Parametric Study of Gear Rattle and the Effect of Flexible Enclosures on Gearbox Vibratory Responses

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PARAMETRIC STUDY OF GEAR RATTLE AND THE EFFECT OF FLEXIBLE ENCLOSURES ON GEARBOX VIBRATORY RESPONSES

by

Joshuah Thomas Racine

A Thesis
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Advisor: Judah Ari-Gur, D.Sc.

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Joshuah Thomas Racine
Gearbox noise emitted from vehicles has become of particular importance to manufacturers due to the significant reduction of engine and masking noises in most modern designs. The reduction of gearbox rattle was the subject of this research using Romax Designer. Both drive and neutral loading conditions were considered for a parametric study of the influence of altering input torque fluctuation, backlash between gears, component rigidities and driveline inertia. Next, an external enclosure was added, where its effect was studied by altering the wall thickness, material properties and fixed boundary positions. Finally, results obtained from Romax Designer for a physical gearbox casing were verified by comparison with experimental results and finite element analysis. The casing was dimensioned using a white light scanner, and tested for its modal characteristics with accelerometer readings taken during hammer tests. Furthermore, gearbox rattle analysis for the three computer simulated models highlighted the importance of driveline resonances in determining increases in drive rattle. Neutral loading conditions, on the other hand, were found to be sensitive to torque fluctuations and driveline inertias. Analysis of the gearboxes with an external casing exhibited significant variation in rattle noise with a sufficiently flexible enclosure. This study provides an outline for designing a complete gearbox for reductions in rattle.
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CHAPTER I

INTRODUCTION

The reduction of gear rattle noise in modern vehicular designs is increasingly important in meeting customer expectations of acoustic comfort. Gear rattle noise is characterized as noise generated by meshed gear teeth that are rotating and that do not transmit power. Due to the perceived link between quiet operation and mechanical quality, emphasis must be placed on defining methods to reduce the magnitude of rattle noise through pre- and post-production modeling and analysis techniques. The focus of this research is to outline the parameters associated with gear rattle, and suggest methods for reducing its prevalence in both theoretical and physical gearbox models.

An investigation has been conducted using three simplified, two-speed gearboxes in order to isolate and study the effects of individual parameters. These parameters include input torque fluctuation magnitude, backlash, driveline inertias, and component rigidities. Previous studies related to the effect of these parameters on neutral rattle noise are extensive, but often neglect the addition of an external gearbox enclosure. In addition, previous literature related to gearbox rattle noise is focused primarily on neutral and coast rattle conditions, rather than drive rattle. The primary software package used for analysis was Romax Designer. Romax Designer allows for the addition of the gearbox housing to the analysis of the dynamic model. The inclusion of the external casing was paramount in obtaining results which are realistic, taking into account the path of vibration from the unladen gear sets to the mounting locations of the gearbox structure. Furthermore,
Romax Designer was used to investigate the nature of drive rattle and neutral rattle for individual gear pair meshes for each of the three aforementioned two-speed gearboxes.

The work presented herein aims to outline a process through which an individual can analyze existing and computer generated gearbox structures for the onset of rattle. A literature survey regarding the parameters related to rattle noise and the numerical techniques used to measure its magnitude is given as a basis for comparison. Efforts were made to validate the dynamic analysis results of Romax Designer, and use the software to emphasize the importance of design complexity.
CHAPTER II

LITERATURE SURVEY OF GEAR RATTLE

Overview

Gear rattle noise is defined as vibrations created from unloaded gears with backlash between meshed teeth (Singh et al. 1989). Before conducting a study on rattle noise in geared systems, it is first appropriate to become acquainted with the parameters linked to the phenomenon. In addition, it is necessary to review the research conducted on gear rattle noise to understand the evolution of experimental and theoretical techniques to measure and define gearbox vibratory responses. This section presents the parameters linked to gear rattle noise along with select research projects, whose conclusions are useful in understanding results obtained from computer simulated and physical gearbox models.

Key Parameters

Many parameters are known to contribute significantly to the advent of rattle noise in unloaded gear meshes. The first of these parameters is backlash, which is characterized by the difference between the width of a gear tooth and the spacing for the gearset. Drag torque is another root cause of gear rattle, and is defined as viscous resistance from lubricants or friction between rotating parts (Fujimoto et al. 1987). One of the largest contributing parameters to the creation of gear rattle noise is the clutch stiffness within a gearbox. In fact, early studies of gear rattle found that optimizing the torsional characteristics of a clutch provided a favorable means to reduce gearbox noise.
(Sakai et al. 1981). Another parameter found to be related to rattle noise in early research of gearbox vibratory responses was the distribution of inertia throughout the transmission. Sakai et al. (1981) found that the inertia of the driven wheel, related to the drag torque of the corresponding component, could be used as a criterion for the initiation of gear rattle. Finally, the transfer of vibration from meshed gear teeth to an observer is an important characteristic in defining the nature of rattle noise for a specified mechanism. In sum, the aforementioned parameters have been studied extensively, being carefully defined and related to the magnitude and characteristics of rattle noise responses from gearboxes.

**Backlash**

Additional clearance between the driving gear tooth and the driven gear for an unloaded meshed pair results in single-sided or double-sided impacts caused by torque or acceleration fluctuations due to the firing configuration of the associated power train. Research into reducing this effect by Seaman et al. (1984) and Rust and Brandl (1990), has shown that reducing backlash between gear teeth, without eliminating it, has a very small impact on the magnitude of gear rattle given current design practices. Both sources maintain that the elimination of backlash between meshed gear teeth would remove gear rattle for the unloaded pair based on equations of motion formulated to analyze rattle phenomena. However, Rust and Brandl (1990) add that the elimination of backlash between meshed gear teeth is, of course, impractical without the installation of additional components, which may reduce the reliability of the geared mechanism and offset the gains made by reducing gearbox noise.
A representation of the contact phases for backlash is shown in Figure 1. Here, the total backlash is represented as the coefficient $b$. A negative value for the coefficient of backlash indicates that there is driven side contact, while the total backlash would be taken as an absolute value. As noted by Trochon (1997) and Wang et al. (2001), single-sided impacts are typically associated with the driving gear tooth profile impacting on the driven side of the driven gear, or within the region of positive backlash. Additionally, double-sided impacts are defined as the "bouncing" of a gear tooth between the positive and negative regimes of backlash. Typically, a gear will be observed impacting multiple times within one backlash regime before converting to another. A final form of backlash is defined by the aforementioned authors as "erratic" impacts between meshed gears, where teeth show chaotic motion. This form of backlash is most commonly related to large magnitudes of gearbox noise as it indicates rapid penetration between the positive and negative backlash regimes.

Figure 1 Backlash regime for contact between meshed gear teeth
Singh et al. (1989) and Padmanabhan et al. (1995a) also make another important assertion related to the non-linearity of backlash. When constructing numerical models, the non-linearity created by simulated backlash results in numerical stiffness due to small time intervals between changes in impact regimes. As such, Padmanabhan et al. (1995a) sought solutions to avoid the numerical stiffness associated with gear rattle problems through various numerical approaches such as the Runge-Kutta method. Furthermore, it is necessary in setting up equations of motion related to gear rattle to properly represent the non-linearity and regimes related to backlash. Using Newton’s law of impact, as described by Pfeiffer (1996), the impulsive nature of backlash is considered time variant as a result of the errors related to manufacturing gear teeth. Singh et al. (1989) and Comparin and Singh (1990) include backlash within their model as clearances between the unloaded gear pairs, and between the clutch hub and driving gear. However, it is also necessary to understand that impacts created within the backlash regimes will influence the remaining contacts within the gear train (Pfeiffer, 1996). Needless to say, the complexities associated with backlash in gear rattle numerical simulations are important for convergence.

**Drag Torque**

The two most common contributing factors to drag torque for a gear pair were defined by Rust and Brandl (1990) as being the drag on the bearing of the unloaded gear and friction from the gearbox lubricant. In order to develop realistic equations of motion for gear rattle simulation, the friction from the gearbox lubricant should be considered a function of viscosity, oil depth and splash effects. The viscosity is mainly influenced by
the bulk temperature of the gearbox, which causes rattle to increase in conjunction with rising temperature (Seaman et al., 1984) and lower viscosity.

Studies conducted by Fujimoto et al. (1987, 2001, 2003) analyzed the effect of temperature on gear rattle extensively. In addition, studies by Sakai et al. (1981) and Singh (1989) developed models with a varying temperature to observe the effect on gear rattle noise. Early assumptions, based on a study by Sakai et al. (1981) were that noise levels in gearboxes are likely to increase with a rising bulk temperature. As a corollary, increased drag torque was assumed to provide drastic reduction or loss of gear rattle noise at a certain threshold. A trivial point, noted by Pfeiffer (1996), states that gear rattle would be removed from geared systems upon reaching the freezing temperature of the gearbox lubricant. These assumptions are used in analytical simulation of gear rattle by Sakai et al. (1981) and Seaman et al. (1984), which were eventually expanded upon by Trochron and Singh (1998). Trochron and Singh (1998) studied the phenomena using a simulated two-stage clutch model, where increased drag torque caused high angular displacements and torques. This forced the operating point of the geared system to a second stage, indicating low lubricant temperatures could be associated with double-sided impacts. Perhaps the most irritating phenomenon described by Trochron and Singh (1998) was the shift between low and high temperatures causing noise to rapidly increase and decrease as the dual-stage clutch transitioned between each stage.

The next, unfortunate side effect of increasing drag torque as a means to reduce gear rattle is the effect it has on the overall efficiency of geared systems. Kim and Singh (2001) note that the most practical transmissions attempt to reduce or minimize drag torque in order to increase the overall efficiency of the drive train. The foundation of the
model proposed by Kim and Singh (2001) is based on three cases for the unladen gear pair. The first case assumes that the driving gears rotated faster than the free-wheeling gear, causing moments of lost contact and drag pulling on the unladen gear. Next, the unladen gear rotated faster than the driving gears, causing a “push” on the unladen gear which again initiates impacts. Finally, the driving gear can rotate at the same speed as the driven gear, indicating no drag torque effects on the wheel. This sequence was used to show that increased drag torque on the driven wheel can be used as a method to reduce gear rattle, and single-sided impacts.

The next step in investigating drag torque on automotive transmission rattle was to allow for vibratory drag. Kim and Singh (2001) studied these effects, ultimately determining that oil sealed bearings were an effective measure in reducing gear rattle, while being highly sensitive to changes in temperature. The most recent studies in controlling drag torque by Fujimoto and Kizuka (2001 and 2003), assumed an optimum range of oil temperatures were effective in minimizing gearbox noise. Overall, it should be assumed that an optimal range of temperatures, related to the viscosity and nature of “push” and “pull” on the shaft bearings and gears minimizes gear rattle noise. In addition, drag torques dragging each gear in opposite directions was not a contributing factor to the attenuated damping coefficients for vibration analysis (Fujimoto and Kizuka, 2003).

**Clutch Stiffness**

Early studies on rattle noise focused attention on the effect of clutch stiffness on the vibratory response of automotive transmissions. In particular, Fudala and Engle (1987), Seaman et al. (1984) and Sakai et al. (1981) found that optimization of the clutch stiffness, by means of increasing the mass moment of inertia of the clutch, was an
effective strategy in reducing rattle noise in manual transmissions. Additionally, Fujimoto et al. (1987) and Ohnuma et al. (1985) noted a change in the structure of rattle due to a phenomenon which occurred when transferring between the stages of a dual-stage clutch. This occurrence is known as the "jump phenomenon" or a large increase in rattle noise during the transition between stages of a multi-stage clutch, and has been the focus of subsequent papers including research conducted by Padmanabhan and Singh (1993). They found that dry friction or tri-stage clutches were superior to their counterparts since the jump phenomenon had a less prevalent effect on the amount of rattle noise emanating from a gearbox. Furthermore, Trochon (1997) offered a technique for measuring the stages of the clutch by use of transition angles. The transition angles are associated with driven and idling conditions, where the first stage is typically optimized by having a relatively small spring coefficient (Fujimoto et al., 1987). It should be noted however, that large variation in spring constants between stages may result in unfavorable rattle noise characteristics. Fudala and Engle (1987) supplied evidence for this assumption, where increased damping and variation in spring rates for their model with a dual stage clutch caused resonance when torsional vibrations were found to be within the region of the natural frequencies of the drive line. Singh et al. (1989) also provided a method for optimization through the use of various analytical models, which altered the hysteresis and spring rates of dual-stage clutches, after first acquiring results related to the dynamic response of the system. This is most suitable, as the dynamic response of each system in question will have a large impact on the nature of rattle noise and jump when related to clutch stiffness.
Distribution of Inertia

The distribution of inertia, mainly in relation to the driven and driving gear locations within transmissions, are another key parameter when defining rattle noise behavior. Sakai et al. (1981), Seaman et al. (1984) and Rust and Brandl (1990) studied the relationship between the driven wheel inertia and the drag torque of the corresponding gear as a criterion for the onset of rattle noise. Sakai et al. (1981) first provided the relationship, which was then used by Seaman et al. (1984) in redistributing inertias within a manual transmission to generate theoretical results. It was found that reducing the inertia of the driven wheel provided favorable results, in that this strategy required less drag torque. Pfeiffer (1996) verified this in his review of rattle literature by proving through experimentation that increased inertia of the driven wheel typically causes an increase in the amount of rattle noise, albeit large wheels are ill-equipped to drive small gears (Karagiannis and Pfeiffer, 1991). It has also been found that increased inertia in dual and single-mass flywheels causes significant reductions in rattle by Comparin and Singh (1990). However, as described earlier in this paper and by Croker et al. (1990), common engineering design techniques require reduced mass in the flywheels of transmissions in order to reduce weight and increase fuel efficiency. This effect is further compounded by the firing sequence of four cylinder engines (deemed more fuel efficient than six and eight-cylinder counterparts), which have larger torque fluctuations and thus increased rattle noise (Fudala and Engle, 1987).

Vibration Transfer

Excitations caused by meshed gear teeth generate sounds which are transferred to surrounding structures through the transfer of vibration. The transfer of vibration related
to the rattle phenomenon is of specific interest in reducing the sound pressure level (SPL) emitted from a gearbox enclosure as it is observed by the customer. The present study will focus on the path of vibration from the internal components of a gearbox to the mounts attached to the gearbox housing. Kostic and Ognjanovic (2007) defined this transfer between the elastic structures of a transmission (gears, shafts, housing, bearings, etc.) as a means to absorb the energy causing disturbances within the system. A system absorbing energy is most efficient when it insulates as many disturbances as possible which are found to excite the natural frequencies of the mechanism. Kostic and Ognjanovic’s (2007) and Sellgren and Akerblom’s (2005) models, unlike most previous work, took into account the effect of a gearbox enclosure rigidity and stiffness. Specifically, Sellgren and Akerblom (2005) stated that an enclosure which is not sufficiently rigid may have the effect of increasing transmission error between meshed gear teeth. The effect of this increase was an intensification of the disturbances between meshed teeth and larger magnitudes of gearbox noise. When reviewing each model it is apparent that the transfer of vibration to the structure of attenuation for geared mechanisms takes into account multiple characteristics related to the nature of gearbox rattle noise.

Other research into the transfer of vibration within gearbox enclosures attempted to control sounds generated within a mechanism. Much like the isolating characteristics of a clutch (Padmanabhan and Singh, 1993), the gearbox housing was used by Guan et al. (2005) as a means to dampen the vibrations emitted from their model by placing an actuating force on the input shaft of a two-gear system. This method was used to isolate the frequencies which could potentially excite the resonant frequencies of the enclosure.
Rust and Brandl (1990) also successfully controlled gearbox noise by stiffening the enclosure using ribs, and damping the noise using a constraining layer around the housing. Finally, Tangasawi et al. (2007) took into account the transfer of vibration to properly analyze gearbox noise during experimentation. In their model, they mounted accelerometers along the path of vibration in order to gain more accurate results from noise, vibration and harshness analysis (NVH) of an automotive manual transmission. Through a review of literature related to the transfer of vibration in gearbox noise analysis, the importance of this parameter can be observed to be an effective means to reducing gear rattle, controlling the dynamic response of a mechanism and properly obtaining experimental results.

Past Rattle Investigations

Numerous approaches to developing equations of motion related to the parameters of gear rattle noise have been proposed. Typically, these models are non-linear and aim to find a suitable rattle criterion that can be utilized to define the dynamics of complex drive trains. Most notably, models accurately describing the system dynamics of a gearbox have been created by lumping inertias within theoretical models to reduce the order of degrees of freedom. Additionally, most gear rattle models are evaluated using only torsional degrees of freedom, neglecting axial and lateral motion of gears, shafts, synchronizers, bearings and the gearbox enclosure. This section will provide a detailed explanation of the methods used to generate gear rattle criterion as well as evaluation methods behind analysis of gear rattle parameters.
**Rattle Criterion**

Sakai et al. (1981) first proposed a rattle criterion which would be the subject of multiple papers on gear rattle. Through the use of both experimental and theoretical models for the analysis of rattle noise generated by an automotive manual transmission, they were able to perform parametric studies associated with the clutch stiffness of the transmission. The model generated assumed a lumped parameter torsional system, which took into account viscous damping and was solved through numerical integration of the linear differential equations. The results showed that optimization of the clutch hysteresis provided a significant reduction in gear rattle noise, which was validated using information obtained from accelerometers on the housing of the transmission being modeled. The criterion for rattle noise in this research stated that noise will be generated if the drag torque on the driven gear is exceeded by the inertial torque on the driven gear.

Research presented by Seaman et al. (1984) used an approach for reducing rattle using the distribution of inertia within an automotive transmission. This model was based on the criterion generated by Sakai et al. (1981), where it was postulated that rattle is a function of drag torque, angular acceleration and inertia for a driven gear. Thus, numerical calculations based on parametric studies of gear rattle were used to redistribute the inertia of the components within a transmission, which were later verified through experimentation. The link between the drag torque and inertia torque of the gear was also used to reduce the “shift effort” for the manual transmission used during experimentation.

Gear rattle was studied by Ohnuma et al. (1985) using differential equations for a four degree of freedom model shown in Figure 2. Numerical integration of the model (with non-linearity generated from the clutch plate torsional characteristics, backlash between gear meshes and clutch hub-spline backlash) was conducted using the Runge-
Kutta-Gill method. From numerical integration, a rattle criterion was defined based on the amplitude of the collision force between meshed gears and the number of impacts during the time it took the driving shaft to fully rotate twice. This criterion showed good correlation with experimental results, based on optimizing the clutch hysteresis using methods previously defined by Sakai et al. (1981). Jump phenomenon is also mentioned by Ohnuma et al. (1985), where changes between two stages of a multistage clutch caused large increases in rattle noise. Overall, the clutch was redesigned using the methods outlined by Sakai et al. (1981) and the authors’ rattle criterion so that it acted as a vibration isolator as opposed to an exciter.

![Figure 2: 4-DOF Neutral Rattle Model](image)

**Symbols**
- $I$: Inertia
- $\theta$: Angular displacement
- $\dot{\theta}$: Angular velocity
- $\ddot{\theta}$: Angular acceleration
- $k$: Spring constant
- $c$: Damping coefficient
- $T$: Torque

Experimental results obtained by Rust and Brandl (1990) both summarized and successfully implemented a parametric design of a transmission system. Using the rattle criterion from Seaman et al. (1984), and an understanding of the effect of backlash, drag torque, angular acceleration, the path of vibration and the distribution of inertia, the authors were able to reduce the level of drive rattle noise for a five speed transmission with a four cylinder internal combustion power train. Rust and Brandl (1990) took into account the torsional resonance of the system, given a path of vibration measured using
laser vibrometers to evaluate vibrations throughout the transmission. Experimental results showed that increasing the mass of the flywheel, adding ribs to the gearbox enclosure and having a lighter free-running gear at a prescribed location was effective in significantly reducing rattle noise (to a non-audible level compared to other sources of noise).

Jump Phenomenon

Using a four-degree of freedom model similar to that studied by Ohnuma et al. (1985), Fujimoto et al. (1987) evaluated the effects of idling rattle during the transition between the first and second stage of a two-stage clutch. The differences between the model proposed by Ohnuma et al. (1985) and the current model, were the addition of damping for the input shaft, and a non-constant drag torque value based on the engine rotational velocity as shown in Figure 3. Taking into account the jump phenomenon, Fujimoto et al. (1987) found that decreasing the spring constant for the first stage reduced the rotational velocity at which jump occurs. In addition, the reduction of the second stage spring constant also reduced rattle noise, providing a basis for the reduction of gearbox noise and jump based on an optimal design for the clutch characteristics (hysteresis and stiffness). The results presented in this paper were based on numerical integration, with no objective explanation given for optimally designing the clutch characteristics of the model.
While research into non-linear vibration analysis of spur and helical gears was extensively studied by many authors, Cheng and Lim (2003) presented a study of backlash induced vibrations on a theoretical hypoid gear pair. The authors' model was a nonlinear time varying (NLTV) lumped parameter system accounting for both the translational and torsional motion of the hypoid gear pair. The effect of backlash, friction coefficients, line of action and time dependent mesh positions were evaluated as in the model. In addition, transmission error was applied to the hypoid gear set to allow for correlation with gear whine analysis. The differential equations of motion were integrated using a 5/6th order Runge-Kutta algorithm, converting from the time to frequency domain using Fast Fourier Transforms. The spectra of FFT were analyzed for various forced responses with varying mean load torques. It was found that varying mesh stiffness during simulations did not cause larger dynamic vibration amplitudes than analysis without variations, even considering increases in transmission error at low load torques. Additionally, the dynamic model formulated by Cheng and Lim (2003) found that low load torques resulted in tooth separation which could be linked to jump phenomenon. While the theoretical model was not related specifically to gear rattle noise, the authors presented a basis for analyzing non-parallel shaft gearbox casings with backlash, accounting for jump phenomenon.
**Rattle Index**

Singh et al. (1989) provided an analytical study of neutral gear rattle for a four degree of freedom lumped parameter model (Figure 4), along with a rattle index in their studies. The rattle index provided by this paper superseded the criterion used by Sakai et al. (1981) and Seaman et al. (1984), which measured increases in rattle noise based on the drag and inertial torque of the output or “driven” gear. However, it was found that the level of rattle noise correlates more appropriately with the angular acceleration of the input gear. Equation 1 shows the basis for the criterion found by Singh et al. (1989), where the rattle level time history is given using subscripts $I$ and $O$ for the input and output gears respectively. The parameters in Equation 1 are $I$, $R$, $T$ and $\dot{\theta}$ which indicate the mass moment of inertia, pitch radius, drag torque and angular acceleration respectively. Rattle will occur given that $\beta(t) \leq -1$. The root mean square value of $\beta(t)$ was then calculated, since it was reasonable to assume the input torque and angular displacements would not be constant over a sampled time interval. Thus, a root mean square value was calculated to allow for proper comparison between gear pairs. Finally, Singh et al. (1989) provided an explanation for numerical errors for non-linear models (based on numerical “stiffness”), a simplified physical model, and an outline for the balance and transition points for clutches for neutral gear rattle problems.

$$\beta(t) \equiv \left( I_o R_I / T_{d0} R_O \right) \dot{\theta}_I(t)$$ (1)
Comparin and Singh (1990) built upon the aforementioned models by Sakai et al. (1981), Seaman et al. (1985) and Fujimoto et al. (1987) by providing an in depth non-linear system of equations for neutral gear rattle studies. Each model used three non-linear stiffness elements, including the multi-stage clutch, and backlash in the clutch spline and gear pair as shown in Figure 5. Additionally, models by Ohnuma et al. (1985), Sakai et al. (1981) and Fujimoto et al. (1987) were linearized neglecting impacts between meshed gear pairs. Overall, the numerical models generated in this paper were found to be suitable, while adding single-sided, double-sided and non-impact regimes for backlash, and non-sinusoidal input torque fluctuations. The mathematical model presented by Comparin and Singh was validated through consistency with digital and analog simulations provided by Fujimoto et al. (1987), Ohnuma et al. (1985) and Sakai et al. (1981), and provided a basis for analytically solving for previously observed non-linear behavior in transmissions.
A study by Padmanabhan and Singh (1993) used a generic three degree of freedom model, with all relevant non-linearities (Figure 1a, p. 610 from source). This research attempted to determine whether a non-linear or linear clutch is superior for gear rattle reduction, and what effects the shape of impact acceleration had on the perception of rattle noise. Numerical integration for the system of equations was performed using the 4/5\textsuperscript{th} order Runge-Kutta method with variable time step sizes. Padmanabhan and Singh (1993) used both filtered and unfiltered frequency responses, and provided evidence that three stage and dry friction clutches were superior to previously evaluated two stage models with “moderate” spring constants for the first stage. Additionally, the procedures outlined by the authors of this research were helpful in diagnostic optimization of clutch designs when taking into account impact acceleration shapes.

A computational tool for comprehensively evaluating gear rattle was created by Wang et al. (2001), using MATLAB and information obtained for a five speed manual transmission. The computational solver program code was created for a model with only torsional degrees of freedom and non-linearities derived for a multi-stage dry friction clutch with a torsional spring rate, alongside damping and pre-loads on the shaft bearings.
and flywheel springs. Also, the model lumped inertias for the unloaded gears, as they were assumed to be the primary sources of rattle noise due to impacts. A rattle criterion was based on a method created by Singh et al. (1989), measuring the relative displacement of the unloaded meshed gears along a line of action, with input torque fluctuations modeled using sinusoidal Fourier series with a predefined amplitude and phase angle. Finally, the rattle index described by Singh et al. (1989) for the RMS acceleration between the rattling pair was also implemented into the computational solver. Results derived from the computational solver indicated that double sided and irregular impacts are related to high magnitudes of rattle noise, and that drive rattle gives way to a larger index value. Future work for the solver aimed to add the effect of synchronizers, and to experimentally validate the current results. The authors also found that differential equations for the clutch non-linearity made the model prone to “stiff” equations of motion as outlined by Padmanabhan and Singh (1993), which necessitated the use of a variable time step Runga-Kutta algorithm to obtain an accurate solution.

Impact Theory

Two mathematical models based on impact theory for a multibody system were presented by Karagiannis and Pfeiffer (1991) based on a patching method and discrete mappings of a prescribed rattling state. The patching method presented involved combining free flight and impact phases, whereas the discrete mapping method described the rattling state by defining impacts based on the preceding impact within a sequence. The discrete mapping method was viewed as being useful for practical applications attempting to estimate parametric influences on drive trains. Experimental results utilized the discrete mapping method for a one-stage model shown in Figure 6, giving good
agreement with theoretical results. Additionally, several real configurations were said to have utilized the calculations to successfully reduce rattle noise for automobiles. Assumptions for the mathematical model included the removal of input excitation fluctuations (only drag torque terms appear for free-flight phase of backlash), and steady-state conditions. The two-stage model formulated for analysis also indicated that rattling noise is not significantly affected by the vibrations of members near the unloaded meshed gear pair. Finally, the theoretical results for the one-stage model were also useful in verifying assumptions from Sakai et al. (1981) and Seaman et al. (1984), since large gears were found to be poorly suited to drive small gears and the magnitude of rattle noise increased in conjunction with input torque excitation amplitudes.

![Figure 6 1-Stage Rattle Model (Karagiannis and Pfeiffer, 1991, Fig. 2)](image)

Figure 6 1-Stage Rattle Model (Karagiannis and Pfeiffer, 1991, Fig. 2)

In order to outline the methods and parameters associated with gear rattle problems, Pfeiffer (1996) reviewed previous studies into the phenomenon. This research emphasized the amplitude of angular vibrations of the drive shaft at the entry of gearbox enclosures, using Newton’s impact law to explain the free-flight of meshed gear teeth during backlash. However, the review does not outline noise loss from the meshed gear set due to the transmission housing, since it was outside of the scope of research.
conducted prior to the review. Additionally, the author outlined a model for one-sided impacts in transmissions, where the driven wheel does not react to the driving gear. Pfeiffer (1996) emphasized the need for modeling the entire drive line, making sure to correctly model gear ratios, geometries, masses, material properties and manufacturing tolerances for future research. Using a simple two shaft gearbox, previous assumptions based on parametric influence on rattle noise were verified, including relationships between the inertia of the driven wheel, backlash, increased gear ratios and viscous effects of lubricants.

A theoretical model created by Pfeiffer and Glocker (2000) attempted to solve problems related to unilateral contact for multiple cases in mechanical engineering. These cases included the landing impact of an airplane, turbine blade damping, assembly processes and rattling in meshed gears. The problem of rattling between meshed gear pairs was considered a function of the drive shaft torsional vibrations, which were considered harmonic. An example gearbox shown in Figure 7, previously created by

Figure 7 5 Stage Non-Linear Model with Reverse (Pfeiffer, 1996, Fig. 1)
Karagiannis and Pfiefer (1991), was used to illustrate the problem of rattling noise where the relative measure of noise generation was calculated by summing the impact pulses given a noise intensity function. The theory used to solve for rattle noise was evaluated using a definite mass matrix with relative position and velocity measurements of the bodies in contact. However, the theory did not take into account impacts, which the authors represented by rewriting the equations of motion and solving at the points of impact. This iterative process was said to provide promising results, albeit unable to converge upon accurate measurements given previous experimental and theoretical models. Additionally, only linearization, augmented Lagrangian methods and variable time step algorithms proved sufficient in generating iterative results for the equations of motion.

