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Elementary School Teachers’ Use of Curricular Resources for Lesson Design and Enactment

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ELEMENTARY SCHOOL TEACHERS’ USE OF CURRICULAR RESOURCES FOR
LESSON DESIGN AND ENACTMENT

by

Napthalin Achubang Atanga

A dissertation submitted to the Graduate College
in partial fulfillment of the requirements
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This study investigated how teachers used curricular resources to teach mathematics with two different curriculum programs, a commercially developed program (Scott Foresman Addison Wesley-Mathematics) and an NSF-funded reform program (Investigations in Number, Data, and Space). This research examines the kinds of curricular resources available to six teachers (three per program), those resources they planned to use, those actually used, ways teachers used curricular resources in association with each other, and types of adaptations made. As a result, I developed insights into capacities teachers need to use curricular resources in a connected way toward the mathematical points of the lesson.

The two programs provided curricular resources with different emphases, which influenced what teachers planned to use and actually used. Scott Foresman Addison Wesley-Mathematics allocated a considerable portion of its resources to problems for skill practice. Teachers who used this program incorporated a significant number of these problems during enactment of the lesson. Investigations in Number, Data, and Space provided many ways students might respond to tasks and questions teachers need to use to assess students’ understanding of key concepts. Teachers who used this program integrated this resource during enactment to promote and assess student thinking.
Ways in which curricular resources could be used as a coherent set toward key ideas of the lesson were not always visible to teachers. Some teachers recognized written mathematical points of the lesson and used available resources to effectively communicate key ideas to students, while others did not. Teachers’ recognition of appropriate mathematical points influenced different types of adaptations they made, which resulted in contrasting levels of emphasis placed on key mathematical ideas, meaning, and storyline, and students’ engagement in the written mathematical points of the lesson.

The results of the study revealed that to use curricular resources in a coherent way to teach to the mathematical points, teachers need to identify the mathematical points in curricular resources; identify relationships among curricular resources toward the mathematical points of the lesson, among activities within and across lessons; and recognize gaps among available curricular resources.
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CHAPTER I

THE RESEARCH PROBLEM

Supporting teachers with appropriate and adequate curricular resources is a way of promoting effective teaching in order to foster significant student learning. The National Council of Teachers of Mathematics (NCTM, 1991) emphasized that changes geared toward improving student learning require that “teachers have long-term support and adequate resources” (p. 2). Curriculum materials\(^1\) containing various kinds of curricular resources (e.g., mathematical tasks and anticipated student thinking) have been designed to support teachers in enacting lessons. It is important to identify kinds of curricular resources embedded in written curriculum materials (i.e., the teacher’s guide) and ways teachers use them as a set during enactment to achieve lesson goals. This is because curricular resources contain main ideas students are to learn and how these ideas need to be explored (Brown, 2009), support teachers with new teaching pedagogies (Fan & Kaeley, 2000), contain suitable conditions for teaching and learning of appropriate content (Apple, 1986), and make explicit a teacher’s pedagogical design capacity (defined later) (Davis & Krajcik, 2005).

Also, teachers’ capacity required to use curricular resources as a set to plan and teach lessons needs to be investigated, because curricular resources provided by curriculum designers have a relationship to each other toward mathematical points of the lesson. So, it is important to understand ways teachers used them as a set towards

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\(^1\) Curriculum materials include instructional resources such as textbooks, teacher’s guide, implementation guide, lesson plans, student artifacts (e.g., worksheets), and other important resources that teachers rely on in order to plan and enact their daily lessons (Forbes & Davis, 2010).
intended learning goals and identify factors responsible for this. This has potentials of making evident capacities and ways teachers need to use curricular resources effectively.

In this study, I identified kinds of curricular resources available for teachers in written curriculum materials and ways teachers used these resources in conjunction with each other to teach mathematics. The ways teachers use curricular resources provided to them at the lesson level within the teacher’s guide in conjunction with each other refers to the manner in which teachers weave them together so that they support each other to meet the mathematical points\(^2\) of the lesson. I also developed insights into capacities teachers need to use curricular resources to design and enact a lesson.

**Curricular Resources**

The conceptualization of resources that support teachers in teaching has been approached from a wide variety of perspectives such as “human and materials resources as well as mathematical, cultural, and social resources” (Adler, 2000, p. 210). Cohen, Raudenbush, and Ball (2003) interpreted resources to include “teachers’ formal qualifications, books, facility, class size and time” (p. 127). Resources have also been seen as digital/technological material (Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013; Lagrange, Artigue, Laborde, & Trouche, 2003). In these conceptualizations, resources that support teachers in teaching mathematics are both within and outside the classroom setting. For those within the classroom setting, Pepin, Gueudet, and Trouche (2013) define *mathematical teaching resources* as “all the resources which are developed and used by teachers (and pupils) in their interaction with mathematics in/for teaching

\(^2\) The mathematical points of the lesson may or may not be articulated by the curriculum authors, in the form of objectives or key contents. More details about this are explained in Chapter IV.
and learning, inside and outside the classroom” (p. 929). Pepin et al.’s (2013) definition of resources includes curriculum materials designed for use by teachers and students. Written curriculum materials contain teacher’s guide which is made up of lessons teachers are to teach.

In this study, I define curricular resources as those valuable support provided to teachers within each lesson in the teacher’s guide. These resources are those provided in each lesson of a curriculum program that teachers consult when preparing to teach every day. It excludes all other resources outside the lesson level such as schemes of work, standards, different kinds of curriculum materials, digital materials, unit/chapter overview at the beginning of a unit, overview of an investigation, and so on. I narrowed down to this because school districts usually recommend a single curriculum program for use per grade in their schools and teachers focus only on what is available to them at the lesson level.

Many researchers (e.g., Brown, 2009; Brown & Edelson, 2003; Schneider & Krajcik, 2002; Stein & Kim, 2009) have contributed to this notion of curricular resources within each lesson in teacher’s guides but it needs elaboration as it is critical to this study. These researchers studied various kinds of resources from written curriculum materials (teacher’s guide) that teachers found valuable to use as they planned and enacted lessons. Although these researchers offered explicit examples of curricular resources, only Brown and Edelson used the term curricular resources. None of these researchers offered a clear definition of curricular resources.

Brown (2009) and Brown and Edelson (2003) divided the notion of curricular resources into three broad categories: representations of tasks, representations of
concepts, and representations of physical objects. According to these researchers, representations of tasks include directions teachers and students are to follow, such as instructions/procedures and lesson structure (for the teacher), problems to solve (for students), and scripts to guide lesson enactment by the teacher and students. Representations of concepts refer to ways in which concepts are represented and organized, as well as the relationship among them. Tools such as diagrams and models, explanations about concepts, and descriptions about relationships between tools and concepts are in this category. Schneider and Krajcik (2002) added assessment to the category of representations of concepts. According to Schneider and Krajcik, assessment is designed with a goal to foster students’ learning of concepts. Assessments also embed concepts to be learned. Representations of physical objects refer to suggested concrete materials listed in written curriculum materials that will eventually be used to visualize concepts. According to Brown, these three types of curricular resources together represent a way in which curriculum materials convey to teachers the main ideas to be explored, materials to be used to support such explorations, and appropriate ways these main ideas are to be represented.

Design transparency, another broad category of curricular resources, is defined as support intended to help teachers understand why certain activities or sequence of lessons is suggested and what students might be thinking. Stein and Kim (2009) extended the notion of curricular resources to include design rationale and anticipated student thinking. Therefore, design transparency is made up of design rationale and anticipated student thinking. Design rationale refers to information that explains the purpose of a lesson or any suggested activity to teachers with the hope that teachers’ enactment may align with
designers’ intent. *Anticipated student thinking* refers to information that draws teachers’ attention to what students might say or do, understand, misunderstand, have difficulties with, and correctly articulate. This helps teachers anticipate and interpret students’ responses in order to plan appropriate moves to foster student understanding.

Stein and Kim (2009) argued that curriculum materials that provide teachers with design rationale and anticipated ways students may approach a task or respond to questions are more likely to foster successful enactment than those that do not. Davis and Krajcik (2005) argued in particular that making design rationale explicit helps teachers see and appreciate the connection among suggested activities as well as why those activities were recommended.

Based on the above perspectives and in terms of mathematics curriculum materials, I use *curricular resources* in this study to mean a set of information provided within the lesson in the curriculum that supports teachers to design and enact lessons, such as mathematical tasks to be explored and how they can be structurally organized, explanation of purpose of any suggested activity, mathematical foci, a variety of ways mathematical concepts to be learned can be represented, ways students might respond to tasks, and specific instructional strategies. In this study, I consider curricular resources to include representations of tasks (Brown, 2009; Brown & Edelson, 2003), representations of concepts (Brown, 2009; Brown & Edelson, 2003; Schneider & Krajcik, 2002), and design transparency (Davis & Krajcik, 2005; Stein & Kim, 2009), all of these found within the lesson teachers are to teach in the teacher's guide and include student pages. I include representations of physical objects, such as materials lists (Brown, 2009), in the
category of representations of concepts because these are part of visuals. Subcategories for each of the curricular resources for this study are provided in Chapter II.

One of the expectations in the work of teaching is for teachers to make use of curricular resources so that intended mathematical learning goals may be achieved. Yet, what teachers do to use these curricular resources together as a set to plan and design instruction is not well understood. Although research studies have investigated teachers’ use of some curricular resources, they have done so in ways that seem as though these curricular resources can be self-supporting, such as using them in effective ways without support from other resources (e.g., Lloyd, 2008; Stein, Grover, & Henningsen, 1996). For example, how teachers use a set of curricular resources toward mathematical points of lesson is missing from research literature. By using “a set of resources,” I intend to focus on teachers’ use of curricular resources in conjunction with each other, rather than as individual curricular resources, to plan and enact lessons. Next, I briefly make a connection between using curricular resources as a set and Brown’s (2002) notion of Pedagogical Design Capacity (PDC), because it includes teachers’ ability to use a set of curricular resources to design and enact instruction.

Brown (2002) defined PDC as a teacher’s ability “to perceive and mobilize existing resources in order to craft instructional episodes” (p. 70). Brown (2009) illustrated PDC by explaining how two teachers (Janet and Bill) with similar knowledge and skill crafted lessons that followed different paths for the same laboratory experiment. Brown said that Janet and Bill both had a firm grasp of scientific processes, but Janet believed that students need to drive classroom events, while Bill believed that students needed more structure and analogies to make concepts they are to learn clearer.
According to Brown, Janet improvised a debate so that students could be exposed to ways scientific knowledge is contested and refined, whereas Bill improvised with additional resources so as to provide greater clarity to students on key scientific concepts. Brown further argued that both teachers exhibited a strong PDC as they identified and interpreted affordances of resources and used them to craft lessons and achieve instructional goals. As such, PDC can be interpreted in this study to include ways in which curricular resources available in the teacher’s guide are woven together by the teacher in a lesson toward intended mathematical points.

**Use of Curricular Resources as a Set**

Harris, Marcus, McLaren, and Fey (2001) have argued that students learn important mathematical concepts and procedures better when they are encountered in problems. According to Brown (2009), various curricular resources in curriculum materials explain the main ideas to be learned, ways in which these main ideas can be explored, and how these ideas can be represented. Therefore, studying teachers’ use of curricular resources in conjunction with other curricular resources available in the lesson is important because teachers need to be able to use a set of resources to achieve lesson goals. Without using resources in such a way, it may be difficult for teachers to take students to their intended learning destinations (lesson goals), especially with reform programs. Furthermore, studying ways in which teachers use curricular resources as a set can help in elaborating dimensions of PDC.

It is important at this point to explain what I mean by “use” of curricular resources as a set. “Use” of curriculum materials has been conceptualized differently by different researchers. Forbes and Davis (2010) define curriculum materials in a much
broader way to include teacher’s guide. Teacher’s guide contains curricular resources provided in each lesson that teachers draw on to use during enactment. “Use” has been seen as ways teachers read and what they learn (Remillard, 1999) and evaluation of teacher’s guide to determine whether or not to follow suggestions (Sosniak & Stodolsky, 1993). Sherin and Drake (2009) found that teachers evaluate the content of their lessons in terms of students, teachers, and parents. Sherin and Drake also argued that teachers use curriculum materials by making adaptations to them. These adaptations results from different kinds of interactions such as when teachers engage in an interpreting available resources to determine their suitability for use in class to foster learning.

Remillard (2005) found four different ways researchers conceive teachers’ use of curriculum materials: following the text, drawing from the text, interpreting the text, and participating with the text. Brown (2009) takes use to mean teachers’ ability to perceive and mobilize resources. Gueudet, Pepin, and Trouche (2013) defined teachers’ use of curriculum materials as teacher documentation work, and the outcomes of teacher documentation. According to Gueudet et al. (2013) as part of the work of teaching when using curriculum materials, teachers interact with resources embedded in them by constantly “selecting, modifying, and creating new resources, in-class and out-of-class” (p. 1004). Gueudet et al. call what teachers do in and out of class as documentation work and its result the document. Gueudet et al.’s study focused on two teachers’ use of resources such as other colleagues. Because teachers may not always be opportune to engage fully with other colleagues as resources in their schools, they interact closely with curricular resources embedded in teacher’s guide they use.
I take “use of curricular resources” in my study to mean ways teachers engage with curricular resources as a set at the lesson level in the teacher’s guide to make evident the mathematical point of the lesson and expose students to opportunities to learn the mathematical point. By engaging curricular resources as a set, I mean establishing coherent interactions and connections between resources toward written mathematical points of the lesson. Defining “use” in this way provided me with insights into capacities teachers need to engage curricular resources as a set. These capacities may subsequently be refined to develop a theory extending the field’s understanding of teachers’ use of curricular resources as a set. This may also lead to unpacking key dimensions of PDC. This is because effective use of curricular resources shows a strong PDC and vice versa.

**Some Issues in Mathematics Education**

Why is it necessary to study teachers’ use of available curricular resources in written curriculum materials in association with each other? What issues may be addressed by studying teachers’ use of available curricular resources as a set? I now frame the significance of this study in terms of some issues in the field of mathematics education for which teachers’ use of available curricular resources in written curriculum materials as a set might provide us with valuable insights. In particular, my argument builds on three important issues in mathematics education: (1) the importance of studying teachers’ use of curricular resources as a set, (2) the lack of theory to conceptualize how teachers use curricular resources as a set, and (3) the need to articulate PDC. An elaboration of each of these issues is provided below.
The Importance of Studying Teachers’ Use of Curricular Resources as a Set

One way of increasing our understanding of how teachers use curriculum materials is investigating ways in which teachers weave various available curricular resources together to design instruction. Some studies have focused on how teachers use individual curricular resources. In this section, I identify gaps among such studies and argue further for the need to study teachers’ use of curricular resources as a set.

Over the years, researchers have studied teachers’ use of curriculum materials (Carter, Gammelgaard, & Pope, 2003; Cooney, 2009; Drake & Sherin, 2006, 2009; Grouws, 2003; Harris, Marcus, McLaren, & Fey, 2000; Hiebert & Wearne, 2003; Lloyd, 2008; Marcus & Fey, 2003; Phillips, 2009; Remillard, 1999, 2000, 2005, Remillard & Bryans, 2004; Sherin & Drake, 2009; Sherin, 2002; Stein & Smith,1998; Stein, Grover, & Henningsen, 1996). From these studies, I identified that greater attention has been given to the study of mathematical tasks (e.g., selection, implementation, and cognitive demand).

Research studies focusing on teachers’ use of mathematical tasks from planning to enactment have examined some key aspects, such as what teachers do (Remillard, 1999, 2000, 2005), kinds of adaptations teachers make (e.g., Choppin, 2009, 2011; Lloyd, 2008; Remillard & Bryans, 2004), the cognitive demand required for tasks (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998), and selection and implementation of tasks (Carter, Gammelgaard, & Pope, 2003; Grouws, 2003; Hiebert & Wearne, 2003; Marcus & Fey, 2003). Studying ways teachers use curricular resources has been investigated by many researchers (e.g., Brown, 2009; Gueudet & Trouche, 2009;
Remillard, 1999, 2000; Remillard & Bryans, 2004). Ways teachers use these resources in conjunction with each other has not been intentionally theorized.

Mathematical tasks should not be considered as if their enactment can support student learning of important mathematical concepts and procedures without support from other curricular resources. This perspective has clearly exhibited limitations in research. For example, Lloyd (2008) found that the teacher in her study made adaptations to the mathematical tasks based on anticipated students’ thinking. However, Lloyd did not explain how the teacher used the two curricular resources together to achieve lesson goals. Also, Stein, Grover, and Henningsen (1996) and Stein and Smith (1998) found that teacher actions could cause the cognitive demand of mathematical tasks to decline. However, Stein et al. (1996) and Stein and Smith did not explore further to examine whether the use of other curricular resources could have promoted better enactment of the mathematical tasks.

Stein and her colleagues in 2008 focused on using mathematical tasks to orchestrate classroom discussion in productive ways and proposed five ways of doing so. One way was to anticipate what students may think about the mathematical task. We see that mathematical tasks are linked to student thinking in some way. For example, mathematical tasks are implemented to develop student thinking and reasoning. Student thinking is anticipated based on mathematical tasks assigned to them. Therefore, student thinking and mathematical tasks are related. In general, curricular resources provided in curriculum materials are related to each other in some way to promote student learning. Because of the interrelatedness of curricular resources evidenced in these researchers’ studies, studying teachers’ use of resources as a set is important.
The Lack of Theory to Conceptualize How Teachers Use Curricular Resources

Many studies have developed different conceptions of ways teachers use curriculum materials. For example, Sherin and Drake (2009) examined parts of curriculum materials teachers read and what they read for; Lloyd (1999) focused on classroom organization and student autonomy; Stein and Kaufman looked at relationship between teachers’ use of educative features; Brown (2002) investigated teachers’ ability to perceive and mobilize curricular resources to craft instructional episodes and name this construct as PDC; Gueudet and Trouche (2009) focused on teacher’s documentation work with resources; Remillard (1999) conceptualized the design and construction arenas as places where teachers interact on a daily with resources embedded in curriculum materials. These theoretical perspectives do not explain ways teachers use curricular resources as a set to achieve lesson goals. Developing theories that explain ways in which teachers use available curricular resources together as a set is needed in the field of mathematics education.

English (2010) identified lack of theory in general as a major challenge to the field of mathematics education. Silver and Herbst (2007) also argued that the field of mathematics education may not advance without careful attention to theory. Similarly, I assume that our understanding of ways in which teachers use curriculum materials may not advance without careful attention to developing theory about teachers’ use of curricular resources as a set toward lesson goals.

It is important to acknowledge that today’s formal theories in mathematics education were once informal or local knowledge. Cochran-Smith and Lytle (1999) identified this informal or local knowledge as knowledge in and of practice. According to
Cochran-Smith and Lytle, knowledge *in* practice is defined as knowledge obtained by teachers as they engage with the process of practice through reflection on their activities, whereas knowledge *of* practice is knowledge acquired by teachers as they interpret and use theory produced by others about the practice of teaching. In other words, teachers gain knowledge *in* practice as they use curricular resources but also gain knowledge *of* practice as they interpret ideas or philosophies advanced by others.

In the context of this study, ways teachers actually use a set of curricular resources at the lesson level in the teacher’s guide of written curriculum materials in association to plan and enact lessons is not well understood. The practical experience (i.e., knowledge *in* practice) that teachers gain as they use available curricular resources as a set remains implicit and hence unknown to researchers. Therefore, teachers’ use of curricular resources as a set needs to be unpacked, which implies understanding teachers’ capacity to curricular resources effectively to teach mathematics. Studying ways in which teachers use curricular resources in association with each other in the lesson and developing insights about capacities revealed as teachers use resources to enact lessons may lead us to an understanding of the construct of PDC. Brown (2009) emphasized that the way teachers put the various existing resources into play is an important part of PDC.

In the section that follows, I explain the importance of this study in relation to PDC.

**The Need to Understand Pedagogical Design Capacity (PDC)**

I argue that understanding teachers’ ability or capacity to use curricular resources in written curriculum materials as a set so that lesson goals are attained is a way to improve the field’s understanding of the construct of PDC. Brown (2009) argued that PDC helps explain differences between two teachers having similar knowledge and
commitment, yet producing different versions of the same lesson. Davis and Krajcik (2005) argued that PDC could stimulate teachers to contribute to discourse about teaching. PDC also helps teachers to make good decisions about which resources to use (Barab & Luchmann, 2003; Brown & Campione, 1996; Hubisz, 2003; Kesidou & Roseman, 2002). PDC also helps teachers determine when and how to use resources, what adaptations to make, and what combination of resources will promote student learning. For example, Seago (2007) found that some teachers made adaptations in ways that significantly changed the original intent of materials, and they termed it “fatal adaptations” (p. 11), which can be attributed in part to poor decisions and judgment on the part of teachers. However, not all adaptations made by teachers are poor. For example, the two teachers from Brown’s study whom he described as “demonstrating strong capacity - pedagogical design capacity” (p. 30) made good decisions in identifying affordances in the resources to use.

Therefore, I assume that understanding how teachers use available curricular resources as a set to enact a lesson can help us gain insights into teachers’ capacities. This study aims to develop some insights into different capacities teachers need to use available curricular resources in the lesson in association with each other to achieve lesson goals.

**Research Questions**

In this study of teacher practice, the overarching research question is: *How do elementary school teachers use various kinds of curricular resources available to them in conjunction with each other to design and enact lessons?* My specific research questions are:
1. What kinds of curricular resources are available for teachers in the lessons they teach?

2. What kinds of curricular resources do the teachers plan to use?

3. What kinds of curricular resources are used in conjunction with each other during enactment of lessons and in what ways?

4. What types of adaptations do teachers make when using these curricular resources, and what makes teachers engage in such adaptations?

5. What insights does teachers’ use of curricular resources reveal about their capacity to use the resources to enact lessons?

Question 1 is posed to provide background for studying questions 2, 3, 4, and 5. For question 1, I identified the various types of curricular resources available in the lessons taught by each teacher in this study. I quantified each available curricular resource in each lesson and across lessons. Also, I identified qualities of curricular resources curriculum materials provided for teachers in the lessons they planned to enact.

For Question 2, I identified curricular resources in teachers’ plans that they were going to use as a set during enactment. I also used follow-up interviews to identify further curricular resources that teachers planned to use. The purpose of question 3 is to identify curricular resources that teachers actually used during enactment. Also, I examined various ways in which teachers used available curricular resources in written curriculum materials as a set to achieve lesson goals. In Question 4, I identified types of adaptations that teachers made when using curricular resources together with other curricular resources, as well as reasons for engaging in such adaptations. I posed question 5 to develop insights of capacities teachers revealed as they used curricular resources. These
capacities could be knowledge, ability, ways of acting, and understanding of mathematical ideas embedded in curricular resources (Kim & Remillard, 2011). To answer question 5, I drew on results from questions 1 through 4 in order to gain insights into what teachers did to use curricular resources together with other available curricular resources to achieve lesson goals written in the curriculum.

**Overview of Design**

This study is an offshoot of a four-year NSF funded project, Improving Curriculum Use for Better Teaching (ICUBiT). The ICUBiT project seeks to understand teacher capacities as they use curriculum materials, conceptualizing PDC.

In this study, I examined teachers’ use of curricular resources using two curriculum programs, *Investigations in Number, Data, and Space* (hereafter called *Investigations*) and *Scott Foresman Addison Wesley-Mathematics* (hereafter referred to as *SFAW-Mathematics*). *Investigations* and *SFAW-Mathematics* contain various types of curricular resources to support teachers. Although *SFAW-Mathematics* contains various types of curricular resources to support teachers and teaching, it is largely traditional, favoring direct instruction and following established practices, whereas *Investigations* has a heavy focus on developing students’ conceptual understanding and reasoning. *Investigations* and *SFAW-Mathematics* are K-5 and Pre-K-6 mathematics curricula, respectively. Using these two distinct curriculum programs, I studied various ways teachers used a combination of curricular resources embedded in each of them to achieve lesson goals.

This study used data gathered from six practicing elementary school teachers in grades 3-5. Three of them used *Investigations* while the other three used *SFAW-
Mathematics. The ICUBiT project collected data from these teachers using curriculum reading logs (CRL) (developed by the ICUBiT project), interviews, and videotapes of lessons. I conducted one additional interview with each of the six teachers to explore questions that could not be answered by the existing data.

Organization of the Dissertation

This dissertation is organized into five chapters. In Chapter I, I have introduced my study, identified a research gap, described the significance of the study, and stated the research questions. In Chapter II, I examine what research literature has articulated about teachers’ use of and interaction with curriculum materials. In Chapter III, I present the design of the study in detail, including the participant teachers and curriculum materials they used, and data collection and analysis methods. In Chapter IV, I provide answers to my research questions. In Chapter V, I discuss my results. I discuss curricular resources available in written curriculum materials and those teachers actually used; patterns of ways teachers use curricular resources, and ways to improve teachers’ capacity to use available curricular resources in written curriculum materials in association with other available curricular resources. To conclude Chapter V and my study, I state the limitations, implications, and related further studies of interest.
CHAPTER II
THEORETICAL FOUNDATIONS

This chapter reviews literature that provides a foundation for the framework used in this study in the following areas: research in the area of teachers’ use of curriculum materials, research studies on curricular resources in particular, and previous work on pedagogical design capacity (PDC).

Teachers’ Use of Curriculum Materials

Literature on teachers’ use of curriculum materials includes the following main aspects: what teachers read, relationships between teachers and curriculum materials, and teachers’ actions as they use curriculum materials (i.e., types of adaptations teachers make). In the following sections, I review each of these aspects.

Teachers’ Reading of Curriculum Materials

One of the basic things teachers do as they start to use curriculum materials is to read its content. By reading, teachers may come into contact with the curriculum designers’ ideas and intentions, an understanding of the scope and sequence of topics, and concepts to pursue in their classrooms. In addition, when teachers read curriculum materials, they may come to learn new content and a wide variety of instructional strategies (Ball & Cohen, 1996; Davis & Krajcik, 2005; Schneider & Krajcik, 2002).

Given these benefits of reading curriculum materials, some researchers have investigated what teachers read and do not read from curriculum materials. Stodolsky (1989), who examined teachers’ use of textbook topics, student pages, and suggestions for teachers, found that some of the teachers did not look at suggestions for teaching, but rather held firmly to the topics in the curriculum materials, especially those found on
student pages. Similarly, Freeman and Porter (1989) found that teachers in their study were more interested in reading student exercises than directives and suggestions about approaching the lesson. Although these studies explained parts of curriculum materials some teachers read, other important questions remained unanswered.

What are specific reasons for reading various parts of curriculum materials? What resources do teachers use from the parts they read to enact a lesson? What are specific reasons for deciding to use those resources in teaching? In Remillard’s (1999) study, two teachers read different parts of the same curriculum for different reasons. Catherine, one of the teachers in the study, attended to suggestions associated with exercises for students and activities on student pages. Catherine rarely attended to suggestions that curriculum materials offered on how to facilitate problem-solving activities so that students could develop an understanding of the concepts embedded in the problems. Remillard found that when Catherine focused on students’ exercises and activities, she was looking for tasks, procedures, or algorithms she could appropriate or steps to follow verbatim or things she could alter to match her understanding. In contrast, Jackie, the other teacher in the study, read instructional suggestions in the teacher’s guide and attended to understanding the concepts students were to learn. Rather than using what the curriculum suggested, however, she invented her own task based on her understanding of what students ought to learn. According to Remillard, this resulted in both teachers having very different patterns of selecting tasks for their students.

From Remillard’s (1999) findings, we see that teachers have an orientation that affects parts of the curriculum they read, which in turn determines instructional paths and the emphasis of their lessons. For example, Catherine decided to appropriate tasks from
the curriculum, whereas Jackie decided to invent her own tasks. In other words, Catherine used tasks or exercises in the curriculum but adapted them to fit her purposes and needs, whereas Jackie created new tasks or exercises. Therefore, based on Jackie’s understanding of mathematical ideas to be learned and relationships to be developed by students, she decided to create her own activities for the lesson to meet her own purposes and needs. Furthermore, Catherine’s attention to procedures could be interpreted to mean that she emphasized memorization, while Jackie’s attention to suggestions for teaching could be interpreted to mean that she emphasized a deep understanding of key aspects to be learned.

Drake and Sherin (2009) and Sherin and Drake (2009) used data gathered from 10 teachers and investigated patterns in teachers’ use of a reform-based elementary mathematics program to understand when teachers read curriculum materials. These studies had two important findings. First, the teachers in their studies read curriculum materials with certain audiences in mind, such as students, parents, or administrators, as well as themselves. In terms of students, teachers asked pertinent questions such as “Did my students understand how to find the partner number?” (Sherin & Drake, 2009, p. 485); in terms of parents, teachers asked, “What will parents think about the ideas introduced in this lesson?” (Sherin & Drake, 2009, p. 485); and for themselves, teachers asked, “Do I understand the mathematics in the lesson?” (Sherin & Drake, 2009, p. 485). Second, the teachers provided rationale for when they read curriculum materials, such as for general overview, details, or both. By reading for general overview, the teachers were trying to get a general sense of what the lesson was aiming at without attending to details. This implies these teachers may not properly evaluate affordances and constraints of the
numerous curricular resources that curriculum materials offer. These researchers found that teachers who read this way often omitted aspects they did not understand during enactment.

By reading for details, the teachers were focused on pedagogical strategies suggested in the curriculum. According to these researchers, some teachers read for details during enactment. The effectiveness of reading for details during instruction and how this could help when teachers are already in action are questionable. What happens if a teacher reads a very difficult concept or suggestion for teaching during instruction and does not understand it? Drake and Sherin (2009) and Sherin and Drake (2009) found that such teachers often omitted suggestions or concepts they did not understand and this omission could stifle the flow of the lesson, disconnecting lesson elements that are put in place to achieve lesson goals and promote students’ understanding. Remillard (2011) and Remillard and Bryans (2004) reported similar results. Stein and Kaufman (2010) argued that teachers who spend time reading teacher support materials carefully before instruction are better prepared to provide appropriate support to students as they go through very difficult and sometimes unmapped terrains of some tasks in their learning to achieve lesson goals.

In summary, the studies described in this section suggest that (a) some teachers read curriculum materials both before and during instruction; (b) teachers read different parts of curriculum materials and focus on different things; and (c) teachers have an orientation that affects parts they read, which in turn determines instructional paths and the emphasis of their lessons. All these influence the kinds of curricular resources
teachers use and how they use them. Also, they influence the kinds of relationships teachers have with curriculum materials they use.

**Relationships Between Teachers and Curriculum Materials**

A growing body of research, particularly in the field of math and science education, has considered curriculum materials as an important tool to usher in innovations and improve instruction and teaching practices (Ball & Cohen, 1996; Brown, 2002, 2009; Davis & Krajcik, 2005; Remillard, 1999, 2000, 2005; Sherin & Drake, 2009). Despite this critical role, various researchers have viewed teachers’ use of curriculum materials differently. These differences have resulted in varying perspectives about how teachers should use or interact with curriculum materials.

Curriculum materials were considered in the past to be complete and teachers had only to follow them strictly and closely in order to orchestrate and improve instruction (Ball & Cohen, 1996; Remillard, 2005). But many researchers differed on what it means to “strictly” or “closely” follow a set of curriculum materials. These diverse interpretations probably led to different conceptions of fidelity of implementation (Jacobs, Hiebert, Givin, Hollingsworth, Garnier, & Wearne, 2006; Chval, Chávez, Reys, & Tarr, 2009; Huntley, 2012; Tarr, McNaught, & Grouws, 2012; Tarr, Chávez, Reys, & Reys, 2006; Brown, Pitvorec, Ditto, & Kelso, 2008). Remillard (2005) synthesized the research literature and identified researchers’ conceptions of the ways teachers use curriculum materials.

Researchers’ conceptions of how teachers use curriculum materials.

Remillard (1999) identified the design and construction arenas as places where most teachers interact with curriculum materials. The design arena occurs during lesson
planning, whereas the construction arena occurs during enactment of lessons. In the
design arena, teachers make a number of decisions concerning what to use or what not to
use from curriculum materials. The construction arena provides an opportunity for
teachers to implement what has been decided in the design arena.

Remillard (2005) identified four different conceptions of how researchers view
teachers’ interaction with curriculum materials in the design and construction arenas:
following or subverting the text, drawing from the text, interpreting curriculum materials,
and participating with the text. These conceptions are based on “different assumptions
about curriculum, teaching, and reader-text interaction” (Remillard, 2005, p. 216).
Remillard argued further that these four conceptions of curriculum use may enable the
field to gain “insights about teaching and curriculum materials depending on whether we
regard teaching as the primary unit of analysis or focus on teachers’ interactions with a
particular curricular resource” (p. 223). Remillard does not claim any of these
conceptions are superior over another. Therefore, these are not in any progression of
complexity or superiority.

*Teachers following or subverting the text.* Remillard (2005) explained that some
researchers see teachers’ use of curriculum materials as following as close as possible
what the text has laid down for teachers. This perspective of curriculum use is closely
related to the concept of fidelity of implementation in which, ideally, a close match is
expected between the written and the enacted lessons. According to Remillard,
researchers who see teachers’ curriculum use from this perspective are most concerned
with ways authors of curriculum materials can achieve greater clarity for teachers.
Teachers drawing from the text. In this conception, researchers see teachers as drawing resources from curriculum materials and incorporating these into their lessons (Remillard, 2005). Researchers who take this perspective depend on the teachers’ ability to use the many resources in enacting their lessons. In this perspective of curriculum use, the focus is to understand the factors that influence decisions teachers make and how these decisions evolve in the classroom during enactment (Remillard, 2005).

Teachers as interpreters of curriculum materials. Another perspective of curriculum use identified by Remillard (2005) holds that teachers bring their own experiences and beliefs of ideas, activities, and concepts embedded in curriculum materials to create their own meanings in an attempt to interpret the intentions of the authors. Within this perspective, Ben-Perez (1990) argued that teachers create meaning mainly from personal knowledge. Ben-Perez further described that when teachers interpret and analyze curriculum materials, they open up potential embedded in curriculum. With this interpretation, Remillard argued that in this perspective of curriculum use, fidelity between written and enacted curriculum is highly unlikely, because interpretations and analysis of curriculum suggestions may lead to a change in ways teachers use them based on their personal knowledge and beliefs about mathematics.

Teachers participating with the text. Remillard (2005) indicated that some researchers see teachers’ use of curriculum materials as engaging in a “dynamic interrelationship” (p. 221). In this case, the use of curriculum materials is seen as a collaboration between teacher and the text. Within this perspective, many researchers concern themselves with the nature of this dynamic interrelationship between teachers
and curriculum materials. In other words, researchers who have this perspective of curriculum use focus on studying and explaining the nature of the participation between curriculum materials and teachers.

**Types of curriculum use.** The different types of curriculum use identified in research, such as thorough piloting, offloading, adaptation, adopting, and adapting, can happen within any of these conceptions identified by Remillard. For example, in participating with the text, a teacher may offload if the curriculum does a better job in meeting his/her needs. Similarly, offloading may occur when teachers interpret curriculum materials. A description of the different types of curriculum use follows.

Thorough piloting (Remillard & Bryans, 2004) means that teachers read and use all parts of curriculum materials, whereas offloading (Brown & Edelson, 2003) occurs when teachers rely heavily on curriculum materials for instructional support and shift curriculum design responsibility completely to curriculum materials. However, reading and using all parts of the curriculum does not imply using all available curricular resources. Although thorough piloting and offloading sound similar, they are indeed different. For example, in thorough piloting, a teacher may use all parts of curriculum materials with or without an understanding of mathematical ideas or relationships embedded in them, because he or she may think all parts are important. In contrast, in offloading, teachers may read and use some parts because they agree with the goals, whereas other parts may not be offloaded. In Brown’s (2009) study, one teacher offloaded some parts of a lesson and not others. The sections of the curriculum material that the teacher felt did a good job in meeting his or her instructional goals were offloaded, whereas other parts were modified based on the teacher’s understanding.
Teachers offload for different reasons. A novice teacher may engage in offloading because of limited understanding of the subject matter, whereas an experienced teacher may engage in offloading because the curriculum supports his or her goals (Brown, 2009). Even when teachers offload some parts of the curriculum and not others as they enact their lessons, the offloaded part may not perfectly match the written curriculum.

In another type of curriculum use, curriculum materials are “experienced in situations” (Connelly & Claudinin, 1986, p. 6). By experienced in situations, Connelly and Claudinin mean in different contexts, teachers may use curriculum materials in ways that are unique. This experience may lead teachers to make their own contributions to the instructional design with whatever they draw from the curriculum. Brown and Edelson (2003) called contributions teachers make to curriculum materials adaptation, whereas Remillard and Bryans (2004) refer to it as adopting and adapting. According to Brown and Edelson, adaptation means that teachers adopt certain resources of the curriculum, but overall they make significant contributions that greatly impact the implementation of lessons. According to Remillard and Bryans, adopting and adapting occur when teachers consider curriculum materials as guides for general structure and content. This means that such teachers will rely on curriculum materials for mathematical tasks, what topics to teach, and how the various topics are sequenced, but overall may move the lesson in directions that agree with their understanding and knowledge. Using curriculum materials as a pool of resources does not explain what teachers interpret or understand from the resources.

Other teachers have interpreted curriculum materials, but this may not always lead to the desired outcome. Stake and Easley (1978), in their study of mathematics and
science lessons in the 1970s, did not see an example of mathematics or science being taught using inquiry. These authors rather found teachers making adaptations to the teacher’s guide that reflected traditional views. According to these authors, although the teachers in their study claimed to be interpreting and understanding the inquiry approach, they were actually interested in finishing the text. Chavez (2003) and Collopy (2003) found similar results when studying teachers’ interpretation of Standards-based curriculum materials. Chavez (2003) concluded that it is possible to adopt a textbook without actually being immersed in its philosophical underpinnings.

Lloyd (1999) argued that “curriculum implementation consists of dynamic relation between teacher and particular curriculum features” (p. 244). This could be interpreted to mean that in using curriculum materials, teachers engage with particular curricular resources embedded in them. McLaughlin (1976) argued that at the end of the interactive process of dynamic interrelationship, both teacher and curriculum material are changed in that the material may no longer be used as designed and the teacher may be planning to enact the lesson in a different way. Therefore, a perfectly close match between written and enacted curriculum may not be possible (Donovan, 1983; Komoski, 1977; Manouchehri & Goodman, 1998).

These different types of curriculum use have not been arranged in any hierarchy. For example, interpretation is not a superior type of use as compared to offloading. This is because a teacher may interpret and yet not meet the mathematical point of the lesson, whereas another may offload and steer instruction toward lesson goals in an appropriate way.
Characterization of teachers’ actions on curriculum materials. Figure 2.1 shows Brown and Edelson’s (2003) Design Capacity for Enactment (DCE) framework. The DCE framework highlights different types of interaction that occur between teachers and curricular resources as teachers “adapt, adopt, or improvise” (Brown, 2009, p. 26), but ways teachers act and what they understand to use these resources are not well articulated.

![Design Capacity for Enactment (DCE) Framework](image)

Figure 2.1. Design Capacity for Enactment (DCE) Framework (Brown and Edelson, 2003, p. 4)

According to Brown (2009) and Brown and Edelson (2003), who studied teachers’ use of curriculum materials in science education, in the DCE framework, curriculum materials bring curricular resources, whereas teachers bring their personal resources. The DCE framework captures the exchange that goes on between these participants (teacher and curriculum materials). Curricular resources as defined in Chapter I refers only to things provided in the written curriculum materials, while teacher resources refers to “teachers’ knowledge, skills, goals, and beliefs and how they influence the ways teachers perceive and appropriate different aspects of curriculum designs” (Brown, 2009, p. 26). In this study, I refer to teacher resources as “capacity,”
which encompasses knowledge, ability, ways of acting, and understanding of mathematical ideas embedded in curricular resources (Kim & Remillard, 2011).

Different researchers have described these interactions between teachers and curriculum materials differently. Remillard and Bryans (2004) used the terms thorough piloting, adopting and adapting, and intermittent and narrow (i.e., type of curriculum use in which a teacher occasionally consults curriculum material and reliance on the material is minimal); Brown and Edelson (2003) used the terms offloading, adaptations, and improvisation (i.e., teachers design their own instructional path and follow, but use the curriculum to provide initial ideas); and Lloyd (1999) used the term adaptations of improvisation (i.e., teacher makes adaptations to an improvisation). All of these different characterizations of teacher actions on the curriculum describe various approaches that may be used to adapt curricular resources.

**Types of adaptations.** Some studies (e.g., Drake & Sherin, 2009; Forbes, 2009; Forbes & Davis, 2010; Sherin & Drake, 2009) have examined the types of adaptations that teachers make as they use curriculum materials. Common types are insertion or addition, deletion or omission, and substitution (Forbes & Davis, 2010). According to Forbes and Davis, insertion occurs when a new element is added to the lesson plan, deletion is when an element from the existing plan is deleted completely, and substitution is when an element in the lesson plan is substituted with a new element. The difference between insertion and substitution is that in the former, an element is added either to make existing elements clearer for understanding of concepts or to increase the level of sophistication, challenging students who have understood the concept further, whereas in the latter a complete replacement takes place.
Less common types of adaptations were relocation, inversion, and duplication (Forbes & Davis, 2010). *Relocation* is moving an existing element from one part of the lesson to another, *inversion* is switching the order of placement of two or more existing elements, and *duplication* is “including an existing element from the lesson plan in another part of the lesson plan” (Forbes & Davis, 2010, p. 826). Based on the above perspectives on types of adaptations used, *addition, omission, substitution, and change of sequence* are used for this study. This is because inversion, relocation, and duplication all result in a change of sequence of the resources being used, meaning the order of use is different from what the curriculum materials propose or suggest.

