Fuzzy Based Forebody Vortex Flow Controller for
the X-29A Aircraft

Jin Suzuki
Western Michigan University

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FUZZY BASED FOREBODY VORTEX FLOW CONTROLLER
FOR THE X-29A AIRCRAFT

by

Jin Suzuki

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirement for the
Degree of Master of Science
Department of Mechanical and Aeronautical Engineering

Western Michigan University
Kalamazoo, Michigan
June 1997
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Jin Suzuki
1997
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I dedicate this thesis to them.

Jin Suzuki
FUZZY BASED FOREBODY VORTEX FLOW CONTROLLER
FOR THE X-29A AIRCRAFT

Jin Suzuki, M.S.

Western Michigan University, 1997

This paper demonstrates the development and performance of A Fuzzy Logic based regulator for an aircraft which is equipped with pneumatic forebody Vortex Flow Controller at high angle-of-attack flight condition. The VFC provides the pilot additional control power at flight conditions where conventional effectors such as rudder and differential flaps lose effectiveness.

Fuzzy Logic Control (FLC) is applied to a mixed bang-bang and conventional type control system. This unique configuration ensures robustness of the system and simplifies the controller design. Performance of the FLC based regulator shows a 10.3% reduction in cost function compared to the system controlled by a Variable Structure Controller. Due to the simplicity of FLC, the designer is able to modify its control law without high level mathematical model of the system.
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INTRODUCTION

The Gulf War proved that “Air Superiority” is one of the most important factors in modern combat. The air superiority is achieved by four main factors, better pilot quality, more number of aircraft, better weapon systems, and better quality of aircraft.

After the Korea War, the prediction was made by many engineers and analysts in the U.S. Air Force that the era of dog-fight is over because of the invention of guided missile. In fact, new aircraft planned at these years were not designed to carry a gun, a classical but the most reliable weapon, and which of that was replaced to missiles. The design concept of the aircraft was called “Missile Carrier” in which aircraft is treated as only a missile platform.

The concept was denied later in the Vietnam War. Pilots are faced with the fact that they need to maneuver as much as they could, since high maneuver is required to evade from missiles. As a result, high maneuver aircraft such as F-14 and F-15 are developed based on the experience.

The efforts to make aircraft more maneuverable have been continued today. The latest technologies such as thrust vector control, high powered engine, composite material application, and digital fight control system push flight limitation beyond previous flight envelope. It is called post-stall maneuver, and maneuverability is referred as agility in most of the time. Generally, the loss of aircraft agility is
remarkable as aircraft increase angle-of-attack. In most cases, one of the reasons is the loss of control effectiveness due to separation of flow and wake effect. Another reason is the increasing of static and dynamic lateral-directional instability.

X-29A Research Aircraft

The X-29A is designed to demonstrate advanced concept and technology of aircraft that is equipped with variable camber wing surfaces, forward swept wing with thin supercritical airfoil, strake flaps, differential flaps and computerized fly-by-wire flight control system (see Figure 1).

![Figure 1. Top View of X-29A.](image)

Aircraft No. 2, one of the two X-29A built for the project, was modified for installation of Vortex Flow Controller, VFC, system. The VFC system consists of two high pressure nitrogen tanks and control valves with two small nozzle jets at forebody of nose section (see Figure 2).
The ability to maneuver fighter-type aircraft at high-angle-of-attack using a tangential slot blowing, or VFC, for controlling the forebody vortex is generally well understood (Valasek and Downing, April 1995). The behavior of forebody vortices largely depends on the angle-of-attack, in other words the higher the angle-of-attack is, the higher and the further the migration of the forebody vortices from the body will be, thus on most occasions this results in a shedding non-symmetric and unstable. The aircraft directional control is obstructed by this von Karman vortex shedding.

![Figure 2. X-29A VFC System External Characteristics (Valasek and Downing 1995).](source)


For example, vortex injection on RHS VFC nozzle results a RHS side force on forebody which provides yawing control in RHS direction. In case of generation yawing moment in LHS direction, the operation is reversed.

Unlike conventional controller, VFC nozzles are bang-bang type control effector so that ordinary feedback gain controller is not suitable for controlling.
Variable Structure Controller, VSC, is developed by Valasek and Downing for closed loop control of VFC nozzles (Valasek and Downing, August 1995). See Figure 3.

Figure 3. Creation of Forces Using Forebody Vortices in a Vortex Flow Control System (Valasek and Downing 1995).


Application of VSC generally improved the regulator performance of the X-29A aircraft and synthesis of continuous and bang-bang effector was successful. As shown in Figure 21, it seems that general responses and damping is improved under VFC nozzle control. However from practical point of view, the excessive switching of nozzle valves is not mechanically desirable. In addition, adjustment of VFC nozzle activity is very complicated and time consuming, because VSC is optimized controller.

Recently, an innovative control theory, “Fuzzy Logic”, got attention among control applications. Many commercial products in Japan proved that Fuzzy Logic perform well in either linear and non-linear application.
The objective of this paper is to develop a fuzzy logic based regulator which controls VFC nozzles with better of similar overall performance. SIMULINK/MATLAB with FUZZY LOGIC TOOLBOX is selected for development tools of fuzzy logic controller. These developing tools are also used for simulation and further analysis.
AIRCRAFT MODEL AND DYNAMICS

Aircraft Dynamics

The objective is to design a fuzzy based closed-loop VFC controller which will drive the system from a specified initial sideslip angle to a command terminal nonzero sideslip angle.

