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Prediction of Shaft Vibration Modes in a Geared Transmission System

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PREDICTION OF SHAFT VIBRATION MODES IN A GEARED TRANSMISSION SYSTEM.

by

Kevin W. Marsh

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
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Department of Mechanical and Aeronautical Engineering

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Kevin W. Marsh
During the development process of heavy-duty truck transmissions, specific gear designs (geometries) are created to meet load requirements and ratio coverage. Few tools are available to investigate the noise generating potential of these transmissions early in the design stages. This research describes the development and application of a software program that predicts transmission countershaft vibration modes. From simple gear and shaft geometries, the program calculates gear mesh forces and bearing forces then construct the system of equations to compute the shaft vibration modes. Because of the way the input data was structured, this software tool can also be used to perform basic analysis on gear tooth interaction and compare two different transmission models for noise and vibration diagnostics. The methods presented in this research can be applied to a variety of simply supported shafts, which make it useful for many different transmission models. Noise and vibration data from two transmissions are presented and compared to the predictions from the program. It was shown that the shaft mode frequencies correspond reasonably well with the frequencies measured in the noise data and that the other diagnostics features in the program will be useful when analyzing a complete transmission system.
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INTRODUCTION

During the development process of heavy-duty truck transmissions, specific gear designs (geometries) are created to meet load requirements and ratio coverage. Few tools are available to investigate the noise generating potential of these transmissions early in the design stages. In order to determine the noise performance of a transmission, the unit is physically tested in an anechoic chamber and then the noise data is analyzed to gain insight into the root causes of any potential noise issues. Shaft resonances are a primary cause of these noise problems. When a shaft vibrates, forces are transmitted to the bearings and into the housing, which ultimately radiates noise. During a transmission noise test sequence certain techniques and experience were used to determine the particular shaft vibration modes that may be contributing significantly to the overall noise level. If a specific shaft mode was found there were only a few options available to remedy the problem. Gear manufacturing processes, such as grinding, could be added to improve the quality of the gear set. The number of teeth on the gears may be changed to tune the forcing function out of the normal operating speed range, thereby moving the excitation away from a particular vibration mode. Bearings also play a significant role in shaft vibration modes and occasionally a bearing with a different radial stiffness can be used to fine-tune the system. All of these options add cost to the product, either directly in manufacturing or indirectly through customer dissatisfaction. The work presented in this report will encompass the development of a software tool to evaluate the noise generating potential of the
entire gear/shaft system by calculating gear mesh forces, bearing forces and shaft vibration modes for each countershaft in the transmission. In addition to its prediction capabilities, this program can also be used as a diagnostic tool to evaluate noise and vibration issues for current products. Gear analysis calculations can be performed on all of the loaded gear sets at once, instead of individually as done now. This software program would ultimately interface with an existing database so that any new or current transmission design could be analyzed.
CHAPTER I

GEARED TRANSMISSION SYSTEM OVERVIEW

Transmission Application

The transmissions being modeled are used in class-8 trucks, which may be more commonly referred to as a semi or 18-wheeler. These transmissions have a capacity of between 1000 lb-ft and 2000 lb-ft of input torque depending on the model. The operating speed range is between 1100 and 2100 rpm with typical cruise speed around 1400 rpm. A driver spends an average of 8 hours per day behind the wheel of these trucks so a noise that may be acceptable to the casual observer, may be considered objectionable by the operator. The overall noise level of a class-8 vehicle has decreased considerably over the years, making the noise contribution of the drive train components stand out more against the background noise level. Because noise has become a more sensitive issue in the past few years, suppliers must look for new techniques to ensure acceptable noise performance from their components.

Power Flow

The geared transmission systems used in this work are twin-countershaft models, which have a front and rear section. The power is transmitted from the engine, through the clutch and into the input shaft of the transmission. The torque is split between the two countershafts, which allows for the use of smaller components.
and a more compact transmission. If a gear is selected that engages the front box, then two front box gears are loaded – the head gear set splits the torque to the countershafts and the second set returns the torque to the mainshaft. If the rear section is engaged, the power flow is duplicated in the back box – the auxiliary drive set splits the torque to the rear countershafts and the second set returns the torque to the output shaft. These transmissions can operate with the front box only, back box only or both sections engaged at the same time. A top-view schematic of the transmission power flow is shown in Figure 1. The inputshaft, mainshaft and outputshaft are not being modeled in this work. The front countershaft and auxiliary countershaft, which are simply supported by bearings, are being modeled because previous work has identified them as the main contributors to the noise peaks.

Figure 1. Transmission Layout and Power Flow
Types of Vibration Modes

Like any multi-degree of freedom spring-mass-damper system, geared transmission systems will contain different vibration modes, each with its own natural frequency. In these complex systems there exists shaft vibration modes, housing (panel) modes as well as modes of other components. The two types of resonance modes that are the focus of this research are countershaft torsional modes and countershaft translational (lateral) modes. The transmission designers have classified two distinct lateral modes depending on how the shaft moves in the bearings. Bounce modes are where the ends of the shaft move up and down in phase with each other. Rock modes are similar to bounce modes except the ends of the shaft move out of phase with each other. Torsional modes are simply the twisting of a shaft in the bearings. The springs in the system are the bearings at the end of each shaft and the shaft itself, flexing in the torsional and lateral direction. As Beacham et al. [1] explains, when these shafts vibrate they generate dynamic forces at the bearings, which excite the transmission housing thereby radiating noise. A pictorial representation of how each of these modes may look is shown in Figure 2. Transmission housing modes, or panel modes, occur when the bearing forces excite a specific mode within the case itself. These panel modes can be significant contributors to the overall noise level of a transmission. It can be difficult to differentiate between what is simply case radiated bearing vibration and what is a unique panel mode. The goal of this research was to investigate specific shaft
vibration modes; panel modes have been mentioned as relevant background information but have not been addressed as part of this work.

![Figure 2. Representation of Shaft Vibration Modes](image)

**Noise Test Procedures**

Extensive noise testing has been performed on these transmissions during the many years they have been in production. A brief description the noise testing procedures and data analysis techniques will be presented here. A typical noise test for these transmissions involves installing it in a noise chamber and recording the overall noise level with six microphones while sweeping the input speed or torque. The microphones are arranged in a hemispherical array (Figure 3) with a 6' radius measured from the centerline of the transmission. The size of noise test chamber is dictated by the need to closely simulate a free-field measurement. Free-field is described by Hassall and Zaveri [2] as being in the open air with no reflective surfaces to interfere with the sound propagation. To approximate the free-field condition, the size of the test chamber was made to be approximately 20 feet long x 20 feet wide x 12 feet high.
Figure 3. Microphone Arrangement in the Anechoic Test Chamber

The data from the microphones are averaged into a single sound pressure level curve (Figure 4) and plotted versus torque or speed. From this curve, the high noise peaks and the RPM at which they occur can be found. To identify the frequency content at each RPM of interest, the discrete speed spectrum (Figure 5) can be plotted. If the numbers of teeth for each loaded gear set are known, the gear mesh frequencies can be calculated. Each of the fundamental gear mesh frequencies and their harmonics are shown on the spectrum, thus identifying which gear set is contributing to the high noise peak. Due to the size of the transmission and thickness of the case, acoustic noise radiated directly from the gear mesh is considered negligible. The motivation for this research project is to determine what the natural frequencies of each shaft are
and then try to design the gear sets and shafts or select bearings to avoid those particular resonant frequencies.

