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**A Comparative Study of the Stress Concentration Factors Arising Between Two Equal, Nearly Intersecting, Perpendicular, Pressurized Holes in an Infinite Solid with Those Arising Due to an Infinite Series of Such Holes**

Paul J. Drogowski

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A COMPARATIVE STUDY OF THE STRESS CONCENTRATION FACTORS  
ARISING BETWEEN TWO EQUAL, NEARLY INTERSECTING,  
PERPENDICULAR, PRESSURIZED HOLES IN AN  
INFINITE SOLID WITH THOSE ARISING DUE  
TO AN INFINITE SERIES OF SUCH HOLES

by

Paul J. Drogowski

A Thesis  
Submitted to the  
Faculty of The Graduate College  
in partial fulfillment of the  
requirements for the  
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Department of Mechanical and Aeronautical Engineering

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Finally, I dedicate this thesis to my many nieces and nephews. Strive, always, to perform to the best of your ability and look to God and your family for your strength.

Paul J. Drogowski

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Paul J. Drogowski, M.S.E.

Western Michigan University, 1995

Finite element analysis is used to determine the stress concentration factors arising from two equal, perpendicular, nearly intersecting, pressurized holes in an infinite solid while varying the distance and pressure ratios between the holes. The results are compared and contrasted with those previously obtained for an infinite series of such holes. A ratio of the distance between the holes to the diameter of the holes at which the holes can be considered to be single holes in an infinite solid is determined. A study is performed to determine a correlation between the location of the region of maximum stress and the hoop, radial and bending stresses within the solid.

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# INTRODUCTION

## Historical Background

Any discontinuity or sudden change in the geometry of a loaded structure will result in a stress concentration, or local region of higher stress, in the region of the discontinuity. The study of stress concentrations is not new to the mechanical engineering field. Extensive research has been performed and documented on plate and round bar subjects.[1,2,3,5] Some research has been performed on the effects of nearly intersecting pressurized holes in infinite solids; However, this work is primarily limited to infinite series of such holes.[4,6,7]

It has been the desire of the aerospace industry to reduce the overall weight of air and space craft. Pressurized fluids are distributed to various actuators through control manifolds. These manifolds typically have a large number of holes of various sizes that pass near one another. Pressures within the holes induce high stresses in the regions of near intersections. In order to minimize the size and weight of these manifolds, it is beneficial to gain an understanding of the effects of these holes on the induced stress levels.

## Significance of Problem

Preliminary studies indicate that the stress concentration factors resulting from two equal, perpendicular, pressurized holes in an infinite solid are less than those resulting from an infinite series of such holes. (See Figures 1 and 2.) If this is the case, then it is possible to realize greater weight savings by studying further the effects of the two equal holes. It is also desirable to know how far apart the holes must be in order to consider each as a single hole in an infinite solid.

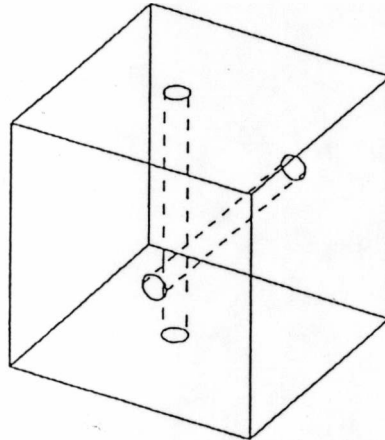


Figure 1. Two Nearly Intersecting Perpendicular Holes.

Kalinka [4] notes that, while keeping the diameters constant, as the distance between the holes increases the location of the maximum stress moves from the point of minimum cross-section to a point on the inside of the high pressure hole. He offers only bending stresses as a cause. This thesis addresses the effects of radial, hoop, and bending stresses on the location of the maximum stress.