In a study by Mason et al. (2007), the authors attempted to compute gear rattle magnitudes in a vacuum pump for forcing frequencies of the input shaft rotational velocity. The study focused on forcing mechanisms within the vacuum pump, including the mounting of gears without a centered axis of rotation (eccentricity) and torque fluctuations. The torque in the system was assumed to vary in a sinusoidal fashion from a mean torque value. The equations of motion for gear rattle analysis were based on Newton’s second law of motion converted into angular coordinates (Pfeiffer, 1996), with a rattle criterion based on the normal backlash values for the geared components. As an effect, the backlash of the system introduced non-linearities in addition to the input velocity excitations. The gear pair mesh was then modeled based on Hooke’s law, and analyzed by bounding the parameters which cause rattle in gears, or teeth to lose contact when meshed. This was done by using eccentricity functions, formulated to measure the
likelihood of rattle, which gave stable solutions for multiple cases. The author expressed interest in finding a means to eliminate all rattle conditions from the system by using periodic functions created for free-flight, positive backlash and negative backlash motions. However, the periodic functions were unsolvable using all three conditions, thus future work was proposed to remove the “noisy” solutions from the system while increasing the number of DOF. None of the results obtained in this paper are confirmed experimentally, while the author is said to complete experimentation to correlate his results with a physical model.

**Numerical Stiffness**

Padmanabhan et al. (1995b) set out to resolve the numerical “stiffness” reported by Singh et al. (1989), Comparin and Singh (1990), Croker et al. (1990), and Padmanabhan and Singh (1993) due to small time steps for periodic impacts of meshed gear teeth. It was found that periodic occurrence of impacts resulted in high frequency transient responses, causing the stiffness matrix to have eigenvalues with large orders of magnitude between the smallest and largest values. Thus, Padmanabhan et al. (1995b) set out to non-dimensionalize the equations of motion related to rattle problems described by the aforementioned authors. The pre-processing suggested by the authors included non-dimensionalizing and coupling the equations of motion, linearizing around an operating point, and reducing the order of the system to remove ill conditioning problems inherent to numerical integration methods. After the pre-processing stage, an appropriate numerical technique was chosen, and used to evaluate the model. Finally, a post processing phase was suggested, using the indices outlined by Padhmanabhan et al. (1993) to ensure vibration isolation from the clutch.
In order to define the state of the art, Padmanabhan et al. (1995a) reviewed literature related to the analysis of gear rattle, and outlined a procedure for developing robust theoretical models for gearbox noise. The authors acknowledged that most work related to rattle noise dealt with neutral gear rattle, with few studies related to coast and drive rattle problems. More specifically, neutral gear rattle problems were primarily concerned with low frequency dynamic excitations, due to torque fluctuations, load reversals and drag torque. Due to the large order of gear rattle problems and given the clearances between each component, numerical integration required the reduction of the order of the model. In addition, as noted by Padmanabhan et al. (1995b), vibro-impact models were prone to numerical stiffness and ill-conditioning creating erroneous analytical data. The authors suggested first formulating a suitable model, selecting an appropriate numerical integration method, and developing performance indices which can be used to evaluate and optimize parameters of interest. Overlapping between each stage of this process was also needed to create an appropriate model, due to difficulties associated with creating non-dimensionalization schemes during pre-processing. To test their process, the three stage method was used for two case studies (reverse idler and clutch-design models) with good results. Additionally, performance indices for evaluating noise control and perception of gear rattle were created during analysis of the clutch-design model. Finally, the authors planned future work to assess the applicability of their methodology to drive and coast rattle problems.

Research conducted by Kim and Singh (2000) concerning gear rattle noise for unloaded gear pairs in the drive rattle mode using a five speed manual transmission V6 turbo diesel engine, used both linear and non-linear analysis of a model defined using two
definitions for degrees of freedom. It was found that higher orders of DOF (10) obtained similar results to a model with less DOF (6), indicating that a reduced order model was capable of effectively representing the physical manual transmission casing through linearization. The reduced order, 6 DOF model made use of non-dimensionalization to reduce the numerical stiffness of the system, and was solved for using a 4/5th order Runge-Kutta algorithm. Theoretical results indicated that a dual-mass flywheel was successful in eliminating impacts in not only the engaged gear pair, but also the unloaded meshed gear pair. However, the dual-mass flywheel introduced low resonant frequencies into the system, due to the small spring rate for the flywheel. The authors suggested that more work was to be done to confirm the accuracy of theoretical results through experimentation, and sought to add oil sealed bearings to a future model in an attempt to control gear rattle noise for low drag torque values.

**Smoothening Functions**

As an extension of the study done by Kim and Singh (2000), Kim and Singh (2001) use the same 6-DOF (Figure 8) and 10-DOF models to compare with experimental results obtained from a light duty truck. The experimental results recorded by the authors plotted time histories of the rotational velocity of the vehicle drive line for multiple loading conditions. As previously recorded by Kim and Singh (2000), the 6-DOF model analytical results deviated only slightly from the 10-DOF model, so it was deemed suitable for non-linear analysis. In order to control error between continuous and discontinuous functions for the clutch, smoothening functions were applied to the hysteresis of the clutch stiffness. Experimental results for the vehicle using oil sealed bearings (nearest to the unloaded gears), showed heavy single sided impacts as the clutch
transitioned to its second stage implying that the component was now acting as a vibration exciter instead of isolator. As a result, oil sealed bearings were found to be detrimental to the vibratory impact response of the higher gears for the experimental model. Future analysis based on adding the vehicle inertias to the drivetrain was suggested to improve prediction methods and results.

In an effort to assess the usefulness of smoothening functions in gear rattle analysis, Crowther et al. (2008) created a computer simulated model using variable step integrations (e.g. Runge-Kutta method). This method was presented as a means to both speed up the solution time for computer simulations and to increase the order of accuracy in solving gear rattle equations of motion. A torsional model of an automotive drive line, with a sinusoidal variance around a mean torque excitation was considered to be sufficient. Using a non-smoothened function and integrating with time steps, it was found that the dynamic response across the gear and spline backlashes exceeded the mean.

**Figure 8 6-DOF Torsional Rattle Model Schematic (Kim and Singh, 2000, Fig. 1)**
torsion associated with the torque input. Thus, impacts were assumed to occur, and the number of time steps within the computational solver was increased, causing divergence between systems with smaller time step tolerances. Using two smoothening functions, correlation between the results was assessed after approximating the absolute value functions and setting parameters to control the extent of smoothening. Using MATLAB, only one case (arc tangent smoothening), provided erroneous results due to over-smoothening of the system response observed by “rounded corners” for the clearance function. The effect of grazing on the normal backlash limits was also found to be problematic for convergence, which necessitated the removal of these regions from the system to obtain a reasonable result. Only some models showed significant improvement, although the authors planned to implement the use of smoothening functions on future models with higher orders of DOF associated with greater numerical stiffness (Padmanabhan and Singh, 1993).

Lubricated Impacts

In order to verify the assumed parametric influences on transmission rattle postulated by Sakai et al. (1981), Seaman et al. (1984), Ohnuma et al. (1985), Fudala and Engle (1987) and Fujimoto et al. (1987), Croker et al. (1990) created an experimental test rig for NVH performance analysis. The test rig model was constructed using an input drive connected to a variable speed DC motor, with a high inertia flywheel (reducing the effect of input torque fluctuations). Using three distinct types of oil over a range of temperatures, viscosity was varied using the three configurations shown in (Croker et al., 1990, Figure 2 p. 123). The experimental results showed a decrease in rattle with proper distribution of inertia, a jump in rattle noise originating due to distinct changes in the
angular acceleration threshold for the clutch housing, and the influence of drag torque on rattle noise (albeit non-linear). The results also showed that periods where teeth became un-meshed did not always ensure audible rattling in unloaded gear pairs.

Using the procedures outlined by Padmanabhan et al. (1995b), Trochon and Singh (1998) evaluated the effect of drag torque on rattle noise for a transmission subjected to cold temperatures. Using a four-degree-of-freedom torsional model, and the 4/5th order Runga-Kutta numerical integration method for the non-dimensionalized equations of motion, analytical results and experimental observations showed significant double-sided impacts for the clutch with very cold lubricants. Assuming the lubricant temperature was close or equal to the temperature outside of the vehicle, the authors provided an explanation based on increases in lubricant temperatures causing pauses in the transitions between the stages of a multi-stage clutch design. As such, objectionable noise was produced, requiring future design studies to provide noise control guidelines for vehicles in low temperature climates.

Fujimoto and Kizuka (2001) offered an improved model for the analysis of gear rattle noise by providing evidence that rattle noise levels obtained during experimentation could be directly correlated with root mean square angular accelerations of the countershaft fluctuation. In addition, the authors found an optimum range of temperatures for transmission lubricants. It was observed that the lubricant temperatures affected the clutch torsional characteristics. As oil temperature was increased, the torque required to transition between stages was decreased for the two-stage clutch, causing a shift into the first stage reducing the overall magnitude of rattle noise. Further increases in temperature caused the motion between gears to become more apt to single-sided or chaotic impacts.
which increased rattle noise. As a result, optimal lubricant temperatures for the experimental transmission were found, and an analytical model was used to confirm the experimental results (Figure 9).

![Graph showing RMS value of C/S angular acceleration (dB) vs. T/M oil temperature (°C)](image)

Figure 9 Simulated and Experimental RMS Countershaft Acceleration (Reprinted with permission from SAE Paper No. 2001-01-1164 © 2001 SAE International, Fujimoto and Kizuka, Fig. 15)

In an extension of the work conducted by Fujimoto and Kizuka (2001), Fujimoto and Kizuka (2003) used the same four-cylinder diesel engine, light-duty truck to measure the vibrations within the backlash regimes where rattle noise was initiated. In order to simplify the experimental model, the authors replaced the idler and reverse gears of the transmission with a representative mass equivalent to the total mass of each component combined as shown in Figure 10. Next, rotary encoders were used to measure the vibratory response at a single location per gear which was then compared to calculated data for the vibrations for each gear mesh pair. Good correlation was observed between the direct measurement of the gear positions and the calculated results, thus allowing the authors to perform accurate experimental measurements. The measurements obtained led them to the conclusion that drag torque should be treated as an offset value for the torque fluctuations within the clutch disc. Additionally, coefficients of attenuation used in the
In order to more aptly explain the influence of oil squeeze between unladen gear teeth pairs Brancati et al. (2005) used a one degree of freedom model to study the effect of lubrication in gearboxes (Figure 11). Models proposed before the authors’ attempt to explain gear rattle noise with dissipative effects to explain damping, in terms of clutch hysteresis or bearing friction. However, dead space between gears was thought to be effected by damping as teeth approached each other with thin layer of lubricant. Thus, contact stiffness was modeled as a function of the location of the driving member. Fast Fourier transforms (FFT’s) of the gear relative motion and acceleration were then used in the analysis of the meshed pairs. Using the index proposed by Singh et al. (1989) for the relative acceleration between driven and driving gears, three cases for lubricant levels were evaluated (full, medium and no oil). The simulated model with no oil overestimated
the level of rattle, while an increase in oil level seemed to decrease the magnitude of rattle noise in the transmission. Additionally, a gearbox full of oil showed little influence of oil viscosity on the level of rattle noise. In sum, oil squeeze effect on rattle noise perception was found to be a function of the lubricant viscosity where the thin layer of film compresses as gear pairs approached each other before impact. In order to reduce rattle noise the authors suggested a mechanism to apply lubricant at unladen gear meshes before contact or additives to increase the adhesive properties of the lubricant. However, an increase in the overall level of lubricant in a gearbox was not suggested, as it would reduce the mechanical efficiency of the geared mechanism.

Due to the scarcity of models investigating the effect of lubricated impacts on gear rattle, Theodossiades et al. (2007) created a study to assess the impact of hydrodynamic lubricant film within meshed gear pairs. The model used for evaluation was a front wheel drive vehicle with the six-speed manual transmission shown in Figure 12. Angular displacements, velocities and accelerations of the input load were added to a computational model based on experimentally obtained time history values. In addition, the hydrodynamic lubricant was assumed since the meshed gear teeth experienced low loads during idle gear rattle. FFT analysis of the respective time histories indicated that
the even engine order frequencies (corresponding to a four-cylinder engine) were dominant in the vibratory response of the system, alongside the torsional natural frequencies and forcing frequencies of the meshed gear teeth. Assuming some of the dominant frequencies from the FFT analysis were from the structural vibratory response, the torsional model was linearized and solved for the normal modes of the system. Conclusions drawn from the given information indicated that the acceleration of the idler gears tended toward a minimum value at optimal temperatures, correlating well with results found by Fujimoto and Kizuka (2001) based on the authors' rattle index. As a result, it was hypothesized that the lubricant in gear teeth impacts acted as non-linear spring dampers, and that the engine order dynamic frequencies dominated the NVH response of the rattle model.

Figure 12 6-Speed Manual Transmission Layout (Courtesy of Elsevier - Theodossiades et al, 2007, Fig. 1)

As an extension of the research conducted by Theodossiades et al. (2007), Tangasawi et al. (2007) expanded upon the methodology of lubricated impacts for idle gear rattle analysis. Specifically, the authors modeled a transmission with 11 DOF, adding the lateral motion of the output shaft gears to the equations of motion from the torsional model presented by Theodossiades et al. (2007). The axial motion of the output shafts supporting the idler gears were neglected, and the input shaft kinematics were
derived from experimental measurements, reducing the order of the non-linear model. An experimental validation model was also used, with a transmission attached to a diesel power train and accelerometers mounted in the path of vibration from the bearings to the mounts of the gearbox enclosure (Figure 13). Information collected from the accelerometers were then compared with the frequency spectrum calculated using the numerical model for the modal frequency of the enclosure. Once the un-damped natural frequencies of the theoretical model were calculated, the lubricant stiffness was added to the linearized model as a Fourier series. Results from the theoretical model indicated that the fluid film damping had little effect on the un-damped natural frequencies. Overall, the numerical results correlated well with the experimental data, while the authors proposed future work to include the effect of the gearbox flexibility to the input shaft lateral motion.

![Image](image.png)

**Figure 13 Accelerometer locations along path of vibration** (Courtesy of Elsevier - Tangasawi et al, 2007, Fig. 4)

**Intuitive Approaches**

A model defined by Fudala and Engle (1987) was presented as a study of coast and drive rattle cases, emphasizing the need to approach rattle problems by evaluating the entire drive train. Using computer simulation and keeping the entire drive line in mind,
they modified the gearbox to move the unloaded meshing frequencies out of the operational range of frequencies for the transmission. Clutch optimization was performed using an approach similar to previous models by Sakai et al. (1981) and Seaman et al. (1984), by altering the component inertia. The meshing frequencies were also moved outside of the operational range of the drive line operational frequencies by changing gear ratios, installing damping springs between the dual-mass flywheel, and considering the firing frequency of the power train or the input load excitation.

Shimizu (1993) provided an approach to evaluate gear rattle for a front-engine, front wheel drive manual transmission automobile with a four cylinder power train. This study was performed to explain why some vehicles with gasoline fueled power trains produced more rattle than similar diesel fueled vehicles, which was contrary to the conventional assumptions for gear rattle problems at the time. Time histories of the rotational accelerations and velocities were taken from two power trains, finding that regions of backlash occurred once per engine rotation, which was followed by a separation between the input shaft rotational velocity and flywheel rotational velocity. This separation was found to cause collisions between the shaft and flywheel, which varied the input velocity on the meshed gear teeth, causing vibratory excitations throughout the drive train. As such, it was determined that off center rotation between the flywheel and clutch contributed significantly to the magnitude of rattle noise for both drive trains. Finally, gear rattle noise was found to be sensitive to angular velocity of the engine, and maximum backlash for the clutch-disc-spline. For every parameter, an increase was correlated to an increase in rattle noise. Shimizu (1993) suggested reducing the distance between the flywheel and input shaft center of rotation shown in Figure 14,
and minimizing backlash for the clutch-disc-spline to reduce rattle noise for similar transmissions.

Figure 14 Center of Rotation FBD (Reprinted with permission from SAE Paper No. 932003 © 1993 SAE International, Shimizu, Fig. 4)

Rivin (2000) presented a study of various backlash eliminating gears as a means to reduce rattle. As noted by Rust and Brandl (1990), components used to eliminate backlash between meshed teeth are prone to decrease the reliability of a mechanism. However, Rivin (2000) studied the use of backlash eliminating gears as well as scissor gears as a means to reduce rattle and provided a simplified model of meshed gear teeth. The simplified model generated by Rivin (2000) assumed that because collisions are defined by short duration pulses with large magnitudes, it is intuitive that increases in the duration between impacts should decrease noise generated during impact. Thus, metal shims were installed at the idle surfaces of the driven gear for a meshed gear pair as a means to reduce rattle noise as shown in Figure 15. A study of the sound pressure levels of the experimentally measured output response of the geared mechanism showed a significant decrease in rattle noise. Future studies would be needed to understand the long term reliability and appropriateness of the proposed design for geared systems with multiple inertias and large input loads or fluctuations.
Derk (2005) provided an intuitive approach to reducing rattle noise for a heavy duty diesel truck operating under normal conditions. This study was conducted on two engine types which were evaluated for compliant methods to reduce rattle noise. Namely, the author aimed to take advantage of a non-linear phenomenon where increased backlash can result in an overall reduction in rattle noise (Figure 16). One transmission had torsional springs installed between the crankshaft and crank gear, and between the cam shaft and remainder of the drive line to isolate rattling for each component. The second transmission could not make use of a torsional spring between the crankshaft and gear, requiring a spring between the idler mesh and crank gear. Evaluation of the sound pressure levels, measured by microphones, and the normalized speed of each component, measured by accelerometers, showed single-sided impacts were removed during normal operating conditions. In addition, vibration isolation between the cam and crankshafts for each transmission reduced fluctuations in the rotational speed of the components and significantly reduced the sound pressure levels for the transmission.
Objective Noise Ratings

In order to remove the need for subjective assessments of drive rattle noise in automobiles, Johnson and Hirami (1991) created an objective system for measuring gearbox noise. The objective rating system was created using three manual transmission passenger vehicles with similar drive lines. As a basis for measurement, angular position sensors of the driveline components were compared with the angular position of the flywheel. Using a similar method to that used by Croker et al. (1990), the Hiblert transform method was applied to measured output responses to demodulate the speed of each unloaded gear and other alternating components, while utilizing low pass filtering to remove the effect of tooth passing frequencies from the angular acceleration measurements. Experimental results obtained by Johnson and Hirami (1991) indicated that rattle is reduced within the rotational cycle as the input rotational velocity is increased, and showed a necessity to define impact cycles which were more realistic than those given by Sakai et al. (1981). Using a newly defined impact cycle (Figure 17), nine stimuli were evaluated for each vehicle, and contour plots from experimental results were

Figure 16 Backlash compliance graph (Courtesy of Elsevier - Derk, 2005, Fig. 1)
plotted against index equations which obtained high levels of correlation between objective and subjective analysis of gear rattle noise.

Figure 17 Elastic Impact Model (Reprinted with permission from SAE Paper No. 911082 © 1991 SAE International, Johnson and Hirami, Fig. 10c)

An objective measurement system created by Heinrichs and Bodden (1999) was used to evaluate a diesel vehicle, which was considered relevant as rattle and knocking in diesel fueled engines are not easily distinguished from each other. A source, system and receiver were set up using excitations from the transmission, structure born vibration path and human perception, respectively. Using stop and go scenarios shown in the top of Figure 18, a transfer function was created as a metric for the perception of rattle noise and the absolute values for frequencies of rattle noise were used to create a Difference Log absolute Spectrum (DLS). Heinrichs and Bodden (1999), found that phase (difference between stop and go scenarios) had no significant effect on the perception of rattle, while the absolute value for the frequencies of rattle vibrations and transfer function were useful in obtaining an objective rating system. The transfer function,
evaluated with an average difference level to the DLS, showed correlation of 0.92 or 92% between experimental results (subjective rating) and the objective rating estimations.

In an extension of the research by Heinrichs and Bodden (1999), Bodden and Heinrichs (1999), provided a further spectral metric for evaluating the time histories of drive rattle conditions. Two phases were defined for this research for stop and go scenarios with engaged and disengaged clutch positions while decelerating (Figure 18). Modulation, specified over a range of time history signals, was used to create a spectrum of noise. Then fast Fourier transforms (FFT) were used to evaluate the absolute value of each frequency for the spectrum. Since the modulation frequencies showed clear peaks at multiples of half of the engine rotating speed, it was found that time histories of the engine pulses could be evaluated against the time history of rattle responses. Polynomial fit curves were then produced, giving a correlation of 0.95 between experimental results of 19 diesel vehicles and objective rating estimations. Thus, the work of Bodden and
Heinrichs (1999) improved upon the research by Heinrichs and Bodden (1999) by making a more robust and accurate methods for evaluating the sound quality of rattle noise in diesel vehicles.

A unique testing rig was evaluated by Forcelli et al. (2004) for its ability to study the NVH characteristics of a drive train without adding engine noise to the system. This was done by using a Virtual Engine Simulator (VES), which was capable of producing torque and speed fluctuations of up to 500 Hz, with maximum angular displacements of ±35°. The VES was capable of controlling the level of input fluctuation using given time histories for angular accelerations to study the nature of rattling in automotive transmissions. Thus, an experimental study was done on the subjective and objective ratings of rattle in a semi-anechoic chamber with dynameters in soundproofed enclosures. For the objective measurements, the engine shaft velocity fluctuation measurements were taken at the flywheel and housing vibrations for the test vehicle by accelerometers. Sound Pressure Levels (SPL) were then recorded with microphones located a meter from the gearbox enclosure. Six conditions were tested and then correlated with the VES. The conclusions drawn from the experimental results were that gear rattle noise was clearly influenced by fluctuations of gearbox input velocity. In addition to the objective test, a subjective test with 30 subjects rating noise from the VES on a scale of 1 to 10, after being presented with a recording of a control rattle noise. This was shown to decrease the normal time for evaluation in typical automotive manufacturing subjective analysis.

Research conducted by Barthod et al. (2004) utilized both subjective and objective ratings for the auditory perception of rattle created by a four stroke combustion engine with a manual transmission. The primary goals of this research were to evaluate
the physical parameters of rattle which affect its perception (using subjective tests) and to find a relationship between the physical parameters of excitation signals and the subjective ratings. Using time histories from experimental results, the authors calculated the rotational velocity fluctuations for the engine second order harmonics. Using a rattle noise of 27 Hz as a reference, a subjective evaluation was conducted as a dissemblance test, using the Taguchi method to truncate the cost and length of the experiment. Six parameters including loudness, localization, the frequency of rattle noise, intermittence, frequency of modulation amplitude and signal rate of modulation were used in the subjective rating of rattle noise. Of the six parameters, loudness was calculated to be the most influential factor. Measurements from accelerometers placed on gearbox components in the path of vibration were used to plot sound pressure levels, with the input acceleration fluctuations taking into account harmonics to produce a more realistic and chaotic behavior throughout the impact regime. Overall, it was found that phase differences in the impact signal result in distinct excitation frequencies given equivalent SPL’s, and that the harmonic phase has a significant effect on the magnitude of rattle noise (Figure 19).

Figure 19 Noise spectra for phase angle $\phi = -42$ and -66 degrees (Barthod et al, 2004, Fig. 20)
As an extension of the research of Barthod et al. (2004), Barthod et al. (2007) evaluated the effect of input harmonics on a gearbox using a different method for objective rating of rattle noise. The model took into account the inertia of gears, backlash, position of the unloaded gears, gear reduction and input torque fluctuations. The theoretical model used is based on the Kelvin Voigt model with two-degrees of freedom shown in Figure 20. Next, damping was added to the model as a non-linear parameter, and a rattle threshold was defined as a function of the angular acceleration input amplitude. Experimental results indicated that harmonics had no effect on the rattle threshold, that there was an optimal range of backlash, and that small rotating inertias of the unloaded gears required large angular accelerations to create rattle noise. However, decreasing backlash had significant effects on rattle noise only to the point of the rattle threshold, after which small (almost negligible) changes in noise were observed. Finally, it was found that increasing an unloaded gear inertia made the gear more sensitive to the variation in engine rotational velocities, and that large backlashes cause significant velocity differences due to a longer range of free-flight phase between meshed gear teeth.
Models with Rigid or Flexible Enclosures

Campbell et al. (1997) created a model utilizing a torsional compliance method to predict the noise characteristics of a vehicle with rear wheel drive and an automatic transmission with a ring gear configuration. Using acceleration measurements to obtain the Sound Pressure Levels inside the vehicle at the tail stock housing, and differential case, the authors determined the characteristics of noise generated in first gear. Experimental results comparing the natural frequencies based on the predictions made through finite element analysis showed good correlation (Figure 21). However, correlation was carefully obtained by modeling each section of the drive line carefully, making proper assumptions based on the input torque and component inertias. Gear tooth stiffness, modeled as spring elements for mating gears, was important in allowing the authors to add forces to the analytical model for validation of excitation measurements. Overall, it was found that certain mode shapes were of particular importance to controlling the resonant characteristics of the gearbox with various enclosures modeled by finite elements. Since this research was primarily concerned with whine noise, transmission error was a primary concern. In addition, the finite element model also aimed to understand the system dynamics of the gearbox by altering component inertias, which is necessary in reducing rattle noise.
Finite element and experimental analysis for transmission noise generated by a gearbox with a flexible enclosure was presented by Sellgren and Akerblom (2005) for Volvo Construction Equipment. The finite element model generated by the authors took into account three submodels, including the housing, pinion system and gear system which were analyzed using ANSYS. Studying the effect of design parameters, gear finishing methods and assembly operations on gearbox vibratory responses, the authors concluded that the gearbox enclosure must be sufficiently stiff to resist mesh misalignments which induce transmission error. Correlation between the simulated finite element results and experimental data found direct correlation between transmission error and gearbox whine noise. In addition, rebuild errors were found to significantly increase or decrease the amount of noise in the system, which proved to be significant in assuming sources of variation outside of manufacturing tolerance errors. The variations in vibratory response output from the experimental models were correlated to the axial bending pre-loading on the bearings. The authors use of the practices outlined by Campbell et al. (1997) proves the effectiveness of finite element modeling of individual components
connected using stiffness values to predict the vibratory response of a gearbox with one or more external enclosures.

Yakoub et al. (2004) added to the state of the art by predicting rattle noise radiating from a manual transmission gearbox with a flexible enclosure. The conventional wisdom up to this point assumed that torsional fluctuations in the rotational speed of the engine were the source of rattle noise, without a means to theoretically confirm this hypothesis. Using computer aided engineering (CAE) software, a theoretical model composed of the rigid bodies was assumed to be concentrated masses and inertias, with the flywheel driven by input velocity excitations. A parametric approach was used to evaluate the contact between gears, introducing a penetration regime for backlash, where each gear was given its own coordinate system. Using a simulated engine response, experimental results were obtained and used to define the fluctuation at the flywheel. State space representations of the equations of motion were then formed, with a finite element model added to define the eigensolutions for the mode shapes of the system. Finally, the complete model, including the flexible enclosure was evaluated in the frequency domain using FFT’s. Results from the theoretical analysis showed the housing vibration for a system with an input velocity fluctuation had larger magnitudes of rattle noise (SPL) than the model without an input excitation (Figure 22). In addition, the penetration regimes were found to be suitable for future research, as the authors expressed interest in using them for parametric analysis of rattle noise at extremes of the dynamic response.
A study based on active control of a gearbox structure was performed by Guan et al. (2005) in an attempt to control gear noise. The active control system in this study was assumed to be linear, since the damping and transmitted loads were very high, and the operating frequencies of the gear meshes were well below the gearbox torsional resonances. Thus, a system composed of two spur gears with an actuation force located on the input shaft, and accelerometers parallel to the gear mesh line of action was created for experimentation (Figure 23). Sound pressure levels were recorded by microphones, and the error signal for the control system was provided via the accelerometers. Experimentation showed that the select structural resonances and an out of phase gear mesh mode were the dominant sources of noise in the system, which were removed from the input signal using low-pass filters in order to control the gear mesh forcing frequencies more efficiently. The actuating force, when used to control only the gear mesh forcing frequencies and some subsequent harmonics, was able to reduce the overall noise of the system significantly (5-8 dB). As a result, the authors planned future work to
utilize active control suppression of noise on gear meshes with higher operating speeds and torsional loads.

