The Effect of Rotor Wake/Exit Guide Vane Interaction on the Total Pressure Losses in an Airfoil Cascade

Aaron Mosebach

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THE EFFECT OF ROTOR WAKE/EXIT GUIDE VANE INTERACTION ON THE TOTAL PRESSURE LOSSES IN AN AIRFOIL CASCADE

by

Aaron Mosebach

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
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Western Michigan University
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December 1995
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Aaron Mosebach
Computational simulations have been performed to study the effects of unsteady rotor wakes on the total pressure losses and the boundary layer characteristics in a compressor cascade. In the numerical simulations, transitional and turbulent flow conditions were modeled at design and off-design inlet flow angles, and for varying rotor wake widths and wake velocity deficits.

First, simulations were performed in the absence of rotor wakes at the inlet of the EGV (Exit Guide Vane) cascade. Second, simulations were performed for different wake width and wake velocity deficit values at the inlet of the EGV cascade. The majority of the numerical simulations in which a rotor wake was specified indicated that the total pressure losses increase as the rotor wake width and wake velocity deficit increase. Flow turning, in general, decreased as the wake depth and wake velocity deficit were increased. For the design simulations, the skin friction was found to decrease as the wake width was increased. In addition, some of the design flow simulations predicted a decrease in the skin friction as the wake velocity deficit was increased. For several of the off-design flow simulations, the skin friction decreased as the wake width was increased. The results from these simulations provide a data base that can be used for improving compressor cascade performance.
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<table>
<thead>
<tr>
<th>English</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a$</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Specific heat at constant volume</td>
</tr>
<tr>
<td>$d$</td>
<td>Wake depth w.r.t the steady inlet velocity</td>
</tr>
<tr>
<td>$e$</td>
<td>Specific energy</td>
</tr>
<tr>
<td>$e_t$</td>
<td>Total energy</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian of the transformation</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$P$</td>
<td>Static pressure</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Effective Prandtl number</td>
</tr>
<tr>
<td>$q$</td>
<td>Dynamic pressure</td>
</tr>
<tr>
<td>$r$</td>
<td>EGV trailing edge radius</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal gas constant</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$s$</td>
<td>Surface arc length of EGV</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number</td>
</tr>
<tr>
<td>$T$</td>
<td>Static temperature</td>
</tr>
<tr>
<td>$u$</td>
<td>Axial component of velocity</td>
</tr>
<tr>
<td>$v$</td>
<td>Circumferential component of velocity</td>
</tr>
<tr>
<td>$V$</td>
<td>Total Velocity</td>
</tr>
</tbody>
</table>
List of Symbols-continued

\( w \)  \hspace{1cm} \text{Wake width w.r.t the blade pitch}

\( X \)  \hspace{1cm} \text{Axial distance}

**Greek**

\( \beta \)  \hspace{1cm} \text{Inlet, exit flow angle}

\( \gamma \)  \hspace{1cm} \text{Ratio of specific heats}

\( \kappa \)  \hspace{1cm} \text{Thermal conductivity}

\( \lambda \)  \hspace{1cm} \text{Second coefficient of viscosity}

\( \mu \)  \hspace{1cm} \text{Effective first coefficient of viscosity}

\( \rho \)  \hspace{1cm} \text{Density}

\( \tau \)  \hspace{1cm} \text{Shear stress}

\( \theta \)  \hspace{1cm} \text{Momentum Thickness}

\( \omega \)  \hspace{1cm} \text{Cascade losses}

**Subscripts**

\( c \)  \hspace{1cm} \text{Chord of EGV}

\( h \)  \hspace{1cm} \text{Energy equation term}

\( i \)  \hspace{1cm} \text{Inviscid}

\( L \)  \hspace{1cm} \text{Laminar quantity}

\( T \)  \hspace{1cm} \text{Turbulent quantity}

\( t \)  \hspace{1cm} \text{Stagnation quantity}

\( v \)  \hspace{1cm} \text{Viscous}

\( x, y \)  \hspace{1cm} \text{First derivative with respect to } x \text{ or } y

\( xx, yy \)  \hspace{1cm} \text{Second derivative with respect to } x, y
List of Symbols-continued

\( \xi, \eta \)  
First derivative with respect to \( \xi \) or \( \eta \)

\( \xi \xi, \eta \eta \)  
Second derivative with respect to \( \xi \) or \( \eta \)

\( -\infty \)  
Inlet/upstream quantity

\( +\infty \)  
Exit/downstream quantity

\( \infty \)  
Freestream quantity

Superscripts

*  
Dimensional quantity

+/-  
Upper/lower surface quantity

~  
Computational domain, relative reference frame quantity
CHAPTER I

INTRODUCTION

During the last decade there have been many studies devoted to evaluating the effects of different sources on flow unsteadiness. These sources include turbulence, rotating stall, flutter, vortex shedding, and blade row interaction (Giles, 1991). Of all of these sources forementioned, blade row interaction is often of primary importance.

There are four different types of flow unsteadiness that are associated with blade row interaction in a turbomachine (Valkov, 1994). First, wake/rotor interaction may cause flow unsteadiness as a result of the stator wakes which are generated by viscosity. These stator wakes are unsteady in the rotor relative frame of reference. The wake interaction with the rotor is a predominantly inviscid process. Conversely, for wake/stator interaction, the rotor wakes are unsteady in the absolute frame of the stator.

Second, potential stator/rotor interaction can cause flow unsteadiness. With potential stator/rotor interaction, there is a non-uniform steady lift distribution that exists along the rotor blades. There also exists a non-uniform steady lift distribution along the stator blades. The trailing edge of the stator experiences the non-uniform pressure field that is moving in the rotor relative frame of reference, causing unsteadiness in the flow. Conversely, the leading edge of the rotor experiences the non-uniform pressure field of the stator blades in the absolute frame of reference causing unsteadiness in the flow. This interaction be-
between the stator and the rotor is inviscid and is appropriately named the potential stator/rotor interaction.

Third, vortices that are shed from an upstream blade row interact with the blades downstream and can cause flow unsteadiness. Vortex shedding is caused by viscous flow passing over the trailing edges of the stator or rotor blades. Fourth, the interaction of moving shock waves produced from a blade upstream with a blade in a row downstream can cause flow unsteadiness.

Physical Problem

Of the four sources of unsteadiness mentioned previously, this thesis details the source of unsteadiness caused by rotor wake/Exit Guide Vane (EGV) interaction. In particular, the effects of changing the rotor wake width and wake velocity deficit on the EGV cascade flowfield and total pressure losses have been investigated. In addition, the effects of the rotor wakes on the EGV boundary layer and the blade loading were studied.

The physical problem of interest in this investigation involves the time-dependent flow, with negligible body forces, of a calorically perfect gas through a two-dimensional EGV blade row. At the upstream boundary of the computational domain of the EGV blade row, rotor wakes were computationally simulated using a moving inlet velocity profile. The velocity distortions convect downstream in the axial direction and also in the circumferential direction, and model the circumferential movement of the rotor blades in the absolute reference frame of the EGV.

The flow analysis was conducted for the EGV blade row operating at the following flow conditions: (a) design and off-design mean flow angles of 40 and
55 degrees, respectively, and (b) with and without transition being specified. The analyses of the flow within the EGV passage based on the previously mentioned flow conditions were first conducted in the absence of simulated rotor wakes (no-wake). The results of the no-wake simulations were then used to initialize the unsteady (wake) solutions where a wake was specified at the inlet boundary of the computational domain of the EGV.

Assumptions

The following assumptions were made in the current analysis. First, this investigation neglected potential flow interaction between the EGV and the rotor. The potential effects represent one of the blade row interaction sources of unsteadiness discussed earlier. Second, since potential effects were neglected, only the flowfield in the EGV passage was computed. A moving rotor wake model was implemented upstream of the EGV blade row at the boundary of the computational domain. This rotor wake model is discussed in detail in the Boundary Conditions chapter of this report. Third, the flow was assumed to be viscous and two-dimensional.

Motivation for the Work

The current work was motivated by the need to generate a design data base which can be used to reduce the total pressure losses within an EGV passage. By determining how wake interaction affects the total pressure losses, a blade design approach can be implemented which tailors blade wakes for performance improvements.
There have been numerous efforts devoted to the study of wake/stator interaction using experimental, computational, and theoretical approaches. Some of these efforts will now be mentioned to lay the necessary foundation for the present study.

A theoretical wake chopping and rectification hypothesis was made by Kerrebrock et al. (1970), which stated that for a compressor, the rotor wakes move towards the pressure surface of the downstream stator blades due to their high momentum. The opposite is true for a turbine. This hypothesis was supported by the work done by Kumar et al. (1971) using inert-gas tracing elements.

Adachi (1974) conducted experiments which evaluated the unsteady pressure distribution and the blade forces due to wake interaction for a stator blade within an axial-flow blower. It was found that as the wake center passed over the blades, a positive pressure peak occurred on the pressure surface and a negative pressure peak on the suction surface. In addition, it was determined that the effects produced from wake interaction increase with the blade loading, the flow capacity, and the number of blades.

Hodson (1990) investigated the effects of blade row interaction on the boundary layer. It was found that the passing wakes affect the laminar-to-turbulent transition point along the blade surface.

An experiment studying three-dimensional wake/stator interaction was conducted by Schulz et al. (1990). They experimentally measured the unsteady pressure, the boundary layer quantities, and the wake movement and development within a stator blade row passage. Through the analysis of their data, it
was found that the fluctuations of the velocity within the wake greatly affect the angle of incidence. This, in turn, affects the pressure distribution on the blade.

Unsteady computational flow simulations using Navier-Stokes flow solvers have been conducted on two-dimensional flows with blade row interaction. In particular, computational research has been conducted for two-dimensional rotor wake/EGV interaction by Barnett et al. (1994), who analyzed how the rotor wakes affect the flowfield in a compressor EGV passage. Some of the objectives of this analysis were: (a) to study the flow within the EGV passage in the absence of simulated rotor wakes at an off-design incidence angle condition where flow separation exists, and (b) to study the flowfield within the passage in the presence of simulated rotor wakes with 5 percent and 30 percent wake velocity deficits at an off-design angle of incidence. The investigation of the flow around the EGV in the absence of simulated rotor wakes was used as the reference by which to study the behavior of the simulated rotor wake/EGV interaction on the boundary layer of the EGV. In addition, the eruption of vorticity observed on the suction surface of the EGV at the laminar separation bubble was investigated (Barnett et al., 1994). The results indicated that the presence of rotor wakes significantly alter the unsteady boundary layer behavior. It was also determined that as the rotor wake convected downstream and was cut by the EGVs, the flow within the passage tended to move from the suction surface on one blade to the pressure surface of the next blade in the blade row. This was attributed to the high circumferential velocity of the rotor wake.