Figure 4. Typical Plot with High Noise Peaks

Figure 5. Typical Frequency Spectrum Identifying Gear Mesh Frequencies
CHAPTER II

CURRENT TECHNOLOGY REVIEW

Transmission Error

All geared systems will contain a forcing function at the fundamental gear mesh frequency or one of its harmonics. The magnitude of this forcing function is determined by the transmission error (TE) of the loaded gear set. Chung et al. [3] defines transmission error as the deviation in output gear rotation for a given constant angular velocity at the input gear. The lower the TE, the lower the noise output will be for a given gear set. James and Douglas [4] and Lim and Houser [5] agree that TE is the key parameter affecting gear noise and they were able to incorporate TE into each of their models.

Gear Mesh Stiffness

Gear mesh stiffness is the spring rate of the loaded gear teeth. When two gears are in contact and one of the shafts is held fixed, mesh stiffness can be thought of as the amount of torque per degree of rotation. As gears roll through mesh, the number of gear teeth that are actually in contact changes. This changing mesh stiffness has a negative effect on the overall TE. If the concern is noise radiated directly from the gear mesh, constant mesh stiffness will be a benefit. However, when similar transmission models were tested with experimental gear sets that had "constant" mesh
stiffness, the overall transmission noise level was actually higher than with standard gearing. Figure 6 shows overall noise level curves for standard production gearing and gearing with optimized TE. The prototype gearing with a actually generated and overall noise level that was 12 dBA higher that the standard production gears. There are machines that can measure the gear sets running in a lightly loaded condition. It is difficult to measure TE for the actual operating conditions for transmissions in this work because of the high torques involved. Software programs are available that can calculate TE from the micro-geometry of the gear tooth profiles, which is typically the method used for most applications.

![Noise Plot of Optimized TE versus Standard Production](image)

**Figure 6. Noise Plot of Optimized TE versus Standard Production**
Modeling Approaches

In works by James and Douglas [4] and Lim and Houser [5], their approaches are similar in that they couple the shaft to be modeled with the driving shaft through the gear mesh. In addition, they combine the torsional and lateral modes by incorporating both sets of equations in one model. Once the vibration modes have been found, the next step taken by James and Douglas [4] was to determine which ones were most critical. In many of the works, finite element analysis was performed to either validate predictions or to investigate case radiated noise.

For this research a more simplified approach was taken. Prediction of uncoupled shaft modes was successful, but in addition, a modeling process was developed and a program was written to apply the techniques to a variety of twin countershaft transmissions.
CHAPTER III

RESEARCH GOALS AND ACCOMPLISHMENTS

There were three main objectives for this research project. First, to predict shaft vibration modes in both new and current transmission models by using only basic gear and shaft geometries. Second, create a diagnostics program that could help with noise and vibration problems in current transmission models. Because the information required for the first two objectives was the same required for gear tooth analysis, the last objective was to create a program that would analyze all of the loaded gear sets in the transmission simultaneously for a wide range of operating conditions. With the current procedures, only one gear set is analyzed at one specific condition.

Much of the work in this research is predicated on equations derived from basic gear geometries (Figure 7), so some time will be spent on their explanation. For more detailed gear information, the reader may review the reference material by Martin [6] or Juvinall and Marsek [7]. At the outset of this research project, the focus was on prediction of shaft vibration modes. As the software program evolved, it was found to be useful for analyzing many other gear tooth motions. Although these gear actions are not used in shaft vibration modes, they are very useful in the diagnostics portion of the program. Once complete, the software program will not only predict the shaft vibration modes in a transmission, but can be integrated into an existing design process to provide a quicker and more comprehensive transmission analysis.
Gear Analysis

Center distance is the distance between the centers of two gears in mesh. For the twin countershft transmission discussed here, it can also be thought of as the distance between the mainshaft and the upper countershft for example. The general shape of the gear tooth is described as an involute. However, for most industrial gears the tooth profile often deviates from a perfect involute for various reasons. To prevent tip root interference the gear teeth are cut deeper in the root and the tips can be chamfered as part of the normal manufacturing process.

Figure 7. Gear Geometry and Nomenclature

The list of gear data required for the calculations are number of teeth, face width, distance from the end of the shaft to the gear face, weight (ounces), mass moment of inertia (in-lb-sec^2), tip chamfer, center distance, normal diametral pitch, pressure angle and orientation angle. Orientation angle is the angle of the line connecting the centers
of the shafts from horizontal, which for these transmissions is about 19°. All of the software screens used for gear analysis are shown in Appendix B.

Two gear teeth rolling through contact define a line of action. The starting point is when they first come in contact and the ending point is when they just leave contact. The angle that the line of action makes with a line drawn perpendicular to a line connecting the centers of the gears is called the pressure angle. The normal diametral pitch is the number of teeth on the gear divided by the diameter of the pitch circle in inches.

End of active profile (EAP) diameter is

\[ EAP = OD - 2 \times TC \]  

(1)

where \( OD \) is the outside diameter of the gear and \( TC \) is the tip chamfer. Diametral pitch transverse \( DP_T \) is

\[ DP_T = DP_N \times \cos(HA_N \frac{\pi}{180}) \]  

(2)

where \( DP_N \) is the normal diametral pitch and \( HA_N \) is the normal helix angle. The transverse pressure angle \( PA_T \) is

\[ PA_T = a \tan \left( \tan \frac{PA_N \frac{\pi}{180}}{\cos(HA_N \frac{\pi}{180})} \right) \frac{\pi}{180} \]  

(3)

where \( PA_N \) is the normal pressure angle.
The transverse base pitch $BP_T$ is

$$BP_T = \frac{\pi}{DP_T} \times \cos \left( \frac{PA_T \times \pi}{180} \right)$$ \hspace{1cm} (4)

The base diameter $BD$ is

$$BD = PD \times \cos \left( \frac{PAT \times \pi}{180} \right)$$ \hspace{1cm} (5)

The operating pitch diameter $PD_O$ is

$$PD_O = CD \frac{T_{MS}}{(T_{MS} + T_{CS})^2}$$ \hspace{1cm} (6)

where $CD$ is the center distance, $T_{MS}$ is the number of teeth on the mainshaft gear and $T_{CS}$ is the number of teeth on the countershaft gear. The operating pressure angle $PA_O$ is

$$PA_O = a \cos \left( \frac{BD}{PD_O} \right) \frac{180}{\pi}$$ \hspace{1cm} (7)

The total length for the line of action $LOA_{TL}$ is

$$LOA_{TL} = \sqrt{CD^2 - \left( \frac{BD_{MS}}{2} + \frac{BD_{CS}}{2} \right)^2}$$ \hspace{1cm} (8)
These gear tooth action equations have been presented here because they are used in subsequent calculations that will be plotted. There are many other gear equations in the software program but are of secondary interest and will not be presented in the report.

