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An Adaptive Kalman Filter for Voltage Sag Detection in Power Systems

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AN ADAPTIVE KALMAN FILTER FOR VOLTAGE SAG DETECTION IN POWER SYSTEMS

by

Hisham Odeh Alrawashdeh

A dissertation submitted to the Graduate College in partial fulfillment of the requirements for the degree of Doctor of Philosophy Electrical and Computer Engineering Western Michigan University April 2014

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The exact values of the noise covariance matrix of the Kalman filter state vector $Q$ and the measured signal noise covariance matrix $R$ must be obtained in order to achieve the optimal performance of the Kalman filter. There have been many techniques and assumptions made to evaluate and compute $Q$ and $R$. The effects of $Q$ and $R$ are investigated in detail in this dissertation. Based on these investigations, the Kalman filter has been modeled to detect the fundamental signal amplitude variations of power system signals. This technique helps in evaluating voltage sags in power systems.

Two algorithms are proposed in this dissertation. In these algorithms, the performance of the Kalman filter is investigated with the assumption that $Q$ and $R$ matrices are unknown and not necessary to evaluate them exactly. In the proposed algorithms, two adaptive Kalman filters are used to detect and to track the variations in the fundamental amplitude of the measured power system voltage signal in order to diagnose voltage sags. Also, the algorithms work for normal frequency variations in power systems.
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Hisham Odeh Alrawashdeh
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CHAPTER 1

INTRODUCTION

1.1 Power System Harmonics

Power system harmonics can be defined as the components of the electrical signals that have multiple frequencies of the fundamental (system) frequency, $f_0$, which is either 50 or 60 Hz [1-6]. However, an electrical signal (such as voltage or current) will have the fundamental component besides the harmonic components. Instead of having pure sinusoidal waveform of the fundamental frequency, there will be a distorted sinusoidal signal.

The harmonic signals can be classified as low or high order harmonics, with respect to their frequency magnitude; where the amplitude of the harmonic signal is usually inversely proportional to the harmonic order. The low order harmonics have larger amplitude values compared with the high order harmonics. For that reason the low order harmonics are more effective in signal distortion and they have the most priority in harmonics detection compared with higher order harmonics.

The main causes of the power system harmonics are the nonlinear devices such as converters, static VAR compensators, transformers, rotating machines, arc furnaces, fluorescent lightings, power electronics loads and unbalanced system conditions [1,2,3]. In transformers, rotating machines, and fluorescent lighting, the source of harmonics is the saturation of the magnet cores in these devices. The rotating machines are designed to operate linearly for economic reason [2,3]. Due to the load variations, this may cause the operating point to be in saturated area. The
order of harmonics in the transformer is usually $6n \pm 1$ \cite{1}, where $n$ is the order of the harmonic.

The harmonics affect the performance of the power systems, and their effects can be divided into short term and long term effects \cite{3}. In the short term the harmonics cause high distortion in the electrical signal, while in the long term there is an increase in the losses and the voltage stress \cite{3}. The effects of the harmonics in electrical apparatus can be summarized as follows \cite{1,2}:

1. Harmonic losses: there are several types of harmonics losses;
   a. Harmonic resonance: the resonance may occur with one of the harmonic frequencies, and this will cause over voltage and high losses in the power system, which may damage the power system components or shorten the life of the power devices.
   b. Copper losses in stator and rotor of the rotating machines especially due to the low order harmonics. The core and the eddy current losses also increase in transformer and rotating machines.
   c. Corona losses: the harmonics will cause over-voltages in overhead transmission lines, which increases the corona discharge that causes extra losses in the power systems.

2. Increase dielectric stress: in cables this will shorten their lives and for overhead transmission lines, this will increase the corona phenomena in power systems. These can increase the chances of breakdown of these lines.

3. Changing the shape and the characteristics of the voltage and the current waveforms: this will affect the performance of the
protection devices, the quality of the consumer devices, the power factor, and the power measurement.

4. Interference between the power system and the communication system: can cause high noise in the communication signal.

The reduction of the harmonics is one of the main concerns in power systems. For that reason, the Kalman filter is widely used in harmonics detection in power systems [12-57].

1.2 Kalman Filter

The foundation of Kalman filter was established in 1960 by R.E. Kalman for discrete time systems and extended to the continuous systems [7]. The Kalman filter is a recursive, linear, real time, and optimal filter to estimate the state of dynamic noisy system. The noise sources in the Kalman filter are assumed to be independent Gaussian white additive noise [7,8,9]. A recursive Filter means that it is suitable for real time applications, where there is no need to store much data; it needs only the current state and the previous state. The optimality of the Kalman filter is due to minimizing the statistical variance matrix of the state error. The optimality of the Kalman filter can be achieved under certain conditions [7]:

- The system is linear and well modeled.
- All noise sources are white Gaussian noise.
- The covariance matrices of the noise are well defined.

The Kalman filter calculation can be divided into two stages; prediction and updating stages. In the prediction stage, the calculation is predicted based on the previous instant state until recent data was measured, and then the calculation enters the updating stage to modify the predicted state [7-9]. Several previous studies
showed the effect of using the Kalman filter and the extended Kalman filter in power system harmonics detection. Some of them mixed the Kalman filter with other optimization methods to overcome some of the Kalman filter drawbacks. The Kalman filter is designed to give the optimal solution under certain conditions, and one of these conditions is the certainty of noise covariance matrices Q and R. If these matrices are not defined very well, they will cause the Kalman filter to obtain suboptimal solution, or it may diverge. The great majority of the previous studies show that the Kalman filter used to detect harmonics in power systems, focused on how to estimate the noise covariance matrices (Q and R) [8-13].

The previous studies that used the Kalman filter in power systems can be divided into four categories;

In category one, studies showed the capability of the Kalman filter in harmonics detection. Even though these studies showed that the Kalman filter is an efficient filter for detecting the harmonics in power systems. The main disadvantages of this category are: (i) there is no sufficient information for the noise covariance matrices; (ii) in each study, the values of noise covariance matrices (R and Q) are different without showing how to determine these values; and (iii) the Kalman filter was used for static case where there is no dynamic change in the system parameters [2-12].

The second category showed that the previous studies used other techniques beside the Kalman filter to estimate the harmonics in power systems in order to overcome some of the drawbacks of the Kalman filter. The main techniques that were used with the Kalman filter are: (i) adaptive linear neural network (KF-Adaline); (ii) fuzzy algorithms; (iii) the least square error method; and (iv) αβ transformation method. The studies showed the flexibility of the Kalman filter to be used with other
techniques. The main drawbacks of this category are the increased complexity of the proposed methods. Even though these studies have good results in harmonic detection, they did not show all cases, such as fundamental amplitude and frequency variations in power systems [13-24].

In the third category, researchers proposed other methods such as: (i) linear-minimum variance filter; (ii) a new algorithm to estimate harmonics based on the $H^\infty$ filter; and (iii) a new algorithm to estimate the harmonics based on a neural network. They showed their methods performance over the Kalman filter. The advantage of these proposed methods showed better results or at least the same as the Kalman filter results but the main gaps in these methods were that each proposed filter was designed for specific applications. Even though better results were obtained in this category than Kalman filter method, the sufficient conditions for Kalman filter optimality were not met [25-34].

The fourth category showed that the previous studies proposed adaptive Kalman filter but in different fields such as: (i) a new algorithm to estimate the motor temperature based on adaptive Kalman filter; (ii) estimate the charge state of lithium-ion batteries; (iii) for navigation systems; (iv) and to track stereo visual schemes. In this category the studies showed that updating the Kalman filter matrices allows the Kalman filter to detect the dynamical change in system parameters. The main disadvantages of this category are: (i) they used very long formula for evaluating the actual value of the noise covariance matrices ($R_{real}$ and $Q_{real}$) to be updated; (ii) the covariance matrices were updated using extra techniques like fuzzy logic; (iii) and there were strong arguments about the importance of updating one or both of the noise covariance matrices or updating the ratio between them [35-57].
1.3 Dissertation Outline

- Chapter 1 introduces the main concepts that will be used in the later chapters.
- Chapter 2 will contain the literature review of the power system harmonics, Kalman filter, adaptive Kalman filter, and fuzzy algorithm.
- Chapter 3 discusses the effect of Q, R and Q/R on the performance of the Kalman filter for static signal under several circumstances.
- Chapter 4 discusses the effect of Q, R and Q/R on the Kalman filter performance for dynamic change in the system fundamental amplitude.
- Chapter 5 proposes two adaptive Kalman filters based on the results obtained from the previous chapters to improve the Kalman filter performance for amplitude detection in power system.
- Chapter 6 contains conclusions and future work.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Harmonic is one of the major problems in power systems, where it is essential to detect the fundamental component and the harmonics in the measured electrical signal. In many applications, the operation of the protection and control devices needs an accurate measurement of the electrical signal. If the measured signal has a high value of harmonics and noise, this may cause a wrong decision in control and protection devices. For that reason, the Kalman filter is widely used in harmonics detection of power systems [12-57]. Several previous research studies showed the effect of using the Kalman filter and the extended Kalman filter in power harmonic detection, and some of the previous studies mixed the Kalman filter with other optimization methods to overcome some of the Kalman filter drawbacks. Zadeh et al. [12] combined the Kalman filter with the least square error method to solve the delay issue in the Kalman filter for the transient period and to reduce its sensitivity to the system information. They showed that the Kalman filter had a delay at about 2.5 cycle in the transient case, where in this period, the change of the system parameters are usually large. Beides and Heydt [13] showed that the Kalman filter was efficient to estimate harmonics in the IEE-14 bus system with different noise ratios. Dash et al. [14] used the αβ transformation to transform the three phase quantities to complex vector. Then, they showed the quality of using the Kalman filter in harmonics estimation under certain level of signal-to-noise ratio. Hali and Girgis [15] used the Kalman filter for noise source identification. The Kalman filter is used to select the optimal location of a limited number of harmonic meters, then based on the location,
the Kalman filter will be able to estimate the harmonics in power systems. Al-Hamadi and Soliman [16] proposed a new approach by assuming the harmonic signal as fuzzy noise with a linear model. Then they used the Kalman filter to identify the critical parameters of the membership functions of the fuzzy noise. The main reason for using the fuzzy model for the harmonics is "Obtaining fuzzy parameters rather than crisp ones yields all possible extreme variations the parameters can take" [16]. Kennedy et al. [17] proposed an adaptive extended Kalman filter to estimate the harmonics in power systems. They used three signals to test the Kalman filter capability for estimating the harmonics under several conditions. They concluded that modifying $Q/R$ is not sufficient, especially when the S/N ratio exceeds 30 dB. They proposed an algorithm to compute $Q$ and $R$ at each iteration. Routray et al. [18] proposed an adaptive extended Kalman filter for power system frequency measurement. Their proposed Kalman filter resets the covariance matrices based on the error and the convergence of the algorithm. They mentioned that the covariance matrices should be reset as long as any one of the parameters changes. Kumar et al. [19] proposed a new technique based on a Kalman filter to estimate the dynamic harmonics in power systems. The proposed method required lower harmonics meters since it used pseudo measurement for the unknown measurement. Joorabian et al. [20] divided the harmonic model to linear and nonlinear models. They used the least square method to solve the linear model where there is no need to have exact parameters to estimate the amplitude of the signal. The nonlinear model is solved using a neural network. Valipour et al. [21] proposed a new method based on a neural network to estimate on-line harmonics data and compared their results with Kalman filter. Saiz and Guadalupe [22] used a Kalman filter for on-line harmonics tracking,
and the fundamental matrix is calculated off line and stored in order to reduce the computation time.