Another computational study of interest was conducted by Valkov (1993) who investigated the unsteady two-dimensional flow that resulted from the interaction of a simulated rotor wake with a stator blade row. In this study, the stator
blade loading, the wake thickness, the wake velocity defect, and the wake reduced
frequency were varied in order to measure their respective effects on the flowfield
within a stator blade passage.

Over the suction surface, the dominant source of flow unsteadiness was
categorized by the evolution of vortices (called B-vortices) in, or at the edge of
the boundary layer near the leading edge of the stator blade. The B-vortices were
formed as a result of the rotor wakes meeting with the leading edges of the stator
blades in the blade row. Three different parameters were found which increased
the strength of the B-vortices. These parameters were the pressure distribution,
the tangential momentum of the wake, and the wake reduced frequency. Stronger
B-vortices were produced in the presence of strong adverse pressure gradients.
The strong adverse pressure gradients caused the B vortices to become distorted
and to detach from the suction surface. As a result, a pair of counter-rotating
vortices formed producing a vortex street of B- and P-vortices. Favorable pressure
gradients caused the production of weaker B-vortices. By increasing the tangential
momentum of the wake or by having lower reduced wake frequencies, the strength
of the B-vortices was increased. Even when strong adverse pressure gradients were
not present, a vortex street of B- and P-vortices was produced further downstream
as a result of the stronger B-vortices formed near the leading edge due to these
two parameters.

Over the pressure surface, unsteady flow behavior was characterized by a
pair of vortices of alternating sign (called $W^+$ and $W^-$-vortices). These vortices,
which represent the dominant source of flow unsteadiness over the pressure surface,
were formed due to the migration of the rotor wake from the middle of the stator
blade passage. The strength of these vortices was found to increase with an
increase in the tangential momentum of the wake. Valkov (1993) mentioned that an increase in the wake thickness was equivalent to a proportional increase in the total momentum of the wake. This forementioned statement is especially relevant to the present study where wake thickness is considered as one of the parameters influencing the total pressure loss. Finally, the unsteady affects caused by the counter-rotating W-vortices were found to be small when compared with the B-vortices produced on the suction surface.

The wake velocity defect was represented by the wake strength in the study by Valkov. The wake strength was found to have minimal impact on the unsteady flow behavior for both the pressure and suction surfaces.

Valkov (1993) identified two different stator blade loading fluctuations which were produced as a result of the rotor wake/stator interaction. First, a strong pressure pulse was produced at the leading edge of the stator blades as a result of the potential flow interaction of the rotor wake with the stator blades. This pulse was negative on the suction surface and positive on the pressure surface. Second, pressure fluctuations were identified over the suction surface. These pressure fluctuations were associated with the previously mentioned vortices that act along the pressure and suction surfaces. On the suction surface negative pressure fluctuations were present. On the pressure surface, the fluctuations were alternating positive/negative.

As the blade loading was increased (higher inlet flow turning angle), strong B-vortices were observed on the suction surface. The increase in the strength of the B-vortices was attributed to the presence of a strong adverse pressure gradient. The result was the presence of stronger negative pressure fluctuations on the suction surface.
On the suction surface, a theoretical control strategy was introduced by Valkov (1993) which could reduce the unsteadiness caused by the formation of the vortices as the rotor wake meets the stator blade leading edge. Non-uniform suction along the suction surface was suggested which would make the boundary layer thinner. By making the boundary layer thinner, particularly in the region from where the vortices are evolving along the suction surface leading edge, the flow unsteadiness should decrease.

For the pressure surface, the rotor wakes move towards the pressure surface within the blade passage forming vortices near the pressure surface. For this case, a control strategy which would eliminate the vortices was not suggested as a feasible method of unsteady flow reduction. Instead, Valkov (1993) proposed non-uniform blowing along the pressure surface as the method for reducing flow unsteadiness. Non-uniform blowing would cause the vortices to convect downstream without getting too close to the pressure surface. As a result, a decrease in the pressure fluctuations along the pressure surface could be achieved.

Studies dealing with isolated airfoil stall have been conducted for helicopter rotors by Carr (1988), and Carr et al. (1992). These studies have helped to resolve the critical length and time scales important to the onset of the stall vortex which causes a loss of overall compression. The relationships that exist between the studies of isolated airfoil stall and turbomachinery compressor stall are still not fully understood. It is known, however, that endwall separation and/or boundary layer separation can decrease the compression of a system causing rotating stall or surge.
Navier-Stokes Equations

The equations of fluid motion which govern the unsteady flow in the EGV passage are the time dependent, two-dimensional, Reynolds-averaged Navier-Stokes equations. These equations are written in Cartesian coordinates and non-dimensional form as

\[ Q_t + (F_i + Re^{-1}F_v)_x + (G_i + Re^{-1}G_v)_y = 0 \]  (1)

where the vector of conserved variables, \( Q \), the inviscid flux vectors, \( F_i \) and \( G_i \), and the viscous flux vectors, \( F_v \) and \( G_v \), are given by

\[ Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e_t \end{pmatrix} \]  (2)

and

\[ F_i = \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ (e_t + P)u \end{pmatrix}, \quad F_v = - \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{hx} \end{pmatrix} \]  (3)
\[
G_i = \begin{pmatrix}
\rho v \\
\rho uv \\
\rho v^2 + P \\
(e_t + P)v
\end{pmatrix} \quad G_v = - \begin{pmatrix}
\tau_{yx} \\
\tau_{yy} \\
\tau_{hy}
\end{pmatrix}
\] (4)

where

\[
\begin{align*}
\tau_{xx} &= 2\mu \frac{\partial u}{\partial x} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\
\tau_{xy} &= \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
\tau_{yx} &= \tau_{xy} \\
\tau_{yy} &= 2\mu \frac{\partial v}{\partial y} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\
\tau_{hx} &= u\tau_{xx} + v\tau_{xy} + \gamma \mu Pr^{-1} \frac{\partial e}{\partial x} \\
\tau_{hy} &= u\tau_{yx} + v\tau_{yy} + \gamma \mu Pr^{-1} \frac{\partial e}{\partial y} \\
e_t &= \rho e + \rho \left( u^2 + v^2 \right) \\
e &= \frac{P}{\rho (\gamma - 1)}
\end{align*}
\]

The second coefficient of viscosity, \( \lambda \), is calculated using Stokes' hypothesis, \( \lambda = -2/3\mu \).

The Navier-Stokes equations of motion were non-dimensionalized in order that parameters such as the Reynolds number could be varied independently. In addition, in order to readily solve the equations of motion, the equations were transformed from physical space \((x,y,z)\) into computational space \((\xi, \eta, \tau)\) through the relations

\[
\xi = \xi(x,y,t)
\]
This transformation simplifies the implementation of the boundary conditions by introducing body-fitted curvilinear coordinates. In addition, this transformation allows for a computational domain to be produced where the grid lines are equally spaced. The grid lines in the physical domain may or may not be unequally spaced. The physical and computational domains for a general grid topology are shown in Figures 1 and 2, respectively. The flow directions in the computational domain can be understood if one visually "opens up" the physical domain grid on the right, where \( J = 1 \), which would be similar to one opening a book and then laying the book down on its pages.

The non-dimensionalized two-dimensional Navier-Stokes equations in body-
fitted coordinate form are written as

$$\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{F}_i}{\partial \xi} + \frac{\partial}{\partial \xi} \left( Re^{-1} \tilde{F}_v \right) + \frac{\partial \tilde{G}_i}{\partial \eta} + \frac{\partial}{\partial \eta} \left( Re^{-1} \tilde{G}_v \right) = 0$$  \hspace{1cm} (6)

where

$$\tilde{Q} = J^{-1} Q$$

$$\tilde{F}_i = J^{-1} \left( Q \xi_t + E_i \xi_x + F_i \xi_y \right)$$  \hspace{1cm} (7)

$$\tilde{G}_i = J^{-1} \left( Q \eta_t + E_i \eta_x + F_i \eta_y \right)$$

In order to reduce the computational time necessary to produce a solution to the Navier-Stokes equations of motion, the thin-layer approximation was applied (Baldwin and Lomax, 1978). The thin-layer approximation considers the streamwise viscous stresses to be negligible. As a result, the transformed non-
dimensionalized governing equations have the form

\[
\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{F}_i}{\partial \xi} + \frac{\partial \tilde{G}_i}{\partial \eta} + \frac{\partial}{\partial \eta} \left( Re^{-1} \tilde{G}_v \right) = 0
\]

(8)

where

\[
\tilde{G}_v = -\begin{pmatrix}
0 \\
K_1 u_\eta + K_2 \eta_x \\
K_1 v_\eta + K_2 \eta_y \\
K_1[P_r^{-1}(\gamma - 1)^{-1}(a^2)_\eta + (q^2/2)_\eta] + K_2 K_3 \\
K_1 = \mu \left( \eta_x^2 + \eta_y^2 \right) \\
K_2 = \frac{\mu}{3} (\eta_x u_\eta + \eta_y v_\eta) \\
K_3 = u \eta_x + v \eta_y \\
q^2 = u^2 + v^2
\end{pmatrix}
\]

(9)

(10)

In the limit, as \( Re \to \infty \), all viscous terms in Eqn. 8 become negligible and the inviscid (Euler) equations of motion are obtained (Dorney, 1992).

