Study of the Structure of $^9$C via Single Neutron Transfer

Scott T. Marley

Western Michigan University, scott.thomas.marley@gmail.com

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STUDY OF THE STRUCTURE OF $^9$C VIA SINGLE NEUTRON TRANSFER

by

Scott T. Marley

A Dissertation
Submitted to the
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requirements for the
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Department of Physics
Advisor: Alan H. Wuosmaa, Ph.D.

Western Michigan University
Kalamazoo, Michigan
June 2012
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WE HEREBY APPROVE THE DISSERTATION SUBMITTED BY

Scott T. Marley

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Physics (Department)

Alan Wuosmaa, Ph.D.
Dissertation Review Committee Chair

Nuclear Physics (Program)

Dean Halderson, Ph.D.
Dissertation Review Committee Member

Michael Farniano, Ph.D.
Dissertation Review Committee Member

K. Ernst Rehm, Dr. habil
Dissertation Review Committee Member

APPROVED

Dean of the Graduate College

Date June 29, 2012
STUDY OF THE STRUCTURE OF $^9$C VIA SINGLE NEUTRON TRANSFER

Scott T. Marley, Ph.D.
Western Michigan University, 2012

This thesis describes a study of the nucleus $^9$C, produced in the single-neutron transfer reaction $^d(^{10}$C,$^t)^9$C using a radioactive $^{10}$C beam. The structure of the neutron-deficient nucleus $^9$C is poorly known. Only a few excited states have been observed and no information exists of their single-particle characteristics. The measured ground-state magnetic dipole moment of $^9$C is anomalously small and could imply large contributions from sd-shell orbitals in the ground-state wave function. The positions of the $^9$C excited states and their single-particle properties are vital to furthering the accuracy of ab initio nuclear models which have excelled in modeling light nuclear systems in the p-shell. To probe the structure of $^9$C the neutron-removing reaction $^{10}$C($^d,t)^9$C, in inverse kinematics, was performed at the ATLAS facility at Argonne National Laboratory.

An “in-flight” radioactive $^{10}$C beam was developed at ATLAS through the $p(^{10}$B,$^{10}$C)$n$ reaction using a 185-MeV $^{10}$B beam incident on a cryogenic hydrogen (H$_2$) gas cell. The resulting 171-MeV $^{10}$C beam had an average intensity of $2.2 \times 10^4$ ions per second and was placed on a $660 \mu$g/cm$^2$ deuterated polyethylene ([CD$_2$]$_n$) target. Tritons were detected and identified in an array of annular double-sided silicon detectors (DSSDs) covering laboratory angles between 7° and 42°. Heavy recoils from particle-bound and unbound states were detected in a set of forward-angle silicon detectors in a $\Delta$E-E configuration.

The $^9$C ground state transition was unambiguously identified in the detector system and angular-distribution data were extracted. The neutron-pickup spectroscopic factor was deduced from a comparison of distorted-wave Born approximation (DWBA) calculations using both traditional bound-state form factors and those derived from ab initio calculations. The comparison between the two results assesses the reliability of applying ab initio calculations to transfer reaction theory and provides insight into the low-lying structure of $^9$C.
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Scott T. Marley
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Chapter 1

Introduction and Motivation

1.1 Introduction

The study of the structure of atomic nuclei is essential for understanding the strong nuclear interaction. The strong force between protons and neutrons allows for certain configurations to be stable forming the basis for the existence of all elemental matter. The strong interaction between nucleons has a range of 1–2 femtometers ($10^{-15}$ m) and its effect within nuclei must be studied using indirect means, in particular by determining the properties of nuclei made up of differing numbers of protons and neutrons. In the century since the discovery of the atomic nucleus, from the pioneering research of Rutherford, Geiger, and Marsden [1] [2], vast amounts of information about atomic nuclei have been acquired by observing both radioactivity and a wide variety of nuclear scattering experiments. The search for patterns and correlations in these data led to theories of nuclear structure. For instance, the observation that certain nuclei, comprised of “magic numbers” (2, 8, 20, 28, 50, 82, and 126) of protons and/or neutrons, were especially well bound compared to nuclei with only an additional nucleon led to the formulation of nuclear shell theory [3]–[6] (Figure 1.1).

The nuclear shell model set the theoretical framework for understanding the structure of nuclei. A nucleus can be modeled by placing the Z protons and N neutrons into orbital shells, similar to electrons in an atom, with in many cases the valence nucleons defining the properties of the nucleus. This approach to nuclear structure explained many of the spins and parities of ground and low-lying excited states of stable nuclei. While the shell model provided a general understanding of many nuclear properties, it did not reproduce experimental values for many nuclear observables (e.g., electromagnetic moments).
These deviations from the simple shell model implied that the properties of nuclear states were mitigated by more complex interactions within the nuclei. The nuclear models that evolved to better explain the observed data, including incarnations of the shell model, have incorporated a more detailed understanding of the nucleon-nucleon interaction and can reproduce the basic properties of many nuclei near stability.

Figure 1.1: Chart of all known nuclear isotopes. The structures of the 254 stable isotopes (black) and adjacent nuclei have been studied extensively. The “magic numbers” of nucleons for nuclei near stability are labeled. Many unstable isotopes (colored) have yet to be produced in sufficient quantity to study their properties. Figure generated from Ref. [7].

In the last two decades, several first-principle approaches to modeling nuclear structure have been developed. These models utilize realistic nucleon-nucleon potentials based on extensive nucleon-nucleon scattering data and, through various computational methods, calculate observables of the nuclear many-body system. These so-called “ab initio” models, have had successes in calculating energy levels, electromagnetic moments, and spectroscopic overlaps in both stable and near-stable nuclei (Figure 1.2) [8]–[12]. However, these calculations are computationally demanding and scale with the number of nucleons in the selected system, thus limiting their application to lighter nuclei (A ≤ 12). The structure of unstable, or short-lived, nuclear isotopes far from stability is — in general — not well reproduced by modern nuclear models and the focus of this research.

In unstable nuclei, the nucleons are less bound by the strong interaction and result in the nucleus decaying to configurations of more stable nuclei through beta- or particle-decay. Beta-unstable nuclei
are still bound by the strong nuclear force, however, the weak nuclear interaction can transform a proton to a neutron ($\beta^+$-decay) or a neutron to a proton ($\beta^-$-decay) depending on which is energetically permitted. Particle-unstable nuclei are not sufficiently bound by the strong interaction and decay through the emission of a proton, neutron, or an alpha particle. Unstable nuclei can have half-lives ranging from nanoseconds to many billions of years. Many of these isotopes, while rare or non-existent terrestrially, can be produced in stellar environments and may play an important role in nucleosynthesis of the elements [13]. Most unstable nuclei have excesses of either protons ($N/Z<1$) or neutrons ($N/Z>1$). Nuclei with large nucleon excesses have very different structures than nuclei closer to stability and are of interest to improve current nuclear models. A theoretical benchmark is to accurately model the nuclei furthest from stability which lie on the so-called nucleon “drip lines” (Figure 1.3). The short-lived nature of these nuclei makes their study extremely difficult or impossible using the experimental techniques used formerly to study the stable nuclear isotopes. In traditional transfer reaction experiments, targets made from stable isotopes were bombarded with beams of light nuclei (e.g., protons, deuterons, etc.) which add or remove nucleons producing the isotope of interest. If the measurement requires that the target be composed of unstable, radioactive nuclei, its half-life must be very long. Otherwise, a beam of radioactive ions, one or two nucleons different from the isotope of interest, must bombard a target comprised of light nuclei (e.g., $p$, $d$, $^3$He). Over the past fifteen years, nuclear research facilities have developed beams of radioactive ions allowing for programs of experiments to be performed to study rare nuclei. Recent advancements of
these methods and techniques have made possible the study of nuclei on or near the drip lines, including the most neutron-deficient particle-stable isotope of carbon: $^9$C.

Figure 1.3: The proton (red) and neutron (blue) drip-lines on the chart of the nuclides. Plot generated from Ref. [7].

1.2 Motivation for the Study of $^9$C

Other than $^3$He, which is stable, $^9$C, with three fewer neutrons than its most abundant stable isotope $^{12}$C, is the most neutron-deficient nucleus known to be particle-bound ($N/Z = \frac{1}{2}$). The $^9$C ground state is stable with respect to proton emission and undergoes beta decay to states in $^9$B with a half-life of $126.5 \pm 0.9$ milliseconds [14]–[16]. Very little is known about the excited states of $^9$C with only a single state having been observed. The magnetic dipole moment of the ground state has been studied and confirmed to be anomalously small compared to theoretical calculations [17] [18]. Studying $^9$C using a single-nucleon transfer reaction would provide spin-parity assignments and spectroscopic information on the populated states which would be important benchmarks for modern nuclear theories.
1.2.1 Previous Studies

Multi-Nucleon Transfer Reactions

The study of $^9$C is extremely difficult as it lies adjacent to the proton drip line. Before the availability of radioactive beams, $^9$C was studied exclusively through complex multi-nucleon transfer reactions. The discovery of $^9$C and several subsequent studies utilized the three-nucleon removing reaction $^{12}$C($^3$He,$^6$He)$^9$C [19]–[22]. Other studies, focused on the beta-decay of $^9$C, used the two-proton transfer reaction $^7$Be($^3$He,$n$) [15] and a variety of neutron knockout reactions (e.g., $^{10}$B(p,2n) [16], $^{12}$C(p,d2n) [14], etc.) to produce $^9$C in the laboratory. The ground state of $^9$C was assigned spin-parity ($J^\pi$) of $\frac{3}{2}^-$ from its beta-decay to states of known spin and parity in $^9$B. Excited states of $^9$C have been observed in the three-neutron transfer studies of Benenson and Kashy [21] and Golokov, et al. [22]. In the former study, the first-excited state of $^9$C was identified at an excitation energy of 2.2±0.1 MeV (Figure 1.4). The latter study also populated the first-excited state and possible additional broad states at 3.3 and 4.3 MeV, however, did not provide widths for these states. Neither study made assignments of spin or parity for any state populated due to the complex nature of the reaction used.

![Figure 1.4: Energy spectra of $^6$He ions for the ground and first-excited states of $^9$C populated in the $^{12}$C($^3$He,$^6$He)$^9$C reaction. Figure from Ref. [21, Fig. 1].](image)
Radioactive Beam Studies

Figure 1.5: Excitation energy spectrum and angular distribution for the \(d(^{8}\text{B},n)^{9}\text{C}\) reaction. Due to the design of the experiment, only the \(^9\text{C}\) ground-state transition was observed. Figures from Ref. [23, Figs. 1 & 2].

The structure of \(^9\text{C}\) has been probed directly through experiments using radioactive beams of \(^8\text{B}\) and \(^{10}\text{C}\). The first single-particle transfer study to populate \(^9\text{C}\) was through the \(d(^{8}\text{B},n)^{9}\text{C}\) reaction [23]. The experiment was sensitive to only \(^9\text{C}\)-recoils in coincidence with neutrons which limited this study to observing the ground-state transition (Figure 1.5). The asymptotic normalization coefficient (ANC) was determined for the reaction and used to calculate the cross section for the \(^8\text{B}(p,\gamma)^{9}\text{C}\) reaction at astrophysical energies. The second experiment to utilize a \(^8\text{B}\) beam performed resonance elastic scattering to probe the proton-unbound excited states of \(^9\text{C}\). The \(R\)-matrix analysis of these data supported a first-excited state with spin-parity of \(J^\pi = \frac{1}{2}^-\) as well as evidence for a broad \(J^\pi = \frac{5}{2}^-\) state at approximately 3.6 MeV in excitation energy [24] (Figure 1.6). The most recent radioactive beam study of \(^9\text{C}\) was through the one-nucleon knockout reaction study performed with by Grinyer et al. [25]. The purpose of the study was to measure the total reaction cross sections for both one-neutron and one-proton knockout reactions to the ground states of \(^9\text{C}\) and \(^9\text{Be}\), with respective \(^{10}\text{C}\) and \(^{10}\text{Be}\) beams, and compare these results to modern \textit{ab initio} theories. The measured reaction cross sections were smaller than those from Variational Monte Carlo and No Core Shell Model (NCSM) calculations by 30–50% [25].

Asymmetries in the \(A=9\) Isobars

The magnetic dipole moment of \(^9\text{C}\) has been measured to be \(-1.3914\pm0.0005\) \(\mu_N\) using the \(\beta\)-NMR technique in two independent studies [17] [18]. The measured magnetic dipole moment is anomalously
small compared to the extreme single particle value ($\mu_{SP}=-1.91\,\mu_N$) and larger than calculations from shell and \textit{ab initio} models. The ground-state magnetic dipole moment of the $^9\text{C}$ mirror nucleus, $^9\text{Li}$, is also not well reproduced. The structure of $^9\text{Li}$ is in better agreement with nuclear theories with its ground and first few excited states having spin-parity assignments explainable in a shell-model framework [26] [27]. The anomalous ground-state magnetic dipole moment of $^9\text{C}$ could suggest contributions from $sd$-shell states in its ground-state wave function and result in the asymmetry in the $^9\text{C}$–$^9\text{Li}$ mirror system [28].

Another mirror asymmetry in the $^{A=9}$ isobars has been identified in the ground state beta-decays of $^9\text{C}$ and $^9\text{Li}$ to high-lying states in $^9\text{B}$ and $^9\text{Be}$, respectively. The strength of the $^9\text{Li}$ beta-decay to the 11.8 MeV state in $^9\text{Be}$ is over four orders of magnitude larger than its mirror decay in $^9\text{C}$ to the 12.2 MeV state in $^9\text{B}$ [29] [30]. The source of this asymmetry has not been discovered and must be present in either the $^9\text{C}$–$^9\text{Li}$ ($J^\pi=\frac{3}{2}^-$, $T=\frac{3}{2}$) or $^9\text{Be}$–$^9\text{B}$ ($J^\pi=\frac{5}{2}^-$, $T=\frac{1}{2}$) mirror levels [30] [31].

The isobaric mass multiplet equation (IMME) was developed as a phenomenological means of modeling the masses of nuclear isobars [32]. The masses are determined by a quadratic function of the isospin projection ($T_z$) of each nucleus:

$$\Delta M = a + b T_z + c T_z^2$$

(1.1)

and has been used successfully to model masses of stable and unstable nuclear isobars. The IMME fit to the $^{A=9}$ ($T=\frac{3}{2}$) isobars requires a small cubic term to properly fit the mass for the $^9\text{C}$ ground state [33] [34]. Similar violation of IMME has also been seen in the $^{A=7}$ ($T=\frac{3}{2}$), $^{A=8}$ ($T=2$) and $^{A=32}$ ($T=2$)
multiplets with little theoretical explanation for the deviations from the model [35] [36].

1.3 Scope of This Study

It is the aim of this research to provide some concrete information on the properties of the low-lying states of $^9$C and clarify the anomalous structure known to be present in its ground state. The properties of this most neutron-deficient nucleus are vital to stringently test modern shell and \textit{ab-initio} nuclear models which represent the most current understanding of the nuclear interaction. The population of states in $^9$C using a neutron-removing transfer reaction and the subsequent extraction of single-neutron spectroscopic information, relative to $^{10}$C, would provide data to test theoretical models. This impetus led to the development of a radioactive $^{10}$C beam at the ATLAS Facility at Argonne National Laboratory for the purpose of studying $^9$C using the $d(^{10}C,t)^9C$ reaction.

The theoretical framework of the study, with treatments of nuclear structure and transfer reactions are presented in Chapters 2 and 3, respectively. The details of the experiments including the specifics of radioactive beam development and the detector system comprise Chapter 4. The data analysis and reduction for the $d(^{10}C,t)^9$C experiments are covered in Chapters 6 and 7. The reduced data set is analyzed with respect to transfer reaction theory in Chapter 8 followed by a discussion of the results presented in Chapter 9. The final chapter summarizes the findings of the study.
Chapter 2

Nuclear Structure

The first purpose of this chapter is to outline the understanding of nuclear structure provided by the shell model and more modern nuclear theories. The emphasis is placed on the simple shell model and the many observed properties of low-lying nuclear states that are reproduced using the independent-particle approach without the use of residual interactions. The general aspects of several \textit{ab initio} methods for modeling the structure of nuclei are included as, at the time of this writing, they represent the most refined approaches of modeling atomic nuclei with twelve or fewer nucleons.

The second purpose of the chapter is to detail how these methodologies, both simple and refined, are able to reproduce the properties of $^9\text{C}$, the subject of this study, and its mirror nucleus $^9\text{Li}$. With only nine nucleons, some basic properties of $^9\text{C}$ should be well represented by the shell model. Also, its light mass also places $^9\text{C}$ in range of several \textit{ab initio} calculations. Several theories have attempted to reconcile the anomalous magnetic moment of the $^9\text{C}$ with varying success. These findings stand untested due to the paucity of data of the single-particle nature of $^9\text{C}$.

2.1 Nuclear Shell Model

2.1.1 Formalism

In a nucleus with $A$ nucleons, the nuclear Hamiltonian is defined as:

$$H = T + V = \sum_{i}^{A} T_i + \sum_{i>j=1}^{A} V_{ij}(r_i, r_j)$$

(2.1)
where \( T_i \) is the kinetic energy operator of a single nucleon and \( V_{ij} \) the two-body nuclear potential, which is summed over all combinations of nucleon pairs. The Schrödinger equation for the nucleus is,

\[
\sum_i^A \frac{\hbar^2}{2m_i} \nabla_i^2 \Psi_A + \sum_{i>j=1}^A V_{ij}(\mathbf{r}_i, \mathbf{r}_j) \Psi_A = E \Psi_A
\]  

(2.2)

where \( \Psi_A \) is the many-body nuclear wave function for the nucleus and \( E \) the energy eigenvalue. The shell model takes into account the motion of the individual nucleons moving in an average potential, \( V_i(\mathbf{r}_i) \), which is defined as the effective one-body component of the two-body operator:

\[
H = H_0 + H_{eff} = \sum_{i=1}^A \left[T_i + V_i(\mathbf{r}_i)\right] + \left[\sum_{i>j=1}^A V_{ij}(\mathbf{r}_i, \mathbf{r}_j) - V_i(\mathbf{r}_i)\right]
\]  

(2.3)

where \( H_0 \), the first term in brackets, is the single-particle Hamiltonian and \( H_{eff} \) is the “residual” interaction that takes into account the two-body component of the nuclear interaction. In the extreme independent-particle approach, \( H_{eff} \) is very small compared to the single-particle operator and can be neglected to first order. However, effective interactions within a nucleus are required to accurately model nuclei and are taken into account in more modern nuclear models (e.g., Large Basis Shell Models, Mean-Field Theories, etc.). Despite its extreme simplicity, the independent particle approximation, which relies on the one-body operator, can explain the observed “magic numbers” and the properties of low-lying states of stable nuclei.

### 2.1.2 The Nuclear Potential

An individual nucleon within the nucleus can be treated as a particle moving in the average central potential \( V_i(\mathbf{r}_i) \) with an orbital angular momentum \( \hbar l_i \), spin angular momentum \( s_i = \pm \frac{\hbar}{2} \), and total angular momentum \( j_i \):

\[
j_i = l_i + s_i
\]

(2.4)

As nucleons and electrons are both spin-\( \frac{1}{2} \) particles (i.e., fermions) an analogy can be made between the motion of electrons in an atom bound by the Coulomb potential, and nucleons confined within a nucleus [6]. The nucleons moving in the potential \( V_i(\mathbf{r}_i) \) can occupy oscillator “shell” states each representing
discrete energies which are solutions to Equation 2.1.1. Nucleons fill each shell in accordance to the Pauli exclusion principle, which forbids fermions from occupying states with identical quantum numbers simultaneously. Here, pairs of nucleons with anti-aligned angular momenta couple to \( J=0 \) \([37], [38]\). The form of the nuclear potential determines the energies of these “single-particle” states as well as the degeneracy of each oscillator shell. The addition of a spin-orbit term, \((l \cdot s)\), to the one-body potential was found to explain all of the experimentally observed magic numbers \([4], [5]\).

The shape of the nuclear potential must be modeled based on the observed properties of the nucleus with an attractive uniform potential within the nuclear interior which falls to zero rapidly at the nuclear surface \([6]\). Harmonic oscillator or square-well potentials can be used as simple approximations of the nuclear potential \((2.1)\). A potential with a more realistic shape at the nuclear surface is the Woods-Saxon potential \([39]\):

\[
V_{WS} = \frac{V_0}{1 + e^{-\frac{r}{a}}}
\]

where \(V_0\) is the well depth, \(R\) the nuclear radius, and \(a\) the diffuseness of the nuclear surface (Figure 2.1). These potentials are able to reproduce the first few magic numbers, however, fail to reproduce those observed in heavier nuclei without the spin-orbit term (Figures 2.1 & 2.2). The dependence of the spin-orbit force on the orbital angular momentum \((l)\) of the nucleon leads to a energy splitting such that states with \(j = l + \frac{1}{2}\) are lower in energy than those with \(j = l - \frac{1}{2}\) and successfully reproduces the magic numbers of 28, 50, 82, and 126 (Figure 2.2). These numbers are “magic” for near stability and do not necessarily apply to unstable nuclei. It has been observed that nuclei with increasing neutron-to-proton
ratios lose the “stable” magic numbers (i.e., shell gaps) which can lead to new magic numbers [40]. In general, for stable and unstable nuclei, the valence nucleons which occupy the unfilled subshells, are responsible for the basic properties of the low-lying states.

### 2.1.3 Single-Particle Properties of Nuclear States

A nucleus has properties which represent the sum of contributions from the constituent nucleons. In the case where a nucleus contains one valence particle outside a closed nuclear shell, the single-particle model can provide an understanding of many simple properties such as spins, parities, excitation energies, and electromagnetic moments. The nuclear spin (J) and parity of a nucleus in its ground state, or lowest energetic eigenvalue, can be represented quite accurately by the independent-particle approach of the shell model. The parity (\(\pi\)) is a quantity which denotes how the nuclear state (\(\Psi_A\)) reflects under coordinate inversion, a requisite property of solutions of the Schrödinger equation (Equation 2.1.1), and related to the total angular momentum of the nuclear state (\(\pi = (-1)^L\)). Thus, a state with an even or odd \(L\) is said to have even or odd parity, respectively.

In the extreme independent-particle approach, a nucleus with even numbers of both protons and neutrons would have a ground-state spin of zero and positive parity (\(J^\pi=0^+\)). An odd-A nucleus would have ground state spin and parity defined by the unpaired nucleon with a single-particle wave function, \(\psi_{njlm}\), with \(J^\pi=\frac{(-1)^L}{\text{odd}}\). The first few excited states of the odd-A nucleus would be represented as the promotion of the unpaired nucleon to higher shell model states (Figure 2.3). The ground and low-lying states of many nuclei can be modeled with this approach, however, even at low energies, their structure can involve complex multi-particle configurations. While these states can be approximated by single-particle states, they are not well modeled by the simple shell model and require the inclusion of effective or realistic interactions to the nuclear Hamiltonian.

The ground-state nuclear magnetic dipole moment reflects the charge currents within the nucleus. Both proton and neutrons have non-zero magnetic moments, and the magnetic dipole moment of a nucleus depends on contributions from all nucleons and is defined as:

\[
\mu = \mu_N \sum_{i=1}^{A} \left[ g_l l_i + g_s s_i \right]
\] (2.6)
Figure 2.2: Schematic of energy levels produced from the shell model using a variety of one-body potentials. The inclusion of a spin-orbit term reproduces the experimentally observed “magic” numbers for nuclei near stability. Figure from Ref. [41, Fig. 1.1].
where $\mu_N$ is the nuclear magneton, with $g_I$ and $g_s$ representing the angular momentum and spin $g$-factors. In the independent-particle approach, nucleons with anti-aligned spins and orbital angular momenta would not contribute to the net magnetic dipole moment producing even-A nuclei with $\mu=0$ and odd-A nuclei defined by the $I$ and $s$ of the unpaired nucleon. The single-particle magnetic dipole moments (i.e., Schmidt values [43]) are extreme values which, while providing a general understanding of odd-A magnetic moments, are rarely obtained by observed values (Figure 2.4). The consistent reduction of measured magnetic dipole moments relative to the Schmidt values (“quenching”) implies that a more modern nuclear interaction is required to model the ground-state structure of nuclei.

