

# MATH IN ACTION

## TABLE OF CONTENTS

### LESSON 1 – WORLD RECORD SPEEDS

#### LINEAR FUNCTIONS WITH PROPORTIONAL RELATIONSHIPS

##### *Focus on: SLOPE*

Lesson 1.1 – On Your Mark, Get Set, Go! Usain Bolt: The fastest man on the planet	Page: 10
Lesson 1.2 – The Fast and the Furious The world record for land speed	Page: 14
Lesson 1.3 – “Safety is in the Speed” Pavel’s speed skating world record	Page: 18
Lesson 1.4 – Team Work Makes the Dream Work Pulling it all together to make sense	Page: 22

### LESSON 2 – HIGH EXPECTATIONS

#### LINEAR FUNCTIONS WITH NON-PROPORTIONAL RELATIONSHIPS

##### *Focus on: Y-Intercept*

Lesson 2.1 – “You Go Girl!” The increase of high school female athletes since 1997	Page: 24
Lesson 2.2 – There’s No Mountain Too High Ski Lift to the Top	Page: 30
Lesson 2.3 – Those Who Don’t Jump Will Never Fly Sky Diving	Page: 34
Lesson 2.4 – There is no “I” in Team Pulling it all together to make sense	Page: 39

## LESSON 3 – GETTING PRICEY

### LINEAR FUNCTIONS – MOVING BETWEEN REPRESENTATIONS

*Focus on: Given Table Information*

**Lesson 3.1 – Hey Hey Hockey Town** **Page: 42**  
**Ticket Prices for a Redwings Game**

**Lesson 3.2 –Getting Fit** **Page: 46**  
**Purchasing a Gym Membership**

## LESSON 4 – WATCHING THE WEIGHT FALL RIGHT OFF

### LINEAR FUNCTIONS – MOVING BETWEEN REPRESENTATIONS

*Focus on: Given Graph Information*

**Lesson 4.1 – Cycling Through Calories** **Page: 51**  
**Burning Calories While Biking**

**Lesson 4.2 – Burn Baby Burn** **Page: 55**  
**Burning Calories While Running**

## LESSON 5 – A PROFESSIONAL’S OPINION

### LINEAR FUNCTIONS – MOVING BETWEEN REPRESENTATIONS

*Focus on: Given Equation Information*

**Lesson 5.1 – Serve it to them** **Page: 60**  
**Paying for Tennis Lessons**

**Lesson 5.2 – Swing with Everything You’ve Got** **Page: 63**  
**Paying for Baseball Lessons**

## LESSON 6- REACHING NEW HEIGHTS

### LINEAR FUNCTIONS – ONE REPRESENTATION

*Focus on: Tables Only*

**Lesson 6.1 – Climbing to the Top** **Page: 67**  
Temperature Change with Elevation

**Lesson 6.2 – Setting High Goals** **Page: 70**  
World Records For Pole Vault

## LESSON 7- BEND IT LIKE BECKHAM

### LINEAR FUNCTIONS – ONE REPRESENTATION

*Focus on: Graphs Only*

**Lesson 7.1 – Whoa, that’s Hot!** **Page: 74**  
Temperature Increase While Playing on Artificial Turf

**Lesson 7.2 – What a Pass!** **Page: 77**  
How Long Will it Take to Pass the Ball to Your Teammate?

## LESSON 8 – FAMILY FUN TIME

### LINEAR FUNCTIONS – ONE REPRESENTATION

*Focus on: Equations Only*

**Lesson 8.1 – Fore!** **Page: 81**  
Putt Putt Golfing

**Lesson 8.2 – Strike!** **Page: 84**  
The Number of Bowling Alleys in the U.S.

## UNIT ASSESSMENT

### PROJECT TIME!

**Final Project – Roll it All Together** **Page: 87**  
Which Roller Skating Company?

# PACING GUIDE

## SPORTS IN ACTION

### FULL UNIT

This unit should take approximately **21 days. ( 4 weeks and a day)**

If needed the pace can be slowed for students to fill a full 5 weeks.

Extra time to work on final project at the end of the unit may also be helpful.

## LESSON 1 – WORLD RECORD SPEEDS

### LINEAR FUNCTIONS WITH PROPORTIONAL RELATIONSHIPS

#### Lesson 1.1 – On Your Mark, Get Set, Go!

Usain Bolt: The fastest man on the planet (1 day)

#### Lesson 1.2 – The Fast and the Furious

The world record for land speed (1 day)

#### Lesson 1.3 – “Safety is in the Speed”

Pavel’s speed skating world record (1 day)

#### Lesson 1.4 – Team Work Makes the Dream Work

Pulling it all together to make sense (1 day)

## LESSON 2 – HIGH EXPECTATIONS

### LINEAR FUNCTIONS WITH NON-PROPORTIONAL RELATIONSHIPS

#### Lesson 2.1 – “You Go Girl!”

The increase of high school female athletes since 1997 (1 day)

#### Lesson 2.2 – There’s No Mountain Too High

Ski Lift to the Top (1 day)

#### Lesson 2.3 – Those Who Don’t Jump Will Never Fly

Sky Diving (1 day)

**Lesson 2.4 – There is no “I” in Team**  
**Pulling it all together to make sense**

**(1 day)**

**LESSON 3 – GETTING PRICEY**

**LINEAR FUNCTIONS – MOVING BETWEEN REPRESENTATIONS**

**Lesson 3.1 – Hey Hey Hockey Town**  
**Ticket Prices for a Redwings Game**

**(1 day)**

**Lesson 3.2 –Getting Fit**  
**Purchasing a Gym Membership**

**(1 day)**

**LESSON 4 – WATCHING THE WEIGHT FALL RIGHT OFF**

**LINEAR FUNCTIONS – MOVING BETWEEN REPRESENTATIONS**

**Lesson 4.1 – Cycling Through Calories**  
**Burning Calories While Biking**

**(1 day)**

**Lesson 4.2 – Burn Baby Burn**  
**Burning Calories While Running**

**(1 day)**

**LESSON 5 – A PROFESSIONAL’S OPINION**

**LINEAR FUNCTIONS – MOVING BETWEEN REPRESENTATIONS**

**Lesson 5.1 – Serve it to them**  
**Paying for Tennis Lessons**

**(1 day)**

**Lesson 5.2 – Swing with Everything You’ve Got**  
**Paying for Baseball Lessons**

**(1 day)**

**LESSON 6- REACHING NEW HEIGHTS**

**LINEAR FUNCTIONS – ONE REPRESENTATION**

**Lesson 6.1 – Climbing to the Top**  
**Temperature Change with Elevation**

**(1 day)**

**Lesson 6.2 – Setting High Goals**

**World Records For Pole Vault**

**(1 day)**

**LESSON 7- BEND IT LIKE BECKHAM**

**LINEAR FUNCTIONS – ONE REPRESENTATION**

**Lesson 7.1 – Whoa, that’s Hot!**

**Temperature Increase While Playing on Artificial Turf (1 day)**

**Lesson 7.2 – What a Pass!**

**How Long Will it Take to Pass the Ball to Your Teammate? (1 day)**

**LESSON 8 – FAMILY FUN TIME**

**LINEAR FUNCTIONS – ONE REPRESENTATION**

**Lesson 8.1 – Fore!**

**Putt Putt Golfing**

**(1 day)**

**Lesson 8.2 – Strike!**

**The Number of Bowling Alleys in the U.S.**

**(1 day)**

**UNIT ASSESSMENT**

**PROJECT TIME!**

**Final Project – Roll it All Together**

**Which Roller Skating Company?**

**(2 days)**

First day for multiple tasks with questions  
Second day to construct presentation poster

## COMMON CORE STATE STANDARDS FOR MATHEMATICS

### MATHEMATICS – HIGH SCHOOL – ALGEBRA

#### ★ A-CED CREATING EQUATIONS

##### Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

#### ★ A-REI REASONING WITH EQUATIONS AND INEQUALITIES

##### Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

#### ★ F-IF INTERPRETING FUNCTIONS

##### Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

##### Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

##### Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

★F-BF BUILDING FUNCTIONS

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context

★ F-LE LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - a. Prove that linear functions grow by equal differences over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
2. Construct linear functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe a quantity increasing by a constant rate using graphs and tables.



# LESSON 1 OVERVIEW

## WORLD RECORD SPEEDS

### GOALS OF THE LESSON

This lesson is designed to get the student's familiar with linear functions that have a proportional relationship between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 1, the focus will be on the idea of slope. Students will use the different contexts as a way of learning what slope is and how to calculate it.

By the end of the lesson, students should be able to explain how to find slope, what it is, what it represents, and the idea that all linear functions have a constant slope!

All of these real world contexts given to help the students learn revolve around world records for speed in a variety of sports. This topics make the tasks and lesson very interesting. The following are the contexts that are used to help students learn about slope:

#### **Lesson 1.1 – On Your Mark, Get Set, Go!**

**Usain Bolt: The fastest man on the planet**

#### **Lesson 1.2 – The Fast and the Furious**

**The world record for land speed**

#### **Lesson 1.3 – “Safety is in the Speed”**

**Pavel's speed skating world record**

After completing all three of these components to the lesson in lesson 1.4. The students will be asked to look at and explore the similarities and differences between the 3 different examples of linear functions. This will be wear conclusions will be made and students will understand the idea that all linear functions have a constant slope.

#### **Lesson 1.4 – Teamwork Makes the Dream Work**

**Pulling it All Together**

# 1.1 ON YOUR MARK... GET SET... GO!

## THE FASTEST MAN ON THE EARTH

The fastest human footspeed on record was seen during the final **100 meters** sprint of the World Championships in Berlin on 16 August 2009 by Usain Bolt. He won the race in **9.58 seconds**.



### TASK 1

We normally don't think about speed per 100 meters. When you think of speed what do you think of?

**Miles per hour**

Convert his speed into units that are more commonly used.

$$\frac{100 \text{ meters}}{9.58 \text{ seconds}} \times \frac{60 \text{ seconds}}{1 \text{ min}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1609.34 \text{ meters}} = \frac{23.3501237 \text{ miles}}{1 \text{ hour}} = \frac{23.35 \text{ miles}}{1 \text{ hour}}$$

Students may need to do this conversion in parts. Help them to break down the problem into smaller pieces.

Ask questions like, "If we have 9.58 seconds, how do we find out how many minutes this is?"

Calculate the distance that Usain could travel in .5 hours, 1 hour, 2 hours, 4 hours, 6 hours, and 9 hours.

$$.5 \text{ hours} \times \frac{23.35 \text{ miles}}{1 \text{ hour}} = 11.675 \text{ miles}$$

$$1 \text{ hours} \times \frac{23.35 \text{ miles}}{1 \text{ hour}} = 23.35 \text{ miles}$$

$$2 \text{ hours} \times \frac{23.35 \text{ miles}}{1 \text{ hour}} = 46.7 \text{ miles}$$

$$4 \text{ hours} \times \frac{23.35 \text{ miles}}{1 \text{ hour}} = 93.4 \text{ miles}$$

$$6 \text{ hours} \times \frac{23.35 \text{ miles}}{1 \text{ hour}} = 140.1 \text{ miles}$$

$$9 \text{ hours} \times \frac{23.35 \text{ miles}}{1 \text{ hour}} = 210.15 \text{ miles}$$

## TASK 2

**Explore the table for Usain's speed.** (*if he could continue to hold this pace forever*)

Time (hrs)	Distance (miles)
.5	11.675
1	23.35
2	46.7
3	70.05
4	93.4
6	140.1
9	210.15

**What do you notice about this table?**

Students might make some of the following conjectures:

"As X increases, Y increases"

**Calculate** *the change in y-values* **for the follow pairs of points:**  
*the change in x-values*

$$(.5, 11.675) \text{ and } (1, 23.35) \quad \frac{23.35 - 11.675}{1 - .5} = \frac{11.675}{.5} = 23.35 \text{ miles per hour}$$

$$(6, 140.1) \text{ and } (9, 210.15) \quad \frac{210.15 - 140.1}{9 - 6} = \frac{70.05}{3} = 23.35 \text{ miles per hour}$$

$$(1, 23.35) \text{ and } (2, 46.7) \quad \frac{46.7 - 23.35}{2 - 1} = \frac{23.35}{1} = 23.35 \text{ miles per hour}$$

$$(2, 46.7) \text{ and } (3, 70.05) \quad \frac{70.05 - 46.7}{3 - 2} = \frac{23.35}{1} = 23.35 \text{ miles per hour}$$

$$(1, 23.35) \text{ and } (4, 93.4) \quad \frac{93.4 - 23.35}{4 - 1} = \frac{70.05}{3} = 23.35 \text{ miles per hour}$$

$$(4, 93.4) \text{ and } (6, 140.1) \quad \frac{140.1 - 93.4}{6 - 4} = \frac{46.7}{2} = 23.35 \text{ miles per hour}$$

$$(2, 46.7) \text{ and } (4, 93.4) \quad \frac{93.4 - 46.7}{4 - 2} = \frac{46.7}{2} = 23.35 \text{ miles per hour}$$

**What do you notice with these different pairs of points?**

All of these calculations are equal!

## SLOPE

We define **SLOPE** as  $\frac{\Delta Y}{\Delta X} = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}}$   
for corresponding x and y values

## TASK 3

On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.

Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down?

Talk about which variable goes on which axis with your students. This will help them to grasp the idea of independent and dependent variables.

## TASK 4: BUILDING AN EQUATION

How far would Usain travel if he ran his speed for 1 hour?  $\frac{23.35 \text{ miles}}{1 \text{ hour}} \times 1 \text{ hour} = 23.35 \text{ miles}$

For 2 hours?  $\frac{23.35 \text{ miles}}{1 \text{ hour}} \times 2 \text{ hours} = 46.7 \text{ miles}$

For 10 hours?  $\frac{23.35 \text{ miles}}{1 \text{ hour}} \times 10 \text{ hours} = 233.5 \text{ miles}$

Supposed Usain runs for “n” hours, how far would he have traveled?

$$\frac{23.35 \text{ miles}}{1 \text{ hour}} \times n \text{ hours} = 23.35n \text{ miles}$$

## TASK 5: CHECK YOUR HYPOTHESIS

On your TI-Nspire, plot your hypothesis function on your graph page.  
If your graph doesn’t match the data change it so that it fits.

**What did you notice about your hypothesis and your collected data?**

**It matches the trend of the points that were plotted and hits all of the points!**

## 1.2 THE FAST AND THE FURIOUS

### THE LAND SPEED RECORD

Land speed refers to the fastest speed of a person in a car on land. On October 15<sup>th</sup>, 1997, Andy Green broke the world record for land speed in Black Rock Desert Nevada, USA. At this time it was recorded that he hit an amazing speed being the first driver of a car to break the sound barrier. During this drive he drove **1 mile** in a mere **4.718 seconds**!



#### TASK 1

Convert his speed into units that are more commonly used.

$$\frac{1 \text{ mile}}{4.718 \text{ seconds}} \times \frac{60 \text{ seconds}}{1 \text{ min}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{763.035184 \text{ miles}}{1 \text{ hour}} = \frac{763 \text{ miles}}{1 \text{ hour}}$$

Again, students may need to do this conversion in parts. Help them to break down the problem into smaller pieces.

Calculate the distance that Andy's car could travel in 2 hours, 7 hours, and 11 hours.

$$2 \text{ hours} \times \frac{763 \text{ miles}}{1 \text{ hour}} = 1526 \text{ miles}$$

$$7 \text{ hours} \times \frac{763 \text{ miles}}{1 \text{ hour}} = 5341 \text{ miles}$$

$$11 \text{ hours} \times \frac{763 \text{ miles}}{1 \text{ hour}} = 8393 \text{ miles}$$

## TASK 2

Explore the table for Andy's car's speed (*if this could be maintained forever*)

Fill in the blanks in the table below.