Turbulence Modeling

The eddy viscosity and the eddy diffusivity formulations were implemented in the solution procedure of the non-dimensionalized Navier-Stokes equations of motion to model turbulent flow behavior. In the eddy viscosity formulation, laminar and turbulent shear stress terms are combined in the momentum equations and the energy equation through an effective viscosity which can be defined as

\[
\mu = \mu_L + \mu_T
\]

(11)
In the eddy diffusivity formulation, laminar and turbulent heat fluxes are combined in the energy equation through an effective thermal conductivity which can be defined as

$$\frac{\kappa}{c_p} = \frac{\mu_L}{Pr_L} + \frac{\mu_T}{Pr_T}$$

(12)

The Baldwin-Lomax algebraic turbulence model (Baldwin et al., 1978) was used to predict the local value of the turbulent viscosity, $\mu_T$, at each point in the flowfield around the EGV. With the Baldwin-Lomax turbulence model, the flowfield is divided into three distinct regions which consist of an inner region, an outer region, and a wake region. The inner region next to the EGV surface is commonly referred to as a laminar sublayer where laminar shear stresses are of the greatest importance (Hoffman, 1989). The outer region is referred to as the turbulent zone where turbulent shear stresses are dominant.
CHAPTER III

BOUNDARY CONDITIONS

Boundary conditions are imposed at the inlet and exit of the computational domain of one EGV blade and also along the blade's surface. The implicit solution of the inlet and exit boundary conditions is based on a quasi-two-dimensional characteristic analysis of linearized Euler equations. The implicit solution can be applied to steady and unsteady flow simulations. Since the quasi-two-dimensional inlet and exit boundary conditions are not strictly non-reflecting (Verdon, 1993), nonphysical reflections may occur at the far-field boundaries which can affect the flowfield and reduce computational efficiency (Giles, 1990). As a result, an explicit, linear, two-dimensional, non-reflecting boundary condition method proposed by Giles is used to calculate the change in the incoming characteristic waves based on the outgoing characteristic waves produced from the implicit quasi-two-dimensional characteristic analysis.

The no-slip boundary condition is imposed along the surface of the EGV blade. The normal derivative of the pressure is assumed to be negligible along the EGV. Finally, a specified heat flux along the EGV blade is assumed to be constant in time (heat flux equals zero for adiabatic).

Periodic boundary conditions are applied at the boundaries of the H-grid in the blade-to-blade direction and also at the boundaries of the O-grid in the streamwise direction. These boundary conditions are solved implicitly. Zonal boundary conditions are applied at the boundaries of the overlaid region of the H-
and O-grids (Rai, 1987). Linear interpolation is used to explicitly determine the conservative flow variables at the zonal boundaries after each time step. Further details on the periodic and zonal boundary conditions can be found in the reference by Dorney (1992).

Along the inlet boundary, a mathematical equation representing a Gaussian velocity profile is used to simulate an incoming rotor wake (Volkov, 1993). The unsteady local velocity at any point along the inlet at any time is easily determined through this equation.

Characteristic Analysis

For viscous flow, the characteristic boundary conditions at the inlet and exit are retained since the flow is assumed to be predominantly inviscid. Through this assumption, the quasi-two-dimensional characteristic analysis can be developed from the unsteady, one-dimensional Euler equations. The non-conservative form of the unsteady, one-dimensional linearized Euler equations can be written as

$$\frac{\partial Q}{\partial t} + A \frac{\partial Q}{\partial x} = 0 \quad (1)$$

where the vector $Q$ and the Jacobian matrix $A$ can be defined as

$$Q = \begin{bmatrix} \rho \\ u \\ P \end{bmatrix}, \quad A = \begin{pmatrix} u & \rho & 0 \\ 0 & u & 1/\rho \\ 0 & \gamma P & u \end{pmatrix} \quad (2)$$

From the non-conservative form of the unsteady, one-dimensional linearized Euler equations the characteristic form of the one-dimensional Euler equations can
be established. The eigenvalues or characteristic values of the Jacobian matrix are calculated by

\[ \det (A - \lambda I) = 0 \]  

(3)

where \( I \) is the identity matrix. Solving Eqn. 3 for the three eigenvalues of the \( A \) matrix gives

\[ \lambda_1 = u \]  

(4)

\[ \lambda_2 = u + a \]  

(5)

\[ \lambda_3 = u - a \]  

(6)

Physically, these three eigenvalues represent three velocities. The first eigenvalue corresponds to the velocity of the particles in the flow. The second and the third eigenvalues represent the acoustic speeds of the medium. One observation concerning these eigenvalues is that the velocity of the particles in the flow can affect the direction of propagation of the eigenvalue or characteristic value. Consider the illustration in Figure 3 where a fluid is flowing subsonically through a converging-diverging nozzle where \( a \) is the speed of sound (Merkle, 1987). The \( u \) and \( u + a \) eigenvalues are positive and propagate from left to right. The \( u - a \) eigenvalue is negative and propagates from right to left. If the velocity of the particles in the flow becomes supersonic, the direction of propagation of the \( u - a \) eigenvalue changes from left to right.

Now that the eigenvalues are known, the left and right eigenvectors must be determined. This is accomplished by first assuming that the eigenvalues of the
A matrix can be diagonalized by the transformation

\[ T^{-1}AT = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \Lambda \quad (7) \]

The right eigenvector matrix, \( T \), can be found by premultiplying both sides of Eqn. 7 by \( T \). Defining the matrix \( T \) as \( t_{ji} \), Eqn. 7 becomes

\[ \begin{bmatrix} u & \rho & 0 \\ 0 & u & 1/\rho \\ 0 & \rho a^2 & u \end{bmatrix} \begin{bmatrix} t_{1i} \\ t_{2i} \\ t_{3i} \end{bmatrix} = \begin{bmatrix} t_{1i} & \lambda_i \\ t_{2i} & \lambda_i \\ t_{3i} & \lambda_i \end{bmatrix} \quad (8) \]

where \( i = 1, 3 \) for the three eigenvalues. The three eigenvalues from Eqn. 7 are

Figure 3. Direction of Propagation of Eigenvalues.
placed into Eqn. 8 which produces

\[
T = \begin{bmatrix}
-1/a^2 & 1/(2a^2) & 1/(2a^2) \\
0 & 1/(2\rho a) & -1/(2\rho a) \\
0 & 1/2 & 1/2
\end{bmatrix}
\] (9)

The matrix of left eigenvectors is then

\[
T^{-1} = \begin{bmatrix}
-a^2 & 0 & 1 \\
0 & \rho a & 1 \\
0 & -\rho a & 1
\end{bmatrix}
\] (10)

The non-conservative form of the Euler equations shown in Eqn. 1 can be written in terms of the characteristic variables by premultipling it by \( T^{-1} \) to produce (Merkle, 1987)

\[
T^{-1} \frac{\partial Q}{\partial t} + T^{-1} \Lambda TT^{-1} \frac{\partial Q}{\partial x} = 0
\] (11)

where \( TT^{-1} \), the identity matrix, has been inserted. This equation can also be written as

\[
\frac{\partial W}{\partial t} + A \frac{\partial W}{\partial x} = 0
\] (12)

where

\[
W = T^{-1} Q
\] (13)

Before defining the characteristic variables, it should be noted that in this form the differential operators have been uncoupled. The equations are no longer coupled with each other except through the vector of primary dependent variables, \( Q \).
The vector, \( W \), contains the characteristic variables

\[
W = [w_1, w_2, w_3]^T
\]

\[
w_1 = -\rho a^2 + P
\]

\[
w_2 = \rho au + P
\]

\[
w_3 = -\rho au + P
\]

A fourth characteristic variable (Giles, 1991) is obtained for two-dimensional flow if there are negligible variations in the y-direction. This variable is

\[
\frac{\partial w_4}{\partial t} + u \frac{\partial w_4}{\partial x} = 0
\]

where

\[
w_4 = \rho av
\]

Each of the characteristic variables is associated with a characteristic wave and an eigenvalue (Dorney, 1992) where,

1. \( w_1 = -\rho a^2 + P \) is a linear entropy wave associated with \( \lambda_1 = u \).
2. \( w_2 = \rho au + P \) is a pressure wave associated with \( \lambda_2 = u + a \).
3. \( w_3 = -\rho au + P \) is a pressure wave associated with \( \lambda_3 = u - a \).
4. \( w_4 = \rho av \) is a vorticity wave associated with \( \lambda_4 = u \).

The developed characteristic information is incorporated into the approximate factorization solution procedure which is described in the chapter on the Numerical Procedure.
Implicit Inlet and Exit Conditions

The implicit inlet and exit boundary conditions are enforced during the third step of the approximate factorization scheme discussed in the Numerical Procedure. Both of these boundary conditions are formulated using the characteristic variable information developed in the preceding section. Further information describing the implementation of the inlet and exit boundary conditions into the approximate factorization scheme can be found in the work of Dorney (1992).

Viscous Surface Boundary Conditions

Along the blade surface, no-slip boundary conditions apply. This viscous boundary condition is incorporated into the implicit approximate factorization scheme through the knowledge that

\[ \Delta(\rho u)_w = \Delta(\rho v)_w = 0 \]  

where \( w \) refers to the surface of the blade. In addition, the normal derivative of the pressure is assumed to be negligible along the blade surface. Finally, a specified heat flux along the blade surface is assumed to be constant in time (adiabatic).

Approximate, Two-Dimensional Boundary Conditions

A methodology to produce approximate, two-dimensional, unsteady boundary conditions has been proposed by Giles (1990). This methodology allows the outgoing pressure, entropy, and vorticity waves to leave the computational domain without producing artificial reflections. In this method, the incoming characteristic waves are based on the outgoing characteristic waves. The outgoing charac-
teristic waves at the inlet and the exit are already known through the previously mentioned implicit characteristic analysis. The incoming characteristic waves are determined by

\[
\frac{\partial}{\partial t} \begin{bmatrix}
w_1' \\
w_2' \\
w_3' \\
w_4'
\end{bmatrix} + \begin{bmatrix}v & 0 & 0 & 0 \\0 & v & (a+u)/2 & (a-u)/2 \\
0 & (a-u)/2 & v & 0 \\
\end{bmatrix} \frac{\partial}{\partial y} \begin{bmatrix}w_1' \\
w_2' \\
w_3' \\
w_4'
\end{bmatrix} = 0
\]

Equation 21, which is solved implicitly, updates the incoming characteristic waves. The implicit values of the incoming characteristic waves are then substituted into an explicit update formulation (Giles, 1990). This explicit update formulation can be written in perturbation form (Dorney, 1992) as

\[
\begin{bmatrix}
w_1' \\
w_2' \\
w_3' \\
w_4'
\end{bmatrix} = \begin{bmatrix}-a^2 & 0 & 0 & 1 \\
0 & 0 & \rho a & 0 \\
0 & \rho a & 0 & 1 \\
0 & -\rho a & 0 & 1
\end{bmatrix} \begin{bmatrix}\delta \rho \\
\delta u \\
\delta v \\
\delta P
\end{bmatrix}
\]

\[
\begin{bmatrix}
\delta \rho \\
\delta u \\
\delta v \\
\delta P
\end{bmatrix} = \begin{bmatrix}-1/a^2 & 0 & 1/2a^2 & 1/2a^2 \\
0 & 0 & 1/2\rho a & -1/2\rho a \\
0 & 1/\rho a & 0 & 0 \\
0 & 0 & 1/2 & 1/2
\end{bmatrix} \begin{bmatrix}w_1' \\
w_2' \\
w_3' \\
w_4'
\end{bmatrix}
\]

where

\[
\delta \rho = \rho - \rho_{\text{inlet/exit}}
\]
\[
\delta u = u - u_{\text{inlet/exit}} \quad (25)
\]
\[
\delta v = v - v_{\text{inlet/exit}} \quad (26)
\]
\[
\delta P = P - P_{\text{inlet/exit}} \quad (27)
\]

The subscripts, \(-\infty\) and \(+\infty\), refer to the steady flow variables at the inlet and exit, respectively. The subscripts, "inlet" and "exit" refer to the known values of the flow variables at the inlet and exit boundary.