Liu and Chen [23] proposed a new model of harmonics. The amplitude and phase are presented as a summation of wavelet functions and the Kalman filter is combined with this model to estimate on-line harmonics. Serna and Maldonado [24] used a Taylor-Kalman filter to estimate the harmonics in order to overcome the sensitivity of the Taylor-Kalman filter by replacing it with k-type Taylor-Kalman-Fourier filter. They compared different types of Taylor-Kalman filters based on the computational complexity and cost. Subudhi and Ray [25] proposed a Kalman filter mixed with an adaptive linear neural network (KF-Adaline) to estimate the harmonics in power systems, and they compared the results with those found by a recursive least square adaptive linear neural network. Based on their results the KF-Adaline has better harmonics estimation. Shatshat et al. [26] proposed a new algorithm (Adaline – based active power filter), and they proved the quality of their algorithm by comparing the results with the Kalman filter. Cardoso and Grundling [27] proposed on-line frequency identification to correct the fundamental matrix in the Kalman filter to have better performance. When linear Kalman filter is used to estimate the harmonics, constant frequency is assumed, which may cause incorrect results. The frequency is determined by using an Amplitude-Frequency-Phase-Locked loop. Pigazo and Moreno [28] modeled the Kalman filter matrices in different ways to minimize the execution time for the harmonics estimation. Gunda and Sarma [29] used a Kalman filter to reduce the harmonics of the adjustable speed drive and showed the performance of the algorithm by comparing their results with that obtained from FFT filter. Sanoo and Dash [30] proposed a new filter using a complex $H^*$filter to estimate the variable frequency and the amplitude of a signal. They
compared their results with those obtained with an extended Kalman filter to estimate the frequency in worse case measurement, and their results showed a better performance of the proposed filter over the extended Kalman filter. Girgis et al. [31] proposed a new algorithm based on the Kalman filter to estimate amplitude and frequency of time varying amplitude signal. Huang et al. [32] proposed a fuzzy algorithm combined with a Kalman filter to estimate the frequency in distorted signals. The value of $R$ matrix is equal to the inverse of the weight matrix. The weight matrix is equal to $W_k = W_{k-1} e^{x_k - u_k}$. The Kalman filter is used to adjust the Kalman gain state covariance matrix by using the residual and rate of residual, in order to reduce the divergence of the filter. They mentioned the importance of resetting the Kalman gain and the state covariance matrix after any change in the parameters (i.e. frequency, amplitude and phase). Macias and Exposito [33] proposed self-tuned Kalman filter to overcome the divergence problem relates to the $Q$ and $R$ matrices. The covariance matrix $Q$ is modified based on the square of the state error and the results of the self-tuned Kalman filter is compared with the results obtained from other methods where $Q$ was kept constant. The self-tuned Kalman filter estimates the amplitude quickly when a sudden change happened. Nesilhan et al. [34] proposed an algorithm based on the Kalman filter to estimate the fundamental frequency, when the fundamental frequency is highly distorted. Sadinezhad and Agelidis [35] proposed a new algorithm based on an artificial neural network to estimate the harmonics in power systems. The algorithm was proposed to be suitable for on-line tracking, and it works for low rate sampling and low computational time. They used the extended Kalman filter to correct the estimated error of the fundamental frequency that may occur in the proposed method. Le et al. [36] investigated statistically the performance of digital filters such as Kalman filter when the noise is not white. They concluded
that "In the case that the Kalman filter is applied to a quasi-stationary signal under harmonic models, the rms errors between the input and the estimated signals are rather small. If a few harmonics of power frequency are interested, it is not necessary to have high model order to reduce computational load" [36]. Sahoo et al. [37] proposed a new algorithm to estimate the amplitude and the frequency of the fundamental signal amplitude and the odd harmonics. They combined $H^*$ to estimate the amplitude and the "Adaline" algorithm to estimate the frequency. They compared their results with the results obtained from the extended Kalman filter. Based on their results, they concluded that the new algorithm is faster than the Kalman filter, and it is applicable to all noise sources, while the Kalman filter is applied only for white noise. Rechka et al. [38] compared the performance of several harmonic digital filters such as DFT, Kalman filter, etc, and based on their results, the Kalman filter has good performance in terms of computational time, detection precision, and robustness.

Yaz et al. [39] compared the performance of the extended Kalman filter with the linear-minimum variance filter based on Monte carlo simulation. The error covariance matrices are unknown, and the standard deviation is considered as a state variable in their model. The number of variables in this presentation becomes very large, and it will double the number of variables in the harmonic model. They assumed standard white noise (zero mean and unity variance). The linear minimum variance filter had better performance compared with the Kalman filter based on their results. Sahoo and Rath [40] proposed a new algorithm to estimate harmonics based on $H^*$ filter, and they compared their results with a Kalman filter. Based on their results, the proposed algorithm had better performance than the Kalman filter.

Mori et al. [41] proposed a new algorithm to estimate the harmonics based on neural network. They compared their results with 10 digital filters, one of which is the
Kalman filter. They concluded that the adaptive time variant algorithms have better performance in harmonics estimation than time invariant algorithms. Borkowski and Bien [42] proposed an algorithm to estimate the frequency based on a discrete Fourier transform. They combined an extended Kalman filter with B-spline resample and algorithm can be used for on line and off line tracking problems. The main feature of this algorithm is the capability for solving the lacking synchronizing problem between the sampling rate and the fundamental frequency.

Aghazadeh et al. [43,44] used a Kalman filter to estimate the fundamental frequency in a power system under several considerations such as: noise, harmonics, frequency changing, and load changing. Najafi et al. [45] used a Kalman filter to estimate the on line harmonics in coupling point between HVDC and HVAC in power systems. Griffo et al. [46] proposed strategy to solve linear quadratic regulator (LQR) problems to estimate the frequency in power systems. They emphasize that the optimal value of the LQR problem depends on the relative value between $Q$ and $R$, not on their absolute values. The first estimation of $Q$ and $R$ matrices are 1 and 0 respectively. The value of $R$ is calculated using a nonlinear inequality equation and the choice of $Q$ value was not clear, but they used this $Q$ to reduce negative sequence in the current.

Dash et al. [47] proposed a new technique based on "Adaline" neural network to estimate the harmonics in a power system. They compared the proposed technique with the Kalman filter and concluded that the new algorithm is simpler than the Kalman filter in real-time calculations. Fusco and Russo [48] proposed a self tuning technique for voltage control in power system. They divided the main tasks of the technique into three tasks. The first task is to measure the fundamental and harmonics component of voltage and current then use the Kalman filter to estimate them. In the
second task, the least square method is used for on line parameter identification. In the third task, the adaptive voltage regulator is used to regulate the voltage to the desired value. Moo and Chang [49] proposed an adaptive controller to estimate the harmonics. They used the steepest descent method to estimate the output voltage and a modified Kalman filter to compensate the delay in DSP. Bueno et al. [50] proposed a new algorithm based on the Kalman filter to reduce the effect of the noise in order to improve the quality of the power system convertor. They discussed the importance of calculating $R$ and mentioned that when $R$ is assumed to be diagonal, it affects the Kalman filter quality for dependent variables. Bittanti and Savaresi [51] suggested a new way to reduce the controlling parameters of the Kalman filter from three to one. They mentioned in their paper that the ratio between $Q$ and $R$ is more important than their real values. Maitelli and Yoneyama [52] used a Kalman filter for parameters identification and at the same time they used another algorithm to tune the covariance matrices. Scala and Bitmead [53] proposed a new extended Kalman filter to estimate a time-varying frequency; they used three control parameters $\varepsilon, q$ and $r$. $Q$ must be positive definite to have bounded positive controllability matrix. They claimed that the effective of $Q$ is more significant than $R$.

Yu et al. [54] proposed a new technique to estimate the value of $Q$ matrix and switched between two values of $Q$ matrix; one for the transient and the other for steady state. At the beginning, they set $Q = 0$ and the value was good in the steady state part, but it diverges when they had a step change in the signal. Then they set $Q=1$ and showed good response in the transient part (step change in signal) but they had large oscillation in the steady state part. They used student statistical model with 95% confident to decide which $Q$ to be used.
Many of the previous studies assumed known value of the noise covariance matrices $Q$ and $R$ while the other used an adaptive Kalman filter to estimate $Q$ and $R$ in detecting harmonics in power systems. Many previous studies used an adaptive Kalman filter in different applications (rather than for power system harmonics). The distorted signal in the Abdelsalam et al. [55,56] algorithm must pass through a hybrid model of wavelet and Kalman filter to define three parameters; amplitude, slope of amplitude, and harmonics indication. A fuzzy controller is then used to determine the power quality disturbances. The $Q$ matrix is set to be 1 in the Kalman model. $R$ is calculated at each iteration, and $R$ is equal to the covariance of the coefficient of the first row of the digital wavelet matrix. Based on the fuzzy inputs, the disturbance in the power system can be classified to normal, interruption, sag, swell, and surge.

Nounou et al. [57] proposed a multi scale fuzzy Kalman algorithm to solve several data filtering problems. In their literature review, they mentioned that the previous fuzzy algorithms used a single scale while a signal may have multi scale data. Jun and Yuzhou [58] proposed an algorithm that combined an extended Kalman filter with a cascade speed observer to improve the performance of a synchronous motor speed observer. Based on their results, the fuzzy controller increases the algorithm robustness more than the conventional methods. Huang et al. [59] proposed a fuzzy adaptive combined with a Kalman filter to estimate the frequency in noisy signals. The fuzzy system was used to adjust the Kalman gain and the state covariance matrix by using the residual and rate of residual in order to reduce the divergence of the filter. They mentioned the importance of resetting the Kalman gain and state covariance matrix after any change in the parameters (that is frequency, amplitude, and phase).
The idea of resetting the Kalman gain matrix using hysteresis was proposed by Dash et al. by using the threshold method. Reynolds [60] proposed a new algorithm for a signal with large noise. The aim of the proposed algorithm is to have a robust technique for noise covariance matrix estimation. The measurement noise estimation can be classified into four types based on Mehra classification and one of them is innovation correlation method. They compared their results with the Belangar method, who used LSR to estimate the covariance matrix. They mentioned that the Belangar method is optimal in the narrow sense. Gao et al. [61] proposed a new algorithm to estimate the motor temperature based on an adaptive Kalman filter. They estimate both $Q$ and $R$ using on-line calculations. Noriega and Pasupathy [62] introduced a method to estimate the error covariance matrices adaptively based on innovations-correlation method. Sangsuk-Iam and Bullock [63] analyzed the divergence and convergence of the Kalman filter of incorrect noise covariance matrices. They concluded that, even though that the covariance matrices in some cases are incorrect, the Kalman filter was asymptotically stable. If the system is modeled accurately, the incorrect covariance matrices will never cause the divergence of the Kalman filter. Gerasimos [64] compared the results obtained from an extended Kalman filter (EKF) with one obtained from a particle filter. For position measurement, the particle filter does not make any assumption about nature noise. Based on results, the particle filter has better performance on the accuracy and the robustness to the nature of noise.

Al-Hamadi and EL-Naggar [65] proposed a new fuzzy-Kalman filter to estimate the synchronous motor parameters that can be obtained from short circuit test. The fuzzy model was used to adjust the error covariance matrices. They compared the result of the proposed filter with the traditional Kalman filter and the
results were good for this algorithm. Other algorithms have been combined with a Kalman filter in different non harmonic detection applications [66-72] such as; a new fuzzy Kalman filter to track stereo visual scheme, an algorithm to estimate the charge state of lithium-ion batteries using a neural network and an extended Kalman filter, and an intelligent algorithm for navigation systems.

2.2 Dissertation Goals

Based on the literature review, there are two main goals for this dissertation. The first goal is to investigate the effect of noise covariance matrices $Q$ and $R$ in the Kalman filter to detect the fundamental signal amplitude in static and dynamic cases.