### 2.1.4 Isospin Symmetry and Independence

Isospin ($T$), is an additional degree of freedom introduced by Heisenberg which defines protons and neutrons as two different states of the nucleon [44]. The nucleon, with isospin equal to $\frac{1}{2}$, has isospin projections ($T_z$) which are analogous to the spin angular momentum projection of a spin-$\frac{1}{2}$ particle, with protons being described as $T_z=+\frac{1}{2}$ and neutrons with $T_z=-\frac{1}{2}$. The total isospin of a nucleus can be defined similarly to that of the total angular momentum ($J$) with the sum of the isospins of the nucleons in a nucleus. The shell model, as well as many modern theories, was formulated under the assumptions of isospin symmetry and isospin independence of the nuclear interaction. Isospin symmetry implies that the nuclear interaction treats “mirror” pairs of nucleons identically while isospin independence infers that
Figure 2.4: Experimental values for magnetic dipole for odd-neutron (top) and odd-proton (bottom) nuclei plotted by the total angular momentum ($j$) of the unpaired nucleon. The lines denoting the extreme single-particle, or Schmidt, values for a given $j$ define the general trend for the observed magnetic moments. Figures from Ref. [6, Figs. 1.4 and 1.5].
any nucleon pair (i.e., \( pp \), \( nn \), and \( pn \)) experiences the same interaction. Experimental evidence for the isospin symmetry and, to lesser degree, independence of the nuclear force is seen in the states of nuclear isobars. Isobars are nuclei with the same number of nucleons (A). In a group of isobars, which differ only in their isospin projection, states with the same spin, parity, total isospin, and relative energy are termed isobaric analogue states (IAS). Mirror nuclei are isobars with conjugate numbers of protons and neutrons which under isospin symmetry should have similar properties. The similarity of the mirror systems is best observed in the ordering of states of a specific spin and parity (Figures 2.3 & 2.5). The presence of isobaric analog states in the self-conjugate nucleus, with equal numbers of protons and neutrons (\( T_z = 0 \)), in integer-isospin multiplets supports the isospin independence of the nuclear interaction (Figure 2.5). In most mirror systems, the relative positions of the excited states differ between odd-proton and odd-neutron mirror states. The shift in the energy of isobaric mirror states, termed the Thomas-Ehrman effect, arises from the differences in the proton and neutron wave functions and a shift due to Coulomb distortions of the A-1 core [45]–[47]. As such, neither assumption of the nucleon-nucleon interaction is exactly true; the Coulomb interaction and higher-order effects break isospin symmetry and an isospin dependence observed in the structure of the deuteron which has a ground state spin and parity of \( J^{\pi} = 1^+ \), as opposed to having all angular momenta coupled to zero. Thus, small isospin symmetry breaking and isospin non-conserving terms must be included in the nuclear interaction to accurately reproduce observed nuclear phenomena.

Figure 2.5: Energy level diagram for three members of the A=18 isobars plotted relative to the ground state of the \( T=0 \) isobar \(^{18}\text{F}\). The mirror-nuclei of \(^{18}\text{O}\) and \(^{18}\text{Ne}\) both have ground and first-excited states in approximately the same position is evidence of some isospin symmetry of the nuclear interaction. The presence of isobaric analogs states (IAS) in \(^{18}\text{F}\), which has equal numbers of protons and neutrons, indicates there some isospin independence in the nuclear interaction. Figure modified from Ref. [48]
2.2 Ab Initio Nuclear Models

Ab initio nuclear models take on the difficult task of solving the nuclear many-body equation exactly from a nuclear interaction based on first principles. All models in utilize realistic two-body (NN) nuclear interactions (e.g., Argonne-v18 [49], CD-Bonn [50], etc.) which are based on nucleon-nucleon scattering data. These interactions include small charge-dependent and isospin-symmetry breaking terms which are necessary in order to reproduce $p-p$ and $p-n$ scattering data. These models have either realistic or effective three-body nuclear interactions which have been found to be necessary in the accurate modeling of $p$-shell nuclei [51].

2.2.1 Quantum Monte Carlo Methods

In the last twenty years, the Variational Monte Carlo (VMC) and Green’s Function Monte Carlo (GFMC) methods have been used to calculate the properties of $s$- and $p$-shell nuclei [51]. While both computational techniques have been utilized over the last 50 years for $A=3$ and 4 systems [52] [53], their more modern approaches are refined for large-scale computing resources required for modeling nuclei up to $A \leq 12$. The success of these methods can be attributed to the development and implementation of a nuclear interaction with realistic two- and three-body potentials. The three-body nuclear potentials (e.g., Urbana-IX [54], Illinois-2–5 [55], etc.) are derived from $NNN$ scattering data (e.g., $p-d$) and adjusted to reproduce the binding energies of $^3\text{He}$ and $t$ [51]. The QMC nuclear Hamiltonian has the general form,

$$H = T + V = \sum_i^A T_i + \sum_{i>j=1}^A V_{ij} + \sum_{i>j>k=1}^A V_{ijk}$$

(2.7)

which is Equation 2.1.1 with an additional three-body potential $V_{ijk}$. The Variational Monte Carlo is used as an approximate method of determining solutions to the $A$-body nuclear system with a given set of quantum numbers. The variational principle [56] is used on a parameterized many-body wave function $\Psi_A$ to obtain the upper bound of of the exact ground state eigenvalue $E_0$:

$$E_V = \frac{\langle \Psi_A | H | \Psi_A \rangle}{\langle \Psi_A | \Psi_A \rangle} \geq E_0$$

(2.8)
where the parameters of $\Psi_A$ are varied to minimize $E_V$ [51]. The resulting wave functions can be used to calculate estimates of electromagnetic moments and single nucleon densities which can be integrated to determine spectroscopic overlaps. The VMC wave functions are used as the initial trial functions for the more precise and computationally demanding GFMC calculations. The Green’s function Monte Carlo method improves on the VMC wave function by propagating out the calculation in imaginary time ($\tau$):

$$\Psi_0 = \lim_{\tau \to \infty} e^{-(H-E_0)\tau}\Psi_T$$

where $\Psi_T$ is the trial function supplied by the VMC and $\Psi_0$ is the exact lowest energy state for a given set of quantum numbers. If the calculation is propagated to a sufficiently large $\tau$ the solution is considered exact and are terminated as they approach this limit to conserve computing resources. The GFMC has been used to successfully calculate nuclear binding energies, electromagnetic moments, and the energies of both bound and unbound excited states (Figure 2.6).

Figure 2.6: GFMC calculations of the energy levels for states in $^8$Be using the AV18/UIX potential. The calculations are propagated in imaginary time ($\tau$) and converge upon their respective energy expectation values. Figure from Ref. [51, Fig. 3].

2.2.2 No-Core Shell Model

No-Core Shell Model theories (NCSM) solve the nuclear many-body equation in a large model space with contributions from all nucleons defining the nuclear wave function. The NCSM utilizes realistic two-body potentials and has begun to implement realistic three-body forces [57][58]. The calculations which do not utilize three-body forces use an effective two- or three-body interaction specific to the sys-
The calculations are performed in a large harmonic oscillator model space and allowed to converge toward the exact solution as much as is computationally possible. The accuracy of NCSM calculations improves when being performed with increasingly large model spaces making a fully converged calculation possible for only the lightest nuclei \((A \leq 4)\) \([58]\). The NCSM approach is still a valid means of obtaining approximate solutions to the nuclear many-body system and has been used in calculations of electromagnetic moments, level spectra, and single nucleon and cluster-overlap functions (Figure 2.7).

![Figure 2.7: Energy levels of \(^{10}\)Be produced by NCSM calculations. The calculated energies converge upon the experimental values when performed in larger harmonic-oscillator model spaces. The \((8–9)\hbar\Omega\) calculations shown here only model the \(2^+\) first-excited state of \(^{10}\)B correctly with all other states far from the experimental values. Figure from Ref. [57, Fig. 9].](image)

\section*{2.3 Summary of \(^9\)C Theoretical Calculations}

\subsection*{2.3.1 Level Structure}

The light mass of the \(^9\)C–\(^9\)Li mirror system has made these nuclei accessible to shell model calculations as well as to a variety of cluster and \textit{ab initio} models. The ordering of the low-lying states of both systems has been calculated and is compared to experimentally observed states in Table 2.1. All models do accurately reproduce the first-excited state of \(^9\)C as being above the two-proton separation energy \((S_{2p}=1.44 \text{ MeV})\). Since no other excited states have been confirmed in \(^9\)C a comparison of the possible mirror states in \(^9\)Li must be made. The calculated \(\frac{5}{2}^-\) and \(\frac{3}{2}^-\) levels from the NCSM are in good agreement with those observed in \(^9\)Li. At the time of this writing there have been no NCSM or GFMC calculations of the \(^9\)C level structure. These theoretical models would benefit from experimental
confirmation of excited states beyond the 2.2 MeV state.

Table 2.1: Summary of experimentally observed and theoretically-calculated energy levels for $^9$C and $^9$Li. The calculations for $^9$C are from WBT and PWT shell-model theories [61] published in Ref. [24, Tables 1 & 2]. The $^9$Li values are from Cohen-Kurath (CK) shell model [62] as well as ab-initio calculations published in Ref. [26, Table 1].

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$^9$C Exp. (MeV)</th>
<th>WBT (p-sd-fp)</th>
<th>PWT (p-shell)</th>
<th>$^9$Li Exp. CK (MeV)</th>
<th>GFMC</th>
<th>NCSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1/2^+$</td>
<td>2.2</td>
<td>1.4</td>
<td>2.2</td>
<td>2.69</td>
<td>3.33</td>
<td>1.5(4)</td>
</tr>
<tr>
<td>$^3/2^-$</td>
<td>3.9</td>
<td>3.6</td>
<td>4.3</td>
<td>3.88</td>
<td>3.0(5)</td>
<td>4.54</td>
</tr>
<tr>
<td>$^5/2^-$</td>
<td>4.1</td>
<td>4.1</td>
<td>5.38</td>
<td>5.45</td>
<td>3.6(5)</td>
<td>5.5</td>
</tr>
</tbody>
</table>

2.3.2 Magnetic Dipole Moment

The ground-state magnetic dipole moments of mirror nuclei are ideal systems to examine the isospin symmetry of the nuclear interaction. The $^9$C–$^9$Li mirror system has an apparent asymmetry in the measured ground-state magnetic dipole moments, -1.3914 $\mu_N$ and 3.4391 $\mu_N$ respectively, where the $^9$C value is 30% smaller than the extreme single-particle (Schmidt) value (-1.91 $\mu_N$). The measured $^9$Li magnetic dipole moment is closer to the Schmidt value (3.79 $\mu_N$) making the observed $^9$C magnetic dipole moment anomalous and differing from any other $p$- or $sd$-shell system. All shell, cluster, and ab initio model calculations produce magnetic moments which are smaller than the measured values for both $^9$C and $^9$Li. A theoretical study performed by Utsuno [28], in an effort to determine the mechanism of the nuclear interaction which brings about this asymmetry, utilized a full $p$-$sd$-shell model calculation with effective interactions describing the N=3 nuclei as well as the structure of $^9$Li. The resulting fit to the measured $^9$C magnetic dipole moment required the ground-state wave function to possess both $l=0$ and $l=2$ components of approximately 40% ([28]). A measurement of the properties of the ground and low-lying excited states in $^9$C would provide much needed information for these theoretical approaches to the ground-state magnetic dipole moment.

2.3.3 Neutron Overlaps

The Variational Monte Carlo and No-Core Shell Model have both calculated the one-neutron density function for the $^{10}$C–$^{9}$C+$n$ system. The integrals of these values over the volume of the nucleus produce
spectroscopic factors which originate from first-principle approaches to the nuclear interaction. The VMC calculations were performed for both $1p_{3/2}$ and $1p_{1/2}$ overlaps. The ab-initio spectroscopic factors along with those from a $p$-shell model are summarized in Table 2.2.

Table 2.2: Summary of the spectroscopic factors for $^{10}$C–$^{9}$C+$n$ calculated from NCSM and VMC one-neutron density functions [25] [63]. Cohen-Kurath spectroscopic factors from a $p$-shell model are listed for comparison [62].

<table>
<thead>
<tr>
<th>System</th>
<th>$C^2S$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CK</td>
</tr>
<tr>
<td>$^{9}$C+$n(p_{3/2})$</td>
<td>1.735</td>
</tr>
<tr>
<td>$^{9}$C+$n(p_{1/2})$</td>
<td>0.255</td>
</tr>
</tbody>
</table>

2.4 Summary

Many approaches to modeling nuclear structure have had success in reproducing observables of nuclei near stability. The failure of these theories to model nuclei far away from stability is an impetus for the study of rare nuclei. $^{9}$C has an anomalous ground-state structure and is within the range of several ab initio nuclear models which currently cannot account for its observed properties. The possibility of sd-shell components in the ground-state wave function of $^{9}$C can be assessed by obtaining experimental data of the spins, parities, and spectroscopic factors the ground and low-lying states of $^{9}$C.
Chapter 3

Single-Nucleon Transfer Reactions

3.1 Introduction

Nuclear structure studies can be performed through a variety of experiments using nuclear reactions where nuclei are produced by the addition or removal of nucleons to or from a known isotope. The typical technique utilizes an accelerated beam of light ions incident on a stationary target. The reaction which occurs can be mediated by either the formation of a compound nucleus, where the beam is absorbed into the target nucleus forming a $A_{\text{beam}}+A_{\text{target}}$ system, or through an exchange of nucleons at the surface of the target nucleus. The latter subset of reactions, designated “direct” reactions, are prompt processes ($\approx 10^{-22}$ s) which depend quite strongly on the initial and final states of the reaction. These reactions include non-elastic processes such as the transfer of one or more nucleons to or from the target as well as inelastic scattering. Transfer reactions have been used heavily over the past sixty years to study the single-particle structure of nuclei. The subset of single-nucleon transfer reactions is very selective and populates states in the residue nucleus which resemble the target nucleus either with or without a nucleon in a single-particle state. The data from these “single-step” reactions are well represented by theoretical models which can be used to interpret the properties of the populated states. The distorted-wave Born approximation (DWBA) reaction theory utilizes the optical model to calculate transfer reaction yields and is used to extract spectroscopic information from the experimental data. Simple single-nucleon transfer reactions such as $(d,p)$ are well modeled, with more complex reactions (e.g., $(d,t)$, $(t,\alpha)$, etc.) still being well-represented with the appropriate optical-model potentials. In the following chapter, the basic information regarding single-particle transfer reactions is presented as well as the considerations that must be taken into account when performing them in inverse kinematics. The components of the
DWBA theory are described as well as their application to the \((d,p)\) and \((d,t)\) reactions. The use of single-nucleon transfer reactions in the study of unstable nuclei is a viable experimental technique which is utilized in the current study of \(^9\)C via the \(d(\text{^{10}C},t)^9\text{C}\) reaction.

### 3.2 Reaction Kinematics

Single-nucleon transfer reactions are direct reactions which involve the exchange of a proton or neutron from a target nucleus by an energetic beam of ions. A beam with mass \(A_b\) can add ("stripping") or remove ("pickup") a single nucleon from the target \((A_t)\). The reaction products consist of the ejectile \((A_b\pm1)\) and the recoil or residue \((A_t\mp1)\). A given transfer reaction can take place if the center-of-mass energy \((E_{cm})\) of the beam–target system is larger than the \(Q\)-value of the reaction. The center-of-mass energy and \(Q\)-value are defined as:

\[
E_{cm} = \frac{M_t}{M_t + M_b} E_b
\]

\[
Q = M_{before} - M_{after} = [M_{beam} + M_{target}] - [M_{ejectile} + (M_{residue} + E_x)]
\]

where \(E_b\) is the beam energy in MeV, the isotope masses are in MeV/\(c^2\) and \(E_x\) is the excitation energy of the residue its final state. The resultant energies and angles of the reaction products are dictated by the conservation of energy and momentum. The kinematics of transfer reactions performed at energies of 5–20 MeV per nucleon only experience small relativistic effects \((0.1<\beta<0.2)\), and thus their kinematics can be treated non-relativistically.

In the laboratory frame, the ejectile with a mass \(M_{ej}\) is emitted at an angle \(\theta_{ej}\) and energy \(E_{ej}\) with the residue having its own set of values \((M_{res}, \theta_{res}, E_{res})\). The ejectile and residue energies are functions of their respective angles \((E_i(\theta_i))\) with solutions being more simple to express in the center-of-mass frame and then translate to the laboratory frame. The velocity of the center-of-mass frame, \(V_{cm}\), defined as:

\[
V_{cm} = \frac{M_{beam}}{M_{beam} + M_{target}} V_{beam}
\]

and is applied as a Galilean transformation between the laboratory and center-of-mass systems. In the center-of-mass system, the net momentum is zero and thus, for a two-body final state, the relationship
between the ejectile and residue angles are supplementary:

\[ \theta_{cm,ej} + \theta_{cm,res} = \pi \]  

(3.4)

with their respective center-of-mass velocities \((V_{ej} \text{ and } V_{res})\) being constants relative to \(V_{cm}\) [64]. The constant ratios of \(V_{cm}\) and each reaction product’s center-of-mass velocity \((K)\) are defined as:

\[ K_{ej} = \frac{V_{CM}}{V_{ej}} = \left[ \frac{M_b M_{ej} E_{cm}}{M_t M_{res} (E_{cm} + Q)} \right]^{1/2} \]  

(3.5)

\[ K_{res} = \frac{V_{CM}}{V_{res}} = \left[ \frac{M_b M_{res} E_{cm}}{M_t M_{ej} (E_{cm} + Q)} \right]^{1/2} \]  

(3.6)

These ratios define the range of possible laboratory angles where each reaction product can be observed in the laboratory frame. The measurement of the observed yield of the ejectiles over a range of laboratory angles, or angular distribution, can be used to determine properties of the populated state in the residual nucleus.

Traditional transfer reactions, studying nuclei at or near stability, are performed with light ion beams \((e.g. \ p, \ d, \ etc.)\) bombarding stable, heavy targets. In the modern transfer-reaction studies of unstable nuclei, heavy beams of short-lived radioactive ions are incident on light targets. Transfer reactions performed in either “normal” or “inverse” kinematics are equivalent and governed by the same kinematic equations. However, performing a measurement in inverse kinematics can present difficulties that are not present in the normal kinematic case. The following examples are of the neutron-stripping reaction \(^{10}\text{C}(d,t)^{9}\text{C}\) at 17.1 MeV per nucleon with velocity transformations and energy-angle functions presented for both normal and inverse kinematic reactions.

**Normal Kinematics**

As an example of normal kinematics, the \(^{10}\text{C}(d,t)^{9}\text{C}\) reaction is performed with a 34.2-MeV deuteron beam incident on a hypothetical \(^{10}\text{C}\) target. In reality, a target comprised of \(^{10}\text{C}\) \((t_{1/2} = 19.29 \text{ s})\) cannot exist and is used here in the normal kinematics example strictly for comparison with that of the feasible inverse kinematic reaction. The beam is lighter than the target \((M_b < M_t)\), and thus the velocity of the center-of-mass frame is a small fraction of the beam velocity \((V_{cm} = \frac{1}{6} V_{beam})\). The ejectile velocity in
the center-of-mass frame is greater than \( V_{cm} \) \((K_{ej}<1)\) and can be emitted over all laboratory angles with an energy which varies monotonically with angle (Figure 3.1). The residue has a center-of-mass velocity smaller than \( V_{cm} \) and a maximum laboratory angle defined by the relation [64]:

\[
\tan \theta_{\text{max, res}} = \left[K_{res}^2 - 1 \right]^{-\frac{1}{2}}
\]  

(3.7)

The measurement of the angular distribution of the triton is a straightforward procedure, however the detection and identification of the residue is made difficult by its low energy.

**Inverse Kinematics**

In inverse kinematics, the \( d(\text{^10C, } t)\text{^9C} \) reaction is performed with a 171-MeV \(^{10}\text{C}\) beam incident on a deuterium target. The beam is heavier than the target \((M_b > M_t)\), and thus the velocity of the center-of-mass frame is a the majority of the beam velocity \((V_{cm} = \frac{5}{6}V_{beam})\). The center-of-mass velocities of all reaction products are smaller than \( V_{cm} \). Thus, the \( 2\pi \) range of the ejectile and residue in the center-of-mass frame is compressed into a forward-focused “cone” in the laboratory frame with maximum angles given by Equation 3.7. The energy-angle relationship for both products possess two solutions, each corresponding to a different \( \theta_{cm} \), for each laboratory angle (Figure 3.2). In the present scenario, a
Figure 3.2: Velocity transformation diagram and plot of ejectile and residue energy vs. laboratory angle for the \(d^{(10}C,t)^9C\) reaction in “inverse” kinematics performed at 17.1 MeV per nucleon. The black and blue dots indicate the two kinematic solutions possible for each laboratory angle.

measurement of the angular distribution of the tritons emitted in the laboratory requires a detector array with high angular resolution positioned at forward angles. The residues possess more energy than in the normal kinematics case and can be detected by various methods at angles even more forward-focused than the ejectiles.

The kinematics of the single-particle transfer reaction must considered carefully in order to obtain angular distributions data from the reaction of interest. Angular distributions of ejectiles can provide information about the properties (\(e.g.,\) spins, parities, and spectroscopic information, etc.) of states populated in the residue. The shape of the measured angular distribution can provide some of this information, which can be deduced using simple scattering analysis involving plane waves.

### 3.3 Plane Wave Born Approximation

In the early study of near-stable nuclei via \((d,p)\) reactions, it was discovered that the angular distribution of the emitted protons had a strong dependence on the angular momentum \(l\) transferred in the reaction [65] [66]. The theory developed to model this aspect of transfer reactions approximates the initial and final states of the system as plane waves. For the \((d,p)\) stripping reaction the initial
(deuteron-target) state is defined as [67][68]:

$$\Psi_i = e^{i\mathbf{k}_i \cdot \mathbf{r}} \psi_t$$

(3.8)

where \( \mathbf{k}_i \) is the momentum of the incident deuteron, \( \mathbf{r} \) the position, and \( \psi_t \) the A-body wave function of the target. Similarly, the final (proton-residue) state is defined as:

$$\Psi_f = e^{i\mathbf{k}_f \cdot \mathbf{r}} \psi_r$$

(3.9)

where both \( \mathbf{k}_f \) and \( \mathbf{r} \) define the momentum and position of the proton and \( \psi_r \) is the residue wave function with A+1 nucleons. The residue wave function can be approximated as a transferred neutron in a single-particle state about the target nucleus with angular momentum \( l = l_t \). The scattering amplitude, \( f(\theta) \), of the reaction is given by the first Born approximation:

$$f(\theta) = -\frac{\mu}{2\pi \hbar^2} \int e^{i\mathbf{q} \cdot \mathbf{r}} \psi_f V(r) \psi_i d^3r$$

(3.10)

where \( \mathbf{q} = \mathbf{k}_f - \mathbf{k}_i \), \( \mu \) is the reduced mass, and \( V(r) \) the nuclear potential. The form of \( V(r) \) can be approximated as a delta function at the nuclear radius (R). Integrating Equation 3.10 over all coordinates, the scattering amplitude for the plane-wave Born approximation reduces to the relationship:

$$f(\theta) \approx j_{l_t}(qR) \approx j_{l_t} \left( \sqrt{k_i^2 + k_f^2 - 2k_i k_f \cos \theta} R \right) \approx j_{l_t} \left( 2kR \sin \left( \frac{\theta}{2} \right) \right)$$

(3.11)

where \( j_{l_t} \) is the spherical Bessel function. The resulting reaction cross section can be determined by evaluating the square of the scattering amplitude:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

(3.12)

The angular distribution for the proton can be calculated using Equations 3.11 and 3.12. Thus, the shape of the angular distribution is dependent on the amount of angular momentum transferred between initial and final single-particle states (Figure 3.3). If the ground state of the target is of known spin and
parity and the $l$-value of the transferred nucleon determined from its angular distribution, the spin-parity of the populated state in the residue can be inferred.