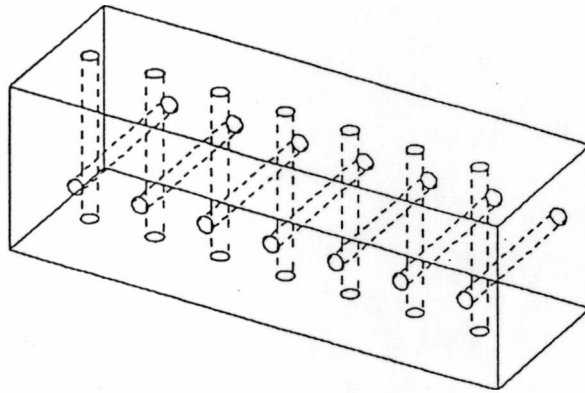


Figure 2. Two Infinite Lines of Perpendicular Holes.

### Technical Background

Typically a stress concentration factor is defined as the ratio of the actual stress to a theoretically calculated nominal stress in the region of interest. This is a convenient way to deal with the many possible geometries and loading conditions that have been studied in the past. However, due to the simplicity of the loading of pressurized holes, it is much more convenient to define the stress concentration factor for the holes addressed in this thesis as the ratio of the actual von Mises stress to the maximum fluid pressure within the holes.

One can easily show that the stress concentration factor of a single pressure hole is 1.73 when it is defined in this way. Therefore, one would expect that as the distance between the two holes increases, the stress concentration factor would approach this value.

## RESEARCH METHODOLOGIES

Simple solutions exist for many stress concentration problems. However, these simple solutions apply mainly to flat plate or round bar problems.[5] In order to determine the stresses within one of the infinite solids as described above, it is necessary to use experimental methods (i.e. photoelastic stress analysis or strain gaging) or a numerical method, such as the finite element approach. The finite element method is an accurate, relatively inexpensive approach to the solution of this problem and was employed in this research. The finite element method has been used in the related studies.[1,4,6,7]

It is obvious that two planes of symmetry exist within the model shown in Figure 1. Advantage was taken of this symmetry, and models were created as shown in Figure 3. The finite element research consisted of two main parts, the comparative study and the infinite hole separation determination.

### Comparative Study

Finite element models were created with hole separation distance ( $t$ ) to hole diameter ( $D$ ) ratios from 0.01 to 0.20 in steps of 0.01. Using a commercial finite element software package on a PC, analyses were performed on these models with pressure ratios of 0.0, 0.1, 0.2, ..., 1.0.

In order to assure sufficient accuracy, convergence studies were performed on several models prior to analysis. The convergence studies were performed by refining the mesh until the stress observed for a given load converged.

### Infinite Hole Separation Determination

Finite element models were created using  $t/D$  ratios from 0.20 to 6.00 at various intervals. Using a commercial finite element software package on a PC, analyses were performed on these models with pressure ratios of 0.0, 0.1, 0.2, ..., 1.0.

In order to assure sufficient accuracy, convergence studies were performed on several models prior to analysis. Infinite separation distance was determined to be the distance at which the stress concentration factor is within 5% of 1.73 and is constant with respect to the pressure ratio.

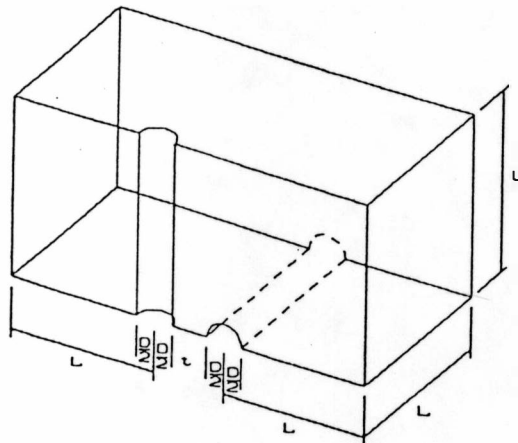


Figure 3. Reduced Geometry for Two Perpendicular Holes.

## RESULTS

### Comparative Study

The comparative study revealed that for every  $t/D$  ratio and pressure ratio combination the stress concentration factor is lower for the two holes in an infinite solid than for the infinite series of holes in an infinite solid. It also showed that as the  $t/D$  ratio increased, the location of the region of maximum stress moved from the low pressure side of the region of thinnest cross section toward a region away from the thinnest cross section in the higher pressure hole for lower pressure ratios. It also showed that for larger  $t/D$  ratios, as the pressure ratio approaches unity, the region of maximum stress moves toward the region of thinnest cross section.

