Active Control of Aerial Refueling Drogue

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ACTIVE CONTROL OF AERIAL REFUELING DROGUE

by

Taeseung Kuk

A dissertation submitted to the Graduate College
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
Mechanical Engineering
Western Michigan University
April 2014

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ACTIVE CONTROL OF AERIAL REFUELING DROGUE

Taeseung Kuk, Ph.D.
Western Michigan University, 2014

An active stabilization method for an aerial refueling drogue used in probe-drogue refueling system is presented in this study. The present refueling drogue exhibits an unstable motion due to various turbulent natures, which results in high failure rates and sometimes catastrophic accidents. Although it is considered a routine procedure for human pilot, the workload for mid-air refueling is highly demanding and autonomous operation is by no means a simple task. With rapidly growing use of unmanned aerial vehicles, autonomous aerial refueling is identified as one of essential technologies in future aviation. The present research is a pioneering work in active drogue stabilization that includes conceptual design of a controllable drogue, comprehensive dynamic wind tunnel experiments including a scale model prototype manufacturing, mathematical modeling and feedback control, and hardware-in-the-loop simulation.

A set of four aerodynamic control surfaces is implemented to minimize the level of modification necessary for the current hose-drogue refueling system. An inertial measurement unit is adopted as a primary sensor to detect the drogue motion. To verify the system performance, a four degrees-of-freedom dynamic test rig system is fabricated for wind tunnel experiment. This system is composed of a rigid link and the controllable drogue, and a representative mathematical model is constructed based on the multi-body dynamic analysis with experimentally measured data. A feedback control law is
implemented to a custom designed avionics hardware. An artificial gust generator is fabricated to create turbulent flow field in the wind tunnel, and is quantified by a five-hole probe system. The results from the closed-loop, hardware-in-the-loop simulation test indicates that the proposed concept reduces the drogue motion up to 90%.
ACKNOWLEDGEMENTS

A refueling drogue is prone to instability subject to external disturbances as seen on many publically available aerial refueling video clips. The mathematical model proposed by my advisor, Dr. Kapseong Ro, precisely captures this instability, and it motivated this study for the model-based control synthesis.

First of all, I would like to thank Dr. Ro for his instruction and guidance. Secondly, I extend my sincere gratitude to my dissertation committee members, Drs. James W. Kamman, Koorosh Nagshineh, and Ikhlas Abdel-Qader for their time and effort. Especially, the instructions in and outside the classroom by Dr. Kamman were essential for the completion of this research. I also appreciate Department of Mechanical & Aerospace Engineering for the financial support during my doctoral study.

I would not forget CEO Joowon Park, CEO Wooseok Kang, and my friend Meyonghoon Seong for being always there to help me with any technical assistance. In addition, I really thank Dr. Jingon Kim, and Dr. Sam Ok Koo for great encouragement whenever I felt disappointed. I also should mention my parents, sister, brother-in-law, nephew and niece for providing all sorts of tangible and intangible support during last 7 years. I would also like to thank to my friends: Dr. Seonhoo Kim, Dr. Changjin Yoon, Dr. Seongsik Shin, Seungki Choi, Jeongho Moon, Shinje Cho, and Byungjun Park. It is a huge supporting power for me to talk with them.

I love aircraft and enjoy developing it. Although this research is apt to be used for a military purpose, I wish that this technique be used for peaceful UAV applications.
Lastly, I try to be a wise person who can see through not only engineering but also person, society, world and universe for progress and balance.

Taeseung Kuk
TABLE OF CONTENTS

ACKNOWLEDGEMENTS ........................................................................................................... ii
LIST OF TABLES ................................................................................................................... viii
LIST OF FIGURES ................................................................................................................. x

CHAPTER

1. INTRODUCTION .................................................................................................................. 1

1.1 Historical perspectives of aerial refueling ................................................................. 1

1.1.1 Beginning of aircraft and aerial refueling ............................................................... 1

1.1.2 Aerial refueling in jet age ....................................................................................... 3

1.1.3 Modern aerial refueling ......................................................................................... 6

1.2 Literature review ........................................................................................................... 8

1.2.1 Dynamic modeling of hose-drogue refueling system ........................................... 9

1.2.2 Various methods for drogue stabilization ............................................................. 13

1.3 Problem description .................................................................................................... 14

1.3.1 Problem statement .............................................................................................. 14

1.3.2 Contribution of the present work ......................................................................... 15

2. DESIGN AND FABRICATION .................................................................................. 17

2.1 Design overview ......................................................................................................... 17

2.1.1 ASDRS design ................................................................................................... 18

2.1.2 System configuration ......................................................................................... 21

2.1.3 System dimension ............................................................................................. 24
CHAPTER

2.2 ASDRS hardware ......................................................................................................26
  2.2.1 Test system overview ..................................................................................26
  2.2.2 Actively stabilized drogue ........................................................................27
  2.2.3 Avionics .....................................................................................................31

2.3 ASDRS software ..................................................................................................34
  2.3.1 Interface control document ......................................................................34
  2.3.2 Drogue operation software .....................................................................38
  2.3.3 Ground control system software ...........................................................39

3. DYNAMIC MODELING ..........................................................................................41
  3.1 Determination of inertial properties ..............................................................41
  3.2 Equations of motion .......................................................................................44
    3.2.1 Reference frames, angles and position vectors ..................................44
    3.2.2 Kinematics ............................................................................................49
    3.2.3 Equations of motion ..............................................................................51
  3.3 External loads ....................................................................................................55
    3.3.1 Drogue aerodynamic loads ..................................................................56
    3.3.2 Bar aerodynamic loads ........................................................................58
    3.3.3 Gust loads ..............................................................................................60
  3.4 Actuator dynamics ...........................................................................................71

4. NUMERICAL SIMULATION AND CONTROL LAW SYNTHESIS ..............72
  4.1 Numerical simulation .......................................................................................73
CHAPTER

4.1.1 Open-loop wind tunnel test .................................................... 76
4.1.2 Simulation and model tuning ............................................... 80

4.2 Linear model .......................................................................... 84
  4.2.1 State-space model .............................................................. 84
  4.2.2 Transfer function model .................................................. 86

4.3 Control law synthesis ............................................................. 93

4.4 Comments on transfer functions via system identification .......... 100

5. RESULTS ANALYSIS ................................................................ 103
  5.1 Test descriptions .................................................................... 103
  5.2 Response to vertical continuous wave gust ............................ 104
  5.3 Response to horizontal continuous wave gust ....................... 110
  5.4 Response to the vertical single wave gust .............................. 116
  5.5 Response to horizontal single wave gust ............................... 122
  5.6 Summary ............................................................................. 127

6. CONCLUSION AND FUTURE RESEARCH ............................. 128
  6.1 Summary and conclusion ......................................................... 128
  6.2 Future research .................................................................... 130

REFERENCES ............................................................................. 132

APPENDICES
  A. Drogue Code ........................................................................ 136
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPENDICES</td>
<td></td>
</tr>
<tr>
<td>B. Coefficients of Kane's Equation</td>
<td>146</td>
</tr>
<tr>
<td>C. Five Hole Probe</td>
<td>148</td>
</tr>
<tr>
<td>D. State-Space Model</td>
<td>150</td>
</tr>
<tr>
<td>NOTATIONS</td>
<td>156</td>
</tr>
</tbody>
</table>
LIST OF TABLES

1.1 Comparison between the flying-boom and the probe-drobe refueling method ........... 6
1.2 List of US Patents on drogue stabilization.................................................................. 13
2.1 ASDRS Design Requirements .................................................................................... 18
2.2 Drogue dimension....................................................................................................... 24
2.3 AHRS ICD (AHRS to Controller) .............................................................................. 35
2.4 Uplink ICD (GCS to Controller)................................................................................. 36
2.5 Wings control methods ............................................................................................... 37
2.6 Downlink ICD (Controller to GCS)............................................................................ 38
3.1 Measured inertial properties ....................................................................................... 44
3.2 Partial velocities for use with Kane’s Equation............................................................ 52
3.3 Gust signal characteristics........................................................................................... 66
3.4 List of gust signal cases .............................................................................................. 66
4.1 Linearized state-space model at 500Pa ................................................................. 85
4.2 Input and output of transfer functions ....................................................................... 86
4.3 Transfer functions by small perturbation method .................................................. 87
4.4 PD controller gain ..................................................................................................... 99
List of Tables -- Continued

4.5 Transfer function by SI technique ................................................................. 101

5.1 Closed-loop test plan matrix ................................................................. 104

5.2 Nonlinear simulation results for the continuous vertical gust input .............. 109

5.3 Wind tunnel test results for the continuous vertical gust input ....................... 110

5.4 Nonlinear simulation results for the continuous horizontal gust input ............ 115

5.5 Wind tunnel test results for the continuous horizontal gust input ................. 115

5.6 Nonlinear simulation results for the single wave vertical gust ....................... 121

5.7 Wind tunnel test results for the single wave vertical gust input ...................... 121

5.8 Nonlinear simulation results for the single wave horizontal gust input .......... 126

5.9 Wind tunnel test results for the single wave horizontal gust input ............... 126
LIST OF FIGURES

1.1 First aerial refueling ...................................................................................................... 2
1.2 Grappled-line looped hose method for aerial refueling ................................................. 3
1.3 Probe-drogue aerial refueling method for jet airplane .................................................. 4
1.4 Probe-drogue based aerial refueling systems ............................................................... 8
1.5 Autonomous high-altitude refueling program by NASA ............................................. 12

2.1 System development procedure .................................................................................. 17
2.2 ASDRS V1.0 test ........................................................................................................ 19
2.3 Configuration of ASDRS V1.0 .................................................................................. 19
2.4 Comparison of conventional drogue and actively stabilized drogue ......................... 22
2.5 Wing control mode ..................................................................................................... 23
2.6 Drawing of actively stabilized drogue ......................................................................... 25
2.7 ASDRS Test Rig Configuration .................................................................................. 27
2.8 Drogue manufacturing process .................................................................................. 28
2.9 Drogue with 4 wings ................................................................................................. 29
2.10 Wing manufacturing process ................................................................................... 30
2.11 Wing calibration ...................................................................................................... 31
List of Figures -- Continued

2.12 Wing position from the front view ................................................................. 31

2.13 ASDRS avionics system configuration .......................................................... 32

2.14 ASDRS avionic system components ............................................................ 33

2.15 Assembled ASDRS avionics unit ................................................................. 34

2.16 Communication configuration of ASDRS avionics ...................................... 35

2.17 LabVIEW front panel for ground control station ....................................... 40

3.1 Configuration of rigid link system ............................................................... 42

3.2 Bifilar torsional pendulum method .............................................................. 43

3.3 Moment of Inertia measurement ................................................................. 43

3.4 Rigid link system coordinate for the wind tunnel test ............................... 45

3.5 Rigid link system coordinate for the numerical simulation ....................... 47

3.6 Definition of external loads ........................................................................ 55

3.7 Drogue Lift Coefficient .............................................................................. 57

3.8 Drogue Drag Coefficient ............................................................................ 57

3.9 Drogue Moment Coefficient ....................................................................... 58

3.10 Aerodynamic force of Bar ......................................................................... 59

3.11 Tangential and normal force coefficients for Bar .................................... 59
List of Figures -- Continued

3.12 Shape of 5-hole probe ............................................................................................... 60
3.13 Block diagram of 5-hole probe .................................................................................. 61
3.14 Manufacture and calibration of 5-hole probe............................................................. 61
3.15 Pitot port calibration data ......................................................................................... 62
3.16 Side port calibration data ........................................................................................ 62
3.17 Vertical port calibration data .................................................................................... 63
3.18 Artificial gust generator ........................................................................................... 64
3.19 Configuration of artificial gust generator ................................................................. 64
3.20 Commands for the artificial gust generator ............................................................... 65
3.21 Measured gust properties at 600pa and 1000Pa......................................................... 67
3.22 Peak value of single wave gust input ....................................................................... 68
3.23 $\text{RMS}_{AC}$ value of continuous wave gust input .................................................... 68
3.24 Drogue trajectories subject to the single wave vertical gust .................................... 69
3.25 Drogue trajectories subject to the single wave horizontal gust ............................... 69
3.26 Drogue trajectories subject to the continuous wave vertical gust ............................ 70
3.27 Drogue trajectories subject to the continuous wave horizontal gust ....................... 70
3.28 Actuator step response ............................................................................................ 71
List of Figures -- Continued

4.1 Input command signal for the open-loop test ............................................................. 77
4.2 Angle responses to vertical command ........................................................................ 77
4.3 Open-loop responses of Angle 3 to vertical command ............................................... 78
4.4 Open-loop response of the drogue Z position to vertical command ......................... 78
4.5 Open-loop response of Angle 2 by horizontal command ......................................... 79
4.6 Open-loop response of Angle 4 to horizontal command ......................................... 79
4.7 Comparison of Angle 1 and Angle 5 at steady conditions......................................... 81
4.8 Comparison of Angle 1 responses to vertical command ......................................... 82
4.9 Comparison of Angle 5 responses to vertical command ......................................... 82
4.10 Comparison of Angle 2 responses to horizontal command ..................................... 83
4.11 Comparison of Angle 6 responses by horizontal command .................................... 83
4.12 Pole-Zero map of vertical channel transfer function .............................................. 88
4.13 Pole-Zero map of horizontal channel transfer function ......................................... 88
4.14 Vertical channel step response of the perturbation model ....................................... 90
4.15 Horizontal channel step response of the perturbation model .................................. 91
4.16 Damping ratio of Poles and Zeros of the perturbation model ................................. 91
4.17 Frequency of Poles and Zero of the perturbation model ......................................... 92
List of Figures -- Continued

4.18 Transfer function gain of the perturbation model ..................................................... 92

4.19 Controller block diagram .......................................................................................... 94

4.20 Root-Locus Diagram with a PD controller ............................................................... 95

4.21 Bode plot with PD controller .................................................................................... 96

4.22 Single wave gust responses ....................................................................................... 97

4.23 Continuous wave gust responses .............................................................................. 98

4.24 Gain scheduling curve ............................................................................................. 100

4.25 Pole-Zero map of horizontal channel transfer function via SI technique .......... 102

5.1 Drogue acceleration for the vertical continuous wave gust ..................................... 106

5.2 Drogue trajectory for the vertical continuous wave gust ......................................... 108

5.3 ASDRS performance comparison for the continuous vertical gust input ............... 110

5.4 Acceleration comparisons for the horizontal continuous wave gust .................... 112

5.5 Trajectory comparisons for the horizontal continuous wave gust .......................... 114

5.6 ASDRS performance comparison for the continuous horizontal gust input .......... 115

5.7 Acceleration comparisons for the single wave vertical gust ................................... 118

5.8 Trajectory comparisons for the single wave vertical gust ....................................... 120

5.9 ASDRS performance comparison for the single wave vertical gust input ............... 121
List of Figures -- Continued

5.10 Acceleration comparisons for the single wave horizontal gust .............................. 123

5.11 Trajectory comparisons for the single wave horizontal gust ................................. 125

5.12 ASDRS performance comparison for the single wave horizontal gust ...................... 127
CHAPTER 1

INTRODUCTION

1.1 Historical perspectives of aerial refueling

1.1.1 Beginning of aircraft and aerial refueling

Since the Wright brothers’ first successful human flight, aeronautical engineers have been pushing the aircraft performance limit. There are several key performance parameters: speed, altitude, endurance and range. More powerful engine and better aerodynamic design enables to design faster and higher aircrafts. Development of a jet engine accelerated the new record racer. From 1950’s to 1980’s, a lot of jet aircraft had been developed for new speed and altitude record breaks. Nowadays, the speed and altitude record racers were declined as they reached the physical limitations arising from aerodynamics heating, huge drag force, and propulsion system capability. Modern aircraft performance is bounded for 18 km altitude and Mach 2.5 except for a few of specialized aircrafts. In other aspects of aircraft performance, range and endurance are also key indices. These performance indices are determined by aircraft weight, fuel capacity, aerodynamic and propulsion properties. Basically, longer flight needs more fuel. An obvious solution is to refuel during the flight. It is called ‘Aerial Refueling’ or ‘Inflight Refueling’.

Engineers tried to overcome the range and endurance limitations using aerial refueling. Capt. Lowell H. Smith and Lt. John P. Richter performed the first aerial
refueling on 27 June 1923 [1] shown in Fig. 1.1. The DH-4B biplane remained aloft over the skies of Rockwell Field in San Diego, CA for 37 hours. The tanker hung down the hose and nozzle on the air, and the receiver crew caught the dangling nozzle by hands, plugged in the fuel tank, and refilled the while flying. This performance was almost acrobatic. After the first aerial refueling, numerous aerial refueling attempts were made for new endurance records and an aircraft could endure more than twenty days on air. But the method was too complicated and dangerous for a practical purpose.

![First aerial refueling](image)

**Figure 1.1 First aerial refueling**

First practical aerial refueling system was Grappled-Line Looped-Hose method by Cobham’s Flight Refuel Ltd [2]. Fig. 1.2 shows the in-flight photo of the method with the connection procedure. This method was commercially used for crossing the Atlantic Ocean. From August 5 to October 1, 1939, sixteen cross Atlantic flight were made by Empire flying boats, with fifteen crossings using looped-hose method. After sixteen crossings, further trials were suspended due to the World War II.

During World War II, the range of the allied forces’ fighters was shorter than that of the bombers. Many bombers were shot down by German fighters without the fighters’
cover in the skies of Europe. They considered the aerial refueling method, but it was too difficult to adapt on the fighters in 1940s. Grappled-line Looped-hose method is only one practical refueling method in 1940s. This limitation was overcome by a jettisonable fuel tank and development of long range fighters like P-51. But, the air force needed to solve this problem for the future warfare.

Figure 1.2 Grappled-line looped hose method for aerial refueling

1.1.2 Aerial refueling in jet age

At the end of World War II, a German jet-fighter appeared. The speed and altitude of the fighter jet whelmed the propeller based fighter performance. After World War II, many countries developed jet planes. Flying fast and high aspects of jet plane satisfied the performance demands, but short range and endurance due to high fuel consumption rate appeared as a critical limitation of jet planes. So engineers looked for the solutions
from aerial refueling. The looped-hose method required the large internal space for tankers and receivers, which was impossible to apply on the jet fighters. Cobham FRL improved disadvantages of the looped-hose method, resulting in Probe-Drogue method.

**Figure 1.3 Probe-drogue aerial refueling method for jet airplane**

The probe-drogue method [2], shown in Fig. 1.3, consists of hose-drogue on a tanker and a probe on a receiver. A drogue is a shuttlecock shape device installed at the end of hose, and it produces drag force for stability. A probe is a long pipe installed on the nose section of the receiver, and it couples with the drogue for refueling. The refueling process begins with the tanker deploying the hose with the drogue. The receiver pilot flies toward the drogue and couples with the probe. This procedure was adopted on a modified Lancaster tanker and a modified Meteor III, and the first flight test conducted in 1949 with the record of more than 12 hours and 3600 miles. Figure 1.3 shows the photo of the probe-drogue method using Lancaster tanker and Meteor III receiver.

After the WWII, US NAVY and USAF searched for aerial refueling methods. In the earlier days, they adapted the probe-drogue method. While US NAVY was satisfied with the probe-drogue method and continued to improve the system, USAF looked into a different procedure. In the late 1940s, USAF requested new aerial refueling methods for a
faster fuel transfer rate to Boeing Aircraft Company. Accordingly, Boeing developed the flying-boom method which uses a controllable telescoping pipe installed on the rear of the tanker to transfer the fuel to the receiver. USAF compared two refueling methods and selected the flying-boom method. This was because USAF long distance nuclear bomber fleets were the most important interest of the Air Force with the beginning of the Cold War era. The pilot of a large bomber less favored the probe-drogue method as it required difficult maneuvering flight. Moreover, the flying-boom method transfers the fuel at a much faster rate per coupling, which was a big advantage for the bomber fleets. Thus, USAF selected and continued to use the flying-boom method for their refueling purpose.

On the other hand, US NAVY chose the probe-drogue method for their refueling method and made improvements over the time. The probe-drogue method allows multiple points refueling as well as use of buddy-stores. These advantages are important for NAVY for increasing the capability and flexibility to carry out NAVY specific missions. The carrier based fighter fleets are majority for US NAVY, while a large tanker with flying boom can’t take off on a carrier.

Consequently, both the probe-drogue method and the flying-boom method have been used around the world. Each method has advantages and disadvantages. The flying-boom method is easy for the receiver since a designated boom operator in the tanker controls the boom for connection. The flying-boom method is typically used to refuel bombers and transport planes for its fast fuel transfer rate, but requires different settings to refuel fighters. It also cannot be used for fueling helicopters and multiple aircrafts. It is more expansive and complicated. In contrast, the probe-drogue method can refuel helicopters and multiple aircrafts, and it is much cheaper and simpler. However, the
method is less stable in turbulence flight condition thus requiring significant flying effort by the receiver pilot. The fuel transfer rate is much slower comparing to that of the flying-boom method. Table 1.1 shows the comparison between the flying-boom and the probe-drogue refueling method [3].