The second goal is to propose an adaptive Kalman filter to detect and track the fundamental signal amplitude in power systems when the dynamic change occurs. This will help to diagnose some problems in power systems such as voltage sag problem.
CHAPTER 3

KALMAN FILTER INVESTIGATION FOR STATIC SIGNAL

3.1 Kalman Filter for Voltage Sag Problem

In most of practical applications, control devices operate based on the measured signals, where the measured signal usually has a noisy portion. The noise may affect the decision of the control devices and for that reason several types of filters are used to get rid of the noise; one such filter is the Kalman filter.

The Kalman state vector $X_k$ at any instant can be calculated as follows:

$$
X_k = G_k X_{k-1} + B_k U_k + W_k
$$
$$
Z_k = C_k X_k + V_k
$$

(3.1)

where $X_k$ is the state vector, $G_k$ is the transition matrix, $B_k$ is the input control vector, $W_k$ is the process noise, (assumed to be white Gaussian noise with zero mean and covariance matrix $Q_k$), that is $N(0, Q_k)$, $Z_k$ is observation of the state $X_k$, $C_k$ is the observation matrix and, $V_k$ is the observation noise, (also assumed to be Gaussian white noise with zero mean and $R_k$ covariance matrix, that is $N(0, R_k)$).

The Kalman filter calculation can be divided into two stages: prediction and updating stages. In the prediction stage, the state is predicted based on the previous instant until a recent data is measured. The calculation enters the updating stage to modify the prediction state as follows:
For the prediction stage, the equations are:

\[ \begin{align*}
X_{k|k-1} &= G_k X_{k-1|k-1} + B_{k-1} U_{k-1} \\
P_{k|k-1} &= G_k P_{k-1|k-1} + G_k^T Q_k \end{align*} \]  
(3.2)

For the updating stage, the equations are:

\[ \begin{align*}
Y_k &= Z_k - C_k X_{k|k-1} \\
K_k &= P_{k|k-1} C_k^T \left( C_k P_{k|k-1} C_k^T + R_k \right)^{-1} \\
X_{k|k} &= X_{k|k-1} + K_k Y_k \\
P_{k|k} &= [I - K_k C_k] P_{k|k-1} \end{align*} \]  
(3.3)

where \( k \) is the discrete time, \( X_{k|k-1} \) is the state vector in current state given previous state, \( K_k \) is Kalman gain in discrete time, \( P_{k|k-1} \) is the state covariance matrix of current state given previous state, \( P_{k-1|k-1} \) is the previous state of the state covariance matrix, \( P_{k|k} \) is the current state of state covariance matrix, \( I \) is the unit matrix, and \( X_{k|k} \) is the current estimated value by the Kalman filter.

The Kalman filter will be used in this dissertation to estimate the fundamental component amplitude of a signal containing harmonics and noise. Based on the amplitude, there are several problems in the power system that can be diagnosed such as the voltage sag problem where the voltage decreases due to load increases even under normal operating conditions. If the sagging continues, the voltage can collapse. It is important to model the power system in order to estimate the amplitude of the fundamental.
3.2 Harmonics Modeling for Kalman Filter

The system can be modeled in two ways: (i) the first model includes only the fundamental component in the system matrices and harmonic components are ignored, and (ii) the second model includes fundamental component and the harmonic components to have better estimation of the fundamental signal. In the second model the size of the system matrices are larger than the first model since there are more harmonics in the second model. In power systems, the order of harmonics is odd and there are no even harmonics.

Generally, the state variables in discrete time $X(k)$ is chosen as follows:

For the Fundamental component (that is $n=1$) the signal state variable $X(k)$ at any instant will be:

$$X(k) = A \cos(wkT),$$  \hspace{1cm} (3.4)

where $A$ is a signal amplitude, $k$ is discrete time, $T$ is sampling time, and $w$ is the fundamental frequency.

For the next state the signal becomes:

$$X(k+1) = A \cos(w(k+1)T),$$  \hspace{1cm} (3.5)

Geometry Conversion can be used to decompose equation (3.5) as:

$$X(k+1) = A \cos(wkT)\cos(wT) - A \sin(wkT)\sin(wT)$$

Let $A \cos(wkT) = X_1(k)$, $A \sin(wkT) = X_2(k)$, and $\cos(wT)$ and $\sin(wT)$ are constants.
\[ X_1(k + 1) = A \cos (w k T) \cos (w T) - \sin (w k T) \sin (w T) \]

\[ X_1(k+1)= \cos (w T)X_1(k) - \sin (w T)X_2(k) \] \hspace{1cm} (3.6)

\[ X_2(k + 1) = A \sin (w(k + 1)T) = A \sin (w k T + w T) \]

\[ X_2(k + 1) = A \cos (w k T) \sin (w T) - \sin (w k T) \cos (w T) \]

\[ X_2(k + 1) = \sin (w T)X_1(k) + \cos (w T)X_2(k) \] \hspace{1cm} (3.7)

The state equations will be as follows:

\[ X(k + 1) = G X(k) + B U(k) + W_k \]

\[ Y(k + 1) = C X(k) + D U(k) + V_k \] \hspace{1cm} (3.8)

where \( G \) is the transition matrix.

Substituting (3.6) and (3.7) into (3.8) leads to:

\[
\begin{bmatrix}
X_1(k+1) \\
X_2(k+1)
\end{bmatrix}
= \begin{bmatrix}
\cos(wT) & -\sin(wT) \\
\sin(wT) & \cos(wT)
\end{bmatrix}
\begin{bmatrix}
X_1(k) \\
X_2(k)
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0
\end{bmatrix}
U(k).
\] \hspace{1cm} (3.9)

where \( G = \begin{bmatrix}
\cos(wT) & -\sin(wT) \\
\sin(wT) & \cos(wT)
\end{bmatrix} \), \( B = \begin{bmatrix}
0 \\
0
\end{bmatrix} \)

For the third harmonics, the state variables at any instant will be:

\[ X_3(k) = A \cos (3 w k T) , \]

\[ X_4(k) = A \sin (3 w k T) \] \hspace{1cm} (3.10)
The next state for the signals in (3.10):

\[ X_3(k+1) = A \cos(3wkT) \cos(3wT) - \sin(3wkT) \sin(3wT) \]

\[ X_4(k+1) = \cos(3wT)X_3(k) - \sin(3wT)X_4(k) \]  \hspace{1cm} (3.11)

\[ X_4(k+1) = A \sin(3wkT) \cos(3wT) + \cos(3wkT) \sin(3wT) \]

\[ X_4(k+1) = \cos(3wT)X_4(k) + \sin(3wT)X_3(k) \]  \hspace{1cm} (3.12)

Now by substituting (3.11) and (3.12) into (3.12) leads to:

\[
\begin{bmatrix}
X_3(k+1) \\
X_4(k+1)
\end{bmatrix} =
\begin{bmatrix}
\cos(3wT) & -\sin(3wT) \\
\sin(3wT) & \cos(3wT)
\end{bmatrix}
\begin{bmatrix}
X_3(k) \\
X_4(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix} U(k) \]  \hspace{1cm} (3.13)

Combining (3.9) and (3.13), the signal state variable becomes:

\[
\begin{bmatrix}
X_1(k+1) \\
X_2(k+1) \\
X_3(k+1) \\
X_4(k+1)
\end{bmatrix} =
\begin{bmatrix}
\cos(wT) & -\sin(wT) & 0 & 0 \\
\sin(wT) & \cos(wT) & 0 & 0 \\
0 & 0 & \cos(3wT) & -\sin(3wT) \\
0 & 0 & \sin(3wT) & \cos(3wT)
\end{bmatrix}
\begin{bmatrix}
X_1(k) \\
X_2(k) \\
X_3(k) \\
X_4(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} U(k) \]  \hspace{1cm} (3.14)

And the transition matrix for first and third harmonics (G):

\[
G =
\begin{bmatrix}
\cos(wT) & -\sin(wT) & 0 & 0 \\
\sin(wT) & \cos(wT) & 0 & 0 \\
0 & 0 & \cos(3wT) & -\sin(3wT) \\
0 & 0 & \sin(3wT) & \cos(3wT)
\end{bmatrix}
\]
In general the state variable for the signal with \( n \)th order harmonic at any instant can be as follows:

\[
X(k) = [x_1(k) \ x_2(k) \ x_3(k) \ \ldots \ x_n(k)]^T
\]

\[
x_1(k) = A \cos(wkT + \theta_1)
\]

\[
x_2(k) = A \sin(wkT + \theta_1)
\]

\[
x_3(k) = A \cos(3wkT + \theta_3)
\]

\[
x_4(k) = A \sin(3wkT + \theta_3)
\]

\[
\vdots
\]

\[
x_{n-1}(k) = A \cos(nwkT + \theta_n)
\]

\[
x_n(k) = A \sin(nwkT + \theta_n)
\]

where \( n \) is the order of harmonic.

\[
G = \begin{bmatrix}
G_1 & Z & Z & \ldots & Z \\
Z & G_3 & Z & \ldots & Z \\
Z & Z & G_5 & \ldots & Z \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
Z & Z & Z & \ldots & G_n
\end{bmatrix}
\]

The jth transition matrix \( G_j \) is given as:

\[
G_j = \begin{bmatrix}
\cos(jwT) & -\sin(jwT) \\
\sin(jwT) & \cos(wjT)
\end{bmatrix}
\]

\[
Z = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
B = [0 \ 0 \ \ldots \ 0]^T \text{ with } n \times 1
\]
\[
C = \begin{bmatrix}
1 & 0 & 1 & 0 & \cdots & 0
\end{bmatrix} \text{ with } 1 \times n \tag{3.20}
\]

\[D = 0 \tag{3.21}\]

This model can be used to detect the fundamental signal amplitude with any harmonic order by adjusting the \(C\) matrix.

In chapter 2, several previous studies discussed the importance of estimating the noise covariance matrices \(Q\) and \(R\) in Kalman filter performance. There were strong arguments regarding which is more effective in the Kalman filter performance: \(Q\), \(R\) or the ratio between them. The effect of the noise covariance matrices and the ratio between them will be discussed in details in this chapter for static signal.

### 3.3 Effect of Noise Covariance Matrices

In this section the amplitude of the fundamental component is kept constant (as a static signal) and the first model (as mentioned in section 3.2) will be used in the Kalman filter for simplicity to detect the amplitude of the fundamental component. The noise covariance matrices will be assumed unknown in order to examine the effect of the noise covariance matrices and their ratio carefully.
3.3.1 Effect of Measurement Noise Covariance Matrix \( (R) \)

Consider the following typical example of an electrical signal in (3.22),

\[
y(t) = 1.414 \cos(100\pi t + \pi/6) + 0.3 \cos(300\pi t + \pi/5) + 0.1 \cos(500\pi t + \pi/8) \quad (3.22)
\]

To model the electrical signal in (3.22), only the fundamental component will be used in the Kalman filter predicted stage to be used as a reference for any dynamic changes that may occur in the measured signal.

The fundamental component in (3.22) is:

\[
X(k) = 1.414 \cos(100\pi t + \pi/6) \tag{3.23}
\]

using equation (3.9) and choosing \( T = 0.1 \) msec as a sampling time, the state variable for the fundamental component in (3.23) is as follows:

\[
\begin{bmatrix}
X_1(k+1) \\
X_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
0.9995 & -0.0314 \\
0.0314 & 0.9995
\end{bmatrix}
\begin{bmatrix}
X_1(k) \\
X_2(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
U(k) \tag{3.24}
\]

so the transition matrix \( G \) and matrix \( B \) in (3.24) are respectively given as:

\[
G = \begin{bmatrix}
0.9995 & -0.0314 \\
0.0314 & 0.9995
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

Consider the signal in (3.22) with fundamental and third harmonics components.