Figure 3.3: Angular distributions from plane-wave calculations for the $(d,p)$ neutron-transfer reaction. The shape of the angular distribution changes with the amount of $l$-transferred to the neutron. Figure from Ref. [68, Fig. 3.2]

The plane-wave Born approximation is a simple approach to modeling single-particle transfer reactions. In general, the observed angular-momentum dependence of emitted ejectiles can be explained, however, many aspects of the transfer are ignored to reach these conclusions. Specifically, the form of the scattering potential $V(r)$ set to be a delta function at the radius $R$ which is known to be unrealistic. Additionally, all particles are assumed to have “straight” trajectories ($k_i$ and $k_f$) both before and after the transfer reaction with no account for the motion of the initial (beam-target) and final (ejectile-residue) states in their respective averaged potentials. The distorted-wave Born approximation utilizes the optical model to account for these factors and has had much success as a reaction-theory model for transfer reactions.

3.4 The Optical Model

The optical model is used to provide a more realistic potential for nucleus-nucleus scattering. An optical model potential is defined as an average potential of a projectile nucleus moving relative to all
Figure 3.4: Optical-model fits to angular distributions of deuteron elastic scattering performed on light and intermediate mass targets. Figure from Ref. [69, Fig. 1].
nucleons present in the target. The basic form of optical model potential is:

\[ U_0 = V + iW = -V_0 f(r, r_0, a) - iW_0 f(r, r_w, a_w) \]  
(3.13)

where \( f(r, r_0, a) \) and \( f(r, r_w, a_w) \) are Woods-Saxon potentials (Equation 2.5). The real potential \( (V) \) represents the elastic scattering of the beam while the imaginary potential \( (W) \) absorbs particle flux into the nucleus and represents all inelastic channels. A spin-orbit term can be included in the optical model potential:

\[ U = U_0 + \left( \frac{\hbar}{mc} \right)^2 \frac{d}{dr} U_{so}(l \cdot s) \]  
(3.14)

where \( U_{so} \) has the same form as Equation 3.13. To describe all interactions present in the system, a Coulomb interaction is included with the general form:

\[
V_C(r) = \begin{cases} 
  \frac{Z_b Z_t e^2}{r} & (r \geq R_C) \\
  \frac{Z_b Z_t e^2 [3-(\frac{r}{R_C})^2]}{2R_C} & (r < R_C)
\end{cases}
\]

where the potential is that of a point charge outside of the Coulomb radius, \( R_C \), and a uniform sphere of charge \( (Z_t) \) within. The optical model parameters for a scattering system are determined by adjusting the parameters to fit angular distributions of elastic scattering data (Figure 3.4).

### 3.5 The Distorted-Wave Born Approximation

The distorted-wave Born approximation uses the optical model to more accurately represent the interactions present in the entrance and exit channels of the transfer reaction. Four optical-model potentials are used to describe the scattering system: two bound-state potentials, an incoming scattering potential, and an outgoing scattering potential. The bound-state potential for the incoming channel describes the nucleon to be transferred, bound in a single-particle state \( j_i \) about an inert “core” of the target (pickup) or the beam (stripping) nucleus. Similarly, the bound-state potential for the outgoing channel describes the transferred nucleon bound in its final state \( j_f \) about the core of the beam (pickup) or target (stripping) nucleus. The incoming and outgoing potentials describe the average potential
(nuclear and Coulomb) of the respective beam–target and ejectile–residue systems (Figure 3.5). As mentioned in the previous section, optical-model parameters are determined from elastic-scattering data for the system of interest. Bound-state optical model parameters, for single-nucleon transfer, are derived from proton or neutron scattering data. The resulting optical model well depths (e.g., $V_0$, $W_0$, $V_{so}$, etc.) are then adjusted to reproduce the single-nucleon binding energy of the nucleus [70].

Figure 3.5: Schematic illustration of the incoming and outgoing channels of the single-neutron pickup reaction $d^{(10}C,t)^{9}C$ as seen in the center-of-mass frame.

3.5.1 Transition Amplitude

The transition amplitude for the distorted-wave Born approximation is similar to the scattering amplitude of the PWBA (Equation 3.10), however, it is a much more detailed representation of the transfer reaction system. The general form is [68]:

$$T_{DWBA} = \langle \chi_i^{(-)}|V_{eff}|\psi_f \chi_i^{(+)} \rangle$$  \hspace{1cm} (3.15)

where $\chi_i^{(+)}$ and $\chi_i^{(-)}$ are the scattering solutions (eigenfunctions) for the incoming (beam) and outgoing (ejectile) distorted waves, $\psi_i$ and $\psi_f$ are the initial and final nucleon-core wave functions, $V_{eff}$ is the effective two-body interaction for the nuclei in the entrance (prior form) or exit (post form) channels [68], and the bra-kets imply that the enclosed quantities are integrated over all variables. The calculation of the transition amplitude is performed by the separation of the scattering wave functions into radial and
angular components and solving Equation 3.15 as a sum over partial-waves:

\[
T_{DWBA} = \sum_{lm} \int \int d^3r_f d^3r_i \chi_f^{(-)}(k_f, r_f) < \psi_f | V_{eff} | \psi_i > \chi_i^{(+)}(k_i, r_i) \tag{3.16}
\]

The sum is performed for all \( l \) in the infinite series, however, converges after critical value of \( l \) representing a distance where the integrand goes to zero due to the fall-off of the bound-state wave function. The bracketed term contains the bound-state form factor which itself is an integral over all internal coordinates of the two bound-state systems and contains the wave function of the transferred nucleon as well as the overlap of the target-residue systems. The differential cross section is proportional to the square of the transition amplitude produced by the distorted wave calculations:

\[
\frac{d\sigma}{d\Omega}_{DWBA} \approx |T_{DWBA}|^2 \tag{3.17}
\]

3.5.2 Spectroscopic Factors

The residue in a single-nucleon transfer is similar to the target with either an additional (stripping) or missing (pickup) nucleon. The DWBA calculations for the transfer system produce a theoretical calculation of the transfer assuming the initial and final state are “pure” single-particle states. The spectroscopic factor, \( S \), is the comparison of the experimentally observed transfer cross section to that produced from the distorted-wave calculations and is a measure of how much the residue looks like the target plus or minus a nucleon in a single-particle state. In general, the spectroscopic factor is defined as:

\[
\frac{d\sigma}{d\Omega}_{Exp} = S \frac{d\sigma}{d\Omega}_{DWBA} \tag{3.18}
\]

Absolute Spectroscopic Factors

The shape of the angular distributions are well represented via DWBA calculations, and while more robust than the plane-wave Born approximation, the absolute scale of the calculated cross sections are sensitive to the reaction-theory model. Specifically, the radial parameters of the bound state optical-model potentials determine the distance at which the nucleon transfer is modeled to occur and small changes can affect the magnitude of the calculated cross section. Thus, absolute spectroscopic factors
are not entirely reliable in the comparison of experimental and DWBA calculations. The comparison between the spectroscopic factors from different excited states within the residue are a more reliable probe of the single-particle nature of the populated states.

The \( (d,p) \) Reaction

The \( (d,p) \) reaction is one of the simplest and more well-understood single-neutron stripping reactions. Along with the \( (d,n) \) reaction they represent the straightforward addition of a nucleon to a nucleus. The experiments are simple to perform on stable targets in normal kinematics and have been used in inverse kinematics studies with radioactive beams incident on deuterium-containing materials. The distorted-wave Born approximation excels at modeling the neutron-transfer which can be attributed to the simple overlap between the deuteron and proton wave functions. Additionally, the entrance and exit channels of the reaction are well represented by the optical model potentials developed from extensive proton and deuteron elastic scattering studies. The \( (d,p) \) reaction is still widely used to determine the spin, parity, and single-particle nature of states in nuclei, including neutron-rich nuclei away from stability (Figure 3.6).

Figure 3.6: Angular distributions and DWBA fits for \(^{16}\text{C}\) states populated in the \( d(^{15}\text{C},p)^{16}\text{C} \) reaction at 8.2 MeV/A. Figure from Ref. \[71, \text{Fig. 2}\].
The \((d,t)\) Reaction

The \((d,t)\) reaction is a neutron-pickup process which can be used in the place of the simpler \((p,d)\) reaction. The difference in the masses of the deuteron and the triton make the \((d,t)\) reaction less inelastic \((Q<0)\) than the \((p,d)\) by approximately 4 MeV which makes it experimentally more favorable in laboratories where higher energy beams are not available. The \((d,t)\) reaction is a more complex process that can be attributed to the presence of an additional neutron — which does not get transferred — that must be accounted for in all aspects of modeling the reaction. The DWBA does model the pickup reaction sufficiently well providing that the proper choice of optical-model parameters are utilized (Figure 3.7). The reaction has been used to study a variety of light and intermediate mass nuclei and was chosen over the \((p,d)\) and \((^3\text{He},\alpha)\) reactions in the current study of \(^9\text{C}\).

![Angular distribution and DWBA fits to the ground and first-excited of \(^{12}\text{C}\) populated in the \(^{13}\text{C}(d,t)^{12}\text{C}\) reaction at 14.5 MeV/A. Figure from Ref. [72, Fig. 4]](image-url)
3.6 Summary

The properties of states in nuclei can be studied using single-particle transfer reactions. The shape of angular distributions of ejectiles emitted in the reaction can provide spin and parity assignments to states populated in the residue. The comparison of observed cross sections to those produced by distorted-wave calculations of transfer between single-particle states can yield spectroscopic factors which are a measure of the single-nucleon overlap of the target-residue system. The experimental considerations taken to obtain triton angular distributions from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction are presented in following chapter. The subsequent data analysis, reduction, and comparison to distorted-wave transfer reaction theory is detailed in Chapters 5–7.
Chapter 4

The $d^{(10}\text{C},t)^9\text{C}$ Experiment

4.1 Introduction

The $d^{(10}\text{C},t)^9\text{C}$ experiment was performed at the ATLAS facility at Argonne National Laboratory. The goal of the experiment was to populate states in $^9\text{C}$ through the $d^{(10}\text{C},t)^9\text{C}$ reaction in inverse kinematics and measure their excitation energies, spin-parities, and to extract spectroscopic factors. A radioactive $^{10}\text{C}$ beam was developed for this experiment at the ATLAS In-Flight Radioactive Beam facility and was used in two successful runs of ATLAS Experiment #1333. The first was an eight-day run in February 2011 and the second, a four-day run, in May 2011. The latter experiment was granted as refund time due to a failure of the tandem Van de Graaff accelerator which ended the February 2011 run early. There was no change in the detectors or electronics between the two experiments. The two experiments did differ in that the May 2011 experiment used thicker targets and that the radioactive $^{10}\text{C}$ beam was produced at a slightly different energy ($169$ MeV) than during the February 2011 run ($171$ MeV). In addition, the difference in the calibration constants obtained from alpha source data performed between the two experiments warranted a separate alpha calibration of each data set. This change in calibration was likely due to radiation damage to the detectors during the February 2011 beam time. The analysis of all data sets is presented in Chapters 5–7.
4.2 $^{10}$C Beam Development

4.2.1 Radioactive Beams at ATLAS

Argonne National Laboratory has been a pioneer in nuclear-physics research since its founding in 1946. The Argonne Tandem Linear-Acceleration System (ATLAS) was commissioned in 1985 and was the first superconducting linear accelerator for heavy ions at Coulomb-barrier energies. ATLAS can produce beams of stable ions from protons to uranium at energies up to 19 MeV per nucleon. Stable beams can be delivered to six different beam lines hosting a wide variety of nuclear physics experiments such as HELIOS [73] and GAMMASPHERE [74] (Figure 4.1). Radioactive ion beams (RIBs) have been available at ATLAS for nearly two decades through both the “two-accelerator” [75] and “in-flight methods” [76]. The so-called “two-accelerator” method was not used in the current study, though is outlined in this section for completeness. The majority of RIBs at ATLAS have been produced through the “in-flight” method which was the only means of creating the $^{10}$C beam necessary for this study.

Figure 4.1: Floor plan of the ATLAS facility at Argonne National Laboratory. All $d(^{10}$C,$t)^{9}$C experiments in this study were performed in the Split-Pole Spectrograph target area.
4.2.2 Two-Accelerator Method

The two-accelerator method is a technique for producing beams of long-lived radioactive isotopes and is similar to the production of stable beams. The standard source material used in the ATLAS SNICS (Source of Negative Ions by Cesium Sputtering) source is replaced with long-lived radioactive material such as $^{56}$Ni ($t_{1/2} = 6.075$ d) or $^{14}$C ($t_{1/2} = 5700$ y). A small sample of the material is placed in a concave metal cathode and bombarded by Cs$^+$ ions. Singly negative ions (A$^-$) of the source material are produced through electron exchange reactions with the cesium and are then extracted from the source by the negative bias of the cathode [77].

The ions are focused into an intense, low-energy beam and transported to the ATLAS FN-Tandem Van de Graaff accelerator. The tandem Van de Graaff accelerator consists of two linear charging columns, low energy (LE) and high-energy (HE), with a shared central positive terminal. The LE column accelerates the negative ions to the terminal where the beam passes through a carbon “stripping” foil where electrons are removed from the ions in the beam, changing them from negatively to positively charged ions. These ions are then accelerated in the HE column permitting the production of ion beams with energies up to many tens of MeV [78]. At the exit of the tandem accelerator there is a second stripping foil which can be used to further increase the charge state of the beam before injecting it into the ATLAS linear accelerator.

The ATLAS linear accelerator consists of a series of superconducting RF resonator cavities that are designed for accelerating positively charged heavy-ion beams. The Cu-Nb resonators are housed in cryostats to provide the insulation and refrigeration necessary for their operation at liquid helium temperatures. The ATLAS resonator designs uses a “split-ring” geometry which allows for efficient acceleration of each beam pulse. The electric fields produced inside a single resonator can be many MV/m and the most recent designs can accelerate ions up to $\beta = 0.3c$. The two-accelerator method has been successfully used to develop beams of $^{18}$F, $^{44}$Ti, and $^{56}$Ni [75]–[81].

4.2.3 In-Flight Method

The majority of radioactive beams produced at ATLAS have been developed using the “in-flight” technique [76]. A stable primary beam is used to bombard a production target and, via a secondary
reaction, produce a beam one or two nucleons removed from stability. This method is necessary for the production of short-live isotopes that have half-lives on the order of milliseconds. Stable beam intensities at ATLAS typically exceed $10^{11}$ s$^{-1}$ and radioactive beam intensities range from $10^3$ to $10^6$ s$^{-1}$. In-flight beam intensities depend on several factors including the production reaction cross section and kinematics, the quality and intensity of the primary beam, and the transmission of the beam to target.

The production reactions for ATLAS in-flight beams are currently limited to reactions on a gas target or a beryllium foil. The gas production target is used for hydrogen- and helium-induced reactions and kept at cryogenic temperatures ($\approx 90$ K) and pressures up to 1.4 atm. The gas cell is a cylindrical volume 2.5 cm in diameter and a nominal 3.71 cm in length housed in a double-walled stainless steel vessel cooled by a continuous flow of liquid nitrogen (LN$_2$). The beam enters and exits the target volume through windows made of 1.9 mg/cm$^2$ HAVAR™ foils. This design provides a thick, light target suitable for many single-nucleon transfer reactions (e.g., ($d,p$), ($d,n$), ($d,^3$He), ($^3$He,$n$), etc.) and charge exchange reactions such as ($p,n$). In addition to the reaction Q-value, the energy loss of both beams in the volume and the windows of the gas cell defines their final energies. Table 4.1 lists the energies of some examples of primary and secondary beams as they pass through the gas production target.

<table>
<thead>
<tr>
<th>Production Reaction</th>
<th>Beam Species</th>
<th>$E_{before}$ (MeV)</th>
<th>$E_{center}$ (MeV)</th>
<th>$E_{after}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7$Li($^7$Li,$p$)$^8$Li</td>
<td>$^7$Li</td>
<td>80</td>
<td>78.4</td>
<td>76.8</td>
</tr>
<tr>
<td>$Q=-0.192$</td>
<td>$^8$Li</td>
<td>-</td>
<td>76.5</td>
<td>74.6</td>
</tr>
<tr>
<td>$^10$B($^10$B,$n$)$^{11}$C</td>
<td>$^{10}$B</td>
<td>185</td>
<td>181.4</td>
<td>178.4</td>
</tr>
<tr>
<td>$Q=-4.490$</td>
<td>$^{10}$C</td>
<td>-</td>
<td>176</td>
<td>171.5</td>
</tr>
<tr>
<td>$^11$N($^11$N,$n$)$^{14}$O</td>
<td>$^{14}$N</td>
<td>223</td>
<td>216</td>
<td>208.8</td>
</tr>
<tr>
<td>$Q=-5.926$</td>
<td>$^{14}$O</td>
<td>-</td>
<td>208.4</td>
<td>198.6</td>
</tr>
</tbody>
</table>

The primary beam that exits the production cell has a lower energy due to reactions with the target gas and energy loss in the windows. The radioactive beam exits the gas cell in a cone which is determined by the maximum angle of the production reaction. This angle, $\theta_{max}$, is a consequence of the production reaction being performed in inverse kinematics (see Equation 3.7) and must be taken into account when using an in-flight beam. A superconducting solenoid located after the production target is then used to focus the radioactive beam. The opening aperture of the solenoid defines the maximum angle that an in-flight beam can be accepted. After the solenoid, a single RF-resonator is used to optimize its longitudinal...
and transverse emittance of the beam. A dipole magnet is used to separate the radioactive beam from the stable beam by applying a magnetic field that selects the radioactive ion beam in its highest charge state. Neglecting relativistic effects, the magnetic rigidity of an ion beam traveling through a uniform magnet field is defined by:

$$B\rho = \frac{mv}{q} = \frac{\sqrt{2mE}}{q}$$  \hspace{1cm} (4.1)$$

where $B$ is the magnetic field of the dipole, $\rho$ the cyclotron (bending) radius, $m$ the mass of the beam, $q$ the charge state, and $v$ the velocity of the beam perpendicular to the magnetic field. The optimum cyclotron radius for transmission to an experimental beam line, $\rho_0$, defines the optimum magnetic field setting:

$$B = \frac{mv}{q\rho_0}$$  \hspace{1cm} (4.2)$$

Selecting the magnetic field for the in-flight beam,

$$B_2 = \frac{m_2v_2}{q_2\rho_0}$$  \hspace{1cm} (4.3)$$

gives a different bending radius for the stable beam,

$$\rho_1 = \frac{m_1v_1}{q_1B_2}$$  \hspace{1cm} (4.4)$$

since the ions differ in mass, charge, and energy. The relation between the radii,

$$\frac{\rho_1}{\rho_0} = \frac{m_2v_2q_1}{m_1v_1q_2}$$  \hspace{1cm} (4.5)$$

determines in which direction the dipole deflects the primary beam relative to the in-flight beam. As a result, the magnetic field will deflect most the primary beam at its maximum energy into slits which collimate the chosen experimental beam line after the magnet.
Section 4.2.4: \(^{10}\text{C}\) Beam Development

An in-flight radioactive beam of \(^{10}\text{C}\) was developed at ATLAS using the \(p(^{10}\text{B},^{10}\text{C})n\) reaction. This beam was first developed at ATLAS in the Spring of 2009 for the purpose of studying \(^9\text{C}\) via one neutron transfer. The \(d(^{10}\text{C},t)^9\text{C}\) reaction has a negative \(Q\)-value (-15.02 MeV) and will require a \(^{10}\text{C}\) beam greater than 11 MeV per nucleon to populate \(^9\text{C}\) excited states up to 4 MeV in excitation energy. For the beam development run, ATLAS Experiment \#1088X, a \(^{10}\text{B}\) beam of approximately 117 MeV was first tuned to a Faraday cup on a diagnostic target ladder two meters upstream of the Split-Pole Spectrograph (SPS). The cryogenic production cell, filled with H\(_2\) gas, was then placed in the beam line and bombarded with the stable beam. After the gas cell, the degraded \(^{10}\text{B}\) beam had an energy of 112 MeV and the average energy of the \(^{10}\text{C}\) beam was approximately 105 MeV. To ensure that the beam line elements downstream of the production cell were optimized for the rigidity of the primary beam \(^{10}\text{B}^{5+}\) the degraded stable beam was first tuned to the diagnostic Faraday cup. To select the radioactive beam, the magnetic separator and other elements were then scaled to the rigidity of the radioactive beam \(^{10}\text{C}^{6+}\).

The scaling parameter, \(S\), is the ratio between the two rigidities. Using Equation 4.1:

\[
S = \frac{B_2 \rho_0}{B_1 \rho_0} = \frac{q_1}{q_2} \sqrt{\frac{m_2 E_2}{m_1 E_1}}
\]  

(4.6)

For the \(^{10}\text{C}\) development run, the scaling factor was 0.807 and differed by less than a percent in the experimental runs at higher beam energies. After the separator, any \(^{10}\text{B}\) present must satisfy the rigidity
setting for the radioactive beam \( (B_2\rho_0) \) and have an energy:

\[
E = \left(\frac{q_1}{q_2}\right)^2 \frac{m_2}{m_1} E_2
\]  

which for the development run was approximately 73 MeV. The resulting \(^{10}\text{C}\) beam, at small primary beam intensity, is tuned to the diagnostic ladder and optimized with two silicon surface barrier detectors in a \( \Delta E\)-\( E \) telescope configuration. The different species present in the beam can be identified by comparing the energy loss of an ion in the thin “\( \Delta E \)” detector (75 \( \mu \)m) with the remaining energy deposited in the thick residual-energy detector (2.0 mm). This technique is discussed in more detail in the Section 5.4.3. The E-residual detector signal was used to differentiate the two beams and to optimize the ratio of \(^{10}\text{C}\) to the \(^{10}\text{B}\) still present (Figure 4.3).

Figure 4.3: A particle identification plot from the diagnostic \( \Delta E\)-\( E \) telescope showing both \(^{10}\text{C}\) and \(^{10}\text{B}\) beams. This spectrum was the first taken during the development run as the \(^{10}\text{B}\) beam is more intense than the radioactive \(^{10}\text{C}\) beam.

**RIB Sweeper**

The Radioactive Ion Beam Sweeper is a device designed to further suppress ions from the primary beam relative to the in-flight beam. It takes advantage of the time structure of the ATLAS stable beams as well as that the beams have previously been magnetically separated with the radioactive beam traveling at a different velocity than the stable beam. The sweeper is a one-meter long RF-cavity located immediately downstream of the magnetic dipole separator in the Split-Pole Spectrograph beam line. The
sweeper applies a transverse, RF-electric field to the cavity which deflects all ions present. The phase of the sweeper RF is optimized such that the deflection of the radioactive beam is minimized and that of the primary beam is at maximum (Figure 4.4). This method was sufficient to reduce the $^{10}\text{B}$ beam from over 75% of the count rate to less than 0.5% during the development run. For the February 2011 and May 2011 experiments, the average purity of the $^{10}\text{C}$ beam was no lower than 85%.