Figure 23 Active control system for spur gear pair (Courtesy of Elsevier - Guan et al, 2005, Fig. 1)

An analysis of the affect of a gearbox wall, or casing, was conducted by Kostic and Ognjanovic (2007) on noise emitted from a transmission. It was established that previous work on predicting and reducing gearbox noise was focused on optimizing the macro or micro geometry of gears, such as the work by Sellgren and Akerblom (2005). Sellgren and Akerblom (2005) noticed that transmission error is increased to gearboxes with limited rigidity, causing the authors to take particular care to view the enclosure as both a vibration isolator and amplifier of transmission noise excitations. Using finite element analysis, the natural frequencies and mode shapes of the gearbox housing were found and correlated with experimental results. Measurements of the vibratory response of the gearbox housing were performed using a modal hammer while recording the excitation time histories. The time histories were then analyzed using an FFT frequency analyzer, and modal testing was conducted with intentional defects in the gear geometry with the upper casing removed and installed. Experimental results showed intense impacts at the location where a defective tooth came into contact with another member.
In addition, more pronounced defects caused more intense impact. Comparison of the results between the open and closed gearbox enclosure with defective teeth showed that the gearbox emitted noise to the interior and exterior of the housing. More importantly, the gearbox showed marked differences between sound levels of frequencies related to the natural frequencies of the gearbox components, providing confirmation to the notion that the natural frequencies of the gearbox were the dominant sources of noise for transmissions.

In an extension of the work by Sellgren and Akerblom (2005), Sellgren (2008) addressed the design challenges in completely defining the drive line for Volvo Construction Equipment. After validating the theoretical model with test rig data in current and previous research, the author found certain challenges in dealing with gearbox leakages and proper modeling of the automotive disc brakes. Finite element analyses of the disc brakes and differential casing address each issue appropriately, adding to the complexities of the system. The model created by Sellgren (2008) expands normal torsional vibration models by including component inertias for a large percentage of the drive line. In this research the author’s primary concern was addressing the complexities of generating a proper analytical model, while ensuring reduced efforts in prototyping by identifying critical system parameters to the vibratory response of an automobile. Future work, as well as the research presented was assumed to be paramount in decreasing uncertainty in predicting gearbox noise.

A study by Korde and Wilson (2009) assesses the use of advanced CAE tools to create efficient and durable transmission designs while reducing cost and development time for automotive manufacturers. The authors note that traditional methods for dealing
with rattle noise in the manufacturing process include building a prototype transmission, and any other necessary components to test the NVH characteristics of the transmission through experimentation. However, this is both costly and ultimately ineffective in reducing noise given current design practices. Due to this, the authors use the Romax Designer software package to analyze a theoretical model with an external enclosure as shown in Figure 24. The theoretical, full-system transaxle model is validated through durability, efficiency and gear whine analysis performed prior to rattle analysis. Thus, the software package used by the authors is capable of determining multiple parameters of interest related to gearbox noise. Given suitable design procedures, manufacturing gearboxes using the outlined procedure was determined to be cost effective, while reducing time required to prototype suitable transmission casings to meet customer expectations of acoustic comfort.

Figure 24 Romax gearbox model with and without enclosure ("Courtesy of the American Gear Manufacturers Association" Körde and Wilson, 2009, Fig. 1)
CHAPTER III

PROBLEM STATEMENT

There are two main objectives for this project. First, the effect of an enclosure on individual parameters of gearbox rattle must be evaluated to expand upon current research. Second, a parametric study must be conducted on the interactive effects of altering the dimensions and dynamics of the gearbox structural components and loads. In order to meet each objective, numerical models of conceptual two-speed gearboxes, both with and without an enclosure, were evaluated using computer simulation. A two-speed gearbox configuration has been chosen to reduce the complexities of the interactions between each component, making the models suitable to study the effect of individual parameters. The individual parameters effecting gearbox rattle that will be studied include the inertia and rigidities of the gears and other components, backlash, positioning, and the input load excitation.

The work conducted for this research was done in a manner which applies accepted practices for analyzing gear rattle, while relating parametric trends with previous research. First, the input load excitation was assumed to be sinusoidal around a mean operating torque, which is standard upon reviewing the studies by Comparin and Singh (1990) and Crowther et al. (2008), among others. As the input load excitation stems from the clutch and flywheel properties, careful consideration was given to modeling these components. Thus, the inertias and rigidities of a flywheel and clutch are applied to each model, paying close attention to the proportional mass of each component. Each model is designed with helical gears, given lower magnitudes of noise.
when compared to spur gears (Juvinal and Marshek, 2006). Hypoid, bevel and worm gears are not considered for the research due to the limitations of the software package used for analysis. Only gearboxes with two operating speed ratios and parallel shafts were investigated for gearbox rattle.

In total, three gearbox configurations were considered. The first has two helical gear pairs with an input and output shaft rotating in opposite directions. The second gearbox utilizes two idler gears, placed between the main shafts enabling the input and output rotational directions to coincide. Finally, the third gearbox design has a helical gear pair, and a helical group with a pinion, wheel and idler. The third design facilitates both forward and reverse motion of the gearbox, at varying speed ratios.

A final goal of this research is to correlate analytical and experimental results for a validation gearbox with an external enclosure. The validation gearbox chosen for this research maintains most of the same parameters as the aforementioned configurations. The gearbox is a small-size two-speed mechanism with an external plastic enclosure. In addition, the internal gears are spur gears and the gearbox has been analyzed without a lubricant. Thus, the gearbox is thought to be more prone to large magnitudes of rattle due to reduced contact ratios, without the viscous damping effect of lubricated impacts. Experimental measurements of stresses and strains at prescribed locations along the gearbox enclosure have been correlated with theoretical predictions.

The procedures outlined by this research provide a basis for reducing rattle in pre-production gearbox designs through computer simulation. Using the software package described herein, gearboxes were created with the addition of a flexible enclosure given that they fall within the design limitations (parallel shafts, etc.). Individual design
parameters were isolated and prioritized based on their influence over the magnitude of rattle, giving an optimal design for manufacture.

**Problem Formulation**

In order to visualize the problem at hand, and for the sake of simplicity, a sample two-speed gearbox is shown in Figure 25. The primary components for the gearbox, as shown, include the flywheel, clutch housing, gear pairs, bearings and shafts. One of the gears in each pair is rigidly attached to its associated shaft while the other is allowed to rotate freely unless engaged using a carrier type synchronizer arrangement. The synchronizer arrangement avoids additional inertias being added to the system, as would normally be the case with a manual transmission set up. Additionally, each gearbox uses the definition of first and second gear for the highest and lowest speed ratios, respectively. Since each pair includes helical gears, taper roller bearings are placed at both ends of the input and output shafts to absorb axial forces generated by the meshed gear teeth. The clutch housing is applied to the model as an inertia which is rigidly connected to the input shaft. This housing includes the lumped inertias of the clutch including the friction plate(s), springs, and external enclosure. Finally, the flywheel is applied to the model as a predefined mass and inertia connected to the clutch. The gearbox, as shown, has added flexibility due to the proximity of the gears to each other. As such, the spacing between the gear pairs can be studied as a means to reduce gearbox rattle.
In addition to the aforementioned simplifications, other principal assumptions are necessary in analysis of the two-speed gearboxes. These assumptions include static characteristics of the clutch, sinusoidal input load fluctuation, constant mesh stiffness and identical gear tooth profiles. Manufacturing tolerance deviations in the profile of each gear tooth are not included in the model, and would otherwise be assumed negligible. In addition, the input load excitation, as described in the subsequent section, is based on sinusoidal fluctuations related to the firing frequency of an internal combustion engine. Finally, the mesh stiffness used to derive the non-linear mesh force and backlash characteristics of the model is assumed to be constant. It is also important to note that the gearbox configuration shown in Figure 25 is just one of the three arrangements presented. However, the fundamental definitions and components associated with the gearbox shown in Figure 25 can be related to each design.
Another consideration in generating a model for the gearbox is the addition of an external enclosure. Seen in Figure 26 is a sample enclosure for the gearbox casing shown in Figure 25. The primary distinction in analysis between the two gearbox configurations is that the model seen in Figure 26 offers the inclusion of the inertia and stiffness of the enclosure. Additionally, during analysis of the model seen in Figure 25, the bearings are assumed to be rigidly fixed whereas the bearings in the model shown in Figure 26 are attached to the gearbox housing. This allows user defined boundary conditions to be applied to the enclosure, creating an additional source in the path of vibration from the meshed gear teeth to the outside. As such, more realistic results can be obtained from analysis of the dynamics of the system.
Gearbox Configurations

Based on the above description of a two-speed gearbox configuration, three theoretical gearboxes were created. Throughout this paper these gearboxes will be referred to as the basic two-speed, reverse idler and dual idler configurations. The first, or basic two-speed, gearbox configuration has an input and output shafts with four gears and bearings downstream the flywheel and clutch inertias. Compared to the basic two-speed configuration, the reverse idler configuration has an additional shaft, gear and set of bearings, which accommodate both forward and reverse motion through the addition of an idler on the left gear set. The final gearbox configuration, or dual idler, has two idler shafts placed between the input and output shafts, both with gears in mesh with their respective pinion and wheel. During rotation, the dual idler configuration input and output shafts are rotating in the same direction. These three configurations are suitable given the goals of the research since they are relatively compact, quiet and include all relevant component inertias and masses.

Basic Two-Speed

The basic two-speed gearbox configuration has the fewest internal components, and thus is the least complex design of the three aforementioned assemblies. Figure 27 shows the plan of the gearbox, including the flywheel, clutch, bearings, gears and shafts. The flywheel and clutch are the leftmost components shown in Figure 27, and are represented by a given mass, width, diameter and polar inertia (rigidly mounted to the input shaft). The two gear sets have 3:1 and 2:1 speed ratios for first and second gear, respectively. Finally, the bearings shown at both ends of the input and output shafts are Koyo HM81649/10 taper roller bearings.
Figure 27 Basic Two-Speed Gearbox (w/o casing)

Table 1 Basic Two-Speed Gearbox Component Definitions

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>Mass (kg)</th>
<th>Coordinate X</th>
<th>Coordinate Y</th>
<th>Coordinate Z</th>
<th>Polar Inertia (kgm²)</th>
<th>Length (mm)</th>
<th>Diameter (mm)</th>
<th>Bore (mm)</th>
<th>Young's Modulus (MPa)</th>
<th>Density (kg/m³)</th>
<th>Poisson's Ratio</th>
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<td>0</td>
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<td>22.5</td>
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<td>20</td>
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<td>3</td>
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<td>0</td>
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</tr>
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<td>Input Bearing 2</td>
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<td>0</td>
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<td>14400</td>
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<td>7800</td>
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</table>

Given in Table 1 are the positions of each component in a Cartesian coordinate system, along with the basic geometry and material properties of the parts. A section number is designated for each step in the input and output shafts, which is used to clearly designate the components relative location with respect to the shaft geometry. The
diameter given for the gears represents the reference pitch diameter, while the diameter for the bearings is indicative of the largest measured diameter of the component. Similarly the length for the gears is the face width, while the length for the bearings is the largest measurement in the direction parallel to the longitudinal direction of the shafts, or along the z-axis.

Reverse Idler

As mentioned previously, the reverse idler configuration has an additional shaft, gear and set of bearings when compared to the basic two-speed configuration. The addition of the idler shaft allows the output shaft to rotate in both directions. When the output shaft rotates in the same direction as the input shaft, the gearbox is subject to a speed ratio of 2:1. Conversely, when the input and output shaft rotate in opposite directions the speed ratio is 3:1. It should be noted that the speed ratios are equivalent to those of the previous gearbox for second and first gear, respectively.

Figure 28 Reverse Idler Gearbox (w/o casing)
Shown in Table 2 are the Cartesian coordinates for each component shown in Figure 28. Again, the polar inertia, length, weight, diameter and bore are shown for each component and the assembly is assumed to be manufactured from steel. The flywheel and clutch for this configuration also have the same polar inertia, weight, length, diameter and bore as the previous basic two-speed configuration. The idler shaft has been placed in such a way as to reduce the number of errors and warnings given by Romax Designer, as well as to ensure that arrangement is sufficiently stiff to resist the reaction forces of the pinion and wheel. This was done by adjusting the module of each gear set until a minimal number of warnings were observed, and then making small adjustments to the relative positions of the idler shafts.

Table 2 Reverse Idler Component Definitions

<table>
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<tr>
<th>Component</th>
<th>Mass (kg)</th>
<th>Coordinate X (mm)</th>
<th>Coordinate Y (mm)</th>
<th>Coordinate Z (mm)</th>
<th>Polar Inertia (kgm^2)</th>
<th>Length (mm)</th>
<th>Diameter (mm)</th>
<th>Bore (mm)</th>
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<td>18</td>
<td>2.05E+05</td>
<td>7800</td>
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</table>
Dual Idler

The third and final configuration is the most complex of the three designs, as it has the most components. When compared to the reverse idler configuration, the dual idler configuration includes an additional shaft, gear and set of bearings. Figure 29 shows the dual idler configuration where the first and second gear sets have speed ratios of 3:1 and 2:1 respectively. Unlike the previous two designs, the bearings on the idler shafts are FAG 30204A taper roller bearings. However, the taper roller bearings attached to the input and output shafts are still Koyo HM81649/10 bearings. This bearing configuration is necessary so that the face of the bearings on the idler shafts do not interfere with any of the other internal components, and to accommodate the shaft geometry. Once again, the flywheel and clutch have the same dimensions, weights and inertias as the reverse idler and basic two-speed configurations.

Figure 29 Double Idler Gearbox Configuration (w/o casing)
Shown in Table 3 are the Cartesian coordinates for the components shown in Figure 29. As with the previous configurations, each component is assumed to be manufactured from steel. Similar to the configuration of the reverse idler, the helical gears are placed in such a manner as to have the reaction forces applied to the idler gear “pulling” the gear toward the plane tangent to the input and output shafts. As such, the largest component of the reaction force is applied in the positive x-direction. The idler shafts are positioned within the gearbox to minimize the number of warnings associated with the detailed gear geometry of the assembly from Romax Designer. Finally, it should be noted that the output shaft has a 5 mm or greater bore throughout to increase its flexibility (allowing for a more appropriate safety factor given the input loads).

Table 3 Double Idler Component Definitions

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>Mass (kg)</th>
<th>Coordinate (mm)</th>
<th>Polar Inertia (kg m^2)</th>
<th>Length (mm)</th>
<th>Diameter (mm)</th>
<th>Bore (mm)</th>
<th>Material (Steel) Properties</th>
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</tr>
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<td>180</td>
<td>44</td>
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Input Load Excitation

As the input load excitation is the primary contributing factor to the magnitude and nature of rattle noise (Heinrichs and Bodden, 1999), attention must be given to its formulation. The total engine torque $T_E$, for an internal combustion engine, fluctuates between minimum and maximum values occurring during the compression and ignition cycles of the power train, respectively. These values are reached periodically about a mean torque value, based on a sinusoidal Fourier series expansion as described by Seaman et al. (1984) and Trochon (1997). The period $T_P$ of the varying torque $T_p$ is based on the firing frequency of the power train driving the input of a given gearbox. Thus, using the angular engine velocity $\dot{\theta}_E$ (rad/sec) one finds that the varying torque is defined in the time domain as seen in Equation 2. The coefficients in this equation are the amplitude of the fluctuating torque $T_{Pn}$ associated with the phase angle $\varphi_p$ and the $n^{th}$ order harmonics. Given an engine speed of 1800 rpm ($\omega_E = 30$ Hz), and a $T_{P1}$ of 125 N·m, with a phase angle $\varphi_p$ of 0 degrees, one would calculate the output response shown in Figure 30a with the inclusion of the first, second and third order harmonics. Figure 30b gives a Fourier Transform of the input excitation signal, showing the frequencies correlating to the harmonics are 30, 60 and 90 Hz for $T_{Pn}$ amplitudes of 125, 62.5 and 31.25 N·m respectively.

$$T_P(t) = \sum_{n=1}^{\infty} T_{Pn} \sin(n\dot{\theta}_E t + \varphi_p)$$

A final term involved in the excitation response, or damping of gear rattle is drag torque. The drag torque terms, when summed, are assumed to be equivalent to the mean torque $T_M$ associated with the engine driving the input shaft for coast and neutral rattle. As stated by Trochon (1997), the drag torque terms are usually separated into expressions.
related to the clutch friction and damping for the input and output gears. Usually, the latter terms are determined during experimentation and are related to oil viscosity. These terms are accounted for during analysis using Romax Designer by applying an ISO 14179-1 (US) friction model to the gears and bearings, associated with the user defined bulk gearbox temperature and lubricant fill level.

![Figure 30 Engine Torque Excitation (a) and Fourier Transform (b)](image)

**Backlash**

Backlash between meshed teeth, as previously described, is associated with the tolerances in the design and calculated using Romax Designer's gear rattle module. The gear mesh stiffness ($k_G$) and mesh force ($F_G$) are output and given as shown in Figure 31, with a region of backlash based on the angular position of the input and output gear. The mesh stiffness is defined using Hertzian contact and the bending stiffness of the meshed gear teeth. The angular displacement of the input and output gear are given using the equation shown in Figure 31, where the subscripts P and G denote the pinion and gear respectively. The clearances between the pinion and gear can be derived from the detailed geometry of the gears, along with the relative spacing of the gearbox shafts. Using this information, the non-linear gear mesh force is applied to the model given a region of normal backlash and constant gear mesh stiffness.
In order to analyze the results generated by Romax Designer’s gear rattle module, the key characteristics of the outputs must be defined. These characteristics include outputs such as the gear rattle index, time-varying relative displacement between gears and natural frequencies of the full order dynamic system. As there is no analytical reference to the level of sound generated during the impact regime of meshed gear teeth, indices have been generated to obtain the relative magnitude of rattle. The formulation of such indices were presented by Singh et al. (1989), and used by various other authors including Padmanabhan et al. (1995) and Wang et al. (2001). In addition to the rattle index, Romax Designer generates an output plot of the relative displacement of gear teeth over a specified time interval, and given a required sampling rate. This output is useful in determining the type of impacts between gears, and the frequencies of interest within the dynamic model. Finally, the mode shapes and natural frequencies of the gearbox are paramount in determining the components which have a significant effect on rattle. More specifically, modal analysis of the gearbox for cases of drive rattle is important in determining resonant frequencies influencing the overall rattle of the theoretical model.
Gear Rattle Index

A gear rattle index stated by Padmanabhan et al. (1995) and Wang et al. (2001) will be used in this paper as a means to measure the magnitude of rattle generated by each theoretical gearbox. The index, stated by Wang et al. (2001) is shown in Equation 3, which is derived from the root mean square values of acceleration between meshed gear teeth. This index is calculated in Romax Designer, and can be used in determining increases or decreases in rattle intensity for individual gear pairs by comparing the relative accelerations between two meshed gears.

\[ R_i = \left( \frac{\ddot{\theta}_{\text{gear,rms}}}{\ddot{\theta}_{\text{flywheel,rms}}} \right); \ \dot{\theta}_{i,rms} = \left[ \frac{1}{\tau} \int_0^\tau \dot{\theta}_i^2 dt \right]^{1/2} \]  

(3)

It is important to note that the sound pressure level, or subjective noise derived from the system is not being considered here. Instead, the intensity of rattle is based on the angular accelerations of meshed gear teeth. Thus, the absolute value for the gear rattle index in relation to indices for the same gear pair, given parametric changes in the dynamic model, can be used to determine an increase or decrease in rattle. For instance, a gear rattle index of 1.0 for a pinion and gear pair increasing to 10.0 for the same pair due to a change in the dynamic model indicates a significant increase in rattle.

Relative Displacement of Gears

Using Romax Designer, backlash regimes are given as the variation of displacement between two meshed gear teeth. Shown in Figures 32 and 33 are examples for the relative displacements between two unladen gear pairs. For these figures, zero indicates that the driving gear tooth is at half the distance of normal backlash from the two driven teeth the driving gear is meshed with. Positive displacement indicates that the
driving gear teeth are displacing in the direction of the driven side of the gear teeth of the driven gear. Conversely, negative displacement indicates that the driving gear teeth are displacing in the direction of the driving side of the gear teeth of the driven gear (coast side). As previously indicated in the description of backlash in the Literature Survey, backlash regimes are categorized into three subgroups. The first is shown in Figure 32, where the driving gear teeth are oscillating between the negative and positive regions of normal backlash for the first two tenths of a second. However, this response changes phase and slowly settles into single-sided impacts on the driven side of the driven gear. Responses of this nature will henceforth be referred to as single-sided backlash regimes.

![Backlash Regime](image)

Figure 32 Single Sided Impact Backlash Regime

Another subgroup of backlash responses, discussed in the Literature Survey is displayed in Figure 33. Here, the driving gear teeth oscillate between the region of negative and positive displacement within the region of normal backlash. In addition, there are distinct periods of time at which the driving gear teeth remain in the regions of
positive or negative displacement. This type of response is referred to as a double-sided impact backlash regime. The third and final subgroup of backlash responses is not shown within this section due to its nature. Any response not associated with single-sided or double-sided impacts is considered chaotic. As such, there is no single example of a chaotic response, although the nature of such responses will be analyzed and discussed in future sections of this paper. However, it is useful to again note that chaotic backlash regimes are most readily associated with large magnitudes of gear rattle.

![Backlash Regime](image)

**Figure 33 Double Sided Impact Backlash Regime**

The relative displacement between two meshed gear teeth is an important output parameter in analyzing gearbox rattle. As discussed in subsequent sections, analysis of the backlash regime is useful in determining the dominant frequencies associated with gear rattle. In addition, inferences as to the magnitude of rattle can be made by observing backlash regimes (independent of the aforementioned subgroups) for a given meshed gear pair. These inferences are associated with backlash regime output responses related
to the elastic behavior of gears. More specifically, it is possible for the displacement of the driving gear to exceed to the region of normal backlash. This presumably indicates that the driving gear is elastically deforming the driven gear teeth, thus causing large magnitudes of rattle.
CHAPTER IV

DYNAMIC ANALYSIS VALIDATION

Validation of Romax Designer’s dynamic analysis module for an imported gearbox enclosure was of particular interest in establishing confidence in conclusions drawn for theoretical gearboxes with the addition of an external casing. Specifically, the natural frequencies and modal characteristics obtained from Romax Designer were needed for correlation with experimental results attained for a small-size production gearbox, and with finite element analysis performed through Abaqus. As will be shown later, experimental results indicated that Romax’s predictions of the modal characteristics of a small-size production gearbox were accurate. Dimensioning of the detailed features of the gearbox enclosure chosen for validation was conducted using a white light scanner, while appropriate material properties were obtained by precise measurement of the volume and weight of the gearbox components. The external enclosure was then subjected to hammer tests, where the frequency response was recorded over a range of 0-2500 Hz. Finally, the solid element mesh used for analysis in Romax Designer was imported into Abaqus with the assistance of Altair Hyperworks Hypermesh preprocessing software package. Using the same mesh with tetrahedron shaped solid elements, the mode shapes and natural frequencies calculated by Romax Designer and Abaqus were compared. The reliability of Romax Designer in predicting the NVH characteristics of gearboxes with multiple internal components was evaluated based on the accuracy in calculating the modal characteristics of the validation gearbox and subsequent models with an external enclosure.
Validation Gearbox

Acquisition of the noise, vibration and harshness characteristics of a two-speed gearbox was the primary concern of this research. In particular, the characteristics of gear rattle of simplified two-speed gearboxes with the addition of the rigidity and inertia of an external enclosure was vital in expanding upon pre-production methods for manufacturing transmissions with reduced sound pressure levels. In order to verify the finite element results of an external enclosure within Romax Designer, comparisons were made with test results. Due to this, a small size two-speed gearbox was acquired. This gearbox, shown in Figure 34, has three parallel shafts, six gears and bearings, and a two-piece plastic enclosure. The model shown below was obtained for its likeness to the theoretical models generated in subsequent sections, which was helpful in formulating conclusions based on assumed advantages and drawbacks of the model.

![Figure 34 Traxxas Two-Speed Gearbox](image)

As previously stated, three steps were used in gathering accurate and reasonable results for the model shown above. First, the model was accurately dimensioned using the
most precise tools available to the researcher. Then, the physical model was subjected to hammer tests to obtain frequency response output values. Finally, the values obtained from NVH analysis of the external enclosure were compared to results obtained through dimensions found in the first step.

**Dimensional Metrology**

Due to the small scale of the acquired validation gearbox, proper dimensioning of the system was crucial to the obtaining of accurate modal analysis results using computer simulation. In addition, the complex geometry of the casing (relative to that of the gears, shafts and bearings) required the use of a white light scanner. As a result, the Advanced Topometric Sensor (ATOS) digitizing system was chosen to provide accurate measurements of the gearbox shown in Figure 34. The ATOS system was particularly useful in creating measurements at a high resolution over a short period of time. These measurements were made based on triangulation principles, where two-dimensional photographs taken by two cameras at specified angles created several thousand data points for each of several readings. Next, the data points were transferred into a common global coordinate system using reference points. These reference points, seen in Figure 34, were small circular markers adhered to the external and internal surface of the gearbox enclosure.

After multiple measurements were made, a cross-sectional representation of the gearbox was generated as shown in Figure 35. These cross-sections were then imported into Pro-Engineer and SolidWorks to generate a CAD model of gearbox casing based on the nodal coordinates. The cross sections were defined using planes perpendicular and parallel to the front face of the gearbox, as well as planes parallel to the primary
mounting surface and perpendicular to the longitudinal direction of the parallel shafts. The spacing between the spline curves generated by these sections was taken as 2 mm perpendicular to each of the aforementioned planes. This was done in order to create an accurate model, while reducing the complexity and number of nodal coordinates to be considered. The nodal coordinates of these curves were then used to define a cartesian coordinate system and an appropriately dimensioned shape for the final model.

![Figure 35 Cross-Sectioned Part (Left) and CAD Model (Right)](image)

**Experimental Results**

Experimental analysis conducted on the gearbox casing was performed using a unidirectional piezoelectric accelerometer placed on the exterior surface of the front and back casing sections (Figure 36). The software used for this analysis was Smart Office Analyzer (version V4.0 B2952 CD 7.02), with integrated circuit piezoelectric (ICP) sensors from PCB for signal conditioning of measurements made by the accelerometer and hammer. The hammer measurements were taken in mV/N, with the accelerometer readings taken in mV/g. The sensitivity for the dB readings for the hammer (SN 6373) was set to 1.1 mV/N while the sensitivity for the accelerometer (SN 13879) was set to
0.988 mV/g as instructed by the manufacturer (PCB Piezotronics, 2010). Data was collected at a sampling rate of 12.8 kHz with a resolution of 3.125 Hz by both instruments, which was assumed sufficient based on the rigidity of the gearbox casing or large intervals between natural frequencies (> 100 Hz). The DAQ card used to condition the signals was a National Instruments Card (NI 9234) mounted to a hi-speed USB carrier (NI USB-9162), which was connected to the laptop running Smart Office Analyzer.

![Figure 36 Accelerometer Location on Front (Left) and Back (Right) Casing Sections](image)

During the experiment, the accelerometers were mounted in a single location using bee’s wax, while the casing was suspended from a table ledge by a strand of string. This setup is assumed to simulate free boundary conditions for all points along the surface of the casing. The nodal positions of the impact locations are given in Cartesian coordinates in Table 4, and are shown in Figures 37 and 38 for the front and back casing, respectively. Figure 37 also gives the position of the x, y and z axes, where the y plane is tangent to the furthest protruding point of the front casing. Coordinates given in Table 1 are all relative to the axes shown in Figure 37, and the highlighted nodes are the locations of the accelerometer for each of the tests.
Table 4 Nodal Impact Coordinates

<table>
<thead>
<tr>
<th>Node</th>
<th>x</th>
<th>y</th>
<th>z</th>
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</thead>
<tbody>
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<tr>
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<td>8</td>
<td>-38.00</td>
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<td></td>
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<td>16.38</td>
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<td><strong>11.45</strong></td>
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<tr>
<td>9</td>
<td>-17.75</td>
<td>-16.90</td>
<td>67.06</td>
</tr>
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</table>

Figure 37 Front Casing Impact Coordinates
The outputs of interest for this experiment include the Frequency Response Function (FRF) and Phase plots. An FRF plot is a representation of the system output response for a given input excitation signal (acceleration divided by the hammer gage force). Each plot was created using an average of 3 impacts, while maintaining favorable coherence and impulse during the process of striking the casing sections. The primary assumptions in obtaining experimental results are that the casing can be considered unconstrained and that there is not a significant mass contribution from the accelerometer or adhesive to skew the results. In addition, no external noise is assumed to have leaked into the system from the string used to suspend the casing or the laboratory environment. Seen in Figures 39 and 40 are FRF plots for 3 of the 17 nodal coordinates used to take
measurements. These positions were chosen, as they are assumed to show all of the high energy peaks of interest over the range of 0-2500 Hz. Figure 39 shows two readings as none of the FRF plots obtained during experimentation had each of the 6 peaks shown (circled in red). These peaks indicate that there are 6 potential frequencies for the front casing (1040.63, 1150, 1421.88, 1768.75, 1971.88, and 2406.25 Hz), and 4 potential frequencies for the back casing (834.38, 1100, 1890.63 and 2290.63 Hz). The remaining FRF plots can be seen in Appendix C, along with their associated phase plots indicating the experimental mode shape characteristics of the casing.