Sliding velocity is the relative speed at which the gear teeth slide together. First, the radius of curvature for both the driving and driven gear teeth must be calculated. Only the equation for the driving gear is shown.

\[ ROC = \sqrt{\left(\frac{EAP^2 - BD^2}{2}\right)} \]  
(9)

Next, the rolling velocity is calculated.

\[ RV = ROC \times \pi \times RPM \]  
(10)

Finally, the sliding velocity is the difference between the rolling velocities of the mating gears.

\[ SV = RV_{DG} - RV_{DN} \]  
(11)

where \( RV_{DG} \) is the rolling velocity of the driving gear and \( RV_{DN} \) is the rolling velocity of the driven gear. A plot of sliding velocity is shown (Figure 8). Slip ratio is simply the ratio of the sliding velocity to the rolling velocity, calculated for both the driving and driven gear set. A plot of the slip ratio is shown (Figure 9).
Operating Conditions:
1500 RPM
1200 LB-FT

Model-1 17th Gear (0.86:1)

Model-2 3rd Gear (4.88:1)

RED: driving gear
BLUE: driven gear

Figure 8. Plot of Gear Tooth Sliding Velocities

Figure 9. Plot of Gear Tooth Slip Ratios
The gear tooth sliding calculations were a slight deviation to the main goal of this research project. Although not directly related to the shaft vibration modes, the procedures used to perform these calculations on all loaded simultaneously will be a significant benefit in future transmission analyses.

Noise and Vibration Diagnostics

In addition to vibration prediction, considerable emphasis was placed on creating a noise and vibration diagnostics portion of the program. When a transmission is returned for a noise-related issue, the current approach is to calculate a single gear mesh frequency for a specific operating condition. With this new program, the transmission model is chosen, a gear position is selected and the frequencies are plotted for all loaded gear sets over a wide range of conditions. This allows for a much quicker analysis by looking at all forcing functions and operating conditions at once.

Occasionally, bearing faults are found to be the primary forcing function instead of a gear mesh and currently, there are no convenient methods employed to compute those frequencies. With this program the bearing geometry data, Eschman et al. [8], can be entered into the model and the frequencies calculated for any selected gear position and operating speed. Now with a simple menu pick, bearing frequencies, gear mesh frequencies and shaft frequencies can be found.

From the beginning, this tool was conceived to help analyze existing noise and vibration problems, as well as predicting potential new ones. With this new
program, all of the loaded gear sets can be calculated simultaneously and plotted for a range of operating conditions. The benefit to this type of analysis is that the entire transmission can be operated on as a whole, actually shifting gears and plotting the results. This research will ultimately be used with a master transmission database to solve or troubleshoot many different gear-related issues. All of the software program screens used for noise and vibration diagnostics are shown in Appendix C.

Pitch line velocity (Figure 10) is simply the rotating speed of the gear’s pitch diameter. This parameter is used to determine the noise generating potential of a gear set. In general, the faster the pitch line velocity, the more noise the gear set will generate. The pitch line velocity in feet per minute is

\[ PLV = \frac{\pi \times PD}{12} \times rpm \]  

(12)

Figure 10. Plot of Pitch Line Velocities
Gear mesh frequencies (Figure 11) are one of the most important factors used in transmission noise and vibration diagnostics because they are the primary forcing functions when a noise complaint is encountered. Gear mesh frequencies are found by

\[ GMF = \frac{T_{DG} \times rpm}{60} \]  

(13)

where \( T_{DG} \) is the number of teeth on the driving gear.

![Gear Mesh Frequencies](image)

**Figure 11. Plot of Gear Mesh Frequencies**

As mentioned above, the fundamental gear mesh frequency is the primary forcing function for a gear set. A few secondary forcing functions described by Lang [9], assembly phase passing frequency and hunting tooth frequency, are also worth noting.
These frequencies are much lower than the gear mesh frequency and although they do not create primary noise peaks, they can lead to sidebands. The number of assembly phases $N_{ap}$ for a gear set is equal to the highest common integer factor between the numbers of teeth on each gear. Therefore, the assembly phase passing frequency (Figure 12) is found by

$$f_{ap} = \frac{GMF}{N_{ap}} \tag{14}$$

where $GMF$ is the gear mesh frequency for the gear set. The hunting tooth frequency (Figure 13) is calculated by

$$f_{ht} = \frac{N_{ap} \times GMF}{T_{MS} \times T_{CS}} \tag{15}$$

where $T_{MS}$ is the number of teeth on the mainshaft gear and $T_{CS}$ is the number of teeth on the countershaft gear.
Figure 12. Plot of Assembly Phase Passing Frequencies

Figure 13. Plot of Hunting Tooth Frequencies
Model Formulation Procedure

The approach used in this research was to formulate a linear, lumped-mass model that would predict shaft vibration modes in both torsional and translational (lateral) motion. There are several shafts in the transmissions; only the front countershafts and auxiliary countershafts are modeled. The front countershafts have gears that are pressed on and keyed to the shaft as well as gears that are integral with the shaft. Auxiliary countershafts have integral gears and gears that are pressed on and welded. Torsional and lateral stiffness for the model are calculated using the diameter of the shafts alone. Changes in stiffness due to a gear that is pressed on or welded to the shaft are not taken into account. For gears that are integral with the shaft, nominal shaft stiffness is used.

The unique approach for this research was to create a software program that would accommodate a variety of different transmission models. By developing a method to section each shaft and attach the gears, many different shafts can be modeled. The information required for modeling the shaft assembly is gear weight, position of each gear, shaft diameter and rotational mass moment of inertia. Additional gear geometry information is required to compute the gear mesh forces and bearing forces.

One difficult part about the modeling process was gathering detailed shaft and bearing geometries. The bearing geometries are required for the diagnostics portion of this program; the gear and shaft geometries are required for modeling the vibration modes. To simplify the use of this program a modeling procedure and data sheet were
created, which a designer, with access to design software, can fill in. Once the data sheets are complete, the information can be entered directly into the modeling program. The gear data entry screen (Figure 14) is the main interface to building a model. It is shown here to acquaint the reader with the program layout. All of the other software program screens used to build a transmission model are shown in Appendix A.

Figure 14. Gear Data Entry Screen to Build a Transmission Model
Gear Mesh Forces

The gear data required for shaft vibration modeling are weight, mass moment of inertia and some basic gear geometry parameters. The inertia is used in the torsional model and the weight in the translational model. Currently, a transmission design sheet is used to summarize the key gear parameters in a particular transmission. Data from the design sheet can be entered directly into the vibration model. An additional datasheet was created to capture the gear weights, inertias and locations on the shaft.

The main forces used in the calculations are tangential forces, which generate torque at the gear, and radial or separation forces, which push the gears apart. Because a linear modeling approach was chosen, friction forces are assumed negligible in this research. As a gear pair rolls through mesh, the same numbers of teeth are not always in contact. For part of the rotation there is one tooth pair in contact and for the remaining part of the rotation there are two or more tooth pairs in contact. The changing number of teeth in contact during mesh results in a changing mesh stiffness.

The goal of this research was to create a flexible, intuitive software program that can predict shaft vibration modes. In order to apply the proper forces on each shaft, the gear forces must be calculated for every operating condition. The input conditions (torque and speed) are known; the gear forces are not. The program user need only select a gear position and enter the operating conditions at the input of the
transmission for the gear and bearing forces to be calculated. A schematic of the gear force arrangement for a twin-countershaft transmission has been drawn (Figure 15).