Although these distinctions by Forbes and Davis (2010) give us an understanding of how to characterize each teacher’s actions on curriculum materials, emphasis is on the number and types of curriculum materials used. This is evident in one of their questions: “How many and what types of curriculum materials do preservice elementary teachers use and what adaptations do they make?” (p. 821). Although this study investigated some resource use within curriculum material, they were particularly interested in the frequency of student worksheets used. However, there are more curricular resources within a curriculum material than just student worksheets.

Choppin (2009, 2011), studying teachers’ adaptations and use of curriculum materials, argued that research should examine what teachers do with resources embedded within a single curriculum. Rather than a heavy emphasis on the kinds of programs teachers use, with minimal focus on resources within each, as in the study by Forbes and Davis, Choppin advocates for an in-depth study of ways in which the various kinds of resources within a curriculum material are used. I extend Choppin’s idea by
suggesting that ways in which teachers use the various kinds of curricular resources in conjunction with other available curricular resources within a curriculum program be investigated. I suggest this extension because ways in which teachers weave these curricular resources together may greatly influence what students learn and the achievement of lesson goals.

To summarize, research on curriculum use provides us with researchers’ conceptions of ways teachers use curriculum materials. It also provides us with knowledge about types of curriculum use that may be triggered by offloading, appropriation, improvisation, invention, and adaptations. Also, these studies provide insertion (or addition), deletion (or omission), substitution, and change of sequence as types of adaptations to use in this study.

**Curricular Resources**

Because this study focuses on how teachers use curricular resources, it is important to review literature to answer the following questions. What are curricular resources? How useful are curricular resources? Which of them have been studied thus far? What are some main findings of studies that examined teachers’ use of curricular resources? Researchers have contributed to the notion of curricular resources from different perspectives. Ruthven (2011) interpreted curricular resources to include digital tools, textbooks, and schemes of work to be used by teachers. This interpretation by Ruthven means that teachers have to make use of many resources outside curriculum materials while teaching. This interpretation is similar to that of Forbes and Davis (2010), which considers curricular resources to include a number of curriculum materials that teachers use in preparing for teaching. Also, Land (2011) identified *Investigations* and
Cognitively Guided Instruction as curricular resources. In contrast, Choppin (2009) argued that teachers ought to have a mastery of what a particular curriculum offers them in terms of curricular resources, as most school districts adopt just a single curriculum for each subject in each grade.

Brown (2002, 2009), Brown and Edelson (2003), Davis and Krajcik (2005), and Schneider and Krajcik (2002), all in the field of science education, discussed curricular resources in terms of things provided in written curriculum material. According to Brown (2009) and Brown and Edelson (2003), curricular resources are seen in terms of how tasks and the main concepts to be learned by students are represented, whereas Davis and Krajcik (2005) and Stein and Kim (2009) discuss them in terms of making designers’ intent explicit. In Chapter I, I defined curricular resources as a set of information provided in the curriculum that supports teachers to enact lessons, such as mathematical tasks to be explored and how they can be structurally organized, explanation of the purpose of any suggested activity, mathematical foci, a variety of ways mathematical concepts to be learned can be represented, ways students might respond to tasks, and specific instructional strategies.

Generally, curricular resources as defined above are intended to support teachers during teaching in a number of ways. For example, curricular resources may support teachers with appropriate representations to use in teaching and understanding the mathematical ideas embedded in them (Davis & Krajcik, 2005; Schneider & Krajcik, 2002; Stein & Kim, 2009); help teachers anticipate what students might be thinking or how students will respond to activities designed for classroom use and provide challenging questions to ask students (Ball & Cohen, 1996; Collopy, 2003; Heaton, 2000;
support teachers’ learning of subject matter, both common and specialized content (Ball, Thames, & Phelps, 2008; Wang & Paine, 2003); help teachers establish connections among units, topics, and lessons as teachers enact lessons during the year (Ball & Cohen, 1996); and provide a rationale for why designers suggested an activity or why these activities, lessons, or units are sequenced in a particular way (Heaton, 2000; Petish, 2004). Stein and Kim (2009) argued that making rationales visible to teachers may help them to bridge units, topics, lessons, and even representations emphasizing connections that may encourage students to see mathematics as a connected discipline. Davis and Krajcik (2005) argued that using these curricular resources could promote a teacher’s PDC by including additional text (e.g., annotations) that explains possible ways in which a resource could be used to achieve intended lesson goals.

In this study, I adopted three broad categories of curricular resources: representations of tasks (Brown, 2002, 2009; Brown & Edelson, 2003), representations of concepts (Brown, 2002, 2009; Brown & Edelson, 2003; Schneider & Krajcik, 2005), and design transparency (Ball & Cohen, 1996; Davis & Krajcik, 2005; Stein & Kim, 2009). I now elaborate on these categories, identifying subcategories and their sources, and including how they could support teachers.

**Representations of Tasks**

Brown (2009) proposed representations of tasks as a broad category for curricular resources. This category refers to ways in which tasks are represented in the curriculum. Tasks are important because many have come to see them as an avenue for learning meaningful mathematical ideas (Harris et al., 2000; NCTM, 1989, 1991), as an important and critical part of teaching and learning of mathematics (Brousseau, 1997; Christiansen
& Walther, 1986), and as a tool for developing reflection in the teaching of mathematics (Grevholm, 2009). Brown (2009) and Brown and Edelson (2003) provided subcategories of representations of tasks, such as (a) instructions and procedures for teachers and students to follow, (b) the way classroom activities should be structured and paced, (c) scripts for enactment by teachers and students and (d) problems to solve.

Instructions and procedures for teachers and students to follow include a sequence of directions or actions (including questions to ask) and a list of vocabulary to be learned. These represent the tasks in different ways. Suggested questions may be directed at key mathematical ideas to be represented, stimulating deep teacher reflections on what suggested questions are getting at when provided ahead of time. I classify any lists of vocabulary to be learned by students as representations of tasks because they are indications that these vocabularies are to be learned within the task. However, I consider the definitions of these terms to include mathematical explanations, as explained below.

Participation structure and time allocation provide information about the kind of student engagement suggested (individual, pair, small group, or whole group) and anticipated time needed for a task. Participation structure is more about ways in which students should work to complete given tasks. Similarly, anticipated time is more about optimal duration for completing assigned tasks. I classify participation structure and time as subcategories of representations of tasks since these are directions provided to teachers in terms of time management and classroom organization. When teachers know this, they can pair or group students in advance and plan to support struggling students within the anticipated time. This may enable teachers to plan what can be done with the suggested student engagement and anticipated time.
Problems to solve/representation of problems refers to problems that students are asked to solve both in class and at home. As I examined the two curriculum materials (Charles et al., 2008; Wittenberg et al., 2008) used in this study, I found four different kinds of problems that students are expected to solve. Teachers, depending on what they need at a particular time, appropriate, adopt, modify, or simply draw from the text what they will use. Table 2.1 shows a summary of different sources of subcategories of representations of tasks.

Table 2.1

Sources of Representations of Tasks

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<tr>
<td>Scripts for enactment by teachers and students</td>
<td>What to do? Script for enactment by teachers and students</td>
<td>Questions for teachers to ask students</td>
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<tr>
<td>Problems to solve</td>
<td>Tasks, sample test problems, homework problems, differentiation problem (reteaching, practice, enrichment, problem solving)</td>
<td>Tasks, differentiation problems (intervention and extension problems), homework problems</td>
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<td>Participation structure/time</td>
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</tbody>
</table>

Synthesizing representations of tasks from the different sources (Brown, 2009; Brown & Edelson, 2003; Charles et al., 2008; Wittenberg et al., 2008), subcategories to be used in this study include directions for teachers and students to follow, participation structure/time, and problems to solve/representations of problems. I put problems to solve under three purposes, namely, reinforcement of mathematical concepts learned,
exploration and development of mathematical concepts, and review of mathematical concepts learned in previous lessons. Table 2.2 provides subcategories and descriptions of representations of tasks.

Table 2.2

Representations of Tasks (RT)

<table>
<thead>
<tr>
<th>Curricular resources</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directions for students and teachers to follow (RT1)</td>
<td>These include: Guidelines to follow for teachers and students, explicit questions to ask students, and vocabulary list.</td>
</tr>
<tr>
<td>Participation structure/time (RT2)</td>
<td>This is the organization of classroom activities/duration. These include: Individual work, partner work, small groups, whole class and time-indicators.</td>
</tr>
<tr>
<td>Problems/representation of problems (RT3)</td>
<td>Suggested problems assigned for students to solve in class which may include representation of a problem. These include problems for:</td>
</tr>
<tr>
<td></td>
<td>• Reinforcement of mathematical concepts learned</td>
</tr>
<tr>
<td></td>
<td>• Exploration and development of mathematical concepts</td>
</tr>
<tr>
<td></td>
<td>• Review of mathematical concepts learned in previous lessons</td>
</tr>
</tbody>
</table>

Representations of Concepts

Brown (2009) and Brown and Edelson (2003) identified different ways that concepts students are to learn can be represented. Proper representations of concepts may enhance student learning. Concepts to be learned are sometimes embedded in mathematical problems students are to solve. These problems contain representations of concepts that students have to unpack to be able to solve. Representations of concepts help teachers and students focus on core mathematical content to be learned. Brown
identified relationships, diagrams, models, explanations, descriptions, and analogies as ways a concept can be represented. Schneider and Krajcik (2002) identified assessment as another way a concept can be represented. Table 2.3 shows the subcategories of representations of concepts identified from different sources.

Table 2.3

Sources of Representations of Concepts

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Grading rubric,</td>
<td>Assesment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Designated informal assessment</td>
<td>Questions to ask</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(check, talk about it)</td>
<td>when observing</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>students at work</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Relationships

Diagrams

Models

Models

Explanations

Mathematical Explanations

Descriptions

Descriptions

The works of Brown (2009), Brown and Edelson (2003), and Schneider and Krajcik (2002) deal with science education, but their subcategories of representations of
concepts can be modified for use in this study. For example, explanations as a subcategory identified by Brown can be used as mathematical explanations in this study. Similarly, diagrams and models can simply be used as visuals. Putting these ideas together, I identified five subcategories of representations of concepts for use in this study: (1) visuals, (2) mathematical explanations, (3) descriptions of representations, (4) relationships, and (5) assessments. Descriptions of each of these subcategories are provided in Table 2.4.

Visuals are a way that a concept being learned is represented in students’ solutions or strategies presented by the curriculum. This can be by way of diagrams, models, material lists, and figures. Visuals are helpful for teachers and students to understand meaning of a concept. For example, place-value blocks are visuals that can help both teachers and students see different parts, showing how a number is composed (i.e., thousands, hundreds, tens, and ones). Visuals can show the different stages of a concept and stimulate student thinking. For example, a visual may actually show different ways of representing a concept depicting different levels of understanding the mathematical concept from simple to complex. This could help teachers focus on key ideas that help students identify critical aspects of the concept being represented.

Mathematical explanations are statements provided by curriculum materials that describe or define important mathematical concepts and conventions, facts, or relationships to teachers and students. A purpose of these explanations may be to improve teachers’ learning of mathematical concepts to teach. Proper mathematical explanations may enable teachers to convey accurate mathematical ideas to students. When teachers
have a firm grasp of these explanations, they may develop confidence and their ability to drive classroom discussion or activities toward lesson goals. Mathematical explanations

Table 2.4

*Representations of Concepts (RC)*

<table>
<thead>
<tr>
<th>Curricular resources</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visually (RC 1)</td>
<td>These are used to represent a concept in students’ solutions or strategies presented by curriculum materials. These also include suggested material lists. Examples of visuals include: Diagrams, Model, Tables, and Figures.</td>
</tr>
</tbody>
</table>
| Mathematical explanations (RC 2)              | These are sentences that explain or define important mathematical concepts to teachers and students. These include:  
  - Definition of concepts or terms;  
  - Sentences that explain the mathematics (embedded in tasks, visuals, etc.), mathematical facts, or conventions. |
| Descriptions of representations (RC 3)        | These are sentences that describe representations used in the curriculum.                        |
| Relationships (RC 4)                          | These are sentences that explain the connections embedded in curriculum materials.  
  - Connections made in the curriculum material among activities, representations, strategies, concepts, etc.  
  - Indications of how specific lessons or topics connect to previous/future lessons or topics.  
  - Statements that explain how a concept is modeled in a representation. |
| Assessment (RC 5)                             | This provides a lens through which teachers can assess students’ learning. It could be in the form of:  
  - Grading rubric (Test-taking practice rubric/sample student work)  
  - Questions/focus for teachers to think about when observing what students are doing  
  - Designated informal assessment (e.g., check, talk about it) |
can also be conveyed to teachers through definitions of vocabulary students are to learn. Proper use of vocabulary conveys exact meaning to both students and teachers that represents key ideas in a task.

Descriptions of representations offered in curriculum materials could include describing a representation and explaining its usefulness or how a concept is embedded in a representation. They could support understanding ways of constructing representations should teachers and students need to do so. Descriptions of representations may provide teachers with support by explaining when and how representations can be used, as well as help them “see” a concept in a representation. This could facilitate teachers’ understanding in moving between representations.

Relationships are statements that explain the connections among strategies, mathematical ideas, or concepts embedded in curriculum materials. For example,

Depending on the strategy chosen, the answer appears in different places on the number line. Marisol is thinking about the problem as removing 446 from 1,300. In her representation (page 139), the answer is the number landed on when all parts of the smaller number have been removed in a series of jumps. In Sabrina’s and Richard’s strategies, the answer is a combination of jumps themselves (300 + 500 + 54). (Wittenberg et al., 2008, Gr. 4, U5, p. 140)

These sentences show the relationships among the strategies used by Marisol, Sabrina, and Richard by locating where the answers are.

Statements about relationships serve two different purposes. First, they may provide support that could develop teachers’ understanding of how the topics, concepts, and lessons are connected. Second, they may provide support that could enable teachers to make these connections as lessons are being enacted. Whereas connections between concepts and representations may be a resource for teachers at the lesson level, connections between topics and lessons may be a much broader and comprehensive
resource. Within each lesson, making connections may foster the use of multiple representations, whereas connections among units, topics, and lessons can foster a greater organizational agenda for teachers.

Assessment allows a lens through which teachers can evaluate students’ learning. Assessment is important as it provides a means by which teachers can monitor student learning to determine whether students have an understanding of the concepts being taught. As I examined curriculum materials used in this study, I identified three kinds of assessment-related resources, at least one of which is included in any written curriculum program. First, a grading rubric for test-taking practice and sample student work for the test are used to provide ways in which students might represent the concepts learned in their thinking. Second, some curriculum materials provide questions or focal points to guide teachers when they observe students at work. Outcomes of these questions may lead teachers to decide whether a student needs an intervention or an extension (a challenge). Third, designated informal assessments are used at some point in a lesson to quickly check students’ overall understanding. This checks students’ understanding by examining what students say or do about the concepts learned.

These assessments are designed to determine whether students appropriately understand the concepts. Teachers are expected to consider general questions (e.g., “Can students accurately model or draw $8 \times 6$?” [Wittenberg et al., 2008, Gr.4, U1, p. 50]) when observing students at work. Specific questions, such as the following, are sometimes provided to make explicit to teachers reasons for main questions:

If they draw an $8 \times 6$ array, can they identify the groups and how many in each group? Do they know how to find the total number of arrays? If they show 8 groups of 6 cubes or draw a picture of 6 groups of 8, can they identify what the 8
and the 6 represents in their model or picture? (Wittenberg et al., 2008, Gr.4, U1, p. 50)

Through these specific questions, teachers may eventually come to understand the designers’ intent about concepts to be learned.

**Design Transparency**

Do curriculum designers provide reasons for their guidance in the written lessons? Do they make them explicit? Design transparency provides reasons for design decisions.

Curriculum designers, in developing lessons, make certain decisions that might enable students to understand intended key ideas or concepts. However, these decisions are often hidden from teachers. Remillard (2000) argued that teachers’ guides often contain “steps to follow, problems to give, actual questions to ask, and answers to expect” (p. 347) without explaining to teachers why these are important or what might be achieved. Ball and Cohen (1996) also commented that curriculum materials “often offer carefully designed lessons, models, and activities, but teachers’ guide rarely discusses the strengths and weaknesses of particular designs. The developers’ pedagogical judgments thus remain hidden from teachers as they adapt, omit, or augment the materials” (p. 7). Davis and Krajcik (2005) argued that making rationales for decisions visible is one way that curriculum materials could move beyond simply adding new ideas to teachers’ repertoires and, instead, help them to integrate their knowledge base and make connections between theory and practice—taking advantage of how curriculum materials are situated in teachers’ work. (p. 5)

Therefore, making design decisions explicit can empower teachers in making informed pedagogical decisions.

Stein and Kim (2009) investigated transparency of two curriculum materials: *Investigations* and *Everyday Mathematics (EM)*. According to Stein and Kim, a
transparent curriculum material is defined as one containing “explanations for why a particular task or route through a teaching-and-learning territory was selected, including how that task or route might lead to students’ understanding of worthwhile mathematical processes and ideas” (p. 47). Transparency and rationale are similar in that by providing rationale, the curriculum is being transparent and vice versa. Stein and Kim also examined *Investigations* and *EM* to determine whether teachers were supported in anticipating ways students would respond to questions, prompts (e.g., actual student work such as “students’ drawings, invented strategies, or representations” [p. 47]), and specific interpretations of tasks by students. These researchers found that *Investigations* is more transparent than *EM*. Stein and Kim further explained that *Investigations* explains design decisions to teachers, particularly identifying and making reference to important mathematical ideas students are pursuing, whereas *EM* tells teachers what to do without letting them know why it is important. Also, *Investigations* supports teachers with anticipated ways students might respond or react to a task more than *EM* does. Therefore, based on the above mentioned studies, design transparency has subcategories as shown in Table 2.5.

Table 2.5

*Sources of Design Transparency*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Make visible designers’ pedagogical judgment</td>
<td>Making rationales for decisions/transparency</td>
<td>Transparency</td>
</tr>
<tr>
<td>Offer a range of student work to teachers</td>
<td>Anticipated student thinking</td>
<td>Anticipated student thinking</td>
</tr>
</tbody>
</table>
My synthesis of the different sources mentioned above identified two subcategories of design transparency: rationale/transparency and anticipated student thinking. Descriptions of these subcategories are provided in Table 2.6.

Table 2.6

*Design Transparency (DT)*

<table>
<thead>
<tr>
<th>Curricular resources</th>
<th>Descriptions</th>
</tr>
</thead>
</table>
| **Rationale/Transparency (DT 1)** | This is the basic reason offered for doing something such as a task, etc.  
|                                   | • Lesson objectives/math focus points/key concepts; Provision of reasons/description of a task, an activity, etc;  
|                                   | • Making explicit why certain suggested activities, representations, instructional approaches are pedagogically appropriate.                                                                                       |
| **Student thinking (DT 2)**       | This describes what students might say or understand, which might be correct or incorrect. This includes:  
|                                   | • Anticipated student difficulties/errors/misconceptions;  
|                                   | • Anticipated adequate student strategies/explanations/understanding/student actual work                                                                                                                        |

Rationale/transparency refers to the curriculum designers’ explanations as to why an instructional move, a task, an activity, a lesson, a sequence of activities, or a unit is suggested. It may help teachers understand how a suggested activity may facilitate the learning of key mathematical concepts in the lessons (Davis & Krajcik, 2005; Stein & Kim, 2009).

Rationale/transparency further describes the connection with the mathematical purpose of the lessons; may provide reasons for a particular sequencing of activities used in the lesson or topic, thereby helping teachers in making connections among topics, activities, or lessons (Schneider & Krajcik, 2002); and explains why a question or an
instructional move is suggested, thereby helping teachers during lesson enactment (Davis & Krajcik, 2005).

Anticipated student thinking refers to a set of ways in which students might approach a task or problem to be solved. This kind of resource may provide teachers with difficulties, errors, and misconceptions students might face or make so that before enactment, teachers can plan ways of addressing these when they occur (Stein & Kim, 2009). These kinds of curricular resources also allow teachers to predict strategies students can use and articulate during planning (Davis & Krajcik, 2005; Stein & Kim, 2009) so as to develop appropriate ways of supporting student learning during enactment of lessons.

In summary, I have elaborated on what constitutes curricular resources and what elements are of interest in this study, and explained ways in which they can specifically support teachers to enact lessons. Tables 2.2, 2.4, and 2.6 provide us with subcategories of representations of tasks, representations of concepts, and design transparency, respectively, for this study. In the section that follows, I will examine which of these curricular resources has been studied in research so far, what has been studied about these resources, and what needs to be studied about them.

**What Do We Know From Research About Using Curricular Resources?**

Of all the curricular resources identified above, mathematics education research has given explicit attention to mathematical tasks. Although anticipated student thinking has been a huge part of research in education, it has not been attended to as a resource. It has been incorporated into many research studies as an influential component. For anticipated student thinking, attention has been given to hypothetical paths students might
take when working on a task (Bakker & Gravemeijer, 2003; Simon, 1995), ways to develop student thoughts during enactment (e.g., Breyfogle & Herbel-Eisenmann, 2004; Sherin, Louis, & Mendez, 2000; Smith, Hughes, Engle, & Stein, 2009; Stein, Engle, Smith, & Hughes, 2008), and different ways teachers perceive and mobilize students’
mathematical thinking (e.g., Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996;

Mathematical tasks have come to be seen as centers for learning meaningful mathematics. Harris, Marcus, McLaren, and Fey (2001) stated, “The emerging view suggests that students will learn important mathematics more effectively if they encounter the concepts and techniques of the subject through carefully organized collaborative investigations of mathematically rich problems” (p. 310). Kahan and Wyberg (2003) recommended that using mathematical tasks in problem-solving teaching has at least three benefits. First, students can develop an understanding about key concepts and ideas in mathematics as they engage and make sense of the task they face. Second, efficient mathematical strategies can be developed to solve problems. Third, students’ interests and motivation to engage in mathematics can be strongly promoted. Grevholm, Millman, and Clarke (2009) also argued that mathematical tasks have function, form, and focus that are intended to “inspire, challenge, and motivate students” (p. 1).

Having students encounter concepts and techniques through mathematically rich problems may help develop students’ reasoning and understanding of mathematical concepts. According to Harris et al. (2001), mathematically rich tasks should provide
teachers with an insight of students’ previous learning and an appropriate level of challenge for students (i.e., neither too easy nor too hard; with previous learning, students make substantial progress), support students in resolving issues about the task collaboratively, and expose students to new concepts and problem-solving strategies.

For a long time, mathematical tasks have been greatly investigated by different researchers. Emphasis has been on studying adequate ways students engage with academic tasks in classrooms (Doyle & Carter, 1984), ways of assigning classroom tasks that match students’ attainment (Bennett & Desforges, 1988), students’ cognitive plans (i.e., memory, procedure, understanding or comprehension, and an opinion) as critical to successful learning from mathematical tasks (Marx & Walsh, 1988), selection of mathematical tasks (Grouws, 2003; Marcus & Fey, 2003), and implementation of mathematical tasks in reform classrooms (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998).

Stein et al. (1996) and Stein and Smith (1998) identified factors linked to maintaining the cognitive demand of mathematical tasks as proper scaffolding of students’ thinking and reasoning; students monitoring their own progress; modeling of high performance by teachers or capable students; pressing for justification, explanation, and meaning through questioning, and appropriate feedback; building tasks with emphasis on prior learning; drawing conceptual connections often; and allowing students sufficient time to work on the task ahead of them. As these factors are intended to build students’ capacity to think and reason mathematically, Smith, Bill, and Hughes (2008) identified five ways to do this: “make sense of the mathematical ideas that you want them to learn; expand on, debate, and question the solutions being shared; make connections
among the different strategies that are presented; look for patterns; and begin to form generalizations” (p. 134).

None of these studies explicitly attended to ways teachers can use anticipated student thinking while enacting tasks. Stein and her colleagues argued that within mathematical tasks, students can be encouraged to think and reason mathematically and that proper questions can be used to arrest the decline of cognitive demand of mathematical tasks. In all of these studies, we see that mathematical tasks and student thinking cannot be separated from one another. This is because one of the primary goals of mathematical tasks is to develop student thinking. These studies are therefore suggesting that looking at how teachers use curricular resources individually may be a limited way of using available curricular resources embedded in curriculum materials. Therefore, teachers’ use of available curricular resources in written curriculum materials in association with other curricular resources should be studied.

**Pedagogical Design Capacity (PDC)**

Brown (2002) defined PDC as a teacher’s ability “to perceive and mobilize existing resources in order to craft instructional episodes” (p. 70). In this definition, Brown does not elaborate on specifics of “ability” and “existing resources.” I consider curricular resources (defined and elaborated in Chapter I and in Chapter II, Tables 2.2, 2.4, and 2.6) that are embedded in curriculum materials to be “existing resources” in this study. To perceive and mobilize curricular resources happens both during lesson planning and enactment. For lesson planning, I describe mainly the kind of curricular resources teachers planned to use. For lesson enactment, I describe curricular resources teachers
actually used and ways in which they use resources as a set. From this, I gained some insights about capacities teachers need to use curricular resources as a set.

According to Sleep (2012), during lesson planning, “to perceive” is to read (curriculum materials), make sense of curriculum suggestions and content, and evaluate affordances and limitations of curricular resources, whereas “to mobilize” is to articulate the mathematical points, orient instructional activities, and use curricular resources to plan the lesson. Also, Sleep identified that during lesson enactment, “to perceive” is for teachers to read (students), make interpretations of student responses, and evaluate student thoughts, whereas “to mobilize” is to orient activities to the mathematical points, emphasize key mathematical ideas, manage multiple purposes, connect student understanding, and leverage resources in the curriculum.

From Brown’s (2009) illustration of how two teachers demonstrated a strong PDC, “ability” can be taken to mean what teachers can do best to achieve lesson goals. For example, Brown describes how Janet and Bill were experts at managing student-centered classrooms and clarifying concepts using analogies and representations, respectively. Based on these strengths, these two teachers used different curricular resources to achieve lesson goals written in the curriculum. Their different strengths led them to be able to interpret key affordances within the curriculum material and to use those to design their lesson.

In this study, I looked at capacity in terms of whether teachers identify key mathematical ideas in curricular resources, relationships among the resources around written lesson goals, relationships among activities within a lesson and among lessons, and ways teachers use these relationships to achieve lesson goals they articulate, which
may or may not be same as written lesson goals. To investigate ways teachers use the
identified relationships, I looked for the types of adaptations teachers make, what
activates the use of available curricular resources, and purpose of acting (i.e., what
teachers want to achieve) after being activated, to use curricular resources in a connected
way. Therefore, I define teachers’ capacity to use curricular resources as capacity needed
to identify affordances and limitations in curricular resources and to create relationships
among available curricular resources in order to maximize affordances and minimize
limitations when using curricular resources as a set to enact lessons that help students
learn the key mathematical ideas.

My focus on teachers identifying key mathematical ideas in written lesson goals
and relationships among curricular resources is to examine whether they identify
affordances and limitations of curricular resources as a set and how teachers use them to
support each other during enactment. Choppin (2009) argued that achieving the original
purposes of a lesson might enable teachers to create or nurture greater opportunities for
students to learn. Choppin also argued that teachers’ identification of relationships among
curricular resources, activities, and lessons is enhanced by an understanding of the key
mathematical ideas intended for student learning. Choppin added that understanding
these relationships might help improve teachers’ curricular knowledge to use particular
curriculum material; teachers’ knowledge of strengths and weaknesses of each resource,
activity, and lesson; and how these resources, activities, and lessons could be used to
support each other so that optimal opportunities for student learning are created.

Brown (2009) recommended that interpreting key affordances of curricular
resources is a critical step in using them. Identifying these affordances and limitations
determines the kinds of adaptations teachers may make to enhance student learning. These adaptations, in turn, foster teachers’ use of curricular resources as a coherent set. Failure to identify these relationships and establish them during enactment may lead to teachers “treat[ing] each content idea as discrete” (Schneider & Krajcik, 2002, p. 238). Many researchers (e.g., Stein & Kim, 2009) advocated that in teaching any mathematical content, teachers must relate it to other mathematical content. This relationship might empower teachers to use curricular resources in a connected way.

PDC is critical to studying teachers’ use of curricular resources as it may empower teachers to use identified relationships among these resources in productive ways (i.e., toward written lesson goals). Brown also argued that the use of curricular resources can help develop a teacher’s PDC. In other words, by continually using curricular resources, teachers may begin to identify key relationships among these resources embedded in written curriculum materials, developing their ability to use them in a connected way towards written lessons goals. According to Brown (2009), PDC can enable researchers to make clear distinctions among teachers with very identical knowledge, skills, and commitment. Therefore, studying teachers’ use of curricular resources is critical for PDC and vice versa. In other words, high PDC is needed for effective curriculum use, which, in turn, helps develop PDC.

**Characterizing Teachers’ Use of Curricular Resources**

Sleep (2012) identified seven central tasks that teachers engage in to “steer instruction toward the mathematical point” (p. 938). Sleep lists the central tasks as attending to and managing multiple purposes, spending instructional time on mathematical work, spending instructional time on the intended mathematics, making
sure students are doing the mathematical work, developing and maintaining a mathematical storyline, opening up and emphasizing key mathematical ideas, and keeping a focus on meaning. These central tasks can provide a way to describe how teachers use curricular resources in conjunction with other resources toward written mathematical points. For example, curricular resources might be used in a way that develops a meaningful storyline toward key mathematical points emphasized in the curriculum. Therefore, these central tasks fit the purpose of my study. Because some of these central tasks are closely related to each other, I modified them, as shown in Table 2.7, to fit my study based on the work of Sleep.

The four central tasks in the left column of Table 2.7 describe ways by which teachers’ use of curricular resources toward written intended mathematical points are characterized. These moves reveal the overall purpose for which curricular resources are being used. For example, the questions that teachers use, asking students to explain and justify their answers, may challenge student thinking with the aim to engage students in doing the mathematics associated with the lesson. The descriptions in the right column illustrate opportunities for teachers to use curricular resources. For example, in summarizing student work, teachers may use different student thinking to identify key mathematical ideas, establish relationships among them, and relate them to lesson goals of the day. These descriptions also have the potential to reveal different capacities teachers possess in using curricular resources in a connected way. In this study, I used the information in Table 2.7 as theoretical background to characterize ways teachers weave curricular resources toward mathematical points written in the curriculum. The
descriptors were used to characterize individual teachers’ ways of using curricular resources.

Table 2.7

*Characterizing Teachers’ Use of Curricular Resources*

<table>
<thead>
<tr>
<th>Characteristic Moves</th>
<th>Descriptions</th>
</tr>
</thead>
</table>
| Making sure students are actively engaged in doing the mathematics | • Asking questions that engage students in mathematical reasoning  
• Not doing the work for students (i.e., not asking and answering own questions)  
• Asking students to explain their thoughts and justify their reasoning.  
• Focusing students on problems that expose them to different aspects of the concept to be learned |
| Making connections between current work and past or future work (Mathematical Storyline) | • Making use of previous learning to construct new knowledge  
• Making mathematical connections across the activities within and across lessons  
• Summarizing students’ work  
• Deliberate progression of mathematical ideas  
• Explaining the mathematical storyline to students |
| Emphasizing key mathematical ideas                               | • Providing accurate definitions of mathematical terms  
• Using these mathematical terms often  
• Using multiple similar examples to emphasize a concept or an idea or provide clear explanation of key concepts  
• Making inputs on key mathematical ideas that students do not raise (e.g., incorrect answer, incorrect strategy, appropriate strategies, using a target question to introduce other ideas such as “what if”) |
| Emphasizing meaning                                               | • Using representations in ways that develop meaning  
• Using multiple representations and emphasizing meaning  
• Connecting answers back to the problem situation to establish meaning |
CHAPTER III

METHODOLOGY

In this chapter, I describe the setting for this study, which includes the ICUBiT Project, curriculum materials, and participant teachers. I also describe tools used to collect data, various data sources, and, finally, how the collected data were analyzed, along with analytical tools used.

Setting

ICUBiT Project

As mentioned in Chapter I, my study is an offshoot of the Improving Curriculum Use for Better Teaching (ICUBiT) Project, a four-year NSF-funded study. I now present the goals of the ICUBiT project, explain how the goals of my study are related to the project goals, and elaborate in depth on an aspect I am pursuing in trying to understand the construct of pedagogical design capacity (PDC).

The ICUBiT project started in the 2009-2010 school year in two institutions in the U.S.: Western Michigan University and the University of Pennsylvania. In trying to identify key dimensions of PDC, the project has been engaged in three principal activities, two of which are directly related to my study. First, the project engages in analyzing five curriculum programs to understand the kind of support for teachers each program provides. Teachers’ use or nonuse of available curricular resources can lead us to understand their capacities or abilities to use curriculum to design instruction. This curriculum analysis can also help us to identify curricular resources unique to particular curriculum materials that influence teachers as these materials are used. This analysis
may lead the ICUBiT project to an understanding of teachers’ PDC in relation to identifying strengths in available supports for use.

This analysis involves the teacher’s guides of *Scott Foresman Addison Wesley-Mathematics*, *Everyday Mathematics*, *Math in Focus*, *Math Trailblazers*, and *Investigations in Number, Data, and Space*, ranging from Standards-based to commercially developed curricula. Seven codes to analyze support for teachers in these curriculum materials were developed. The codes are intended to capture referential information, directions given to teachers or students, design transparency, student thinking, explanations of mathematical ideas, decision-making opportunities, and hybrids (i.e., situations where two or more of the above mentioned codes are strongly represented at the same time). These codes are largely applied to sentences intended to describe how curriculum materials clearly provide support to teachers.

The ICUBiT project also studies ways teachers use curriculum materials in order to identify design opportunities for teachers as they enact their lessons and to propose a conceptual model of PDC that guides future research and professional development. The ICUBiT project collected data from 25 teachers, five for each of the five curriculum programs mentioned previously, including videotaped lessons and interviews.

Generally, the ICUBiT project is focused on understanding ways teachers use support provided for them in curriculum materials and to develop a conceptual model of PDC. By using a subset of the data collected by the project, I focused my study on teachers’ use of various curricular resources to design and enact lessons. Then I developed insights about capacities teachers need to use curricular resources in their written curriculum materials as a set. This elaboration of using curricular resources as a
set will contribute to understanding project goals at a fine grain level. This is geared
toward supporting Brown’s (2009) argument that teachers need to mobilize additional
resources to support their adaptations and sustain the enactment of their lessons, a key
aspect of PDC.

Curriculum Materials

This study focuses on six elementary school teachers’ use of available curricular
resources from two curriculum materials, *Investigations* and *SFAW-Mathematics*, two of
the five curriculum programs in the ICUBiT project. *Investigations* (second edition) and
*SFAW-Mathematics* (diamond edition) were both published in 2008 by Pearson
Education, Inc. *Investigations* is an elementary mathematics curriculum program
developed with funding from the NSF to help teachers implement reform
recommendations. *SFAW-Mathematics* is a commercially developed mathematics
curriculum program, which claims to “make math simpler to teach, easier to learn, and
more accessible to every student” (Charles et al., 2008, p. T4). One reason for using these
two curriculum programs in this study is that *Investigations* is Standards-based (i.e.,
NSF-funded) and *SFAW-Mathematics* is commercially developed. Although some reform
recommendations are incorporated into *SFAW-Mathematics*, the teaching that it promotes
is still direct and relies heavily on the teacher. As such, the two programs may include
different kinds of resources for the teacher to enact lessons. Other reasons for the
suitability of each of these two curricula for this study are explained next.

*Investigations in Number, Data, and Space (Investigations)*

*Investigations* is used for this study partly because of the nature of its
mathematical tasks, its mathematical foci, and the kinds of classroom discourse that are
promoted. Mathematical tasks in *Investigations* have high cognitive demand. Stein and Kim (2009) found that mathematical tasks in *Investigations* are either doing mathematics or procedures with connections to a concept, with none at the other lower levels of cognitive demand. Because these mathematical tasks have very high cognitive demands, elementary school teachers are often challenged in the process of managing them in their classrooms, as students may approach the tasks in unpredictable and bizarre ways (Stein & Kaufman, 2010). Although managing mathematical tasks in *Investigations* may be challenging, support is usually provided to help teachers identify important ideas to focus on. For example, Stein and Kim found that the authors of *Investigations* provide rationale and support to teachers on how students might respond to the tasks. This finding by Stein and Kim seems to suggest that providing rationale and support for teaching is a step toward directing teachers’ attention to the mathematical foci of the tasks.

*Investigations* identifies the mathematical foci of student activities for teachers. By providing rationale for why a mathematical task is suggested, *Investigations* often explains to teachers the mathematics embedded in it. Stein and Kim (2009) found that providing rationale helps teachers to identify connections among suggested activities and develop teachers’ deep understanding of the mathematics they teach. I assume that with such a deep understanding of the mathematical ideas, teachers may become more capable of steering a classroom discussion in productive ways that could foster student learning. Yet, how teachers use design rationales and other resources to “steer” (Sleep, 2012, 938) classroom discussions toward mathematical foci of the lessons is not known.

*Investigations* promotes classroom discourse. According to Stein and Kim (2009), student learning in *Investigations* is located in the interaction between the teacher and
students. This suggests that classroom discourse is both central and critical to promoting student learning with *Investigations*. An approach used by *Investigations* to achieve student learning through classroom discourse is to provide opportunities for students to present their ideas and justify them, and to allow others to critique those ideas, thereby refining them. Yet, it is challenging to promote such a discourse in the classroom.

As explained above, *Investigations* provides support intended to assist teachers in the enactment of challenging mathematical tasks. Teachers are expected to use resources together to achieve lesson goals. Understanding and using these resources is an important part of teaching that needs to be investigated. Therefore, *Investigations* is used in this study to explore ways in which teachers use curricular resources as a set to achieve intended lesson goals.

**Scott Foresman Addison Wesley-Mathematics (SFAW-Mathematics)**

*SFAW-Mathematics* is used in this study for various reasons. It is one of the most widely used, commercially developed curriculum materials. The authors of *SFAW-Mathematics* have incorporated some reform recommendations, but teachers are not given clear directions on what to do in order to meet such recommendations.

Although enormous curricular resources have been included, guidance on how teachers use these resources is minimal. For example, the first two pages of every lesson in *SFAW-Mathematics* provide features such as Investigating the Concept, Spiral Review, and attempted connections of mathematics to other disciplines such as English Language/Literature and Social Studies, but guidance on how teachers should use these resources to achieve intended lesson goals is almost non-existent. Furthermore, *SFAW-Mathematics* has various types of curricular resources intended to support teachers in
achieving their lesson goals, such as problem solving and differentiation options. Differentiated instructional options such as math and literature, English language learners, reteaching, and math and science to meet students’ varying learning needs are provided. Moreover, how to use these many resources reasonably in one lesson is not clearly addressed.

In addition, research studies (e.g., Kaufman & Stein, 2010) on teachers’ use of curriculum materials have focused mainly on NSF-funded NCTM Standards-based curriculum programs. As such, little is known about how teachers use commercially developed programs like SFAW-Mathematics, even though these curriculum materials are being widely used in many classrooms in the U.S.

With minimal guidance on how to use curricular resources that are intended to support teachers enacting lessons in SFAW-Mathematics, do teachers see connections or relations between these resources and lesson goals? Since we do not know the answer to this question, it is important to study how teachers use the range of curricular resources in SFAW-Mathematics to enact lessons and achieve lesson goals.

**Teacher Participants**

This study uses data gathered from six teachers in the Midwestern U.S. Three of the teachers, Maria, Lisa, and Jennifer, used *Investigations*; the other three teachers, Caroline, John, and Dan, used SFAW-Mathematics. Following is a description of some of the characteristics of these teachers based on introductory interviews (explained under interviews below) with each of them.

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3 All names of teachers in this study are pseudonyms.
The focus of this study is on the ways in which teachers use available curricular resources in their written curriculum materials in association with each other to achieve lesson goals. The data used were from teachers who have experience teaching from the curriculum, rather than from beginning teachers who are just learning how to use curriculum materials. The teaching experience of the six teachers ranged from 11 to 25 years and from Head Start through grade 8. They had also taught a wide variety of subjects such as music, literature, science, and mathematics in different grades and been exposed to different types of curriculum programs, ranging from Standards-based to commercially developed. For example, Maria used traditional curriculum materials before using Investigations in her current school. Similarly, Caroline used Everyday Mathematics before using SFAW-Mathematics. All six teachers indicated what aspects they liked about the programs they used as well as aspects that they thought needed to be modified to fit their specific situations. For those parts they did not like, the teachers had made modifications to meet either district or state standards. Table 3.1 provides demographic information of these teachers, including the number of years of teaching, other curriculum programs they had used before, number of years using the current program, subjects they have taught, and grade levels they have taught.

