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Prediction of Airfoil Stall in Icing Conditions Using Wing Surface Pressures

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PREDICTION OF AIRFOIL STALL IN ICING CONDITIONS USING WING SURFACE PRESSURES

by

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This thesis presents work on a Normalized Lift Coefficient instrumentation system that predicts airfoil stall in icing conditions. The system uses four surface pressures, measured aft of the ice formation, from which the aircraft's normalized lift coefficient is determined. The pressure port locations were selected such that the calibration algorithm remains nearly constant as the leading edge ice shape and thickness change. Data collected in the Western Michigan University Wind Tunnel is presented for three ice formations varying in both shape and thickness. The ice and no ice conditions were compared and a set of four ports was found that holds the calibration algorithm nearly constant regardless of ice configuration. An error analysis was also performed to determine the validity of the data. A program called LEWICE developed at NASA Glenn was also used to computationally analyze what is happening to the four ports that were found through the wind tunnel testing.
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INTRODUCTION

An airplane's stall margin (SM) can be defined as the percentage of the airplane's lift coefficient ($C_L$) remaining to be used, as shown in equation (1). The Normalized Lift Coefficient ($C_{LN}$) is defined in equation (2). When stall margin is 100%, the aircraft has its entire lift coefficient available for use. When stall margin is 0% the aircraft has none of its lift coefficient available and the aircraft has reached stall.\(^1\,^2\)

\[ SM = (1 - C_{LN}) \cdot 100\% \]  \hspace{1cm} (1)

\[ C_{LN} = \frac{C_L}{C_{L_{max}}} \]  \hspace{1cm} (2)

Currently available cockpit instrumentation systems display a normalized angle of attack or normalized airspeed, $C_{LN}$. These systems require the pilot to manually adjust the airplane's approach angle of attack or approach speed based on flight manual data combined with the calculated aircraft weight and flap configuration. The accuracy of these systems is dependent on calibrations performed for known aircraft configuration changes, such as landing flaps. Changes in the wing airfoil shape due to ice formation are not accounted for in the system calibrations and are therefore errors. If an airplane accumulates significant ice, the actual stall margin
will decrease with no indicated change in the cockpit. As the actual stall margin approaches zero the airplane will have a reduction in its performance and control margins. If the aircraft is being flown by an autopilot, these reduced margins most likely will go undetected by the flight crew. If the pilot and autopilot have accurate stall margin data, the ice formation can be detected early, allowing the crew to respond in a timely manner.

This thesis presents initial work on an in-flight stall margin instrumentation system that maintains accuracy in spite of wing leading edge ice formations. The system uses four surface pressures, measured aft of the ice formation, from which the aircraft's normalized lift coefficient ($C_{LN}$) is determined. The pressure port locations are selected such that the calibration algorithm remains nearly constant as the leading edge ice shape and thickness change. A 2-D airfoil pressure model was used to obtain the surface pressure distribution for a range of angles of attack and ice formations. This initial pressure data was collected in Western Michigan University’s wind tunnel using an airfoil approximating a NACA 0018, as this model was readily available. A software package, LEWICE, was obtained from NASA Glenn Research Center and was used to predict four ice shapes. These shapes were cut from a foam block, using a hot wire, and attached to the model leading edge for testing. Analysis of the data has identified port locations that provide $C_{LN}$ data within 11% of the actual value, with and without ice. These initial experiments have shown great promise but the concept needs to be duplicated on an airfoil more commonly used for wings and more ice shapes also need to be tested.
BACKGROUND THEORY

Stall Margin Theory

Stall Margin (SM) is defined as a function of Normalized Lift Coefficient ($C_{LN}$), as shown in equations (1) and (2). For the system being presented in this paper, $C_{LN}$ will be determined from a defined pressure coefficient ($C_p$). $C_p$ is defined in equation (3), where $P_1$, $P_2$, $P_3$, and $P_4$ are wing surface pressures, measured at four different port locations. By substitution the $C_p$ can also be written in terms of the wing surface pressure coefficients.\textsuperscript{1,2} Given that $C_p$ is a function of angle of attack ($\alpha$), flap deflection ($\delta_f$), ice shape (IS) and port location (PL), then it follows that $C_p$ will also be a function of these parameters, as shown in equation (4). An example of the port locations is shown in Figure 1.