Figure 39 Front Casing FRF for Nodal Positions 2 (black) and 3 (grey)

Figure 40 Back Casing FRF for Nodal Position 4
Analytical Results

The next and final step in the comparison of experimental and computational results for the validation gearbox casing was to complete the dynamic analysis of a CAD model generated using the dimensions obtained from the ATOS white light scanner. Using the procedures outlined in Appendix B, a solid element mesh of the CAD model for each of the casing sections was generated using Hypermesh. The elements were 4-node tetrahedron shaped solid elements, which are well suited for use in meshing CAD models with an irregular geometry. Once the solid element mesh was generated, it was imported into Romax Designer given the Procedures outlined in Appendix A. Here, instead of performing modal analysis with bearing attachments, an accelerometer with a small mass was connected to each casing given a fixed radius and search criterion. Thus, the mass contribution from the accelerometer was kept to a minimum, which should result in comparable results from Romax Designer’s Dynamic Analysis module and the experimental findings in the previous section.

Figure 41 Fundamental Frequency Mode Shape of Front Casing (ROMAX Designer)
Figure 42 Fundamental Frequency Mode Shape of Back Casing (ROMAX Designer)

Seen in Figures 41 and 42 are the displaced model at the fundamental frequencies of the gearbox enclosure sections. The representation of the mode shapes from Romax Designer correlated well with the information derived from the phase plots given in Appendix C. Additionally, the natural frequencies from experimentation (found from the average of peaks from the FRF plots in Appendix C) and modal analysis results from Romax Designer are compared in Table 5. These natural frequencies were derived by using the density found by measuring the weight of the physical component and dividing it by the volume of the CAD model. The mass of the front casing was found to be 22.2374 grams, while the mass of the back casing was found to be 36.0108 grams using a digital weighing scale. Overall, the calculated density of the part is 1404 kg/m³ which was similar to various nylon synthetic polymers. Further, a modulus of elasticity of 3000 N/mm² with a Poisson’s ratio of 0.37 was found to be accurate in the comparison of natural frequencies. These figures are similar to those of a polyoxymethylene plastic
(acetal), which has an $E = 2757$ N/mm$^2$, $v = 0.38$ and density of 1430 grams/mm$^3$. Overall, the results given in Table 5 show that the frequencies for the front casing matched well, with a maximum deviation between results of 4.003 percent. The back casing had fewer mode shapes within the frequency range of interest, with a larger maximum deviation of 7.02 percent between the experimental and computational results.

Table 5 Frequency Comparison

<table>
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<th>Mode</th>
<th>Romax (Hz)</th>
<th>Experimental (Hz)</th>
</tr>
</thead>
<tbody>
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<td>1040.63</td>
</tr>
<tr>
<td>2</td>
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<td>1150.00</td>
</tr>
<tr>
<td>3</td>
<td>1493.80</td>
<td>1421.88</td>
</tr>
<tr>
<td>4</td>
<td>1751.63</td>
<td>1768.75</td>
</tr>
<tr>
<td>5</td>
<td>1974.54</td>
<td>1971.88</td>
</tr>
<tr>
<td>6</td>
<td>2351.00</td>
<td>2406.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>Romax (Hz)</th>
<th>Experimental (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>892.25</td>
<td>834.38</td>
</tr>
<tr>
<td>2</td>
<td>1028.04</td>
<td>1100.00</td>
</tr>
<tr>
<td>3</td>
<td>1832.50</td>
<td>1890.63</td>
</tr>
<tr>
<td>4</td>
<td>2247.78</td>
<td>2290.63</td>
</tr>
</tbody>
</table>

The results shown herein are helpful in generating confidence in the dynamic analysis results obtained from Romax Designer. Specifically, the mode shapes and natural frequencies of the unconstrained gearbox casing sections are similar, but not exact. Deviations from a precise solution could be attributed to the accuracy of the CAD model, external noise contributions to the experimental results, invalid assumptions or a number of other unknown characteristics. However, these results in addition to those of the subsequent section are considered sufficient in assuming accurate results can be derived from Romax Designer.
Comparative Dynamic Analysis Results

In addition to the correlative studies of experimental results obtained for the validation gearbox, finite element results generated by Romax Designer were compared to results obtained by using Abaqus. Consistency between each model was assured since the finite element mesh was the same, given each model was prepared in Altair HyperWorks Hypermesh software package and exported in the appropriate file format (.dat for ROMAX and .inp for Abaqus). In addition, the models used the same aforementioned material properties, and element definitions with free boundary conditions applied to each surface of the members. Comparisons between each result relied upon drawing similarities between the predicted mode shapes and natural frequencies of the programs respective models.

Figure 43 Fundamental Frequency Mode Shape of Front Casing (Abaqus)
Figures 43 and 44 show the results obtained by Abaqus for the fundamental mode shape of both the front and back casings respectively. Comparing these results to those shown in Figures 41 and 42, it is clear that Abaqus predicts the same modal response from the gearbox casings. Similarly, Table 6 shows the frequencies predicted by Romax Designer and Abaqus. Slight variations were seen between each software package, which is due to the modeling restrictions of Romax Designer. Romax's dynamic analysis for gearbox casings requires that at least one condensation node be applied to the model. Normally, this assumes that the gearbox casing is fixed to the cylindrical face of the bearings holding the shafts within a mechanism. However, this would require that the dynamic model include the inertia and rigidity of the bearing condensation nodes. Romax also allows for the definition of an accelerometer for the prediction of NVH characteristics of the dynamic system. For the sake of brevity, and since the Abaqus model has no defined bearings or internal components, the Romax model was defined
without condensation nodes at the bearings. Instead, a zero mass cylindrical accelerometer with a radius of 2 mm and face width of 2.5 mm was fixed at the locations used during experimentation of the validation gearbox. Once this was completed, Romax Designer was capable of running a modal frequency analysis on the gearbox casing.

Table 6 Correlation of Results between Romax Designer and Abaqus

<table>
<thead>
<tr>
<th>Mode</th>
<th>Front Casing</th>
<th>Natural Frequency (Hz)</th>
<th>Back Casing</th>
<th>Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1034.34</td>
<td>1029.60</td>
<td>1892.25</td>
<td>1889.44</td>
</tr>
<tr>
<td>2</td>
<td>1114.00</td>
<td>1110.50</td>
<td>1028.04</td>
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</tr>
<tr>
<td>3</td>
<td>1493.80</td>
<td>1482.30</td>
<td>1832.50</td>
<td>1832.10</td>
</tr>
<tr>
<td>4</td>
<td>1751.63</td>
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<td>2247.78</td>
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</tr>
<tr>
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<td>6</td>
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<td></td>
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</tr>
</tbody>
</table>

The similarity between the frequencies calculated by Abaqus and Romax (Table 6), with a difference of less than 1%, indicated that the modeling technique utilized herein is appropriate. As such, the work done in this section, as well as the one preceding it, indicated that Romax Designer’s dynamic analysis of solid element meshes for gearbox enclosures was both accurate and robust. Results obtained throughout this paper were based on this assumption, given the work which has been presented. In addition, comparative studies between the predicted mode shapes of gearbox enclosures for theoretical models imported in Romax and Abaqus were conducted to demonstrate the validity of this assumption.
CHAPTER V

PARAMETRIC ANALYSIS

Four parameters were evaluated for their influence on gearbox rattle, including component rigidities, inertias, input load excitation and backlash. First, the input load excitation was studied for its effect due to the resonant frequencies of the gearbox assembly for drive rattle noise. Next, backlash was varied by designating a tooth thinning factor within the tolerance regions accepted in Romax Designer. In addition, component inertias were varied by changing the reflected inertia of the system by means of altering the synchronizer arrangement of the gearbox. Finally, the rigidity was varied by means of changing the dimensions of the driveline components while maintaining a constant mass. Each of these parameters was first evaluated for gearboxes without an external enclosure. After the effects of each parameter were carefully outlined and compared between the theoretical gearboxes, further analysis was conducted on the effects of adding the inertia and rigidity of a gearbox housing.

Input Load Variation

The input load variation of each of the three theoretical gearbox assemblies can be directly related to the clutch stiffness and engine firing frequencies as described in the problem statement. Romax Designer allows for the harmonic response of the gearbox input load fluctuation to be considered by means of defining an alternating torque amplitude (about a mean value), phase angle and multiplication factor for the $n^{th}$ order harmonic. Taken into consideration are the first three harmonics of a firing frequency common to a four cylinder internal combustion engine given by Equation 4, where N.C.
is the number of cylinders. Thus, the amplitude of each harmonic is halved from the preceding fluctuating torque value. In order to study the effect of the amplitude of the torques and their subsequent harmonics, a program created in MATLAB was used to take a Fast Fourier Transform (FFT) of the response associated with the relative displacement between two meshed gears. This method was proven to be particularly useful in determining the dominant resonant frequencies for drive rattle, based on the magnitude of peaks for the FFT’s of the backlash response for unladen gear pairs. Given in Figure 45 is the input load fluctuation for the basic two-speed, reverse idler and dual idler configurations for a 1000 rpm (16.66 Hz) input shaft speed at 7.59 hp in first gear \((T_{p1} = 125 \text{ N} \cdot \text{m}, T_m = 54.05 \text{ N} \cdot \text{m} \text{ and } \phi_p = 0)\).

\[
\Omega_{\text{fluctuation}} = \frac{N.C.}{2} \cdot \Omega_{\text{input}}
\]  

Torque Excitation on: Input Power

![Figure 45 Input Load Fluctuations (0.1 sec time interval)]

Basic Two-Speed Configuration

As a basis for comparison when varying torque amplitudes in future tests, each gearbox was subjected to a benchmark run. Shown in Figures 46-48 are the benchmark rattle output values for the basic two-speed gearbox configuration for neutral, first gear
and second gear respectively. It can be seen that values for first and second gear are given for a range of input speeds from 1000-4750 rpm (16.7-71.67 Hz), while the neutral case were run over a range of 500-1250 rpm (8.33-20.83 Hz). Additionally, the input horsepower values range from 7.59 to 80.05 hp for first and second gear and 1.01-10.88 hp for neutral. These values were derived by allowing Romax Designer to create a horsepower curve for a given range of input speeds, and then scaling the output values in order to reach a range which is within a specified safety factor (ex. SF = 1.5) from failure for medium tensile strength steel components. The designated life cycle for each of the three gearboxes was 1000 hrs, at a bulk engine temperature of 70°C.

![Figure 46 Basic Two-Speed Rattle Response (Neutral)](image)

![Figure 47 Basic Two-Speed Rattle Response (First Gear)](image)
As the unladen gear sets were of primary interest in evaluating drive rattle, FFT’s will be taken for these responses exclusively. As a result, Figures 47b and 48a will be evaluated for rattle sensitivity given drive rattle conditions, while both gear pairs are considered for neutral gearbox rattle sensitivity (Figures 46a-b). One observation of note is that the rattle index tends to dissipate as the horsepower and input speed was increased for neutral rattle conditions, while under drive rattle conditions there were peaks at specific engine speeds and input power loads. More specifically, the gear rattle index tends to increase rapidly at 1500 and 4500 rpm (25 and 75 Hz) in first gear and 1250 and 3000 rpm (20.83 and 50 Hz) in second gear. These input speeds can most readily be associated with firing frequencies of 50, 100, 150, 300 and 450 Hz in first gear and 41.66, 83.33, 100, 125, 200 and 300 Hz in second gear. Additionally, the subsequent meshing frequencies were of interest and were calculated using the formula in Equation 5 where \( n \) is the number of gear teeth for a given shaft speed \( \Omega_s \) (rad/sec). Since the number of teeth for both pinions is 15, the meshing frequencies in first gear are 375 and 1125 Hz, with 312.5 and 750 Hz as the second gear forcing frequencies at the respective input shaft speeds.
\[ \omega_{\text{mesh}} = 2\pi n \Omega_5 \] (5)

Given in Figures 49 and 50 are Fourier transforms of the backlash response for first gear at 1500 rpm and 4500 rpm respectively. In addition, Fourier transforms of the backlash regime for second gear at 1250 rpm and 3000 rpm are given in Figures 51 and 52 respectively. Figure 49, given for first gear at 1500 rpm, clearly shows peaks at 50, 100 and 150 Hz, with additional peaks at roughly 200, and 360 Hz. Figure 50, for first gear at 4500 rpm, shows peaks at 75, 150, 200, 300, 375 and 450 Hz. While most of these frequencies can be accounted for by means of evaluating the input speed and harmonics of the fluctuating torque values, others are associated with the natural frequencies of the gearbox. Shown in Table 7 are the natural frequencies of the two-speed gearbox, given for first and second gear along with neutral loading conditions.

![Fourier Transform of Backlash Response](image)

Figure 49 FFT of Backlash Regime in First Gear for Two-Speed Benchmark (1500 rpm = 25 Hz)
Figure 50 FFT of Backlash Regime in First Gear for Two-Speed Benchmark (4500 rpm = 75 Hz)

From the values given in Table 7, one can determine that the second and fourth modes (187.5 and 368.0 Hz) influence the backlash response for a 1500 rpm (25 Hz) input speed. In addition, there is a small peak at a frequency within the range of the third bending mode (262.5 Hz), albeit significantly less pronounced than the peaks at other frequencies. There is a contribution from the meshing frequency at 375 Hz, resonating with the fourth mode frequency (368.0 Hz). For Figure 50, the first, second, and fourth bending modes (165.4, 187.5 and 368.0 Hz) appear with large peaks over the frequency range of interest. Furthermore, the first mode resonates with the second harmonic of the torque input as the amplitude is largest at 150 Hz, without the separation of peaks between these two frequencies as in Figure 49. This resonance is thought to be the reason for the distinct increase in rattle for a 4500 rpm (75 Hz) input speed.
Table 7 Two-Speed Gearbox Modal Frequencies

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<th>Mode</th>
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<th>First Gear</th>
<th>Second Gear</th>
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<td>Bending</td>
<td>Torsional</td>
</tr>
<tr>
<td></td>
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<td>Bending</td>
<td>Torsional</td>
</tr>
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<td>21</td>
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</table>

Figure 51 FFT of Backlash Regime in Second Gear for Two-Speed Benchmark (1250 rpm = 20.83 Hz)
The peaks seen in Figure 51 are at 41.66, 83.33, 125, and 171.5 Hz. As with the previous case, the first three frequencies are influenced solely by the input torque fluctuation and its subsequent harmonics. The fourth high energy peak is associated with the first mode (171.5 Hz) of two-speed gearbox for second gear. While there appears to be no influence from resonance, there is a large contribution from the first mode shape within the backlash regime. In Figure 52, there is a large peak shown for a frequency of 200 Hz, with much smaller contributions coming from the 100, 300, 397.6, 503.9 and 600 Hz frequencies. The large magnitude for the peak at 200 Hz shows the influence of the second torque fluctuation harmonic resonating with the 194.5 Hz natural frequency of the two-speed gearbox. As such, there is a large increase in rattle at this input speed. Under drive conditions, it would be beneficial to stiffen or weaken the gearbox in order to create a significant difference between the input fluctuation frequencies and the natural
frequencies should the system operate under these conditions for extended periods of
time.

After evaluating the effect of resonance on drive rattle conditions for the basic
two-speed configuration, the next step was to evaluate the effect of reducing the
amplitude of $T_{p1}$ from 125 N•m to 62.5 N•m. Figures 53 and 54 indicate that a reduction
in the amplitude of $T_{p1}$ in neutral conditions causes a significant reduction (20.1-92.45%)
in the gear rattle index. Physically, this reduction in rattle would be linked to an increase
in the flywheel and clutch inertias, which has been previously indicated by the works of
Fudala and Engle (1987), Seaman et al. (1984), and Sakai et al. (1981) among others. In
addition, since large torque fluctuations are common to neutral rattle conditions
(Trochon, 1997) the significant reduction in rattle indices observed in the lower input
speed ranges (500-750 rpm) highlight the importance of tuning the torsional
characteristics of the flywheel and clutch for neutral rattle conditions.

![Figure 53 Two-Speed Neutral Torque Fluctuation Rattle Response (First Gear Set)](image)

Figure 53 Two-Speed Neutral Torque Fluctuation Rattle Response (First Gear Set)
Figures 55 and 56 show the rattle response for first gear after reducing the amplitude of $T_{Pl}$ from 125 N•m to 62.5 N•m. Figure 56 represents the unladen gear set, where the rattle response is changed in nature, but the magnitudes are relatively close to each other (0.0345-39.6% deviation). However, Figure 55 shows a significant increase in whine over the entire range of input speeds for the laden gear set. An increase of 6.09 to 44.95% in gear rattle index is observed in Figure 55, indicating that whine in the laden gear pair is significantly increased for a reduction in torque fluctuation amplitude. In addition, the rattle response of the laden gear pair must be influenced by both the reduction in torque amplitude as well as the backlash response of the unladen gear set. The rattle response in Figure 55, is technically whine (as the set is laden), although there is the possibility of rattle in laden gears given impacts on the coast side of the gear pair. The responses shown for the two gear sets are indicative of the fact that larger torque amplitudes are not necessarily analogous to greater magnitudes of rattle within the unladen gear set for drive rattle conditions.
Figures 57 and 58 show the rattle response for second gear after reducing the amplitude of $T_{P1}$ from 125 N•m to 62.5 N•m. Figure 58 represents the laden gear set, where the whine response again shows an overall increase in rattle as seen at most of the sampled data locations. Again, the unladen gear set shows less significant deviation between rattle indices over the range of input speeds as seen in Figure 57. However, an overall increase in rattle due to an increase in input torque fluctuation amplitude can be perceived. Figures 59 and 60 give FFT’s of the backlash response associated with the 125
N\text{m} and 62.5 N\text{m} input torque fluctuations, respectively. These figures are very similar, with fewer frequencies of interest shown in Figure 50. As Figure 59 has an increased number of peaks, which would result in impacts occurring more frequently (either periodically, or on the drive and coast side of the backlash regime) due to an increased number of contributing frequencies. Another possible explanation for this phenomenon lies within the relative displacement between the laden gear teeth. The 125 N\text{m} response, as shown in Figure 61 occasionally displaces to the driven side of the mesh. This phenomenon is not seen within the response for the laden gear set with a 62.5 N\text{m} input (Figure 62), which may account for an increase given larger angular accelerations for the laden gear set. As a result, the coupling between the laden gear and unladen sets may account for the additional frequencies in the FFT spectra, as well as an increase in rattle due to non-linearities in the dynamic system.

![1st Gear Pinion and Wheel](image)

Figure 57 Two-Speed 2nd Gear Torque Fluctuation Rattle Response (First Gear Set)
Figure 58 Two-Speed 2nd Gear Torque Fluctuation Whine Response (Second Gear Set)

Figure 59 FFT of Backlash Regime of Unladen Gear Set for 125 N•m $T_{p1}$ (2500 rpm = 41.67 Hz)
Fourier Transform of Backlash Response

Figure 60 FFT of Backlash Regime of Unladen Gear Set for 62.5 N•m $T_F$ (2500 rpm = 41.67 Hz)

Figure 61 Laden Gear Displacement for 125 N•m Torque Fluctuation (Second Gear)
Shown in Figures 63-65 are the benchmark rattle output values for the reverse idler gearbox configuration for neutral, first gear and second gear, respectively. As with the previous case, the values for first and second gear are given for a range of input speeds from 1000-4750 rpm, while the neutral case is run over a range of 500-1250 rpm. Additionally, the input horsepower values range from 7.59 to 80.05 hp for first and second gear and 1.01 to 10.88 hp for neutral. As the second gear set is unloaded in first gear, it can be seen that there is a peak at 4000 rpm (66.66 Hz) for the idler and wheel pair. This input speed correlates with forcing frequencies of 133.33 Hz, 266.66 Hz and 400 Hz along with meshing frequencies of 1000 and 1333.33 Hz. In second gear, peaks are seen at 2500 rpm and 4000 rpm. At 2500 rpm (41.66 Hz) the forcing frequencies are 83.33, 166.67 and 250 Hz with two meshing frequencies of 625 and 833.33 Hz. Along with the forcing frequencies and meshing frequencies of each of the input speeds taken into consideration are the natural frequencies of the system given in Table 8.
Figure 63 Reverse Idler Rattle Response (Neutral)

Figure 64 Reverse Idler Rattle Response (First Gear)
Figure 65 Reverse Idler Rattle Response (Second Gear)

Table 8 Reverse Idler Gearbox Modal Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
</tr>
</thead>
<tbody>
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<td>Bending</td>
<td>Torsional</td>
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<tr>
<td>1</td>
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<td>2</td>
<td>180.10</td>
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<tr>
<td>3</td>
<td>192.30</td>
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<td></td>
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<tr>
<td>4</td>
<td>227.40</td>
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</tr>
<tr>
<td>5</td>
<td>232.70</td>
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</tr>
<tr>
<td>6</td>
<td>349.20</td>
<td>521.00</td>
<td></td>
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<tr>
<td>7</td>
<td>407.90</td>
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<td>8</td>
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<tr>
<td>12</td>
<td>998.90</td>
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</tr>
<tr>
<td>13</td>
<td>1008.70</td>
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</table>
Given in Figures 66 and 67 are Fourier transforms of the backlash response for first gear at 4000 rpm (66.66 Hz) for the Idler and Pinion as well as Idler and Wheel meshes respectively. While Romax Designer gives the relative displacement between both gear meshes, it can be seen from these Figures that the dominant frequencies in the range of interest are the same for both pairs, since they belong to the same gear set. Reviewing the data seen in Figure 66 and Table 8, the dominant amplitude of the FFT is accounted for given the interaction between the fourth bending mode (253.2 Hz) in first gear and the second harmonic of the input torque amplitude (266.66 Hz). Also shown in Figure 66 are two small peaks at the first and third harmonics of the input torque fluctuation. Finally, a peak is shown at 800 Hz, which is within the range of the fifth bending mode (753.2 Hz). These results are significant due to the fact that two bending modes cause an increase in rattle within the system. While one can tune the torsional characteristics of gearbox housings (clutch and flywheel), it is important to account for axial and bending deformations due to an input torque fluctuation for drive rattle conditions. More importantly, numerical models of rattle conditions must account for these degrees of freedom instead of reducing the equations of motion to decrease the time needed to differentiate using computer simulation.
Given in Figure 68 is a Fourier transform of the backlash response for second gear at 2500 rpm (41.66 Hz). Reviewing the data seen in Figure 68 and Table 8, the dominant amplitude of the FFT is accounted for given the interaction between the fourth bending
mode (239.7 Hz) in second gear and the second harmonic of the input torque amplitude (266.66 Hz). Also shown in Figure 68 are peaks at the first and second harmonic of the torque amplitude, first torsional mode (297.7 Hz), and fifth bending mode (577.7 Hz). Finally, a peak is shown near one of the second axial mode (757.1 Hz) of the gearbox with roughly the same amplitude as the fifth bending and first torsional mode.

Figure 68 FFT of Backlash Regime of 1st Pinion and Gear in Second Gear (2500 rpm = 41.67 Hz)

Given in Figure 69 is a Fourier transform of the backlash response for second gear at 4000 rpm (66.67 Hz). Reviewing the data seen in Figure 69 and Table 8, the dominant amplitude of the FFT is accounted for given the interaction between the fourth bending mode (239.7 Hz) in second gear and the second harmonic of the input torque amplitude (266.66 Hz). Also shown in Figure 69 are peaks at the first and second harmonic of the torque amplitude (133.33 and 266.67 Hz), and sixth bending mode (641.1 Hz). Finally, three peaks are seen at 800 Hz, 950 Hz and 1200 Hz. The 800 Hz frequency is reoccurring from Figure 67 for the same input speed, and could be the result of the
contribution from the second axial mode (757.1 Hz) or a sub-harmonic of the input torque function. However, the two remaining frequencies are unaccounted for given meshing and natural frequencies of the system.

![Fourier Transform of Backlash Response](image)

**Figure 69 FFT of Backlash Regime of 1st Pinion and Gear in Second Gear (4000 rpm = 66.67 Hz)**

After evaluating the effect of resonance on drive rattle conditions for the reverse idler configuration, the next step was to evaluate the effect of reducing the amplitude of $T_{P1}$ from 125 N•m to 62.5 N•m. Figures 71 and 72 indicate that a reduction in the amplitude of $T_{P1}$ in neutral conditions causes a significant reduction (24.1-29.97%) in the gear rattle index. It is also important to note that no significant reduction or increase in whine is observed in Figure 70 for the mesh between the idler and pinion. Since the inertial difference between the two gears is small and the gears are loaded the mesh is not as sensitive to increases in angular accelerations. Additionally, the clearances between the pinion and idler as well as their inherent rigidity (both are rigidly connected to the
shaft, while the wheel has free rotation) reduce the ability for large increases in rattle threshold for the range of input speeds.

**Figure 70 Reverse Idler Neutral Torque Fluctuation Whine Response (Second Idler and Pinion)**

**Figure 71 Reverse Idler Neutral Torque Fluctuation Rattle Response (Second Idler and Wheel)**
Figures 73 through 75 show the rattle response for first gear after reducing the amplitude of $T_{P1}$ from 125 N·m to 62.5 N·m. Figure 75 represents the laden gear set, where the rattle response is changed in nature, but the magnitudes are relatively close to each other (0.794-17.2% deviation). Figures 73 and 74 show a significant difference between rattle responses for the unladen gear set. A reduction of 8.23 to 22.9% in gear rattle index is observed in Figure 74, indicating that rattle in the unladen idler and wheel pair is significantly reduced for a reduction in torque fluctuation amplitude. However, an increase of 12.6 to 32.1% in rattle indices is observed for the idler and pinion gear mesh (no synchronizer and thus loaded). The increase in rattle index, or ratio of relative accelerations for reduced torque amplitudes between the idler and pinion indicates that the idler shaft excites the system when not sufficiently loaded. As such, the relative acceleration of the gear mesh, compared to the input acceleration response is larger for the 62.5 N·m input.
Figure 73 Reverse Idler 1st Gear Torque Fluctuation Whine Response (Second Idler and Pinion)

Figure 74 Reverse Idler 1st Gear Torque Fluctuation Rattle Response (Second Idler and Wheel)

Figure 75 Reverse Idler 1st Gear Torque Fluctuation Whine Response (First Gear Set)
Figures 76 through 78 show the rattle response for second gear after reducing the amplitude of $T_{P1}$ from 125 N·m to 62.5 N·m. Figure 76 shows a decrease (4.77-8.47%) in rattle indices values for the idler and pinion gear mesh. As this pair is linked to the source of the torque fluctuation, an overall decrease in rattle given smaller fluctuation amplitudes is appropriate. Figure 77 represents the laden idler and wheel mesh, where the rattle response is changed in nature, but the magnitudes are not significantly different (0.391-5.15% deviation). Figure 78 shows a significant difference between rattle responses for the unladen gear set. A reduction of 10.6 to 20.3% in gear rattle indices is observed in Figure 78, indicating that rattle in the unladen gear mesh between the pinion and wheel is significantly reduced for a decrease in torque fluctuation amplitude. Overall, these results are similar to the previous observations from Figures 73 through 75, in that the unladen gear mesh has reduced rattle for decreased torque fluctuation amplitudes. In addition, the laden gear mesh(s) observes small changes in rattle indices over the range of input speeds due to the rigidity of the gear set. Physically, this indicates that a reduction of torque amplitude is favorable for drive rattle conditions given the addition of the rigid idler shaft.

![Graph showing rattle response for 2nd Gear Idler and Pinion](image)

**Figure 76** Reverse Idler 2nd Gear Torque Fluctuation Whine Response (Second Idler and Pinion)
Figure 77 Reverse Idler 2nd Gear Torque Fluctuation Whine Response (Second Idler and Wheel)

Figure 78 Reverse Idler 2nd Gear Torque Fluctuation Rattle Response (First Gear Set)

**Dual Idler Configuration**

Shown in Figures 79-81 are the benchmark rattle output values for the reverse idler gearbox configuration for neutral, first gear and second gear, respectively. As with the aforementioned cases, the values for first and second gear are given for a range of input speeds from 1000-4750 rpm, while the neutral case is run over a range of 500-1250 rpm. Additionally, the input horsepower values range from 7.59 to 80.05 hp for first and
second gear and 1.01 to 10.88 hp for neutral. As the first gear set is unloaded in second gear, it can be seen that there is a peak at 2000 rpm (33.33 Hz) for the idler and wheel pair. This input speed correlates with forcing frequencies of 66.66 Hz, 133.33 Hz, and 200 Hz along with meshing frequencies of 400 and 500 Hz. In first gear, peaks are seen at 4250 rpm (70.83 Hz), with a peak in the laden set at 2000 rpm (similar to that from the second gear loading conditions). At 4250 rpm the forcing frequencies are 141.66, 283.33 and 425 Hz with two meshing frequencies of 850 and 1062.5 Hz. Along with the forcing frequencies and meshing frequencies of each of the input speeds taken into consideration are the natural frequencies of the system given in Table 9.

