

ON EULERIAN IRREGULARITY AND DECOMPOSITIONS IN GRAPHS

Eric Andrews, Ph.D.

Western Michigan University, 2014

An Eulerian walk in a connected graph G is a closed walk that contains every edge of G at least once, while an irregular Eulerian walk in G is an Eulerian walk that encounters no two edges of G the same number of times. The minimum length of an irregular Eulerian walk in G is called the Eulerian irregularity of G and is denoted by $EI(G)$.

For a nontrivial connected graph G of size m , it is shown that $\binom{m+1}{2} \leq EI(G) \leq 2\binom{m+1}{2}$ and that $EI(G) = 2\binom{m+1}{2}$ if and only if G is a tree of size m . A necessary and sufficient condition has been established for all pairs k, m of positive integers for which there is a nontrivial connected graph G of size m with $EI(G) = k$. A formula for the Eulerian irregularity of a graph in terms of the size of certain even subgraph of the graph has been established. Furthermore, Eulerian irregularities are determined for all graphs of cycle rank 2 and all complete bipartite graphs as well as all prisms, grids and powers of cycles. Some general results on Eulerian irregularities of circulants are also presented.

For a set S of graphs and a graph G , a decomposition $\mathcal{D} = \{H_1, H_2, \dots, H_k, R\}$ of G is called an S -maximal k -decomposition if $H_i \cong H$ for some $H \in S$ for each integer i with $1 \leq i \leq k$ and R contains no subgraph isomorphic to any subgraph in S . Let $\text{Min}(G, S)$ and $\text{Max}(G, S)$ be the minimum and maximum k , respectively, for which G has an S -maximal k -decomposition. A set S of graphs without isolated vertices is said to possess the intermediate decomposition property if for every connected graph G and each integer k with $\text{Min}(G, S) \leq k \leq \text{Max}(G, S)$, there exists an S -maximal k -decomposition of G . All graphs of size 3 or less are determined that possess the intermediate decomposition property. Furthermore, the sets of graphs having size 3 that possess the intermediate decomposition property are determined as well as some sets of graphs having having size more than 3.