Time (hrs)	Distance (miles)
0	0
1	763
2	1526
3	2289
7	5341
9	6867
11	8393

How can you find the missing X values?

You need to analyze the table. Notice how the Y values are changing as the X values change in order to find the appropriate value for the missing X values.

Students might get stuck on this. In that case have them continue to move along and find the slopes for multiple pairs of points. Hopefully that will help them to see that the slope is constant and that they will be able to figure out the missing X values.

What do you notice about this table?

As X increases, Y increases.

When X is 0 Y is also 0.

Remember, in the last lesson we mentioned the idea of slope as  $\frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}}$

Calculate the slopes for the following:

From 0 hours to 1 hour  $\frac{763-0}{1-0} = \frac{763}{1} = 763 \text{ miles per hour}$

Help students to think about the idea of slope as the change in Y over the change in X. They need to find the corresponding Y values for the X values given.

From 2 hours to 3 hours  $\frac{2289-1526}{3-2} = \frac{763}{1} = 763 \text{ miles per hour}$

From 1 hour to 7 hours  $\frac{5341-763}{7-1} = \frac{4578}{6} = 763 \text{ miles per hour}$

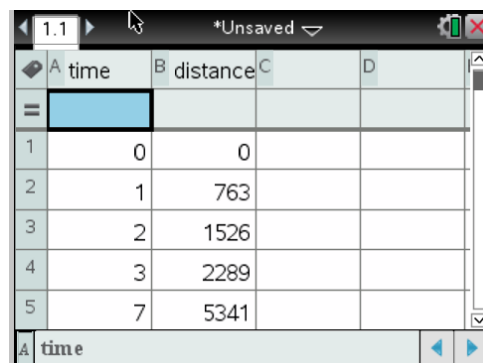
From 3 hours to 9 hours  $\frac{763-0}{1-0} = \frac{763}{1} = 763 \text{ miles per hour}$

What do you notice with these different slopes?

They are all the same!

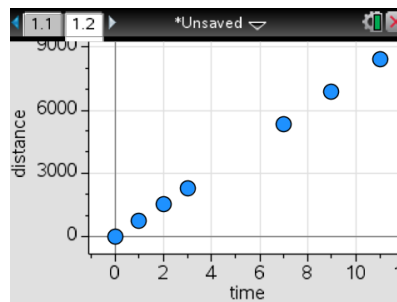
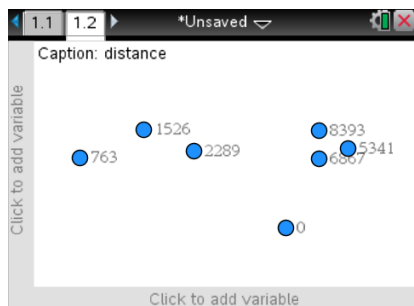
### TASK 3

On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.



	A time	B distance	C	D
=				
1	0	0		
2	1	763		
3	2	1526		
4	3	2289		
5	7	5341		

Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down?





## TASK 4

How far could Andy and his car travel in 1 hour?

$$\frac{763 \text{ miles}}{1 \text{ hour}} \times 1 \text{ hour} = 763 \text{ miles}$$

In 7 hours?

$$\frac{763 \text{ miles}}{1 \text{ hour}} \times 7 \text{ hour} = 5341 \text{ miles}$$

In 4 hours?

$$\frac{763 \text{ miles}}{1 \text{ hour}} \times 4 \text{ hour} = 3052 \text{ miles}$$

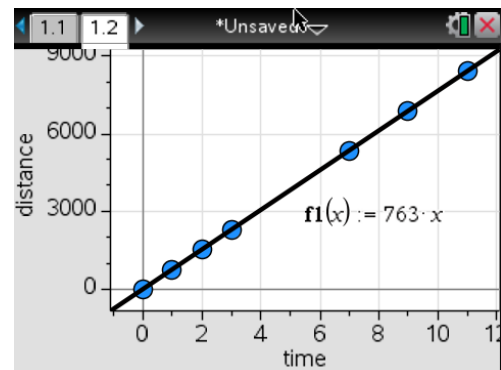
Suppose Andy drives his crazy fast car for “h” hours, how far would he travel?

$$\frac{763 \text{ miles}}{1 \text{ hour}} \times h \text{ hours} = 763h \text{ miles}$$

## TASK 5

On your TI-Nspire, plot your hypothesis function on your graph page.

If your graph doesn't match the data change it so that it fits.



What did you notice about your hypothesis and your collected data?

It matches the trend of the points that were plotted and hits all of the points

## 1.3 "SAFTEY IS IN THE SPEED"

### WORLD RECORD 500 METER SPEED SKATING

As Ralph Waldo Emerson has said "In skating over thin ice, our safety is our speed", this is also the mindset for many speed skaters. Pavel Kulizhnikov is a Russian speed skater. In 2015, he became the first speed skater to finish the **500 meter** in under 34 seconds with a world record of **33.98 seconds**.



#### TASK 1

Convert his speed into units that are more commonly used.

$$\frac{500 \text{ meters}}{33.98 \text{ seconds}} \times \frac{60 \text{ seconds}}{1 \text{ min}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{1609.34 \text{ meters}} = \frac{32.9155658}{1 \text{ hour}} = \frac{33 \text{ miles}}{1 \text{ hour}}$$

Again, students may need to do this conversion in parts. Help them to break down the problem into smaller pieces.

Calculate the distance that Pavel could travel in 3 hours, 4 hours, 6 hours, and 9 hours.

$$3 \text{ hours} \times \frac{33 \text{ miles}}{1 \text{ hour}} = 99 \text{ miles}$$

$$4 \text{ hours} \times \frac{33 \text{ miles}}{1 \text{ hour}} = 132 \text{ miles}$$

$$6 \text{ hours} \times \frac{33 \text{ miles}}{1 \text{ hour}} = 198 \text{ miles}$$

$$9 \text{ hours} \times \frac{33 \text{ miles}}{1 \text{ hour}} = 297 \text{ miles}$$

## TASK 2

Explore the table for Pavel's speed (*if this could be maintained forever*)

Fill in any missing spaces

Time (hrs)	Distance (miles)
0	0
3	99
4	132
6	198
7	231
9	297

What do you notice about this table?

Students might make some of the following conjectures:

"As X increases, Y increases"

"When X equals 0, Y equals 0"

Calculate the slopes for the following:

From 0 hours to 3 hours  $\frac{99-0}{3-0} = \frac{99}{3} = 33 \text{ miles per hour}$

From 3 hours to 6 hours  $\frac{198-99}{6-3} = \frac{99}{3} = 33 \text{ miles per hour}$

From 6 hours to 7 hours  $\frac{231-198}{7-6} = \frac{33}{1} = 33 \text{ miles per hour}$

From 3 hours to 4 hours  $\frac{132-99}{4-3} = \frac{33}{1} = 33 \text{ miles per hour}$

From 4 hours to 6 hours  $\frac{198-132}{6-4} = \frac{66}{2} = 33 \text{ miles per hour}$

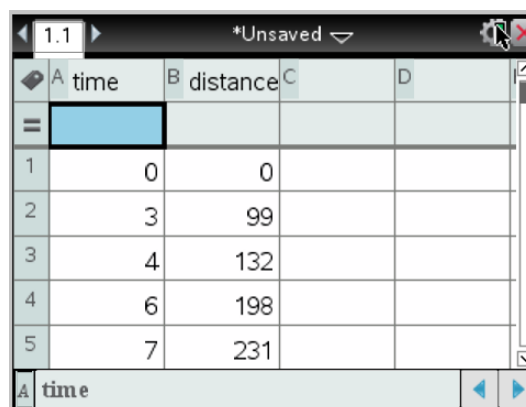
From 7 hours to 9 hours  $\frac{297-231}{9-7} = \frac{66}{2} = 33 \text{ miles per hour}$

What do you notice about these different slopes?

**They are all the same!**

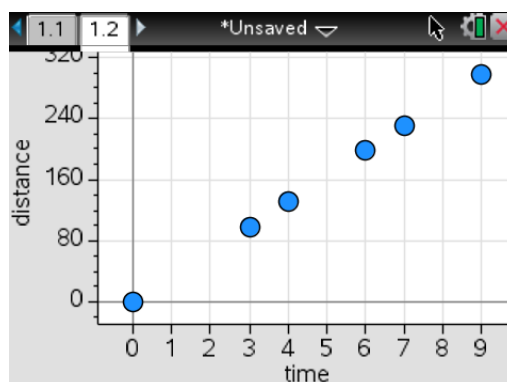
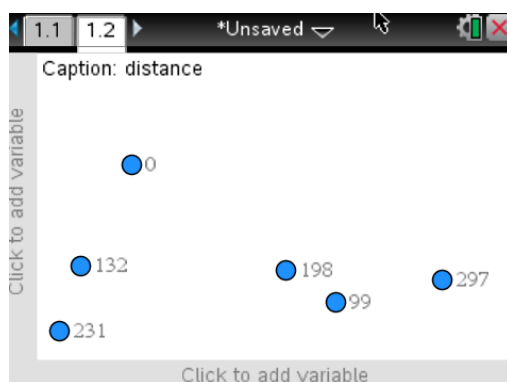
### TASK 3

On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.



	A time	B distance	C	D
=				
1	0	0		
2	3	99		
3	4	132		
4	6	198		
5	7	231		

Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down?



## TASK 4

How far could Andy and his car travel in 1 hour?

$$\frac{33 \text{ miles}}{1 \text{ hour}} \times 1 \text{ hour} = 33 \text{ miles}$$

In 2 hours?

$$\frac{33 \text{ miles}}{1 \text{ hour}} \times 2 \text{ hour} = 66 \text{ miles}$$

In 12 hours?

$$\frac{33 \text{ miles}}{1 \text{ hour}} \times 12 \text{ hour} = 396 \text{ miles}$$

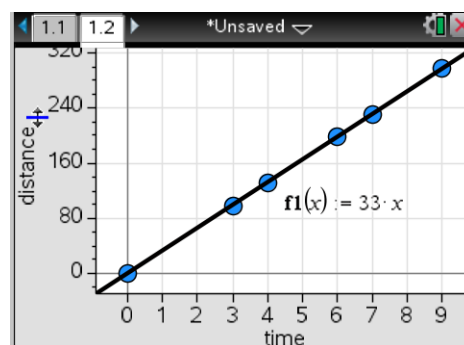
Suppose Andy drives his crazy fast car for “z” hours, how far would he travel?

$$\frac{33 \text{ miles}}{1 \text{ hour}} \times z \text{ hour} = 33z \text{ miles}$$

## TASK 5

On your TI-Nspire, plot your hypothesis function on your graph page.

If your graph doesn't match the data change it so that it fits.



What did you notice about your hypothesis and your collected data?

It matches the trend of the points that were plotted and hits all of the points!

# 1.4 TEAM WORK MAKES THE DREAM WORK

## BRINGING IT ALL TOGETHER

In the last 3 lessons we have talked about a few super speedy world records.

Review the tables and graphs for each of the 3 world records we studied earlier.

### RUN

Time (hours)	Distance (miles)
.5	11.675
1	23.35
2	46.7
3	70.05
4	93.4
6	140.1
9	210.15

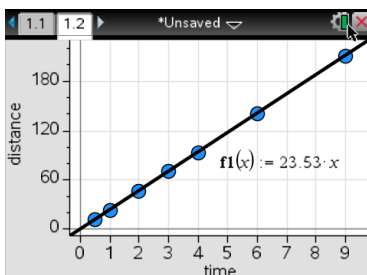
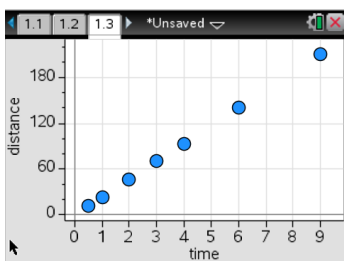
### DRIVE

Time (hours)	Distance (miles)
0	0
1	763
2	1526
3	2289
7	5341
9	6867
11	8393

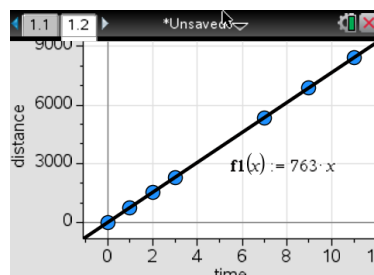
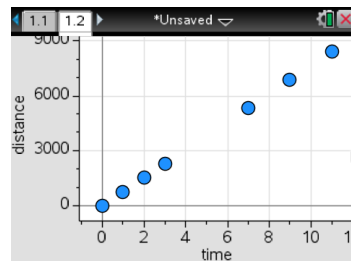
### SKATE

Time (hours)	Distance (miles)
0	0
3	99
4	132
6	198
7	231
9	297

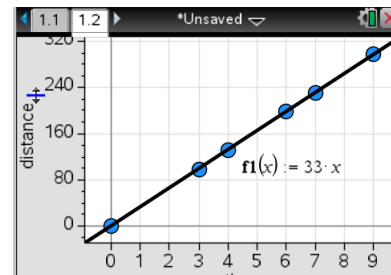
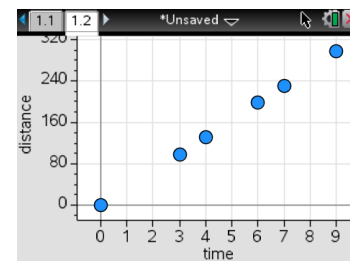
### RUN



### DRIVE



### SKATE



**What do you notice about all three of these situations?**

The following are possible student generated responses:

They are all going up as time increases.  
They all look like the distances get bigger as the time gets longer.  
They all hit the point (0,0).  
Each situation has its own slopes  
All three of the equations are some number times X (whatever the time is).  
Each situation keeps the same slope the whole time.  
They all look like lines.  
They all have different points and slopes.  
They all increase by different amounts.  
They are all talking about speed.  
They all have constant slopes!

**What is similar?**

They are all going up as time increases.  
They all look like the distances get bigger as the time gets longer.  
They all hit the point (0,0).  
All three of the equations are some number times X (whatever the time is).  
Each situation keeps the same slope the whole time.  
They all look like lines.  
They are all talking about speed.  
They all have constant slopes!

**What is different?**

Each one has a different slope.  
They all have different points and slopes.  
They all increase by different amounts.

**All of these are situations that can be represented by linear functions.**

**From this...**

**What can we say is true about ALL linear functions from our three examples?**

**ALL LINEAR FUNCTIONS HAVE A CONSTANT RATE OF CHANGE (SLOPE). THAT IS WHAT MAKES IT LINEAR!**

# LESSON 2 OVERVIEW

## HIGH EXPECTATIONS

### GOALS OF THE LESSON

This lesson is designed to get the student's familiar with linear functions that have a non-proportional relationship between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 2, the focus will be on the idea of the y-intercept of a linear function. Students will use the different contexts as a way of learning what a y-intercept is and how to calculate it. They will also get a review from lesson 1, and continue to see slope.

By the end of the lesson, students should be able to explain how to find the y-intercept of a linear function, what it is, what it represents, and the idea that a y-intercept represents the initial value for a given situation.