Figure 79 Double Idler Rattle Response (Neutral)
Figure 80 Double Idler Rattle Response (First Gear)

Figure 81 Double Idler Rattle Response (Second Gear)
Table 9 Dual Idler Gearbox Modal Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial</td>
<td>Bending</td>
<td>Torsional</td>
</tr>
<tr>
<td>1</td>
<td>122.60</td>
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<td></td>
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<tr>
<td>2</td>
<td>164.40</td>
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</tr>
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<td>3</td>
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<td></td>
</tr>
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<td>7</td>
<td>459.30</td>
<td>753.20</td>
<td>821.20</td>
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<td>8</td>
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<td>10</td>
<td>679.60</td>
<td>871.50</td>
<td>883.70</td>
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<td>11</td>
<td>748.10</td>
<td>909.70</td>
<td>963.90</td>
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<tr>
<td>12</td>
<td>779.80</td>
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<td>904.70</td>
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<tr>
<td>14</td>
<td>991.00</td>
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</tr>
<tr>
<td>15</td>
<td>1158.30</td>
<td></td>
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</tr>
</tbody>
</table>

Given in Figure 82 is an FFT of the backlash response for second gear at 2000 rpm (33.33 Hz) for the first idler and pinion. As with the reverse idler design, FFT’s of both gear pairs within a given idler mesh have the same dominant frequencies in the range of interest. As a result, only one FFT is shown for the sake of brevity from here on. Reviewing the data seen in Figure 82 and Table 9, the dominant amplitude of the FFT is accounted for given the interaction between the first bending mode (225.8 Hz) in first gear and the third harmonic of the input torque amplitude (200 Hz). Also shown in Figure 82 is a small energy peak at 600 Hz, which is within the range of the fourth bending mode (560.9 Hz) of the gearbox in second gear. The overall response has larger peak amplitudes within the lower range of frequencies (less than 300 Hz), with an additional peak at twice the initial range of interest.
Given in Figure 83 is a Fourier transform of the backlash response for first gear at 2000 rpm (33.33 Hz) for the second idler and pinion. Reviewing the data seen in Figure 83 the dominant amplitude of the FFT is accounted for given the first harmonic of the input torque amplitude (66.66 Hz). Also shown in Figure 83 are peaks at the two remaining harmonics of the torque input fluctuation, and first torsional mode (390.9 Hz). There is also a subsequent peak near 400 Hz, which is likely caused by the meshing frequency. This response is unique in that the harmonics do not cause resonance with the driveline natural frequencies, but a large increase in rattle index is observed due to resonance between the meshing and natural frequencies of the gearbox structure.
Fourier Transform of Backlash Response

Figure 83 FFT of Backlash Regime of Second Pinion and Idler in First Gear (2000 rpm = 33.33 Hz)

Given in Figure 84 is a Fourier transform of the backlash response for first gear at 4250 rpm (70.83 Hz) for the second idler and pinion. Reviewing the data seen in Figure 84 the dominant amplitude of the FFT is accounted for given the first harmonic of the input torque amplitude (141.66 Hz). Also shown in Figure 84 are peaks at the remaining harmonics of the torque input fluctuation, the first and second torsional modes (390.9 and 583.2 Hz) and the second bending mode (264.3 Hz). In addition, there is a high energy peak seen for the second harmonic due to the proximity of this frequency to the second bending mode at 264.3 Hz. The remaining possible resonance in the system is with the first torsional mode and third harmonic of the input torque fluctuation (390.9 and 425 Hz respectively).
Figure 84 FFT of Backlash Regime of Second Pinion and Idler in First Gear (4250 rpm = 70.83 Hz)

Figures 85 through 88 show the rattle response for neutral after reducing the amplitude of $T_{p1}$ from 125 N•m to 62.5 N•m. Figure 85 shows an increase (8.04-11.76%) in rattle index values for the idler and wheel gear mesh. Figures 86 and 88 show small deviations in rattle indices over the specified range of input speeds, indicating again that the inertial difference between the gear pair results in an insensitivity to rattle. Finally, Figure 87 shows an overall increase of 12.82 to 32.07% in rattle indices from the 125 N•m input torque fluctuation amplitude to the 62.5 N•m torque amplitude. This is consistent with the increase in the idler and wheel meshed pair in the second gear set, although it is contradictory to previous results for neutral conditions. For the previous gear trains, an overall reduction is observed for a decrease in torque amplitude for meshes with a large inertial difference. Shown in Figures 89 and 90 are Fourier transforms of the backlash response between the idler and wheel at 125 N•m and 62.5 N•m respectively. As with the previous case for second gear for the two-speed gearbox, the response with
the larger rattle index has fewer peaks in the frequency band after the torque fluctuation peaks. Thus, given more contributing frequencies, the gear indices tend to be smaller for unladen gear pairs with a large inertial difference and added rigidity due to the idler shafts (which may act as an isolator or exciter for the system).

Figure 85 Dual Idler Neutral Torque Fluctuation Rattle Response (Second Idler and Wheel)

Figure 86 Dual Idler Neutral Torque Fluctuation Whine Response (Second Idler and Pinion)
Figure 87 Dual Idler Neutral Torque Fluctuation Rattle Response (First Idler and Wheel)

Figure 88 Dual Idler Neutral Torque Fluctuation Whine Response (First Idler and Pinion)
Figures 89 through 94 show the rattle response for second gear after reducing the amplitude of $T_{p1}$ from 125 N•m to 62.5 N•m. Figures 91 and 92 represent the laden gear sets, which exhibit only small deviations (0.063-12.023%) between both input torque sets, which exhibit only small deviations (0.063-12.023%) between both input torque.
amplitudes. This is consistent with results from previous gearbox structures. In addition, the rattle indices associated with the unladen gear pairs show an overall increase, with the largest increase occurring between the gear pairs with the largest inertial difference. The idler and wheel pair rattle response is shown in Figure 93 with deviations ranging between -4.37 and 72.76%. Similarly, the rattle response shown in Figure 94 for the idler and pinion of the right gear set shows deviations between -12.96 and 43.74 percent. Again, the largest deviations are associated with the gear pairs with the largest inertial difference, and driven rattle conditions observe an overall increase in rattle given larger torque fluctuation amplitudes. As such, it can be assumed that the loaded pair (idler and pinion) from the first gear set, act as an exciter due to the idler shaft being sufficiently stiff and being downstream the laden gear set.

![Figure 91 Dual Idler 2nd Gear Torque Fluctuation Whine Response (Second Idler and Wheel)](image-url)
2nd Gear Idler and Pinion

Figure 92 Dual Idler 2nd Gear Torque Fluctuation Whine Response (Second Idler and Pinion)

1st Gear Idler and Wheel

Figure 93 Dual Idler 2nd Gear Torque Fluctuation Rattle Response (First Idler and Wheel)
Figures 95 through 98 show the rattle response for first gear after reducing the amplitude of $T_{p1}$ from 125 N•m to 62.5 N•m. Figures 97 and 98 represent the laden gear sets, which exhibit only small deviations (0.241-19.98%) between both input torque amplitudes. Again, this is consistent with results from previous gearbox structures. In addition, the rattle indices associated with the unladen gear pairs (Figures 95 and 96) show an overall increase in rattle. However, this increase is small, which indicates that this gear set is not as sensitive to increases or decreases in torque fluctuation amplitude for rattle. This result is significant, as it shows a potential robustness to torque fluctuation amplitude that the previous gearboxes do not have. As there are larger inertial differences between the 1st gear set (175, 44 and 6702 kgmm$^2$ for the pinion, idler and wheel respectively) than the 2nd gear set (280, 75 and 1718 kgmm$^2$ for the pinion, idler and wheel respectively), and the speed ratio is smaller for the 2nd gear set, the insensitivity could be expected. Also, it should be noted that the loaded gear set is upstream the rattling pair (second idler and wheel).
Figure 95 Dual Idler 1st Gear Torque Fluctuation Rattle Response (Second Idler and Wheel)

Figure 96 Dual Idler 1st Gear Torque Fluctuation Whine Response (Second Idler and Pinion)
Figure 97 Dual Idler 1st Gear Torque Fluctuation Whine Response (First Idler and Wheel)

Figure 98 Dual Idler 1st Gear Torque Fluctuation Whine Response (First Idler and Pinion)

Backlash

Fine tuning of the backlash between meshed gear teeth was another parameter considered for reduction of the onset of neutral and drive rattle. Through the increase of a tooth thinning factor from 0.01 to 0.25, the backlash was increased as a percentage of the module (~ 1%) of the gear sets in each gearbox to study the overall effect of this parameter on the magnitude of neutral and driven gear rattle. For reference, the backlash
values associated with each tooth thinning factor are found in the Tables of Appendix E. Data collected for the rattle indices were at peak locations for each of the three gearbox structures as outlined in the torque fluctuation section. Thus, for the basic two-speed gearbox configuration the input speed was set to 500, 4500 and 3000 rpm (8.33, 75 and 50 Hz) for neutral, first gear and second gear respectively. The reverse idler configuration was run at 500 and 4000 rpm for neutral (8.33 and 66.67 Hz), and first and second gear respectively. Finally, the dual idler configuration was run at 500, 2000 and 4000 rpm (8.33, 33.33 and 66.67 Hz) for neutral, first gear and second gear respectively. The peak values used were chosen, since it was assumed that larger changes in rattle would be observed at the resonant frequencies of the gearbox structure for drive rattle conditions and the largest difference between the mean torque and amplitude of the torque fluctuation for neutral rattle.

Basic Two-Speed Configuration

The first of two tests run for the basic two-speed configuration was to observe the change in rattle indices when altering the backlash between the first gear pinion and wheel. The tooth thinning factor was set from 0.01 to 0.25 at intervals of 0.01 for the first gear wheel giving the results shown in Figures 99 through 104. In neutral, rattle indices reduce as the backlash is increased in both gear sets as shown in Figures 99 and 100. In Figures 101 and 102 the rattle index reduces as the backlash is increased in the unladen second gear set, and displays a chaotic nature for the laden first gear set. Finally, shown in Figures 103 and 104 are the rattle indices for second gear, where the unladen set is the first gear set. Each figure shows a steady increase in gear rattle index over the range of tooth thinning factors. These results indicate that the unladen gear set observes an increase in rattle index as the backlash is gradually increased. Additionally, rattle indices
either reduce or are chaotic in nature when backlash is increased in laden gear pairs. Finally, when there are no laden pairs and with backlash increasing in one of two gear pairs, the rattle index reduces in magnitude until it begins fluctuating around a mean value. These results indicated that the coupled nature of the basic two-speed gearbox may result in a decrease in rattle given larger backlash between meshed gear teeth. This is consistent with results documented by Derk (2005) for gear trains with large torque fluctuations, where backlash suddenly reduces and remains relatively constant at a given input load excitation and input speed.

Figure 99 Basic Two-Speed Rattle Response for Backlash in Neutral (First Gear Set)
2nd Gear Pinion and Wheel

Figure 100 Basic Two-Speed Rattle Response for Backlash in Neutral (Second Gear Set)

1st Gear Pinion and Wheel

Figure 101 Basic Two-Speed Whine Response for Backlash in 1st Gear (First Gear Set)

2nd Gear Pinion and Wheel

Figure 102 Basic Two-Speed Rattle Response for Backlash in 1st Gear (Second Gear Set)
The decrease in rattle indices for tooth thinning factors increasing from 0.01 to 0.25 may also be explained by observing the spectra of Fourier transforms in Figures 105 and 106. Figure 105 has larger peaks at low frequency spectra indicating longer impacts for smaller tooth thinning factors. As a result, the rattle index for a tooth thinning factor of 0.01 is larger than the rattle index for a tooth thinning factor of 0.25. This is consistent with previous results observed in the Torque Fluctuation section for the two-speed and dual idler gearbox configurations.
Reverse Idler Configuration

Backlash was also tested for the reverse idler configuration, where the addition of the idler driveline component changed the characteristics for rattle responses seen for the basic two-speed configuration. Shown in Figures 107 through 109 are the rattle responses for neutral, given an increase in tooth thinning factors from 0.01 to 0.25 by intervals of
0.01 for the first gear set. Unlike the previous study, the tooth thinning factor is applied to the idler for neutral and first gear drive rattle conditions. In neutral, both rattle responses for the wheel show a shape resembling an exponential decay, where the pinion and idler mesh for the second gear set have increasing rattle indices over the range of tooth thinning factors. The difference in rattle response for the idler and pinion mesh can be attributed to the fact that the given mesh is the only mesh without a free running gear (both gears are integral with their respective shafts). Thus, the free running wheels show a decrease in rattle, due to a larger number of contributing frequencies for a tooth thinning factor of 0.01, and more time between impacts for a tooth thinning factor of 0.25.

Figure 107 Reverse Idler Rattle Response for Backlash in Neutral (First Gear Set)
Figures 108 through 112 show the rattle responses for first gear, with a tooth thinning factor applied to the second gear idler. As with the neutral rattle responses, the rattle indices decrease rapidly and remain relatively constant above a tooth thinning factor of 0.10 (Figures 110 and 112). The idler and pinion mesh for second gear exhibit much larger increases in rattle indices for increased backlash between the mesh than in the neutral rattle responses (63.50% compared to 21.56% increase respectively). This change in response is indicative of the addition of a loaded gear pair for first gear conditions. Seen in Figures 110 and 112 are larger rattle indices over the range of tooth
thinning factors for both meshes including the gear with the largest inertia. As the rattling pair is the second gear idler and wheel, it stands to reason that the tooth thinning factors applied to the idler allow the member to observe longer periods between impacts, decreasing the overall rattle indices. In addition, the rattle index increases between the idler and pinion as this pair is more sensitive to the increase in thinning factors (smaller tooth size). Additionally, the gear pair for Figure 111 is loaded, and is influenced by additional input frequencies imposed by the unloaded idler and wheel mesh (previous results indicate the idler acts as an exciter in first gear).

![1st Gear Pinion and Wheel](image1)

Figure 110 Reverse Idler Whine Response for Backlash in First Gear (First Gear Set)

![2nd Gear Idler and Pinion](image2)

Figure 111 Reverse Idler Whine Response for Backlash in First Gear (Second Idler and Pinion)
The final loading conditions evaluated for the reverse idler configuration were for second gear, with a tooth thinning factor applied to the first gear wheel. Again, the increase in backlash for the first gear set causes a sharp decrease in rattle threshold for the gear set as shown in Figure 113. Figures 114 and 115 exhibit relatively constant whine responses for the entire range of tooth thinning factors, with the largest decrease in rattle indices seen below a tooth thinning factor of 0.05. As the second gear set is more rigid given the addition of the idler configuration, the difference in rattle response, and resistance of the laden gear mesh to changes in magnitude for rattle indices is to be expected given previous observations. Thus, for each gear pair the region of backlashes evaluated goes beyond the region of increasing rattle threshold and exhibit behaviors similar to those documented by Derk (2005) for gear trains with large torque fluctuations.
Figure 113 Reverse Idler Rattle Response for Backlash in Second Gear (First Gear Set)

Figure 114 Reverse Idler Whine Response for Backlash in Second Gear (Second Idler and Pinion)

Figure 115 Reverse Idler Whine Response for Backlash in Second Gear (Second Idler and Wheel)
Dual Idler Configuration

The final gearbox evaluated for the effect of increasing backlash was the dual idler configuration. Shown in Figures 116 through 119 are the rattle responses for the first and second gear sets under neutral rattle conditions. It is important to note that for the dual idler configuration, the tooth thinning factor was applied to the idlers in the unladen gear pair in drive rattle conditions and the second gear idler for neutral rattle conditions. As a result, the relative change in magnitude for backlash for the dual idler configuration is less than that of the previous configurations. Figures 118 and 119 exhibit small increases in rattle indices for increases in backlash (20.7 to 37.3%), with the idler and wheel mesh displaying the largest increase in rattle threshold (having the largest inertial difference). Similarly, only small changes in rattle indices are exhibited for the first gear set, with a maximum decrease of 30.173% in rattle indices observed for the first gear idler and pinion (Figure 117). Both of the rattling pairs (idler and wheel meshes) show gradual increases in rattle threshold, as the clearances are changed incrementally. However, the loaded pairs (idler and pinion meshes) show only small changes in rattle indices over the range of input speeds.

Figure 116 Dual Idler Rattle Response for Backlash in Neutral (First Idler and Wheel)
Figures 120 through 123 are the rattle responses for first gear, given an increased tooth thinning factor for the second gear idler. Both unladen gear sets show an overall
increase in rattle indices for the range of tooth thinning factors applied to the second gear idler (Figures 122 and 123). The largest increase in magnitude (78.9%) for rattle indices is given for the gear pair with the largest inertial difference (Figure 122), with the inclusion of the gear which is not integral with the shaft. The laden gear set again shows the largest increase in rattle for the pair with the largest inertial difference (Figure 120), with an insensitivity to rattle observed for the pinion and idler pair (Figure 121). All of the gear meshes indicate that the range of tooth thinning factors applied to the second gear idler does not exceed the specified value needed to observe the phenomenon similar to that documented by Derk (2005). Again, this can be attributed to the fact that the relative change in magnitude for backlash for the dual idler configuration is significantly smaller than that for the previous configurations. Additionally, the structure is more rigid with the addition of the bearings applied to the ends of each idler shaft.

![Graph](image)

*Figure 120 Dual Idler Whine Response for Backlash in First Gear (First Idler and Wheel)*
Figure 121 Dual Idler Whine Response for Backlash in First Gear (First Idler and Pinion)

Figure 122 Dual Idler Rattle Response for Backlash in First Gear (Second Idler and Wheel)

Figure 123 Dual Idler Whine Response for Backlash in First Gear (Second Idler and Pinion)

Shown in Figures 124 through 127 are the rattle responses in second gear, with a tooth thinning factor applied to the first gear idler. Again, the unladen set has an increase
in rattle indices for the entire range of tooth thinning factors applied to the first gear idler. Also, the largest increase in relative magnitude for rattle index is shown in Figure 124 for the pair with the largest inertial difference (79.1%). Figures 126 and 127, given for the laden gear set, display insensitivity to changes in tooth thinning factor for the first gear idler. However, the largest deviations in rattle index magnitudes are again displayed for the gear pair with the largest inertial difference. Overall, each of the three loading conditions for the dual idler configuration display increases in rattle for the range of increasing backlash. Again, this is attributed to the fact that the changes in backlash are finer than with the previous two configurations, while the gearbox is inherently more rigid than the aforementioned basic two-speed and reverse idler gearboxes. In addition, relative change in rattle indices is distinctly larger for the gear pair with the largest speed ratio (and subsequently larger reflected inertias upstream the load). This indicates, as with the previous observation, that rattle is proportional to the relative ratio of linear accelerations between the unladen gear pair (larger for increased gear ratio).

![1st Gear Idler and Wheel](image124)

Figure 124 Dual Idler Rattle Response for Backlash in Second Gear (First Idler and Wheel)
Figure 125 Dual Idler Whine Response for Backlash in Second Gear (First Idler and Pinion)

Figure 126 Dual Idler Whine Response for Backlash in Second Gear (Second Idler and Wheel)

Figure 127 Dual Idler Whine Response for Backlash in Second Gear (Second Idler and Pinion)
Component Inertias

The alteration of inertias is documented as being one of the most effective means of affecting rattle in automotive manual transmissions. Most notably, Fudala and Engle (1987), Pfeiffer (1996), Kim and Singh (2000) and Barthod et al. (2004, 2007) have studied the effects of increasing or decreasing the inertia of the driven members of an unladen gear pair and flywheel. This section will focus on increasing or decreasing the reflected inertias within a system by means of altering the synchronizer arrangement within each of the three gearbox structures. This particular technique has been tested as an effective means to reduce gearbox rattle noise by Rust et al. (1990), who modified the synchronizer arrangement of a 5-speed manual transmission. This technique is beneficial analytically, as it avoids changing multiple parameters significantly in order to increase or decrease gearbox rattle. In addition, the assumptions of the aforementioned authors can be comparatively evaluated by means of significantly altering the driven and driving inertias of the unladen gear set.

Basic Two-Speed Configuration

The first gearbox tested for the effect of altering component inertias within a gearbox structure was the basic two-speed configuration (Figure 27). Shown in Figures 128 and 129 are the rattle responses in first gear for the second and first gear sets respectively. The unladen gear pair shown in Figure 129 shows a distinct increase in rattle index given a larger driving inertia. Overall, the increase in the second gear set rattle index with a synchronizer at the pinion is a minimum of 2.86 times that of the alternate configuration. In addition, it can be seen in Figure 128 that the rattle index is relatively unaffected for the laden gear set except in the regions of 3500-4500 rpm.
(58.33-75 Hz), which is near the resonant frequency of the unladen gear pair discussed in
the previous section. Thus, these results agree with the assertions of Karagiannis and
Pfeiffer (1991) that small driven inertia for a pinion tends to increase the threshold for
rattle noise within a gearbox structure. For reference, the natural frequencies of the
system with this configuration are given in Table 10, where an overall increase is
observed for the range between the lowest input speed (16.7 Hz) and largest meshing
frequency (1187.5 Hz).

![Graph 1](image1)

**Figure 128 Two-Speed Gearbox 1st Gear Inertia Variance Whine Response (First Gear Set)**

![Graph 2](image2)

**Figure 129 Two-Speed Gearbox 1st Gear Inertia Variance Rattle Response (Second Gear Set)**
Table 10 Two-Speed Gearbox Modal Frequencies (Second Gear Pinion Synchronizer)

<table>
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<td>Torsional</td>
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Shown in Figures 130 and 131 are the rattle responses in second gear for the first and second gear sets respectively. Similar to the previous test, the synchronizer arrangement has been moved from the gear to pinion for the unladen gear set. Here, Figure 131 represents the laden gear set, which displays an increase in rattle indices for the range of input speeds given. Similarly, the rattle indices for the unladen gear set increase over the range of input speeds given in Figure 130. However, the difference in magnitude is much greater for the unladen gear set, as with the previous results. This is to be expected, as the driven gear has a sufficiently large inertia compared to that of the driving wheel which has increased the threshold for rattle in this gear pair. Overall, a minimum difference in magnitude of 618% is observed in Figure 130, which is indicative of much larger magnitudes of rattle for a similar physical model. Again, Table 11 is given for reference, where the peaks at 1500 rpm (25 Hz) and 2500 rpm (41.67 Hz) in Figure 130 can be explained by resonance between the meshing (375 and 625 Hz) and natural frequencies of the gearbox structure.
Figure 130 Two-Speed Gearbox 2nd Gear Inertia Variance Rattle Response (First Gear Set)

Figure 131 Two-Speed Gearbox 2nd Gear Inertia Variance Whine Response (Second Gear Set)

Table 11 Two-Speed Gearbox Modal Frequencies (First Gear Pinion Synchronizer)

<table>
<thead>
<tr>
<th>Mode</th>
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<th>First Gear</th>
<th>Second Gear</th>
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Reverse Idler Configuration

The next gearbox tested for the effect of altering component inertias within a gearbox structure was the reverse idler configuration (Figure 28). Shown in Figures 132 through 134 are the rattle responses in first gear for the first and second gear sets. The unladen gear pairs are shown in Figure 133 and 134, where the pinion and idler mesh has magnitudes of rattle indices that are both greater and less than when the synchronizer arrangement is on both wheels. Only slight changes in rattle indices are seen in this mesh, as the gears (driven or driving) are relatively similar in pitch radius and inertia. The wheel and idler pair, shown in Figure 134 has significantly larger values of rattle indices for gearbox with synchronizers on both gears. Finally, Figure 132 shows the laden gear mesh, where the response is only significantly different when evaluated at 3250 and 3500 rpm (54.17 and 58.33 Hz). The large decrease in rattle for Figure 134 is thought to be the effect of the first gear load being placed further downstream and reaching the output before being used to power the second gear idler and wheel.

Figure 132 Reverse Idler 1st Gear Inertia Variance Whine Response (First Gear Set)
Figure 133 Reverse Idler 1st Gear Inertia Variance Whine Response (Second Idler and Pinion)

Figure 134 Reverse Idler 1st Gear Inertia Variance Rattle Response (Second Idler and Wheel)
Table 12 Reverse Idler Gearbox Modal Frequencies (Second Gear Pinion Synchronizer)

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<td>Frequency (Hz)</td>
<td>Frequency (Hz)</td>
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<tr>
<td>14</td>
<td>1151.30</td>
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</table>

Shown in Figures 135 through 137 are the rattle responses for second gear when the synchronizer arrangement is placed on the first gear pinion. Figures 135 and 137 show an overall decrease in rattle indices given this configuration, where no significant change is seen in Figure 136. These results are similar to the previous results for the alteration of inertias within the reverse idler configuration, when considering the unladen gear set response observes decreasing rattle indices, while smaller deviations are seen in the laden gear pairs. Also, as with the previous case, the change in rattle indices for the idler and pinion gear pair are not significantly altered. As the relative inertias of the gears are similar and the pitch radii of the pinion and idler are comparable, the set is less sensitive to changes in angular acceleration when laden. One final observation of note is that due to decreases in rattle indices at all sampling locations for the first gear set, the second gear set also observes a decrease in the ratio between the root mean square driven and driving angular accelerations.
1st Gear Pinion and Wheel

Input Speed (RPM)

2nd Gear Idler and Pinion

Input Speed (RPM)

2nd Gear Idler and Wheel

Input Speed (RPM)

Figure 135 Reverse Idler 2nd Gear Inertia Variance Rattle Response (First Gear Set)

Figure 136 Reverse Idler 2nd Gear Inertia Variance Whine Response (Second Idler and Pinion)

Figure 137 Reverse Idler 2nd Gear Inertia Variance Whine Response (Second Idler and Wheel)
Table 13 Reverse Idler Modal Frequencies (First Gear Pinion Synchronizer)

<table>
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Dual Idler Configuration

The final gearbox evaluated for alterations of driveline inertias on rattle was the dual idler configuration (Figure 29). Figures 138 through 141 are given for the rattle responses of the first and second gear sets. Figures 138 and 139 present the results for the laden gear set where no significant deviation in rattle indices are observed for the idler and wheel mesh and small decreases in rattle indices are observed for the idler and pinion mesh. The unladen idler and wheel mesh shown in Figure 140 displays an overall decrease in rattle indices for all input angular velocities. Figure 141 shows smaller decreases in rattle magnitudes for the pinion and idler mesh which is reasonable given the small inertial difference between the driveline components. The decrease in rattle indices may also be linked to the fact that the second gear idler and pinion are loaded with a synchronizer arrangement on the second gear wheel. When switching the arrangement to the pinion, a smaller ratio of angular accelerations is observed (load is now taken from the first gear set across the wheel and idler), and the idler can act as a more effective rattle isolator.
Figure 138 Dual Idler 1st Gear Inertia Variance Whine Response (First Idler and Wheel)

Figure 139 Dual Idler 1st Gear Inertia Variance Whine Response (First Idler and Pinion)

Figure 140 Dual Idler 1st Gear Inertia Variance Rattle Response (Second Idler and Wheel)
2nd Gear Idler and Pinion

![Graph showing gear rattle index vs input speed](image)

**Figure 141** Dual Idler 1st Gear Inertia Variance Whine Response (Second Idler and Pinion)

**Table 14** Dual Idler Modal Frequencies (Second Gear Pinion Synchronizer)

<table>
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<tr>
<th>Mode</th>
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<th>Neutral Bending (Hz)</th>
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<th>Second Gear Bending (Hz)</th>
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Shown in Figures 142 through 145 are the rattle responses for second gear when the synchronizer arrangement is moved from the first gear wheel to the pinion. As with the previous results, decreases are observed for both unladen gear pairs. Additionally, the idler and wheel mesh has significantly larger differences in rattle index magnitudes than the idler and pinion mesh. Figure 145 shows the rattle indices for the laden idler and
wheel with decreasing rattle indices given for the synchronizer arrangement on the first gear pinion. Conversely, the rattle indices for this arrangement exceed those of the baseline configuration for the idler and pinion mesh. However, the deviation between the rattle indices for the same input speed is less than that of the idler and wheel mesh (206.7% to 3580.1% comparatively). This configuration is different than that of the previous set, as the second gear wheel is upstream the laden set, which has a load placed along the mesh prior to the output power exiting the system.

![Graph](image1)

**Figure 142 Dual Idler 2nd Gear Inertia Variance Rattle Response (First Idler and Wheel)**

![Graph](image2)

**Figure 143 Dual Idler 2nd Gear Inertia Variance Whine Response (First Idler and Pinion)**
Figure 144 Dual Idler 2nd Gear Inertia Variance Whine Response (Second Idler and Wheel)

Figure 145 Dual Idler 2nd Gear Inertia Variance Whine Response (Second Idler and Pinion)

Table 15 Dual Idler Gearbox Modal Frequencies (First Gear Pinion Synchronizer)

<table>
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<th>Mode</th>
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<th>First Gear</th>
<th>Second Gear</th>
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<td>1113.90</td>
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<td>903.60</td>
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<td>1341.00</td>
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<td>15</td>
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</tr>
<tr>
<td>16</td>
<td>1280.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rigidity

The resistance to elastic deformation, or increased rigidity of the internal components of each of the three theoretical gearbox assemblies was tested by altering their cross-sectional dimensions. In particular, the cross-section of the input and output shaft was altered, making sure to keep the mass of the component constant and in order to increase rigidity. By altering the natural frequencies of the gearbox assembly, drive rattle resonance can be controlled with relation to the structure borne frequencies related to the input fluctuation and meshing frequencies of the gearbox. Alteration of component rigidities was used as a method to improve upon the resonant characteristics of the gearbox assemblies, as described in the preceding sections of this paper.

