield indicates that the electrode has been completely removed from that side of the shield. Just as an exposed sensor, the shield itself receives EMI, however, this EMI is conducted through the topside of the shield’s electrode (and not the sensors) to an electrical ground common to the sensor and the instrumentation. Figure 119 shows this electrical diagram of shield grounding.

Figure 119. Electrical Grounding Diagram.

This common grounding (ground in the electrical outlet) ensures that ground loop interference is not created in an attempt to alleviate the EMI. A ground loop provides for additional current flow in wiring that carries the signal to be measured, thereby
contaminating the signal (Yeager et al. 1998). Figure 120 shows a ground loop present due to more than one grounding connection between instrumentation (Keithley 1996).

Figure 120. Ground Loop Present.

Figure 121 shows the absence of a ground loop due to the sharing of a single ground connection by all instrumentation.

Figure 121. Ground Loop Not Present.

By having an understanding for the need to avoid ground loop currents while isolating the PVDF sensor from EMI, accurate measurements can be made using the sensor. Figure 122 shows a diagram of the experimental setup for recording and analyzing data from a sensor on the beam.
Figure 122. Experimental Setup on the Beam for Signal Recording and Analyzing.

The beam is excited and the input force is measured as detailed in Chapter IV. A sensor is connected to a PCB Model 462A Charge Amplifier in order to measure all the charge from the sensor, before the signal is recorded and analyzed by Fourier Analysis (Thompson 2002). The charge amplifier has very large input impedance on the order of 1 GΩ. This large input impedance prevents charge measured from the sensor from bleeding away (dissipation through instrumentation). As the charge output of a PVDF sensor is small, any portion of sensor output charge that bleeds away, greatly alters the charge output measured. The charge amplifier scales the charge measurement according to the sensitivity of the transducer wired to its input and the expected operating range. The PVDF manufacturer has documented this particular sample as having a sensitivity of
23 pC per Newton (see Appendix H) so the amp sensitivity is set to 23 pC per Newton (AMP 1994). According to the charge amp operations manual, any transducer sensitivity between 10 and 110 must have the output divided by a factor of 100. The operating range value is set to its lowest value of 50 Newton per volt. Therefore, a voltage that is sent to the TEAC tape recorder, and then to the DSA, (for frequency analysis), from a sensor yields a sensor-to-DSA sensitivity of \[
\frac{sensitivity \times range}{100} = 11.5 \text{ pC per volt.}
\] Therefore, the voltage displayed by the DSA is multiplied by this 11.5 pC per volt sensitivity to yield the charge value.

The electrical connections and setup for experimental sensors and their instrumentation for the rectangular plate follow the same setup as described for the beam. Figure 123 shows the experimental setup and signal flow diagram for the plate.

![Diagram of experimental setup](image)

**Figure 123.** Experimental Setup on the Plate for Signal Recording and Analyzing.
Results For Beams

Figure 124 shows a diagram of experimental sensor shapes and their positions on the beam used to verify the simulation. Each shape and position was randomly selected from those used in the analytical verification detailed in Chapter V.

![Diagram of Sensor Shapes](image)

Figure 124. Diagram of Sensors for Experimental Beam Simulation Verification.

Figure 125 through Figure 128 show the experimental results for the sensor shapes of Figure 124. These plots show the comparison of the experimental sensor results and the simulated results of the same sensor.
Figure 125. Experimental and Simulated Triangular Beam Sensor Results.

Figure 126. Experimental and Simulated Diamond Beam Sensor Results.
Figure 127. Experimental and Simulated Rectangular Beam Sensor Results.

Figure 128. Experimental and Simulated Quadratic Beam Sensor Results.
Figure 125 through Figure 128 show good correlation between the simulated and the experimental sensor responses. The frequency location of structural resonances is consistent for both plots, however, the magnitude deviates between the two across the frequency range since the experimental results are always lower. However, the manufacturers sensitivity value could be off by a certain amount which could explain the differences present in this data.

Results For Rectangular Plate

As for the beam, the simulation of sensors on the rectangular plate is experimentally verified. As such, a random selection of sensor shapes and positions were chosen from those used in the analytical verification of the sensor simulation. Figure 129 through Figure 132 show the experimental sensor shapes and positions used for the experimental verification of the simulated results.

![Diagram of a rectangular plate with a diamond-shaped sensor](image)

Figure 129. Experimental Diamond Shaped Sensor for Rectangular Plate Verification.
Figure 130. Experimental Triangular Sensor for Rectangular Plate Verification.

Figure 131. Experimental Rectangular Sensor for Rectangular Plate Verification.
Figure 132. Experimental Quadratic Sensor for Rectangular Plate Verification.

Figure 133 through Figure 136 show the experimental results for the simulated sensors in Figure 129 through Figure 132. The experimental results are plotted with the simulated results to show the comparison.

Figure 133. Experimental and Simulated Plate Diamond Sensor Results.
Figure 134. Experimental and Simulated Plate Triangular Sensor Results.

Figure 135. Experimental and Simulated Plate Rectangular Sensor Results.
Figure 136. Experimental and Simulated Plate Quadratic Sensor Results.

The experimental results for the plate show very limited correlation in shape and magnitude with the simulation. The following discussion of errors attempts to address these correlation limitations in the experimental plate data as well as in the experimental beam data.

Discussion of Errors

It can be seen from the results that although the simulation does predict the frequency location of structural resonances as the experimental sensors show, it deviates in magnitude from the experimental results. This deviation in magnitude between the simulated and the experimental results is attributed to several factors.

