The Common Core State Standards in Mathematics for Place Value: Developmentally Appropriate or Not?

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THE COMMON CORE STATE STANDARDS IN MATHEMATICS FOR PLACE VALUE:
DEVELOPMENTALLY APPROPRIATE OR NOT?

by
Sarah N. Hughey

A thesis submitted to the Graduate College
in partial fulfillment of the requirements
for the degree of Master of Arts
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Western Michigan University
April 2020

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THE COMMON CORE STATE STANDARDS IN MATHEMATICS FOR PLACE VALUE: DEVELOPMENTALLY APPROPRIATE OR NOT?

Sarah N. Hughey, M.A.
Western Michigan University, 2020

This exploratory case study examines the developmental appropriateness of the Common Core State Standards in Mathematics (CCSSM) for place value for second through fourth-grade students. A correlation analysis was performed on end of year scores from the 2018-19 school year on a standardized norm-referenced test and a conceptually-based interview assessment from the Math Recovery program (AVMR) on 137 students from one school in the Midwest. An item analysis was also performed on AVMR assessments for 70 students from the 2019-20 school year. The results showed that second and third-grade students did not display the expected amount of growth in conceptual place value, all students under-utilized efficient mental computation strategies, such as compensation and transformation, and early overuse of the standard algorithms had a negative impact on place value understandings. Recommendations related to these results include the addition of standards related to the compensation and transformation strategies, the delay of certain standards in second and third grade, and the removal of the standard algorithms for addition and subtraction from the elementary grades.
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Sarah N. Hughey
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CHAPTER 1
INTRODUCTION

The Common Core State Standards in Mathematics (CCSSM) determine the learning goals and guide the curriculum for students across the United States. In 2009, lawmakers came together to develop the Common Core State Standards (CCSS) in order to standardize learning expectations throughout the country. Before this, most states developed their standards on the National Council of Teachers of Mathematics Standards (2000), which were organized in grade bands rather than specific grade levels. This resulted in a great deal of variation across states (Reys, 2006), and many believed that this lack of specificity resulted in a lowering of expectations. Working together with teachers, educational leaders, researchers, and organizations throughout the country, state policymakers came together to create a common set of standards for both language arts and mathematics in order to raise student achievement and to prepare all students for success in college and a globalized economy (Common Core State Standards Initiative, n.d., 2019b).

At first, the CCSSM were generally looked upon favorably, gaining high amounts of support by the general public and looked at optimistically by various groups such as teachers and researchers (Cheng, Henderson, Peterson, & West, 2019). In particular, researchers described how cognitive demand and higher-order thinking were emphasized more by the CCSSM than what was required by previous state standards (Cobb & Jackson, 2011; Porter, McMaken, Hwang, & Yang, 2011). An example of this new emphasis was the focus on conceptual number sense for elementary school students and the placement of the standard algorithms for addition,
subtraction, multiplication, and division in the fourth, fifth, and sixth grades (versus the typical approach of starting instruction on the standard algorithms in first grade). This last change was particularly celebrated by many due to research showing that teaching algorithms too early can undo earlier learning in number sense (see Kamii & Dominick, 1998, as an example). Schmidt and Houang (2012) analyzed previous state standards from 2009 and similarities they shared with the CCSSM and found that students in states with higher rates of similarities tended to have higher rates of mathematical achievement. In addition, they found that the CCSSM were similar to the standards of countries with high student achievement in mathematics. Furthermore, leaders in organizations such as the National Association for the Education of Young Children and the National Association of Early Childhood Specialists (NAEYC & NAECS-SDE, 2010) released statements showing strong support for both the language arts and mathematics standards for early grades and expressed enthusiasm for the learning progressions seen in them. While not everyone was in full support of the CCSS, with researchers actively debating their merit and having nuanced views of the initiative (i.e., Rakow, 2012), the standards did generally garner support.

Despite the fact that researchers, policymakers, and the general public viewed standardizing expectations across the country fairly positively, criticism seems to be building against the CCSS as a whole. According to Cheng et al. (2019), support for the CCSS has drastically fallen since 2013. In 2018, the EdNext poll showed that 45% of the general public support the CCSS versus 65% in 2013, with 38% of the general public actively opposing the use of the CCSS. This is in spite of the poll showing positive public perceptions of standards for reading and math, as long as the term “Common Core” was not mentioned. Teachers in particular show rising rates of opposition to the implementation of the CCSS, reaching 51%
disapproval in 2018 (a 7-point increase from 2018) and dropping from over 75% of teachers showing support in 2013 to 43% in 2018 (Cheng et al., 2019, p. 19). These results are in spite of the fact that the idea of common nationwide standards as opposed to individualized standards for each state still enjoys a high approval rate among the general public (61%) (Cheng et al., 2019, p. 20).

So what could be causing this high level of dissatisfaction? One cause may be that the standards are difficult to enact instructionally, and thus, meeting the goal of raising student achievement is not being realized in many districts. To answer this specifically in regards to mathematics, student achievement in mathematics remains less than satisfactory after several years of implementation of the CCSSM. According to the National Assessment of Educational Progress (NAEP), which tests students on a nationwide level, fourth-grade students had an initial drop in average scores between 2013 and 2015 (from 242 to 240, both of which are not a proficient score). This remained stagnant between 2015 and 2017. A similar pattern occurs in the eighth-grade mathematics scores. Before 2013, scores were showing a pattern of improvement in both grades (The Nation’s Report Card, n.d.). This is significant because almost all states implemented the CCSSM during the 2013 and 2015 period, yet the nationwide NAEP scores do not seem to show improvement. While an initial lack of improvement can be expected when new changes in standards occur, with further delays in their alignment with pedagogy and curricular materials, the fact that scores remained low in 2017 is concerning.

When examining specific scores by state, scores on tests developed to align to the Common Core State Standards (CCSS) do not show results meeting the CCSS’ goal of making every student proficient in mathematics (Koretz, 2017). Using Michigan as an example, Michigan’s Center for Educational Performance and Information (CEPI) (2019) shows that at the
end of the 2017-18 school year the majority of Michigan’s students at every grade level from third grade through eighth grade were not able to score at the proficiency level on the math section of the Michigan Student Test of Educational Progress (M-STEP). This is a summative exam developed to test proficiency with standards (Michigan Department of Education, 2018). Overall, approximately 62.5% of students did not score at the proficient level or above, with the number of students proficient in each grade level decreasing as the grade level increased. The 2018-19 scores showed similar results, with approximately 61.3% of students not scoring at the proficient level or above. A breakdown of these results can be seen in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Grade</th>
<th>2017-18</th>
<th>2018-19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Third</td>
<td>54%</td>
<td>53%</td>
</tr>
<tr>
<td>Fourth</td>
<td>58%</td>
<td>58%</td>
</tr>
<tr>
<td>Fifth</td>
<td>66%</td>
<td>65%</td>
</tr>
<tr>
<td>Sixth</td>
<td>65%</td>
<td>65%</td>
</tr>
<tr>
<td>Seventh</td>
<td>64%</td>
<td>64%</td>
</tr>
<tr>
<td>Eighth</td>
<td>66%</td>
<td>N/A</td>
</tr>
<tr>
<td>Total</td>
<td>63%</td>
<td>61%</td>
</tr>
</tbody>
</table>

(Michigan’s Center for Educational Performance and Information, 2019)

Lackluster results can also be seen in the Programme for International Student Assessment, an international exam organized by the Organisation for Economic Co-operation and Development that compares proficiency levels among 15-year-old students in different
countries. On the 2018 PISA, the United States scored below average on mathematics overall when compared to countries in the Organisation for Economic Co-operation and Development (OECD) (which share political and economic connections and commonalities), with no significant improvement in scores since 2003 (Pál, Marec, & Schwabe, 2019). When analyzing specific proficiency levels in mathematics, Pál, Marec, and Schwabe (2019) found that the United States scored below the OECD average in consideration to the percent of students reaching Level 2 and Level 5, with drastic differences between the United States’ percentages and those of top performers. As an example, compare how 8% of students in the United States scored at Level 5 or higher to 44% in Beijing, Shanghai, Jiangsu and Zhejiang (China) and 37% in Singapore. Given that the developers of the CCSSM aimed to raise expectations in education to therefore improve achievement, the lack of advancement in rankings on the PISA could show that something is amiss and that researchers should explore why this is happening.

In sum, student achievement has not shown the improvement many hoped for after the implementation of the CCSSM. Several factors may have caused this. For instance, instruction may not have necessarily changed in the way developers of the standards hoped it would after the implementation of the standards (Garland, 2014). This may be exacerbated by the fact that the standards for mathematical practice are listed separately from the content standards and are not directly found in the standards themselves (see the Standards for Mathematical Practice, Common Core State Standards Initiation, 2019d). Also, some suggest the standards may be out of alignment with research on mathematics education that has been conducted since their implementation (e.g., Decker & Roberts, 2015; Fisher, Dobbs-Oates, Doctoroff, & Arnold, 2012; Hackenberg, Norton, & Wright, 2016; Park & Cho, 2017; Peters & De Smedt, 2018; Yilmaz,
Another consideration may be that the standards themselves are high quality but that the timing and placement of the standards may not be appropriate for each age and grade level.

This study explores this last possibility, that of inappropriate placement of the standards, and if students can reasonably be expected to attain the learning goals set at each grade level in light of the concept of backwards design or the Understanding by Design (UbD) approach to instruction. According to Wiggins and McTighe (2005), this approach claims that effective planning for instruction depends on a design process that takes learning goals and assessments into account before other considerations. If there is something wrong with the learning goal, then this can compromise the entire design process and lead to less student understanding and learning when instruction occurs. Therefore, it is of utmost importance to have learning goals and standards that align with current research on learning trajectories for mathematics and that are then placed at a grade level where students can reasonably meet them in light of their age and cognitive development. If evidence shows that the placement of standards may not be appropriate, an attempt to rearrange them may lead to higher student achievement levels on tests that show long-term results such as the NAEP and the PISA. On the other hand, if the placement of standards appears correct after analysis, then the other considerations of why student achievement is not increasing can be examined with more clarity.

This study will focus on only a small portion of the large set of CCSSM content standards—those dealing with place value instruction and how this appears in the CCSSM. This topic was chosen for its central focus in the elementary school curriculum, being addressed as early as kindergarten with tens and ones and the teen numbers, and extending throughout the remaining grades as numbers of expanded. Place value instruction focuses on student understandings behind the value of each digit in a number and how students use this
understanding in other areas of mathematics, especially when they are first learning how to add and subtract multi-digit numbers. These early understandings of place value critically affect how a student later performs in areas such as multiplication, division, and algebraic reasoning. In addition, research shows that long-term damage to student understanding in mathematics can occur if formal or standard algorithms for addition and subtraction are presented to children before they have a firm understanding of earlier place value concepts (Clarke, 2005). Therefore, determining whether the standards related to place value are developmentally appropriate for each grade level can provide insights into both the attainability of these standards themselves and how students may perform with later standards related to different concepts.

The CCSSM place value standards for grades 2-4 will be examined in order to answer the following research questions:

1. Are the CCSSM place value standards for grades 2-4 developmentally appropriate and attainable for students?

2. What are the most common student errors with place value that hinder progress towards conceptual place value understanding?