All of these real world contexts given to help the students learn revolve around a variety of sports. This topics make the tasks and lesson very interesting. The following are the contexts that are used to help students learn about y-intercepts:

#### **Lesson 2.1 – You Go Girl!**

**The increase of high school female athletes since 1997**

#### **Lesson 2.2 – There's No Mountain Too High**

**Ski Lift to the Top**

#### **Lesson 2.3 – Those Who Don't Jump Will Never Fly**

**Sky Diving**

After completing all three of these components to the lesson in lesson 2.4, the students will be asked to look at and explore the similarities and differences between the 3 different examples of linear functions. This will be where conclusions will be made and students will understand the idea of a y-intercept as the initial value of the function. They will also be introduced to the "Slope-Intercept" form of an equation for a linear function in this lesson.

#### **Lesson 2.4 – There is no "I" in Team**

**Pulling it all together to make sense**



## 2.1 YOU GO GIRL!

### SPORTS PARTICIPATION

In **1997**, about **2.6 million** girls competed in high school sports. The number of girls competing in high school sports has increased by **an average of 0.06 million** per year in the years **1997 to 2008**.



#### TASK 1

How many girls competed in 2008?

$$\frac{.06 \text{ million}}{1 \text{ year}} = \frac{y-2.6}{11-0}$$

$$\frac{.06 \text{ million}}{1 \text{ year}} = \frac{y-2.6}{11}$$

$$.06 * 11 = y - 2.6$$

$$.66 = y - 2.6$$

$$.66 + 2.6 = y$$

$$3.26 = y$$



So there will be 3.26 million female athletes in high school sports in the year 2008 ( 11 years after 1997)

How did you find that number?

The slope is  $\frac{.06 \text{ million}}{1 \text{ year}}$

Remember that: **SLOPE** as  $\frac{\Delta Y}{\Delta X} = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}}$   
for corresponding x and y values

We can use the point (0, 2.6) to represent the number of female athletes in 1997 and the point (11, y) for the year 2008 and the unknown number of female athletes (in millions) that year

Students might need help to see (0, 2.6) as representing 1997. Why not (1997, 2.6)?

Then we can set up an equation to represent the slope using the two points and then solve to get the missing y value for the number of female athletes in 2008.

Could you find the number of female athletes in the year 2004? What would it be?

Yes, we can use the same process as in the last question.

(0, 2.6) and (7, y) are the two points being considered.

$$\frac{.06 \text{ million}}{1 \text{ year}} = \frac{y-2.6}{7-0}$$

$$\frac{.06 \text{ million}}{1 \text{ year}} = \frac{y-2.6}{7}$$

$$.06 * 7 = y - 2.6$$

$$.42 = y - 2.6$$

$$.42 + 2.6 = y$$

$$3.02 = Y$$

So there will be 3.02 million female athletes in high school sports in the year 2004 (7 years after 1997)

Discuss with students that they could have used (11, 3.26) as their other point instead of (0, 2.6) if they wanted since linear functions have a constant rate of change.

At what year will there be 3.38 million female student athletes in high school sports?

we can use the same process as in the last question, except we are focusing on the x coordinate of point with an unknown, rather than the y.

(0, 2.6) and (x, 3.38) are the two points being considered.

$$\frac{.06 \text{ million}}{1 \text{ year}} = \frac{3.38-2.6}{x-0}$$

$$\frac{.06 \text{ million}}{1 \text{ year}} = \frac{.78}{x}$$

$$.06 * x = .78$$

$$.06x = .78$$

$$X = \frac{.78}{.06}$$

$$X = 13$$

So there will be 3.38 million female athletes in high school sports in the year that is 13 years after 1997, or 2010.

Again, the students could have used (11, 3.26) as their other point instead of (0, 2.6) if they wanted since linear functions have a constant rate of change.

**Do you think that there is a limit to this pattern? OR will this trend continue forever?**

Some students might say "No, there is no limit". They might say this because this is a linear function and there is a constant rate of change. They might think that since the situation is linear that the slope has to continue on forever, and forever.

Some students might think that this trend has a limit. They could say this because there will eventually be a time where the trend could predict more female high school athletes than the actual number of females in high school at the time. Or the trend could predict that 100% of the females in high school will participate in school sports, which is not likely due to the fact that some people prefer to not participate in sports.

**TASK 2**

**Explore the table for the number of female athletes in school sports.  
Fill in the blank spaces.**

Time (years since 1997)	Female Athletes (in millions)
0	2.6
1	2.66
4	2.84
5	2.9
7	3.02
9	3.14
13	3.38

**What do you notice about this table?**

At time 0, the number of female athletes is 2.6 (million)

There slope for this situation is .06 (given in the beginning of the lesson)

**Is there anything that seems to be different from the tables and information from the last lesson on "World Record Speeds"?**

At time 0, the number of female athletes is 2.6 (million)

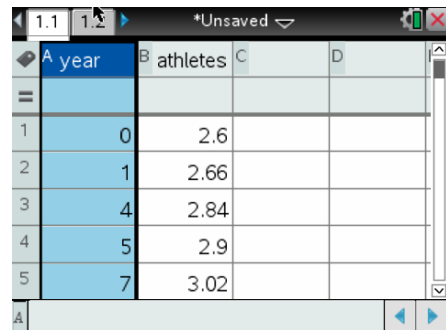
In the "World Record Speeds" lesson, every time that time was at 0, the distance was also 0.

Do you think that this situation has a constant slope?  
 If yes, how did you decide that and what is the slope?  
 If no, how did you decide that?

Yes, as you can calculate from one point to the next, the slope comes out to be .06 million per year.

### TASK 3

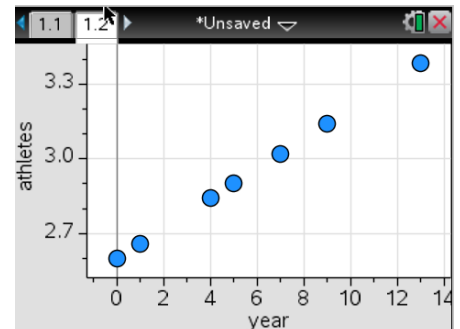
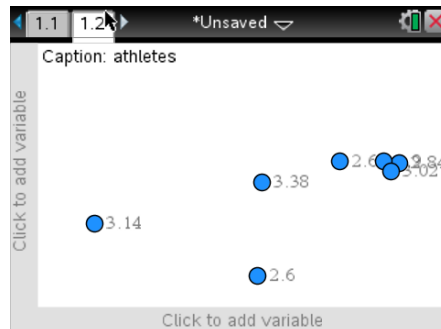
On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.



	A year	B athletes	C	D
1	0	2.6		
2	1	2.66		
3	4	2.84		
4	5	2.9		
5	7	3.02		

Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down? Justify your reasoning.

The bottom (independent variable) should be years = Time  
 The side (dependent variable) should be athletes (in millions)

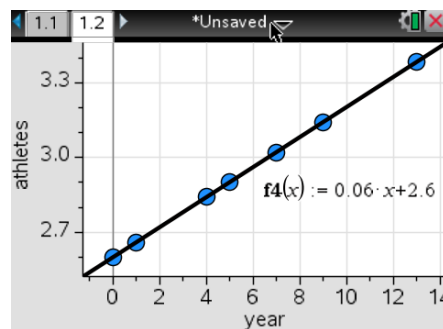
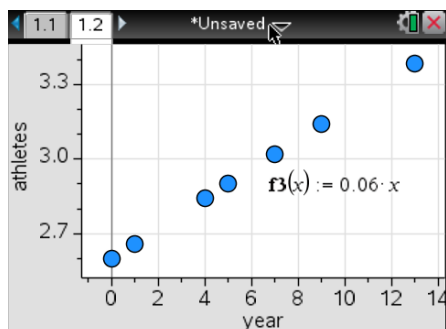


Looking at this graph, how does it differ from the ones from lesson 1? (Usain's run, fastest car, speed skating examples)

This graph looks different in the bottom left corner. It does not pass through the point (0,0).

## BUILDING AN EQUATION

On your TI-Nspire, plot your hypothesis function on your graph page.  
If your graph doesn't match the data change it so that it fits.



Many students will probably try  $.06x$  as their equation. They have the right thought process since that is the slope of this linear situation. This is also what all of the previous examples from the last lesson looked like since they were proportional relationships. The students should notice that if they try  $f(x) = .06X$  as their equation, it will not graph the line containing these points.

You might need to step in and remind them of the previous questions. "How is this situation different than those from the first lesson?" Hopefully they will see/remember that at time 0, this situation does not have 0 as the outcome. So they need to change their equation.

What did you notice about your hypothesis and your collected data?

Eventually the line fits with data, when you use  $f(x) = .06x + 2.6$ .

You might need to have a conversation with the students about why this equation is different than those from the first lesson. Do not tell them too much detail. Let them figure it out on their own and with the help of the next couple of days for this lesson.

## 2.2 THERE'S NO MOUNTAIN TOO HIGH

### SKIING

A ski lift raises the skiers up **130 feet every minute**. Even in the ski lodge, at the bottom of the ski lift, is **620ft above sea level**. The top of the mountain is 1420 ft about sea level.



### TASK 1

How long would it take for you to reach the top of the mountain?

(0, 620) and (x, 1420) are the two points we want to consider.

$$\frac{130 \text{ ft}}{1 \text{ min}} = \frac{1420 - 620}{x - 0}$$

$$\frac{130 \text{ ft}}{1 \text{ min}} = \frac{900}{x}$$

$$130 * x = 900$$

$$130X = 900$$

$$X = \frac{900}{130}$$

$$X = 6.923$$

So it will take 6.923 minutes until you reach the top of the mountain (1420 ft).

How did you find that number?

The slope is  $\frac{130 \text{ ft}}{1 \text{ min}}$

Remember that: SLOPE as  $\frac{\Delta Y}{\Delta X} = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}}$   
for corresponding x and y values

We can use the point (0, 620) to represent the height you are at, at the bottom of the ski lift and the point (x, 1420) for the time it takes to get to the top of the mountain (1420 ft)

Then we can set up an equation to represent the slope using the two points and then solve to get the missing  $x$  value for the time it takes to get to the top of the mountain

If you rode the ski lift for 5 mins, what altitude would you be at?

$(0, 620)$  AND  $(5, y)$  are the points we would want to think about with this problem.

$$\frac{130 \text{ ft}}{1 \text{ min}} = \frac{y - 620}{5 - 0}$$

$$\frac{130 \text{ ft}}{1 \text{ min}} = \frac{y - 620}{5}$$

$$130 * 5 = y - 620$$

$$650 = y - 620$$

$$650 + 620 = y$$

$$1270 = y$$

So after 5 mins on the ski lift, you will be at an altitude of 1270 ft.

Do you think that there is a limit to this pattern? OR will this trend continue forever?

Yes, there is a limit to this situation. The mountain only goes up to a certain height, therefore the ski lift can only go as high as the mountain goes.

## TASK 2

Explore the table for the number of female athletes in school sports.

Fill in the blank spaces.

Time (mins)	Altitude (in feet)
0	620
1	750
4	1140
5	1270
6.1538	1420

What do you notice about this table?

"This table has much less values that the previous ones did"

"At time = 0, there is a height, it is not 0."

Is there anything that seems to be different from the tables and information from the last lesson on "World Record Speeds"?

At time 0, the number of female athletes is 2.6 (million)

In the "World Record Speeds" lesson, every time that time was at 0, the distance was also 0.

Do you think that this situation has a constant slope?

If yes, how did you decide that and what is the slope?

If no, how did you decide that?

Yes, as you can calculate from one point to the next, the slope comes out to be 130 feet per minute.

### TASK 3

On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.

	A time	B height	C	D
=				
1	0	620		
2	1	750		
3	4	1140		
4	5	1270		
5	6.1538	1420		

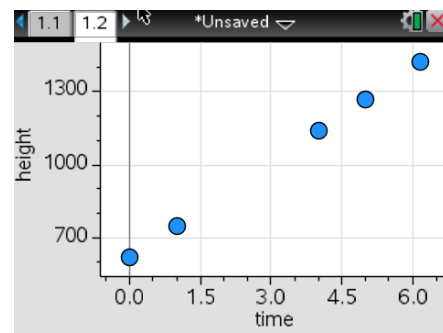
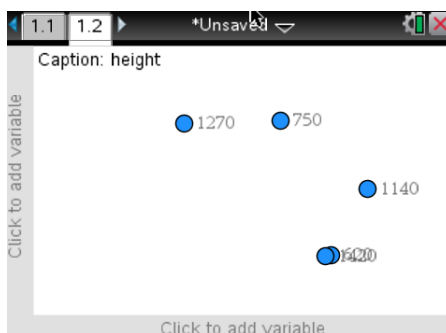
Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down? Justify your choices.

The bottom (independent variable)

should be time (in mins)

The side (dependent variable)

should be height (altitude in ft)





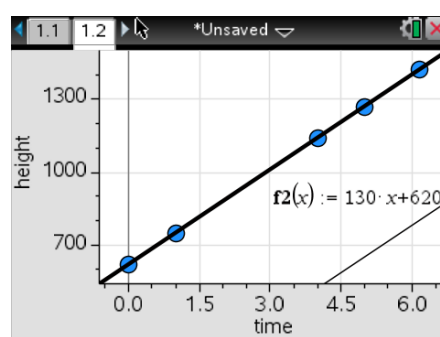
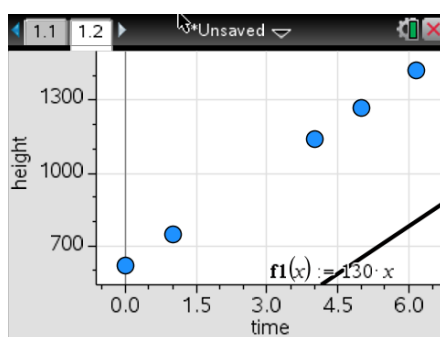
Looking at this graph, how does it differ from the ones from lesson 1? (Usain's run, fastest car, speed skating examples)

This graph looks different in the bottom left corner. It does not pass through the point (0,0).

## BUILDING AN EQUATION

On your TI-Nspire, plot your hypothesis function on your graph page.

If your graph doesn't match the data change it so that it fits.



Many students will probably try  $130x$  as their equation. They have the right thought process since that is the slope of this linear situation. This is also what all of the previous examples from the last lesson looked like since they were proportional relationships. The students should notice that if they try  $f(x) = 130x$  as their equation, it will follow the path of the points we put in from the table.

What did you notice about your hypothesis and your collected data?

Eventually the line fits with data, when you use  $f(x) = 130x + 620$

Again, you might need to have a conversation with the students about why this equation is different than those from the first lesson. Do not tell them too much detail. Let them figure it out on their own and with the help of the next couple of days for this lesson.

## 2.3 THOSE WHO DON'T JUMP WILL NEVER FLY

### SKYDIVING

For a standard sport Skydiving experience, a person will jump from a plane that is flying at **12,500 feet** above the ground level. From there the person experiences “free fall” at, on average, **115 mph until they reach 2,500 feet** above the ground where they must pull their parachute cord.



#### TASK 1

Thinking about this rate, you probably won't be falling for hours out of the plane. Convert this rate to feet per second.

$$\frac{115 \text{ mles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ mins}} \times \frac{1 \text{ min}}{60 \text{ secs}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} = \frac{168.86 \text{ feet}}{1 \text{ sec}} = \frac{169 \text{ feet}}{1 \text{ sec}}$$

Students may need to do this conversion in parts. Remind them that that is okay to do.

What altitude will you be at after 30 seconds into the fall?