\begin{equation}
C_p = \frac{P_1 - P_2}{P_3 - P_4} = \frac{Cp1 - Cp2}{Cp3 - Cp4}
\end{equation}

\begin{equation}
C_p = f_1(\alpha, \delta_f, IS, PL)
\end{equation}

$C_{LN_{\text{actual}}}$ is defined in equation (5), where ($\alpha_s$) is the stall angle of attack for the airfoil configuration. $C_{LN_{\text{indicated}}}$ is related to $C_{LN_{\text{actual}}}$ through the calibration function ($f_2$) of $C_p$ and flap deflection, as is shown in equation (6).
Let $C_{P_{x0}}$ be defined as $C_{P_{x}}$ in the calibration configuration, as shown in equation (7). The calibration configuration is the wing configuration when there are no ice formations.

$$C_{P_{x0}} = f_{1}(\alpha, \delta_f, IS = 0, PL) = f_{1}(\alpha, \delta_f, 0, PL)$$  \hspace{1cm} (7)$$

In the calibration configuration, $C_{L_{N,\text{actual}}}$ is equal to $C_{L_{N,\text{indicated}}}$ and both are equal to a function ($f_2$) of $C_{P_{x0}}$ and flap deflection, as is shown in equation (8). Note that the function of flap deflection is a first order effect and needs to be considered in the baseline calibration ($f_2$).
Function \((f_2)\) typically would have the form shown in equation \((8a)\).

\[
f_2(C_{p_0}, \delta_f) = A_0(\delta_f) + A_1(\delta_f) \cdot C_{p_0} + A_2(\delta_f) \cdot (C_{p_0})^2
\]  

(8a)

As ice is accumulated, the wing configuration changes and \(C_{L_{\text{N,indicated}}}\) is no longer a function of \(C_{p_0}\) alone. It can now be expressed as a new function \((f_3)\) of the ice shape, flap deflection angle, and \(C_{p_0}\), as is shown in equation \((9)\).

\[
C_{L_{\text{N,indicated}}} = f_3(C_{p_0}, IS, \delta_f)
\]  

(9)

Now let the function \((f_3)\) have the form shown in equation \((10)\), where \(C_{p_0}\) is a function \((f_1)\) of only angle of attack, flap deflection, and port location. The new functions \((m_1)\) and \((m_2)\) are functions of only flap deflection, ice shape and port location.

\[
C_{L_{\text{N,indicated}}} = \frac{m_1(\delta_f, IS, PL)}{m_2(\delta_f, IS, PL)} \cdot f_A(C_{p_0}, \delta_f)
\]  

(10)

Assuming port locations can be selected such that functions \((m_1)\) and \((m_2)\) are equal, this equation can be re-written, as is shown in equation \((11)\), where \(C_{L_{\text{N,indicated}}}\) is only a function of \(C_{p_0}\). Note that the function of flap deflection here is a second
order effect, as the baseline flap effects have been accounted for in the calibration function \( f_2 \).

\[
C_{LN\_indicated} = f_4(C_{p_0}, \delta_f) \tag{11}
\]

Since \( C_{LN\_indicated} \) is only a function of \( C_{p_0} \) and second order flap deflection in both equations (8) and (11), functions \( f_2 \) and \( f_4 \) must be equal, as shown in equations (12).

\[
f_2(C_{p_0}, \delta_f) = f_4(C_{p_0}, \delta_f) \tag{12}
\]

Because both \( C_{LN\_indicated} \) is equal to \( f_2 \), and \( C_{LN\_actual} \) is equal to \( f_4 \), and \( f_2 \) equals \( f_4 \) then \( C_{LN\_indicated} \) with ice must equal \( C_{LN\_actual} \), equation (13).

\[
C_{LN\_actual} = C_{LN\_indicated} \tag{13}
\]

Given the assumptions as to the form of \( f_3 \), and the selection of port locations that make \( m_1 \) and \( m_2 \) equal, \( f_2 \) will equal \( f_4 \). Thus \( C_{LN\_indicated} \) is equal to \( C_{LN\_actual} \) independent of ice shape and flap deflection.
Actual Conditions