![Figure 4.4: A schematic diagram of the Radioactive Ion Beam Sweeper. The RF-electric field is applied at the optimal phase, which deflects most of the primary $^{10}\text{B}$ beam and not the in-flight $^{10}\text{C}$ beam.](image)

**4.3 Scattering Chamber**

The detector system for the $d(^{10}\text{C},t)^{9}\text{C}$ experiment was housed in a 60 cm × 30 cm cylindrical scattering chamber (Figure 4.5) located immediately downstream of the ConFlat™ cross containing the diagnostic ladder. To define the maximum beam spot dimensions a set of four tantalum slits on micrometers (“four-jaw” slits) were present in the diagnostic cross. For all experiments these slits were set to make an aperture of 5 mm × 5 mm square. To protect the detector system from low-energy particles, a 2.0 mm tantalum shield with a 1.0 cm aperture was placed at the entrance to the main scattering chamber. The Spit-Pole Spectrograph scattering chamber located downstream of main scattering chamber was used to contain the Monitor-Detector Array (Section 4.5.4) during the experiments (Figure 4.5). The main scattering chamber and all detectors were aligned using an optical telescope located upstream of the scattering chambers which was itself aligned to a fixed point level with the beam height of the accelerator.
4.4 Targets

The \(d^{(10}\text{C},t)^{9}\text{C}\) experiments used two different types of targets: deuterated polyethylene (\([\text{CD}_2]_n\)) and natural carbon (\(\text{C}_{\text{nat}}\)). A selection of \([\text{CD}_2]_n\) targets were used as deuteron targets in both experimental runs. The primary target for the February 2011 experiment was a 660 \(\mu\text{g/cm}^2\) \([\text{CD}_2]_n\) foil. Two thicker \([\text{CD}_2]_n\) foils (1.1 mg/cm\(^2\) and 1.7 mg/cm\(^2\)) were used for the shorter May 2011 run in an attempt to increase statistics. The use of thicker targets increases the effects of energy loss and straggling to the outgoing particles of interest. In the case of the \(d^{(10}\text{C},t)^{9}\text{C}\) reaction at 17 MeV/A, the tritons emitted have energies greater than 7 MeV and lose less than a percent of their energy in the \([\text{CD}_2]_n\) target [82]. Table 4.2 lists the calculated energy loss of tritons in \([\text{CD}_2]_n\) for several energies. The energy lost by the \(^{10}\text{C}\) beam in the target is a small effect due to the high energy and low mass of the beam as well as the low mass of the targets. Simulations with the TRIM code calculate the average energy loss of a 171-MeV \(^{10}\text{C}\) beam in the thickest \([\text{CD}_2]_n\) target to be approximately 2.0 MeV [82].

Natural carbon targets were used to address the backgrounds from the \(^{12}\text{C}\) contaminant in the \([\text{CD}_2]_n\) targets. In the February 2011 experiment, an extensive set of data was collected with a \(^{10}\text{C}\) beam incident on a 1.2 mg/cm\(^2\) carbon target. It was determined that for the \(d^{(10}\text{C},t)^{9}\text{C}_{gs}\) reaction there is negligible background from the carbon contaminant in the \([\text{CD}_2]_n\) targets. The background contributions to the \(^{9}\text{C}\) ground- and excited-state results are discussed in Sections 5.5.3 and 5.6.3 respectively.

The stoichiometry of the thickest \([\text{CD}_2]_n\) target was tested both before and after the May 2011
experiment. The target underwent the two scattering experiments at the Western Michigan University Accelerator Laboratory in Kalamazoo, MI. A beam of 4.5-MeV protons was scattered off the target and detected in a 150 µm silicon surface-barrier detector at several laboratory angles. The active area of the detector was defined by a 4.76 mm aluminum collimator located 16.0 cm from the target. The proton beam was monitored by integrating the charge on a Faraday cup located at zero degrees. In addition to the 1.7 mg/cm$^2$ CD$_2$ target, scattering data were taken for the 660 µg/cm$^2$ [CD$_2$]$_n$ and 1.6 mg/cm$^2$ C$_{nat}$ targets. An analysis and summary of these data is described in Section 6.2.

Table 4.2: Triton energy losses and radial straggling in 850 mg/cm$^2$ of deuterated polyethylene. Calculations performed with the SRIM code [82].

<table>
<thead>
<tr>
<th>$E_{triton}$ (MeV)</th>
<th>$dE/dx$ (MeV/mg/cm$^2$)</th>
<th>$E_{loss}$ (MeV)</th>
<th>Radial Straggling (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.139</td>
<td>0.118</td>
<td>7.87</td>
</tr>
<tr>
<td>10</td>
<td>0.105</td>
<td>0.089</td>
<td>6.50</td>
</tr>
<tr>
<td>15</td>
<td>0.076</td>
<td>0.065</td>
<td>1.36</td>
</tr>
<tr>
<td>20</td>
<td>0.060</td>
<td>0.051</td>
<td>5.30</td>
</tr>
<tr>
<td>25</td>
<td>0.050</td>
<td>0.043</td>
<td>2.20</td>
</tr>
</tbody>
</table>

4.5 Detector System

4.5.1 Introduction

The $d^{(10}\text{C},t)^{9}\text{C}$ experiment required a detector system with sufficient acceptance and resolving power to detect and identify tritons in coincidence with carbon recoils. Several silicon detectors in $\Delta E$-$E$ telescope configurations were used to measure the energies and angles of coincident particles as well as to determine their mass (A) and nuclear charge (Z) (Figure 4.6). This technique is similar to that used in previous transfer reaction studies with radioactive beams at ATLAS, which utilized arrays of segmented silicon detectors [26] [83] [84].

4.5.2 DSSD Array

Light ions ($p, d, t$, etc.) were detected and identified in four annular double-sided silicon detectors (DSSDs) arranged in two $\Delta E$-$E$ telescopes. The telescope closest to the target (D1) was comprised of two Micron S1 DSSDs with thicknesses of 500 µm ($D1\Delta E$) and 1040 µm ($D1E$). The telescope furthest from the target (D2) was comprised of two Micron type S2 DSSDs with thicknesses 65 µm ($D2\Delta E$) and
Figure 4.6: A diagram of the shielding and detector system used in the \( d^{(10}C,t)^{9}C \) experiments.

1000 \( \mu \)m (D2\( \Delta E \)). The S1 detectors had an active inner radius of 24 mm, an outer radius 48 mm, and a total of 64 radial segments each a quarter of an annulus and 1.5 mm wide. The S2 detector had an active inner radius of 11 mm, an outer radius of 35 mm, and a total of 48 radial segments each 0.5 mm wide and almost a full annulus. The opposite side of both detector types were comprised of 16 azimuthal segments as illustrated in Figure 4.7.

Figure 4.7: A close-up image of the 48 annular segments (“rings”) of the D2\( \Delta E \) detector. The 16 azimuthal segments (“wedges”) of detector D1\( \Delta E \) can been seen in the foreground. Photo by J. C. Lighthall.

The full segmentation of each DSSD could not be utilized due to the limited number of preamplifier channels available. As a solution the signals of certain adjacent, geometrically relevant segments were combined into a single channel. For a single DSSD, raw signals are fed out from each segment through aluminum wires bonded to the detector surface to multi-pin connectors on the DSSD PC boards. The signals for all segments are then connected to a converter PC board where the four radial signals of each annulus are combined into a single channel. After this conversion, both S1- and S2-type detectors
have 16 radial channels ("rings") and 16 azimuthal channels ("wedges"). The wedge signals of the two E-residual DSSDs were further combined to make four signals, each being a contiguous quarter of the detector, to reduce the number of overall channels. The resulting 104 channels from the DSSD array were then connected to the scattering chamber feedthroughs and then to preamplifier circuits. Figure 4.8 depicts the number of pins, connector types, and signal conversion scheme for each DSSD detector.

Figure 4.8: The signal-conversion scheme for the DSSD Array. The total number of detector channels was reduced to the number of preamplifier channels available.

The DSSD telescopes were mounted in the scattering chamber on a single aluminum mount with all detectors parallel to the target plane. In order to maximize the angular coverage of each ejectile telescope, the separation between the ΔE and E detectors was minimized. This was done by mounting the detectors with the connector side of the PC board facing away from one another (Figure 4.9). The minimum separation distance between DSSD-ΔE and DSSD-E detectors was approximately 8mm for both telescopes. In total, the DSSD detectors covered laboratory angles between 7.2° and 42.7°. The effective coverage for the individual telescopes was between 24.8° and 39.7° for telescope D1, and 7.6° and 21.8° for telescope D2.

### 4.5.3 Recoil-Detector Array

Heavy beam-like recoils were detected in an array of eight planar silicon detectors arranged in four quadrant ΔE-E telescopes. The recoil telescopes were mounted on a copper mount and placed in the rear of the scattering chamber at a distance of 39.4 cm from the target (Figure 4.10). The recoil-detector
array subtended angles between $1.3^\circ$ and $7.0^\circ$ in the laboratory and had a square central aperture of approximately $1.0 \text{ cm} \times 1.0 \text{ cm}$. The azimuthal coverage of each quadrant was $82^\circ$ and thus the azimuthal coverage of the recoil-detector array was $\Delta \phi = 1.64\pi$ radians. The $\Delta E$ detectors were approximately $80 \mu\text{m}$ thick and the $E$-residual detectors were $1000 \mu\text{m}$ thick except one (E4) that was $1500 \mu\text{m}$. The eight signals from the recoil detectors were connected to a single feedthrough on the top of the main scattering chamber and then to preamplifier circuits.
4.5.4 Monitor-Detector Array

To monitor the beam intensity and purity, a ΔE-E telescope was placed in a second scattering chamber approximately one meter downstream from the target in order to measure the elastic scattering of the radioactive beam and its contaminants from a gold target. The Monitor-Detector Array was comprised of two annular Micron model LL1 detectors with thicknesses of 145 μm (ΔE<sub>mon</sub>) and 492 μm (E<sub>mon</sub>). These detectors were mounted on an aluminum fixture and placed at the rear of the Split-Pole Spectrograph scattering chamber (Figure 4.11). The mount consisted of a 4.76 mm thick aluminum front piece with a 1.0 cm center hole. To prevent degraded beam from reaching the detectors, the front piece was shielded with a 0.25 mm-thick tantalum collimator centered about the hole and with the same diameter aperture. A gold target foil of either 200 or 500 μg/cm<sup>2</sup> was clipped to the downstream face of the front piece, occluding the center hole. The detectors were mounted on an aluminum back piece at a distance of 10.16 cm. The detectors were clamped into place using two pieces of Lexane to minimize the distance between ΔE<sub>mon</sub> and E<sub>mon</sub>. The telescope covered laboratory angles from 5.0° to 9.2° relative to the gold target.

![Figure 4.11: The Monitor-Detector Array that was mounted in the Split-Pole Spectrograph scattering chamber and used to measure the elastic scattering of the beam from a gold foil. Photo by J. C. Lighthall.](image)

The LL1-type monitor detectors are segmented into quadrants on one side and a single segment on the other. Since no azimuthal information was required to accurately monitor the beam, the four quadrants were combined into one signal. The aluminum bonding wires connecting two of the four quadrants on the
$\Delta E_{\text{mon}}$ detector had their bonds broken during shipping leaving the monitor telescope with an efficiency of at most 50%. The efficiency of the monitor telescope is addressed in the analysis chapter using Monte Carlo simulations (Chapter 6). During the February 2011 experiment, the target holder in the SPS scattering chamber obscured the beam during the run. The aluminum target holder frame was thick enough to stop the $^{10}$C and $^{10}$B beams allowing only light breakup products to encounter the gold foil and scatter into the telescope. While the particle identification is not of direct use, the absolute number of counts in the monitor detectors is a quantity essential to the measurement. Thus, the most accurate monitor of the beam for the February 2011 experiment is the $^{10}$C elastic-scattering data from the $^{12}$C in the $[\text{CD}_2]_n$ target. This Monitor-Detector Array coverage issue was remedied before the May 2011 run which has two sets of beam-flux integration data.

4.6 Signal Processing and Data Acquisition

4.6.1 Introduction

The signals from the silicon detectors used in the $d(^{10}\text{C}, t)^9\text{C}$ experiments required processing before the data could be recorded, organized, and eventually analyzed. Analog signals were required to measure the energy deposited by ions as well as the timing response of individual detector segments. These signals are used as inputs for Analog-to-Digital and Time-to-Digital Converters (ADCs and TDCs) which digitize them into the data stream of a data acquisition system (DAQ). Digital signals are used as logic to trigger the DAQ and to serve as inputs to rate-meters and/or scalers. The following section details the scheme of the electronics that produced these signals and how they were used in the experiments.

4.6.2 Signal Amplification and Electronics Scheme

The signals produced from ions passing through or stopping in the silicon detectors used in the experiments required two stages of amplification. The first stage, or preamplification, was implemented at the feedthroughs of the scattering chambers and produced step-like signals with fast rise-times ($\approx$10–20 ns), exponential fall times (time constant $\approx$50 $\mu$s), and amplitudes in the millivolt range. The second stage of signal amplification was done by shaping amplifiers which take the step-like energy signals and
output an amplified Gaussian-shaped waveform. After the shaping amplifiers, the signals have amplitudes in the volt range (1–10V) and are then connected as inputs to ADCs to readout detector energy signals to the data acquisition system.

The shaping amplifier modules also produce a logic “trigger” signal when one or more channels is above a set threshold. Trigger signals are NIM (Nuclear Instrumentation Module) standard pulses which are nominally zero volts for a logic-false (0) bit and -0.8 V for a logic-true (1) bit [85]. The trigger signals are used to indicate when different elements of the detector system have particle or calibration data. These signals are then used to generate a master or “event” trigger for the data acquisitions system. For both \(d^{10}\text{C},t\)\(^9\text{C}\) experiments there were six different elements to the event trigger: DSSD-telescopes D1 and D2, the Recoil-Detector array, the Monitor-Detector Array, the diagnostic \(\Delta E\)-E telescope, and a pulser. All telescopes triggers were the logical-OR of the \(\Delta E\)- and E-detectors except for the Recoil-Detector array that used the logical-OR of all four \(\Delta E\) detectors. The reason for using a logical-OR for most trigger sources was to include particles which produce a signal in only one detector in a telescope (e.g., low-energy particles, etc.). Figure 4.12 shows the different trigger sources used to make the event trigger for the DAQ.

4.6.3 Data Acquisition System

The SCARLET data acquisition system, developed by Dr. Ken Teh in the Physics Division at Argonne National Laboratory, was used in the experiments. The SCARLET DAQ is an expandable, Ethernet-based system that is designed to work with CAMAC modules specifically for experiments utilizing silicon and other charged particle detectors with high trigger rates (kHz–MHz). For this experiment, a single SCARLET system was used which controlled two CAMAC crates containing eight Phillips 7164 ADCs, a Phillips 7168H TDC, and a Lecroy 2251 Scaler Module. The central unit of the system was the SCARLET event-builder, a single Linux server used by the experimenter to initiate and control data acquisition. Once acquisition is started, an event trigger is distributed to the two CAMAC crates via secondary, dedicated Linux servers called Read-On-Command (ROC). The ROCs communicate with the CAMAC crates and initiate charge integration and digitization of the signals from the detectors. During this process, the SCARLET system outputs a “busy” signal which is the primary source of the system-
Figure 4.12: Electronics diagram for the $d^{10}\text{C},t)^9\text{C}$ experiments. The event trigger for the SCARLET data acquisition system was designed for use in all stages of the experiment (i.e., calibration, beam development, and beam-on-target measurements) without having to alter the trigger electronics. The trigger had six components from the detector array, diagnostic telescope, and a pulser. The trigger could be inhibited by the “busy” signal from the DAQ system or the ATLAS accelerator out-of-lock signal.
wide veto signal which inhibits the electronics from triggering the DAQ. Figure 4.13 is a schematic of
the SCARLET data acquisition system.

Figure 4.13: Diagram of the SCARLET data acquisition system used in both February and May 2011
experiments.
Chapter 5

Data Analysis

5.1 Introduction

The object of data analysis is to go from the raw data produced by the experiment, to a quantity such as a nuclear half-life, excitation energy, or reaction cross section. This process is accomplished in two stages. In the first stage, the raw data are manipulated in such a way that only the data pertinent to the quantity of interest are selected. In the second stage these data are used to extract the measured quantity using a known mathematical relationship (e.g., nuclear decay, Q-Value, etc.). This chapter covers the former stage of data analysis for the $d(^{10}\text{C},t)^{9}\text{C}$ experiments which includes the processes of data handling, calibration of the detector system, particle identification, and the selection of the $^{9}\text{C}$ ground and excited state transitions. The latter stage of analysis where final-state angular distribution data extracted and compared to theory is the content of Chapters 6 and 7, respectively. The methods shown in this chapter were used for both February and May 2011 data sets unless indicated otherwise.

5.2 Data Handling

5.2.1 SCARLET Data to ROOT TTrees

The on-line data analysis was performed with the DAPHSORT client on the ANL Physics Division MUSIC cluster. DAPHSORT made the analysis of large data sets slow and difficult to perform remotely. The solution to this difficulty was to convert all data from the archived SCARLET format into data structures using the ROOT data analysis libraries [86]. A ROOT file was made for each SCARLET run file with data sorted into a number of structured ROOT TTree objects. A ROOT TTree, or “tree”,...
Figure 5.1: Structure and member data of ROOT TTree objects used in the analysis for the $d(^{10}\text{C},t)^{9}\text{C}$ experiments.

The object has a hierarchical structure resembling a tree with an instance having “branches” and “leaves” of data. In addition to the user-defined form the ROOT TTree object allows for data compression and enhanced access speed [86]. Once converted into files of trees, any number of file groups could be made and analyzed in comparably little time (e.g., minutes vs. hours). The general ROOT files used in both analyses contained three TTree objects containing all of the information from the archived SCARLET data. The trees and their constituent data are detailed below:

**ADC Data**

The “adcdata” tree contains all of the data from eight Phillips 7164 Analog-to-Digital Converters (ADC) and one Phillips 7168H Time-to-Digital Converter (TDC). Also included are several pieces of information from the SCARLET event buffers (e.g., run file number, UNIX time of event, length of event, etc.). The tree stores all of these data in four “branches”: run information (dat), TAC and TDC ($c10time$), DSSD Array ($dssd$), and Recoil- and Monitor-Array data ($recmon$). The member data of these branches are shown in Figure 5.1.

**Scaler Data**

The “scalerdata” tree contains the data from the Lecroy 2251 Scaler Module. A scaler buffer was produced by the SCARLET event builder every ten seconds and placed into the data stream. This TTree has a single branch, a fourteen element array ($scl[14]$) containing the following information: the time since acquisition began (in seconds), the time since the last scaler buffer (in seconds), and the twelve scaler channels. This branch allows any of the monitored rates to be plotted versus the relative and —
with the timedata tree — absolute time of events.

**Time Data**

The “timedata” tree contains the relevant time information from each run file. The time of the first \( t_{\text{start}} \) and last \( t_{\text{stop}} \) acquisition events as well as their difference \( t_{\text{tot}} \) are recorded in a single “branch”. These three times, stored in the Unix format\(^1\), are set by the SCARLET event builder with a resolution of one second. While these time data do not provide event-by-event discrimination, it is sufficient to provide an absolute time on data acquired within the resolution and to generate scaler-like rates for any data from the “adcdata” tree.

### 5.3 Data Reduction

The ROOT files described in the previous section contain an entry in each tree for every triggered DAQ event. The multiple sources of the event trigger (Figure 4.12) imply that not all events are pertinent to every part of the analysis. To this end, the “adcdata” tree was divided into three subsets. This task was done by creating new trees each satisfying the conditions of a particular type of event: a monitor, singles, or coincidence data event. The conditions for each subset are listed in Table 5.1. The subset TTree objects were then saved to a separate, smaller file which included the time and scaler trees. In total, these subsets were a quarter of all data acquired.

<table>
<thead>
<tr>
<th>Subset Name</th>
<th>Criteria</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor Data</td>
<td>( \text{anyMON} \land \neg \text{anyDSSD} \land \neg \text{anyREC} )</td>
<td>A monitor detector event</td>
</tr>
<tr>
<td>Singles Data</td>
<td>( \neg \text{anyMON} \land (\text{anyDSSD} \oplus \text{anyREC}) )</td>
<td>Either a single DSSD or RECOIL event</td>
</tr>
<tr>
<td>Coincidence Data</td>
<td>( \neg \text{anyMON} \land \text{anyDSSD} \land \text{anyREC} )</td>
<td>A DSSD-RECOIL coincidence event</td>
</tr>
</tbody>
</table>

Table 5.1: The three subsets of detector data sorted from the “adcdata” TTree object. The defining criteria are written in symbolic logic where “!” is a logical NOT, \( \land \) is a logical AND, \( \lor \) is a logical OR, and \( \oplus \) is an exclusive-OR (XOR) operator. All elements listed are assumed to be above an array-specific threshold.

\( ^1 \)A time format defined as the number of seconds, excluding leap seconds, since midnight Thursday, January 1\(^{st} \), 1970.
Coincidence Data

The coincidence subset, having data in both DSSD and Recoil-Detector Arrays, contains all of the important information for the transfer-reaction measurement. An event with a triton of proper energy and angle in the DSSD Array, coincident with a $^9$C in the Recoil-Detector Array, is a distinct signature of the $d(^{10}\text{C},t)^9\text{C}_{gs}$ reaction. These coincidence data would also contain events of transfer to proton-unbound excited states in $^9\text{C}$. A $^9\text{C}$ nucleus produced in an excited state can decay via one- or two-proton emission to states in $^8\text{B}$ and $^7\text{Be}$. Tritons detected in coincidence with these decay products, with the proper energy and angle, are evidence of populating $^9\text{C}$ excited states via the $d(^{10}\text{C},t)^9\text{C}$ reaction.

Singles Data

The subset of “singles”, or non-coincident data, contains events with particles in either the DSSD or Recoil-Detector Arrays. These data contain information about the beam and reactions for which the detector system is not sensitive to in coincidence. In the singles subset, the recoil detectors are sensitive to elastic and inelastic scattering of the $^{10}\text{C}$ beam on the $^{12}\text{C}$ and deuterium in the \([\text{CD}_2]_n\) target which provides a means of monitoring the beam intensity over time. The elastic scattering of the less intense $^{10}\text{B}$ and $^7\text{Be}$ beams can be used to measure the purity of the in-flight beam. All light particles detected in the DSSD Array as singles can be used with coincidence data to identify particle groups and provide sufficient statistics to create particle gates for different species ($p, d, t, \text{etc.}$).

Monitor Data

The subset of monitor data was made to determine the absolute normalization of the beam. The monitor telescope was sensitive only to elastic scattering of the in-flight and contaminant beams on a Au target. By integrating the number of $^{10}\text{C}$ particles detected and identified in the $\Delta\text{E-E}$ monitor telescope, the total in-flight beam flux can be calculated. The absolute cross section of the $d(^{10}\text{C},t)^9\text{C}$ reaction can then be calculated using this quantity. These calculations are part of the analysis of all beam-flux integration data discussed in Section 6.3.
5.4 Energy Calibration

The calibration of the detector system was performed with several different sources of different ion species and energy. The DSSD Array was positioned and had amplifier gains set to detect and identify tritons from 7 to 26 MeV, and was calibrated with a selection of radioactive sources. Alpha particles of a known energy stop in a DSSD producing a raw spectrum with energy peaks that are used to calibrate the detector (Figure 5.2). The centroids of the peaks are found with a peak-fitting algorithm and the relationship between energy and raw channel is determined by a linear fit:

\[ E = P_1 \text{channel} + P_0 \] (5.1)

with \( E \) being the calibrated energy in MeV, \( P_1 \) the slope of the line in MeV/channel, and \( P_0 \) the offset also in MeV. The Recoil-Detector Array was designed to detect and identify heavier ions as energetic as the beam (~17 MeV/A) and was calibrated with an in-situ calibration technique described in Section 5.4.3. While these methods are sufficient to calibrate the detectors to first order, there are multiple effects that must be taken into account.