Both the simulated and the experimental results rely on the manufacturers commercially published values of the stress/charge coefficients, $e_{mn}$, for the PVDF material. The simulation uses these coefficients as seen in the numerical formulations of...
Eq. (13) and Eq. (14). The experimental results depend on the stress/charge coefficients when setting up the charge amplifier. The charge amplifier requires the user to specify the sensitivity of the transducer wired to its inputs, as well as the expected operating range. However, it is felt that these values vary as a function of position within the material. This is analogous to the variability observed in the material modulus of metals within a random selection of steel samples thought to be identical (Baumeister 1978). Therefore, as the charge amplifier sensitivity cannot account for this variability, the experimental and simulated results will differ. In addition, an assumption was made that the spatial sensitivity $F(x,y)$ is constant throughout the material. However, it is possible that this is not the case which can provide for sensitivity variability within the material. This in turn provides for variability in the results.

In addition, the PVDF material is anistropic, or rather the piezoelectric properties of the material vary with respect to the material direction. This is a concern only for two dimensional structures where straining occurs in two directions. As discussed previously, the stress/charge coefficient $e_{31}$ is the piezoelectric coefficient acting in the $3$ plane in the $1$ direction, and the stress/charge coefficient $e_{32}$ is the piezoelectric coefficient acting in the $3$ plane in the $2$ direction. This is tensor notation representative of the material coordinates of Figure 137.
If the 1 and the 2 directions are not oriented parallel to the structure’s $x$ and $y$ directions respectively, then a skew angle is developed between the $e_{31}$ and $e_{32}$ coefficients yielding a non-zero $e_{36}$ component, where the 6 direction is a rotation about the 3 axis (a rotation in the 12 plane). However, if the 1 and 2 directions are oriented parallel to the structure’s $x$ and $y$ directions, then only two stress/charge coefficients are present. The charge amplifier used to obtain results from the experimental sensors is calibrated to the sensitivity of the 1 direction only. Therefore, the experimental sensor signals are calibrated to the 1 direction only. It is important to remember that according to Eq. (12), the sensor output is in response to strains in two directions with two different sensitivities. In addition, the manufacturer does not commercially publish $e_{mn}$ data for the 32 direction, however, they claim that it is typically 10% of the 31 direction. The case of multiple sensitivities lends itself to being misrepresented by single sensitivity calibrations.

With regard to skew angle, it is possible that the 31 direction is not perfectly parallel to the roll axis of the material as it is assumed to be. As such, the machine edge
of a roll of PVDF purchased could possibly be parallel not to $e_{31}$ but rather to some skew angle of $e_{36}$. Furthermore, it is not known whether $e_{31}$ runs perfectly straight along a length of PVDF. Each of these unknowns could account for the experimental deviations from theoretical simulated results.

The phase angle between the simulated and the experimental results can be brought into phase by multiplying the experimental data by a $-1$. This $-1$ can come from the polarity of the film with respect to the sign associated with positive transverse vibration displacements. Knowing the polarity of the films surface (top or bottom) with respect to positive or negative bending can provide for experimental and simulated results that are in phase with each other without having to compensate this by multiplying by a $-1$. 

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CHAPTER VII
USE OF SIMULATION FOR EVALUATION OF FABRICATION ERRORS

The numerical techniques of the simulation offer the flexibility to manipulate the shape vectors that describe the sensors on the structure such that the sensors appear to possess errors. These errors can be practically occurring shaping or placement errors.

Results for Shaping Errors

The simulation is used to evaluate the influence of shaping errors associated with discontinuities in the sensor shape. As such, a rectangular (zero order) sensor on the beam at \( \left( 0, \frac{1}{2} L \right) \) is simulated having a “notch-like” error on its edge. The notch is centered at the midpoint of the sensor and extends into the sensor from the top. The depth of the notch \( n \) is varied from 2mm, to 5mm, to 7.5mm, to 10mm, where the width of the notch \( w \) is held constant at 69.3 mm. Figure 138 shows where this notch error appears and the corresponding dimensions.

![Figure 138. Shaping Error in Sensor Shape.](image-url)
Figure 139 shows the simulated sensor charge output results with the 2 mm notch depth error in comparison to the same sensor charge output results without a notch error. In addition, Figure 140 shows the simulated percentage difference (linear) between the results for the 2 mm notch depth error and the same sensor without a notch error. The same plots are developed for the 5 mm, the 7.5 mm, and the 10 mm notch depths. Figures 141-142 show these results using the 5 mm notch depth, Figures 143-144 show these results using the 7.5 mm notch depth, and Figures 145-146 show these results using the 10 mm notch depth. As anticipated, as notch area increases so does the difference in sensor charge output from charge output of the same sensor without an error. In addition, Figure 147 shows an experimental comparison of the rectangular sensor charge output with the 10 mm notch error and without the 10 mm notch error.

Figure 139. Charge Output Comparison for 2 mm Notch Depth.
Figure 140. Percentage Difference for 2 mm Notch Depth.

Figure 141. Charge Output Comparison for 5 mm Notch Depth.
Figure 142. Percentage Difference for 5 mm Notch Depth.

Figure 143. Charge Output Comparison for 7.5 mm Notch Depth.
Figure 144. Percentage Difference for 7.5 mm Notch Depth.

Figure 145. Charge Output Comparison for 10 mm Notch Depth.
Figure 146. Percentage Difference for 10 mm Notch Depth.