Under actual flight conditions there will be deviations from the assumptions made in the theory. The indicated $C_{LN}$ may not be totally independent of the ice shape or flap deflection. There will be some error related to the ice ($E_{r1}$) and some error related to the second order flap deflection ($E_{r2}$), as shown in equation (14).

$$C_{LN\_indicated} = C_{LN\_actual} + E_{r1}(IS) + E_{r2}(\delta_f)$$  \hspace{1cm} (14)

Figure 2 presents data from a stall margin system installed on Western Michigan University’s flight test aircraft. Each curve is $C_{p_x}$ versus $C_{LN}$ curves for different flap angle settings. The ports were located on this airplane to maximize the sensitivity of $C_{p_x}$ to changes in $C_{LN}$ and not to compensate for flap deflection. The current system uses a multivariate calibration similar to the one presented in equation (8a). If the calibration function were to ignore the affect of flap deflection, a $C_{LN}$ error of 6% would result. It is important to note that flap deflection angles are repeatable. This repeatability allows a different calibration to be used for each setting, which will minimize $E_{r2}(\delta_f)$. 
Figure 2. Cessna 182 Stall Margin Flight Test Data.

The error, $E_{r_1}(IS)$, due to ice shape cannot be removed by this method because of the non-repeatable characteristic of ice formations. Aircraft icing is a random phenomenon and therefore has to be compensated for through other means. The method presented in this thesis minimizes the affect of ice by the careful selection of the pressure sensor port locations. Computational methods of analysis have difficulty predicting the pressure coefficients at high angles of attack with horned ice shapes. They will also have difficulty determining the stall angle of attack due to the separated flow produced during stall. For these reasons, an experimental approach was taken to evaluate the proposed theory.
EXPERIMENTAL ANALYSIS

Experimental Method

Wind Tunnel Model Setup

A NACA 0018 pressure model was constructed for the Western Michigan University wind tunnel. The model had a one foot chord and spanned the entire forty-two inch test section. This was done to eliminate 3-D effects on the wing thereby simulating 2-D flow. The model had twenty pressure ports on the upper wing surface and twenty ports on the lower wing surface.

The wind tunnel at Western Michigan University is not capable of producing ice, thus another method had to be developed to predict and model the leading edge ice shapes. For this purpose a computational ice simulation software package called LEWICE, developed at NASA Glenn Research Center, was used.

LEWICE is a 2D code that uses panel methods to determine the flow solution. LEWICE also uses an integral boundary layer code, a water particle trajectory code, and a heat and mass transfer code. LEWICE places a set number of water particles a distance far in front of the airfoil. The program then proceeds to calculate the trajectory of these particles to determine which particles will strike the airfoil and
which will pass by. The program then calculates how the water will freeze on the
airfoil based on inputs to the program such as particle size, and air temperature. From
this method a layer of ice is built up on the airfoil. This method is time stepped to get
the multiple layers of ice that determine the final ice shape. Figure 3 shows the
different shapes used in this test.

Figure 3. Computationally Determined Ice Shapes.

Each ice shape was formed using the same airspeed, droplet size and angle of
attack, while varying the temperature and duration in the icing conditions. These
parameters were used to simulate an aircraft in a cruise flight configuration. Table 1
shows the parameters used to produce each ice shape.

Both horned and smooth ice shapes were modeled, however, configuration (1)
was not tested due to model vibration problems. For the baseline wing configuration
and configurations (2), (3), and (4), surface pressure data was collected over a range
of angles of attack. The ports under the ice formation were not used, as they may not
be available under actual icing conditions.
Table 1
Parameters for Ice Shape Formation

<table>
<thead>
<tr>
<th></th>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
<th>Configuration 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (deg C)</td>
<td>-4.85</td>
<td>-17.8</td>
<td>-17.8</td>
<td>-4.85</td>
</tr>
<tr>
<td>Airspeed (m/s)</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>Angle of Attack (deg)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Droplet Size (mm)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Time in flow (sec)</td>
<td>600</td>
<td>1200</td>
<td>600</td>
<td>300</td>
</tr>
<tr>
<td>Pressure at infinity (N/m^2)</td>
<td>100000</td>
<td>100000</td>
<td>100000</td>
<td>100000</td>
</tr>
<tr>
<td>Relative Humidity (%)</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Repeatability Study

Each configuration was tested multiple times to test for repeatability. The Cp versus x/c was compared for each run to determine the repeatability of the data. A sample of this comparison is shown in Figures 4 and 5. The data was determined to be repeatable and the analysis was continued.