Figure 5.2: The calibration of a detector spectrum with two peaks of reference.
5.4.1 Energy Defect

When an ion with energy \( E_{\text{particle}} \) is stopped in a biased silicon detector it produces an energy pulse. The pulse is a result of the motion of the electron-hole pairs produced by the ionization of the bulk silicon. In silicon detectors, the energy required to produce an electron-hole pair (\( \epsilon \)) has been found to depend on the nuclear charge (\( Z \)) of the detected ion [91]–[93]. The difference in the value of \( \epsilon \) between proton and helium isotopes is on the order of 0.3% with an average value of 3.6 eV per electron-hole pair. The amplitude of the signal produced is proportional to the energy deposited in the silicon detector. The detected energy \( E_{\text{detector}} \) is not equal to the initial ion energy due to energy lost in the silicon detector as well as in any foils or source material encountered. Early experimental use of silicon surface-barrier detectors revealed that heavy ions stopped in a silicon detector will produce a smaller amplitude pulse than a lighter ion with the same energy. This “pulse height defect” or “energy defect” is the net energy loss of the ion due to different processes [88][89]. The contributions to the energy defect can be summarized in the following equation:

\[
E_{\text{detector}} = E_{\text{particle}} - \Delta E_w - \Delta E_n - \Delta E_r \tag{5.2}
\]

where \( \Delta E_w \) is the energy lost in the window or “dead layer” of the detector, \( \Delta E_n \) is the energy lost in elastic collisions with nuclei the detector medium, and \( \Delta E_r \) is the energy lost due to the recombination of electron-hole pairs in the wake of the ionizing particle. The energy lost by a particle via “electronic stopping”, or inelastic collisions with bound electrons, in the fiducial volume of the silicon detector results in \( E_{\text{detector}} \). The detector is insensitive to the electronic stopping in the dead layer producing what can be a large component of the overall defect. The energy loss due to electronic stopping is determined by the Bethe Formula (non-relativistic case, for \( \frac{v}{c} \ll 1 \)):

\[
\frac{dE}{dx} = \frac{4\pi n Z^2}{m_e v^2} \left( \frac{e^2}{4\pi \varepsilon_0} \right)^2 \ln \left( \frac{2 m_e v^2}{I} \right) \tag{5.3}
\]

where \( E \) is the particle’s energy, \( x \) is the distance traveled by the particle, \( n \) and \( I \) the electron density and the mean excitation potential of the medium, respectively. The energy lost by an ion is directly proportional to its mass and the square of its nuclear charge as well as inversely proportional to its energy.
\[
\frac{dE}{dx} \propto \frac{Z^2}{v^2} \approx \frac{Z^2 m}{2E}
\]

(Equation 6.4).

For a pair of ions with the same energy and differing mass and/or charge, the heaviest, most highly charged ion will lose more energy in the dead layer of the silicon detector.

Figure 5.3: A graph of electronic and nuclear stopping powers for alpha particles in silicon. Data from Ref. [87].

All charge-ionization detectors, including silicon detectors, are insensitive to energy lost by an ion via nuclear collisions. The “nuclear stopping” in the detector does not ionize the bulk medium and thus does not contribute to the charge collection. This energy loss is still small compared to that from electronic stopping by two to three orders of magnitude (Figure 5.3). Nuclear stopping increases at the very end of a particle’s range at very low energy where collisions are more probable. The defect due to the recombination ($\Delta E_r$) is important in the detection of heavy ions ($A \geq 100$) and is below the attainable resolution for light ions ($1 \leq Z \leq 17$) at low and intermediate energies [89]–[93]. In the detector calibrations for this study, both the nuclear stopping and recombination contributions to the defect are considered small enough to be neglected. The energy lost before the ion encounters the detector as well as the dead layer losses are considered for both calibration methods.
5.4.2 DSSD Array Calibration

The calibration of the DSSD Array was performed using two different alpha-emitting radioisotope sources: $^{228}\text{Th}$ and $^{148}\text{Gd}-^{244}\text{Cm}$. These sources provide alpha particles at energies ranging from 3.18 to 8.79 MeV (Figure 5.4). The sources were placed in various positions in the target chamber in order to illuminate all of the individual detectors (Figure 5.5). The accumulated spectra of alpha particle energies were used to derive calibration constants for the DSSDs. This process utilized a peak-fitting program to determine the centroids of the raw energy peaks, and then a linear fit was performed to match to the known alpha energy values. The maximum energy lost by a 5.42 MeV-alpha particle in the thickest dead layer in either DSSD (2.0 $\mu$m) is approximately 400 keV at large angles ($\sim$40°). The energy lost by a 7 MeV triton in the same thickness dead layer is approximately 65 keV ($\sim$1%) and thus the dead layer effects were not corrected for in the DSSD calibration.

There were sufficient statistics to perform a pixel calibration on the DSSD-ΔE detectors which treats each ring-wedge segment as an individual detector. This method removes contributions from segments with poor resolution that can be present in a “strip” calibration. For the February 2011 run, the DSSD-E detectors did not have sufficient alpha data for a full pixel calibration due to the compact geometry of the array, thus strip calibrations were done for all rings and wedges. The constants before and after the February 2011 experiment changed less than 5% with the exception of those for detector D2ΔE (Figure 5.7). A likely source for this change is radiation damage from the scattered primary or in-flight beam.
5.4.3 Recoil-Detector Array Calibration

The “Punch-through” Method

Detector arrays with ∆E-E telescope configurations designed to detect and identify high energy ions (>100 MeV) can be difficult to calibrate with radioactive sources. In many cases, the thin ∆E-detector can completely obscure the thick E-residual detector from alpha particles. Without the use of radioactive sources, the only ways to obtain an energy calibration are either through the use of an electronic pulser to simulate the detector signals into the preamplifier inputs, or to perform an in-situ calibration with a beam-induced source. The “punch through” calibration method can be used for a ∆E-E telescope in an experiment where several different identifiable ion species, over a range of energies, both stop in and penetrate the two detectors. The energy lost in each detector is primarily due to electronic stopping described by the Bethe Equation (Equation 5.3). If the energy lost in the ∆E-detector is plotted against the energy lost in the E-residual detector then ions are grouped in hyperbolic curves proportional to:

\[
\frac{dE}{dx}(E - \frac{dE}{dx}) \propto Z^2 \frac{ln(v^2)}{v^2} m \frac{v^2}{2} \approx \frac{Z^2 m}{2E_x} \]  

(5.5)
assuming $\frac{dE}{dx} \ll E$ and the energy dependence $x \approx 0.8$. The maximum energy of an ion that stops in the E-residual detector ($E_{pt}$) is indicated as a single point on a particle identification spectrum (Figure 5.6). Ions with energies greater than $E_{pt}$ will have both smaller overall telescope energies as less energy is deposited in the two detectors. These loci can be calculated with the Monte Carlo energy loss code “Stopping Radiation In Matter” (SRIM) using the known detector thicknesses, mass, and nuclear charge of the ions. The SRIM calculation utilizes a model developed by Ziegler, Biersack, and Littmark which allows for the calculation of ion stopping powers to an accuracy of 5% [94] [95]. At its “punch-through” locus, an ion provides a calibration point for both $\Delta E$ and E-residual detectors. This method was ideal for the calibration of Recoil-Detector Array given the many different heavy ions produced during the $d^{(10}C, t)^{9}C$ experiments. The energy lost in the dead layers of the Recoil-Detector Array were small in comparison to energy of the incident ions ($\leq 1\%$) and was not accounted for in the punch-through calibration.

Figure 5.6: An example of the “punch-through” calibration method for a $\Delta E$-E telescope using three different ion species. The punch-through locus ($E_{pt}$) for each ion provides an energy for both E and $\Delta E$ detectors.
5.5 The $d(^{10}\text{C},t)^{9}\text{C}$ Ground State Reaction

5.5.1 Coincidence Timing

An event from the coincidence data set is known to have at least one particle detected in the DSSD Array and another in the Recoil-Detector Array. The actual time difference in which these two particles encounter their respective detectors, relative to one another, is referred to as the coincidence time of the event. Selecting events which have realistic coincidence timing is an effective technique to reduce the backgrounds due to random coincident processes. From the ROOT data alone this is known within the resolution of the data acquisition system ($<1\mu$s). The actual time is approximately 2–5 nanoseconds for the $d(^{10}\text{C},t)^{9}\text{C}$ reaction in the experimental geometry. To measure this very small time interval with higher precision, the timing signals from all pertinent channels are processed and delayed and placed into either a TAC or TDC.

TAC Spectrum

The time between ions detected in the DSSD array and those detected in the recoil detectors was measured by a Time-to-Amplitude Converter (TAC). The TAC received a start signal from the logic-OR of the D1ΔE and D2ΔE rings and the a stop signal from the logic-OR of all four recoil ΔE-detectors. A one-dimensional spectrum of difference in time between these two signals was made with the full range of 400 nanoseconds (Figure 5.7). This spectrum was used during the on-line analysis of the experiment to determine the subset DSSD-Recoil coincidences to be evaluated. In the off-line data analysis, coincidence data were identified through different criteria (Table 6.1) making this method redundant. Thus the TAC spectrum was used as a measure of time between a particle depositing energy in a DSSD detector and another encountering a recoil ΔE-detector in the same event.

The TAC spectrum was found to be an insufficient measure of the coincidence timing as it was inconsistent over the entire data set. Over the course of the February 2011 experiment, a subset of TAC events with known DSSD-Recoil coincidences — including known $^{9}\text{C}$-triton events— were found to have values of 0.5% of full scale. These events come from instances where the TAC receives a START signal from the DSSD Array and does not receive a STOP signal from a recoil ΔE-detector. A possible reason for this is that one or more ΔE-detector signals fell below its discriminator threshold. A cause for this
change could be radiation damage to the thin (80 μm) ΔE-detectors by neutrons or scattered beam. The overall DSSD-Recoil coincidence rate changed little during the February 2011 experiment, while the distribution of TAC measurements with the “zero” values increased relative to coincidences within the general TAC gate (Figure 5.8). This supports the notion that radiation damage contributed to this inefficiency. As a result, the TAC data were not used as a criterion for the identifying the $d(^{10}\text{C},t)^{9}\text{C}$ reaction. The count rates for the experiment were not high and the coincidence condition on the ROOT data was sufficient to select these data.
5.5.2 Particle Identification

The products from the \((d^{10}C,t)^9C\) reaction to the ground state of \(^9C\) were detected and identified in the detector system. Tritons of the proper energy and angle were detected and identified in the DSSD telescopes in coincidence with \(^9C\) recoils in the particle-bound ground state \((t_{1/2}=126 \text{ ms})\). These events were identified by applying particle and timing gates on the coincidence data acquired with a \(^{10}C\) beam bombarding a deuterated polyethylene \(([CD_2]_n)\) target. The process of identifying the tritons and \(^9C\) recoils is detailed in this section.

![Figure 5.9: \(\Delta E\) vs. \(E\) particle identification spectrum for a recoil telescope quadrant with ions are grouped by proton number \((Z)\). The “general” gate on all carbon-like ions is shown in black.](image)

**Carbon Recoil Identification**

The heavy, beam-like products from many different reactions were detected in the Recoil-Detector Array in coincidence with at least one particle in the DSSD Array (Figure 5.9). The resolution of the recoil telescopes was sufficient to resolve the separate isotopes \(^8\)B from \(^{10}\)B and \(^7\)Be from \(^9\)Be. Identification was made simple due to the absence of the particle-unbound isotopes \(^9\)B and \(^8\)Be. The recoil telescopes could not resolve \(^9\)C from the \((d,t)\) reaction and scattered \(^{10}\)C ions due to the small difference in energy loss between these species in the thin \(\Delta E\)-detectors. A general two-dimensional gate on all carbon-like ions was made for each of the four recoil particle-identification spectra (Figure 5.9). Events satisfying this selection criterion were used to examine coincident light particles present in the DSSD array.
Triton Identification

Light ions from different nuclear reactions were detected in the DSSD Array and their mass (A) and nuclear charge (Z) were identified from ΔE vs. E particle identification spectra. Protons, deuterons, and tritons, in addition to stable helium isotopes, are clearly identified in both DSSD telescopes (Figure 5.10a and b). Those particles that were coincident with carbon recoils were selected by applying the general carbon gate to these spectra. Figure 5.10c and 5.10d show that the resulting spectra are primarily protons and tritons with all other possible ejectiles suppressed. The greatest source of protons in the DSSD Array is from fusion-evaporation reactions of the in-flight beam on the $^{12}$C in the [CD$_2$]$_n$ target. The production of tritons in the $^{10}$C+$^{12}$C system is seen to be small when compared to those from the $d(^{10}$C,t)$^{9}$C reaction. Backgrounds to the triton gates are discussed in Section 5.5.3.
Figure 5.11: $\Delta E$ vs. $E$ particle identification spectrum for a recoil telescope quadrant gated on coincident triton ejectiles. The $^9\text{C}$ recoils from the $d(^{10}\text{C},t)^9\text{C}$ can be seen within the $^9\text{C}$-gate (red) being a subset of the “general” carbon gate (black).

$^9\text{C}$ Identification

In order to completely identify all reaction products of interest, gates were made for all tritons identified in each DSSD telescope. Spectra from singles and coincidence data sets were used to make the two-dimensional triton gates as the total number of tritons in either data set were less numerous that of either deuterons or protons (Figure 5.10c & d). In order to check the consistency of the general carbon gate in identifying tritons from the $d(^{10}\text{C},t)^9\text{C}$ reaction, these gates were used to identify recoil events coincident with tritons. Figure 5.11 shows that $^9\text{C}$ ions are clearly identified lying in the lower part of the general carbon gate in the triton-gated spectra of the recoil detectors. A refined $^9\text{C}$ gate for each recoil telescope was made as a replacement for the general carbon gate. The combination of both and $^9\text{C}$-recoil gates permit an unambiguous identification of the $d(^{10}\text{C},t)^9\text{C}_{gs}$ reaction.

Additional Triton Identification

Most tritons from the $d(^{10}\text{C},t)^9\text{C}$ reaction, emitted within the acceptance of the DSSD array, fall within the two-dimensional triton gate. Some tritons encounter a DSSD-$\Delta E$ segment with sufficient thickness to stop inside the detector. These events do not have a residual energy to plot in the particle identification spectra and exist outside the triton gates. This inefficiency in the DSSD Array affects a small portion of the angular coverage for tritons produced in the ground state transition. Tritons emitted
at angles between 21° and 24.5° with energies from 11.5 to 13.7 MeV stop in the D1ΔE detector. This phenomenon can best be seen the histogram of laboratory angle versus triton energy where the unshaded triton spectrum has a gap in energy and angle (Figure 5.12). Those counts present in the red-shaded region of Figure 5.12 are tritons which stopped in the D1ΔE detector and were selected by a different set of criteria.

![Figure 5.12: A histogram of laboratory angle vs. triton energy for the d(10C,t)9Cgs reaction produced from both 9C- and triton-gates. The dashed line is the calculated kinematic curve for the tritons. The two shaded regions define the gaps in the detector acceptance due to geometric (blue) and detector thickness (red) inefficiencies.](image)

These “stopped” triton events were identified by analyzing the subset of coincidence events without D1E signals and applying stringent gates on both ejectile and recoil spectra. First, the laboratory angle vs. energy spectrum for particles stopped in the DSSD-ΔE detectors is gated on the region of interest (Figure 5.12). A complementary gate is also placed on the total recoil energy vs. total ejectile energy plot for 9C- and triton-gated coincidences which has the same range in ejectile energies (Figure 5.13). In addition to the 9C recoil gate, a condition is placed on the data such that the particles detected in the DSSD and Recoil-Detector Array must be coplanar \( (\phi_{\text{ejectile}} \approx \phi_{\text{recoil}} + \pi) \) due to conservation of momentum. All tritons detected in the February 2011 experiment, including the “stopped” tritons satisfying the listed conditions, are plotted in Figure 5.12.
5.5.3 Sources of Background

The sources of background in the identification of the \(d^{(10\text{C},t)^9\text{C}}_{gs}\) reaction are either from the contaminants in the in-flight beam or the \(^{12}\text{C}\) component of the \([\text{CD}_2]_n\) target. Any reaction that produces a triton- or \(^9\text{C}\)-like signal in the detector system would be significant and must be accounted for in the analysis. As detailed in Section 4.2.4, the in-flight \(^{10}\text{C}\) beam was accompanied by less intense beams of \(^{10}\text{B}\) and \(^{7}\text{Be}\). There are only a few reactions between the contaminant beams and the \(^{12}\text{C}\) or deuterium in the \([\text{CD}_2]_n\) target which produce an appreciable background for the \(d^{(10\text{C},t)^9\text{C}}_{gs}\) reaction.

The triton-producing transfer reactions \(d^{(10\text{B},t)^9\text{B}}\) \((Q=-2.18\ \text{MeV})\) and \(d^{(7\text{Be},t)^6\text{Be}}\) \((Q=-4.42\ \text{MeV})\) do not provide a significant source of tritons with energies large enough \((E \geq 7\ \text{MeV})\) to be present in the triton gates. \(^9\text{C}\) ions can only be produced through complex two- or three-nucleon transfer reactions which have very small cross sections and thus are strongly suppressed compared to the \((d,t)\) reaction.

To address \(^{12}\text{C}\)-induced backgrounds, the same analysis performed on the \(^{10}\text{C}+[\text{CD}_2]_n\) data to identify the \(d^{(10\text{C},t)^9\text{C}}\) reaction was done for the \(^{10}\text{C}+^{12}\text{C}_{nat}\) coincidence data set. The two DSSD particle-identification spectra gated on the general carbon gate show that mostly protons and a small number of deuterons are produced from reactions on the \(^{12}\text{C}\) contaminant. Tritons are not observed within the triton gate coincident with any \(Z=6\) recoil. Figure 5.14 compares the background spectra with a natural
Figure 5.14: $^9$C-gated particle identification spectra for both DSSD telescopes from data taken with $^{10}$C beam incident on CD$_2$ (blue) and natural carbon (red) targets. There are no processes that generate a triton background due to the beams or $^{12}$C target contaminant.

carbon target to the [CD$_2$]$_n$ target data.

5.6 $^9$C Excited States

5.6.1 Detection

A major goal of this study was to populate $^9$C in its low-lying excited states via the $d(^{10}\text{C},t)^{9}\text{C}$ reaction. All known and predicted excited states of $^9$C are unbound to one- and two-proton decay ($S_p = 1.3$ MeV, $S_{2p} = 1.47$ MeV). Identification of a $^9$C excited state thus requires the detection of its decay products in the Recoil-Detector Array in coincidence with energetic tritons from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction in the DSSD Array. The detector system was sensitive only to the products of the $^9$C decay and lacked the stopping power and angular resolution required to detect and identify the protons in order to discern their decay mechanism. The decay of the excited $^9$C recoil can result in secondary or tertiary trajectories for the final-state daughter which may be outside the acceptance of the Recoil Detector Array. For $^9$C excited-state decay, the three-, four-, or possibly five-body final states ($^8\text{B}_{gs}+p+t$, $^7\text{Be}_{gs}+2p+t$, or $\alpha+^3\text{He}+2p+t$) as well as the kinematic effect of a smaller maximum angle for the triton must be taken into account. A discussion of simulations for all final-state detection efficiencies is presented in Chapter 6.

An excited state of $^9$C can undergo two-proton decay through either prompt or sequential decay mechanisms. A prompt two-proton decay would involve the emission of both protons, either at large
angles (∼180°) or at the same angle in a so-called “di-proton” configuration, relative to the 7Be daughter. In a sequential decay, the 9C excited state would emit a single proton to an energetically accessible excited state in 8B and then a second proton decay to a state in 7Be. Previous studies have been able to determine the decay mechanisms of the 8C ground state as well as unbound states of 10C using resonance decay spectroscopy which requires high-resolution reconstruction of all reaction products [96] [97]. In order for a nucleus (A,Z) to undergo “true” prompt two-proton decay, there must be an absence of energetically accessible states in the intermediate system (A-1, Z-1) [98]. Since this is not the case for 9C excited states, it is assumed that all two-proton decays are sequential and decay through intermediate states in 8B.

**The 9C First Excited State**

The only confirmed excited state of 9C is at 2.2 MeV with a width of 100±20 keV [21]. There exist several states in 8B and 7Be which are accessible via one- or two-proton decay (Figure 5.15). In the decay of 9C2.2 there are two channels that have a final state of 8Bgs+p. The first being the one-proton decay to the final state, shown in blue in Figure 5.15. The second is the single-proton decay to the 770 keV (Γ=35.6 keV) first-excited state of 8B followed by a gamma decay to the ground state. The gamma decay branch has been measured and is approximately 7x10⁻⁵ and thus the 8B0.77 state is far more likely to proton decay to states in 7Be. Since the 8B ground state is particle-bound, all sequential two-proton decays for the 9C2.2 state go through the narrow 8B0.77 resonance which result in a 7Be0+2p final state. These channels include, a secondary proton decay to the 7Be ground state or the 429 keV first-excited state, both shown in red in Figure 5.15. The latter has a 100% gamma branch to the 7Be ground state. Thus, by populating the 9C2.2 state with the d(10C,t)9C reaction the only two possible final states are 8Bgs+p and 7Be0+2p coincident with tritons.

**Excited States Below 4.4 MeV**

All excited states in 9C between the two-proton and 3He separation energies (S3He = 4.397 MeV) can decay through the same channels to the same two final states. All 9C excited states that are at higher excitation energies than the first-excited state can decay through two additional intermediate states in 8B (Figure 5.16). The resonances at 2.32 MeV (Jπ=3+, Γ=350 keV±30) and 3.5 MeV (Jπ=2−, Γ=8±4
Figure 5.15: The energy levels and decay channels of interest for the 2.2 MeV first-excited state of $^9$C.

MeV) in $^8$B become accessible at $^9$C excitation energies of 3.15 and 2.8 MeV respectively. However there are no additional accessible states in $^7$Be and all two-proton decays have a final state of $^7$Be$_{gs}+2p$. Since all final states are the same, the discerning factor in the identification of which state in $^9$C was populated is the energy of the tritons in coincidence with the $^8$B and $^7$Be recoils.

Figure 5.16: Energy levels and states of interest in the one- (blue) and two-proton (red) decay of excited states between 2.2 and 4.39 MeV in $^9$C. A hypothetical state at 3.3 MeV ($\Gamma=500$ keV) is depicted and can decay through the wide 3.5 MeV ($\Gamma=8\pm4$ MeV) resonance to the ground states of $^8$B and $^7$Be. The $^8$B$_{3.5}$ state shown has a width of 4 MeV.

Excited States Above 4.4 MeV

Any excited state in $^9$C at an excitation energy greater than 4.4 MeV is unbound to $^3$He emission to the ground state of $^6$Be. The $^6$Be ground state is proton unbound ($\Gamma=92$ keV) and is known to decay
through the $^5\text{Li}+p$ channel to the $\alpha+2p$ final state. All other aforementioned one- and two-proton decays are also possible (Figure 5.17).