We need to consider the following points from the linear function to solve this. (0, 12500) and (30, Y)  
Students will need help to realize that the slope is negative. Might need to have a discussion about that.

$$\frac{-169 \text{ feet}}{1 \text{ sec}} = \frac{Y-12500}{30-0}$$

$$\frac{-169 \text{ feet}}{1 \text{ sec}} = \frac{Y-12500}{30}$$

$$\underline{-169 \text{ feet} \times 30} = \underline{Y-12500}$$

$$-5070 = Y - 12500$$

$$12500 - 5070 = Y$$

$$7430 \text{ feet} = Y$$

After 30 secs of falling in the sky, the diver will be at an altitude of 7430 feet above the ground.

What time will you need to pull the parachute cord?

We will need to consider the following points to do this calculation: (0, 12500) and (X, 2500)

$$\frac{-169 \text{ feet}}{1 \text{ sec}} = \frac{2500-12500}{x-0}$$

$$\frac{-169 \text{ feet}}{1 \text{ sec}} = \frac{-10000}{x}$$

$$-169 * X = -10000$$

$$X = 59.17 \text{ secs}$$

At about 59 seconds, you will need to pull the parachute cord.

What is the slope to represent this situation?

The slope for this function is -169 feet per second. This is something that is new for the students. Discuss with the students why we must make it negative.

What makes this problem/situation different than the rest of the linear situations we have explored this unit?

The slope is negative. So the height will continue to decrease over time.

Will this trend continue forever? Provide your reasoning.

No, this pattern can only continue until you reach the 2,500 feet above the ground and pull the parachute cord. After this the fall will be much slower in order to fall safely to the ground. The rate of decrease of altitude will decrease after you pull the parachute cord. If the fall continued at the -169 ft per second, you would hit the ground too hard and splat! Ou

## TASK 2

Explore the table for the number of female athletes in school sports.

Fill in the blank spaces.

Time (secs)	Altitude (in feet)
0	12500
10	10810
30	7430
45	4895
50	4050
57	2867
About 59	2500

What do you notice about this table?

The slope is negative. As the time increases, the altitude decreases.

We also have to stop sooner, because this situation only will follow this pattern for about 60 seconds.

What information about this linear situation can you find out from this given table?

The slope.

The starting value.

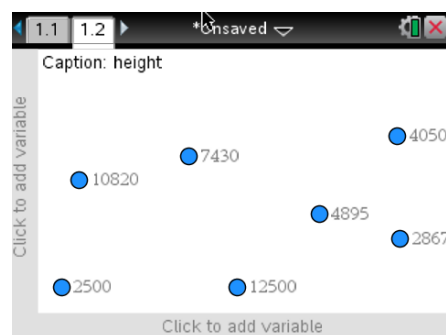
## TASK 3

On your TI-Nspire, open up a list and spreadsheets page and enter the information from the previous table.

	A time	B height	C	D
1	0	12500		
2	10	10820		
3	30	7430		
4	45	4895		
5	50	4050		

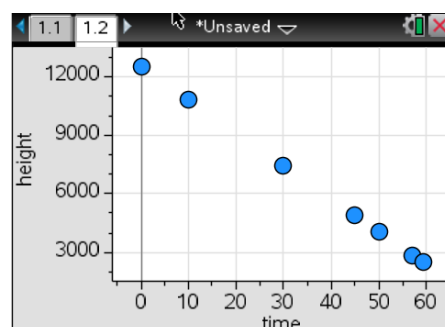
Then plot these points on a data and statistics page. What variable goes on the bottom and what variable goes on the side up and down? Justify your answer.

Time will be the independent variable and go on the bottom.  
Height will be the variable on the side because height is dependent on time.



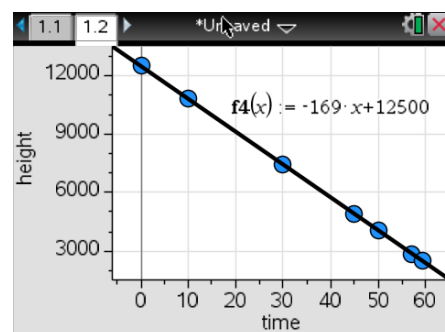
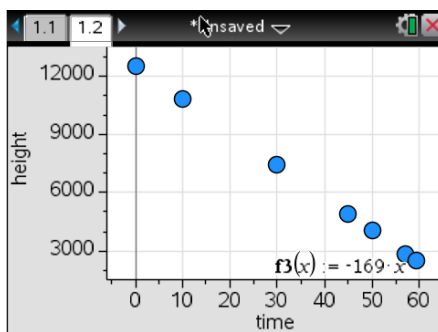
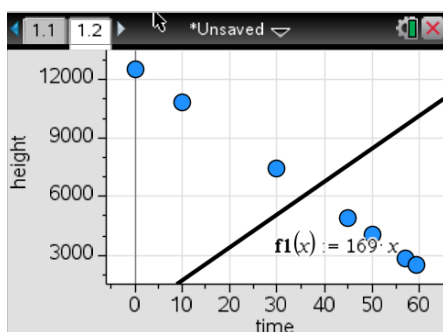
Looking at this graph, how does it differ from the ones from previous situations in previous lessons?

This graph tilts down due to its negative slope.  
It also doesn't start at (0,0) like the ones in the first lesson.



## BUILDING AN EQUATION

On your TI-Npsire, plot your hypothesis function on your graph page.  
If your graph doesn't match the data, change it so it fits.



The first graph is if students try to use a positive slope of 169 ft/sec without a starting value. This will be common since these are the kinds of equations that they learned in lesson 1. The second graph would be if the students tried  $y = -169x$  as their equation. This is good because they are recognizing the negative slope, however, they are still assuming its proportional with this equation, making it go through the point (0,0). The third graph is of what we want students to get to.  $Y = -169X + 12500$ . This includes the negative slope and the starting height of 12500 feet.

**What did you notice about your hypothesis and your collected data?**

Again, you might need to have a conversation with the students about why this equation is different than those from the first lesson. Do not tell them too much detail. Let them figure it out on their own and with the help of the next couple of days for this lesson. The idea of the y-intercept or initial value will be brought out during lesson 1.4.

## 2.4 THERE IS NO "I" IN TEAM

### BRINGING IT ALL TOGETHER

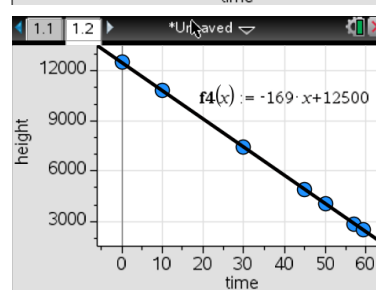
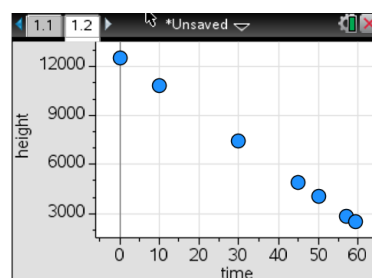
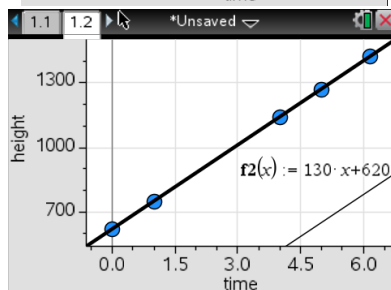
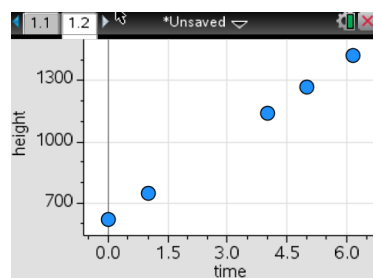
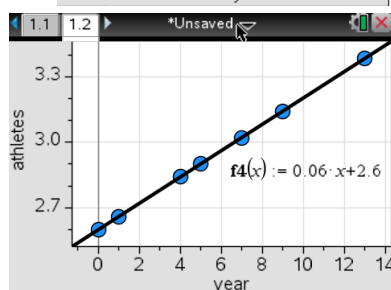
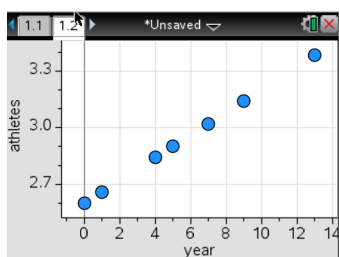
In the last 3 lessons we have talked about a few different experiences with linear functions.

**Review the tables and graphs for each of the situations we studied earlier.**

Time (yrs since 1997)	Female athletes (in millions)
0	2.6
1	2.66
4	2.84
5	2.9
7	3.02
9	3.14
13	3.38

Time (hours)	Distance (miles)
0	620
1	750
4	1140
5	1270
6.158	1420

Time (hours)	Distance (miles)
0	12500
10	10810
30	7430
45	4895
50	4050
57	2867
59.17	2500



**What do you notice about all three of these situations?**

The following are possible student generated responses:

Each situation has its own slopes

All three of the equations are some number times X (whatever the time is) and add in another number.

Each situation keeps the same slope the whole time.

They all look like lines.

They all have different points and slopes.

They all have constant slopes!

**What is similar?**

They are all going up as time increases.

They all look like the distances get bigger as the time gets longer.

All three of the equations are some number times X (whatever the time is) and then add another number.

The number being added in the equation is where the line starts.

Each situation keeps the same slope the whole time.

They all look like lines.

They all have constant slopes!

**What is different?**

Each one has a different slope.

They all have different points and slopes.

They all increase by different amounts.

Two of them are increasing while the last one is decreasing.

Now let's compare the equations from each of these situations.

Girls in Sports

Ski lift

Sky Diving

$$Y = 0.06x + 2.6$$

$$Y = 130x + 620$$

$$Y = -169x + 12500$$

**What do you notice about these equations?**

**What is similar?**

All of the equations include the slope.

All of the equations have another number being added.

The number being added is the number you started with!



What is different?

One of the equations has a negative slope.

From this...

**$Y = m \cdot x + b$**  is called **Slope Intercept Form** for a linear equation. Where  $m$  represents the slope of the line and  $b$  represents the  $y$ -intercept, or initial value of the function.

# LESSON 3 OVERVIEW

## GETTING PRICY

### GOALS OF THE LESSON

This lesson is designed to get you familiar with linear functions that have a proportional and non-proportional relationships between the input and output. The goal is to use real world contexts that fit this to help gather information on properties and characteristics of linear functions. In lesson 3, the focus will be on the representation of tables. You will use a table representation of a linear function to find out information about the linear situation and then construct a graphical representation and an equation to represent the situation.

By the end of the lesson, you should be able to explain how to find and calculate slope from a table, locate the y-intercept from a table, and show how to create another representation for the linear function when only given a table.

All of these real world contexts, given in the beginning, will not provide any information on the slope or y-intercept of the function. It will just give a story line to follow and help make sense of the table. These are used to help you learn revolving around situations that are more relevant to your lives than a traditional text. This topics make the tasks and lesson very interesting. The following are the contexts that are used to learn about linear function with only given table representations of the data.

#### **Lesson 3.1 – Hey Hey Hockey Town** **Ticket Prices for a Redwings Game**

#### **Lesson 3.2 – Let's Get Fit!** **Purchasing a Gym Membership**

# 3.1 HEY HEY HOCKEY TOWN!

## DETROIT RED WINGS GAME

Tickets for hockey games between the Detroit Red Wings and the Chicago Blackhawks can get very pricy. For a game in the Joe Lois Arena in Detroit, you always want a good seat!



The following is a table that shows the relationship between the number of tickets bought and the price to pay. These tickets are in section 107, row 3 (right behind the opponent’s team bench).

Fill in any blank spaces

Tickets	Price to Pay
0	0
1	360.61
3	1081.83
6	2,163.66
8	2,884.88
11	3,966.71
15	5,409.15

What do you notice about this table?

This table has the point (0,0) in it. The last lesson did not. -> starting value is 0. (y-intercept)

As X increases so does Y.

We can find the slope from the table.

Slope is \$360.61 per ticket.

What can you conclude about the price of tickets in terms of the number of tickets bought?

The more tickets you buy, the more expensive the price is.

You have to pay \$360.61 for each ticket that you buy.

If you don't buy any tickets, you don't have to pay anything.

#### TASK 1

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.

Remember: A now next formula uses the previous term to find the next term.

For example: How much would you have to pay for 4 tickets, based on the price for 3 tickets?

$$A_n = A_{n-1} + 360.61$$

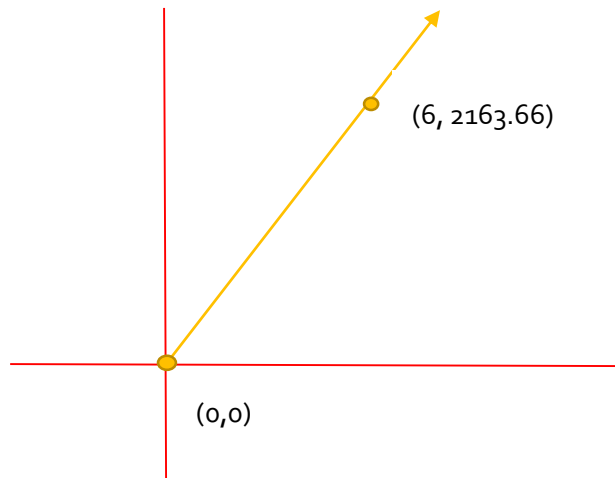
$$A_0 = 0$$

"you have to add 360.61 to the price if you want to buy one more ticket"

## TASK 2

What would a graph for this function look like?  
Graph this situation by hand.

Would this be a continuous graph  
or a group of distinct points?



## TASK 3

Write an equation to represent this data.

$$Y = 360.61X$$

How did you come up with this equation?

From the table, we can see that this situation goes through the point  $(0,0)$ , so it is proportional. That means that the y-intercept, or starting value, is 0.

We can calculate the slope for this situation from the table. The slope comes out to be \$360.61 per ticket.

We can then use slope intercept form of linear functions to write an equation for this situation.

$y = m(x) + b$  is slope intercept form for a linear function. Where  $m$  is the slope and  $b$  is the y intercept. Thus we obtain the equation  $y = 360.61X + 0$ .

Show that your equation works for this situation.

(same information from previous question)

## 3.2 LET'S GET FIT!

### GYM MEMBERSHIP

Let's Get Fit is a very successful gym, however due to the high quality there is also a high demand to get into the gym.



The following is a table that shows the relationship between the number of months that you want to be a member at the gym, and the price to pay.

Fill in any blank spaces

Months of Membership	Price to Pay
0	350
1	380
3	440
6	530
12	710
16	830
24	1070

What do you notice about this table?

This table has the point (0,350) in it. The last lesson did not. -> starting value is 350. (y-intercept)

As X increases so does Y.

We can find the slope from the table.

Slope is \$30 per month.

What can you conclude about the price of tickets in terms of the number of tickets bought?

The longer you want to stay a member, the more expensive the price is.

You have to pay \$30 for each month that you stay a member.

There is an initial price to become a member. You must pay \$350 to become a member.

#### TASK 1

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.

Remember: A now next formula uses the previous term to find the next term.

For example: How much would you have to pay for 4 tickets, based on the price for 3 tickets?

$$A_n = A_{n-1} + 30$$

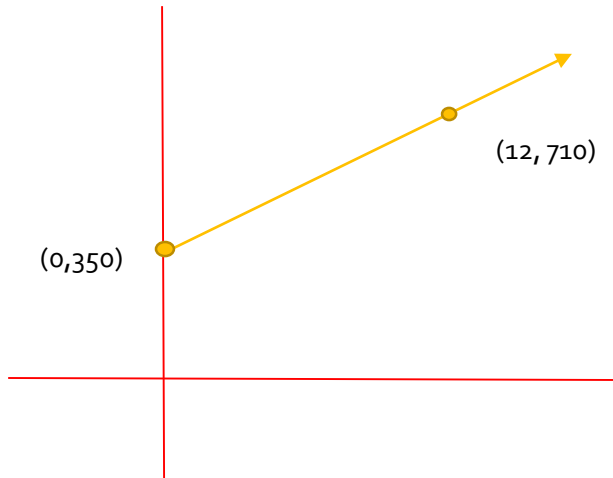
where  $A_0 = 350$

"you have to add 30 to the price if you want to stay a member for another month. And pay that initial fee of \$350"

## TASK 2

What would a graph for this function look like?