“Developmentally appropriate” in this study will refer to whether or not the standards are attainable given the abilities of a vast majority of students at each grade level, especially in connection with what can be reasonably expected given a student’s age and stage of cognitive development. Furthermore, “attainable” will refer to the CCSSM’s proposal that all students should be able to show a proficient understanding of the standards (Common Core State Standards Initiative, n.d., 2019b, 2020a).
CHAPTER 2
LITERATURE REVIEW

Place Value

It is important to distinguish the different approaches to place value instruction. According to Wright, Ellemor-Collins, and Tabor (2012), conventional instruction in place value often involves presenting multi-digit numbers in terms of their digits instead of their value. For example, in 21, the “2” is discussed as a two in the tens column instead of its value or how many tens it represents. Students solve questions involving place value with rehearsed or memorized conventions and procedures (p. 83). A prime example of instruction utilizing conventional place value is teaching the standard algorithms for addition and subtraction. Before the implementation of the CCSSM, most instruction in the early grades related to place value utilized the standard algorithms, and this typically led to assumptions that students automatically learned place value as they were taught the algorithms for addition and subtraction. For example, students are taught to “borrow” from a larger place value in subtraction problems requiring regrouping, and both numbers involved are discussed solely in terms of its digit instead of its value. See Figure 1 for an example and the typical explanation that accompanied subtraction regrouping problems.
“Five is bigger than three, so you have to borrow a one from the four and add it to the three. Four turns into three, and three turns into thirteen. Thirteen minus five equals eight, and three minus one equals two.”

Figure 1. The standard algorithm for subtraction with 43-15.

However, current research suggests that educators need to focus on “conceptual” place value before introducing students to conventional place value (Wright et al., 2012, p. 83). This consists of discussing numbers in terms of their “full value” and in solving questions using constructed mental strategies that utilize number relationships, as opposed to using single-digit numbers without consideration to their place value in a memorized procedure or a standard algorithm. An example of a student using conceptual place value would be one who can increment groups of ones, tens, and hundreds in their full value when given an addition problem, perhaps by drawing these increments as “jumps” on an empty number line or by verbally describing the value of each digit as they mentally add parts of each number together. For example, a student using conceptual place value for the previous problem might think the following: 43 – 10 is 33, 33 – 3 is 30 and 30 – 2 is 28.

A student using conceptual place value would also tend to solve the numbers from “left to right” or starting with the larger numbers instead of the algorithm’s insistence on starting with the ones place (Kamii & Dominick, 1998). Instead of teaching procedures, instructors focused on conceptual place value gradually increase the complexity of addition and subtraction problems (e.g., moving from using materials to a bare number task, increasing the range of numbers used, and designing questions that make increments and decrements more challenging)
until the student can flexibly use multiple strategies and can choose the most effective one for each problem (Ellemor-Collins & Wright, 2011). Table 2 below, from Wright et al. (2012), further differentiates the conceptual place value and conventional place value approaches (p. 83).

Table 2

*Wright, Ellemor-Collins, and Tabor’s (2012) Descriptions of Conceptual and Conventional Place Value*

<table>
<thead>
<tr>
<th>Conceptual Place Value</th>
<th>Conventional Place Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers are presented and discussed in their full value: 20 as twenty or two tens; 21 as twenty-one, or twenty and one.</td>
<td>Numbers may be explicitly presented or discussed in terms of digits: 20 has 2 in the tens column; 21 has 1 in the ones column.</td>
</tr>
<tr>
<td>Tasks involve increments/decrements in sequence.</td>
<td>Typically, tasks are not presented as a sequence of increments/decrements.</td>
</tr>
<tr>
<td>Solving tasks essentially involves inquiry or problem-solving.</td>
<td>Solving tasks might require following a convention or rehearsing a given procedure.</td>
</tr>
<tr>
<td>Answering tasks might involve using knowledge of the number sequence.</td>
<td>Answers are unlikely to relate tasks to the number sequence.</td>
</tr>
<tr>
<td>Answers do not involve exchanging units. For example, students solve 195 and ten more is 205, but do not need to explain this by trading 10 tens for 1 hundred.</td>
<td>Answers involve explicitly exchanging or trading: 10 ones for 1 ten, 10 tens for 1 hundred.</td>
</tr>
<tr>
<td>Attention is on structuring numbers around dynamic relationships of ones, tens and hundreds.</td>
<td>Attention is on manipulating numbers in terms of the formal place value system.</td>
</tr>
<tr>
<td>The aim is to cultivate strong mental strategies.</td>
<td>The aim is to prepare students to use the standard algorithms.</td>
</tr>
</tbody>
</table>
Mental Computation

Mental computation is a key part of conceptual place value problem-solving and involves being able to use the base-ten system to mentally increment and decrement by ones, tens, hundreds, etc. in multiple ways (Wright et al., 2012, p. 77). Encouraging these types of strategies broadens students’ understanding of our number system and requires them to think more deeply about place value. Gürbüz and Erdem (2016) connect these mental computations to long-term mathematical reasoning, describing how being flexible in strategies and being able to make various connections with place value between different numbers is crucial to higher-order thinking and later mathematical skills.

Wright et al. (2012) describe several student strategies for conceptual place value, such as the following:

- **Jump:** Students start at one number and then break apart the other number in order to move forward for addition or go backwards for subtraction. There can be variations on the jump strategy, such as over-jumping (adding or subtracting more in order to use an easier “friendly number” and then correcting the answer afterwards) and jumping to a decuple (adding or subtracting to a multiple of ten and then taking away groups of tens and ones) (pp. 100-101) (see Figure 2).
Figure 2. The jump strategy with 43-15 on an empty number line (two variations).

- Split: Students add or subtract the numbers in the tens place and the ones place separately and then join them together at the end of the problem (p. 100-101). This strategy most closely mirrors the standard algorithms but can lead to errors if students do not understand how to deal with problems where the amount to take away in a particular place is larger than what one starts with (see Figures 3 and 4).

\[
\begin{align*}
63 &= 60 + 3 \\
21 &= 20 + 1 \\
60 + 20 &= 80 \\
3 + 1 &= 4 \\
80 + 4 &= 84
\end{align*}
\]

Figure 3. The split strategy with 63+21.
Figure 4. The split strategy incorrectly applied with 43-15.

- **Split-jump:** Students begin with the split strategy, usually with the tens place, and then finish the question with the jump strategy. This is more often used when the question involves regrouping (pp. 100-101) (see Figure 5).

Figure 5. The split-jump strategy with 43-15.

- **Compensation:** The over-jumping strategy noted above is a form of compensation, where students solve a similar problem that is easier for them and then correct for what the problem is in reality. One or both numbers may be changed during this process (pp. 100-101) (see Figure 6).
Figure 6. The compensation strategy with 54-29.

- Transformation: Students change both numbers in the problem in order to convert one or both of them into “friendly numbers” that are easier to work with. This is different from compensation in that the answer does not require correction in the end. It actually creates a new problem that is equivalent to the original problem (pp. 101-101) (see Figures 7 and 8).

Figure 7. The transformation strategy for 54-29, which is changed to 55 – 30, to maintain a constant difference.
Other authors describe strategies related to mental computation and conceptual place value differently. These strategies are often referred to as invented strategies or as informal strategies. Carpenter, Franke, Jacobs, Fennema, and Empson (1997) utilize the term invented strategies in their longitudinal study showing that students who began place value instruction with them showed a greater understanding of the base-ten system and were more successful at solving novel problems compared to students who started with the standard algorithms. These authors described the invented algorithms that they saw as sequential invented strategies, combining-units invented strategies, compensating invented strategies, and other invented strategies (Carpenter et al., 1997, p. 9). Russell (2000) describes these as alternative solutions to the standard algorithms or to traditional strategies and also incorporates conceptual place value strategies in her definition of computation fluency. She believes that students can obtain computational fluency if they are encouraged to develop and use strategies that are closely tied to conceptual place value.

Developing conceptual place value understanding is critical for establishing a solid foundation for students to build on as they continue to explore numbers. For example, Russell (2000) describes how a strong understanding in number sense is needed in order to connect place
value concepts to standard algorithms and synthesize knowledge in a way that leads children to reason about and accurately use the algorithms. Otherwise, they will compartmentalize their mathematical understandings in ways that both hinders future achievement in more complex ideas and causes errors when using the algorithms. In addition, Thompson and Bramald (2007) reveal that children need “a substantial amount of time developing links” between verbal and non-verbal systems related to place value and that poor mathematical achievement may be related to exposure to algorithms before students have developed a clear understanding of place value (p. 11). Clarke (2005) also describes how early exposure to standard algorithms before conceptual understanding in place value is developed can cause long-term harm in student understanding, and Carpenter et al. (1997) confirm this with their findings that students who have mastered invented strategies before standard algorithms avoid “buggy algorithms” (p. 6). Therefore, it is of utmost importance that students have ample time to develop their understanding of conceptual place value and develop efficient mental computation strategies based on that understanding.

The CCSSM

The CCSSM are organized by grade level and domains. The domains for grades K-5 are Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations–Fractions, Measurement and Data, and Geometry. When reading the list of standards, one will first see a broad goal with listed standards that start with “CCSS.MATH.CONTENT” followed by a grade level, domain, letter, and number. If we take CCSS.MATH.CONTENT.1.NBT.C.4 as an example, the “1” refers to first grade, “NBT” refers to the domain, “C” means that the standard falls under the third broader learning target under the domain for first grade, and “4” means that it is the fourth listed standard out of all of the
standards under NBT listed for that first grade. Some may have an additional letter at the end of
the standard, such as CCSS.MATH.CONTENT.1.NBT.2.A, which means that the standard is an
umbrella for further concepts or examples, and “A” in this case means that it is referring to the
first addendum of that standard. For the purposes of this study, standards will be referred to their
shortened names, where CCSS.MATH.CONTENT and addendums are dropped. For example,
CCSS.MATH.CONTENT.1.NBT.2 is referred to as 1.NBT.2, with consideration to all
addendums versus addressing each one individually).

In regards to the interpretation and use of the CCSSM, many consider the standards to be
a proxy for a learning trajectory. While the definition and use of the phrase “learning trajectory”
can vary widely between researchers, an accepted definition for mathematics is the pattern in
how a child’s thoughts about a subject become more sophisticated over time (Lobato & Walters,
2017). According to Sarama and Clements (2009), learning trajectories are comprised of three
parts: a goal, a developmental path (or increasingly more sophisticated levels of understanding
and thinking), and instructional tasks that help them master a level on the developmental path
before moving onto the next (p. 64).

The standards provide two parts of this model of a learning trajectory in topics such as
place value by setting grade-level goals and describing how student understanding of this subject
should develop over time as they progress through the grades. Confrey, Maloney, and Corley
(2014) describe how the authors of the CCSSM (including themselves) used research on learning
trajectories as the basis for how to write and model the standards, also requesting researchers to
submit examples of trajectories that could be used in the document. These authors also expanded
the definition of a learning trajectory to “clusters and sequences of standards and their related
descriptors” and proposed a six-step process to connect the learning trajectories emphasized in
the standards to those developed from empirical research, with the ultimate goal of guiding curriculum development (Confrey et al., 2014, pp. 720, 722, & 731).

The CCSSM provides, then, a learning trajectory on all relevant topics for elementary school students. It also provides basic “milestones” for each topic, which can either help or hinder student understanding depending on whether or not these milestones are placed in grades where most students can reasonably attain them in light of their age and development. There clearly was discussion and consideration of the attainability of the learning goals at each grade level and to the time needed for students to develop a deep understanding of the ideas (versus setting a faster pace where students could attain certain standards sooner).