Using equation (3.14), it can be shown that the state variable for the system in (3.22) is given by:

\[
\begin{bmatrix}
X_1(k+1) \\
X_2(k+1) \\
X_3(k+1) \\
X_4(k+1)
\end{bmatrix} =
\begin{bmatrix}
0.9995 & -0.0314 & 0 & 0 \\
0.0314 & 0.9995 & 0 & 0 \\
0 & 0 & 0.9995 & -0.0941 \\
0 & 0 & 0.0941 & 0.9995
\end{bmatrix}
\begin{bmatrix}
X_1(k) \\
X_2(k) \\
X_3(k) \\
X_4(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
U(k)
\]
where \( G = \begin{bmatrix} 0.9995 & -0.0314 & 0 & 0 \\ 0.0314 & 0.9995 & 0 & 0 \\ 0 & 0 & 0.9995 & -0.0941 \\ 0 & 0 & 0.0941 & 0.9995 \end{bmatrix} \) and \( B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \)

For measured or observation state:

\[
Z(k) = CX(k) + DU(K)
\]

The measurement state for signal in (3.22) is given by:

\[
Z(K) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \\ X_3(k) \\ X_4(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} U(k),
\]

Where \( C = [1 \ 0 \ 1 \ 0 \ 1] \), and \( D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \)

The Fig. 3.1(a) shows the graphical representation of a fundamental signal component given by equation (3.22). The Fig.3.1 (b) shows the fundamental components and harmonics and Fig.3.1(c) shows the addition of noise to the signal in (3.22). The Kalman filter will be used to detect the amplitude of the fundamental component of this signal. The real value of the measurement noise covariance matrix for this signal \( R_{real} \) is 0.1 but it will be assumed unknown in the Kalman filter model in order to figure out the importance of its value in the Kalman filter performance. In order to distinguish between the real values of noise covariance matrices and their assumed values in Kalman filter, the real values of \( R \) and \( Q \) will be called \( R_{real} \) and \( Q_{real} \). They will be kept constant in this chapter. The assumed values of \( R \) and \( Q \) will
be varied and the Kalman filter will be used to detect the amplitude of the fundamental component of (3.22).

![Graphs](image)

Fig. 3.1 (a) Plot of the Fundamental Signal, (b) Fundamental Signal with Harmonics and (c) the Fundamental Signal with Harmonics and Noise

Figure 3.2 shows the fundamental signal magnitude estimated by the Kalman filter for \( R \) equal to \( R_{\text{real}} \) \((R=R_{\text{real}}=0.1)\). The figure shows that the Kalman filter converges quickly to the actual value of the fundamental signal amplitude from arbitrary initial value.
Figure 3.3 shows the error between the actual value of the fundamental signal and the estimated value using Kalman filter for different initial values. In this figure $R = R_{\text{real}}$ and $Q = Q_{\text{real}}$. The initial value of the signal affects on the estimated value at the beginning of the operation of the Kalman filter, but the Kalman filter converges to the actual value of the fundamental signal amplitude eventually. The difference between the actual value of the fundamental and the estimated value by Kalman filter is called an “error signal”.

Fig. 3.2 Fundamental Signal Estimated by Kalman Filter for $R = R_{\text{real}}=0.1$

Fig. 3.3 The Difference between the Actual Value of the Fundamental Signal and the Estimated Value using Kalman Filter for Different Initial Values
The Fig 3.4 shows the effect of changing $R$ from 0.01 to 0.1 (that is 10 times) but with arbitrary initial values. There is hardly any difference between the actual value of the fundamental amplitude and the estimated value of the fundamental amplitude using a Kalman filter. The error signal is negligible.

Fig.3.4 The Amplitude of the Actual Value of the Fundamental Signal and the Estimated Amplitude using a Kalman Filter for Different Values of $R$ chosen from 0.01 to 0.1

Similarly the Fig 3.5 shows the effect of changing $R$ from 0.1 to 100 (that is 1000 times) with arbitrary initial values. Here the error signal and the steady state error increase as the value of $R$ increases.

Fig.3.5 The Amplitude of the Actual Value of the Fundamental Signal and the Estimated Value using Kalman Filter for Different Values of $R$ chosen from 0.1 to 100
It is noticed from Figs. 3.4 and 3.5 that the difference between the actual amplitude of the fundamental component and the estimated amplitude using the Kalman filter at the beginning of the operation (during transient state) is higher than at the steady state. This is due to the initial values. For more investigation of the effects of $R$ and by taking the initial values into consideration, the minimum and maximum values of $R$ in Figs. 3.4 and 3.5 (that $R=0.01$ and $R=100$) will be now examined in detail in Fig. 3.6 and Fig. 3.7 respectively; ten initial values of the Kalman state variables are chosen randomly from the interval [-2 2]. As it is shown in these figures, the initial values strongly affects on the Kalman filter performance in case of $R$ equals to 100 more than for the case of $R=0.01$.

![Fig. 3.6 The Amplitude of the Actual Value of the Fundamental Signal and the Estimated Value using Kalman Filter for Different Values of Initial Values for $R = 0.01$](image-url)
Fig. 3.7 The Amplitude of the Actual Value of the Fundamental Signal and the Estimated Value using Kalman Filter for Different Initial Values for \( R = 100 \)

The Fig. 3.8 shows another value of \( R = 0.0001 \) is chosen where the ratio between this value and \( R_{\text{real}} \) is 1000 (\( R_{\text{real}}/R = 1000 \)), which is the same ratio used in Fig. 3.7 (\( R/R_{\text{real}} = 100/0.1 = 1000 \)). Ten initial values are also chosen randomly from the same interval \([-2, 2]\). The Fig. 3.8 shows that the steady state error approaches zero with a small ripple. This means that as the value of \( R \) is very small, the initial value slightly affects on Kalman filter estimation.

Fig. 3.8 The Amplitude of the Actual Value of the Fundamental Signal and the Estimated Value using Kalman Filter for Different Values of Initial Values for \( R = 0.0001 \)
The Fig. 3.9 (a) shows the signal when $R_{\text{real}}$ changes to 100 ($R_{\text{real}}=100$). The signal in this case will have large portion of noise, which means that the amplitude of the measured signal increases because of this noise. In order to examine the effect of $R$ in this case, ten different initial values from interval [-2 2] are selected randomly and two values of $R$ are chosen; one is less than the $R_{\text{real}}$ by 100 times ($R = 1$) and the other is greater than $R_{\text{real}}$ by the same ratio ($R = 10000$). The results are shown in Fig. 3.9 (b) and Fig. 3.9(c), respectively. When $R = 1$, the response of the Kalman filter is independent of the initial value but it takes more time to approach the actual value of the fundamental signal. For $R = 10000$, the Kalman filter is highly dependent on the initial values, and it converges to another value instead of the actual value of the fundamental signal amplitude, which caused high steady state error.

![Fig. 3.9 (a) The Measured Signal when the Real Value of the $R_{\text{real}}$ is 100, (b) the Actual Amplitude of Fundamental and the Estimated Value using Kalman Filter for $R = 1$, and (c) the Actual Amplitude of Fundamental and the Estimated Value using Kalman Filter for $R = 10000$](image-url)
3.3.2 Effect of the Process Noise Covariance Matrix ($Q$)

The effect of $Q$ will now be examined for a constant $R$ (that is $R = R_{\text{real}} = 0.1$). The Fig 3.10 shows the estimated amplitude value by the Kalman filter compared with the actual value of the fundamental amplitude for different values of $Q$. As shown in Fig. 3.10, the performance of the Kalman filter is getting worse in estimating the amplitude as the value of $Q$ is increased.

![Fig. 3.10 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Different Values of $Q$, and $R = R_{\text{real}}$](image)

To discuss the effect of $Q$ in more detail, ten initial values of Kalman state variables are selected randomly from interval $[-2, 2]$ and two values of $Q$ ($Q=10^{-10}$ and $Q=10^{-5}$) are tested.
Q=0.1) used in Fig. 3.11. The performance of the Kalman filter is good for $Q=10^{-10}$ and worse for $Q=0.1$.

Fig. 3.11 (a) The Estimated Amplitude by Kalman Filter for $Q = 10^{-10}$ and $R = R_{\text{real}}$, and (b) The Estimated Amplitude by the Kalman Filter for $Q = 0.1$ and $R = R_{\text{real}}$. 
The Fig 3.12 shows the estimated amplitude of the fundamental signal by the Kalman filter for $Q = 0.1$ and different values of $R$. It is noticed that when $R$ is small, the estimation of the fundamental amplitude in the Kalman filter is worse (see Fig. 3.12 (a)). As $R$ increases, the performance of the Kalman filter improves until a certain value of $R$. The Kalman converges to the actual value slowly as shown in Figs. 3.12 (e and f). For example, for $R=0.01$, there is a significant amount of noise in the estimated fundamental amplitude. Also, for $R=100$, there is small ripple in the estimated fundamental amplitude and the steady state error approaches zero.

Fig. 3.12 The Estimated Amplitude by Kalman Filter for Different Values of $R$ and $Q=0.1$
Similarly, the performance of the Kalman filter for \( Q = 10^{-10} \) is also examined in Fig. 3.13 with the same different values of \( R \). The estimated value of the fundamental signal amplitude by the Kalman filter converges to the solution rapidly when the value of \( R \) is small and around the actual value (see Figs. 3.13 (a) and (b)). As \( R \) increases from 0.1 to 100,000, the Kalman filter becomes slow and the steady state error increases (see Figs. 3.13 (d)-(f)).

![Fig. 3.13 The Actual Value and the Estimated Value by Kalman Filter for Different Values of R and Q = 10^{-10}](image)

Based on the results shown in Fig. 3.12 and Fig. 3.13, the performance of the Kalman filter can be improved by controlling the ratio between \( Q \) and \( R \) matrices. By controlling the ratio between \( Q \) and \( R \), one control parameter can be used to adjust this ratio instead of using two control parameters (\( Q \) and \( R \)). The effect of \( Q/R \) ratio will be examined in the next section.
3.3.3 Effect of Ratio between the Noise Covariance Matrices (Q/R)

Fig. 3.14 shows the estimated amplitude of the fundamental signal by the Kalman filter for different values of the ratio $Q/R$. The response of the Kalman filter in estimating the fundamental amplitude can be divided into two parts: (i) when the ratio $Q/R$ is very small (see Figs. 3.14 (a and b)), the response is affected by individual values of $Q$ and $R$; and (ii) as the ratio $Q/R$ increases, the performance of Kalman filter is not affected by the individual value of $Q$ and $R$ in steady state portion.

![Fig. 3.14 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Different Values of $Q/R$, and $R_{\text{real}} = 0.1$](image)
The Fig. 3.15 shows the estimated amplitude of the fundamental signal by the Kalman filter for constant $Q/R$ ($Q/R = 10^{-9}$) and different values of $R$ ($R_{\text{real}}$ is still constant at 0.1). It is clear from Fig. 3.15 that as the value of $R$ increases, the Kalman filter has different responses even though that $Q/R$ is constant. When Fig. 3.15 is examined carefully, the steady state error increases significantly for $R = 1000$ and above (see Figs. 3.15 (d, e and f)) where this value is much larger than the real value ($R_{\text{real}} = 0.1$) by 10000 times.

![Fig. 3.15 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Different Values of $Q$ and $R$ with Constant $Q/R = 10^{-9}$](image-url)
In Fig. 3.16, the real value of the measurement noise covariance ($R_{\text{real}}$) is changed to 100 (very noisy environment). This figure shows that the estimated fundamental signal amplitude by the Kalman filter for different values of $Q$ and also for different noise covariance ratio $Q/R$. When the results in this figure are examined, it is seen that the performance of the Kalman filter to estimate the amplitude of the fundamental signal has a poor response.