Table 1.1 Comparison between the flying-boom and the probe-drogue refueling method

<table>
<thead>
<tr>
<th>TYPE</th>
<th>FLYING-BOOM</th>
<th>PROBE-DROGUE</th>
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| Configuration | Tanker – Boom  
|            | Tanker – Hose – Drogue               |                                     |
|            | Receptacle – Receiver                | Tanker – Probe – Receiver           |
| Advantage  | ✓ Higher fuel flow rates             | ✓ Simple & cheaper tanker           |
|            | ✓ More easy to connect               | ✓ Multipoint hose-drogue            |
|            | ✓ Cost of boom operator              | ✓ Helicopter refuel                 |
|            | ✓ Complexity of tanker               | ✓ No boom operator                  |
|            | ✓ One receiver refueling at a time.  |                                     |
|            | ✓ No helicopter refuel               |                                     |
| Disadvantage | ✓ Low fuel flow rate                 | ✓ Drogue subjects to turbulence     |
|            | ✓ Drogue subject to damage by poor flying technique |
| Service    | ✓ US Air Force                       | ✓ US Navy                           |
|            | ✓ Japan                              | ✓ US Marine Company                 |
|            | ✓ Netherland                         | ✓ NATO                              |
|            | ✓ Israel                             | ✓ Russia                            |
|            | ✓ Turkey                             | ✓ Many other countries              |

1.1.3 Modern aerial refueling

The flying-boom method and the probe-drogue method are concurrently used around the world. The selection is made according to the types of fighter plane. The fighters that are designed for USAF are compatible with the flying-boom method. Thus,
F-4, F-15, F-16 and F-22 all use the flying-boom method. Those countries which operate these fighters thus need the tanker with flying-boom. As a result, USAF, Israel, Netherland, Turkey and Japan operate these tankers. But it is much more costly and less effective compared to the probe-drogue method. To overcome these limitations, three solutions are invented for F-16. First, Israel developed a fixed-probe on F-16’s nose. Second, Cobham designed an extension tank that includes a retractable probe. Third, Lockheed developed F-16 Block Conformal Air Refueling Tank System (CARTS) per a customer’s request.

On the other hand, US NAVY, USMC, and many other countries adopted the probe-drogue method for several reasons. It is cheaper and effective for fighter fleets with multiple refueling points, and it can be applied to existing transport aircraft and helicopter fleet. It can be also operated on a carrier based tankers. In addition, it is possible to add a buddy store for a fighter to a fighter refueling.

Nowadays, USAF operates more than 80% of the tanker fleet in the world. Thus, flying-boom method is the dominant method of aerial refueling service. However, it is difficult for USAF to keep the large tanker fleet with its own budget after the Cold War. To share the budget and cost for operating tanker fleets, some of the USAF flying-boom tankers adapt the boom-drogue to provide the service for US NAVY, USMC, and NATO. For its advantages, the probe-drogue method is being examined by the US Congress to be considered for USAF’s refueling method as well.

Fig. 1.4 shows some photos of components and different arrangements for the probe-drogue refueling system.
1.2 Literature review

Aerial refueling has been performed routinely over the past 90 some years, yet not much significant research and development work was carried out until recently. Process automation exploded in many engineering fields with the advancements in electronics and information technologies. Now, unmanned aerial vehicles (UAVs) dominating the sky, autonomous refueling attracted significant interests from the research and development community [4~7]. In this section, public domain literature was reviewed focusing upon the probe-drogue refueling. There are two research areas. First, modeling

Figure 1.4 Probe-drogue based aerial refueling systems
and simulation for the hose-drogue with a tanker and a receiver’s flow-field is substantially studied. Secondly, the drogue stabilization methods subject to external disturbances has attracted for research and development. Although significant research is carried out and in progress for the detection, control and guidance of the tanker and the receiver, it is not included in this review as it falls beyond the scope of the present study.

1.2.1 Dynamic modeling of hose-drogue refueling system

Dynamic modeling and simulation of the aerial refueling hose-drogue dynamics is composed of two major components. The first component is a dynamic model for the hose-drogue dynamics. The dynamic model represents the hose and the drogue physical motion characteristics. There are two types of modeling approach. One approach is the elasto-dynamic hose model based on finite element method (FEM), and the other approach is the lumped mass hose model with rigid link kinematics. The second component is the flow field model between the tanker and the receiver. Three methods were developed; the finite volume/element method (FVM/FEM) based computational fluid dynamic (CFD) analysis, the vortex panel method, and the Helmholtz horseshoe-vortex model.

The earliest work on analytical modeling of the refueling hose-drogue dynamics was by Eichler, which dates back to 1978 [8]. The hose-drogue system is modeled based on a classical 2-D cable theory with a constant drag force acting on the drogue end, and a set of governing equations of motion was derived in partial differential equation form. Three dynamic effects were studied: wingtip vortex, wing vibration, and wind gusts. The report concludes that; 1) the effect of the vortex was negligible, 2) the wing vibration showed some gain effect at the drogue end ranging from 0.33Hz ~ 6.8Hz at resonant
frequencies, 3) the wind gust effects showed a range of maximum amplitude from 1.1ft to 5.5ft for the nominal sine wave.

Bloy et al developed a static model of the hose-drogue to provide data in a real-time flight simulation of air-to-air refueling [9]. Finite element method was applied to the hose with a constant drag force acting on the drogue end. The model predicts; 1) the interference effect of the flow around the nose of receiver on the pre-contact position of the drogue, 2) the hose shape and the loads induced on the receiver probe during contact. The results from the static model were compared with the dynamic model by Eichler [8], and it concludes that the static model is less satisfactory at high receiver closure rates.

Zhu et al developed a new beam element model for the finite element analysis to overcome the cable theory when applied to model refueling hose-drogue dynamics [10]. When the hose slackens under dynamic loading, the classic cable theory suffers the singularity problem. A new three-node locking-free curved beam element was developed for finite element modeling of refueling hose-drogue. Analyses based on numerical simulation results were presented including; 1) the oscillation of hose due to the disturbance from the tanker and the vortex-induced velocity, and 2) a receiver coupling with a hose reel malfunction.

John C. Vassberg et al conducted the researches about KC-10’s refueling drogue from 2002 to 2005. These research [11~14] utilized the DACVINE code for a tanker and a receiver’s flow filed. The DACVINE code is a high order panel method. It calculates a flow-field around the KC-10 tanker and F/A-18D receiver airplane. Two airplanes’ flow fields superpose, and this flow field affects the hose-drogue movement. Especially, the combined flow-field shows the aerodynamic effectiveness of receiver’s fore-body effect.
toward the drogue movement. The hose model is the lumped mass model with longitudinal elasticity. These papers show ‘hose whip’ phenomena by the receiver contact. To reduce the hose whip phenomenon, a reel take-up system was suggested to control the hose tension after the contact [12, 13]. A number of closure techniques have been analyzed and the requirements for optimal probe-drogue coupling were discussed [14].

NASA Dryden flight research center focused upon autonomous aerial refueling flight tests since 2004 [4~7]. The flight test research conducted with DARPA in 2006 was autonomous aerial refueling demonstration (AARD) using two modified F/A-18 aircraft [5]. In these test flights, vision based sensors were used to detect the drogue motion and to guide the receiver. After this demonstration, NASA performed autonomous high-altitude refueling (AHR) program using two modified RQ-4 global hawk UAVs, shown in Fig. 1.4, in 2012 [15].

Autonomous refueling project is now in progress for unmanned X-47B by US NAVY, Northrop Grumman Corporation, and DARPA. These series of research indicate the importance of the autonomous aerial refueling technique for future unmanned aerial vehicles (UAVs) [16].

Kamman et al [17, 18] studied modeling of towed cable since 1990. This cable model was adopted to develop a hose-drogue model. Ro et al [19~24] carried out a series of research associated with the hose-drogue refueling system since 2007. Their research started with the characterization of drogue aerodynamic through wind tunnel testing [19], followed by modeling and simulation work for the hose-drogue dynamics. The hose was divided into a finite number of rigid segments which were connected by ball-and-socket
jointed. Each segment mass was lumped at each joint with a refueling drogue mass added to the last segment end [20]. The flow-field model by Ro et al includes two elements. The first element is the horse-shoes vortex model for tanker and receiver wake vortex and wing tip vortex. The second element is the CFD result around the fore-body portion of the receiver aircraft. The CFD result is an important around the receiver nose during coupling. As the receiver closes in to the drogue, the drogue would move up and away from a steady position due to the receiver’s fore-body effect. The phenomenon was modeled by combining the flow-field model, i.e. superposing the CFD results and the tanker horse-shoe vortex model. The post contact hose whipping motion was also investigated and an alternative hose-tension control scheme was suggested. Overall, the numerical simulation analysis showed excellent agreement with the NASA flight test results [20].

Figure 1.5 Autonomous high-altitude refueling program by NASA
1.2.2 Various methods for drogue stabilization

The results from the hose-drogue dynamic modeling and simulation as well as the flight tests by NASA signify the instability of the drogue motion. This instability has been overlooked for the last 60 years as the human pilot deals with it reasonably with efforts and training. Recently, increasing number of UAVs and never ending requirements for them encourages looking into the autonomous aerial refueling technologies for UAVs [25]. The probe-drogue method would be most likely for UAVs’ refueling, but the unpredictable drogue motion makes it difficult for autonomous operations. For this reason, drogue stabilization was considered. Table 1.2 summarizes the related US Patents [26~29].

Table 1.2 List of US Patents on drogue stabilization

<table>
<thead>
<tr>
<th>Inventor</th>
<th>Stabilization Concept</th>
<th>Patent No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith Aerospace LLC (GE)</td>
<td>1) Rotating mass - passive</td>
<td>US20100237196A1</td>
</tr>
<tr>
<td></td>
<td>2) Controllable wing - active</td>
<td></td>
</tr>
<tr>
<td>GE Aviation Systems LLC</td>
<td>Controllable duct</td>
<td>US8186623B2</td>
</tr>
<tr>
<td>UAV Refueling Inc.</td>
<td>Compressed air thruster</td>
<td>US6926049B1</td>
</tr>
<tr>
<td>The Boeing Company</td>
<td>Canopy control</td>
<td>US8162261B2</td>
</tr>
</tbody>
</table>

As GE Aviation System and Smith Aerospace merged in 2007, GE Aviation owns three patents. The first stabilization concept is by a rotating mass via ramming air turbine. This rotating mass with turbine increases the stability of the drogue via gyroscopic effect. It is a passive stabilization method and may increase the inertia but not damping property [26]. The second is to attach controllable wings on the drogue for stabilization. This idea is almost the same concept as the present research. The third is a controllable duct for stabilization. A set of air duct is placed on the drogue to provide aerodynamic control
force [27]. The fourth is by air thruster using compressed air. Four thrusters installed around the drogue hub generate lateral force [28]. The fifth is via manipulating the drogue canopy as asymmetric canopy shape makes required side force [29].

While these patents were just casting of possible concepts, Boeing actually carried out some experimental study to measure the lateral force via drogue canopy manipulation in the wind tunnel [30]. Using the experimentally obtained data, a simple rigid-link dynamic model of the hose-drogue system was developed. This model was used to conceptually study the active drogue control based on the Linear Quadratic Regulator (LQR) design technique.

1.3 Problem description

1.3.1 Problem statement

A refueling drogue is prone to instability subject to external disturbances. While the human pilot handles this with repetitive training and intelligence, it imposes significant technical challenges for refueling UAVs. Autonomous refueling of UAVs has been studied largely from the point of view for the receiver guidance problem. Flight tests were carried out by NASA for autonomous refueling using various vision based sensors, but the success rate was questionable.

The objective of this research is to carry out a comprehensive investigation of active drogue stabilization concept. This includes conceptual design of a controllable drogue and a prototype manufacturing, mathematical modeling based on analyses and experiment, model-based control law synthesis, and validation and verification via the nonlinear closed-loop simulation test and the dynamic wind tunnel testing of the prototype.
1.3.2 Contribution of the present work

The present research is a pioneering work in active drogue stabilization. The study includes design, test and evaluation of a controllable drogue based on analyses and wind tunnel experiments.

The main contributions of the present study are:

- proposition of design guidelines for actively stabilized drogue refueling system (ASDRS) using aerodynamic control surfaces
- experimental verification of the proposed concept through a prototype fabrication and dynamic wind tunnel testing

Within the above main contribution, the detailed original contributions may be summarized as follows:

- understanding of proper aerodynamic control surface configuration and arrangement via wind tunnel testing
- a novel design method for onboard avionic system for active stabilization of a refueling drogue
- a novel dynamic testing method in a wind tunnel to study hose-drogue motion under turbulent condition
- development of a mathematical model of the representative hose-drogue refueling system for wind tunnel testing
- design and validation of feedback control law for active drogue stabilization based on model-based control law synthesis

The result of this study may serve two applications. The experimental results presented in this study are based on approximately 1/3 scale of the actual full size drogue
used in manned aircraft refueling operation. Therefore, the results from this study may actually be used for autonomous refueling of small size UAVs. On the other hand, it can be used as a starting point to design and fabricate a full size ASDRS for the current manned aircraft refueling system.
CHAPTER 2

DESIGN AND FABRICATION

2.1 Design overview

Design of an engineering system is a complicate process, and it must be followed by requirement analysis and systematic development procedure. In this research, the system development procedure is established as shown in Fig 2.1. As indicated, it is an iterative procedure and the results presented in this research are based on the second version of the system design. The first version of the system was to mainly verify the ASDRS design concept based on the aerodynamic control surface in the preliminary design stage [23]. Major changes in the second version are discussed in detail shortly.

![Figure 2.1 System development procedure](image-url)
2.1.1 ASDRS design

Most important and practical design requirement in a nutshell for the ASDRS design is minimizing the cost necessary for being implemented into the exiting hose-drogue-probe refueling system as suggested by US NAVY [25], while reducing the motion subject to external disturbances. Setting this as the goal of the ASDRS design, four requirements are established as summarized in Table 2.1.

<table>
<thead>
<tr>
<th></th>
<th>ASDRS Design Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reduction of the magnitude and frequency of the drogue motion</td>
</tr>
<tr>
<td>2</td>
<td>New drogue must be receptible in the current drogue pod</td>
</tr>
<tr>
<td>3</td>
<td>New drogue must be a standalone system</td>
</tr>
<tr>
<td>4</td>
<td>System operation and status monitoring should be wireless</td>
</tr>
</tbody>
</table>

The first requirement is the terminal goal, which is obvious that the new design must reduce the drogue motion subject to external disturbance. Secondly, the new design must be receptible to the current pod, whether it is an external pod or internal to a tanker, because the drogue should be kept inside in stowed condition. This requires that the size of aerodynamic control surface size be limited and must be foldable. In addition, to avoid a potential collision between the control surfaces on the drogue and a receiver fore-body as well as to be able to be stowed in the pod, the size of the control surfaces should be limited at most to the outer diameter of the existing drogue. Thirdly, the new drogue system must be an independent system. This means that the new drogue is a simple drop-in replacement of the existing drogue, so the system hardware and software should be a standalone unit. The fourth requirement should be remote accessibility and real-time monitoring of the system operation.
With these front end design requirements, a preliminary study was carried out primarily to investigate the feasibility of aerodynamic control surface for active drogue stabilization. Fig. 2.2 and 2.3 shows the overall system setup for the preliminary study, and it is referred as the design version 1.0 hereafter.

![Figure 2.2 ASDRS V1.0 test](image)

![Figure 2.3 Configuration of ASDRS V1.0](image)

The key characteristics of the version 1.0 are as follow:
• control surface span longer than the drogue outer diameter

• motion variables were measured via four sets of potentiometers, and the signals were differentiated numerically for control purpose

• all computations, command and control were performed in the ground control station computer

• control law design based on experimentally obtained (via system identification technique) transfer function model

Some important discoveries made from the version 1.0 tests can be summarized as:

• aerodynamic control surface provide sufficient control authority for drogue stabilization

• the control surface arrangement influences the drogue stabilization; saltire configuration (×) showed superior performance to cross (+) configuration

• control signal delay from the ground control station computer is substantial and influences closed-loop control design

Based on the results from the version 1 design study, a new ASDRS drogue is fabricated, and major upgrades are made to the test rig focusing upon the design requirements listed in Table 2.1. The new ASDRS system with the upgraded test rig referred as the design version 2.0 hereafter. Fig. 2.3 shows the overall system setup for the design version 2.0. The key characteristics of the version 2.0 design are summarized as follow:

• control surface span within the drogue outer diameter
• standalone onboard avionic system based on inertial measurement unit (IMU) for active drogue stabilization
• quantification of turbulence produced by the gust generator
• mathematical modeling of the ASDRS test rig based on multi-body dynamic analysis, and development of numerical simulation program
• model-based control law design and implementation into the onboard embedded system

2.1.2 System configuration

As discussed in Chapter 1, there are five different ideas patented for active drogue stabilization, yet it is common that the actuation energy source being the aerodynamic dynamic pressure. This study focuses on the aerodynamic control surface or controllable wing for actuation device.

Fig. 2.4 shows the configuration of a conventional drogue and an actively stabilized drogue studied in this research. The conventional drogue composes of a front hub cap, rear hub cap, struts, and canopy. The front hub cap connects with a refueling hose. The rear hub cap holds the struts, and it couples with a probe on the receiver plane. The struts make a shuttlecock shape to guide the probe for connection, and the assembly is being deployed from and stowed in the pod. The canopy is made of fabric materials to be stowed inside the drogue pod, and it generates a drag force. This drag force exerts tension to the hose end, while providing stabilizing effect on the drogue. Thus, it is often referred as the aerodynamically stabilized drogue. This is true in a passive and static sense, but should be distinguished from the ASDRS. When the conventional drogue is subject to atmospheric turbulence and approaching receiver plane, it undergoes random
dynamic motion often resulting in mission failure. The idea of the ASDRS is to actively stabilize this random dynamic motion. Aerodynamic control surface or simply wing being the most efficient device in moving fluid it is believed to be the most efficient means of actuation for the ASDRS.

![Comparison of conventional drogue and actively stabilized drogue](image)

**Figure 2.4 Comparison of conventional drogue and actively stabilized drogue**

In this study, a set of four foldable wings are implemented around the rear hub cap. These wings are individually actuated by a servo motor powered by onboard batteries, and they are setup to generate three directional forces and rolling moment. Based on the design version 1.0 testing results, the saltire configuration (×) is adopted. Fig. 2.5 illustrates the coordination of each surface to generate four different modes of actuation: drogue roll control, horizontal and vertical control, and drag control. Primary control of interest for active drogue stabilization is the horizontal and vertical control since the drogue motion under external disturbance is dominant in the lateral plane. Therefore, the roll and the drag mode control are excluded in the present study, and the dynamic wind tunnel tests are all performed with the drogue roll motion being fixed. Controlling the drogue rolling motion would be interesting but it requires further
understanding of hose and drogue coupling nature. The drogue indeed undergoes rolling motion but it is generally limited during deploying and stowing, and stays relatively stationary during refueling process. The drag control mode is also interesting and needs further research for its use because increasing the drag on the drogue would increase the tension in the hose resulting in the vertical position change of the drogue relative to the tanker [20].

![Wing control mode](image.png)

**Figure 2.5 Wing control mode**

For active control, sensing the drogue motion is crucial. Since accurate measurement of displacements and velocities is difficult to achieve during flight condition, acceleration feedback is considered for this study. Active drogue stabilization then becomes a problem of regulating the acceleration associated with the drogue dynamic motion under external disturbance. Since the drogue undergoes 3-dimensional motion, an attitude and heading reference system (AHRS) is essential. An AHRS is a fusion sensor system that measures accelerations and rotations in three orthogonal axes and computes in real-time the velocity and the attitude when it undergoes dynamic motion. Since the gravitational acceleration needs to be compensated for acceleration feedback control for drogue stabilization, inertial measurement unit (IMU) alone is not sufficient. For this reason, an AHRS is adopted as the primary sensing device as it
enables to compensate the gravitational acceleration from the drogue dynamic motion. In this research, a micro-electro-mechanical- system (MEMS) based AHRS is chosen, which is manufactured by SBG System in France.

2.1.3 System dimension

The dimension of ASDRS in this study is chosen as approximately a 1/3 scale of the full size refueling drogue currently used for manned aircraft. This is to satisfy multiple purposes. The first and the most important purpose is to experimentally verify the ASDRS concept, so the drogue size chosen is able to be tested in the Advanced Design Wind Tunnel (ADWT) at Western Michigan University. Its test section is 4ft width and 3ft height, so the drogue and the dynamic test rig should be fitted in the test section. Secondly, the current design is to be directly targeted for application to autonomous aerial refueling between UAVs; particularly for those small UAVs of 6ft to 8ft wingspan and weighing 30lb to 60lb.

Table 2.2 Drogue dimension

<table>
<thead>
<tr>
<th>Drogue Dimension</th>
<th>Unit (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strut outer diameter</td>
<td>230.8</td>
</tr>
<tr>
<td>Canopy outer diameter</td>
<td>223.6</td>
</tr>
<tr>
<td>Canopy inner diameter</td>
<td>140</td>
</tr>
<tr>
<td>Hub diameter</td>
<td>63.5</td>
</tr>
<tr>
<td>Hub length</td>
<td>167</td>
</tr>
<tr>
<td>Strut length</td>
<td>152.7</td>
</tr>
<tr>
<td>Length</td>
<td>320</td>
</tr>
<tr>
<td>Wing Span</td>
<td>228.3</td>
</tr>
<tr>
<td>Wing chord</td>
<td>46</td>
</tr>
<tr>
<td>Avionics bay diameter</td>
<td>60.9</td>
</tr>
<tr>
<td>Avionics bay length</td>
<td>75</td>
</tr>
</tbody>
</table>
Asides from the drogue dimension, the drag is produced by the drogue is also an important parameter. If the drag to weight ratio is too small, the wings for ASDRS must have high angle of attack to produce necessary control force. This might cause the wings to stall and lead the drogue to be unstable. If the drag to weight ratio is too large, the drogue needs large canopy size to maintain an appropriate tow angle. Based on the previous research [19, 20], the drogue is designed to produce 5lbs of drag at 70mph and to weight approximately 1.5lbs. Table 2.2 details the drogue dimension, and Fig. 2.6 shows a cross sectional view of the drogue for this study.