**Conceptual Place Value in the CCSSM**

The standards for conceptual place value understandings and its application with addition and subtraction are listed below in order to help analyze the timing of the learning trajectory used in the CCSSM for this domain. Other standards for place value application are available, such as those in third grade for multiplication and rounding, but the focus here is on how the standards expect students to develop their foundations in place value given how critical this will be for their understanding in more advanced topics. This discussion is also limited to kindergarten through third grade due to the placing of learning goals that directly relate to early understandings in place value. In addition, the standards most relevant to early place value understanding analyzed in this study are found mainly in the Counting and Cardinality (CC) and Number and Operations in Base Ten (NBT) domains (Common Core State Standards Initiative, 2019a, 2019c). Table 3 provides a summary of the CCSSM standards dealing with place value.
### Table 3

**A Summary of Place Value in the CCSSM, Kindergarten Through Third Grade**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Expectation</th>
</tr>
</thead>
</table>
| Kindergarten (5-6 years) | - Count to 100 by ones and by tens  
- Understand that numbers from 11 to 19 are composed of a ten and ones  
- Compose and decompose numbers 11 to 19 by place value and record this in an equation |
| First Grade (6-7 years) | - Understand that the digits in a two-digit number represent amounts of tens and ones  
- Within 100, add a two-digit and a one-digit numbers together and a two-digit number with a multiple of ten; be able to model this, use non-count-by-one strategies, use a written method to show this, and explain the reasoning used in these problems  
- Find ten more or less of a two-digit number without counting and provide an explanation of the reasoning behind this, using models, non-count-by-one strategies, and written methods  
- Subtract multiples of 10 from multiples of ten within 10-90 |
| Second Grade (7-8 years) | - Fluently add and subtract within 100, including regrouping, using multiple non-count-by-one strategies  
- Understand the value of digits in a three-digit number  
- Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form  
- Add and subtract 10 or 100 from any number between 100 and 900  
- Explain why addition and subtraction strategies work, using place value and the properties of operations |
| Third Grade (8-9 years) | - Fluently add and subtract within 1000 using multiple, non-count-by-1 strategies  
- Standard algorithm is a fourth grade standard |

The CCSSM begins place value instruction in kindergarten, expecting students to master two related concepts by the end of the year: counting by ones and tens and being able to decompose numbers up to 20 by groups of tens and ones. This can be seen in K.CC.1, “Count to
100 by ones and by tens,” and K.NBT.1, “Compose and decompose numbers from 11 to 19 into
ten ones and some further ones, e.g., by using objects or drawings, and record each composition
or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers
are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.”

In first grade, the CCSSM expect students to expand these skills with numbers up to 100
and to apply them to addition and subtraction. One standard focuses on continuing to develop the
students’ ideas of tens and ones: 1.NBT.2, which states, “Understand that the two digits of a two-
digit number represent amounts of tens and ones.” In contrast, three standards focus on student
understandings in multi-digit addition and subtraction. These are 1.NBT.4, “Add within 100,
including adding a two-digit number and a one-digit number, and adding a two-digit number and
a multiple of 10, using concrete models or drawings and strategies based on place value,
properties of operations, and/or the relationship between addition and subtraction; relate the
strategy to a written method and explain the reasoning used. Understand that in adding two-digit
numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a
ten,” 1.NBT.5, “Given a two-digit number, mentally find 10 more or 10 less than the number,
without having to count; explain the reasoning used,” and 1.NBT.6, “Subtract multiples of 10 in
the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using
concrete models or drawings and strategies based on place value, properties of operations, and/or
the relationship between addition and subtraction; relate the strategy to a written method and
explain the reasoning used.”

The standards introduce second-grade students to three-digit numbers and operations,
with mastery expected in adding and subtracting numbers up to 100 by the end of the year. One
standard directly relates to adding and subtracting within 100: 2.NBT.5, “Fluently add and
subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.” Four more standards relate to three-digit numbers; these are 2.NBT.1, “Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones,” 2.NBT.3, “Read and write numbers to 1000 using base-ten numerals, number names, and expanded form,” 2.NBT.B.7, “Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds,” and 2.NBT.8, “Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.” 2.NBT.9 also provides a broad cover for student explanations in place value by stating, “Explain why addition and subtraction strategies work, using place value and the properties of operations.”

The main standard in third grade in relation to adding and subtracting using place value is 3.NBT.A.1, “Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.” The standard algorithms for addition and subtraction are not addressed until fourth grade, but many educators and curriculum designers may interpret 3.NBT.A.1 to direct them to teach students the standard algorithm. Others interpret the word “algorithms” in this standard as student-invented algorithms instead of the standard algorithms for addition and subtraction.

The progression of place value topics in the CCSSM seem to align with the three dimensions of conceptual place value instruction as described by Wright et al. (2012): extending
the range of numbers, making increments and decrements more complex, and distancing the setting (the use and visibility of materials) (p. 80). It seems that the CCSSM treats the progression of place value standards intentionally with each of these three concepts in mind, as the standards address the same skills while increasing the level of sophistication asked of the students in each domain. To summarize, first grade standards extend the range and complexity of the CPV questions asked of Kindergarten students while maintaining the setting/materials, second-grade students keep the range but increase the complexity and take away the setting, and third grade requires the same types of strategies required by the second-grade standards while extending the range to 3-digit numbers, which therefore increases the complexity of the problems and requires further distance from the setting (since the third grade standard assumes that no materials are needed to understand how to use the hundreds place in CPV questions). Table 4 also summarizes the connection between the CCSSM’s treatment of CPV and Wright et al. (2012).

Table 4

*Place Value Standards in the CCSSM and the Three Dimensions of CPV as Described by Wright, Ellemor-Collins, and Tabor (2012)*

<table>
<thead>
<tr>
<th>Grade</th>
<th>Extending the range</th>
<th>Increasing the complexity of increments/decrements</th>
<th>Distancing the setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Third</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>
The place value standards can also be compared to the CPV learning trajectories as proposed by Wright, Stanger, Stafford, and Martland (2015). These researchers differentiate the CPV learning trajectories between jump and split strategies, emphasizing that starting students with the jump strategy can help avoid some pitfalls in CPV such as when the addition of the ones place in the split strategy goes over 9 (pp. 136, 159-160, 176-177). Both learning trajectories can be seen in Table 5 below.

Table 5

<p>| CPV Learning Trajectories as Proposed by Wright, Stanger, Stafford, and Martland (2015) |
|---------------------------------|---------------------------------|</p>
<table>
<thead>
<tr>
<th>Jump Strategy Trajectory (pp. 159-160)</th>
<th>Split Strategy Trajectory (pp. 176-177)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward and backward number word sequences by tens on and off the decuple</td>
<td>Review addition and subtraction in the ranges 1 to 10 and 1 to 20</td>
</tr>
<tr>
<td>Adding from a decuple and subtracting to a decuple</td>
<td>Higher decade addition and subtraction without and with bridging the decuple, using addition and subtraction facts in the range 1 to 10</td>
</tr>
<tr>
<td>Adding to a decuple and subtraction from a decuple</td>
<td>Review counting by tens forwards and backwards on and off the decuple</td>
</tr>
<tr>
<td>Incrementing and decrementing by tens off a decuple</td>
<td>Adding and subtracting 10 to a number</td>
</tr>
<tr>
<td>Adding a 1-digit number to a 2-digit number without bridging the decuple</td>
<td>Adding and subtracting decuples to a number</td>
</tr>
<tr>
<td>Adding a 1-digit number to a 2-digit number involving partitioning and bridging the decuple</td>
<td>Higher decade addition and subtraction, bridging the decuple</td>
</tr>
<tr>
<td>Subtracting a 1-digit number from a 2-digit number without bridging the decuple</td>
<td>Review partitioning and combining 2-digit numbers</td>
</tr>
</tbody>
</table>
### Table 5—Continued

<table>
<thead>
<tr>
<th>Jump Strategy Trajectory (pp. 159-160)</th>
<th>Split Strategy Trajectory (pp. 176-177)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtracting a 1-digit number from a 2-digit number involving partitioning and bridging the decuple</td>
<td>Partitioning and combining involving non-standard forms</td>
</tr>
<tr>
<td>Incrementing flexibly by tens and ones with tens strips</td>
<td>Adding two 2-digit numbers without regrouping</td>
</tr>
<tr>
<td>Introducing the empty number line (ENL)</td>
<td>Subtraction involving two 2-digit numbers without regrouping</td>
</tr>
<tr>
<td>Adding tens to a 2-digit number</td>
<td>Adding two 2-digit numbers with regrouping</td>
</tr>
<tr>
<td>Subtracting tens from a 2-digit number</td>
<td>Subtraction involving two 2-digit numbers with regrouping</td>
</tr>
<tr>
<td>Adding two 2-digit numbers without regrouping</td>
<td>Review jump strategy for addition and subtraction</td>
</tr>
<tr>
<td>Subtraction involving two 2-digit numbers without regrouping</td>
<td>Give tasks where children can choose either a jump or a split strategy and discuss children’s solutions</td>
</tr>
<tr>
<td>Adding two 2-digit numbers with regrouping</td>
<td>Place value tasks</td>
</tr>
<tr>
<td>Subtraction involving two 2-digit numbers with regrouping</td>
<td></td>
</tr>
<tr>
<td>Addition and subtraction involving two 2-digit numbers using other strategies</td>
<td></td>
</tr>
</tbody>
</table>

**Viewpoints on the CCSSM**

While the CCSSM standards for place value generally align with the conceptual place value literature in terms of the order of the learning goals themselves, there is a debate within the field of mathematics education on the developmental appropriateness of the CCSSM in general.
A prominent example of a researcher opposed to the CCSSM is Kamii (2015), who has identified several standards at the lower elementary level that are inappropriate. While she generally regards the entire CCSSM for kindergarten through third grade as developmentally inappropriate, Kamii specifically points out 21 standards as examples of expectations that she finds questionable or outright unacceptable, including these standards with connections to conceptual place value:

- K.CC.1: “Count to 100 by ones and by tens.”
- K.NBT.A.1: “Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18=10+8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.”
- 1.NBT.B.2 and 1.NBT.B.2.B: “Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.”
- 2.NBT.B.5: “Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.”
- 2.NBT.B.6: “Add up to four two-digit numbers using strategies based on place value and properties of operations.”
- 2.NBT.B.7: “Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship...”
between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.”

- 3.NBT.A.2: “Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.”

Kamii (2015) has conducted studies in which she found students lacked proficiency with these grade-level expectations pertaining to place value and concluded that these standards introduce content too soon or are not appropriate for children at all. She examined them in light of a constructivist ideology using Piaget’s theories, arguing that having children carry out mathematical tasks sooner than is developmentally appropriate causes them to rely more on socio-conventional knowledge and memorized statements rather than a true understanding of the mathematical concepts at hand. Furthermore, she argued that certain ideas should not be a focus of instruction, but should be developed by children themselves as they age. In summary, Kamii believes that the placement of many of the place value standards are too early for kindergarten through third grade students and leads to unwanted consequences such as unlearning previously constructed knowledge.

In a similar vein, Katz (2015) argues that pushing formal instruction too soon will harm a student’s intellectual dispositions (abilities related to reasoning skills and connected to a child’s natural inclinations and interests). Citing two longitudinal studies that show formal early instruction does not provide long-term results, she argues that academic goals and instruction should not be the priority of preschool and Kindergarten lessons. Instead, schools should focus
on experiences and projects that emphasize a child’s intellectual dispositions, while encouraging basic academic skills as a way to support these initiatives. This conflicts with the direction of the CCSSM due to the fact that there are standards that address formal mathematical ideas in Kindergarten.

On the other hand, Clements, Fuson, and Sarama (2017), who contributed to the writing of the CCSSM and who are associated with several foundations supporting the reformation of early childhood education in the U.S., criticize those who oppose the CCSSM, especially when it comes to Kamii (2015). They criticize her theoretical perspective and methodologies as outdated and inaccurate, claiming that her viewpoint and others who share similar thoughts are incorrect and “damaging to children” (Clements et al., 2017, p. 157). They summarize their own research and others that show that children can understand mathematical content earlier than Kamii proposes. Referring to the early education concept of developmentally appropriate practice (DAP), they define developmentally appropriate instruction as instruction that acknowledges the child’s background and current knowledge and then enables them to reach “challenging and achievable” goals (Clements et al., 2017, p. 155), and they define efforts to portray the CCSSM as developmentally inappropriate as the cause of inequities and low student achievement rates. In particular, their claim about inequities causing difficulties in mathematical achievement is also seen in other research (i.e., Morgan, Farkas, Hillemeier, & Maczuga, 2016). Clements et al. (2017) then claim that those who assert that the CCSSM are developmentally inappropriate do not understand the literature available about how young students learn and have developmentally inappropriate views themselves when they place limitations on what students can achieve due to their age.
Other researchers in support of the CCSSM also have strong viewpoints about those who doubt the appropriateness of the CCSSM. Zimba (2015), one of the main writers behind the CCSSM, strongly refutes arguments stating that the CCSSM are developmentally inappropriate, describing that the development team utilized research on early education and that experts in the field provided input on the standards. He debunks what he views as myths about the CCSSM by citing several studies showing that early mathematics education is linked to positive outcomes later in life and detailing false dichotomies that opponents of the standards use. For example, he does not believe that mathematics and play are polar opposites given that academics can raise the quality of children’s play. Researchers aligned with the stances similar to Zimba’s believe that the development process behind the CCSSM involved great care in making sure that the standards were developmentally appropriate for each grade level (as an example, see NAEYC & NAECS-SDE, 2010).