Wake Model

A mathematical equation representing a Gaussian velocity profile is used to simulate an incoming rotor wake (Valkov, 1993). This velocity profile represents the velocity distribution in the rotating frame across the wake. The equation representing the velocity distribution in the rotating frame can be written as (Valkov, 1993)

\[
\frac{w(y_w)}{W} = 1 - Ae^{-By_w^2} \quad (28)
\]

where \(A\) is the velocity defect, \(B\) is the wake thickness, and \(y_w\) is the distance from the wake centerline. The velocity profile of the rotor wake in the rotating frame is illustrated in Figure 4.

Equation 28 represents an inlet boundary condition which is specified to represent an incident vortical gust. Through the work of Verdon (1993), the dependent variables can be determined. The pressure and the density at the inlet maintain their no-wake (Case 0) time-averaged values. As a result, the flow unsteadiness is caused only by the vortical gust and is not due to pressure or entropic disturbances (Dorney, 1992). The unsteady local velocity components of
the vortical gust are added to the time-averaged components of the velocity along the inlet for the no-wake (Case 0) case to produce the total velocity along the inlet. Once the $u$ and $v$ velocities are known at the inlet, all of the dependent variables are known and the characteristic variables can be determined.
CHAPTER IV

PHYSICAL MODEL

An overlaid O-II computational grid arrangement was used to discretize the compressor blade row. In this arrangement, an algebraically generated H-grid was used in the regions upstream of the leading edge, downstream of the trailing edge, and in the inter-blade region of the EGV within the blade row. The H-grid consists of $300 \times 61$ (streamwise x tangential) grid points. The O-grid, which was generated through using elliptic partial differential equations, consisted of $551 \times 81$ grid points. The O-grid, which was body fitted to the surface of the EGV, was overlaid upon the H-grid. The outer boundary of the O-grid was not necessarily coincident with the grid lines of the H-grid. In addition, there was an overlap region between the grids where the equations of motion were solved on both the O- and H-grids (see Figure 5).

The geometry of the EGV consisted of combining the thickness distribution of a NACA 0012 airfoil which is given by (Barnett et al., 1994)

$$T(x) = H_T[2.969x^{1/2} - 1.260x - 3.516x^2 + 2.843x^3 - 1.015x^4], \ 0 \leq x \leq 1, \ (1)$$

with the thirteen percent arc camber distribution represented by

$$C(x) = H_C - R + [R^2 - (x - 0.5)^2]^{1/2}, \ 0 \leq x \leq 1 \quad (2)$$

where $H_C > 0$ is the midchord height of the camber line and $R = (2H_C)^{-1}(0.25 +
$H_C$ is the chamber line radius. The original NACA wedge-shaped trailing edge was replaced with a round trailing edge. Based on the previous information, the blade surface coordinates were represented by

$$ (X, Y)_{\text{B}}^\pm = [x \mp 0.5T(x)\sin \theta, C(x) \pm 0.5T(x)\cos \theta], 0 \leq x \leq 1, \quad (3) $$

where $\theta = \tan^{-1}(dC/dx)$, and the $+$ and $-$ signs refer to the upper and lower surfaces of the blade, respectively. Other information pertinent to the physical model is shown in Table 1 below.
Table 1

Design Parameters of the EGV

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade stagger angle</td>
<td>15 degrees</td>
</tr>
<tr>
<td>Gap-chord ratio (G)</td>
<td>0.60</td>
</tr>
<tr>
<td>Design condition flow angle ($\beta_1$)</td>
<td>40 degrees</td>
</tr>
<tr>
<td>Off-design condition flow angle ($\beta_1$)</td>
<td>55 degrees</td>
</tr>
<tr>
<td>Unit Reynolds number (Re)</td>
<td>300,000/in</td>
</tr>
<tr>
<td>Exit static to inlet total pressure ($P_2/P_{i1}$)</td>
<td>0.9625</td>
</tr>
</tbody>
</table>
CHAPTER V
NUMERICAL PROCEDURE

For the two-dimensional analysis of the rotor wake/EGV interaction, a
time-marching, implicit, third-order spatially accurate, second-order temporally
accurate, approximate-factorization finite-difference integration (Beam et al., 1977)
method is used. In the approximate-factorization integration method, finite-
difference approximations are applied to the derivatives of the non-linear Navier-
Stokes equations of motion. The primary dependent variables contained within
the vector $\dot{Q}$ are then solved for at a given instant in time in terms of all of the
discrete points contained in the computational grid. Newton iterations (Burden
et al., 1985) are globally applied at each time step for the primary dependent flow
variables to eliminate linearization errors and increase stability (Dorney, 1992). In
addition, the Steger-Warming flux-vector splitting scheme is applied. This scheme
expresses the split flux Jacobian matrices, which are represented in the approxi-
mate factorization solution procedure discussed below, in terms of the eigenvalues
and eigenvectors.

The non-dimensionalized, Navier-Stokes equations expressed in terms of
the body-fitted curvilinear coordinate system with the incorporation of the thin-
layer approximation can be written as

$$\frac{\partial \dot{Q}}{\partial \tau} + \frac{\partial \dot{E}_i}{\partial \xi} + \frac{\partial \dot{G}_i}{\partial \eta} + \frac{\partial}{\partial \eta} (Re^{-1}\ddot{G}_v) = 0$$

(1)
Using an implicit scheme, Eqn. 1 is then written as

\[
\frac{\dot{Q}^{n+1} - \dot{Q}^n}{\Delta \tau} + \left( \frac{\partial \tilde{F}_i}{\partial \xi} \right)^{n+1} + \left( \frac{\partial \tilde{G}_i}{\partial \eta} \right)^{n+1} - \left( \frac{\partial \left( Re^{-1} \tilde{G}_v \right)}{\partial \eta} \right)^{n+1} = 0
\]  

(2)

where the time derivative was approximated by backward finite difference formulation (Hoffman, 1989). The following approximations are then used to linearize Eqn. 2

\[
\tilde{F}^{n+1}_i = \tilde{F}^n_i + \left( \frac{\partial \tilde{F}_i}{\partial \dot{Q}} \right) \Delta \dot{Q}
\]  

(3)

\[
\tilde{G}^{n+1}_i = \tilde{G}^n_i + \left( \frac{\partial \tilde{G}_i}{\partial \dot{Q}} \right) \Delta \dot{Q}
\]  

(4)

\[
\tilde{G}^{n+1}_v = \tilde{G}^n_v + \left( \frac{\partial \tilde{G}_v}{\partial \dot{Q}} \right) \Delta \dot{Q}
\]  

(5)

where \( \frac{\partial \tilde{F}_i}{\partial \dot{Q}} \), \( \frac{\partial \tilde{G}_i}{\partial \dot{Q}} \), and the \( \frac{\partial \tilde{G}_v}{\partial \dot{Q}} \) are the fluid Jacobian matrices defined as \( \frac{\partial \tilde{F}_i}{\partial \dot{Q}} = \tilde{A} \), \( \frac{\partial \tilde{G}_i}{\partial \dot{Q}} = \tilde{B} \), and \( \frac{\partial \tilde{G}_v}{\partial \dot{Q}} = \tilde{B}_v \). Equations 3, 4, and 5 are then substituted into Eqn. 2 to produce

\[
\frac{\Delta \dot{Q}}{\Delta \tau} + \frac{\partial}{\partial \xi} (\tilde{F}^n_i + \tilde{A} \Delta \dot{Q}) + \frac{\partial}{\partial \eta} (\tilde{G}^n_i + \tilde{B} \Delta \dot{Q}) - \frac{\partial}{\partial \eta} (\tilde{G}^n_v + \tilde{B}_v \Delta \dot{Q}) = 0
\]  

(6)

or

\[
(I + \Delta \tau \left[ \frac{\partial}{\partial \xi} (\tilde{A}) + \frac{\partial}{\partial \eta} (\tilde{B}) - \frac{\partial}{\partial \eta} (\tilde{B}_v) \right]) \Delta \dot{Q} = -\Delta \tau \left( \frac{\partial \tilde{F}^n_i}{\partial \xi} + \frac{\partial \tilde{G}^n_i}{\partial \eta} - \frac{\partial \tilde{G}^n_v}{\partial \eta} \right)
\]  

(7)

The approximate factorization procedure is introduced into Eqn. 7 which reduces the pentadiagonal coefficient matrix into two tridiagonal matrices (Hoff-
man, 1989). Equation 7 then becomes

$$\left( I + \Delta \tau \left[ \frac{\partial}{\partial \xi} (\tilde{A}) \right] \right) \left( I + \Delta \tau \left[ \frac{\partial}{\partial \eta} (\tilde{B}) - \frac{\partial}{\partial \eta} (\tilde{B}_v) \right] \right) \Delta \tilde{Q} =$$

$$-\Delta \tau \left[ \frac{\partial \tilde{F}_{i_1}^n}{\partial \eta} + \frac{\partial \tilde{G}_{i_1}^n}{\partial \eta} - \frac{\partial \tilde{C}_{i_1}^n}{\partial \eta} \right]$$

Equation 8

The inviscid fluxes and Jacobian matrices in Eqn. 8 are split according to the sign of the eigenvalues produced in the characteristic analysis which produces

$$\left( I + \Delta \tau \left[ \frac{\partial}{\partial \xi} (\tilde{A}^+ + \tilde{A}^-) \right] \right) \left( I + \Delta \tau \left[ \frac{\partial}{\partial \eta} (\tilde{B}^+ + \tilde{B}^-) - \frac{\partial}{\partial \eta} (\tilde{B}_v) \right] \right) \Delta \tilde{Q} =$$

$$-\Delta \tau \left[ \frac{\partial (\tilde{F}_{i_1}^+ - \tilde{F}_{i_1}^-)^n}{\partial \eta} + \frac{\partial (\tilde{G}_{i_1}^+ - \tilde{G}_{i_1}^-)^n}{\partial \eta} - \frac{\partial \tilde{C}_{i_1}^n}{\partial \eta} \right]$$