Regarding curriculum programs they used before this study, some teachers liked the practice problems provided in them, whereas others liked the emphasis on students’ mathematical understanding. For example, Caroline and John explained that practice problems were helpful for them as they assigned them to students. In contrast, Maria and Lisa used practice problems to highlight key mathematical ideas students are to learn.
Although all of these four teachers mentioned state standards regarding decisions they made on what to emphasize during enactment, they had different reasons.

Table 3.1

*Summary of Years of Teaching and Curriculum Programs Used*

<table>
<thead>
<tr>
<th>Name of teacher</th>
<th>Previous elementary math curriculum programs</th>
<th>Grades taught</th>
<th>Subject taught before in non-elementary classrooms</th>
<th>Current Curriculum program</th>
<th>Current grade teaching</th>
<th>Number of years with current curriculum program</th>
<th>Total number of years teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maria</td>
<td>CD&lt;sup&gt;a&lt;/sup&gt;</td>
<td>K, 1, 4, 5</td>
<td>Investigations</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Jennifer</td>
<td><em>Basal, SFAW-Mathematics</em></td>
<td>Head Start, 2-7</td>
<td>Mathematics</td>
<td>Investigations</td>
<td>4</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>Lisa</td>
<td><em>Houghton Mifflin series, Connected Math</em></td>
<td>1-8</td>
<td>Mathematics</td>
<td>Investigations</td>
<td>5</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>Caroline</td>
<td><em>Everyday Mathematics</em></td>
<td>4, 6, 7, 8</td>
<td>Language Arts, Computer Science</td>
<td><em>SFAW-Mathematics</em></td>
<td>4</td>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>John</td>
<td>CD&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3, Middle School</td>
<td>Music, Mathematics</td>
<td><em>SFAW-Mathematics</em></td>
<td>4</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>Dan</td>
<td><em>Connected Math</em></td>
<td>6, 7, 8</td>
<td>Science, Mathematics</td>
<td><em>SFAW-Mathematics</em></td>
<td>5</td>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

<sup>a</sup> Some teachers could not remember the names of the curriculum programs used before, but explained they were traditional. These are denoted CD to mean Commercially Developed.

Caroline and John used review and practice of problems principally to teach and prepare students for state assessment; Maria and Lisa consulted with these standards only to make up for what they thought was lacking in the curriculum after emphasizing student understanding.
Each of these teachers had a unique experience with the different curricula they had used in the past. They had also used various available curricular resources as they planned and enacted their lessons, and they expressed their ideas about what impacted them most positively as well as what they modified to use and meet lesson goals. The effectiveness of the modifications the teachers made rests in their capacity to use available curricular resources.

Data Collection

Data were collected from the six teachers through interviews, curriculum reading logs (CRLs), and classroom observations (videotapes of their lessons). Data for this study were collected in spring 2012. These teachers were observed in fall 2011 as well; however, this study uses data only from spring 2012, because the teachers became more comfortable with the presence of the ICUBiT project team in their classrooms and therefore exhibited their usual teaching practices. In the following sections, I explain each data source and its rationale: introductory interviews, CRLs, classroom observations, post-lesson interviews, and final interviews.

Introductory Interviews

An introductory interview was conducted with each of the six teachers before classroom observations (see Appendix A for the introductory interview protocol). Besides some basic demographic information, this interview focused on the following: information about each teacher’s experience with curriculum materials, lesson preparation, what a typical mathematics lesson looks like in their classrooms, and resources that influenced their planning for and teaching of mathematics. This interview
was designed to gather some overall characteristics of these teachers’ use of curriculum materials before observations began.

Teachers were asked to explain what they each focused on or modified about those curriculum materials they were currently using. For those items they modified, they were asked to explain reasons behind their decisions. Furthermore, teachers were asked to identify other resources that they used to plan their lessons that were different from the teacher’s guide. Teachers were asked questions about how they prepared for teaching a lesson. The teachers also described from their perspectives what a typical mathematics lesson looked like in their classroom. Their descriptions included what they normally did when they prepared and taught a lesson, their emphasis in teaching, and how they responded to students’ progress. Each of these teachers also mentioned the resources drawn on outside the program used.

**Curriculum Reading Logs (CRL)**

The ICUBIT project developed curriculum reading logs (CRLs) to identify parts of the teacher’s guide that teachers read during planning and planned to use during instruction, and parts that influenced their planning. Before classroom observations, each teacher completed CRLs for the lessons observed, using different-colored highlighters.

On a copy of lessons (from the teacher’s guide), teachers were asked to indicate four main things. First, teachers were asked to highlight parts of the lessons they read as they prepared to teach in yellow. Skimmed parts of the lesson were not to be highlighted. If the reading was not sequential, teachers were asked to number the different parts they read from 1 to \( n \), with 1 being the first part and \( n \) being the \( n \)th part read in that order. Second, teachers were asked to highlight parts of the lesson they planned to use during
enactment with blue. This could include student pages, tasks, lesson instruction, homework assignments, materials lists, and other instructional resources in the teacher’s guide. Third, teachers were asked to highlight any other part of the teacher’s guide different from those already mentioned that influenced their planning in orange. This included resources they adapted or planned to adapt, as well as those they found useful in their planning. Fourth, teachers were asked to list other parts of the curriculum material not included in the copy of the given lessons, or other resources that influenced their planning. All of these provided a general sense of each teacher’s plan for enacting a lesson.

This provided insight into the teachers’ thoughts about various curricular resources and what they planned to use overall, even if they did not accurately complete the CRLs. Interviews were used along with CRLs to determine curricular resources teachers actually planned to use. These provided an overall instructional plan for each lesson and a glimpse of each teacher’s attention to available curricular resources.

**Classroom Observations**

Each lesson observed was videotaped and teachers were the focus. The goal was to capture how each of these teachers made use of curricular resources during enactment. During spring 2012, each teacher was observed for three consecutive lessons to see how teachers build up lessons using resources in their written curriculum material together with other resources, rather than isolated use of curricular resources. Table 3.2 shows the names of the teachers, grade levels, units, and lessons that were observed.
### Table 3.2

**Lessons Observed**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Curriculum program</th>
<th>Grade</th>
<th>Unit/Chapter</th>
<th>Topics</th>
</tr>
</thead>
</table>
| Maria   | *Investigations*   | 3     | 5            | 4.2: Multiply or Divide  
|         |                    |       |              | 4.3: Writing Story Problems  
|         |                    |       |              | 4.4: Solving Multiplication and Division Problems  
| Jennifer| *Investigations*   | 4     | 4            | 3.1: Making Right Angles  
|         |                    |       |              | 3.2: More or Less Than 90 Degrees?  
|         |                    |       |              | 3.3: Assessment: Building Angles  
| Lisa    | *Investigations*   | 5     | 6            | 1.3: Decimals on the Number Line  
|         |                    |       |              | 1.4: Decimals In Between  
|         |                    |       |              | 1.5: Assessment: Decimal Problems  
| Caroline| *SFAW-Mathematics* | 4     | 7            | 7-12: Finding Averages  
|         |                    |       |              | 7-13: Dividing by Multiples of 10  
|         |                    |       |              | 7-14: Dividing with Two-Digit Divisors  
| John    | *SFAW-Mathematics* | 4     | 7 & 8        | 7-15: Equestrian Competitions Review  
|         |                    |       |              | 8-1: Relating Solids and Plane figures  
| Dan     | *SFAW-Mathematics* | 5     | 5            | 5-1: Collecting Data from a Survey  
|         |                    |       |              | 5-2: Bar Graphs  
|         |                    |       |              | 5-3: Line Graphs  

*a* A lesson on problem solving (word problems) involving multiplication and division

**Post-lesson Interviews**

Post-lesson interviews were conducted by two members of the ICUBiT project team who observed the lessons, within 1-2 weeks after the three consecutive lessons were
taught. See Appendix B for the post-lesson interview protocol. Before these interviews were conducted, the classroom videos and CRLs were examined in order to pose questions that required further explanations from each of the teachers regarding decisions made when using curriculum materials. These interviews had three main parts: questions about the CRL in relation to the week observed, about the week in general, and about the week in specific.

Regarding the use of CRLs, questions concerned patterns of how these teachers read the curriculum. This part had four foci. First, these teachers were asked to explain what they read and why. Second, if teachers were inconsistent in their reading (i.e., across the lesson teachers did not have a pattern), they were asked to explain reasons for that. Third, teachers were asked to explain parts they decided to use and how these were determined. Fourth, if the teachers highlighted parts that contained important information such as design transparency or mathematical explanations, they were asked whether this information helped them in their planning or teaching and in what ways.

Questions about the week observed in general focused on in-the-moment decisions that teachers made during enactment as well as their reasons for these decisions. Two principal questions were asked. First, was enactment of the lessons different from what had been planned? If it was, teachers were asked to explain what they changed and why. Second, teachers were also asked if there was anything they would like to do differently and why.

Questions about the lessons observed aimed at understanding typical and non-typical teacher moves that were identified. To inquire about teachers’ decisions in observed moves, three critical questions were asked. First, teachers were asked about
what was skipped that was a part of their plan, what they added that was not planned, and alterations they made to curriculum suggestions. Second, when any of these were observed, teachers were further asked if they had guidance from the curriculum materials to make such modifications. Third, teachers were asked if there were other ways they used curriculum materials different from those observed. The purpose of this third question was to identify other patterns of curriculum use that were not identified during observations.

**Final Interviews**

I conducted one last interview with each of the six teachers in fall 2013. See Appendix C for an example of the final interview protocol. I developed these questions after analyzing all three lessons from each teacher, identifying things that needed further clarifications in terms of teachers using curricular resources in conjunction with other resources. This interview focused on questions about using curricular resources that the existing interviews did not clarify. These questions had three benefits. First, they provided me with an understanding of why some curricular resources in teachers’ guides are not consistently used. These questions enabled me to determine factors that probably are inherent in the curricular resources or limitations on the part of teachers that hinder their productive use of resources in conjunction with other available curricular resources. Second, teachers were asked questions about the relationships among different parts of a lesson in meeting lesson goals, lessons, representations used, and specific curricular resources and intended lesson goals articulated by them. For example, a teacher was asked to identify intended lesson goals in a representation used in the curriculum. Third,
teachers were to identify particular points within a lesson where specific written lesson goals were achieved.

Data Analysis

Developing insights into ways teachers use a set of available curricular resources in their written curriculum materials (i.e., teacher’s guide) requires a synthesis of data from multiple sources (CRLs, classroom observations, and interviews). I analyzed CRLs for curricular resources available in each written lesson and the teacher’s overall plan for instruction. I analyzed classroom videos for curricular resources embedded in curriculum materials that each teacher actually used, ways in which these resources were used in conjunction with other available curricular resources, and the types of adaptations each teacher made. Interviews were analyzed for types of adaptations (to confirm type of adaptations observed during enactment of the lessons), reasons why teachers made these adaptations during the enactment of lessons, and relationships between resources and intended lesson goals (final interview). Below I explain how data from each of these sources were analyzed and how the results were summarized to answer my research questions.

Curriculum Reading Logs (CRLs)

CRLs were analyzed for all three lessons observed per teacher to identify kinds of resources, i.e., representations of tasks, representations of concepts, and design transparency, available in each lesson (see Chapter II, under curricular resources) within the lessons in the teacher’s guide. Student pages for each lesson included in the teacher’s guide were also analyzed in terms of the categories identified. All curricular resources were coded by sentences except visuals and representations of problems that are non-
textual. I coded sentence by sentence to determine the proportion of each broad category and subcategories of curricular resources provided at the lesson level in the teacher’s guide. This enabled me to determine the kinds of curricular resources emphasized by each written curriculum program at the lesson level. In addition, I also identified the number and types of problems used in curriculum materials. A problem could contain at least one sentence.

**Representations of tasks (RT).** Codes for this category were: directions to teachers and students (RT1), participation structure/time (RT2), and problems to solve/representation of problems (RT3), as explained in Chapter II (see Table 2.2). These codes were drawn from previous studies (e.g., Brown, 2009) and modified as elaborated in Chapter II. These codes were confirmed after I examined some lessons in curriculum materials for this study (Charles et al., 2008; Wittenberg et al., 2008). Details of these codes are provided in Table 2.2.

**Directions to teachers and students (RT1).** These are explicit directions provided in curriculum materials for teachers and students to follow. An example of a typical sentence that provides directions for teachers and students is “Ask each group to present the list of multiplication combinations that its members prepared in the previous activity” (Wittenberg et al., 2008, Grade 4, Unit 1, p. 62). I coded the sentence above as RT1 because it directs teachers as to what to tell each group to do.

**Participation structure/time (RT2).** Participation structures (e.g., whole group and pairs) and time allocation may appear as icons or sentences. I coded each icon as a sentence. When two structures are suggested for a given activity or task, I coded each of them separately because sometimes the curriculum actually makes use of two structures
in one task. Therefore, whenever a sentence suggests participation structure, time, or both, I coded it as RT2. For example, I coded “for the next 30 minutes, students work with partners to solve problems on Student Activity Book pages 25-27” (Wittenberg et al., 2008, Grade 3, Unit 3, p. 77) as RT2. This is because it suggests time and participation structure. I coded this sentence only once because it contains both participation structure and time representing the same subcategory to avoid inflating the number of sentences.

**Problems to solve/representations of problems (RT3).** These are problems that students are to solve in class or at home. From analyzing the two curriculum programs for this study, I identified different types of problems students are expected to engage with as assigned by the teacher. These different types of problems are listed in Table 2.2. These problems are used for reinforcement of mathematical content taught in the current lesson, exploration and development of mathematical concepts students are to learn, and review of mathematical concepts learned in previous lessons. I coded the number of sentences associated with problems as RT3. Then I counted the number of distinct problems associated with these sentences. Lastly, I classified the problems according to their various purposes in a lesson. I counted representations found in problems together with visuals as these are non-textual.

**Representations of concept (RC).** This code highlights different ways a mathematical concept can be represented in curriculum materials to promote student learning of mathematics. In Chapter II, I identified visuals (RC1), mathematical explanations (RC2), descriptions of representations (RC3), relationships (RC4), and assessment (RC5) as codes for representing concepts. I developed these codes using previous studies (e.g., Brown, 2009; Schneider & Krajcik, 2002). After examining some
lessons in the two curriculum programs for this study (Charles et al., 2008; Wittenberg et al., 2008), these codes were confirmed. A summary is shown in Table 2.4.

Visuals (RC1). Visuals are figures (e.g., diagrams and models). Figures are defined as non-textual representations in the form of symbols, tables, graphs, or a combination thereof that are continuous and convey a single overarching idea (from ICUBiT coding guide). Figures may illustrate what has to be written on the board, possible student thinking, moves teachers can make, or clarification of particular concepts. I coded each visual separately as it may be conveying a distinct mathematical idea or concept. I did not code photographs and pictures that did not represent a concept students are learning.

Mathematical explanations (RC2). These are explanations of mathematical concepts provided to teachers and students. For example, mathematical explanations can be provided as definitions of some mathematical concepts or vocabulary, and explanations of some mathematical facts or conventions. For example, I coded “the specific numbers used to make an estimate determine whether an estimate is an overestimate or an underestimate” (Charles et al., 2008, Grade 4, Volume 1, p. 5) as RC2 because the sentence explains the mathematical concepts of overestimation and underestimation that students are expected to learn.

Descriptions of representations (RC3). These are sentences in the curriculum that describe representations (e.g., diagrams and models) used to illustrate mathematical concepts students are expected to learn. For example, “a place-value chart can help you read and write numbers. Each group of 3 digits, starting from the right, forms a period. The periods are separated by commas” (Charles et al., 2008, Grade 4, Volume 1, p. 5) is a
description of the place-value representation. I coded each sentence in the quotation above as RC3 because each provided a description of place-value charts.

**Relationships (RC4).** These are sentences that provide relationships among suggested activities, anticipated student strategies, as well as among topics or lessons within and across grade levels. For example:

When students use a number line representation for a subtraction problem that involves distance (e.g., finding the difference between 76 and 126), it is important that they recognize that the solution is a series of jumps that they made from one number to the other (24 + 26 = 50). This is different from a number line representation of a removal situation, such as subtracting 76 from 126 in parts, when the jumps represent the parts of the number line being removed, and the solution is the number landed on (50). (Wittenberg et al., 2008, Grade 3, Unit 3, p. 127)

The second sentence in the quotation above explains how different the two strategies are:

“jumps made on a number line from one number to another” and “jumps that represent the parts of the number line removed.” I coded each sentence about relationships separately. In the above example, I coded the second sentence as RC4. In some cases, these relationships are signaled by providing a list of related topics or concepts in bulleted form. I coded each bullet or numbering as a separate sentence.

**Assessment (RC5).** Assessments are also ways in which mathematical concepts are represented in the curriculum. For example, “(1) there are two 1s in 1.516. Does each have the same value? Explain. (2) In word form, how do you read the decimal point?” (Charles et al., 2008, Grade 5, Volume 1, p. 8) is an assessment of student understanding of place value. I coded each of these sentences as RC5. Also, some test problems in *SFAW-Mathematics* have a grading rubric. The grading rubric illustrates mathematical concepts that students are supposed to learn. In these grading rubrics, responses and
strategies that contain the important mathematical concepts are emphasized and rewarded appropriately with full or partial credit.

**Design transparency (DT).** These are sentences that make explicit curriculum designer’s intent, purpose of an activity, and what students might be thinking. This type of curricular resource is analyzed using two codes: rationale/transparency (DT1) and anticipated student thinking (DT2), as presented in Chapter II. The two codes were identified from previous research (Ball & Cohen, 1996; Davis & Krajcik, 2005; Stein & Kim, 2009) and confirmed using the two curriculum programs (Charles et al., 2008; Wittenberg et al., 2008) used for this study. Table 2.6 provides a summary for design transparency.

**Rationale/Transparency (DT1).** These are sentences or statements that explain to teachers the purpose or goals of a lesson and purpose of an activity, a task, or an action. For example, I coded “the Math Workshops today and tomorrow allow time for students to practice and refine their multiplication strategies” (Wittenberg et al., 2008, Grade 5, Unit 1, p. 94) as DT1 because it provides reasons for students to engage with the particular Math Workshops.

**Anticipated student thinking (DT2).** These are sentences that provide teachers with anticipated ways students might approach a task or strategies they might use. For example, I coded each sentence in the following quotation as DT2: “I saw five groups of dots. There were 4 dots in each group. I know that $5 \times 4 = 20$, so there are 20 dots altogether” (Wittenberg et al., 2008, Grade 4, Unit 1, p. 60).

**Multiple codes.** I assigned multiple distinct codes to some sentences because they may represent more than one curricular resource in strong ways that cannot be ignored.
For example, I assigned multiple codes to “if students have trouble determining the solution or forget to write the solution after drawing the picture, then ask them to go back and reread the question, looking at their drawing, and then write the solution” (Charles et al., 2008, Grade 4, Volume 1, p. 147). I coded it as DT2 and RT1 because it provides anticipated difficulties students might face and explicitly indicates what teachers need to do, respectively.

**Summarizing CRL coding data.** To answer question 1, “What kinds of curricular resources are available for teachers in the lessons they teach?” once CRLs were coded in terms of kinds of curricular resources available, my subsequent analysis went through four stages. First, for each lesson taught, I identified various kinds of curricular resources available. Second, across the three lessons for each teacher I identified patterns in various kinds of available curricular resources. Third, patterns of various kinds of available curricular resources per curriculum were identified. Fourth, using the above analysis, I described distinctive features of various kinds of available curricular resources per curriculum and across curricula. These results were used to develop insights into capacities teachers need to use curricular resources. In addition, the analysis results of each lesson were used as the basis for analyzing its corresponding videotaped lesson.

In particular, to answer question 2, I coded parts of the CRLs teachers highlighted in blue to identify various kinds of curricular resources that each teacher planned to use in teaching. I also used teacher responses in the interview to check the accuracy of the parts they planned to use and then adjusted the portions of curricular resources that teachers actually planned to use. I then summarized those data and developed patterns as explained in question 1.
**Classroom Observations**

Enacted lessons were analyzed for the kinds of curricular resources that were actually used by each teacher and ways in which they were used in association with other resources. For each lesson observed, I looked at my analysis of the CRLs to remind myself of the specific curricular resources available and the overall plan for instruction. After noting these curricular resources, I began coding the transcript of classroom videos with a direct focus on whether these specific curricular resources were used during the enactment of the lesson and, if so, how were they used. I coded the transcript of each lesson while watching the video as well.

Transcripts for all enacted lessons for each teacher were coded with the Using Curricular Resources Coding Protocol (UCRCP) that is presented in Table 3.3. In using these curricular resources during enactment, some of them may be explicitly visible, whereas others may not be observable in part because of their nature. For example, observing whether rationale for using a task was used may be difficult unless verbalized explicitly by the teacher. Therefore, only those curricular resources that are observable were the focus of coding. The UCRCP is adapted from the ICUBiT Design Decision Coding Protocol (DDCP). My analysis of enacted lessons followed three major steps. First, my analysis aimed at identifying patterns in each teacher’s extent and quality of using available curricular resources as a set. As I watched the classroom videos and looked through the codes in Table 3.3, I focused on observable curricular resources each teacher used. I began by identifying whether an available curricular resource was used, omitted, or substituted.
Table 3.3

**Using Curricular Resources Coding Protocol (UCRCP)**

<table>
<thead>
<tr>
<th>a.</th>
<th><strong>Representations of task, representations of concepts, and design transparency</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td><strong>Use</strong>-The teacher makes use of available curricular resources during enactment.</td>
</tr>
<tr>
<td>1.</td>
<td><em>Used</em>-The teacher makes use of available curricular resources as recommended in the teacher’s guide.</td>
</tr>
<tr>
<td>2.</td>
<td><em>Changed</em>-The teacher makes use of available curricular resources in teachers’ guide but modifies it.</td>
</tr>
<tr>
<td>3.</td>
<td><em>Change of sequence</em>-Teacher makes use of available curricular resources but changed the recommended sequence in the teacher’s guide (within and between lessons).</td>
</tr>
<tr>
<td>4.</td>
<td><em>Added</em>-Teacher makes use of available curricular resources in the teacher’s guide but added another resource for clarification or reinforcement.</td>
</tr>
<tr>
<td>ii.</td>
<td><strong>Omission</strong>-The teacher does not use curricular resource suggested in the teacher’s guide.</td>
</tr>
<tr>
<td>iii.</td>
<td><strong>Substitution</strong>-Teacher substitutes an available curricular resource with other resources.</td>
</tr>
</tbody>
</table>

| b. | *Curricular resources used together* – Extent to which curricular resources are used. Extent of use is denoted by the numbering i, ii, and iii. Each extent of use is qualified by 1, 2, and 3 below. For example, if overall, many available curricular resources were actually used during enactment; I classified the extent of use as *full use*, and coded it as bi. |

If overall curricular resources are not used in conjunction with other available curricular resources towards written mathematical points; I classified the quality of use as *minimally connected* coded it a bi3, which means *full use* and *minimally connected*.

**Extent of use**

i. *Full use*-overall, many available curricular resources were actually used during enactment.

ii. *Moderate use*-overall, some available curricular resources were actually used during enactment.

iii. *Minimal use*-overall, few of the available curricular resources were actually used during enactment.

**Quality of use**

1. *Highly connected*-overall, available curricular resources are used in conjunction with other resources toward written mathematical points.

2. *Moderately connected*-overall, some available curricular resources are used in conjunction with other resources towards written mathematical points while others are not.

3. *Minimally connected*-overall, curricular resources are not used in conjunction with other available curricular resources towards written mathematical points.

---

*a* These codes are assigned at the end of coding each lesson.
Whenever an available curricular resource was actually used during instruction, I coded that as used, changed, change of sequence, or addition, depending on how it was used. I coded a used curricular resource as “used” if it was used overall as recommended in the teacher’s guide. I also counted it as used once, even if mentioned several times by the teacher. In this way, I noted the number of kinds of curricular resources visibly used by each teacher.

A curricular resource received a code of “changed” if it was used with modifications during enactment. For example, if students were to solve all problems in a set, but during enactment the teacher assigned only a subset of those problems to students, then the available curricular resource (problems to solve, RT3) has been used, but with modification. Furthermore, a lesson goal may require that teachers use the inverse relationship between some operations to solve problems. If the problems are actually solved, but independently of each other without using the inverse relationship between the operations, then lesson goals have been utilized with modification because the mathematical point of the lesson has not been met. If an available curricular resource was used, but at a time different from the curriculum’s suggestion, perhaps either earlier or later in the same lesson or across lessons, then I coded it as a “change of sequence.”

If a resource from outside the curriculum was being used, then I coded it as an “addition.” If at any point in a lesson an available observable curricular resource was supposed to be used but was not being used during that lesson, I coded it as an “omission.” In addition, if an available observable curricular resource was not used and a new resource was used in its place, then I coded it as a “substitution.” This new resource could be from other materials the teacher was using. All these are adaptations that could
happen as teachers use curricular resources. I used UCRCP (i.e., Table 3.3, part a) to code types of adaptations made during the enactment of lessons.

Second, after analyzing each lesson, I classified the teacher’s extent and quality of using available curricular resources in individual lessons. The extent of using available curricular resources is classified as full use, moderate use, and minimal use. If, overall, many available curricular resources were actually used during enactment, I classified the extent of use as full use. For example, Jennifer actually used 63.5% of curricular resources available to her during enactment. In particular, Jennifer used 54.5%, 77.8%, and 69.5% of representations of tasks (RT), representations of concepts (RC), and design transparency (DT), respectively, available to her. Therefore, I classified her extent of use as full use, as these curricular resources were substantially used across all three broad categories.

If, overall, some available curricular resources were actually used during enactment, I classified the extent of use as moderate use. For example, Maria actually used 32.5% of curricular resources available to her in the lessons she taught. In particular, Maria used 30.1%, 57.1%, and 25% of representations of tasks (RT), representations of concepts (RC), and design transparency (DT), respectively, available to her. The broad categories of curricular resources were not substantially used during enactment. Therefore, I classified her extent of use as moderate. If, overall, a few available curricular resources were actually used during enactment, I classified the extent of use as minimal use.

For each extent of use, I also determined the quality of using curricular resources as highly connected, moderately connected, and minimally connected. If, overall, in
enacting lessons many curricular resources were used in conjunction with other resources toward written lesson goals, I classified the quality of use as *highly connected*. Note that in this case various curricular resources support the lesson goals in a highly *connected* way. All resources provided at the lesson level in the teacher’s guide aim at the lesson goals and are closely related to support each other. For example, in one of the lessons taught by Caroline, the authors of *SFAW-Mathematics* provided two strategies for solving $1,800 \div 20$. Allison said, “$1,800 \div 20$ is the same as $180 \div 2$. So $1,800 \div 20$ is 90” (Charles et al., 2008, Grade 4, Volume 3, p. 406), and Anthony said, “What times 20 equals 1,800? I know $90 \times 20 = 1,800$. So $1,800 \div 20$ is 90” (Charles et al., 2008, Grade 4, Volume 3, p. 406). During enactment, Caroline explained the mathematical ideas embedded in each of the strategies. She said,

If you look at what Allison did, she said that 1,800 divided 20 is the same as 180 divided by 2…she just got rid of the extra 0's, like we did there where we crossed them out...My preference you leave your original problem like we've been modeling and you show me that you're crossing out. I want to see those 0's still there. Just like we modeled up there and what you have in your notebook. And so she knows 1,800 divided by 20 is 90. Ok? (Caroline, enacted lesson)

For Anthony’s strategy, Caroline said,

If you look at what Anthony says, he's saying what times 20 equals 1,800? Well, he knows that's 90 times 20, because he knows that multiplication is the reciprocal or inverse operation of division; and so for some of us we think in terms of multiplication when we work our division problems. We say to ourselves like Trinity modeled for us earlier, 5 times 6 is 30. So she knew that 30 divided by 6 was 5. So she did the inverse operation like Anthony is showing us, doing a simpler multiplication problem to help us find our division answer. (Caroline, enacted lesson)

Here, available curricular resources are visuals (student sample work), anticipated student thinking (what students might say), and mathematical explanations (e.g., Allison’s strategy). The mathematical point of this lesson is “basic facts and place-value
patterns can help you find quotients like $2,100 \div 70$” (Charles et al., 2008, Grade 4, Volume 3, p. 406). Caroline used the visual (student sample work) to provide mathematical explanations of strategies Allison and Anthony, students in the teacher’s guide articulated. The visual contained mathematical explanations (e.g., $1,800 \div 20$ is the same as $180 \div 2$. So $1,800 \div 20$ is 90) to support the key ideas. Anticipated student thinking provided an explanation contained in the visual that Caroline connected to what was done in class. For Anthony’s method, Caroline explicitly said it is multiplication being used to solve division problems. Then she explained that the inverse relationship between the two operations is being used to solve the problem. Therefore, Caroline used these curricular resources in a way that supported each other toward the written mathematical point of the lesson. The visual did not explain the use of inverse relationship between multiplication and division to solve problems. Yet Caroline explicitly made this known to students to connect to what they did previously. She minimized the limitations of the visual and maximized its affordances (explained the key mathematical ideas it contained). This is typical of how Caroline used curricular resources in association with other resources. Therefore, I classified Caroline’s quality of use as highly connected toward mathematical points of the lesson.

If, overall, some of the curricular resources were used in conjunction with other curricular resources toward written mathematical points of the lesson, then I classified the quality of use as *moderately connected*. For example, some curricular resources are used in ways that support each other toward key mathematical ideas of the lesson, as explained in Caroline’s case. Others are used in ways that do not support each other toward lesson
goals (as explained next for Maria). In these cases, I classify the quality of use as moderately connected.

If, overall, curricular resources were not used in conjunction with other available curricular resources toward lesson goals, I classified the quality of use as minimally connected. For example, in one of the lessons taught by Maria, the authors of Investigations suggest that teachers discuss problems 2 and 3 with students, highlighting the inverse relationship between multiplication and division to solve problems. In addition, they suggest teachers should ask the following questions:

What is same about these problems? What is different? What information do you know in problem 2? What about problem 3? What do we call each of these types of problems? (Wittenberg et al., 2008, Grade 3, Unit 5, p. 123)

and then

Listen for students to identify problem 2 as a division problem because we are told the total number of muffins and are asked to find how many equal bags can be made. Problem 3 is a multiplication problem because we are told how many packs of yogurts were bought and are asked to find how many were bought all together. (Wittenberg et al., 2008, Grade 3, Unit 5, p. 123)

During enactment, Maria highlighted problems 2 and 3 for discussion, but had the following conversation with students:

**Teacher:** Now, ladies and gentlemen...look at question number 2 and question number 3...Do you notice anything special about question number 2 and question number 3? Then keep looking...What do you notice?

**Student:** The top one’s 5 and the bottom one’s 20 because 20 divided by 4 is 5 and then 5 times 4 is 20.

**Teacher:** Good. Adding and subtracting are exact opposites, right? So are multiplication and division so these have the same set of numbers in them it’s just that this one is the inverse or opposite of the one right above it. Kind of cool. So if you solve this one and you solve the same numbers you automatically know the answer without having to even solve them. They’re part of the same...?

**Student:** Fact family.

**Teacher:** Fact family, absolutely.
The curricular resources Maria actually used in this excerpt were problems to solve and directions to follow (what do you notice?). Maria used the problems and asked questions concerning what students noticed about numbers 2 and 3. The question Maria asked led students to “fact family” and did not bring out attributes of multiplication and division problems. The question also did not support the use of inverse relationship between multiplication and division to solve these problems, which is a mathematical point of this lesson. The mathematical point Maria emphasized in the interaction shown above is “fact family,” which is different from that in the curriculum. Therefore, Maria did not use the resources to provide clear explanations of key mathematical ideas. Hence, Maria did not use the curricular resources in ways that supported each other to emphasize key mathematical ideas of the lesson. This is typical of how Maria used curricular resources and, hence, I classified Maria’s quality of use as minimally connected toward mathematical points of the lesson. Other curricular resources could have helped Maria enact the lesson better, but she did not use them (see explanation in Chapter IV).

To answer question 3, after my initial analysis of enacted lessons, I characterized each lesson using the extent of using curricular resources and the connectedness of the resources used, such as full use and highly connected, moderate use and highly connected, and moderate use and minimally connected. In this way, nine categories were possible combinations that could be observed; however, I am not claiming that all of these were seen in this study. But for any of these categories, I grouped all lessons classified as such. For example, all lessons whose quality of use I classified as “highly connected” were grouped together. In this category, I identified characteristics of ways teachers use curricular resources in association with other resources toward written
mathematical points. I identified this by comparing the lessons to identify typical approaches taken by teachers as they use curricular resources in association with other resources toward the mathematical point written in curriculum programs and kinds of relationships established among resources by teachers during enactment.

Third, using the analysis from steps one and two above, I looked for patterns in the kinds of curricular resources actually used and the quality of use around mathematical points provided in the curriculum. For each quality of use identified in this study, I made comparisons to determine similarities and differences among teachers and describe ways they use curricular resources toward written mathematical points of their lesson (using Table 2.7). I used these results to gain insights into capacities teachers need to use curricular resources in conjunction with other curricular resources to design instruction toward mathematical points.

**Interviews**

I analyzed the interviews (introductory, follow-up, and final) in order to further clarify patterns in the types of adaptations teachers made during enactment that were captured in observation data. I also investigated reasons for making adaptations during the enactment of lessons. To do this, I read through the interview transcripts several times and identified emerging themes as codes and coded them. From the final interviews, I also investigated whether teachers saw relationships between available curricular resources and lesson goals they articulated.

**Coding for adaptations.** Types of adaptations teachers made were identified during enactment of lessons and were captured using Table 3.3 (part a). However, I used
interviews to confirm adaptations that teachers made that I had identified from observations, which helped me triangulate data sources to support or dispute my claims.

**Coding reasons for adaptations.** Using interview transcripts, I coded reasons teachers decided to make changes. I read follow-up interview transcripts of all the teachers several times, looking for emerging themes from the reasons teachers gave for making changes. Table 3.4 presents the codes of common reasons I found in the interview transcripts for why teachers made changes. Reasons for adaptations were coded in chunks, as they were embedded in teachers’ explanations of what they were doing. Also, these reasons can be understood only when viewed as a group of sentences rather than individual sentences. I read and coded each transcript carefully to identify the reasons teachers gave for making changes in their lessons.

Table 3.4

*Codes for Reasons of Adaptations*

<table>
<thead>
<tr>
<th>Reasons for adaptations</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student thinking (ST)</td>
<td>To match up with what students are capable of doing</td>
</tr>
<tr>
<td>Practice/MEAP/other colleagues (PM)</td>
<td>To equip students for future challenges</td>
</tr>
<tr>
<td>Student engagement (SE)</td>
<td>To keep monitoring task engagement</td>
</tr>
<tr>
<td>Prior experience (PE)</td>
<td>To make use of past learning</td>
</tr>
<tr>
<td>Time/Materials</td>
<td>Insufficient time/material to enact everything in curriculum</td>
</tr>
</tbody>
</table>

**Summarizing interview results.** To answer question 4, I identified typical types of adaptations teachers made during classroom enactment; reasons provided for these
adaptations during introductory, post-lesson, and final interviews; and perceived relationships among curricular resources articulated by teachers during the final interviews that likely influenced the types of adaptations made. By perceived relationships above, I mean the relationships that teachers identified during planning that probably led to adaptations being made. These perceived relationships were identified during the final interviews as teachers were asked about how they think the various kinds of available curricular resources were related to the lesson goals they articulated.

**Characterizing Teachers’ Capacity to Use Curricular Resources**

To answer question 5, I looked at the results of analysis across the various kinds of curricular resources available to the teachers, the types of adaptations made when using these curricular resources during enactment, typical reasons provided for making the adaptations, types of relationships among curricular resources that teachers used during enactment of lessons, and typical ways in which teachers used the curricular resources during enactment of lessons. This enabled me to propose insights into capacities teachers needed to use curricular resources in association with each other toward written lesson goals.
CHAPTER IV

RESULTS

In this chapter, I present results of my analysis to answer the five research questions for this study. I state each research question followed by the findings that answer it.

Available Curricular Resources in the Lessons Analyzed

In this section, I provide results for question 1: What kinds of curricular resources are available for teachers in the lessons they teach? To answer this question, I analyzed the kinds of curricular resources in nine written lessons per curriculum that were taught by the teachers observed. My analysis revealed that in both curriculum programs used for this study, representations of tasks and design transparency have greatest and least percentages, respectively. All subcategories of the three kinds of curricular resources were available for the lessons analyzed in both curriculum programs (see Tables in Appendix D). In spite of this availability, descriptions of representations are minimally available for both programs; in each program descriptions of representations were available in only one of the nine lessons analyzed.

Table 4.1 shows proportions of curricular resources available in the written lessons analyzed for both curriculum programs used for this study. I describe the availability of various subcategories for representations of tasks (RT), representations of concepts (RC), and design transparency (DT).

Representations of Tasks (RT)

SFAW-Mathematics contains a higher percentage of representations of tasks among available curricular resources (75.6%) than Investigations (62.8%).
Table 4.1

<table>
<thead>
<tr>
<th>Curricular Resources</th>
<th>Investigations</th>
<th>SFAW-Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Grand Total</td>
<td>% a</td>
</tr>
<tr>
<td>RT1</td>
<td>450</td>
<td>38.7</td>
</tr>
<tr>
<td>RT2</td>
<td>77</td>
<td>6.6</td>
</tr>
<tr>
<td>RT3</td>
<td>204</td>
<td>17.5</td>
</tr>
<tr>
<td>RT (Total)</td>
<td>731</td>
<td>62.8</td>
</tr>
<tr>
<td>RC1</td>
<td>89</td>
<td>7.1</td>
</tr>
<tr>
<td>RC2</td>
<td>80</td>
<td>6.9</td>
</tr>
<tr>
<td>RC3</td>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>RC4</td>
<td>25</td>
<td>2.1</td>
</tr>
<tr>
<td>RC5</td>
<td>46</td>
<td>4.0</td>
</tr>
<tr>
<td>RC (Total)</td>
<td>243</td>
<td>19.4</td>
</tr>
<tr>
<td>RC (Total-sen) b</td>
<td>154</td>
<td>13.2</td>
</tr>
<tr>
<td>DT1</td>
<td>105</td>
<td>9.0</td>
</tr>
<tr>
<td>DT2</td>
<td>174</td>
<td>15.0</td>
</tr>
<tr>
<td>DT (Total)</td>
<td>279</td>
<td>24.0</td>
</tr>
<tr>
<td>Grand Total</td>
<td>1253</td>
<td>100</td>
</tr>
<tr>
<td>Grand Total (sen) b</td>
<td>1164</td>
<td>100</td>
</tr>
</tbody>
</table>

% is calculated on the total number of sentences. For example, % for RT1 for Investigations is (450/1164)*100 = 38.7%; % for visuals is calculated with respect to Grand Total. That is, (89/1253)*100 for visuals (RC1) in Investigations.

Grand Total (sen) = Grand Total minus number of visuals (RC1) because these are non-textual.

The representations of task for SFAW-Mathematics are mostly problems, whereas in Investigations they are mainly directions for teachers and students to follow. Although both programs contain greater percentages of this resource, each curriculum program emphasized its different subcategories in terms of percentages.

**Directions for teachers and students to follow (RT1).** Table 4.1 shows that Investigations contains a higher percentage of guidance to teachers and students (38.7%) than SFAW-Mathematics (25.2%). A lot of the directions for teachers and students to follow in SFAW-Mathematics are in the optional parts of each lesson. The main parts of
each lesson actually contain minimal guidance to teachers and students. The guidance these programs provide to teachers differs in terms of mathematical points and rationales.

In *Investigations*, guidance provided to teachers and students sometimes contains mathematical points to be achieved, while this is rare in *SFAW-Mathematics*. For example, in an *Investigations* lesson that involved solving multiplication and division story problems taught by Maria, the authors provided the following guidance to the teacher: “Before assigning the problems, discuss how to determine which type of problem each is” (Wittenberg et al., 2008, Grade 3, Unit 5, p. 122). This guidance provided by authors of *Investigations* to teachers gives them insight into mathematical points embedded in the discussion (e.g., determine whether a problem is a multiplication or division story problem).

In *Investigations*, guidance is often provided to teachers along with a rationale. For example, from *Investigations*, teachers are asked to “encourage students to act out the action of each problem, using cubes or drawings. Doing so will help students recognize that the division situation have them starting with an amount that gets divided into equal groups” (Wittenberg et al., 2008, Grade 3, Unit 5, p. 122). This guidance provides a rationale to teachers for encouraging students to act out the action of each problem. Such guidance to teachers with a rationale is uncommon in *SFAW-Mathematics*.

**Participation structure and time (RT2).** How students should engage with assigned activities (i.e., individually, in groups, or whole group) and suggested optimal time needed to complete them are also provided in both programs. For example, Table 4.1 shows that *Investigations* provides a greater percentage of participation structure and time (6.6%) than *SFAW-Mathematics* (3.5%). The two programs differ in terms of the
availability of RT2 throughout every lesson. Authors of *Investigations* allocate participation structure and time for every activity they suggest, while *SFAW-Mathematics* does not. Authors of *SFAW-Mathematics* allocate this resource to all optional activities except for “Getting Started,” but not for those in the main part of the lesson. Figure 4.1 and Figure 4.2, in *SFAW-Mathematics*, show that all activities under “Reaching All Learners” have participation structure and time, while the warm up, teach (learn), practice, and assess sections—which constitute the main parts of the lesson—have none.