Graph this situation by hand.



What similarities do you notice with your graph and the table that you started with?

The graph shows the y-intercept (0, 350) which is clearly given by the table.

The graph shows the slope of the line between the points.

## TASK 3

Write an equation to represent this data.

$$Y = 30X + 350$$

How did you come up with this equation?

From the table, we can see that this situation goes through the point (0, 350), so it is non-proportional. That means that the y-intercept, or starting value, is 350.

We can calculate the slope for this situation from the table. The slope comes out to be \$30 per month.

We can then use slope intercept form of linear functions to write an equation for this situation.

$y = m(x) + b$  is slope intercept form for a linear function. Where  $m$  is the slope and  $b$  is the y intercept. Thus we obtain the equation  $y = 30X + 350$ .



Show that your equation works for this situation.

(same information from previous question)

# LESSON 4 OVERVIEW

## WATCHING THE WEIGHT FALL RIGHT OFF

### GOALS OF THE LESSON

This lesson is designed to get the student's familiar with linear functions that have a proportional and non-proportional relationships between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 4, the focus will be on the representation of graphs. Students will use a graphical representation of a linear function to find out information about the linear situation and then construct a tabular representation and an equation to represent the situation.

By the end of the lesson, students should be able to explain how to find and calculate slope from a graph, locate the y-intercept from a graph, and show how to create another representation for the linear function when only given a graph.

All of these real world contexts given in the beginning, will not provide the students with any information on the slope or y-intercept of the function. It will just give them a story line to follow and help make sense of the graph. These are used to help the students learn revolving around situations that are more relevant to their lives. This topics make the tasks and lesson very interesting. The following are the contexts that are used to help students learn about linear function with only given graphical representations of the data.

#### **Lesson 4.1 – Cycling Through the Calories** **Burning Calories While Biking**

#### **Lesson 4.2 – Burn Baby Burn** **Burning Calories While Running**

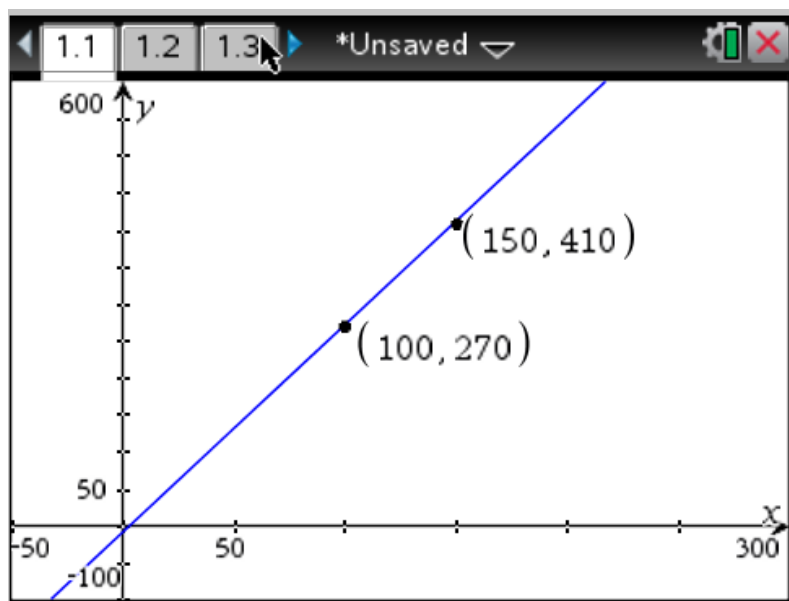
## 4.1 CYLCING THROUGH THE CALORIES

### BURNING CALORIES WHILE BIKING

According to the American Heart Association, the number of calories a person can burn while biking at 12mph depends on the weight of that person (in pounds)



The following is a graph that shows the relationship between a person's weight (in pounds) and the number of calories that they will burn while biking at 12 mph.



What variable is along the bottom of the graph (x- axis) and what variable goes along the y-axis? Justify your answer.

The x-axis represents the independent variable. For this situation, this is the weight of the person. The x-values represent a person's weight in pounds.

The y-axis represents the dependent variable, or the one that depends on the other. In this case, the number of calories burned depends on the amount that you weigh. So the y-values represent the number of calories burned.

What conclusions about this linear situation can you conclude from the graph?

As  $x$  increases, so does  $y$ . So as a person's weight increases they will burn more calories per hour while biking at 12mph.

Since as  $x$  increases,  $y$  increases, we know that the slope of this function is positive. We can also calculate the slope by using the slope formula. Slope =  $\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

For this situation the slope is  $\rightarrow \frac{410 - 270}{150 - 100} = \frac{140}{50} = \frac{2.8 \text{ calories}}{\text{pound}}$

We also can see from the graph that this situation is not proportional because the  $y$ -intercept is not at  $(0,0)$ .

Will the trend of this graph continue on forever in both directions? Explain your reasoning.

No. Think about it, the  $y$ -intercept is negative. That is saying that if you weighed 0 pounds, that you would burn a negative 10 calories, which means you would gain 10 calories for every hour that you bike at 12mph. That makes no sense.

It also makes no sense to even weigh 0 pounds. There is a clear understanding that there is a limit where a person can no longer weigh that little and still be able to bike at 12mph. This also is the same for the other direction. There is a point where you will weigh too much to probably be able to move, or live, let alone bike at 12mph. For example, a person weighing 700 pounds probably is incapable of biking.

### TASK 1

Based on this, create a table that represents the data from the graph.


What variable goes in the left column?

The left side represents the independent variable. For this situation, this is the weight of the person. The  $x$ -values represent a person's weight in pounds.

What variable goes in the right column?

This side represents the dependent variable, or the one that depends on the other. In this case, the number of calories burned depends on the amount that you weigh. So the  $y$ -values represent the number of calories burned.

Are there any restrictions for what you can put in for the x values?

Yes, there is a clear understanding that there is a limit where a person can no longer weigh that little and still be able to bike at 12mph. This also is the same for the other direction. There is a point where you will weigh too much to probable be able to move, or live, let along bike at 12mph. For example, a person weighing 700 pounds probably is incapable of biking.

## TASK 2

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.

$$A_n = A_{n-1} + 2.8$$

$$\text{where } A_0 = -10$$

“You have to add 2.8 calories to the number of calories you will burn if you add another pound to your weight. You also need to subtract the initial amount which is 10 calories.”

## TASK 3

Write a Y= equation to represent this data.

$$Y = 2.8x - 10$$

How did you come up with this equation?

From the graph, we can see that this situation goes through a point along the y-axis that is not (0,0), so it is non- proportional. That means that the y-intercept is not 0. It ends up being -10.

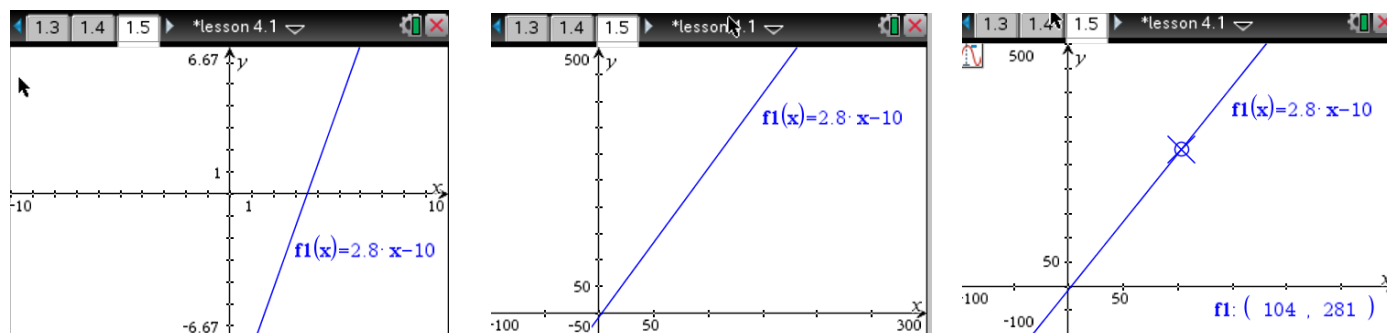
We can calculate the slope for this situation from the table. The slope comes out to be 2.8 calories per pound.

We can then use slope intercept form of linear functions to write an equation for this situation.

$y = m(x) + b$  is slope intercept form for a linear function. Where m is the slope and b is the y intercept. Thus we obtain the equation  $y = 2.8x - 10$ .

Show that your equation works for this situation. Prove by graphing your equation and using the Trace function on your TI-nspire.

(Same information as in previous question). But prove on calculator.



The first graph is the initial view of the graph when the student plugs in their equation.

The second graph is after adjusting the window to see more data that fits with this situation.

The third graph is what they students will see when they use the trace function on their graph. Notice and point out that the coordinate that they are on on the line is shown in the bottom corner.

The students will then check to see if this fits the data that they were given from the original graph.

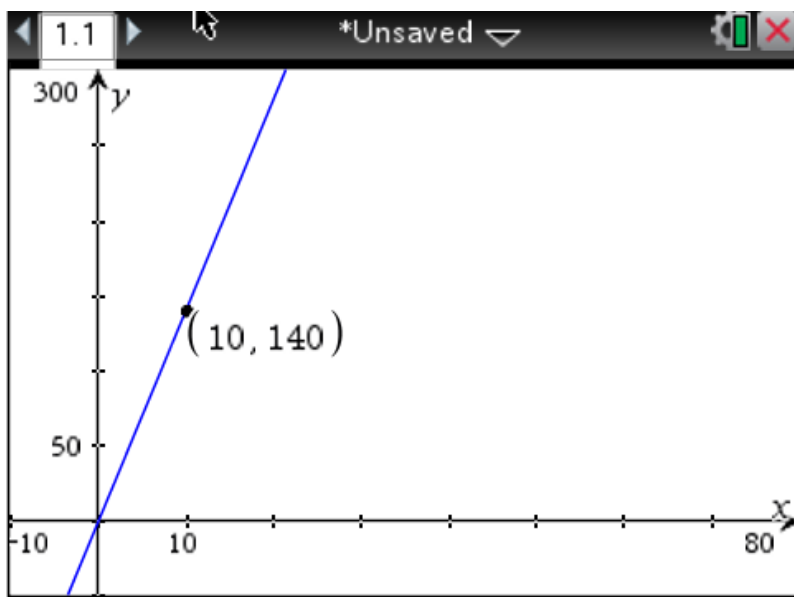
## 4.2 BURN BABY BURN

### BURNING CALORIES WHILE RUNNING

According to calculations done by the USA Track and Field group, a jogger will burn a certain number of calories per minute while running at 9 miles per hour.



The following is a graph that shows the relationship between the length of time a person is running at 9mph and the number of calories that they burn.



What variable is along the bottom of the graph (x- axis) and what variable goes along the y-axis? Justify your answer.

The x-axis represents the independent variable. For this situation, this is the weight of the person. The x-values represent a person's weight in pounds.

The y-axis represents the dependent variable, or the one that depends on the other. In this case, the number of calories burned depends on the amount that you weigh. So the y-values represent the number of calories burned.

What conclusions about this linear situation can you conclude from the graph?

As  $x$  increases, so does  $y$ . So as a person running at gmph will burn more calories the longer they continue to run.

Since as  $x$  increases,  $y$  increases, we know that the slope of this function is positive. We can also calculate the slope by using the slope formula. Slope =  $\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

For this situation the slope is  $\rightarrow \frac{140-0}{10-0} = \frac{140}{10} = \frac{14 \text{ calories}}{1 \text{ minute}}$

We also can see from the graph that this situation is proportional because the  $y$ -intercept is at  $(0,0)$ .

Notice that only one point is labeled on the graph. How could we find the slope for this situation based on the graph?

If you look closely at the graph you will be able to notice that the graph crosses through the point  $(0,0)$ . Thus the students can use the given point and the point  $(0,0)$  to find the slope of the function.

Will the trend of this graph continue on forever? Explain your reasoning.

As long as the person could maintain that pace of running this trend will continue on.

### TASK 1

Based on this, create a table that represents the data from the graph.


What variable goes in the left column? Justify

The left side represents the independent variable. For this situation, this is the weight of the person. The  $x$ -values represent a person's weight in pounds.

What variable goes in the right column? Justify

This side represents the dependent variable, or the one that depends on the other. In this case, the number of calories burned depends on the amount that you weigh. So the  $y$ -values represent the number of calories burned.



## TASK 2

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less ticket.

$$A_n = A_{n-1} + 14$$

$$A_0 = 0$$

"You have to add 14 calories to the number of calories you will burn for every minute more you run at 9mph."

## TASK 3

Write an equation to represent this data.

$$Y = 14x$$

How did you come up with this equation?

From the graph, we can see that this situation goes through the point  $(0,0)$ , so it is a proportional relationship. That means that the y-intercept is 0.

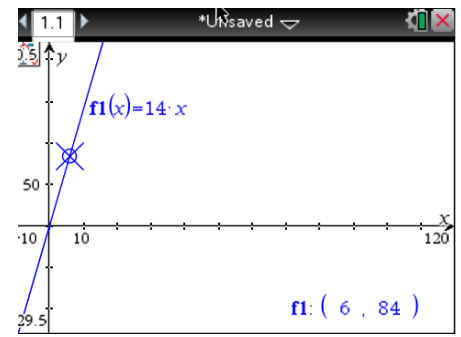
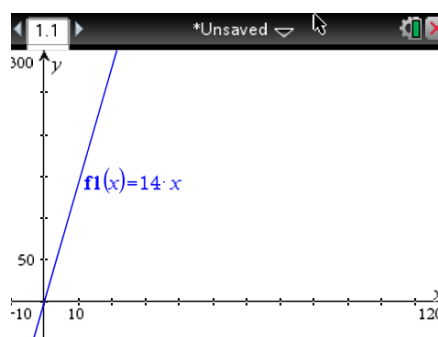
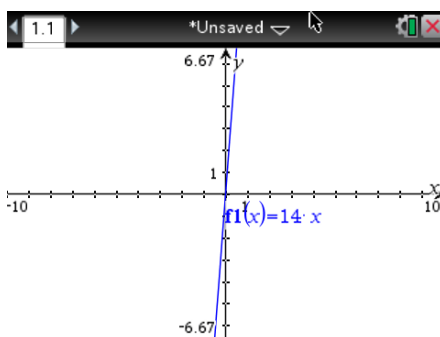
We can calculate the slope for this situation from the table. The slope comes out to be 14 calories per minute.

We can then use slope intercept form of linear functions to write an equation for this situation.

$y = m(x) + b$  is slope intercept form for a linear function. Where  $m$  is the slope and  $b$  is the y intercept. Thus we obtain the equation  $y = 14x + 0$ . Or  $y = 14x$ .

Show that your equation works for this situation. Prove by graphing your equation and using the Trace function on your TI-nspire.

(Same information as in previous question). But prove on calculator



The first graph is the initial view of the graph when the student plugs in their equation.

The second graph is after adjusting the window to see more data that fits with this situation.

The third graph is what they students will see when they use the trace function on their graph. Notice and point out that the coordinate that they are on on the line is shown in the bottom corner.