Fig. 3.16 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Different values of $Q$ and $Q/R$, and $R_{\text{real}} = 100$
3.4 Analysing the Results

The results obtained from the previous figures in section 3.2 show how the value of $Q$ and $R$ strongly affect the Kalman filter performance. It is shown that the performance of the Kalman filter in estimating the fundamental signal amplitude depends on the right and well modelled system and the measured value (output of the system). In a very noisy environment, the Kalman filter must give more weight to the predicted value than the measured value and vice versa. Therefore, $Q$ and $R$ can be considered as weights for two signals; predicted and the measured signals. If the value of $R$ in the Kalman filter model is less than the $R_{real}$, the Kalman filter gives more weight to the measured signal, and this will cause more noise in estimated signal. If $R$ value is larger than $R_{real}$, the Kalman filter gives more weight to the predicted signal than the measured value, and this may cause a problem if the system is not modelled very well. Similarly for $Q$, if $Q$ is very small, the Kalman filter gives more weight to the predicted value than the measured value. If $Q$ is larger than $Q_{real}$, the Kalman filter gives more weight to the measured signal than to the predicted value.

The estimated fundamental amplitude of the system model depends strongly on the initial value. If the Kalman filter gives more weight to the predicted value based on $Q$ and $R$ values, the steady state error occurs because of the initial values of the state variables.
The two initial values \([1.221, 0.7050]^T\) are selected to investigate the effect of the Kalman filter on the predicted value based on \(Q\) and \(R\) values. The Fig. 3.17 shows the estimated actual value of the fundamental signal amplitude by the Kalman filter model for the system in equation (3.22) at \(Q = 10^{-10}\) and different values of \(R\). If \(R\) is small, the estimated amplitude of the fundamental signal amplitude has very little variation (see Figs. 3.17 (a) and (b)). For large values of \(R\), the estimated amplitude of the fundamental signal amplitude is constant and the steady state error approaches to zero (see Figs. 3.17 (c), (d), (e) and (f)).

![Fig. 3.17 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for \(Q= 10^{-10}\) and Different Values of \(R\) Starting from Initial Condition \([1.221, 0.7050]^T\)](image)

For that reason the ratio between \(Q\) and \(R\) is very important and can be used to control the Kalman filter performance.
The Fig. 3.18 shows the semi-log plots of the actual value of the fundamental signal amplitude and estimated fundamental amplitude value from the Kalman filter with different Q and R: note that Q is logarithm of base 10.

Fig. 3.18 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Several Values of \( R \) and \( Q \), and \( R_{real}=0.1 \)
Fig. 3.19 shows the estimated value for $R = 1000$ and $Q = 0.0032$ for different random initial values in the interval $[0 \ 2]$. The Kalman filter response converges successfully to the actual value of the fundamental signal in this figure and it is not affected by the initial values.
Fig. 3.19 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Random Initial Value lie in the Interval [0 2], $R = 1000$, and $Q = 0.0032$. For Fig. 3.20, the real value of the measurement noise covariance matrix is changed to 100 ($R_{real} = 100$). $Q$ and $R$ are changed to different values. This figure shows that even though the signal is too noisy, there are still values of $Q$ and $R$ rather than the real values that can be used to estimate the fundamental signal. Based on the results shown in Fig. 3.20, the estimated value by the Kalman filter is correct at $R=10^5$ and $Q=0.09$. 

![Graph showing actual and estimated values](image-url)
To investigate the performance of the Kalman filter for these values \((R=10^5, Q=0.09)\), fifty initial values from the interval \([0 2]\) are selected randomly. The estimated values by the Kalman filter are shown in Fig. 3.21. The results show that the estimated amplitude is around the actual value but the steady state error still exists.
The Kalman filter will be used to estimate the fundamental signal amplitude of the signal in (3.22) containing odd harmonic components. Actually in power systems only the odd harmonics are used. The Fig. 3.22 shows the signal containing all odd harmonic components until 51st harmonics with and without noise.
Fig. 3.22 Signal Contained Fundamental Signal and the First 51 Odd Harmonics with and without Noise of $R_{\text{real}}=0.1$.

The Fig.3.23 and Fig.3.24 show the estimated amplitude by the Kalman filter for different values of $R$, but $Q$ is kept constant. The Kalman filter shows good response for several values of $R$ near to the real value ($R_{\text{real}} = 0.1$). When $R = 1000$, the Kalman filter could not estimate the actual value correctly.
Fig. 3.23 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter of the Square Signal for Several Values of $R$, and $Q$ is kept constant.
Fig. 3.24 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter of the Square Signal for Several High Values of \( R \), and \( Q \) is kept constant.

From the previous results in this section, it was found that the performance of the Kalman filter can be improved by setting the value of \( Q \) to 0.0032 and the value of \( R \) to 1000. Fig. 3.25 and Fig. 3.26 show the estimated value of fundamental signal by Kalman filter with multiple values of \( Q = 0.0032 \). The results in these figures show that the Kalman filter estimates successfully the fundamental signal amplitude for \( Q = 0.0032 \) comparing to the other values of \( Q \). Even though the second signal is totally different from the first signal, they still have the same \( Q/R \) for \( R = 1000 \).
The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter of the Square Signal for Several Values of \( Q \), and \( R \) is kept constant.
Fig. 3.26 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter of the Square Signal for Several Values of $Q$, and $R$ is kept constant.
3.5 Effect of Fundamental Frequency Variation

The fundamental frequency is assumed to be constant in the system model making a linear system but if the fundamental frequency is not constant, the system is non-linear and an extended or unscented Kalman filter can be used. Usually in normal operation, the system fundamental frequency is almost constant for power system stability; small variations of the fundamental frequency are allowed between [49.5 to 50.5] Hz. To investigate the effect of the fundamental signal frequency variations on the Kalman filter estimation, the fundamental frequency of signal in (3.22) will be varied to different values between [49.2-50.5] Hz but the frequency value in Kalman filter model will be kept on 50 Hz.

Figure 3.27 shows the estimated fundamental signal amplitude by the Kalman filter for different values of the fundamental signal frequency with $R = 0.1$ and $Q = 10^{-10}$. The estimated amplitude by the Kalman filter is strongly affected by the frequency variation, because the frequency variation influences the predicted value of the Kalman filter model. In this case the estimated amplitude can be improved by reducing the weight of the predicted value inside the Kalman filter model and by increasing $Q$ or increasing the weight of the measured signal by or decreasing $R$. 
Fig. 3.27 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman Filter for Several Values of Fundamental Frequency, $R=0.1$ and $Q=10^{-10}$

Figures 3.28 and 3.29 show the estimated amplitude of the fundamental signal when the frequency of the measured signal is 49.5 Hz (the fundamental signal inside the Kalman filter model is still 50 Hz). In Fig. 3.28, the value of $R$ is equal to the real value (that $R_{\text{real}}=0.1$) and the value of $Q$ is changed but in Fig. 3.29, the value of $Q$ is $10^{-10}$ and the value of $R$ is changed. From Fig. 3.28, it is noticed that, as the value of $Q$ is increased, the Kalman filter gave more weight to the measured signal, which will
overcome the problem of the fundamental frequency in the Kalman filter model. When the value of $Q$ increases, the weight of the measured signal will increase and the Kalman filter will estimate the output based more on the measured signal than the predicted value. This will cause noise in the estimated signal similar to the measured noise.

![Graph](image)

Fig. 3.28 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman for Several Values of $Q$, $R=0.1$ and $f=49.5$Hz.
In Fig. 3.29, when the value of $R$ decreases, the Kalman filter will give more weight to the measured value to estimate the fundamental signal amplitude than the predicted signal. This will be helpful until the certain value of $R$, and then the estimated output will have a noisy portion as the measured signal (see when $R=10^{-7}$).

For frequency variations, the results show that the performance of the Kalman filter can be improved by balancing $Q$ and $R$ instead of using their exact values.

Fig. 3.29 The Actual Value of the Fundamental Signal and the Estimated Value by Kalman for Several Values of $R$, $Q=10^{-10}$ and $f=49.5\text{Hz}$.
CHAPTER 4

KALMAN FILTER INVESTIGATION FOR DYNAMIC SIGNAL

The main objective of the Kalman filter in this dissertation is to detect the amplitude of the fundamental signal in order to determine the voltage sag problem. In normal operation, the amplitude of the fundamental signal varies until a certain limit before the voltage collapses. In this chapter the amplitude of the fundamental signal will be changed to different levels, and the Kalman filter will be used to estimate the output. The effect of the noise covariance matrices will also be investigated for such signal. The same signal in chapter 3 will be used here, and is given by:

$$y(t) = 1.414 \cos(100\pi + \pi/6) + 0.3\cos(300\pi + \pi/8) + 0.1\cos(500\pi + \pi/8)$$  \hspace{1cm} (4.1)

4.1 Effect of Updating the Measurement Noise Covariance Matrix ($R$)

The real value of the noise covariance is kept constant as in section 3.3.1 ($R_{real} = 0.1$). It was shown in section 3.3.1 that the Kalman filter was successfully used to estimate the value of the fundamental amplitude when $R = 0.1$ and $Q = 10^{-10}$. These values will be used for dynamic change in the fundamental amplitude. Figure 4.1 shows what happens when there is a sudden change in the actual value of fundamental amplitude at $t = 0.4$ sec. The result shows that the Kalman filter converges to the actual value at the beginning, but when the sudden change occurs, the Kalman filter is unable to converge back to the new value of the actual fundamental signal amplitude.
Fig. 4.1 The Estimated Amplitude of the Fundamental Signal by Kalman Filter For Sudden Change in the Amplitude at 0.4sec, $R = 0.1$, $Q = 10^{-10}$ and $R_{\text{real}} = 0.1$

Previous studies mentioned that it is important to update the noise covariance matrices when any of the system parameters change. To study the effect of updating the noise covariance matrices, the Fig. 4.2 shows that the Kalman filter starts from $R = R_{\text{real}} = 0.1$ and $R$ is updated to several values at the instant dynamic change occurs in actual value at $t = 0.4$sec. The value of $Q$ at this point will be kept constant ($Q = 10^{-10}$). It is seen from Figs. 4.2(a)-(b), when $R$ is updated to smaller value, the Kalman filter performance is enhanced and converges to the actual value, but if it is updated to a larger value, the response of the Kalman filter is poor as shown in Figs. 4.2 (c) to (d).
Fig. 4.2 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, $R$ is Updated at 0.4 sec from 0.1 to Several Values, $Q = 10^{-10}$ and $R_{real}=0.1$

In similar fashion, Fig. 4.3 shows that $R$ is 0.001 at the beginning of the Kalman filter operation and the value of $R$ is updated to several values at $t = 0.4$. 
When the starting value of $R$ is 0.001, the Kalman filter converges to the actual value. As the value of $R$ decreases at the instant of the dynamic change occurs, the Kalman filter converges to the new real value successfully. If $R$ increases at the time the dynamic change occurs, the Kalman filter is unable to track the fundamental signal amplitude.
Fig. 4.3 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, \( R \) is Updated at 0.4 sec from 0.001 to Several Values, \( Q = 10^{-10} \) and \( R_{\text{real}} = 0.1 \)

Figure 4.4 shows that the Kalman filter converges slowly for \( R = 100 \). As the value of \( R \) decreases at 0.4 sec (the instant of the dynamics change), the Kalman filter estimates the amplitude correctly as shown in Figs. 4.4 (a)-(b) but incorrectly when the value of \( R \) increases as shown in Figs. 4.4 (c)-(d). The amplitude transient and the
settling time strongly depend on the decreasing value of \( R \). When \( R \) decreases sharply, the Kalman filter converges quickly to the actual value with high amplitude transient.

