

CLASSIFYING SPACES OF SYMMETRIC GROUPS
AND WREATH PRODUCTS

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This thesis was motivated by a desire to better understand the structure of classifying spaces of symmetric groups. The results contained in this thesis fall into two categories: general results about stable splittings or the groups we will work with, and specific results about the stable splittings for $B\mathfrak{S}_8$ and $B(Z_2 \wr Z_2 \wr Z_2)$, completed at the prime 2. Regarding the splittings, $Z_2 \wr Z_2 \wr Z_2$ is the largest 2-group outside of some specific families to have its classifying space completely split, and this document offers an example of the theory on splittings developed by Martino and Priddy applied to the symmetric group \mathfrak{S}_8 , a group which is not a p -group.

Amongst the general results are group theoretic results about the n -fold iterated wreath product of Z_p with itself, denoted $Z_p^{\wr n}$. We explore the structure of its automorphism group and its maximal elementary abelian subgroups. Using this information, we determine the number of original summands of $BZ_p^{\wr n}$ and which of those summands appear in $B\mathfrak{S}_{p^n}$. We also relate their cohomological structure to an algebra of Dickson invariants.

We also present two examples involving linkage. It has been hypothesized that all linkage is strong linkage; we present a counterexample. It was also thought that isomorphic summands of a classifying space would either all be linked in that space or all be linked in the spaces for some collection of subgroups. Again, we identify a counterexample.