A high angle-of-attack trimmed condition, where the VFC nozzle is effective as well as differential flaps and rudder, is selected and shown in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Trim Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>α₁</td>
<td>degree</td>
<td>40</td>
</tr>
<tr>
<td>H₁</td>
<td>feet</td>
<td>38000</td>
</tr>
<tr>
<td>V₁</td>
<td>feet/sec</td>
<td>338</td>
</tr>
<tr>
<td>q̄</td>
<td>psf</td>
<td>37</td>
</tr>
<tr>
<td>δc</td>
<td>degree</td>
<td>-24.4</td>
</tr>
<tr>
<td>δstf</td>
<td>degree</td>
<td>12.8</td>
</tr>
<tr>
<td>δtf</td>
<td>degree</td>
<td>20.7</td>
</tr>
</tbody>
</table>
The lateral/directional linear state equation for the generic X-29A aircraft at this flight condition is:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{p} \\
\dot{r} \\
\dot{\phi} \\
\dot{\psi}
\end{bmatrix}
= 
\begin{bmatrix}
0.043 & 0.64 & -0.77 & 0.073 & 0. \\
-8.39 & 0.78 & -0.69 & -0.74E-5 & 0. \\
0.14 & 0.063 & -0.11 & -0.17E-5 & 0. \\
0. & 1. & 0.84 & 0. & 0. \\
0. & 0. & 1. & 0. & 0.
\end{bmatrix}
\begin{bmatrix}
\beta \\
p \\
r \\
\phi \\
\psi
\end{bmatrix}
+ 
\begin{bmatrix}
-0.0034 & 0.0013 \\
0.86 & -0.17 \\
0.092 & -0.12 \\
0. & 0. \\
0. & 0.
\end{bmatrix}
\begin{bmatrix}
\delta_{df} \\
\delta_r \\
\delta_m
\end{bmatrix}
\] (1)

Note, all angular quantities in the equation is in radians. Three control effectors are available in this model, two continuous (\(\delta_r\) or rudder and differential \(\delta_{df}\) of differential flaps) and one bang-bang (VFC or \(\delta_{vn}\)).

The lateral/directional flight control system throughout this pare has sampling rate of 10 Hz, instead of using the nominal X-29A flight control system sampling rate of 40 Hz. The actuator model in all the following simulation model is reduced from the real X-29A actuators. The design specifications are also identical for comparison which is:

- 5% setting time in sideslip angle: 5 seconds
- Maximum sideslip angle: 10 degrees
maximum body axis yaw rate - 30 degree/second
maximum bank angle - 30 degrees

Initial condition of a sideslip angle of five degrees is selected because it is sufficiently large angle at this flight condition to excite the Dutch roll and saturate differential flaps and rudder but prevent the aircraft from departing controlled flight.

To develop feedback gain and to calculate performance index, following control weighting matrices are prepared and feedback gains are designed based on the cost function (described in later section) and weighting matrices in the present reference (Valasek and Downing, August 1995). The weighting matrices which are given in the reference is:

\[ Q_{11} = 32.8 \quad Q_{22} = 2.0 \quad Q_{33} = 3.65 \quad Q_{44} = 3.65 \]
\[ Q_{55} = 20 \quad R_{11} = 85 \quad R_{22} = 2.5 \]

The feedback gains obtained from these matrices are:

\[ K = \begin{bmatrix} -0.92 & 2.08 & -2.06 & 0.23 & 0.081 \\ 5.22 & -2.59 & -4.25 & -0.49 & -2.62 \end{bmatrix} \] (2)

For comparison, the values of weighting matrices and feedback gains are held through FLC design.
DESIGN EXAMPLE

Cost Function

To optimize the control system numerically, mathematical function, called cost function or performance index, is applied. The design procedure for optimization is to minimize the cost function in which equation is given as:

\[ J = \frac{1}{2} \sum_{k=0}^{\infty} \left( X_k' \hat{Q} X_k + U_k' \hat{R} U_k + 2 X_k' M U_k \right) \]  

(3)

where M, Q, and R are weight matrices, X_k and U_k are sampled output and input data respectively. Designer has a freedom to choose the weight matrices M, Q and R. In general, the selection of the weight matrices are done by trial and error approach. Feedback gain for continuous effector is obtained from the cost function and weighting matrices. The cost function is also used to indicate the general performance of the system. The smaller the cost function is, the better the controller.

For comparison, nominal controller, Design 1, and Variable Structure Controller, Design 2, which are designed in the reference (Valasek and Downing, 1995) as Design 1 and Design 2, are reconstructed in the SIMULINK/MATLAB software.
Nominal Controller

Nominal controller, Design 1, is a simple feedback gain controller, which controls continuous effector, rudder and differential flaps. The system is designed by Valasek and Downing and this is reconstructed in SIMULINK for comparison and verification. Figure 4 shows a diagram of Design 1 in SIMULINK.

![Block Diagram of Design 1 in SIMULINK.](image)

The "Inport 1" is a dummy inport which may use for command input for future analysis. The "Effector" block simulate real X-29A actuator. The "State-Space1" block corresponds to aircraft model. Feedback signal is sampled by 10Hz which is done by the "10Hz Sampling" block. Feedback gain matrix is located right after that. Gain, K, is multiplied to feedback signal in the "Matrix Gain" block. Time history data is stored in the "Output" block for later analysis. The "costdd" block automatically sample the output and input data every 0.1 second and calculates cost.
function during simulation. Local cost functions are saved in the “costo” block for total cost calculation (Valasek and Downing, 1995).

Variable Structure Controller

Variable Structure Controller, Design 2, is also designed by Valasek and Downing to control VFC nozzles (Valasek and Downing, 1995). SIMULINK model reconstructed for this system is shown in Figure 5. General arrangement of blocks are same as that of Design 1 except for the “controller” block.

Figure 5. Block Diagram of Design 2 in SIMULINK.

Figure 6 shows a detail design of the “controller” block. The “in_1” is import block from feedback line, and the “out_1” block is the output in which command signal is sent. Feedback signal is split into two lines, one goes to continuous effector and the other goes to VFC control. The conventional feedback-gain controller, as designed in Design 1, still controls continuous effector by the “K”, gain, block.
Three blocks are in parallel connection, “continuous plus”, “continuous” and “continuous minus” blocks (see Appendix B for detail program listing).

![Diagram of Controller Block](image)

**Figure 6.** Detail Design of “Controller” Block of Design 2 in SIMULINK

Each of the blocks has aircraft model and weight matrices in its own for cost calculation. The block automatically samples the data with a sampling rate of 10 Hz. All blocks, “continuous plus”, “continuous”, and “continuous minus”, calculate cost function in each cycle with different VFC command, +1, 0, and -1 respectively. These costs are compared in the “selec” block (see appendix C for detail program listing), VFC command which has minimum cost is selected as a VFC command for next cycle. Thus, controller selects optimized VFC command to result minimum cost in each sampling cycle.
Fuzzy-Logic, Overview

Many people misunderstood Fuzzy-logic because of its term "fuzzy". As described by McNeil and Freiberger, "Fuzzy-logic is not a logic that is fuzzy, but logic that describes and tames fuzziness." (McNeil and Freiberger, 1993, pp. 12).