Fig. 4.4 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, \( R \) is Updated at 0.4 sec from 100 to Several Values, \( Q = 10^{-10} \) and \( R_{\text{real}}=0.1 \)
4.2 Effect of Updating the Process Noise Covariance Matrix (Q)

The Figs. 4.5 to 4.7 show what happens when Q is updated to several values starting from \( Q = 10^{-10} \), \( Q = 10^{-8} \) and \( Q = 10^{-6} \) respectively with \( R = R_{real} = 0.1 \) (kept constant). Based on the results shown in these figures, the performance of the Kalman filter can be improved by updating Q. For larger Q values, the Kalman filter will give more weight for the measured signal, and this allows the Kalman filter to sense the dynamic change of the signal which will improve the estimated fundamental signal amplitude. However, updating the Q matrix will be useful to a certain limit because when the weight of the measured signal increases, this will cause some noise in the estimated signal as shown in Figs. 4.5 to 4.7 (c) - (d).
Fig. 4.5 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4 sec, $Q$ is Updated at 0.4 sec from $10^{-10}$ to Several Values, $R = 0.1$ and $R_{real} = 0.1$

The Figs. 4.6 to 4.7 show similar results as observed for Fig. 4.5.
Fig. 4.6 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, $Q$ is Updated at 0.4 sec from $10^{-8}$ to Several Values, $R = 0.1$ and $R_{real} = 0.1$
Fig. 4.7 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4 sec, \( Q \) is Updated at 0.4 sec from \( 10^6 \) to Several Values, \( R = 0.1 \) and \( R_{real} = 0.1 \).

Fig 4.8 shows what happens for estimating the amplitude by the Kalman filter when \( Q \) is updated from \( Q = 10^{-10} \) to different values with \( R = 100 \) and \( R_{real} = 0.1 \). Note that \( Q \) will be updated to different values at the instant when the dynamic change occurs at \( t = 0.4 \). The results shown in Fig. 4.8 emphasize the previous results.
shown in Figs 4.5 to 4.7, that increasing $Q$ until a certain limit at the instant when the
dynamic change occurs will let the Kalman filter sense the measured signal variations
and enhance the estimated fundamental signal amplitude.
Fig. 4.8 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4 sec. $Q$ is updated at 0.4 sec from $10^{-10}$ to Several Values, $R = 100$ and $R_{real} = 0.1$
Fig 4.9 shows the estimated amplitude by the Kalman filter for small $R (R = 0.01)$ and this value of $R$ will let the Kalman filter give more weight to the measured signal. The value of $Q$ is updated from $10^{-10}$ to different values at the instant of the amplitude changes at $t = 0.4$ sec. The result is similar to one shown in Fig. 4.8. Note that $Q$ at the beginning of the simulation (before the dynamic change occurs) gives good performance.

Fig. 4.9 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, $Q$ is Updated at 0.4 sec from $10^{-10}$ to Several Values, $R = 0.01$ and $R_{\text{req}}=0.1$
In Fig. 4.10, the value of $Q$ at the beginning of Kalman filter operation is 0.1. This value is considered a large value for $Q$ to make the Kalman filter give more weight to the measured signal, which has already a noise. Figure 4.10 shows when $Q$ is updated at the instant of dynamic change to small values and large values. It is seen from this figure that decreasing $Q$ will give balance weight between measured and predicted signals.

Fig. 4.10 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, $Q$ is Updated at 0.4 sec from 0.1 to Several Values, $R = 0.01$ and $R_{real}=0.1$

Updating either $Q$ or $R$ will improve the Kalman filter performance at the instant when the dynamic change occurs. This improvement is restricted to a balance
between the measured signal and the predicted signal. Increasing the weight of the measured signal at that instant of dynamic change occurs will be helpful to detect the fundamental amplitude changes. This can be done by decreasing $R$ or increasing $Q$. Decreasing $R$ or increasing $Q$ above a certain value will lead to noise in the estimated signal similar to the noise in the measured signal.

4.3 Effect of Updating both of the Noise Covariance Matrices ($Q$ and $R$)

From the previous figures in sections 4.1 and 4.2, it can be concluded that either decreasing $R$ or increasing $Q$ will let the Kalman filter interact with the actual signal to estimate the fundamental signal amplitude changes. The question is, which is more effective; decreasing $R$ or increasing $Q$?

To investigate this, Fig. 4.11 shows the estimated amplitude of the Kalman filter when both $R$ and $Q$ updated. In the first subplot $R$ is kept constant ($R=0.01$) and $Q$ is increased by 1000 times ($Q$ from $10^{-10}$ to $10^7$). In the second subplot $R$ is decreased by 10 times and $Q$ is increased by 100 times ($R$ from $0.01$ to $0.001$ and $Q$ from $10^{-10}$ to $10^8$). In the third subplot $R$ is decreased by 100 times and $Q$ is increased by 10 times ($R$ from $0.01$ to $0.0001$ and $Q$ from $10^{-10}$ to $10^{-9}$) and in fourth subplot $R$ is decreased by 1000 times and $Q$ is kept constant ($R$ from $0.01$ to $10^{-5}$ and $Q = 10^{-10}$). As is shown in Fig. 4.11, decreasing $R$ is more effective to enhance the Kalman filter performance than increasing $R$ for the same ratio.
Fig. 4.11 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Sudden Change in the Amplitude at 0.4sec, Q and R are Updated to Several Values, and \( R_{rel}=0.1 \)

Fig. 4.12 shows two dynamic changes occur in the signal at \( t = 0.4 \) sec and \( t = 0.8 \) sec. The same values of \( Q \) and \( R \) that has been used in Fig. 4.11 are used in this figure, where \( Q \) and \( R \) are updated only at 0.4 sec. However, there was no updating of the values of \( R \) and \( Q \) at \( t = 0.8 \) sec, that is \( Q \) and \( R \) are kept constant after \( t = 0.4 \) sec.
The results show that the estimated signal improves when the value of $Q$ and $R$ are updated at $t = 0.4$ sec but keeping these value constant will not be sufficient to estimate the second amplitude changing at $t = 0.8$ sec as in shown in Figs. 4.12 (c) to (d).

Figures 4.13 (a) to (d), show different multiples of the values of $Q$ and $R$ updated by the same factor for both of the dynamic changes. For example, Fig. 4.13(c) shows that at $t = 0.4$ sec, $R$ is updated from 0.01 to 0.0001 (factor of $10^{-2}$) and at $t = 0.8$ sec $R$ is updated from $10^{-4}$ to $10^{-6}$ (same factor of $10^{-2}$), but $Q$ is updated from $10^{-10}$ to $10^{-9}$ at $t=0.4$ sec (factor of $10^{-1}$) and from $10^{-9}$ to $10^{-8}$ at $t = 0.8$ sec (same factor $10^{-1}$). From this figure, the results show that the estimated amplitude by the Kalman filter after the second change is not the same as the first one. This is because the individual values of $Q$ and $R$ before the second dynamic change occurs are not the same as the values of $Q$ and $R$ before the first dynamic change. For that reason, updating values of $Q$ and $R$ by the same ratio for next dynamic change will not improve the Kalman filter performance unless the values of $Q$ and $R$ are returned to their original values before the next dynamic change occurs.
Fig. 4.12 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Two Sudden Change in the Amplitude at 0.4 sec and 0.8 sec, $Q$ and $R$ are Updated at 0.4 sec to Several Values, and $R_{rel}=0.1$
Fig. 4.13 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Two Sudden Change in the Amplitude at 0.4sec and 0.8 sec, $Q$ and $R$ are Updated at 0.4 sec and 0.8sec by the Same Ratio, and $R_{real}=0.1$

In Fig. 4.14, the values of $Q$ and $R$ are first updated at 0.4 sec, and then returned to their original values at 0.5 sec. They are updated again at 0.8 sec by the
same ratio. The results in this figure show that the estimated amplitude is improved by returning $Q$ and $R$ values to their original values (the values that show good response in the steady operation).

Fig. 4.14 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for Two Sudden Change in the Amplitude at 0.4sec and 0.8 sec, $Q$ and $R$ are Updated at 0.4 sec and 0.8sec by the Same Ratio $Q$ and $R$ are Returned to their Original Values at 0.5sec, and $R_{ref}=0.1$
4.4 Effect of Frequency Variation

It is a major challenge to use Kalman filter to determine the amplitude of fundamental of the measured signal if the system model is non-linear when the frequency variations are considered. To investigate the effect of the frequency variations, the frequency of the measured signal will vary between 49 Hz to 51 Hz, while the fundamental frequency of the system in the Kalman filter model must be kept constant at the nominal frequency 50 Hz.

Fig. 4.15 shows what happens when \( Q \) is kept constant (\( Q=10^{10} \)), \( R \) is updated from 0.01 to \( 10^{-5} \), and the actual fundamental amplitude is changed at \( t = 0.4 \) sec for different signal fundamental frequencies. Fig. 4.15 shows that when the frequency is not equal to the nominal frequency, the estimated amplitude by the Kalman filter diverges from the real value even before the dynamic change occurs. When \( R \) is updated to small values at the instant when the dynamic change occurs at \( t = 0.4 \) sec, the weight of the measured signal increases and makes the Kalman filter sense this dynamic change and the frequency variation. This leads to improve the estimated fundamental signal amplitude. It is worth mentioning here that if the value of \( R \) is \( 10^{-5} \) from the beginning of the simulation, the Kalman filter is less sensitive to the frequency variation while updating \( R \) is necessary to detect the amplitude changing.
Fig. 4.15 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude at 0.4 sec, the Fundamental Signal has Different Values, $R$ is Updated at 0.4 sec from $10^{-5}$, $Q=10^{-10}$, and $R_{real}=0.1$
Figure 4.16 shows the effect of changing the fundamental frequency from nominal value 50 Hz to different values at instant when the dynamic change in amplitude occurs at $t = 0.4\text{sec}$. $R$ is updated to different values at this instant, and $Q$ is kept constant ($Q=10^{10}$) which improves the estimated amplitude by Kalman filter under such condition. The result in this figure shows that updating $R$ gives better results in estimating the actual fundamental signal amplitude than updating $Q$.

Fig.4.16 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude and the Frequency at 0.4sec, $R$ is Updated at 0.4 sec from $0.01$ to $10^{-5}$, $Q =10^{10}$, and $R_{real}=0.1$
The Fig. 4.17 shows the same effect as in Fig. 4.16. In Fig. 4.17 $R$ is kept constant ($R=0.001$) and $Q$ is updated from $10^{-10}$ to $10^{-7}$ at $t = 0.4$ sec. The result shows that the estimated amplitude is improved. By comparing the results obtained from Fig.4.16, it is noticed that updating $R$ gives good performance.

Fig. 4.17 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude and the Frequency at 0.4sec, $Q$ is Updated at 0.4 sec from $10^{-10}$ to $10^{-7}$, $R = 0.001$, and $R_{real} = 0.1$
4.5 Effect of the State Covariance Matrix ($P$)

Even though the real values of the measured signal noise covariance matrix $R$ and the process noise covariance matrix $Q$ are not changing, the predicted values of these matrices must be updated for detecting the amplitude changes. There must be a better way to let the Kalman filter interact with the signal changing without the need to update $Q$ and $R$ values. Returning to the Kalman filter (3.3), the state covariance matrix ($P$) has been used to detect the Kalman filter convergence. In literature the value of $P$ matrix is set to a large value at the beginning of the Kalman filter operation. If the Kalman filter converges, the value of $P$ approaches zero to indicate that the Kalman filter has converged. This only works for steady state signals. However, updating $P$ matrix may be necessary to let the Kalman filter detects the dynamic signal changes.

Figure 4.18 shows the effect of updating $P$ from $[0 \ 0; 0 \ 0]$ to $[0.1 \ 0; 0 \ 0.1]$ while keeping $Q$ and $R$ constant (that is $Q = 10^{-10}$ and $R = 0.01$) at $t = 0.4$ sec for different values of measured signal fundamental signal amplitude. Results from Fig. 4.18 show that updating $P$ allows the Kalman filter to detect the fundamental amplitude correctly.
Fig. 4.18 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude at 0.4 sec to Different Values, $P$ is Updated at 0.4 sec to Several Values, $Q = 10^{-10}$, $R = 0.001$ and $R_{real} = 0.1$

Figure 4.19 shows that $P$ is updated to several values. If $P$ is updated to larger values, there is transient in the amplitude and it takes a short time for the amplitude to
reach steady state. But if it is updated to small values, the amplitude transient is decreased but the time to reach steady state increases.