Figure 2.6 Drawing of actively stabilized drogue
2.2 ASDRS hardware

2.2.1 Test system overview

The goal of this research is not only the conceptual analysis and design of actively stabilized drogue but also, and more importantly, the experimental verification of the proposed concept. In addition to design and fabrication of the ASDRS prototype, a test rig system is also designed and fabricated to verify the ASDRS functions. This test rig represents a hose-drogue refueling system in terms of a rigid bar with the drogue connected at one end while the other end is connected to the ceiling of the wind tunnel test section. Each connection is composed of universal joint thus the test rig has total 4 degrees-of-freedom system, and potentiometers are installed at each joint to measure the relative angles. From these angles, the position and the attitude of the drogue can be calculated. For the design version 1.0 study, these signals were numerically differentiated to obtain the acceleration of the drogue to investigate the feasibility of the wing based active stabilization of the drogue motion. In addition to the rigid bar-drogue system, an artificial gust generator is the important component of the test-rig system. The gust generator creates unsteady flow field in the wind tunnel test section to induce the unsteady drogue motion.

It is important that the required experiments to verify the proposed ASDRS must be conducted in a coordinated manner. All commands, communications, controls, and data collection must be well planned. Fig. 2.7 shows a schematic diagram of the test rig system configuration for this study. All collected data are sent to ground control station (GCS) to be recorded and analyzed for the ASDRS performance evaluation. Also, the GCS transmits the control servo signals to the ASDRS onboard controller. In the
following sections, fabricating the necessary components of the ASDRS test system is briefly discussed.

![ASDRS Test Rig Configuration](image)

**Figure 2.7** ASDRS Test Rig Configuration

### 2.2.2 Actively stabilized drogue

The drogue main body was first to be fabricated for this study, and the manufacturing process is illustrated in Fig. 2.8. Each component is made out of aluminum from lathe, milling and laser cutting process as shown in Fig. 2.8(a), and the front hub cap with weight saving holes shown in Fig. 2.8(b). The drogue has 12 struts as shown in Fig. 2.8(c), and the drogue canopy is made using fabric material as shown in Figs. 2.8(d) and 2.8(e). Each strut is connected together via circular pins and flexible wires, and pin connected to the rear hub cap. The assembled drogue without the front hub cap is shown in Fig. 2.8(f). The wings are installed on the rear hub cap portion. An avionics unit is placed between the front and the rear hub cap.
Figure 2.8 Drogue manufacturing process

The proposed ASDRS has a set of four wings with symmetric airfoil section. They are made of ply-woods, metal, and servo actuators as shown in Fig. 2.9 and components are shown in Fig. 2.10. Ply-woods were laser cut as shown in Fig. 2.10(a) to make up wing components. Each wing is composed of two portions; stabilizer and control surface as shown in Fig. 2.10(b). These two parts are connected by a hinge pin and a servo actuator. Fig. 2.10(c) shows an assembled wing with a servo except covering
skin. The servo actuator, shown in Fig. 2.10(d) has 0.8kg-cm torque and 0.10sec/60degree speed. The servo is placed inside the control surface, not in the stabilizer portion to allow a larger control surface area for the given span and chord. Fig. 2.10(e) shows the four assembled wings. Calibration of each wing for deflection amount in angle to command input in pulse-width-modulation (PWM) signal is very important, and the calibration jig is also made as shown in Fig. 2.11, and a linearized calibration curve is obtained. The max angle of wing deflection is set to ±25degree. Four modes of command signals are programmed with respect to the drogue fixed coordinate system as shown in Fig. 2.12.
Figure 2.10 Wing manufacturing process
2.2.3 Avionics

The ASDRS avionic system is composed of AHRS unit, a radio-frequency (RF) modem, a microprocessor based controller, data logger, voltage regulator and batteries as shown in Fig. 2.13. The system resides in the avionics compartment of a cylindrical shape with 2.5in and 75mm, which is placed between the front and the rear hub cap.
The controller is a commercial-off-the-shelf (COTS), open-source electronics prototyping platform made by Arduino. There is a variety of platforms available, and Ardupilot Mega shown in Fig. 2.14(a) is chosen for this study because this board is typically used for unmanned vehicle autopilot system. The control board has 4 serial ports, 8 PWM inputs and outputs (I/O), 40 digital I/O, and 16 analog inputs. The serial ports are connected to data logger, AHRS unit, and RF modem. The eight PWM input and output ports are used to read the remote control (RC) receiver’s servo signals, and generate 4 control signals for servo actuators on each ASDRS wing. The AHRS sensor is made by SBG System shown in Fig. 2.14(c). It is connected with the control board via serial communication (RS-232) at 115.2 kbps baud rate. The sensor outputs are attitudes, angular rates, and accelerations with 100Hz update rate. The communication between a notebook computer based ground control station (GCS) and the ASDRS avionics is via RF modem shown in Fig. 2.14(d). It operates on 915MHz radio frequency with 250 kbps max baud rate, and communicates with the control board at 57600bps baud rate via bi-directional link. The data logger shown in Fig. 2.14(b) records all monitoring signals on a
micro-SD memory card at 25Hz update rate. The battery shown in Fig. 2.14(f) is a two cell lithium-polymer battery pack of 8.4volt and 800mAh capacity. Since the AHRS, the controller board, the RF modems and the servo actuators all operates on 5volt, a voltage regulator shown in Fig. 2.14(e) was used for the avionic system power source. The power system can supply the electric power more than one hour.

Figure 2.14 ASDRS avionic system components
Fig. 2.15 shows the assembled avionic unit for the ASDRS. The rear side avionic unit has 4 connectors for 4 servo actuators. Each connector has 3 wires; ground, power, and PWM signal. The power switch is located on the side of avionics unit. The avionic unit is covered with 2 pieces of thin ply-wood skin.

![Assembly of the avionic unit for ASDRS](image)

**Figure 2.15 Assembled ASDRS avionics unit**

### 2.3 ASDRS software

#### 2.3.1 Interface control document

Interface control document (ICD) defines the data communication format that among different digital devices. The communication for the ASDRS test system occurs among the AHRS unit, the controller unit, and the GCS computer, thus the ASDRS test system has three ICDs. Fig. 2.16 shows the means of communication among three components. The max output rate of the AHRS is 100Hz, but it is set to deliver at 50Hz to match the servo actuator PWM output frequency. Since the drogue’s natural frequency was found to be less than 10Hz from the design version 1.0 study, the controller main loop is also set to run at 50Hz. However, the communication between the onboard avionic system and the GCS is set at 25Hz of the RF modem speed.

The AHRS unit measures the three axes angular rates, accelerations, and magnetic heading of the drogue, and computes the attitude of a drogue. The ARHS outputs are the
time, the attitudes, the angular rates, and the accelerations of the drogue, and it the
ccontroller board receives these data according to Table 2.3 via a serial communication
(RS-232). The data are composed of the header, the body, and the ender, all in 48 bytes
length. The data type of the body is 4 byte single precision floating point follow IEEE
754 except the time. The AHRS data is converted for the gravity compensated
acceleration signal in the control board for feed-back purpose.

![Communication configuration of ASDRS avionics](image)

**Figure 2.16 Communication configuration of ASDRS avionics**

<table>
<thead>
<tr>
<th>Table 2.3 AHRS ICD (AHRS to Controller)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start byte</td>
</tr>
<tr>
<td>Header</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>Body</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>28</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>46</td>
</tr>
<tr>
<td>Ender</td>
</tr>
</tbody>
</table>
The uplink is defined as the data communication from GCS to the drogue with its data format is given in Table 2.4. It contains the command signals to 4 servos on the wings and 4 control gain values. The data type for this command is 2 byte unsigned integer. The wing command is composed of 4 different types control signals; PWM control, calibrated wing control, mode control and autonomous control. Table 2.5 shows these control types. The PWM control directly operates the each wing through PWM value. The servo actuators follow the PWM value within 1ms to 2ms. The wing control operates on each wing angle using the calibration data, and the output command is the desired wing angle in degree according to eq. 2.1. The mode control is the coordinated signals to the servo in relation to the vertical, horizontal, roll and drag mode controls of the drogue motion. The GCS sends these signals according to eq. 2.2, and the controller converts the signal to each servo actuators. The each mode angle is transformed to the wings angle according to eq. 2.3. The autonomous control is for the wing to be controlled by the internal control law by the ASDRS control logics.

**Table 2.4 Uplink ICD (GCS to Controller)**

<table>
<thead>
<tr>
<th></th>
<th>Start Byte</th>
<th>End Byte</th>
<th>Data Type</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Header</td>
<td>0</td>
<td>0</td>
<td>Byte</td>
<td>0xFF</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td>0x00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td></td>
<td>0xFE</td>
</tr>
<tr>
<td>Body</td>
<td>3</td>
<td>4</td>
<td>Unsigned Integer</td>
<td>wing 1</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6</td>
<td></td>
<td>wing 2</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>8</td>
<td></td>
<td>wing 3</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>10</td>
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<td>wing 4</td>
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<td>11</td>
<td>12</td>
<td>Integer</td>
<td>Gain 1</td>
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<td></td>
<td>13</td>
<td>14</td>
<td></td>
<td>Gain 2</td>
</tr>
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<td>16</td>
<td></td>
<td>Gain 3</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>18</td>
<td></td>
<td>Gain 4</td>
</tr>
<tr>
<td>Ender</td>
<td>19</td>
<td>19</td>
<td>Byte</td>
<td>0xAA</td>
</tr>
</tbody>
</table>
Table 2.5 Wings control methods

<table>
<thead>
<tr>
<th>Control methods</th>
<th>Range</th>
<th>Neutral point</th>
<th>Unit</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM control</td>
<td>1000 ~ 2000</td>
<td>1500</td>
<td>µs</td>
<td>Each wing PWM control</td>
</tr>
<tr>
<td>wing control</td>
<td>12500 ~ 17500</td>
<td>15000</td>
<td>0.01 deg</td>
<td>Each wing angle control</td>
</tr>
<tr>
<td>mode control</td>
<td>22500 ~ 27500</td>
<td>25000</td>
<td>0.01 deg</td>
<td>each mode control</td>
</tr>
<tr>
<td>auto control</td>
<td>30000 ~</td>
<td>-</td>
<td>-</td>
<td>embedded controller</td>
</tr>
</tbody>
</table>

Wing_command = 15000 + 100* Wing_angle

Mode_command = 25000 + 100* Mode_angle

Wing_1 = Horizontal_mode + Vertical_mode + Roll_mode + Drag_mode
Wing_2 = -Horizontal_mode + Vertical_mode + Roll_mode - Drag_mode
Wing_3 = -Horizontal_mode - Vertical_mode + Roll_mode + Drag_mode
Wing_4 = Horizontal_mode - Vertical_mode + Roll_mode - Drag_mode

The downlink is defined as the data communication from the drogue to GCS with the data format given in Table 2.6. This 40 byte data packet includes the drogue control methods, the status, the attitude, the angular rates, the accelerations, the wing angle commands, the horizontal command, the vertical command, and the time. The control method follows GCS commands, and this signal is returned to GCS for monitoring purpose. The drogue status represents the AHRS and the telemetry’s status. The attitude, the angular rates, and the accelerations come from the AHRS, and the 4 PWM signal represent the control methods for wing control methods. These signals are recorded on GCS computer for post analysis. GCS records all these data in addition to the gust generator outputs and the rigid bar angles from the potentiometers.
Table 2.6 Downlink ICD (Controller to GCS)

<table>
<thead>
<tr>
<th>Start byte</th>
<th>end Byte</th>
<th>Data Type</th>
<th>Data</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Header</td>
<td></td>
<td>Byte</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0xFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0x00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0xFE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>Control method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>Status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Body</td>
<td></td>
<td>Integer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>roll</td>
<td>rad/100</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>pitch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>yaw</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>Q</td>
<td>rad/sec/100</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>ax</td>
<td>m/sec²/100</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>ay</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>az</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>PWM 1</td>
<td>µs</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>PWM 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>28</td>
<td>PWM 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>PWM 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>Integer</td>
<td>Horizon mode</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>34</td>
<td>Vertical mode</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>36</td>
<td>-</td>
<td>reserve</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>38</td>
<td>Unsigned Integer</td>
<td>time</td>
<td>sec/100</td>
</tr>
<tr>
<td>Ender</td>
<td>39</td>
<td>0xAA</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3.2 Drogue operation software

The control logic to stabilize the drogue motion is developed using Arduino integrated development environment (IDE). The Arduino code is in C++ or C, and it is called the sketch in Arduino IDE. The source code is located at Appendix A. The main loop of the sketch is running at 50Hz to be synchronized with the AHRS and the servo actuator update rate. Every 2 cycles, the main loop receives and transmits the data with GCS.
Since the acceleration signals measured by the AHRS include the gravitational acceleration, it must be compensated to obtain the horizontal and vertical acceleration of the drogue motion only due to external disturbance. This compensation operation is done in the controller board after calculating the gravity vector using the attitude information from the AHRS. This compensated acceleration represents the pure drogue acceleration, and these signals are to be regulated according to the control logic to be discussed in the next chapter.

2.3.3 Ground control system software

GCS is based on a notebook computer which runs Laboratory Virtual Instrument Engineering Workbench (LabVIEW) software. A LabVIEW program is composed of two parts; a front panel and a block diagram. The front panel is graphical user interface window that displays and controls a process. The block diagram represents a source-code or the logics of the program.

GCS software is the LabVIEW program that has 4 major functions; controlling the gust generator, measuring the rigid bar angle, controlling and measuring the drogue motion, and recording and displaying the complete data of the ASDRS test system. The first function of GCS is the gust generator control. The gust generator has two plates that are connected with two servo actuators. GCS controls these plates using a PWM signal generator which is connected to the GCS notebook computer USB ports. The second function is measuring and displaying the rigid bar angle from the joint mounted 4 potentiometers. The third function is monitoring and controlling the drogue motion via the RF modem. When the drogue control is in active stabilization mode, GCS adjusts the internal controller gains and send these values via the RF modem. The fourth function is
recording and displaying the data associated with the drogue motion, gust generator control, and the rigid link angles. Each data has different sampling rates. All these data are sampled at different rates, so all data are down sampled to the drogue sampling rate of 25Hz for recording. The recorded data are used to analyze the drogue characteristics.

Fig. 2.17 shows the GCS’s a front panel, which is composed of the graph display window, the status indicators, the control buttons and the commands windows. The graph displays show the Y-Z plane trajectory of the drogue motion, which is calculated from the rigid bar angle measured by the joint mounted potentiometers. In addition, the Y and Z direction acceleration time history of the drogue motion is displayed also, which comes from the AHRS via the RF modem. The indicators represent each system status. The control buttons and command windows provide the users to control the properties and procedures of each device.

Figure 2.17 LabVIEW front panel for ground control station
CHAPTER 3

DYNAMIC MODELING

In this chapter, a mathematical model of the ASDRS test-rig system is developed based on multi-body dynamic analysis. This model is later used for a model-based control law design in Chapter 4. For the ASDRS version 1.0 study, necessary transfer functions were obtained from the experimentally obtained data using system identification technique, and feedback control laws were developed based on these transfer functions. For the version 2.0 study, the mathematical model developed in this chapter is used for the control law development, and also the closed-loop nonlinear simulation for verification before actually implementing the control laws into the drogue onboard avionics. The mathematical model plays an important role in developing the control law comparing to just using experimentally determined transfer functions via system identification technique. This will be discussed in Chapter 4 and 5.

3.1 Determination of inertial properties

Shown in Fig. 3.1, the ASDRS test-rig system is composed of a rigid bar and a drogue that are connected by two universal joints at points O and P1. To develop an analytical model for numerical simulation, the first step in modeling process is to determine necessary inertial properties; size, mass, center of gravity, and moment of inertia of each component. The mass moment of inertia matrix for the bar and the drogue are defined in eq. 3.1 and 3.2. The axial component $I_{xx}$ of the bar is assumed to be zero as its diameter is relatively small comparing to its length. Also, since it is axisymmetric
$I_{yy}$ and $I_{zz}$ of the bar and the drogue has the same values and the products of inertia are all equal to zero. The mass moment of inertia matrix for the drogue takes the same format due to its axisymmetric configuration except it has a finite value of the axial component.

$$
\begin{bmatrix}
0 & 0 & 0 \\
0 & I_{yy1} & 0 \\
0 & 0 & I_{yy1}
\end{bmatrix}
$$

(3.1)

$$
\begin{bmatrix}
I_{xx2} & 0 & 0 \\
0 & I_{yy2} & 0 \\
0 & 0 & I_{yy2}
\end{bmatrix}
$$

(3.2)

Each moment of inertia value is measured via experimental method called the Bifilar torsional pendulum method.[31] In this method, two thin wires are connected to a test object in parallel as shown in Fig. 3.2, and the oscillation period $T$ is measured followed by applying some torsional displacement. This oscillation period is used to
calculate the moment of inertia of the object in the axis parallel to the wire using the formula given by

\[ I = \frac{mgD^2T^2}{16\pi^2h} \]  

(3.3)

where \( m \) and \( g \) represent the mass and the gravitational acceleration, and \( D \) and \( h \) are the distances in the test shown as shown in Fig. 3.2.

Figure 3.2 Bifilar torsional pendulum method

Figure 3.3 shows the photos of measuring the oscillation period for the bar and the drogue, and all measured and computed inertial properties are given in table 3.1.

Figure 3.3 Moment of Inertia measurement
Table 3.1 Measured inertial properties

<table>
<thead>
<tr>
<th>Value</th>
<th>Bar</th>
<th>Drogue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>10mm</td>
<td>228.6mm</td>
</tr>
<tr>
<td>Length</td>
<td>925mm</td>
<td>320mm</td>
</tr>
<tr>
<td>C.G point (From pivot)</td>
<td>480mm</td>
<td>141mm</td>
</tr>
<tr>
<td>Mass</td>
<td>0.2714kg</td>
<td>0.7548kg</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>0</td>
<td>0.0658kg-m$^2$</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>1.249kg-m$^2$</td>
<td>0.235kg-m$^2$</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>1.249kg-m$^2$</td>
<td>0.235kg-m$^2$</td>
</tr>
</tbody>
</table>

3.2 Equations of motion

3.2.1 Reference frames, angles and position vectors

Three reference frames are introduced for the ASDRS test-rig system; the earth-fixed frame, and the two body-fixed frames that are attached to the bar and the drogue, respectively. The Cartesian coordinates for the earth-fixed frame are defined as shown in Fig. 3.4. The Cartesian coordinates for the bar and the drogue are defined according to its asymmetry; with the origin at the center of gravity (CG, the point G1 for the bar and the point G2 for the drogue), the X-axis along the axis of symmetry and the Y-axis and the Z-axis are pointing to the right and to vertically down.