![Figure 4](image_url)

Figure 4. Configuration 3 Repeatability Test for an Angle of Attack of 4 Degrees.
Determining the Stall Angle of Attack

To determine $C_{LN}$, the stall angle of attack for each ice configuration needed to be known. The pressure model produced too great a load for the available wind tunnel balance and the pressure tubing would have fouled the balance, thus the stall angle of attack could not be determined from the force data. Tufts, placed on the wing upper surface at several chord locations, were observed to determine the angle of attack for the flow separation. The wing was considered to be stalled when the flow had separated over the aft 50% of the chord. This stall angle of attack was measured and was used in equation (15) to determine $C_{LN}$ for each angle of attack and ice formation tested. The stall angles of attack for each configuration are listed in Table 2.

$$C_{LN} = \frac{\alpha}{\alpha_{stall}}$$  \hspace{1cm} (15)
Table 2

Stall Angles of Attack for Each Ice Configuration

<table>
<thead>
<tr>
<th></th>
<th>Stall Angle(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Ice</td>
<td>9</td>
</tr>
<tr>
<td>Configuration 2</td>
<td>4.8</td>
</tr>
<tr>
<td>Configuration 3</td>
<td>5</td>
</tr>
<tr>
<td>Configuration 4</td>
<td>6</td>
</tr>
</tbody>
</table>

**Pressure Port Analysis**

A non-trivial $C_{p_x}$ was computed for each combination of the pressure ports not located under the ice formation. Then the relationship between $C_{p_x}$ and $C_{LN}$ was examined. To qualify, $C_{LN}$ had to be between 0.5 and 1.0. This range of $C_{LN}$ represents the normal landing and takeoff values. Also the slope of the $C_{LN}$ vs $C_{p_x}$ curve was required to be greater than a minimum threshold to ensure sufficient sensitivity to changes in $C_{LN}$. The port sets that met this criterion were compared against the other ice shapes and the baseline data.
Experimental Results

This analysis yielded a set of port locations that maintained nearly the same relationship, regardless of ice configuration. The x/c locations for this set of ports are listed below page.

(1) Upper Wing Surface

P1 (x/c = 0.22)

P3 (x/c = 0.41)

(2) Lower Wing Surface

P2 (x/c = 0.68)

P4 (x/c = 0.41)

The pressures measured at these ports are combined in equation (16) to give $C_p_x$.

$$C_p_x = \frac{P_1 - P_2}{P_3 - P_4}$$  \hspace{1cm} (16)

Figure 6 shows the $C_{l_n}$ vs $C_p_x$ relationship, for this set of ports. The relationship is shown for the no ice baseline and ice configurations (2), (3), and (4), which are labeled C2R4, C3R2, and C4R1, respectively.
Error Analysis

Figure 6 shows that for a given $C_p$ there is a $C_{LN}$ error of around 11% between the four curves. An uncertainty analysis was done on the data for these ports to see if the uncertainty in the measurements explained the 11% error. Figure 7 shows the same graph as in Figure 6, but this time the uncertainty for the baseline is also shown. The baseline uncertainty was plotted because the baseline curve is the calibration that would be used for the actual instrumentation. If all the ice configuration data fell within this baseline uncertainty, then the error in each port reading would be due to instrumentation error and would not be due to ice configuration. The method used to determine the uncertainty is shown in Appendix A.
The Cp and angle of attack data is shown in Appendix B. The $C_p$ and $C_{LN}$ data for these ports, and the calculated uncertainties are shown in Appendix C. The uncertainty for the baseline configuration encompasses most of the data from the other configurations, with some exceptions at higher normalized lift coefficients. To lessen the uncertainty further pressure data would need to be taken at a higher dynamic pressure ($Q$). It was not possible to take data at a higher $Q$ due to the lack of structural integrity of the pressure model. Initially, the 11% error would be acceptable because the $C_{LN}$ indicated to the pilot would be better than no indication at all. Ideally, the error would need to be in the plus or minus 2% range.