![Figure 15. Gear Mesh Forces and Shaft Arrangement](image)

Each countershaft will have two loaded gear sets. The first gear set, often referred to as the head set, splits the torque from the input shaft into the two countershafts. The second gear set, selected by the shift lever, returns the torque to the mainshaft. The equation for tangential gear mesh force in the headset is:

$$F_{T1} = \frac{12T_{IN}}{R_1/S}$$  \hspace{1cm} (16)
where $T_{IN}$ is the input torque, $R_1$ is the pitch radius of the main drive gear and $S$ is the number of countershafts, which in this case is two. The tangential gear mesh force for the second gear set is:

$$F_{T2} = \frac{-12T_{CS}}{R_3}$$

(17)

where $T_{CS}$ is the torque in the countershaft and $R_3$ is the pitch radius of the countershaft driving gear. The difference in sign is due to the change in driving gear. In the first gear set, the mainshaft gear is driving the countershaft. For the second gear set, the countershaft gear is driving the mainshaft. Radial or separation forces are simply the tangential forces multiplied by the tangent of the pressure angle. For the headset, the radial forces are:

$$F_{R1} = -|F_{T1} \tan(\Phi_{T1})|$$

(18)

and for the second gear set are:

$$F_{R2} = -|F_{T2} \tan(\Phi_{T2})|$$

(19)

Radial forces are always negative because they are always pointing outward.

**Bearing Forces**

Once the gear mesh forces are obtained, the bearing forces can be calculated by creating a free body diagram of the countershaft. The gears modeled in this work are spur gears; the force equations for bearings, Norma FAG [10], have not been expanded to incorporate helical gears. During transmission operation very few, if any, forces are constant. There are engine torsional vibrations that cause the input torque
to fluctuate. Varying gear mesh stiffness causes the tangential forces to change in both magnitude and direction. For this research, the gear mesh forces are assumed to be constant and are resolved into Y and Z directions to compute bearing forces. The methods used to compute bearing forces in the Y-direction for a given load condition are shown. The shaft used for this example is broken into nine sections, each section having a weight $W$ and distance from the endpoint $L$ (Figure 16).

The bearing geometry data is only used for the noise and vibration diagnostics portion of the program. Radial bearing stiffness is a non-linear characteristic of bearings and is used in the translational shaft model. As the radial forces at the bearing increase, the stiffness becomes greater. James and Douglas [4] created a 6-by-6-stiffness matrix for the bearings used in their transmission model. This approach allowed the stiffness to be linearized, which could then be applied to the harmonic analysis. Lim and Houser [5] simply applied a value to the bearing stiffness for use in a parametric study. For this research, the emphasis was on the overall transmission modeling process. Actual bearing geometries are difficult to gather and many bearing manufacturers consider that information proprietary. The forces are calculated at each bearing location, but a constant bearing stiffness of 9.5e6 lbs/inch was used. Historical data on similar transmission noise testing indicated that this would be a good nominal value for this study.
Taking the sum of moments about point A, \( W_2 L_2 + W_3 L_3 + W_4 L_4 + W_5 L_5 + W_6 L_6 + W_7 L_7 + W_8 L_8 + W_9 L_9 + F_1 L_3 + F_2 L_6 - BY_2 L_9 = 0 \) \hspace{1cm} (20)

\[ F_1 = F_{mg} Y_1 \times t_{y1} + F_{sep} Y_1 \times s_{y1} \] \hspace{1cm} (21)

\[ F_2 = F_{mg} Y_2 \times t_{y2} + F_{sep} Y_2 \times s_{y2} \] \hspace{1cm} (22)

Next, sum the forces in the Y-direction and solve for the bearing forces.

\[ BY_2 = \frac{\sum W_i + F_1 L_3 + F_2 L_6}{L_9} \] \hspace{1cm} (23)

\[ BY_1 = \sum W_i + F_1 + F_2 - BY_2 \] \hspace{1cm} (24)

Figure 16. Free-Body Diagram of Countershaft
Sectioning Shafts

A sample front countershaft can be partitioned to create the data for this model (Figure 17). In a similar fashion, auxiliary countershafts as well as shafts in other transmissions can be modeled. Each bearing journal is a section, each gear hub is a section, and any uniform shaft section by itself is counted. If a gear hub is large, then it can be considered separate from the gear. Design software with the transmission data can be used to get detailed shaft information. However, a good ruler can be used to measure the gears and shafts to within +/- 1 mm, which is adequate for this research. Both methods were used for the transmission models, and the ruler method was much quicker. Appendix D contains all of the software program screens used in shaft model prediction.

Figure 17. Sample Countershaft Sectioning
To demonstrate how the software program would work to build a shaft, the shaft data entry screen is shown (Figure 18).

Figure 18. Shaft Data Entry Screen
Torsional Vibration Model

A separate model was developed for the torsional and lateral displacement, but the same concepts apply to both. To begin, the shafts need to be broken into logical sections by using a print of the shaft assembly or measuring each gear position with a ruler. A section can be a bearing journal, gear hub or significant change in shaft diameter. If a part of the shaft has no gear attached, it will be considered its own section. Next, a graphical representation of the shaft is drawn with a mass moment of inertia for each section and stiffness between each section (Figure 19).

![Figure 19. Representation of Torsional Shaft Mode](image)

Because no lateral motion is assumed for the torsional model, the bearing stiffness does not enter into the equations. The equations of motion (Figure 20) can then be written for each shaft in a free, unloaded state by applying an angular displacement, $\Theta_i$, to each section. For the development of this system of equations it was assumed that $\Theta_1 > \Theta_2 > \Theta_3 > \Theta_4 > \Theta_5 > \Theta_6 > \Theta_7$. For a shaft that is partitioned into N sections, there will be N equations of motion. Once the equations are written, they can be rearranged into matrix form to be solved as a simple eigenvalue problem. An $N \times N$ matrix is created from the original N equations.
\[ J_1 \ddot{\Theta}_1 + k_1 (\Theta_1 - \Theta_2) = 0 \]
\[ J_2 \ddot{\Theta}_2 - k_1 (\Theta_1 - \Theta_2) + k_2 (\Theta_2 - \Theta_3) = 0 \]
\[ J_3 \ddot{\Theta}_3 - k_2 (\Theta_2 - \Theta_3) + k_3 (\Theta_3 - \Theta_4) = 0 \]
\[ J_4 \ddot{\Theta}_4 - k_3 (\Theta_3 - \Theta_4) + k_4 (\Theta_4 - \Theta_5) = 0 \]
\[ J_5 \ddot{\Theta}_5 - k_4 (\Theta_4 - \Theta_5) + k_5 (\Theta_5 - \Theta_6) = 0 \]
\[ J_6 \ddot{\Theta}_6 - k_5 (\Theta_5 - \Theta_6) + k_6 (\Theta_6 - \Theta_7) = 0 \]
\[ J_7 \ddot{\Theta}_7 - k_6 (\Theta_6 - \Theta_7) = 0 \]

Figure 20. System of Equations for Torsional Vibration Model

The J values are the mass moment of inertias of each section. This value is easily calculated for a simple shaft, but for a gear it is a bit more difficult to compute. For a solid shaft, the mass moment of inertia is

\[ J = \frac{md^2}{8} \]  \hspace{1cm} (25)

where \( m \) and \( d \) are the mass and diameter of the shaft section respectively. There are a few ways to determine the mass moment of inertia for a gear. If the gear has been drawn in a 3D graphics program, it can generally be computed in software. To measure the value directly, a torsional pendulum test fixture can be used. The gear is placed on a thin, horizontal disc that is suspended from very long, small wires. By displacing the disc a small angle and allowing it to rotate freely, the period of oscillation is measured and averaged through five cycles. Using the gear mass and
period of oscillation, the mass moment of inertia can be computed. The last and least accurate way is to assume the gear is a uniform disc with a hole in the center and calculate the inertia directly. Because the gears used in these models have varying web thickness and the hubs often extend beyond the face width, this method is not recommended. However, the more uniform the gear hubs and webs are, the more accurate the calculation will be. Gears that are integral with the shaft fall into this category. If the gear is integral with the shaft, compute the weight and mass moment of inertia as if it were a ring pressed on the shaft. Use the shaft diameter as the inside diameter, the pitch diameter as the outside diameter and the gear face width as the thickness. The weight of each gear must also be measured or calculated. Once calculated, this information can be put into the gear data sheet to simplify the final data entry into the software.