The test-rig has 4 degrees-of-freedom, and the four independent variables to describe the system motions are the four angles to represent the attitude of the bar and the drogue. These angles, as shown in Fig. 3.4, are in fact directly measured via 4 potentiometers installed on each universal joint. With the angles and the coordinate system being defined as described above, the coordinate transformation matrices among these frames can be obtained as follows:
Figure 3.4 Rigid link system coordinate for the wind tunnel test

Reference: \( R(x, y, z) = \begin{bmatrix} i \\ j \\ k \end{bmatrix} \) \( \iff \) Bar: \( B(x', y', z') = \begin{bmatrix} i' \\ j' \\ k' \end{bmatrix} \)
where \( i' \) \( j' \) \( k' \) = \( L_{\text{Bar}} \cdot \begin{bmatrix} i \\ j \\ k \end{bmatrix} \) \( (3.4) \)

\[
L_4 = \begin{bmatrix} C_1 & 0 & -S_1 \\ 0 & 1 & 0 \\ S_1 & 0 & C_1 \end{bmatrix} \quad Y \text{-axis: } \theta_1 \text{ rotation} \quad (3.5)
\]

\[
L_2 = \begin{bmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z \text{-axis: } \theta_2 \text{ rotation} \quad (3.6)
\]

\[
L_{\text{Bar}} = L_2 L_4 = \begin{bmatrix} C_1 C_2 & S_2 & -S_1 C_2 \\ -C_1 S_2 & C_2 & S_1 S_2 \\ S_1 & 0 & C_1 \end{bmatrix} \quad (3.7)
\]

Reference: \( R(x, y, z) = \begin{bmatrix} i \\ j \\ k \end{bmatrix} \) \( \iff \) Drogue: \( D(x'', y'', z'') = \begin{bmatrix} i'' \\ j'' \\ k'' \end{bmatrix} \)
where \( i'' \) \( j'' \) \( k'' \) = \( L_{\text{Drogue}} \cdot \begin{bmatrix} i \\ j \\ k \end{bmatrix} \) \( (3.8) \)

\[
L_3 = \begin{bmatrix} C_3 & 0 & -S_3 \\ 0 & 1 & 0 \\ S_3 & 0 & C_3 \end{bmatrix} \quad Y \text{-axis: } \theta_3 \text{ rotation} \quad (3.9)
\]
\[ L_4 = \begin{bmatrix} C_4 & S_4 & 0 \\ -S_4 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_4 \text{ rotation} \quad (3.10) \]

\[ L_{\text{Drogue}} = L_4 \cdot L_5 \cdot L_2 \cdot L_1 = \begin{bmatrix} -C_1(S_2S_4 - C_2C_3C_4) - S_1S_3C_4 & C_2S_4 + S_2C_3C_4 & S_1(S_2S_4 - C_2C_3C_4) - C_1S_3C_4 \\ S_1S_3S_4 - C_1(S_2C_4 + C_2C_3S_4) & C_2C_4 - S_2C_3S_4 & S_1(S_2C_4 - C_2C_3S_4) + C_1S_3S_4 \\ S_1C_3 + C_1C_2S_3 & S_2S_3 & C_1C_3 - S_1C_2S_3 \end{bmatrix} \quad (3.11) \]

Using the coordinate transformation matrices, the necessary position vectors are defined as follows:

\[ R_{G1} = L_{\bar{\text{Bar}}}^T \begin{bmatrix} -l_1 \\ 0 \\ 0 \end{bmatrix} = l_1 \begin{bmatrix} -C_1C_2 \\ -S_2 \\ S_1C_2 \end{bmatrix} \quad (3.12) \]

\[ R_{P1} = L_{\bar{\text{Bar}}}^T \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} = l_2 \begin{bmatrix} -C_1C_2 \\ -S_2 \\ S_1C_2 \end{bmatrix} \quad (3.13) \]

\[ R_{G2} = R_{P1} + L_{\text{Drogue}}^T \begin{bmatrix} -l_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_2C_1C_2 + l_1(C_1(S_2S_4 - C_2C_3C_4) + S_1S_3C_4) \\ -l_2S_2 - l_1(C_2S_4 + S_2C_3C_4) \\ l_2S_3C_2 - l_1(S_1(S_2S_4 - C_2C_3C_4) - C_1S_3C_4) \end{bmatrix} \quad (3.14) \]

\[ R_{P2} = R_{P1} + L_{\text{Drogue}}^T \begin{bmatrix} -l_4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_2C_1C_2 + l_1(C_1(S_2S_4 - C_2C_3C_4) + S_1S_3C_4) \\ -l_2S_2 - l_1(C_2S_4 + S_2C_3C_4) \\ l_2S_3C_2 - l_1(S_1(S_2S_4 - C_2C_3C_4) - C_1S_3C_4) \end{bmatrix} \quad (3.15) \]

In the next section, a mathematical model is developed based upon multi-body dynamic analysis using Kane’s equations. If the angles for the drogue attitude as defined in Fig. 3.4 are used to derive the equations of motion (EOM), the derivation process are much more complicated and it results in complex expressions for the final EOM. This may result in a longer calculation time and numerical errors in computer simulation. For these reason the angles for the drogue attitude are defined with respect to the earth-fixed
frame as shown in Fig. 3.5. The attitude angles for the bar remain the same as $\theta_1$ and $\theta_2$ for both the test and the simulation, but the attitude angles of $\theta_3$ and $\theta_6$ defined for the drogue attitude with respect to the earth fixed frame. In addition, the drogue is allowed to rotate relative to the bar, so $\theta_7$ is defined for the drogue roll in the EOM derivations and the test-rig system can be considered a 5 degrees-of-freedom system in this sense. The roll motion is not considered for this study, but the mathematical model does include this motion so that it can be used for the future study.

![Figure 3.5 Rigid link system coordinate for the numerical simulation](image)

Now, with the angles and the coordinate system being defined as described above the coordinate transformation matrices among these frames can be obtained as follows:

Reference: $R(x, y, z) = \begin{bmatrix} i \\ j \\ k \end{bmatrix}$ $\Leftrightarrow$ Bar: $B(x', y', z') = \begin{bmatrix} i' \\ j' \\ k' \end{bmatrix}$

$$L_1 = \begin{bmatrix} C_1 & 0 & -S_1 \\ 0 & 1 & 0 \\ S_1 & 0 & C_1 \end{bmatrix} \quad Y-axis: \theta_1 \ rotation$$

$$L_2 = \begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix} \quad Z-axis: \theta_2 \ rotation$$

$$L_3 = \begin{bmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_6 \ rotation$$

$$L_4 = \begin{bmatrix} C_4 & 0 & S_4 \\ 0 & 1 & 0 \\ -S_4 & 0 & C_4 \end{bmatrix} \quad Y-axis: \theta_3 \ rotation$$

$$L_5 = \begin{bmatrix} C_5 & 0 & S_5 \\ 0 & 1 & 0 \\ -S_5 & 0 & C_5 \end{bmatrix} \quad Z-axis: \theta_6 \ rotation$$

$$L_6 = \begin{bmatrix} C_6 & -S_6 & 0 \\ S_6 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_7 \ rotation$$

$$L_{12} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_2 \ rotation$$

$$L_{13} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_6 \ rotation$$

$$L_{14} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Y-axis: \theta_3 \ rotation$$

$$L_{15} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_6 \ rotation$$

$$L_{16} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_7 \ rotation$$

$$L_{23} = \begin{bmatrix} C_2 & 0 & 0 \\ 0 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_3 \ rotation$$

$$L_{24} = \begin{bmatrix} C_2 & 0 & 0 \\ 0 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_6 \ rotation$$

$$L_{25} = \begin{bmatrix} C_2 & 0 & 0 \\ 0 & C_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Y-axis: \theta_2 \ rotation$$

$$L_{26} = \begin{bmatrix} C_2 & 0 & 0 \\ 0 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_7 \ rotation$$

$$L_{34} = \begin{bmatrix} C_3 & 0 & 0 \\ 0 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_3 \ rotation$$

$$L_{35} = \begin{bmatrix} C_3 & 0 & 0 \\ 0 & C_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Y-axis: \theta_6 \ rotation$$

$$L_{36} = \begin{bmatrix} C_3 & 0 & 0 \\ 0 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_7 \ rotation$$

$$L_{45} = \begin{bmatrix} C_4 & 0 & 0 \\ 0 & C_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Y-axis: \theta_2 \ rotation$$

$$L_{46} = \begin{bmatrix} C_4 & 0 & 0 \\ 0 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z-axis: \theta_7 \ rotation$$

$$L_{56} = \begin{bmatrix} C_5 & 0 & 0 \\ 0 & C_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad X-axis: \theta_6 \ rotation$$

47
\[
L_2 = \begin{bmatrix} 
C_2 & S_2 & 0 \\
-S_2 & C_2 & 0 \\
0 & 0 & 1 
\end{bmatrix} \quad \text{Z-axis: } \theta_2 \text{ rotation} \tag{3.18}
\]

\[
L_{\text{Bar}} = L_2 L_4 = \begin{bmatrix} 
C_1 C_2 & S_2 & -S_1 S_2 \\
-C_1 S_2 & C_2 & S_1 \\
S_1 & 0 & C_1 
\end{bmatrix} \begin{bmatrix} 
i' \\
j' \\
k' 
\end{bmatrix} = L_{\text{Bar}} \begin{bmatrix} 
i \\
j \\
k 
\end{bmatrix} \tag{3.19}
\]

\[
\text{Reference: } R(x, y, z) = \begin{bmatrix} i \\
j \\
k 
\end{bmatrix} \quad \Leftrightarrow \quad \text{Drogue: } D(x'', y'', z'') = \begin{bmatrix} i'' \\
j'' \\
k'' 
\end{bmatrix} \tag{3.20}
\]

\[
L_3 = \begin{bmatrix} 
C_5 & 0 & -S_5 \\
0 & 1 & 0 \\
S_5 & 0 & C_5 
\end{bmatrix} \quad \text{Y-axis: } \theta_3 \text{ rotation} \tag{3.21}
\]

\[
L_6 = \begin{bmatrix} 
C_6 & S_6 & 0 \\
-S_6 & C_6 & 0 \\
0 & 0 & 1 
\end{bmatrix} \quad \text{Z-axis: } \theta_6 \text{ rotation} \tag{3.22}
\]

\[
L_7 = \begin{bmatrix} 
1 & 0 & 0 \\
0 & C_7 & S_7 \\
0 & -S_7 & C_7 
\end{bmatrix} \quad \text{X-axis: } \theta_7 \text{ rotation} \tag{3.23}
\]

\[
L_{\text{Drogue}} = L_7 \cdot L_6 \cdot L_5 = \begin{bmatrix} 
C_5 C_6 & S_6 & -S_5 C_6 \\
S_5 S_7 - C_5 S_6 C_7 & C_6 C_7 & C_5 S_7 + S_5 S_6 C_7 \\
S_5 C_7 + C_5 S_6 S_7 & -C_6 S_7 & C_5 C_7 - S_5 S_6 S_7 
\end{bmatrix} \begin{bmatrix} 
i'' \\
j'' \\
k'' 
\end{bmatrix} = L_{\text{Drogue}} \begin{bmatrix} 
i \\
j \\
k 
\end{bmatrix} \tag{3.24}
\]

Using the coordinate transformation matrices defined as above, the necessary position vectors are defined as follows:

\[
R_{G_1} = L_{\text{Bar}}^T \begin{bmatrix} -l_1 \\
0 \\
0 
\end{bmatrix} = I_1 \begin{bmatrix} -C_1 C_2 \\
-S_2 \\
S_4 C_2 
\end{bmatrix} \tag{3.25}
\]
\[
R_{p1} = L_{\text{bar}}^T \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} = l_2 \begin{bmatrix} -C_2C_2 \\ -S_2 \\ S_1C_2 \end{bmatrix}
\] (3.26)

\[
R_{G2} = R_{p1} + R_{G2/P1} = R_{p1} + L_{\text{drogue}}^T \begin{bmatrix} -l_3 \\ 0 \\ 0 \end{bmatrix} = l_2 \begin{bmatrix} -C_2C_2 \\ -S_2 \\ S_1C_2 \end{bmatrix} + l_3 \begin{bmatrix} -C_6C_6 \\ -S_6 \\ S_5C_6 \end{bmatrix}
\] (3.27)

\[
R_{p2} = R_{p1} + R_{P2/P1} = R_{p1} + L_{\text{drogue}}^T \begin{bmatrix} -l_3 \\ 0 \\ 0 \end{bmatrix} = l_2 \begin{bmatrix} -C_2C_2 \\ -S_2 \\ S_1C_2 \end{bmatrix} + l_3 \begin{bmatrix} -C_6C_6 \\ -S_6 \\ S_5C_6 \end{bmatrix}
\] (3.28)

### 3.2.2 Kinematics

With the reference frame defined in the previous section, the angular velocities and the angular accelerations can be found as follows:

\[
\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \theta_2S_1 \\ \theta_2C_1 \end{bmatrix}
\] (3.29)

\[
\dot{\begin{bmatrix} \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix}} = \begin{bmatrix} L_6 \cdot L_5 \end{bmatrix} \begin{bmatrix} \dot{\theta}_7 \\ \dot{\theta}_8 \\ \dot{\theta}_9 \end{bmatrix} = \begin{bmatrix} \theta_6S_5 + \theta_7C_5C_6 \\ \theta_6 + \theta_7S_6 \\ \theta_6C_5 - \theta_7S_5C_6 \end{bmatrix}
\] (3.30)

\[
\frac{d}{dt} (R_{\theta}^{\text{bar}}) = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 - \dot{\theta}_4 \dot{\theta}_5 \end{bmatrix}
\] (3.31)

\[
\frac{d}{dt} (R_{\theta}^{\text{drogue}}) = \begin{bmatrix} \theta_6S_5 + \theta_7C_5C_6 - \dot{\theta}_5S_5C_6 - \dot{\theta}_6\dot{\theta}_7C_6 \\ \theta_5 + \dot{\theta}_7S_6 + \dot{\theta}_6\dot{\theta}_7C_6 \\ \theta_6C_5 - \dot{\theta}_5S_5C_6 - \dot{\theta}_5\dot{\theta}_7C_6 + \dot{\theta}_6\dot{\theta}_7S_6 \end{bmatrix}
\] (3.32)

Next, the inertial velocities and accelerations of the point G1, P1, G2, and P2 shown in Fig. 3.5 can be found by using the vector differentiation rule in two reference frames [32], and the results are summarized as follows:
\[ V_{G1} = R \bar{\omega} \times R_{G1} = l_1 \begin{bmatrix} \dot{\theta}_1 C_2 + \dot{\theta}_2 C_1 S_2 \\ -\dot{\theta}_2 C_1 \\ \dot{\theta}_C C_2 - \dot{\theta}_S S_2 \end{bmatrix} \]  
(3.33)

\[ V_{p1} = R \bar{\omega} \times R_{p1} = l_2 \begin{bmatrix} \dot{\theta}_1 C_2 + \dot{\theta}_2 C_1 S_2 \\ -\dot{\theta}_2 C_2 \\ \dot{\theta}_C C_2 - \dot{\theta}_S S_2 \end{bmatrix} \]  
(3.34)

\[ V_{G2/p1} = R \bar{\omega} \times R_{G2/p1} = l_3 \begin{bmatrix} \dot{\theta}_5 S_5 C_6 + \dot{\theta}_6 C_5 S_6 \\ -\dot{\theta}_6 C_6 \\ \dot{\theta}_C C_6 - \dot{\theta}_S S_6 \end{bmatrix} \]  
(3.35)

\[ V_{p2/p1} = R \bar{\omega} \times R_{p2/p1} = l_4 \begin{bmatrix} \dot{\theta}_5 S_5 C_6 + \dot{\theta}_6 C_5 S_6 \\ -\dot{\theta}_6 C_6 \\ \dot{\theta}_C C_6 - \dot{\theta}_S S_6 \end{bmatrix} \]  
(3.36)

\[ V_{G2} = V_{p1} + V_{G2/p1} \]  
(3.37)

\[ V_{P2} = V_{p1} + V_{P2/p1} \]  
(3.38)

\[ a_{G1} = R \bar{\alpha} \times R_{G1} + R \bar{\alpha} \times V_{G1} = l_1 \begin{bmatrix} \ddot{\theta}_1 C_2 + \ddot{\theta}_2 C_1 S_2 + (\dot{\theta}_1^2 + \dot{\theta}_2^2)C_1 C_2 - 2\dot{\theta}_1 \dot{\theta}_2 S_1 S_2 \\ -\dot{\theta}_2 C_1 + \dot{\theta}_2^2 S_2 \\ \dot{\theta}_C C_2 - \dot{\theta}_S S_2 - (\dot{\theta}_1^2 + \dot{\theta}_2^2)S_1 C_2 - 2\dot{\theta}_1 \dot{\theta}_2 C_2 \end{bmatrix} \]  
(3.39)

\[ a_{p1} = R \bar{\alpha} \times R_{p1} + R \bar{\alpha} \times V_{p1} = l_2 \begin{bmatrix} \ddot{\theta}_1 C_2 + \ddot{\theta}_2 C_1 S_2 + (\dot{\theta}_1^2 + \dot{\theta}_2^2)C_1 C_2 - 2\dot{\theta}_1 \dot{\theta}_2 S_1 S_2 \\ -\dot{\theta}_2 C_2 + \dot{\theta}_2^2 S_2 \\ \dot{\theta}_C C_2 - \dot{\theta}_S S_2 - (\dot{\theta}_1^2 + \dot{\theta}_2^2)S_1 C_2 - 2\dot{\theta}_1 \dot{\theta}_2 C_2 \end{bmatrix} \]  
(3.40)

\[ a_{G2/P1} = R \bar{\alpha} \times R_{G2/P1} + R \bar{\alpha} \times V_{G2/P1} \]

\[ = l_3 \begin{bmatrix} \ddot{\theta}_5 S_5 C_6 + \ddot{\theta}_6 C_5 S_6 + (\dot{\theta}_6^2 + \dot{\theta}_5^2)C_5 C_6 - 2\dot{\theta}_5 \dot{\theta}_6 S_5 S_6 \\ -\dot{\theta}_6 C_6 + \dot{\theta}_6^2 S_6 \\ \dot{\theta}_C C_6 - \dot{\theta}_S S_6 - (\dot{\theta}_5^2 + \dot{\theta}_6^2)S_5 C_6 - 2\dot{\theta}_5 \dot{\theta}_6 C_5 S_6 \end{bmatrix} \]  
(3.41)
\[ a_{P2/P1} = R D^{\text{Drogue}} \times R_{P2/P1} + R^{\text{Drogue}} \times V_{P2/P1} \]
\[ = l_4 \left[ \ddot{\theta}_6 S_6 C_6 + \ddot{\theta}_5 C_5 S_5 + (\ddot{\theta}_6^2 + \dot{\theta}_6^2) C_5 C_6 - 2\dot{\theta}_5 \dot{\theta}_6 S_5 S_6 \right] \]
\[ - \dot{\theta}_5 C_5 - \ddot{\theta}_6 S_6 \]
\[ + \left[ \ddot{\theta}_5 S_5 C_5 - \ddot{\theta}_6 S_5 S_6 - (\ddot{\theta}_5^2 + \dot{\theta}_5^2) S_5 C_6 - 2\dot{\theta}_5 \dot{\theta}_6 C_5 S_6 \right] \]
\[ (3.42) \]

\[ a_{G2} = a_{P1} + a_{G2/P1} \]
\[ (3.43) \]

\[ a_{F2} = a_{P1} + a_{F2/P1} \]
\[ (3.44) \]

### 3.2.3 Equations of motion

There are different approaches to obtain equations of motion for a dynamic system. In this study, the EOM are derived based on the Kane’s equations which can be expressed as

\[ \sum_{i=1}^{N_a} \left( m_i a_{Gi} \cdot \frac{\partial V_{Gi}}{\partial u_k} \right) + \sum_{i=1}^{N_a} \left[ (I_{Gi} \cdot a_{Bi}) + (\omega_{Bi} \times H_{Gi}) \right] \cdot \frac{\partial \omega_{Bi}}{\partial u_k} = F_{uk} \]
\[ (3.45) \]

The Kane’s equations are similar to those from D’Alembert’s principle, but one major difference is the use of the independent generalized speeds. D’Alembert’s principle uses the generalized coordinates, and this may result in a different number of equations than the number of degree-of-freedom having the additional constraint equations to represent the relationship for the dependent coordinates. In contrast, the Kane’s equation introduces the generalized speeds with the partial velocities which are the same number as the degrees-of-freedom of the dynamic system.

In this research, the EOM are developed using the Kane’s equations [32]. The rigid link system has 5 independent generalized coordinates and the 5 independent generalized speeds for Kane’s equations are \( \dot{\theta}_1 , \dot{\theta}_2 , \dot{\theta}_3 , \dot{\theta}_6 , \) and \( \dot{\theta}_7 . \) Table 3.2 shows the generalized speeds and their partial derivatives to be used for deriving the EOM using the Kane’s equation.
### Table 3.2 Partial velocities for use with Kane’s Equation

<table>
<thead>
<tr>
<th></th>
<th>( V_{G1} )</th>
<th>( V_{G2} )</th>
<th>( \omega_{\text{Bar}} )</th>
<th>( \omega_{\text{Drogue}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\theta}_1 )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_1} \left[ \begin{array}{cc} S_1 C_2 \ 0 \ C_1 C_2 \end{array} \right] )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_1} \left[ \begin{array}{cc} S_1 C_2 \ 0 \ C_1 C_2 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Bar}}}{\partial \dot{\theta}_1} = \left[ \begin{array}{c} 0 \ 1 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Drogue}}}{\partial \dot{\theta}_1} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
</tr>
<tr>
<td>( \dot{\theta}_2 )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_2} \left[ \begin{array}{cc} C_1 S_2 \ -C_2 \ -S_1 S_2 \end{array} \right] )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_2} \left[ \begin{array}{cc} S_2 C_6 \ 0 \ C_2 C_6 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Bar}}}{\partial \dot{\theta}_2} = \left[ \begin{array}{c} S_1 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Drogue}}}{\partial \dot{\theta}_2} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
</tr>
<tr>
<td>( \dot{\theta}_3 )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_3} \left[ \begin{array}{cc} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_3} \left[ \begin{array}{cc} S_2 C_6 \ 0 \ C_2 C_6 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Bar}}}{\partial \dot{\theta}_3} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Drogue}}}{\partial \dot{\theta}_3} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
</tr>
<tr>
<td>( \dot{\theta}_4 )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_4} \left[ \begin{array}{cc} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_4} \left[ \begin{array}{cc} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Bar}}}{\partial \dot{\theta}_4} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Drogue}}}{\partial \dot{\theta}_4} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
</tr>
<tr>
<td>( \dot{\theta}_5 )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_5} \left[ \begin{array}{cc} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_5} \left[ \begin{array}{cc} C_5 S_6 \ -C_6 \ -S_5 S_6 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Bar}}}{\partial \dot{\theta}_5} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Drogue}}}{\partial \dot{\theta}_5} = \left[ \begin{array}{c} S_5 \ 0 \ C_5 \end{array} \right] )</td>
</tr>
<tr>
<td>( \dot{\theta}_6 )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_6} \left[ \begin{array}{cc} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial}{\partial \dot{\theta}_6} \left[ \begin{array}{cc} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Bar}}}{\partial \dot{\theta}_6} = \left[ \begin{array}{c} 0 \ 0 \ 0 \end{array} \right] )</td>
<td>( \frac{\partial \omega_{\text{Drogue}}}{\partial \dot{\theta}_6} = \left[ \begin{array}{c} C_5 C_6 \ S_6 \ -S_5 C_6 \end{array} \right] )</td>
</tr>
</tbody>
</table>

The moment of inertia matrix for the bar and the drogue needs to be transformed to the earth-fixed coordinate system using the matrix transformation rule as shown below:

\[
\begin{align*}
I_{\text{Bar}}^B &= I_{\text{Bar}}^T \cdot I_{\text{Bar}}^T \\
I_{\text{Drogue}}^B &= I_{\text{Drogue}}^T \cdot I_{\text{Drogue}}^T
\end{align*}
\]  