![Figure 7. $C_p$ Versus $C_{LN}$ for the Selected Port Locations With Baseline Error.](image)
COMPUTATIONAL ANALYSIS

The pressure distributions for the ice and no ice configurations were also studied using LEWICE. This analysis provided insight into how the port locations, selected from the wind tunnel data, provided nearly the same relationship between $C_{p_x}$ and $C_{LN}$, regardless of ice shape.

LEWICE is capable of producing $C_p$ versus $x/c$ data multiple angles of attack and for all the configurations of ice. This data is only acceptable to an extent. At higher angles of attack the data diverges and is very erratic. For angle of attacks from 0 degrees to 5 degrees, a pressure distribution was obtained for the ice and no ice configurations. An analysis of the data, for a given angle of attack, revealed the pressures at ports P2, P3, and P4 remained nearly unchanged. Port P1, on the other hand, varied as the ice configuration changed. The results for angles of 3, 4 and 5 degrees are shown in Figures 8, 9, and 10. Angles 0, 1 and 2 are not shown because the change in $C_p$ at P1 is too small to effectively graph. The locations of each port are circled on the graphs. The pressure data in these figures are presented in the form of the non-dimensionalized pressure coefficient ($C_p$).
Figure 8. LEWICE Cp Distribution for an Angle of Attack of 3 Degrees.

Figure 9. LEWICE Cp Distribution for an Angle of Attack of 4 Degrees.
Several observations can be made from Figures 8, 9 and 10:

1. Port P1 has the greatest sensitivity to ice configuration and angle of attack because of its location on the upper surface and proximity to the leading edge of the airfoil. The remaining ports are sensitive to angle of attack and dynamic pressure changes. They serve to eliminate the effects of changes in dynamic pressure.

2. Because ports P2, P3 and P4 have nearly the same pressure for each ice shape, at a given angle of attack, their locations serve to adjust relative percentage change caused by the change in P1.
3. The closer P1 is to the leading edge the greater the pressure difference due to ice configuration will be and therefore the instrumentation will be more sensitive to the change in the wing leading edge configuration.

4. There may be more than the one set of ports discovered in the wind tunnel testing. The figures show that the pressure remains unchanged for multiple x/c locations. This indicates that P2, P3, and P4 may be able to be moved. The wind tunnel pressure model may have been able to pick up more port locations had there been more pressure ports at different locations on the model.
RESULTS AND CONCLUSIONS

Analysis of Icing Effect

Consider an aircraft that is in a cruise flight, holding a constant angle of attack. As ice accumulates on the leading edge of the wing, the $C_p$ decreases. This is seen in the experimental data and the LEWICE analysis, as shown in Figures 8, 9, and 10. If the $C_{LN}$ remained unchanged, this effect would result in a deviation from the baseline $C_p$ versus $C_{LN}$ calibration curve. This is depicted in Figure 11, by the vertical vector. In reality, an airplane flown at a constant angle of attack, would have an increase in $C_{LN}$ because of the decrease in stall angle of attack associated with the leading edge ice formation. This is depicted in Figure 11 by the horizontal vector. This brings the actual $C_{LN}$, obtained at the $C_p$ measured with ice, back to the baseline calibration curve. In effect, the change in the $C_p$ due to ice is offset by the change in the $C_{LN}$ caused by the ice. As was pointed out in the previous section, moving the ports will alter the effect of ice on the $C_p$. This would allow the designer to match the effect of the ice on the $C_{LN}$.
Figure 11. $C_{LN}$ Versus $C_p$ for a General Calibration Equation.