![Participation Structure and Time Diagram](image)

*Figure 4.1. Optional activities in SFAW-Mathematics With Participation Structure and Time (Charles et al., 2008, Grade 4, Volume 3, p. 412B)*
Problems to solve (RT3). The authors of SFAW-Mathematics allocate a greater percentage of problems among available resources (46.9%) than the authors of Investigations (17.5%). This is because lots of short review and practice problems such as warm-up and spiral reviews are provided in SFAW-Mathematics, whereas these are
minimal in *Investigations*. All three categories of problems, which are grouped according to their purposes—reinforcement of the lesson content or practice (RF), exploration/development of mathematical concepts (EDC), and ongoing review of previously learned mathematical content (OR)—are provided in *SFAW-Mathematics* and *Investigations*. Despite this, both curriculum programs exhibited varying emphases.

Table 4.2 shows the different percentages of problems that each of these curriculum programs emphasize.

**Table 4.2**

*Types of Problems Available*

<table>
<thead>
<tr>
<th></th>
<th>Investigations</th>
<th>SFAW-Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>% (^a)</td>
</tr>
<tr>
<td>Number of Sentences</td>
<td>204</td>
<td>100.0</td>
</tr>
<tr>
<td>Number of Problems</td>
<td>99</td>
<td>100.0</td>
</tr>
<tr>
<td>Problems by Purpose</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RF</td>
<td>26</td>
<td>26.3</td>
</tr>
<tr>
<td>EDC</td>
<td>54</td>
<td>54.5</td>
</tr>
<tr>
<td>OR</td>
<td>19</td>
<td>19.2</td>
</tr>
</tbody>
</table>

\(^a\)% is calculated on number of problems available. For example, RF for *Investigations* is \((26/99)\times100 = 26.3\)

In *Investigations*, the greatest and least percentage of problems is allocated to those that students explore to develop their understanding of mathematical concepts (54.5%) and review of mathematical content learned in previous lessons (19.2%), respectively.

The percentage of problems used for reinforcement in *SFAW-Mathematics* (59.3%) is more than twice that for *Investigations* (26.3%). This is because in *SFAW-
Mathematics, more problems for reinforcement of concepts and relatively short warm-up computational-based exercises are provided (345) than in Investigations (26). The percentage of problems used for review of previously learned mathematical content in SFAW-Mathematics (34%) is almost twice that of Investigations (19.2%). Figure 4.3 and Figure 4.4 show examples of problems for reinforcement of mathematical concepts learned in SFAW-Mathematics and Investigations, respectively.

Figure 4.3. Problems for Reinforcement of Mathematical Concepts in SFAW-Mathematics (Charles et al., 2008, Grade 4, Volume 3, p. 410)
Figure 4.4. Problems for Reinforcement of Mathematical Concepts in *Investigations* (Wittenberg et al., 2008, Grade 4, Unit 4, p. 93)

The percentage of problems for exploration and development of mathematical concepts students are learning used in *Investigations* (54.5%) is more than eight times that of *SFAW-Mathematics* (6.7%). This is because more problems for exploration and development of mathematical concepts are provided in *Investigations* (54 out of 99) than in *SFAW-Mathematics* (39 out of 582). Also, problems for EDC are provided in every lesson in *Investigations*, while this is not the case for *SFAW-Mathematics*. Figure 4.5 and Figure 4.6 are examples of problems for exploration and development of mathematical concepts provided in *SFAW-Mathematics* and *Investigations*, respectively.
**Figure 4.5.** Problems for Exploration/Development of Mathematical Concepts in *SFAW-Mathematics* (Charles et al., 2008, Grade 4, Volume 3, p. 406)

**Activity**

**What’s the rule?**

**a.** Use a calculator to find each quotient. Look for a pattern in the number of zeros.

\[
\begin{align*}
60 \div 3 &= 20 \\
120 \div 4 &= 30 \\
300 \div 5 &= 6 \\
60 \div 30 &= 2 \\
120 \div 40 &= 3 \\
300 \div 50 &= 6 \\
600 \div 30 &= 20 \\
1,200 \div 40 &= 30 \\
3,000 \div 50 &= 60
\end{align*}
\]

**b.** Find each quotient without a calculator. Then check your answers with a calculator.

\[
\begin{align*}
48 \div 8 &= 6 \\
21 \div 7 &= 3 \\
20 \div 4 &= 5 \\
480 \div 8 &= 60 \\
210 \div 7 &= 30 \\
200 \div 4 &= 50 \\
480 \div 80 &= 6 \\
210 \div 70 &= 3 \\
200 \div 40 &= 5 \\
4,800 \div 80 &= 60 \\
2,100 \div 70 &= 30 \\
2,000 \div 40 &= 50
\end{align*}
\]

**c.** Describe a rule that tells how to find the quotient. Cross out any zeros that both the dividend and the divisor have in common and then divide.

---

**Figure 4.6.** Problems for Exploration/Development of Mathematical Concepts in *Investigations* (Wittenberg et al., 2008, Grade 3, Unit 5, p. 122)

**Name**

**Date**

**Story Problems** (page 1 of 2)

Solve each problem and show how you solved it.

1. A robot has 4 hands. Each hand has 6 fingers. How many fingers does the robot have altogether?

2. We made 20 muffins for the bake sale. We put the muffins in bags to sell. We put 4 muffins in each bag. How many bags of muffins did we have to sell?

3. We bought 3 packs of yogurt cups. Each pack had 4 yogurt cups. How many yogurt cups did we buy?

4. Five children have one back of 35 movie tickets to share equally. Each movie costs one ticket. How many movies can each child see?

5. Before school my mother gave me a pack of 24 new pencils. When I got to school, I wanted to share them equally among my three friends and me. How many pencils will each of us get?

6. Benjamin drew a picture of 7 pentagons. Each pentagon has 5 sides. How many sides are there in all?
Therefore, the authors of *Investigations* emphasize problems to explore and develop students’ mathematical understanding, while the authors of *SFAW-Mathematics* use problems mainly to practice, reinforcing mathematical skills learned.

**Representations of Concepts (RC)**

As shown in Table 4.1, the authors of *SFAW-Mathematics* allocate a slightly higher percentage of representations of concepts among available curricular resources (17.3%) than their *Investigations* counterparts (13.2%).

**Visuals and representations of problems (RC1).** Visuals are non-textual and therefore analyzed separately. Table 4.3 shows frequency and percentages of visuals available in both programs. Approximately the same percentage of visuals and representations found in problems were available in the lessons analyzed for both curriculum programs. The total number of visuals is higher in *SFAW-Mathematics* than in *Investigations* because the former program used more visuals in many problems it provides to teachers and students than the latter program. In *SFAW-Mathematics*, visuals are also used to illustrate definitions and show different images of three-dimensional shapes (see Figure 4.2).

Table 4.3

**Visuals Available to Teachers**

<table>
<thead>
<tr>
<th></th>
<th>Investigations</th>
<th></th>
<th>SFAW-Mathematics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>%</td>
<td>Frequency</td>
<td>%</td>
</tr>
<tr>
<td>Visuals available outside of problems</td>
<td>34</td>
<td>38.2</td>
<td>81</td>
<td>38.6</td>
</tr>
<tr>
<td>Visuals available in Problems</td>
<td>55</td>
<td>61.8</td>
<td>129</td>
<td>61.4</td>
</tr>
<tr>
<td>Total</td>
<td>89</td>
<td>100</td>
<td>210</td>
<td>100</td>
</tr>
</tbody>
</table>
Mathematical explanations (RC2). In SFAW-Mathematics, sentences that explain mathematics to students make up 7.3% of the overall sentences, while in Investigations, 6.9% is allocated for that, as shown on Table 4.1. Although the percentages are similar, both programs present these resources differently in terms of development of the mathematical ideas.

Mathematical explanations that the authors of SFAW-Mathematics provide are mainly definitions of mathematical vocabulary, as shown in Figure 4.2, and standard mathematical procedures for teachers and students to follow for computational purposes, as shown in Figure 4.7.

Figure 4.7. An Example of Mathematical Procedure Provided in SFAW-Mathematics (Charles et al., 2008, Grade 4, Volume 3, p. 404)
These definitions and mathematical procedures/strategies are presented to teachers and students directly without reference to how they were developed and are mostly void of meaning. As such, these curricular resources may not support teachers and students to develop an insightful understanding of these mathematical definitions and procedures.

Although definitions are provided in *Investigations*, standard mathematical procedures are not offered to teachers and students. In cases where definitions of terms are provided, students are expected to engage in an exploration to develop an intuitive understanding of the term before the teacher formally introduces it. For example, in one of the lessons analyzed from *Investigations*, rather than offer a conventional definition for multiplication and division problems, the authors guide teachers to the underlying mathematical idea as follows:

Listen for student understanding of the difference between multiplication and division. For example, do the problems students make for $18 \div 3$ begin with the quantity 18 and divide into 3 equal groups or groups of 3? Do the problems for $6 \times 3$ involve 6 groups of 3 or 3 groups of 6? (Wittenberg et al., 2008, Grade 3, Unit 5, p. 126)

In this excerpt, the authors of *Investigations* provide a meaning of multiplication and division situations. In other words, for multiplication situations, one knows the number of groups and the number of items in each group, and for division situations one begins with a larger quantity and divides it into equal groups or groups of the same quantity.

The authors of *Investigations* provide students with strategies to use that are based on meaning of operations. Strategies based on “meaning of operations” refer to strategies that focus on using conceptual ideas such as those mentioned above for multiplication and division problems. These meanings are developed as students explore concepts in the
strategies. In another lesson from *Investigations*, the authors suggest a discussion of where to place 1.25 on a number line. In this discussion, the authors offer a conceptual strategy and suggest that teachers ask students the following questions: “… is it more or less than 1.2? How do you know? Is it more or less than 1.3? How do you know?” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 40). These questions provide students with a meaning of decimals and size of number to use each time ordering decimal numbers is assigned. Although both programs offer strategies to students and teachers as explained above, the kind of meaning associated with strategies in *Investigations* is rare in *SFAW-Mathematics*. Therefore, *SFAW-Mathematics* places a heavy demand on its teachers to provide meaning to students when using standard definitions and procedures, while *Investigations* teachers are supported in the course of developing strategies based on conceptual understanding.

**Descriptions of representations (RC3).** Descriptions of representations, although available, were minimally provided in both programs. Table 4.1 shows that both *SFAW-Mathematics* and *Investigations* allocated the same percentage (0.2%) of this resource to teachers. Only in one and two of the lessons analyzed for this study in *Investigations* and *SFAW-Mathematics*, respectively, did teachers receive support of this kind.

**Relationships (RC4).** Table 4.1 shows that a slightly higher percentage of relationships among units, lessons, activities, mathematical ideas, or representations provided in *SFAW-Mathematics* (3.9%) than in *Investigations* (2.1%). However, these relationships are more explicit in *Investigations* than *SFAW-Mathematics*. For example, a
lesson analyzed for this study begins with the following statements that the teacher is expected to say to students:

In our last few sessions, you were placing the Power Polygons next to each other to see what kinds of new polygons you could make. Today, we are going to do something similar, but instead of focusing on the shape, we are going to be paying attention to the angles. (Wittenberg et al., 2008, Grade 4, Unit 4, p. 89)

The first sentence in this quotation explicitly provides information on what had been done, while the second sentence explicitly provides information on how current learning will build on previous learning, establishing relationships between lessons and activities.

In contrast, RC4s in *SFAW-Mathematics* are mostly topics reviewed in problems. In all lessons analyzed for this study from *SFAW-Mathematics*, spiral review sections contain lists of topics provided that are related to review problems such as “multiplying and adding money, problem solving skills, multi-step problems” (Charles et al., 2008, Grade 4, Volume 3, Unit 8, p. 434A). Therefore, teachers using *Investigations* can be more empowered to make connections among activities, mathematical ideas, lessons, etc., with such resources than their *SFAW-Mathematics* counterparts. As such, *SFAW-Mathematics* seems to place a demand on teachers to make connections on their own.

**Assessments (RC5).** A slightly higher percentage of assessments is provided in *SFAW-Mathematics* (5.9%) than *Investigations* (4.0%). Assessment of facts and conceptual understanding is provided in *SFAW-Mathematics*, while only assessment of conceptual understanding is provided in *Investigations*. In *SFAW-Mathematics*, the authors allocate assessment problems mainly to examine students’ recall capacity of mathematical ideas and procedures explained by the teacher. These authors also assess students’ ability to recall facts and information usually provided in the text. For example, Figure 4.2 shows an informal assessment, “Talk About It.” The assessment question
focuses on students’ ability to look at the visuals above it and identify the solids in the question.

Assessments in *SFAW-Mathematics* also target students’ understanding of mathematical concepts, which often emphasize useful relationships between mathematical ideas or representations. *SFAW-Mathematics* includes grading rubrics for test-taking practice problems. Although the test items may not usually be conceptual, the grading rubric emphasizes that teachers pay attention to an understanding of the concept. These grading rubrics illustrate different levels of students’ responses and allocation of points along with sample answers. Figure 4.8 shows a Test-Taking Practice problem and grading rubric that gets at the depth of a concept with various levels of points based on the level of conceptual articulation.

*Figure 4.8. Test-Taking Practice and its Grading Rubric Provided in SFAW-Mathematics* (Charles et al., 2008, Grade 4, Volume 3, pp. 436-437)
In contrast, *Investigations* offers assessments that focus mainly on students’ understanding of mathematical concepts, by including questions that dig deeper into students’ mathematical understanding. For example, in one of the lessons analyzed for this study, the following questions to assess students’ understanding of a right angle are provided: “Do students easily recognize the shape of a right angle? Do they use the corners of a square such as shape A or the corners of their papers as tools for measuring right angles?” (Wittenberg et al., 2008, Grade 4, Unit 4, p. 91). In particular, teachers are expected to use these questions to assess students’ thinking when observing them at work. Of all the assessments in the lessons analyzed for this study, the authors of *SFAW-Mathematics* and *Investigations* provided 39.4% and 100%, respectively, of their assessments with focus on students’ understanding of mathematical concepts. *Investigations* did not include assessment resources that target simply students’ recall abilities.

**Design Transparency (DT)**

Curricular resources in the category of design transparency help teachers understand the rationale of guidance and directions provided in curriculum materials, which can promote effective enactment of the lessons. The present study includes rationale/transparency and anticipated students’ thinking in this category. For lessons analyzed for this study, the authors of *Investigations* included a higher percentage of design transparency (24%) than the authors of *SFAW-Mathematics* (7.1%); this is the case in each of the two subcategories as well, as shown in Table 4.1. The percentage of anticipated student thinking (15%) in *Investigations* is more than three times that of *SFAW-Mathematics* (4.4%). This difference is because anticipated student thinking is
provided in every lesson of *Investigations*, while this is not the case in *SFAW-Mathematics*. Similarly, *Investigations* and *SFAW-Mathematics* contained 9% and 2.7%, respectively, of rationale/transparency. Also, in *Investigations*, rationale/transparency is provided for many of the activities and guidance to teachers and students. In *SFAW-Mathematics*, the rationale/transparency provided is mainly in lesson goals.

**Rationale/transparency (DT1).** Table 4.1 shows that a higher percentage of sentences in *Investigations* is allocated to rationale/transparency than in *SFAW-Mathematics*. Two main kinds of rationale/transparency emerged in my analysis. These are rationale for a lesson, an activity, or a problem, and rationale for teacher action.

In both curriculum programs, rationale for each lesson in the form of math focus points or objectives (i.e., indicating mathematical goals of the lessons) is provided. The authors of *Investigations* further indicate within each lesson where particular objectives are addressed. In addition, they provide rationale/transparency for why an activity or an action is suggested, whereas the authors of *SFAW-Mathematics* do not. For example, in one of the lessons analyzed for this study, the authors of *Investigations* suggest that students make smaller angles on their desks with two pencils, along with a statement that “the purpose here is for students to revisit how angles are created by degrees of turns as the sides, or rays, pivot from the vertex” (Wittenberg et al., 2008, Grade 4, Unit 4, p.-100). Also, in *Investigations*, rationale is provided for using particular problems such as “the problems on *Student Activity Book* pages 23-25 are used to assess students in representing and ordering decimals” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 51). This kind of rationale, which alerts the teacher to watch out for students’ understanding
of the mathematical ideas of representing and ordering decimals and to assess them appropriately, is absent in *SFAW-Mathematics*.

Furthermore, the authors of *Investigations* suggest teacher moves or actions and then provide rationale for those moves or actions as explained in RT1. Such rationales often communicate to teachers the mathematical points students are to learn. This kind of action-rationale support to teachers is not seen in *SFAW-Mathematics*.

**Anticipated student thinking (DT2).** Curricular resources of student thinking are provided in three distinct and yet related areas: ways students might approach a task (or What Students Might Say), suggested questions that encourage conceptual ways student might respond to a task, and difficulties students might face. Let me explain these kinds of resources and ways in which they are provided in each program.

The ways students might approach a task are provided in *Investigations* in the form of what the students might try. For example, in a lesson where students are asked to find decimal equivalence, it is stated, “Some students may also be interested in the ten-thousandths grid and may come up with equivalents to ten thousandths (e.g., one tenth is equivalent to 1,000 ten thousandths)” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 38). In *Investigations*, material resources that students might need to fully articulate their thinking are also provided to teachers. The ten-thousandths grid in the above example signals to teachers ahead of time to prepare the necessary material. Such support is not seen in the lessons analyzed from *SFAW-Mathematics*.

Both curriculum programs provide ideal and desired strategies students are to learn. The programs provide this in the form of “What Students Might Say” with little photographs of children, showing the ideas are actually theirs. Figure 4.9 and Figure 4.10
shows “What “Students Might Say” in *Investigations* and *SFAW-Mathematics*, respectively. Anticipated student thinking shown in Figure 4.9 is presented in *Investigations* when students are asked to share their ideas.

**Students might say:**

“I know all of the angles in the green triangle are 60 degrees because two of the little angles in shape O fit into each one and those are 30 because three of them fit into the right angle.”

“I know from yesterday that the small angles in shape O are each 30. And two of those fit into it, so it is 60 degrees.”

*Figure 4. 9. “What Students Might Say” Presented in *Investigations* (Wittenberg et al., 2008, Grade 4, Unit 4, p. 103)*

“What Students Might Say” shows mathematical ideas students may or are likely to use. It shows previous learning that could be articulated by students. In addition, it reveals relationships between mathematical ideas students can draw from to develop their understanding of what is currently being taught. For example, in Figure 4.9, the second student is drawing from previous learning to find the size of another angle. Also, in Figure 4.10, the student is making use of compatible numbers to estimate a quotient in this division problem. Therefore, “What Students Might Say” has the potential of informing teachers about important and significant student prior knowledge to activate so that desired learning is fostered. In addition, it also communicates desired student thinking for current topics that are expected.

Student ideal and desired mathematical understandings are provided along with questions in both *SFAW-Mathematics* and *Investigations*. Figure 4.11 shows these questions embedded in “Investigating the Concept” in *SFAW-Mathematics*. Figure 4.12
shows an example of questions provided in *Investigations* that reflect desired thinking regarding ordering decimals.

*Figure 4.10. “What Students Might Say” in SFAW-Mathematics (Charles et al., 2008, Grade 4, Volume 3, p. 409)*
Figure 4.11. Suggested Questions That Provide Anticipated Student Thinking in “Investigating the Concept” in SFAW-Mathematics (Charles et al., 2008, Grade 5, Volume 2, p. 262A)

Whereas in SFAW-Mathematics only one or two questions are provided to the teacher, many questions are provided in Investigations. These questions provide teachers with a useful tool to check students’ understanding. In Investigations, main questions are sometimes provided together with subquestions. These subquestions provide a breakdown of the main questions to give teachers a sense of the components of key mathematical ideas students might be thinking of. Such detailed subquestions are absent in SFAW-Mathematics. Also, in SFAW-Mathematics, desired responses to these questions are provided to teachers, which is absent in Investigations. This is because these questions are mostly fact-oriented, rather than process-oriented (see the example in Figure 4.12), and so anticipated processes in response to the questions may be difficult to provide.
The subquestions provided in *Investigations* might convey to teachers the different parts of a concept and process that students are to learn. This can, in turn, develop teachers’ mathematical knowledge. However, it can be challenging to teachers with limited mathematical knowledge who are themselves wondering about their answers. The sample responses provided in *SFAW-Mathematics* may help teachers struggling with the mathematical content.

Authors of *SFAW-Mathematics* also explain how students might respond to problems assigned to them. For test-taking practice problems, Figure 4.8 shows an example of a grading rubric in *SFAW-Mathematics*. In grading rubrics, the authors of *SFAW-Mathematics* anticipate student thinking that reveals different levels of their understanding of the concept being learned and for which they will receive full or partial credit. The authors of *Investigations* do not explain anticipated student thinking in this way.
In addition to assessing students’ mathematical understanding, both curriculum programs communicate difficulties students might face when interacting with concepts they are to learn. Also, suggested interventions or moves teachers might take to help resolve these difficulties are provided. In *Investigations*, these anticipated student difficulties and suggested teacher moves to remedy them are provided within the main text. For example, one *Investigations* lesson stated, “If students are having difficulties placing the numbers with 5 in the hundredths place, such as 0.35, ask them to show you how they would represent the numbers on the hundredths grid on *Student Activity Book* page 16” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 41).

In contrast, the *SFAW-Mathematics* lessons provide anticipated student difficulties and errors mainly in “Ongoing Assessment” and “Check” (error intervention), as shown in Figure 4.13. These appear in the margins of *SFAW-Mathematics* lessons on a regular basis. These “Ongoing Assessments” and “Error Interventions” are mostly about facts and procedures, rather than thinking and reasoning.

![Figure 4.13. Student Difficulties Provided in *SFAW-Mathematics* (Charles et al., 2008, Grade 5, Volume 2, p. 260)](image-url)

*Figure 4.13. Student Difficulties Provided in *SFAW-Mathematics* (Charles et al., 2008, Grade 5, Volume 2, p. 260)*
These student difficulties might communicate to teachers the ideas that students might struggle with that need careful attention. They might also signal that careful planning is needed to help foster appropriate student learning to overcome anticipated difficulties. In addition, they communicate mathematical skills and definitions that likely need to be examined to foster student learning.

Curricular Resources Teachers Plan to Use

This section presents results for research question 2: What kinds of curricular resources do the teachers plan to use? The availability of the various curricular resources mentioned in question 1 raises an interesting question: How much of available curricular resources do teachers plan to use? I analyzed these aspects in CRLs, based on what participant teachers indicated as parts they were going to use to teach the lessons and the interviews with teachers about their plans. I present the kind and amount of available curricular resources these teachers planned to use and how they planned to use them.

Table 4.4 shows the various kinds of curricular resources available to all six teachers in the lessons they taught, those they planned to use, and the percentages they planned to use. These percentages are calculated on the number of each curricular resource available per category and per curriculum.

_SFAW-Mathematics_ and _Investigations_ teachers planned to use 44.2% and 41.2%, respectively, of the available curricular resources, as shown on Table 4.4. This slight difference in percentage is because _SFAW-Mathematics_ teachers planned to use more problems 67% (390 out of 582) than _Investigations_ teachers 60.6% (60 out of 99) (see Table 4.5). Many problems planned to use suggest more sentences teachers planned to use, as each problem contains at least one sentence. Table 4.4 shows that the number of
sentences associated with problems in *SFAW-Mathematics* is more than five times that of *Investigations* (see RT3). In addition, more problems are available in *SFAW-Mathematics* than in *Investigations*, as explained in the previous section (see Table 4.2).

For each broad category of curricular resources, using the summary data in Table 4.4, I present percentages for subcategories per curriculum that teachers planned to use. Then, I compare the percentage of the subcategories teachers planned to use across curricula. Lastly, I identify themes and compare the kinds of curricular resources teachers in this study planned to use per curriculum program.

Table 4.4

<table>
<thead>
<tr>
<th>Curricular Resource</th>
<th>Investigations</th>
<th></th>
<th></th>
<th>SFAW-Mathematics</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Available CRs</td>
<td>CRs planned to Use</td>
<td>%</td>
<td>Available CRs</td>
<td>CRs planned to Use</td>
<td>%</td>
</tr>
<tr>
<td>RT1</td>
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<td>40.4</td>
<td>586</td>
<td>111</td>
<td>18.9</td>
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<tr>
<td>RT2</td>
<td>77</td>
<td>46</td>
<td>59.7</td>
<td>82</td>
<td>14</td>
<td>17.1</td>
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<tr>
<td>RT3</td>
<td>204</td>
<td>99</td>
<td>48.5</td>
<td>1089</td>
<td>705</td>
<td>64.7</td>
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<tr>
<td>RT (Total)</td>
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<td>327</td>
<td>44.7</td>
<td>1757</td>
<td>830</td>
<td>47.2</td>
</tr>
<tr>
<td>RC1</td>
<td>89</td>
<td>54</td>
<td>60.7</td>
<td>210</td>
<td>81</td>
<td>38.6</td>
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<tr>
<td>RC2</td>
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<td>33</td>
<td>41.3</td>
<td>170</td>
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<td>3</td>
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<td>91</td>
<td>54</td>
<td>59.3</td>
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<td>RC (Total-sen)</td>
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<td>179</td>
<td>44.5</td>
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<td>24.8</td>
<td>62</td>
<td>5</td>
<td>8.1</td>
</tr>
<tr>
<td>DT2</td>
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<td>31.0</td>
<td>103</td>
<td>24</td>
<td>23.3</td>
</tr>
<tr>
<td>DT (Total)</td>
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<td>80</td>
<td>28.7</td>
<td>165</td>
<td>29</td>
<td>17.6</td>
</tr>
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<td>41.2</td>
<td>2534</td>
<td>1119</td>
<td>44.2</td>
</tr>
<tr>
<td>Grand Total (sen)³</td>
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<td>462</td>
<td>39.7</td>
<td>2324</td>
<td>1038</td>
<td>44.7</td>
</tr>
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</table>

³Grand Total (sen) = Grand Total for sentences *minus* number of visuals (RC1) because these are non-textual.
<table>
<thead>
<tr>
<th>Problems by purpose</th>
<th>Investigations</th>
<th>SFAW-Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of problems available</td>
<td>Number of problems planned to use</td>
</tr>
<tr>
<td>Total</td>
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<td>20</td>
</tr>
<tr>
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<td>54</td>
<td>28</td>
</tr>
<tr>
<td>OR</td>
<td>19</td>
<td>12</td>
</tr>
</tbody>
</table>

<sup>a</sup>RF is problems for reinforcement of mathematical concepts learned, EDC is problems for exploration and development of mathematical concepts, OR is ongoing review problems.

Table 4.5 *Number and Percentage of Problems All Six Teachers Planned to Use*
Representations of Tasks (RT)

Although the overall percentages of available representations of tasks that *Investigations* and *SFAW-Mathematics* teachers planned to use are very similar, their emphases within the subcategories vary greatly.

**Directions for teachers and students to follow (RT1).** *Investigations* teachers planned to use more of the guidance provided for teachers and students (40.4%) than their counterparts who used *SFAW-Mathematics* (18.9%). This is because the guidance for teachers and students to follow in *SFAW-Mathematics* is mostly in the optional activities, as explained in the results for question 1 (see Figure 4.1). *SFAW-Mathematics* teachers planned to use these optional activities only minimally. Figure 4.2 shows part of the main section of the lessons with minimal guidance to teachers and students. In contrast, in *Investigations*, guidance is provided for all activities. These activities are sequenced for teachers, as explained in the results for question 1. *Investigations* teachers planned to use most of the activities and hence the corresponding suggested guidance. Some of these guidance and directions are associated with the rationale for why they are suggested.

*Investigations* teachers planned to use the guidance that is supported by a rationale, while their *SFAW-Mathematics* colleagues did not. For example, Maria, one of the *Investigations* teachers, planned to use an example of guidance provided in question 1 that contains the rationale for why students should act out the action of each problem when solving it. *SFAW-Mathematics* teachers did not plan to use guidance with rationale because these are not provided in the main text of the curriculum, as also explained in the previous section.
Participation structure and time (RT2). Table 4.4 shows that *Investigations* and *SFAW-Mathematics* teachers planned to use 59.7% and 17.1%, respectively, of available participation structure and time resources. This difference in percentage is again because *Investigations* teachers planned to use most of the sequenced activities, as explained in the previous section; each suggested activity has participation structure and time. Lisa explained during the follow-up interview how she planned to use the suggested time in her teaching.

**Interviewer:** How do you decide how to manage or use class time for activities—for introduction, discussion, group work, closing, individual work? How do you manage and make those decisions?

**Lisa:** If it's a new concept, I'll take more for my introduction. If it’s something we’ve been reviewing, my introduction is usually a quick review, and then I give them more time, so that they can come ask me questions or sit with me and I can work with them. Closing depends. It’s usually real short, kind of a quick review, close it up, “How did it go?” If I see a lot of kids struggled, then that’s my note for the next day to make my introduction a little longer, maybe less individual work…

In the excerpt, Lisa explained that she planned to manage her time based on the nature of what students are to learn. If it is a new concept students are to learn, then she planned to use more time to introduce it. Lisa planned to use the last few minutes of her lesson for a quick review, assessment, and then decisions regarding the next lesson. She planned to have a recap of the main ideas and assess whether students understood the concept they were to learn or still struggled with it. Assessment helps Lisa to know what to focus on in the introduction of the next class.

In contrast, teachers who used *SFAW-Mathematics* minimally planned to use optional activities that contain this resource and there is no such resource for the main part of the lesson.
Problems to solve (RT3). The percentage of problems teachers planned to use according to the purpose varies greatly between these two curriculum programs.

Teachers who used *Investigations* planned to emphasize exploration and development of mathematical concepts, whereas *SFAW-Mathematics* teachers focused on review of previously learned mathematical concepts. Table 4.5 shows that 46.7% and 10.0% of the problems *Investigations* and *SFAW-Mathematics* teachers, respectively, planned to use were for exploration and development of mathematical concepts. In contrast, 45.6% and 20.0% of the problems *SFAW-Mathematics* and *Investigations* teachers, respectively, planned to use were for review of mathematical concepts students learned previously. Teachers who used *SFAW-Mathematics* explained they planned to use these problems often to refresh students’ understandings of what was learned previously. For example, John said,

> Well, Spiral Review, like I said before, I always use that. It’s part of their morning work. They do that pretty much every day, and it, the thing I like about it is it brings back subjects and topics that they haven’t touched since last year in some cases, and in a lot of other cases it’s stuff from September, and it just helps keep it fresh in their minds. (John, follow-up interview)

The differences in the types of problems planned to use can be explained in terms of availability and sequencing of these problems. Table 4.5 shows that 54 of the 99 problems available in all nine lessons analyzed from *Investigations* are those for exploration and development of mathematical concepts. Since all activities and problems are sequenced in *Investigations*, as explained previously, these teachers were more likely to plan to use them as such. For *SFAW-Mathematics*, sequencing may also be a main reason teachers planned to use review problems. This is because every lesson in *SFAW-Mathematics* opens with problems that review previously learned mathematical concepts.
All three SFAW-Mathematics teachers planned to use spiral review problems at the beginning of their lessons, as explained in the excerpt above.

Table 4.5 reveals some interesting patterns about the problems teachers planned to use from those available in the written curriculum materials. First, Investigations and SFAW-Mathematics teachers planned to use almost the same percentage of problems available to them. Although these percentages are almost the same, the number of problems teachers planned to use varied greatly. Investigations and SFAW-Mathematics teachers planned to use 60 out of 99 (60.6%) and 390 out of 582 (67%) problems, respectively. Jennifer, one of the Investigations teachers, was exposed to many problems for her lessons on angles and she planned to use a substantial number of them (92.2% – 47 out of 51 problems); Lisa, another Investigations teacher, planned to use the least number of available problems (20.6% – 7 out of 34 problems). All SFAW-Mathematics teachers planned to use a substantial number of problems available to them.

Second, Investigations teachers seemed to emphasize reinforcement of mathematical concepts when compared to their counterparts who used SFAW-Mathematics. Table 4.5 shows that Investigations teachers planned to use 76.9% of problems for reinforcement (practice problems) available to them, whereas SFAW-Mathematics counterparts planned to use only 50.1%. This result has to be interpreted with care because Investigations provides fewer problems for reinforcement of mathematical concepts than SFAW-Mathematics. For example, Investigations and SFAW-Mathematics teachers planned to use 20 out of 26 and 173 out of 345 problems for reinforcement, respectively. Among Investigations teachers, Jennifer and Maria planned to use the greatest (55% – 11 out of 20) and least (0% – 0 out of 20) number of problems
for reinforcement, respectively. Among *SFAW-Mathematics* teachers, Caroline and John planned to use the greatest (68.2% – 118 out of 173) and least (8.1% – 14 out of 173) number of problems for reinforcement of mathematical concepts, respectively.

Third, *SFAW-Mathematics* teachers planned to use all 100% of the problems for exploration and development of mathematical concepts (EDC) available to them. This indicates that, when available, *SFAW-Mathematics* teachers may plan to incorporate them into their teaching. For example, problems for EDC were available in one, two, and one lessons for Caroline, Dan, and John, respectively. All three *SFAW-Mathematics* teachers indicated that they planned to use them. However, this type of problem was not available in every lesson in *SFAW-Mathematics*.

**Representations of Concepts (RC)**

Table 4.4 shows percentages of subcategories of representations of concepts teachers planned to use from those available to them in their teacher’s guide. The percentage of available representations of concepts *Investigations* and *SFAW-Mathematics* teachers planned to use are very similar. In spite of this similarity, there is variation in the different subcategories teachers planned to use, as explained next.

**Visuals and representations of problems (RC1).** *Investigations* teachers were more likely to plan to use visuals than their *SFAW-Mathematics* counterparts. Table 4.4 shows that *Investigations* teachers planned to use 60.7% (54 out of 89) of visuals available to them, whereas *SFAW-Mathematics* teachers planned to use 38.6% (81 out of 210). Although the proportion is higher for *Investigations*, more visuals are provided in *SFAW-Mathematics* as explained above. Also, many visuals in *SFAW-Mathematics* are
provided with problems. As SFAW-Mathematics teachers planned to use many problems (explained previously), they also incorporated the associated visuals into their plan.

**Mathematical explanations (RC2).** Table 4.4 shows that the percentage of mathematical explanations teachers who taught from either program planned to use is very similar (44.9% and 42.5%). However, Investigations teachers planned to focus on conceptual meaning while SFAW-Mathematics teachers planned to focus on definitions and conventional mathematical procedures/strategies.

Most of the mathematical explanations that SFAW-Mathematics teachers planned to use from those available were definitions of vocabulary and mathematical strategies. John indicated he planned to use all the definitions of vocabulary in Figure 4.2 while Caroline indicated she planned to use mathematical procedures/strategy for students to follow in Figure 4.7. In contrast, mathematical explanations that Investigations teachers planned to use included some definitions, justifications of strategies, mathematical ideas students should learn from an activity, and mathematical generalizations. For example, Jennifer planned to use mathematical ideas in a suggested discussion, which focused students on determining the size of acute angles by relating them to 90 degrees. In this activity, the authors of Investigations state that “students should come away from this discussion knowing that the sum of the smaller (acute) angles that make up a right angle must be 90 degrees” (Wittenberg et al., 2008, Grade 4, Unit 4, p. 104). This quote identifies an important mathematical idea students should take away from the discussion, and Jennifer indicated that she was going to use this support in teaching the lesson.
Jennifer explained during the follow-up interview that she planned to use this mathematical idea (cited above) to suggest different things students should try in order to get to the desired mathematical understanding for the discussion.

…suggest different combinations for them to try, because some of the kids just really, visually, can’t look at something and go, “Okay, I think this is, this," you know, “is a 30, and this is a 60,” or even the 90’s. I still have some kids that can’t even tell me what a 90 degree angle is, even though we talk about the corner of the paper and a square. They’ll tell me a square is, but then if you hand them a triangle that’s got it, they, they don’t get it. So, having to help them with those, adapting with offering different possibilities for them to try, I think is mainly the support that I’m giving them at this point for the ones that are really having a hard time with it… (Jennifer, follow-up interview)

**Descriptions of representations (RC3).** Table 4.4 shows that *SFAW-Mathematics* teachers planned to use 75% (3 out of 4) of descriptions of representations provided, while *Investigations* teacher planned to use 0% (0 of the 3) of those available. This resource was available in only two of the lessons in *SFAW-Mathematics* and in one of the lessons in *Investigations*.

**Relationships (RC4).** *SFAW-Mathematics* teachers planned to use a higher percentage (59.3%) of curricular resources on relationships among activities, lessons, and grade levels available to teachers than their *Investigations* counterparts (28%). However, such relationships in *SFAW-Mathematics* are mostly indicating topics reviewed through review problems. Two of the three *SFAW-Mathematics* teachers indicated they planned to use these relationships in all lessons. In other words, they indicated on their CRLs the list of topics to be reviewed would be used. During follow-up interview, Dan explained he planned to use this resource to communicate to students that they could employ ideas from other topics learned previously to solve problems.

**Interviewer:** On number 6 of that same Spiral Review, it says that the area of the rectangle is 72, the length is twice the width. You brought up
factor trees in solving that problem and discussed the different dimensions you could have for 72. Could you talk about this and what made you decide to do that?

**Dan:** Well, after we figured out it was not 72 times 72, we, um, we then realized, “Okay, what numbers times what equals 72?” And then I was like, “Oh, man! This would be a good idea to show the prime factorization of 72, and then you can figure out the factors for breaking it down.” So that’s how I want the students thinking, too, so I wanted to show them on the white board how to break down 72, which they have learned before, and how you can use those prime numbers to make up the different factors.

This kind of plan Dan had could promote students using mathematical ideas in a connected way to develop and build a strong understanding of processes.

Teachers who used *Investigations* planned to use relationships that provide explicit connections between lessons. In other words, they planned to state this to students. For example, in one of the lessons Jennifer taught, she indicated in her plan to use explicit connections between lessons provided. Jennifer planned to state what was done previously and how what has to be done currently builds on previous learning. These kinds of relationship are embedded in the guidance provided to teachers.

**Assessments (RC5).** *SFAW-Mathematics* teachers planned to use a slightly higher percentage of this resource from those available to them (36.5%) than their *Investigations* (32.6%) counterparts. *SFAW-Mathematics* teachers planned to assess mathematical facts while *Investigations* teachers planned to assess students’ understanding of conceptual ideas. For example, all assessments *SFAW-Mathematics* teachers planned to use required simple recall of mathematical ideas to determine students’ understanding of the mathematical concepts (see “Talk About It” in Figure 4.2). John explained during final interview how he planned to use this particular “Talk About It.”

… I definitely do the Talk About It, just so they can see that, um, all the solids aren’t necessarily completely made up of plane figures…and it’s one of the things that helps them lead to…learn that a cone doesn’t have any, um, doesn’t have any
vertices. It’s one of those things the cone … they instantly go to one, I said, and then, and that’s where I can say, “what is a vertex?” And they say, “Well, it’s where—” “Show me the edge.” “Uh,” and then they go, “There are no edges.” “So if it doesn’t have any edges, it can’t have a vertex.”… “Yes, it has that point at the top”… “Yes, it’s a point,” but it’s not necessarily a vertex. So then that’s one of those things where we can get and, once again, go over exactly what it is, even though it looks like this and the, and the pyramids, it looks like it’s the same thing, so shouldn’t it be the same thing?… (John, final interview)

In this explanation, John planned to use “Talk About It” to illustrate that not all solid figures are made up of plane figures. He planned to discuss examples and non-examples of each type. He planned to use these examples and non-examples to emphasize critical attributes of each, highlighting similarities and differences between them (e.g., a cone and pyramid). He also planned to illustrate key ideas students were to learn and then assess their understanding of the mathematical points.

None of the SFAW-Mathematics teachers planned to assess students’ conceptual ideas, which is, in large part, due to the availability of such resources in SFAW-Mathematics. In contrast, all assessments Investigations teachers planned to use assessments aimed at students’ understanding of mathematical concepts (see an example in Figure 4.12). This is because in Investigations only assessments that focus on students’ understanding of conceptual ideas are available.

**Design Transparency (DT)**

Table 4.4 summarizes subcategories of design transparency (DT) that teachers planned to use from those available in the teacher’s guides of both curriculum programs. Investigations teachers planned to use a higher percentage (28.7%) of this resource than their SFAW-Mathematics counterparts (17.6%). This is, in part, because more of this resource is available in Investigations (279 sentences) than in SFAW-Mathematics (165 sentences).
Rationale/Transparency (DT1). *Investigations* and *SFAW-Mathematics* teachers planned to use 24.8% and 8.1%, respectively, of rationale. *Investigations* teachers planned to use math focus points, and the rationale for why an activity or an action is suggested. *SFAW-Mathematics* teachers planned to mainly focus on lesson objectives. *SFAW-Mathematics* teachers planned to use lesson objectives because these were the only ones available, as explained in the result for question 1. *Investigations* teachers planned to use lesson objectives (math focus points), and the rationale for an activity, and suggested teacher and student actions as these were available, as explained in the results for question 1.