Rigid Cross-Sections

Shown in Figures 146 and 147 are cross-sections of the input and output shafts of the basic two-speed gearbox configuration respectively. The hollow cross sections shown at the bottom of each figure were designed for added rigidity by increasing the moment of inertia, while keeping the mass of the shaft constant for all sections (same cross-sectional area). Consequently, the natural frequencies of the system with the more rigid of the two shafts are given in Table 16. Comparisons between these frequencies and those in Table 7 show an increase for all three load conditions, with significant increases in the lower ranges of frequencies. Significant differences between rattle indices were expected due to a maximum increase of 33.2% for the fundamental frequencies, changing the resonant characteristics for driven rattle. Additionally, the slight change in inertia for the input and output shafts was expected to influence the rattle response for neutral, first and second gear.
Figure 146 Two-Speed Base (Top) and Rigid (Bottom) Input Shaft Cross Sections

Figure 147 Two-Speed Base (Top) and Rigid (Bottom) Output Shaft Cross Sections
Table 16 Two-Speed Gearbox Modal Frequencies (Bored Shafts)

| Mode | Neutral | | | First Gear | | | Second Gear | | |
|------|---------|------|------|-------------|------|------|-------------|------|------|-------------|
|      | Axial   | Bending | Torsional | Axial | Bending | Torsional | Axial | Bending | Torsional |
| 1    | 221.90  |         |         | 222.60  |         |         | 228.20   |         |         |         |
| 2    | 231.40  |         |         | 239.80  |         |         | 261.70   |         |         |         |
| 3    | 330.50  |         |         | 334.60  |         |         | 336.50   |         |         |         |
| 4    | 398.60  |         |         | 401.70  |         |         | 384.30   |         |         |         |
| 5    | 413.20  |         |         | 413.50  |         |         | 414.30   |         |         |         |
| 6    | 475.10  |         |         | 543.30  |         |         | 459.50   |         |         |         |
| 7    | 554.60  |         |         | 601.30  |         |         | 561.70   |         |         |         |
| 8    | 679.10  |         |         | 753.60  |         |         | 711.60   |         |         |         |
| 9    | 741.10  |         |         | 791.00  |         |         | 726.80   |         |         |         |
| 10   | 834.90  |         |         | 1008.60 |         |         | 835.90   |         |         |         |
| 11   |         | 930.10  |         |         | 1020.30 |         |         | 922.30   |         |         |
| 12   | 1019.90 |         |         | 1047.10 |         |         | 1170.30  |         |         |         |
| 13   | 1065.90 |         |         | 1103.60 |         |         | 1254.00  |         |         |         |
| 14   | 1165.60 |         |         | 1175.10 |         |         | 1297.40  |         |         |         |

Shown in Figures 148 and 149 are cross-sections of the input and output shafts for the reverse idler configuration respectively. As with the previous design practice for the two-speed gearbox, hollow cross-sections were used to increase the overall rigidity of the reverse idler configuration over the frequency range between the input speed and maximum meshing frequency of the gearbox structure. Table 17 is given for the natural frequencies of the gearbox with hollow shaft members, which can be compared to Table 8 from the Torque Fluctuation section. Again, an overall increase in natural frequencies is observed for the gearbox structure with a maximum difference in fundamental frequencies of 21.8%. This was again expected to cause a significant difference in rattle indices for the rattle response in neutral, first and second gear.
Figure 148 Reverse Idler Base (Top) and Rigid (Bottom) Input Shaft Cross Sections

Figure 149 Reverse Idler Base (Top) and Rigid (Bottom) Output Shaft Cross Sections
Table 17 Reverse Idler Gearbox Modal Frequencies (Bored Shafts)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial</td>
<td>Bending</td>
<td>Torsional</td>
</tr>
<tr>
<td>1</td>
<td>197.90</td>
<td>205.70</td>
<td>209.90</td>
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<tr>
<td>2</td>
<td>209.30</td>
<td>213.90</td>
<td>216.70</td>
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<td>226.40</td>
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<td>10</td>
<td>587.50</td>
<td>797.00</td>
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<tr>
<td>11</td>
<td>768.50</td>
<td>998.20</td>
<td>841.80</td>
</tr>
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</table>

Shown in Figures 150 and 151 are cross-sections of the input and output shafts for the dual idler configuration respectively. As with the previous design practice for the reverse idler gearbox, hollow cross-sections were used to increase the overall rigidity of the dual idler configuration over the frequency range between the input speed and maximum meshing frequency of the gearbox structure. Table 18 is given for the natural frequencies of the gearbox with hollow shaft members, which can be compared to Table 9 from the Torque Fluctuation section. Again, an overall increase is observed for most natural frequencies of the gearbox structure with a maximum difference in fundamental frequencies of 9.6%. This was again expected to cause a significant difference in rattle indices for the rattle response in neutral, first and second gear. Finally, it is important to note that the first two frequencies in second gear actually observe a decrease for the dual idler configuration. In addition, this gearbox observes the smallest change in rigidity by comparison to the two previous gearbox configurations.
Figure 150 Dual Idler Base (Top) and Rigid (Bottom) Input Shaft Cross Sections

Figure 151 Dual Idler Base (Top) and Rigid (Bottom) Output Shaft Cross Sections
Table 18 Dual Idler Gearbox Modal Frequencies (Bored Shafts)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial</td>
<td>Bending</td>
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</tr>
<tr>
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<td>339.70</td>
<td>660.70</td>
<td>677.40</td>
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<tr>
<td>6</td>
<td>388.30</td>
<td>724.80</td>
<td>741.40</td>
</tr>
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<td>7</td>
<td>486.80</td>
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</tr>
<tr>
<td>15</td>
<td>1323.20</td>
<td></td>
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</tr>
</tbody>
</table>

Two-Speed Configuration

Using the cross-sections of the shafts shown in Figures 146 and 147, the effect of increased rigidity was tested on the two-speed gearbox configuration. Given in the Figures below are the rattle responses for neutral, first and second gear with the upper and lower shaft cross sections for the solid (baseline) and hyphenated lines respectively. Under neutral rattle conditions, increased rigidity in the input and output shaft causes an increase in the magnitude of rattle indices in both unladen gear pairs (Figures 152 and 153). The pair with the largest deviating magnitudes is the pair with the largest speed ratio. This is to be expected, as the RMS acceleration between the pinion and wheel would be greatest for the first gear pair. In addition, the increase in rotational inertia of the shaft would have the largest effect on the gear set with the smallest pinion, as either pinion is integral with the input shaft.
The rattle responses for the laden and unladen gear sets in first gear are given in Figures 154 and 155 respectively. The second gear pinion and wheel mesh (unladen gear set) has larger magnitudes of rattle indices for the more rigid of the two gearboxes as shown in Figure 155. Additionally, peaks in the unladen gear mesh are observed at 2750 and 3500 rpm (45.833 and 58.33 Hz), compared to 2500 and 3500 (41.67 and 58.33 Hz) for the baseline rattle response in first gear. The frequencies associated with the first and second input speed are 91.66, 183.33, 275, and 687.5 Hz and 116.66, 233.33, 350 and
875 Hz respectively. Possible resonant frequencies for both input speeds include the second and third modes (239.8 and 334.6 Hz) in first gear for the 3500 rpm input.

![1st Gear Pinion and Wheel](image1)

**Figure 154 Two-Speed Rigidity Whine Response for 1st Gear (First Gear Set)**

![2nd Gear Pinion and Wheel](image2)

**Figure 155 Two-Speed Rigidity Rattle Response for 1st Gear (Second Gear Set)**

The rattle responses for the unladen and laden gear sets in first gear are given in Figures 156 and 157 respectively. Again, the increased rigidity increased the threshold for rattle in the unladen gear set, where two peaks are observed at 2500 and 3750 rpm (41.67 and 62.5 Hz). As with the previous case, the 2500 rpm input speed is associated with frequencies of 91.66, 183.33, 275 and 687.5 Hz. The 3750 rpm input speed is associated with forcing frequencies of 125, 250 and 375 Hz with a meshing frequency of
937.5Hz. For an input speed of 2500 Hz, resonance is possible for the first torsional and seventh bending modes (261.7 and 711.6 Hz). Additionally, the second peak can occur for resonance with the first and second torsional modes (261.7 and 922.3 Hz), as well as the third bending mode (384.3 Hz) as found in Table 16. As the first torsional mode is common for both input speeds, it is assumed to be the cause for the increase in rattle threshold at the aforementioned input speeds. FFT’s of the backlash responses are given in Figure 158 for the 2500 and 3750 rpm input speeds respectively, confirming this assumption.

![Figure 156 Two-Speed Rigidity Rattle Response for 2nd Gear (First Gear Set)](image1)

![Figure 157 Two-Speed Rigidity Rattle Response for 2nd Gear (Second Gear Set)](image2)
Reverse Idler Configuration

Using the cross sections shown in Figures 148 and 149 for the input and output shafts respectively, the rattle responses shown in Figures 159 through 167 were collected for neutral, first and second gear loading conditions. Figures 159 through 161 are given for neutral loading conditions, where less significant deviations in rattle indices are seen over the range of input speeds (0.019 to 10.49%). This is to be expected as the reverse
idler configuration is subject to smaller increases in natural frequencies than the two-speed configuration. Additionally, the reverse idler configuration has fewer natural frequencies below the highest meshing frequency of the gearbox. Thus, smaller changes to the dynamics of the system (inertia and rigidity), in addition to large torque fluctuations result in a system which is insensitive to changes in rattle threshold for this parametric study.

**Figure 159 Reverse Idler Rigidity Rattle Response for Neutral (First Gear Set)**

<table>
<thead>
<tr>
<th>RPM</th>
<th>Baseline Gearbox</th>
<th>Rigid Gearbox</th>
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<tbody>
<tr>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>625</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1250</td>
<td></td>
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</tr>
</tbody>
</table>

**Figure 160 Reverse Idler Rigidity Whine Response for Neutral (Second Idler and Pinion)**
Figure 161 Reverse Idler Rigidity Rattle Response for Neutral (Second Idler and Wheel)

Another result of interest when studying the reverse idler configuration due to changes in the rigidity of the system is seen in Figures 162 through 164. Specifically, Figures 163 and 164 show that the response seems to shift or move in a direction of increasing natural frequencies. This is related to the fact that the increases in natural frequencies are on the order of deviations between increased input speeds. The difference between each sample point is roughly 4.16 Hz, or 8.33, 16.67 and 25 Hz when related to the input forcing function. Evaluating the mode shapes of the frequencies associated with Tables 8 and 17, similar modes are within this tolerance region explaining the subsequent phenomenon.

Figure 162 Reverse Idler Rigidity Whine Response for 1st Gear (First Gear Set)
Figures 165 through 167 show similar characteristics to those in Figures 162 through 164 for second and first gear loading conditions, respectively. More specifically, the rattle responses seem to shift to a region of higher natural frequencies, reflecting the additional stiffness of the system. In addition, only small deviations in the magnitude of rattle indices are observed (0.78 to 19.19%, not accounting for shift in frequencies), which is thought to be the result of the inherent rigidity of the system. Overall, the three loading conditions display similar trends in that the additional rigidity does little to decrease the rattle threshold of the system. However, the peaks shown in the drive rattle
responses are shifted to a region of higher natural frequencies. Depending on the design requirements, these results would be useful in setting a range of optimal input speeds.

Figure 165 Reverse Idler Rigidity Rattle Response for 2nd Gear (First Gear Set)

Figure 166 Reverse Idler Rigidity Whine Response for 2nd Gear (Second Idler and Pinion)

Figure 167 Reverse Idler Rigidity Whine Response for 2nd Gear (Second Idler and Wheel)
Dual Idler Configuration

Using the cross-sections seen in Figures 150 and 151 for the input and output shaft respectively, the effect of increased rigidity was studied for the dual idler configuration. While the two-speed gearbox shows an overall increase in natural frequencies from the lowest input speed to the largest meshing frequency, the dual idler configuration shows significant increases in natural frequencies (>10% deviation) above 500 Hz for neutral, first gear and second gear. Additionally, there is no decrease in the number of natural frequencies below the highest meshing frequencies as with the reverse idler configuration (13 compared to 11 for the baseline and rigid model respectively). As a result, the rattle responses for the dual idler configuration do not display the same relative increases in rattle indices as indicated by Figures 168 through 171 for neutral loading conditions. Figures 168 and 169, show decreases in rattle indices for the change in rigidity for both idler and wheel meshes of the first and second gear sets respectively. Conversely, an increase in rattle indices are shown in Figure 171 for the idler and pinion mesh of the second gear set. This increase is different from the response for the idler and wheel meshes due to the fact that both the idler and pinion are integral with the shaft. Additionally, the idler and pinion mesh has a pinion driving a smaller gear, and the rigidity of the input shaft is more significantly altered by changes to the cross-sectional configuration. As a result, the response is akin to that of the two-speed gearbox, albeit less pronounced as the inertial difference between the pinion and idler is not as great as that of the idler and wheel. Similarly, the pinion and idler mesh from Figure 169 has the smallest change in rattle indices as it is the mesh with the smallest difference in inertia (0.34 to 10.49% compared to 5.29 to 23.27%). Finally, the reduction in rattle indices for the idler and wheel meshes is thought to be the result of the fact that no significant
increase in natural frequencies are seen in neutral rattle conditions (<5%). In fact, some of the contributing natural frequencies (below 600 Hz), actually add flexibility to the gearbox.

Figure 168 Dual Idler Rigidity Rattle Response for Neutral (First Idler and Wheel)

Figure 169 Dual Idler Rigidity Whine Response for Neutral (First Idler and Pinion)

Figure 170 Dual Idler Rigidity Rattle Response for Neutral (Second Idler and Wheel)
Figures 172 through 175 give the rattle responses for first gear, given changes to the overall rigidity of the dual idler configuration. The laden gear set, shown in Figure 172 and 173, indicate a slight increase in rattle indices over the range of input speeds. As with the neutral rattle response for the unladen second gear set, Figure 175 shows an increase in rattle threshold for the idler and pinion mesh. Furthermore, the idler and wheel mesh shows an overall decrease in rattle indices for the idler and wheel mesh shown in Figure 174. These results are comparable to the previous results, after taking into consideration the laden first gear set. To reiterate, the ratio of rattle indices is typically smaller for the laden gear set, as the synchronizer is engaged for the wheel and the response is dominated by the forced input fluctuation and is typically devoid of double sided impacts. Figure 174 also exhibits a behavior which was observed for the reverse idler configuration in that a peak of the baseline response at 4250 rpm (70.833 Hz) has been shifted to a higher input speed. This is reasonable given that the dual idler configuration does exhibit increases in rigidity for the range of natural frequencies associated with increased input speeds (158.33-475 Hz).
Figure 172 Dual Idler Rigidity Whine Response for 1st Gear (First Idler and Wheel)

Figure 173 Dual Idler Rigidity Whine Response for 1st Gear (First Idler and Pinion)

Figure 174 Dual Idler Rigidity Rattle Response for 1st Gear (Second Idler and Wheel)
Figure 175 Dual Idler Rigidity Whine Response for 1st Gear (Second Idler and Pinion)

Figures 176 through 179 give the rattle responses for second gear, given changes to the overall rigidity of the dual idler configuration. The laden gear set, shown in Figures 178 and 179, indicates an increase in rattle threshold for the idler and pinion mesh and a decrease for the idler and wheel mesh. The ratio between the baseline and rigid model rattle indices is markedly increased compared to the previous two data sets (ex. 16.81% to 26.09% average for the 2nd gear idler and wheel in first and second gear, respectively), which is due to the relative difference in inertia between the gear set. With a smaller speed ratio, the inertial difference between the pinion, idler and wheel is greater, thus leading to the results shown in Figures 178 and 179. The unladen gear set, shown in Figures 176 and 177 display similar trends to those from neutral loading conditions. However, the addition of a laden gear set results in a resonant peak at 4000 rpm (66.67 Hz), which has possible resonance with the second bending mode (263.0 Hz) as well as the third and fourth axial modes (783.6 and 826.4 Hz). Figure 180 for the Fourier transform of the backlash response of the pinion and idler mesh of the first gear set shows that the second bending mode (263.0 Hz) interaction with the second harmonic of the input speed fluctuation (266.7 Hz) causes the distinct increase in rattle at 4000 rpm.
Additionally, there is an interaction between the laden meshing frequency of 800 Hz and the third and fourth axial modes.

**Figure 176 Dual Idler Rigidity Rattle Response for 2nd Gear (First Idler and Wheel)**

**Figure 177 Dual Idler Rigidity Whine Response for 2nd Gear (First Idler and Pinion)**

**Figure 178 Dual Idler Rigidity Whine Response for 2nd Gear (Second Idler and Wheel)**
Figure 179 Dual Idler Rigidity Whine Response for 2nd Gear (Second Idler and Pinion)

Fourier Transform of Backlash Response

Figure 180 FFT of Rigid Dual Idler Gearbox in 2nd Gear Unladen Gear Set (4000 rpm = 66.67 Hz)
CHAPTER VI

EFFECT OF EXTERNAL ENCLOSURE

Typical models used to evaluate gear rattle, such as the reduced order torsional models created by Fujimoto et al. (1987), Singh et al. (1989) and Padmanabhan and Singh (1993) have been created to evaluate the parameters associated with gear rattle, neglecting the addition of an external enclosure. Traditional practices for further evaluation of the path of vibration from the source to the receiver involve objective and subjective assessments of gearbox rattle noise. Heinrichs and Bodden (1999) and Forcelli et al. (2004) presented such studies, which were conducted for post production transmissions taking measurements at various locations through the path of vibration and using test subjects to rate the level of rattle. Thus, evaluation of rattle was achieved primarily through experimentation, with analytical validation used as a means to pinpoint an explanation for perceived reductions. Further work, conducted by Campbell et al. (1997), added the effect of housings attached to driveline components to account for the natural frequencies associated with the gearbox structure for a computer simulated gear whine model. Such work has been built upon by Sellgren and Akerblom (2005), and Kostic and Ognjanovic (2007) who account for the gearbox housing on the noise spectra of transmissions. The current research will try to adapt these methods, adding the housing inertia and stiffness to the gear rattle models from the Parametric Analysis. Using similar computer simulation techniques outlined by Korde and Wilson (2009), Romax Designer is utilized to evaluate the effect of a solid element mesh generated for the external enclosures of three simplified two speed gearboxes. The bearings are no longer assumed
to be grounded, and the external boundary conditions will be defined by the casing structure. In addition, Fast Fourier Transforms (FFT) of the backlash responses of unladen gear sets were used as a means of comparison to evaluate the dominant frequencies associated with the gearbox housing. The added benefit to this design technique was twofold. First, the models which were evaluated were able to more accurately represent a physical model significantly influenced by the housing stiffness when under large loads and torque fluctuations (Korde and Wilson, 2009). Second, the procedures outlined herein aim to reduce the monetary investment of subjective or objective analysis of post production transmissions by circumventing the process through the use of computer simulation.

**Gearbox Casings**

The three gearbox configurations described in Chapter II: Problem Statement are shown with their respective gearboxes in Figures 181 through 183 (dimensions in Appendix F). Each layout shown has a side profile of the gearbox, with the imported solid element mesh. The solid element mesh was imported into Romax Designer given the procedures outlined in Appendix A, and were then connected to the internal components of the gearbox by the cylindrical face of the bearing elements. Thus, the path of vibration originates at the unladen gear set and continues to the shafts, through the bearings and eventually to the fixed supports of the gearbox housing and the surrounding atmosphere. Each gearbox configuration has fixed boundary conditions at the mounting locations as indicated by red markers in Figures 181 through 183 (right). The benchmark mounting locations of the gearbox housings were similar to those from the gearbox described in Chapter III: Dynamic Analysis Validation. The natural frequencies for these configurations are shown in Tables 19 through 21, which can be used to determine
resonance for the following analyses. Alterations in the rigidity of each of the three
gearboxes shown below were tested through changes in the elastic modulus, density,
internal component rigidities and locations of fixed boundary conditions. Additionally,
some tests were conducted on the gearboxes after increasing or decreasing the wall
thickness. Through these methods, conclusions regarding the effect of including the
gearbox casing stiffness and inertias on the sampled gear rattle indices were used to
formulate methods for reducing gearbox rattle.

Figure 181 Two-Speed Gearbox Casing Layout Side (Left) and Top (Right) View
Figure 182 Reverse Idler Gearbox Casing Layout Side (Left) and Front (Right) View

Figure 183 Dual Idler Gearbox Casing Layout Side (Left) and Top (Right) View
### Table 19 Basic Two-Speed Gearbox Modal Frequencies (with Enclosure)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>Axial</td>
<td>Bending</td>
</tr>
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<td>1</td>
<td>146.40</td>
<td>144.80</td>
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<tr>
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### Table 20 Reverse Idler Gearbox Modal Frequencies (with Enclosure)

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Table 21 Dual Idler Gearbox Modal Frequencies (with Enclosure)

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Two-Speed Gearbox

Shown in Figures 184 and 185 are the rattle responses for the two-speed gearbox configuration given the addition of an external enclosure with the aforementioned fixed boundary condition locations. Both the first and second gear set show a decrease in rattle indices over the range of input speeds, indicating that the gearbox acts to isolate the amount of rattle within the gearbox structure. Furthermore, the effect of the gearbox enclosure is pronounced enough to emphasize the importance of including its rigidity for dynamic analysis (≤ 39.37% reduction). It should also be noted that the response seen in Figure 184 shows the sampled data points intersecting at intervals of 150 rpm, or 2.5 Hz. Overall, the isolating effect is similar to that documented by Sellgren and Akerblom.
(2005), where the gearbox acts to reduce the amount of perceivable rattle with a closed casing.

The first gear rattle responses are shown in Figures 186 and 187, showing an overall reduction in rattle indices over the range of input speeds (3.54 to 56.81%). The unladen set, shown in Figure 187, has more distinct differences in rattle index magnitudes at low input speeds. In addition, there are two sudden decreases in rattle indices at 2000 and 3000 rpm (33.33 and 50 Hz) while the remaining values are constant at roughly 0.7.
Again, these responses indicate that the level of rattle has been decreased for the system, with the gearbox casing acting as a vibration isolator.

Figure 186 Two-Speed 1st Whine Response with External Casing (First Gear Set)

Figure 187 Two-Speed 1st Rattle Response with External Casing (Second Gear Set)

The second gear rattle responses are shown in Figures 188 and 189, showing an overall reduction in rattle indices over the range of input speeds (1.33 to 54.63%). The unladen set, shown in Figure 188, has less variance in the rattle response for the gearbox with a casing than that for the gearbox without a casing. While the gearbox again acts as an isolator for second gear loading conditions, there are some input speeds where the rattle index for the gearbox with a casing exceeds that of the gearbox without a casing. Not coincidently the 2750 rpm input speed (45.833 Hz) has a resonant frequency of 275
Hz which is close to the fourth mode (274.9 Hz). In addition, the meshing frequency (687.5 Hz) is close to the thirteenth mode (684.3 Hz). Thus, resonance again is crucial in determining regions where the rattle response will experience sudden increases, or where the casing has an unfavorable influence on the perceived of the gearbox structure.

![Graph 1](image1)

Figure 188 Two-Speed 2nd Rattle Response with External Casing (First Gear Set)

![Graph 2](image2)

Figure 189 Two-Speed 2nd Whine Response with External Casing (Second Gear Set)

**Reverse Idler Gearbox**

To further study the effect of an external enclosure on gear rattle, the reverse idler configuration was evaluated with the casing shown in Figure 182. The resulting rattle responses for neutral loading conditions are seen in Figures 190 through 192. The results
show less variance between the rattle response including an external enclosure and the baseline rattle response for the reverse idler (0.018 to 7.54%). However, at higher input speeds, there is some indication that the casing has a noticeable effect on the level of rattle within the gearbox structure.

Figure 190 Reverse Idler Neutral Rattle Response with External Casing (First Gear Set)

Figure 191 Reverse Idler Neutral Whine Response with External Casing (Second Idler and Pinion)
Further analysis of the reverse idler configuration with the addition of an external enclosure is conducted for first gear loading conditions, with results shown in Figures 193 through 195. Here, the casing has an even less significant effect on the rattle response (0.336 to 4.25% variation), with almost identical responses for the gearbox both with and without an external casing. As a result, it can be assumed that the casing (as shown in Appendix E) is inefficient in either isolating or exciting the gearbox rattle response.
A final test evaluating the effect of an external casing on the reverse idler configuration is conducted for second gear loading conditions. Figures 196 through 198 show the rattle responses for second gear, where the unladen gear pair results are shown in Figure 196. Again, the rattle responses are not significantly altered given the addition of an external enclosure (0.0285 to 4.169% variation for rattling pair). All three rattle responses indicate that the casing has an unpronounced effect on rattle for the reverse idler configuration. As the internal components for the reverse idler are stiffer than the two-speed configuration, and previous results (Chapter IV – Parametric Analysis) indicate that the idler shaft acts as a vibration isolator, the casing has no noticeable effect.
on the rattle threshold for this computer simulated model. Furthermore, the casing is
decidedly stiffer than that of the two-speed gearbox. As such, the gearbox casing has
approximately the same boundary conditions as those imposed for the configuration
without an external enclosure as it is sufficiently rigid. These assumptions will be further
evaluated in a case study following the results for the dual idler configuration.

Figure 196 Reverse Idler 2nd Rattle Response with External Casing (First Gear Set)

Figure 197 Reverse Idler 2nd Whine Response with External Casing (Second Idler and Pinion)
Dual Idler Gearbox

The dual idler configuration rattle responses for neutral loading conditions, given the addition of the enclosure shown in Figure 183 are shown in Figures 199 through 202. As with the previous study for the reverse idler configuration, there is no pronounced difference between the rattle responses with or without an external enclosure. The largest variations between the rattle indices are seen in the first gear set (maximum of 9.94% variation), which is subject to the largest speed ratios. As such, variations in the relative accelerations between gears are inherently more sensitive for the gear set with the largest speed ratio. However, these variations cannot be assumed to be significant, given the relative changes in magnitudes between configurations (0.11 to 9.94% with an average 2.24% deviation).
Figure 199 Dual Idler Neutral Rattle Response with External Casing (First Idler and Wheel)

Figure 200 Dual Idler Neutral Rattle Response with External Casing (First Idler and Pinion)

Figure 201 Dual Idler Neutral Rattle Response with External Casing (Second Idler and Wheel)
Further results collected for the dual idler configuration indicate that first gear loading conditions result in only small differences in rattle indices for the structure with and without a casing (average of 1.43% deviation). Figures 203 through 206 show the rattle responses for first gear loading conditions where Figure 205 has results for the rattling gear pair. In all of the responses, there is a noticeable difference between the gearbox without an enclosure and the configuration with a casing at 4250 rpm (70.833 Hz), where the meshing frequency is in the range of the thirteenth mode shape (1062.5 and 1057.6 Hz respectively). However, the difference is not overly pronounced except for the idler and pinion response for the engaged gear set (Figure 204), with no noticeable effect on the rattling pair.
Figure 203 Dual Idler 1st Whine Response with External Casing (First Idler and Wheel)

Figure 204 Dual Idler 1st Whine Response with External Casing (First Idler and Pinion)

Figure 205 Dual Idler 1st Rattle Response with External Casing (Second Idler and Wheel)
The second gear rattle responses for the dual idler configuration with an external enclosure are shown in Figures 207 through 210. Figures 207 and 208 show the unladen set, with Figure 207 representing the rattling pair. As with the previous case, some noticeable difference in rattle indices are seen at a high frequency input rotational speed (4500 rpm or 75 Hz). Overall, this configuration is similar to the reverse idler configuration in that the idler shafts are rigid and act as vibration isolators, where the influence of an external enclosure is less pronounced than with the two-speed configuration. Additionally, the casing is stiffer than the two-speed configuration casing which is another possible root cause for the lack of rattle reduction given an external enclosure.
Figure 207 Dual Idler 2nd Rattle Response with External Casing (First Idler and Wheel)

Figure 208 Dual Idler 2nd Whine Response with External Casing (First Idler and Pinion)

Figure 209 Dual Idler 2nd Whine Response with External Casing (Second Idler and Wheel)
Case Study #1: Increased Rigidity in Basic Two-Speed Gearbox

In order to build upon the conclusions drawn from the results from this section, a study was conducted on the effect of increasing the rigidity of the internal components for the basic two-speed gearbox. One assumption from the previous section was that the lack of variation between the rattle responses for the reverse and dual idler configuration was due to the rigidity of internal components. Thus, using the cross sections from Figures 146 and 147 and the casing and boundary conditions shown in Figure 181, the rigidity was increased in the basic two-speed gearbox to attempt to obtain similar decreases in variation between rattle responses. The results for neutral loading conditions for this test are given in Figures 211 and 212. Clearly, the variation between the rattle responses is much smaller (0.026 to 15.52% compared to 0.060 to 39.37% variation) when compared to the difference in rattle responses in Figures 184 and 185 for the first and second gear sets respectively. It can be reasonably assumed for neutral loading conditions, that a gearbox with rigid internal components and a semi-flexible casing has a
similar rattle response to the casing without a casing and fixed boundary conditions applied at the cylindrical faces of the shaft bearings.