Figure 147. Experimental Results with and without a Notch Error of 10 mm.
The simulation was also used to evaluate the influence of a shaping discontinuity within the bounds of the sensor shape itself. The same rectangular (zero order) sensor used in the previous shaping error evaluation is used in this evaluation as well. The error simulation uses a square inner area error as seen in Figure 148. The error square is evaluated for sides of length \( s \) equal to 5 mm, 10 mm, 15 mm, and 20 mm each positioned along the horizontal middle line of the sensor at one-eighth of the sensors length. Figure 140 shows where this error is evaluated.

![Figure 148. Shaping Error in Sensor Inner Area.](image)

Figure 149 shows the comparison of the simulated sensor charge output results with the 5 mm error square in comparison to the same sensor charge output results without an error square. In addition, Figure 150 shows the percentage difference (linear) between the results for the sensor with a 5 mm error square and the same sensor without an error square. Figures 151-152 show these results for the 10 mm error square, Figures 153-154 show these results for the 15 mm error square, and Figures 155-156 show these results for the 20 mm error square. As with the notch error, the more area the error square consumes, the greater the difference from the same sensor without an error. In addition, Figure 157 shows experimental results for the rectangular sensor charge output with a 20 mm error square and without a 20 mm error square.
Figure 149. Charge Output Comparison for 5 mm Error Square.

Figure 150. Percentage Difference for 5 mm Error Square.
Figure 151. Charge Output Comparison for 10 mm Error Square.

Figure 152. Percentage Difference for 10 mm Error Square.
Figure 153. Charge Output Comparison for 15 mm Error Square.

Figure 154. Percentage Difference for 15 mm Error Square.
Figure 155. Charge Output Comparison for 20 mm Error Square.

Figure 156. Percentage Difference for 20 mm Error Square.
Figure 157. Experimental Results with and without a 20 mm Error Square.

In conclusion, these shaping errors do not affect the frequency at which a resonance occurs nor the magnitude which it posses below 900 Hz. From an active control standpoint, the conclusions that can be drawn from these results show that the effectiveness of a controller is not reduced. This is due to the fact that the controller is concerned about the frequency at which resonance occurs (frequency for largest sound radiation) and the magnitude it posses (quantity of sound radiated). The controller is not especially concerned with off resonant activities of the structure, which seems to be the only affected portion of PVDF sensing due to shaping errors.

Results for Placement Errors

The simulation was also used to evaluate the influence of error in sensor placement using the same sensor as in the previous two shaping error evaluations on the beam. The sensor position was offset by 5 mm, 10 mm, 15 mm, and 20 mm in the
positive x direction (see Figure 16). Figure 158 shows the sensor charge output for a 5 mm offset in comparison to the same sensor without any offset in its placement. Figure 159 shows the percentage difference (linear) between the offset sensor charge output and the same sensor without an offset in its position. Figure 160-161 show the same plots for the 10 mm offset, Figure 162-163 show the same plots for the 15 mm offset, and Figure 164-165 show the same plots for the 20 mm offset. As seen before, the larger the placement error the greater the difference there exists from the same sensor without an error. In addition, Figure 166 shows experimental results for the same rectangular sensor with a placement error of 20 mm and without a placement error of 20 mm.

Figure 158. Charge Output Comparison for 5 mm Offset.
Figure 159. Percentage Difference for 5 mm Offset.

Figure 160. Charge Output Comparison for 10 mm Offset.
Figure 161. Percentage Difference for 10 mm Offset.

Figure 162. Charge Output Comparison for 15 mm Offset.
Figure 163. Percentage Difference for 15 mm Offset.

Figure 164. Charge Output Comparison for 20 mm Offset.
Figure 165. Percentage Difference for 20 mm Offset.

Figure 166. Experimental Results with and without a 20 mm Placement Error.
As predicted by Clark et. al. 1996, placement errors effect the modal contribution to sensor charge output. As the location of a sensor is altered, the modal contribution at the new location, in comparison to the original location, is different. Therefore, the results obtained are in agreement with conclusions drawn from other works. As for shaping errors, placement errors do not affect the location of a resonant frequency, however, they do affect the magnitude at the location of a resonance. Placement errors provide for an inaccurate prediction of the magnitude at which a resonance occurs. From an active control point of view, the effectiveness of a controller to attenuate radiated sound is reduced due to the ill predicted sound radiation. These shaping and placement type errors posses the ability to affect controller performance.

In order to determine the validity the of experimental error results, a repeatability study was performed to determine the variability in experimental measurements. Four rectangular (zero order) sensors were placed at \( \left( 0, \frac{1}{2}L \right) \) on the beam length and their charge output was measured. Each of the four sensors was made from a new piece of PVDF using the techniques outlined in Chapter VI, and each time a sensor was to be measured, all equipment was shut down and reinitialized to measure the charge output. Therefore, using new sensors each time, the variability in the experimental results accounts for all variability possible in sensor fabrication, material properties, and data acquisition. Figure 167 shows the variability results for the rectangular sensors. Figure 168 shows the percentage difference (linear) between the sensor trials each relative to trial one.
Figure 167. Repeatability Study Results.

Figure 168. Percentage Difference of Repeatability Study Relative to Trial One.
The results of the study show that the deviation between a sensor with an error and the same sensor without an error is related to the presence of deviations (edge, notch, placement, etc.) added to the sensor. The error deviation is outside of repeatable measurements. Figures 169-171 show the experimental errors of shape and placement relative to the maximum and minimum range from the repeatability study. The upper and lower limits of the range are the maximum and minimum values at each frequency taken from all four plots at each frequency of Figure 161.

![Figure 169. Experimental Notch Relative to the Repeatability Range.](image)

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Figure 170. Experimental Notch Relative to the Repeatability Range.

