

NILPOTENT ORBITS ON INFINITESIMAL  
SYMMETRIC SPACES

Joseph A. Fox, Ph.D.

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Let  $G$  be a reductive linear algebraic group defined over an algebraically closed field  $k$  whose characteristic is good for  $G$ . Let  $\theta$  be an involution defined on  $G$ , and let  $K$  be the subgroup of  $G$  consisting of elements fixed by  $\theta$ . The differential of  $\theta$ , also denoted  $\theta$ , is an involution of the Lie algebra  $\mathfrak{g} = \text{Lie}(G)$ , and it decomposes  $\mathfrak{g}$  into  $+1$ - and  $-1$ -eigenspaces,  $\mathfrak{k}$  and  $\mathfrak{p}$ , respectively. The space  $\mathfrak{p}$  identifies with the tangent space at the identity of the symmetric space  $G/K$ . In this dissertation, we are interested in the adjoint action of  $K$  on  $\mathfrak{p}$ , or more specifically, on the nullcone  $\mathcal{N}(\mathfrak{p})$ , which consists of the nilpotent elements of  $\mathfrak{p}$ . The main result is a new classification of the  $K$ -orbits on  $\mathcal{N}(\mathfrak{p})$ .