Table 1 shows the stress concentration factors that were obtained during the comparative study. Table 2 shows the percent change in the stress concentration factors from the infinite series of hole to the two holes. Figures 4, 5, and 6 show plots of the stress concentration factors for  $t/D$  ratios of 0.01 to 0.20 as obtained during this research.

Table 1

Stress Concentration Factors Arising Due to Two Equal, Nearly  
Intersecting, Perpendicular, Pressurized Holes in an Infinite  
Solid With  $t/D$  Ratios From 0.01 to 0.20

---

$P_{min}/P_{max} \setminus t/D$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.0	45.8	22.5	15.0	11.2	8.96	7.45	6.36	5.54	4.90	4.40
0.1	41.8	20.8	13.9	10.4	8.39	7.01	6.01	5.26	4.67	4.20
0.2	37.9	19.0	12.8	9.72	7.84	6.58	5.67	4.99	4.45	4.02
0.3	34.0	17.2	11.7	8.97	7.29	6.16	5.34	4.72	4.24	3.85
0.4	30.1	15.5	10.6	8.24	6.76	5.75	5.03	4.48	4.04	3.70
0.5	26.2	13.8	9.65	7.53	6.24	5.37	4.73	4.25	3.86	3.56
0.6	22.3	12.1	8.63	6.85	5.75	5.00	4.45	4.03	3.70	3.44
0.7	18.5	10.4	7.65	6.19	5.28	4.66	4.20	3.85	3.56	3.33
0.8	14.8	8.88	6.72	5.58	4.86	4.35	3.97	3.68	3.45	3.25
0.9	11.3	7.41	5.87	5.03	4.48	4.08	3.78	3.55	3.35	3.20
1.0	8.24	6.13	5.14	4.56	4.16	3.87	3.64	3.45	3.30	3.17

$P_{min}/P_{max} \setminus t/D$	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.0	3.99	3.64	3.34	3.09	2.87	2.67	2.57	2.50	2.44	2.39
0.1	3.83	3.50	3.23	2.99	2.79	2.61	2.46	2.39	2.34	2.29
0.2	3.68	3.38	3.13	2.91	2.72	2.56	2.41	2.32	2.27	2.22
0.3	3.54	3.27	3.04	2.84	2.67	2.52	2.38	2.26	2.21	2.17
0.4	3.42	3.17	2.97	2.79	2.63	2.49	2.37	2.27	2.17	2.13
0.5	3.31	3.09	2.91	2.75	2.61	2.49	2.38	2.28	2.19	2.12
0.6	3.22	3.03	2.87	2.73	2.60	2.50	2.40	2.31	2.24	2.17
0.7	3.15	2.98	2.84	2.72	2.61	2.52	2.44	2.36	2.29	2.23
0.8	3.10	2.96	2.84	2.73	2.64	2.56	2.49	2.42	2.36	2.31
0.9	3.07	2.95	2.85	2.76	2.68	2.61	2.55	2.49	2.44	2.39
1.0	3.06	2.97	2.88	2.81	2.74	2.68	2.63	2.58	2.53	2.49

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Table 2

Percent Differences Between the Stress Concentration Factors Arising Due to Two Equal, Nearly Intersecting, Perpendicular, Pressurized Holes in an Infinite Solid With  $t/D$  Ratios From 0.02 to 0.20 and an Infinite Series of Such Holes [4,7]