Figure 171. Experimental Notch Relative to the Repeatability Range.
CHAPTER VIII
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The ultimate goal of this research was to develop a numerical simulation for the prediction of charge output for the piezoelectric material PVDF (Polyvinylidene Fluoride) based on the surface displacements it experiences on a vibrating beam and rectangular plate. The development of a simulation was motivated by the need to determine sensor output based on its shape without having to make the sensor one thinks will measure volume displacement. In addition to sensor shape and placement optimization, such a simulation can provide for the prediction of sensor fabrication errors in shaping and placement. According to the review of literature, analytical predictions of sensor placement error exist only for beams with pinned conditions, however, the simulation can predict shaped PVDF sensor output based on shaping and placement errors for any class of boundary conditions. This is accomplished by simply altering points in the sensor shape vector and integration limits.

Using numerical tools available in the mathematical software Matlab®, experimental structural displacements were used to predict sensor charge output based on the sensor shape and position on the structure. The simulation has excellent correlation with corresponding analytical predictions using experimental point displacements for the beam. Having analytically verified the simulated prediction of sensor charge output, a variety of sensor shapes were fabricated for the experimental beam where their
experimental response was compared against the simulated. Excellent agreement was found when comparing the trends of the experimental output of the sensors with that of their simulated output for the beam. The simulated and experimental sensors both predict structural resonances (bending modes) at the same frequencies, however, the magnitude deviates. The discrepancies in magnitude are attributed to the variability of the anistropic stress/charge coefficients, $e_{mn}$, of the PVDF material. The variability of these values is analogous to the variability of the material modulus of structural material throughout a structural member. However, no prediction of the stress/charge coefficient variability as a function of position on a sensor is available. In addition, using shape and position errors, the simulation was able to show a deviation in misshaped and ill positioned sensor output from a corresponding correctly shaped and positioned sensor.

For the plate, there exists excellent correlation between the simulation and the corresponding analytical predictions using experimental point displacements, just as with the beam. However for the experimental sensors on the plate, significant deviations were encountered when compared to the simulation.

Recommendations for Future Work

The literature review had shown that shaped PVDF sensors could be used for the measurement of total or local surface volume displacement in active noise control and active structural acoustic control settings. However, these sensors were required to incorporate either the boundary conditions of the structure being sampled, or they required some form of a supplementary controller algorithm or other sensor to assist in the measurement of the structures total or local volume displacement. Therefore, further
research work could concentrate on the development of shaped PVDF sensors that could measure a structure's local or total volume displacement without any supplements. In addition, active controllers could implement sensors that could locally sample a structure's volume displacement while providing for effective active noise suppression when compared to total volume displacement sampling. As such, a prediction of shaped PVDF sensor output can provide an opportunity to solve for such sensor shapes and their locations on structures.

As identified in the review of relevant literature, there exists the need to correlate sensor shape and position on a structure such that it measures the volume displacement without the use of supplementary controller algorithms or sensors. As such, the simulation could solve for the sensor shape based on a known volume displacement signal as a function of frequency. Such a signal can be determined using techniques outlined in the experimental setup and procedural discussions of this work. Matlab® offers optimization tools that would be useful in determining the shape and position of a sensor given a desired output. As discussed previously, there exists the opportunity for local volume displacement sensors to provide noise suppression that is as effective as that using total volume displacement sensing. Given this, a total or local volume displacement signal could be substituted into an optimization routine to produce a vector of coordinate positions that describe the area over which the corresponding sensor would cover. This allows this sensor position and shape to be realized experimentally with confidence.
There are many unknown characteristics of PVDF that remain possible contributors to deviation between simulated and experimental results. A thorough investigation of the PVDF material properties should be performed. Such an investigation should address the variability of the stress/charge coefficients as a function of position on a film's surface area. In addition, the investigation should consider the consistency of the orientation of the $31$ and the $32$ piezoelectric coefficients along the length of the PVDF film. Previously, attempts were made to determine these coefficients using tensile specimens. However, this was done before the issues with signal confidence were addressed such as shielding. Having the ability to achieve a confident signal from a sensor, these tensile specimens should be revisited in an effort to experimentally determine the $e_{mn}$ coefficients.

In an effort to address the deviations between the simulated and experimental results for the plate, the second partial derivative calculations (in the simulation) of the transverse vibration deflections of the plate should be addressed. As the beam results were successful, this points to the contribution of the $y$ direction. The beam results did not use contributions in the $y$ directions and this is the only new contribution to the simulation to accommodate two dimensions.
REFERENCES

AMP. (1994). *Basic Design Kit*. AMP Incorporated, P/N 0-1004308-0 REV:E.


The following flow diagram helps to show what the user must input and what the simulation develops and saves when using the simulation files.

**USER INPUTS**

All experimental accelerometer data from DSA:
- Xpoint1 – Xpoint(end)
- Ypoint1 – Ypoint(end)
- Zpoint1 – Zpoint(end)

**MATLAB EXECUTABLE FILES**

- **Filename:** Loader.m
  - **APPENDIX B**

- **Filename:** Interpolation.m
  - **APPENDIX C**

- **Filename:** Second_Derivative.m
  - **APPENDIX D**

**GENERATED OUTPUTS**

- Displacement Arrays
- Filenames:
  - top_Dx
  - top_Dy
  - top_Dz

- Displacement Arrays
- Filenames:
  - Dx_interp
  - Dy_interp
  - Dz_interp

- Displacement Arrays
- Filenames:
  - fxx fx ux
  - fyy fy vy

**Position Coordinates for the Sensor Shape**
- \( a_1 \) - starting x coordinate
- \( a_2 \) - ending x coordinate
- \( c_m \) - center height coordinate
- \( c_1 \) - starting y coordinate
- \( c_2 \) - ending y coordinate
- \( F(x) \) - sensor shape eqn.