Anticipated student thinking (DT2). Table 4.4 shows that *Investigations* and *SFAW-Mathematics* teachers planned to use 31.0% and 23.3%, respectively, of resources of anticipated student thinking. There is also difference in the quality of anticipated student thinking the teachers planned to use.

The teachers who used *SFAW-Mathematics* planned to use mainly “What Students Might Say” (see Figure 4.10), “Ongoing Assessment” (see Figure 4.13), and “Error Intervention” (see Figure 4.13). For example, Caroline explained during the final interview that she planned to use “Ongoing Assessments” and “Check” because these tell her misconceptions students might have. As such, this gives her an opportunity to plan ahead of time ways to help students understand what they ought to learn should misconceptions and errors arise in class. She planned to watch out for those misconceptions in order to address them in the lesson.

I know what to be watching for and I have just little tips. It’s kind of like in science, what the misconceptions are. If you know what the misconceptions are ahead of the way, then you’re watching for them and then you’re able to address them along the way. So I will familiarize myself with the Ongoing and the Check,
just before I teach the lesson, usually the night before, to have it fresh in my mind, so I’m watching for them… (Caroline, final interview)

None of the SFAW-Mathematics teachers planned to use anticipated conceptual student responses to test-taking practice problems available in the form of a grading rubric (see Figure 4.8 for an example). A grading rubric was available in one, two, and two lessons taught by Caroline, Dan, and John, respectively.

Investigations teachers planned to use the following: ways students might approach a task (What Students Might Say, see Figure 4.9), difficulties students might face, and suggested questions that reveal conceptual ways students might respond to a task in one lesson or the other (see Figure 4.12), as these are available in Investigations.

Jennifer, an Investigations teacher, explained during the final interview how she planned to use What Students Might Say:

….I use the students might say page to remind myself of the goals of the lessons and where I need my students to be with their thinking and understanding of the concepts. If I find a student isn’t following the concepts I know I have to …have some more interactions with that child to determine the disconnection that may have developed or ideas that were missed…..These pages also help me to know if students have a deep understanding quickly that I may need to give them a new challenge or we may be able to move on to another section more quickly… (Jennifer, final interview)

In this excerpt, Jennifer explained she planned to use anticipated student thinking (What Students Might Say) to know the depth of mathematical thinking and reasoning expected of students, assess whether students have a misunderstanding of key ideas, and decide whether to challenge them further. In effect, Jennifer planned to use this form of anticipated student thinking to understand the progression of key mathematical ideas students are to learn and to develop ways to help them move toward that point.
Teachers in this study planned to use curricular resources to identify misconceptions and design moves to help students should the misconceptions occur in class. They also planned to emphasize key mathematical ideas, promote appropriate learning for students, and assess whether students have a clear understanding of concepts. However, characteristics of ways they actually use curricular resources in association with other resources may differ. The frequency and kinds of curricular resources they actually use may also differ from those in Table 4.4.

**Curricular Resources Teachers Actually Used and Ways in Which They Used Them**

The kinds of curricular resources teachers actually used and ways they used them are provided in this section to answer question 3. The analysis of the available curricular resources teachers actually used is based on those that were explicitly observed in classroom teaching. For example, the guidance of asking, “Is it more or less than 1.2?” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 40) was considered explicitly observable during enactment only when the teacher asked such questions. I considered this guidance actually used when Lisa asked, “Is it bigger than 60?... Is it less than 92?” Also, I consider rationale used only when the teacher articulated it. For example, if a teacher asks students to perform an action and provides a rationale for it as suggested in the curriculum, then it is clear that it is used. In some cases, I could not conclude if a resource was used when I did not observe it during enactment. In addition, some curricular resources may not be observed during enactment and it is possible to conclude they were not used. For example, if a set of problems was not being assigned to students during enactment, I concluded the teacher did not use them. Therefore, I present the results of question 3 based on curricular resources that could possibly be observed during
enactment. First, I present trends in available curricular resources I observed these six teachers actually use compared to those they planned to use. Second, I describe attributes of typical ways teachers actually used curricular resources in conjunction with other resources.

**Patterns in Available Curricular Resources Teachers Actually Used**

Table 4.6 shows the percentage of available curricular resources the six teachers in this study planned to use and actually used. For teachers who used *Investigations* there is a slight increase and for teachers who used *SFAW-Mathematics*, there is a decrease in the percentage of curricular resources they actually used when compared to those in their plans. Table 4.6 shows that *Investigations* teachers actually used a greater percentage of representations of concepts and design transparency than indicated on their plans.

Table 4.6

*Percentage of Curricular Resources Teachers Planned to Use and Actually Used from Those Available in the Written Lessons*

<table>
<thead>
<tr>
<th>Curricular Resources</th>
<th>Investigations</th>
<th>SFAW-Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Curricular Resources planned to use</td>
<td>Curricular Resources actually used</td>
</tr>
<tr>
<td>RT1</td>
<td>40.4</td>
<td>38.7</td>
</tr>
<tr>
<td>RT2</td>
<td>59.7</td>
<td>51.9</td>
</tr>
<tr>
<td>RT3</td>
<td>48.5</td>
<td>-</td>
</tr>
<tr>
<td>RT (Total)a</td>
<td>43.3</td>
<td>40.6</td>
</tr>
<tr>
<td>RC1</td>
<td>60.7</td>
<td>36.0</td>
</tr>
<tr>
<td>RC2</td>
<td>41.3</td>
<td>67.5</td>
</tr>
<tr>
<td>RC3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RC4</td>
<td>28.0</td>
<td>64.0</td>
</tr>
<tr>
<td>RC5</td>
<td>32.6</td>
<td>63.0</td>
</tr>
<tr>
<td>RC (Total)</td>
<td>44.9</td>
<td>53.9</td>
</tr>
<tr>
<td>DT1</td>
<td>24.8</td>
<td>37.1</td>
</tr>
<tr>
<td>DT2</td>
<td>31.0</td>
<td>52.3</td>
</tr>
<tr>
<td>DT (Total)</td>
<td>28.7</td>
<td>46.6</td>
</tr>
<tr>
<td>Grand Total</td>
<td>39.8</td>
<td>45.3</td>
</tr>
</tbody>
</table>

*aPercentage of RT minus problems to solve*
This increase for *Investigations* may be accounted for by mathematical explanations and anticipated student thinking, while the decrease for *SFAW-Mathematics* might be explained by visuals and relationships. For example, *Investigations* teachers planned to use 41.3% of available mathematical explanations, but actually utilized 67.5% of them during enactment. Also, *Investigations* teachers planned to use 31.0% of anticipated student thinking, but actually incorporated 52.3% during enactment. On the other hand, *SFAW-Mathematics* teachers planned to use 81 of the 210 visuals, but actually utilized only 55 of them during enactment. Also, *SFAW-Mathematics* teachers included 59.3% of available relationships in their plans, but actually incorporated only 25.3% of them during enactment. Next, I compare the percentage of the different categories and subcategories actually used to those that teachers indicated in their plans. I also provide examples of kinds of curricular resources these teachers actually used during enactment.

**Representations of tasks (RT).** The percentage of representations of tasks that *Investigations* and *SFAW-Mathematics* teachers actually incorporated into their lessons shows a slight decrease from those planned to use. This is because they actually used a smaller percentage of this resource than they had planned to. For example, *Investigations* teachers planned to use 40.4% of available directions but actually incorporated only 38.7% of them. *Investigations* teachers actually used 51.9% of available participation structure and time, when they had planned to use 59.7%. Similarly, *SFAW-Mathematics* teachers had planned to use 18.9% of available directions, but actually engaged only 14.8% of them.

**Directions for teachers and students to follow (RT1).** The decrease in percentage of directions to follow for *Investigations* teachers can be attributed to Maria. Maria did
not use most of the directions the curriculum suggested to teachers. For example, when
the curriculum suggested that teachers discuss two particular problems in story contexts
in the whole group, certain questions were suggested that teachers ask students, but Maria
did not ask them. The curriculum suggested that teachers, after asking those questions,
listen for characteristics of the different problem types; Maria did not. She did not
because she designed her own route, which was “identifying key words” in the problems.

In spite of this, overall, *Investigations* teachers used suggested directions so that
students would experience the mathematics they were to learn, while their *SFAW-
Mathematics* counterparts simply focused on procedures during enactment. For example,
Maria, an *Investigations* teacher, used guidance that asked students to act out the actions
of problems so that they could understand the meaning of division. During enactment
Maria often asked students to close their eyes and imagine they were sharing things out
equally.

Ok, close those eyes, you and your four best friends standing outside the movie
theatre, Mom bought you a book of 35 movie tickets. You’re gonna share them
equally. So I’ve got 5 kids, 35 movie tickets and I’m gonna share them equally.
Ok? (second enacted lesson)

In Maria’s directions, students had an opportunity to understand the division context. On
the other hand, the guidance John, a *SFAW-Mathematics* teacher, used simply directed
students in a step-by-step manner to carry out an activity, as explained below.

Now, here's what I'd like you to do with your pencil. What I'd like you to do is,
toward the middle of your page...draw along one line like that. So you're drawing
along one side of one square. Now what I’d like you to do is draw down the other
two sides of the square so it looks like this. So the open part of the square that you
haven't traced is pointing toward you. Now what I’d like you to do is from this
corner where I'm pointing right now, go and do the next square. And then from
there, go straight down along the line and then once again we will have traced
three sides of the square. (John, third enacted lesson)
The directions John gave students in this excerpt are suggested under “Investigating the Concept” in one of the lessons he taught. He strictly controls the formation of a net of a cube, and hence a cube, in his directions.

**Participation structure and time (RT2).** Table 4.6 shows that *Investigations* teachers actually used a slightly lower percentage of this resource during enactment, when compared to those in their plans while *SFAW-Mathematics* teachers actually used the same percentage of this resource. Although *SFAW-Mathematics* teachers showed consistency in using this resource, they actually employed far less of it than their colleagues who used *Investigations*. For example, *SFAW-Mathematics* and *Investigations* teachers actually used 17.1% (14 out of 82) and 51.9% (40 out of 77) of participation and time suggestions, respectively. *SFAW-Mathematics* teachers used a smaller percentage because most of the optional activities that contained this resource (see Figure 4.1) were omitted, and the main part of the lesson does not include participation and time suggestions. On the other hand, *Investigations* teachers actually assigned most of the suggested activities that contained this resource and, consequently, followed suggested participation structure and time. But the decrease in percentage use can be attributed to Maria, who did not engage students in activities they ought to do, thereby not using this resource. For example, Maria planned to use the activity “Different Ways to Write Problems” but did not. Also, she planned to use the math workshop in which students practice multiplication and division, but she did not. As such, she did not use the suggested participation structure and time for these activities.

**Problems to solve (RT3).** Table 4.7 shows percentages of problems teachers actually used during enactment in relation to those in their plans and available in the
teacher’s guide. *Investigations* and *SFAW-Mathematics* teachers actually used 66.7% and 56.4%, respectively, of the problems they incorporated into their plans. Of the problems available in the curriculum, *Investigations* and *SFAW-Mathematics* teachers actually used 40.4% and 36.9%, respectively.

Table 4.7

*Percentage of Problems Teachers Actually Used From Those Planned to Use and Available*

<table>
<thead>
<tr>
<th></th>
<th><em>Investigations</em></th>
<th></th>
<th><em>SFAW-Mathematics</em></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from those planned to use</td>
<td>from those available</td>
<td>from those planned to use</td>
<td>from those available</td>
</tr>
<tr>
<td>Problems used</td>
<td>66.7</td>
<td>40.4</td>
<td>55.1</td>
<td>36.9</td>
</tr>
<tr>
<td>Purpose of problems</td>
<td>RF</td>
<td>65.0</td>
<td>50.0</td>
<td>71.7</td>
</tr>
<tr>
<td></td>
<td>EDC</td>
<td>78.6</td>
<td>40.7</td>
<td>66.7</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td>41.7</td>
<td>26.3</td>
<td>36.5</td>
</tr>
</tbody>
</table>

\(a\) RF (Reinforcement of the lesson Content or Practice), EDC (Exploration/Development of Concepts), OR (Ongoing Review)

\(b\) % is calculated for each type of problem from those available. For *Investigations*, percentage of EDC actually used from those available is \((22/54)*100 = 40.7\) and for RF we have \((13/26)*100 = 50\).

With respect to problems that the teachers planned to use, *Investigations* teachers actually used a greater proportion of those for exploration/development of mathematical concepts, whereas *SFAW-Mathematics* teachers’ actually used problems for reinforcement of mathematical concepts. From the problems available in the teacher’s guide, *Investigations* and *SFAW-Mathematics* teachers seemed to show interest in those for reinforcement and EDCs, respectively. However, EDC problems are minimally provided in *SFAW-Mathematics*. Table 4.7 shows that *Investigations* teachers devote more attention to problems for exploration and development of mathematical concepts.
(EDC) with less emphasis on those that review mathematical concepts learned previously. This may be because most of the problems in *Investigations* are for EDCs. Problems for review of previously learned mathematical concepts are minimally provided in *Investigations*. From problems *Investigations* teachers planned to use, they actually assigned 78.6% (22 out of 28) of those for EDC and 41.7% (5 out of 12) suggested for review.

Jennifer and Lisa are at the upper and lower ends, respectively, for using EDC and review problems. For example, Jennifer and Lisa actually used 59.1% (13 out of 22) and 13.6% (3 out of 22) of those problems for EDC, respectively, from those they planned to incorporate. Jennifer incorporated 80% (4 out of 5) and Lisa 0% (0 out of 5) of all review problems in the observed *Investigations* lessons.

Although from available problems in the curriculum, *Investigations* teachers actually show an emphasis for reinforcement (50%) over EDCs (40.7%), focus on review is still very small (26.3%). This result has to be interpreted with care because *Investigations* provide more problems for EDCs (54 out of 99) than those for reinforcement of mathematical concepts learned (26 out of 99). Jennifer and Maria are at the upper and lower ends in their use of problems for reinforcement. For example, Jennifer actually used 76.9% (10 out of 13) and Maria 0% (0 out of 13) of problems for reinforcement/practice. Maria had reinforcement problems in the lessons she taught, but did not plan to nor actually use them.

Table 4.7 shows that in terms of problems *SFAW-Mathematics* teachers planned to use, emphasis was on reinforcement of mathematical concepts with less focus on review of previously learned content. From the problems *SFAW-Mathematics* teachers
planned to use, 71.7% (124 out of 173) of those for reinforcement were actually incorporated during enactment. Caroline used a greater percentage of problems in the lessons she taught as compared to other SFAW-Mathematics teachers. For example, Caroline actually assigned 71.8% (89 out of 124) of problems for reinforcement of mathematical concepts, while John incorporated 5.6% (7 out of 124) of them. Also, SFAW-Mathematics teachers incorporated 36.5% of review problems during instruction. Of these, Dan asked students to solve 43.1% of review problems (28 out of 65), while Caroline assigned 23.7% (15 out of 65).

From Table 4.7, with respect to all the problems available in the lessons SFAW-Mathematics teachers taught, the focus seems to be placed on EDC with less emphasis on review of mathematical concepts, although care has to be taken in interpreting this result because EDC problems are minimally available in SFAW-Mathematics teacher’s guide. For example, SFAW-Mathematics teachers actually used 66.7% of available problems for EDC. Caroline, a SFAW-Mathematics teacher actually incorporated 61.5% (24 out of 39) of the available problems for EDC, while Dan, another SFAW-Mathematics teacher assigned none of the EDC problems. However, these EDC problems are not available in all lessons taught by SFAW-Mathematics teachers. For example, EDC problems were available only in one of Caroline’s and John’s lessons and in two of Dan’s lessons. Although each of the SFAW-Mathematics teachers actually started their lessons with review problems, only 32.8% of those available were incorporated into their instruction. A reason for this is because none of the SFAW-Mathematics teachers assigned “Mixed Review and Test Prep” questions at the end of practice problems in each lesson. Also,
two of the SFAW-Mathematics teachers used either Problem of the Day or Spiral Review and Test Prep, while one of them incorporated both, which fall under Spiral Review.

**Representations of concepts (RC).** Table 4.6 shows there is an overall increase in percentage of available representations of concepts Investigations teachers actually used and a decrease in percentage actually used by SFAW-Mathematics teachers. The percentage of visuals actually used decreased for both Investigations and SFAW-Mathematics from those indicated in teachers’ plans. Mathematical explanations showed an increase for both programs. Investigations teachers incorporated a higher percentage of relationships and assessments during enactment while SFAW-Mathematics teachers actually used a lower percentage of these resources than those they had planned to incorporate. I now discuss each subcategory of representations of concepts and provide explanations for either an increase or decrease in the percentage actually used.

**Visuals and representations of problems (RC1).** The decrease in percentage of this resource is greater for Investigations than SFAW-Mathematics teachers, as shown in Table 4.6. The decrease in the actual number of visuals used is greater for Investigations than for SFAW-Mathematics. For example, SFAW-Mathematics and Investigations teachers actually used 55 out of 210 and 32 out of 89 visuals, respectively. SFAW-Mathematics and Investigations teachers had planned to employ 70 out of 210 and 54 out of 89 visuals, respectively. This shows that the decrease in actual number of visuals is 15 for SFAW-Mathematics and 22 for Investigations.

This decrease in percentage for Investigations teachers can be attributed to Maria and Jennifer, who used 0 out of 5 and 22 out of 42, respectively, that they had included in
their plans. Maria explained during follow-up interview that she omitted the use of Figure 4.15 because it emphasized notations, which she had already incorporated. She said,

I actually did end up skipping this. I did talk about the notation, but I did end up skipping this. So, it was just more or less for, for reason of flow of the lesson at that point, but, like I said, it did end up being something that I did end up not coming back to. (Maria, follow-up interview)

In Maria’s explanation, using that again would have interrupted students’ thinking as she had engaged them in identifying key words. This omission is a clear indication that the teacher did not see the mathematical point communicated in Figure 4.15.

In spite of the decrease in use of visuals on the part of Jennifer, she actually incorporated the greatest number of visuals into her lessons among *Investigations* teachers. Jennifer used visuals for exploration and development of mathematical concepts as well as for reinforcement of main ideas learned.

Among the *SFAW-Mathematics* teachers, the decrease in percentage actually used can be attributed to Caroline and John. These teachers, respectively, incorporated 13 out of 18 and 15 out of 29 visuals they had indicated in their plans. John explained during the follow-up interview why he did not use some visuals that he originally planned to use.

**Interviewer:** At the beginning of this 8.1 lesson, you asked students to draw a net of a cube and cut it out and make the cube. What made you decide to do that at the start of the lesson?

**John:** Uh, basically so they could see plane figures becoming a solid figure, and see it laid out in front of them. “What is this?” Well, it’s obviously six squares. There’s no doubt about it, what it is. And when they see that take shape and make it themselves and, “Oh, now I have a cube here,” and they could see those faces, they’re all squares. And then, from that, on the next page, without doing the nets of the rectangular prism and rectangular pyramid, they can see, “Oh, if we lay this out and we would fold this up,” and that way they could see, they could see visually what this is going to become without actually having to do it, whereas if they’d done it with the cube, they can see, oh, they can extrapolate and see how it’s going to work.
Interviewer: So could you talk about why you skipped this part?

John: Um, basically because we had done something like it. We had, we had made the cube, which I knew was going to be much easier than doing the rectangular pyramid. That was one of those things that, outlining that and drawing that and, that had disaster written all over it. So if I did the cube, they could see the squares and see how the plane figures relate to the solid figures and then visualize, basically visualize what was going to go on there.

In John’s explanations, he was avoiding repetition of the same activity that could become boring to students. John felt the idea of transforming a 2D shape into a 3D shape had been sufficiently covered with cubes. So, students could extend this idea to a rectangular pyramid. This would save class time as well. Also, *SFAW-Mathematics* teachers incorporated visuals during the main part of their lessons to provide explanations and also when they assigned problems to students.

**Mathematical explanations (RC2).** The increase in percentage of mathematical explanations for *Investigations* teachers is greater than their counterparts who used *SFAW-Mathematics*, as shown on Table 4.6. However, *SFAW-Mathematics* teachers used a greater amount (or portion) of this resource than their *Investigations* counterparts. For example, *Investigations* and *SFAW-Mathematics* teachers appeared to actually use 67.5% and 55.3%, respectively, of available mathematical explanations, rather than the 41.3% and 42.4% they had planned to use, as shown in Table 4.6. The increase in percentage for *SFAW-Mathematics* teachers can be attributed to Caroline and John. These two teachers used 22.4% and 14.1%, respectively, of this resource, when they had indicated in their plan to employ 11.8% and 9.4%. Both Caroline and John, while reviewing problems at the start of every lesson, emphasized mathematical ideas students had learned. John said, …basically looking at process that they may have forgotten, they may have missed. They may, like for example, zeroes in the quotient. There’s one that I knew that, ‘cause it had come up in other lessons where there’s a zero in a
quotient, and it’s, and they’ve put, and their answer’s 15, not 105. And so, so that’s something, one of those areas that I wanted to be sure I focused on. The dividing of money, basically I just wanted to remind them, move the decimal point straight up and you’re going to be just fine… (John, follow-up interview)

By reminding students of what had been forgotten, John and Caroline provided a lot mathematical ideas when they engaged with spiral review problems at the beginning of each lesson. For Investigations teachers, the increase in percentage is because of Maria and Jennifer. They respectively used 18.8% instead of 11.3%, and 38.8% instead of 21.3%, of the mathematical explanations they planned to incorporate. Additional resources appeared in Jennifer’s lessons, as she provided mathematical strategies that neither the curriculum nor students suggested. For example, in relating two polygons E to form 90 degrees, Jennifer suggested the standard division algorithm to find the size of angle E. Jennifer explained during the follow-up interview that students had influenced her to propose that the standard division algorithm be used to divide 90 degrees by 2. She explained:

It depends on the student and what their particular needs are. I know I have several students in here that can do mental math really quickly and easily. I have students that are very visual and very tactile, and so I have to give them a lot more support in other ways, because they visually can’t, um, divide 90, and just that concept scares them to pieces … (Jennifer, follow-up interview)

Students’ need seems to influence Jennifer to add many other mathematical explanations not suggested by the curriculum. For example, Jennifer used the standard division algorithm to visually show students how to find the size of angle E rather than mentally do it.

Two kinds of mathematical explanations, definitions and mathematical strategies, were used by both SFAW-Mathematics and Investigations teachers. However, SFAW-Mathematics used conventional definitions, while their Investigations counterparts went
through examples to define concepts. For example, John used the definitions in Figure 4.2, reading them as written to students, while Maria incorporated examples to provide the meaning of multiplication and division problems as she helped students write their own story problems. Also, *SFAW-Mathematics* teachers used mathematical strategies without justifications, while their *Investigations* counterparts justified their strategies. For example, Caroline actually used the mathematical strategy in Figure 4.7, and emphasized the steps outlined to find the average or mean. On the other hand, Jennifer used the suggested mathematical strategies that students in her class employed in finding the size of an angle, and then provided a justification for why the strategy worked as she superimposed polygons to determine the relationship between angles. Although not provided by the curriculum, as mentioned above, Jennifer added a mathematical explanation that the sum of internal angles of a triangle is 180 degrees to justify her approach.

**Descriptions of representations (RC3).** Table 4.6 shows that *Investigations* teachers neither planned nor used this resource during enactment. However, this resource was provided in only one of Lisa’s three lessons and was not provided for the other *Investigations* teachers. Representations were described in two of Dan’s lessons and in none of the others in *SFAW-Mathematics*. Although Dan planned to use just 75% (3 out of 4) of the sentences that describe representations, he actually incorporated all of them during enactment. Dan used these descriptions when he explained the construction of line graphs and stem-and-leaf plots in his first and third lessons, respectively.

**Relationships (RC4).** The increase in the percentage of relationships, shown in Table 4.6, that *Investigations* teachers actually incorporated into their lessons during
enactment is because they used 64% of this resource instead of the 28% indicated in their plans. Jennifer is solely responsible for this increase, as she actually used 52% of the relationships when she included only 4% in her plan, because she established more connections among the lessons, activities, and angles than she had planned to. Jennifer explained during the follow-up interview why she changed her plan to enact a discussion that contained relationships to be established, when she initially did not intend to.

**Interviewer:** In the second lesson, although you read the beginning of this, you didn’t plan to do this discussion, but then you did start using that in class to discuss obtuse and acute angles. So what made you decide to do that?

**Jennifer:** Um, I think because, um—[pause] it’s a good shape to show both on it, and they, I think I decided not to use the trapezoid because [pause] they were opposite angles and the trapezoid, the two obtuse are on the top and the two acute are down at the bottom. So I think having them opposite, um, makes them maybe a little more visually identifiable. I mean, I know the kids can see the two on the top are bigger than the two on the bottom, but then, I’ve noticed a lot of times that they start thinking, “Okay, well, this one on the bottom’s as big as the one on the top.” I don’t know if that really makes sense, but in their mind they’re twisting it, or they’re not putting the square on there the right way and they’re not quite sure how to do it. And I think that gives better parallel lines, because you have the two that aren’t parallel on the trapezoid. So I think that might be why I decided to grab it.

In Jennifer’s explanations, she wanted to put her own design into the lesson, but after reflecting on her idea and identifying some limitations, she changed her plan and decided to follow what the curriculum suggested. Jennifer thought using a trapezoid would be better to explain obtuse and acute angles, but later realized the kind of confusion that this might cause students, because both obtuse angles are at the top and both acute angles at the bottom. She therefore thought having the angles opposite, as suggested in the curriculum, may be more visible for students. So, she decided to return to the curriculum suggestion to use a parallelogram. To use this quadrilateral, the curriculum actually
suggested that teachers say, “Yesterday we were thinking about how to measure angles that are smaller than a right angle or less than 90 degrees” (Wittenberg et al., 2008, Grade 4, Unit 4, p. 95). During enactment, Jennifer actually said, “Yesterday we started working with our polygons and talking about right angles, and then we also got into some conversation about angles that are smaller than 90 degrees and some that are larger than 90 degrees” (second lesson); hence, she made connections across lessons. The decrease for *SFAW-Mathematics* teachers is because 59.3% of this resource was included in their plan, but only 25.3% was actually incorporated during enactment of lessons. The decrease in the portion of RC4 used is evenly distributed among all *SFAW-Mathematics* teachers.

*SFAW-Mathematics* teachers struggled to establish relationships between mathematical lessons, while their *Investigations* counterparts used what the curriculum program offered. For example, whenever Dan went over review problems with students, he deliberately reminded them of previous content (list of topics, as explained in question A) and asked if those ideas could be used to solve the problem in question. This happened because relationships provided in *SFAW-Mathematics* were implicit, as explained in question 1, and teachers had to figure these out themselves. In contrast, Jennifer made connections between lessons, which is provided by *Investigations*, thereby making the mathematical storyline of the lessons visible to students.

*Assessments (RC5).* Table 4.6 shows that there is an increase and decrease in the percentage of this resource actually used by *Investigations* and *SFAW-Mathematics* teachers, respectively. The increase in *Investigations* is because teachers actually used 63% instead of the 32.6% in their plans. In particular, Lisa used 37% of this resource
when she did not plan to incorporate any, and Jennifer used all that she wanted to (26.1%). Lisa explained during the follow-up interview why she used Ongoing Assessments often.

**Interviewer:** Do you use the Ongoing Assessment?

**Lisa:** Yep, because I have a lot of kids, I have five LD (*learning disability*) students this year. So I want to see if they’ve got core idea, and I’ve got the kids that *do* understand it, so I want to push them further. See what their suggestions are. …

According to Lisa, she had five LD students and that probably influenced her to use ongoing assessments more. She used them often to detect struggles of her LD students and challenge those advanced students. To detect the struggles, she wanted to identify the level of understanding of the core mathematical ideas that her LD students had learned and the difficulties they faced. To challenge advanced students, she wanted to extend the mathematical understanding and reasoning of those already grounded into the concept being learned.

The decrease in assessments actually used by *SFAW-Mathematics* teachers can be explained by the fact that 36.5% were included in the plan to be used, but only 24.1% were incorporated into the lesson from those available. In particular, Dan actually used 13.1% out of what he planned to incorporate. Also, other *SFAW-Mathematics* teachers showed a slight increase or decrease in the percentage of assessments they actually used. For example, Caroline and John planned to use 5.1% and 6.6%, but actually used 5.8% and 5.1%, respectively, of this resource. Dan explained during the final interview the parts of the assessments he did not use.

**Interviewer:** What's important about “Talk About It” that you want to use it?

**Dan:** Well, I think, um, like I said the vocabulary, it kind of introduces the vocabulary. It always shows the diagram and then you can
explain to the students what the diagram means and then you can answer the questions as a class.

**Interviewer:** It seems in every lesson you plan to use Talk About It.
**Dan:** Yeah, try to.
**Interviewer:** And this Check activity, as well.
**Dan:** Sometimes the activity, sometimes not. Depends on the activity.

According to Dan, he incorporated “Talk About It” into every lesson taught to introduce the vocabulary. Because it is associated with the diagrams in the lessons Dan taught, he used it. However, he only sometimes used “Check,” an assessment that comes immediately before the practice in every lesson, although he always planned to. This might, in part, be responsible for the decrease in assessments he actually used in his lessons.

*Investigations* teachers did not assess students’ recall abilities because these were not available, as explained in question 1. *SFAW-Mathematics* teachers used only assessments that focused on students’ recall ability. For example, John used “Talk About It” (see Figure 4.2) and ”Check” questions to assess students’ ability to recall the mathematical facts learned before proceeding to practice. In contrast, *Investigations* teachers used only questions that assessed students’ conceptual understandings (see example in Figure 4.12 that Lisa actually used). However, these are the only kinds of assessment in *Investigations*. Although conceptual assessments are available in some of the lessons the *SFAW-Mathematics* teachers taught, none was used. Most of the teachers cited lack of time as a hindrance.

**Design transparency (DT).** Teachers in general used this category of resources more than they planned to. *Investigations* teachers planned to use 28.7% (80 out of 279) of the sentences that provide design transparency and actually incorporated 46.6% (130 out of 279). Also, *SFAW-Mathematics* teachers indicated in their plans that they would
use 17.6% (29 out of 165 sentences that provide design transparency), but actually engaged with 39.4% (65 out of 165).

**Rationale (DT1).** Table 4.6 shows a higher percentage in rationale for *SFAW-Mathematics* teachers (50.0%) than for their counterparts who used *Investigations* (37.1%). This is due to the fact that John and Dan actually used a greater percentage of this resource than they indicated in their plans. For example, John planned to use none, but ended up incorporating 25.8% of this resource available to *SFAW-Mathematics* teachers. Also, Dan planned to use 1.6%, but actually incorporated 14.5%. Lisa and Jennifer, who used *Investigations*, also incorporated a greater percentage of this resource into their lessons than they had indicated. Lisa used 11.4% but planned to use only 9.5%, and Jennifer incorporated 21.9% into her lesson when just 9.5% was in her plans.

These teachers probably did not realize how much of the rationale resource they were going to use. Also, the teachers already knew the goals students had to achieve for their lessons and so did not particularly highlight them on their CRLs. Yet, during enactment, they actually used them. For example, John did not highlight any lesson goal that he was going to use. During enactment, he actually did state the goals of the lessons and how they would be achieved. Also, in Jennifer’s second lesson, she did not plan to state the goals of previous and current lessons according to her CRLs, but she did during enactment. However, particular reasons for these differences are not known because the follow-up and final interviews did not specifically ask for these explanations.

Both *SFAW-Mathematics* and *Investigations* teachers stated lesson goals during enactment, but *Investigations* teachers made connections to other lessons, while their *SFAW-Mathematics* counterparts did not. For example, Dan, who used *SFAW-
Mathematics, in his first lesson said, “We’re going to start doing a lot with graphs, graphs, collecting data, doing surveys,” which are the written goals for that lesson. On the other hand, Jennifer, who used Investigations, incorporated lesson goals in a way that connected previous lessons, as indicated in the written lesson. She stated,

So we've been working on polygons and we've been using them to make a different...polygons...today we’re gonna look at the angles that are made when we put two or more of the polygons together to learn how to tell what kind of an angle it is… (Jennifer’s first lesson)

**Anticipated student thinking (DT2).** The increase in percentage of this resource for both programs, shown in Table 4.6, can be explained by the percentages used by Caroline and John for SFAW-Mathematics and Lisa and Jennifer for Investigations. For example, Caroline actually employed 23.3% instead of the 16.5% anticipated student thinking she planned to engage with. Caroline explained under anticipated student thinking that she used error interventions more to prepare ahead of time ways to help students overcome difficulties that the curriculum program identified they could face. These error interventions contain possible difficulties and challenges students might face.

Also, Jennifer actually incorporated 27.6% instead of the 6.9% of anticipated student thinking she planned to. Jennifer explained previously (under mathematical explanations) that because some of her students are visual learners, she used these anticipated strategies often, which involved the use of visuals to promote student understanding of key mathematical ideas that they communicate. Therefore, she probably realized during enactment that her students didn’t understand the key mathematical ideas and so decided to use more anticipated student thinking.

Two distinct ways of how to use student thinking were exhibited by these teachers. Caroline, who used the desired anticipated student thinking in Figure 4.10,
simply explained all the mathematical ideas embedded in them. Also, Error Interventions and Ongoing Assessments were used often by Caroline and Dan. They explained the errors students might face and cautioned them to be careful. In contrast, Jennifer probed students to bring out anticipated thinking. When these ideas surfaced, she picked them up and explained the details of the mathematical ideas embedded in them. Most of the questions to anticipate students thinking were used by all *Investigations* teachers.

**Ways in Which Teachers Used Curricular Resources in Conjunction With Each Other**

Table 4.8 shows the extent and quality of use of available curricular resources for all 18 lessons analyzed in this study. I classified the extent of use of available curricular resources as full use and moderate use. I also classified quality of use as highly, moderately, or minimally connected. I provide examples to illustrate each extent and quality of use. Then for each quality of use, I identified ways in which teachers used available curricular resources in conjunction with each other during enactment to achieve the goals of the lesson. In fact, I will use three different terms to refer to the goals. First, I will use *written goals* to mean the goals elaborated by the curriculum, including objectives, math focus points, key ideas, etc. Second, I will refer to *teacher goals* to indicate those identified by the teacher. Third, I will use *mathematical point* to describe the mathematics embedded in a task/activity/problem/lesson that is critical for students to learn. The written goals may or may not articulate the mathematical point of the lesson.
Table 4.8

Extent and Quality of Use of Curricular Resources

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Curriculum program</th>
<th>Grade</th>
<th>Unit/Chapter</th>
<th>Topics</th>
<th>Extent of use of curricular resources</th>
<th>Quality of use of curricular resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maria</td>
<td>Investigations</td>
<td>3</td>
<td>5</td>
<td>4.2: Multiply or Divide</td>
<td>Moderate Use</td>
<td>Minimally Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.3: Writing Story Problems</td>
<td>Moderate Use</td>
<td>Minimally Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.4: Solving Multiplication and Division Problems</td>
<td>Moderate Use</td>
<td>Minimally Connected</td>
</tr>
<tr>
<td>Jennifer</td>
<td>Investigations</td>
<td>4</td>
<td>4</td>
<td>3.1: Making Right Angles</td>
<td>Full Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.2: More or Less Than 90 Degrees?</td>
<td>Full Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.3: Assessment: Building Angles</td>
<td>Full Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td>Lisa</td>
<td>Investigations</td>
<td>5</td>
<td>6</td>
<td>1.3: Decimals on the Number Line</td>
<td>Moderate Use</td>
<td>Minimally Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.4: Decimals In Between</td>
<td>Moderate Use</td>
<td>Minimally Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5: Assessment: Decimal Problems</td>
<td>Moderate Use</td>
<td>Minimally Connected</td>
</tr>
<tr>
<td>Caroline</td>
<td>SFAW-Mathematics</td>
<td>4</td>
<td>7</td>
<td>7-12: Finding Averages</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7-13: Dividing by Multiples of 10</td>
<td>Moderate Use</td>
<td>Moderately Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7-14: Dividing with Two-Digit Divisors</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td>John</td>
<td>SFAW-Mathematics</td>
<td>4</td>
<td>7 &amp; 8</td>
<td>7-15: Equestrian Competitions *</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Review</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8-1: Relating Solids and Plane figures</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td>Dan</td>
<td>SFAW-Mathematics</td>
<td>5</td>
<td>5</td>
<td>5-1: Collecting Data from a Survey</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5-2: Bar Graphs</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5-3: Line Graphs</td>
<td>Moderate Use</td>
<td>Highly Connected</td>
</tr>
</tbody>
</table>

* A lesson on problem solving (word problems) involving multiplication and division
**Extent of use of curricular resources.** Table 4.8 demonstrates that I found two extents of use of available curricular resources, moderate and full, in this study. I explain each of them next. I classified each teacher’s overall extent of use of available curricular resources in the lessons they taught. Details of this classification are shown in Table 4.9.

Table 4.9

*Percentage of Each Broad Category Actually Used Per Teacher*

<table>
<thead>
<tr>
<th></th>
<th>RT</th>
<th>RC</th>
<th>DT</th>
<th>Actually used</th>
<th>Extent of use after removing optional parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maria</td>
<td>30.1</td>
<td>57.1</td>
<td>25</td>
<td>32.5</td>
<td>Moderate Use</td>
</tr>
<tr>
<td>Lisa</td>
<td>33.4</td>
<td>51.9</td>
<td>51.1</td>
<td>40.6</td>
<td>Moderate Use</td>
</tr>
<tr>
<td>Jennifer</td>
<td>54.5</td>
<td>77.8</td>
<td>69.5</td>
<td>63.5</td>
<td>Full Use</td>
</tr>
<tr>
<td>Caroline</td>
<td>22.6</td>
<td>52.6</td>
<td>53.6</td>
<td>35.5</td>
<td>Moderate Use</td>
</tr>
<tr>
<td></td>
<td><em>(76.7)</em> a</td>
<td><em>(92.7)</em></td>
<td><em>(81.1)</em></td>
<td><em>(84.8)</em></td>
<td>Full use</td>
</tr>
<tr>
<td>Dan</td>
<td>7.8</td>
<td>42.8</td>
<td>18.6</td>
<td>20.7</td>
<td>Moderate Use</td>
</tr>
<tr>
<td></td>
<td><em>(26.9)</em></td>
<td><em>(72.0)</em></td>
<td><em>(26.7)</em></td>
<td><em>(47.5)</em></td>
<td>Moderate Use</td>
</tr>
<tr>
<td>John</td>
<td>15.7</td>
<td>31.6</td>
<td>39.4</td>
<td>23.7</td>
<td>Moderate Use</td>
</tr>
<tr>
<td></td>
<td><em>(37.5)</em></td>
<td><em>(77.5)</em></td>
<td><em>(68.4)</em></td>
<td><em>(53.4)</em></td>
<td>Full Use</td>
</tr>
</tbody>
</table>

a % in parentheses indicates % of each broad category actually used after curricular resources in the optional parts have been removed.

**Moderate use.** I classified the extent of use for Maria, Lisa, Caroline, Dan, and John as moderate use, as available curricular resources to these teachers in the three broad categories were not substantially used. By substantial use, I mean a large percentage of at least two of the broad categories were used. Maria and Lisa, who taught with *Investigations*, actually used lower percentages because they omitted many suggested curricular resources as they enacted each lesson.
For Caroline, Dan, and John, lower percentages of available curricular resources were actually used because they did not use most of the optional parts of their lessons, shown in Figure 4.1. In particular, the optional parts of their lessons contained most of the directions and participation structure and times, which were actually not used. This explains, in part, why the percentage for representations of tasks is particularly low for these three teachers.

However, when available curricular resources in the optional parts of SFAW-Mathematics were taken out of the calculations, Caroline and John had an extent of use classified as full use, as shown in Table 4.9. Although Dan showed improvements in the use of these curricular resources, the overall actual use was still not substantial. Dan was particularly low in using representations of tasks (RT) and design transparency (DT) and high in representations of concepts (RC). This is because during enactment, Dan focused on the main ideas that students were to learn, making sure he explained them carefully. He did not often incorporate Ongoing Assessments and Error Interventions that contained anticipated student thinking (difficulties students might face). These results show that the optional parts had a great impact on the overall use of curricular resources for SFAW-Mathematics teachers. I removed the optional parts from the calculations only from SFAW-Mathematics because this is a regular component of that curriculum and the teachers all consistently did not use it much. Also, no clear directions are provided by the authors of SFAW-Mathematics about how to use the optional parts and establish relationship between these and the main part of the lesson.

**Full use.** I classified the extent of use of available curricular resources in all lessons taught by Jennifer as full use. This is because she actually used about 63.5% of
the curricular resources available to her in the lessons she taught. In particular, Jennifer actually used 54.5%, 77.8%, and 69.5% of representations of tasks, representations of concepts, and design transparency, respectively, available to her. These percentages are quite substantial for each of these broad categories of curricular resources, and this occurred because Jennifer used most of the suggested activities in the curriculum together with associated resources.

**Quality of use of curricular resources.** Two levels of quality of using resources, minimally connected and highly connected, are shown in Table 4.8.