Equation 9

Equation 9 can be solved by the following procedure:

1. Determine the residual as

$$\Delta \tilde{Q}^* = -\Delta \tau \left[ \frac{\partial (\tilde{F}_{i_1}^+ - \tilde{F}_{i_1}^-)^n}{\partial \xi} + \frac{\partial (\tilde{G}_{i_1}^+ - \tilde{G}_{i_1}^-)^n}{\partial \eta} - \frac{\partial \tilde{C}_{i_1}^n}{\partial \eta} \right]$$

Equation 10

2. Make a solution sweep in the $\eta$ coordinate direction

$$\left( I + \Delta \tau \left[ \frac{\partial}{\partial \eta} (\tilde{B}^+ + \tilde{B}^-) - \frac{\partial}{\partial \eta} (\tilde{B}_v) \right] \right) \Delta \tilde{Q}^{**} = \Delta \tilde{Q}^*$$

Equation 11

3. Make a solution sweep in the $\xi$ coordinate direction

$$\left( I + \Delta \tau \left[ \frac{\partial}{\partial \xi} (\tilde{A}^+ + \tilde{A}^-) \right] \right) \Delta \tilde{Q} = \Delta \tilde{Q}^{**}$$

Equation 12

Finite difference approximations were applied to the derivatives of the equations in the solution procedure discussed above. Application of these approx-
imations to the derivatives is called discretization. The split inviscid flux vectors
were discretized using Osher's scheme (Chakravarthy et al., 1985) and the viscous
flux vector was discretized using standard central differences. A first-order back­
ward difference approximation was applied to the positive upwind contribution of
the inviscid flux Jacobian, and a first-order forward difference approximation was
applied to the negative downwind contribution of the inviscid flux Jacobian. The
viscous flux Jacobians were discretized using standard central differences. Inform­
ation describing the calculation of the flux vectors and the Jacobian matrices
using the finite-difference approximation methods mentioned above can be found

In the solution procedure above, the flux Jacobian matrix, $\tilde{A}$, and was split
as

$$\tilde{A} = \tilde{A}^+ + \tilde{A}^-$$  \hspace{1cm} (13)

where

$$\tilde{A}^+ = \tilde{T}\Lambda^+\tilde{T}^{-1}$$  \hspace{1cm} (14)

$$\tilde{A}^- = \tilde{T}\Lambda^-\tilde{T}^{-1}$$  \hspace{1cm} (15)

$\Lambda^+$ represents a matrix containing the positive eigenvalues of the flux Jacobian
matrix, $\tilde{A}$. $\Lambda^-$ represents a matrix containing the negative eigenvalues of the flux
Jacobian matrix, $\tilde{A}$. $\tilde{T}$ and $\tilde{T}^{-1}$ represent the eigenvectors. The determination
of the eigenvalues and the eigenvectors for the flux Jacobian matrix, $\tilde{A}$, are dis­
ussed in detail in the chapter on the Boundary Conditions. The splitting of the
flux Jacobian matrix $\tilde{B}$ is handled in a similar manner to the splitting of the flux
Jacobian matrix $\tilde{A}$. In addition, the solution of the eigenvalues and the eigenvec­tors for the flux Jacobian matrix, $\tilde{B}$, is done in a manner similar to the solution
of the eigenvalues and the eigenvectors for the flux Jacobian matrix, $\tilde{A}$.

Consider an example where a supersonic ($u > a$) flow is moving within the EGV passage and the Steger-Warming (Steger et al., 1979) flux-vector splitting method is applied. The positive and negative eigenvalues which are associated with the previously discussed positive and negative flux Jacobian matrices can be defined by

$$\lambda_i = \lambda_i^+ + \lambda_i^-$$

(16)

where

$$\lambda_i^+ = \frac{\lambda_i + |\lambda_i|}{2}, \quad \lambda_i^- = \frac{\lambda_i - |\lambda_i|}{2}$$

(17)

With $\lambda_1 = u$, $\lambda_2 = u + a$, $\lambda_3 = u - a$, and $\lambda_4 = u$, the diagonal positive and negative eigenvalue matrices have the form

$$T^{-1}\tilde{A}^+T = \Lambda^+ = \begin{bmatrix} u & 0 & 0 & 0 \\ 0 & u + a & 0 & 0 \\ 0 & 0 & u - a & 0 \\ 0 & 0 & 0 & u \end{bmatrix}$$

(18)

and

$$T^{-1}\tilde{A}^-T = \Lambda^- = 0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(19)

Since the Navier-Stokes equations were transformed into the curvilinear body-fitted coordinate system, the eigenvectors and the matrices containing the positive and negative eigenvalues are also transformed. For the case above where
supersonic flow was considered, the four eigenvalues may or may not be positive due to the transformation from the Cartesian coordinate system to the generalized curvilinear body-fitted coordinate system. If the Navier-Stokes equations are not transformed, the eigenvalues would all be positive for the supersonic case where \( u > a \) as shown above.
CHAPTER VI

NUMERICAL RESULTS

In an effort to better understand how unsteady wakes affect the performance and the boundary layer of an EGV, numerical simulations have been performed. In the numerical simulations, transitional and turbulent flow conditions were modeled at design and off-design mean inlet flow angles for varying wake widths and wake velocity deficits. For the transitional flow conditions, the point of transition on the suction surface of the EGV was specified at $s/s_{te} \approx 0.20$, where $s$ is the arclength measured along the suction surface of the EGV from the leading edge and $s_{te}$ is the arclength measured along the suction surface from the leading edge to the trailing edge.

The numerical simulations were first performed for the case where a rotor wake was not specified at the inlet of the EGV cascade. Once these simulations were completed, numerical simulations where performed for cases in which different wake width and wake velocity deficit values were specified at the inlet of the EGV cascade. These values of the wake width and the wake depth were based on the range of values used in the studies by Giles (1988, 1990) and Manwaring and Wisler (1993). The width of the wake, $w$, was varied from 0.10 to 0.30 of the cascade gap and the velocity deficit, $d$, was varied from 0.10 to 0.30 of the freestream velocity.

The numerical simulations were run on a Digital Equipment Corporation Alpha 3000-300LX workstation. The simulations without a specified wake at the
inlet were run for 25 cycles at 500 iterations per cycle. The simulations where
a wake was specified at the inlet were run for 10 cycles at 500 iterations/cycle.
A paper discussing the results from the numerical simulations was submitted for
presentation at the 1996 AIAA/ASME/SAE/ASEE Joint Propulsion Conference,
to be held in Orlando, Florida (Dorney et. al, 1996).

The matrix of test cases performed is shown in Tables 2 and 3. Case 0
represents the “baseline” no-wake simulation and Cases 1 through 9 represent the
simulations where the rotor wake widths and rotor wake velocity deficits were
specified. Cases 1 through 5 represent the cases where the wake width was varied
and Cases 6 through 9 represent the cases where the wake depth (wake velocity
deficit) was varied. The time-averaged results for the test cases presented in
Tables 2 and 3 were averaged over four wake passing cycles.

Table 2

<table>
<thead>
<tr>
<th>Design Test Cases</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta_1 = 40 \text{ deg}$ Turbulent</th>
<th>$\beta_1 = 40 \text{ deg}$ Transitional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 0</td>
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<td>$d = 0.00, w = 0.00$</td>
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<tr>
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<td>$d = 0.10, w = 0.10$</td>
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<td>$d = 0.10, w = 0.15$</td>
</tr>
<tr>
<td>Case 3</td>
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<td>$d = 0.10, w = 0.20$</td>
</tr>
<tr>
<td>Case 4</td>
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<td>$d = 0.10, w = 0.25$</td>
</tr>
<tr>
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<td>$d = 0.10, w = 0.30$</td>
</tr>
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<td>$d = 0.15, w = 0.20$</td>
</tr>
<tr>
<td>Case 7</td>
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<td>$d = 0.20, w = 0.20$</td>
</tr>
<tr>
<td>Case 8</td>
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<td>$d = 0.25, w = 0.20$</td>
</tr>
<tr>
<td>Case 9</td>
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<td>$d = 0.30, w = 0.20$</td>
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</tbody>
</table>
Table 3

Off-Design Test Cases

<table>
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<th></th>
<th>$\beta_1 = 55 \text{ deg}$</th>
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</thead>
<tbody>
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<td>Turbulent</td>
<td>Transitional</td>
<td></td>
</tr>
<tr>
<td>Case 0</td>
<td>$d = 0.00, \ w = 0.00$</td>
<td>$d = 0.00, \ w = 0.00$</td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
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<td>$d = 0.10, \ w = 0.10$</td>
<td></td>
</tr>
<tr>
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<td>$d = 0.10, \ w = 0.15$</td>
<td>$d = 0.10, \ w = 0.15$</td>
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<tr>
<td>Case 3</td>
<td>$d = 0.10, \ w = 0.20$</td>
<td>$d = 0.10, \ w = 0.20$</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>$d = 0.10, \ w = 0.25$</td>
<td>$d = 0.10, \ w = 0.25$</td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td>$d = 0.10, \ w = 0.30$</td>
<td>$d = 0.10, \ w = 0.30$</td>
<td></td>
</tr>
<tr>
<td>Case 6</td>
<td>$d = 0.15, \ w = 0.20$</td>
<td>$d = 0.15, \ w = 0.20$</td>
<td></td>
</tr>
<tr>
<td>Case 7</td>
<td>$d = 0.20, \ w = 0.20$</td>
<td>$d = 0.20, \ w = 0.20$</td>
<td></td>
</tr>
<tr>
<td>Case 8</td>
<td>$d = 0.25, \ w = 0.20$</td>
<td>$d = 0.25, \ w = 0.20$</td>
<td></td>
</tr>
<tr>
<td>Case 9</td>
<td>$d = 0.30, \ w = 0.20$</td>
<td>$d = 0.30, \ w = 0.20$</td>
<td></td>
</tr>
</tbody>
</table>

No-wake (Case 0)

Since the primary goal of the study was to investigate the effects of wake width and wake velocity deficit on the cascade losses, it is important to understand what causes changes in the total pressure across the EGV passage. For a calorically perfect gas, it can be shown that

\[
R \ln \frac{P_{o2}}{P_{o1}} = c_p \ln \frac{T_{o2}}{T_{o1}} - (S_2 - S_1)
\]

for a constant specific heat. Total pressure losses can be attributed to changes in total temperature and entropy.