The students will then check to see if this fits the data that they were given from the original graph.

# LESSON 5 OVERVIEW

## A PROFESSIONAL'S OPINION

### GOALS OF THE LESSON

This lesson is designed to help students to become familiar with linear functions that have a proportional and non-proportional relationships between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 5, the focus will be on the representation of equations. Students will use an equation representation of a linear function to find out information about the linear situation and then construct a tabular representation and a graphical representation the situation.

By the end of the lesson, students should be able to explain how to find and calculate slope from an equation, locate the y-intercept from an equation, and show how to create another representation for the linear function when only given an equation.

All of these real world contexts given in the beginning, will not provide the students with any information on the slope or y-intercept of the function. It will just give them a story line to follow and help make sense of the equation. These are used to help the students learn revolving around situations that are more relevant to their lives. This topics make the tasks and lesson very interesting for students. The following are the contexts that are used to help students learn about linear function when only provided with an equation representation of the data.

#### **Lesson 5.1 – Serve it to them**

##### **Paying for Tennis Lessons**

#### **Lesson 5.2 – Swing with Everything You've Got**

##### **Paying for Baseball Lessons**

## 5.1 SERVE IT TO THEM

### PAYING FOR TENNIS LESSONS

Getting a professional's opinion can really help to increase your skill level in any sport. Due to this many professional athletes will be paid a lot to help train other players. A tennis pro is paid a certain amount of money per hour for a private lesson for members in a community club.



The following is an equation that describes the price a professional tennis player will receive ( $P(x)$ ) based on the number of hours they work and train members from the community club.

$$P(x) = 40x$$

What conclusions about this linear situation can you conclude from the equation?

You can clearly see the slope for the function in the equation. The slope is 40 dollars per hour. You can also clearly see that this linear function has a y-intercept at (0,0) because the equation doesn't have anything being added to it after the  $40x$ . This also shows that this is a proportional relationship.

Can we plug in any number for  $x$  into the equation? Can we have any value of  $x$  for this situation? Justify your answers.

Mathematically yes. We can plug in any number into the equation for  $x$  and still get a value out for  $y$ . However we can not have negative numbers for  $x$  values in this situation. It would make no sense to have a person working for negative hours. That makes no sense. There might also be a point in time where this situation can no longer have super big  $x$  values. No one can work for 70 hours straight without breaks. So this situation has some limitations for the domain ( $x$ -values).

## TASK 1

Based on this, create a table that represents the data from the graph.


What variable goes in the left column?

The left side represents the independent variable. For this situation, this is the number of hours. The x-values represent the time that the lessons last for.

What variable goes in the right column?

This side represents the dependent variable, or the one that depends on the other. In this case, the amount that the pro player makes depends on the number of hours they work. So their payment depends on the number of hours.

What similarities do you notice with your graph and the equation that you started with?

You can calculate all of the values for the table very easily from the equation. You can plug in a number for x and then solve for y.

You can clearly see the slope for the function in the equation and you can also calculate the slope from the values in the table. They should match up since these both represent the same linear situation.

The table can also show you the y-intercept value (if the students chose to put that coordinate in their table).

## TASK 2

Create a now-next formula for how to find the price you would need to pay for n lessons based on the price you would pay for one less lesson.

$$A_n = A_{n-1} + 40$$

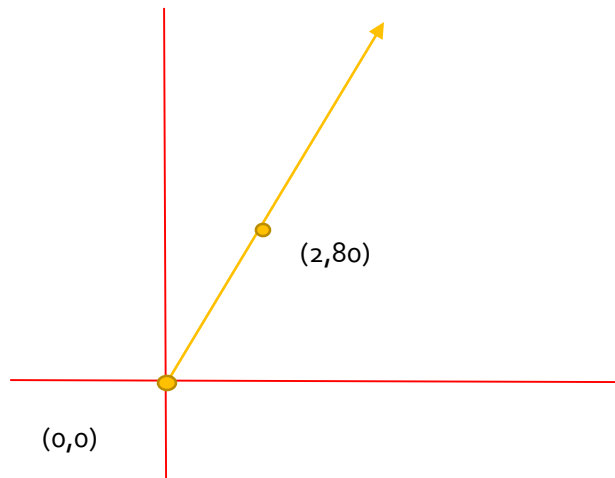
$$A_0 = 0$$

"You have to add 40 dollars to the amount that the pro will make for every hour that they teach a tennis lesson."

### TASK 3

What would a graph for this function look like?

Graph this situation by hand.



What similarities do you notice with your graph and the equation that you started with?

You can clearly see the slope for the function in the equation and you can also calculate the slope from a pair of points on the graph. The slope is also noticeably positive and steep, which can be seen from the equation since the slope is 40. They should match up since these both represent the same linear situation. The graph can also show you the y-intercept value since the line will go through the point  $(0,0)$ .

## 5.2 SWING WITH EVERYTHING YOU'VE GOT

### PAYING FOR BASEBALL LESSONS



Getting a professional's opinion can really help to increase your skill level in any sport. Great Lake's Baseball offers but private and group lessons to help people work on and better their skills. A major private lesson includes a session with experienced and premier Major League Baseball coaches.



The following is an equation that describes the price to pay ( $P(x)$ ) for a sessions with Major League Baseball coach based on the number of sessions that you want to attend.

$$P(x) = 55x + 20$$

What conclusions about this linear situation can you conclude from the equation?

You can clearly see the slope for the function in the equation. The slope is 55 dollars per session. You can also clearly see that this linear function has a y-intercept at (0,20) because the equation has a +20 after the 55x.

This also shows that this is a non-proportional relationship.

Can we plug in any number for x into the equation? Can we have any value of x for this situation? Justify your answers.

Mathematically yes. We can plug in any number into the equation for x and still get a value out for y. However we can not have negative numbers for x values in this situation. It would make no sense to have a person going to a negative number of sessions. That makes no sense.

## TASK 1

Based on this, create a table that represents the data from the graph.


What variable goes in the left column?

The left side represents the independent variable. For this situation, this is the number of sessions. The x-values represent the time that the lessons last for.

What variable goes in the right column?

This side represents the dependent variable, or the one that depends on the other. In this case, the amount that the person must pay depends on the number of sessions they go to. So the price depends on the number of hours.

What similarities do you notice with your graph and the equation that you started with?

You can calculate all of the values for the table very easily from the equation. You can plug in a number for x and then solve for y.

You can clearly see the slope for the function in the equation and you can also calculate the slope from the values in the table. They should match up since these both represent the same linear situation.

The table can also show you the y-intercept value (if the students chose to put that coordinate in their table).

## TASK 2

Create a now-next formula for how to find the price you would need to pay based on the price you would pay for one less session.

$$A_n = A_{n-1} + 55$$

$$A_0 = 20$$

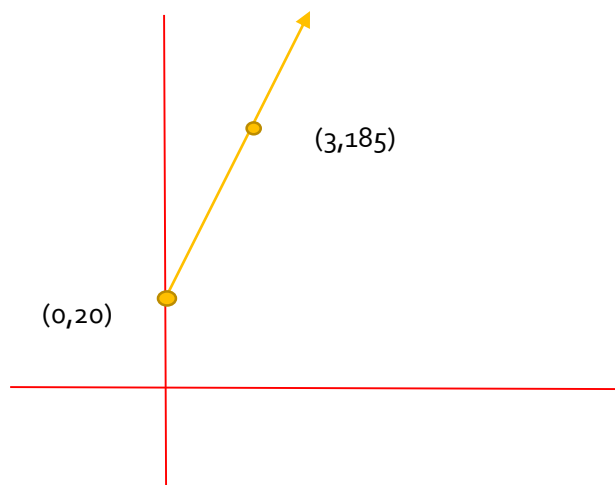
"You have to add 55 dollars to the amount that you will pay for every session that you want to attend"



## TASK 3

What would a graph for this function look like?

Graph this situation by hand.



What similarities do you notice with your graph and the equation that you started with?

You can clearly see the slope for the function in the equation and you can also calculate the slope from a pair of points on the graph. The slope is also noticeably positive and steep, which can be seen from the equation since the slope is 55. They should match up since these both represent the same linear situation. The graph can also show you the y-intercept value since the line will go through the point  $(0, 20)$ .

# LESSON 6 OVERVIEW

## REACHING NEW HEIGHTS

### GOALS OF THE LESSON

This lesson is designed to get the student's familiar with linear functions that have a proportional and non-proportional relationships between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 6, the focus will be on the representation of tables. Students will be given a table with only one row filled in to represent the sports context that they were given at the beginning on the lesson. The students will use this to create their own table that will represent the linear situation that they were given. This lesson emphasizes the idea of moving within one representation.

By the end of the lesson, students should be able to explain how proportional and non-proportional linear relationships are shown in tables. They should also be able to identify multiple linear functions (in the form of tables) that could represent the given sports context.

All of these real world contexts given in the beginning, will not provide the students with any information on the slope or y-intercept of the function. It will just give them a story line to follow and help make sense of the table. These are used to help the students learn revolving around situations that are more relevant to their lives. These topics make the tasks and lesson very interesting for the students. The following are the contexts that are used to help students learn about the concepts of a linear function with only given a table representation of the data.

#### **Lesson 6.1 – Climbing to the Top** **Temperature Changes with Elevation**

#### **Lesson 6.2 – Setting High Goals** **World Records for Pole Vault**

# 6.1 CLIMBING TO THE TOP

## TEMPERATURE CHANGE WITH ELEVATION

A mountain climber feels that the air temperature decreases as his elevation above sea level increases.



Let the following table represent the drop in temperature, in degrees Fahrenheit, that the mountain climber is experiencing while climbing to a higher elevation.

Complete the table with 6 other possible points that could fulfil this situation.

Elevation	Temperature
2000	60

There will be a variety of answers for possible points to fulfil this situation and fill the table. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a negative slope pattern since the temperatures decreases as the altitude increases. It is also important that if students do put in a point to show the initial temperature (y-intercept) that the y value is not 0 since this is a non-proportional relationship.

**Is this a proportional or non-proportional linear relationship? Justify your answer.**

A proportional situation goes through the point  $(0,0)$ . So a non-proportional situation will not go through that point, making the  $y$ -intercept some other value besides 0. On the table you would see a point that would be something along the lines of  $(0, 90)$ . This is a non-proportional relationship because at 0 ft. above sea level, you will have a temperature that is not 0 degrees.

**How did you chose the values that you added to the table?**

It will be important to discuss with the students that there is an infinite number of possible lines that could go through the one given point and be a non-proportional linear function. Therefore they can really fill in any possible points they want as long as the points follow a linear non-proportional relationship. The students must also make sure that the students recognize that the slope of this situation is negative. They must follow this pattern as well.

**Is there a limit to this pattern?**

Yes. We there will be a limit for the elevations for the problem. The lowest elevation will be at the bottom of the mountain and the highest elevation is at the top of the mountain. There is also a range of values that can represent the temperature of the air. There is a point where it can't get too cold or too hot. For example, you wouldn't see a temperature of 150 degree Fahrenheit on a mountain.

**Given the values that you filled in, what would you say would be the corresponding  $y$  value to this  $x$  value if it was added to your table?**

6500	
------	--

Again, there will be a variety of answers for possible points to fulfil this situation and fill the table since there are an infinite amount of possible lines going through one point and that are non-proportional. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a negative slope pattern since the temperatures decreases as the altitude increases. It is also important that if students do put in a point to show the initial temperature ( $y$ -intercept) that the  $y$  value is not 0 since this is a non-proportional relationship. As long as students follow these rules and patterns of the linear non-proportional situation, their point should be fine.

A student claimed that this was another point represented in their table from the information given at the beginning on the lesson:

2000	60
3000	75

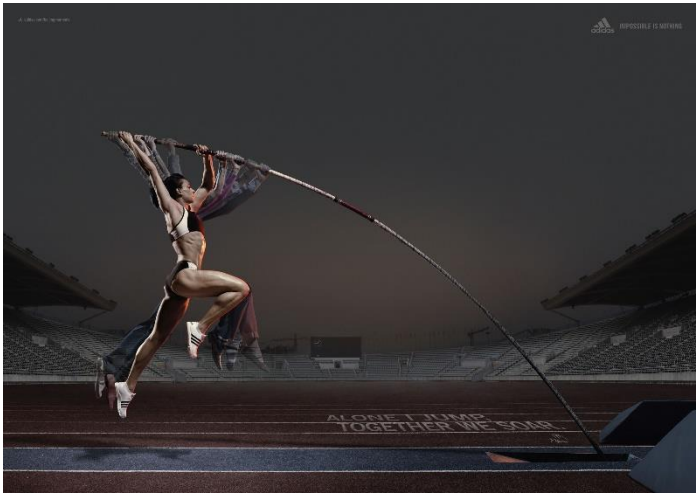
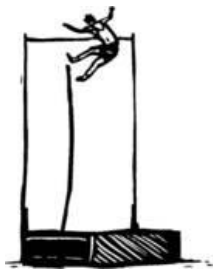
Does this point make sense to be a possible point on our table? Why or why not?

No, this could not be a possible point on the table. Looking at this point the table suggests that the temperature increase as elevation increases. This is not the case, as the problem mentions that the temperature decreases as the altitude increases. So this point would not make sense with this situation. The y value in this point should be less than 60 degrees since the elevation has increased from 2000 feet about sea level to 3000 feet about sea level.

# 6.2 SETTING HIGH GOALS

## POLE VAULT RECORDS

Over the years, the men’s world record for pole vaulting has continued to increase.



Let the following table represent the men’s world record for pole vault over the years since 1900.

Complete the table with 6 other possible points that could fulfil this situation.

Year	Height in meters
57	4.78

There will be a variety of answers for possible points to fulfil this situation and fill the table. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the pole vault record increases as the years go on. It is also important that if students do put in a point to show the initial world record (y-intercept) that the y value is not 0 since this is a non-proportional relationship.

Is this a proportional or non-proportional linear relationship? Justify your answer.

A proportional situation goes through the point  $(0,0)$ . So a non-proportional situation will not go through that point, making the y-intercept some other value besides 0. On the table you would see a point that would be something along the lines of  $(0, 2)$ . This is a non-proportional relationship because at the year 1900 the pole vault world record was more than 0 meters high.

How did you chose the values that you added to the table?

It will be important to discuss with the students that there is an infinite number of possible lines that could go through the one given point and be a non-proportional linear function. Therefore they can really fill in any possible points they want as long as the points follow a linear non-proportional relationship. The students must also make sure that the students recognize that the slope of this situation is positive. They must follow this pattern as well.

Is there a limit to this pattern?

Yes. We there will be a limit for the height of the world record for the problem. The world record started at some height that was greater than 0 meters. So there would never be a height of 0 meters, or a height of negative meters. There is also a limit on how high the record could get. The record will never reach something crazy like 100ft because that would be very unsafe for the athletes. There is also a limit on the years. At one point in the past, pole vault was not a thing. Therefore there couldn't be a height for the record if the sport was not started.

Given the values that you filled in, what would you say would be the corresponding y value to this x value if it was added to your table?

100	
-----	--

Again, there will be a variety of answers for possible points to fulfil this situation and fill the table since there are an infinite amount of possible lines going through one point and that are non-proportional. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the record height has increased as the years have increased. It is also important that if students do put in a point to show the initial temperature (y-intercept) that the y value is not 0 since this is a non-proportional relationship. As long as students follow these rules and patterns of the linear non-proportional situation, their point should be fine.

A student claimed that this was another point represented in their table from the information given at the beginning on the lesson:

57	4.78
60	4.5

Does this point make sense to be a possible point on our table? Why or why not?