Human always deal with fuzziness to distinguish members of a class from non-members such as:

*quite beautiful, about right, a kind of,...*

and so on. When we use those term, there lies the basic concept that all the states are expressed in degrees. Traditional logic, on the other hand, require crisp boundary to distinguish classes. For example to express room temperature condition, we have two expressions "HOT" and "COMFORTABLE". In traditional logic, boundary line is required to distinguish those two conditions. Suppose, 80° F is boundary line, then temperature higher than 80° F is "HOT" and lower than 80° F is "COMFORTABLE". Now questions arise "What about the temperature of 79.5° F?" or "Is there any difference between 79.5° F and 80° F?". This problem is not complex at all for human logic. We may simply say about the room temperature of 79.5° F;

It is kind of HOT.
Fuzzy-logic reflects these linguistic expressions by using the concept of degrees.

It is 0.8 HOT out of 1.0

It is 0.2 COMFORTABLE out of 1.0

It is noted that it is common to express degrees in maximum scale of 1.0. In real life, the classes are not simply “right or wrong” or “back or white”. They are sometimes “right and wrong” or in other words “sort of right, but it could be wrong”. By using degree concept, we can avoid such classical boundary-type class distinction by applying Fuzzy-logic.

Fuzzy Logic Controller

The process of Fuzzy Logic Controller, FLC, consists of three main step, fuzzification, decision making, and defuzzification (Figure 7). A crisp input value is transformed to degree of classes at fuzzification process. The output value, expressed in degree of classes, is determined at decision making process. Finally, degree of classed in output is transformed back to crisp number at defuzzification process. Detail of the steps are described in the following section.
Fuzzification

The idea of fuzzification is to change the crisp number to the grade of linguistic classes. Fuzzification process is done graphically by using Membership Function shown in Figure 8.

Figure 8. An Example of Membership Function With Input of $T = 79.5^\circ F$. 
Recalling the example of the room temperature discussed in previous section, the input is the temperature which is, in this case, 79.5°F. The classes "HOT" and "COMFORTABLE" are called as "linguistic variable". Each linguistic variable is expressed in the function, which is called membership function (MF). There are several types of MFs used today. Figure 9 shows variety of popular MFs.

![Variety of Membership Functions](image)

Figure 9. Variety of Membership Functions.

The selection of MF is done by a designer based on design objective of the system and system characteristics. As shown in the figure 8, x-axis corresponds to input, in this case temperature, its range is called "universe of discourse" and y-axis corresponds to degree of class in 0 to 1.

The example in Figure 8, input temperature value of 79.5°F is applied to x-axis and it is projected to MFs. Crisp input value is transformed into degree of linguistic variable from each MF.
Those can be read at the y-value of each MF as a degree of each linguistic variable:

\[
\text{HOT} = 0.8 \\
\text{COMFORTABLE} = 0.2
\]

Decision Making Process (Fuzzy Rule)

The decision making process is "the very soul of the process." (McNeil and Freiberger, 1993, pp. 111) in fuzzy logic. In this process, knowledge of skilled operator is reflected to control process. The output is determined based on input condition and algorithm, which is called "Fuzzy rule".

When human make a decision to control a system, it is natural that we use if-then type algorithm based on input and output conditions. For example, when we control Air Conditioner in a room, simple form of our decision should be made as follows;

\[
\begin{align*}
\text{IF} & \quad \text{temperature} = \text{VERY HOT} \quad \text{then} \quad \text{turn A/C on to MAXIMUM} \\
\text{IF} & \quad \text{temperature} = \text{MEDIUM HOT} \quad \text{then} \quad \text{turn A/C on to MEDIUM} \\
\text{IF} & \quad \text{temperature} = \text{CONFORTABLE} \quad \text{then} \quad \text{turn A/C OFF}
\end{align*}
\]

However, control system in real life is highly non-linear and very complex. Normally, the condition or environment what we use for decision making is not only a instant condition, but also a rate of change condition. Based on these conditions, a skilled operator is able to predict the next instant to react or to prepare for the future status while controlling a system. Yager suggests that the error - change of error, e-\(\Delta e\), type control should be used for FLC for better performance as well as human
decision making (Yager, 1994). Based on e-Δe control, the example of fuzzy rule above is modified as;

IF T = VERY HOT and ΔT = POSITIVE LARGE then A/C = MAXIMUM
IF T = MEDIUM and ΔT = POSITIVE LARGE then A/C = MAXIMUM
IF T = MEDIUM and ΔT = NEGATIVE LARGE then A/C = OFF

Note T denotes temperature and ΔT denotes rate of temperature change.

For convenience, FAM-table (shown in Figure 10) is widely used for development of fuzzy rule. In Figure 10, horizontal column represents rate of change and vertical column represents present condition.

<table>
<thead>
<tr>
<th>input property</th>
<th>ΔT</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF property</td>
<td>Negative</td>
</tr>
<tr>
<td></td>
<td>Large</td>
</tr>
<tr>
<td>Comfortable</td>
<td>Zero</td>
</tr>
<tr>
<td>Slight Hot</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>Small</td>
</tr>
<tr>
<td>Medium</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>Large</td>
</tr>
<tr>
<td>Very Hot</td>
<td>OFF</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
</tr>
</tbody>
</table>

Figure 10. FAM Table.

The each block in FAM table is filled up based on designer’s knowledge and experience. Of course some system does not meet or does not need all the conditions
stated in FAM table due to system characteristics, performance required or process speed requirement, that it is not always necessary to fill out all the blocks in FAM table.

The linguistic variable introduced at fuzzification step is necessary to do the human-style decision making process. Detail description is shown in later section as an example.

Generally, initial design of FLC does not work properly, thus it requires some modification for input/output MF and fuzzy rules. This process is called as “Tuning”. Tuning process is conducted based on designer’s knowledge and experience also.

Design Example: Process and Development of FLC

To show a detail fuzzy control process, it may be simpler to show a design example of FLC. In this chapter, design example of the Automated Air Conditioning System using FLC is presented.