For the undisturbed simulation (Case 0), the total temperature change across the EGV for all flow conditions (design and off-design, with and without transition) was zero. The known total temperature change across the EGV reveals
that

\[(\Delta h_t)_{abs} = C_p(\Delta T_t)_{abs} = 0\]  \hspace{1cm} (2)

and also that the work is zero across the EGV. Thus, changes in the total pressure across the EGV were not a function of changes in the total temperature for the undisturbed simulation.

All of the entropy contours which are shown for the design and the off-design Case 0 simulations in Figures 6-13 display a trail of shed turbulent vortices in the wake and clearly indicate that changes in entropy are occurring for all flow conditions. The trail of turbulent vortices in the wake not only reveal that the entropy is not constant, but also indicate that the compression process is not isentropic. The shed vortices emanate at the trailing edge and extend downstream. The formation of these vortices in the wake can be attributed to the reversal of flow on the suction surface caused by an adverse pressure gradient and the blunt trailing edge. An adverse pressure gradient can be defined as

\[
\frac{dP}{dx} > 0
\]  \hspace{1cm} (3)

The point of separation on the EGV can be shown as the point where

\[
\left(\frac{dU}{dy}\right)_{y=0} = 0
\]  \hspace{1cm} (4)

Figures 14-17 illustrate the Mach contours for the design and off-design Case 0 simulations. In these figures, flow separation is observed on the suction surface. In addition, the wakes from the trailing edge of the EGV extend downstream. For the design transitional and turbulent Mach number plots (see
Figure 6. Unsteady Entropy Contours for Case 0, Transitional, $\beta_1 = 40$ Deg.

Figure 7. Unsteady Entropy Contours for Case 0, Turbulent, $\beta_1 = 40$ Deg.
Figure 8. Unsteady Entropy Contours for Case 0, Transitional, $\beta_1 = 55$ Deg.

Figure 9. Unsteady Entropy Contours for Case 0, Turbulent, $\beta_1 = 55$ Deg.
Figure 10. Unsteady Entropy Contours for Case 0, Transitional, $\beta_1 = 40$ Deg.

Figure 11. Unsteady Entropy Contours for Case 0, Turbulent, $\beta_1 = 40$ Deg.
Figure 12. Unsteady Entropy Contours for Case 0, Transitional, $\beta_1 = 55$ Deg.

Figure 13. Unsteady Entropy Contours for Case 0, Turbulent, $\beta_1 = 55$ Deg.
Figures 14-15), the shape of the vortex sheets emanating from the trailing edge are quite similar. With the off-design transitional and turbulent Mach number plots (see Figures 16-17), the vortex sheets emanating from the trailing edge of the EGV are not similar in shape. This lack of similarity is also displayed when comparing the off-design transitional and turbulent entropy contours (see Figures 12-13). Near the leading edge of the EGV, higher Mach numbers exist for the off-design transitional and turbulent Mach number contours when compared to the design transitional and turbulent Mach number contours. In addition, higher Mach numbers exist in the wake region of the EGV at the off-design transitional and turbulent conditions when compared to the design flow conditions.

The skin friction coefficient can be defined as

\[ \bar{C}_f = \frac{\tau_w}{\frac{1}{2} \rho \infty V_{\infty}^2} \] (5)
Figure 15. Unsteady Mach Contours for Case 0, Turbulent, $\beta_1 = 40$ Deg.

Figure 16. Unsteady Mach Contours for Case 0, Transitional, $\beta_1 = 55$ Deg.
Figure 17. Unsteady Mach Contours for Case 0, Turbulent, $\beta_1 = 55$ Deg.

where

$$\tau_w = \mu \left( \frac{du}{dy} \right)_{y=0}$$  \hspace{1cm} (6)

The point of flow separation on the EGV suction surface is found by determining where the velocity gradient becomes negative due to a reversal of the flow direction. The velocity gradient at the surface can be defined as

$$\left( \frac{du}{dy} \right)_{y=0} = \frac{u_2 - u_1}{y_2 - y_1}$$  \hspace{1cm} (7)

For both the design and off-design skin friction distributions shown for Case 0 in Figures 18-21, the flow separates earlier for the turbulent flow condition when compared to the transitional flow condition. The separation point for the design turbulent Case 0 simulation was located at approximately 73.5 percent axial chord, while it was located at approximately 75 percent axial chord for the design transitional Case 0 simulation. The separation point for the off-design turbulent
Case 0 simulation was located at approximately 46 percent axial chord, while it was located at approximately 60 percent axial chord for the off-design transitional Case 0 simulation. The distance between the separation points for the turbulent and transitional flow conditions increases as the mean inlet flow angle is increased from 40 to 55 degrees. Note, in the figures, "P.S." denotes the pressure surface of the EGV and "S.S." denotes the suction surface of the EGV.

On the suction surface for the transitional skin friction distributions (see Figures 18 and 20), the skin friction decreases before the transition point, as it would along a flat plate. The skin friction then increases through transition because of the change from laminar to turbulent flow. Finally, the skin friction begins to decrease with distance again up to the EGV trailing edge. In addition, large fluctuations in the skin friction coefficient appear to be occurring before the point of transition on the suction surface at 20 percent axial chord for the off-
Figure 19. Skin Friction Coefficient Distribution for Case 0, Turbulent, $\beta_1 = 40$ Deg.

Figure 20. Skin Friction Coefficient Distribution for Case 0, Transitional, $\beta_1 = 55$ Deg.
Figure 21. Skin Friction Coefficient Distribution for Case 0, Turbulent, $\beta_1 = 55$ Deg.

design transitional skin friction distribution. The fluctuations can be attributed to the flow separation.

The skin friction distributions for a flat plate have been provided in Figure 22. These distributions were based on the same unit Reynolds number of $300,000$/in which was used in the numerical simulations. For the transitional skin friction coefficient distribution, laminar flow exists up to the location of transition, and turbulent flow exists beyond the transition location. The equations which are representative of the skin friction coefficient distributions for laminar and turbulent flow (for a flat plate) are

$$
\bar{C}_f = \frac{0.664}{Re_x^{0.5}}
$$

(8)
Figure 22. Skin Friction Coefficient Distributions for a Flat Plate.

for laminar flow and

$$\bar{C}_f = \frac{0.577}{(Re_x)^2}$$ \hspace{1cm} (9)

for turbulent flow where $Re_x$ represents the unit Reynolds number.

For the pressure surface of the EGV, the skin friction coefficient appears to be at a minimum at approximately mid-chord for all flow conditions. There is a mild decrease in the skin friction coefficient from the EGV leading edge to approximately mid-chord and then a mild rise in the skin friction coefficient from the mid-chord to the trailing edge for all flow conditions.

Finally, there is a local spike at the trailing edge of the skin friction distributions for all flow conditions. This spike can be attributed to the fact that the O-grid lines near the trailing edge are forced to conform to the shape of the EGV, causing discontinuities. Local spikes at the trailing edge are also observed for the momentum thickness distributions and the surface pressure coefficient dis-
tributions yet to be discussed.

The momentum thickness deals with the momentum flux deficiency due to shear in the boundary layer along the EGV. If an inviscid flow were being considered, the momentum thickness would be zero. This is commonly expressed as

$$\rho U_\infty^2 \theta = \rho \int_0^\delta (U_\infty - u(y))u(y) \, dy \quad (10)$$

where $\theta$ is the momentum thickness for a "displaced" inviscid velocity profile, $\delta$ is the distance from the surface (boundary layer thickness) where $u = 0.99U_\infty$, and $U_\infty$ is the velocity at the edge of boundary layer (inviscid velocity). The term, $\rho U_\infty^2 \theta$, in Eqn. 10 represents the momentum flow under frictionless conditions, whereas the other term represents the momentum flow due to shear in the boundary layer. This is graphically illustrated in the velocity profile shown in Figure 23, where the shaded region represents the reduction of flow caused by the shear layer interaction in the boundary layer. To reduce a potential flow by the same amount, the surface would have to be displaced.

Figures 24-27 present the momentum thickness distributions for the design and off-design Case 0 simulations. There is an increase in the momentum thickness on the suction surface from the leading edge to the trailing edge of the EGV due to shear in the boundary layer at all the flow conditions. The momentum thickness is much greater on the suction surface for the off-design Case 0 simulations compared to the design Case 0 simulations. In addition, for the design and off-design Case 0 simulations, the momentum thickness on the suction surface is greater for the turbulent flow conditions compared to the transitional flow conditions. For the pressure surface, there is a less dramatic increase in the momentum thickness at
The surface pressure coefficient distributions are shown in Figures 28 and 29 for the Case 0 design and off-design simulations, respectively. In this investigation, the pressure coefficient is defined as

\[ C_P = 2.0 \frac{(P - P_{11})}{(\rho_1 q_1^2)} \]  

(11)

The area between the pressure and suction surfaces shown in Figure 28 and Figure 29 indicates that the EGV is a fore-loaded airfoil.

On the suction surface for the design (transitional and turbulent) pressure coefficient distributions, the pressure coefficient decreases with distance until the point of peak suction is reached (see Figure 28). At the point of peak suction, the flow velocity is at a maximum. Beyond the point of peak suction, there is a pressure rise, which indicates the presence of an adverse pressure gradient. The
Figure 24. Momentum Thickness Distribution for Case 0, Transitional, $\beta_1 = 40$ Deg.

Figure 25. Momentum Thickness Distribution for Case 0, Turbulent, $\beta_1 = 40$ Deg.
Figure 26. Momentum Thickness Distribution for Case 0, Transitional, $\beta_1 = 55$ Deg.

Figure 27. Momentum Thickness Distribution for Case 0, Turbulent, $\beta_1 = 55$ Deg.
surface pressure coefficient, at the point of peak suction, appears to be about the same for both the transitional and turbulent flow conditions at $\alpha = 40$ degrees. For the off-design (transitional and turbulent) pressure coefficient distributions, the point of peak suction is at the leading edge of the EGV (see Figure 29). At this point, the transitional flow condition has a slightly higher peak suction compared to the turbulent flow condition. From the leading edge, the pressure rises as distance increases, which indicates the presence of an adverse pressure gradient.