No, this could not be a possible point on the table. Looking at this point the table suggests that the record for pole vault decreased over time. This is not possible. This is not the case, as the problem mentions that the pole vault record has gotten worse over time. So this point would not make sense with this situation. The y value in this point should be greater than 4.78 since the world record has gotten better since 1957.



# LESSON 7 OVERVIEW

## BEND IT LIKE BECKHAM

### GOALS OF THE LESSON

This lesson is designed to get the student's familiar with linear functions that have both a proportional and non-proportional relationships between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 7, the focus will be on the representation of graphs. Students will be given a graph with only one point plotted to represent the sports context that they were given at the beginning on the lesson. The students will use this to create their own graph that will represent the linear situation that they were given. This lesson emphasizes the idea of moving within one representation.

By the end of the lesson, students should be able to explain how proportional and non-proportional linear relationships are shown in graphs. They should also be able to identify multiple linear functions (in the form of graphs) that could represent the given sports context.

All of these real world contexts given in the beginning, will not provide the students with any information on the slope or y-intercept of the function. It will just give them a story line to follow and help make sense of the graph. These are used to help the students learn revolving around situations that are more relevant to their lives. This topics make the tasks and lesson very interesting. The following are the contexts that are used to help students learn about the concepts of a linear function when only provided with a graphical representation of the data.

#### **Lesson 7.1 – Whoa, that's Hot!**

**Temperature Increase While Playing on Artificial Turf**

#### **Lesson 7.2 – What a Pass!**

**How Long Will it Take to Pass the Ball to Your Teammate?**

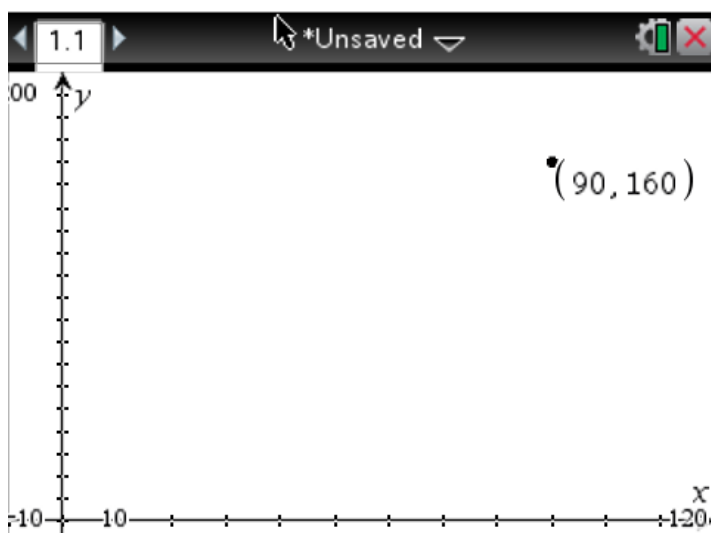
## 7.1 WHOA, THAT'S HOT!

### TEMPERATURE INCREASE WHILE PLAYING ON ARTIFICIAL TURF

A soccer player can always explain to you that playing on a turf field in the summer is way hotter than playing on a grass field. Turf can get so hot in the sun that you cannot walk on it barefoot.



The following is a graph that represents the increase of the temperature, in degrees Fahrenheit, that the soccer player can notice of the turf compared to the air temperature at that time.



What variable goes on the bottom? Justify.

The left side represents the independent variable. For this situation, this is the number of hours. The x-values represent the time that the lessons last for.

What variable goes on the side (up and down)? Justify.

This side represents the dependent variable, or the one that depends on the other. In this case, the amount that the pro player makes depends on the number of hours they work. So their payment depends on the number of hours.

Draw a possible graph for this situation knowing that the point (90, 160) is on the graph and that this is a non-proportional linear relationship. (You can draw it on the graph provided)

There will be a variety of answers for possible graphs to fulfil this situation. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the temperature of the turf increases as the air temperature increases. It is also important that the students do not let their graph cross the origin (0,0) because this is a non-proportional linear relationship.

From a graphical perspective what does it mean to be a non-proportional linear function? What does this look like on a graph?

A proportional situation goes through the point  $(0,0)$ . So a non-proportional situation will not go through that point, making the y-intercept some other value besides 0. On the graph you would see the line crossing the y-axis at some place other than the origin  $(0,0)$ .

How did you create your graph?

It will be important to discuss with the students that there is an infinite number of possible lines that could go through the one given point and be a non-proportional linear function. Therefore they can really create any line that has a constant rate of change, has a positive slope, does not go through the origin, and goes through the given point  $(90, 160)$ .

Is there a limit to this pattern? Justify.

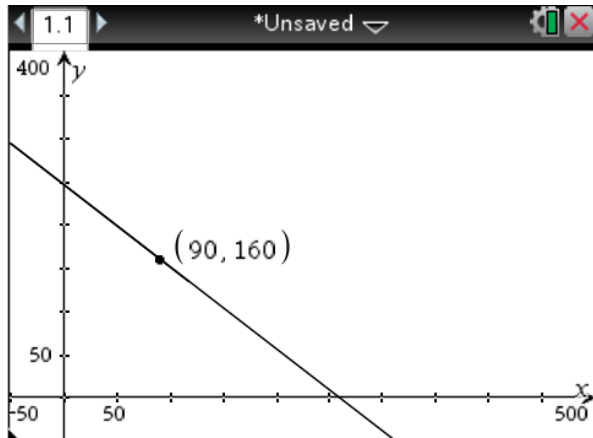
Yes. There will be a limit for the temperature of the turf. I do not think that the turf would get to a temperature of something crazy like 700 degrees. There will also probably be a low that we will never hit. For example, we probably will not see a temperature of the turf at -100 degrees. Thus there are some limitations as we get to extreme temperatures. The same patterns follow for the air temperature as well.

Given the graph you created for this situation, what would you say would be the corresponding y value to this x value if it was found on your graph?

$(65, ?)$

Again, there will be a variety of answers for possible points to fulfil this situation and fill the following ordered pair since there are an infinite amount of possible lines going through one point and that are non-proportional. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the temperature of the turf increases as the temperature of the air increases.

A student claimed that this was another graph that could represent the information given at the beginning on the lesson:



Does this make sense to be a possible graph for this situation? Why or why not?

No, this could not be a possible graph for this situation. Looking at this graph, it suggests that the temperature of the turf decreases as the air temperature increases. This is not the case, as the problem mentions that the temperature of the turf increases as the air temperature increases. So this graph would not make sense with this situation. The graph would need to have a positive slope and still go through the point (90, 160).

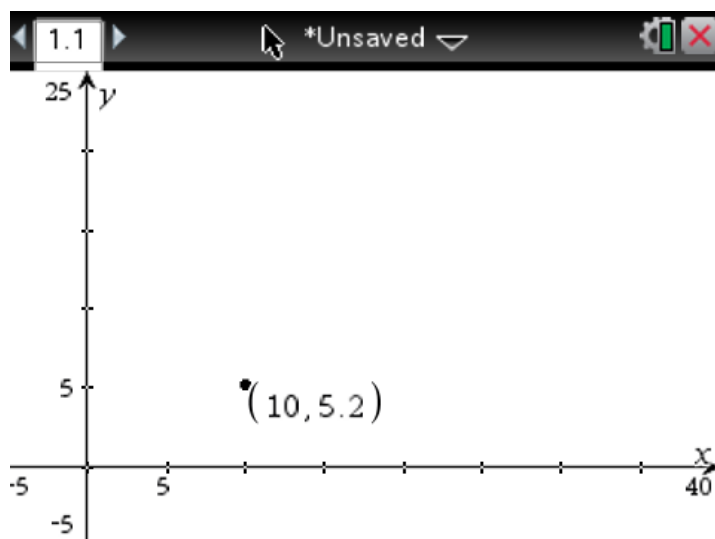
## 7.2 WHAT A PASS!

### HOW LONG WILL IT TAKE TO PASS THE BALL TO YOUR TEAMMATE?

It is important to make accurate passes to our teammates in a soccer game if you want your team to keep possession of the ball for the game. An accurate pass requires you to get the ball to your teammate.



The following is a graph that represents the distance the ball will travel in meters, based on the time it took to get there (in seconds), assuming the ball travels at a constant speed.



What variable goes on the horizontal axis? Justify.

The left side represents the independent variable. For this situation, this is the number of hours. The x-values represent the time that the lessons last for.

What variable goes on the vertical axis? Justify.

This side represents the dependent variable, or the one that depends on the other. In this case, the amount that the pro player makes depends on the number of hours they work. So their payment depends on the number of hours.

Draw a possible graph for this situation knowing that the point (10, 5.2) is on the graph. (You can draw it on the graph provided)

There will be a variety of answers for possible graphs to fulfil this situation. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the distance the ball travels increases when the time it is traveling increases. It is also important that the students make their graph cross the origin (0,0) because this is a proportional linear relationship.

**Is this situation proportional or non-proportional? Justify your answer.**

A proportional situation goes through the point  $(0,0)$ . When the ball has been hit and traveled for 0 seconds, it should have traveled 0 feet. So this is a proportional linear relationship. Since this is a proportional relationship this graph must go through the origin. On the graph you would see the line crossing the y-axis at the origin  $(0,0)$ .

**Does your graph extend forever in both directions? Justify your answer.**

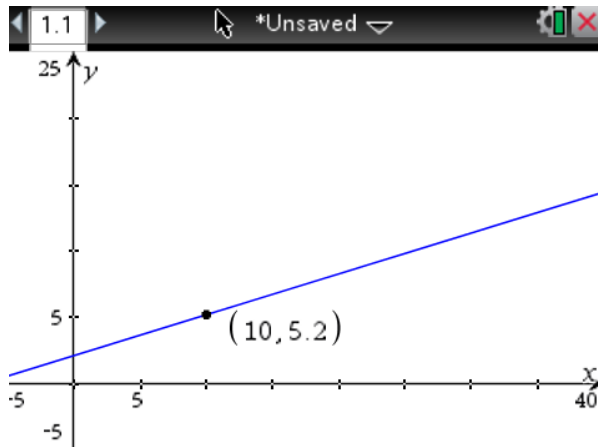
Assuming that the ball travels at a constant speed and that it could maintain that speed forever, we could say that there are no upper limits for this function. However, there is a limit for the lowest amount of time that the ball could travel. Would it make sense for a ball to travel from a negative number of seconds? No. So the lowest amount of time that the ball could travel for is 0 seconds.

**Given the graph you created for this situation, what would you say would be the corresponding y value to this x value if it was found on your graph?**

**$(15, ?)$**

Again, there will be a variety of answers for possible points to fulfil this situation and fill the following ordered pair since there are an infinite amount of possible lines going through one point and that are proportional. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the distance the ball travels increases as the time increases.

A student claimed that this was another graph that could represent the information given at the beginning on the lesson:



Does this make sense to be a possible graph for this situation? Why or why not?

No, this could not be a possible graph for this situation. Looking at this graph, it suggests that the temperature of the turf decreases as the air temperature increases. This is not the case, as the problem mentions that the distance of the ball increases as the time is traveling increases. The graph would need to have a positive slope and still go through the points (0,0) and (10, 5.2).

# LESSON 8 OVERVIEW

## FAMILY FUN TIME

### GOALS OF THE LESSON

This lesson is designed to provide students with opportunities to become familiar with linear functions that have both proportional and non-proportional relationships between the input and output. The goal is to use real world contexts that fit this to help students gather information on properties and characteristics of linear functions. In lesson 8, the focus will be on the representation of equations. Students will be given an equation to represent the sports context that they were given at the beginning on the lesson. The students will use this to create equivalent equations that will represent the linear situation that they were given. This lesson emphasizes the idea of moving within one representation.

By the end of the lesson, students should be able to explain how proportional and non-proportional linear relationships are shown in equations. They should also be able to identify multiple linear functions (in the form of equations) that could represent the given sports context.

All of these real world contexts given in the beginning, will not provide the students with any information on the slope or y-intercept of the function. It will just give them a story line to follow and help make sense of the equation. These contexts are used to help the students learn revolving around situations that are more relevant to their lives. These topics make the tasks and lesson more interesting and engaging for the students. The following are the contexts that are used to help students learn about the concepts of a linear function when only given an equation representation of the data.

#### **Lesson 8.1 – Fore!**

##### **Putt Putt Golfing**

#### **Lesson 8.2 – Strike!**

##### **The Number of Bowling Alleys in the U.S.**



## 8.1 FORE!

### PUTT PUTT GOLFING

Craig's Cruisers is a great place to go to have some fun! One of their main attractions is the Putt Putt Golf course that they have. Who doesn't love a good game of Putt Putt?



The following is an equation that represents the price for a game of mini golf per hole that is played.

$$P(x) = \frac{1}{2}x$$

What is represented by the independent variable? Which variable is it? Justify.

The independent variable is the variable that doesn't get changed based on the other one. In this case, the number of holes is the independent variable because that can be chosen randomly and was not determined by any other variables. In the equation, the number of holes played is represented by  $x$ .

What is represented by the dependent variable? Which variable is it? Justify.

The dependent variable is the variable that depends on the other variable. It will be determined based on what the other variable is. In this case, the price you will pay for Putt Putt is the dependent variable because it is determined based on the number of holes you wish to play. In the equation, the price you must pay is determined by  $P(x)$ .

Write one equation that is equivalent to this equation that could also represent this situation. There will be a variety of answers for possible equations to fulfil this situation. The biggest thing to look for is to make sure that their equation is also a proportional relationship. Since the given equation is in the form  $y=mx + o$ , the initial value (or price for a game without playing any holes), their equation must follow this pattern and have a  $y$ -intercept at  $o$ .

Is this situation proportional or non-proportional? Justify your answer.

A proportional situation goes through the point  $(0,0)$ . When you play 0 holes of putt putt golf, you pay \$0. So this is a proportional linear relationship. Since this is a proportional relationship the equation, in the form of  $y=mx+b$ , must have a value of 0 for  $b$ .

Determine if the graph of this situation is a line? Justify your answer.

Yes. This situation is represented by a line. We know this because all lines are linear functions. This situation is represented by a linear function because it has a constant rate of change (slope). This is shown in the initial formula of  $P(x) = \frac{1}{2} X$ . This is in slope- intercept form of an equation for a linear function, where the slope is  $\frac{1}{2}$ . Thus the graph that represents this situation is a line.

Does your graph extend forever in both directions? Justify your answer.

No. There is a limit to the variable that represents the number of holes you can buy to play. It would make no sense to pay for to play a negative number of holes on a putt putt gold course. There would also be a limit for how many holes you could pay for. If there are only 18 hours per golf course, would you really pay to play 1000 holes? And would you have the time to do this?

Given the equation you created for this situation, what would you say would be the corresponding  $y$  value to this  $x$  value?

$(9, ?)$

Again, there will be a variety of answers for possible points to fulfil this situation and fill the following ordered pair since there are an infinite amount of possible lines going through one point and that are proportional. The biggest thing to look for is to make sure that their points follow a linear pattern (constant rate of change) and that it follows a positive slope pattern since the price to pay increases as the number of holes increases.

Given your equation, what will be the price to pay for a full game of Putt Putt Golf, assuming that there are 18 holes on the course?

There will be a variety of equations used that the students came up with the represent this function that are all equivalent to the original given equation. Therefore as long as their equations are actually equivalent to the original equation, they will represent the exact same line and should provide the same price to pay for 18 holes as the initial given equation. Thus there answer should be \$9 for a full game of Putt Putt Golf.

Does this match the amount you must pay to play a full 18 holes of Putt Putt Golf from the initial given equation? Justify your answer.