Two of the goals of the CCSSM are to improve coherence and focus, which in turn improves implementation of learning trajectories across the country and helps improve student learning by “aligning curriculum, instruction, and assessment” (Confrey et al., 2014, p. 731). When the standards-based trajectories are synthesized with outstanding research on learning trajectories and intentionally utilized in instruction, the quality of curriculum and outcomes improves (Confrey et al., 2014). Therefore, many researchers support the CCSSM as a whole and claim that better alignment to instruction would be the next steps for improving mathematics education in the country versus rethinking the standards themselves.

Given this divide in mathematics educators’ and researchers’ thinking, more research is needed to determine if the standards are attainable for the majority of students, which this study aims to help address. In connection with this need, early proponents of the CCSS stated that the
CCSS “must be a living document” that can and should be revised frequently in light of ongoing research (Confrey & Krupa, 2012; Confrey et al., 2014). It is therefore important that researchers continuously conduct studies that analyze the appropriateness and the impact of the CCSS, with open discussion about whether the CCSS should be updated and if any changes are needed. Since mathematical achievement has stagnated under the CCSSM, these standards should be re-examined in order to discuss if the learning goals themselves are inappropriately paced or if other changes and supports are needed to improve mathematics education in the United States.

**Instructional Approaches to Place Value**

Clearly, while standards are necessary to provide a guide for important content and grade level expectations, they are not in and of themselves going to affect student achievement. The approach curriculum designers and educators take to implementing the standards are the key as to whether or not the standards can attain their purpose in raising student achievement. In particular, developers of the CCSSM frequently argue that the standards themselves are high-quality, but are misinterpreted by curriculum designers and are not taught with fidelity by educators (i.e., Garland, 2014). Therefore, the curriculum and instructional methods that students are exposed to provide an important context for considering the impact of the standards.

One aspect that can help researchers consider whether the instructional approach implement the standards with fidelity is if instruction incorporates the CCSSM’s eight mathematical practices. The mathematical practices are listed separately from other standards and focus on addressing instruction for all students (versus how the standards discussed previously address content goals). The mathematical practice standards address the role of the student in learning mathematics, with the assumption that educators will guide students in doing
this and provide opportunities where these practices can occur. According to the Standards for Mathematical Practice (2019d), these practices are:

- “Make sense of problems and persevere in solving them”
- “Reason abstractly and quantitatively”
- “Construct viable arguments and critique the reasoning of others”
- “Model with mathematics”
- “Use appropriate tools strategically”
- “Attend to precision”
- “Look for and make use of structure”
- “Look for and express regularity in repeated reasoning”

The developers of the CCSSM emphasize that incorporating these practices into all grade levels provides a balance of conceptual and procedural understanding and that a heavy reliance on procedures may show a lack of understanding, leading to less engagement in these practices (Common Core State Standards Initiative, 2019d). It should also be noted that the practices themselves are seen as guidance for curriculum itself and the types of tasks that are chosen for instruction.

These standards have some similarities with the recommendations for developing conceptual place value. For example, both the “reasoning abstractly and quantitatively” and “use appropriate tools strategically” standards emphasize encouraging students to use coherent representations while problem solving. This can be seen in the recommended instructional approaches and tasks from Wright et al. (2015) when they detail methods using tools such as the empty number line, which is often used by students to represent their mental computation strategies using conceptual place value ideas (p. 136). The “reasoning abstractly and
quantitatively” standard also encourages the use of strategies based on understanding of the properties of the operations, which aligns with the types of strategies developed with conceptual place value activities that encourage children to be flexible in the strategies they can use (Wright et al., 2015, pp. 140-141).

While this study did not include an analysis of instruction on place value, some assumptions can be made based on the types of strategies students chose to solve addition and subtraction problems. Curriculum and instruction that maximizes the potential use of the mathematical practices in the CCSSM can aid in the study of the standards themselves, as it better guarantees that the standards are being taught as intended. Given the alignment between research on place value instruction and standards related to mathematical practices, an assumption can be made that students who are instructed in this way will have higher understandings in conceptual place value and can better attain grade-level expectations.
CHAPTER 3

METHODOLOGY

This exploratory case study aims to determine if the CCSSM standards for place value are developmentally appropriate by comparing norms-based test scores and an assessment that shows the level of mathematical understanding that a student has attained. A comparison between these two types of data can provide valuable insight into whether a learning target is developmentally appropriate or not. For example, if students who generally score at the top of the nation in mathematics (say, at the 90th percentile) routinely fail to attain an expected level of understanding, researchers and policymakers should question the appropriateness of the related grade-level standard. On the other hand, if most students regardless of their standing on a norms-based test show proficiency with a standard, one can safely say that educators can reasonably expect all or most students to attain the related understandings or skills associated with that standard.

Setting

Student data was drawn during the 2018-2019 and 2019-2020 school years from second, third, and fourth grade students who attended a Title I elementary school serving about 300 students in southwestern Michigan. These grade levels were chosen due to the constraints of one of the assessments used (the NWEA MAP assessment), which is not administered in Kindergarten or first grade and because this school serves students up to fourth grade. All students from the school who had valid end-of-year scores on both of the chosen assessments were included in this study. When it comes to the demographics of this school, the majority
(approximately 60%) of the students in this school are white, with almost equal proportions of African American, Asian, Hispanic/Latino, and multiracial students comprising the rest of the student body. Less than one percent of students come from another demographic such as American Indian or Native Hawaiian. About 7% of the students are English Language Learners (ELLs), and approximately 20% of the students have an Individualized Education Program (IEP) (Michigan’s Center for Educational Performance and Information, 2018).

The curriculum used by this school is the Bridges in Mathematics curriculum. The Bridges curriculum is used in K-5 mathematics education and was created to be aligned with the CCSSM, especially when it comes to the eight standards for mathematical practice. The curriculum is explicit with its connections to the mathematical practices, both summarizing the curriculum’s implementation of these practices at the beginning of each grade level’s teacher’s guides, as learning targets that are listed in each lesson, and as an integral part to the formative and summative assessments teachers use to evaluate their students. An example of the connections between this curriculum and the mathematical practices are found in the introduction to the Bridges in Mathematics Grade 4 Teachers Guide (The Math Learning Center, 2019) and is shown in Table 6 below.

While this table describes the connections in terms of student behavior, the lessons are also explicit in its scripts and instructions to teachers with how to make sure that these practices are intentionally used in the classroom.
Table 6

*The Math Learning Center’s (2019) Description of the Connections Between the Mathematical Practices and the Grade 4 Curriculum*

<table>
<thead>
<tr>
<th>CCSS Standard for Mathematical Practice</th>
<th>Characteristics at Grade 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make sense of problems and persevere in solving them (4.MP.1)</td>
<td>Fourth graders consider the meaning of a problem and look for appropriate, efficient ways to solve it. They use concrete and visual models as well as expressions and equations to represent, understand, and solve problems. They try different approaches when necessary, evaluate whether their solutions make sense in the context of the problem, and use alternative methods to check their answers.</td>
</tr>
<tr>
<td>Reason abstractly and quantitatively (4.MP.2)</td>
<td>Fourth graders connect the specific quantity represented by a number to written symbols. They make abstract representations of problems as they solve them, for example by writing equations, but can also think about those symbols in relation to the problem to make sense of the quantities in context.</td>
</tr>
<tr>
<td>Construct viable arguments and critique the reasoning of others (4.MP.3)</td>
<td>Fourth graders refine their mathematical communication skills by using words (written and spoken) and symbols (equations and expressions) to clarify their thinking. They support the representations they have made with sketches of objects, and they explain and justify their own strategies and solutions. They also ask specific questions to better understand and evaluate other students’ reasoning.</td>
</tr>
<tr>
<td>Model with mathematics (4.MP.4)</td>
<td>Fourth graders represent mathematical situations with numbers, words, sketches, actions, charts, graphs, equations, arrays, and ratio tables. They learn to connect these models and explain the connections among them. They use models not only as a way to represent problems, but also as tools for solving them and developing deeper understanding of mathematics.</td>
</tr>
<tr>
<td>CCSS Standard for Mathematical Practice</td>
<td>Characteristics at Grade 4</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>----------------------------</td>
</tr>
<tr>
<td>Use appropriate tools strategically (4.MP.5)</td>
<td>Fourth graders learn to consider the tools, both concrete and abstract, at their disposal and to select the ones that will be most useful to them in solving a particular mathematical problem or performing a particular task. For example, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. To use tools strategically, students must understand the requirements of the task, their own needs and strengths, and the capabilities of the tools available to them.</td>
</tr>
<tr>
<td>Attend to precision (4.MP.6)</td>
<td>Fourth graders are increasingly able to be clear and precise in communicating mathematically, both in writing and in discussion. They specify units of measure and are careful to use the correct language to describe operations and symbols. They also take care to measure, draw, and label with precision.</td>
</tr>
<tr>
<td>Look for and make use of structure (4.MP.7)</td>
<td>When considering mathematical situations and solving problems, fourth graders seek out patterns and notice structure. They use what they notice to solve problems and develop deeper conceptual understandings.</td>
</tr>
<tr>
<td>Look for and express regularity in repeated reasoning (4.MP.8)</td>
<td>Fourth graders notice repetition when solving problems and use that repetition to develop more efficient strategies for solving similar problems. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms.</td>
</tr>
</tbody>
</table>

In second grade, the jump and split strategies are emphasized for computation. The split strategy in particular is emphasized through Bridges’ focus on helping students understand three-digit numbers through decomposition (breaking numbers apart by place value) and with applications of the expanded form for those numbers (e.g., the expanded form of 642 is
600+40+2). However, the jump strategy is also emphasized, with a heavy focus on using open
number lines to help students visualize the jumps in a problem (The Math Learning Center,
2019, pp. ii-iii). To highlight this, the jump strategy is the first addition strategy students see
modeled without materials and written out with more formal notations (i.e., The Math Learning
Center, 2019, Grade 2 Unit 2 Module 3, pp. 29-36). Students then practice jump strategies on
open number lines and through a number of lessons before being posed a question that
encourages them to discover the split strategy with materials (i.e., The Math Learning Center,
2019, Grade 2 Unit 3 Module 1, p. 22). There is also a focus on compensation strategies near the
end of the school year after split and jump strategies have been reinforced (i.e., The Math
Learning Center, 2019, Grade 2 Unit 7 Introduction, pp. v-vii).

The third grade Bridges curriculum immediately reviews the jump and split strategies at
the start of the school year and continually reinforces them throughout the year, both in terms of
extending the range of numbers and by continuing to build distance from materials (The Math
Learning Center, 2019, Grade 3 Unit 1 Introduction). It also continues to develop compensation
strategies, while introducing and focusing heavily on transformation, to the point where teachers
are encouraged to prioritize transformation strategies over compensation ones (The Math
Learning Center, 2019, Grade 3 Unit 3 Introduction). These transformation strategies are called
“Give and Take” for addition and “Constant Difference/Distance” for subtraction.