**Instrumentation Comparison**

Figure 12 shows a comparison of the pressure based system to a non-pressure based system. A $C_{LN}$ equal to 0.6 was used for comparison, as this would constitute a standard aircraft approach to landing. The no ice baseline relationship between $C_{LN}$ versus $C_p$ was used to calculate the $C_{LN}$ as a function of $C_p$. The $C_{LN}$ (ideal) represents the ideal, no error relationship. The $C_{LN}$ (flown with instrumentation) represents the $C_{LN}$ that would be indicated with ice formed on the leading edge, still using the pressure based instrumentation. The $C_{LN}$ (flown without instrumentation)
represents the $C_{LN}$ that would result if the angle of attack were held constant as it would be for the baseline at a $C_{LN}$ equal to 0.6. This would be the result if a pilot were using a normalized lift indicator that obtained its inputs from angle of attack and flap deflection and not wing pressures.

Under the worst possible tested case, the largest $C_{LN}$ obtained with the pressure based instrumentation would be 0.7, with cockpit display indicating 0.6. If a conventional $C_{LN}$ system were used, the largest $C_{LN}$ obtained would be 1.17 (stalled), with the cockpit display indicating 0.6. This means that if a conventional system was used the pilot would stall the aircraft even though the cockpit display is indicating a $C_{LN}$ of 0.6. If the pressure based system was used the pilot would have held the $C_{LN}$ at
0.7 while the cockpit display indicates a $C_{\text{LN}}$ of 0.6. The pressure-based system provided a more accurate $C_{\text{LN}}$ indication to the pilot.

**Conclusion**

Provided that the relationship for $C_{\text{p}_x}$ between ice and no ice conditions hold true for a wider range of ice shapes on typical wing airfoils, an instrumentation system could be made available to pilots. Such a system would give pilots information in icing conditions, well beyond that available today. Currently the pilot does not know how much lift coefficient is available in reserve. The pilot flying in icing conditions is in a test pilot role and could easily enter a stall while flying airspeeds that would otherwise be normal. Even with de-icing equipment, icing conditions may exist that can have a serious impact on the airplane's aerodynamics. The system under development would allow the pilot to safely extract the airplane from the icing conditions and make a landing, knowing the margin from stall.

To prevent the pressure ports from being obstructed by ice, they would need to be heated. The ports would also need a prevision to separate water and prevent blockage. The performance of the system with ice farther aft on the wing will need to be demonstrated. Future work should include testing additional ice shapes and roughness on more typical wing airfoil sections. They should be tested experimentally and computationally. A computational method to predict the general port locations
before wind tunnel testing also needs to be developed. After verification of the 2-D work, 3-D testing over a variation of wing configurations and icing conditions will be needed.
Appendix A

MathCAD Error Calculation Program
CLn Error Analysis

CLn Information

\[ \alpha_{\text{BaselineR}6} := (0.70, 0.85, 2.07, 3.05, 4.12, 5.05, 6.05, 7.07, 8.05, 9.05, 10.10) \]

\[ \alpha_{\text{stallBaselineR}6} := 8.8 \]

\[ \alpha_{\text{C}2R4} := (1.025, 2.074, 3.026, 4.076, 5.076, 6.053, 7.078, 8.127, 9.055, 10.104) \]

\[ \alpha_{\text{stallC}2R4} := 4.8 \]

\[ \alpha_{\text{C}3R2} := (0.84, 1.19, 2.00, 3.01, 3.99, 5.02, 6.04, 7.05, 8.09, 9.14, 10.12) \]

\[ \alpha_{\text{stallC}3R2} := 5 \]

\[ \alpha_{\text{C}4R1} := (0, 1.049, 2.074, 3.124, 4.076, 5.101, 6.077, 7.151, 8.079, 9.104, 10.129) \]

\[ \alpha_{\text{stallC}4R1} := 6 \]

Error Calculations

\[ \omega_{\Delta \alpha} := 0.1 \quad i := 0..9 \quad j := 0..10 \]

\[ d\text{CLn}d\alpha(\alpha s) := \frac{1}{\alpha s} \quad d\text{CLn}\alpha(\alpha, \alpha s) := \frac{-\alpha}{\alpha s^2} \]

\[ \omega_{\text{CLn}}(\alpha, \alpha s) := \sqrt{d\text{CLn}d\alpha(\alpha s)^2 + d\text{CLn}\alpha(\alpha, \alpha s)^2} \cdot \omega_{\Delta \alpha} \]