**Minimally connected.** I classified Maria’s and Lisa’s quality of use of available curricular resources in all lessons they taught as minimally connected. This is because most of the curricular resources these teachers identified and used were not engaged in association with each other in ways that the written goals of their lessons could be achieved. This minimal connectivity was based on students not doing the mathematics, lack of appropriate mathematical content for students to learn, and lack of adequate mathematical storyline. These themes are highly interrelated and not mutually exclusive. I describe each of these ways and use examples from Maria’s and Lisa’s lessons for illustration. In addition, I provide a list of available curricular resources that each of these teachers used, resources they added, and those not used. Furthermore, I explain whether the use of available curricular or additional resources helped to move the lessons toward the written goals. I also explain whether or not using any curricular resource hindered the lesson from being steered toward the written goals.

**Students not doing the mathematics.** Teachers ought to provide opportunities for student to engage in doing the mathematics (NCTM, 2000). This could include asking
students to justify their reasoning and exposing them to problems that show different aspects of the concept they are to learn. When these descriptors and others (see Table 2.7) are significantly absent, then students are not doing the mathematics that they are supposed to do.

The excerpt below occurred after students had solved problems assigned to them from Student Activity Book pages 42-43 (see Figure 4.6), either individually or in pairs. In this part of the lesson, the teacher was to engage students in a discussion; the written goals for this are “using the inverse relationship between multiplication and division to solve problems and identifying characteristics of these problems” (Wittenberg et al., 2008, Grade 3, Unit 5, pp. 122-123). During this discussion, the curriculum suggests that problems 2 and 3 in Figure 4.6 be highlighted and the above written goals discussed. After this discussion, students were to begin writing their own story problems in context and then put them together to form the class multiplication/division book. Written goals for the next lesson included “writing and solving multiplication and division problems in context” (Wittenberg et al., 2008, Grade 3, Unit 5, p. 125).

In contrast to the suggestions in the curriculum, Maria’s own goals for this discussion were for students to identify key words to determine whether it is a multiplication or division problem, to solve the problem, and then to write their own problem. We see that Maria’s goals differed from written goals in two ways. First, students had to identify “key words,” which are not suggested in the teacher’s guide, and, second, the key ideas Maria focused on did not contain the use of inverse relationship between the operations to solve problems. Maria led a whole class discussion after students solved these problems, as shown in the excerpt below.
In this excerpt, Maria used the following available curricular resources: problems to solve (explorations and development of mathematical concepts), directions to follow (“how are we gonna solve this it?” encourages students to act out each problem using cubes or drawings), visuals (representations), representations (mental visualization of multiplication and division contexts), and student strategies and anticipated student thinking (e.g., skip counting). Maria did not use directions such as “display the chart that you divided into columns from multiplication and division charts (M39)” (Wittenberg et al., 2008, Grade 3, Unit 5, p. 123) and the chart M39 itself. The resource Maria added is directions to follow (e.g., what’s our key word?).

**Maria:** …We made 20 muffins for the bake sale. We put the muffins in bags to sell, we put 4 muffins in each bag, how many bags of muffins did we have to sell? *Now close your eyes again.* 20 muffins on the table, ok? I’m taking those 20 muffins and then putting 4 muffins in this bag, 4 muffins in this bag, 4 muffins in this bag, 4 muffins in this bag…Now *what’s our key word on this one?*

**Student:** In each bag

**Maria:** In each bag. Right? We could do four muffins in each bag if we wanted to but that putting them in groups, into bags, tell us that this one’s gonna be what, Sam? Multiplication or division?

**Student:** Division

**Maria:** Division. Ok? So we’ve got a division problem on this one we know. Now how are we gonna solve it?

**Student:** Draw 5 circles.

**Maria:** Ok, draw 4 circles and then I pass them out. Ok so we’re gonna go 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 and then what’s our answer, Alex?

**Student:** 5

**Maria:** 5 because that’s how many are in…?

**Student:** Each bag

**Maria:** In each basket, right? So we’ve got 5 bags of muffins to sell. Now what’s our number sentence on that one? *What’s our number sentence on that one,* Alison?

**Student:** 20 divided by…

**Maria:** Ok so this is divided so it is gonna be 20 divided.

**Student:** By 4

**Maria:** Remember it’s always the other number in our problem.
Student: Equals 5
Maria: Equals 5 and remember that equals 5 is like the punctuation at the end of a sentence. Right? So now we bought 5 packs of yogurt cups, each pack had 4 yogurt cups, how many yogurt cups did we buy?..Ok so close those eyes again, 5 packs of yogurt cups, you know how yogurts are they come all attached in little pack of 4? Ok so I’ve got 5 of those, I’ve got a pack of 4, a pack of 4, a pack of 4, a pack of 4. It wants to know how many yogurt cups did we buy. Ok, open your eyes, what’s our key word on this one?

Student: How many did we buy in all.
Maria: In all...So it’s kind of asking how many did we buy in all or all together. Right? Which tells us it’s what, Wyatt? Multiply or divide?

Student: Uh, multiply.
Maria: It’s a multiply. Ok? So how do I solve this one?

Student: Um, 5 times 4.
Maria: Ok so our number sentence is going to 5 times 4. Miss Fantasia, what are you doing?

Student: Counting on my fingers.
Maria: Counting what on your fingers, love? Are you counting by 4’s? What number would be easier to count by?

Student: 5.
Maria: By 5’s. How many times?

Student: 4.
Maria: 4 times so go ahead. Oh wait...count for me.

Student: 5, 10, 15...20.
Maria: 20? This is our answer because that would have been our 4 times. Right? Good job, Fantasia. Ok so then, did that...what’s our punctuation up here, Fantasia? You said it’s 5 times 4.

Maria read each problem to the students for exploration and development of concepts. Then she provided directions for the students to follow as she repeatedly asked questions such as, “What’s our key word?” This question is an additional direction that focused students to identify key words from the problems that could tell them whether they have to multiply or divide. When students provided a key word that Maria agreed with, she accepted it without asking them to justify their reasoning. Maria did this for every problem that she solved with students. Maria did not ask students to justify why they thought the suggested key word indicated a multiplication or division problem. She
also did not use questions to elicit desired attributes of each problem type. By not having students highlight such attributes, Maria’s key word question did not support the written goals of the lesson. The question about key words, rather, took students away from a written goal in this lesson. Therefore, as Maria did not ask students to justify their reasoning for why a problem is multiplication or division, the characteristics of these situations were not identified. As such, Maria did not use the questions and problems in a connected way toward a mathematical point of the lesson.

Also, after students identified the key word, Maria asked, “How are we gonna solve this one?” and then directed students to close their eyes and imagine the problem, acting it out. The direction for students to act out the problem and visualization was suggested by the curriculum (as explained in the results for question 1). After this visualization, Maria went ahead and solved problems using representations. Maria drew circles and passed out the 20 muffins so that 5 were in each bag (although 4 were to be in each bag), acting out the problem. The circles supported the direction to follow, as they provided a visual representation for Maria to make tally marks in each of them to represent putting the muffins into bags as shown in Figure 4.14. In this figure, the circles helped students to identify the answer to the problem as 5. The visualization, the picture, and the questions Maria asked supported each other to get at the answer to the problem.

*Figure 4.14. Maria's Use of Representations to Solve Problems (Image Captured in Maria’s Lesson)*
As such, the circles, questions, visualization were connected to solve the problem, a small part of the written goal of this lesson. It is worth noting that although 5 is the correct answer, Maria’s representation (see Figure 4.14) was wrong and did not match the problem. Maria ought to have 4 in each bag and then have 5 bags. Maria used the above-mentioned curricular resources in an attempt to develop meaning which unfortunately did not match the problem. The resources mentioned above were not connected toward most of the written goals of the lesson. For example, the rationale suggested in the curriculum for acting out each problem was so that students could identify the attributes of each problem type (see question 1). Maria did not ask questions that could focus students toward this point of the lesson. Although an answer to the problem was found, as explained previously, the curricular resources used above were not appropriately connected to engage students to bring out attributes of division or multiplication problems, a key idea of the lesson.

Therefore, Maria exposed students to problems that could lead them to learn different mathematics not identified in the lesson. For example, during enactment, she highlighted problems 2 and 3 that could have been used to discuss written goals of the lesson, but she did not focus on the key ideas. Also, she did not ask questions that could prompt students to explain their thoughts and justify their reasoning. In addition, Maria did not ask questions that could engage students in mathematical reasoning toward the written goals of the lesson. Hence, I describe Maria’s use of curricular resources as not engaging students to doing meaningful mathematics work toward the written goals of the lesson. As such, I classified Maria’s quality of using available curricular resources as minimally connected toward the written goals because available curricular resources were
not used in association with others to achieve them. These written goals are the same as the mathematical points of the lessons Maria taught.

*Lack of appropriate mathematical content for students to learn.* In every mathematics lesson, teachers ought to ensure that students are exposed to appropriate mathematical content. Maria highlighted two problems (see minimally connected example in Chapter III) and directed students with the question “What do you notice?” As explained earlier (in Chapter III), the discussion Maria organized led students to identify “fact family,” which was not a written goal for this lesson as it did not highlight the inverse relationship between multiplication and division. The question Maria asked and problems 2 and 3 were connected to “fact family” and to neither attributes of multiplication and division problems nor the use of inverse relationship between the two operations to solve problems.

In this lesson, the curriculum provides the following directions for teachers to follow that Maria could have used to lead students to the characteristics of these two problems, but she did not.

Listen for students to identify Problem 2 as a division problem because we are told the number of muffins and are asked to find how many equal bags can be made. Problem 3 is a multiplication problem because we are told how many packs of yogurts were bought and are asked to find how many cups were bought altogether. (Wittenberg et al., 2008, Grade 3, Unit 5, p. 123)

Without appropriately using these directions, it was difficult for Maria to lead students to the attributes of multiplication and division problems. Maria used visualization and key words to distinguish these two kinds of problems. But her focus on key words, such as “share equally or how many in each…,” for example, to refer to division problems, blocked the key understanding that in division situations we are told the total number and
then asked to find how many are in each bag or how many bags can be formed from being discussed. Therefore, in using problems and questions, Maria connected these resources to “fact family” rather than toward the attributes of multiplication and division situations. As such, key ideas written in this lesson were not explained to students by Maria as she used these problems and questions.

Another key idea that students were to learn is the use of inverse relationship between the two operations to solve problems. Figure 4.15 shows a visual (chart) the curriculum suggests teachers could use to illustrate how the inverse relationship between these two operations can be employed to solve problems. Maria neither used Figure 4.15 nor encouraged students to think about how this inverse relationship between multiplication and division could be used to solve problems. Although the curriculum did not provide clear explanations of how Figure 4.15 could be used to illustrate ways the inverse relationship between the two operations might be used to solve problems, Maria did not add that. She also did not use Figure 4.15 to add characteristics of each problem situation.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Number in Each Group</th>
<th>Product</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>4 muffins</td>
<td>20</td>
<td>20 ÷ 4 = ___</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>or ___ × 4 = 20</td>
</tr>
<tr>
<td>5 packs</td>
<td>4 yogurt cups</td>
<td>?</td>
<td>3 × 4 = ___</td>
</tr>
</tbody>
</table>

*Figure 4.15. Visual to Illustrate Inverse Relationship Between Multiplication and Division and Their Attributes (Wittenberg et al., 2008, Grade 3, Unit 5, p. 124)*

As such, Maria did not raise the level of mathematical content in the curriculum. Therefore, Maria did not input key ideas that were not clearly explained. Hence, the
directions, problems, and visualization were not connected by Maria toward the written goals of the lesson.

Therefore, in using problems and questions, Maria did not provide accurate meaning of multiplication and division situations. As such, she did not provide clear explanations of key mathematical concepts. Maria did not also input key ideas that were not raised by the curriculum, as explained above. Based on these conclusions, I describe Maria’s use of curricular resources in conjunction with other resources as characterized by a lack of key ideas or appropriate content for students to learn meaningful mathematics. As explained above, Maria used problems and questions that could lead her to key ideas, but she focused on fact family and key words rather than the mathematical points of the lesson (i.e., attributes of multiplication and division situations, and the use of the inverse relationship between these operations to solve problems). Consequently, Maria did not use available curricular resources in a connected way toward the mathematical points. Hence, I classified Maria’s quality of use as minimally connected toward the mathematical points of the lesson.

Lack of mathematical storyline. By a mathematical storyline, I mean following a deliberate progression and making connections among mathematical ideas toward the mathematical points over a course of lessons. When these are absent, there will be no storyline for students to follow and they are likely to see mathematics as set disjoint skills. The lesson from which the excerpt below is taken focuses on assessing students’ understanding of the following mathematical points (written goals): “representing decimal fractions as parts of an area, identifying decimal fractions, and percent equivalents, and ordering decimals and justifying their order through reasoning about
This lesson assesses students’ mathematical understanding of ordering decimals using decimal problems. Before assigning decimal problems to students, this lesson opens up with a game “Smaller to Larger” in which students place decimal numbers “in increasing order (least to greatest) from left to right in each row and in increasing order from top to bottom in each column” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 50). Lisa assigned students the problems on Student Activity Book page 25 to solve individually. During this work time, Lisa was to assess students’ understanding of the mathematical points listed above. The conversation in the excerpt below occurred toward the end of the third lesson Lisa taught. Lisa led a whole group discussion of problem 5 before asking students to solve problems 6 and 7.

In this excerpt, available curricular resources to Lisa are problems to solve, anticipated student thinking (e.g., find the percent), and rationale (solve the problems). Lisa added from outside the written lessons the following resources: directions to follow (what do you do to figure out this problem?), mathematical explanations (moving decimal point two places to the right), and visual (H-10). Many other curricular resources were available in this lesson, but Lisa did not use them:

- **Rationale** – (this assessment addresses Benchmarks 1 and 2 for this unit: Benchmark 1: read, write, and interpret decimal fractions to thousandths. Benchmark 2: order decimals to the thousandths)
- **Visuals** (hundredths and thousandths grid)
- **Anticipated student thinking** (e.g., interesting things students might do)
- **Directions to follow** (e.g., encourage students to use hundredths and thousandths grids to solve the problems on Student Activity Book page 25 (Wittenberg et al., 2008, Grade 5, Unit 6, pp. 53-54)
Lisa: Ok, two things today in your books, page 25 and 26. 25 you’ll notice there are 3 story problems. Mitch and Hanna, have gardens that are the same size. Mitch planted 0.250 of his garden with tomatoes. Hanna planted 3/8th’s of her garden with tomatoes. Who planted more and how do you know? Now, you’re comparing a decimal and a fraction but aren’t they the same thing?

Student: Yeah.

Lisa: Ok. What do you have to do in order to figure out this problem? Brandon?

Student: Find the percent.

Lisa: You could find the percent. What else could you do?

Student: Move the decimal.

Lisa: Alex?

Student: Find the decimal.

Lisa: Find the decimal for what?

Student: The 3/8ths.

Lisa: Ok, so we know Mitch planted 0.250. Hanna, we’ve got to figure out what 3/8th’s is equal to. How do you do that?

Student: Look at the…

Lisa: Is there a resource somewhere that tells you?

Student: Right there.

Lisa: Ok. Well I’m not getting up to get it, I don’t need the resource.

Student: I’ve got still the H-10 [this H-10 chart was constructed in a previous lesson. It contained fractions and their decimal equivalences]

Lisa: You’ve still the H-10. Hannah!

Student: What did you ask?

Lisa: I’ll wait. One per group.

Student: 37 ½.

Lisa: Hannah said that 3/8th’s – thank you—boys sit down now. One per group, no. One per group. You sit down, you’ve got one. I’ll wait. Who’s got it? Who knows the decimal for 3/8ths? James?

Student: 37 ½.

Lisa: Ok. So, who planted more?

Student: Umm…Hanna.

Lisa: Hanna with 37 ½ or Mitch with 25?

Student: 37 ½.

Lisa: Ok so we would put Hanna without an H but you have to explain why.

Lisa assigned the problems on Student Activity Book pages 23-25 (Wittenberg et al., 2008, Grade 5, Unit 6, pp. 53-54) and asked students to solve only numbers 5 to 7.

Then she realized that students had to compare fractions and decimals, a mathematical
idea she did not introduce. So, she directed students with the question, “What do you do to figure out this problem?” A student responded, “Find the percent,” but this student thinking, which was anticipated in the curriculum, was not pursued by Lisa. She further asked, “What else could you do?” and another student’s proposal to find decimals was taken up. Lisa further prompted students to use a visual (chart) constructed in a previous lesson to find the decimal equivalence for $\frac{3}{8}$.

The chart helped to provide a solution to the problem. The questions Lisa asked supported the use of the chart to find a solution as an alternative to the first student’s proposal to find percent. The problems provided an opportunity for the questions to be asked because the numbers in it were to be in a common form before being compared. The chart supported both the problem and the questions in that it provided a visual used to solve the problem. Therefore, the chart, the questions, and problem were used in a way that supported each other to get a meaningful solution. Students used the decimal for $\frac{3}{8}$ to compare with 0.250 and determine who of Hanna and Mitch planted more of their garden with tomatoes. As such, Lisa connected the solution back to the problem situation to establish meaning.

However, the questions, the chart, and the problems were not used in such a way that students could interpret decimal to thousandths using representations, a written goal to be assessed. In providing the solution to problem 5, Lisa did not use the above-mentioned resources in a connected way toward representing decimal fractions as part of an area, another mathematical point of the lesson. Although students concluded which portion is bigger by looking at the decimal numbers, students lacked an understanding of a visual representation of how much bigger is $\frac{3}{8}$ than 0.250. Therefore, I classified Lisa’s
quality of use of the above-mentioned resources as minimally connected to the mathematical points of the written lesson stated above. From the way Lisa used that chart, it is clear that she focused on answers without paying attention to the rationale for this problem.

From the rationale provided above, we see that students were expected to use all problems on *Student Activity Book* pages 23-25 to read, write, and interpret decimal fractions to thousandths and also order decimals to the thousandths. Students were also supposed to use the visuals (hundredths and thousandths grid) to convert the decimal in the problem to fractions and then compare them. However, the problems were partially assigned to students. In the first four problems, students were to use visuals (grid papers) to convert decimals to percentages and then fractions. These problems could have helped Lisa address benchmark 1 (read, write, and interpret decimal fractions to thousandths). Using this experience from these problems, the students could have easily solved problems 5 to 7. In other words, the students could have found it much more mathematically rewarding to use those visuals (grids) to convert 0.250 into a fraction and then determine which of 0.250 and $\frac{3}{8}$ is bigger. Unfortunately, this did not happen.

Therefore, in using the above-mentioned curricular resources, Lisa did not deliberately move this activity toward the written mathematical points of the lesson. Lisa did not make mathematical connections between problems 1 to 4 and the problems students were to solve, 5 through 7. Also, Lisa did not use previous learning (shading of grids to represent decimal numbers) to construct students’ understanding of comparing decimal numbers in thousandths. Therefore, I describe Lisa’s use of resources as lacking a storyline toward the mathematical points of the lesson.
**Highly connected.** I classified Caroline’s, Dan’s, Jennifer’s, and John’s quality of use of available curricular resources in all lessons as highly connected toward the mathematical points of the lesson. This is because many available curricular resources were used in conjunction with other resources toward lesson goals. This high connectivity was achieved by emphasizing key mathematical ideas, emphasizing meaning, and developing the storyline. I use examples from lessons these teachers taught to describe each of these ways. In addition, I provide a list of available curricular resources that each of these teachers used and other resources they added. Furthermore, I explain whether the use of available curricular or additional resources supported each other toward the lesson goals.

*Emphasizing key mathematical ideas.* To emphasize key mathematical ideas, I mean making sure the main content of the lesson is provided to students in a clear and understandable way. The excerpt below is from a lesson taught by John, who used *SFAW-Mathematics*. In this lesson, the topic is “relating solid and plane figures.” Written goals are “a plane figure has two dimensions: length and width; and a solid figure has three dimensions: length, width, and height” (Charles et al., 2008, Grade 4, Volume 3, p. 434). The written goals and the topic communicate different mathematical ideas. While the written goals simply define plane and solid figures, the topic provides a deep mathematical idea that could be explored. This is an unusual way of stating lesson goals in *SFAW-Mathematics* because often an active verb is used. In addition, the curriculum provides a key idea for the lesson as “there is a unique relationship between solid figures and flat shapes” (Charles et al., 2008, Grade 4, Volume 3, p. 434). This key idea the curriculum provides resonated with the topic of the lesson.
The main part of the lesson provides a number of three-dimensional shapes (e.g., rectangular prism, triangular prism, and rectangular pyramids) as examples of solid figures with flat surfaces and used the cube to define a face, an edge, and a vertex. Also, the curriculum provided examples of three-dimensional shapes (e.g., cone, cylinder) with curved surfaces. The excerpt below is at the beginning of this lesson where John leads students to construct a cube and then used it to provide definitions of vocabularies to be learned and establish a relationship between solid and plane figures.

John stated his goal for this lesson as “Today we're going to relate two different types of figures together. What we call plane figures and what we call solid figures.” The goal stated by John is very different from the written goals (objectives) in that those stated by John were more encompassing, while those in the written lesson were shallow. John’s goal for the lesson was in line with the key idea mentioned above and agreed with the topic of the lesson. John steered his lesson toward both the written goals stated above and his goal, as explained later.

Available curricular resources John used in the excerpt below were directions to follow, visuals (graph paper), mathematical explanations (definitions of vocabulary), rationale (written goals), and key idea (relationship between solid and plane figures). Resources added by John include rationale (goal he stated), and directions to follow (e.g., how to construct the net of a cube and then a cube).

**John:** Today we're going to relate two different types of figures together. What we call plane figures and what we call solid figures. What I'd like you to do is with your graph paper, starting up as you can see them; I am kind of near the top of my page. …Now, first of all, the lines that you have on your graph paper are all making what type of shape?

**Student:** Square.
John: A bunch of squares. Now, a square is an example of a plane figure. Meaning it's flat. It's one surface. It has basically what we call two-dimensions. It has length and it has width, now the square. Now, here's what I'd like you to do with your pencil...draw along one line like that. So you're drawing along one side of one square...draw down the other two sides of the square so it looks like this. So the open part of the square that you haven't traced is pointing toward you...do the next square. And then from there, go straight down along the line and then once again we will have traced three sides of the square.

John: What I'd like you to do now...is go down and this across two squares and do it on the other side as well. And then across the bottom like that. Now question: We have just made this shape...How many squares make up this shape?

Student: Six.

John: Six, we have six squares inside it, right? Very good. Now what I'd like you to do is cut out that shape?

Student: Mr. John, like cut out the body?

John: So just start from any side you want

Student: I cut off mine.

John: Now one of you is one step ahead of us. What I'd like you to do is along the lines that you did not draw...fold along those lines...you're going to fold them all in the same direction. So you're going to fold them all up, so this one we're going to fold up so it...it's now standing up... Now, you're going to have one side that's standing a little taller than all the others and you're going to keep folding that in the same direction. So that’s what you're left with looks something like this...

Student: That's masking tape, right?

John: You have six squares that made up the cube. So we have turned six plane figures, in other words flat figures, into a solid figure that has now three dimensions. We have length, width and height. For example I could relate this another way, the surface of your desk is a plane figure. The actual top, top of your desk because it has some thickness, right? That is a solid figure. The surface on top is a plane figure, the whole top of your desk, the wood itself is a solid figure. We have turned 6 plane figures into a cube, which is a solid figure.

John began this lesson by stating what he wants to accomplish as mentioned above. Then he provided directions on the construction of the net of a cube using a visual (graph paper). In directing students, John asked questions such as, “The lines that you
John used provided an opportunity for this question to be asked because it contained unit squares. Students said they form squares and John added, “Now, a square is an example of a plane figure. Meaning it's flat. It's one surface. It has basically what we call two-dimensions. It has length and it has width, now the square.” As John defined a square, he pointed at its flat surface and dimensions, length and width. This established a one-to-one correspondence between the definitions of a square, line segments that make up a square, and the dimensions. Therefore, John used available curricular resources here to provide accurate definition of a square and its dimensions, length and width. As John used curricular resources, such as a graph paper, rationale for lesson, directions to follow, and mathematical explanation (definition of a square and its dimensions) in association with each other to highlight a mathematical point of the lesson, he “steered his instruction” (Sleep, 2012, p. 938) toward the first part of the written goals.

The question John asked also directed students to squares that were being put together to form the net of a cube. This is because John wanted the students to see that this visual (net of a cube) is being made up of squares. In addition, John asked the question, “How many squares make up this shape (the net of a cube)?” The net of a cube John constructed supported this question as it provided a visual representation that students could see and understand that six squares have been put together to form a net. John continued to provide directions on the construction of a cube from its net. These directions helped transformed the visual (net of a cube) into another visual, a cube itself. John then said, “You have six squares that made up the cube. So we have turned six plane figures, in other words flat figures, into a solid figure that has now three dimensions. We
have length, width and height.” John did two things in this statement. First, he used the cube to establish a relationship between plane and solid figures, a goal he stated at the beginning of the lesson, which is in line with the key idea stated in the curriculum and consequently mathematical points of the lesson. Second, while holding the cube, he pointed at the dimensions—length, width, and height, establishing a one-to-one correspondence between them and the solid figures. Therefore, John used visuals (net of a cube and cube) and directions to follow (questions) to relate plane and solid figures as well as illustrating the three dimensions—length, width, and height—“to steer his instruction” (Sleep, 2012, p. 938) toward both his goal, written goals, and the key idea provided in the curriculum.

Given these two things that John did in the quotation provided above, we see that John deliberately made the content of the lesson richer, from simply providing definitions to establishing a relationship between the two kinds of figures (plane and solid). John used the curricular resources provided above to extend the definitions students ought to learn toward a much deeper mathematical point. I describe John’s move here as providing a clear explanation of key mathematical concepts students are to learn.

John later used the visual he created (cube) to provide other definitions that students were to learn, such as a face, an edge, and a vertex. As John held the cube, he said,

Squares. So it’s 6, the 6 faces of your cube are all squares. So a flat...so in flat surfaced figures, which is what we’re going to be talking about today for the most part, flat surface is a face. Your cubes have six faces those six faces are all squares. Yes?

John pointed at the six squares and called them faces of a cube. This mathematical explanation is supported by the question that asked for the number of squares in the net of
a cube. He emphasized that the faces must be flat because the shapes that formed them are flat, relating it to the number of squares that make up the net of a cube. John further defined an edge and a vertex:

Ok, so an edge, look at the next highlighted part, it says: An edge is a line segment where two faces meet. Everyone hold up your cube. Run your finger along an edge. Very good, very good, that is an edge. Notice two faces come together in other words, any place you folded that fold line is an edge. Any place where you folded them and those faces came together you created an edge. So those fold lines you started out with that guided you on your folding those became edges. The last one is a vertex, a vertex is where three or more edges meet, the plural is vertices. So, point on your cube to a vertex. Jamian, pick it up, point to a vertex.

John established accurate definitions of a face, an edge, and a vertex, pointing at each one of them on the constructed cube and explaining how they are formed. We see that the visual (constructed cube) helped the illustration and definition of the vocabulary students were to learn. Therefore, John provided accurate definitions of mathematical terms as he used the visual he constructed.

In providing accurate definitions of mathematical vocabulary students were to learn, however, John did not engage the students in mathematical thinking and reasoning. He funneled their thinking (Wood, 1998) by the nature of the questions he asked. He did not ask students to explain their thoughts and justify their reasoning. For example, when he asked students how many squares make up the net of a cube, he did not probe them further for justification. He rather engaged in a detailed explanation about how plane figures are transformed into solid figures. Therefore, John did most of the mathematical work for the students at this point. However, overall, John emphasized the key ideas students were to learn using available curricular resources, as explained above.
From the description above, John used resources in a mutually supportive way that moved the instruction toward both his goal and key ideas written in the curriculum. In particular, John used visuals in a way that supported his introduction of the vocabulary students were to learn. He used the visuals to provide accurate definitions of mathematical vocabulary, clear explanations of key mathematical concepts, and therefore emphasized key mathematical ideas of the lesson. John clearly moved his lesson toward the goal he stated and those written in the curriculum as he used these curricular resources together. Hence, I classified John’s quality of use of curricular resources in association with other resources toward the key idea written in the curriculum as highly connected.

*Emphasizing meaning.* By emphasize meaning I mean developing students’ mathematical understanding of concepts they are to learn in an orderly and systematic way that makes sense. For example, in the above excerpt, John made a connection between plane and solid figures. He established a way of constructing solid figures using plane figures. As explained above, John used the six squares that made up the net of a cube to construct a cube. He said each square on the net of a cube is now a flat surface on the cube. John showed students how a two-dimensional figure is transformed into a three-dimensional figure and illustrated to students how the length and width of a square now gets transformed into length, width, and height of a cube. This transformation process helped developed an understanding of how a cube is formed. The explicit connections John made between a square and a solid figure (a cube) developed an understanding of faces, edges, and vertices of solid figures. Therefore, I describe John’s use of lesson
goals, visuals, and mathematical definitions in a connected way to develop meaning of the key idea of the lesson, the unique relationship between solid figures and flat shapes.

Also, John used the visual (constructed cube) to connect back to its net. He made sure students identified the six squares in the net of a cube on the constructed cube. He introduced vocabulary by saying that those six squares are now called faces of the cube. Then he pointed at meeting points of two faces and called it an edge. He further counted the number of edges in a cube and added the formation of vertices from edges. Therefore, John used the visual (constructed cube) to develop and emphasize meaning of each vocabulary these students were to learn. I describe John’s use of curricular resources in a connected way towards mathematical point as emphasizing what each vocabulary means and how they are formed. Hence, I describe John’s use of curricular resources towards the key ideas of the lesson as emphasizing meaning of the construction of solid figures and vocabularies.

*Developing storyline.* To assess the development of storyline, I looked at how the teachers made use of previous learning (within the same lesson or a previous lesson) to construct new knowledge for students. This is so that students develop an understanding of how mathematical ideas are connected to solve problems and construct new knowledge.

The excerpt below is from a lesson Caroline, who used *SFAW-Mathematics* taught and has title “Dividing with Two-Digit Divisors.” The written goals for this lesson are “estimate quotients with two-digit divisors, and use models to find quotients” (Charles et al., 2008, Grade 4, Volume 3, p. 408). The key idea of this lesson is “you can divide with two-digit divisors like you did with one-digit divisors, but your estimate is
even more important” (Charles et al., 2008, Grade 4, Volume 3, p. 408). Therefore, in this lesson, the written mathematical point is finding a good estimate of the quotient when dividing with two-digit divisors. The curriculum presents compatible numbers and rounding as ways to estimate the quotient. After finding an estimate, students have to check using a long division algorithm if the quotient is an overestimation or an underestimation. Then the curriculum suggests what has to be done in each of these cases (see Figure 4.10).

Caroline stated her goals for the lesson as “do some estimating to help us find a possible quotient.” Caroline’s goal is similar to those written in the curriculum as she placed emphasis on estimating the quotient when dividing with a two-digit divisor. She actually “steered her lesson toward the mathematical point of the lesson” (Sleep, 2012, p. 938), as explained below.

Available curricular resources that Caroline used in this excerpt are problem for reinforcement of mathematical idea, visual (student sample work), anticipated student thinking (e.g., what students might say), rationale (e.g., lesson goals, why use compatible numbers), relationships, and mathematical explanations (e.g., strategy to find compatible numbers). The resource Caroline added is a visual (reference sheet). Caroline did not use directions to follow (e.g., have students identify the compatible numbers in the warm-up exercise) and problems to solve (warm-up exercises) that are provided in the curriculum.

Caroline: All right, my divisor is going to be 38 and my dividend is going to be 271…Ok, boys and girls, if we look on page 408. Ok? There's one way and another way. We are going to do some estimating to help us find a possible quotient and…then we're going to multiply, which is the inverse of division, that's why we've been practicing some of our multiplication problems to take a look to see how close we are.
Caroline: Now if you look at example A on page 408, one way it says to use compatible numbers and division. So he did some estimating, so I'm looking, 38 is close to what?

Student: 40.

Caroline: 40. And 271, if I do multiples of 4, boys and girls, what multiple of 4 is closest to 27? Brianna?

Student: 6.

Caroline: 6 was too low, 8 is too high. Boys and girls, grab the blue reference sheet [a sheet that contains numbers and their multiples] in the middle of your...move your finger across multiples of 4. You should have 4, 8, 16, 20, 24, 28, 32. What's closest to 27?

Student: 28.

Caroline: 28, so I'm gonna change my 38 to 40 and that's 271 to 280. Ok, I am finding compatible numbers that are multiples. Now, we can do mental math, how many times does 40 go into 280? If I were to think of in my head 280 divided by 40, I would do ...4 goes into 28, 7 times. So, what we just did is we found compatible numbers...so that we could get a potential quotient. This is our first step, 7. Now, if you look at page 408, then the next thing we would do is we would say about 7 would be the right answer. So now what I'm gonna do is take my actual number 38 and my actual 271 and I'm gonna do 7 times 38 off to the side.

Caroline: I have 266 as my answer. Now I'm going to look at my number up here. Is 271 more than 266?

Student: Yes.

Caroline: Yes. So I know 7 is the right quotient. Now I can put 266 here because I've already done 7 times 38. I did it right here and I can subtract. Ok? Ok? So the first step is doing your compatible numbers to get an idea of what your quotient is. We're just making a guess, but it's an educated guess because I used compatible numbers to get me in the ballpark. Then I take the quotient that I think it is and I multiply by it my actual divisor. I look at that product, what my answer is and I say, "Can I subtract from this?" If I chose 8 it would be too high wouldn't it? If chose 6 it might be way too low. So, let me finish, ok? Can I take 6 from 1, boys and girls?

Caroline assigned the problem 271 ÷ 38 and provided a rationale for the lesson, as stated previously. This rationale supported the problem by making it relevant, because “dividing by a multiple of 10,” a method used in the previous lesson, could not be used to estimate the quotient directly. Caroline then referred students to a process involved in finding
compatible numbers, illustrated in an earlier activity in this same lesson, to estimate the quotient in the problem. The compatible number strategy established a relationship between activities within a lesson and supported the rationale for estimating the quotient. This is because it provided a way for estimating a quotient.

Caroline asked an additional question: “What numbers are close to 38 and 271?” This additional direction supported the strategy of finding compatible numbers, as it created an opportunity to make known what they are. Students responded that a compatible number for 38 is 40 but had a difficult time determining one for 271. Caroline introduced a visual (reference sheet constructed by the class in a previous lesson) that contained numbers and their multiples. Since 40 had been given as a compatible number for 38, Caroline asked further what multiple of 4 is close to 27. As students struggled with this, Caroline asked them to use the reference sheet to determine this. Caroline used the reference sheet, containing numbers and their multiples, to help move the compatible number strategy forward. Caroline then said that 38 and 271 had compatible numbers of 40 and 280, respectively. She explained that finding compatible numbers so that one is a multiple of the other reduces the process of estimating a quotient to mental math, a rationale provided in the curriculum.

The reference sheet helped students find a multiple of 40 close to 271, 280, but hindered Caroline from connecting to the immediate previous lesson by making use of “dividing by multiples of 10” at that point to find the quotient. The curriculum suggested that teachers have students identify compatible numbers in the warm-up exercise, but this did not happen. Not using the warm-up exercises and the direction to find compatible numbers created a missed opportunity to construct new knowledge from previous
learning at that point. However, the procedure of dividing by multiples of 10 was used later in this same lesson to divide the compatible numbers.

Establishing that 280 is a multiple of 40 is important in that it completely reviews the compatible number strategy of estimating quotient. It also establishes that 40 and 280 are compatible numbers from 38 and 271, respectively. Figure 4.16 shows that Caroline used the method “dividing by multiples of 10” learned in the immediate previous lesson to find the estimated quotient for the problem. The method used in the previous lesson provided an easier way to estimate the quotient through the compatible number strategy. Caroline used this estimated quotient and the standard division algorithm to complete the problem.

*Figure 4.16. Caroline’s Demonstration of Division by Multiples of 10 (Image Captured in Caroline’s Lesson)*

We see that Caroline deliberately made connections to previous activities within the lesson and methods used in a previous lesson toward estimating the quotient when dividing with a two-digit divisor, the mathematical point. Therefore, the problem, the compatible number strategy, directions to follow, reference sheet, and the “dividing by multiple of 10” strategy learned in a previous lesson were used in a connected way toward the written mathematical point (i.e., finding a good estimate of the quotient when dividing by a two-digit divisor).

The standard division algorithm is used to determine whether the estimated quotient is appropriate. Caroline concluded that, for this problem, the estimated quotient
was okay since 271 is greater than 266 (the product of 7 and 38). The long division algorithm and the conclusion about the appropriateness of the quotient are all mathematical ideas Caroline got from the student strategy in Figure 4.10 provided in the lesson. This anticipated student thinking provided different mathematical ideas, such as determining whether the quotient is appropriate when compared to the dividend and the long division algorithm, that were used to complete the problem. Caroline deliberately used these resources in a way that moved the lesson toward the key idea of the lesson.

Caroline then summarized the steps involved in estimating the quotient when dividing by a two-digit divisor. The summary contained key ideas that Caroline went through in class, which are also embedded in the anticipated student thinking (see Figure 4.10). Knowing that students were not yet exposed to all cases of the key idea (i.e., underestimation and overestimation of quotients) they were to learn, Caroline used other problems to illustrate further situations of the procedures. These situations could result in either an overestimation or underestimation of the quotient and the kinds of adjustments that can be made. In a particular example where the quotient was an underestimate, Caroline said,

This was an excellent example...because 6 right away I knew was too small but I wanted to show what would happen if you chose an estimate too small. Ok? You're going to have a remainder that is greater than your divisor and we know whenever we have a remainder greater than our divisor that means our quotient up here is not big enough. Ok? If I'm doing 2 into 135, 135 is closer to 140, boys and girls, than it is to 125. It's 15 away from 140 where it's only err, 15 away from 135 where it's only 5 away from 140. So right away, in my compatible numbers there was a better choice but sometimes we miss the better choice. Ben? So if you miss the better choice that's why we always check our remainders. Our remainders always have to be less than our divisors. Ok? Remainders always need to be less than the divisors.
These mathematical explanations provided by Caroline supported students’ understanding of what to do, as she further highlighted critical areas to focus on, including what to do when the quotient is too small, and an understanding of key ideas needed in dividing with two-digit divisors. These mathematical ideas included comparing the remainder and the divisor. If the remainder is bigger than the divisor, then add 1 to the estimated quotient to get a second estimate and perform the long division algorithm again. This addition of 1 is removing one whole divisor from the remainder and adding it to the quotient, so that the remainder is less than the divisor. If the product of the estimated quotient and the divisor is greater than the dividend, then 1 has to be reduced from the estimated quotient to get another estimate of the quotient. This reduction of 1 is subtracting one whole divisor from the product of the estimated quotient and the divisor, so that the remainder is less than the dividend.

Caroline provided most of the explanations as she used the above-mentioned curricular resources in a connected way toward the mathematical point of the lesson. Although Caroline focused students on problems that exposed them to different parts of the mathematical procedure they were to learn, as explained above (underestimation and overestimation of quotients), she did not engage students in mathematical reasoning. Student responses were simply funneled into yes or no answers, as shown in the excerpt above. Even with these, students were not asked to justify their responses. Therefore, she did most of the mathematical work for the students.

In spite of these two shortcomings, the above explanations, overall, reveal that Caroline used available curricular resources together with the added resource in ways that established connections across activities within a lesson (the use of compatible number
strategy) and across lessons (the use of dividing by multiples of 10 and then the standard division algorithm). As such, she used previous student learning to help them construct new knowledge. Also, she made a summary of important mathematical steps to estimate a quotient when dividing by two-digit divisors. She exposed students to important models (underestimation and overestimation) to find estimate of quotients. Therefore, I concluded that Caroline developed a storyline with available curricular resources.

**Types of Adaptations Teachers Make and What Influenced Them**

This section provides answers to question 4: What types of adaptations do teachers make when using these curricular resources, and what makes teachers engage in such adaptations? The following types of adaptations were made by teachers in this study: omission, use and change, use and change of sequence, and use and addition. I describe examples of these adaptations as well as reasons for such adaptations from the teachers’ and researcher’s points of view.

**Omission**

Every teacher in this study omitted either some of the available curricular resources or certain components of the lesson. For example, Maria omitted some lesson goals, suggested representations or charts, an entire lesson (session 4.4, Wittenberg et al., 2008, Grade 3, Unit 5, pp.129-132), and the suggested material list (Wittenberg et al., 2008, Grade 3, Unit 5, p. 127). Lisa often omitted parts of suggested classroom activities or an entire classroom activity from the lessons she taught. Also, the three SFAW-Mathematics teachers typically omitted certain components of their lessons that contained several curricular resources. For example, activities in *Reaching All Learners* (see Figure 4.1) that were omitted by all SFAW-Mathematics teachers contained directions to follow,
participation structure and time, visuals, mathematical explanations, etc. Based on what teachers said during the interviews, they omitted these curricular resources or components from their lessons for the following reasons: repetition of content, insufficient materials, teacher knowledge, and mathematical value. It is also worth noting that multiple reasons can contribute to different types of adaptations teachers make. Therefore, these reasons may overlap each other.