First of all, input and output MFs is designed. The design of MF completely depends on the designer’s experience and knowledge. There are two ways to obtain the expert knowledge and experience. One is the case that the designer oneself is already an expert of the system or become an expert of the system. The other way is that the designer get the expert knowledge by interviewing a person who is already an expert.

MFs of two input variables, T (temperature) and ΔT (change of temperature), and output, A/C command, for the system are designed (Figure 11).
Universe of discourse, or operation range of input/output, is determined based on designer's knowledge. Each input is divided into four MFs:

ZE = Zero
PS = Positive Small
PM = Positive Medium
PL = Positive Large

Which of those indicate intensity and direction of MF.

Universe of discourse of output is determined due to limitation of the system or output signal range. In this example, output range is 0 - 100 %. Same procedure of output MF design method is taken as it is done for input MF design.
Of course, this is not a final design of MF. Because it is possible and usually happens that these MF designs may not be suitable to the system or does not perform well as expected, even though the designs are based on expert knowledge. Adjustment of MF and Fuzzy-rules (described in next step) is done without any complex steps after first simulation of the controller. This step is called "Tuning".

In this example, arbitrary input numbers of $T = 79.5 \, ^\circ F$ and $\Delta T = 2.5 \, ^\circ F/sec$ are applied to inputs. The input values are on the MFs of PM and PS for $T$-input and PS and PM on $\Delta T$-input. Fuzzification process transforms the input numbers from single number to fuzzy-term for each input which are shown in Figure 12. Degree of MFs for each input is shown in Table 2.

### Table 2
An Example of Membership Function and Degree of Inputs

<table>
<thead>
<tr>
<th>Input</th>
<th>$T$</th>
<th>Degree</th>
<th>$\Delta T$</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>0.1</td>
<td></td>
<td>PS</td>
<td>0.49</td>
</tr>
<tr>
<td>PM</td>
<td>0.9</td>
<td></td>
<td>PM</td>
<td>0.51</td>
</tr>
</tbody>
</table>

FAM-table constructed for the system is shown in Figure 13. There are four fuzzy rules that correspond to the input conditions:

1. If $T = PS$ and $\Delta T = PS$ then $A/C = PS$
2. If $T = PS$ and $\Delta T = PM$ then $A/C = PM$
3. If $T = PM$ and $\Delta T = PS$ then $A/C = PM$
4. If $T = PM$ and $\Delta T = PM$ then $A/C = PL$
Figure 12. Fuzzy Controller Process Example in Air Conditioning Control Design.
The degree of output MF is determined by "Implication Method". In most fuzzy application min-implication method is used because of its simplicity. Figure 12 shows a detail process of min-implication method. The smaller degree of MF from the inputs is directly projected to corresponded output MF in which the relation is stated in Fuzzy rules. Four output degrees of MFs in this example are obtained through the process. The results are shown in Table 3.

The mathematical expression of min-implication method is:

\[ R(y, x) = \min [A(x):B(y)] = \min [A(x)\cdot B(y)] \]

where

\[ x \in X \quad y \in Y \]  \hspace{1cm} (4)

Composition process combines those degrees of output MFs to obtain single output number which is shown in Figure 12. This composition method is called min-
max method which simply combines output MF graphically with degree defined by implication method.

Table 3
An Example of Implication Process

<table>
<thead>
<tr>
<th>Rule #</th>
<th>Output MF</th>
<th>Output Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PS</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>PM</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>PM</td>
<td>0.49</td>
</tr>
<tr>
<td>4</td>
<td>PL</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Mathematical expression of max-min composition method is:

\[ B(y) = \max \min [A(x) : R(y,x)] \]

\[ x \in X \quad y \in Y \quad (5) \]

Finally, defuzzification process produces single output number. As proceeded in fuzzification process, defuzzification process is also conducted graphically. Mean-of-Maxima, MOM, and Center-of-Area, or centroid, are the most typical defuzzification methods among FLC applications today. Graphical result for the example is shown in Figure 12. As shown in Figure 12, MOM method determines output by calculating the centroid of MF area with the largest grade of linguistic variable. Therefore, MOM method produces discrete output signal and it is useful for switching-type device such as, on-off or high-medium-small-off. In this example,
0.51 from PL is selected as the largest MF and the output value calculated is 100%.

Equation of MOM is;

\[ Y_{MOM}(C) = \frac{\sum y_k}{n} \]  \hspace{1cm} (6)

Centroid method is sometimes called the center of gravity method or center of area method. The output is defined by calculating the center of gravity of entire output MF grade obtained from composition method. This method works well for continuous device since smooth transition in output signal is obtained in real time operation. The output value calculated by COM method is 78.7%. The mathematical expression of COM method is given as:

\[ Y_{COM}(C) = \frac{\sum C(y_k)y_k}{\sum C(y_k)} \]  \hspace{1cm} (7)
DEVELOPMENT OF FUZZY LOGIC CONTROLLER
FOR THE X-29A AIRCRAFT

In this paper, FLC is designed to control only VFC nozzles, while conventional linear feedback controller controls continuous controllers, rudder and differential flaps, to avoid complexity of the controller. In Reference (Valasek and Downing, 1995) Variable Structure Controller was successfully developed to synthesize continuous and discrete effectors. However, the output shows that the controller forces frequent switching to VFC nozzle system and it is not simple to adjust its timing of control since it is linear optimized controller. From mechanical point of view, it is not desirable for nozzle mechanism to function close-open process so frequently and rapidly. The goal of FLC design in this paper is to achieve continuous nozzle function and less repeating without losing total system performance of aircraft control.

The development of FLC can be divided into three main steps: (1) System characteristics analysis, (2) Development of membership function and fuzzy rules, and (3) Simulation and tuning.

To develop FLC, it is necessary for designer to obtain expert knowledge of the system. There are two situations to get “expert knowledge”. If there is someone who already know the system characteristics well, then designer is able to get the knowledge by interviewing him or her. Another approach is, that the designer oneself
become an expert of the system. Since it is not possible to interview to test pilot who flew X-29 aircraft, later approach is used for this paper. SIMULINK tool box for MATLAB software is used to examine the aircraft dynamics and its input responses.