Results

\[ \omega_{\text{C}4R1j}, i := \omega_{\text{CLn}}(\alpha_{\text{C}4R1j}, i, \alpha_{\text{stallC}4R1}) \]

\[ \omega_{\text{C}2R4j}, i := \omega_{\text{CLn}}(\alpha_{\text{C}2R4j}, i, \alpha_{\text{stallC}2R4}) \]

\[ \omega_{\text{C}3R2j}, i := \omega_{\text{CLn}}(\alpha_{\text{C}3R2j}, i, \alpha_{\text{stallC}3R2}) \]

\[ \omega_{\text{BaselineR}6j}, i := \omega_{\text{CLn}}(\alpha_{\text{BaselineR}6j}, i, \alpha_{\text{stallBaselineR}6}) \]
Cp Error Analysis

CP Information

\[
\begin{align*}
\omega_{C4R1} &= \begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
0 & 0.017 & 0.017 & 0.018 & 0.019 & 0.02 & 0.022 & 0.024 & 0.026 & 0.028 & 0.03
\end{bmatrix} \\
\omega_{C2R4} &= \begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
0 & 0.021 & 0.023 & 0.025 & 0.027 & 0.03 & 0.034 & 0.037 & 0.041 & 0.044 & 0.049
\end{bmatrix} \\
\omega_{C3R2} &= \begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
0 & 0.02 & 0.021 & 0.022 & 0.023 & 0.026 & 0.028 & 0.031 & 0.035 & 0.038 & 0.042
\end{bmatrix} \\
\omega_{BaselineR6} &= \begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
0 & 0.011 & 0.011 & 0.012 & 0.013 & 0.013 & 0.014 & 0.015 & 0.015 & 0.016
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\omega_{C4R1} &= \begin{bmatrix}
-0.787 & -0.89 & -0.909 & -0.957 & -1.029 & -1.075 & -1.101 & -1.156 & -1.105 & -1.085 \\
-0.5 & -0.528 & -0.537 & -0.577 & -0.611 & -0.636 & -0.654 & -0.674 & -0.655 & -0.648 \\
-0.291 & -0.293 & -0.254 & -0.223 & -0.194 & -0.18 & -0.139 & -0.139 & -0.113 & -0.109 \\
-0.127 & -0.125 & -0.118 & -0.096 & -0.081 & -0.073 & -0.074 & -0.086 & -0.064 & -0.087
\end{bmatrix} \\
\omega_{C2R4} &= \begin{bmatrix}
0.029 & -0.797 & -0.887 & -0.96 & -1.026 & -1.115 & -1.143 & -1.219 & -1.209 & -1.078 & -1.048 \\
-0.464 & -0.49 & -0.543 & -0.578 & -0.606 & -0.664 & -0.66 & -0.681 & -0.652 & -0.71 \\
-0.75 & -0.334 & -0.307 & -0.264 & -0.2 & -0.178 & -0.134 & -0.124 & -0.105 & -0.151 & -0.143 \\
1.484 & -0.135 & -0.134 & -0.118 & -0.093 & -0.085 & -0.073 & -0.07 & -0.079 & -0.148 & -0.188
\end{bmatrix} \\
\omega_{C3R2} &= \begin{bmatrix}
-0.629 & -0.687 & -0.75 & -0.821 & -0.846 & -0.9 & -0.944 & -0.979 & -1.017 & -1.148 & -1.377 \\
-0.428 & -0.458 & -0.483 & -0.541 & -0.538 & -0.564 & -0.589 & -0.614 & -0.65 & -0.638 & -0.652 \\
-0.366 & -0.349 & -0.311 & -0.3 & -0.248 & -0.233 & -0.192 & -0.159 & -0.171 & -0.151 & -0.135 \\
-0.16 & -0.159 & -0.128 & -0.137 & -0.11 & -0.114 & -0.086 & -0.11 & -0.078 & -0.088 & -0.091
\end{bmatrix}
\end{align*}
\]
Baseline \( R_6 := \) 
\[
\begin{pmatrix}
0.625 & -0.89 & -0.977 & -1.059 & -1.111 & -1.248 & -1.258 & -1.469 & -1.571 & -1.586 & -1.731 \\
0.531 & -0.484 & -0.478 & -0.521 & -0.534 & -0.632 & -0.618 & -0.757 & -0.812 & -0.822 & -0.878 \\
0.25 & -0.345 & -0.288 & -0.258 & -0.203 & -0.196 & -0.116 & -0.097 & -0.038 & -0.033 \\
1.876 & -0.114 & -0.092 & -0.084 & -0.069 & -0.069 & -0.016 & -0.044 & -0.031 & -0.006 & -0.019
\end{pmatrix}
\]