(3.46)  

(3.47)

Then, the kinematics of the rotational motion of the bar and the drogue can be obtained as follows:
\[
\left( ^{R}I_{\text{Bar}} \cdot ^{R}a_{\text{Bar}} \right) + \left( ^{R}q_{\text{Bar}} \times ^{R}H_{\text{Bar},G1} \right)
= I_{yy1} \left[ -\ddot{\theta}_{1}C_{1}S_{2} + \dot{\theta}_{2}S_{4} + \dot{\omega}_{2}S_{2}C_{2} + 2\theta_{2}\ddot{\theta}_{1}C_{1}S_{2} \right]
\]
\[
= \left[ \dot{\theta}_{1}C_{2}^2 - \dot{\theta}_{2}\sin(2\theta_{2}) \right]
\]
\[
\ldots
\]

\[
\left( ^{R}I_{\text{Drogue}} \cdot ^{R}a_{\text{Drogue}} \right) + \left( ^{R}q_{\text{Drogue}} \times ^{R}H_{\text{Drogue}} \right)
= I_{xx2} \left( \dot{\theta}_{5}S_{6} + \ddot{\theta}_{5}S_{6} + \dot{\theta}_{6}\sin(2\theta_{5}) + \dot{\theta}_{6}C_{6} \right)
\]
\[
= I_{yy2} \left( \ddot{\theta}_{5}C_{6}^2 + \dot{\theta}_{5}\theta_{6}\sin(2\theta_{5}) \right)
\]
\[
I_{xx2} \left( \dot{\theta}_{5}S_{6} - \ddot{\theta}_{5}C_{5}S_{6} - \dot{\theta}_{5}\theta_{6}S_{6} - \dot{\theta}_{5}\theta_{6}C_{5}S_{6} \right) + I_{yy2} \left( \ddot{\theta}_{5}S_{6} - \dot{\theta}_{6}S_{6} + \dot{\theta}_{6}C_{6} \right)
\]

The external loads acting on the test-rig system are the drag force on the bar, and the drag force on the drogue, and the control moment acting on the CG of the drogue due to the control surface deflections. The details on modeling of the external loads are discussed in the next section. Finally, followed by formulating the Kane’s equations of eqs. (3.50~54), a set of five 2nd order nonlinear differential equations can be obtained as follow:

\[
\left[ m_{1}\frac{\partial V_{i1}}{\partial \theta_{1}} + m_{2}\frac{\partial V_{i2}}{\partial \theta_{1}} \right]
+ \left[ \left( ^{R}I_{\text{Bar}} \cdot ^{R}a_{\text{Bar}} \right) + \left( ^{R}q_{\text{Bar}} \times ^{R}H_{\text{Bar}} \right) \right] \cdot \frac{\partial \omega_{2}}{\partial \theta_{1}}
+ \left[ \left( ^{R}I_{\text{Drogue}} \cdot ^{R}a_{\text{Drogue}} \right) + \left( ^{R}q_{\text{Drogue}} \times ^{R}H_{\text{Drogue}} \right) \right] \cdot \frac{\partial \omega_{2}}{\partial \theta_{1}}
= \left( F_{x1}i + F_{y1}j + F_{z1}k \right) \frac{\partial V_{i1}}{\partial \theta_{1}} + \left( F_{x2}i + F_{y2}j + F_{z2}k \right) \frac{\partial V_{i2}}{\partial \theta_{1}}
\]

\[
+ \left( M_{x1}i + M_{y1}j + M_{z1}k \right) \frac{\partial \omega_{2}}{\partial \theta_{1}}
\]

\[
+ \left( M_{x2}i + M_{y2}j + M_{z2}k \right) \frac{\partial \omega_{2}}{\partial \theta_{1}}
\]

53
\[
\begin{align*}
&\left[ m_{aG1} \cdot \frac{\partial V_{G1}}{\partial \theta_2} + m_{aG2} \cdot \frac{\partial V_{G2}}{\partial \theta_2} \right] \\
&\quad + \left\{ \left[ (I_{Bar} \cdot \alpha_{Bar}) + (\omega_{Bar} \times H_{Bar}) \right] \cdot \frac{\partial \omega_{Bar}}{\partial \theta_2} + \left[ (I_{Drogue} \cdot \alpha_{Drogue}) + (\omega_{Drogue} \times H_{Drogue}) \right] \cdot \frac{\partial \omega_{Drogue}}{\partial \theta_2} \right\} \\
&= \left( F_{xi} \hat{\mathbf{i}} + F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G1}}{\partial \theta_2} + \left( F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G2}}{\partial \theta_2} + \left( M_{xi} \hat{\mathbf{i}} + M_{yi} \hat{\mathbf{j}} + M_{zi} \hat{\mathbf{k}} \right) \frac{\partial \omega_{Drogue}}{\partial \theta_2}
\end{align*}
\]

\[
\begin{align*}
&\left[ m_{aG1} \cdot \frac{\partial V_{G1}}{\partial \theta_3} + m_{aG2} \cdot \frac{\partial V_{G2}}{\partial \theta_3} \right] \\
&\quad + \left\{ \left[ (I_{Bar} \cdot \alpha_{Bar}) + (\omega_{Bar} \times H_{Bar}) \right] \cdot \frac{\partial \omega_{Bar}}{\partial \theta_3} + \left[ (I_{Drogue} \cdot \alpha_{Drogue}) + (\omega_{Drogue} \times H_{Drogue}) \right] \cdot \frac{\partial \omega_{Drogue}}{\partial \theta_3} \right\} \\
&= \left( F_{xi} \hat{\mathbf{i}} + F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G1}}{\partial \theta_3} + \left( F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G2}}{\partial \theta_3} + \left( M_{xi} \hat{\mathbf{i}} + M_{yi} \hat{\mathbf{j}} + M_{zi} \hat{\mathbf{k}} \right) \frac{\partial \omega_{Drogue}}{\partial \theta_3}
\end{align*}
\]

\[
\begin{align*}
&\left[ m_{aG1} \cdot \frac{\partial V_{G1}}{\partial \theta_5} + m_{aG2} \cdot \frac{\partial V_{G2}}{\partial \theta_5} \right] \\
&\quad + \left\{ \left[ (I_{Bar} \cdot \alpha_{Bar}) + (\omega_{Bar} \times H_{Bar}) \right] \cdot \frac{\partial \omega_{Bar}}{\partial \theta_5} + \left[ (I_{Drogue} \cdot \alpha_{Drogue}) + (\omega_{Drogue} \times H_{Drogue}) \right] \cdot \frac{\partial \omega_{Drogue}}{\partial \theta_5} \right\} \\
&= \left( F_{xi} \hat{\mathbf{i}} + F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G1}}{\partial \theta_5} + \left( F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G2}}{\partial \theta_5} + \left( M_{xi} \hat{\mathbf{i}} + M_{yi} \hat{\mathbf{j}} + M_{zi} \hat{\mathbf{k}} \right) \frac{\partial \omega_{Drogue}}{\partial \theta_5}
\end{align*}
\]

\[
\begin{align*}
&\left[ m_{aG1} \cdot \frac{\partial V_{G1}}{\partial \theta_7} + m_{aG2} \cdot \frac{\partial V_{G2}}{\partial \theta_7} \right] \\
&\quad + \left\{ \left[ (I_{Bar} \cdot \alpha_{Bar}) + (\omega_{Bar} \times H_{Bar}) \right] \cdot \frac{\partial \omega_{Bar}}{\partial \theta_7} + \left[ (I_{Drogue} \cdot \alpha_{Drogue}) + (\omega_{Drogue} \times H_{Drogue}) \right] \cdot \frac{\partial \omega_{Drogue}}{\partial \theta_7} \right\} \\
&= \left( F_{xi} \hat{\mathbf{i}} + F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G1}}{\partial \theta_7} + \left( F_{yi} \hat{\mathbf{j}} + F_{zi} \hat{\mathbf{k}} \right) \frac{\partial V_{G2}}{\partial \theta_7} + \left( M_{xi} \hat{\mathbf{i}} + M_{yi} \hat{\mathbf{j}} + M_{zi} \hat{\mathbf{k}} \right) \frac{\partial \omega_{Drogue}}{\partial \theta_7}
\end{align*}
\]

Now, substituting the partial velocity expressions given in Table 3.2 in these equations along with the accelerations, and the angular velocities and the angular acceleration obtained in Eq. 3.29 ~ Eq. 3.32, and Eq. 3.39 ~ Eq. 3.41, respectively. Finally, these equations are arranged into a matrix form for computer simulation given by Eq. 3.55. Each coefficient of C and R matrices are described at appendix B.
\[
\begin{pmatrix}
C_{11} & C_{12} & C_{15} & C_{16} & C_{17} \\
C_{21} & C_{22} & C_{25} & C_{26} & C_{27} \\
C_{51} & C_{52} & C_{55} & C_{56} & C_{57} \\
C_{61} & C_{62} & C_{65} & C_{66} & C_{67} \\
C_{71} & C_{72} & C_{75} & C_{76} & C_{77}
\end{pmatrix}
\begin{pmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\ddot{\theta}_5 \\
\ddot{\theta}_6 \\
\ddot{\theta}_7
\end{pmatrix}
= 
\begin{pmatrix}
R_1 \\
R_2 \\
R_5 \\
R_6 \\
R_7
\end{pmatrix}
\] (3.55)

\[
[C][\ddot{\theta}] = [R] \quad \Rightarrow \quad [\ddot{\theta}] = [C]^{-1}[R]
\] (3.56)

### 3.3 External loads

The external loads acting on the test-rig system are the gravitational and aerodynamic force. The loads on the bar are modeled as single resultant force acting on its center of gravity due to the gravitational and the aerodynamic force. The loads on the drogue are modeled as single resultant force acting on its CG due to the gravitational and the aerodynamic force, and the aerodynamic moment about its CG. Therefore, there are total 6 components of the external forces and 3 components of external moment acting the test-rig system as shown in Fig. 3.6.

![Figure 3.6 Definition of external loads](image-url)
Now, these loads are expressed in the form of

$$\begin{bmatrix} F_{x1} \\ F_{y1} \\ F_{z1} \end{bmatrix} = D_{Bar} + \begin{bmatrix} 0 \\ 0 \\ m_{Bar}g \end{bmatrix}$$ (3.57)

$$\begin{bmatrix} F_{x2} \\ F_{y2} \\ F_{z2} \end{bmatrix} = D_{Drogue} + \begin{bmatrix} 0 \\ F_{Horizontal} \\ F_{Vertical} + m_{Drogue}g \end{bmatrix}$$ (3.58)

$$\begin{bmatrix} M_{x2} \\ M_{y2} \\ M_{z2} \end{bmatrix} = M_{Drogue} + \begin{bmatrix} 0 \\ M_{Horizontal} \\ M_{Vertical} \end{bmatrix}$$ (3.59)

### 3.3.1 Drogue aerodynamic loads

The aerodynamic loads on the drogue are measured in the ADWT at WMU. The drogue was subject to a range of incidence angles (angle of attack) and dynamic pressures. Defining the lift coefficient, the drag coefficient, and the moment coefficient about its CG using

$$C_L = \frac{Lift}{Q \cdot S}$$ (3.60)

$$C_D = \frac{Drag}{Q \cdot S}$$ (3.61)

$$C_{M}^{cg} = \frac{Moment^{cg}}{Q \cdot S \cdot D_{outer}}$$ (3.62)

where the drogue’s surface area $S$ is defined by drogue front view projection are which is 0.024m$^2$, these coefficients can be computed and shown in Figs. 3.7, 3.8 and 3.9, respectively. Note that the forces and the moment in this test represent the clean drogue aerodynamic loads without control input. From these tests, the angle of attack is found to be a dominant variable for the drogue aerodynamic loads.
Figure 3.7 Drogue Lift Coefficient

Figure 3.8 Drogue Drag Coefficient
3.3.2 Bar aerodynamic loads

The aerodynamic forces on the bar is estimated using the published aerodynamic data in [20], where the fluid dynamic loads on a slender bar of a circular cross-section are given in terms of tangential and normal forces as shown in Fig.3.10. To utilize the data for the modeling and numerical simulation, the tangential length, the angle of attack, and the diameter of the bar must be calculated. In addition, tangential and normal Reynolds number must be calculated using Eq. 3.69 and Eq. 3.70. These Reynolds numbers are then used to calculate the tangential and normal pressure coefficients using the data shown in Fig. 3.11. Finally, the tangential and the normal load on the bar can be calculated using Eq. 3.71 and Eq. 3.72 given below.
\[ l_t = \frac{\pi \cdot D_{Bar}}{2 \sin(\alpha)} \]  

(3.63)

\[ \text{Re}_t = \frac{V \cdot l_t}{v_k} \]  

(3.64)

\[ \text{Re}_n = \frac{\text{norm}(V_n) \cdot D_{Bar}}{v_k} \]  

(3.65)

\[ D_t = \frac{1}{2} \rho V_t^2 \pi D_{Bar} l_{Bar} C_t \]  

(3.66)

\[ D_n = \frac{1}{2} \rho V_n^2 \pi D_{Bar} l_{Bar} C_n \]  

(3.67)
3.3.3 Gust loads

The free steam flow in the wind tunnel is close to steady laminar flow. However, the objective of this study is to verify the performance of active drogue stabilization subject to external disturbances. Therefore, creating turbulent flow conditions in the wind tunnel and characterizing the level of turbulence intensity is important to quantify the performance of active drogue stabilization. To achieve this, an artificial gust generator is fabricated, and the turbulent flow field profile is measured via a 5-Hole Probe system.

3.3.3.1 5-Hole Probe

To measure the turbulence intensity level in the wind tunnel produced by the artificial gust generator, a 5-hole probe is fabricated for this study as shown in Fig. 3.12. It basically measures the 3 components of a flow velocity. It has 5 front holes and 2 side holes, and these holes are connected to 4 pressure sensors as shown in Fig. 3.13. The pressure sensors P1, P2 and P3 are differential type pressure sensors that measure the differential pressure between two pressure ports. The pressure sensor P4 is a static pressure sensor that measures the absolute pressure through the static hole. The readings from these pressure sensors are connected to a computer via a data acquisition system with a sampling rate 2000 samples/sec and 14bit resolution. The computer then records and calculates the 3 components of a flow vector from these pressure readings. Figs. 3.14 and 3.15 shows the manufacturing and calibration process of the 5-hole probe.

![Figure 3.12 Shape of 5-hole probe](image_url)
Figure 3.13 Block diagram of 5-hole probe

Figure 3.14 Manufacture and calibration of 5-hole probe

The 5-Hole Probe is calibrated in the wind tunnel within the range from -20° to 20° angle of attack (AoA) and angle of side slip (AoS), and the measured pressure coefficients are shown in Fig. 3.15 to Fig. 3.17. The details on the 5-hole probe operation principle and the related equations are described in Appendix C.
Figure 3.15 Pitot port calibration data

Figure 3.16 Side port calibration data
3.3.3.2 Artificial gust generator

The flow in the wind tunnel is in general steady and laminar type with very low turbulence strength. To induce a turbulent condition in the wind tunnel for the ASDRS performance test, an artificial gust generator as shown in Fig. 3.18 is fabricated. It is installed in the front portion of the tunnel test section ahead of the drogue. The gust generator consists of two flat plates and actuators as shown in Fig. 3.19. The deflection angle is ranging from 0° to 30°. Two plates are controlled by two servo actuators with 20kg-cm torque and 0.18sec/60° speed powered by 4 cell Ni-MH battery pack. The GCS controls the servos through a PWM signal generator.

The gust generator is programmed to produce 4 different types of gust using 2 types of the plate deflection. The first type is a continuous wave gust and a single wave gust. The continuous wave gust is produced by a continuous sine wave deflection of the plate, while the single wave gust is produced by a deflection of plate in response to ‘1-cos’ type
servo actuator signal. This type of gust signal is typically used for the aircraft gust load analysis [33]. The second type of the deflection creates a vertically dominating gust and a horizontal dominating gust. The vertically dominating gust is produced by deflecting the two plates in the same direction, while the horizontal gust is by deflecting the two plates in reverse direction with a continuous wave condition. In case of a single wave with the horizontal gust, only one plate deflection is commanded for asymmetric flow. Figure 3.20 shows the plate deflection commands according to the gust type.

![Figure 3.18 Artificial gust generator](image1)

![Figure 3.19 Configuration of artificial gust generator](image2)
### Table 3.3

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Vertical mode</th>
<th>Horizontal mode</th>
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<tbody>
<tr>
<td>Single wave</td>
<td>![Graph](Plate-Right Plate-Left)</td>
<td>![Graph](Plate-Right Plate-Left)</td>
</tr>
<tr>
<td>Continuous wave</td>
<td>![Graph](Plate-Right Plate-Left)</td>
<td>![Graph](Plate-Right Plate-Left)</td>
</tr>
</tbody>
</table>

*Figure 3.20 Commands for the artificial gust generator*

#### 3.3.3.3 Gust measurement and profiling

The natural frequency of the drogue motion changes as the dynamic pressure (wind tunnel speed) changes. Therefore, to clearly observe the gust effect on the drogue motion, it is necessary to change the level of gust intensity accordingly. This is achieved by adjusting the gust generator frequency for vertical/horizontal gust in accordance with the drogue natural frequency at each dynamic pressure. Table 3.3 shows the frequency and the amplitude of the gust generator depending upon the gust type and dynamic pressure, and resulting 20 cases of the gust profiles are defined in Table 3.4.
Table 3.3 Gust signal characteristics

<table>
<thead>
<tr>
<th>Dynamic Pressure (Pa)</th>
<th>Frequency(Hz)</th>
<th>Amplitude(Deg)</th>
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</thead>
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<tr>
<td></td>
<td>Vertical</td>
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<td>0.75</td>
<td>0.65</td>
</tr>
<tr>
<td>700</td>
<td>0.87</td>
<td>0.7</td>
</tr>
<tr>
<td>800</td>
<td>1</td>
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</tr>
<tr>
<td>900</td>
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</tr>
<tr>
<td>1000</td>
<td>1.2</td>
<td>0.82</td>
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Table 3.4 List of gust signal cases

<table>
<thead>
<tr>
<th>Dynamic Pressure (Pa)</th>
<th>Single wave</th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>Horizontal</td>
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<tr>
<td>600</td>
<td>case 1</td>
<td>case 2</td>
</tr>
<tr>
<td>700</td>
<td>case 5</td>
<td>case 6</td>
</tr>
<tr>
<td>800</td>
<td>case 9</td>
<td>case 10</td>
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<td>case 14</td>
</tr>
<tr>
<td>1000</td>
<td>case 17</td>
<td>case 18</td>
</tr>
</tbody>
</table>

Each cases defined in Table 3.4 are being measured for the duration of 10 seconds, and the measured gust wind vector are shown in Fig. 3.21 at the dynamic pressure of 600Pa and 100Pa, respectively, for illustration purpose. Here, \( u \), \( v \), and \( w \) represent the wind vector component in X, Y, and Z direction of the earth-fixed coordinates. As expected, \( w \) component is a major variable in the vertical mode, and \( v \) component is a major variable in the vertical mode while \( u \) component appears for both the horizontal and the vertical gust.
To characterize the single wave gust type, the peak value of the wind vector from the steady state value was measured for the range of dynamic pressure and plotted as shown in Fig. 3.21. It can be observed that the peak value of the $u$ velocity component is the largest among others followed by the $w$ component in both the horizontal and vertical gust type. The $v$ component remains relatively stationary indicating that the side component velocity changes are minor. However, in the horizontal gust case, the magnitude of $w$ component gets substantially reduced comparing to the vertical gust case which results in clearly observable lateral displacements of the drogue motion. In cases of the continuous wave gust, the root mean square (RMS) values of the wind vector change
for the ranges of the dynamic pressure are computed and plotted in Fig. 3.23. The change patterns for the horizontal and the vertical wind component are similar to that of the single wave gust.

<table>
<thead>
<tr>
<th></th>
<th>Vertical mode</th>
<th>Horizontal mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single wave</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Peak value</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.22 Peak value of single wave gust input

<table>
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<th>Vertical mode</th>
<th>Horizontal mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous wave</td>
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<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>RMS value</td>
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<td></td>
</tr>
</tbody>
</table>

Figure 3.23 RMS_{AC} value of continuous wave gust input

Finally, Fig. 3.24~27 shows the trajectory of the drogue position subject to the different types of the gust defined above. These trajectories are obtained from the potentiometers data, and it exhibits somewhat chaotic patterns at low dynamic pressure range. However, at 900Pa and 1000Pa, the trajectories are more stable and distinct.
Figure 3.24 Drogue trajectories subject to the single wave vertical gust

Figure 3.25 Drogue trajectories subject to the single wave horizontal gust
Figure 3.26 Drogue trajectories subject to the continuous wave vertical gust

Figure 3.27 Drogue trajectories subject to the continuous wave horizontal gust
3.4 Actuator dynamics

For a feedback control law design, modeling of actuator dynamics is important. The ASDRS uses 4 servo actuators to control the wings. The response time for the actuator is relatively fast comparing the drogue natural frequency, but it leads substantial delay effect in the feedback loop. Thus it is modeled as a first order transfer function from the experimentally measured time response shown in Fig. 3.28, and it is expressed as

\[ a(s) = \frac{1}{0.055s + 1} \]  

(3.68)

This transfer function is then converted into the discrete model with 0.01s sampling time for numerical simulation and feedback control design, and it takes the form of

\[ a_d(z) = \frac{0.1667(z + 1)}{z - 0.6667} \quad \text{ (} T_s = 0.01\text{sec) } \]  

(3.69)

Figure 3.28 Actuator step response
CHAPTER 4

NUMERICAL SIMULATION AND CONTROL LAW SYNTHESIS

There are many factors that influence the performance of a feedback control system. These can be hardware components such as actuators and sensors, and different control logics, etc. However, the most important factor for the feedback control system design should be the thorough understanding of a given system dynamics that needs to be controlled. Typically, the system dynamics are analyzed via mathematical modeling and computer simulation, and/or sometimes via experiments by building a prototype. A mathematical model evolves with the system development process as more and more information is available. It may start from a purely analytical work, but eventually it may become a training tool such as commercial flight simulators used for pilot training. In this research, the mathematical model for the ASDRS test-rig system incorporates as much experimentally obtained data as possible. In addition, through numerical simulations, the model is also tuned to match the experimentally obtained data as closely as possible.