---

$P_{min}/P_{max} \setminus t/D$	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.0	5.2	5.0	5.2	5.5	5.5	5.7	5.8	5.9	6.5
0.1	5.9	5.8	6.1	6.3	6.5	6.7	6.7	6.7	6.8
0.2	6.7	6.8	7.1	7.4	7.6	7.8	7.8	7.8	8.0
0.3	7.7	8.0	8.3	8.6	8.9	9.1	9.3	9.4	9.4
0.4	9.0	9.4	9.9	10.3	10.5	10.7	11.0	11.1	10.9
0.5	10.6	11.2	11.7	12.2	12.5	12.7	12.8	13.0	13.0
0.6	12.7	13.4	14.0	14.4	14.7	14.8	15.1	15.1	15.0
0.7	15.4	16.2	16.7	17.0	17.2	17.3	17.3	17.3	17.3
0.8	18.8	19.5	19.8	20.0	20.0	19.9	19.8	19.7	19.5
0.9	23.2	23.4	23.4	23.2	22.8	22.6	22.2	22.0	21.7
1.0	29.3	28.6	27.4	26.5	25.8	25.0	24.4	24.0	23.4

$P_{min}/P_{max} \setminus t/D$	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.0	4.5	4.6	4.7	4.7	4.9	8.1	8.8	8.6	8.4	8.2
0.1	5.5	5.6	5.6	5.7	5.7	7.0	9.8	9.8	9.7	9.5
0.2	6.8	6.8	6.9	6.9	7.0	7.0	9.0	10.4	10.5	10.5
0.3	8.3	8.4	8.4	8.5	8.5	8.5	8.6	10.9	11.7	12.2
0.4	10.1	10.1	10.2	10.3	10.3	10.3	10.3	11.9	14.7	15.3
0.5	12.1	12.1	6.8	12.2	12.2	12.2	12.2	13.0	14.8	17.3
0.6	14.2	14.2	14.2	14.2	14.2	14.2	14.1	14.2	15.9	17.5
0.7	16.5	16.4	16.3	16.2	16.1	16.0	15.9	15.8	16.8	17.8
0.8	18.7	18.5	18.4	18.2	18.0	17.8	17.6	17.4	17.9	18.5
0.9	20.8	20.5	20.3	20.0	19.7	19.4	19.1	18.9	18.9	19.0
1.0	22.8	22.3	21.9	21.5	21.2	20.8	20.5	20.1	19.8	19.5

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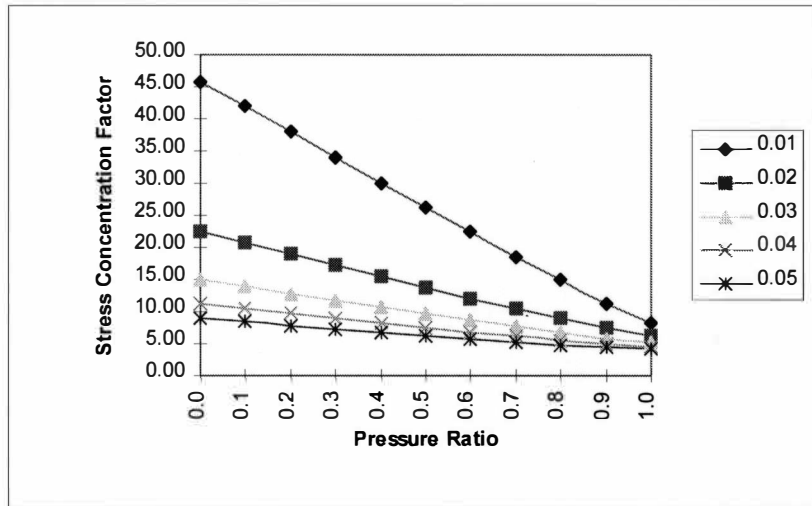


Figure 4. Plot of the Stress Concentration Factors Arising Due to Two Equal, Nearly Intersecting, Perpendicular, Pressurized Holes in an Infinite Solid With  $t/D$  Ratios From 0.01 to 0.05.