Fig. 4.19 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude at 0.4 sec, $P$ is Updated at 0.4 sec to Several Values, $Q = 10^{-10}$, $R = 0.001$ and $R_{rel} = 0.1$
In Figure 4.20 and 4.21, the signal frequency is 49.5 Hz and 50.5 Hz respectively. $P$ is updated at 0.4sec to several values in these figures. The results from these figures show that updating $P$ improves the Kalman filter capability for detecting the amplitude changing. The value of $P$ approaches zero quickly after the dynamic change occurs but the estimated amplitude in steady state region after $t = 0.4$ sec will be depend on the values of $Q$ and $R$.

Fig. 4.20 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude at 0.4sec, $P$ is Updated at 0.4 sec to Several Values, $f = 49.5$ Hz, $Q = 10^{-10}$, $R = 0.001$ and $R_{real}=0.1$
Fig. 4.21 The Estimated Amplitude of the Fundamental Signal by Kalman Filter for a Sudden Change in the Amplitude at 0.4 sec, $P$ is Updated at 0.4 sec to Several Values, $f = 50.5$ Hz, $Q = 10^{-10}$, $R = 0.001$ and $R_{real} = 0.1$

Figure 4.22 shows that the amplitude and $P$ are changed at $t = 0.4$ sec with constant values of $Q$ and $R$ ($Q=10^{-10}$ and $R=10^{-5}$), where these values of $Q$ and $R$ improved the estimated amplitude for frequency variation. The results of Fig. 4.22 show that updating $P$ improves the Kalman filter performance for the dynamic change detection, and the values of $Q$ and $R$ improve the Kalman filter estimation in the steady state interval. Figure 4.23 shows that two dynamic changes in the fundamental amplitude of the measured signal at $t = 0.4$ sec and $t = 0.8$ sec. In this figure $P$ is updated at both instants to several values, where the values of $Q$ and $R$ are the same as in Fig. 4.22. The results in Fig. 4.23 show that the Kalman filter performance to track the amplitude has been improved by updating $P$ matrix.
It can be seen from the results in section 4.5 that the values of $R$ and $Q$ that improved the Kalman filter performance for dynamic amplitude detection are not equal to the real values. Therefore, it is not important to know the exact values of $Q$ and $R$ matrices. It is only necessary to balance these matrices to adjust the weights of the measured signal in the Kalman filter calculation to get better estimation.

Fig. 4.22 The Estimated Amplitude of the Fundamental Signal by Kalman filter for a Sudden Change in the Amplitude at 0.4sec, $P$ is Updated at 0.4 sec to Several Values, $f = 50.5$ Hz, $Q = 10^{-10}$, $R = 10^{-5}$ and $R_{real}=0.1$
Fig. 4.23 The Estimated Amplitude of the Fundamental signal by Kalman Filter for Two Sudden Change in the Amplitude at 0.4 sec and 0.8 sec, $P$ is Updated at 0.4 sec and 0.8 sec to Several Values, $f = 50.5 \text{ Hz}$, $Q = 10^{-10}$, $R = 10^{-5}$ and $R_{real}=0.1$.
5.1 First Proposed Adaptive Kalman Filter Algorithm

Based on the results obtained in the previous chapters, the proposed algorithm will depend on updating the process covariance matrix $P$ in order to detect and track the amplitude variations. Updating $P$ matrix at dynamic changes will improve the estimated fundamental signal amplitude by the Kalman filter. Two important factors affect the estimated fundamental amplitude; (i) the required increment of $P$, and (ii) the instant time to update $P$.

Consider the same electrical signal $y_s(t)$ that used of (3.22)

$$y_s(t) = 1.41\cos(100\pi t + \frac{\pi}{6}) + 0.3\cos(300\pi t + \frac{\pi}{5}) + 0.1\cos(500\pi t + \frac{\pi}{8})$$

The real value of the noise covariance is set to 0.1 ( i.e $R_{real} = 0.1$).

First, Fig. 5.1 shows the amplitude of the fundamental signal estimated by the Kalman filter for different updated values of $P$. When $P$ is updated to smaller values, the response of the Kalman filter is slow. When $P$ is updated to larger values, the response becomes faster but causes overshoot at the instant of updating as shown, for example in Fig 5.1(f). If the overshoot is ignored, any larger values of $P$ will improve the Kalman filter performance.
Fig. 5.1 Amplitude of the Fundamental Signal Estimated by Kalman Filter for Sudden Amplitude Change at 0.4 sec, \( P \) is Updated to Several Values at 0.4 sec, and \( R_{real}=0.2 \)

Second, consider the instant time at which \( P \) is updated where the dynamic change occurs in the fundamental signal amplitude to update the \( P \) matrix. If the exact
time \( t_r \), of the dynamic change is known, it will be the suitable time for updating the state covariance matrix but it will require extra calculation and other techniques (such as fuzzy logic) to determine \( t_r \). However, in the proposed algorithms, the \( P \) matrix will be updated periodically irrespective of the dynamic changes in the fundamental signal amplitude. Figure 5.2 shows the fundamental signal estimated by the Kalman filter when the state covariance matrix \( P \) is updated periodically. The amplitude of the fundamental in the measured signal is kept constant in this figure. The result shows that updating \( P \) value periodically will not prevent the Kalman filter from estimating the fundamental amplitude for static signal. The updating \( P \) causes transients in the fundamental amplitude at the instant of \( P \) update. Fig. 5.3 shows the estimated fundamental signal amplitude if the transients are ignored. Fig. 5.4 shows the mean value of the estimated fundamental signal amplitude by the Kalman filter.
Fig. 5.2 The Estimated Amplitude by Kalman Filter for Different Periodic Values of $I_{ir}$
Fig. 5.3 Estimated Amplitude by Kalman Filter after Ignoring the Estimated Values at the Updating Time for Different $t_{tr}$ Time

Fig. 5.4 The Average Values of the Estimated Amplitude by Kalman Filter for Different Updating $t_{tr}$ Time
Fig. 5.5 shows several values for $t_{tr}$ and the instant $t = 0.4$ sec when there are dynamic changes in the fundamental amplitude in the measured signal. The proposed algorithm is able to track the changes in the fundamental amplitude of the measured signal. The Fig. 5.6 shows the mean values from Fig. 5.5. The results from these figures show that the estimated amplitude is strongly dependent on the value of $t_{tr}$. As the value of $t_{tr}$ is increased, this causes a delay in the proposed algorithm to track the dynamic changes even though the proposed algorithm is successful in tracking the fundamental amplitude as shown in first and second subplot of Fig 5.5. There are no delays as shown in third and fourth subplot of Fig 5.5.

![Fig. 5.5 Estimated Amplitude by Kalman Filter after Ignoring the Estimated Values at the Updating Time for Different $t_{tr}$ Time](image-url)
The fundamental frequency (50 Hz) usually changes due to load changes; under normal conditions, the frequency could be vary up to 49.5 Hz. The Kalman filter cannot estimate the fundamental signal amplitude if the frequency is varying because the system becomes non-linear; the Kalman filter works for linear systems. The Fig. 5.7 shows the results when the frequency changes to different values at $t = 0.4$ sec. The proposed algorithm would track and detect the fundamental signal amplitude but the normal Kalman filter could not track the fundamental signal amplitude.
Fig. 5.7 Estimated Amplitude by Kalman Filter and the Proposed Algorithm when the Frequency is changed at $t = 0.4$ sec

Figure 5.8 shows the estimated amplitude using a Kalman filter and the first proposed algorithm for dynamic change in the amplitude and the fundamental frequency at $t = 0.4$ sec. The proposed algorithm shows very good performance for amplitude detection, but the normal Kalman filter could not estimate the fundamental amplitude after $t = 0.4$ sec.
The last experiment for investigating the proposed algorithm performance for the signal that has large harmonics order with noise if the fundamental frequency is changed. Fig 5.9 shows a square signal that contains the fundamental signal and all
odd harmonics with and without the measurement noise. Figure 5.10 shows the estimated amplitude of the fundamental signal of the square wave by the Kalman filter and the first proposed algorithm for two frequencies 50Hz and 49.5 Hz. The Fig.5.10 shows that the Kalman filter is able to estimate the amplitude of the fundamental component when the fundamental frequency is equal to 50 Hz, but it could not estimate it when the frequency is changed to 49.5 Hz at t = 0.5. However, the proposed algorithm has good response in spite of the harmonic order and after the frequency changes from 50 Hz to 49.5 at t = 0.4.

Fig. 5.9 Square Wave Signal With and Without Noise
5.2 Second Proposed Adaptive Kalman Filter Algorithm

Fig 5.1 shows that updating $P$ value to a higher value when the dynamic change occurs will help the Kalman filter to detect and track the fundamental signal amplitude of the measured signal. The problem is to determine the transient time $t_r$ where the dynamic changes occur in the measured signal in order to update $P$. As shown in the first proposed algorithm, $P$ is updated periodically. The updating time period of the $P$ matrix must be very small to decrease the delay in the response of the Kalman filter in detecting the dynamic changes. The second proposed algorithm will use two Kalman filters: the first Kalman filter is used to determine the necessary time for updating the $P$ matrix due to frequency or amplitude changing and a second Kalman filter is used for amplitude detection.

A new algorithm is proposed where two Kalman filters will be needed. The Fig.5.11 shows the block diagram for the second proposed adaptive Kalman filter.
The first Kalman filter will be used to sense if dynamic changes are happening while the second Kalman filter will be used to track the signal in order to diagnose the voltage sag problem. The first Kalman filter will be used to determine the necessary time for updating the $P$ matrix in the second Kalman filter.

The second Kalman filter will be used to update $P$ matrix to higher value than the $P$ matrix of the first Kalman filter in order to detect and track the fundamental amplitude after the dynamic changes occur. However, the $P$ matrix for the first Kalman will be updated periodically and must be selected such that the output variations of the first Kalman filter is bounded within narrow limits. For good response, it is better if the Kalman filter updates of the $P$ matrix can be achieved as soon as a dynamic change occurs. Therefore, the periodic time of the first Kalman filter is selected to be small after several tries; 0.01 sec or $\frac{1}{2}$ cycle for 50 Hz.

![Fig. 5.11 The Block Diagram of the Second Proposed Adaptive Filter](image)

Fig. 5.11 The Block Diagram of the Second Proposed Adaptive Filter

Fig. 5.12 shows the results for different factors for updating the $P$ matrix in the first Kalman filter for constant periodic time ($t = 0.01$ sec). Fig.5.12 shows the best factor for updating the $P$ matrix in order to keep the output variations of the first Kalman filter bounded is 2.5 at $t = 0.01$ sec, as a periodic time.
Fig. 5.12 Estimated Amplitude by Kalman Filter for Fix Periodic Time and Different Updating Values

By comparing the estimated output of the second and the first Kalman filters in the normal operation, the difference between them will be within this narrow limit. When a dynamic change in the measured signal occurs, the output of the first Kalman filter will interact with the signal at this instant more than the second Kalman filter. This is because the updating $P$ matrix causes the amplitude difference between the first and second Kalman filter to exceed the narrow limit and this will be used as an indicator for an amplitude change in the signal. When the difference between the first and the second Kalman filter exceeds this bounded limit, the $P$ matrix in the second filter must be updated to track the signal variation.

The first Kalman filter will update the $P$ value while the second will keep it constant. The Fig. 5.13 shows the amplitude variation between the second and the first Kalman filters for amplitude change at 0.3 sec. The $P$ matrix in the first Kalman filter
is updated every 0.01 sec with a factor of 1.05 from the previous value of the $P$ and
the fundamental amplitude of the measured signal is changed at 0.3 sec to several
values. When the amplitude is constant, the amplitude variation is bounded within
0.01 limits. When the amplitude is changed at 0.3 sec, the variation between the two
Kalman filters increases and exceeds the normal limit (0.01). This will be used to
detect the amplitude change in the signal to update the $P$ matrix in the second Kalman
filter.