Control law synthesis is a process of designing control logics for a given dynamic system to meet the desired specifications. In most cases, the control law synthesis starts with a mathematical model of a given dynamic system. The form of this mathematical model can be a set of nonlinear differential equations or a set of transfer functions. If the model is in the form of nonlinear differential equations, these equations may need to be linearized at an equilibrium point or an operating condition. Then the dynamic model
becomes a linear system operating around the equilibrium condition, and the model can be represented in terms of a linear state-space model or a transfer function. The linear models can be also obtained by running experiments. Once the input and the output characteristics are obtained, necessary transfer functions can be constructed using the system identification technique. These linear models are typically referred as ‘control design model’ for which a feedback control law is developed.

In this chapter, the mathematical model of the ASDRS test-rig system developed in Chapter 3 is numerically simulated using MATLAB, and the results are compared experimentally obtained data. The model is then tuned to closely match with the experimentally obtained data. By numerically linearizing the model at different operating conditions, a set of linear model in the state-space format is obtained and transformed into a set of transfer functions. To cross validate these numerically obtained transfer functions, the system identification technique is used to extract necessary transfer functions. However, these experimentally obtained transfer functions are found to be not suitable for control law synthesis for this study, so the control law design is based upon the numerically obtained transfer functions. Therefore, hereafter, the control design model for this study refers to the transfer function model obtained via numerical linearization of the nonlinear dynamic model of the ASDRS test-rig system.

4.1 Numerical simulation

The equations of motion (EOM) for the ASDRS test-rig system derived in Chapter 3 are in the form of 5 nonlinear 2nd order differential equations. These equations can be solved numerically using a digital computer. There are several different numerical methods and the Runge-Kutta 4th order method with a fixed-step size is used for this
study. The fixed-step is set to 0.01sec for numerical simulation. Now, in order to solve the 2\textsuperscript{nd} order differential equations numerically, they need to be converted into a set of 1\textsuperscript{st} order differential equation. Therefore, the numerical simulation of the ASDRS test-rig system becomes integrating the set of 10 1\textsuperscript{st} order differential equations numerically with respect to time.

The 5 variables in the equations of motion shown in (3.55) are in fact describing the status of the ASDRS test-rig system at a given time, representing the angles of the bar and the drogue. Transforming the 2\textsuperscript{nd} order differential equation into the 1\textsuperscript{st} order, the auxiliary variables being introduced are in fact the rate terms for these angles. Thus, the 10 variables completely represent the current status the ASDRS test-rig system and these variables are called state variables and written in a vector form (called the state vector) as

\[
x = \begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4 \\
\theta_5 \\
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\dot{\theta}_4 \\
\dot{\theta}_5 \\
\end{bmatrix} = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
\end{bmatrix}
\]  

\text{(4.1)}

In addition, the right-hand side of eq. (3.55) includes the external load terms arising from the horizontal force, the vertical force, the rolling moment and the drag force produced by the active control wings along with the horizontal gust input and the vertical gust input. Therefore, the input variables can be collected into a vector form (called the input vector) as
The drag mode and the roll mode controls are not considered for this study, so the primary 4 input variables are $U_{\text{Horizontal}}$, $U_{\text{Vertical}}$, $U_{\text{Roll}}$, and $U_{\text{Drag}}$ which are produced by the coordinate deflection of wings as discussed in Chapter 3.

With the state vector and the input vector being defined above, the set of 10 first order differential equations are

$$
\dot{x} = \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\dot{\theta}_4 \\
\dot{\theta}_5 \\
\dot{\theta}_6 \\
\dot{\theta}_7 \\
\dot{\theta}_8 \\
\dot{\theta}_9 \\
\dot{\theta}_10
\end{bmatrix} = \begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8 \\
\dot{x}_9 \\
\dot{x}_{10}
\end{bmatrix} = \begin{bmatrix}
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10}
\end{bmatrix}
\begin{bmatrix}
f_1(x,u) \\
f_2(x,u) \\
f_3(x,u) \\
f_4(x,u) \\
f_5(x,u)
\end{bmatrix} \quad (4.3)
$$

Along with the state and the input vector, the output vector can be defined. The output variables of interest are the inertial position of the point $P_2$ as

$$
y = \begin{bmatrix}
P_{2x} \\
P_{2y} \\
P_{2z}
\end{bmatrix} \quad (4.4)
$$
4.1.1 Open-loop wind tunnel test

In order to verify and refine the mathematical model, the open-loop wind tunnel tests are carried out. These allow for comparing the responses of the mathematical model and the actual system response subject to the identical test input, and may be iteratively used to tune up some relevant parameters in the mathematical model until the responses are close. The control forces and moments produced by the wing deflections were very difficult to measure in the ADWT due to small magnitudes, which were beyond the limit of the current balance system. Therefore, the control influence coefficients or control derivatives are estimated through the open-loop dynamic wind tunnel testing.

For the open-loop test, a simple square shape input as shown in Fig. 4.1 is used as a test input signal. The test inputs for the ASDRS are applied to the vertical command and to the horizontal command, and the responses are the attitude angles of the drogue. These are shown in Fig. 4.2 ~ Fig.4.4 for the responses to the vertical command, and Fig. 4.5 ~ Fig. 4.6 for the responses to the horizontal command. Here Angle 1 and Angle 2 represent the bar attitude, and Angle 3 and Angle 4 for the drogue attitude. These tests were conducted at 6 different dynamic pressure conditions ranging from 500Pa to 1000Pa with 100Pa increment. From these graphs, the effect of dynamic pressure is clearly discernable in both the static and the dynamic responses. The vertical command affects Angle 1 and Angle 3. The horizontal command primarily affects Angle 2 and Angle 4.
Figure 4.1 Input command signal for the open-loop test

Figure 4.2 Angle responses to vertical command
Figure 4.3 Open-loop responses of Angle 3 to vertical command

Figure 4.4 Open-loop response of the drogue Z position to vertical command
Figure 4.5 Open-loop response of Angle 2 by horizontal command

Figure 4.6 Open-loop response of Angle 4 to horizontal command
4.1.2 Simulation and model tuning

Using the open-loop dynamic responses of the ASDRS test-rig described in the previous section, the mathematical model is verified and tuned so that the response of the mathematical model is close to the test response. The response comparisons and the model tuning process are discussed next.

4.1.2.1 Static response matching

At a given dynamic pressure, the bar and the drogue system establishes a steady condition after going through some transient changes followed by a command input. This steady condition corresponds to an equilibrium point of the mathematical model. This point can be numerically found, or by running a nonlinear simulation until the solution reaches to a steady-state condition. In this study, the latter method is used to find the equilibrium point of the mathematical model for the range of dynamic pressure.

Figure 4.7 shows the comparison of the simulation and the open-loop test for Angle 1 and Angle 5 for the ranges of dynamic pressure. There exists relatively small difference which may be stemming from numerical errors, measurement errors, and the wind tunnel flow conditions, etc. However, these differences are small enough and the mathematical model generally reflects the system response in static sense. Therefore, no mathematical model tuning work is carried out to match the steady state condition.
4.1.2.2 Dynamic response matching

Followed by the static response comparison, the dynamic responses are compared. The responses to the vertical and the horizontal command are compared for the ranges of dynamic pressure. Unlike the static case, some discernable discrepancies are present in damping and control force characteristics reflected in the response magnitude. Fig. 4.8 and Fig. 4.9 compare the dynamic response to the vertical input command for the ranges of dynamic pressure, while Fig. 4.10 and Fig.11 compare the response to the horizontal input command after iterative matching process. The primary parameters during the tuning up process are the aerodynamic damping values and the control force coefficients as these are very difficult to identify via the analysis and available wind tunnel test capabilities. By changing these values along with the dynamic response characteristics, the values were estimated to produce the final response comparisons are shown in these figures.
Figure 4.8 Comparison of Angle 1 responses to vertical command

Figure 4.9 Comparison of Angle 5 responses to vertical command
Figure 4.10 Comparison of Angle 2 responses to horizontal command

Figure 4.11 Comparison of Angle 6 responses by horizontal command
4.2 Linear model

Most of the model-based control law synthesis method is based upon the assumption that the system dynamics are linear. This seems like major restrictions for practical applications, but many physical systems that are inherently nonlinear behave like a linear system locally at an operating point. This is true for the ASDRS test-rig system. In this section, a linear model of the test-rig system in terms of transfer function is obtained via numerical linearization process. This transfer function represents the input and the output behavior of the drogue at a particular operating condition which in this case is a given dynamic pressure.

4.2.1 State-space model

The first step in obtaining a linear model is to find an equilibrium condition. This is mathematically equivalent to find a solution to the systems of nonlinear algebraic equations. Solving these nonlinear equations involves numerical optimization process, and requires an initial guess to start with. Alternatively, the equilibrium condition can be found by solving the nonlinear differential equations numerically until reaching to a steady-state solution. Again, an initial condition must be provided for this process to start. For this study, the steady-state values are measured from the wind tunnel test for the ranges of dynamic pressure, and the steady-state solutions of differential equations are obtained by using the experimentally obtained steady-state value as the initial condition. Fig. 4.7 illustrates one such solution for Angle 1 and Angle 5.

Once an equilibrium value of the state space form is found, the gradients of the state equations with respect to each state variables and input variables are calculated iteratively, and thus forming up the state matrix and the input matrix of coefficients.
\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]

(4.5)

Here, the output equations are the inertial position of \( P_2 \). Table 4.1 shows these matrices for the dynamic pressure at 500Pa, and the matrices for the other dynamic pressure conditions are listed in Appendix D.

**Table 4.1 Linearized state-space model at 500Pa**

<table>
<thead>
<tr>
<th>A</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
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<td>y2</td>
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<td>0</td>
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<th>u6</th>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>y2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
4.2.2 Transfer function model

A transfer function is a single-input and single-output (SISO) linear model expressed in Laplace domain. The state space model obtained from the mathematical model has 6 inputs and 3 outputs, and by applying Laplace transformation the state space model is converted into a 3 by 6 transfer function matrix (TFM) representing multi-input multi-output (MIMO). However, the control surface configuration is set up to produce like a SISO system for each channel. In other words, the vertical movement is controlled by the vertical control input, and the horizontal movement is controlled by the horizontal control input. Thus, the linear model of the ASDRS test-rig system is indeed composed of two SISO transfer functions with the drogue Y-axis and Z-axis position being the outputs and the horizontal and the vertical deflection being the inputs as summarized in Table 4.2. These transfer functions take different coefficients depending upon the range of dynamic pressure, and these are listed in Table 4.3. The denominator of these transfer function takes the $4^{th}$ order form since the vertical input-output channel is separated from the horizontal input-out channel owing to the symmetry of the system and the control actuation configuration. Fig. 4.12 and Fig. 4.13 show the pole-zero maps of the vertical channel and the horizontal channel transfer function, respectively, for the ranges of dynamic pressure.

### Table 4.2 Input and output of transfer functions

<table>
<thead>
<tr>
<th></th>
<th>Vertical mode</th>
<th>Horizontal mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>$U_{\text{Vertical}}$ : Vertical command</td>
<td>$U_{\text{Horizontal}}$ : Horizontal command</td>
</tr>
<tr>
<td>Output</td>
<td>$P_{2Z}$: Z Position of Drogue</td>
<td>$P_{2Y}$: Y Position of Drogue</td>
</tr>
</tbody>
</table>
Table 4.3 Transfer functions by small perturbation method

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>$G_v(s)$: vertical mode</th>
<th>$G_h(s)$: Horizontal mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>(\frac{0.065598(s^2 + 5.181s + 100.1)}{(s^2 + 1.124s + 21.84)(s^2 + 4.805s + 93.92)})</td>
<td>(\frac{0.045684(s^2 + 5.055s + 50.66)}{(s^2 + 1.307s + 18.26)(s^2 + 4.843s + 52.95)})</td>
</tr>
<tr>
<td>600</td>
<td>(\frac{0.065513(s^2 + 5.239s + 115.2)}{(s^2 + 1.144s + 23.99)(s^2 + 4.853s + 107.8)})</td>
<td>(\frac{0.060107(s^2 + 5.054s + 61.02)}{(s^2 + 1.269s + 20.61)(s^2 + 4.903s + 63.51)})</td>
</tr>
<tr>
<td>700</td>
<td>(\frac{0.065452(s^2 + 5.28s + 129.2)}{(s^2 + 1.162s + 26.36)(s^2 + 4.888s + 120.5)})</td>
<td>(\frac{0.075832(s^2 + 5.033s + 70.96)}{(s^2 + 1.261s + 23.06)(s^2 + 4.932s + 73.82)})</td>
</tr>
<tr>
<td>800</td>
<td>(\frac{0.065403(s^2 + 5.312s + 143.8)}{(s^2 + 1.181s + 29.07)(s^2 + 4.915s + 133.7)})</td>
<td>(\frac{0.092782(s^2 + 5.018s + 81.3)}{(s^2 + 1.265s + 25.78)(s^2 + 4.949s + 84.72)})</td>
</tr>
<tr>
<td>900</td>
<td>(\frac{0.065369(s^2 + 5.334s + 157.9)}{(s^2 + 1.2s + 31.89)(s^2 + 4.935s + 146.6)})</td>
<td>(\frac{0.1109(s^2 + 5s + 91.38)}{(s^2 + 1.275s + 25.56)(s^2 + 4.959s + 95.56)})</td>
</tr>
<tr>
<td>1000</td>
<td>(\frac{0.065341(s^2 + 5.351s + 171.3)}{(s^2 + 1.218s + 34.68)(s^2 + 4.95s + 158.8)})</td>
<td>(\frac{0.13016(s^2 + 4.979s + 101)}{(s^2 + 1.288s + 31.31)(s^2 + 4.966s + 106.1)})</td>
</tr>
</tbody>
</table>
Figure 4.12 Pole-Zero map of vertical channel transfer function

Figure 4.13 Pole-Zero map of horizontal channel transfer function
Observing the pole-zero maps above, there are two pairs of complex conjugate poles and one pair of complex conjugate zeroes. The slow pair of complex conjugate poles represents the first mode of the drogue motion. The drogue motion associated with this mode is the horizontal (or the vertical) motion in phase with the bar’s horizontal (or the vertical) motion. The fast pair of complex conjugate poles represents the second mode of the drogue motion, which describes the drogue motion relative to the bar in the horizontal (or the vertical) direction. Figs. 4.14 and 4.15 show the response to the step command input for the ranges of dynamic pressure. Overall, the response patterns are very similar in both the vertical and the horizontal direction.

Examining the effect of dynamic pressure is important for control design synthesis, especially in scheduling gains of a fixed format controller. Figs. 4.16 ~ 4.18 show the damping ratio and the frequency changes for the ranges of the dynamic pressure. As the dynamic pressure increase, the damping ratio the system decreases while the frequency increases. These are expected nature of the hose-drogue system since the aerodynamic loads increases with increasing dynamic pressure.

It is interesting to note that the changes of the transfer function gain exhibit different trend in the vertical and the horizontal channel as shown in Fig. 4.18. The horizontal transfer function gain is rapidly increasing with increasing dynamic pressure than the vertical. The gain at the dynamic pressure of 1000Pa is 2.8 times higher than that of 500Pa. The vertical transfer function gain remains almost constant for the ranges of the dynamic pressure. This implies that the horizontal channel control gets very sensitive with increasing dynamic pressure. This is due to the fact that there is no force acting on the hose-drogue system in the horizontal channel at the equilibrium. The horizontal force
acting on the system is only generated by the wings other than from external disturbances, and this aerodynamic control force increases with the dynamic pressure. In contrast, the forces acting on the vertical plane is composed of the drag of the drogue, the weight of the drogue, and the tension in the hose (or the bar in the ASDRS test-rig system). These are in a static equilibrium, and the control force generated by the wings on the drogue is relatively small comparing to these forces. The vertical control force does increase with the dynamic pressure, but so does the drag drogue resulting in no significant control effective changes for the ranges of the dynamic pressure.

Figure 4.14 Vertical channel step response of the perturbation model
Figure 4.15 Horizontal channel step response of the perturbation model

Figure 4.16 Damping ratio of Poles and Zeros of the perturbation model
Figure 4.17 Frequency of Poles and Zero of the perturbation model

Figure 4.18 Transfer function gain of the perturbation model
4.3 Control law synthesis

The transfer function models of the ASDRS test-rig system obtained in the previous section represent the output of the horizontal (or the vertical) displacement of the drogue to the horizontal (or the vertical) control force. However, the drogue displacement is practically hard to measure in actual flying scenario without expensive external aids, and this research suggested an IMU based sensor system to measure the level of acceleration as the drogue motion output. With the accelerations as the measurement variables, the control design is posed as a regulator (or disturbance rejection) design problem.

The representative transfer functions from the control input to the drogue accelerations can be obtained from the transfer function models obtained from the previous sections by expressing

\[ G_a(s) = s^2 \cdot G(s) \]  

where \( G(s) \) here are those transfer functions with the position output. With this acceleration transfer function, the closed-loop feedback control design can be expressed as the block diagram shown in Fig. 4.19. Here, \( G_a(s) \) represents the drogue acceleration dynamics, \( C(s) \) represents the controller to be designed, \( A(s) \) represents the actuator dynamics as described in the previous chapter, and the input to the system is the disturbance input caused by the disturbance.
For the controller design, a proportional-derivative (PD) controller of the form

\[
C(s) = k_p + k_ds = k_p \left(1 + \frac{k_d}{k_p} s\right)
\]  \hspace{1cm} (4.7)

is considered, and the corresponding Root-Locus Diagram with a PD controller is shown at the dynamic pressure of 800Pa in Fig. 4.20 for an illustration. Basically, the controller highly influences the first mode of the drogue motion, while it makes the pole corresponding to the second mode to terminate at the neighboring zero, and the actuator pole to terminate at the controller zero.

Looking at the Bode Diagram shown in Fig. 4.21, the PD controller reduces the magnitude of the peak gain and the corresponding frequency; the peak gain reduced from \(-9.33\)dB to \(-11.6\)dB at the corresponding frequency from 5.83 rad/sec to 5.45 rad/sec for the vertical channel, and from \(-5.58\)dB to \(-5.45\)dB at the corresponding frequency from 5.80 rad/sec to 4.89 rad/sec for the horizontal channel.
(a) Vertical mode  
(b) Horizontal mode

Figure 4.20 Root-Locus Diagram with a PD controller
The corresponding closed-loop time domain simulation responses to the single wave gust as described in Chapter 3 are shown in Fig. 4.22, and to the continuous wave gust in Fig. 4.23. It is apparent that the PD controller reduces the peak acceleration and the corresponding frequency. In the vertical channel, the peak acceleration is decreased from 0.146 m/sec$^2$ to 0.117 m/sec$^2$, which is 20% reduction. In the horizontal channel, the peak acceleration is decreased from 0.239 m/sec$^2$ to 0.193 m/sec$^2$, which is 19% reduction. Observing the responses to the continuous wave gust in Fig. 4.23 the vertical mode response’s peak value is decreased from 0.32 m/sec$^2$ to 0.164 m/sec$^2$, which is 49% reduction, while in the horizontal channel it is decreased from 0.421 m/sec$^2$ to 0.235 m/sec$^2$ m/sec$^2$, which is 46% reduction.
Figure 4.22 Single wave gust responses
Figure 4.23 Continuous wave gust responses
Different controller gains are obtained for the ranges of dynamic pressure by examining the each corresponding Root-Locus Diagrams, and the designed gain values are listed in Table 4.9, and also shown in Fig. 4.24. The controller gains programmed to read the necessary gain values for a given dynamic pressure. This type of method is the foremost method for handling nonlinear parameter variations in flight control systems, known as Gain Scheduling.

Regarding the scheduled gain values based on the actual wind tunnel tests, the derivative action on the acceleration signal is very sensitive to the sensor noise so the corresponding gain value (k_d) is fixed at 0.1. It was observed that increasing the value excited the 2nd mode motion and made the drogue unstable. It was also interesting to observe the effect of dynamic pressure increases on the sensitivity of the scheduled gain values. As the dynamic pressure increases, the drogue response to the change of a gain value becomes much more sensitive.

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Gain</th>
</tr>
</thead>
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<tr>
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<td>Horizontal Mode</td>
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<tr>
<td></td>
<td>K_p</td>
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<tr>
<td>600pa</td>
<td>4</td>
</tr>
<tr>
<td>700pa</td>
<td>3.2</td>
</tr>
<tr>
<td>800pa</td>
<td>2.9</td>
</tr>
<tr>
<td>900pa</td>
<td>2.4</td>
</tr>
<tr>
<td>1000pa</td>
<td>2.3</td>
</tr>
</tbody>
</table>
4.4 Comments on transfer functions via system identification

In the design version 1.0 study, the System Identification (SI) method is used to obtain the control design model in a transfer function format. The drogue positions are measured via the joint mounted potentiometers using the kinematic equations in Chapter 3, and they are numerically differentiated twice to obtain the acceleration signal. Then, the acceleration to the control command transfer function is obtained using the SI method.