Bridges then introduces the standard algorithm as well and presents it side-by-side with
other CPV methods. The fourth grade curriculum largely follows the third grade model by
presenting addition and subtraction strategies for problems up to 1,000. Although discussion of
the standard algorithms is held off until near the middle of the year in fourth grade, they are
taught as though the students were not exposed to them in third grade, and teachers are
discouraged from focusing on them to the detriment of other strategies (The Math Learning Center, 2019, Grade 4 Unit 4 Introduction).

All teachers at this school are also trained in Math Recovery® and are directed to use this program with all students, especially those who are struggling with CPV. The Math Recovery® program follows the ideology of Wright et al. (2012) in developing student understanding of conceptual place value as opposed to traditional place value by gradually extending the range of problems, distancing the setting, and increasing the complexity of problems. Students are encouraged to explore numbers and develop mental computation strategies with the goal of getting them to reach grade level expectations. Math Recovery® tends to focus on student dialogue and debate to think about and model math concepts, as well as on student-driven partner work and small group work through tasks such as math games and story problems. These activities are particularly helpful as they are geared towards a student’s specific level of mathematical understanding as opposed to whole-group and grade-level activities that may or may not meet a student where they are. Phillips, Leonard, Horton, Wright, and Stafford (2003) describe the interventions found in this program as high quality and efficient at improving student mathematical attainment, also noting that teachers using the Math Recovery® program tend to change their overall pedagogical approach to reflect more constructivist approaches to conceptual place value (as compared to using more direct approaches with traditional place value).

**Instruments**

Two assessments were used for this study: The Northwest Evaluation Association Measures of Academic Progress (NWEA MAP) 2-5 Mathematics assessment and the U.S. Math Recovery Council’s conceptual place value Add+VantageMR (CPV AVMR) assessments. The
NWEA MAP 2-5 Mathematics assessment was used to provide norms-based results on both overall mathematical ability and on four different domains of mathematical knowledge (NWEA, 2019). It is a standardized test that is used on a nationwide level, and the four domains tested are Measurement and Data, Operations and Algebraic Thinking, Geometry, and Number and Operations. Student responses to questions provide a raw score, and these points are then converted to percentiles. The CPV AVMR assessments were developed to help teachers diagnose what part of the learning trajectories students are currently on in regards to their understanding of the values of digits in multi-digit numbers and how to add and subtract (U.S. Math Recovery Council, 2019). These assessments are aligned with the CCSSM’s expectations; Table 7 summarizes the alignment between the CCSSM and the scores, or constructs, that students receive after taking these assessments. As compared to the NWEA, these assessments are more of a diagnostic tool used by the teacher and are selected by individual districts who would like to use it as a part of their assessment program. The CPV AVMR assessment gives students construct scores from 0 to 5 based on their overall performance, which will be used for this study.

Table 7

*Alignment Between the CCSSM Standards and CPV Constructs*

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>K.CC.1, K.CC.4, K.NBT.1</td>
<td>Constructs 0 and 1</td>
</tr>
<tr>
<td>1.NBT.2, 1.NBT.4, 1.NBT.5, 1.NBT.6</td>
<td>Construct 2</td>
</tr>
<tr>
<td>2.NBT.1, 2.NBT.3, 2NBT.5, 2.NBT.8, 2.NBT.9</td>
<td>Constructs 3 and 4</td>
</tr>
<tr>
<td>3.NBT.2</td>
<td>Construct 5</td>
</tr>
</tbody>
</table>
The CPV AVMR assessments are divided into five different progress monitoring sheets. The first one determines if a student is a construct 0 or 1, which refers to whether a student is beginning to understand that a unit of ten is ten ones and can start to work with units of tens without having to count by ones as long as materials are available (Calhoun Intermediate School District, n.d.). According to both AVMR and the CCSSM, students are expected to reach this level of understanding by the end of kindergarten (Calhoun Intermediate School District, 2016). In particular, this first assessment helps teachers determine whether the student may be ready for place value instruction or if they need to work on an earlier topic first, such as basic addition and subtraction or how to structure numbers.

The second progress monitoring sheet checks to see if students have progressed to construct 2, where students have a stronger understanding of place value with materials and can complete addition and subtraction problems that simultaneously ask them to use tens and ones, which is expected by the end of first grade according to the CCSSM and the alignment between the standards and AVMR. (Students who can only solve problems without materials using the standard algorithm are also considered to be at construct 2, as they are typically not able to explain how to use the full value of each of the digits in a number in a given operation without having additional support from materials.) The next two progress monitoring sheets check to first see if students can use one place value strategy to add and subtract within 100 without materials (construct 3) and then to see if students can use multiple strategies for adding and subtracting within 100 and can solve problems involving regrouping (construct 4). Both of these constructs are addressed in second grade. The last progress monitoring sheet determines whether a student can extend construct 4 understandings to three-digit numbers and problems within
Examples of items from these progress monitoring sheets are:

- Place Value Progress Monitoring 0-1: Presenting bundles of ten sticks and single sticks to the student and asking them, “Get 40 sticks.” This checks for student understanding related to ten being a single unit made out of a group of ones. The interviewer will also hide increasing amounts of bundles/sticks under a cover after presenting them to the student to see if the student can add by tens and ones.

- Place Value Progress Monitoring 1-2: The interviewer repeats the exercise of presenting sticks and bundles to the student, hiding them under a cover, and then asking for the total amount beneath the cover, but the complexity of the questions is increased (ex. simultaneously adding ones and tens, going across the decuple).

- Place Value Progress Monitoring 2-3: This assessment takes away the setting/materials and presents bare number tasks (i.e., 63+21 and 43-15) to the student to check for at least one viable mental strategy that uses conceptual place value.

- Place Value Progress Monitoring 3-4: This assessment asks two of the same questions and two similar questions compared to the last progress monitoring sheet (the main difference being that 54-29 is the regrouping subtraction question) that students have to use more than one mental strategy in an efficient way in order to pass.

- Place Value Progress Monitoring 4-5: Similar to the last two progress monitoring sheets, but this tests students on three-digit questions (i.e., 342+120 and 304-198) (Calhoun Intermediate School District, n.d.).
Data Collection

Reports containing NWEA MAP data and the CPV AVMR construct scores from the 2018-2019 school year were provided by the school in the study. The scores from the NWEA MAP test were automatically calculated and organized into reports by the NWEA company, which were then given to the school. The CPV AVMR constructs scores were determined by individual teachers near the end of the 2018-2019 school year and then were self-reported to a math interventionist who personally organized the data in a report for the principal and math coordinator for later reference. Only students who had scores on both assessments were used for this study. There were 44 second-grade, 37 third-grade, and 56 fourth-grade students included for the correlation. Each student’s score had to be pulled from each report and connected in a separate document. Once the MAP and construct scores were matched, the data was de-identified and then used to perform a correlation analysis between the two assessments.

In order to avoid inter-rater reliability and teacher inflation issues, 32 third-grade students and 38 fourth-grade students were also interviewed during the 2019-2020 school year using the CPV progress monitoring sheets from Calhoun Intermediate School District (n.d.). The author conducted all of the interviews as a routine part of the school day. Each interview lasted from approximately five minutes to twenty minutes depending on how many progress monitoring questions needed to be conducted to determine the student’s construct level. Students were shown each question and asked to verbally answer it and describe their thinking, without writing down their work or otherwise being given any materials (with exception to the popsicle sticks and bundles of ten sticks provided for the earliest test questions). As students described their methods, the interviewer determined which strategy the student used and had the option of scribing what the student said, along with noting any other actions the student attempted (e.g.,
tracing imaginary lines with fingers on the table as if he or she had paper to conduct the standard algorithm on) or asking further questions in the event that the student’s description or answers did not clearly provide a strategy or construct level.

The data was de-identified, and then each progress monitoring sheet was coded for construct, strategy used, and errors present. Once this information was processed and compiled together, quantitative data was gathered on the number of students at each construct in each grade and on the types of errors that the majority of students (those at a construct 3 and 4) presented while problem-solving. Since the data was de-identified as it was collected it was not used in the correlation analysis to other sets of data such as the NWEA MAP test scores. A timeframe for the data collection events is shown below in Table 8.

Table 8

*Data Collection Timeline*

<table>
<thead>
<tr>
<th>Activity</th>
<th>Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-of-year administration of the 2018-19 CPV AVMR assessments, 137 students</td>
<td>April-May 2019</td>
</tr>
<tr>
<td>District collection of end-of-year CPV AVMR construct scores for 2018-19, 137 students</td>
<td>June 2019</td>
</tr>
<tr>
<td>Spring 2019 (end-of-year) NWEA MAP administration, 137 students</td>
<td>May 2019</td>
</tr>
<tr>
<td>Collection of end-of-year 2018-19 NWEA MAP scores, 137 students</td>
<td>October 2019</td>
</tr>
</tbody>
</table>
Analysis

Two correlation analyses were conducted. The first was on the overall scores that students received on the NWEA MAP test and their construct levels in CPV. This was done in order to see whether students who scored above the 33rd percentile on the NWEA MAP test also maintained grade-level scores on the CPV AVMR assessments. The 33rd percentile was chosen since it is approximately half a deviation away from the mean and can show researchers if most students who are close to average, average, or are above average can meet the standards. Students who scored below the 33rd percentile will be reported in order to determine if the CCSSM’s claim that 100% of all students must show proficiency on the standards is attainable. The results from this comparison are beneficial because the NWEA MAP 2-5 Mathematics assessment uses what Koretz (2008) describes as a relative inference (how the students compare to each other) while the CPV AVMR assessment uses an absolute inference (whether students can pass a specific standard without regard to how their peers perform). Students who score higher using the relative inference of the NWEA MAP 2-5 Mathematics assessment but cannot meet the absolute inference of the CPV AVMR assessment may reveal that the demands of the standards may be developmentally inappropriate.

Both raw scores and percentiles were used to do the correlation, because a difference in one point on the raw NWEA MAP score could lead to a larger difference in a student’s percentile. In addition, the percentage of students proficient according to their CPV construct was determined at six different ranges of percentiles in order to see if comparing them to each other would provide any insights beyond the overall correlation for the grade. These ranges were 0-100 (all students), 0-32 (students far below average as compared to other students who took the NWEA MAP test in the nationwide norms-based data set), 33-50 (students below average and
exactly average), 51-84 (students above average), 84-100 (students high above average), and 33-100 (students below average, average, above average, and high above average).

The item analysis of the 2019-2020 AVMR CPV assessments included a narrative incorporating both quantitative and qualitative data which summarized the patterns seen in the progress monitoring sheets for each grade. These assessments revealed where students were located on the CPV “learning trajectory” in comparison to the expectations set forth by the CCSSM standards written for each grade level. They also revealed patterns in student problem-solving according to strategies used and errors made. To find these patterns, the third-grade and fourth-grade progress monitoring sheets and data were separated, and the number of constructs at each grade level was determined.

The progress monitoring sheets were first reviewed for any strategies used and then any errors that occurred with each strategy attempted. A summary of patterns seen in third- and fourth-grade students was written for students scoring below construct 3 and at construct 5, as the number of samples falling under this category was small enough to look at each assessment and synthesize them in a qualitative summary. There was a larger number of third-grade students scoring at construct 3 and fourth-grade students scoring at constructs 3 and 4, so their results were coded to aid with an analysis of any patterns occurring. Third-grade students scoring at construct 3 were also coded despite the smaller number of samples to provide an easier comparison to fourth-grade students scoring at construct 3. The categories involved in the coding of strategies for both grades were jump, split, split-jump, compensation, algorithm, and other while the categories used to code errors were inflexibility, subtraction regrouping errors occurring with the split strategy, regrouping errors with the split-jump strategy, standard
algorithm errors, miscalculation across the decuple, and other. Strategies and errors coded as “other” were considered for inclusion as a qualitative description in the summary of results.