On the pressure surface, there is an increase in pressure as the distance along the chord increases for the design (transitional and turbulent) pressure coefficient distributions (see Figure 28). For the off-design (transitional and turbulent) pressure coefficient distributions, there is a decrease in the pressure coefficient near the leading edge which flattens out at approximately 40 percent of the axial chord (see Figure 29).
The frequency of the trailing edge vortices in the wake of the EGV was determined for the Case 0 simulations. If a velocity-measuring instrument is placed in the wake, one cycle of oscillation corresponds to the distance between two vortices (Panton, 1984). The frequency of this oscillation is known as the vortex shedding frequency. By non-dimensionalizing the vortex shedding frequency by the freestream velocity and the EGV trailing edge radius, the Strouhal number was determined, and is defined as

$$St = \frac{f(r)}{V_\infty}$$  \hspace{1cm} (12)$$

The Strouhal numbers for the design (transitional and turbulent) flow conditions were 0.0045 and 0.0065, respectively. For the off-design (transitional and turbulent) flow conditions, the Strouhal numbers were 0.0017 and 0.0026, respectively.

It is important to mention that all of the time-averaged flow quantities
produced for the Case 0 simulations and for the cases where a wake was specified at the EGV inlet were averaged over four wake passing cycles. Therefore, the effects of unsteady vortex shedding and the wake-passing frequency are included in the predicted results. As mentioned previously, changes in entropy can be attributed to unsteady vortex shedding and can affect the losses. According to Valkov (1993), changing the wake-passing frequency influences the unsteady flow over the suction surface. Since the unsteady flow behavior over the suction surface is influenced, the losses may also be affected.

The goal of the study was to examine the effects of the wake width and the wake velocity deficit on the losses in a compressor EGV passage at several different flow conditions. The losses were defined as

\[ \omega = \frac{2(P_{t_1} - P_{t_2})}{\rho_1 V_1^2} \]  

The losses for the Case 0 design transitional and turbulent simulations were 0.03173 and 0.03346, respectively. For the Case 0 off-design transitional and turbulent flow conditions, the losses were 0.02428 and 0.02471, respectively.

The predicted flow turning for the design and off-design Case 0 simulations was determined for various flow conditions. The predicted flow turning can provide information concerning the losses. For the design transitional and turbulent flow simulations, the flow turning angle values were 43.47 and 43.19 degrees, respectively. For the off-design transitional and turbulent flow simulations, the flow turning angle values were 57.94 and 55.47 degrees, respectively. The transitional flow simulations achieve more flow turning which agrees with the losses mentioned previously. The greater flow turning angles for the transitional flow conditions,
when compared to the turbulent flow conditions, can be attributed to a change in the effective shape of the EGV. The effective shape of the EGV is changed through changes in the boundary layer thickness. In the design and off-design momentum thickness distributions for Case 0 shown in Figures 24-27, a thinner boundary layer is displayed for the transitional flow conditions when compared to the turbulent flow conditions.

Simulations With a Specified Wake (Cases 1-9)

The results showing the losses and the predicted flow turning are presented in Tables 4 and 5 for the different unsteady test cases. In these tables, the Case 0 simulations have also been included. The results shown in the tables will be discussed in detail, with the design flow conditions being presented first.

Table 4

Predicted Time-averaged Losses

<table>
<thead>
<tr>
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<th>$\beta_1 = 40$ $deg$</th>
<th>$\beta_1 = 40$ $deg$</th>
<th>$\beta_1 = 55$ $deg$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Turbulent</td>
<td>Transitional</td>
<td>Turbulent</td>
<td>Transitional</td>
</tr>
<tr>
<td>Case 0</td>
<td>0.03346</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.03643</td>
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<td>0.03682</td>
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</tr>
<tr>
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<td>0.04162</td>
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<td>0.03906</td>
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Table 5

Predicted Flow Turning, $\Delta \beta$ (deg)

<table>
<thead>
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</tr>
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<td>Transitional</td>
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<tr>
<td>Case 9</td>
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<td>42.55</td>
</tr>
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Design Flow Conditions, $\beta_1 = 40$ Degrees

In Figure 30, the time-averaged losses are shown as a function of the wake depth and wake width. The turbulent simulations, in general, show greater time-averaged losses compared to the transitional simulations. There are large variations in the losses for the turbulent simulations. Even though large variations exist, there appears to be a trend which shows a small increase in the losses as the wake width and wake depth are increased. For the transitional simulations, there are smaller variations in the losses as the wake depth and wake width increase. In general, the losses become greater as the wake width and the wake depth are increased. The predicted flow losses for the design flow conditions are also tabulated in Table 4.

In Figure 31, the time-averaged flow turning angle is shown as a function of
the wake width and the wake depth. The transitional simulations show a greater amount of flow turning compared to the turbulent simulations. This trend agrees with the plot of the time-averaged losses (see Figure 30) where the transitional simulations show smaller time-averaged losses compared to the turbulent simulations. The turbulent and transition simulations reveal a general decrease in the flow turning as the wake depth is increased. Increasing the wake width does not significantly affect the flow turning. The predicted flow turning for the design flow conditions can be seen in Table 5.

A perturbation skin friction coefficient was defined in order to show the variations from the Case 0 simulations. The perturbation skin friction coefficient is defined as

\[
\bar{C}_f = (C_f - \bar{C}_f)/\bar{C}_f
\]  

(14)

where \(C_f\) is the time-averaged skin friction coefficient for the simulations where
a wake is specified, and $\bar{C}_f$ is the time-averaged skin friction coefficient for the Case 0 simulations. The perturbation skin friction coefficient distributions along the pressure and suction surfaces of the EGV were investigated in order to reveal the effects of the unsteady rotor wakes on the EGV boundary layer.

In Figures 32 and 33, the perturbation skin friction coefficient distributions are shown for the pressure surface where the wake width is varied. For both the transitional and turbulent simulations, the skin friction decreases as the wake width increases. For the transitional simulations, the skin friction coefficient values are lower than the Case 0 skin friction coefficient values. For the turbulent simulations, the skin friction coefficient values are lower than the Case 0 skin friction coefficient values except near the trailing edge. Near the trailing edge, the Case 0 skin friction coefficient values are higher for wake widths below 0.20. In addition, there is little variation in the skin friction values at all wake widths for
transitional flow.

In Figures 34 and 35, the perturbation skin friction coefficient distributions are shown for the suction surface where the wake width is varied. For both the transitional and turbulent simulations, the skin friction decreases up to the point of separation as the wake width is increased. In addition, large fluctuations are exhibited in the flow separation region. For the turbulent simulations, the skin friction values are slightly higher than the no-wake skin friction values for smaller wake widths near the leading edge. As the axial chord distance increases up to the separation point, the skin friction values for all wake widths are less than the no-wake values. Beyond the point of separation, the skin friction values are greater than the no-wake skin friction values. For the transitional simulations, the skin friction values are lower than the Case 0 skin friction coefficient values up to the point of separation. Beyond the point of separation, the skin friction values
Figure 33. Pressure-surface Perturbation Skin Friction for Cases 1-5, Transitional, $\beta_1 = 40$ Deg.

are greater than the no-wake skin friction values. A small fluctuation is observed at the location of transition in Figure 35 for all wake widths.

In Figures 36 and 37, the perturbation skin friction coefficient distributions are shown for the pressure surface where the wake depth is varied. For the turbulent simulations, the skin friction values are generally greater than the no-wake values as the wake depth is varied. As the wake depth is increased, the skin friction values, in general, decrease and approach the no-wake skin friction values. For the transitional simulations, the skin friction increases with the wake depth over the first 50 percent of the axial chord. In addition, the skin friction values are greater than the no-wake skin friction values for all wake depths.

In Figures 38 and 39, the perturbation skin friction coefficient distributions are shown for the suction surface where the wake depth is varied. In the turbulent simulations, the skin friction values remain greater than the no-wake skin friction
Figure 34. Suction-surface Perturbation Skin Friction for Cases 1-5, Turbulent, $\beta_1 = 40$ Deg.

Figure 35. Suction-surface Perturbation Skin Friction for Cases 1-5, Transitional, $\beta_1 = 40$ Deg.
Figure 36. Pressure-surface Perturbation Skin Friction for Cases 6-9, Turbulent, \( \beta_1 = 40 \text{ Deg.} \)

Figure 37. Pressure-surface Perturbation Skin Friction for Cases 6-9, Transitional, \( \beta_1 = 40 \text{ Deg.} \)
values up to approximately 50 percent of the axial chord for all wake depths. Beyond the 50 percent axial chord location and up to the point of flow separation, the skin friction drops below the no-wake skin friction for all wake depths. Downstream of the flow separation region, the skin friction is greater than the no-wake skin friction, and there are small fluctuations in the skin friction for all wake depths.

In the transitional simulations, the skin friction increases as the wake depth is increased up to the transition point. At the point of transition, a small decrease in the skin friction is noticed. Beyond the location of transition, and up to the point of separation, the skin friction values for all of the wake depths remain relatively constant. An increase of the skin friction values as the wake depth is increased is still observed after transition until the point of flow separation is reached. Beyond the separation point, large fluctuations in the skin friction are displayed for all wake depths. In addition, the skin friction values are generally greater than the Case 0 values as the wake depth is varied.

In Figure 40, the time-averaged separation location as a function of the wake width and the wake depth is presented for both the turbulent and transitional simulations. There is little variation in the separation location for varying wake widths and wake depths. Flow separation occurs at around 72 percent of the axial chord for the turbulent simulations. For the transitional simulations, flow separation occurs at approximately the 74.5 percent axial chord location.

Off-Design Flow Conditions, $\beta_1 = 55$ Degrees

The time-averaged losses as a function of the wake width and the wake depth are shown for the off-design flow conditions in Figure 41. In the transitional
Figure 38. Suction-surface Perturbation Skin Friction for Cases 6-9, Turbulent, \( \beta_1 = 40 \) Deg.

Figure 39. Suction-surface Perturbation Skin Friction for Cases 6-9, Transitional, \( \beta_1 = 40 \) Deg.
simulations, the time-averaged losses remain fairly constant for the different wake widths and wake depths. This indicates that the effects of the passing rotor wakes on the losses is minimal and that the transition location and flow separation regions are dominating the losses. The losses in the turbulent simulations vary considerably as the wake width and the wake depth are varied. An increasing trend in the losses is observed for increasing wake widths and wake depths.

The time-averaged flow turning as a function of the wake width and the wake depth is shown in Figure 42. The transitional simulations show a greater amount of flow turning when compared to the turbulent simulations for varying wake widths and wake depths. This agrees with the results presented in Figure 41, which indicate that the transitional simulations exhibit smaller cascade losses compared to the turbulent simulations. In addition, for the transitional and turbulent simulations, there is a general decrease in the flow turning as the wake
depth and wake width are increased.