![Graph](image1)

**Figure 211** Rigid Basic Two-Speed Neutral Rattle Response with External Casing (First Gear Set)

![Graph](image2)

**Figure 212** Rigid Basic Two-Speed Neutral Rattle Response with External Casing (Second Gear Set)

Further studying the effect of rigid internal components with first gear loading conditions again shows smaller variance when comparing Figures 213 and 214 with Figures 186 and 187 for the first and second gear sets respectively (0.1314 to 44.2462% compared with 3.34 to 56.81%). Additionally, it is seen that the gearbox natural frequencies have an influence on the overall response when observing the difference in peak rattle indices in Figure 214 for the rattling pair. Reviewing the results for second gear in Figures 215 and 216, the increased rigidity has a smaller overall effect on
changing the rattle response between the model with and without a casing. This can be attributed to the reduced speed ratio, and further increases in rigidity as observed by the natural frequencies in Table 22.

Figure 213 Rigid Basic Two-Speed 1st Gear Whine Response with External Casing (First Gear Set)

Figure 214 Rigid Basic Two-Speed 1st Gear Rattle Response with External Casing (Second Gear Set)
Table 22 Case Study #1 Basic Two-Speed Gearbox Modal Frequencies

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Figure 215 Rigid Basic Two-Speed 2nd Gear Rattle Response with External Casing (First Gear Set)
Case Study #2: Varied Mounting Locations for Basic Two-Speed Gearbox

Another test conducted for the basic two-speed gearbox was to change the fixed locations of the casing shown in Figure 181 to the faces nearest the bearings as shown with the red markers in Figure 217. The objective of this analysis was to prove that boundary conditions imposed in a fashion similar to that of the gearbox with a casing would provide similar results, while having minimal effects from the flexibility of the gearbox enclosure (Table 7 compared with Table 23 natural frequencies). Figures 218 and 219 are given for neutral loading conditions in the first and second gear sets, respectively. These results show less variation between rattle responses than either of the previous studies for the two-speed configuration with the addition of an external enclosure (average of 3.86% deviation). Still, some noticeable differences are observed for the unladen pairs indicating that the casing has some effect on the perceived rattle of the system.
Figure 217 C.S. #2 Basic Two-Speed Casing Front (left) and Top (right) Layout

Table 23 Case Study #2 Basic Two-Speed Gearbox Modal Frequencies

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The rattle responses for first and second gear are shown in Figures 220 through 223, with the unladen pairs given by Figures 221 and 222. The rattling pair for first gear loading conditions again shows slight differences in the rattle responses for the gearbox with and without the casing stiffness in Figure 221 (4.06% average). However, differences are noticeable in the range of 2500 to 4000 rpm (41.66 to 66.67 Hz), where the casing is shown increasing the rattle threshold for the gearbox enclosure. This effect is significant, as it shows the flexibility of the system can increase the level of rattle given improper selection of mounting locations for the casing. However, these effects may not be consistent between loading conditions as indicated by Figure 222 for the unladen pair...
in second gear. Here, the rattle response shows an overall reduction in indices over the range of input speeds (0.082 to 14.3%), where the gearbox is less sensitive at higher input speeds. These results show that many parameters must be taken into effect given the alteration of boundary conditions. Certain factors must be considered, and knowledge of the operational constraints and conditions are essential in obtaining favorable reductions in rattle. Here, it is clear that the first study for the two-speed casing provides favorable rattle responses given flexible internal components and boundary conditions applied at mounting conditions as shown in Figure 181.

Figure 220 C.S. #2 Two-Speed 1st Gear Whine Response with External Casing (First Gear Set)

Figure 221 C.S. #2 Two-Speed 1st Gear Rattle Response with External Casing (Second Gear Set)
Case Study #3: Flexible Dual Idler

The third and final analysis run was for a more flexible dual idler configuration with the addition of an external enclosure. After confirming that the basic two-speed configuration could be stiffened to reduce the variation between rattle responses for the gearbox with and without a casing, it was appropriate to reduce the stiffness of the internal components of the dual idler. The resulting responses should not only show significant variation between rattle responses for the gearbox with and without a casing; there should also be a notable reduction in rattle indices over the range of input speeds for
neutral, first and second gear loading conditions. Figure 224 shows the flexible dual idler, which has aluminum shafts ($E = 68.9$ MPa with density of $2700$ kg/m$^3$) with increased cross sectional area at the gears for the output shaft, and larger bores throughout the shaft components. Additionally, the gearbox can be seen with the casing, which has the same dimensions as those for the previous dual idler configuration analysis, but was assigned aluminum material properties.

Figure 224 Flexible Dual Idler with (left) and without (right) a casing

The resulting responses, as shown in Figures 225 through 228 for neutral loading conditions, have notable reductions in rattle indices over the range of input speeds (save for select data points). Exceptions to this are found in Figures 227 and 228 at an 800 rpm input speed (13.33 Hz), Figure 226 at 1050 and 1100 rpm (17.5 and 18.33 Hz), and Figure 225 at 950 rpm (15.83 Hz). However, these increases in rattle seem to be related to a shift in the rattle response, rather than an overall increase. Subsequently, there are marked decreases (2.62 to 18.43% variation) in rattle indices for the gearbox with a semi-
flexible casing indicating the enclosure added stiffness and inertia has isolated some of the rattle.

Figure 225 C.S. #3 Dual Idler Neutral Rattle Response with Casing (First Idler and Wheel)

Figure 226 C.S. #3 Dual Idler Neutral Whine Response with Casing (First Idler and Pinion)

Figure 227 C.S. #3 Dual Idler Neutral Rattle Response with Casing (Second Idler and Wheel)
Further analysis of the flexible dual idler configuration with the addition of an aluminum casing for first gear loading conditions presents similar results to those for neutral loading conditions. The unladen gear pair (2nd Idler and Wheel) in Figure 231, has a marked decrease in rattle indices for the gearbox with an aluminum casing, while the remaining responses indicate no significant change, some reduction, or peaks in rattle indices at select resonances (≤ 15.9% variation) associated with the frequencies in Tables 24 and 25. Here, Table 24 gives the natural frequencies of the gearboxes without an enclosure, and Table 25 has the natural frequencies for the flexible dual idler with the addition of an aluminum casing. These results are more favorable than those for the previous analysis for the dual idler configuration, as the casing has a noticeable influence on the rattle response (0.0125 to 5.437% reduction). In addition, the gearbox has been designed in such a way as to reduce the overall weight of the structure. This was done by reducing the amount of material used (bored shafts), and by using an alternative material (aluminum) for the shafts. Finally, the decreased stiffness of the idler shafts allows said components to act in such a way as to isolate some of the rattle within the gearbox, instead of exciting the system as indicated by previous results.
Table 24 Case Study #3 Flexible Dual Idler Gearbox Modal Frequencies (without Casing)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial</td>
<td>Bending</td>
<td>Torsional</td>
</tr>
<tr>
<td>1</td>
<td>102.70</td>
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<td>2</td>
<td>119.00</td>
<td>185.90</td>
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<td>124.40</td>
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<td>217.50</td>
<td>407.90</td>
<td>327.00</td>
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<td>5</td>
<td>220.20</td>
<td>484.50</td>
<td>448.70</td>
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<td>6</td>
<td>305.30</td>
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<td>326.40</td>
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<td>9</td>
<td>466.70</td>
<td>900.00</td>
<td>811.10</td>
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<td>10</td>
<td>577.90</td>
<td>985.80</td>
<td>903.20</td>
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<td>11</td>
<td>613.40</td>
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<td>12</td>
<td>897.70</td>
<td>1181.00</td>
<td>1196.40</td>
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<td>18</td>
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</table>

Table 25 Case Study #3 Flexible Dual Idler Gearbox Modal Frequencies (with Casing)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Neutral</th>
<th>First Gear</th>
<th>Second Gear</th>
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<tr>
<td></td>
<td>Axial</td>
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<td>101.50</td>
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<td>250.00</td>
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<td>288.80</td>
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<td>790.00</td>
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<td>788.70</td>
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<td>20</td>
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</table>
Figure 229 C.S. #3 Dual Idler 1st Gear Whine Response with Casing (First Idler and Wheel)

Figure 230 C.S. #3 Dual Idler 1st Gear Whine Response with Casing (First Idler and Pinion)

Figure 231 C.S. #3 Dual Idler 1st Gear Rattle Response with Casing (Second Idler and Wheel)
Figure 232 C.S. #3 Dual Idler 1st Gear Whine Response with Casing (Second Idler and Pinion)

Shown in Figures 233 through 236 are the rattle responses for the third and final loading condition, or second gear. Figure 233 represents the rattling gear pair, where the results are dissimilar to the previous trends for first gear and neutral loading conditions. While variation between the rattle responses is significant, an overall increase in rattle is seen with the addition of a flexible aluminum casing (up to 24.31%). However, these increases are found at resonant peaks, indicating an inherent sensitivity to rattle in second gear. More specifically, resonance occurs for the first, second and fourth bending mode (119.0, 149.7, 250.0 Hz) for the 1750, 2250 and 2750 rpm input speeds (29.166, 37.5 and 45.83 Hz). As such, it can be seen again that driveline resonance is of particular importance when considering drive rattles. Further refinement of the dual idler configuration suggested would be required for second gear loading conditions, as the system is inherently sensitive to rattle given the numerous peaks in rattle index due to driveline resonance. This is indicated both by the magnitude of rattle indices for the unladen gear pair, and by the number of peaks within the rattle response.
Figure 233 C.S. #3 Dual Idler 2nd Gear Rattle Response with Casing (First Idler and Wheel)

Figure 234 C.S. #3 Dual Idler 2nd Gear Whine Response with Casing (First Idler and Pinion)

Figure 235 C.S. #3 Dual Idler 2nd Gear Whine Response with Casing (Second Idler and Wheel)
Figure 236 C.S. #3 Dual Idler 2nd Gear Whine Response with Casing (Second Idler and Pinion)
CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Trends for transmission rattle levels due to variations in the input torque fluctuation, gear backlash, rigidity and reflected inertia were studied through the use of Romax Designer. Increases in backlash were found to increase the rattle threshold of models with small clearances (up to 373.8%), while sudden decreases in rattle threshold (up to 60.53%) were found for large increases in clearances between gears (given large amplitudes for fluctuating torque). Additionally, the reduction of amplitudes of the input torque fluctuation was found to decrease the level of rattle indices for the two-speed gearbox by up to 48.04%. Conversely, these effects were reduced or reversed given the isolating effects of one or more rigid idler shafts, and resulted in up to a 44.95% increase in rattle indices. Further, increases in rigidity were linked to distinct changes (up to 26.09%) in rattle threshold when taking into account the ability of the structure to both isolate and excite vibratory responses. Finally, the alteration of the reflected inertia of the system was found to be the most influential and complex parameter linked to the increase in rattle indices. Changes in the inertia of the drive line were found to increase the rattle indices of the system by up to 3580%. While the use of flexible idler shafts may be less common or desirable from a design standpoint, a study related to their effect on the vibratory response of the gearbox structure should be the focus of future research.

The second section of the research, dedicated to studying the effects of an external enclosure, points out methods which are beneficial in the reduction of gearbox rattle. Optimization or careful consideration regarding changes in the fixed locations of the gearbox structure significantly alters the level of rattle within the system. Clearly,
boundary conditions which are similar to those imposed by Romax Designer (fixed bearings) for rigid enclosures result in a rattle response which is similar in nature to models without an external enclosure. However, assuming a flexible enough enclosure (reduction of >5% in fundamental frequencies of gearbox), the rattle response can be significantly altered based on the resonant qualities of the driveline components. When boundary conditions are applied in such a way as to allow flexibility within the system, the rattle threshold can be reduced substantially, or up to 56.81% (where 5% reduction or less is considered insignificant). In addition, internal components which have been optimized for the input loads and weight are beneficial since overly rigid shafts, bearings and gears hinder potential reductions in rattle given the addition of an external enclosure.

Overall, it should be apparent that the reduction of rattle within the two-speed gearboxes can be achieved through many means. The gearbox must be optimized for weight and resistance to failure given the input load conditions. Factors which should be considered at this design stage include the clearances between gears, structural rigidity, reflected inertias of the system and input load fluctuation. After taking these factors into consideration, an external enclosure can be designed to reduce the level of rattle by being sufficiently flexible to alter the vibratory response while avoiding resonance with other driveline components or failure due to fatigue. Of course, other iterations will be necessary after completing these two stages in order to reach a suitable gearbox structure. In addition, these design considerations will have to be compatible with other NVH studies conducted on the same computational model. The added benefit to this method is that it avoids costly post-production correction of the gearbox design. The root causes of
rattle sensitivity can be established and corrected before manufacturing the structural components.

Future studies based on this research are planned to create more complete two-speed gearboxes, which build upon the current complexity of the three computer simulated models. These additions include the application of lubricated impacts, clutch hysteresis, pre-loaded bearings, mesh impact damping, a splined clutch and flywheel arrangement and vehicular or external inertias. Studies prior to those discussed herein implied that the addition of lubricants allows for an optimal range of gearbox bulk temperatures for the reduction of rattle. Further, the addition of a clutch-hub-spline arrangement is likely to introduce lower natural frequencies into the dynamic model. External inertias must also be added to the models to account for their effect on rattle sensitivity. Another design strategy that was not fully explored, but found to have a significant effect on rattle was the alteration of bearing types, or pre-loading bearings to dampen the vibratory response of the shafts. Finally, a clutch hysteresis factor, global damping coefficient and mesh impact damping constant can be applied to the system, given recent additions to the rattle module in Romax Designer. It should be noted that some of these design parameters are more aptly considered given a physical model for study. The clutch stiffness, hysteresis, global damping, and mesh impact damping would be more appropriately measured given pre-existing components such as a comparably sized flywheel and clutch (capable of sustaining the loads from a pre-existing powertrain). This can be accomplished using the model from Chapter III: Dynamic Analysis Validation, or another model which falls within the limitations of the current study (two-speed gearbox with an external enclosure).
This research presents a sound foundation for future work in the area of gearbox rattle. Both computational and analytical techniques have been used to study the vibratory response of physical and simulated two-speed gearboxes. These techniques were used to define the scope of the research and present topics of interest when designing pre-production gearbox structures.
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APPENDIX A

Procedure for importing casing into Romax Designer
Using the solid element mesh created in Appendix B, Romax Designer can be used to run static and dynamic analysis on a gearbox with an external enclosure. In order to import the elements into Romax Designer, the Gearbox Assembly worksheet must be open, which will provide the option in the toolbar as shown in Figure A.1. Choosing to “Add New Assembly/Component” the component option is checked and the Gearbox Casing is chosen as shown in Figure A.2. Using the dialog box shown in Figure A.3, the bearing element attachments to the gearbox casing can be chosen to create a stiffness matrix. Next, the Gearbox Casing worksheet can be activated by double clicking on the option shown in Figure A.4. After doing this, the menu in Romax Designer’s toolbar is seen as shown in Figure A.5. Choosing to “Import and Position FE Data”, the graphical user interface shown in Figure A.6 is presented. Using the rotation options and applying the proper translations to the gearbox, the bearing elements can be properly attached to the mesh for the external enclosure. After this, the eigenvectors and mass matrices for the gearbox enclosure are generated, and used for static and dynamic analysis of the enclosure.

Figure A.1 Romax Designer Components Toolbar Menu
Figure A.2 Component List

Figure A.3 Gearbox Stiffness Connections
Figure A.4 Root Assembly List

Figure A.5 Properties Menu
Figure A.6 FE Positioning

Figure A.7 Bearing Nodal Connections
Using the Properties menu shown in Figure A.4, the bearings elements can be attached to the by selecting Edit Node Connections. Once this option has been selected the dialog box shown in Figure A.7 appears, where Romax Designer can be used to Estimate nodal connections based on the appropriate search criteria. Once the surrounding nodes from the mesh have been identified, and each bearing is positioned correctly, all the node connections can be viewed by checking the box beside “Display all node connections” (Figure A.7) and clicking OK. Next, the gearbox casing mesh is used to create a condensed finite element model by selecting the option shown in the Analysis menu shown in Figure A.8. Once “Condense FE Model…” has been chosen, the dialog box in Figure A.9 appears. The boundary conditions defined in the NASTRAN output file (*.dat) can be applied by checking the box beside the currently defined constraints and load cases. A default of 20 modes is selected for Dynamic Analysis, and dynamic reduction of the FE model is done by selecting OK. Figure A.10 shows the dialog box that appears during dynamic analysis of the external enclosure.

![Figure A.8 Analysis Menu](image_url)
Condense Finite Constraints = -·· : ,: ' - ... ·••i•�•:,,

Currently defined constraints and load cases:

- Restraint case number 1

Dynamic analysis

- Run dynamic reduction
  - Number of modes: 20

Analysis options

- Solve using only first order elements
- Condense all FE components in gearbox

Figure A.9 Condense Finite Element Model Dialog Box

Figure A.10 Dynamic Reduction of FE Model
Using the Analysis menu shown in Figure A.8, Static Deflections of the finite element model can be approximated using Romax Designer. Again, the boundary conditions for the gearbox enclosure should be selected under Constraints and each loadcase should be solved for using bearing deflections. A similar analysis window to that shown in Figure A.10 appears, followed by the analysis results shown in Figure A.11. In order to switch between the loadcases, the pull down menu under Selected Loadcase can be used. Also, to re-run the analysis, Edit Settings must be chosen. This may be appropriate after choosing to edit the casing material or repositioning the enclosure using the options in the Properties menu (Figure A.5).

Figure A.11 Static Deflection Dialog Box
The final step in analyzing the response of the gearbox enclosure is to analyze the modal frequencies of the structure. From the Analysis menu shown in Figure 8, modal frequency analysis can be conducted by selecting the option for Component Mode Shape Analysis. The dialog box shown in Figure A.13 is similar to that in Figure A.11, except condensation nodes can be selected and Dynamic Analysis can be run for a particular number of nodes or over a specified frequency range. The condensation nodes correlate to the bearing connections and can be used to fix these locations when appropriate. After running the dynamic analysis (clicking on the Solve button), an error report should appear with zero warnings or errors as shown in Figure A.14. After the dialog box in
Figure A.14 is closed, the results for the dynamic analysis will appear as shown in Figure A.15. Using the scroll bar under the selected modes, each mode shape can be seen, with its corresponding natural frequency in hertz.

![Component Mode Shape Analysis Dialog Box](image)

Figure A.13 Component Mode Shape Analysis Dialog Box
Romax FE Solver Version 1.113

Figure A.14 Error Report

Component mode shape analysis complete

Results on:
- Stress
- Strain

Available values:
- Displacement Magnitude
- X Displacement
- Y Displacement
- Z Displacement
- Radial Displacement
- Angular Displacement
- Rotation Magnitude
- X Rotation

Figure A.15 Dynamic Analysis Results
After running analysis on the static and dynamic characteristics of the gearbox enclosure, dynamic analysis of the gearbox with all of its internal and external components can be completed. In order to do this, the gearbox assembly window must be activated from the root assembly menu as shown in Figure A.4. Next, Gearbox Dynamic Analysis is chosen from the Analysis menu shown in Figure A.16, and the dialog box in Figure A.17 is given. Using the Analysis Settings, the number of modes or range of natural frequencies can be chosen for the appropriate load case. Once the dynamic analysis is complete, the dialog box in Figure A.18 appears, listing the natural frequencies for the coupled and uncoupled mode shapes. In addition, right clicking on a natural frequency presents the option to view the mode shape related to the frequency. Shown in Figure A.19 is a cutaway view of a gearbox enclosure for one of its modal frequencies. By increasing or decreasing the mode number, each mode shape can be viewed after refreshing.
Figure A.17 Gearbox Dynamic Analysis

Figure A.18 Dynamic Analysis Results
Figure A.19 Gearbox Mode Shape Display
APPENDIX B

Solid Element Meshing of Gearbox Enclosure in Hypermesh
In order to import a gearbox casing component, Romax Designer requires that a finite element mesh be imported from NASTRAN, ANSYS or Abaqus. Altair Hyperworks’ Hypermesh software is capable of exporting a finite element mesh in the file format of each of these FEA software packages, and is a versatile tool for pre-processing CAD models for analysis. For this reason, Hypermesh was used to generate the solid element mesh for each gearbox casing. The file format deemed most convenient for use with Romax Designer was NASTRAN long format, with the extension *.dat. For the purposes of this research, the solid element mesh was generated using tetrahedron shaped elements, each with 4 nodes. Also, since each casing was created in SolidWorks given metric units, the density and modulus of elasticity are defined using the units kg/m\(^3\) and MPa respectively. The first step in creating the mesh in Hypermesh after opening the program is to import an IGES file created in a CAD software package as shown in Figure B.1.

![Figure B.1 Altair Hypermesh Import Graphical User Interface](image)

The next step in generating a mesh using Hypermesh is to select the 3D option as shown in Figure B.2. The user must then decide to mesh the CAD component by selecting the tetramesh
option in the menu that appears. Next, the part ID associated with the CAD component is to be designated as shown in Figure B.3. After the component is highlighted, the mesh is generated making sure to input an appropriate element size, and selecting 2D tria elements with 3D tetra shaped elements (not mixed or quad). After generating the mesh as shown in Figure B.4, one must make sure that no error messages are displayed before accepting or rejecting the mesh.

Figure B.2 3-Dimensional Tetramesh of CAD Model
After the mesh has been generated, Hypermesh can then be used to define the material definitions and properties of the solid elements. Figure B.5 shows how to create a material, giving it a easily distinguishable name, and selecting a card image which has not already been defined.
For the ease of use, the material is defined with the type ALL, and given that no more than one material per casing is used, MAT1 will suffice as a card image. Then, as shown in Figure B.6, one must right click and choose to conduct a “card edit” on the Material, which presents the user with a window as shown in Figure B.7. Using the interface shown in Figure B.7 the density, modulus of elasticity and Poisson’s ratio are defined in metric units (as previously noted).

Figure B.5 Creating a Material
After the material has been properly defined, the element properties must also be created and assigned to the component. Using the pull down menu from the top of the screen, one must create
a property and define it as shown in Figure B.8. A name is given to the property and the material created in the previous steps is assigned to this property. Additionally, the property is given the card image PSOLID, with the type ALL. After the property has been created, it can then be assigned to the component. This is done as shown in Figure B.9, which should result in the message shown (all elements properly assigned their respective material properties).

![Figure B.8 Component Properties](image)

Figure B.8 Component Properties
68753 elements were selected. All elements were successfully assigned proper

**Figure B.9 Material and Property Assignment**

Figures B.10-12 show the final step in the pre-processing stage using Hypermesh. The boundary conditions of the mesh are applied as constraints at the mounting locations of the gearbox enclosure. As shown in Figure 10, the component is viewed with shaded elements and meshed lines so that the mounting surface is more easily distinguishable. Then, as shown in Figure B.11, the boundary conditions are created by selecting the option "constraints" within the
BC's menu. Finally, the boundary conditions are applied to the surfaces by choosing the “surfs” option and selecting the appropriate mounting conditions (Figure B.12).

Figure B.10 Mesh with Shaded Elements
After pre-processing of the solid element mesh is complete, the file must be exported in NASTRAN long format. As shown in Figure B.13, under the file selection options, the File Type is designated as NASTRAN with the Template as Long Format. Finally, the File option is filled out such that the output file is given as with the extension *.dat. It is important to note under export options that the “All” option is selected from the pull down menu, with prompts for invalid elements and overwriting a file with the same file name extension. Once the .dat file is exported, one should review the text for the output file and make sure that it contains all of the pertinent information to the model before attempting to import the casing mesh into Romax Designer for static and dynamic analysis.
Figure B.13 NASTRAN File Export
APPENDIX C

NVH Experimental Results
Front Casing FRF & Phase Plots

Node 1

Figure C.1 Node 1 FRF & Phase Plot for Front Casing
Figure C.2 Node 2 FRF & Phase Plot for Front Casing

Figure C.3 Node 3 FRF & Phase Plot for Front Casing
Figure C.4 Node 4 FRF & Phase Plot for Front Casing
Figure C.5 Node 5 FRF & Phase Plot for Front Casing

Figure C.6 Node 6 FRF & Phase Plot for Front Casing
Figure C.7 Node 7 FRF & Phase Plot for Front Casing
Figure C.8 Node 8 FRF & Phase Plot for Front Casing

Back Casing FRF & Phase Plots

Figure C.9 Node 1 FRF & Phase Plot for Front Casing
Figure C.10 Node 2 FRF & Phase Plot for Front Casing
Figure C.11 Node 3 FRF & Phase Plot for Front Casing

Figure C.12 Node 4 FRF & Phase Plot for Front Casing
Figure C.13 Node 5 FRF & Phase Plot for Front Casing
Figure C.14 Node 6 FRF & Phase Plot for Front Casing

Figure C.15 Node 7 FRF & Phase Plot for Front Casing
Figure C.16 Node 8 FRF & Phase Plot for Front Casing
Figure C.17 Node 9 FRF & Phase Plot for Front Casing
APPENDIX D

Gearbox Mode Shapes of Interest
Baseline Two-Speed Gearbox

Figure D.1 Baseline Two-Speed First Neutral Bending Mode 166.0 Hz

Figure D.2 Baseline Two-Speed First Neutral Torsional Mode 742.9 Hz
Figure D.3 Baseline Two-Speed First Neutral Axial Mode 1159.5 Hz

Figure D.4 Baseline Two-Speed First 1st Gear Bending Mode 165.4 Hz
Figure D.5 Baseline Two-Speed First 1st Gear Torsional Mode 575.4 Hz

Figure D.6 Baseline Two-Speed First 1st Gear Axial Mode 1157.7 Hz
Figure D.7 Baseline Two-Speed First 2nd Gear Bending Mode 171.5 Hz

Figure D.8 Baseline Two-Speed First 2nd Gear Axial Mode 1165.4 Hz
Figure D.9 Baseline Two-Speed First 2nd Gear Torsional Mode 4730.9 Hz

Baseline Reverse Idler Gearbox

Figure D.10 Baseline Reverse Idler First Neutral Bending Mode 163.7 Hz
Figure D.11 Baseline Reverse Idler First Neutral Axial Mode 349.2 Hz

Figure D.12 Baseline Reverse Idler First Neutral Torsional Mode 407.9 Hz
Figure D.13 Baseline Reverse Idler First 1st Gear Bending Mode 172.5 Hz

Figure D.14 Baseline Reverse Idler First 1st Gear Torsional Mode 286.7 Hz
Figure D.15 Baseline Reverse Idler First 1st Gear Axial Mode 655.1 Hz

Figure D.16 Baseline Reverse Idler First 2nd Gear Bending Mode 172.3 Hz
Figure D.17 Baseline Reverse Idler First 2nd Gear Torsional Mode 297.7 Hz

Figure D.18 Baseline Reverse Idler First 2nd Gear Axial Mode 477.5 Hz
Baseline Dual Idler Gearbox

Figure D.19 Baseline Dual Idler First Neutral Axial Mode 122.6 Hz

Figure D.20 Baseline Dual Idler First Neutral Bending Mode 164.4 Hz
Figure D.21 Baseline Dual Idler First Neutral Torsional Mode 459.3 Hz

Figure D.22 Baseline Dual Idler First 1st Gear Bending Mode 240.4 Hz
Figure D.23 Baseline Dual Idler First 1st Gear Torsional Mode 390.9 Hz

Figure D.24 Baseline Dual Idler First 1st Gear Axial Mode 624.9 Hz
Figure D.25 Baseline Dual Idler First 2nd Gear Bending Mode 225.8 Hz

Figure D.26 Baseline Dual Idler First 2nd Gear Torsional Mode 334.1 Hz
Figure D.27 Baseline Dual Idler First 2nd Gear Axial Mode 821.2 Hz

Two-Speed Gearbox Inertia Test with First Gear Pinion Synchronizer

Figure D.28 First Gear Pinion Two-Speed First Neutral Bending Mode 186.6 Hz
Figure D.29 First Gear Pinion Two-Speed First Neutral Torsional Mode 435.0 Hz

Figure D.30 First Gear Pinion Two-Speed First Neutral Axial Mode 1467.0 Hz
Figure D.31 First Gear Pinion Two-Speed First 1st Gear Bending Mode 186.7 Hz

Figure D.32 First Gear Pinion Two-Speed First 1st Gear Torsional Mode 437.4 Hz
Figure D.33 First Gear Pinion Two-Speed First 1st Gear Axial Mode 1463.7 Hz

Figure D.34 First Gear Pinion Two-Speed First 2nd Gear Torsional Mode 222.1 Hz
Figure D.35 First Gear Pinion Two-Speed First 2nd Gear Bending Mode 311.6 Hz

Figure D.36 First Gear Pinion Two-Speed First 2nd Gear Axial Mode 1463.0 Hz
Two-Speed Gearbox Inertia Test with Second Gear Pinion Synchronizer

Figure D.37 Second Gear Pinion Two-Speed First Neutral Bending Mode 180.2 Hz

Figure D.38 Second Gear Pinion Two-Speed First Neutral Torsional Mode 196.3 Hz
Figure D.39 Second Gear Pinion Two-Speed First Neutral Axial Mode 1373.3 Hz

Figure D.40 Second Gear Pinion Two-Speed First 1st Gear Bending Mode 173.3 Hz
Figure D.41 Second Gear Pinion Two-Speed First 1st Gear Torsional Mode 282.3 Hz