Finally, a comparison was conducted between the CPV AVMR construct scores from the 2018-2019 and the 2019-2020 school years. The previous year’s second-grade scores were compared to the current year’s third-grade scores while the previous year’s third-grade scores were compared to the current year’s fourth-grade scores. This was done because the data from the 2018-2019 school year was self-reported by seven different people and then used for evaluation purposes, so this comparison was performed to help check for inter-rater reliability and inflation concerns.
Correlation Analysis

Second-Grade Results

The correlation between AVMR CPV construct scores and the base MAP score for second graders was 0.81. Although most second-grade students did not attain proficiency on CPV, meaning they did not achieve a construct 4 on the AVMR CPV assessment, the correlation is still high in that students who scored high on the MAP test typically still did better on the AVMR CPV assessment as compared to those scoring lower on the MAP test. In other words, there was still a trend that those who did better than peers on one test did better than others on the second test. The correlation between the AVMR CPV construct scores and MAP percentiles was 0.68. This correlation is lower because the percentile rankings are less specific than the base MAP score, but there is still a positive connection between how a student scores on both tests.

When the number of students proficient with CPV is analyzed by percentile ranges, as seen in Table 9, it appears that second-grade students struggled as an overall group with this domain. Of the 29 second-grade students who scored above the 33rd percentile on the NWEA MAP, six of those students (approximately 14%) showed proficiency on the AVMR CPV assessment, meaning they achieved a construct 4. Two of those six students scored in the below average and average range (33rd to 50th percentiles) on the NWEA MAP while four scored in the above average range (51st to 84th percentiles). While none of the students who scored below the 33rd percentile on the NWEA MAP showed proficiency on the CPV, as expected, curiously,
neither of the two students who scored high above average (85\textsuperscript{th} percentile or above) were proficient on the AVMR CPV assessment.

Table 9

*Percentage of Students Proficient on the AVMR CPV Assessment: Second Grade (n=44)*

<table>
<thead>
<tr>
<th>Percentile Range (NWEA MAP)</th>
<th>Number of Students in the Percentile Range</th>
<th>Number Proficient On AVMR CPV</th>
<th>Percentage Proficient on AVMR CPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>44</td>
<td>6</td>
<td>14%</td>
</tr>
<tr>
<td>0-32</td>
<td>15</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>33-100</td>
<td>29</td>
<td>6</td>
<td>21%</td>
</tr>
<tr>
<td>33-50</td>
<td>14</td>
<td>2</td>
<td>14%</td>
</tr>
<tr>
<td>51-84</td>
<td>13</td>
<td>4</td>
<td>31%</td>
</tr>
<tr>
<td>85-100</td>
<td>2</td>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Third-Grade Results**

The correlation between AVMR CPV construct scores and the base MAP score for third graders was 0.74. The correlation between the AVMR CPV construct scores and the MAP percentile rankings was 0.69. Again, this means that students who scored higher on one test also scored higher on the other test as compared to their peers in this study, although students in this study tended to not meet grade-level expectations on the AVMR CPV assessment (construct 5 for third grade). Of the 37 third-grade students, 14 students (about 38\%) scored above the 33rd percentile on the NWEA MAP and 14 students were proficient on the AVMR CPV assessment. In contrast, only 2 of the 23 (8.7\%) students scoring below the 33rd percentile were proficient on the AVMR CPV assessment. This is seen in Table 10 below.
Table 10

*Percentage of Students Proficient on the AVMR CPV Assessment: Third Grade (n=37)*

<table>
<thead>
<tr>
<th>Percentile Range (NWEA MAP)</th>
<th>Number of Students in the Percentile Range</th>
<th>Number Proficient On AVMR CPV</th>
<th>Percentage Proficient on AVMR CPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>37</td>
<td>14</td>
<td>38%</td>
</tr>
<tr>
<td>0-32</td>
<td>23</td>
<td>2</td>
<td>9%</td>
</tr>
<tr>
<td>33-100</td>
<td>14</td>
<td>12</td>
<td>86%</td>
</tr>
<tr>
<td>33-50</td>
<td>5</td>
<td>4</td>
<td>80%</td>
</tr>
<tr>
<td>51-84</td>
<td>9</td>
<td>8</td>
<td>89%</td>
</tr>
<tr>
<td>85-100</td>
<td>0</td>
<td>0</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Fourth-Grade Results**

The correlation between the NWEA MAP and the AVMR CPV for fourth graders was 0.75. The correlation between the AVMR CPV construct scores was 0.62. This continues the pattern seen in the other two grades where higher performance on one test as compared to peers in this study was connected to relatively higher performance on the other even though relatively few students reached proficiency on the AVMR CPV assessment, which is still considered a 5 for fourth grade. Data for 56 students was provided. As seen in Table 11, the majority of students scoring above the 33rd percentiles were able to reach proficiency on the AVMR CPV assessment, although six students out of the 28 (21.43%) students who were below the 33rd percentile on the NWEA MAP test did show proficiency on the AVMR CPV.
Table 11

*Percentage of Students Proficient on the AVMR CPV Assessment: Fourth Grade (n=56)*

<table>
<thead>
<tr>
<th>Percentile Range (NWEA MAP)</th>
<th>Number of Students in the Percentile Range</th>
<th>Number Proficient On AVMR CPV</th>
<th>Percentage Proficient on AVMR CPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>56</td>
<td>30</td>
<td>54%</td>
</tr>
<tr>
<td>0-32</td>
<td>28</td>
<td>6</td>
<td>21%</td>
</tr>
<tr>
<td>33-100</td>
<td>28</td>
<td>24</td>
<td>86%</td>
</tr>
<tr>
<td>33-50</td>
<td>12</td>
<td>10</td>
<td>83%</td>
</tr>
<tr>
<td>51-84</td>
<td>13</td>
<td>11</td>
<td>85%</td>
</tr>
<tr>
<td>85-100</td>
<td>3</td>
<td>3</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Summary of the Correlation Analysis Findings**

The findings seem to suggest that the place value standards at the third and fourth grades are attainable for students scoring at the 33rd percentile or above on the NWEA MAP, with the second-grade standards being out of reach for most students no matter how their mathematical abilities compared to peers on the norms-based test. However, a correlation analysis like this in and of itself is not sufficient to determine which standards are attainable and which ones should have their placements reconsidered. Instead, it provides an initial analysis to indicate that some of the grade-level expectations may need to be reconsidered. The additional examination of student performance on individual items on the CPV assessments will help determine general trends by grade level and what may be considered developmentally appropriate for each group.
Item Analysis

Third-Grade Students

The majority of third-grade students assessed during the 2019-2020 school year (21 students, or approximately 64%) scored at a construct three on the CPV assessment. Students were placed at this level if they did not need to use materials to solve addition and subtraction problems within 100 that did not involve regrouping and if they used at least one place value strategy to solve the problems (versus counting backwards or forwards by ones). This means that the majority of third-grade students performed similarly to what is expected from a second-grade student at the beginning or middle of the year. These results are shown in Table 12 below.

Table 12

*Percentage of Third-Grade Students at Each Construct for CPV (n=33)*

<table>
<thead>
<tr>
<th>Construct</th>
<th>Number of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Kindergarten: does not recognize ten as a group of ones; solves problems by counting by ones)</td>
<td>5</td>
<td>15%</td>
</tr>
<tr>
<td>1 (Kindergarten: recognizes ten as a group of ones but still relies on count by one strategies)</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>2 (First grade: can utilize ten as a unit in addition and subtraction problems when materials are provided)</td>
<td>1</td>
<td>3%</td>
</tr>
<tr>
<td>3 (Second grade: can use one mental strategy based on place value to solve two-digit addition and subtraction problems without materials)</td>
<td>21</td>
<td>64%</td>
</tr>
<tr>
<td>4 (Second grade: Solves two-digit addition and subtraction problems using multiple mental strategies)</td>
<td>5</td>
<td>15%</td>
</tr>
<tr>
<td>5 (Third grade: Solves three-digit addition and subtraction problems using an efficient mental strategy for the problem at hand)</td>
<td>1</td>
<td>3%</td>
</tr>
</tbody>
</table>
A total of 6 students (approximately 18% of all third-grade students in the study) performed below a second-grade level (construct 0-2), with the five students at construct 0 performing below an end of kindergarten level and the student at construct 1 performing at an end of first grade level. Three of the construct 0 students required materials to count by one and did not show a conceptual understanding of how to use groups of 10 while the other two students could not count by ones with materials. One of these two students seemed to have memorized how to verbally say the forward number sequence with tens despite not accurately counting materials by ones. The construct 2 student could solve addition problems up to 100 without materials, including with regrouping, but needed materials to attempt to solve any subtraction problems.

With third-grade students scoring at a construct 3, most could not move on to a construct 4 due to inflexibility with strategies or errors with the subtraction regrouping question (54-28). Fifty-seven percent of third-grade students at construct 3 were inflexible with strategies, and all students coded as such relied solely on the split strategy. Two-thirds of students at construct 3, whether or not they showed flexibility with strategies on the other questions, were unable to successfully solve the regrouping subtraction question due to attempting to use the split strategy with it and not recognizing or correctly utilizing the negative number that results from it (e.g., stating that 4-9=5 instead of -5 and then performing 30+5 instead of 30-5).

Three third-grade students scoring at a construct 3 (approximately 14%) attempted to be flexible with strategies by using the split-jump strategy (e.g., starting the problem by splitting both numbers by place-value and then switching to a jump strategy partway through the problem) with the regrouping problems but struggled to perform the steps correctly. With 54-29,
for instance, one student first did 50-20=30 but then added 9 to perform 39-4 instead of adding 4 to then perform 34-9. Four students (approximately 19% of students at construct 3) showed an understanding of how to perform a place value strategy but miscalculated their final answer. All of these errors related to adding or subtracting across a decuple. As an example, students performing a split-jump strategy with 54-29 accurately would have solved it as 50-20=30, 30+4=34, and 34-9=25, but some students became stuck on 34-9 and inaccurately solved it as 24 and 22.

Two students (approximately 10% of students at construct 3) attempted to use the standard algorithm during the interview. One student stated that a tutor said to always use the standard algorithm with regrouping and did it correctly with addition. However, the student then stated that it wasn’t possible to do this with the subtraction regrouping problem and proceeded to successfully figure out how to solve it with the split strategy. The other student attempted to duplicate the standard algorithm with subtraction regrouping but still solved 54-29 as 35 instead of 25 and also stated that there was no other way to solve the problem. These attempts were made despite the fact that paper and pencil were not provided; these students traced the standard algorithm with their fingers on the table and verbally described the standard algorithm when asked to explain. A combined table of the construct 3 errors made by third-grade students is seen in Table 13 below.
Table 13

**Typical Errors Made by Third-Grade Students at a Construct 3 for CPV (n=21)**

<table>
<thead>
<tr>
<th>Error</th>
<th>Number of students</th>
<th>Percentage of third-grade students at a construct 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflexibility</td>
<td>12</td>
<td>57%</td>
</tr>
<tr>
<td>Regrouping error: split strategy with subtraction</td>
<td>14</td>
<td>67%</td>
</tr>
<tr>
<td>Regrouping error: split-jump strategy with subtraction</td>
<td>3</td>
<td>14%</td>
</tr>
<tr>
<td>Standard algorithm without an alternative strategy</td>
<td>2</td>
<td>10%</td>
</tr>
<tr>
<td>Miscalculations across the decuple</td>
<td>4</td>
<td>19%</td>
</tr>
</tbody>
</table>

When examining grade-level expectations as shown by the Michigan State Standards & Math Recovery Alignment Document from Calhoun Intermediate School District (2016), it should be expected that most third-grade students should perform at construct 4 near the middle of the school year and at construct 5 by the end of the year. However, only 5 third-grade students (approximately 15%) in this study performed at this level. Students were considered to be a construct 4 in this study if they could accurately and flexibly solve problems within 100 but could not flexibly and accurately solve all of the three-digit addition and subtraction problems. Table 14 shows the typical errors made by third graders at construct 4.