Step 2. is the main part of the FLC development phase. Detail of the process is described in next section.

FLC developed at first time usually does not work properly. It is like a situation, that even though a person knows how to ride bicycle theoretically, he/she needs some practice to ride it naturally. The coordination of input process, decision making and output process for riding bicycle is adjusted through trial-and-error period. The FLC also requires the adjustment step based on first run of the simulation to obtain proper control of the system. This step is called “Tuning”.

Analysis of System Characteristics and Input Selection for FLC

To examine the characteristics of X-29A aircraft with VFC input, open-loop simulation model in SIMULINK is constructed as shown in Figure 14.

Input signal of Rudder and differential flaps are set to zero, to simulate no activity of rudder and differential flaps, and signal “VFC input” is able to produce a step signal of -1 or 1. Aircraft model is simulated by a full state space equation, same as the one in equation (1). Time history data of \( \beta \), \( d\beta \), \( p \), \( r \) and \( \phi \) are saved into “To Workspace” block which is used for further analysis. To save the angle unit of input data of “To Workspace” in degree (since all angle components in State-Space...
equation are in radian), the block “Rad-Deg” is added right after “State-Space1” block.

The result of 5 seconds simulation with step input of VFC = +1 is shown in Figure 15. The nozzle is activated by the VFC signal of +1 from signal generator at t = 1 second and it continues until t = 5 seconds. The reaction of β, dβ, and r starts immediately after the VFC activation. It is noted that a slight delay is found for reaction of p and φ. This delay can be described from state equation (equation (1)). In p and φ state equation, VFC input state, δvn, is multiplied to zero so that δvn has no direct control in p and φ states. Therefore, we can say that the reactions of p and φ are not a result of VFC input command but the reactions are actually induced from β and r states in p and φ state equation. To maintain a process speed of controller, β, dβ and r are selected for controller inputs of FLC.
Figure 15. Reactions of Open Loop Analysis.
Figure 15—Continued. Reaction of Open Loop Analysis
Design of MFs

At the initial stage of FLC design, in general, the designer has no clue about the range of universe of discourse and shape of MF. Characteristics of Design 2 (Figure 21) is carefully analyzed to estimate the maximum range of $\beta$, $d\beta$ and $r$. To ensure stability of the system, the universe of discourse must not be smaller than system input, $\beta$, $d\beta$ and $r$, possibly obtained in actual operation.

The range of $\beta$, $d\beta$ and $r$ are set as:

$$-10^\circ < \beta < 10^\circ, -15^\circ < d\beta < 15^\circ, \text{ and } -10^\circ < r < 10^\circ$$

for initial design.

The number of MFs used in each input is limited by process speed that is allowed for the system, on the other hand, use of higher number of MFs ensure smooth and detailed control. Using 7 MFs in a input is common configuration among most of FLC application because of its good balance between process speed and controllability. As a starting point, 7 MFs are also applied to inputs, $\beta$, $d\beta$ and $r$ also, which are:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PL</td>
<td>Positive Large</td>
</tr>
<tr>
<td>PM</td>
<td>Positive Medium</td>
</tr>
<tr>
<td>PS</td>
<td>Positive Small</td>
</tr>
<tr>
<td>ZE</td>
<td>Zero</td>
</tr>
<tr>
<td>NS</td>
<td>Negative Small</td>
</tr>
<tr>
<td>NM</td>
<td>Negative Medium</td>
</tr>
<tr>
<td>NL</td>
<td>Negative Large</td>
</tr>
</tbody>
</table>

The position and shape of MF also has a significant effect in control system. Again, at initial stage of MF design, designer does not have any knowledge and
experience to select suitable MF, and there is no empirical rule of the MF design either. Yager suggests to use the triangular MF placed in equal distance for initial stage of design (Yager, 1994). Of course this MF may not be suitable for the system, however it is usually a “good enough” MF and we are able to modify this properly at tuning stage. Initial MFs designed are shown in Figure 16.

Since the system uses discrete output device, output needs only 3 output signals, -1, 0, and +1. To obtain right output signal, center of area of each MF must be located at the output signal required so that controller feeds a proper discrete signal. The shape and the range of MF do not make any differences to output signal. For simplicity, narrow and triangular shape MF is selected (Figure 16).

Development of Fuzzy Rule

Once again, knowledge and experience of skilled operator is required to build fuzzy rule sets, also. For designer who do not have those, Yager suggests some example of fuzzy rule sets as a template (Yager, 1994). This templates are useful in some cases when the dynamics and characteristics of the template is matched to the system to be designed. Unfortunately, none of these sets works for the VFC system. To construct the rule sets from scratch, time history plots of open loop examination developed in previous section (Figure 15) and output plot of Variable Structure System (Figure 21) are carefully analyzed to find out that how VFC input results the change in output properties. The basic algorithm based on VFC input = +1 is:
Figure 16. MF Design for Design 3.
if $p = \text{Positive Large}$ and $d\beta = \text{Positive Large}$ then $VFC = +1$

If $r = \text{Positive Large}$ then $VFC = +1$

For initial design, rate control, $d\beta$ and $r$, is the primary consideration for regulator design. Initial design of fuzzy rule is shown as FAM table form in Figure 17.

Selection of Defuzzification Method

Due to the limitation of VFC function, bang-bang control, MOM method is obviously selected for defuzzification method so that the FLC yields the discrete output signal of -1, 0 and +1 only.

Construction of Design 3 in SIMULINK

Initial FLC designed in this chapter is named “Design 3”. A block diagram of Design 3 is constructed for simulation in SIMULINK (shown in Figure 18). Overall design of the block diagram is identical to the one of Design 2 except for feedback section.