Now the torsional stiffness can be computed for each section. Using a modulus of elasticity $E = 30e6$ psi, and Poisson’s Ratio $\nu = 0.3$, the shear modulus, $SM$, is found.

$$SM = \frac{E}{2(1+\nu)}$$

(26)

The polar moment of inertia, $I_p$, can then be found for each shaft section.

$$I_p = \frac{\pi d^4}{32}$$

(27)
Finally, the torsional stiffness for each section, $k_i$, is calculated using the section length $L_i$. These stiffness values are then plugged into the equations of motion.

$$k_i = SM \frac{I_i}{L_i}$$  \hspace{1cm} (28)

The system of equations are rearranged into matrix form in order to solve them as a simple eigenvalue problem. The mass matrix is a diagonal matrix of $J$ values (Figure 21). The stiffness matrix is created simply by arranging the stiffness terms in matrix form (Figure 22).

$$
\begin{bmatrix}
J_1 \\
J_2 \\
J_3 \\
J_4 \\
J_5 \\
J_6 \\
J_7
\end{bmatrix}
$$

**Figure 21. Diagonal Mass Matrix for Torsional Model**

$$
\begin{bmatrix}
-k_1 \\
-k_2 \\
-k_3 \\
-k_4 \\
-k_5 \\
-k_6 \\
-k_7
\end{bmatrix}
$$

**Figure 22. Stiffness Matrix for Torsional Model**
Once the mass and stiffness matrices are found, the eigenvalues (natural frequencies) and eigenvectors (mode shapes) are found by using the eig function in Matlab. Each column of the eigenvector matrix defines a mode shape and each element of the diagonal eigenvalue matrix represents a lambda value that is converted to natural frequency in Hertz.

\[ f_n = \frac{\sqrt{\lambda}}{2\pi} \]

(29)

The final output is a plot of the first four mode shapes and natural frequencies for the selected shaft (Figure 23).

![Figure 23. Sample Plot for Torsional Modes](image)
Lateral Vibration Model

The translational (lateral) model is analogous to the torsional model except that the stiffness is in bending rather than in torsion. The shaft sections are the same as in the torsional model so a similar schematic of the translational model can be drawn (Figure 24). The modulus of elasticity $E$, polar moment of inertia $I_p$ and section length $L_i$ are the same as those in the torsional model.

![Figure 24. Representation of Lateral Shaft Model](image)

The bending stiffness, $k_i$, is then found for each section. In this equation, $w$ represents the unit loading on each section.

$$k_i = \frac{384EI_p}{5wL_i}$$ (30)

One additional factor in the translational model is that the bearing stiffness must now be accounted for. For the systems modeled in this research, the bearings are located at the end of each simply supported shaft. Bearing stiffness is a non-linear value that can be very difficult to compute. Because the focus of this research was on shaft modeling and not bearing modeling, it was decided that a single, nominal value of stiffness would be used. Based on some early internal research done with these types
of transmissions, a radial bearing stiffness of 9.5e6 lb/in. The lateral equations of motion for a shaft with seven sections are shown (Figure 25).

\[
\begin{align*}
M_1 \ddot{y}_1 + k_{b1}(y_1) + k_{i2}(y_1 - y_2) &= 0 \\
M_2 \ddot{y}_2 - k_{i2}(y_1 - y_2) + k_{23}(y_2 - y_3) &= 0 \\
M_3 \ddot{y}_3 - k_{23}(y_2 - y_3) + k_{34}(y_3 - y_4) &= 0 \\
M_4 \ddot{y}_4 - k_{34}(y_3 - y_4) + k_{45}(y_4 - y_5) &= 0 \\
M_5 \ddot{y}_5 - k_{45}(y_4 - y_5) + k_{56}(y_5 - y_6) &= 0 \\
M_6 \ddot{y}_6 - k_{56}(y_5 - y_6) + k_{67}(y_6 - y_7) &= 0 \\
M_7 \ddot{y}_7 - k_{67}(y_6 - y_7) + k_{b2}(y_7) &= 0
\end{align*}
\]

Figure 25. System of Equations for Lateral Vibration Model

Again, these equations can be rearranged and written in matrix form in order to solve this as a typical eigenvalue problem. The mass matrix (Figure 26) and the stiffness matrix (Figure 27) are shown for the lateral vibration model.

\[
\begin{bmatrix}
M_1 \\
M_2 \\
M_3 \\
M_4 \\
M_5 \\
M_6 \\
M_7
\end{bmatrix}
\]

Figure 26. Mass Matrix for the Lateral Vibration Model
\[
\begin{bmatrix}
(k_b + k_{12}) & -k_{12} & 0 & 0 \\
-k_{12} & (k_{12} + k_{23}) & -k_{23} & 0 \\
0 & -k_{23} & (k_{23} + k_{34}) & -k_{34} \\
0 & 0 & -k_{34} & (k_{34} + k_{45}) & -k_{45} \\
0 & 0 & 0 & -k_{45} & (k_{45} + k_{56}) & -k_{56} \\
0 & 0 & 0 & 0 & -k_{56} & (k_{56} + k_{67}) & -k_{67} \\
0 & 0 & 0 & 0 & 0 & -k_{67} & (k_{67} + k_{b2})
\end{bmatrix}
\]

Figure 27. Stiffness Matrix for the Lateral Vibration Model

In a similar fashion to the torsional model, the eigenvalue problem is solved and the mode shapes are plotted (Figure 28).
Unloaded Condition

Translational mode shapes were calculated for both loaded and unloaded conditions. For the unloaded condition, the weight of each section was only that of the individual gear. To determine the unit load, $w$, the weight of the gear was divided by the length of each section.

Loaded Condition

For the loaded condition, gear forces were applied to the shaft section where there was a loaded gear (Figure 29). The radial, or separation force for each loaded gear on the shaft was added to the weight of the gear. The combined weight of the gear and the force were divided by the section length to arrive at the unit load $w$. Since the noise testing was performed at one particular load, the shaft vibration modes were also calculated at that same load. A comparison between the loaded and unloaded translational modes will be discussed in the analysis portion of this research.

Figure 29. Front Countershaft Assembly with Gear Forces
CHAPTER IV

NOISE AND VIBRATION DATA ANALYSIS

According to Beacham et al. [1], Chung et al. [3] and Houser and Lim [5], the noise radiated from the transmission case is the result of high bearing forces generated from shaft resonance modes. The main goal of this research was to predict the countershaft modes that contribute to the resonances and thus, the high noise peaks. When these modes are excited, the resultant noise peaks can clearly be seen.

Currently, no specific techniques are employed in the test procedures to indicate what particular shaft modes are encountered. In the past, the shaft modes have been identified through extensive noise and structural response testing. As an additional exercise, four accelerometers were used to acquire case vibration near the countershaft bearing locations while acquiring the normal noise data on transmission Model #1. By examining the magnitude and phase relationships of the vibration data at the frequencies of specific noise peaks, an attempt was made to determine if any of the shaft modes can be specifically identified. For the other transmission models used in this research, only the noise data was used to compare the results of shaft vibration mode predictions.