Four of the students seemed unable to transfer knowledge of how to use jump or split-jump strategies to three-digit numbers even after showing proficiency with these strategies on problems related to two-digit numbers. Two students (40% of those at construct 4) only used the split strategy and then used it inaccurately with the subtraction regrouping problem. Another student realized that she used the split strategy inaccurately with the subtraction regrouping
problem and then attempted to use the standard algorithm but did not know how to use it correctly, also stating that she did not know another method to solve this problem. Two other students attempted to use the split-jump strategy with the subtraction regrouping problem but could not solve it accurately. It is difficult to make general assumptions due to the small amount of data points with third-grade students who score at a construct 4 for CPV, but most of these students fell back on depending on the split strategy when they began to work with three-digit numbers.

Table 14

*Typical Errors Made by Third-Grade Students at a Construct 4 for CPV (n=5)*

<table>
<thead>
<tr>
<th>Error</th>
<th>Number of students</th>
<th>Percentage of third-grade students at a construct 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflexibility</td>
<td>2</td>
<td>40%</td>
</tr>
<tr>
<td>Regrouping error: split strategy with subtraction</td>
<td>3</td>
<td>60%</td>
</tr>
<tr>
<td>Regrouping error: split-jump strategy with subtraction</td>
<td>1</td>
<td>20%</td>
</tr>
<tr>
<td>Standard algorithm without an alternative strategy</td>
<td>1</td>
<td>20%</td>
</tr>
<tr>
<td>Miscalculation: split-jump strategy</td>
<td>1</td>
<td>20%</td>
</tr>
</tbody>
</table>

Both the construct 3 to 4 progress monitoring sheet and the construct 4 to 5 progress monitoring sheet included problems (54-29 and 304-198) that specifically aimed to see if students would use the compensation or transformation strategies. These problems included a number close to a decuple, which lends itself to both of these strategies. Figures 6 and 7 from chapter 3 show both strategies used with 54-29 while Figures 9 and 10 below demonstrate them
for 304-198. However, no third-grade students attempted to utilize these strategies for either problem.

Figure 9. The compensation strategy with 304-198.

Figure 10. The transformation strategy with 304-198.

One student performed at a construct 5 level, which is considered above-grade level given that this study was conducted partway through the year. This student used both the jump and split strategies to solve problems. However, even though the student scored a construct 5, this does not necessarily mean this student could not benefit further from CPV instruction before
moving onto the standard algorithm given he did not use a compensation or transformation strategy.

**Fourth-Grade Students**

In contrast to the third-grade data, the fourth-grade students in this study had a broader range of scores. Four students (approximately 11%) scored at a construct 5, which students are expected to perform at when they arrive to fourth grade. The majority of students performed at a construct 3 (12 students, or roughly 32%) or a construct 4 (13 students, or about 34%). One student performed at a construct 0 while the rest of the students were almost split between construct 1 (5 students, or about 13%) and construct 2 (3 students, or about 8%). At first glance, it may seem like the fourth-grade data set shows increased student mobility as the constructs increase as compared to the third-grade data set (e.g., fewer students at a construct 0 and seemingly fewer students stagnating at a construct 3), but the percentage of students performing as expected according to their grade level is lower (approximately 10.5% in fourth grade as compared to about 18% in third grade). Results are shown in Table 15.

Fourth-grade students scoring at a construct 3 or below attempted to use the standard algorithm during the test at a higher rate than seen in the third-grade data. One of the students performing at a construct 1 incorrectly attempted to solve a problem using the standard algorithm when given a question to test for the possibility of being able to perform at a construct 2 or 3. This student’s strategy, which is shown below in Figure 11, could possibly show that the student had been coached on using the standard algorithm with regrouping problems without first developing CPV understandings, and this led to the development of a “buggy algorithm.”
Table 15

*Percentage of Fourth-Grade Students at Each Construct for CPV (n=38)*

<table>
<thead>
<tr>
<th>Construct</th>
<th>Number of students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0  (Kindergarten: does not recognize ten as a group of ones; solves problems by counting by ones)</td>
<td>1</td>
<td>3%</td>
</tr>
<tr>
<td>1  (Kindergarten: recognizes ten as a group of ones but still relies on count by one strategies)</td>
<td>5</td>
<td>13%</td>
</tr>
<tr>
<td>2  (First grade: can utilize ten as a unit in addition and subtraction problems when materials are provided)</td>
<td>3</td>
<td>8%</td>
</tr>
<tr>
<td>3  (Second grade: can use one mental strategy based on place value to solve two-digit addition and subtraction problems without materials)</td>
<td>12</td>
<td>32%</td>
</tr>
<tr>
<td>4  (Second grade: Solves two-digit addition and subtraction problems using multiple mental strategies)</td>
<td>13</td>
<td>34%</td>
</tr>
<tr>
<td>5  (Third grade: Solves three-digit addition and subtraction problems using an efficient mental strategy for the problem at hand)</td>
<td>4</td>
<td>11%</td>
</tr>
</tbody>
</table>

![Figure 11. Misapplication of the standard algorithm with 63+21.](image)

Another student could only solve bare number tasks using the standard algorithm, with inconsistent accuracy (especially with the subtraction regrouping problem), leading to a construct 2 score. Four students performing at a construct 3 (a third of the students scoring at this level) switched to the standard algorithm when presented with the subtraction regrouping problem.
One of the students was able to fully perform it with accuracy but could not describe another method for solving the subtraction regrouping problem; the rest had issues with accuracy and flexibility when prompted for an alternative strategy.

Along with the standard algorithm errors, students scoring at constructs 0-2 showed errors typical at those levels (e.g., relying on materials and count by one strategies, struggling to switch between adding ones and tens). An exception is a student who was initially asked a three-digit addition problem to determine where to start testing her construct level. She successfully used the split strategy and correctly described each digit’s value when talking through her thought process. But when asked how to solve 678-153, she told the researcher that she was confused on how to solve 8-3, as she did not know whether it would result in a negative number or not. She needed to use materials to solve any subtraction problems due to confusion over negative numbers, resulting in a construct 2 score.

Beyond errors with the standard algorithm, fourth-grade students scoring at construct 3 struggled with flexibility with strategies. This happened at a higher rate than the third grade data. They also focused on the split strategy, although one fourth-grade student used the jump strategy for each problem given. Errors with subtraction regrouping and the split strategy and moving across the decuple continued to occur with fourth-grade students performing at a construct 3. A summary of the results is shown in Table 16.
Table 16

*Typical Errors Made by Fourth-Grade Students at a Construct 3 for CPV (n=12)*

<table>
<thead>
<tr>
<th>Error</th>
<th>Number of students</th>
<th>Percentage of fourth-grade students at a construct 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflexibility</td>
<td>10</td>
<td>83%</td>
</tr>
<tr>
<td>Regrouping error: split strategy with subtraction</td>
<td>5</td>
<td>42%</td>
</tr>
<tr>
<td>Standard algorithm without an alternative strategy</td>
<td>4</td>
<td>33%</td>
</tr>
<tr>
<td>Miscalculations across the decuple</td>
<td>3</td>
<td>25%</td>
</tr>
</tbody>
</table>

In contrast to students scoring at a lower construct, none of the students scoring at a construct 4 or 5 mentioned the standard algorithm. All students used the split strategy for problems where this strategy would be efficient and then, for the most part, used the jump or split-jump strategy for regrouping problems. Two students attempted an alternative strategy for these questions. A construct 5 student successfully used a compensation strategy with 304-198. On the other hand, a construct 4 student attempted to solve 304-198 with the transformation strategy but did so incorrectly (i.e., with 302-200 instead of 306-200). This is potentially a misapplication of the compensation strategy for addition, where the opposite operation is performed on both numbers instead of increasing or decreasing both by the same amount. Students often demonstrate this mistake when they have been taught the compensation strategy in a procedural way or have been asked to memorize rules instead of being taught to understand and visualize the distance between numbers in an addition and subtraction problem.

Two construct 4 students used a “jump up” strategy successfully with two-digit subtraction problems (where they add from the lesser number until they reach the greater number...
in order to find the difference). See Figure 12. One student explicitly described this strategy as using a “number line in my head” and traced it on the table while explaining the jump up strategy. Even though the subtraction regrouping problem was difficult, the students solved it correctly (with one student at first refusing to do the problem and needing encouragement to give it a try). They later became confused on the construct 4 to 5 progress monitoring assessment and were not able to solve three-digit problems accurately with this strategy.

![Diagram](image)

**Final answer to 54-29: 20+4+1=25**

*Figure 12. An example of the “jump up” strategy seen on 54-29.*

Overall, fourth-grade students performing at a construct 4 seemed more willing to try a strategy other than the split strategy with the three-digit regrouping problem but showed higher rates of errors in terms of completing the steps in the split-jump strategy and with moving across the decuple when using either the jump strategy or the split-jump strategy, especially when subtracting 90. Results are shown in Table 17.
Table 17

*Typical Errors Made by Fourth-Grade Students at a Construct 4 for CPV (n=13)*

<table>
<thead>
<tr>
<th>Error</th>
<th>Number of students</th>
<th>Percentage of fourth-grade students at a construct 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflexibility</td>
<td>2</td>
<td>15%</td>
</tr>
<tr>
<td>Regrouping error: split strategy with subtraction</td>
<td>1</td>
<td>8%</td>
</tr>
<tr>
<td>Regrouping error: split-jump strategy with subtraction</td>
<td>4</td>
<td>31%</td>
</tr>
<tr>
<td>Miscalculations across the decuple</td>
<td>3</td>
<td>23%</td>
</tr>
</tbody>
</table>

**Discrepancy Between Correlation Analysis and Item Analysis**

There is a discrepancy between the data found in the correlation analysis, which used data on student constructs that was self-reported by multiple teachers, and in the item analysis. The second and third-grade students used in the correlation analysis matched with the third and fourth-grade students used in the item analysis, with few exceptions (e.g., a student attending the school earlier moved away, students with high absenteeism rates being unavailable for an interview). For the most part, the data was expected to be similar between the two sets, with students generally showing an increase in a construct or two from the previous year. Tables 18 and 19 below compare the constructs reported in each data set.
The data between the two years essentially appear similar. If student constructs in both data sets were collected and reported in an accurate manner, this would mean that students are generally not showing growth this year. If this is considered to be unlikely, then data like this would result from either the previous year’s data being inflated or the current year’s data being
deflated. Given that the teacher self-reported data was initially collected to be used in monitoring and evaluation purposes, it is more likely that inflation had occurred. This will be discussed in greater detail below.

There is an extreme difference between the 2018-2019 third-grade data and the 2019-2020 fourth-grade data sets. It appears that there is a negative correlation between the two sets; generally, higher amounts of students in each construct in the third-grade data is associated with lower amounts in the fourth-grade data, and vice versa. Again, since the third grade data came from teacher self-reported data that was initially used for monitoring and evaluation purposes, it is likely that inflation had occurred.

Koretz (2017) notes that student data can often be subject to Campbell’s law, which hypothesizes that data used for decision-making or evaluative purposes is often subject to inflation and corruption. In particular, Koretz states that education data and student scores are extremely vulnerable to this when teachers feel any pressure to report higher scores, and he states that score inflation tends to appear more with math scores as compared to other subjects such as reading. Therefore, drawing a conclusion from the correlation analysis in this study is challenging; any future attempts to do a correlation analysis like this should only use data collected as part of the study and none that are reported by the teachers or schools involved.
CHAPTER V
CONCLUSIONS

This study was conducted to determine the developmental appropriateness of the CCSSM and to analyze the errors students often make that hinder their progress in learning conceptual place value (CPV). Both of these questions and their answers are critical to raising mathematical achievement in the United States given how foundational CPV understandings are to mathematical concepts learned in future grades. Despite the changes the CCSSM has brought to CPV instruction, much of which is recommended in the literature (i.e., delaying teaching the standard algorithms in order to develop deeper understanding and flexibility with mental math strategies), improvement in mathematical achievement has not been apparent. The findings from this study suggest that there may be adjustments needed in the standards in regards to the development of students’ CPV and engagement with the standard algorithms.