**Upper and Lower Sensor Vectors**
- \( \text{shape1\_sym (upper)} \)
- \( \text{shape2\_sym (lower)} \)
- \( a_1 \) - starting x coordinate
- \( a_2 \) - ending x coordinate
- \( c_m \) - center height coordinate
- \( c_1 \) - starting y coordinate
- \( c_2 \) - ending y coordinate
USER INPUTS | MATLAB EXECUTABLE FILES | GENERATED OUTPUTS
---|---|---
| | | Displacement Arrays Filenames:
| | | $f_{xx}$ $f_{x}$ $u_{x}$
| | | $f_{yy}$ $f_{y}$ $v_{y}$
| | Upper and Lower Sensor Vectors
| | $shape1\_sym$ (upper)
| | $shape2\_sym$ (lower)
| | $a_{1}$ - starting $x$ coordinate
| | $a_{2}$ - ending $x$ coordinate
| | $c_{m}$ - center height coordinate
| | $c_{1}$ - starting $y$ coordinate
| | $c_{2}$ - ending $y$ coordinate
| | Filename: $Integrate.m$
| | | Axial Integration Result Filenames
| | | $bending\_x$
| | | $inplane\_x$
| | | $bending\_y$
| | | $inplane\_y$
| | Experimental Sensor Being Simulated
| | $Appropriate\_filename$
| | | Filename: $Sensor\_Output.m$
| | | Comparison Plot of Simulated and Experimental Sensor

APPENDIX F

APPENDIX G
Appendix B

Matlab® Code for Point Loader File
The follow Matlab® code loads in the experimental structural accelerometer per unit force FRF measurements and scales them with the proper calibration values. These FRF data are then used to calculate the vibration displacements (meters). Once the vibration displacements have been calculated, the arrays containing these displacement values are reshaped to represent the actual sampling grid/array used on the structures surface. Finally, the reshaped vibration displacement arrays are saved for further processing.

```matlab
clear all
clc
xcorrect=(9.81/0.01029)/((1/0.05211)*4.448222); %calibration correction factor to m/s^2/N
ycorrect=(9.81/0.00979)/((1/0.05211)*4.448222); %calibration correction factor to m/s^2/N
zcorrect=(9.81/0.00996)/((1/0.05211)*4.448222); %calibration correction factor to m/s^2/N
NumberOfpoints=91;

%--------Load accelerometer/force transfer functions for all three directions--------

% load in x direction
Dx=zeros(1600,91);
for i=1:NumberOfpoints
    FileToLoad=strcat('Xpoint',num2str(i));
    LoadindCommand=['load',' ',FileToLoad];
    eval(LoadindCommand);
    F=02i1x;
    Dx(:,i)=-(xcorrect*02i1(2:1601))./((2*pi*F(2:1601)).^2);
end

% load in y direction
Dy=zeros(1600,91);
for i=1:NumberOfpoints
    FileToLoad=strcat('Ypoint',num2str(i));
    LoadindCommand=['load',' ',FileToLoad];
    eval(LoadindCommand);
    F=02i1x;
    Dy(:,i)=-(ycorrect*02i1(2:1601))./((2*pi*F(2:1601)).^2);
end

% load in z direction
Dz=zeros(1600,91);
for i=1:NumberOfpoints
    FileToLoad=strcat('Zpoint',num2str(i));
    LoadindCommand=['load',' ',FileToLoad];
    eval(LoadindCommand);
    F=02i1x;
    Dz(:,i)=-(zcorrect*02i1(2:1601))./((2*pi*F(2:1601)).^2);
end
```

150
F = 0.211x;
Dz(:,i) = -(z_correct*0.211(2:1601))./((2*pi*F(2:1601)).\(^2\));
end

clear FileToLoad LoadingCommand o2i1 o2i1x i n j

% Reshape vibration displacement arrays to match the size of the beam
for i = 1:1:1250
    top_Dx(:,i) = reshape(Dx(:,i),7,13);
    top_Dy(:,i) = reshape(Dy(:,i),7,13);
    top_Dz(:,i) = reshape(Dz(:,i),7,13);
end

save top_Dx top_Dx
save top_Dy top_Dy
save top_Dz top_Dz
Appendix C

Matlab® Code for Interpolation
The following Matlab® code loads in the reshaped vibration displacement arrays and interpolates them according to the sufficient size necessary. The resulting interpolated vibration displacement arrays are save for further processing. The last interpolation scheme provides a three dimensional plotting of the transverse vibration displacement to allow one to view the resulting interpolated array for convenience.

clear
clc

%Look at all the frequencies for the interpolated displacement matrix
reslx=1/3;
resly=1/5;
fit_type='cubic';
[x,y]=meshgrid(1:1:13,1:1:7);
[fx,fy]=meshgrid(1:reslx:13,1:resly:7);
save reslx reslx
save resly resly

%X-axis
load top_Dx
Vx=zeros(6*(1/resly)+1,13*(1/reslx)+1,1250);
for g=1:1250
    disp(['frequency ',num2str(8*g)])
    Vx=interp2(x,y,top_Dx(:,:,g),fx,fy,fit_type);
    Dx_interp(:,:,g)=Vx(:,:,);
end
save Dx_interp Dx_interp
clear top_Dx Dx_interp Vx