From Design 2, controller for bang-bang effector is replaced from VSC to FLC while feedback gain is held in the same configuration. Detail design of the “Controller” block is shown in Figure 19.
FAM table form

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<td>+1</td>
</tr>
</tbody>
</table>

Text form

1. If (beta is PM) and (d(beta is NL) then (vfc is pu) (1)
2. If (beta is PS) and (d(beta is NL) then (vfc is pu) (1)
3. If (beta is ZE) and (d(beta is NL) then (vfc is pu) (1)
4. If (beta is NS) and (d(beta is NL) then (vfc is pu) (1)
5. If (beta is NM) and (d(beta is NL) then (vfc is pu) (1)
6. If (beta is NL) and (d(beta is NL) then (vfc is pu) (1)
7. If (beta is NM) and (d(beta is NL) then (vfc is pu) (1)
8. If (beta is NL) and (d(beta is NM) then (vfc is pu) (1)
9. If (beta is NM) and (d(beta is PL) then (vfc is pu) (1)
10. If (beta is NS) and (d(beta is PL) then (vfc is nu) (1)
11. If (beta is ZE) and (d(beta is PL) then (vfc is nu) (1)
12. If (beta is PS) and (d(beta is PL) then (vfc is nu) (1)
13. If (beta is PM) and (d(beta is PL) then (vfc is nu) (1)
14. If (beta is PL) and (d(beta is PL) then (vfc is nu) (1)
15. If (beta is PM) and (d(beta is PM) then (vfc is nu) (1)
16. If (beta is PL) and (d(beta is PL) then (vfc is nu) (1)
17. If (r is NL) then (vfc is nu) (1)
18. If (r is PL) then (vfc is pu) (1)
19. If (r is NM) then (vfc is nu) (1)
20. If (r is PM) then (vfc is pu) (1)

Figure 17. Fuzzy Rule for Design 3 in FAM Table Form and Text Form.
The "in_1" block is the input port and the "out_1" block is the output port of "Controller" block, fuzzy controller, "Fuzzy Inference System" block, is connected for VFC control. Now, Design 3 is ready for simulation and tuning.

Figure 18. Block Diagram of Design 3 in SIMULINK.

Figure 19. Detail Design of "Controller" Block of Design 3 in SIMULINK.
The performance of Design 1, linear feedback system, and Design 2, Variable Structure Controller, by SIMULINK are presented in Figure 20 and Figure 21. Design 2 demonstrates that activation of VFC reduced both rudder and differential flap activity and improved response of all other states except for bank angle which is more displaced than Design 1. This bank angle displacement can be eliminated by modifying simply increasing the weight on the bank angle state. However, no modification is made because bank angle displacement remains within the design specification. The cost of Design 2 is also improved from 29.77 to 13.15 in 10 seconds run which is 56% improvement over Design 1. Note that Design 2 has frequent open-close valve activity on each VFC nozzle as shown in Figure 21.

The performance of initial FLC, Design 3, is presented in Figure 22. In Figure 22, thick lines represent response of Design 3 while thin line represents the one of Design 2. Coincidentally, the cost of Design 3 is identical to Design 2 which is 13.15. The activity of VFC nozzles are reduced to three times. However, there is one impulsive burst at second nozzle activation. Other states are very similar to Design 2 but yaw rate response and rudder activity. Rudder activity is reduced at second peak. Design 3 has slight different characteristics of yaw rate response comparing to Design 2. It should be noted that two spikes in yaw rate response are found between $t = 1$ second and $t = 2$ seconds in Design 3. These reactions exactly match to VFC nozzle
Figure 20. Design 1, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 29.77.
Figure 20—Cont. Design 1, Generic X-29A, 0.35/40k, $T = 0.1$ sec, Cost = 29.77.
Figure 21. Design 2, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 13.15.
Figure 21—Cont. Design 2, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 13.15.
Figure 22. Design 3, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 13.15.
Figure 22—Cont. Design 3, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 13.15.
activity. Therefore, it is obvious that the second impulsive burst of VFC nozzle activity is driven by too sensitive fuzzy rule in yaw rate state. In order to reduce sensitivity, fuzzy rule 19 and 20 (Figure 17) are deleted. New FLC with less rule is named as Design 4 (Figure 23).

Design 4 performance is presented in Figure 24 with thick line and Design 3 response is plotted by thin line. Design 4 has a cost of 13.11, which is 0.3% improvement over Design 2 and Design 3. The response change from fuzzy rule modification can be clearly seen in yaw rate response and VFC nozzle state. As expected, there is no excess and frequent switching VFC nozzle activity any more. Each nozzle functions only once through 10 seconds run and all the reactions of states still remain within design requirement without loosing its performance. A improvement is found in rudder activity, which is reduced comparing to Design 2. This reduction provides additional control capability in controlling by rudder. Additional benefit of the Design 4 is the process speed. Less number of fuzzy rule in FLC provides faster process speed.

Initial objective of the design of controller, which is to eliminate excess activity of VFC nozzle, is accomplished by Design 4. However, characteristics of the responses are yet like a typical linear controller, which is damping and overshooting. One of the advantage of FLC is that human like controlling that usually results less damping and overshooting comparing to linear automatic controller. Through the design steps of FLC, characteristics of aircraft dynamics are getting understood much better than initial stage of design. This is very common phenomenon through FLC
Figure 23. Fuzzy Rule for Design 4 in FAM Table Form and Text Form.
Figure 24. Design 4, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 13.11.
Figure 24—Cont. Design 4, Generic X-29A, 0.35/40k, $T = 0.1 \text{ sec}$, $\text{Cost} = 13.11$. 
design steps. At this point, it is realized that there is correlation between side-slip rate state and yaw rate state. Fuzzy rule of yaw rate control, which are rule 17 and 18 in Design 4, are reconstructed to two conditioned rule which generates Design 5. The performance and characteristics of reaction of Design 1 is re-examined to construct the rules. This time, more attention is made for differential flaps and rudder saturation. According to a state-equation, it is clear that input of $\delta_{\text{ru}}$ has similar characteristics as $\delta_{r}$ has. The saturation of rudder in Design 1 happens when both $d\beta$ and $r$ reached to their peak in opposite phase. Based on these facts, symmetric shape of FAM table can be constructed. However, from the design experience of Design 3 and Design 4, it is known that the sensitivity of $r$ should be lower. Thus, control low is more concentrated in $d\beta$ control which is shown in Figure 25.