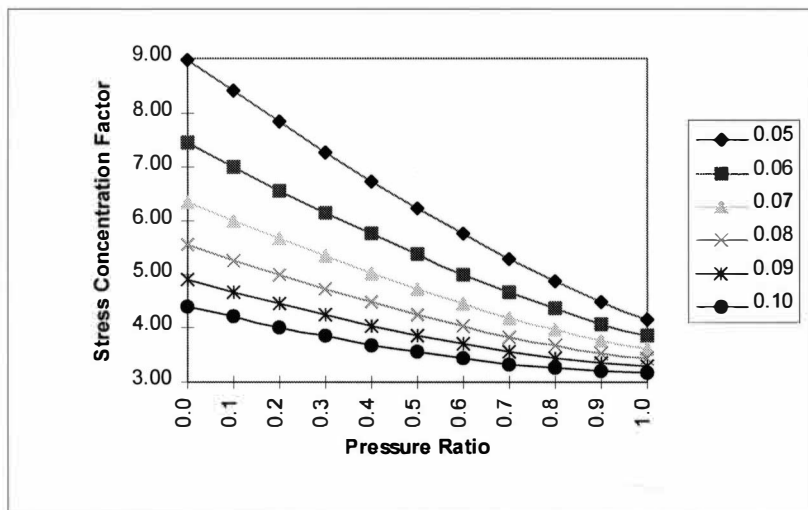


Figure 5. Plot of the Stress Concentration Factors Arising Due to Two Equal, Nearly Intersecting, Perpendicular, Pressurized Holes in an Infinite Solid With  $t/D$  Ratios From 0.05 to 0.10.

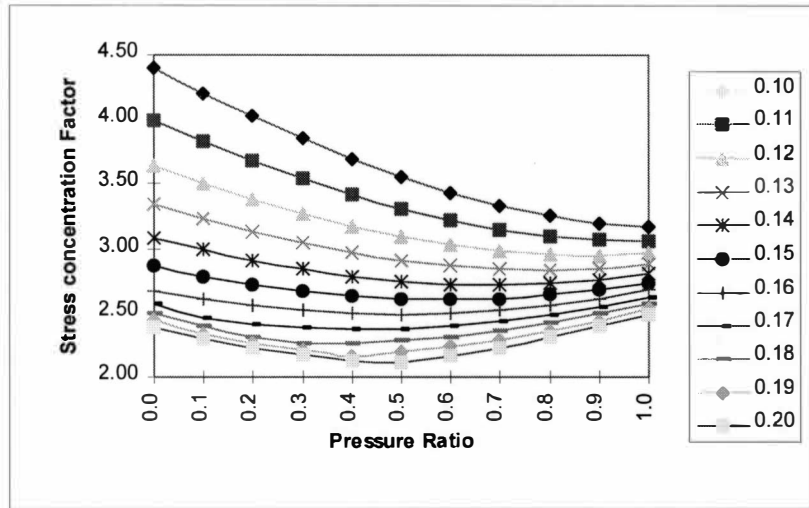


Figure 6. Plot of the Stress Concentration Factors Arising Due to Two Equal, Nearly Intersecting, Perpendicular, Pressurized Holes in an Infinite Solid With  $t/D$  Ratios From 0.10 to 0.20.

#### Infinite Hole Separation Determination

It was found that two holes in an infinite solid can be considered to be infinitely far apart when the  $t/D$  ratio reaches 2.50. Table 3 shows the stress concentration factors obtained during this portion of the research.



## DISCUSSION

The stress concentration factors arrived at during the research documented herein are the products of numerical analyses and are meant to be used as guides for design. Every material and forming operation has the ability to affect the final stresses seen within a structure. Therefore, any design that is generated with the use of these data should be tested to verify the strength and durability of the final product.

### Comparative Study

The two hole configuration has lower stress concentrations than the infinite series configuration. Upon reviewing Table 2, one readily observes that this is true for all configurations addressed herein. This should not be surprising. It is obvious that the infinite solid with the infinite series of perpendicular holes has a smaller ratio of material area to void area in the plane of holes than the infinite solid with the two perpendicular holes. When one plane of the infinite series of holes is pressurized, the pressure tends to push the solid apart perpendicular to this plane of holes for the entire area of this plane; Whereas, when one of the two perpendicular holes is pressurized, the surrounding material, which goes on infinitely, tends to offer more resistance to deformation. Furthermore, the plane of holes has a larger area on which the applied pressure is able to act than does the single set of holes. This larger area

results in a larger force acting perpendicular to the plane of holes than would be seen in the single set of holes. Therefore, a higher stress concentration factor is observed for the infinite series of perpendicular holes than for a single set of perpendicular holes.