In the present study, the drogue acceleration signals are readily available from the onboard IMU sensors, so the SI method can be readily applied to the drogue acceleration signals measured by the IMU sensor to the control command. To compare and cross-validate the control design model, the SI method is again used to obtain the control design model in the present study.

There are several different SI methods, but all require a good prior knowledge on the system dynamic characteristics to obtain a quality representative dynamic model. In addition, the process involved is iterative to ‘identify’ or ‘construct’ the best matching model. From the mathematical model analysis, the 4th order transfer function model order
is chosen to begin with the SI methods. Table 4.9 shows the transfer functions via the SI methods for the ranges of dynamic pressure.

**Table 4.5 Transfer function by SI technique**

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Vertical Channel, $G_v(s)$</th>
<th>Horizontal Channel, $G_H(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>$\frac{1.324s^3 + 2.936s^2 + 1.488s - 0.693}{s^3 + 15.88s^3 + 139.7s^2 + 697.7s + 2898}$</td>
<td>$\frac{0.6598s^3 + 1.05s^2 + 0.898s + 0.09038}{s^3 + 9.789s^3 + 58.19s^2 + 207.3s + 558.1}$</td>
</tr>
<tr>
<td>600</td>
<td>$\frac{2.347s^3 + 31.73s^2 - 31.83s - 3.282}{s^3 + 38.08s^3 + 429.2s^2 + 1792s + 1.148e04}$</td>
<td>$\frac{1.597s^3 + 7.708s^3 + 2.23s - 0.4557}{s^4 + 23.33s^3 + 155.5s^2 + 690.9s + 2838}$</td>
</tr>
<tr>
<td>700</td>
<td>$\frac{2.381s^3 + 11.85s^2 - 13.35s + 7.206}{s^4 + 22.98s^3 + 246.8s^2 + 1004s + 6226}$</td>
<td>$\frac{1.054s^3 + 9.742s^3 + 6.011s - 0.09038}{s^3 + 14.65s^3 + 162.6s^2 + 434.9s + 2928}$</td>
</tr>
<tr>
<td>800</td>
<td>$\frac{0.7972s^3 + 14.75s^2 - 12.38s - 36.9}{s^4 + 11.02s^3 + 271s^2 + 623.3s + 7433}$</td>
<td>$\frac{0.9839s^3 + 7.416s^2 + 2.851s - 3.754}{s^3 + 14.65s^3 + 162.6s^2 + 434.9s + 2928}$</td>
</tr>
<tr>
<td>900</td>
<td>$\frac{1.015s^3 + 18.71s^2 - 24.24s - 9.284}{s^4 + 25.33s^3 + 330.9s^2 + 1250s + 8816}$</td>
<td>$\frac{4.191s^2 + 1.875s^2 - 0.1778s + 0.6664}{s^4 + 44.83s^3 + 91.03s^2 + 1125s + 183.4}$</td>
</tr>
<tr>
<td>1000</td>
<td>$\frac{0.7677s^3 + 12.49s^2 - 9.348s + 6.892}{s^4 + 15.46s^3 + 281.4s^2 + 756.1s + 8815}$</td>
<td>$\frac{0.9898s^3 + 5.626s^2 + 6.936s - 0.7031}{s^4 + 8.662s^3 + 123.2s^2 + 287.7s + 2198}$</td>
</tr>
</tbody>
</table>

It is interesting to observe that the transfer functions via the SI methods has three zeros while the control design model has four zeros including two zeros corresponding to the pure differentiator. This may be due to the condition that the SI method requires the transfer function to be rational. Furthermore, due to the noisy nature of acceleration signal the pure differentiation action is not reflected on the transfer function zeros. At some dynamic pressure conditions, the non-minimum phase zeros are present as shown in Fig. 4.23 and Fig. 4.24. In general, it is difficult to observe the patterns and the trends of the known test-rig system properties from these plots. It is suspected that the test data
include noises and errors from poor steady wind quality as well as measurement system. The SI method requires high quality experimental data, and improvements of test system may be needed. As a result, the transfer functions via the SI methods are found to be not suitable to the control design model for this study.

![Pole-Zero Map](image)

**Figure 4.25** Pole-Zero map of horizontal channel transfer function via SI technique
CHAPTER 5

RESULTS ANALYSIS

In Chapter 4, the proportional-derivative (PD) control is designed for the ASDRS, and the controller gains are found at a discrete number of dynamic pressure points. These gains are scheduled for the ranges of dynamic pressure, and the resulting control logics are programmed into the onboard controller. This chapter discusses the results obtained from the closed-loop nonlinear simulation tests and the dynamic wind tunnel tests.

5.1 Test descriptions

To study the performance of the ASDRS with the proposed hardware and software, a series of the closed-loop tests are carried out. There are two primary variables for the closed-loop tests to be run; the dynamic pressure and the gust input type.

The ranges of dynamic pressure for the closed-loop tests are from 600Pa to 100Pa. This range is determined by considering the drogue position in the ADWT and the typical flight speed of the small UAVs. The minimum dynamic pressure of 600Pa is chosen for the drogue not to touch the bottom of the wind tunnel test section. The highest dynamic pressure of 1000Pa is chosen since this corresponds to the typical maximum speed of 150km/h for most small UAVs. The closed-loop tests are thus carried out for 5 discrete dynamic pressure points from 600Pa to 1000Pa with an increment of 100Pa.

The other primary variable for the closed-loop test is the type of gust. Atmospheric turbulence in nature is very random and typically described by statistical methods. A
typical gust load analysis for aircraft starts with a simple single wave gust, or so called ‘1-cos’ gust, and a continuous random gust based on statistical analysis. For the present study, a similar pattern of wind gusts used for aircraft gust load analysis is generated by the artificial generator as described in Chapter 3. There are 4 types of gust input used for the closed-loop tests; vertical continuous wave gust, vertical single wave gust, horizontal continuous wave gust, and horizontal single wave gust.

Using the two primary variables described above, the test plan matrix is established as shown in Table 5.1. There are total 20 test cases, and two types of the closed-loop tests are carried out for each test case; the nonlinear closed-loop numerical simulation test, and the closed-loop ASDRS dynamic wind tunnel test. The results are compared and discussed in the following sections.

<table>
<thead>
<tr>
<th>Dynamic Pressure (Pa)</th>
<th>Continuous wave gust</th>
<th>Single wave gust</th>
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<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>Horizontal</td>
</tr>
<tr>
<td>600</td>
<td>TEST-01</td>
<td>TEST-02</td>
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<tr>
<td>700</td>
<td>TEST-05</td>
<td>TEST-06</td>
</tr>
<tr>
<td>800</td>
<td>TEST-09</td>
<td>TEST-10</td>
</tr>
<tr>
<td>900</td>
<td>TEST-13</td>
<td>TEST-014</td>
</tr>
<tr>
<td>1000</td>
<td>TEST-17</td>
<td>TEST-18</td>
</tr>
</tbody>
</table>

### 5.2 Response to vertical continuous wave gust

To understand the effectiveness of the proposed ASDRS, the level of drogue acceleration and its trajectory are carefully examined. Fig. 5.1 shows the Y and Z
direction acceleration of the drogue from the simulation and the wind tunnel test at each
dynamic pressure condition when vertical continuous wave gust was applied. Basically,
the artificial gust generator induces the wind speed variation in all three directions and
these are quantified via the 5-hole probe as described in Chapter 3. The measured wind
variations (profiles) are used to run the nonlinear closed-loop simulation. Also, the
continuous wave gust frequency is set close to the natural frequency of the drogue motion
to induce a resonance condition.

Observing Fig. 5.1, both Y and Z accelerations are similar in magnitudes at lower
dynamic pressure while Z acceleration gets larger than Y at higher dynamic pressure. The
acceleration level from the nonlinear simulation is overall greater than the wind tunnel
test. This difference may be due to not only the model inaccuracies as discussed in
Chapter 4, but also the errors in measuring the gust velocity components. Nevertheless,
the acceleration level from the nonlinear simulation and the wind tunnel test is in general
comparable in magnitude. Furthermore, the Z-acceleration increases as the dynamic
pressure increases for the both tests. The acceleration level is reduced with the ASDRS
controller except for the Y acceleration at 900Pa.
Figure 5.1 Drogue acceleration for the vertical continuous wave gust
Figure 5.2 shows the drogue trajectory on the Y-Z plane. It is interesting to observe that at lower dynamic pressure the drogue trajectory from the wind tunnel test exhibits more random characteristics than those at higher dynamic pressure. At higher dynamic pressure, the vertical motion is dominant as expected. This may be due to the characteristics of the wind speed variations produced by the artificial gust generator. When the trajectories from the simulation are compared to the test, the magnitude of the drogue trajectory from the simulation is smaller than that of the test. This could be again due to the model error associated with the damping estimation and the errors from underestimating the wind variation components.
Figure 5.2 Drogue trajectory for the vertical continuous wave gust
The numerical results are summarized in Table 5.2 and 5.3, and graphically shown in Fig. 5.3. In general, the acceleration level of the drogue is decreased by the controller except a few cases. The vertical acceleration is reduced up to 81% in the wind tunnel test results. The horizontal acceleration reduction is not so noticeable because the magnitude caused by the gust is relatively small. On the other hand, it is interesting to observe that the horizontal acceleration level is increased in the simulation test. The gust response may be too fast and outside the controller bandwidth. This should be due to the lack of modeling data as discussed earlier, and further research should be carried out to improve the model fidelity.

Aside from the acceleration magnitude, it is worth while looking at the trajectory of the drogue motion. The travel distance here represents the total displacement made by the drogue during the given sample time in Control ON and OFF conditions. Overall, the distance traveled by the drogue upon the vertical continuous gust input is reduced in both tests upon the gust input. At low dynamic pressure, the reduction rate is relatively small because the motion is slow and undergoes somewhat random trajectory. The reduction magnitude generally increases as the dynamic increases.

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Continuous vertical gust</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Ay_rms (m/sec^2)</td>
<td>Az_rms (m/sec^2)</td>
</tr>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
</tr>
<tr>
<td>600</td>
<td>0.522</td>
<td>0.471</td>
</tr>
<tr>
<td>700</td>
<td>0.577</td>
<td>0.571</td>
</tr>
<tr>
<td>800</td>
<td>0.717</td>
<td>0.645</td>
</tr>
<tr>
<td>900</td>
<td>0.770</td>
<td>0.788</td>
</tr>
<tr>
<td>1000</td>
<td>1.013</td>
<td>0.879</td>
</tr>
</tbody>
</table>

Table 5.2 Nonlinear simulation results for the continuous vertical gust input
Table 5.3 Wind tunnel test results for the continuous vertical gust input

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Ay_rms (m/sec^2)</th>
<th>Az_rms (m/sec^2)</th>
<th>Travel distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Reduction (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.094</td>
<td>0.115</td>
<td>−18.352</td>
</tr>
<tr>
<td>700</td>
<td>0.253</td>
<td>0.277</td>
<td>−8.668</td>
</tr>
<tr>
<td>800</td>
<td>0.091</td>
<td>0.099</td>
<td>−7.615</td>
</tr>
<tr>
<td>900</td>
<td>0.157</td>
<td>0.131</td>
<td>19.517</td>
</tr>
<tr>
<td>1000</td>
<td>0.121</td>
<td>0.126</td>
<td>−3.842</td>
</tr>
</tbody>
</table>

(a) Acceleration reduction

(b) Distance reduction

Figure 5.3 ASDRS performance comparison for the continuous vertical gust input

5.3 Response to horizontal continuous wave gust

Similar to the vertical continuous wave gust, the acceleration and the trajectory of the drogue are also examined under the horizontal continuous wave gust. Shown in Fig. 5.4 are the acceleration responses in Y and Z direction. As expected, the Y-acceleration is dominant in this case. Looking at the acceleration levels for the dynamic pressure of 600Pa, 800Pa, and 1000Pa in the wind tunnel test results, the Y direction acceleration are quite large since the frequency of the wave gust was set at the natural frequency of the drogue. The magnitude is relatively small at 700Pa, which may be caused by miss
matching of the resonant frequency, or some unknown factors such as some errors in the
gust generator, non-linear friction effect, etc.

The nonlinear simulation test shows that the controller works well throughout the
dynamic pressure ranges for the horizontal continuous gust situations. The Z acceleration
level is somewhat larger compared to that of the wind tunnel test, and again it may be due
to the modeling inaccuracies from various frictions, damping effects, and the erroneous
gust component measurements. This is clearly reflected on the drogue trajectory in the Y-Z plane discussed next.
Figure 5.4 Acceleration comparisons for the horizontal continuous wave gust

Fig. 5.5 shows the drogue motion trajectory in the Y-Z plane under the horizontal continuous gust. As expected, the horizontal movement is dominant. The trajectory is generally an oval shape for both the nonlinear simulation test and the wind tunnel test. At 700Pa, as discussed earlier, the drogue trajectory from the wind tunnel test exhibits somewhat different patterns for the same reasons described earlier. In general, the ASDRS significantly reduces the drogue motions upon the horizontal continuous gust case.
Figure 5.5 Trajectory comparisons for the horizontal continuous wave gust

Again, the numerical results are summarized in Table 5.4 and 5.5, and graphically shown in Fig. 5.6. In this case, the horizontal acceleration is reduced up to 90% in the wind tunnel test results. The reduction amount is affected by the acceleration value at the Control OFF condition. If the Control OFF condition value is large enough, the acceleration is reduced largely. In other words, the control system may not able to eliminate a small fluctuation of the drogue acceleration. The Z-acceleration values are relatively small for continuous horizontal gust, so the Z-acceleration reduction amount is also smaller than Y-acceleration reduction.
In terms of the drogue travel distance, the ASDRS reduces it up to 81%. Again in overall sense, the drogue Y trajectory decrease more than 50% with the control except at 700Pa test. The controller did not perform as expected at this dynamic pressure conditions, and needs further investigations.

Table 5.4 Nonlinear simulation results for the continuous horizontal gust input

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Ay_rms (m/sec^2)</th>
<th>Az_rms (m/sec^2)</th>
<th>Travel distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Change (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.914</td>
<td>1.531</td>
<td>-40.259</td>
</tr>
<tr>
<td>700</td>
<td>0.975</td>
<td>2.122</td>
<td>-54.059</td>
</tr>
<tr>
<td>800</td>
<td>1.305</td>
<td>2.055</td>
<td>-36.484</td>
</tr>
<tr>
<td>900</td>
<td>1.203</td>
<td>2.887</td>
<td>-58.342</td>
</tr>
<tr>
<td>1000</td>
<td>1.359</td>
<td>3.131</td>
<td>-56.592</td>
</tr>
</tbody>
</table>

Table 5.5 Wind tunnel test results for the continuous horizontal gust input

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Ay_rms (m/sec^2)</th>
<th>Az_rms (m/sec^2)</th>
<th>Travel distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Change (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.573</td>
<td>5.384</td>
<td>-89.355</td>
</tr>
<tr>
<td>700</td>
<td>0.610</td>
<td>1.008</td>
<td>-39.456</td>
</tr>
<tr>
<td>800</td>
<td>1.432</td>
<td>15.28</td>
<td>-90.633</td>
</tr>
<tr>
<td>900</td>
<td>1.670</td>
<td>4.625</td>
<td>-63.896</td>
</tr>
<tr>
<td>1000</td>
<td>1.402</td>
<td>9.703</td>
<td>-85.555</td>
</tr>
</tbody>
</table>

Figure 5.6 ASDRS performance comparison for the continuous horizontal gust input
5.4 Response to the vertical single wave gust

In this and the next section, the drogue acceleration and its trajectory subject to the vertical single wave gust input are carefully examined. It is to be noted that the artificial generator is not able to generate a pure vertical wind component. Thus, the vertical single wave gust input indeed induces both horizontal and vertical wind component, and consequently the drogue responses in both Y and Z direction.

Figure 5.7 shows the acceleration responses to the vertical single wave gust. The wind tunnel test shows some high frequency contents and continuing oscillatory response, while the simulation responses show rapid peak response in the beginning. This may be due to persisting wind tunnel turbulence after the single wave gust input from the gust generator, while the simulation inputs are set to zero after the gust input. For this reason, the simulation response shows relatively smooth pattern than the wind tunnel test. It also makes different Y-acceleration response pattern between the simulation and the wind tunnel test. At lower dynamic pressure, the Y-acceleration response has pretty large magnitude, thus resulting in relatively a random patterned motion. The Y-acceleration is decreased at higher dynamic pressure. The Z-acceleration shows decaying oscillatory drogue motion. Overall, the Z-acceleration responses show similar frequency and damping characteristics.
<table>
<thead>
<tr>
<th></th>
<th>Y-Acceleration</th>
<th>Z-Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 Pa</td>
<td><img src="image" alt="Y-Acceleration Graph" /></td>
<td><img src="image" alt="Z-Acceleration Graph" /></td>
</tr>
<tr>
<td>700 Pa</td>
<td><img src="image" alt="Y-Acceleration Graph" /></td>
<td><img src="image" alt="Z-Acceleration Graph" /></td>
</tr>
<tr>
<td>800 Pa</td>
<td><img src="image" alt="Y-Acceleration Graph" /></td>
<td><img src="image" alt="Z-Acceleration Graph" /></td>
</tr>
<tr>
<td>900 Pa</td>
<td><img src="image" alt="Y-Acceleration Graph" /></td>
<td><img src="image" alt="Z-Acceleration Graph" /></td>
</tr>
</tbody>
</table>
Fig. 5.7 Acceleration comparisons for the single wave vertical gust

Fig. 5.8 shows the drogue trajectory for the vertical single wave gust input. There are noticeable differences between the simulation and the wind tunnel test results. This is again due to the single wave gust input used for the simulation, which does not reflect the persisting turbulence in the wind tunnel after the gust input is applied. In addition, the initial condition would be a possible cause for these differences. The initial condition used in the nonlinear simulation represents an exact stationary condition, but for the wind tunnel test the drogue undergoes a minute motion caused by turbulence.

Looking at the trajectory at 600Pa, the difference is apparent as the wind tunnel test shows the motion in Y-direction. The trajectory pattern of the simulation and the wind tunnel test is relatively similar for 700Pa, 800Pa, and 900Pa, but shows some difference at 1000Pa. This is due to relatively small wind gust magnitude comparing the wind tunnel speed, so the drogue does not respond to the gust input.
Figure 5.8 Trajectory comparisons for the single wave vertical gust

The numerical results are summarized in Table 5.6 and 5.7, and graphically shown in Fig. 5.9. The response decays much faster with the ASDRS controller. The Z-acceleration reduces up to 61% by the controller. The Y-acceleration is not reduced because the vertical directional gust is the major component, and the Y-direction gust is relatively smaller and random comparing to the Z direction gust. The controller does not perform well at some cases, and the Y-acceleration reduction is not consistent in all cases. However, the trajectory reduction overall is close to 20% to 30% for the vertical single wave gusts except at 500 Pa.
Table 5.6 Nonlinear simulation results for the single wave vertical gust

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Ay_rms (m/sec^2)</th>
<th>Az_rms (m/sec^2)</th>
<th>Travel distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Change (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.127</td>
<td>0.123</td>
<td>3.712</td>
</tr>
<tr>
<td>700</td>
<td>0.160</td>
<td>0.158</td>
<td>0.915</td>
</tr>
<tr>
<td>800</td>
<td>0.094</td>
<td>0.178</td>
<td>−47.066</td>
</tr>
<tr>
<td>900</td>
<td>0.192</td>
<td>0.178</td>
<td>7.509</td>
</tr>
<tr>
<td>1000</td>
<td>0.168</td>
<td>0.151</td>
<td>11.249</td>
</tr>
</tbody>
</table>

Table 5.7 Wind tunnel test results for the single wave vertical gust input

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Ay_rms (m/sec^2)</th>
<th>Az_rms (m/sec^2)</th>
<th>Travel distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Change (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.043</td>
<td>0.045</td>
<td>−4.753</td>
</tr>
<tr>
<td>700</td>
<td>0.100</td>
<td>0.173</td>
<td>−42.263</td>
</tr>
<tr>
<td>800</td>
<td>0.052</td>
<td>0.072</td>
<td>−28.145</td>
</tr>
<tr>
<td>900</td>
<td>0.051</td>
<td>0.059</td>
<td>−12.785</td>
</tr>
<tr>
<td>1000</td>
<td>0.061</td>
<td>0.055</td>
<td>12.144</td>
</tr>
</tbody>
</table>

Figure 5.9 ASDRS performance comparison for the single wave vertical gust input

(a) Acceleration reduction
(b) Distance reduction
5.5 **Response to horizontal single wave gust**

The drogue acceleration response to horizontal single wave gust is shown in Fig. 5.10. Unlike the single wave vertical gust, the gust generator produces a noticeable horizontal wind component although the vertical wind component still exists. The acceleration response shows a decaying oscillatory nature of the drogue motion, and the controller enhances active damping effect. The acceleration magnitude from the simulation and the tunnel test are overall comparable.
Fig. 5.11 shows the drogue trajectory for the single wave horizontal gust input. The trajectory from the wind tunnel test shows more random pattern than those to the other types of gust input. This may arise from the deficiency of the current artificial gust generator characteristics as it is not able to generate a dominant horizontal wind component. Nevertheless, the ASDRS reduced the drogue motion in most cases.
Figure 5.11 Trajectory comparisons for the single wave horizontal gust

The numerical results are again summarized in Table 5.8 and 5.9, and graphically shown in Fig. 5.12. With the ASDRS controller, the reduction in the Y-acceleration ranges from 15% to 80% in the wind tunnel test results, while it ranges from 38% to 57% in the simulation tests. For the Z-acceleration, the controller performance is doubtful as the wind tunnel test reveals the increased level of acceleration up to 41% at 900Pa while 76% reduction at 600Pa. In contrast, the reduction magnitude remains around 60% in the simulation results. This is due to relatively small magnitude of the Z directional
acceleration so that the sensor may not able to detect the drogue acceleration from noise signals.