**Repetition of content.** Some teachers mentioned that repetition of content in their curriculum material caused them to omit activities that re-emphasized mathematical ideas. However, whether there was actual repetition is questionable. In one case, omitting an activity because mathematical content was repeated was harmful while in another it was beneficial. When omission was harmful, some key ideas students were to learn were lost. For example, Jennifer skipped a whole-class discussion on smallest and biggest angle because she felt these ideas had already been discussed in a previous activity. Jennifer had had a discussion with her students that focused on finding the size of acute angles by relating them to 90 degrees. The goal of the discussion on “smallest and biggest angle” is “measuring acute angles by relating them to 90 degrees.” Although the goals for both discussions are the same, the former focused on familiar acute angles, such as 30 degrees, 45 degrees, and 60 degrees, while the latter emphasized smaller ones like 1 degree and formation of angles by degrees of turns as the sides, or rays, pivot from a vertex. The curriculum suggested students should have a sense of smaller acute angles, not only the familiar ones, and how they are formed. Jennifer therefore went on to other components of the lesson, as explained during follow-up interview:

**Interviewer:** In the second lesson, you skipped the whole group closing discussion about small and large angles, larger than 90 and smaller
than 90. You mentioned previously that you usually don’t have
time for a discussion after students have finished their work. What
makes you decide to do that?

Jennifer: Um, I think sometimes we’ve talked about it enough that I feel
comfortable that the majority of the kids really understand what’s
going on, and that a discussion may just take up more time…

This could be inferred to mean that if Jennifer engaged students in the discussion of
smallest and biggest angles, she might be repeating what had been done in a previous
activity. However, in skipping this discussion, Jennifer’s students did not have a sense of
smaller acute angles and how they are formed in general through turns about a pivot.
Although Jennifer had an elaborate discussion with her students on measuring acute
angles by relating them to 90 degrees, opportunities to have a sense of very small angles
and their formation were lost.

John realized that the curriculum broke some mathematical content into pieces
and covered it in two different places. He sufficiently covered it in the first instance and
omitted the subsequent occurrence, as he explained:

…you probably noticed, I skipped a few of them, like, .. basically [there] was one
that was kind of a rehash of the day before’s. ‘Cause there are spots in the
curriculum where it’s obvious they’ve broken into two lessons something that
should probably take two days, that it’s basically the same material. And so, some
of those I did skip. (John, follow-up interview)

Therefore, when these teachers saw that mathematical content is repeated, they omitted
subsequent activities. However, care must be taken to largely maintain the mathematical
points of the lesson. For example, in John’s lesson, he made sure that the intended
content was studied.

Insufficient materials. Teachers explained they omitted certain curricular
resources due to insufficient materials. For example, Dan planned to use “Investigating
the Concept,” which contained a number of curricular resources such as directions to
follow, visuals, and mathematical explanations, but did not because of an insufficient number of cubes available for the activity. As this activity was not used verbatim, the specific curricular resources mentioned above were omitted. Two number cubes labeled 1-6 were to be given to each pair of students. Each pair of students had to construct a table with six rows and toss their cubes to generate two-digit numbers with a tens digit of 1 written in the first row, a tens digit of 2 written in the second row of their tables, and so on. A stem-and-leaf plot was to be constructed from the data students generated.

Although Dan did not use suggested cubes and other specific above-mentioned curricular resources, a stem-and-leaf plot was still constructed. He asked his students to “think of a number 1 through 50. Ok? Just think of a number 1 through 50, all right. Everybody have a number?” He used these randomly generated numbers from his students to construct a stem-and-leaf plot and explained the key mathematical ideas behind it that were written in the curriculum. The written key idea is “stem and leaf plots are a concise way to organize many numbers by place value” (Charles et al., 2008, Grade 5, Volume 2, p. 270). Therefore, Dan maintained the mathematical content of student work, although the cubes and the other above-mentioned curricular resources were omitted.

**Teacher knowledge.** Some teachers in this study omitted certain curricular resources because of limited knowledge about the written mathematical points, which was revealed during the interviews. For example, Maria explained she omitted the chart illustrating the inverse relationship between multiplication and division (see Figure 4.15) because it contained a standard division notation. I infer that due to Maria’s lack of knowledge about the mathematics embedded in this visual, she omitted it. This visual
could have led Maria to identify characteristics of multiplication and division problems as well as provide an explanation of how the inverse relationship between both operations can be used to solve problems. In contrast, Maria’s intent to omit this visual was to focus students on learning and understanding materials relevant to their study, identifying key words to determine whether to multiply or divide, finding solutions, and writing their own story problems. In the excerpt below, Maria explained this notation would have interrupted the flow of activity and consequently student thinking to get to the goals of the lesson she articulated.

…because that kind of goes with the notation stuff. I just wanted them focusing on writing their own problems. So, um, you know, putting it into a chart and all that kind of stuff I felt kind of interrupted the flow of what they were doing. That is something we actually ended up skipping….So, it was just more or less for, for reason of flow of the lesson at that point… (Maria, follow-up interview)

According to Maria, she omitted this chart because she saw the information on the chart as related to the standard division notation only. Maria did not see how the chart (see Figure 4.15) could be used to illustrate the attributes of multiplication and division situations as well as use the inverse relationship between the operations to solve problems. Therefore, her lack of understanding could be responsible for this omission.

Lisa also omitted an activity from a lesson in which students were to shade grids and find decimals, fractions, and percent equivalents, as explained previously. She said:

…shading the grids I didn’t do again, because we did that prior to lesson 1.3. We did it as a whole group and then separately, and they did it well. So I left that out, so I didn’t have to do it again… (Lisa, follow-up interview)

According to Lisa, shading the grids had been done in a previous lesson and was not worth doing again since students did that very well. Her intent was to focus students on learning the “important mathematical ideas” she articulated, and to reduce repetition and
monotony so students would not be bored. Lisa did not see how the grids could have been used to identify decimal, fraction, and percent equivalents, written mathematical points of the lesson. Therefore, Lisa’s limited understanding of written mathematical points the representations can illuminate may be responsible for the omission.

Mathematical value. When some teachers did not see mathematical value in a curricular resource or certain component of a lesson, they omitted it. For example, Maria gave absence of opportunities to practice mathematical skills as a reason for omitting an entire lesson that contained valuable curricular resources. Her intent was to reinforce students’ ability to write their own story problems using the list of key words the class generated. She stated, “And when we learn a game, if it’s a valuable game and it helps them in those practices, I think that that’s important, but this one is just not one that I think is quite as valuable…” (Maria, follow-up interview). According to Maria, lesson (session) 4.4 was omitted because it was all about games. If games lead students to practice, then it’s important—otherwise not. I initially inferred that Maria did not understand the mathematical ideas embedded in the games in session 4.4 when planning her lessons. However, during the final interview, Maria proved the contrary and articulated the details of the games in session 4.4 she omitted as follows:

...the Missing Factor array cards have the multiplication sentence on the front and the product of that multiplication problem on the back. But then also on the back they have one of those, um, multiplication numbers on the back of them. And so what the kids have to do is they have to figure out, “Okay, if this product is 16, then if one of my factors is 4, what would my other factor be?” And so then they have to through somehow figure out, “Okay, well if I count by 4’s.” So here again, it’s reinforcing that fact that multiplication is putting together of equal groups, because if this is 4 along this side, then this one next to it can’t be 6, ‘cause it’s not any bigger than it, you know. It gives the kids a visual reminder and, um, support, I guess, that they can actually see it…I mean, it’s that it’s reinforcing that if I’m trying to find this 16, I have to count by 4 until I land on
that 16. So it’s tearing apart of groups and putting together groups. Does that make sense? (Maria, final interview)

I think she articulated the mathematics in the game during the final interview because an opportunity to think carefully about it again was provided. Maria explained the mathematical ideas of multiplication and division embedded in the game “Missing Factors” that she omitted, but she did not bring those ideas to life in class to reach the written mathematical point of the lesson. Therefore, the relationship between key ideas embedded in the games or written lesson goals were probably not clear to Maria at the time she was teaching the lesson and hence she omitted it.

Also, one of the three SFAW-Mathematics teachers explained that some curricular resources have not worked for her in the past when she attempted to use them. Therefore, she omitted them from her plan. For example, Problem of the Day in SFAW-Mathematics had not worked for Caroline, and so she does not use them often. She explained that

…. I found myself last year trying to do the Problem of the Day, and I found that it just didn’t work for me, so that’s one part of the curriculum that I really don’t use, is the Problem of the Day. Occasionally I find a really good problem and I’ll incorporate it into what we’re doing. But, um, that’s a part of the curriculum that I have not used as much, is the Problem of the Day… (follow-up interview)

I infer in this explanation that when Caroline evaluates problems to use for practice or review and does not see worthwhile mathematical depth, she omits them. My inference is based on her assertion that occasionally whenever she finds good problems, she incorporates them into her teaching. I interpret “good problems” to mean worthwhile problems from her perspective.

In summary, teachers can omit resources for various reasons, but such omission should be determined with careful attention to the written mathematical points that may be lost with such adaptation. While some teachers are sensitive enough to maintain an
eye on written mathematical points, making sure their students learn what is designed for them, others may not. For example, without cubes and other curricular resources in the activity Dan did not use, he made up for materials he omitted. His students still had an opportunity to construct a stem-and-leaf plot and see it as a concise way to organize data by place value.

In contrast, without maintaining an eye on the written mathematical points of lessons, these omissions can create missed opportunities for students to learn key concepts. Maria’s students were not encouraged to use the inverse relationship between multiplication and division to solve problems and had tremendous difficulties writing their own stories, as explained in the previous section. Lisa’s students also did not have an opportunity to read, write, and interpret decimal fractions to thousandths, important benchmarks that were to be accomplished, as explained in the previous section. In addition, Jennifer’s students did not get a sense of small angles together with ways they are formed in general.

Use and Change

Teachers used some curricular resources but made changes to them. According to these teachers, changes were influenced by students’ prior experience and their thinking. For example, Maria used lesson goals, but changed them to “identify key words and writing and solving multiplication/division problems in context.” The written goals are “using the inverse relationship between multiplication and division to solve problems, understanding division as the splitting of a quantity into equal groups; writing and solving multiplication/division problems in context” (Wittenberg et al., 2008, Grade 3, Unit 5, pp. 121-125). Similar to both goals is writing multiplication and division
problems in context. Maria changed “understanding division as the splitting of a quantity into equal groups” to “identify key words to determine whether a problem is a division or multiplication situation.” She also changed “using the inverse relationship between multiplication and division to solve problems” to just solving problems. As such, the use of inverse relationship between the two operations became irrelevant in Maria’s lessons.

During follow-up interview, Maria explained her focus on key words:

**Interviewer:** In the three lessons we observed, you emphasized that students should find key words to solve the stories for multiplication, division. What makes you decide to focus on that, and what influence does the curriculum have on that decision?

**Maria:** The curriculum doesn’t have a whole lot of influence on that decision. That comes from what I know these students need. We have talked for years about finding those key words and things like that in their math problems. Well, then, my teammate teaches all the literacy and she has the kids do what she calls “prove the answers.” So when they read something, and when they find the answer to a question, they have to actually underline it in the text and number it, like the answer to number 1 was here.

In Maria’s explanation, her emphasis on finding key words was due to students’ needs and past experiences. I infer that, according to Maria, asking students to find key words worked in the past and so she changed the math focus points to reflect that. In addition, Maria intended that students could also use key words to justify which operation is needed to solve a problem. With these changes, Maria’s lessons paid less attention to written mathematical points students were to learn. Consequently, it was difficult for Maria’s students to write their own story problems. Also, many of Maria’s students had difficulty solving multiplication or division problems on their own without explicit help from her.

Lisa used a suggested representation, a number line, but changed it to just the interval from 0 to 1 rather than from 0 to 2 as suggested in the curriculum. In addition,
the interval between 0 and 1 and between 1 and 2 in the suggested number line is divided into tenths. Lisa did not divide the interval for 0 to 1 into tenths as suggested. This partial use of the number line truncated the discussion of locating 1.25 that was to be generated. This was a huge step, as it completely changed the configuration of subsequent activities or lessons. Furthermore, Lisa used the decimal numbers provided in the curriculum but changed the set from 0.3, 0.5, 1.25, and 1.8 to 0.3, 0.5, 1.25, 0.05, and 0.8. Lisa explained that student difficulties and confusion influenced her:

Starting slow and working up. Trying to get them to feel good about what they do know. If I would have put 2 up there, I think it would have scared them. So, we’re building slow, slowly in this one, again, so that they’re feeling more successes… So I want to try and build their confidence levels first… Um, because they have a hard time with this book knowing the difference between what 0.05 is and 0.5, or 0.2, 0.02. So I’m always trying to throw those in there, that this is a nickel versus two quarters, and I always try to take it back to money, ‘cause they like money. Yep. So trying to get them to constantly see the difference between the two… (Lisa, follow-up interview)

The goal for using decimals such as 0.02 and 0.2 as the curriculum suggests is to know which is greater and how students figured out that. Although Lisa’s goal for using these numbers was to avoid confusion, it was somehow related to the written lesson goal, “ordering decimals and justifying their order through reasoning about decimal representations, equivalents, and relationships” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 44). However, Lisa did not explicitly discuss which is bigger and why with students during enactment.

Jennifer used available curricular resources but introduced a change in the suggested use. Jennifer cited student thinking as her motivation for changes she made. In a lesson, she spent time with students to solve one of the problems that they were going to work on individually. She explained the reason as follows:
Interviewer: On page 39, the equilateral triangle, the test, how many degrees and how do you know? Before the students began with their individual work, you went through that first problem together with them. What made you decide to do that?

Jennifer: Typically I do like to do at least one problem to kind of set them up so that they have an idea of what they really need to start thinking about and doing. I know I have students that will be able to do that really quickly, but sometimes, um, they don’t always do “how do you know?” They’ll just go, “Okay, well, it’s this, and I’m done.” “Well, okay, that’s great, but how do you know?” And so I try to model for them what my expectations are, so that’s usually why I do one, or other examples.

I infer from Jennifer’s explanation that she wanted to focus students to understand what the problems required and the kind of mathematical thinking that should be employed. As such, modeling what exactly is expected of them is a step in fostering desired student thinking to find the size of required angles. My inference is based on Jennifer’s assertion that students don’t usually pay attention to the “how do you know?” which prompts them to explain their thinking. She engaged students in solving the first problem together so that they could clearly and fully provide their thinking. This was so that students could solve subsequent problems individually, paying attention to explanations needed.

Use and Change of Sequence

Both Maria and Lisa used curricular resources embedded in their teacher’s guides but changed the sequence, using them earlier than suggested. For example, Lisa used homework problems as an in-class activity for students. In addition, Lisa used the “Decimal In Between Game” earlier (immediately after Ordering Tenths and Hundredths) than expected. According to Lisa, student and parent understanding of mathematical concepts of the lessons influenced her to change the sequence of the curricular resources suggested in curriculum material.
Lisa had experienced previously that parents had difficulties with student homework, and so students did not finish what they ought to do. Students returned to school the following day without the homework done, as Lisa explained: “I know I don’t send homework home, because parents have a hard time with the math homework. So until I get kids knowing how to do stuff, I know I’m not sending that home…” (Lisa, follow-up interview). I infer that the underlying idea in the statement above is that students and parents did not have a firm understanding of the mathematical ideas learned at school. As such, parents could not help and students’ homework was not completed. This experience influenced Lisa to change and have students do homework in class in order to complete their assignment. In Lisa’s explanation, she may again ask students to take work home when they show considerable understanding of concepts learned in class and can engage with them independently and fairly well. However, doing homework at school definitely means some suggested activities for students to do in class might be omitted. It might also mean suggested activities may not be properly used. This can suggest that mathematical richness might be lost in order to create time for the homework in class.

In addition, when students indicated they understood concepts learned, Lisa often challenged them almost immediately. These challenges often led to a change of sequence of activities, using them earlier than suggested, as Lisa explained:

They seemed to have a good grasp of the pink cards, or the decimal card A, so I wanted to challenge them right off the bat and say, “Okay, now, let’s see what you do with this group,” and get them ready for those cards, ‘cause they knew the games coming up had those cards anyway, so let’s see where they’re going to go. So I’ll pull in extensions if I feel they’re ready for it, and they seemed to be, so I did it. (follow-up interview)
Set A cards contain decimal numbers in tenths and hundredths. These are relatively easy to order on a number line. Set B cards contain decimal numbers in thousandths and these are difficult to order together with set A cards on a number line without the use of grids to present a visual image of each and establish relationships among them. Because of this difficulty, the curriculum suggests a discussion on which is greater before an extension of the activity to include set B cards.

Set A and set B cards were supposed to be put together in lesson 1.4 as an extension activity, but Lisa used the set B cards earlier in lesson 1.3. According to Lisa, this happened because students understood arranging and ordering the decimal cards of set A. So Lisa felt it was the right time to challenge them on the spot with set B in order to further stimulate their understanding. However, Lisa’s change of sequence did not take into consideration the mathematical points of intermediary activities. For example, the mathematical point of a discussion activity on “ordering tenths and hundredths,” is stated in the curriculum as “ordering decimals and justifying their order through reasoning about decimal representations, equivalents, and relationships” that came after set A cards were used, was omitted. Omitting this goal means loss of an opportunity for students to fully learn how to provide justification using decimal representations, equivalents, and relationships as decimals are ordered.

Use and Addition

Caroline, Dan, Jennifer, and John used available curricular resources, but also brought in additional resources from outside the curriculum. According to them, such decisions were influenced by student difficulty, desire to provide additional mathematical ideas, and practice of mathematical skills.
**Student difficulty.** The teachers made additions to available curricular resources that promoted and developed deep student understanding of mathematical concepts learned previously to avoid confusion. For example, when students were challenged with solving the problem $97.29 \div 100$, Dan used an additional representation (apple context) not provided in the curriculum to support students’ understanding of dividing by powers of 10. As students were challenged with this concept, Dan asked students to figure out what each person would get when 97 apples are divided among 100 people. Dan explained this addition during follow-up interview:

**Interviewer:** In lesson 5.1, when checking answers to the Spiral Review, number 4, which was $97.29$ divided by $100$, you mentioned to think about 97 apples divided to 100 people. Could you talk about this, and what was your intention in doing that?

**Dan:** I just wanted Zack to understand division and how you were taking bigger number...“Find the quotient using——” Yeah, I wanted him to see that it would be close to 1, if you divided, so, that, I didn’t really get to the concept of close to 1, but, um, that’s where I was going to go with that.

The meaning of division in the apple context actually helped students understand that with 97 apples shared among 100 people, each person gets a little less than one apple. So, the decimal point in that division problem should be placed appropriately to get a number less than 1.

**Desire to provide additional mathematical ideas.** Sometimes, the additions made as the teachers used available curricular resources were driven by their desire to provide students with additional mathematical ideas that probably would not be immediately useful. For example, Jennifer told students that the sum of the interior angles in any triangle is 180 degrees. Jennifer used this mathematical fact to prove that the measures of acute angles found in polygon E (and similarly for polygon L), which are
triangles, add up to 180 degrees. Jennifer explained why she did this during the follow-up interview.

**Interviewer:** So in lesson 3.1, we noticed that you mentioned a few things that weren’t indicated in the lesson. For example, that the interior angle sum of a triangle is 180 degrees, or on the second day, that you mentioned if one angle in a triangle is 90 degrees, then the other two have to sum to 90 degrees. So a relation of that still sum of 180 degrees. So what made you decide to include those things?

**Jennifer:** It’s just something I’ve done, and I do, and it’s just, I know it and I guess, um—and I, we’ve, I’ve talked to them about perpendicular, parallel, and, um, the straight line is 90, or 180 degrees, and that it’s all based on a circle. I’ve talked to them about that and the different degrees in a circle. And, so, I know it’s higher but I figure the more they hear it, the more it’ll sink in.

It could be inferred that Jennifer wanted to provide students with advanced mathematical ideas even if the curriculum was silent about it at that point. Jennifer is the only one who used an added mathematical fact more than once. She summed the size of the angles of a triangle that she found to confirm it was indeed 180 degrees. This additional mathematical information provided by Jennifer exposed students to another way the sum of interior angles in any triangle could be used. This fact that she introduced could also be used to find the size of an angle in a triangle when two others are known, although she did not employ it this way herself.

**Practice of mathematical skills.** Teachers made modifications because they wanted students to practice mathematical skills learned in lessons. For example, John added more problems in a lesson he taught because of the need to provide additional practice opportunities to students, as explained in the follow-up interview:

*Just some extra practice. Um, after we do some reteaching, the kids, a lot of students still need practice on just how the process works. So I did a lot of extra single-digit divisors with 3-digit dividends, just so they could do the divide and multiply, subtract, and just get that process down and get comfortable with numbers that they could work with more easily than having the 2-digit divisor and...*
the 3-digit dividend. So, so we did some extra work with that, with those. (John, follow-up interview)

This practice was aimed at automatizing the process of dividing a three-digit dividend by a single-digit divisor. According to John’s explanation, this was in preparation for more difficult procedures students were to learn using larger numbers. Because John knew about the difficulties students might face, he built a firm foundation to support what is to come by providing additional practice problems for more experience.

**Mathematical storyline.** Teachers made modifications as they used curricular resources but did not think about the entire mathematics students were to learn. As such, opportunities for students to learn intended mathematical concepts were greatly reduced. For example, when Maria changed the math focus points for her lesson to include key words, students did not discuss the inverse relationship between multiplication and division in-depth. Although it is absolutely necessary and important for teachers to use and add certain curricular resources in order to support students’ understanding of mathematical concepts when designing a lesson using a curriculum, they need to consider the mathematics across lessons—building on previous lessons and meeting goals in each lesson to develop a storyline of mathematics. The modifications Maria made impacted the math focus points negatively in that opportunities for students to fully learn all the key ideas embedded in her lessons were greatly obscured.

In contrast, other teachers (e.g., Jennifer, Dan, and John) made adaptations to support students’ understanding of concepts learned and prepared them for eventual difficulties they might face. These teachers identified connections among activities in a lesson and across lessons they taught, mathematical ideas embedded in an activity, the depth of mathematical ideas students were to learn, and curricular resources they could
use to support students’ understanding of mathematical concepts. For example, in the
final interview, Jennifer identified the key ideas across the three parts of session 3.1 as
well as the relationship among them.

**Interviewer:** What main ideas do you plan on highlighting through these three
parts of the lesson (session 3.1)? How and why?

**Jennifer:** The main ideas that I focus on are of course the core math concepts
as well as the skills of identifying evidence that supports their thinking and being able to justify their answers. I begin in this
lesson with the ways to make and identify a right angle. I accomplish this by using the edge of a sheet of paper so students
have a common object to use anywhere to help them identify a right angle. Students can use the paper first to find right angles
around the room and then use the polygons to match up corners with the paper after they have been able to locate right angles
around them. Then we move on to making a right angle with the
power polygons so the students can find ways to make a 90 degree
angle without the use of paper. Students will use the tan (O)
polygon as well as the green (N) triangle, the orange (L), green
(G), or (E). Students will need to be able to present their thinking
to others and use knowledge of the angles and degrees to support
their responses.

**Interviewer:** What is the relationship between these ideas?

**Jennifer:** Each of these lessons builds upon the previous lesson until students
are able to identify and determine what is needed to build a right
angle from power polygon. The student also is required to identify
the degrees of the acute angles that build the right angle so future
lessons will begin to build larger obtuse angles by using an acute
angle to be added to the right angle.

Jennifer’s explanations of main ideas she would highlight was in agreement with the
math focus points of the first lesson (session 3.1) she taught, identifying and making right
angles. In addition, she identified curricular resources and ways she would make use of
them in class to foster student understanding of the key ideas. For example, Jennifer
planned to use the edges of a sheet of paper and combine polygons to match the edges
and determine which combination fits to conclude a right angle has been formed. She
also identified the relationship between the key ideas embedded in a lesson as well as
how those in future lessons will build on. With this focus on what is yet to be built, Jennifer made sure the current foundation was properly laid.

The results suggest that teachers think carefully about the modifications they want to make and the written mathematical points of the entire lesson, making sure that the mathematics students are to learn are largely maintained. Teachers should also make sure they understand the relationships among available curricular resources as well as how these, as a group, relate to mathematical points of the lesson before they make adaptations.

**Some Insights into Teachers’ Capacity to Use Curricular Resources**

I draw from the results of questions 1, 2, 3, and 4 to develop insights into teachers’ capacity to use available curricular resources to enact lessons. These results helped me to answer question 5: What insights does teachers’ use of curricular resources reveal about their capacity to use the resources to enact lessons? I describe these insights and provide examples to illustrate each.

Investigating teachers’ use of available curricular resources in conjunction with each other to achieve lesson goals revealed the following insights about their capacity: identifying the mathematical points in lessons they taught, identifying the mathematical point of problems, identifying the mathematical point in representations, identifying relationships among curricular resources towards mathematical point of the lesson, identifying relationships among activities within and across lessons, and identifying gaps. I now describe each of these insights separately, although they are often very closely related.
Identifying the Mathematical Points in Lessons They Taught

Some teachers (2 out of the 6) in this study did not identify accurate mathematical points of lessons they taught, while others did. This determined what curricular resources teachers decided to use and how they used them. For example, in the lessons taught by Maria, her goals focused on identifying key words to determine whether to multiply or divide, find solutions, and write their own story problems. Written goals were “using the inverse relationship between multiplication and division to solve problems, using multiplication combinations to solve division problems, using and understanding division/multiplication notations, understanding division as the splitting of a quantity into equal groups, writing and solving multiplication/division problems in context” (Wittenberg et al., 2008, Grade 3, Unit 5, pp. 121-125). The goals articulated by Maria were similar to and different from those written in the curriculum, as explained in a previous section. Maria introduced the use of “key words” to identify whether a problem was multiplication or division, which was never a part of the written lesson goals. She suggested key words such as “altogether, how many in all, etc.” and “share equally, how many in each group, etc.” to determine multiplication and division problems, respectively. Her attention was focused on searching for key words inside problem statements to determine whether to multiply or divide before finding a solution. This focus greatly lowered the quality of the goals she articulated, because “altogether,” for example, could be used for addition and even division story problems. Maria also did not discuss how the inverse relationships between the two operations could be used to solve problems. The goal for understanding attributes of division and multiplication problems, which also offered a meaning of the operation, was not properly pursued by Maria. This
is because her emphasis on key words greatly reduced the focus on meaning. All of these suggest that Maria did not accurately capture the mathematical points of the lessons she taught. As such, Maria did not maximize affordances of curricular resources she used and this took her away from mathematical points of the lesson.

In contrast, Jennifer fully identified mathematical points of the lessons she taught and emphasized them during enactment. The written goal for an activity Jennifer used was “measuring acute angles by relating them to 90 degrees” (Wittenberg et al., 2008, Grade 4, Unit 4, p. 92). During enactment, Jennifer stated her goal for this activity as “We're gonna talk about how...much an angle is from knowing that our right angle is 90 degrees...So we're gonna talk about: How do we determine how many degrees each angle is by using our power polygons?...” Jennifer’s goal included materials (power polygons) that would be used to determine the size of angles, which are not explicit in the written goal. Jennifer went on to use these materials in this activity. This shows that Jennifer understood the mathematical points of the lesson and how it could be accomplished.

As such, Jennifer maximized affordances as she used curricular resources the curriculum material suggests toward mathematical points of the lesson. For example, Jennifer used directions to follow, mathematical explanations, visuals, and anticipated student thinking (e.g., what students might say, student strategies). She used directions the curriculum suggests such as putting together two polygons E and showing they make a right angle with the help of shape A (a square). This reminded students of a way to identify that a right angle has been formed. She used a mathematical strategy the curriculum suggests to show that the angles in polygon E are the same (i.e.,
superimposing the two polygons E on each other). She also utilized students’ strategies to show that three polygons O form 90 degrees as well as their congruency. In addition, she used the superimposition of polygons strategy to show that two polygons O fit in polygon L, and determined the size of its angles. This superimposition strategy communicated to students that 90 degrees have to be divided by the number of congruent angles combined to form a right angle. In other words, 90 degrees is the dividend, while the number of congruent angles that form a right angle is the divisor. Figure 4.17 shows three polygons O being put together to form 90 degrees.

*Figure 4.17. Putting Three Polygons O to Form 90 Degrees (Image Captured in Jennifer’s Lesson)*

Therefore, proper identification of mathematical points of the lesson helped Jennifer determine the kinds of curricular resources to use and how to use them. It also influenced her “to steer instruction” (Sleep, 2012, p. 938) toward them. Jennifer identified the mathematical points of the lesson and mobilized appropriate curricular resources toward them. In doing this, Jennifer’s students could determine the size of an angle by relating it to a right angle. This can be seen in a strategy a student used that Jennifer highlighted, saying, “So I’m seeing three L’s and I notice on someone’s page that they used 3 of these and they got 3 of those to make...” Jennifer later elaborated this strategy in class, maximizing affordances of available curricular resources, such as
polygons L, mathematical strategy (showing angles are congruent), and questions to move her lessons toward the mathematical points.

**Identifying the Mathematical Point of Problems**

While it may be relatively easy to identify mathematical point of problems assigned in *SFAW- Mathematics*, those in *Investigations* are very challenging. For example, problems provided in *SFAW- Mathematics* typically directly point at practicing the mathematical strategies/methods learned, and this may not task the teacher much. The situation in *Investigations* is a little different. For example, in a lesson Maria taught, the authors of *Investigations* provide three pairs of problems (see Figure 4.6). Each pair has exactly the same numbers, with one a division problem and the other a multiplication problem. Problems 2 and 3 are particularly highlighted by the curriculum material so that teachers can discuss attributes of each type.

Because Maria emphasized identification of key words in each problem to determine the type, she solved them independently of one another. During enactment, Maria highlighted problems 2 and 3 as the curriculum suggests, but emphasized that the numbers in these problems were members of the same fact family, as explained in a previous section, rather than emphasizing the relationship between these operations. The attributes of multiplication and division problems were also not extracted from the discussion. Utilizing the inverse relationship between multiplication and division to solve problems was lost completely. This suggests Maria did not accurately identify the mathematical points of the problems used in this lesson.

This misidentification of mathematical points of problems provided in the curriculum greatly affected Maria’s lessons. For example, as Maria emphasized the
identification of key words, she used the six problems to determine which operation to use and what number sentences to use, without much discussion on using the inverse relationship between multiplication and division to solve problems. As a consequence, after three days of teaching, many of Maria’s students could not write their own story problems. In addition, not knowing the mathematical points of problems may be influenced by her identification of goals as well. In order words, the goals she identified determined what she would emphasize when using problems and vice versa.

**Identifying the Mathematical Point in Representations**

Representations are sometimes difficult to read and understand. The mathematical points they contain may not be easy to identify and articulate. Identifying mathematical points in representations may influence teachers’ use of other curricular resources. For example, Lisa omitted the use of visuals (grids) in her lesson because she did not understand the mathematical points that could be explored with the grids. The curriculum suggests that students solve problems 1 to 7 in the *Student Activity Book*, as explained in minimally connected use of curricular resources. The goals for using these problems, as written in the curriculum, are to “read, write, interpret decimal fractions to thousandths, and order decimals to the thousandths” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 53). Although Lisa did not clearly state her goal for these problems, the focus was on solving for an answer, as explained in minimally connected use of curricular resources.

Problems 1 through 4 have grids for students to shade decimal numbers and convert them into fractions and percentages. Problems 5 through 7 provide an opportunity for students to use the knowledge in the first four problems and order decimals. Then the students determine which is bigger and use the context of the problem
to establish the meaning of the bigger decimal number. Lisa did not identify the mathematical points that could be highlighted in the grids and she omitted them. As a result of this omission, Lisa did not use other curricular resources such as rationale (goals for solving problems 1 through 7) and anticipated student thinking (what students might do). In addition, she also did not meet the mathematical points of solving these problems (minimally connected use of curricular resources for details). Lisa did not recognize affordances of these visuals (grids). I make this inference because Lisa explained her reason for not using the grids as avoiding repetition, since students shaded them previously. Also, Lisa did not identify that the chart she used to convert the fraction to decimal did not provide a visual representation of area covered by the numbers 0.250 and $\frac{3}{8}$. It also did not provide an opportunity for students to justify their reasoning using equivalents and relationships, a mathematical point of the lesson.

In contrast, Dan identified the mathematical point in a representation and used it together with other available resources within the lesson in the teacher’s guide toward mathematical points. A written objective for this lesson Dan taught is “read and interpret given line graphs” (Charles et al., 2008, Grade 5, Volume 2, p. 266). As Dan used the representation, he minimized its limitations. For example, the line graph in Figure 4.18 that Dan used had horizontal and vertical axes, but they were not labeled as x- and y-axes.

![Line Graph](image)

*Figure 4.18. Line Graph Available to Dan in the Second Lesson He Taught* (Charles et al., 2008, Grade 5, Volume 2, p. 266)
The line graph shows only an increasing and not decreasing trend. In order to illustrate the vocabulary (axes, increasing and decreasing trends) that students were to learn, Dan added lines to the visual, as shown in Figure 4.19.

Figure 4.19. Line Graph to Illustrate Axes and Trend (Image Captured in Dan’s Lesson)

Then Dan described how to determine both increasing and decreasing trends with respect to the x- and y-axes. He said compared to the x-axis, if y is increasing, then it is an increasing trend. With reference to the lines Dan added (see Figure 4.19), he said if y is decreasing with respect to the x-axis, then it is a decreasing trend. This description with respect to the x- and y-axes was not provided by the curriculum. The curriculum explains the determination of increasing trend as “if the part of a line between two points is rising from left to right, the data numbers are increasing” (Charles et al., 2008, Grade 5, Volume 2, p. 266). This approach did not make reference to the line graph and hence was disconnected from the suggested representation. Dan’s description was more powerful than what the curriculum provided in that it put together available curricular resources to explain the concept of trend. Therefore, because Dan clearly understood the mathematical point embedded in a representation, he mobilized the labeling of both axes, mathematical definitions (of x- and y-axes), and the additional line to illustrate decreasing trend in a coherent way toward the mathematical points.
Identifying and focusing on meaning in representations may influence the kinds of curricular resources teachers mobilize. Some of the teachers in this study identified and developed meaning when using representations, while others did not. For example, Lisa used a representation (i.e., a chart) constructed in a previous lesson to convert $\frac{3}{8}$ into the decimal 0.375 without emphasizing the meaning of these numbers. Lisa did not have students explain how they determined the equivalent forms of $\frac{3}{8}$ as a decimal number. By failing to do this, Lisa did not identify what the representation she used could not achieve (e.g., representing 0.250 and $\frac{3}{8}$ as part of an area) in relation to the written lesson goals, using these visual relationships to order the numbers and justify which is bigger. Figure 4.20 shows other resources such as hundredths and thousandths grids that could have been mobilized and used to solve problems 5 to 7, but were not.

*Figure 4.20. Grids Available to Lisa to Solve Problems 5 to 7 (Wittenberg et al., 2008, Grade 5, Unit 6, pp. 52-53)*
These visuals (e.g., hundredths and thousandths grids) could have helped Lisa “steer her instruction” (Sleep, 2012, p. 938) to “determine tenths, hundredths, and thousandths equivalents in both fraction and decimal forms” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 54), which is the mathematical points of the lesson.

**Identifying Relationships Among Curricular Resources Toward the Mathematical Point of the Lesson**

Identifying and using relationships among curricular resources around lesson goals influences the way in which teachers use resources. Some teachers in this study identified relationships among curricular resources and used them in powerful ways. Teachers who did not identify relationships among curricular resources used them partially without targeting the mathematical points of the lesson. For example, the authors of *Investigations* suggest that by acting out each of the problems in the *Student Activity Book*, students might understand that, in a division situation, an amount is shared by equal groups. The mathematical point of this action (directions to follow) is for attributes of division problems to be identified. Maria indeed asked students to act out these problems, but did not focus on their characteristics, as suggested. Rather, she emphasized the identification of “key words” to determine whether to multiply or divide. Maria’s focus on identifying key words hindered full enactment of the relationship curricular resources in a connected way toward the mathematical points of the lesson.

In contrast, the example from Jennifer’s lesson provided above (under Identifying the Mathematical Points in Lessons They Taught) clearly illustrates that she identified the relationship among curricular resources, such as visuals, questions, and the mathematical strategy of proving congruency (see Figure 4.17) in achieving the mathematical points of the lesson.
Identifying Relationships Among Activities Within and Across Lessons

Making connections among activities within and across lessons also influences teachers’ use of available curricular resources. Some of the teachers in this study made connections and maintained or reinforced written goals, whereas others lost part or all of it. For example, Lisa changed the sequence of “Decimal In Between Game,” using it earlier than suggested. This change resulted in a loss of the written goal “ordering decimals and justifying their order through reasoning about decimal representations, equivalents, and relationships” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 44) for the discussion of “which is greatest.” In addition, available curricular resources in this activity, such as directions to follow, visuals (grids), anticipated student thinking (what students might say), and mathematical explanations, were not used.

Lisa made this change to connect the activity “Ordering Tenths and Hundredths” using visuals (decimal card set A) to the “Decimal In Between Game,” where card set B were engaged, as explained in types and reasons for adaptations (i.e., use and change of sequence). According to Lisa, her assessment of students’ understanding influenced her decision to change the sequence of these activities. This change of sequence caused Lisa not to use the available curricular resources mentioned above. In particular, Lisa’s students did not have an opportunity to use visuals (grids) to reason with and provide justification for their ordering of decimals, a mathematical point of this lesson. Rather, Lisa used the idea of moving decimal points two places to the right and reading decimal numbers in terms of cents (money context). This alternative route deprived students of the main learning experience. Moving the decimal point two places to the right does not illuminate the meaning and size of decimals.
Alternatively, Caroline made connections to activities within and across lessons and maintained the mathematical point of the lessons. Caroline used the mathematical ideas learned previously to move her lesson forward. For example, as Caroline taught dividing with two-digit divisors, she used the mathematical strategy of finding compatible numbers to estimate the quotient (see Figure 4.10). The authors of the curriculum did not make reference to the compatible number strategy, but Caroline explicitly mentioned it as being used to estimate the quotient. Also, after finding compatible numbers, Caroline explicitly explained that “dividing by multiples of 10” learned in a previous lesson is used to figure out this quotient. Caroline added that this is the reason she demanded compatible numbers to be multiples. The visual (Figure 4.10) was limited in that connections to the sources ideas were not explained. However, Caroline used the visual (anticipated student thinking, see Figure 4.10) and explicitly stated sources of mathematical ideas embedded in it and hence “steered instruction toward the mathematical point of the lesson” (Sleep, 2012, p. 938).

**Identifying Gaps**

Identifying gaps in curriculum materials may enhance teachers’ mobilization of curricular resources. These gaps could be between curricular resources such as written lesson goals and key ideas. For example, John identified this gap and bridged it. The lesson John taught had the goals “a plane figure has two dimensions: length and width; and a solid figure has three dimensions: length, width, and height” (Charles et al., 2008, Grade 4, Volume 3, p. 434). The key idea that the written lesson indicates is “There is a unique connection between solid figures and flat shapes,” (Charles et al., 2008, Grade 4, Volume 3, p. 434). Written goals and key idea are very different. John saw this difference
and led students to construct a net of a cube. Then he transformed the net into a cube and explained that a plane figure with six squares is now a cube. Each plane figure (square on the net) is now a face of a cube. He further explained how the two dimensions of a plane figure have now been transformed into three dimensions of a solid figure. In doing this, John bridged the gap between the written lesson goals and key ideas to accomplish much more. Therefore, John identified a limitation of the written lesson goals to meet the key ideas of the lesson. This influenced the kinds of curricular resources John mobilized and used in order to minimize this limitation. As explained in the section, highly connected use of curricular resources, John used the curricular resources in ways that support each other toward the mathematical points of the lesson.
CHAPTER V
DISCUSSION AND CONCLUSIONS

Discussion

In this study, I used the notion of curricular resources (e.g., Brown, 2009; Davis & Krajcik, 2005; Schneider & Krajcik, 2002; Stein & Kim, 2009) to analyze curriculum materials. In the following, I discuss qualities of curricular resources available in each curriculum program and teachers’ enactment of them. I use research literature to briefly discuss implementation of recommendations for curriculum design. Also, I discuss some of the challenges faced during analysis of curricular resources found at the lesson level in teacher’s guide. I discuss ways teachers used these curricular resources in written curriculum materials in conjunction with each other toward lesson goals. Finally, I discuss insights into teachers’ capacities identified and suggest ways to improve them.

Curricular Resources Available in Written Lessons and Those Used

I discuss availability and use of curricular resources regarding the following: provision of rationale, balancing different types of problems, balancing different types of assessments, potential of anticipated student thinking, and relationships among curricular resources. The quality of each of these resources available to teachers and those they actually used is also discussed. I end the discussion by explaining the challenging nature of analyzing curricular resources.

Provision of rationale. Many researchers (e.g., Davis & Krajcik, 2005; Remillard, 2000) have recommended the provision of rationale in curriculum materials as a step toward motivating teachers to use curricular resources embedded in the curriculum.
In this study, I found rationale provided for each lesson (lesson goals), suggested actions for teachers and students, and recommended activities. I now discuss each of these.