%Y-axis
load top_Dy
Vy=zeros(6*(1/resly)+1,13*(1/reslx)+1,1250);
for g=1:1250
    disp(['frequency ',num2str(8*g)])
    Vy=interp2(x,y,top_Dy(:,:,g),fx,fy,fit_type);
    Dy_interp(:,:,g)=Vy(:,:,);
end
save Dy_interp Dy_interp
clear top_Dy Dy_interp Vy

%Z-axis
load top_Dz
Vz=zeros(6*(1/resly)+1,13*(1/reslx)+1,1250);
for g=1:1250
    Vz=interp2(x,y,top_Dz(:,:,g),fx,fy,fit_type);
    mesh(real(Vz))
title(['Frequency = ',num2str(8*(g)),' (Hz)'])
xlabel('x'); ylabel('y'); zlabel('z');
pause(0.001)
Dz_interp(:,g)=Vz(:,);
end
save Dz_interp Dz_interp
clear top_Dz Dz_interp Vz
Appendix D

Matlab® Code for Second Partial Derivatives
The following Matlab® code loads in the interpolated reshaped vibration
displacement data arrays and calculates all the first and second partial derivatives. The
first and second partial derivatives are save for further processing.

clear all
cIc

load Dx_interp
load Dy_interp
load Dz_interp
load reslx
load resly

dx=610e-3/(1/reslx*12);  %finite length in x direction
dy=350e-3/(1/resly*6);  %finite length in y direction

%Calculate second order partial differentials of the displacement
for f = 1:1:1250
    disp([num2str(f), ' of 1250'])
    [fx(:, :, f), fy(:, :, f)] = gradient(Dz_interp (:, :, f), dx, dy);
    fxx(:, :, f) = gradient(fx(:, :, f), dx);
    [fxy(:, :, f), fyy(:, :, f)] = gradient(fy(:, :, f), dx, dy);
    [ux(:, :, f), uy(:, :, f)] = gradient(Dx_interp (:, :, f), dx, dy);
    [vx(:, :, f), vy(:, :, f)] = gradient(Dy_interp (:, :, f), dx, dy);
end

save fxx fxx
save fyy fyy
save fx fx
save fy fy
save ux ux
save vy vy
Appendix E

Matlab® Code for Sensor Shape Generation
The following Matlab® code generates the sensor shape vector discretized to the interpolated reshaped vibration displacement array. For every shape and its location on a structure that is to be simulated, the function must be manually entered. For the actual code present, a diamond shape is generated. It is important to note that the code does not automatically create the shape vector according to the discretized surface of the structure. At this point, the user must compensate for this in the independent variable $x$ or $y$ direction vector of the shape function. The shape vectors and shape position coordinates are saved for further use in the integration routine.

```matlab
clear all
clc
Lx=610e-3; %Width in x
Ly=350e-3; %Length in y

%Calculate the sensor shape as a function of y
a1=1*Lx/2;
a2=7*Lx/9;
cm=1*Ly/8;
c1=cm-(.75*.0254);
c2=cm+(.75*.0254);
xshape_l=[a1:(Lx/36):(a1+a2)/2];
xshape_r=[(a2+a1)/2:(Lx/36):a2];
shapel_l(1,:size(xshape_l,2))=(c2-cl)/(a2-al)*(xshape_l-al)+cm;
shape2_l(1,:size(xshape_l,2))=-(((c2-cl)/(a2-al))*(xshape_l-al))+cm;
shapel_r(1,:size(xshape_r,2))=(c2-cl)/(al-a2)*(xshape_r-a2)+cm;
shape2_r(1,:size(xshape_r,2))=-(((c2-cl)/(a2-a2))*(xshape_r-a2))+cm;

%Plot the test shape
plot(xshape_l,shapel_l,'k-',...
xshape_l,shape2_l,'r-',...
xshape_r,shapel_r,'b-',...
xshape_r,shape2_r,'g-');grid

xlabel('x');ylabel('y');title('Arbitrary Shapes on a Structural Surface');
legend('F_1(x) Left','F_2(x) Left','F_1(x) Right','F_2(x) Right');

save shape1_r shape1_r
save shape1_l shape1_l
save shape2_r shape2_r
save shape2_l shape2_l
```

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save xshape_r xshape_r
save xshape_l xshape_l
save a1 a1
save a2 a2
save c1 c1
save c2 c2
save cm cm
save Lx Lx
save Ly Ly
Appendix F

Matlab® Code for Integration of Second Partial Derivatives
The following Matlab code calculates the integral of the second partial derivatives (calculated previously) over the surface the sensor enclosed by the shape vector (calculated previously). It loads in the previously save first and second partial derivatives. Then, it integrates each displacement array for every frequency over the surface of the sensor defined by the shape vector that is loaded as well. The resulting arrays are save for further processing. This particular example of an integration routine integrates the diamond shaped sensor.

clear all
clc

global X Y fxx fyy ux vy;

load fxx
load fyy
load ux
load vy
load reslx
load resly
load shape1_sym_r
load shape2_sym_r
load shape1_sym_l
load shape2_sym_l
load xshape_r
load xshape_l
load Lx
load Ly

yr=[0:Ly/(6*5):Ly];
xr=[0:Lx/(12*3):Lx];
[X,Y]=meshgrid(xr,yr);
for f=1:1:1250
  disp(['Frequen cy No. ',num2str(f), ' of 1250 Diamond Long 1 of 2'])
  for e=1:1:size(xshape_r,2)-1
    bending_strip_x_l(f,e,:)=dblquad(@(del2x,xshape_l(1,e),xshape_l(1,e+1),shape2_sym_l(1,e+1),shape1_sym_l(1,e+1),1e-6,'quad',f);
    inplane_strip_x_l(f,e,:)=dblquad(@(delux,xshape_l(1,e),xshape_l(1,e+1),shape2_sym_l(1,e+1),shape1_sym_l(1,e+1),1e-6,'quad',f);
    bending_strip_y_l(f,e,:)=dblquad(@(del2y,xshape_l(1,e),xshape_l(1,e+1),shape2_sym_l(1,e+1),shape1_sym_l(1,e+1),1e-6,'quad',f);
    inplane_strip_y_l(f,e,:)=dblquad(@(delvy,xshape_l(1,e),xshape_l(1,e+1),shape2_sym_l(1,e+1),shape1_sym_l(1,e+1),1e-6,'quad',f);