Fig. 5.13 Amplitude Variation Between the Second and the First Kalman Filter for
Different Signal Amplitude Changing, $P$ is increased 1.05 every 0.01 sec

In the second proposed algorithm, as soon as the amplitude variation between
the first and the second Kalman filters exceeds the normal limit, the second Kalman
filter will update the $P$ matrix to higher value to interact fast with the dynamic change.
The two differences between the first and the second Kalman filters are; ($i$) the $P$
matrix is updated periodically in the first Kalman filter with small variations to keep
these variation within bounded limit; and ($ii$) while the second difference is the
updating value in the second Kalman filter is higher to achieve a faster response for tracking the fundamental amplitude of the measured signal when dynamic changes occur.

Fig. 5.14 shows the strength of the performance of the proposed algorithm when two adaptive Kalman filters are used to detect and track the fundamental amplitude of the measured signal when dynamic changes occur at $t = 0.5$ sec.

![Fig. 5.14 Estimated Amplitude by the Proposed Adaptive Kalman Filter for Different Amplitude Changing at t = 0.5 sec](image)
In Fig. 5.15, the second proposed adaptive Kalman filter shows good response in tracking and detecting the fundamental amplitude of the measured signal when two dynamic changes occur at $t = 0.3$ sec and $t = 0.6$ sec.

Fig.5.15 Amplitude Estimated by the Proposed Adaptive Kalman Filter for Dynamic Changing in the Signal Amplitude
The signal fundamental frequency is kept constant (50 Hz) for all the previous results. In normal operation in power systems, the fundamental frequency varies within ±0.5 Hz due to load variations. Fig. 5.16 shows the estimated fundamental signal amplitude of the measured signal by the second proposed adaptive Kalman filter when the amplitude and frequency change from 50 Hz to different values at $t = 0.5$. The results show in this figure that, for small frequency variations, the second proposed filter shows a good response when the frequency changes by less than 0.5 Hz. When the frequency is equal to 49.5 Hz, the estimated amplitude by the proposed algorithm starts to decrease due to the frequency change before the amplitude change occurs at $t = 0.5$ sec. This causes the difference between the second and the first Kalman filter to exceed the bounded limit and lead to the update of the $P$ matrix in the second Kalman filter to correct the estimated amplitude. When the fundamental amplitude of the measured signal changes, the $P$ matrix is also updated to keep signal tracking. However, when the frequency is decreased to 49 Hz as shown in Fig.5.16, the estimated amplitude by the second Kalman filter is less than the real value. The first Kalman filter constantly triggers the second Kalman filter. The second Kalman filter updates the $P$ matrix in order to keep tracking of the fundamental signal amplitude. Based on the results shown in Fig. 5.16, the second proposed filter shows a good response for normal operation of the power system where the frequency is within ±0.5 Hz range. This multi-triggering from the first Kalman filter can be used as an alarm when there are large variations in frequency.
Fig. 5.16 The Estimated Amplitude by the Proposed Adaptive Kalman Filter for Amplitude Changing at $t = 0.5$sec and Different Signal Frequency Values
5.2.1 Effect of Signal to Noise Ratio (SNR) on Second Algorithm

The signal to noise ratio (SNR) is considered one of the measures of performance. In this section, the performance of the second proposed algorithm for different levels of noise (high or low noise) will be examined. The Figs.5.17 (a) to (g) show the performance of the second proposed algorithm to detect and track the fundamental signal amplitude for different values of the signal to noise ratio (SNR) of the measured signal. The results show that the proposed algorithm has a very good performance when the SNR has high value (that is, low noise) as shown in Figs. 5.17 (a) to (d). For low SNR (that is, high noise), the proposed algorithm also has a good performance, but there are some overshoots on the estimated output (see Figs. 5.17 (e) to (g)). These overshoots are bounded to a certain limit and come from updating $P$ matrix several times by the second Kalman filter of the proposed algorithm. If these overshoots are ignored, the second proposed algorithm will be efficient in detecting and tracking the fundamental signal amplitude even with very high noise.
Fig. 5.17 The Estimated Fundamental Amplitude by the Second Proposed Algorithm for Different Values of SNR
Fig. 5.17 (Continued)
Fig. 5.17 (Continued)
Fig. 5.17 (Continued)
CHAPTER 6

FUTURE WORK AND CONCLUSION

6.1 Contributions

The major contributions of this dissertation are:

- The effect of the noise covariance matrices \((R, Q)\) under static and dynamic cases (i.e. change in fundamental amplitude of measured signal) is investigated. It is found that balancing between \(R\) and \(Q\) can improve the Kalman filter estimation when the real values of these matrices are unavailable and therefore, there is no need for extra techniques to evaluate them.

- When a dynamic change occurs in the system parameter, it is found that updating the Kalman filter covariance matrices lets the Kalman filter interact better with the signal changing, such as the state covariance matrix \(P\).

- Two new adaptive Kalman filters are proposed to detect and track any dynamic change in the fundamental amplitude of the measured signal, which helps in diagnosing the voltage sag problem.

- The proposed algorithms show good performance in detecting and tracking the fundamental amplitude of measured signal even under a higher signal to noise ratio SNR and small frequency variations, where the simple Kalman filter cannot be used in such case.

- The proposed algorithms show robustness against SNR; they can track the fundamental amplitude changing even though the SNR is very small.
One proposed algorithm can be used as alarm for users when severe changes in the fundamental frequency occur.

6.2 Conclusion

Based on the literature review, previous research on Kalman filter estimation of fundamental amplitude in noisy and harmonic signals in power systems can be divided into four categories. First, the Kalman filter is used with fixed noise covariance matrices $Q$ and $R$, and the results show that the Kalman filter performance is very good and better than many existing filters. Second, the Kalman filter is modified to overcome some of the Kalman filter drawbacks in other applications. Third, the focus is on the importance of estimating the real value of the noise covariance matrices to guarantee the optimality of Kalman filter. Fourth, the focus is on the importance of updating the noise covariance matrices for any parameter changes in the system. In many of the previous research studies there were strong arguments for the importance of the noise covariance matrices; $Q$, $R$, and the ratio between them.

The effects of noise covariance matrices are discussed in details for static and dynamic signals. A typical example of an electrical signal was used, where the signal contained fundamental and odd orders of harmonics. Two real values of the measured signal noise covariance matrix ($R_{\text{real}} = 0.1$ and $R_{\text{real}} = 100$) are used for static and dynamic signals. It is concluded from the present work that balancing between $Q$ and $R$ values improves the Kalman filter performance. When the value of $R$ is decreased, then the Kalman filter gives more weight to the measured value than the predicted value. This enhances the Kalman filter capability to detect any changes in the measured signal, but this will lead to the presence of noise in the estimated signal. If the value of $Q$ is decreased, the Kalman filter gives more weight to the predicted
signal value, which helps the Kalman filter to get rid of the noise in the estimated signal. The estimated signal will depend strongly on the system model and cause delay in the Kalman filter response to detect the variations in the measured signal. When $R_{\text{real}} = 0.1$, there are several values of $Q$ to balance a single values of $R$. When $R_{\text{real}} = 100$, there is a single value of $Q$ to balance some values of $R$. The Kalman filter show very good response in detecting the amplitude of signal containing the first 51 odd harmonics. The Kalman filter shows poor response when the fundamental signal frequency is changed because the system becomes nonlinear under the frequency variation condition.

When the fundamental signal amplitude is changed, the Kalman filter shows poor response in detecting this change, even though that the Kalman filter shows good response before the change occurs. Updating $Q$, $R$ or both can improve the Kalman filter performance to detect the fundamental amplitude changes. Updating the value of $R$ to small values gives better results compared with that obtained from updating $Q$.

The $Q$ and $R$ matrices must be updated for each fundamental signal amplitude change. If the values of $Q$ and $R$ are updated but the ratio between them is kept constant, the Kalman filter will not give the same results for dynamic changes in the fundamental signal amplitude. However, $Q$ and $R$ values must return to their origin values but the same ratio before the updating. The Kalman filter response to detect the fundamental signal amplitude can be improved by increasing the state covariance matrix $P$ (where value of $P$ becomes zero when Kalman filter converges). This prevents the Kalman filter from interacting with the fundamental signal amplitude variations.

Updating $P$ at the instant where the fundamental signal amplitude variations occur allows the Kalman filter to detect and track the fundamental signal amplitude. If
$P$ is updated to high values, there is overshoot in the estimated fundamental signal amplitude but the settling time is low. If $P$ is updated to low value, the overshoot is small but the settling time increases.

Updating $P$ at the time when the fundamental amplitude of the measured signal changes improves the Kalman filter performance to detect and track the fundamental signal amplitude. It is not easy to determine the time when measured signal changes occur. Therefore, two adaptive Kalman filters are proposed to track the fundamental amplitude of the measured signal based on the Kalman filters only. The first proposed algorithm updates the $P$ value periodically; the period time can be chosen based on the knowledge of the system. If system changes rapidly or slowly, a small periodic time can be selected. In this proposed method, the value of $P$ must be updated to a higher value, which causes high overshoot in the estimated fundamental signal amplitude. If the overshoot is ignored at the instant where $P$ is updated, the Kalman filter performance improves. This algorithm also shows good response in tracking and detecting the fundamental signal amplitude under normal fundamental frequency variations. In this method when $P$ is updated, the Kalman filter interacts with the signal again to detect any fundamental signal variations. If the value of $P$ is updated periodically, it helps the Kalman filter to correct the estimated fundamental amplitude.

If the fundamental signal frequency is assumed to be constant in the Kalman filter, the model is linear, otherwise the system becomes nonlinear and an extended or unscented Kalman filter should be used. In normal operation the frequency variation is usually kept within narrow limits, where the frequency variation must not exceed $\pm 0.5 \text{Hz}$, [49.5 to 50.5 Hz]. The Kalman filter cannot estimate fundamental signal amplitude correctly under such frequency variation. Updating $P$ matrix periodically in
the first proposed algorithm improves the estimated output by Kalman filter for normal frequency variation.

The second proposed algorithm uses two Kalman filters, one of them is called the first Kalman filter and the other is called the second Kalman filter. Both Kalman filters update the value of $P$ in different ways. In the first Kalman filter, the value of $P$ is updated to small values periodically but with shorter period. This causes the estimated output of the Kalman filter to vary within small narrow limits (threshold limit). When the amplitude is changed, the estimated amplitude by the first Kalman filter exceeds the threshold limit, and this will be used as an indicator to trigger the second Kalman filter to update the $P$ value to detect the dynamic change. The value of $P$ is higher in the second Kalman filter compared to the value of $P$ in the first Kalman filter in order to get fast response in tracking and detecting the fundamental signal amplitude change.

The second proposed algorithm shows a very good response, especially when the fundamental signal amplitude changes suddenly. The main advantage of the second proposed algorithm over the first one is that the output overshoot is low, while the first proposed algorithm is simpler and requires one Kalman filter. It is worth mentioning here that the values of $Q$ and $R$ use in both two proposed algorithms are not the same as real values. It is enough to use small value of $R$ to let the Kalman filter interact with the signal variation.

6.3 Future Work

In future work, the simpler Kalman filter that was used in this work will be replaced by an extended Kalman filter in the two proposed algorithms to sense wide
range frequency variations. This means extra investigations of the effects of $Q$, $R$, and updating $P$ on the performance of two proposed algorithms.

Two models of Kalman filter for power systems are described in Chapter 3. The first model includes only the fundamental amplitude signal and the second model includes the fundamental and all the other harmonics. In this dissertation only the first model is used. In future work, the second model may be used. However, the sizes of Kalman filters matrices for the second model will be large, but the performance of the Kalman filter in the second model will be much better.
REFERENCES