Figure D.42 Second Gear Pinion Two-Speed First 1st Gear Axial Mode 1385.9 Hz
Figure D.43 Second Gear Pinion Two-Speed First 2nd Gear Bending Mode 181.4 Hz

Figure D.44 Second Gear Pinion Two-Speed First 2nd Gear Torsional Mode 501.6 Hz
Figure D.45 Second Gear Pinion Two-Speed First 2nd Gear Axial Mode 1373.4 Hz

Reverse Idler Gearbox Inertia Test with First Gear Pinion Synchronizer

Figure D.46 First Gear Pinion Synchro Reverse Idler First Neutral Bending Mode 174.2 Hz
Figure D.47 First Gear Pinion Synchro Reverse Idler First Neutral Torsional Mode 373.4 Hz

Figure D.48 First Gear Pinion Synchro Reverse Idler First Neutral Axial Mode 491.9 Hz
Figure D.49 First Gear Pinion Synchro Reverse Idler First 1st Gear Bending Mode 173.3 Hz

Figure D.50 First Gear Pinion Synchro Reverse Idler First 1st Gear Torsional Mode 558.5 Hz
Figure D.51 First Gear Pinion Synchro Reverse Idler First 1st Gear Axial Mode 590.5 Hz

Figure D.52 First Gear Pinion Synchro Reverse Idler First 2nd Gear Bending Mode 175.9 Hz
Figure D.53 First Gear Pinion Synchro Reverse Idler First 2nd Gear Torsional Mode 362.2 Hz

Figure D.54 First Gear Pinion Synchro Reverse Idler First 2nd Gear Axial Mode 780.6 Hz
Figure D.55 Second Gear Pinion Synchro Reverse Idler First Neutral Bending Mode 167.4 Hz

Figure D.56 Second Gear Pinion Synchro Reverse Idler First Neutral Axial Mode 284.2 Hz
Figure D.57 Second Gear Pinion Synchro Reverse Idler First Neutral Torsional Mode 429.0 Hz

Figure D.58 Second Gear Pinion Synchro Reverse Idler First 1st Gear Bending Mode 173.1 Hz
Figure D.59 Second Gear Pinion Synchro Reverse Idler First 1st Gear Torsional Mode 386.6 Hz

Figure D.60 Second Gear Pinion Synchro Reverse Idler First 1st Gear Axial Mode 624.1 Hz
Figure D.61 Second Gear Pinion Synchro Reverse Idler First 2nd Gear Bending Mode 173.5 Hz

Figure D.62 Second Gear Pinion Synchro Reverse Idler First 2nd Gear Torsional Mode 347.5 Hz
Figure D.63 Second Gear Pinion Synchro Reverse Idler First 2nd Gear Axial Mode 515.0 Hz

Dual Idler Gearbox Inertia Test with First Gear Pinion Synchronizer

Figure D.64 First Gear Pinion Synchro Dual Idler First Neutral Axial Mode 137.0 Hz
Figure D.65 First Gear Pinion Synchro Dual Idler First Neutral Bending Mode 161.8 Hz

Figure D.66 First Gear Pinion Synchro Dual Idler First Neutral Torsional Mode 530.2 Hz
Figure D.67 First Gear Pinion Synchro Dual Idler First 1st Gear Bending Mode 240.1 Hz

Figure D.68 First Gear Pinion Synchro Dual Idler First 1st Gear Torsional Mode 484.4 Hz
Figure D.69 First Gear Pinion Synchro Dual Idler First 1st Gear Axial Mode 798.7 Hz

Figure D.70 First Gear Pinion Synchro Dual Idler First 2nd Gear Bending Mode 225.9 Hz
Figure D.71 First Gear Pinion Synchro Dual Idler First 2nd Gear Axial Mode 653.3 Hz

Figure D.72 First Gear Pinion Synchro Dual Idler First 2nd Gear Torsional Mode 1708.3 Hz
Dual Idler Gearbox Inertia Test with Second Gear Pinion Synchronizer

Figure D.73 Second Gear Pinion Synchro Dual Idler First Neutral Axial Mode 109.0 Hz

Figure D.74 Second Gear Pinion Synchro Dual Idler First Neutral Bending Mode 129.7 Hz
Figure D.75 Second Gear Pinion Synchro Dual Idler First Neutral Torsional Mode 899.3 Hz

Figure D.76 Second Gear Pinion Synchro Dual Idler First 1st Gear Bending Mode 239.7 Hz
Figure D.77 Second Gear Pinion Synchro Dual Idler First 1st Gear Torsional Mode 420.1 Hz

Figure D.78 Second Gear Pinion Synchro Dual Idler First 1st Gear Axial Mode 751.6 Hz
Figure D.79 Second Gear Pinion Synchro Dual Idler First 2nd Gear Bending Mode 225.2 Hz

Figure D.80 Second Gear Pinion Synchro Dual Idler First 2nd Gear Axial Mode 709.2 Hz
Figure D.81 Second Gear Pinion Synchro Dual Idler First 2nd Gear Torsional Mode 904.4 Hz

Two-Speed Gearbox Rigidity Test

Figure D.82 Two-Speed Gearbox Rigidity Test First Neutral Bending Mode 221.9 Hz
Figure D.83 Two-Speed Gearbox Rigidity Test First Neutral Torsional Mode 475.1 Hz

Figure D.84 Two-Speed Gearbox Rigidity Test First Neutral Axial Mode 1165.6 Hz
Figure D.85 Two-Speed Gearbox Rigidity Test First 1st Gear Bending Mode 222.6 Hz

Figure D.86 Two-Speed Gearbox Rigidity Test First 1st Gear Torsional Mode 239.8 Hz
Figure D.87 Two-Speed Gearbox Rigidity Test First 1st Gear Axial Mode 1175.1 Hz

Figure D.88 Two-Speed Gearbox Rigidity Test First 2nd Gear Bending Mode 228.2 Hz
Figure D.89 Two-Speed Gearbox Rigidity Test First 2nd Gear Torsional Mode 261.7 Hz

Figure D.90 Two-Speed Gearbox Rigidity Test First 2nd Gear Axial Mode 1170.3 Hz
Reverse Idler Gearbox Rigidity Test

Figure D.91 Reverse Idler Gearbox Rigidity Test First Neutral Bending Mode 197.9 Hz

Figure D.92 Reverse Idler Gearbox Rigidity Test First Neutral Axial Mode 361.8 Hz
Figure D.93 Reverse Idler Gearbox Rigidity Test First Neutral Torsional Mode 490.1 Hz

Figure D.94 Reverse Idler Gearbox Rigidity Test First 1st Gear Bending Mode 205.7 Hz
Figure D.95 Reverse Idler Gearbox Rigidity Test First 1st Gear Torsional Mode 276.2 Hz

Figure D.96 Reverse Idler Gearbox Rigidity Test First 1st Gear Axial Mode 318.1 Hz
Figure D.97 Reverse Idler Gearbox Rigidity Test First 2nd Gear Bending Mode 209.9 Hz

Figure D.98 Reverse Idler Gearbox Rigidity Test First 2nd Gear Torsional Mode 371.6 Hz
Figure D.99 Reverse Idler Gearbox Rigidity Test First 2nd Gear Axial Mode 566.1 Hz

Dual Idler Gearbox Rigidity Test

Figure D.100 Dual Idler Gearbox Rigidity Test First Neutral Axial Mode 123.1 Hz
Figure D.101 Dual Idler Gearbox Rigidity Test First Neutral Bending Mode 180.0 Hz

Figure D.102 Dual Idler Gearbox Rigidity Test First Neutral Torsional Mode 486.8 Hz
Figure D.103 Dual Idler Gearbox Rigidity Test First 1st Gear Bending Mode 235.6 Hz

Figure D.104 Dual Idler Gearbox Rigidity Test First 1st Gear Torsional Mode 441.5 Hz
Figure D.105 Dual Idler Gearbox Rigidity Test First 1st Gear Axial Mode 660.7 Hz

Figure D.106 Dual Idler Gearbox Rigidity Test First 2nd Gear Bending Mode 247.7 Hz
Figure D.107 Dual Idler Gearbox Rigidity Test First 2nd Gear Torsional Mode 387.6 Hz

Figure D.108 Dual Idler Gearbox Rigidity Test First 2nd Gear Axial Mode 590.9 Hz
Basic Two-Speed Gearbox with Steel Enclosure

Figure D.109 Two-Speed Gearbox w Steel Casing First Neutral Bending Mode 146.4 Hz

Figure D.110 Two-Speed Gearbox w Steel Casing First Neutral Axial Mode 366.8 Hz
Figure D.111 Two-Speed Gearbox w Steel Casing First Neutral Torsional Mode 1767.0 Hz

Figure D.112 Two-Speed Gearbox w Steel Casing First 1st Gear Bending Mode 144.8 Hz
Figure D.113 Two-Speed Gearbox w Steel Casing First 1st Gear Axial Mode 347.4 Hz

Figure D.114 Two-Speed Gearbox w Steel Casing First 1st Gear Torsional Mode 553.2 Hz
Figure D.115 Two-Speed Gearbox w Steel Casing First 2nd Gear Bending Mode 149.5 Hz

Figure D.116 Two-Speed Gearbox w Steel Casing First 2nd Gear Axial Mode 360.0 Hz
Reverse Idler Gearbox with Steel Enclosure

Figure D.117 Two-Speed Gearbox w Steel Casing First 2nd Gear Torsional Mode 3861.0 Hz

Figure D.118 Reverse Idler Gearbox w Steel Casing First Neutral Bending Mode 157.2 Hz
Figure D.119 Reverse Idler Gearbox w Steel Casing First Neutral Axial Mode 299.6 Hz

Figure D.120 Reverse Idler Gearbox w Steel Casing First Neutral Torsional Mode 575.5 Hz
Figure D.121 Reverse Idler Gearbox w Steel Casing First 1st Gear Bending Mode 163.6 Hz

Figure D.122 Reverse Idler Gearbox w Steel Casing First 1st Gear Torsional Mode 239.2 Hz
Figure D.123 Reverse Idler Gearbox w Steel Casing First 1st Gear Axial Mode 342.9 Hz

Figure D.124 Reverse Idler Gearbox w Steel Casing First 2nd Gear Bending Mode 165.9 Hz
Figure D.125 Reverse Idler Gearbox w Steel Casing First 2nd Gear Axial Mode 318.9 Hz

Figure D.126 Reverse Idler Gearbox w Steel Casing First 2nd Gear Torsional Mode 389.4 Hz
Dual Idler Gearbox with Steel Enclosure

Figure D.127 Dual Idler Gearbox w Steel Casing First Neutral Axial Mode 96.9 Hz

Figure D.128 Dual Idler Gearbox w Steel Casing First Neutral Bending Mode 150.0 Hz
Figure D.129 Dual Idler Gearbox w Steel Casing First Neutral Torsional Mode 583.0 Hz

Figure D.130 Dual Idler Gearbox w Steel Casing First 1st Gear Bending Mode 177.7 Hz
Figure D.131 Dual Idler Gearbox w Steel Casing First 1st Gear Axial Mode 482.0 Hz

Figure D.132 Dual Idler Gearbox w Steel Casing First 2nd Gear Bending Mode 178.3 Hz
Figure D.133 Dual Idler Gearbox w Steel Casing First 2nd Gear Axial Mode 439.6 Hz

Figure D.134 Dual Idler Gearbox w Steel Casing First 2nd Gear Torsional Mode 583.7 Hz
Basic Two-Speed Gearbox with Steel Enclosure Fixed and Rigid Internal Components

Figure D.134 Rigid Two-Speed Gearbox w Casing First Neutral Bending Mode 200.6 Hz

Figure D.135 Rigid Two-Speed Gearbox w Casing First Neutral Torsional Mode 552.1 Hz
Figure D.136 Rigid Two-Speed Gearbox w Casing First Neutral Axial Mode 1986.2 Hz

Figure D.137 Rigid Two-Speed Gearbox w Casing First 1st Gear Bending Mode 204.4 Hz
Figure D.138 Rigid Two-Speed Gearbox w Casing First 1st Gear Torsional Mode 534.7 Hz

Figure D.139 Rigid Two-Speed Gearbox w Casing First 1st Gear Axial Mode 652.9 Hz
Figure D.140 Rigid Two-Speed Gearbox w Casing First 2nd Gear Bending Mode 206.8 Hz

Figure D.141 Rigid Two-Speed Gearbox w Casing First 2nd Gear Axial Mode 374.4 Hz
Figure D.142 Rigid Two-Speed Gearbox w Casing First 2nd Gear Torsional Mode 414.9 Hz

Basic Two-Speed Gearbox with Steel Enclosure Fixed at Bearings

Figure D.143 Two-Speed Gearbox w Casing fixed at Bearings First Neutral Bending Mode 165.9 Hz
Figure D.144 Two-Speed Gearbox w Casing fixed at Bearings First Neutral Torsional Mode 741.2 Hz

Figure D.145 Two-Speed Gearbox w Casing fixed at Bearings First Neutral Axial Mode 1155.4 Hz
Figure D.146 Two-Speed Gearbox w Casing fixed at Bearings First 1st Gear Bending Mode 165.3 Hz

Figure D.147 Two-Speed Gearbox w Casing fixed at Bearings First 1st Gear Torsional Mode 574.8 Hz
Figure D.148 Two-Speed Gearbox w Casing fixed at Bearings First 1st Gear Axial Mode 1153.8 Hz

Figure D.149 Two-Speed Gearbox w Casing fixed at Bearings First 2nd Gear Bending Mode 171.4 Hz
Figure D.150 Two-Speed Gearbox w Casing fixed at Bearings First 2nd Gear Torsional Mode 266.7 Hz

Figure D.151 Two-Speed Gearbox w Casing fixed at Bearings First 2nd Gear Axial Mode 1161.2 Hz
Flexible Dual Idler Gearbox

Figure D.152 Flexible Dual Idler First Neutral Axial Mode 102.7 Hz

Figure D.153 Flexible Dual Idler First Neutral Bending Mode 119.0 Hz
Figure D.154 Flexible Dual Idler First Neutral Torsional Mode 432.7 Hz

Figure D.155 Flexible Dual Idler First 1st Gear Bending Mode 157.6 Hz
Figure D.156 Flexible Dual Idler First 1st Gear Torsional Mode 249.9 Hz

Figure D.157 Flexible Dual Idler First 1st Gear Axial Mode 603.2 Hz
Figure D.158 Flexible Dual Idler First 2nd Gear Bending Mode 180.5 Hz

Figure D.159 Flexible Dual Idler First 2nd Gear Torsional Mode 291.5 Hz
Flexible Dual Idler Gearbox with Aluminum Casing

Figure D.160 Flexible Dual Idler First 2nd Gear Axial Mode 639.4 Hz

Figure D.152 Flexible Dual Idler w Casing First Neutral Axial Mode 95.7 Hz
Figure D.153 Flexible Dual Idler w Casing First Neutral Bending Mode 115.7 Hz

Figure D.154 Flexible Dual Idler w Casing First Neutral Torsional Mode 220.2 Hz
Figure D.155 Flexible Dual Idler w Casing First 1st Gear Bending Mode 118.3 Hz

Figure D.156 Flexible Dual Idler w Casing First 1st Gear Torsional Mode 269.7 Hz
Figure D.157 Flexible Dual Idler w Casing First 1st Gear Axial Mode 1013.9 Hz

Figure D.158 Flexible Dual Idler w Casing First 2nd Gear Bending Mode 119.0 Hz
Figure D.159 Flexible Dual Idler w Casing First 2nd Gear Axial Mode 364.7 Hz

Figure D.160 Flexible Dual Idler w Casing First 2nd Gear Torsional Mode 517.2 Hz
APPENDIX E

Backlash for Tooth Thinning Factors
Basic Two-Speed Gearbox Backlash Values

Table E.1 – Backlash Values for Varied Tooth Thinning Factors in Basic Two-Speed Gearbox

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Reverse Idler Gearbox Backlash Values

Table E.2 – Backlash Values for Varied Tooth Thinning Factors in Reverse Idler Gearbox

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<th>Second Idler and wheel</th>
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### Dual Idler Gearbox Backlash Values

#### Table E.3 – Backlash Values for Varied Tooth Thinning Factors in Dual Idler Gearbox

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<td>First Idler and Pinion</td>
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<td>Second Idler and wheel</td>
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Betreff: Copyright Request - Gear Rattle Figures
Datum: Tue, 15 Mar 2011 21:45:51 -0400 (EDT)
Von: Joshuah Thomas Racine <joshuah.t.racine@wmich.edu>
An: service@hirzel.de
CC: Judah Ari-Gur <judah.ari-gur@wmich.edu>
March 15, 2011

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Name                        Date

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Graduate Research Assistant
Department of Mechanical Engineering
Western Michigan University
4601 Campus Drive (Room F-234)
Kalamazoo, MI 49008
269.598.4996
Brancati et al., 2005

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Cc: Judah Ari-Gur
Subject: Copyright Request - Brancati et al. (2005)
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Name Date

Sincerely,

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Graduate Research Assistant

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Thanks,

Joshuah Racine, B.Sc.
Graduate Research Assistant
CAViDS TRAT Team
Department of Mechanical Engineering
Western Michigan University
269.598.4996

From: "Beth Darchi" <DarchiB@asme.org>
To: "joshuah.t.racine@wmich.edu" <joshuah.t.racine@wmich.edu>
Sent: Friday, March 11, 2011 11:25:36 AM
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Sent: Friday, March 11, 2011 10:32 AM
To: Ivette Rodriguez
Cc: Judah Ari-Gur; josh.racine@gmail.com
Subject: Copyright Request - Comparin and Singh 1990

March 11, 2011

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Joshuah Racine, B.Sc.

Graduate Research Assistant

Department of Mechanical Engineering

Western Michigan University

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Singh, R., Xie, H. and Comparin, R.J. (1989). Analysis of Automotive Neutral Gear Rattle. Journal of Sound and Vibration, 131 (2), pp. 177-196 (Figure 1 p. 178)


The reason I am using these figures is to outline the analysis and simulation practices common to rattling mechanisms. The figures will be used in my thesis for a literature review of work that has been done relating to reducing gear rattle noise in manual transmission vehicles. My thesis is titled: Parametric Study of Gear Rattle and the Effect of Flexible Enclosures on Gearbox Vibratory Responses and will be used as partial fulfillment for my degree of Master in Science in Mechanical Engineering from Western Michigan University on April 31, 2011. My thesis will be roughly 350-400 pages long in both electronic and physical formats. The sources will receive full credit in the manuscript.

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_________________________________________  ____________________________
Name                                      Date

Sincerely,

Joshuah Racine, B.Sc.

Graduate Research Assistant

Department of Mechanical Engineering

Western Michigan University

4601 Campus Drive (Room F-234)
Good afternoon,

Thank you again for your follow-up. After speaking with Dr. Bernhard, he has given his approval for the figures from the two items noted below to be used in your master's thesis, as you indicate that they will be fully cited in the manuscript. If you have any additional questions, please do not hesitate to contact our office.

Regards,
Wendi Frohna

----------
Wendi Frohna
Officer Assistant
to Vice President for Research, Dr. Robert J. Bernhard
Office of the Vice President for Research
University of Notre Dame
317 Main Building
Notre Dame, IN 46556
P: 574-631-3902
F: 574-631-8441
wfrohna@nd.edu

-----Original Message-----
From: Joshuah Thomas Racine [mailto:joshuah.t.racine@wmich.edu]
Sent: Monday, March 21, 2011 11:13 AM
To: Wendi Frohna
Cc: Judah Ari-Gur
Subject: Re: Copyright Request - Kim and Singh (2000) and Bodden and Heinrichs (1999)

Wendi,

I still have not received an e-mail from Dr. Bernhard, so if you could please follow up with my request for copyright clearance it would be greatly appreciated. Thank you very much for your time.
Sincerely,

Joshuah Racine, B.Sc.
Graduate Research Assistant
CAViDS TRAT Team
Department of Mechanical Engineering
Western Michigan University
269.598.4996

----- Original Message ----- 
> From: "Wendi Frohna" <Wendi.L.Frohna.1@nd.edu>
> To: "Joshuah Thomas Racine" <joshuah.t.racine@wmich.edu>
> Cc: "Judah Ari-Gur" <judah.ari-gur@wmich.edu>
> Sent: Friday, March 11, 2011 3:50:26 PM
> Subject: RE: Copyright Request - Kim and Singh (2000) and Bodden and Heinrichs (1999) Dear Joshua,
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>
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> Wendi
>
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> 
> March 11, 2011
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> 
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Charles S. Fischer
Vice President, AGMA Technical Division

-----Original Message-----
From: Joshuah Thomas Racine [mailto:joshuah.t.racine@wmich.edu]
Sent: Friday, March 11, 2011 3:38 PM
To: Mary Ellen Doran
Cc: Judah Ari-Gur
Subject: Copyright Request - Korde and Wilson (2009)

March 11, 2011

American Gear Manufacturers Association
1001 N. Fairfax Street
Fifth Floor
Alexandria, VA 22314-1587

To whom it may concern,

I would like to request your permission to include the figure indicated from the following item in my thesis:

The reason I am using this figure is to give an example gear rattle model. The figure will be used in my thesis for a literature review of work that has been done relating to reducing gear rattle noise in manual transmission vehicles. My thesis is titled: Parametric Study of Gear Rattle and the Effect of Flexible Enclosures on Gearbox Vibratory Responses and will be used as partial fulfillment for my degree of Master in Science in Mechanical Engineering from Western Michigan University. The source will receive full credit in the manuscript.

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If you could, please respond to this e-mail or send a letter as formatted herein to the address provided below. Thank you for your time and attention to this matter.

Charles S. Fischer 14 March 2011

Name Date

Sincerely,

Joshuah Racine, B.Sc.
Graduate Research Assistant
Department of Mechanical Engineering
Western Michigan University
4601 Campus Drive (Room F-234)
Kalamazoo, MI 49008
269.598.4996

Pfeiffer, 1996

Dear Mr. Racine,

it's allowed to you to include the figure stated below in your thesis:

Pfeiffer, F. (1996). Rattling in Gears - A Review. VDI Berichte 1260, pp. 719-737 (Figure 1, p. 721), VDI Verlag GmbH

Therefore, please notice down the above dates into your thesis.
Kindly regards

Wolfgang Bittner

Wolfgang Bittner
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VDI Verlag GmbH
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40001 Düsseldorf

Tel: 0049/(0)211/6188445
Fax: 0049/(0)211/618897445
e-mail: wbittner@vdi-nachrichten.com
Internet: www.vdi-nachrichten.com

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Geschäftsführer: Raymond Johnson-Ohla

-----Ursprüngliche Nachricht-----
Von: Grahe, Harald
Gesendet: Donnerstag, 17. März 2011 09:30
An: Bittner, Wolfgang
Betreff: WG: Copyright Request - Gear Rattle Figures

-----------------------------------------------------------------
Harald Grahe
Schriftenreihen
VDI Verlag GmbH
Postfach 10 10 54, 40001 Düsseldorf
Tel: +49 (0)211 61 88 - 476
Fax: +49 (0)211 61 88 - 97 476
Mail: hgrahe@vdi-nachrichten.com
Internet: http://www.vdi-nachrichten.com

Besucheranschrift:
VDI-Platz 1, 40468 Düsseldorf

-----Ursprüngliche Nachricht-----
Von: Bittner, Wolfgang
Gesendet: Donnerstag, 17. März 2011 08:48
An: Grahe, Harald  
Betreff: WG: Copyright Request - Gear Rattle Figures

Wolfgang Bittner  
Vertriebsabwicklung Schriftenreihen

VDI Verlag GmbH  
Postfach 10 10 54  
40001 Düsseldorf

Tel: 0049/(0)211/6188445  
Fax: 0049/(0)211/618897445  
e-mail: wbittner@vdi-nachrichten.com  
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An: Bittner, Wolfgang  
Betreff: WG: Copyright Request - Gear Rattle Figures

Sehr geehrter Herr Bittner,

können Sie Herrn Racine helfen?

Danke im Voraus!

Mit freundlichen Grüssen

Christiane Güntner

---

Christiane Güntner  
Technik und Wissenschaft  
Sekretariat VDI-Gesellschaft Bauen und Gebäudetechnik

Verein Deutscher Ingenieure e.V.  
VDI-Platz 1  
40468 Düsseldorf

Tel.: +49 (0) 211 62 14-5 77  
Fax: +49 (0) 211 62 14-1 77
Willkommen beim VDI auf der Hannover Messe

Willkommen zum 25. Deutschen Ingenieurtag

Willkommen auf der ISH

-----Ursprüngliche Nachricht-----
Von: Joshuah Thomas Racine [mailto:joshuah.t.racine@wmich.edu]
Gesendet: Mittwoch, 16. März 2011 02:50
An: gbg@vdi.de
Cc: Judah Ari-Gur
Betreff: Copyright Request - Gear Rattle Figures

March 15, 2011

Verein Deutscher Ingenieure e.V.
Gesellschaft Bauen und Gebäudetechnik
Postfach 10 11 39
40002 Düsseldorf

To Whom it may Concern,

I would like to request your permission to include the following figure stated below in my thesis:

Pfeiffer, F. (1996). Rattling in Gears - A Review. VDI Berichte 1260, pp. 719-737 (Figure 1, p. 721)

The reason I am using this figure is to outline the analysis and simulation practices common to rattling mechanisms. The figure will be used in my thesis for a literature review of work that has been done relating to reducing gear rattle noise in manual transmission vehicles. My thesis is titled: Parametric Study of Gear Rattle and the Effect of Flexible Enclosures on Gearbox
Vibratory Responses and will be used as partial fulfillment for my degree of Master in Science in Mechanical Engineering from Western Michigan University on April 31, 2011. My thesis will be roughly 350-400 pages long in both electronic and physical formats. The sources will receive full credit in the manuscript.

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If you could, please respond to this e-mail or send a letter as formatted in this e-mail to the address provided below. Thank you for your time and attention to this matter.

Name __________________________ Date __________________________

Sincerely,

Joshuah Racine, B.Sc.
Graduate Research Assistant
Department of Mechanical Engineering
Western Michigan University
4601 Campus Drive (Room F-234)
Kalamazoo, MI 49008
269.598.4996

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Terri Kelly

Intellectual Property Rights Administrator

SAE International

Phone: 001.724.772.4095; Fax: 001.724.776.9765

E-mail: terri@sae.org

-----Original Message-----
From: Joshuah Thomas Racine [mailto:joshuah.t.racine@wmich.edu]
Sent: Friday, March 11, 2011 2:53 PM
To: copyright
Cc: Judah Ari-Gur
Subject: Copyright Requests - Gear Rattle SAE Papers

March 11, 2011

SAE Copyright Administrator

SAE International

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Warrendale, PA 15096-0001 - USA

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Campbell, B., Stokes, W. and Steyer, G. (1997) Gear Noise Reduction of an Automatic Transmission through Finite Element Dynamic Simulation. SAE paper 971966 (Figure 7 p. 765)

Fujimoto, T. and Kizuka, T. Predictive Calculation of Idling Rattle in Manual Transmissions – Based on Experimental Measurements of Gear Vibrations Occurring in Backlashes. SAE paper 2003-01-0678, 2003 (Figure 1 p. 4)
Fujimoto, T. Kizuka, M. Muramatsu, and S. Yahata. (2001). An Improvement of the Prediction Method of the Idling Rattle in Manual Transmission. JSAE Annual Congress (Figure 15 p. 10)

Fujimoto, T., Chikatani, Y. and Kojima, J. Reduction of Idling Rattle in Manual Transmission. SAE paper 870395, 1987 (Figure 13 p. 7)

Johnson, O. and Hiramai, N. (1991). Diagnosis and Objective Evaluation of Gear Rattle. SAE paper 911082 (Figure 10b p. 288)

Ohnuma, S., Yahata, S., Inagawa, M. and Fujimoto, T. Research on Idling Rattle of Manual Transmission. SAE paper 850979, 1985 (Figure 2 p. 2)

Rivin, E.I. Analysis and Reduction of Rattling in Power Transmission System. SAE paper 2000-01-0032. 2000 (Figure 9 p. 6)

Shimizu, T. Mechanism of the Idle Gear Rattle Synchronized with Engine Rotation. SAE paper 932003, 1993 (Figure 4 p. 10)


The reason I am using these figures is to outline the analysis and simulation practices common to rattling mechanisms. The figures will be used in my thesis for a literature review of work that has been done relating to reducing gear rattle noise in manual transmission vehicles. My thesis is titled: Parametric Study of Gear Rattle and the Effect of Flexible Enclosures on Gearbox Vibratory Responses and will be used as partial fulfillment for my degree of Master in Science in Mechanical Engineering from Western Michigan University on April 31, 2011. My thesis will be roughly 350-400 pages long in both electronic and physical formats. The sources will receive full credit in the manuscript.

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If you could, please respond to this e-mail or send a letter as formatted in this e-mail to the address provided below. Thank you for your time and attention to this matter.

Name

Date
Sincerely,

Joshuah Racine, B.Sc.
Graduate Research Assistant
Department of Mechanical Engineering
Western Michigan University
4601 Campus Drive (Room F-234)
Kalamazoo, MI 49008
269.598.4996