Figure 26 indicates that side-slip reaction and side-slip rate reaction are improved in damping and overshooting comparing previous controller. It seems that other states, roll rate, yaw rate and bank angle, show more displacement than Design 4, but it last only during first 4 seconds. As shown in the Figure, after 4 seconds damping and total characteristics are generally improved. This fact is appeared in total cost 11.92 which is improved 9% over Design 4. VFC nozzle activity is appeared only once for each nozzle through 10 seconds run. Total time of VFC nozzle activation by Design 5 is absolutely longer than Design 4 which means more gas is consumed by Design 5.
FAM table form

<table>
<thead>
<tr>
<th>PL</th>
<th>PM</th>
<th>PS</th>
<th>ZE</th>
<th>NS</th>
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</tr>
</tbody>
</table>

Text form

1. If (beta is PM) and (dbeta is NL) then (vfc is pu) (1)
2. If (beta is PS) and (dbeta is NL) then (vfc is pu) (1)
3. If (beta is ZE) and (dbeta is NL) then (vfc is pu) (1)
4. If (beta is NS) and (dbeta is NL) then (vfc is pu) (1)
5. If (beta is NM) and (dbeta is NL) then (vfc is pu) (1)
6. If (beta is NL) and (dbeta is NL) then (vfc is pu) (1)
7. If (beta is NM) and (dbeta is NM) then (vfc is pu) (1)
8. If (beta is PL) and (dbeta is PL) then (vfc is nu) (1)
9. If (beta is NS) and (dbeta is PL) then (vfc is pu) (1)
10. If (beta is ZE) and (dbeta is PL) then (vfc is pu) (1)
11. If (beta is PS) and (dbeta is PL) then (vfc is nu) (1)
12. If (beta is PM) and (dbeta is PL) then (vfc is nu) (1)
13. If (beta is PL) and (dbeta is PL) then (vfc is pu) (1)
14. If (beta is PM) and (dbeta is PM) then (vfc is nu) (1)
15. If (beta is PM) and (dbeta is NM) then (vfc is nu) (1)
16. If (beta is PI) and (dbeta is PM) then (vfc is nu) (1)
17. If (dbeta is NL) and (r is PL) then (vfc is pu) (1)
18. If (dbeta is NM) and (r is PL) then (vfc is pu) (1)
19. If (dbeta is NS) and (r is PL) then (vfc is pu) (1)
20. If (dbeta is NL) and (r is PM) then (vfc is pu) (1)
21. If (dbeta is NM) and (r is PM) then (vfc is pu) (1)
22. If (dbeta is NS) and (r is PM) then (vfc is pu) (1)
23. If (dbeta is NL) and (r is PS) then (vfc is pu) (1)
24. If (dbeta is NM) and (r is PS) then (vfc is pu) (1)
25. If (dbeta is NL) and (r is ZE) then (vfc is pu) (1)
26. If (dbeta is NM) and (r is ZE) then (vfc is pu) (1)
27. If (dbeta is NL) and (r is NS) then (vfc is pu) (1)
28. If (dbeta is NM) and (r is NS) then (vfc is pu) (1)
29. If (dbeta is PL) and (r is NL) then (vfc is nu) (1)
30. If (dbeta is PL) and (r is NL) then (vfc is nu) (1)
31. If (dbeta is PS) and (r is NL) then (vfc is nu) (1)
32. If (dbeta is PL) and (r is NM) then (vfc is nu) (1)
33. If (dbeta is PM) and (r is NM) then (vfc is nu) (1)
34. If (dbeta is PS) and (r is NM) then (vfc is nu) (1)
35. If (dbeta is PL) and (r is NS) then (vfc is nu) (1)
36. If (dbeta is PM) and (r is NS) then (vfc is nu) (1)
37. If (dbeta is PS) and (r is ZE) then (vfc is pu) (1)
38. If (dbeta is PM) and (r is ZE) then (vfc is nu) (1)
39. If (dbeta is PL) and (r is ZE) then (vfc is nu) (1)
40. If (dbeta is PL) and (r is PS) then (vfc is nu) (1)

Figure 25. Fuzzy Rule for Design 5 in FAM Table Form and Text Form.
Figure 26. Design 5, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 11.92.
Figure 26—Cont. Design 5, Generic X-29A, 0.35/40k, T = 0.1 sec, Cost = 11.92.
In control system analysis, stability of the system is expressed mathematically that stability analysis of a control system is considered an essential factor as a common sense step. The one of the reasons that there are not many fuzzy logic applications exist in this country, even it is applied to many products and succeed in the market in Japan, is lack of the stability proof. A part of this is true and also not true in today’s fuzzy system.

Stability analysis method for a fuzzy logic system exits already for some cases. Altrock indicates that some fuzzy controllers can be treat as a “nonlinear multivariable controller” or “multiband controller”. Then, all stability analysis method for these conventional controller is applicable to the fuzzy logic controllers. Maeda and Murakami suggest a stability analysis method using “phase planes”, and Li and Yonezawa suggest “hyper stability theorem”.

Unfortunately, these analytical method is often impossible and not a general method in most case due to the complex nonlinearities of a fuzzy logic system. Instead of applying analytical method, numerical or trial-and-error method is applied to analyze a stability of fuzzy logic system among industries. The method is very simple. Once first model of fuzzy logic controller is developed, designer actually run the controller in simulation or prototype testing under the operating situations being
expected. Based on the result obtained from testing, designer would modify fuzzy rule or MF until he/she getting an acceptable controller.

It seems that the numerical stability method for a fuzzy logic system does not ensure stability as well as conventional method or analytical method does. However, is analytical stability analysis really guarantee stability? In simulation it does, but it may not be stable in prototype testing because of inaccurate mathematical model.

In real life, model is usually so complex and highly nonlinear that it is hard to construct very accurate mathematical model. As a result, adjustment of the control system is often required after prototype testing.

Similar steps can be seen through the development of fuzzy logic system. Stability in first design, before prototype testing, is secured through the design step of MF and fuzzy rules. MF determines stability in terms of input range. Multi-conditioned decision making process (in fuzzy rule, it is expressed as “IF condition is A and B then do ....”).) in fuzzy rule determines the rule of principle action which is usually consistent in different situations under operation. Tuning stage can be considered as the modification stage in conventional stability analysis. Of course, the numerical method is time consuming and it is not clear to guarantee stability. But, it is the only method available today and reasonable enough as described above. Therefore, in this paper, no stability analysis has been conducted for the fuzzy logic system.
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A fuzzy logic controller has been applied to regulator design for an aircraft which is equipped with forebody vortex flow control effectors. A design procedure of mixed configuration controller design, conventional and fuzzy logic, is developed and presented. The controller design is applied to generic X-29A aircraft which is simulated using the MATLAB/SIMULINK software. From the results, it is concluded that:

1. Fuzzy logic control is applicable to mixed bang-bang and conventional type control systems without major modification or additional sensors and feedback signals.