Teachers handle lesson goals in different ways during enactment. Some focus on key ideas students are to learn, while others rarely do. For example, in a lesson Maria taught from *Investigations*, one of the math focus points identified a key idea students should take away as understanding the characteristics of division, which is splitting of a quantity into equal groups. However, Maria did not appropriately identify this key idea of the lesson she taught. Her focus on students “identifying key words” distracted her from pursuing the attributes of multiplication and division situations. During the interviews, I inferred from Maria’s explanations that she thought “identifying key words” could help capture the attributes of multiplication and division problems. However, this was not the case, as many of her students were unable to write their own story problems. In contrast, an objective of a lesson taught by John from *SFAW-Mathematics* focuses on identifying the dimensions of plane (length and width) and solid figures (length, width, and height). The mathematical point of this lesson was to relate plane and solid figures as identified under the “key idea” in that same lesson, showing how 2D shapes are transformed into 3D shapes. John identified this key idea and emphasized it during enactment. In *SFAW-Mathematics*, the mathematical point is not often well articulated by lesson goals, but emphasized under the heading “key idea” on the third page of every lesson.

Recommended actions for teachers and students to follow are provided in both curriculum programs. However, rationale for why actions are suggested is often present in *Investigations*, and rarely in *SFAW-Mathematics*. This rationale often communicates the mathematical point to teachers. For example, as mentioned previously, the authors of
Investigations suggested in a lesson Maria taught that students should act out the action of each problem, because doing so might encourage them to recognize division situations as starting with a bigger number that gets divided into equal groups. The mathematical point of this suggested action (i.e., students understanding attributes of division situations) is made known to teachers. Maria used this direction of acting out the problems but did not maximize the affordance of the rationale provided because of the emphasis on key words.

Recommended teacher and student actions are provided within suggested activities. Both curriculum programs provide activities for students to engage with. However, only the authors of Investigations provide rationale to teachers for activities they suggest. In every activity provided in Investigations, mathematical points to be achieved are assigned so that teachers know what is targeted. In contrast, this is absent for all optional activities suggested by the authors of SFAW-Mathematics. For example, in Figure 4.2 and Figure 4.11, no reason is provided for the activities. In particular, the title of the latter figure is “making a double bar graph.” Questions such as, Why are students making a double bar graph? and What mathematical idea will students take away from this double bar graph? are not answered for teachers in these activities. This means the mathematical points of optional activities suggested in SFAW-Mathematics are not being communicated to teachers.

Remillard (2000) recommended that rather than just provide guidance to teachers on how to implement a task, curriculum designers should consider supporting them with an understanding of mathematical ideas embedded in these resources. I interpret this as making the mathematical point of these tasks and guidance provided very visible to
teachers. According to Shkedi (1998), this can make teachers autonomous in implementing curriculum materials. Providing teachers with insightful rationale that clearly highlights the mathematical points to be achieved can promote effective enactment, making meaningful connections within and across lessons. Wang and Paine (2003) found that when mathematical points are provided within the lesson, teachers make deeper connections to achieve broader goals.

**Balancing different types of problems.** The results of this study show that problems for reinforcement and review of mathematical concepts in *SFAW-Mathematics* and *Investigations* are featured in all lessons analyzed. Although *SFAW-Mathematics* teachers actually used a greater percentage of problems for exploration and development of concepts (EDC) available to them, only a very small proportion (see Table 4.2) are provided in this curriculum. In addition, these EDC problems were not included in every *SFAW-Mathematics* lesson analyzed in this study. In *SFAW-Mathematics* where EDC problems are absent, a mathematical strategy is suggested to teachers and students to practice the procedure using problems. Problems contain mathematical points to be achieved and these influence the use of other resources such as mathematical explanations and assessments.

The mathematical points of problems used in these programs are sometimes difficult to identify. This difficulty differs for the programs partly because of what each of them tries to establish. Problems for EDCs in *SFAW-Mathematics* target establishing standard mathematical rules students are to follow (see Figure 4.5). The mathematical point is laid out in such a way that teachers can easily identify intended rules or patterns (see Figure 4.5). In contrast, EDC problems posed enormous challenges to teachers who
used *Investigations*. This is partly because EDC problems used in *Investigations* target complete development of mathematical concepts and formation of intuitive definitions emanating from experiences students are exposed to. As such, the mathematical points of these EDC problems may be difficult to identify and understand. For example, Maria (who used *Investigations*) did not understand the mathematical points of the problems displayed in Figure 4.6. She emphasized just solving the problems rather than discussing how the inverse relationships between multiplication and division could be used to provide solutions. In addition, Maria focused on students identifying key words to determine whether it is a multiplication or division problem rather than identifying their attributes. Also, Lisa focused on the answer to solving problems rather than the mathematical point to be achieved. Therefore, making visible the mathematical points of problems used in curriculum programs would be helpful to teachers. In other words, curriculum designers should explicitly provide the targeted mathematical points along with some problems and should highlight other resources that could be used to move the lesson toward these mathematical points.

Although Brown (2009) captured problems students are to solve as a subcategory of representations of tasks, he did not specify what these problems should focus on. Davis and Krajcik (2005) designed heuristics for curriculum materials but did not mention problems as a support that curriculum designers could provide to teachers. Harris et al. (2001) recommended three kinds of problems that could be used in curriculum materials for introducing, exploring, and extending the concept. Harris et al. further suggested that these extension problems could include potential challenges students might face and generalizations they could make. These three problem types are similar to what I call
problems for “exploration and development of mathematical concepts.” While the authors of *Investigations* include these problems in all lessons I analyzed for this study, their *SFAW-Mathematics* counterparts did not. Also, the two programs emphasize different problem types. *Investigations* and *SFAW-Mathematics* emphasize problems for EDCs and reinforcement of mathematical concepts, respectively.

Results of this study also revealed that the quality of mathematical explanation each curriculum program provides reflects the type of problem each one emphasizes. *Investigations* promotes the development of factual and conceptual knowledge in mathematical content because of the EDC problems it emphasizes. In contrast, *SFAW-Mathematics* emphasizes problems that reinforce mathematical concepts and knowledge about mathematical facts and procedures. However, development of most mathematical facts and procedures in *SFAW-Mathematics* is minimally emphasized, and typically these problems are used for teacher demonstration. Therefore, balancing the different types of problems in each curriculum program should be considered by their respective authors so that students can be exposed to rich conceptual and procedural mathematical knowledge (Harris et al., 2001). Davis and Krajcik (2005) recommended that “curriculum materials should support teachers in developing factual and conceptual knowledge of science content…” (p. 12). Although Davis and Krajcik’s recommendations are in science, they can also apply to mathematics. This could be taken to mean that mathematics curriculum materials should support teachers in developing factual and conceptual knowledge in mathematical content.

In addition, the problem types used in these two curriculum programs influence the kinds of assessment they promote. For example, *Investigations* focuses on the use of
EDC problems and so promotes assessment of conceptual mathematical understanding. Questions that dig into different aspects of a concept that students are to learn are provided so that teachers assess deep conceptual understanding. In contrast, *SFAW-Mathematics* assesses students’ to recall facts and standard mathematical procedures, as its problems emphasize reinforcement of these. Therefore, both programs should consider balancing the problem types they use so that students’ understanding of mathematical concepts can be fully developed and assessed.

Balancing different types of assessments. The authors of *SFAW-Mathematics* and *Investigations* provide both factual and conceptual assessments. The authors of *Investigations* provide mostly conceptual assessments to teachers in the form of questions (see Figure 4.12), while their counterparts for *SFAW-Mathematics* provide both factual and conceptual assessments, with strong emphasis on assessing students’ ability to recall mathematical facts (see Figure 4.2, “Talk About It”). Although *SFAW-Mathematics* provides assessments of concepts, it offers them in the form of grading rubrics for test-taking practice (see Figure 4.8). In these grading rubrics, conceptual responses that are expected from students are provided.

While *Investigations* does not contain grading rubrics for test-taking practice, *SFAW-Mathematics* also does not provide conceptual assessments in the form of questions to teachers. The location of these assessments may impact their usability as well. For example, assessment of conceptual understanding (in the form of a grading rubric), when provided in *SFAW-Mathematics*, appears at the end of a lesson. This might convey to teachers that these assessments are optional and therefore not necessarily important. The teachers who used *SFAW-Mathematics* in this study neither read nor used
the grading rubric for test-taking practice (see Figure 4.8) that emphasizes conceptual mathematics ideas students should understand, whereas all *Investigations* teachers actually incorporated at least one conceptual assessment per lesson. In *Investigations*, conceptual assessment questions appear in the main flow of every lesson.

Conceptual assessment questions in *Investigations* are provided as “Ongoing Assessments.” This assessment takes place while students engage in mathematical tasks assigned to them. Ongoing assessment in *SFAW-Mathematics* is provided in two places within a lesson: under “Investigating the Concept” (see Figure 4.11) and then in the margins of every lesson with respect to the “Talk About It” or “Check” (see Figure 4.13). Second, ongoing assessment questions in *SFAW-Mathematics* are also provided within activities (see Figure 4.11), but are rarely used by teachers. Even when teachers use this activity, the assessment is not important to them. For example, Caroline used this activity when she taught “Finding Averages” but was more interested in the mathematical procedure than the ongoing assessment questions, as she did not ask any of the questions. Furthermore, Ongoing Assessments and Check in *SFAW-Mathematics* embedded within activities focus on mathematical facts rather than on understanding of the concept students are to learn. The Ongoing Assessment and Check in *SFAW-Mathematics* are in response to mathematical facts that students can neither recall nor distinguish from others. On the other hand, ongoing assessments within the main flow of each *Investigations* lesson include difficulties or errors students might encounter and ways teachers can help.

Although *SFAW-Mathematics* includes questions in the main flow of every lesson, these focus on students’ ability to recall facts. *Investigations* does not provide
assessments in the form of a grading rubric. I think a grading rubric, when provided and consulted, might be informative regarding the kind of thinking expected of students that must be emphasized during enactment. Therefore, both programs should consider balancing various types of assessments so that what is not appropriately emphasized in one form might be reemphasized in another. This could lead to appropriate and complete assessment of student learning.

**Potential of anticipated student thinking.** Many researchers (e.g., Davis & Krajcik, 2005) recommended that curriculum materials include anticipated ways students may respond to a topic. Both programs have included anticipated mathematical understandings students are expected to articulate. While *SFAW-Mathematics* focuses on standard or conventional mathematical understanding (see Figure 4.10), *Investigations* emphasizes responses that reflect conceptual grounding on the part of students (see Figure 4.9).

The results of this study show that student thinking has been anticipated in the following ways: what students might say (see Figure 4.9 and Figure 4.10 for *Investigations* and *SFAW-Mathematics*, respectively), interesting ways students might approach a task, errors and difficulties students might encounter together with intervention moves (see Figure 4.13 for *SFAW-Mathematics*), and a grading rubric for test-taking practice problems (see Figure 4.8 for *SFAW-Mathematics*). I discuss these different types of anticipated student thinking in terms of uniqueness, connections between them, sources of errors and difficulties students might face, and mathematical points they communicate.
Some forms of anticipated student thinking are unique to particular curriculum programs. Anticipated student thinking in the form of questions that dig into students’ conceptual understanding are unique to Investigations (see Figure 4.12). These provide ways students might respond to an understanding of a mathematical concept in the form of questions asked. For example, the authors of Investigations anticipate that students will correctly place the numbers on the number lines marked with tenths. Then they provide the second main question in Figure 4.12 to have teachers determine whether students have that understanding. In other words, these ongoing assessment questions provided in Investigations are the points of observation. When some students do not have the desired understanding, Investigations usually suggests further teacher moves to help them. All Investigations teachers in this study used this kind of resource.

Unique to SFAW-Mathematics is a grading rubric for test-taking practice (see Figure 4.8). These grading rubrics anticipate different levels of ways students may respond to test questions to be awarded full or partial credit. This ought to inform SFAW-Mathematics teachers of deep understanding to foster in preparing students for taking tests. Unfortunately, none of the SFAW-Mathematics teachers actually used this resource.

The relationships between mathematical ideas in what students say are not explained to teachers in both curriculum programs. For example, in Figure 4.9 (from Investigations), relationships between mathematical ideas that both students articulated are not explained. How the mathematical idea of the first student can be identified in the second student’s thinking is not explained. This raises questions such as, which of the two students’ thinking is more sophisticated and why? Which of them is more efficient and why? Answers to these questions, when provided in the curriculum, may enhance
teachers’ use of anticipated student thinking to foster appropriate student learning. Similarly, the relationship between the student ideas in Figure 4.10 (from SFAW-Mathematics) is also not explained to teachers. In this study, none of the teachers (from both curriculum programs) actually made connections among the mathematical ideas in student thinking as they used them.

Davis and Krajcik (2005) suggested that curriculum developers could anticipate student thinking within the teacher’s guides to “help teachers recognize the importance of students’ ideas and help teachers identify likely student ideas within a topic. Curriculum materials should help teachers gain insights into how they might be able to deal with the ideas in their teaching…” (p. 11). Also, Ball and Cohen (1996) recommended that curriculum materials “could probe and comment on specific subject matter elements evident in students’ ideas, questions, responses, and writing” (p. 7). These curriculum programs have provided anticipated ways students might think about the subject matter, but haven’t really commented on the specific mathematical ideas embedded in what students say. Therefore, these recommendations and suggestions have been only partly incorporated in Investigations and SFAW-Mathematics.

Both curriculum programs identify errors students might make and difficulties they may face together with recommended teacher moves to remedy these situations. Using anticipated student thinking in this way makes teachers more reactive than proactive. Although reacting to difficult situations that arise in the classroom is part of the task of teaching, teachers ought to design instruction for learning to occur in the first place. Both curriculum programs anticipated errors and difficulties student might encounter. However, both programs did not propose ways some of these common errors
and difficulties could be avoided from happening in their teaching. I do not mean that all errors can or should be avoided, because they can be great sources for learning. I suggest that activities should be designed to help students develop deep understanding of concepts so that more common errors are resolved through conceptual teaching. A way to motivate teachers to be proactive might be to identify sources of common errors and challenges students may encounter and suggest additional resources to use in teaching so that those situations minimally arise and can be used as learning opportunities when they do.

Anticipated student thinking is often provided in the form of visuals. Forms of visuals used in both programs include shapes (see Figure 4.2), charts (see Figure 4.15), and graphs (see Figure 4.17). These forms of visuals carry mathematical explanations that teachers need to understand and communicate to students. The mathematical point of each visual ought to be visible to teachers. For example, Figure 4.15 contains two important mathematical points that were not visible to Maria, the teacher who taught this lesson. This probably caused her to omit this representation, which had a huge consequence in the achievement of written lesson goals.

**Connections between activities and lessons.** The results of this study show that the authors of *Investigations* are more explicit in the connections they provide between activities and lessons than their counterparts of *SFAW-Mathematics*. The authors of *Investigations* indicate where a mathematical idea is first introduced or where it will be developed further. For example, in a lesson taught by Jennifer, the authors of *Investigations* suggest that teachers “explain to students that in the next session, they will go on to find the measure of other angles using the Power Polygons” (Wittenberg et al.,
2008, Grade 4, Unit 4, p. 93). In other cases, these authors indicate where a similar activity was or will be encountered. For example, in a lesson taught by Lisa, the authors of *Investigations* state that “Decimal In Between is a new activity, although students played a similar game in unit 4” (Wittenberg et al., 2008, Grade 5, Unit 6, p. 45). In contrast, the lessons I analyzed for *SFAW-Mathematics* provide only a list of topics that could be reviewed when review problems are used, indicating implicit connections are made.

Although both curriculum programs signaled some form of connections, they did not always specify important mathematical points connecting activities, lessons, units, and grade levels. In the second example provided above, the authors of *Investigations* did not specifically say how similar or different the Decimal In Between Game is to a game played in Unit 4. They also did not specify what mathematical idea from the game played in Unit 4 would be used in the Decimal In Between Game. Furthermore, they do not say how the Decimal In Between Game advances the mathematical ideas of the game in Unit 4. With these unspecified threads of connections in curriculum materials, much of the work goes back to the teachers, placing a very heavy demand on them to link ideas and lessons into a coherent way.

Ball and Cohen (1996) recommended that “teacher’s guides could also help teachers to consider ways to relate units during the year” (p. 7). As such, the authors of both curriculum programs may be more explicit than implicit in making connections among activities, lessons, units, and grade levels to help teachers more within the lessons. This is because most teachers focus on what is written at the lesson level rather than elsewhere when planning to teach. Making these connections within the lessons might be
more helpful for teachers to see and establish the links. In addition, these authors should clearly state how activities are related in terms of the development of key mathematical ideas. Curriculum designers may also clearly communicate how similar and different related mathematical ideas are in the lessons they design. This kind of support might enable teachers to teach mathematics using curricular resources in a connected way and developing them from least to most sophisticated. This might convey the message that the mathematical ideas are connected and should not be treated in isolation.

**Coding challenges.** Coding for these curricular resources was very challenging because of their nature and the different units of analysis. Some of the resources were understandable as sentences while others as chunks. Also, some of the curricular resources contain more than one resource and this had to be determined. For example, an anticipated student thinking is about the mathematics students are to learn. Therefore, it was difficult to allocate a single code in such situations. Multiple codes were therefore assigned in order to capture curricular resources available to teachers in lessons they teach. Problems students were to solve were coded differently because it made sense to see them as different problem types rather than in terms of number of sentences. Visuals are non-textual and hence coded and analyzed differently. This resulted in different unit of analysis for problems and visuals. This multiple layer of analysis was incorporated into this study to enable me capture ways different resources are provided to teachers.

**Ways Teachers Use Curricular Resources in Conjunction with Each Other**

In this study, I categorized two levels of the quality of using curricular resources in conjunction with each other as minimally and highly connected. These reveal insights into the capacities teachers exhibited as they used curricular resources to move toward
written lessons goals. I discuss ways teachers use curricular resources and insights, in terms of mathematical points and relationships between activities within and across lessons to orchestrate appropriate student learning. Then, I discuss ways curriculum materials could foster teachers’ capacities to use curricular resources in a highly connected way toward written mathematical points.

**Mathematical points.** A big responsibility in using curricular resources is to identify the mathematical points embedded in them. These results show that teachers must begin identifying appropriate mathematical points that curricular resources communicate and then deliberately “steer instructions toward them” (Sleep, 2012, p. 938). Without proper identification of written mathematical points, it is unlikely that teachers will be able to foster intended student learning. Sleep (2012) recommended that teachers must articulate appropriate mathematical points, orient instructional activities, and move lessons toward those key ideas. Some teachers in this study taught to the mathematical point by incorporating the three interdependent types of teaching work identified by Sleep, while others did not. I discuss mathematical points embedded in curricular resources, such as problems, representations, and rationale, because these were most commonly used by all teachers in this study.

**Problems to solve.** Mathematical points embedded in problems students are to solve should be identified appropriately by teachers. Maria and Lisa did not identify the mathematical points in problems suggested in *Investigations*. As such, they generated moves that “steered their instruction” (Sleep, 2012, p. 938) away from the intended key ideas students were to learn. This heavily impacted the outcome of their lessons.
In contrast, Jennifer, who used the same curriculum program as Maria and Lisa, identified the written mathematical point embedded in problems (e.g., using polygons and relating them to 90 degrees in order to find the size of some angles). She articulated the mathematical point and “steered her instruction toward it” (Sleep, 2012, p. 938). In doing this, Jennifer suggested mathematical strategies that students did not bring up. She deliberately moved her lesson toward the written mathematical points of the problem.

Therefore, in order to use problems so that adequate student learning is achieved, teachers should ask themselves questions such as, What mathematical points are embedded in the lesson? Are the key ideas I am about to emphasize using these problems what students ought to learn? Reflecting on questions such as these may be helpful for teachers to identify appropriate mathematical points that problems convey. To orient classroom activities judiciously and “steer instruction toward mathematical points” (Sleep, 2012, p. 938) embedded in problems, teachers ought to ask questions that require students to think about what the suggested problems require, and purposely suggest what is not voiced by students to move instruction toward written mathematical points.

**Suggested representations.** Suggested representations in this study are much related to problems students are to solve. For example, the visual in Figure 4.15 is related to the mathematical points embedded in problems. The representation support the problems in that they have the potential to make visible the mathematical points communicated in the lesson. The attributes of multiplication and division problems as well as the inverse relationship between these operations are contained in Figure 4.15. Identifying written mathematical points in a representation might help teachers move their lessons toward those in the problems assuming the curriculum is well written and
the visuals and the problems are mutually supportive toward the same mathematical points. Figure 5.1 draws on the cases of Maria and Lisa to show what happens when teachers do not understand and identify the mathematical point of curricular resources in general -they omit curricular resources whose mathematical points are not accessible to them and take alternative paths that may lead to other goals.

Figure 5.1. Adaptation Influenced by Limited Understanding of Mathematical Point

**Rationale.** In this study, I found that in *Investigations*, design rationale contains mathematical points that teachers need to identify and clearly articulate before using them. Jennifer identified the mathematical point of her lessons and highlighted them in her interaction with students. She actually deliberately exposed her students to these mathematical points. For example, Jennifer superimposed two polygons E to show they were equal. She gave students insights about what to use to find the size of angle E. She intentionally put the two polygons E together to form a right angle and used polygon A (a square) to show that 90 degrees is formed. Jennifer used polygon A to motivate students to relate those angles to 90 degrees before finding their measure, the mathematical point of the activity. Because Jennifer identified the mathematical points, she deliberately
moved students toward them. Therefore, deliberate teacher moves to “steer instruction toward written lesson goals” (Sleep, 2012, p. 938) depend hugely on whether appropriate mathematical points are identified and articulated.

To summarize this section, this study revealed that understanding the mathematical points that curricular resources communicate, and the paths to be taken to reach them, determines greatly how resources are used. Absence of this understanding may pose difficulties for teachers to arrive at desired mathematical points, as explained previously for Maria and Lisa. Sleep (2012) made a similar recommendation for lessons in general. Sleep further argued that taking learners to intended learning is a deliberate action. Stein and Kim (2009), in analyzing *Investigations* and *Everyday Mathematics*, found that student learning in the former curriculum program is situated at teacher-student interaction. This means that, in “steering instruction” (Sleep, 2012, p. 938), teachers must make deliberate moves to direct students toward written mathematical points. These moves could be either asking stimulating questions to provoke students’ mathematical thinking toward planned learning, or providing additional ideas that students need to consider as they engage in reasoning about the problems assigned to them, or both.

**Relationships Among Activities Within and Across Lessons**

In this study, I found that one way teachers make use of available curricular resources in association with each other is by relating activities within and across lessons. Some teachers successfully did this, while others did not. For example, Jennifer successfully made connections between activities within a lesson and also used ideas previously learned to make the mathematical storyline visible to students. In doing this,
she stated previous learning and how current lessons build on it. A reason for this may be attributed to the fact that Jennifer identified the mathematical point of each lesson and activity and determined how each one builds on the other to move learning forward.

On the contrary, when teachers did not identify how activities are related within and across lessons, they made changes that negatively impacted the overall learning experience. Figure 5.2 draws from the example of Lisa to show a way teachers establish relationships between activities within and across lessons and its impact.

Figure 5.2. Impact of Changing Sequence of an Activity

This kind of relationship established between activity 3 and activity 1 without carefully thinking of activity 2 however, proved “fatal” (Seago, 2007, p. 11) because activity 2 was lost along with the mathematical point embedded in it.

These results extend the work on the types of adaptations teachers make as they use curriculum materials (e.g., Brown, 2002; Brown & Edelson, 2003; Choppin, 2009, 2011; Forbes & Davis, 2010; Sherin & Drake, 2009). These studies primarily focused on just identifying types of adaptations made, together with reasons. My study adds that adaptations influenced by connections between activities should be carefully evaluated to
determine whether mathematical points are still maintained. Also, teachers must decide whether mathematical points of intermediary activities have been properly incorporated so that key ideas are not lost.

**Ways Curriculum Materials Can Support Teachers’ Use of Curricular Resources Toward Appropriate Mathematical Points**

I discuss ways curriculum materials could help teachers make better use of curricular resources such as representations and problems, as well as types of adaptations (change of sequence), to preserve the mathematical points of lessons.

The misidentification of mathematical points can be attributed to lack of knowledge on the part of the teachers and lack of clarity from the curriculum materials. I infer that this misidentification may be due to lack of knowledge, because even in situations where curriculum materials clearly stated the key ideas that should be taken away from a teacher-student interaction, the teachers still did not understand that. For example, in a lesson Maria taught, the authors of *Investigations* said teachers should listen for the characteristics of multiplication and division situations, but Maria did not. The curriculum actually provided these characteristics, but Maria did not take that route.

On the other hand, lack of clarity on the part of curriculum materials may have hindered teachers from identifying the mathematical points embedded in suggested curricular resources. Hence, providing curricular resources and making explicit reference about what can be accomplished with it can boost teachers’ understanding of how they can be used to proceed to the written mathematical points.

Also, at the beginning of the problems teachers are to use, the curriculum could explicitly state the mathematical points that should be addressed as they are incorporated into the plan. For example, at the top of the problems Maria used, the authors could have
said, “These problems are used to explore the inverse relationship between multiplication and division and also identify characteristics of these problems.” The authors of curriculum materials could also explicitly explain to teachers how the inverse relationship between the operations can be used to solve problems. This could have been very supportive for Maria as she enacted the problems. If an idea is to be used from somewhere else in the curriculum materials, it should also be stated clearly. For example, in a lesson Lisa taught, she used problems 5-7, omitting problems 1-4. However, ideas from the first set of problems are to be used to solve the latter ones (see Figure 4.20). Because Lisa may not have understood this, she omitted problems 1-4, which were vital to solving problems 5-7 and meeting the written mathematical points. A statement at the beginning of problems 5-7, such as “Using mathematical ideas from problems 1-4, solve problems 5-7,” might have been helpful. This could have deterred Lisa from using a chart constructed in a previous lesson. In this way, curriculum programs could be more deliberate in making more visible the mathematical points embedded in curricular resources.

This study also revealed that the types of adaptations teachers make impact their use of curricular resources toward the intended mathematical points. For example, Lisa changed the sequence of activities in her lessons and completely lost the mathematical points of the intermediary activities (see Figure 5.2). Therefore, curriculum designers could anticipate different types of adaptations teachers might make and suggest modifications that could preserve the mathematical value of intermediary activities and the lesson in general and still foster appropriate learning.
Conclusion

To conclude, I summarize the contribution of this study to the field of mathematics education, provide limitations of the study, and indicate directions for future research.

Summary of Contributions

It is worth noting that *Investigations* is more difficult to teach from than *SFAW-Mathematics*. This is because *SFAW-Mathematics* is more direct than *Investigations*. Also, in *Investigations* student learning is at the interaction between teachers and students (Stein & Kim, 2009). In other words, *SFAW-Mathematics* lays out the content to teach in a more direct way that is easily visible to teachers than *Investigations* does. Although worthwhile mathematical content is presented in *Investigations*, it is embedded in mathematical tasks and therefore difficult for teachers with less mathematical knowledge to identify. Because student learning takes place during teacher-student interaction (Stein & Kim, 2009), when teachers do not identify key ideas that students are to learn inside the suggested resources in order to enact the task, exposing students to an adequate learning experience becomes extremely difficult.

This study contributes to a better understanding of curricular resources in general by examining those unique to the two curriculum programs used in this study. This study also contributes to an understanding in the field of mathematics education of ways teachers use curricular resources in association with each other resources toward written mathematical points. Two qualities, minimally and highly connected, were identified in this study. Characteristics of the ways teachers use curricular resources have been identified and suggestions have been made to foster highly connected use of those
resources. Teachers with minimal connections were not able to identify written mathematical points embedded in curricular resources. They hardly overcame the limitations of resources. In addition, the teachers did not recognize relationships between curricular resources toward written mathematical points.

Teachers who used curricular resources in a highly connected way began by identifying the appropriate written mathematical points that curricular resources communicate. They made up for limitations existing in curricular resources (e.g., Dan’s lesson). These teachers also maximized affordances conveyed by curricular resources. In addition, these teachers established one-to-one correspondence between curricular resources as they used them. Also, they recognized connections between activities within and across lessons, as explained previously.

This study has provided the mathematics education community with insights into what teachers do with curricular resources as a set in highly connected and minimally connected ways toward written mathematical points. These insights set a path to develop further understanding of key dimensions of pedagogical design capacity (PDC). Understanding mathematical points embedded in curricular resources, the connections between them, their affordances and limitations as well as ways to maximize strengths and minimize weaknesses, and connections between activities with a focus on mathematical points of intermediary activities are some insights into key dimensions of PDC identified in this study. These insights have the potential of leading the mathematics education community to develop a theory that can be used to understand capacities teachers need to perceive and mobilize curricular resources in a productive way toward written mathematical points of each lesson. This, in turn, further develops an
understanding in the field of mathematics education of ways teachers use curriculum materials. This study also identifies areas in which curriculum designers, teacher educators, and professional development experts can help teachers develop capacities needed to use curricular resources effectively.

**Curriculum designers.** In this study, I gained different insights into capacities teachers need to use curricular resources. As such, teachers need different kinds of support to use curricular resources toward written mathematical points of lessons. While some teachers minimized limitations, maximized affordances, and made connections between curricular resources toward written mathematical points they identified that were the same as those written in the curriculum, others did not. This could mean that curriculum developers should design different curriculum materials for different categories of teachers based on their ability to use curriculum resources. It could also be taken to suggest that curriculum writers incorporate different levels of capacities into curriculum design to help teachers with low capacity. For example, teachers whose quality of use of curricular resources I classified as minimally connected could benefit from curriculum materials in which curricular resources have been annotated. Curriculum materials with annotations should contain information that draws teachers’ attention to affordances of curricular resources. This is because greater transparency is needed to communicate the written mathematical points embedded in curricular resources to teachers who have a hard time identifying them. Also, curriculum designers should explicitly communicate to teachers how curricular resources support each other toward written mathematical points.
Davis and Krajcik (2005) and Stein and Kim (2009) used the word *transparent* to describe this kind of curriculum material. More transparency is needed to make visible pedagogical affordances of resources (Davis & Krajcik, 2005). This transparency on the part of curriculum designers could include suggestions as to when teachers should take certain actions. Curriculum designers could use these results to rethink the quality of curricular resources they provide to better support teachers to enact their lessons. In this way, authors of curriculum materials could better support the development of capacities that teachers need to use curricular resources.

**Teacher education.** Brown (2009) described curricular resources as containing important contents as well as how these contents could be explored. If students are to learn meaningful mathematics, then teachers must be able to understand written mathematical points embedded in curricular resources and guide exploration appropriately so that meaningful learning is encountered. Therefore, how teachers use these curricular resources to expose students to appropriate learning opportunities is important. As such, teacher educators could include assessment of curricular resources as an integral part of training preservice teachers. This might include analyzing curriculum materials to understand the curricular resources they provide and examining their affordances and limitations. In addition, it could include understanding how curricular resources are related toward mathematical points written in the curriculum. It might also include examining different curriculum materials, both commercially developed and reform-oriented so that teachers are exposed to a variety of options. It may further include ways to maximize affordances and minimize limitations of curricular resources so that teachers might create, increase, or nurture opportunities for students to learn and
achieve written lesson goals. This might prepare preservice teachers adequately to teach from any curriculum materials they may be given during their teaching career.

Incorporating the study and use of curricular resources into teacher training programs may help prepare teachers to make optimal use of the variety of curriculum programs they might encounter in their teaching career. Furthermore, assessing curricular resources to identify mathematical points embedded in them might help support teachers in developing Knowledge of Curriculum Embedded Mathematics (Kim & Remillard, 2011).

**Professional development (PD).** PD, in a way, supports teacher education programs by ensuring continuous quality teaching so that students can benefit most. Although it is geared to improve student learning, the first beneficiary is the teacher. Brown (2009) recommended that “in addition to receiving support in learning subject matter and ways of teaching content, which many have long advocated, teachers also require support in exploring which resources to use and how to use them” (p. 33). I expand on the latter part of Brown’s recommendation as an extension of how this study could be useful to PD programs. PD might focus on using curricular resources to develop factual and conceptual knowledge, aspects found to be lacking in some commercially developed curriculum materials. Extending the assessment and utilization of curricular resources to practicing teachers serves to highlight their significance. Furthermore, PD might also continue to build teachers’ capacity to fully recognize the potential of curricular resources and use them as curriculum designers suggest.

**Limitations**

Although the results of this study have potential for wide applicability, they must be used with care for the following reasons. First, I did not analyze the effectiveness of
the ways teachers used curricular resources in relation to creating or nurturing opportunities for students to learn and student achievement. Second, the small sample size for teachers and the short duration of the study makes it hard to make extensive generalizations. Also, analyzing only two curriculum materials out of many in the field makes it hard to conclude that the curricular resources identified and used in this study apply in all situations. Third, my analysis is limited to lessons as written in the teacher’s guide. In other words, I did not analyze curricular resources provided outside of lessons for everyday teaching, such as unit/chapter overview, teacher notes, dialogue boxes, and student math handbook (for *Investigations*) and how teachers use them. This is because the teachers who participated in this study explained they focus only on what is written at the lesson level in the teacher’s guide.

**Directions for Future Research**

Although the findings of this study can be used by many in the field of mathematics education, more still needs to be known. I do not claim the curricular resources I used for this study are all that curriculum materials provide. Therefore, a broader study with many curriculum materials and a greater number of lessons within each program is needed to make a generalization about resources embedded in them and those that still need to be considered. Also, this study gives us significant insights into capacities teachers exhibited as they used curricular resources, which I do not claim are exhaustive. As such, further study over a longer period of time, maybe within an academic year or over several years, with many teachers using many different curriculum programs, both reform and commercially developed, is needed. Such a study might confirm, reject, or refine some of the insights identified in this study. It might also refine
suggestions to improve ways teacher use curricular resources in association with other resources. As a consequence, this suggested study might lead to a theory to characterize teachers’ capacity to use curricular resources in written curriculum materials in association with other resources toward written mathematical points and hence identify more refined components of PDC.

Furthermore, these teacher capacities to use curricular resources in a connected way raised questions as to whether they create or nurture opportunities to learn for students and can influence student achievements. Because these could not be readily answered by this study, further research is needed to establish the relationship among teacher capacity to use curricular resources, opportunities to learn, and student achievement. Findings from such studies might further inform teacher education and PD programs on teachers’ capacities that need development and reinforcement.
REFERENCES


Brown, M. W., & Edelson, D. C. (2003). Teaching as design: can we better understand the ways in which teachers use materials so that we can better design materials to support changes in practice? Evanston, IL: Center for learning technologies in Urban Schools, Northwestern University (Available at: http://letus.org/PDF/teaching_as_design.pdf).


Komoski, P. K. (1977). Instructional materials will not improve until we change the


Appendix A

Introductory Interview Protocol
Background

1. How many years have you taught?
2. Which grades have you taught?
3. Which curriculum packages have you used in the past?
4. What are you using now?

Current Curriculum Materials

Opinion about curriculum materials / package

5. For how long have you been using these (current) curriculum materials?
6. What aspects of the curriculum materials do you like? Dislike?
7. What do you believe is the major emphasis or the philosophy of these curriculum materials?
8. How do these ideas/curricular goals compare to your own ideas and goals?
9. [If the teacher hasn’t addressed other curriculum packages prior to now, ask this question] How does these curriculum materials compare to others you have used in the past?

How curriculum materials are used

10. What does a typical lesson look like for you?
11. How do you prepare for a lesson and how do you use the teachers’ guide in doing so?
   a. [Follow-up questions] Does the teachers’ guide help you in understanding the mathematical focus of a lesson? How does it do so, or not do so?
   b. Does the teachers’ guide help you in organizing the timeline of a lesson (i.e., what you and the students will do at given moments of the lesson)? How does it do so, or not?
12. Are there other specific things, the teachers’ guide helps you to do? Do you use the teachers’ guide during instruction? How? If not, why not?
13. Do you go back to the teachers’ guide after you teach? If so, what do you do with it?
14. Do you refer to (or consult) any other resources that are part of [the purchased curriculum package, e.g., Everyday Math] when planning or teaching a lesson? If so, How?
15. Are there other resources elsewhere [e.g., provided by the district or the department, or researched by the teacher] that you regularly consult and that are not part of the curriculum for developing your lesson plans? If so, how do you use these materials?
16. When you have a question about the curriculum or curriculum materials [i.e., “curriculum” broadly-defined here, as including all purchased curricular resources and any district/departmental or other materials], what do you do? How does your school or district support your use of the curriculum [again, “curriculum” here is broadly-defined]?
Appendix B

Follow-Up Interview Protocol
(A) About the week in general
1. I observed three lessons two weeks ago when you taught ---. How typical was the week in terms of teaching? In terms of using the curriculum materials?
2. Is there anything unusual or specific of the week that I need to know about?
3. How did you feel about the students’ responses to the lessons?
4. Do you remember anything that you did differently from what you planned? If so, what did you do and why?
5. What would you do differently next time? Why?

(B) About the CRL
1. Here is what you highlighted in a copy of the lessons (CRL).
   a. Tell me about parts you highlighted in yellow. What parts do you usually read and why?
   b. (When applicable) I noticed that sometimes you read [a section in the lesson, e.g., Ongoing Assessment] and sometimes not. Tell me about it. Is there a particular reason for that?
   c. Tell me about parts you highlighted in blue. How do you determine what to use in your lesson from the parts you read?
   d. You have some parts highlighted in orange – meaning parts that influenced your planning or parts that you adapted. Tell me about these parts.
2. You also have some parts not highlighted in this copy. What parts do you usually not read and why?
3. Is this how you read the curriculum regularly? Or, is this reading very particular to this week? Why?
4. I noticed you highlighted [choose a portion/portions in the CRL on Codes 2 (rationale), 3 (student thinking), and/or 4 (mathematics)]. Did this help your planning or teaching? How?
5. How has your curriculum use changed over your career? [Probe specifics. Ask this question in the first follow-up interview only.]

(C) About the week in specific (with or without video clips)
1. As I looked at the videotapes of your lessons, I noticed [choose a few moments related to Codes 2, 3, and 4, such as emphasizing a particular mathematical idea for this question and repeat the same set of questions]. Tell me about what happened. What made you decide to do that?
2. Ask a combination of the following questions:
   a. I also noticed you skipped this part of what you planned. Tell me about what you were thinking. What made you decide to do that?
   b. I also noticed that you added this part that was not planned. Tell me about what you were thinking. What made you decide to do that?
   c. I noticed again that you used --- instead of --- that was suggested in the curriculum. Tell me about how you made that decision.
3. We just talked about ways of using curriculum, such as skipping, adding, and changing to the lesson. How typical was this?
4. Does the curriculum provide any guidance about making these kinds of altering lessons - adding or skipping parts of the lesson, or choosing options?
5. Are there any other ways you use the curriculum that are different from those you described today?
6. Is there anything else you would like to add?
Appendix C

Final Interview Protocol
**Final Interview question:**

“This interview is about the lessons observed in spring 2012 (grade 3, unit 5, and sessions 4.2, 4.3, 4.4, and 4.5 of *Investigations in Number, Data, and Space*) and I would like you to assume that you teach these lessons again and answer the questions about your potential decisions.”

1. (Session 4.2) At what points in this lesson are each of the three Math Focus Points addressed?
   a. What makes you think so?
   b. How do you plan to bring that to live in the lesson?

2. (Session 4.2). As you plan to teach this lesson, do you think each of the following elements support students’ learning of lesson goals? If so, in what ways?
   a. Student might say (p. 123)
   b. Math notes (session 4.2, p. 123)
   c. Representations (Table on p. 124)

3. (Session 4.2) How are you going to relate the three parts of this lesson? What ideas are you taking from activity 1 to use in the discussion and then to reinforce with the homework problems?

4. (Session 4.2) As you plan to teach this lesson, how do you plan on using *Student Activity Book* pages 42-43 to support students to understand and use the inverse relationship between multiplication and division to solve problems?
   a. What problems do you plan on highlighting to demonstrate this inverse relationship to students? In what ways will you do this?
   b. How will you use these problems to enable students understand this inverse relationship between multiplication and division?

5. (Session 4.3) At what points in this lesson are each of the Math Focus Points addressed?
   a. What makes you think so?

6. (Session 4.3) As you plan to teach this lesson, how would you plan on using the materials listed at the top of page 127 (that is, “things that come in groups” “cubes,” “graph paper,” or drawings to help students write multiplication and division problems?
   a. How will you bring out the attributes of multiplication and division problems using these materials?

7. What is critical for students to know about division and multiplication problems so that they can create their own problems? What are main differences between the two?

8. How would you plan on using session 4.4 to help students create and solve multiplication and division problems?
   a. What main ideas in session 4.4 do you plan on using to help students create multiplication and division problems? How do you plan on doing that?
Appendix D

Number of Curricular Resources Available for the Lessons Observed in This Study
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<sup>a</sup> These are number of sentences associated with problems only
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Appendix E

HSIRB Approval Form
Date: April 15, 2013

To: Ok-Kyong Kim, Principal Investigator
    Naphthali Atunga, Student Investigator for dissertation

From: Amy Naugle, Ph.D., Chair

Re: HSIRB Project Number 12-04-11

This letter will serve as confirmation that your research project titled “Assessing Teachers’ Pedagogical Design Capacity and Mathematics Curriculum Use – Follow up” has been approved under the expedited category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note: This research may only be conducted exactly as in the form it was approved. You must seek specific board approval for any changes in this project (e.g., you must request a post approval change to enroll subjects beyond the number stated in your application under “Number of subjects you want to complete the study.”) Failure to obtain approval for changes will result in a protocol deviation. In addition, if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

Reapproval of the project is required if it extends beyond the termination date stated below.

The board wishes you success in the pursuit of your research goals.

Approval Termination: April 18, 2014