Figure 5.17: Energy levels and states of interest in the decay of $^9\text{C}$ excited states above 4.4 MeV in excitation energy. A hypothetical 5 MeV state ($\Gamma=500$ keV) is shown to decay by proton decay(s) or $^3\text{He}$ emission to $^8\text{B}_{gs}+p$, $^7\text{Be}_{gs}+2p$, or $\alpha+^3\text{He}+2p$ final states.

### 5.6.2 Particle Identification

Figure 5.18: A particle identification spectra for a Recoil-Detector Array telescope with (right) and without triton gates (left) applied. Note the presence of a small number of $^8\text{B}$ and $^7\text{Be}$ recoils (black) in coincidence with tritons.

Three sets of particle gates were used to identify $^9\text{C}$ excited states from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction. Since the reaction was performed in inverse kinematics (Section 3.2), the tritons produced in populating $^9\text{C}$ excited states are higher in energy than those from the ground-state transition. The triton gates used to identify the ground-state transition were sufficient to select tritons from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction for states up to 5 MeV in excitation energy. Two-dimensional recoil gates were made to select the possible
\(^8\)B and \(^7\)Be decay daughters. Finally, a gate on the total energy of the final state was implemented with energies ranges for the daughters determined from Monte Carlo simulations of the decay (Chapter 6).

![Histogram of laboratory angle vs. triton energy for all final states detected in the \(d^{(10}\text{C},t)^9\text{C}\) experiments. The small number of tritons coincident with \(^8\)B (circles) and \(^7\)Be (triangles) are shown in addition to those coincident with \(^9\)C recoils. The dashed lines represent the kinematic calculations for triton energy for transfer to the ground, 2.2 MeV first-excited state, and a hypothetical state at 3.3 MeV.]

**Proton Identification**

Based on the measured width of the \(^9\)C first excited state, \(\Gamma = 100\) keV, the lifetime of the state is on the order of \(10^{-19}\)s and therefore decays while still in the \([\text{CD}_2]_{\text{n}}\) target. Simulations of the sequential decay have protons being emitted over an angular range covered by both DSSD and Recoil-Detector Arrays, at energies between 8 and 20 MeV, are discussed in Section 6.2. The DSSD Array was not optimized for detecting protons of this high energy, many of which punch through the DSSD-E detectors. At the higher calculated energies, some protons would be indiscernible from deuterons on a particle identification plot. The Recoil-Detector Array is even less optimized for detecting protons. The low amplifier gains used, while ideal for heavier ions at high energy, limited the energy resolution at which hydrogen isotopes can be detected and identified. As a result, protons where not used to identify the final states of \(^9\)C produced in the \(d^{(10}\text{C},t)^9\text{C}\) reaction.
5.6.3 Sources of Background

The possible sources of background for $^8\text{B}+t$ and $^7\text{Be}+t$ final states are similar to those for the $^9\text{C}_{gs}+t$ channel. The processes that can produce a triton background are identical. The backgrounds for the $^8\text{B}$ and $^7\text{Be}$ recoils can be produced through multi-nucleon transfer reactions of $^{10}\text{C}$ on deuterium and $^{12}\text{C}$. The $^{12}\text{C}(^{10}\text{C},^{14}\text{N})^{8}\text{B}(Q=-10.09\ \text{MeV})$ and $^{12}\text{C}(^{10}\text{C},^{15}\text{O})^{7}\text{Be}(Q=-2.93\ \text{MeV})$ reaction have forward-focused kinematics, yet the $^8\text{B}$ and $^7\text{Be}$ recoils produced are much higher in energy than those from daughters of $^9\text{C}$ excited-state proton decay. The $d(^{10}\text{C},\alpha)^{8}\text{B}$ and $d(^{10}\text{C},^{5}\text{Li})^{7}\text{Be}$ reactions have positive $Q$-values (3.49 MeV and 1.39 MeV, respectively) and emit recoils within the acceptance of the Recoil-Detector Array. Any of the recoil products of these reactions would need to be detected with a triton of proper energy and angle to constitute a significant background to the $d(^{10}\text{C},t)^{9}\text{C}_{ex}$ measurement. Figure 5.21 compares the DSSD particle identification spectra from both a $^{10}\text{C}$ beam incident on $[\text{CD}_2]_n$ and carbon targets. The only background event measured was in the $^7\text{Be}+t$ channel with a 22.4 MeV triton detected at a laboratory angle of 30.7 degrees. While the triton is in the general triton gate, its energy and angle are inconsistent with tritons produced with $^9\text{C}$ excited states and thus is not a significant background.
Figure 5.21: DSSD particle identification spectra gated on $^8$B (a & b) and $^7$Be (c & d) recoils for both $^{10}$C + CD$_2$ (blue) and $^{10}$C + C$_{nat}$ (red) data sets. There is no background in the $^8$B-gated triton spectra and only a single count in the $^7$Be-gated triton spectra from reactions on the natural carbon target. A background triton was detected at an energy and angle less than that for tritons populating $^9$C excited states in the $d(^{10}$C,$t)^9$C reaction.

5.7 Summary

In the first stage of the data analysis for this study, through data reduction, detector calibration, and other extensive selection criteria, the $d(^{10}$C,$t)^9$C reaction was unambiguously identified in the detector system. Additionally, a small number of events were detected in coincidence with tritons consistent with the population of proton-unbound excited states of $^9$C via the $d(^{10}$C,$t)^9$C reaction (Figure 5.22). The number of events is small, placing a great emphasis on the efficiency with which the reaction products, from all final states, are detected. These detection efficiencies will be accounted for prior to determining the cross section of the ground-state transition (Chapter 6) and an evaluation of the excited-state data is covered in the computational data reduction (Chapter 7).
Figure 5.22: A plot of $^9$C excitation energy for all final states detected and identified in the $d(^{10}\text{C},t)^{9}\text{C}$ experiment. The $^9$C ground state data are in red with all $^8\text{B}$ and $^7\text{Be}$ data in blue.
Chapter 6

Computational Data Reduction

6.1 Introduction

This chapter covers the second stage of the analysis where the small set of $^9$C ground-state transition data from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction are reduced to an angular distribution. In order to calculate the angular distribution, several experimental quantities (e.g. target thickness, beam intensity, etc.) and their uncertainties must be determined. Also, the final-state detection efficiencies for the ground and low-lying excited-states in $^9$C produced in the $d(^{10}\text{C},t)^{9}\text{C}$ reaction must be simulated for the detector system. The resulting ground-state angular distribution and excited-state data can then be compared to reaction theory calculations, which are presented in Chapter 7.

6.2 Detection Efficiencies

The efficiency with which a detector can detect and identify a particle is a necessary quantity in many nuclear physics measurements. In order to evaluate the total number of actual events, the sum of all observed events in a detector must be corrected by its detection efficiency, $\epsilon_i$, for a specific particle:

$$ N_{\text{actual}} = \frac{N_{\text{observed}}}{\epsilon_i(E, \theta, \phi)} \quad (6.1) $$

where the efficiency can depend on the energy and angle of the incident particle due to the detector geometry. In the case of a silicon detector, the detection efficiency is unity for all particles with an energy $E$ above the threshold and emitted within the solid angle subtended by the detector. Thus,
the experimental detection efficiency is set by the electronics channels configured for each detector or segment. The overall efficiency for detecting two particles in coincidence depends on the efficiency of detecting each particle individually. The “coincidence efficiency” of detector A for detecting a particle in detectors A and B is defined as,

$$\epsilon_{\text{coinc}}(E, \theta, \phi) = \frac{N_{AB}}{N_A} = \frac{N_{AB}}{N_B}$$

with $N_{AB}$ representing the number of observed A+B coincidences and $N_A, B$ being the total number of particles detected in detector A or B. Detection and coincidence efficiencies can be determined through computer simulations by modeling the geometry and response of the detector system on an event-by-event basis. A series of Monte Carlo simulations were performed for the detector system to determine the coincidence efficiencies for all final states of interest in the $d(^{10}\text{C},t)^{9}\text{C}$ experiment. In addition, the detection efficiencies of the Monitor- and Recoil-Detector Arrays for the beam-flux integration reactions were estimated with similar simulations.

### 6.2.1 Monte Carlo Simulations

The Monte Carlo simulations were executed using a C/C++ code written specifically for modeling silicon detectors in nuclear physics experiments. A series of random events from the reaction of interest is generated over a range of laboratory angles using a kinematics program. The events are equally distributed over the selected range of center-of-mass angles. The energies and angles of all reaction products are recorded and taken as an input to the Monte Carlo program where several realistic effects are simulated. The size of the beam spot ($5 \times 5 \text{ mm}^2$) is taken into account by translating the reaction point to a different position on the target plane and calculating new angles $\theta'$ and $\phi'$ (Figure 6.1). The experimental detector geometry (e.g., position, segmentation, thickness, etc.) is used to select only the events emitted within the active solid angle of each detector. A particle is considered “detected” when its energy is above the energy threshold parameter of the detector; these parameters are set to the values appropriate for each detector used in the experiments. Typically energy thresholds were set to experimental values. The calculated energy deposited in the detector is smeared by a Gaussian function to approximate the actual detector energy resolution (FWHM $\sim 50$ keV). Figure 6.1 shows the simulation
of the $d(^{10}\text{C},t)^{9}\text{C}_{gs}$ reaction and via the detection of both tritons and $^{9}\text{C}$ recoils. The determination of the coincidence efficiencies for reactions with three- or four-body final states requires a separate Monte Carlo code for final states where the $^{9}\text{C}$ recoil decays through particle emission (Section 6.6). The program models the two-body production of a triton and a $^{9}\text{C}$ excited state recoil via the $d(^{10}\text{C},t)^{9}\text{C}$ reaction and the subsequent sequential proton decays to states in $^{8}\text{B}$ or $^{7}\text{Be}$. The number of events is chosen to be sufficiently high ($10^5$) such that the statistical uncertainty of the coincidence efficiency is small and a minor contribution to the total uncertainty of the calculated reaction cross section.

Figure 6.1: Depiction of a Monte Carlo simulation of $d(^{10}\text{C},t)^{9}\text{C}_{gs}$ reaction. An event from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction is generated on the beam axis and then offset to a random position on the target plane within the area of the beam spot profile. The new geometry defines whether or not the triton and/or $^{9}\text{C}$-recoil are detected. Ions which are detected have their energies smeared by a Gaussian distribution to simulate the resolution of the detector.

6.2.2 $d(^{10}\text{C},t)^{9}\text{C}$ Coincidence Efficiencies

The DSSD-Recoil coincidence detection efficiencies were simulated for all final states in the $d(^{10}\text{C},t)^{9}\text{C}$ reaction up 3.3 MeV in excitation energy (Figure 6.2). The recoil-coincidence detection efficiency for the ground-state transition producing a two-body final state ($^{9}\text{C}_{gs} + t$) was approximately 80%. The detection efficiency would be larger ($\sim 89\%$) if beam spot was smaller ($1.0$ mm$^2$). In the simulations, larger beam spots systematically result in smaller detection efficiencies due to the broadening of the reaction kinematics (Figure 6.1). As discussed in Section 5.6.1, all excited states below 4.4 MeV can decay to the same final states of $^{8}\text{B}_{gs}+p$ and $^{7}\text{Be}_{gs}+2p$. Detection efficiencies for both the observed first excited state at 2.22 MeV ($\Gamma= 100$ keV) and a hypothetical 3.3 MeV ($\Gamma= 300$ keV) state were
Figure 6.2: A simulated plot of laboratory angle vs. energy for tritons detected in coincidence with $^9$C recoils from the $d(^{10}$C,$t)^9$C reaction. The spectra are from the population of the $^9$C ground state (black), the 2.2 MeV first-excited state (red), and a hypothetical state at 3.3 MeV (blue).

determined. The coincidence detection efficiency for the first-excited state was approximately 85% for the $^8$B channel and 86% for the $^7$Be channel. These coincidence efficiencies vary slightly (<1%) for a state at an excitation energy of 3.3 MeV. It was also possible for the protons emitted by the decaying recoil nucleus to be detected by the Recoil-Detector Array which was determined using the Monte Carlo simulations. The detection of protons in coincidence with the $^8$B or $^7$Be recoil was simulated to be 12% and 5.5%, respectively for the first excited state and 5.7% and 1.4% for the 3.3 MeV state. All of the simulated detection efficiencies are summarized in Table 6.1.

6.2.3 Beam-Flux Monitor Detection Efficiencies

The detection efficiencies for the detector arrays monitoring the radioactive beam intensity were also determined by Monte Carlo simulations (Table 6.2). Both the Recoil- and Monitor-Detector Arrays described in Sections 5.5.3 and 5.5.4 were simulated in the same manner to determine the detection efficiencies for the reactions of interest. The Monitor-Detector Array measured the elastic scattering of the $^{10}$C beam from a gold foil. The dead segments of the Monitor $\Delta E$-detector set the maximum detection efficiency at 50%. The detection efficiency of the Monitor-Detector Array is sensitive to the size of the beam spot due to the small solid angle it subtends at forward angles. The smearing effect of a
Table 6.1: Summary of simulated coincidence efficiencies of the detector system used in the $d(^{10}\text{C},t)^{9}\text{C}$ experiments. Bold figures in brackets denote the total DSSD-Recoil coincidence efficiency for the detection signature.

<table>
<thead>
<tr>
<th>$^{9}\text{C}$ Excited State (MeV)</th>
<th>Final State</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$[^8\text{C}+t]$</td>
<td>$0.828(3)$</td>
</tr>
<tr>
<td>0</td>
<td>$[^8\text{B}+t]$</td>
<td>0.732</td>
</tr>
<tr>
<td></td>
<td>$[^8\text{B}+p+t]$</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+t]$</td>
<td>0.395</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+p+t]$</td>
<td>0.405</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+2p+t]$</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+t]$</td>
<td>$0.854(3)$</td>
</tr>
<tr>
<td>2.2</td>
<td>$[^8\text{B}+t]$</td>
<td>0.808</td>
</tr>
<tr>
<td></td>
<td>$[^8\text{B}+p+t]$</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+t]$</td>
<td>0.587</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+p+t]$</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+2p+t]$</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+t]$</td>
<td>$0.850(3)$</td>
</tr>
<tr>
<td>3.3</td>
<td>$[^8\text{B}+t]$</td>
<td>0.808</td>
</tr>
<tr>
<td></td>
<td>$[^8\text{B}+p+t]$</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+t]$</td>
<td>0.587</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+p+t]$</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+2p+t]$</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>$[^7\text{Be}+t]$</td>
<td>$0.850(3)$</td>
</tr>
</tbody>
</table>

5 $\times$ 5 mm$^2$ beam spot reduces the detection efficiency for the $^{197}\text{Au}(^{10}\text{C},^{10}\text{C})^{197}\text{Au}$ reaction to 32%. A smaller beam spot of 2 $\times$ 2 mm$^2$ would yield an efficiency of approximately 43%. The Recoil-Detector Array monitored the elastic scattering from the $^{12}\text{C}$ in the $[\text{CD}_2]_n$ target and was impacted less by the size of the beam spot. The detection efficiency of the Recoil-Detector Array at maximum can be 91% over its solid angle. The effect of a 5 $\times$ 5 mm$^2$ beam spot reduces the efficiency to approximately 87% for the $^{12}\text{C}(^{10}\text{C},^{10}\text{C})^{12}\text{C}$ reaction. These detection efficiencies are utilized in determining the $^{10}\text{C}$ beam intensity in Section 6.4.

Table 6.2: Summary of simulated detection efficiencies for elastic scattering reactions used for beam-flux integration.

<table>
<thead>
<tr>
<th>Detector Array</th>
<th>Reaction</th>
<th>$\epsilon_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor</td>
<td>$^{197}\text{Au}(^{10}\text{C},^{10}\text{C})^{197}\text{Au}$</td>
<td>$0.323(3)$</td>
</tr>
<tr>
<td>Recoil</td>
<td>$^{12}\text{C}(^{10}\text{C},^{10}\text{C})^{12}\text{C}$</td>
<td>$0.871(8)$</td>
</tr>
</tbody>
</table>

6.3 Target Thicknesses

An accurate measurement of a reaction cross section requires the thickness of the target to be well known. The thicknesses of all targets used in the $d(^{10}\text{C},t)^{9}\text{C}$ experiments were measured at ANL at the time of their manufacture using an energy-loss technique. With this technique, alpha particles from a collimated $^{241}\text{Am}$ source bombard a target foil and the degraded particles are detected in silicon
surface-barrier detector. The average energy lost by the alpha particles in the foil is used to estimate the target thickness. As the method depends on energy loss, the stopping power of the material must be known. The technique is widely used due to the simplicity of the experimental apparatus required and the accuracy of the measurements of the target thickness (≤5%) [99] [100]. The energy-loss method is, however, insensitive to the stoichiometry of the material which is of interest in experiments performed in inverse kinematics with light targets where a depletion of the target material can occur at intermediate beam currents [101]. An alternate technique of determining the target thickness is to measure the elastic scattering of ions from a foil and calculate the density of target nuclei. The elastically scattering cross section depends on the elements and/or isotopes in the material making it possible to evaluate the stoichiometry of the target. The elastic-scattering cross sections for the various target constituents must be known as well as the incident beam current and the precise geometry of the detector used. A series of proton scattering experiments described in Section 4.4 were used as an additional measure of the [CD$_2$]$_n$ target thicknesses.

### 6.3.1 Proton Elastic Scattering

The proton elastic scattering experiments were performed with the previously measured 660 μg/cm$^2$ and 1.7 mg/cm$^2$ [CD$_2$]$_n$ targets as well as a 1.6 μg/cm$^2$ natural carbon foil. Proton-energy spectra were acquired at a minimum of three laboratory angles ranging from 50° to 140°. Figure 6.3 shows spectra from all three targets with peaks from elastic scattering from both $^{12}$C and deuterium. The natural carbon target lacks the strong proton and deuterium peaks present in the two [CD$_2$]$_n$ targets. The integral of each peak, produced from elastically-scattered protons, was used to calculate the density of the target nuclei present. In general, the number of target nuclei per unit area, or areal density, can be calculated from the following equation:

$$N_t = \frac{N_s}{\epsilon N_b} \frac{1}{\Delta\Omega \frac{d\sigma}{d\Omega}(\theta)}$$  \hspace{1cm} (6.3)

where $N_s$ is the total number of elastically scattered protons detected, $N_b$ is the total number of integrated beam particles incident on the target, $\epsilon$ is the efficiency of the detector (e.g., dead time/live time), $\Delta\Omega$ is the solid angle subtended by the silicon detector, and $\frac{d\sigma}{d\Omega}(\theta)$ is the differential cross section at the
laboratory angle $\theta$. The proton elastic-scattering cross sections on deuterium and carbon targets were obtained from references [102] [103]. The deuterium cross sections for each angle at a proton energy of 4.5 MeV were linearly interpolated from the extensive $d(p,p)d$ data performed at 3.998 MeV and 5.0 MeV in reference [103]. All published and calculated cross sections are in the center-of-mass frame, which changes the form of Equation 6.3:

$$N_t = \frac{N_s \Delta \Omega}{N_0} \frac{J}{\Delta \Omega \frac{d\sigma}{d\Omega}(\theta_{cm})}$$ (6.4)

with $J$ being the Jacobian which is defined as:

$$J = \frac{\sin^2(\theta_{lab})}{\sin^2(\theta_{cm})} \cos(\theta_{cm} - \theta_{lab})$$ (6.5)

The areal density ($A_t$), in units of g/cm$^2$, for each component of the target is calculated by multiplying Equation 6.4 by the number of target nuclei per unit mass (g):

$$A_t = N_t M_t \frac{M_t}{N_A}$$ (6.6)

where $M_t$ is the molar mass and $N_A$ is Avogadro’s constant.

Figure 6.3: A plot of three energy spectra from elastic scattering measurements performed with a 4.5-MeV proton beam from the Tandem Van de Graaf Accelerator Laboratory at Western Michigan University. Spectra from the 660$\mu$g/cm$^2$ [CD$_2$]$_n$ (red), 1.7 mg/cm$^2$ [CD$_2$]$_n$ (blue), and a 1.6 mg/cm$^2$ (black) target are shown.
The target thicknesses were measured in two separate proton scattering experiments, performed both before and after the May 2011 \( d(^{10}\text{C},t)^{9}\text{C} \) experiment. The first experiment measured the target thicknesses of the two \([\text{CD}_2]_n\) targets, the thinnest (660 \( \mu \text{g/cm}^2 \)) being the primary target for the February 2011 experiment. The second experiment measured the target thicknesses of the thick, 1.7 mg/cm\(^2\), \([\text{CD}_2]_n\) target as well as a 1.6 mg/cm\(^2\) natural carbon target. The 1.7 mg/cm\(^2\) \([\text{CD}_2]_n\) foil’s target thickness was measured before and after the May 2011 experiment to determine if there was any change in the deuterium content. The carbon target was used as an additional check of the method and to compare results to the thick \([\text{CD}_2]_n\) target. The largest contributing factor to proton-detection efficiency (\(\epsilon\)) was the dead time of the acquisition system (\(\sim 2.5\%\)) and thus determined to be 97.5% for all measurements.

6.3.2 Uncertainty

The uncertainties in the measurement of the target thicknesses via proton scattering come with equal parts from systematic and statistical errors. The propagation of the individual uncertainties for a target thickness measured at \(\theta_{\text{cm}}\) can be calculated with the following relation, using the variables from Equation 6.3:

\[
\frac{\delta N_t}{N_t(\theta_{\text{cm}})} = \sqrt{\left(\frac{\delta N_s}{N_s}\right)^2 + \left(\frac{\delta N_b}{N_b}\right)^2 + \left(\frac{\delta \Omega}{\Delta \Omega}\right)^2 + \left(\frac{\delta \sigma_{\text{diff.}}}{\sigma_{\text{diff.}}}\right)^2}
\]  

(6.7)

The sole contribution to the statistical error in the measurement is \(\delta N_s\) which varied with each spectrum acquired, and was never greater than one percent (\(10^4\) counts). The remaining three terms constitute the systematic error measured in the target thicknesses. The largest of these is the uncertainty in the solid angle, 3\%, which is attributed to the uncertainty in the distance between the collimator and silicon-surface barrier detector. The incident beam intensity was determined with an ORTEC Model 439 current integrator and with the settings used had an uncertainty of 0.3\% [104]. The uncertainties in the experimentally determined elastic-scattering cross sections were approximately 0.5–1.0\% for elastic scattering from deuterium and 4\% from carbon [102] [103]. Thus, for the calculated deuterium and carbon areal densities in Table 6.3 an error of 3.3\% and 5.1\% is assigned, respectively. These errors were added in quadrature (6.1\%) to determine the total effective target thicknesses, mass ratio, and stoichiometry ratios.
Table 6.3: Summary of measured target thicknesses using elastic proton scattering. The \([\text{CD}_2]_n\) targets listed were used in the \(d(^{10}\text{C},t)^9\text{C}\) experiments.