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for f=1:1:1250
disp(['Frequency No. ',num2str(f),' of 1250 Diamond Long 2 of 2'])
for e=1:1:size(xshape_r,2)-1
bending_strip_x_r(f,e,:)=dblquad(@del2x,xshape_r(l,e),xshape_r(l,e+1),shape2_sym_r(l,e),shapel_sym_r(l,e),le-6,'quad',f);
inplane_strip_x_r(f,e,:)=dblquad(@delux,xshape_r(l,e),xshape_r(l,e+1),shape2_sym_r(l,e),shapel_sym_r(l,e),le-6,'quad',f);
bending_strip_y_r(f,e,:)=dblquad(@del2y,xshape_r(l,e),xshape_r(l,e+1),shape2_sym_r(l,e),shapel_sym_r(l,e),le-6,'quad',f);
inplane_strip_y_r(f,e,:)=dblquad(@delvy,xshape_r(l,e),xshape_r(l,e+1),shape2_sym_r(l,e),shapel_sym_r(l,e),le-6,'quad',f);
end
end
bending_x=sum(bending_strip_x_r,2)+sum(bending_strip_x_l,2);
inplane_x=sum(inplane_strip_x_r,2)+sum(inplane_strip_x_l,2);
bending_y=sum(bending_strip_y_r,2)+sum(bending_strip_y_l,2);
inplane_y=sum(inplane_strip_y_r,2)+sum(inplane_strip_y_l,2);
save bending_x bending_x
save inplane_x inplane_x
save bending_y bending_y
save inplane_y inplane_y

In order to integrate an array of data, a subroutine (or m file) is utilized for all first and second partial derivatives for both the x and y direction. The following code is the m file routine is supplement the integration routines.

The second partial with respect to x

function w=del2x(x,y,f)
global X Y fxx;
w=interp2(X,Y,fxx(:,:,f),x,y,'cubic');

The second partial with respect to y

function w=del2y(x,y,f)
global X Y fyy;
w=interp2(X,Y,fyy(:,:,f),x,y,'cubic');

The first partial with respect to x

function w=delux(x,y,f)
global X Y ux;
w=interp2(X,Y,ux(:,:,f),x,y,'cubic');

The first partial with respect to $y$

function w=delvy(x,y,f)
global X Y vy;
w=interp2(X,Y,vy(:,:,f),x,y,'cubic');
Appendix G

Matlab® Code for the Determination of Sensor Charge Output
The following Matlab® code calculates the simulated sensor charge output. It loads in the previously saved solution arrays to the integration routine. It multiplies the x direction data by $e_{31}$ and multiplies the y direction data by $e_{32}$. Then it multiplies the bending terms by the bending moment arm and finally the sum of each of these is computed. In addition to computing the simulated sensor charge output, it also loads in the corresponding experimental sensor that was simulated. The experimental data is scaled with the proper calibration values and then both the simulated sensor charge output and the corresponding experimental sensor charge output are plotted together to observe their correlation. For convenience, the simulated sensor charge output is calculated for the combination of in-plane and bending terms, for only bending terms, and for only in-plane terms depending on what the user needs to compare to the experimental data.

```matlab
clear all
clc
load bending_x
load bending_y
load inplane_x
load inplane_y

e31=.216;
e32=.0216;
hp=(.25*.0254);
hs=28e-6;
arm=-(hp+hs)/2;

Approx_Sol_x=e31.*(arm*bending_x+inplane_x);
Approx_Sol_y=e32.*(arm*bending_y+inplane_y);
q_total=Approx_Sol_x+Approx_Sol_y;
q_itotal=e31.*inplane_x+e32.*inplane_y;
q_btotal=arm*(e31.*bending_x+e32.*bending_y);

%sExperimental Sensors with PCB charge amplifier
%charge amp cal
sensitivity=23e-12/100; %C/N
range=50; %N/V
```

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cal=-sensitivity*range*10; %C/V with TEAC gain

load Pltdia
F=o2i1x;
pvdf_top=o2i1;
clear o2i1 x o2i1

pvdf=cal*pvdf_top/((1./0.05211)*4.448222*.25); %Calibration of PVDF FRF signal C/N with TEAC gain

% Compare Sensor to Simulation
subplot(2,1,1),plot(F(2:1251,:), (180/pi*(angle(pvdf(2:1251,:)))),'k-', ... 
    F(2:1251,:), (180/pi*(angle(q_btotal))),'r-');grid
title('Simulated and Experimental Charge Outputs - Diamond');
xlabel('Frequency (Hz)');ylabel('Phase Angle (degrees)');
axis([0 10000 -200 200])
subplot(2,1,2),plot(F(2:1251,:), 20*log10(abs(pvdf(2:1251,:))),'k-', ... 
    F(2:1251,:), 20*log10(abs(q_btotal)),'r-');grid
xlabel('Frequency (Hz)');ylabel('Charge Output dB');
legend('Experimental','Simulated');
axis([0 10000 -250 -180])
pause

subplot(2,1,1),plot(F(2:1251,:), (180/pi*(angle(pvdf(2:1251,:)))),'k-', ... 
    F(2:1251,:), (180/pi*(angle(q_btotal))),'r-');grid
title('Simulated and Experimental Charge Outputs - Diamond');
xlabel('Frequency (Hz)');ylabel('Phase Angle (degrees)');
axis([0 1600 -200 200])
subplot(2,1,2),plot(F(2:1251,:), 20*log10(abs(pvdf(2:1251,:))),'k-', ... 
    F(2:1251,:), 20*log10(abs(q_btotal)),'r-');grid
xlabel('Frequency (Hz)');ylabel('Charge Output dB');
legend('Experimental','Simulated');
axis([0 1600 -240 -190])
Appendix H