Again, there will be a variety of equations used that the students came up with to represent this function that are all equivalent to the original given equation. Therefore as long as their equations are actually equivalent to the original equation, they will represent the exact same line and should provide the same price to pay for 18 holes as the initial given equation. Thus the answer should be \$9 for a full game of Putt Putt Golf.

A student claimed that this was another equation that could represent the information given at the beginning of the lesson:

$$Y = \frac{4}{2}x + 7$$

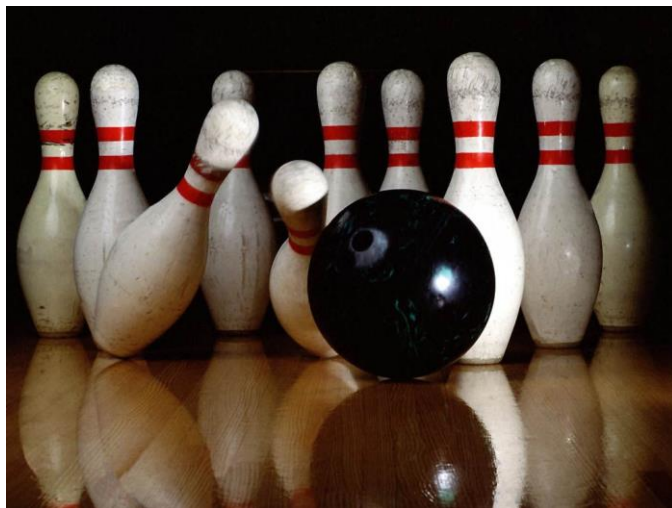
Does this make sense to be a possible equation for this situation? Why or why not?

No, this could not be a possible equation for this situation. Looking at this equation, it suggests that you must pay \$7 to play 0 holes of golf, which we know is not true, since this is a proportional relationship. This equation also claims that you must pay  $\frac{4}{2}$  or \$2 for every hole that you wish to play. This is not equal to the price that the original equation gave us (\$1.5 per hole).

## 8.2 STRIKE!

### THE NUMBER OF BOWLING ALLEYS IN THE US

Bowling has been a very popular sport throughout the world for many centuries. The sport was then brought to America when settlers began coming to the New World. But is it still as big of a sport now as back in the day?



The following is an equation that represents the number of bowling alleys in the United States since the year 1997.

$$B(x) = -215x + 7611$$

What is represented by the independent variable? Which variable is it? Justify.

The independent variable is the variable that doesn't get changed based on the other one. In this case, the years since 1997 is the independent variable since nothing can affect the year that it is. The years just keep on coming and moving along no matter what. The independent variable is represented by  $x$ .

What is represented by the dependent variable? Which variable is it? Justify.

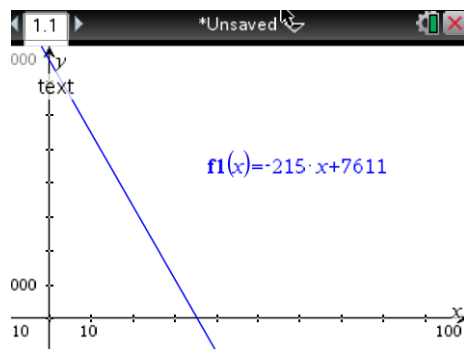
The dependent variable is the variable that depends on the other variable. It will be determined based on what the other variable is. In this case, the number of bowling alleys in the U.S. depend on the year it is. This is represented by  $B(x)$ .

Write one equation that is equivalent to this equation that could also represent this situation. There will be a variety of answers for possible equations to fulfil this situation. The biggest thing to look for is to make sure that their equation is also a nonproportional relationship. Since the given equation is in the form  $y = mx + b$ , the initial value (or number of bowling alleys in the year 1997), their equation must follow this pattern and have a  $y$ -intercept at 7611.

Is this situation proportional or non-proportional? Justify your answer.

A proportional situation goes through the point (0,0). So a nonproportional linear function does not go through the point (0,0). At 0 years after 1997, or at 1997, there was more than 0 bowling alleys in the US. So this is a nonproportional linear relationship. Since this is a nonproportional relationship the equation, in the form of  $y=mx+b$ , must have a value of 7611 for b.

Graph the equation. Justify the shape of your graph.



This graph represents a line. This is due to the fact that this relationship has a constant rate of change (slope).

Given the equation you created for this situation, what would you say would be the corresponding y value to this x value?

( 27, ? )

Again, there will be a variety of answers for possible equations to fulfil this situation. However if all of these equations must be equivalent. So they should all come up with the same answer to this question. So at 27 years after 1997, there will be 1,806 bowling alleys in the US.

Given your equation, at what year is it predicted to have no more bowling alleys in the U.S.?

There will be a variety of equations used that the students came up with the represent this function that are all equivalent to the original given equation. Therefore as long as their equations are actually equivalent to the original equation, they will represent the exact same line and should provide the same answer to this questions as if they used the original equation. So they should get the answer of 35.4 years after 1997. So it will be the year, 2032.4. So part way through the year 2032 there supposedly will be no more bowling alleys in the U.S.

Does this match the year that there will be no more bowling alleys in the U.S. from using the initial given equation? Justify your answer.

Again, there will be a variety of equations used that the students came up with the represent this function that are all equivalent to the original given equation. Therefore as long as their equations are actually equivalent to the original equation, they will represent the exact same line and should provide the same information about at what year after 1997, should there be no more bowling alleys in the US.

A student claimed that this was another equation that could represent the information given at the beginning on the lesson:

$$Y = \frac{-645}{3}x + 7611$$

Does this make sense to be a possible equation for this situation? Why or why not?

Yes, this equation is equivalent to the original given equation. The slopes are equivalent since  $= \frac{-645}{3} = -215$  and they y-intercept is the same. Thus this is an equivalent equation to the initial given equation and can also represent the situation.

# ROLL IT ALL TOGETHER

## FINAL PROJECT

Congratulations!! You have been elected as the leader of the celebration committee for your class! This is a very exciting job, and you just got it at the perfect timing! Your teachers want to have a class skating party to celebrate the end of the year. They need your help to plan out this party. This will be your first job in your new position on the committee for the class.

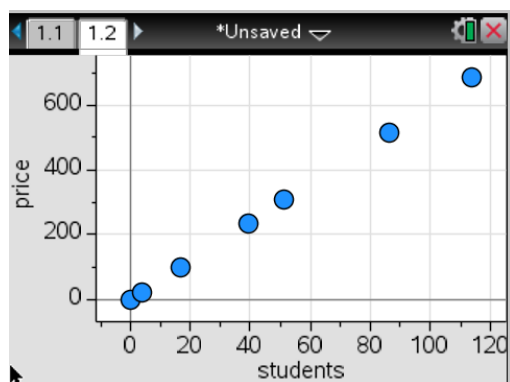


**Your job:** You are in charge of finding a place to rent in-line roller-skates for your class at a reasonable price.

### TASK 1

Use the data given to determine information about the different companies' rental rates. What do you notice about these companies?

#### Roll-Away Skates



#### Gary's Gliding Skates

# of students who attend	Total Price to Pay
0	50
5	75
19	145
25	175
55	325
56	330
120	650

#### Wheelie's Skates and Stuff

Charges \$100 plus \$4 per student

Students will notice that all three companies' information is given through a different representation. This is very important because they will need to use all 4 representations (graph, table, written, equation) for this lesson.

## TASK 2 – COMPARING AND CONTRASTING EQUATIONS

1. For each company, write an equation for the relationship between the numbers of people renting skates and the cost.

Roll Away Skates:

$$Y = 6X$$

Gary's Gliding Skates:

$$Y = 5X + 50$$

Wheelie's Skates and Stuff:

$$Y = 4X + 100$$

How do they differ? Is anything similar between the 3 equations?

The equation for Roll Away Skates does not have an addition number (initial value) added to the equation. This company's linear graph will be a proportional linear function.

Gary's Gliding Skates and Wheelie's Skates and Stuff both are non-proportional linear functions because they also have an additional price to pay when no students show up.

It looks like Roll Away Skates has the highest slope (price per student).  
Wheelie's Skates and stuff has the lowest rate (slope).

When looking at your equations what do the coefficients in front of the x represent? What do these coefficients mean in terms of cost to rent skates?

The number that is presented before the X is the slope of the function. In this scenario, it represents the rate to pay per student. It looks like Roll Away Skates has the highest slope (price per student) at \$6, while Gary's Gliding Skates has a lower rate of \$5, and Wheelie's Skates and stuff has the lowest rate of \$4.

Are these linear functions proportional or non-proportional and why?

Each table has a different initial value (y-intercept). Roll Away Skates is a proportional relationship because when 0 students go, the price to pay for the rink is \$0. The other two functions are both non-proportional because when no students show up to the rink there is still a renting fee for the rink. Gary's Gliding Skates require an initial fee of \$50 and Wheelie's Skates and Stuff require an initial fee of \$100.



## TASK 3: COMPARING AND CONTRASTING TABLES

1. Draw the different tables for the different companies.

Roll Away Skates:


Gary's Gilding Skates:


Wheelie's Skates and Stuff:


What do you notice about all three of these tables? Are there similarities or differences? If so explain.

The three tables all look different. Each table has a different initial value (y-intercept).

Note\* Students may use the same x-values for all three tables, while others do not. Some might also increase x-values based on a constant change while others do not. These might be things that could be discussed as a class as to which way will be more efficient for this project.

If students has negative values for the x-values, it will be very important to discuss why this is not possible for these situations.

Students will probably notice that the y-intercepts are different and that slopes are different from the three tables they create.

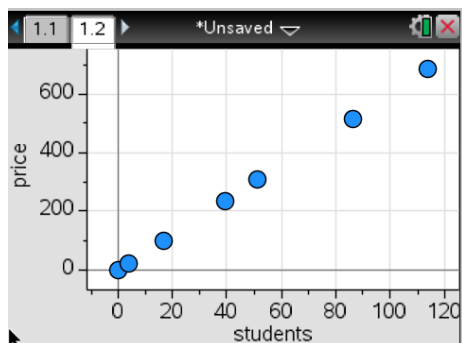
What are the y-intercepts are for each of the equations. What does this mean in terms of the cost to rent skates?

(0, 50) even if no students show up to the party there is still a fee of \$50  
 (0, 0) if not students come to the skating party, then there is no fee  
 (0, 100) if no students show up then there still is a \$100 fee

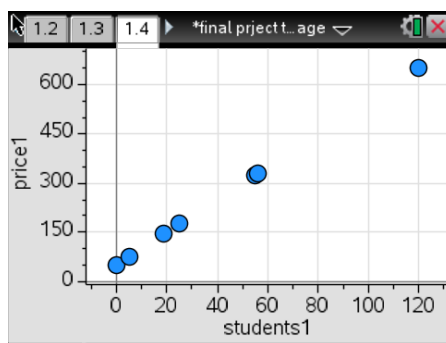
## TASK 4: COMPARING AND CONTRASTING GRAPHS

1. Graph the equations for the three companies.

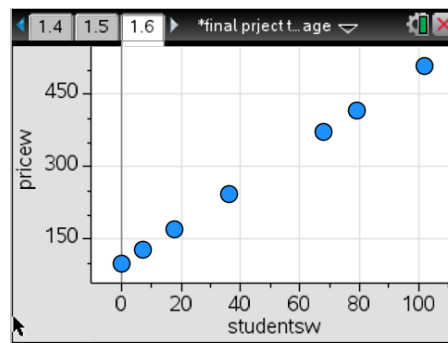
Roll Away Skates:



Gary's Gliding Skates:



Wheelie's Skates and Stuff:



What do you notice about the different graphs? How are they similar? How are they different? Support your claims.

All three of the graphs have positive slopes. So for all three companies the price you must pay increases as more students come to the party to skate. Roll Away Skates has a proportional relationship because it crosses the Y-axis when  $x=0$ . (it passes through the origin). The other two are non-proportional because their y-intercepts are not at the origin. All three graphs have constant slopes so they are all linear functions.

How would you compare the slopes of these three lines?

All three of the graphs have positive slopes. So for all three companies the price you must pay increases as more students come to the party to skate. From the graphs the slopes look very similar, however the slope of Roll Away Skates is steeper than the others because its slope is 6, while the other two are 5 and 4. Therefore the price per student is more when renting from Roll Away Skates.

What do these slopes represent in terms of the different companies?

The slopes of the lines represent the price to pay per student that comes to the skating party. Roll Away Skates has a rate of \$6 per student who comes to the party. Gary's Gliding Skates charges a price of \$5 per student who shows up. Wheelie's Skates and Stuff charges \$4 per student for entry.

## TASK 5: DECISION TIME

Which company would you choose to rent from if 100 students are planning to attend the party? Why?

Roll Away Skates	$Y = 6X$	$\rightarrow$	$6(100)$	$\rightarrow$	\$600
Gary's Gliding Skates	$Y = 5X + 50$	$\rightarrow$	$5(100) + 50$	$\rightarrow$	\$550
Wheelie's Skates and Stuff	$Y = 4X + 100$	$\rightarrow$	$4(100) + 100$	$\rightarrow$	\$500

It would be smartest to rent from Wheelie's Skates if 100 students come to the skating party, since it would only cost \$500 to rent the rink from this company.

Which company would you choose to rent from if 50 students are planning to attend the party? Why?

Roll Away Skates	$Y = 6X$	$\rightarrow$	$6(50)$	$\rightarrow$	\$300
Gary's Gliding Skates	$Y = 5X + 50$	$\rightarrow$	$5(50) + 50$	$\rightarrow$	\$300
Wheelie's Skates and Stuff	$Y = 4X + 100$	$\rightarrow$	$4(50) + 100$	$\rightarrow$	\$300

It would not matter which company you rented from since they all yield the same prices.

Which company would you choose to rent from if only 25 students are planning to attend the party? Why?

Roll Away Skates	$Y = 6X$	$\rightarrow$	$6(25)$	$\rightarrow$	\$150
Gary's Gliding Skates	$Y = 5X + 50$	$\rightarrow$	$5(25) + 50$	$\rightarrow$	\$175
Wheelie's Skates and Stuff	$Y = 4X + 100$	$\rightarrow$	$4(25) + 100$	$\rightarrow$	\$200

If 25 students come to the skating party, then the best company to rent from would be Gary's Gliding Skates because they have the lowest price for 25 students.

If your budget for skate rental is \$250, how many pairs of skates can you rent from each company? How did you determine this?

Roll Away Skates	$250 = 6X \rightarrow$	$41.6666 = X \rightarrow$	$41 = X \rightarrow$	41 pairs of skates
Gary's Gliding Skates	$250 = 5X + 50 \rightarrow$	$200 = 5X \rightarrow$	$40 = X \rightarrow$	40 pairs of skates
Wheelie's Skates and Stuff	$250 = 4X + 100 \rightarrow$	$150 = 4X \rightarrow$	$37.5 = X \rightarrow$	37 pairs of skates

### TASK 6: FINAL PRESENTATION

Last year 58 students came to the skating party, however your teacher's goal is to get more students to go. They are expecting that there could be a max of anywhere from 80- 100 students at the party.

You need to decide which company you should choose to rent from in order to keep the cost at a minimum!

We don't know how many students will show up this year. But trusting your judgment, what is your final decision on which company to rent from for your class party? Why did you choose this company?

**Create a poster to present to your teacher your final decision!**

**Please have the graph, table, equation, and written rule for your chosen company on your poster to back up your decision.**

Answers may vary since the number of students attending is not clear, but it is important that they students use the information from the lesson in order to back up their decision.