2. The fuzzy logic control procedure, which consists of MF and fuzzy rules, does not require detailed mathematical model of the system when applied to mixed bang-bang and conventional type control systems.

Recommendations

Fuzzy logic controllers have a great potential to enhance present control systems. Further studies should be done for fuzzy logic based aircraft applications. Recommendations are as follows:
1. Further studies with actuator rate limit included in the simulation model are needed to make sure that the fuzzy controller does not drive the actuator too hard.

2. A fuzzy logic controller which replaces the linear feedback gains in the continuous and bang-bang type system to enhance system performance should be studied.

3. Demonstrate a fuzzy logic controller in an actual aircraft flight test environment to highlight potential problems and advantages.
Appendix A

Nomenclature
## Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLC</td>
<td>fuzzy logic controller</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>altitude</td>
<td>feet</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
<td></td>
</tr>
<tr>
<td>MF</td>
<td>membership function</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>number of bang-bang variables, also perturbed body axis roll rate</td>
<td></td>
</tr>
<tr>
<td>psf</td>
<td>pounds per square feet</td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>dynamic pressure</td>
<td>pounds/feet²</td>
</tr>
<tr>
<td>r</td>
<td>perturbed body axis yaw rate</td>
<td>degrees/second</td>
</tr>
<tr>
<td>V</td>
<td>velocity</td>
<td>feet/second</td>
</tr>
<tr>
<td>VFC</td>
<td>vortex flow control</td>
<td></td>
</tr>
<tr>
<td>VSC</td>
<td>variable structure controller</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>angle-of-attack</td>
<td>degrees</td>
</tr>
<tr>
<td>β</td>
<td>sideslip angle</td>
<td>degrees</td>
</tr>
<tr>
<td>δ</td>
<td>deflection</td>
<td>degrees</td>
</tr>
<tr>
<td>λ</td>
<td>eigenvalue</td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>bank angle</td>
<td>degrees</td>
</tr>
</tbody>
</table>

### Superscripts
- \( \cdot \): time derivative
- \( * \): trim state
- \( ' \): transpose of matrix or vector
- \( -1 \): inverse of square matrix

### Subscripts
- \( bb \): bang-bang effector
- \( c \): canard
- \( df \): differential flap
- \( r \): rudder
- \( sf \): symmetric flap
- \( stf \): strake flap
- \( l \): trim value
Appendix B

MATLAB M-file Code for “Continuous Plus” Block in Design 2
function(sysp, x0) = cplus7(t,x,u,flag, Ql, Rl, Ml,a, b, c, d, e);

%% This is a file for the block which calculate cost function of continuous
%% plus. Algorithm is based on the file created by Dr. Valasek.
%%
%% This program calls function ' VFCfun.m '.

u_t = u';
offset = 0;
ts = 0.1;

if abs(flag) == 2 % returns to output
    if abs(round((t - offset)/ts) - (t-offset)/ts) < le-8
        xsl_k = [u_t(l) ; u_t(2) ; u_t(3) ; u_t(4)];
        usl_k = [u_t(5) ; u_t(6)];
        ucostl_k = xsl_k'*Ql*xcostl_k + ucostl_k'*Rl*ucostl_k + 2*(xcostl_k'*Ml*ucostl_k);
        sysp = term11 + term21 + term31;
    end
end

% Number of time step
% time increment for Euler's method
% Note 0.1 = step size for simulation

g = 100
h = 0.1 / g

% Inner loop for 4th order Runge-Kutta method

for n = 1:g
    K1 = [a * xsl(:,n) + b * usl_k];
    K2 = [a * (xsl(:,n) + h/2 * K1) + b * usl_k];
    K3 = [a * (xsl(:,n) + h/2 * K2) + b * usl_k];
    K4 = [a * (xsl(:,n) + h * K3) + b * usl_k];
    xsl(:,n+1) = [xsl(:,n) + (h/6) *( K1 + 2*K2 + 2*K3 + K4)];
end

% End of the loop

xcostl_k = xsl(:,g+1);
term11 = 0;
term21 = 0;
term31 = 0;
term11 = [ xcostl_k'*Ql*xcostl_k ];
term21 = [ ucostl_k'*Rl*ucostl_k ];
term31 = [ 2*(xcostl_k'*Ml*ucostl_k) ];
sysp = [ term11 + term21 + term31 ];
else
    sysp = [];
end

elseif flag == 3
    sysp = x;
elseif flag == 4
    ns = (t-offset) / ts;
    sysp = offset + ( 1 + floor(ns+le-13*(1+ns)))*ts;
elseif flag == 0  % initialization
    sysp = [0, 1, 1, 6, 0, 0];
    x0 = 0;
else
    sysp = [];
end
Appendix C

MATLAB M-file Code for "Selec" Block in Design 2
function[sys, x0] = selec(t, x, u, flag);
%
% This is a file for the block which select criterion for VFC command (-1, 0 , 1).
% Algorithm is based on the file created by Dr. Valasek.
%

u_tt = u';
if abs(flag) == 3
  % ************************************************************
  note ************************************************************
  % u_tt(1) = input 1 (cost of continuous plus block or VFC = +1)
  % u_tt(2) = input 2 (cost of continuous block or VFC = 0 )
  % u_tt(3) = input 3 (cost of continuous minus block or VFC = -1)
  % ************************************************************
    if u_tt(1) < u_tt(2) & u_tt(1) < u_tt(3);
      sys = [1 u_tt(1)] ;
    % Note if input 1 is the smallest then output = +1
    elseif u_tt(3) < u_tt(2) & u_tt(3) < u_tt(1) & t > 0.32;
      sys = [-1 u_tt(2)] ;
    % Note if input 3 is the smallest then output = -1
    else
      sys = [0 u_tt(3)] ;
    % Note otherwise output = 0
  end
elseif flag == 0
  sys = [0,0,2,3,0,0];
  x0 = 0;
%else
%  sys = [];
end