<table>
<thead>
<tr>
<th>Target (mg/cm(^2))</th>
<th>Date Tested</th>
<th>(\theta_{\text{lab}})</th>
<th>Areal Densities (mg/cm(^2))</th>
<th>Mass Ratio ((m_D/m_C))</th>
<th>Stoichiometry (D:C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.660 \ [\text{CD}_2]_n)</td>
<td>04/2011</td>
<td>70</td>
<td>0.175(6)</td>
<td>0.24(2)</td>
<td>1.45(9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>0.124(4)</td>
<td>0.21(1)</td>
<td>1.27(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>140</td>
<td>0.155(5)</td>
<td>0.25(2)</td>
<td>1.50(9)</td>
</tr>
<tr>
<td>(1.7 \ [\text{CD}_2]_n)</td>
<td>04/2011</td>
<td>70</td>
<td>0.232(8)</td>
<td>0.25(2)</td>
<td>1.50(9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>130</td>
<td>0.132(4)</td>
<td>0.143(9)</td>
<td>0.85(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>140</td>
<td>0.133(4)</td>
<td>0.149(9)</td>
<td>0.89(5)</td>
</tr>
<tr>
<td>(1.7 \ [\text{CD}_2]_n)</td>
<td>05/2011</td>
<td>70</td>
<td>0.166(5)</td>
<td>0.23(1)</td>
<td>1.35(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>130</td>
<td>0.150(5)</td>
<td>0.17(1)</td>
<td>1.02(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>140</td>
<td>0.147(5)</td>
<td>0.16(1)</td>
<td>0.98(6)</td>
</tr>
<tr>
<td>(1.6 \ C_{\text{naf}})</td>
<td>05/2011</td>
<td>70</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>130</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>140</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

6.3.3 Results

The target thickness measured via elastic scattering are in agreement with alpha energy-loss measurement for the thin \([\text{CD}_2]_n\) target. If it is assumed that the ANL measurements have a 5% uncertainty and the \([\text{CD}_2]_n\) target foils have proper stoichiometry of 2:1, the thin (0.660 mg/cm\(^2\)) target would provide a deuterium target with a thickness of of 0.164\(\pm\)0.008 mg/cm\(^2\). The average measured deuterium thickness from elastic proton scattering was 0.152\(\pm\)0.009 mg/cm\(^2\) which is in good agreement with the alpha energy-loss measurement. Both the 1.7 mg/cm\(^2\) \([\text{CD}_2]_n\) and 1.6 mg/cm\(^2\) natural carbon targets were measured at 60–80% of the total areal density of the ANL measurements. The thick \([\text{CD}_2]_n\) target was measured at the same thickness before and after the May 2011 experiment confirming that the depletion of deuterium target material due to the in-flight \(^{10}\text{C}\) beam was not a measurable effect over the course of the experiment.

6.4 Beam-Flux Normalization

To determine an absolute normalization for cross section calculations of the \(d(^{10}\text{C},t)^9\text{C}\) reaction, the intensity of the radioactive \(^{10}\text{C}\) beam must be known. The elastic-scattering beam-flux integration data, collected from the Recoil- and Monitor-Detector Arrays, must be analyzed to extract this information. For both data sets, the measured target thicknesses, simulated detection efficiencies, and theoretical elastic-scattering cross sections are used to calculated the intensity of the radioactive \(^{10}\text{C}\) beam.
6.4.1 Monitor-Detector Array

The Monitor-Detector Array introduced in Section 4.5.4 was used to monitor the elastic scattering of the beam on a gold target and had an angular coverage from 4.5° to 10.6° in the center-of-mass frame. The sum of all elastically scattered $^{10}\text{C}$ ions was used to calculate the beam intensity using the equation:

$$I_{\text{beam}} = \frac{N_s}{\epsilon} \frac{M_t}{t \sigma A_t N_A}$$

(6.8)

where $I_{\text{beam}}$ is the beam intensity in ions per unit time, $\epsilon$ is the detection efficiency of the monitor $\Delta E$-$E$ telescope, $N_s$ is total number of scattered beam particles detected, $M_t$ is the molar mass of the target, $t$ is the total live time of beam on target, $A_t$ is the areal density of the target, $\sigma$ is the integrated elastic-scattering cross section over the solid angle of the detector, and $N_A$ is Avogadro’s constant. The elastic-scattering cross section for the $^{10}\text{C} + \text{Au}$ elastic system is assumed to be the Rutherford cross section due to the forward angles covered by the detector [111].

As mentioned in Section 4.5.4, the Monitor-Detector Array data for the February 2011 experiment did not provide a reliable measure of the beam intensity. The gold foil was obscured by the center shaft of a target ladder that was mounted in the Spit-Pole Spectrograph scattering chamber. As a result, the $^{10}\text{C}$ beam was implanted in 12.7 mm of aluminum, approximately 50.1 mm from the gold foil. Light ions produced by the beam in the aluminum were transported to the target foil and scattered into the monitor telescope (Figure 6.4). While any precision in the measurement of the beam intensity was lost, the number of light ions detected can be used to compare the relative intensity between the $[\text{CD}_2]_n$ and carbon data sets (Table 6.4). The target ladder was aligned prior to the May 2011 experiment and the Monitor-Detector Array was able to detect and identify elastically scattered $^{10}\text{C}$ beam. Figure 6.4 shows the particle identification spectra from both experiments.

6.4.2 Recoil-Detector Array

The $^{10}\text{C}$ ions elastically scattered from the $^{12}\text{C}$ in the $(\text{CD}_2)_n$ target were detected and identified in the Recoil-Detector Array as an additional measure of the in-flight beam intensity. The $\Delta E$-$E$ recoil telescopes each subtended angles from 2.44° to 12.9° in the elastic center-of-mass frame. In addition
to the target thicknesses and detection efficiency, only the integrated cross section for $^{10}\text{C} + ^{12}\text{C}$ elastic scattering is required to calculate the beam flux. There are no previous measurements of $^{10}\text{C} + ^{12}\text{C}$ elastic scattering. To determine the cross section for the elastic-scattering system, a series of calculations were performed using the reaction code PTOLEMY [109]. Optical-model potentials derived from $^{10}\text{B}$, $^{12}\text{C}$, $^{13}\text{C}$, $^{14}\text{C}$, and $^{14}\text{N} + ^{12}\text{C}$ elastic scattering studies at 10-20 MeV/A were used for the $^{10}\text{C} + ^{12}\text{C}$ calculations [105]–[108]. The elastic-scattering cross section, integrated over the solid angle of the Recoil-Detector Array, had an average value of 22.0 barns and varied by 4% among the selected potentials (Figure 6.5).

The average cross section was used to calculate the $^{10}\text{C}$ beam intensity using Equation 6.4.1.

### 6.4.3 $^{10}\text{C}$ Beam Intensity

The calculated beam intensities for all data sets from both February and May 2011 experiments are listed in Table 6.4. The average $^{10}\text{C}$ beam intensities incident on the $[\text{CD}_2]_n$ targets were $2.2 \times 10^4$ and $1.46 \times 10^4$ $^{10}\text{C}$ ions per second ($^{10}\text{C} \text{ s}^{-1}$) during the February and May 2011 experiments, respectively. These intensities are in good agreement with brief, direct measurements of the radioactive beam during tuning using the $\Delta\text{E-E}$ telescope approximately 30 cm upstream of the target in the diagnostic cross.

The directly measured rates were between $1.7–3.5 \times 10^4$ $^{10}\text{C} \text{ s}^{-1}$ during the February experiment and $0.9–2.1 \times 10^4$ $^{10}\text{C} \text{ s}^{-1}$ for the May 2011 experiment. The beam intensities calculated from the Monitor- and Recoil-Detector Arrays, measured via different elastic-scattering reactions, were in agreement and in the case of the February experiment, provided a necessary redundancy in determining the absolute
Figure 6.5: Calculated elastic scattering data for $^{10}\text{C} + ^{12}\text{C}$ using existing $A=10–14 + ^{12}\text{C}$ potentials. The deviation from the Rutherford component of the cross section (top) as well as the total calculated cross section (bottom) are plotted as a functions of the center-of-mass angle. The range of the abscissa is the acceptance of the recoil detectors in the $^{10}\text{C} + ^{12}\text{C}$ center-of-mass frame.

6.5 $d(^{10}\text{C},t)^{9}\text{C}_{gs}$ Angular Distribution

In order to perform a distorted-wave Born Approximation (DWBA) analysis of the $d(^{10}\text{C},t)^{9}\text{C}$ ground state transition, the angular distribution of the tritons must be produced. As discussed in Section 3.2, the triton angles in the laboratory frame must be translated into the center-of-mass frame and the differential cross section calculated. In this translation, the segmentation of the DSSD Array and beam spot effects introduce some uncertainty in the detected laboratory angle of the triton which must be taken into account prior to calculating the differential cross section. The uncertainties in the detected...
Table 6.4: Summary of calculated $^{10}\text{C}$ beam intensities from beam-flux integration data. The figures in brackets are detector counts per second and used to compare the beam intensities between the [CD$_2$]$_n$ and carbon data sets. All values are in good agreement with zero-degree beam measurements of $1.7-3.5 \times 10^4$ $^{10}\text{C}$ s$^{-1}$ made with a diagnostic $\Delta E$-$E$ telescope during the experiments.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Calculated Beam Intensity ($^{10}\text{C}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp.</td>
</tr>
<tr>
<td>Feb. 2011</td>
<td>CD$_2$ (0.660)</td>
</tr>
<tr>
<td></td>
<td>C (1.031)</td>
</tr>
<tr>
<td>May 2011</td>
<td>CD$_2$ (1.7)</td>
</tr>
</tbody>
</table>

Laboratory angle of the tritons due these effects was simulated with the same Monte Carlo code used in Section 6.2.

Figure 6.6: Illustration of the contribution of detector segmentation to the uncertainty in the calculated center-of-mass angle in the $d(^{10}\text{C},t)^{9}\text{C}$ experiment. The two tritons shown are emitted at different laboratory and center-of-mass angles, yet would be assigned the same $\theta$ and $\theta_{cm}$.

6.5.1 Angular Corrections

The calculation of the center-of-mass angle for a triton emitted from the $d(^{10}\text{C},t)^{9}\text{C}$ reaction must take into account the finite detector segmentation of the DSSD Array and the spatial extent of the beam spot. A triton with energy $E$, laboratory angle $\theta$, and center-of-mass angle $\theta_{cm}$ is detected and identified in one of the DSSD $\Delta E$-$E$ telescopes. The triton is assigned a calibrated energy $E_{det}$ and an angle $\theta_{det}$ which is determined from the position of the geometric center of the DSSD-$\Delta E$ ring. Thus, the calculated center-of-mass angle, $\theta_{cm, det}$, may not be equal to the actual center-of-mass angle of the triton due to
the angular coverage \((\Delta \theta)\) of the DSSD segment and finite beam spot size (Figure 6.6).

To determine the extent of the “smearing” from the DSSD Array segmentation or the size of the beam spot, the Monte Carlo program described in Section 6.2 was used. A large number of \(d^{(10}\text{C},t)^{9}\text{C}\) events were generated from the lower bound of the detector acceptance to the maximum angle of the reaction \((7^\circ – 33^\circ)\). The size of the beam spot was varied, from \(3 \times 3 \text{ mm}^2\) to \(5 \times 5\text{ mm}^2\), to determine the contribution of the beam profile to the angular resolution. The width of the distribution increases with beam spot size, and is comparable to that of the DSSD segmentation. The overall change to the angular distribution is most noticeable at higher laboratory angles \((20^\circ – 33^\circ)\) where the angle subtended by each DSSD segment is large \((\Delta \theta_{lab} \approx 1.5^\circ)\) and the smearing is exacerbated by the kinematic compression near the maximum laboratory angle \((\theta_{max})\) of the \(d^{(10}\text{C},t)^{9}\text{C}\) reaction (Figure 3.2).

Figure 6.7 compares the simulated center-of-mass smearing effects to those observed for the \(d^{(10}\text{C},t)^{9}\text{C}_{gs}\) data set. There is good agreement at forward center-of-mass angles with deviations toward larger angles as the detector segmentation become more coarse. The maximum of the angular distribution lies in the region least affected by the detector segmentation. The distorted-wave Born Approximation calculations are fitted to this peak. Thus, any corrections to the higher-angle data will not affect the results of the DWBA analysis.

![Figure 6.7](image)

**Figure 6.7:** A plot DSSD-\(\Delta E\) ring number vs. the center-of-mass angle for the \(d^{(10}\text{C},t)^{9}\text{C}_{gs}\) reaction from the Monte Carlo simulations (left) and the experiment (right).
6.5.2 Differential Cross Section

The differential cross section for the $^9\text{C}$ ground-state transition was calculated using Equations 6.3, 6.4, and 6.6:

$$\frac{d\sigma}{d\Omega}(\theta_{cm}) = \frac{N_i}{\epsilon_i N_0} \frac{J_i M_t}{\Delta\Omega N_t N_A}$$  \hspace{1cm} (6.9)

where the subscript $i$ denotes a value specific to each bin in the center-of-mass system. The coincidence efficiency, $\epsilon$, was averaged over the DSSD rings utilized for each bin. The additional counts from tritons which stopped in a single DSSD and were not in the normal triton selection criteria were included in the angular distribution (Section 5.5.2). The uncertainty in the differential cross section for the $d(^{10}\text{C},t)^{9}\text{C}$ reaction is dominated by the statistical uncertainty due to the small number of observed $^9\text{C}$-t coincidences and is plotted with the statistical uncertainties (Figure 6.8).

![Figure 6.8](image-url)  
Figure 6.8: Angular distribution for the $d(^{10}\text{C},t)^{9}\text{C}_{gs}$ reaction.

6.6 Summary

The reduction of the $^9\text{C}$ ground-state data set produced an angular distribution for the observed $d(^{10}\text{C},t)^{9}\text{C}_{gs}$ reaction (Figure 6.8). A series of Monte Carlo simulations were used to determine the realistic coincidence detection efficiencies for final states of the ground ($^9\text{C}_{gs}+t$) and excited states ($^8\text{B}_{gs}+t$ and $^8\text{B}_{gs}+t$) of $^9\text{C}$. All final-state coincidence efficiencies for all $^9\text{C}$ excited states below 4.4 MeV in excita-
tion energy were found to be over 80%. The thicknesses of the targets used in the $d(^{10}\text{C},t)^{9}\text{C}$ experiments were measured via proton elastic-scattering at Western Michigan University. The thin $[\text{CD}_2]_n$ target utilized in the acquisition of the $^{9}\text{C}$ ground-state data was determined to have a deuterium thickness of $0.152\pm 0.009 \text{ mg/cm}^2$ which is in agreement with alpha-particle energy loss measurements performed at ANL ($0.164\pm 0.008 \text{ mg/cm}^2$). The average intensity of the in-flight $^{10}\text{C}$ beam was determined to be $2.2\times 10^2 \text{ s}^{-1}$ for the February 2011 experiment using a set of elastic-scattering beam-flux integration data from the Recoil-Detector Array. In the following chapter, the measured $d(^{10}\text{C},t)^{9}\text{C}_{gs}$ angular distribution and the observed excited-state data will be compared to distorted-wave Born Approximation calculations using both traditional inputs as well as those derived from an $ab\ initio$ nuclear model.
Chapter 7

Reaction Theory Analysis

7.1 Introduction

The following chapter describes the analysis of the $^9$C ground and excited-state data from the $d(^{10}$C,$t)^9$C reaction. The ground-state angular distribution is modeled using distorted-wave calculations which are compared to the experimental data. Ground-state spectroscopic factors are extracted by fitting the distorted-wave calculations to the experimental angular distribution. The one-neutron overlap for the $^{10}$C-$^9$C+n and $t-d+n$ systems have been calculated by the Variational Monte Carlo method (Section 2.2.1) which can be compared directly to the spectroscopic factor deduced from the experimental data. The data for $^9$C excited-state are too few to produce an angular distribution. However, they can be compared to the theoretically-calculated yield relative to the ground-state transition. The relative yield can be determined with shell model as well as VMC-derived spectroscopic factors and overlaps providing an additional evaluation of this ab initio method.

7.2 DWBA Analysis

The distorted-wave Born Approximation (DWBA) calculations of the $d(^{10}$C,$t)^9$C$_{gs}$ reaction were performed with the reaction code PTOLEMY [109]. The program was developed at Argonne National Laboratory by M.H. MacFarlane and S. C. Pieper for calculating elastic and inelastic scattering as well as transfer reaction cross sections with a focus on heavy-ion transfer reactions (e.g., $^{208}$Pb($^{16}$O,$^{15}$N)$^{209}$Bi) [109]. PTOLEMY also performs DWBA calculations for light-ion transfer reactions and has been used successfully for transfer reaction studies using radioactive beams [26] [71] [83] [84]. A selection of bound state and optical-model parameters was used to produce the DWBA calculations which were fit to the
maximum of the $^9$C ground-state angular distribution. There were insufficient statistics to perform a DWBA analysis of the $^9$C excited state data, thus only an evaluation of the relative yields of both states populated was performed.

### 7.2.1 Optical Model Parameters

The lack of $^{10}$C+$d$ or $^9$C+$t$ elastic scattering data—from this work or any previous study—required the use of incoming and outgoing potentials from similar transfer systems. A variety of optical-model parameters, obtained from analyses of elastic proton and deuteron scattering from nuclei in the $p$-shell, were used to describe the potentials in for the $d(^{10}$C,$t)^9$C$_{gs}$ reaction. Parameters from ($d$,$t$) and ($d$,$^3$He) reactions on $^{10}$B, $^{12}$C, $^{14}$N, and $^{16}$O were utilized. The optical-model parameters sets for the incoming and outgoing channels are listed in Tables 7.1 and 7.2.

#### Table 7.1: Optical-model parameters used for the incoming channel ($^{10}$C+$d$) in the DWBA calculations of the $d(^{10}$C,$t)^9$C reaction. All potentials are in units of MeV and all lengths in fm.

<table>
<thead>
<tr>
<th>System (ID)</th>
<th>Ref.</th>
<th>$V_0$</th>
<th>$r_0$</th>
<th>$a$</th>
<th>$V_i$</th>
<th>$r_{so}$</th>
<th>$a_{so}$</th>
<th>$V_{a1}$</th>
<th>$r_{a1}$</th>
<th>$a_{a1}$</th>
<th>$R_{c0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{14}$C+$d$ (C1,O1–O2)</td>
<td>[112]</td>
<td>86</td>
<td>1.13</td>
<td>0.697</td>
<td>86</td>
<td>8.22</td>
<td>0.85</td>
<td>0.69</td>
<td>8.62</td>
<td>1.48</td>
<td>0.72</td>
</tr>
<tr>
<td>$^{14}$N+$d$ (N1)</td>
<td>[113]</td>
<td>66.5</td>
<td>1.25</td>
<td>0.752</td>
<td>66.5</td>
<td>1.25</td>
<td>0.68</td>
<td>7</td>
<td>1.25</td>
<td>0.75</td>
<td>–</td>
</tr>
<tr>
<td>$^{14}$N+$d$ (N2)</td>
<td>[113]</td>
<td>68.9</td>
<td>1.25</td>
<td>0.7</td>
<td>68.9</td>
<td>1.25</td>
<td>0.2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.3</td>
</tr>
<tr>
<td>$^{14}$N+$d$ (N3)</td>
<td>[113]</td>
<td>84</td>
<td>1.05</td>
<td>0.8</td>
<td>–</td>
<td>1.28</td>
<td>0.78</td>
<td>7</td>
<td>1.05</td>
<td>0.8</td>
<td>–</td>
</tr>
<tr>
<td>$^{10}$B+$d$ (B1–3)</td>
<td>[114]</td>
<td>84.7</td>
<td>1.11</td>
<td>0.609</td>
<td>5.29</td>
<td>1.37</td>
<td>1.003</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.3</td>
</tr>
</tbody>
</table>

#### Table 7.2: Optical-model parameters used for the outgoing channel ($t+^9$C) in the DWBA calculations of the $d(10$C,$t)^9$C reaction. All potentials are in units of MeV and all lengths in fm.

<table>
<thead>
<tr>
<th>System (ID)</th>
<th>Ref.</th>
<th>$V_0$</th>
<th>$r_0$</th>
<th>$a$</th>
<th>$V_i$</th>
<th>$r_{so}$</th>
<th>$a_{so}$</th>
<th>$V_{a1}$</th>
<th>$r_{a1}$</th>
<th>$a_{a1}$</th>
<th>$R_{c0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t+^{11}$C (C1)</td>
<td>[112]</td>
<td>162.9</td>
<td>1.2</td>
<td>0.72</td>
<td>50.8</td>
<td>1.4</td>
<td>0.88</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$t+^{15}$O (O1)</td>
<td>[112]</td>
<td>161.23</td>
<td>1.2</td>
<td>0.72</td>
<td>46.85</td>
<td>1.4</td>
<td>0.88</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$t+^{15}$O (O2)</td>
<td>[112]</td>
<td>180</td>
<td>1.09</td>
<td>0.78</td>
<td>15.5</td>
<td>2.12</td>
<td>0.47</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$t+^{18}$N (N1-2)</td>
<td>[113]</td>
<td>169</td>
<td>1.14</td>
<td>0.68</td>
<td>32</td>
<td>1.82</td>
<td>0.57</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.4</td>
</tr>
<tr>
<td>$^3$He$+^{9}$B (B1)</td>
<td>[114]</td>
<td>108.1</td>
<td>1.08</td>
<td>0.86</td>
<td>21.7</td>
<td>1.64</td>
<td>0.89</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>20.3</td>
</tr>
<tr>
<td>$^3$He$+^{9}$B (B2)</td>
<td>[114]</td>
<td>69.5</td>
<td>1.18</td>
<td>0.76</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>8.82</td>
<td>1.18</td>
<td>0.76</td>
<td>20.3</td>
</tr>
<tr>
<td>$^3$He$+^{9}$B (B3)</td>
<td>[114]</td>
<td>69.5</td>
<td>1.18</td>
<td>0.76</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>8.82</td>
<td>1.18</td>
<td>0.76</td>
<td>1.07</td>
</tr>
</tbody>
</table>

#### 7.2.2 Bound-State Potentials

Two different types of bound-state parameter sets were used to describe the transferred neutron in $^{10}$C and the triton (Table 7.3). The first is a standard set of Woods-Saxon potentials with their potential depths adjusted to the neutron separation energy (Section 3.5). The second set of bound state potentials were extracted from an ab-initio calculation. The single-neutron density for both $^{10}$C–$^9$C+$n(3/2)$ and
Figure 7.1: Single-neutron density (red) for $^{10}$C($J^\pi=0^+$) and $^9$C($J^\pi=\frac{3}{2}^-$) calculated using the Variational Monte Carlo (VMC) method [115]. Error bars are the statistical error from the VMC calculation. The Woods-Saxon fit to these data (black line) is used as an ab-initio bound-state potential in the DWBA analysis. The green line is the “standard” bound-state potential scaled to match the neutron separation energy of $^{10}$C. At large radii, the VMC calculation deviates positively from the standard overlap by almost a factor of two.

$t-d+p$ systems have been calculated using the Variational Monte Carlo method. For use as a bound-state input for PTOLEMY, these data are fitted with a parametrization to produce a Woods-Saxon potential which yields the theoretically calculated neutron density distribution. The fitted potential does not resemble a standard bound-state potential, with some parameters seeming unphysical, which is a consequence of the fit to the theoretical calculation. Figure 7.1 compares the neutron overlap from the VMC and the parametrized fit to the effective overlap from a standard bound state potential. The VMC-derived overlap is consistently larger than that from the standard potential beginning at a radius of approximately 2.5 fm and similarly for larger radii, thus defining a more diffuse density for the valence neutron compared to the standard potential. The effects of these two very different bound-state potentials on the cross sections produced by the distorted-wave calculations are discussed in Section 7.2.3 as well as in Chapter 8.

Table 7.3: Bound-state potentials used for the DWBA calculations of the $d(^{10}$C,$t)^9$C reaction. The VMC bound states are produced with a Woods-Saxon parameterization fit to the VMC calculated neutron overlap function (Figure 7.1). All potentials are in units of MeV and all lengths in fm.