One must be careful when testing for these shaft modes because they are greatly affected by rebuilding the transmission. When gears are pressed on and off the shafts or when bearing endplays change, the resonant frequency of the shaft modes can shift.
This behavior should be noted and more investigation into this phenomenon will be reserved for future work.

A typical noise data analysis procedure was described in Chapter 1, but will be restated here for clarity. The overall sound pressure level versus speed plot is the first set of data to review. This curve will show if there are any excessive noise peaks and will identify the RPM at which they occur. An RPM spectral map, or waterfall plot, is useful to get a complete picture of the noise peaks for the entire operating speed range. It can identify the relative strengths of all the peaks, thereby focusing on the most critical. For each peak of interest, the corresponding RPM will be used to generate a discrete speed spectrum. This plot will identify the frequencies contained in the peak, which will be compared to the frequencies of the predicted mode shapes.

Case Study: Transmission Model #1

For the first transmission model used in this research, vibration data from four accelerometers were recorded simultaneously with the standard noise data. The motivation was to determine if higher case vibrations were encountered that matched the frequencies of the noise peaks. Second, determine if the vibration data could identify the specific shaft modes at each noise peak. At each end of the front countershaft two accelerometers were fixed to the case, one vertically and one on an axis that intersects the centers of the shafts (Figure 30). The complete transmission mounted in the noise chamber with the accelerometers attached is also shown (Figure 31). All of the noise and vibration data for Transmission Model #1 have been placed together in Appendix E.
Figure 30. Transmission Case Showing Accelerometer Locations

Figure 31. Complete Transmission Setup in Noise Chamber
Two different gear positions that exhibit high noise peaks were selected for the test. One condition has both loaded gears near the same end of the front countershaft and the other condition has one loaded gear near each end of the shaft. These conditions were chosen to see how the load configuration might affect different types of shaft modes.

The data acquisition system used for this vibration research was developed specifically for NVH applications (Figure 32). For noise applications, data sample rates must be at least twice the desired bandwidth. For example, if the frequency range of interest is 10kHz, then the sampling rate must be at least 20kHz. The system used is capable of 100ks/sec on each channel while linked directly to a laptop computer.

![Figure 32. Data Acquisition System Used for Vibration Testing](image-url)
Transmission Model #1 was tested in two different gear positions and the data will be identified by run # 27168 and 27174. The steps taken in the analysis are simply gathering all of the frequencies from the noise data (Table 1) and comparing them to the frequencies of the predicted shaft modes (Table 2). All of the noise and vibration data for Model #1 are shown in Appendix E.

### Transmission Model #1: Run # 27168

<table>
<thead>
<tr>
<th>Input Speed (RPM)</th>
<th>Frequencies of Interest from Noise Data (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>535</td>
<td>513, 1026</td>
</tr>
<tr>
<td>1285</td>
<td>1114, 2227, 3341</td>
</tr>
<tr>
<td>2035</td>
<td>1746, 1951, 3527</td>
</tr>
<tr>
<td></td>
<td>5291</td>
</tr>
</tbody>
</table>

*Table 1. Gear Mesh Frequencies for Run #27168*

<table>
<thead>
<tr>
<th>Predicted Frequencies for Lateral Modes- No Load (Hz)</th>
<th>Predicted Frequencies for Lateral Modes- Load = 1000 lb-ft (Hz)</th>
<th>Predicted Frequencies for Torsional Modes- No Load (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>524</td>
<td>522</td>
<td>1126</td>
</tr>
<tr>
<td>2385</td>
<td>589</td>
<td>2658</td>
</tr>
<tr>
<td>4272</td>
<td>3129</td>
<td>1776</td>
</tr>
<tr>
<td>5466</td>
<td>5277</td>
<td>3095</td>
</tr>
</tbody>
</table>

*Table 2. Predicted Shaft Mode Frequencies for Run #27168*
With gear mesh frequencies, there will often be excitation at the harmonics (integer multiples) as well as the fundamental. Mode shapes, on the other hand, are not harmonic in nature. For example, the frequency of mode 3 or mode 4 will have no correlation to mode 1 or mode 2. Therefore, the assumption is that only one possible mode will be associated with a group of harmonic gear mesh frequencies. By grouping the harmonic gear mesh frequencies, it will be easier to look for and compare mode frequencies (Table 3).

<table>
<thead>
<tr>
<th>Transmission Model #1: Run # 27168</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Speed (RPM)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>535</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1285</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2035</td>
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<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

Table 3. Comparison of Predicted versus Measured Frequencies

The predicted mode frequencies on the right of Table 3 that most closely match the gear meshing frequencies on the left have been highlighted in red. For the first test condition, the predicted mode frequencies show fairly good correlation to the noise data. However, there are some predicted modes that do not necessarily couple in with the noise data. Investigation into the significant and insignificant modes will be a topic of future work.
The second test condition for Model #1 was run at the same input torque but in a different gear position. Again, the gear mesh frequencies (Table 4) from the noise data will be compared to the predicted mode frequencies (Table 5).

**Table 4. Gear Mesh Frequencies for Run #27174**

<table>
<thead>
<tr>
<th>Input Speed (RPM)</th>
<th>Frequencies of Interest from Noise Data (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>665</td>
<td>576, 1153</td>
</tr>
<tr>
<td>935</td>
<td>565, 810, 1130</td>
</tr>
<tr>
<td>1330</td>
<td>803, 1153, 2305, 3458, 4017, 4611</td>
</tr>
<tr>
<td>2035</td>
<td>1764, 3527, 5291</td>
</tr>
</tbody>
</table>

**Table 5. Predicted Shaft Mode Frequencies for Run #27174**

<table>
<thead>
<tr>
<th>Predicted Frequencies for Lateral Modes- No Load (Hz)</th>
<th>Predicted Frequencies for Lateral Modes- Load = 1000 lb-ft (Hz)</th>
<th>Predicted Frequencies for Torsional Modes- No Load (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>524</td>
<td>463</td>
<td>1126</td>
</tr>
<tr>
<td>2385</td>
<td>681</td>
<td>2658</td>
</tr>
<tr>
<td>4272</td>
<td>4027</td>
<td>3095</td>
</tr>
<tr>
<td>5466</td>
<td>4182</td>
<td>1776</td>
</tr>
</tbody>
</table>
Note that the predicted lateral modes for no load are the same for both test conditions. Likewise, the torsional modes are the same. Since no loads were used in these predictions, the modes will be the same. The predicted shaft modes for the loaded condition are unique. Finally, the predicted frequencies were compared to the actual noise data (Table 6).