In Figures 43 and 44, the perturbation skin friction coefficient distributions are shown for the pressure surface as the wake width is varied. For the turbulent simulations, the friction coefficient values are lower than the Case 0 skin friction values for all wake widths. All of the skin friction values approach the Case 0 skin friction coefficient values near the trailing edge of the EGV. For the transitional simulations, the skin friction coefficient values are lower than the Case 0 skin friction coefficient values and also remain fairly constant over the first 25 percent of the axial chord. Beyond the 25 percent axial chord location, the skin friction coefficient values increase and surpass the Case 0 skin friction values at approximately 50 percent of the axial chord. In addition, an increase in the skin friction is shown as the wake width is decreased from the EGV leading edge up to the 50 percent axial chord location.
Figure 42. Time-averaged Flow Turning as a Function of Wake Width and Wake Depth, $\beta_1 = 55$ Deg.

Figure 43. Pressure-surface Perturbation Skin Friction for Cases 1-5, Turbulent, $\beta_1 = 55$ Deg.
In Figures 45 and 46, the perturbation skin friction coefficient distributions are shown for the suction surface where the wake width is varied. The skin friction for the turbulent simulations remains relatively constant from the leading edge until approximately one-quarter of the axial chord. The skin friction values are lower than the Case 0 skin friction values for all wake widths from the leading edge up to the region of flow separation at approximately mid-chord. Beyond the region of flow separation, all of the skin friction values are higher than the Case 0 values. In addition, the skin friction for the turbulent simulations increases as the wake width decreases from the EGV leading edge up to the point of flow separation. The transitional simulations reveal large fluctuations in the skin friction for all wake widths at approximately 15 percent axial chord. According to the suction surface Case 0 off-design transitional skin friction coefficient distribution shown in Figure 20, flow separation occurs at approximately 15 percent axial
Figure 45. Suction-surface Perturbation Skin Friction for Cases 1-5, Turbulent, \( \beta_1 = 55 \) Deg.

Therefore, the region where large fluctuations are occurring at 15 percent chord in Figure 46 is considered to be a region where flow separation is known to exist. Beyond the flow separation region, the skin friction values remain relatively constant and then sharply decrease for all wake widths as flow separation occurs again downstream of mid-chord. The skin friction values are slightly less than the Case 0 values from the leading edge up to the flow separation region at 15 percent axial chord. Beyond this region, the skin friction values remain slightly less than the Case 0 values until the second flow separation region is reached downstream of mid-chord. The skin friction values are greater than the Case 0 values beyond the flow separation region.

In Figures 47 and 48, the perturbation skin friction coefficient distributions are shown for the pressure surface as the wake depth is varied. For the turbulent simulations, the skin friction values are generally greater than the no-wake values.
for smaller wake depths. As the wake depth is increased, the skin friction values surpass the Case 0 skin friction values. For the transitional simulations, the skin friction, in general, becomes larger than the no-wake values near the trailing edge.

The perturbation skin friction coefficient distributions for the suction surface at different wake depths are shown in Figures 49 and 50. For the transitional simulations, flow separation occurs at approximately 15 percent axial chord. Beyond this separation region, the skin friction values for the varying wake widths remain relatively constant until another region of flow separation is approached at approximately 60 percent of the axial chord. The skin friction values are generally greater than the Case 0 skin friction values for all wake depths, except at and near the separation regions. For the turbulent simulations, the skin friction values are approximately the same as the Case 0 values for all wake depths, except at and near the separation region at 60 percent axial chord.
Figure 47. Pressure-surface Perturbation Skin Friction for Cases 6-9, Turbulent, $\beta_1 = 55$ Deg.

Figure 48. Pressure-surface Perturbation Skin Friction for Cases 6-9, Transitional, $\beta_1 = 55$ Deg.
Figure 49. Suction-surface Perturbation Skin Friction for Cases 6-9, Turbulent, \( \beta_1 = 55 \text{ Deg.} \)

Figure 50. Suction-surface Perturbation Skin Friction for Cases 6-9, Transitional, \( \beta_1 = 55 \text{ Deg.} \)
In Figure 51, the time-averaged upstream separation locations as a function of the wake width and the wake depth are shown. As the wake depth is increased, the separation point moves downstream. As the wake width is increased, the flow separation point moves slightly upstream.

The time-averaged downstream separation locations as a function of the wake width and the wake depth are shown in Figure 52. For the transitional and the turbulent simulations, the flow separation point moves further downstream as the wake depth is increased. As the wake width is increased, the flow separation point moves further upstream for the transitional and turbulent simulations. In addition, the separation location for the transitional simulations is approximately 10 percent downstream of where it occurs for the turbulent simulations.

In Figures 53 and 54, the unsteady vorticity contours are shown for a given instant in time for Case 9 of the transitional simulations. The interaction of the
Figure 52. Time-averaged Downstream Separation Location as a Function of Wake Width and Wake Depth, $\beta_1 = 55$ Deg.

Rotor wake/wakes with the vorticity field within the boundary layer of the EGV is revealed. In Figure 55, the unsteady surface pressure coefficient distribution for Case 9 is illustrated. The pressure fluctuations on the suction surface could be attributed to the formation of vortices caused by the passing rotor wakes, similar to those observed by Valkov (1993).
Figure 53. Unsteady Vorticity Contours for Case 9, Transitional, $\beta_1 = 55$ Deg.

Figure 54. Close-up of Unsteady Vorticity Contours for Case 9, Transitional, $\beta_1 = 55$ Deg.
Figure 55. Unsteady Surface Pressure Coefficient Distribution for Case 9, Transitional, $\beta_1 = 55$ Deg.
CHAPTER VII

CONCLUSIONS

A series of numerical simulations were performed to gain insight into the effects of rotor wake/EGV interaction on cascade total pressure losses and boundary layer characteristics. In these numerical simulations, transitional and turbulent flow conditions were modeled at design and off-design operating conditions. In Tables 6-10 shown below, trends are identified based on the results from the numerical simulations. These trends may help in the design of future experiments, and also in the design of compressor blade geometries.

Table 6

General Trends for the Predicted Time-averaged Losses

<table>
<thead>
<tr>
<th>$\beta_1 = 40 , \text{deg}$</th>
<th>$\beta_1 = 40 , \text{deg}$</th>
<th>$\beta_1 = 55 , \text{deg}$</th>
<th>$\beta_1 = 55 , \text{deg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>Transitional</td>
<td>Turbulent</td>
<td>Transitional</td>
</tr>
<tr>
<td>Losses increase as $w$ and $d$ increase</td>
<td>Losses increase as $w$ and $d$ increase</td>
<td>Losses increase as $w$ and $d$ increase</td>
<td>Losses remain relatively constant as $w$ and $d$ are varied</td>
</tr>
</tbody>
</table>
Table 7

General Trends for the Predicted Flow Turning

<table>
<thead>
<tr>
<th>$\beta_1 = 40$ deg</th>
<th>$\beta_1 = 40$ deg</th>
<th>$\beta_1 = 55$ deg</th>
<th>$\beta_1 = 55$ deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>Transitional</td>
<td>Turbulent</td>
<td>Transitional</td>
</tr>
<tr>
<td>- Increasing $w$ does not significantly affect flow turning</td>
<td>- Increasing $w$ does not significantly affect flow turning</td>
<td>- Flow turning decreases as $w$ and $d$ increase</td>
<td>- Flow turning decreases as $w$ and $d$ increase</td>
</tr>
<tr>
<td>- Flow turning decreases as $d$ increases</td>
<td>- Flow turning decreases as $d$ increases</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8

General Trends for the Perturbation Skin Friction Coefficient for $\beta_1=40$ Degrees

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1 = 40$ deg</th>
<th>$\beta_1 = 40$ deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>Transitional</td>
<td>Turbulent</td>
</tr>
<tr>
<td>Pressure Surface</td>
<td>- Skin friction decreases as $w$ increases</td>
<td>- Skin friction decreases as $w$ increases</td>
</tr>
<tr>
<td></td>
<td>- Skin friction values are greater than the Case 0 values for all wake depths</td>
<td>- Skin friction increases as $d$ increases over the first 50 percent of the axial chord</td>
</tr>
<tr>
<td>Suction Surface</td>
<td>- Skin friction decreases as $w$ increases</td>
<td>- Skin friction decreases as $w$ increases</td>
</tr>
<tr>
<td></td>
<td>- Skin friction values are greater than the Case 0 values for all wake depths up to 50 percent chord</td>
<td>- Skin friction increases as $d$ increases</td>
</tr>
</tbody>
</table>
Table 9

General Trends for the Perturbation Skin Friction Coefficient for $\beta_1 = 55$ Degrees

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1 = 55 , \text{deg}$</th>
<th>$\beta_1 = 55 , \text{deg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Turbulent</td>
<td>Transitional</td>
</tr>
<tr>
<td>Pressure Surface</td>
<td>- Skin friction values are lower than Case 0 values for all wake widths</td>
<td>- Skin friction decreases as $w$ increases</td>
</tr>
<tr>
<td></td>
<td>- Skin friction values are greater than Case 0 values for small wake depths</td>
<td>- Skin friction values become larger than the Case 0 values for all wake depths near the EGV trailing edge</td>
</tr>
<tr>
<td>Suction Surface</td>
<td>- Skin friction decreases as $w$ increases</td>
<td>- Skin friction values remain relatively constant for all wake widths between the two regions of flow separation at 15 and 60 percent axial chord</td>
</tr>
<tr>
<td></td>
<td>- Skin friction values are approximately the same as the Case 0 values for all wake depths, except at or near flow separation region</td>
<td>- Skin friction values are greater than the Case 0 values for all wake depths, except at or near the flow separation regions</td>
</tr>
</tbody>
</table>
Table 10

General Trends for the Time-Averaged Separation Location

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1 = 40^\circ$ deg</th>
<th>$\beta_1 = 55^\circ$ deg</th>
</tr>
</thead>
</table>
| Upstream Separation for Transitional and Turbulent Flow | - No upstream separation location | - Separation location moves upstream as $w$ increases  
- Separation location moves downstream as $d$ increases |
| Downstream Separation for Transitional and Turbulent Flow | - Little variation in separation location for varying wake widths and wake depths | - Separation location moves upstream as $w$ increases  
- Separation location moves downstream as $d$ increases |
BIBLIOGRAPHY


Hoffman, K. A., *Computational Fluid Dynamics for Engineers*, The University of Texas at Austin, Austin, TX, 1989.