Documented PVDF Material Properties
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>PVDF</th>
<th>Copolymer</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>Thickness</td>
<td>9, 28, 52, 110</td>
<td>Various</td>
<td>(\mu m) (micron, (10^{-6}))</td>
</tr>
<tr>
<td>(d_{31})</td>
<td>Piezo Strain Constant</td>
<td>23</td>
<td>11</td>
<td>(10^{-12}) \frac{m/m}{V/m} or \frac{C/m^2}{N/m^2})</td>
</tr>
<tr>
<td>(d_{33})</td>
<td>Piezo Strain Constant</td>
<td>-33</td>
<td>-38</td>
<td>(10^{-12}) \frac{m/m}{V/m} or \frac{C/m^2}{N/m^2})</td>
</tr>
<tr>
<td>(\varepsilon_{31})</td>
<td>Piezo Stress constant</td>
<td>216</td>
<td>162</td>
<td>(10^3) \frac{V/m}{N/m^2} or \frac{m/m}{C/m^2})</td>
</tr>
<tr>
<td>(\varepsilon_{33})</td>
<td>Piezo Stress constant</td>
<td>-330</td>
<td>-542</td>
<td>(10^3) \frac{V/m}{N/m^2} or \frac{m/m}{C/m^2})</td>
</tr>
<tr>
<td>(k_{33})</td>
<td>Electromechanical Coupling Factor</td>
<td>12%</td>
<td>20%</td>
<td>(25-29%)</td>
</tr>
<tr>
<td>(k_{i})</td>
<td>Coupling Factor</td>
<td>14%</td>
<td></td>
<td>(25-29%)</td>
</tr>
<tr>
<td>(C)</td>
<td>Capacitance</td>
<td>380 for 28(\mu m)</td>
<td>68 for 100(\mu m)</td>
<td>(\text{pF/cm}^2) @ 1kHz</td>
</tr>
<tr>
<td>(Y)</td>
<td>Young's Modulus</td>
<td>2.4</td>
<td>3.5</td>
<td>(10^9) N/m(^2)</td>
</tr>
<tr>
<td>(V_t)</td>
<td>Speed of Sound</td>
<td>1.5</td>
<td>2.3</td>
<td>(10^3) m/s</td>
</tr>
<tr>
<td>(p)</td>
<td>Pyroelectric Coefficient</td>
<td>30</td>
<td>40</td>
<td>(10^4) C/m(^2) \cdot\text{°K})</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Permittivity</td>
<td>106-113</td>
<td>65-75</td>
<td>(10^{12}) F/m</td>
</tr>
<tr>
<td>(\varepsilon/\varepsilon_0)</td>
<td>Relative Permittivity</td>
<td>12-13</td>
<td>7-8</td>
<td>(10^9) kg/m</td>
</tr>
<tr>
<td>(\rho_m)</td>
<td>Mass Density</td>
<td>1.78</td>
<td>1.82</td>
<td>(10^9) kg/m</td>
</tr>
<tr>
<td>(\rho_e)</td>
<td>Volume Resistivity</td>
<td>(&gt;10^{13})</td>
<td>(&gt;10^{14})</td>
<td>Ohm meters</td>
</tr>
<tr>
<td>(R)</td>
<td>Surface Metallization Resistivity</td>
<td>2.0</td>
<td>2.0</td>
<td>Ohms/square for CuNi</td>
</tr>
<tr>
<td>(R)</td>
<td>Resistivity</td>
<td>0.1</td>
<td>0.1</td>
<td>Ohms/square for Ag Ink</td>
</tr>
<tr>
<td>tan (\delta)</td>
<td>Loss Tangent</td>
<td>0.02</td>
<td>0.015</td>
<td>(\text{@}) 1kHz</td>
</tr>
<tr>
<td>(Y)</td>
<td>Yield Strength</td>
<td>45-55</td>
<td>20-30</td>
<td>(10^9) N/m(^2) (stretch axis)</td>
</tr>
<tr>
<td>(T_c)</td>
<td>Temperature Range</td>
<td>-40 to 80</td>
<td>-40 to 115...145</td>
<td>°C</td>
</tr>
<tr>
<td>(W)</td>
<td>Water Absorption</td>
<td>(&lt;0.02)</td>
<td>(&lt;0.02)</td>
<td>% \text{H}_2\text{O}</td>
</tr>
<tr>
<td>(MV)</td>
<td>Maximum Operating Voltage</td>
<td>750 (30)</td>
<td>750 (30)</td>
<td>V/mil(V/(\mu)m), DC, (\text{@}) 25°C</td>
</tr>
<tr>
<td>(BV)</td>
<td>Breakdown Voltage</td>
<td>2000 (80)</td>
<td>2000 (80)</td>
<td>V/mil(V/(\mu)m), DC, (\text{@}) 25°C</td>
</tr>
</tbody>
</table>