<table>
<thead>
<tr>
<th>Transmission Model #1: Run # 27174</th>
<th>Input Speed (RPM)</th>
<th>Combined Harmonics for Frequencies of Interest</th>
<th>Predicted Frequencies for Lateral Modes-No Load (Hz)</th>
<th>Predicted Frequencies for Lateral Modes-No Load (Hz)</th>
<th>Predicted Frequencies for Torsional Modes-No Load (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>665</td>
<td>576, 1153</td>
<td>524</td>
<td>463</td>
<td></td>
<td></td>
</tr>
<tr>
<td>935</td>
<td>565, 1130</td>
<td></td>
<td>681</td>
<td>1126</td>
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</tr>
<tr>
<td></td>
<td>810</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1330</td>
<td>803, 4017</td>
<td></td>
<td>4027</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1153, 2305, 3458</td>
<td>2385</td>
<td>2658</td>
<td>3095</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4611</td>
<td>4272</td>
<td>4182</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2035</td>
<td>1764, 3527, 5291</td>
<td>5466</td>
<td>1776</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Comparison of Predicted versus Measured Frequencies

For this test condition there is similar correlation in that there are some modes that match the noise data quite well and others that do not couple in. Again, this will be left for future study. However, the unloaded lateral modes appear to be more accurate than the loaded condition. The torsional modes seem to be fairly good predictions below the 2000 Hz range.
Of secondary interest were the case vibration measurements. Four accelerometers were used to record the case vibration near the front countershaft bearings. The vibration data was recorded along with the noise data to look for similar frequencies. All of the noise and vibration data for Model #1 are shown in Appendix E. The vibration data for only one test condition is shown here. From the vibration data, the peaks corresponding to shaft resonances are clearly seen in the time history plot (Figure 33).

![Figure 33. Time History of Noise and Vibration Data](image)

By analyzing the frequency spectrum (Figure 34), the coincident frequencies are identified. In addition, the magnitude and phase relationships (Figure 35) of the vibration data will be analyzed to determine if any particular shaft modes can be identified. The phase is approximately 0° at the highest peak, which could indicate a bounce mode in the shaft.
Figure 34. Frequency Spectrum with Coincident Noise and Vibration Peaks

Figure 35. Magnitude Phase Plot of Vibration Data
Other peaks did not show very consistent phase data. The vibration exercise presented some interesting information for future work, but actual mode prediction was not successful. Another reason it may not have been successful was that the shafts do not necessarily present modes individually. A lateral mode will likely couple in with a torsional mode and trying to differentiate between them with vibration data can be difficult.

**Case Study: Transmission Model #2**

For this transmission model only the noise data were analyzed. Model #2 was chosen because it had a somewhat longer and lighter front countershaft than Model #1. The same basic steps for modeling and data analysis were performed. All of the data on Model #2 is captured in Appendix F. For this transmission model only one gear position was tested. The gear mesh frequencies from the noise data (Table 7) will be compared to the predicted mode frequencies (Table 8).

<table>
<thead>
<tr>
<th>Transmission Model #2: Run # 27933</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Speed (RPM)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>590</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1010</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1920</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Table 7. Gear Mesh Frequencies for Run # 27933
Transmission Model #2: Run # 27933

<table>
<thead>
<tr>
<th>Predicted Frequencies for Lateral Modes - No Load (Hz)</th>
<th>Predicted Frequencies for Lateral Modes - Load = 500 lb-ft (Hz)</th>
<th>Predicted Frequencies for Torsional Modes - No Load (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>545</td>
<td>415</td>
<td>1293</td>
</tr>
<tr>
<td>1921</td>
<td>936</td>
<td>1905</td>
</tr>
<tr>
<td>3188</td>
<td>1026</td>
<td>3339</td>
</tr>
<tr>
<td>4631</td>
<td>3823</td>
<td>5503</td>
</tr>
</tbody>
</table>

Table 8. Predicted Shaft Mode Frequencies for Run # 27933

Comparing the measured frequencies with the predicted mode frequencies shows similar results to Model #1. The loaded modes appear to be more accurate than the unloaded lateral modes. The torsional modes in this case appear to be more accurate at the higher frequencies and not the lower frequencies as in Model #1.

<table>
<thead>
<tr>
<th>Input Speed (RPM)</th>
<th>Frequencies of Interest from Noise Data (Hz)</th>
<th>Predicted Frequencies for Lateral Modes - No Load (Hz)</th>
<th>Predicted Frequencies for Lateral Modes - Load = 500 lb-ft (Hz)</th>
<th>Predicted Frequencies for Torsional Modes - No Load (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>590</td>
<td>403, 806</td>
<td>545</td>
<td>415</td>
<td>1293</td>
</tr>
<tr>
<td>1010</td>
<td>909, 1818</td>
<td>1921</td>
<td>936</td>
<td>1905</td>
</tr>
<tr>
<td>1920</td>
<td>1728, 3456</td>
<td>3188</td>
<td>3823</td>
<td>3339</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2622</td>
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<td>5503</td>
</tr>
</tbody>
</table>

Table 9. Comparison of Predicted versus Measured Frequencies
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The noise and vibration diagnostics portion of the program will be a useful addition to the techniques currently used. It will allow a more comprehensive analysis by having a tool that is much easier to use and one that will look at a more broad range of operating conditions. With this tool, two different transmission models can be analyzed at the same time by plotting the desired parameter for each.

The gear analysis portion of the program was merely an extension of the original concept of this research. Virtually all of the gear geometry data required for the noise and vibration analysis could also be used to compute gear tooth interactions. Although not used in the actual vibration mode predictions, the gear analysis tools will be very helpful when this program is integrated with existing transmission design tools.

Having the ability to predict shaft vibration modes will be of considerable benefit to both designers and noise engineers working with these transmissions. A comprehensive database of transmission models is available today, but it does not have the capability to perform many of the analyses presented in this work. Plans are being made to incorporate the features in this program with the master database of transmission models. The shaft vibration mode prediction portion of the program was successful at demonstrating common methods to utilize this feature across multiple
transmission platforms. Some work will be required to refine the predicted results, but the procedures are in place. For example, the shaft model presented in this research operates on a single shaft only. The operating conditions are calculated throughout the entire transmission, but the shaft is modeled in a free, uncoupled state. Models developed in other work have actually coupled mating shafts via the gear mesh to provide a forced vibration analysis. Now that the modeling procedures have been established, more work will be focused on a coupled shaft analysis. The specific vibration modes modeled are assumed to be independent. If the shaft is vibrating torsionally, the lateral displacements are not factored in. During actual transmission operation, it is very likely that multiple modes are occurring simultaneously. Again, this research was successful at demonstrating techniques that can be applied to a variety of transmission models to compute simple shaft vibration modes.

There were a couple of terms, bearing stiffness and transmission error, presented in this paper that were ultimately not part of the final shaft model. Bearing stiffness is a non-linear value that is dependent on load and is needed when calculating translational displacement modes but does not play a role in torsional modes. Transmission error is a term used to define the quality of gear meshing action as it relates to noise. At the outset of this research, both were thought to be of critical importance. However, since the shaft models developed were in the free, uncoupled state, transmission error could not enter into the equations at all. It was left in the paper as a topic of discussion and a point of future work. As for bearing stiffness, it is used in the translational shaft model. The actual value for stiffness was based on
some internal research performed on similar transmission models. It is known that bearing stiffness has an effect on shaft modes, but due to the fact that the techniques and methods presented in this paper were new, a nominal stiffness value was simply chosen. Rather than spend significant time getting a more accurate value of bearing stiffness, it was decided that the focus should remain on the development of the shaft model.