One also observes that as the pressure ratio increases, the percent differences in the stress concentration factors of the two configurations increases. This is not surprising either. As mentioned above, the infinite solid with the infinite series of holes has larger net forces per material area in the directions perpendicular to the planes of the holes than the solid with a single pair of perpendicular holes. When both planes of infinite series of holes are pressurized, the regions of thinnest cross-section are subjected to higher tensile stresses perpendicular to both plains of holes than are seen in a solid with two perpendicular holes pressurized. As with low pressure ratios, the radial and circumferential stresses can be expected to be similar for both hole configurations. Therefore, a higher stress concentration factor is observed for the infinite series of perpendicular holes than for a single set of perpendicular holes.

#### Infinite Hole Separation Determination

The data given in Table 3 reveals that the stress concentration factors calculated in this research converge to 1.77 when the  $t/D$  ratio approaches 2.50. This value is within 2.9% of the expected stress concentration factor for a single hole in an

infinite solid. The data also shows that, at this  $t/D$  ratio, the stress concentration factor does not change with changes in the pressure ratio. Therefore, it can be concluded that two equal, perpendicular, pressurized holes in an infinite solid can be considered infinitely far apart when the  $t/D$  ratio of the holes is 2.50.

### Stress Concentration Location

As mentioned above, what seems to be a truly strange phenomenon occurs as the  $t/D$  ratio and the pressure ratio change. The location of the region of maximum stress moves.

At low  $t/D$  ratios and low pressure ratios, the maximum stress is found in the low pressure hole in the region of thinnest cross section. A knowledge of thin walled pressure vessels should lead one to expect such. Classical theory teaches that the circumferential and radial stresses can be treated as equal throughout the wall thickness. In the case of the nearly intersecting holes, the thin wall section can also be treated as a non-uniformly loaded beam fixed at both ends with the load increasing sinusoidally toward the center. The stresses induced due to the beam effect are greatest in the center, where the cross-section is a minimum and the curvature is a maximum. The stresses induced by this bending are tensile on the low pressure side of the thin wall and compressive on the high pressure side of the thin wall. The circumferential stresses are all tensile. Obviously, the combination of the bending and the circumferential stresses on the low pressure side of the wall is greater than the

combination of the bending and the circumferential stresses on the high pressure side of the wall. Therefore, it is expected that for small  $t/D$  and pressure ratios the location of the region of maximum stress is in the low pressure hole in the region of thinnest cross-section.

As the pressure ratio approaches unity and the  $t/D$  ratio remains small, the region of maximum stress must extend through the thinnest cross-section to include the entire region of thinnest cross-section from one hole to the other. This, also, is intuitively obvious, as the bending stresses diminish and the compressive stresses approach the fluid pressure.

A region of maximum stress is found in the region of thinnest cross-section in both holes as the  $t/D$  ratio increases and the pressure ratio remains at unity. These regions of maximum stress are separated by a region of lower stress, which increases in size as the  $t/D$  ratio increases. Thick walled pressure vessel theory quickly leads one to expect such a phenomenon.

At low pressure ratios, as the  $t/D$  ratio is increased, the location of the region of maximum stress moves to the high pressure hole on the plane of symmetry that is perpendicular to the high pressure hole and away from the region of thinnest cross-section. This phenomenon can also be understood with the help of thick walled pressure vessel theory. It can easily be shown that the maximum von Mises stress occurs on the interior wall of a thick walled pressure vessel. It is also obvious that as the thickness of the wall increases, the significance of the hole decreases with respect

to the location of the maximum bending stress, i.e. the maximum tensile stress in the high pressure hole due to bending approaches that of the maximum tensile stress in the low pressure hole due to bending. This combined with the maximum circumferential and radial stresses results in the region of maximum stress being found in the high pressure hole.

It was mentioned above that at high pressure ratios, the region of maximum stress is located near the thinnest cross-section. This occurs as the fluid in the low pressure hole begins to support the wall section, thus reducing the effects of bending stresses and increasing the compressive stresses.



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