**Error Calculations**

\[ \omega_{\Delta p} := 0.05 \frac{\text{lbf}}{\text{ft}^2} \quad i := 0 \ldots 9 \quad j := 0 \ldots 10 \]

\[ Q := 7.5 \frac{\text{lbf}}{\text{ft}^2} \]

\[
dCpxdCp1(Cp_2, Cp_3) := \frac{1}{Cp_2 - Cp_3} \quad dCpxdCp2(Cp_1, Cp_2, Cp_3, Cp_4) := \frac{-(Cp_1 - Cp_4)}{(Cp_2 - Cp_3)^2}
\]

\[
dCpxdCp3(Cp_2, Cp_3) := \frac{-1}{Cp_2 - Cp_3} \quad dCpxdCp4(Cp_1, Cp_2, Cp_3, Cp_4) := \frac{(Cp_1 - Cp_4)}{(Cp_2 - Cp_3)^2}
\]

\[ \omega_{Cp}(Cp) := \sqrt{1 + Cp^2 \cdot \frac{\omega_{\Delta p}}{Q}} \]

\[
\omega_{Cpx}(Cp_1, Cp_2, Cp_3, Cp_4) := \sqrt{(dCpxdCp1(Cp_2, Cp_3) \cdot \omega_{Cp}(Cp_1))^2 + 1 + (dCpxdCp2(Cp_2, Cp_3) \cdot \omega_{Cp}(Cp_4))^2 + 1 + (dCpxdCp3(Cp_1, Cp_2, Cp_3, Cp_4) \cdot \omega_{Cp}(Cp_3))^2}
\]

**Results**

\[ \omega_{C4R1_i, j} := \omega_{Cpx}(C4R_1_{i, j}, C4R_1_{i, j}, C4R_1_{i, j}, C4R_1_{i, j}) \]

\[ \omega_{C2R4_i, j} := \omega_{Cpx}(C2R_4_{i, j}, C2R_4_{i, j}, C2R_4_{i, j}, C2R_4_{i, j}) \]
\[ \omega_{C3R2_i,j} := \omega_{Cpx}(C3R2_{0,j}, C3R2_{1,j}, C3R2_{2,j}, C3R2_{3,j}) \]

\[ \omega_{BaselineR6_i,j} := \omega_{Cpx}(BaselineR6_{0,j}, BaselineR6_{1,j}, BaselineR6_{2,j}, BaselineR6_{3,j}) \]

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Appendix B

Cp and Angle of Attack Wind Tunnel Data for the Four Selected Ports
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### BaselineR6

**Stall AOA:** 8.8

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- $CP_x$: -4.45 | 5.58 | 4.66 | 3.71 | 3.15 | 2.70 | 2.44 | 2.22 | 2.15 | 2.02 | 2.03
- $CL_n$: 0.080 | 0.096 | 0.235 | 0.346 | 0.468 | 0.574 | 0.687 | 0.804 | 0.915 | 1.028 | 1.148

### C2R4

**Stall AOA:** 4.8

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<td>-0.528</td>
<td>-0.537</td>
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<td>-0.611</td>
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- $CP_x$: 3.16 | 3.26 | 2.80 | 2.43 | 2.27 | 2.20 | 1.99 | 2.00 | 1.92 | 1.85
- $CL_n$: 0.214 | 0.432 | 0.630 | 0.849 | 1.058 | 1.261 | 1.475 | 1.693 | 1.886 | 2.105
Appendix C

Error Calculation Data for the Four Selected Ports
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REFERENCES

1Hoadley, Conversion of Wing Surface Pressures into Normalized Lift Coefficient, 1979 SAE Transactions.

2Hoadley, Normalized Coefficient of Lift Indicator, US Patent No. 4,235,104