The analytical techniques used in this research encompassed many areas of transmission design. The program has taken complicated gear analysis equations in addition to basic gear mesh frequency equations and combined them into one easy to use program. Several topics of future work were mentioned in this report. The methods used in this software tool provide a strong platform from which to go forward.
REFERENCES


APPENDIX A

Software Program Screens for Transmission Data Input
A 1. Front Panel to Create or Edit a Model

A 2. Gear Data Entry Form to Select Transmission Model and Gear Set
A 3. Gear Position Form to Build Transmission Ratios

A 4. Bearing Data Entry Form
### A 5. Shaft Data Entry Form

**Shaft Data Entry**

- **Select a Shaft:** Front CS Upper
- **Section Number:** Section 3
- **Gear on Section:** Upper CS PTO
- **Bearing on Section:**
- **Save All Shaft Model Data**
- **DONE**

### A 6. Gear Data Sheet

**GEAR INFORMATION**

<table>
<thead>
<tr>
<th>Model:</th>
<th>Gear Tooth Face (inches)</th>
<th>Gear Hub (inches)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Part Number</td>
<td>Description</td>
</tr>
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<td></td>
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</table>

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## SHAFT INFORMATION

<table>
<thead>
<tr>
<th>X dimensions</th>
<th>section diameter</th>
<th>overall length of the shaft</th>
<th>weight of the shaft (oz.)</th>
<th>r - Center Distance</th>
<th>θ - Shaft Orientation Angle</th>
<th>gear OR bearing pin on this section (if any)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sec #</td>
<td>start_x</td>
<td>delta_x</td>
<td>end_x</td>
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<tr>
<td>InputShaft</td>
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<td>Front c/s</td>
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<tr>
<td>Mainshaft</td>
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<tr>
<td>Aux c/s</td>
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A 7. Shaft Data Sheet
<table>
<thead>
<tr>
<th>Bearing Part Numbers</th>
<th>Location in transmission</th>
<th>Type of bearing (ball, TRB, etc)</th>
<th>number of balls or rollers</th>
<th>rollerball diameter</th>
<th>roller length</th>
<th>inner race diameter</th>
<th>outer race diameter</th>
<th>overall contact width of OD</th>
<th>contact angle</th>
<th>dynamic load rating &quot;C&quot; lbf (500 hrs at 33rpm)</th>
<th>radial stiffness</th>
<th>X - coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input shaft</td>
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<td>Aux Drive</td>
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<td>Aux CS rear</td>
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</table>

A 8. Bearing Data Sheet
APPENDIX B

Software Program Screens for Gear Tooth Action
Prediction of Shaft Vibration Modes in a Geared Transmission System

Select Model #1
- Start 100 RPM
- End 3000 RPM
- Start 100 Torque (lb-ft)
- End 1500 Torque (lb-ft)

Diagnostics
- System Frequencies
- Gear Tooth Action

Select Model #2
- Start 300 RPM
- End 2500 RPM
- Start 100 Torque (lb-ft)
- End 1500 Torque (lb-ft)

Process Model #1
Process Model #2

B 1. Front Panel to Select Transmission Models

Model-1
- 17th Gear (0.86:1)

Model-2
- 3rd Gear (4.88:1)

Test Conditions
- 1500 RPM
- 1200 Torque

Calculate

Sliding Velocity
Hertzian Stress
Slip Ratio
PV Factor

B 2. Screen to Select Gear Position and Operating Conditions
B 3. Plot of Gear Tooth Sliding Velocities for all Loaded Gear Sets

B 4. Plot of Slip Ratios for all Loaded Gear Sets
B 5. Plot of Hertzian Contact Stress for all Loaded Gear Sets

B 6. Plot of PV Factor (pressure * velocity) for all Loaded Gear Sets
APPENDIX C

Software Program Screens for Noise and Vibration Diagnostics
Prediction of Shaft Vibration Modes in a Geared Transmission System

Select Model #1

Diagnostics
System Frequencies
Gear Tooth Action

Modeling
Forces and Modes

Select Model #2

Create or Edit a Model

C 1. Front Panel to Select Transmission Models and Operating Conditions

C 2. Plot of Pitch Line Velocities of all Loaded Gear Sets
C 3. Plot of Shaft Frequencies of all Transmission Shafts

C 4. Plot of Gear Mesh Frequencies for all Loaded Gear Sets
C 5. Plot of Hunting Tooth Frequencies of all Loaded Gear Sets

C 6. Plot of Assembly Phase Frequencies of all Loaded Gear Sets
C 7. Plot of Selected Bearing Frequencies for all Transmission Shafts
APPENDIX D

Software Program Screens for Vibration Mode Prediction
D 1. Front Panel to Select Transmission Model and Operating Conditions

D 2. Main Screen to Select Gear Position and Plot Parameters
D 3. Plot of Tangential Gear Mesh Forces for all Loaded Gear Sets

D 4. Plot of Radial (Separation) Gear Mesh Forces for all Loaded Gear Sets
D 5. Plot of Axial Gear Mesh Forces for all Loaded Gear Sets

D 6. Plot of Resultant Gear Mesh Forces for all Loaded Gear Sets
D 7. Plot of Bearing Forces for each Bearing on the Selected Shaft

D 8. Shaft Torques for each Shaft in the Transmission
D 9. Typical Lateral Mode Shape Plot with Frequencies

D 10. Typical Torsional Mode Shape Plot with Frequencies
APPENDIX E

Transmission Model #1: Noise and Vibration Data
E 1. Run # 27168: Overall Sound Pressure Level

E 2. Run # 27168: RPM Spectral Map (Waterfall Plot)
E 3. Run # 27168: Discrete Speed Spectrum at 535 RPM

E 4. Run # 27168: Discrete Speed Spectrum at 1285 RPM
E 5. Run #27168: Discrete Speed Spectrum at 2035 RPM

E 6. Run #27168: Vibration Data at 1285 RPM
E 7. Run # 27168: Time History of Noise and Vibration Data

E 8. Run # 27168: Frequency Spectrums of Noise and Vibration Data
E 9. Run # 27168: Lateral Mode Shapes for Free, Unloaded Condition

E 10. Run # 27168: Lateral Mode Shapes: Load = 1000 lb-ft
E 11. Run # 27168: Torsional Mode Shapes
E 12. Run # 27174: Overall Sound Pressure Level

E 13. Run # 27174: RPM Spectral Map (Waterfall Plot)
E 14. Run # 27174: Discrete Speed Spectrum at 665 RPM

E 15. Run # 27174: Discrete Speed Spectrum at 935 RPM
E 16. Run # 27174: Discrete Speed Spectrum at 1330 RPM

E 17. Run # 27174: Discrete Speed Spectrum at 2035 RPM
Noise and Vibration Peaks at Shaft Resonances.

E 18. Run # 27174: Time History Plot of all Noise and Vibration Data

E 19. Run # 27174: Frequency Spectrum of Noise and Vibration Data
E 20. Run # 27174: Lateral Mode Shapes in the Free, Unloaded Condition

E 21. Run # 27174: Lateral Mode Shapes: Load = 1000 lb-ft
E 22. Run # 27174: Torsional Mode Shapes
APPENDIX F

Transmission Model #2: Noise Data
F 1. Run # 27933: Overall Sound Pressure Level

F 2. Run # 27933: RPM Spectral Map (Waterfall Plot)
F 3. Run # 27933: Discrete Speed Spectrum at 590 RPM

F 4. Run # 27933: Discrete Speed Spectrum at 1010 RPM
F 5. Run # 27933: Discrete Speed Spectrum at 1920 RPM
F 6. Lateral Mode Shapes in the Free, Unloaded Condition

F 7. Lateral Mode Shapes: Load = 500 lb-ft
F 8. Torsional Modes Shapes