Discussion of Comparison Between Correlation Analysis and Item Analysis

Even with inflated data, the 2018-2019 second-grade data set still shows that approximately 86% of all students could not attain grade-level expectations as set by the CCSSM. None of the students scoring at the top of the percentile range showed proficiency with place value and only 31% of students identified as above average showed proficiency with place value. Additionally, the analysis showed that very few students in all three grades performing at the 32nd percentile and below were able to show proficiency with place value standards (0% in second grade, approximately 9% in third grade, and about 21% in fourth grade). These findings suggest that the CCSSM grade level expectations for place value are not fully attainable. While
the inflation in the correlation analysis does not allow for rigorous conclusions, it does show that there is potential for future similar analyses to reveal problems if done with data that is collected separately from data used for evaluation and monitoring purposes.

**Discussion of Item Analysis**

Three key ideas arise from the item analysis data. First, it is alarming to see very few students utilize compensation and transformation strategies despite how frequently these students were exposed to it with the Bridges curriculum. While these strategies are not explicitly required by the CCSSM and do not necessarily need to be used by students in order for them to be successful, it does mean students are not readily accessing highly efficient strategies for certain problems and are losing opportunities for further development of their number sense. As an example, consider the problem $3.00 - 1.99$. Transforming that problem into an equivalent problem, $3.01 - 2.00$, and solving, or using a compensation strategy like taking away $2.00$ and then adding a penny back, are much faster methods than using the split, jump, or standard algorithm methods. They also require a deeper understanding of the operation of subtraction and the properties that underly it. This sets the foundation for understanding equivalence in more abstract context where algebraic symbols are used, so that algebra rules like “what you do to one side you do to the other” are not memorized without meaning. In the long-term, the inability to independently and flexibly use these strategies when the problem lends itself to them can harm overall mathematical achievement and a student’s ability to deepen their mathematical reasoning abilities (Gürbüz & Erdem, 2016).

Second, the proportion of third-grade students scoring a construct 3 is extreme, especially when one considers that the vast majority of these students depended on the split strategy and the curriculum they used encourages students to flexibly use a variety of strategies. The CCSSM
expects students to begin second grade at a construct 2 and then end the school year at a construct 4, so one possibility for these results may be that it typically takes more than one academic year for a student to go from needing materials for CPV questions to showing flexibility in strategies with bare number tasks. If it is the case that third-grade students often remain at construct 3 and overly depend on the split strategy, perhaps expecting students to gain the expected amount of conceptual understanding the year before is resulting in instruction that moves too rapidly leaving gaps in understanding.

Third, the introduction of the standard algorithm seemed to impede student progress with CPV constructs as well as flexibility with strategies. Only one student who initially used the standard algorithm with a problem and failed to solve it could successfully use a different strategy with that same problem. This same student refused to attempt another strategy with a different problem on the assessment that she showed confusion over because she was able to accurately use the standard algorithm earlier and insisted that this method that her tutor showed her had to always be used when possible. With fourth-grade students, any mention of the standard algorithm during the interview had a direct connection to low performance and a lack of foundational understanding in CPV. It may be the case, then, that the introduction of the standard algorithm in the third-grade curriculum alongside lessons encouraging compensation and transformation may have hindered student progress with those mental math strategies. Many other studies (e.g., Carpenter et al., 1997; Clarke, 2005; Kamii & Dominick, 1998) show the pitfalls of introducing the standard algorithm during the elementary years. Overall, this shows a potential argument against the CCSSM’s placement of the standard algorithms for addition and subtraction in fourth grade. While this is already a delay from when the standard algorithms have
been taught historically, the delay may not be sufficient to ensure they do not undermine the mental computation strategies students are learning.

Due to these tendencies in the data, the following three recommendations for changes to the CCSSM seem appropriate and can be developed and considered in future studies. First, the CCSSM standards do not currently demand any specific strategies related to CPV to be taught or used by students. Rather, the wording in the standards, such as the wording found in 2.NBT.B.5, suggest teaching “strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.” However, given the mathematical significance of the compensation and transformation strategies and the reasoning about the operations they develop, these strategies should receive explicit mention and prominence. For example, it would be beneficial to include a 2.NBT.B.5.A addendum which states, “Using materials and models based on place value, use addition and subtract problems utilizing distance such as the compensation and transformation strategies” (Common Core State Standards Initiative, 2019c). Building on this, these strategies should be developed in second and third grade, with the second-grade standards focused on the introduction of these strategies using materials while the third-grade standards focus more on empty number line models or verbal and written reasonings involving place value understandings in light of Wright et al.’s (2012) research on the dimensions of place value instruction.

Second, it may be beneficial to delay by one grade level the second-grade standard 2.NBT.B.5, which asks students to “fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction,” and the third-grade standard 3.NBT.A.2, which extends this range to 1,000. Combined with new standards in the second and third grades asking for explicit focus on the
compensation and transformation strategies (as mentioned previously), this delay would give students more time to fully develop the flexible place value understanding that they will need to have success later in mathematics. The second-grade students struggled to reach expectations and both third- and fourth-grade students from the 2019-2020 data set tended to fixate on the split strategy to the detriment of other strategies. Therefore, especially in regards to regrouping problems, an argument can be made that more time is needed to develop understandings of and flexibility in strategies before materials can be completely removed and before the range of numbers is extended.

Finally, the removal of the standard algorithms from grades K-5 should be considered given both the findings from this study and other relevant research. Even though the placement of the standard algorithms for addition and subtraction in fourth grade is an improvement from what has been done historically, this study aligns with evidence that this may still be too early and lead to the development of buggy algorithms. Along these same lines, the word “algorithm” should be removed from the language in third-grade standard 3.NBT.A.2 to discourage curriculum designers and educators from introducing the standard algorithms too early.

**Additional Implications and Recommendations**

**Implications for Schools**

The correlation analysis used in this study was subject to a large amount of inflation and issues with inter-rater reliability due to the initial use of the data for evaluation purposes. As described by Koretz (2008, 2017), this is seen in almost every instance when teachers and schools are pressured to raise scores or have high scores for any reason. This type of pressure is hindering clear formative assessment of CPV skills. This is problematic, because the purpose of these assessments is to determine where students are on the learning trajectory for CPV and
design appropriate instruction to move them forward. The large discrepancy between the data sets for two different years suggests that this inflation and the difference in inter-rater reliability potentially led to inappropriate lessons and interventions at the start of the year before the new classroom teacher was able to retest the students. This would have then slowed progress right from the beginning of the school year. Given these findings, schools and districts should avoid using formative assessments such as progress monitoring sheets and their related scores for teacher evaluations so that student data remains accurate and can be accurately acted upon.

Schools should also either avoid curriculum that introduces the standard algorithms earlier than what the standards mandate or should consider skipping lessons involving the standard algorithms in the event that their chosen curriculum utilizes them before fourth grade. For example, the Bridges curriculum used in this study has a heavy focus on mental strategies for CPV, which should be viewed as a strength. Instead of switching to a new curriculum entirely, the school in this study could be advised to exclude the third-grade standard algorithm lessons and to reformat second-grade lessons that use a “stacked” number format where the numbers are arranged vertically as this format encourages students to use the standard algorithm. Given how students struggled to maintain CPV understandings after exposure to the standard algorithms, schools should consider delaying the standard algorithms as one step toward raising student mathematical achievement.

Implications for Research

It would be beneficial to have more research performed on the compensation and transformation strategies and why students may either be hesitant to use them or how educators are teaching them. Could it be that these strategies are too difficult or not intuitive enough for students to be able to utilize without heavier exposure than was seen in the curriculum used by
the school in this study? Are the strategies generally being taught too procedurally for students to be able to develop true number sense for them and utilize them efficiently on their own, as may have been evidenced by one of the data points in this study? Or could there be another factor involved? The answer to any of these questions could be directly and immediately beneficial to schools and educators who want to improve their students’ flexibility in regards to CPV strategies.

Additionally, further studies that show why parents and teachers continue to teach the standard algorithms before the standards require them would help better align mathematics instruction to the intentions behind the standards. Inferences can currently be made for why this is the case, such as comfort and knowledge with the standard algorithms given how most adults were educated themselves. Directly uncovering why these algorithms have such a strong cultural hold in our society, given the multitude of other changes that have been accepted, would be helpful. Understanding the reasons behind the continued promotion of the algorithm is critical because even if the standards are developmentally appropriate and aligned with research-based learning trajectories, mathematical achievement can still be undermined if they are not followed due to underlying beliefs that run counter to the recommendations.

**Limitations of Research**

There are several limitations to this study that should be considered when interpreting results and when developing a study that uses a model similar to this one. First, using self-reported data in a study that aims to analyze the appropriateness of a learning goal can cause a weakness in the validity of the data, as seen in this study where inflation occurred because the self-reported data had been initially collected from seven different teachers for evaluation purposes. There is also the question of how reliable the scores are given that seven different
teachers scored the assessments. In a study that seeks to create a correlation between different assessments to help analyze a standard, weaknesses related to validity and reliability are critical and need to be minimized as much as possible. Therefore, any replication of a model like this should prioritize measures that reduce these limitations.

Second, instruction was not analyzed in this study. Although the curriculum and methodologies used by the school are described, the ways teachers at this school utilized them in their instruction was not researched. It is unknown whether or not the teachers were using them with fidelity or as intended by the designers of Bridges and Math Recovery®. Therefore, this had an unknown impact on the students’ level of CPV understandings, whether for better or worse. This causes issues in both how valid an interpretation can be made that answers the research questions and the reliability that would be seen if similar studies were conducted in other schools using these materials. Since instruction overall may or may not support the learning progressions behind the CCSSM, claims about the appropriateness of a particular standard at a specific grade are difficult to make beyond the bounds of this case study.

Third, this study was conducted as a case study on a particular Title I school in the Midwest with a specific setting in terms of student demographics. With a question as broad as whether or not nationwide standards are developmentally appropriate, the generalizability of a study that answers that question is required in order to provide a valid result. However, it is unknown how reliable the results would be if the study was expanded to other schools. In particular, inequity and a lack of resources are often noted as a reason why students struggle with mathematical achievement, so the trends seen in this case study may relate more to the particular setting of the school as opposed to signaling an inherent problem in the standards themselves (i.e., Clements et al., 2017). If the conclusions of this study were replicated with schools with
different settings, resources, and demographics, a stronger claim could be made about the appropriateness of the CCSSM standards.
REFERENCES


Rakow, S. (2012). The Common Core: The good, the bad, the possible. Middle Ground, 16(2), 9-11.


Appendix A

Human Subjects Institutional Review Board
Letter of Approval
Date: October 9, 2019

To: Kate Kline, Principal Investigator
    Sarah Hughey, Student Investigator for thesis

From: Amy Naugle, Ph.D., Chair

Re: IRB Project Number 19-10-14

This letter will serve as confirmation that your research project titled “The Common Core State Standards for Mathematics: Developmentally Appropriate or Not?” has been approved under the exempt category of review by the Western Michigan University Institutional Review Board (IRB). The conditions and duration of this approval are specified in the policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note: This research may only be conducted exactly in the form it was approved. You must seek specific board approval for any changes to this project (e.g., add an investigator, increase number of subjects beyond the number stated in your application, etc.). Failure to obtain approval for changes will result in a protocol deviation.

In addition, if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the IRB for consultation.

The Board wishes you success in the pursuit of your research goals.

A status report is required on or prior to (no more than 30 days) October 8, 2020 and each year thereafter until closing of the study. The IRB will send a request.

When this study closes, submit the required Final Report found at https://wmich.edu/research/forms.

Note: All research data must be kept in a secure location on the WMU campus for at least three (3) years after the study closes.