<table>
<thead>
<tr>
<th>System</th>
<th>Ref.</th>
<th>$V_0$</th>
<th>$r_0$</th>
<th>$a$</th>
<th>$V_i$</th>
<th>$r_i0$</th>
<th>$a_i$</th>
<th>$V_{so}$</th>
<th>$r_{so}$</th>
<th>$a_{so}$</th>
<th>$R_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$C+n (Standard)</td>
<td>[70]</td>
<td>71.4</td>
<td>1.25</td>
<td>0.65</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.25</td>
</tr>
<tr>
<td>$^9$C+n (VMC)</td>
<td>[115]</td>
<td>51.07</td>
<td>3.58</td>
<td>0.598</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>2.6967</td>
<td>3.6</td>
<td>0.55</td>
<td>3.0</td>
</tr>
<tr>
<td>$d+n$ (Standard)</td>
<td>[70]</td>
<td>50</td>
<td>1.25</td>
<td>0.65</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.25</td>
</tr>
<tr>
<td>$d+n$ (VMC)</td>
<td>[115]</td>
<td>147.27</td>
<td>0.7572</td>
<td>0.662</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2.0</td>
</tr>
</tbody>
</table>
7.2.3 DWBA Fits and Spectroscopic Factors

The distorted-wave Born Approximation calculations were performed with all combinations of bound state and optical-model parameters and fitted to the maximum of the $d^{10}\text{C},t)^9\text{C}_{gs}$ angular distribution. A $\chi^2$-fitting algorithm was used to optimize the fits and determine the spectroscopic factors using the relation:

$$ C^2S_{exp} = C^2S_{10C}S_t = \frac{\sigma_{exp}}{\sigma_{DWBA}} $$

(7.1)

where $C$ is the isospin Clebsch-Gordon coefficient for the reaction and is $\sqrt{3}/4$ for $d^{10}\text{C},t)^9\text{C}$, with $S_{10C}$ and $S_t$ representing the spectroscopic factors from the $^{10}\text{C}–^{9}\text{C}+n$ and $t–d+n$ systems. The shape of the angular distribution is well modeled by the $\Delta l=1$ transition to the known $J^\pi=3/2^-$ $^9\text{C}$ ground state for the calculations with all combinations of entrance- and exit-channel optical-model parameters (Figure 7.2). The angular distributions calculated assuming ground-state values of $J^\pi=1/2^+$ $(\Delta l=0)$ and $5/2^+$ $(\Delta l=2)$ are also shown in Figure 7.2 and are in poor agreement with the data, supporting the existing ground-state spin-parity assignment. The spectroscopic factors from the nine optical-model parameter sets with standard bound-state parameters are summarized in Table 7.4.

The $d^{10}\text{C},t)^9\text{C}$ cross sections from distorted-wave calculations using the VMC bound states are larger than those from the standard Woods-Saxon potentials and thus, with $C^2S_{exp}$ as defined in Equation 7.1, yield smaller spectroscopic factors. The ratio of spectroscopic factors produced with the standard WS potentials and those from the VMC are in the last column of Table 7.4. The ratio has an average of 3.4 with a standard deviation from the mean of 0.3 over the nine optical-model parameter sets used in the analysis. A discrepancy similar to this has been observed previously in a study of the $d(^8\text{Li},^3\text{He})^7\text{He}$ and $d(^7\text{Li},t)^6\text{Li}$ reactions where all experimental spectroscopic factors needed to be normalized by a factor 0.32 to compare to VMC-calculated spectroscopic factors.[83]. This is in contrast to a previous comparison of $(d,p)$ spectroscopic factors to those from the VMC which were in agreement within approximately 30% [26]. The consistent enhancement observed in the calculated $d^{10}\text{C},t)^9\text{C}$ cross sections using this method would appear to be due to more than the bound-state form factor derived from Variational Monte Carlo calculations. A possible interpretations of these results in light of the current study are discussed in Chapter 8.
Figure 7.2: DWBA fits to the angular distribution of the $d^{(10}\text{C},t)^9\text{C}_{gs}$ reaction. Calculations were performed with standard bound-state potentials and entrance/exit channel optical-model parameters from Tables 7.1, 7.2 & 7.3. The shape of the angular distribution is consistent with a $\Delta l=1$ (colored lines) transfer to the ground state of $^9\text{C}(J^\pi=3/2^-)$ and resemble neither $\Delta l=0$ (solid black) nor $\Delta l=2$ (dashed black) transitions.

7.3 Relative Yields

The comparison of relative spectroscopic factors between excited states populated in the same reaction is frequently more reliable than absolute spectroscopic information. The limited statistics for events from excited states in $^9\text{C}$ make it impossible to perform a direct comparison between the angular distribution and the calculated angular distribution as done for the ground state. Instead, the relative yields of the $^9\text{C}$ states populated in the $d^{(10}\text{C},t)^9\text{C}$ reaction will be evaluated. An estimate for the expected yield for

<table>
<thead>
<tr>
<th>System</th>
<th>Ref.</th>
<th>$S_{std}$</th>
<th>$S_{VMC}$</th>
<th>$(S_{std}/S_{VMC})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}(d,t)^{14}\text{C}$ (C1)</td>
<td>[112]</td>
<td>1.36</td>
<td>0.46</td>
<td>2.96</td>
</tr>
<tr>
<td>$^{16}\text{O}(d,t)^{18}\text{O}$ (O1)</td>
<td>[112]</td>
<td>1.36</td>
<td>0.44</td>
<td>3.09</td>
</tr>
<tr>
<td>$^{16}\text{O}(d,t)^{18}\text{O}$ (O2)</td>
<td>[112]</td>
<td>1.46</td>
<td>0.37</td>
<td>3.89</td>
</tr>
<tr>
<td>$^{14}\text{N}(d,t)^{16}\text{N}$ (N1)</td>
<td>[113]</td>
<td>1.12</td>
<td>0.31</td>
<td>3.59</td>
</tr>
<tr>
<td>$^{14}\text{N}(d,t)^{16}\text{N}$ (N2)</td>
<td>[113]</td>
<td>1.04</td>
<td>0.29</td>
<td>3.58</td>
</tr>
<tr>
<td>$^{14}\text{N}(d,t)^{16}\text{N}$ (N3)</td>
<td>[113]</td>
<td>1.00</td>
<td>0.28</td>
<td>3.63</td>
</tr>
<tr>
<td>$^{10}\text{B}(d,^3\text{He})^9\text{B}$ (B1)</td>
<td>[114]</td>
<td>0.66</td>
<td>0.21</td>
<td>3.13</td>
</tr>
<tr>
<td>$^{10}\text{B}(d,^3\text{He})^9\text{B}$ (B2)</td>
<td>[114]</td>
<td>0.67</td>
<td>0.20</td>
<td>3.39</td>
</tr>
<tr>
<td>$^{10}\text{B}(d,^3\text{He})^9\text{B}$ (B3)</td>
<td>[114]</td>
<td>0.68</td>
<td>0.20</td>
<td>3.36</td>
</tr>
</tbody>
</table>
a state can be produced by normalizing the theoretical excited-state angular distribution to the ground-state spectroscopic factor and tabulating an estimated number of counts using the simulated efficiency of the detector system for $^8$B and $^7$Be final states (Section 7.2). The estimated yield is dependent on both the distorted-wave reaction-model as well as the structure theory present in the selected form factor. In the evaluation of the $^9$C excited state data, theoretical yields were calculated with both standard and VMC-derived form factors.

In order to compare the estimated and experimental yields, two assumptions must be made to simplify the use of relative spectroscopic factors from theory. First, the $^9$C ground state is considered to be a “pure” $1p_{3/2}$ state with no other $p$- or $sd$-shell components. This assumption does contradict an explanation of the $^9$C ground-state magnetic dipole moment [28]. However for the purpose of a simple estimation of the relative yields, the ground state will be considered an unmixed single-particle state. Second, all of the experimental excited-state counts are from $l=1$ transfer to a “pure” $1p_{1/2}$ state. If the 2.2 MeV first-excited state of $^9$C has a spin and parity of $J^\pi=\frac{1}{2}^-$, as is the assignment of the mirror state in $^9$Li, then there exists some experimental support for the latter assumption. The experimental $Q$-value resolution for the ground state is poor enough that it could be possible that all excited state counts could be from the first-excited state (Figure 6.23). Lastly, since the PTOLEMY reaction code does not support transfer to unbound states, the energy of the $J^\pi=\frac{1}{2}^-$ excited state is artificially adjusted so that the state is bound by 100 keV (i.e., at an excitation energy of 1.2 MeV). The estimated yields calculated under these assumptions with both standard and VMC form factors are listed in Table 7.5. The experimental value of ten counts is in general agreement with the Cohen-Kurath estimate of the spectroscopic factor and, as for the ground-state spectroscopic factor, smaller than both VMC estimates by over a factor of three. A discussion of these estimated yields is presented in Chapter 8.

7.4 Summary

A detailed analysis of the ground state angular distribution for the $d(^{10}$C,$t)^9$C reaction was performed. Distorted-wave calculations of the ground-state transition, using a variety of optical-model parameter sets, are in reasonable agreement with the shape of the experimental angular distribution which is consistent with transfer of a $l=1$ neutron to the $J^\pi=\frac{3}{2}^-$ ground state. The absolute spectroscopic factor
Table 7.5: Summary of estimated yields for a $1p_{1/2}$ state populated in the $d^{(10}C,t)^9$C reaction using both standard and VMC form factors. The yields calculated with standard form factors used relative $1p_{1/2}^{-1}p_{3/2}$ spectroscopic factors from Cohen-Kurath [62] and from VMC calculations [63]. The yield calculated with the VMC-derived form factor for the ground state used the VMC-calculated overlap for $^9$C+n($1p_{1/2}$).

<table>
<thead>
<tr>
<th>System</th>
<th>Ref.</th>
<th>Number of Estimated $1p_{1/2}$ Counts</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Number of Estimated $1p_{1/2}$ Counts</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>[112]</td>
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<td>O1</td>
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<td>[112]</td>
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<td>[113]</td>
<td>$16.9$</td>
</tr>
<tr>
<td>N2</td>
<td>[113]</td>
<td>$15.0$</td>
</tr>
<tr>
<td>N3</td>
<td>[113]</td>
<td>$14.3$</td>
</tr>
<tr>
<td>B1</td>
<td>[114]</td>
<td>$18.0$</td>
</tr>
<tr>
<td>B2</td>
<td>[114]</td>
<td>$15.1$</td>
</tr>
<tr>
<td>B3</td>
<td>[114]</td>
<td>$16.4$</td>
</tr>
<tr>
<td>Average±σ</td>
<td></td>
<td>$13.9±3.4$</td>
</tr>
</tbody>
</table>

was determined for the ground-state transition with both traditional bound-state parameters and from an analysis using a form factor derived from VMC calculations. The VMC-derived form factor for the $d^{(10}C,t)^9$C reaction produces an enhanced absolute cross section relative to the standard form factor. This enhancement is consistent over all optical-model parameter sets used with an average of $3.4±0.3$.

This result is similar to observations from previous study of $(d,t)$ and $(d,^3$He) reactions which also made direct comparisons to VMC overlap functions. The shapes of the bound-state one-neutron densities differ between the standard and VMC bound states with the VMC having a larger contribution at large radii which could possible account for this enhancement.

The theoretical yields for a $1p_{1/2}$ excited state relative to the experimental yield for the $1p_{3/2}$ ground-state were calculated using both form factors and the simulated detection efficiency. The estimated yields produced from calculations using the standard form factor with a VMC-calculated relative spectroscopic factor and that from the VMC-derived form factor using the VMC-calculated $^9$C+n($1p_{1/2}$) spectroscopic factor were very similar at $36.3±8.8$ and $36.0±6.6$ counts, respectively. The estimated yield from standard form factor calculation using the Cohen-Kurath relative spectroscopic factor was $13.9±3.4$ counts. The ten excited-state counts detected and identified during the experiment are a factor of three less than both VMC-based estimates but are in general agreement with the $p$-shell Cohen-Kurath estimate. The accuracy of the one-neutron overlaps calculated by the VMC for $^9$C is discussed in the following chapter.
Chapter 8

Discussion of Results

8.1 Introduction

The following chapter is an interpretation of the results from the current study of the structure of $^9$C through single-neutron transfer. The absolute spectroscopic factor for the ground-state transition from the $d(^{10}$C,$t)^9$C neutron-pickup reaction was determined utilizing multiple sets of optical-model parameters. The theoretical cross sections produced using structure information from Variational Monte Carlo calculations show a consistent enhancement with respect to those determined with standard parameters. The few previous studies which have made this comparison are discussed and a possible source for this enhancement is presented. The applicability of *ab initio* calculations for use in distorted-wave reaction theory is also discussed. The observed $^9$C excited state data were compared to a series of theoretical estimated yields relative to the ground state transition. The results between the relative yields from different structure calculations are presented.

8.2 $^9$C Ground State Spectroscopic Factor

The $d(^{10}$C,$t)^9$C ground-state angular distribution was determined using the data subset comprised of $^9$C recoils coincident with tritons. The absolute scale of the cross sections for the angular distribution was determined by measurements of target thicknesses and the intensity of the in-flight radioactive $^{10}$C beam. The observed data were corrected using coincidence detection efficiencies for the detector system determined by Monte Carlo simulations. The resulting experimental angular distribution was compared to distorted-wave calculations of the $d(^{10}$C,$t)^9$C$_{gs}$ reaction. Using standard optical model parameters for
all bound state and scattering potentials, the absolute spectroscopic factor for the ground state transition is

\[ C^2 S_{\text{exp}} = 1.04 \pm 0.32_{\text{stat.}} \pm 0.3_{\text{sys.}}. \]

The sensitivity of the distorted-wave calculations to the radial parameter of the bound-state potentials precludes a direct comparison between theoretical spectroscopic factors and from those fit to the experimental angular distribution. The following spectroscopic factors from nuclear models are presented as a general comparison. The \( p \)-shell model spectroscopic factors of Cohen and Kurath [62] calculate the one nucleon pickup from \( ^{10}\text{C} \) to the \( 1p_{3/2} \) \( ^9\text{C} \) ground state to be 1.735. The VMC spectroscopic factor for \( d(^{10}\text{C},t)^9\text{C} \) is the product of the \( ^{10}\text{C} \cdot ^9\text{C} + n \) and \( t-d+n \) spectroscopic factors, 1.137 and 1.317 respectively, resulting in a value of 1.50 [63]. A method which does allow for a comparison between theoretical and experimental spectroscopic factors is through the use of VMC-derived bound state parameters (Sections 7.2.2 and 7.2.3).

8.2.1 Variational Monte Carlo Form Factors

In addition to standard bound state potentials, VMC-derived bound state potentials were produced via parameterized fits to the one-neutron overlap function of \( ^{10}\text{C} \) by \( ^9\text{C} \) and \( ^3\text{H} \) by \( ^2\text{H} \). They were used in the DWBA calculations for the \( d(^{10}\text{C},t)^9\text{C} \) ground-state transition. If both structure and reaction theories are reliable, the inclusion of the \textit{ab initio} calculated spectroscopic overlap for both bound states should yield a realistic representation of the data. The theoretical \( d(^{10}\text{C},t)^9\text{C} \) cross sections produced using the VMC-derived bound state potentials were consistently enhanced by a factor of 3.4±0.3 with respect to those distorted-wave calculations utilizing standard bound state parameters. There have been two previous studies [26] [83] which have used VMC-derived bound state potentials to evaluate experimental single-particle transfer cross sections in the described manner and only one has reported enhancements of this magnitude.

The use of VMC bound-state potentials well describes the data from the \( d(^8\text{Li},p) \) reaction as well as the angular distributions from \( ^6,^7\text{Li}(d,p) \) reactions [26] [116]. All theoretical cross sections produced via the “VMC method” were within 30% of the measured data with no need for additional normalization. In the study of the structure of \( ^7\text{He} \) using the \( d(^8\text{Li},^3\text{He})^7\text{He} \) reaction several \( (d,t) \) and \( (d,^3\text{He}) \) reactions with both stable and radioactive beams were observed and analyzed. All experimental cross sections were a
factor of three smaller than those predicted using the VMC method. The observation of the enhancement in the $d(^{10}\text{C},t)^{9}\text{C}$ VMC results of the current study, the presence of an enhancement in the other $(d,t)$ and $(d,^{3}\text{He})$ reactions evaluated in this manner, as well as the apparent absence of enhancement in $(d,p)$ reactions implies a sensitivity of the distorted-wave calculations to some aspect of the VMC-derived form factors.

The reaction-theory analysis in the previous chapter has shown that the enhancement of the theoretical cross-section is consistent over all sets of incoming and outgoing optical model parameters (Table 7.4). Thus, the potentials used to distort the incident deuteron and outgoing triton scattering solutions do not seem to affect the absolute value of the transition amplitude (Equation 3.15). The form factor, along with the partial-wave expansion of radial solutions for the incoming and outgoing distorted-waves, contribute to the transition amplitude in a sum over orbital angular momenta (Equation 3.16). The form factor is a product of the bound state potentials which in the case of the VMC-method are extracted from the VMC-calculated one-neutron density functions. The VMC overlap function for $^{10}\text{C}$ and $^{9}\text{C}+n$ deviates from that described by a standard Woods-Saxon bound-state potential by having a greater density at large radii (Figure 7.1). The asymptotic behavior of the VMC-calculated overlap functions could lead to larger partial-wave contributions from either incoming or outgoing radial scattering eigenfunctions which result in the enhanced theoretical cross sections. The lack of an observed enhancement in the $(d,p)$ reactions suggests that the outgoing triton or $^{3}\text{He}$ distorted-waves, in combination with the enhanced tail of the VMC overlap function, are possibly the source of the discrepancy. Such a result, if the case, calls into question the use of VMC overlaps as inputs to distorted-wave calculations. A recent survey of Green’s function Monte Carlo single-nucleon overlap distributions for light $p$-shell nuclei ($A \leq 7$) [117], has shown that GFMC calculations improve upon the VMC asymptotic behavior while most spectroscopic overlaps are similar to VMC values (Figure 8.1). A comparison between the theoretical cross section using the VMC form factors presented in this study to Green’s function Monte Carlo calculations for the $^{10}\text{C}–^{9}\text{C}+n$ overlap, when available, would be useful in evaluating the reaction-model enhancement.
Figure 8.1: Neutron overlap functions for $^7$He and $^6$He+$n$($1p_{3/2}$) from VMC (black) and and GFMC (green) calculations. The dashed line is the ideal asymptotic form (irregular Coulomb function) of the overlap function which the GFMC calculation resembles more than the VMC. Figure from Ref. [117, Fig. 9].

8.3 $^9$C Excited State Data

The proton-unbound $^9$C excited-states populated in the $d(^{10}$C,$t)^9$C reaction were detected and identified in addition to those from the ground state. The detection efficiency for the $^8$B and $^7$Be recoils in coincidence with tritons was simulated and determined to be effectively equivalent to that for the ground-state transition. The total number of $^9$C excited-state events from both $^8$B+$t$ and $^7$Be+$t$ final states was $10\pm3.2_{\text{stat.}}$. Theoretical estimates of a $1p_{1/2}$ excited state yield relative to the $1p_{3/2}$ ground state were determined using distorted-wave calculations with both standard and VMC-derived bound state potentials and theoretical relative spectroscopic factors from Cohen-Kurath $p$-shell model and VMC theoretical calculations. The standard bound state estimated relative yields are $13.9\pm3.4$ for Cohen-Kurath spectroscopic factors and $36.3\pm8.8$ for VMC spectroscopic factors. The VMC-derived bound state potentials and relative spectroscopic factor produced an estimated relative yield of $36.0\pm6.6$ counts.

The Cohen-Kurath estimate is in agreement with the experimental yield which brings into question the accuracy of the VMC relative spectroscopic factors. In the $^9$C mirror system, $^9$Li, the VMC produces relative spectroscopic factors for the ground and first-excited state that are in general agreement with experiment [26]. In the same study, the Cohen-Kurath spectroscopic factors for the $J^\pi=\frac{1}{2}^-$ ground and
first-excited state were a factor of two less than the VMC calculations. This inconsistency, along with the strict assumptions made regarding $^9$C ground and excited-states, make the model-dependent relative-yield estimates in this work tentative at best and will require comparison to future single-particle studies of the excited states of $^9$C.

8.4 Future Developments

The continuing study of the structure of $^9$C is vital for the development of ab-initio nuclear theories and the understanding of the nuclear interaction in few-nucleon systems. The challenge of studying nuclei at the drip-lines using single-particle transfer is being met with the current generation of radioactive beam facilities. The arrival of next-generation facilities with orders-of-magnitude increase in radioactive beam intensities will provide experimental opportunities to possibly answer some of the questions posited by this current work and other studies of $^9$C. In the interim, there exist many experimental and theoretical avenues to pursue. The detection and identification of the recoils from the proton-unbound excited states of $^9$C is possible with a simple configuration of planar detectors. This technique could be used in the study of other nuclei away from stability and, with small changes in geometry and segmentation would, provide a more useful means to probe the structure of proton-unbound states in nuclei near the proton drip-line (i.e. $^{13}$O). The observed enhancement of the cross sections from distorted-wave calculations using from the VMC form factors warrants further study to evaluate both ab initio and transfer-reaction theories. A transfer reaction study of one or more light isotopes (A ≤ 10) using a variety of single-nucleon transfer reactions, all measured in the same detector system, would allow for a detailed comparison of the different form factors. The continuing development of the Green’s function Monte Carlo method and its comparison to single-nucleon overlaps calculated by the Variational Monte Carlo will also be useful for the understanding of the enhancement. In lieu of new experimental data, the many transfer-reaction studies of p-shell nuclei could be surveyed, calculated using their published optical-model parameters, and compared to DWBA calculations using VMC-derived form factors. The inclusion of other ab initio calculations and distorted-wave codes could clarify the dependence of this enhancement on the reaction and structure models used. Such endeavors could provide the groundwork for the development of ab initio reaction theories.
The structure of the neutron-deficient nucleus $^9\text{C}$ has been studied through a single-neutron transfer reaction. The neutron-pickup reaction $d(^{10}\text{C},t)^9\text{C}$ was performed at an energy of 17.1-MeV per nucleon using a radioactive $^{10}\text{C}$ beam produced via the “in-flight” method. Tritons from the reaction were detected and identified in an array of highly-segmented double-sided silicon detectors while $^9\text{C}$ and heavy recoils from the proton-unbound $^9\text{C}$ excited states were detected in a series of forward-angle silicon detectors in $\Delta E$-$E$ telescope configurations. The ground state transition was unambiguously identified in the detector system as well as events from the proton-unbound excited states of $^9\text{C}$ which decay to the ground states of $^8\text{B}$ and $^7\text{Be}$.

The triton angular distribution for the $d(^{10}\text{C},t)^9\text{C}$ ground-state transition was measured and the absolute scale of the observed cross sections determined by a detailed analysis of the target thicknesses, radioactive beam intensity, and simulated detector efficiencies. The resulting ground-state angular distribution was compared to those from distorted-wave Born approximation calculations using several sets of optical-model parameters. The calculated angular distributions, assuming a $J^{\pi}=\frac{3}{2}^-$ ground state, are in good agreement with the data. The events observed from $^9\text{C}$ excited states were in agreement with a shell-model estimate of the yield of a low-lying $J^{\pi}=\frac{1}{2}^-$ state relative to the ground state in $^9\text{C}$.

The use of bound-state optical-model potentials derived from Variational Monte Carlo calculations in the distorted-wave calculations produces enhanced cross sections for the $^9\text{C}$ ground-state transition relative to standard bound-state potentials. This enhancement is over a factor of three and consistent over all set of incoming and outgoing optical-model parameters. The enhancement from ab-initio form factors has also been reported in previous $(d,t)$ and $(d,^3\text{He})$ studies, but is absent in similar $(d,p)$ evaluations.
This sensitivity of the distorted-wave calculations on the VMC-derived form factor warrants further investigations to evaluate the use of \textit{ab initio} structure calculations in theoretical transfer-reaction studies.
References