The distance reductions are from 3% to -55%. All cases except 900Pa shows the reduced travel distance. At the 900Pa and 1000Pa, the controller reduces the Y-directional movement but the Z-directional movement is increased. It is suspected that the ASDRS control input may excite the 2nd mode oscillation resulting in the frequency increase.

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Single wave horizontal gust</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ay_rms (m/sec^2)</td>
<td>Az_rms (m/sec^2)</td>
<td>Travel distance (m)</td>
</tr>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Change (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.183</td>
<td>0.427</td>
<td>−57.151</td>
</tr>
<tr>
<td>700</td>
<td>0.124</td>
<td>0.250</td>
<td>−50.525</td>
</tr>
<tr>
<td>800</td>
<td>0.087</td>
<td>0.176</td>
<td>−50.528</td>
</tr>
<tr>
<td>900</td>
<td>0.079</td>
<td>0.153</td>
<td>−48.196</td>
</tr>
<tr>
<td>1000</td>
<td>0.071</td>
<td>0.116</td>
<td>−38.731</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q (Pa)</th>
<th>Single wave horizontal gust</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ay_rms (m/sec^2)</td>
<td>Az_rms (m/sec^2)</td>
<td>Travel distance (m)</td>
</tr>
<tr>
<td></td>
<td>Control ON</td>
<td>Control OFF</td>
<td>Change (%)</td>
</tr>
<tr>
<td>600</td>
<td>0.157</td>
<td>0.798</td>
<td>−80.343</td>
</tr>
<tr>
<td>700</td>
<td>0.054</td>
<td>0.161</td>
<td>−66.777</td>
</tr>
<tr>
<td>800</td>
<td>0.053</td>
<td>0.100</td>
<td>−47.508</td>
</tr>
<tr>
<td>900</td>
<td>0.060</td>
<td>0.075</td>
<td>−20.839</td>
</tr>
<tr>
<td>1000</td>
<td>0.070</td>
<td>0.083</td>
<td>−15.013</td>
</tr>
</tbody>
</table>
5.6 Summary

From the closed-loop test results, it can be summarized that the ASDRS in general is able to reduce the drogue motion. However, for same cases, the controller may not function as expected. There are some factors to be further studied. First of all, there are some noticeable discrepancies between the nonlinear simulation and the wind tunnel tests. Secondly, the artificial gust generator may be redesigned to produce more directionally distinctive gust characteristics. The performance of ASDRS may change with different form of control laws as well as other types of active stabilization method.
CHAPTER 6

CONCLUSION AND FUTURE RESEARCH

6.1 Summary and conclusion

This research presents the development of actively stabilized drogue refueling system (ASDRS) for Probe-Drogue Refueling (PDR) method. The refueling drogue in PDR exhibits unstable motion because of the atmospheric turbulence, the tanker wake effect, and the receiver for-body effect. These lead to excessive pilot workload, mission aborts, and possibly catastrophic accidents. For these reasons and particularly with the advent of unmanned aerial vehicles (UAVs), automating the refueling process has attracted increased attention for research and development. The stabilized drogue may provide a solution for the further advancing of UAVs.

The ASDRS proposed in this study makes use of aerodynamic surface as an actuator for active control. A set of four aerodynamic surfaces is attached to a conventional refueling drogue, which are arranged to produce vertical and horizontal force for the drogue. These surfaces are actuated by servo motors commanded by an onboard controller according to the programmed control logics upon the drogue motion. A fusion sensor based on inertial measurement units (IMUs) called the attitude-heading reference system (AHRS) is used to measure the drogue motion.

A prototype ASDRS is designed, fabricated, and tested for the concept verification. The size of the prototype is approximately 1/3 scale of a full size drogue
currently used for manned aircraft refueling. Thus the design guidelines and the study results could be used to manufacture a full size actively stabilized drogue, but the prototype itself is targeted for refueling of the small UAVs.

To verify the performance of the ASDRS prototype, a test-rig system for dynamic wind tunnel is fabricated. This system is composed of a rigid bar, the ASDRS and a gust generator in the wind tunnel, representing a hose-drogue refueling system that is subject to external disturbance. A mathematical model is developed for the test-rig system based on multi-body dynamic analysis, and the model is used to develop a feedback control law for the ASDRS. The control law is implemented into the ASDRS controller, and the closed-loop system test demonstrated up to 90% reduction of the drogue acceleration due to disturbances.

There are two shortcomings associated with the present study that may lead to further investigations. First, there are some discrepancies in the responses between the numerical simulation results from the mathematical model and the wind tunnel experiment results. There may be many contributing factors but amongst the two should be the estimation of control effectiveness of the aerodynamic surfaces and the various nonlinear effects such as aerodynamic damping and joint frictions. Due to the small magnitude and the lack of the wind tunnel balance system, it was not able to accurately measure the forces and moments produced by the control surface deflection. Instead, the model responses were compared to the wind tunnel test results, and the damping coefficients and control influence parameters (control derivatives) were tuned to match the responses to the best to the experimental results. Furthermore, the gust inputs
produced by the artificial gust generator substantially contributes to the discrepancies between the simulation and the test in the ASDRS performance test.

Secondly, the ASDRS performance degrades at some dynamic pressures, and the feedback control input may lead the drogue going into the 2nd mode oscillation. This could be due to the inherent nonlinear nature of the test-rig system, the lack of fidelity in the control design model by consequence, the actuator limitations, the nature of gust characteristics in the wind tunnel, and the lack of robustness of the proposed controller.

6.2 Future research

The proposed ASDRS design concept in this study demonstrated up to 90% reduction of the drogue motion subject to gust, yet opened up several areas to improve and to be studied further.

First, it is important to accurately measure the control effectiveness of the ASDRS controller. Instead of ad hoc matching and tuning of the mathematical model, accurate static measurement of the control forces and moments for control surface deflections should be available for modeling and control law synthesis. Also, various wing arrangements and configurations need to be studied and experimented to achieve the optimal performance.

Secondly, the test-rig system hardware may need further improvements with high quality joints and sensing components, not to mention redesigning the artificial gust generator for distinctive directional wind components. This should lead to a better mathematical model, a better feedback controller design, and accurate ASDRS performance analysis.
Thirdly, the present work does not allow the roll motion of the drogue. The drogue and the rigid bar are connected together by a universal joint, which does not permit the roll motion of the drogue. The drogue indeed exhibits the rolling motion especially during deploying and stowing phase, yet the motion seems minor under disturbances and during coupling. On the other hand, the ASDRS concept presented in this study induces the rolling motion when allowed to roll due to control surface deflections. This roll motion, however, can also be stabilized via the control surfaces as well. Future study must include the roll motion of the drogue based upon the accurate measurement of the roll control effectiveness.

Fourth, it is worthwhile looking at different types of sensing for the drogue motion. The present ASDRS design depends on the acceleration of the drogue, but if an accurate position measurement is available via non-contact sensors such as LIDAR or differential GPS, it may allow the position control of the drogue and also improve the stabilization effect.

Last, as the present ASDRS is targeted for small UAVs applications, performance verification through flight test should be carried out. The flight test should include the tanker alone flight test as well as the receiver plane.

The future sky will be dominated by UAVs with no questions. UAVs evolve very quickly, and the needs are various and very demanding in quantities, qualities, and sizes. Autonomous aerial refueling should be the key technology for some UAVs to fly longer and farther, and also for the safety of existing manned flight operations. The present study suggests one possible concept for autonomous aerial refueling, and the concept has been verified via design, test and evaluation based on analyses and experiments.
REFERENCES


[23] Kuk T., Ro K., “DESIGN, TEST AND EVALUATION OF AN ACTIVELY STABILISED DROGUE REFUELLING SYSTEM”, AIAA Infotech@Aerospace Conference, 2011


APPENDIX

A. DROGUE CODE

/*
* Drogue control system
* By Taeseung Kuk
*/

#include <avr/interrupt.h>
#include <math.h>
#include <avr/io.h>

volatile unsigned int Start_Pulse =0;
volatile unsigned int Stop_Pulse =0;
volatile unsigned int Pulse_Width =0;
volatile int Test=0;
volatile int Test2=0;
volatile int Temp=0;
volatile int Counter=0;
volatile byte PPM_Counter=0;
volatile int PWM_RAW[8] =
{2400,2400,2400,2400,2400,2400,2400,2400};
unsigned int All_PWM=1500;
unsigned int ch_Out[8] = {1500, 1500, 1500, 1500, 1500, 1500, 1500, 1500 } ;

long timer=0;
long timer2=0;

unsigned long time_old1=0;  // sbg timing
unsigned long time_old2=0;  // servo control
unsigned long time_old3=0;  // send_gcs
unsigned long time_old4=0;  // send_wire
unsigned long time_old5=0;  // read_gcs

unsigned long time_now=0;

byte mode =0;
byte state = 0;
byte n=0;
byte sizein1;
byte sizein2;
byte sbg_update=0;
byte gcs_update=0;
byte inByte[48] ; // serial input buffer
byte inByte1[48] ; // sbg serial input data
byte inByte2[20] = { 255, 0, 254, 97, 168, 97, 168, 97, 168, 97,
168, 0, 0, 0, 0, 0, 0, 0, 0, 0, 170 } ; // 3DR radio input data from GCS
byte outByte[40] ; // Data packet for TF and GCS

float sbg_data[9]; // floating type sbg data

float roll, pitch, yaw, ax, ay, az, p, q, r, roll_s;
//int roll_i, pitch_i, yaw_i, ax_i, ay_i, az_i, p_i, q_i, r_i;
float cp, sp, cr, sr, r_ax, r_ay, r_az;
float r_ay_old, r_az_old, r_ay_old2, r_az_old2, r_ay_old3, r_az_old3,
r_ay_old4, r_az_old4, r_ay_old5, r_az_old5;

float horizon, vertical, rolling, drag, horizon_old, vertical_old,
horizon_old2, vertical_old2, horizon_old3, vertical_old3;
unsigned int wing[4] = {41500, 1500, 1500, 1500};

int gain[4], deg[4];

float temp_float;
byte temp_byte;
int temp_int, temp_i;
unsigned int temp_ui;

float r2d= 57.29577951;
float d2r= 0.01745329252;
unsigned int idx=0;

union u_tag
{byte b[4];
 float fval; } u;

union u_tag2
{byte b2[2];
 int int2; } u2;

union u_tag3
{byte b3[2];
 unsigned int uint3; } u3;

void setup()
{
 Init_PWM1(); //OUT2&3
 Init_PWM3(); //OUT6&7
 Init_PWM5(); //OUT0&1
 Init_PPM_PWM4(); //OUT4&5
 // initialize serial:
 Serial.begin(57600); // programer port
 Serial1.begin(115200); // SBG Serial1 on pins 19 (RX) and 18 (TX);
 Serial2.begin(57600); // 3DR radio port
}
void loop()
{
    read_sbg();
    // send_tf1();
    servo_control();
    // send_tf2();
    packet();
    if (state == 1 || state == 11 )
    {
        read_gcs();
        send_gcs();
    }
}

void read_sbg(void)
{
    time_old1 = millis();
    time_now= millis();

    while (((Serial1.available()<47) && ((time_now-time_old1)<21))
    {
        time_now= millis();
        delayMicroseconds(500);  
    }

    for (byte i=0;i<48;i++)
    inByte[i] = Serial1.read();

    {
        for (byte i=0;i<48;i++)
        inByte1[i] = inByte[i];

        for (byte i=0;i<9;i++)
        {
            u.b[0] = inByte1[i*4+8] ;
            u.b[1] = inByte1[i*4+7];
            u.b[2] = inByte1[i*4+6];
            u.b[3] = inByte1[i*4+5];
            sbg_data[i] = u.fval;
        }

        roll  = sbg_data[0]*r2d;
        pitch = sbg_data[1]*r2d;
        yaw   = sbg_data[2]*r2d;
        p      = sbg_data[3]*r2d;
        q      = sbg_data[4]*r2d;
        r      = sbg_data[5]*r2d;
        ax     = sbg_data[6];
        ay     = sbg_data[7];
        az     = sbg_data[8];

        r_ay_old5 = r_ay_old4 ;
        r_ay_old4 = r_ay_old3 ;
r_ay_old3 = r_ay_old2;
r_ay_old2 = r_ay_old;
r_ay_old = r_ay;

r_az_old5 = r_az_old4;
r_az_old4 = r_az_old3;
r_az_old3 = r_az_old2;
r_az_old2 = r_az_old;
r_az_old = r_az;

cp = cos(sbg_data[1]);
sp = sin(sbg_data[1]);
cr = cos(sbg_data[0]);
sr = sin(sbg_data[0]);
r_ax = ax*cp + az*cr*sp + ay*sp*sr;
r_ay = ay*cr - az*sr;
r_az = az*cp*cr - ax*sp + ay*cp*sr + 9.80;

idx++;
if (idx >= 20)
    {idx = 0;}

roll_s = roll_s * 0.9 + 0.1*roll;

switch (state)
{
    case 1:
        state = 2;
        break;
    case 2:
        state = 1;
        break;
    case 11:
        state = 2;
        break;
    case 22:
        state = 1;
        break;
    default:
        state = 1;
}
}

else
{
    sizein1 = Serial1.available();
    Serial1.flush();

    switch (state)
    {
        case 1:
            state = 22;
            break;
        case 2:
state = 11;
break;
case 11:
    state = 22;
break;
case 22:
    state = 11;
break;
default:
    state = 11;
}
}

void servo_control(void)
{
    time_old2 = millis();

    if ( wing[0]<3000)                     // wing pwm control
    {
        mode = 11;

        for (byte i=0; i<4 ; i++)
        {
            ch_out[i] = wing[i] ;
            OutputCh(i, ch_out[i]);
        }
    }
    else if ( 10000<wing[0] && wing[0]<20000  )  // wing deg control
    {
        mode = 22;

        temp_i =(wing[0]-15000) ;
        ch_out[0] = (unsigned int)(1500 -0.2066*temp_i ) ;
        temp_i =(wing[1]-15000) ;
        ch_out[1] = (unsigned int)(1500 -0.2086*temp_i ) ;
        temp_i =(wing[2]-15000) ;
        ch_out[2] = (unsigned int)(1500 -0.2115*temp_i ) ;
        temp_i =(wing[3]-15000) ;
        ch_out[3] = (unsigned int)(1500 -0.2292*temp_i ) ;

        for (byte i=0; i<4 ; i++)
            OutputCh(i, ch_out[i]);
    }
    else if ( 20000<wing[0] && wing[0]<30000 )   // mode control
    {
        mode = 33;

        horizon  = int(wing[0] - 25000)/100 ;
        vertical = int(wing[1] - 25000)/100 ;
        rolling  = int(wing[2] - 25000)/100 ;
        drag     = int(wing[3] - 25000)/100 ;
    }
deg[0] = constrain ((horizon + vertical + rolling + drag)*100, -2500, 2500);
deg[1] = constrain ((horizon - vertical + rolling - drag)*100, -2500, 2500);
deg[2] = constrain ((-horizon - vertical + rolling + drag)*100, -2500, 2500);
deg[3] = constrain ((-horizon + vertical + rolling - drag)*100, -2500, 2500);

ch_out[0] = (unsigned int)(1501 + (deg[0])*(-0.195));
ch_out[1] = (unsigned int)(1516 + (deg[1])*(-0.184));
ch_out[2] = (unsigned int)(1500 + (deg[2])*(-0.2115));
ch_out[3] = (unsigned int)(1500 + (deg[3])*(-0.2292));

for (byte i=0; i<4; i++)
  OutputCh(i, ch_out[i]);
}
else if (30000<wing[0] && wing[0]<40000) // auto mode 1
{
  mode = 44;
  horizon_old2 = horizon_old;
  vertical_old2 = vertical_old;

  horizon_old = horizon;
  vertical_old = vertical;

  // horizon = -1 * (gain[0]/100) * r_ay;
  // vertical = -1 * (gain[2]/100) * r_az;

  horizon = -1 * (gain[0]/100) * r_ay -1.0 * (gain[1]/100) * (r_ay -r_ay_old)*50; // PD controller
  vertical = -1 * (gain[2]/100) * r_az -1.0 * (gain[3]/100) * (r_az -r_az_old)*50; // PD controller
  rolling  = 0;
  drag     = 0;

  deg[0] = constrain ((horizon + vertical + rolling + drag)*100, -2500, 2500);
  deg[1] = constrain ((horizon - vertical + rolling - drag)*100, -2500, 2500);
  deg[2] = constrain ((-horizon - vertical + rolling + drag)*100, -2500, 2500);
  deg[3] = constrain ((-horizon + vertical + rolling - drag)*100, -2500, 2500);

  // ch_out[0] = (unsigned int)(1500 + (deg[0])*(-0.2066));
  // ch_out[1] = (unsigned int)(1500 + (deg[1])*(-0.2086));

  ch_out[0] = (unsigned int)(1501 + (deg[0])*(-0.195));
  ch_out[1] = (unsigned int)(1516 + (deg[1])*(-0.184));
  ch_out[2] = (unsigned int)(1500 + (deg[2])*(-0.2115));
  ch_out[3] = (unsigned int)(1500 + (deg[3])*(-0.2292));

  for (byte i=0; i<4; i++)
OutputCh(i, ch_out[i]);

}  
else if (40000<wing[0])  // auto mode 2
{
    mode = 55;

    horizon_old3 = horizon_old2;
    horizon_old2 = horizon_old;
    horizon_old = horizon;
    vertical_old3 = vertical_old2;
    vertical_old2 = vertical_old;
    vertical_old = vertical;

    //  horizon = -1 * (gain[0]/100) * r_ay -1.0 *
    //  (gain[1]/100) * (r_ay -r_ay_old)*50;  // PD controller
    //  vertical = -1 * (gain[2]/100) * r_az -1.0 *
    //  (gain[3]/100) * (r_ay -r_ay_old)*50;  // PD controller

    horizon = -1 * (gain[0]/100) * r_ay -1.0 * (gain[1]/100) * (r_ay + r_ay_old + r_ay_old2 + r_ay_old3 + r_ay_old4 + r_ay_old5) /6;  // PI controller
    vertical = -1 * (gain[2]/100) * r_az -1.0 * (gain[3]/100) * (r_az + r_az_old + r_az_old2 + r_az_old3 + r_az_old4 + r_az_old5) /6;  // PI controller

    //  horizon = -1* (gain[0]/100) *( 1.282*r_ay +1.442*z^3 + 1.442 *z^2 - 0.9607 *z - 1.121 ) -
    //  1.282 z^3 + 1.442 z^2 - 0.9607 z - 1.122
    //  z^3 - 2.122 z^2 + 1.244 z - 0.1222

    //  vertical = -1* (gain[2]/100) *(0.1405*r_az +0.1492*z^3 + 0.1492 *z^2 - 0.1232 *z - 0.1319 ) -
    //  0.1405 z^3 + 0.1492 z^2 - 0.1232 z - 0.1319
    //  z^3 - 2.622 z^2 + 2.244 z - 0.6221

    roll = 0;
    drag = 0;

    deg[0] = constrain ((horizon + vertical + roll +
    drag)*100, -2000, 2000);
    deg[1] = constrain((-horizon + vertical + roll -
    drag)*100, -2000, 2000);
    deg[2] = constrain((-horizon - vertical + roll +
    drag)*100, -2000, 2000);
    deg[3] = constrain((horizon - vertical + roll -
    drag)*100, -2000, 2000);
ch_out[0] = (unsigned int)(1501 + (deg[0])*(-0.195));
ch_out[1] = (unsigned int)(1516 + (deg[1])*(-0.184));
ch_out[2] = (unsigned int)(1500 + (deg[2])*(-0.2115));
ch_out[3] = (unsigned int)(1500 + (deg[3])*(-0.2292));

for (byte i=0; i<4 ; i++)
    OutputCh(i, ch_out[i]);
}

void packet (void)
{
    outByte[0] = 0xff;
    outByte[1] = 0x00;
    outByte[2] = 0xFE;
    outByte[3] = mode;   // Control mode
    outByte[4] = state;  // SBG & RF status

    for (byte i = 0 ;i<6 ; i++)
    {
        outByte[i*2+5] = highByte(int(sbg_data[i]*100*r2d));
        outByte[i*2+6] = lowByte(int(sbg_data[i]*100*r2d));
    }

    outByte[17] = highByte(int(r_ax*100));
    outByte[18] = lowByte(int(r_ax*100));
    outByte[19] = highByte(int(r_ay*100));
    outByte[20] = lowByte(int(r_ay*100));
    outByte[21] = highByte(int(r_az*100));
    outByte[22] = lowByte(int(r_az*100));

    for (byte i = 0; i<4 ; i++)
    {
        outByte[i*2+23] = (highByte(ch_out[i]));
        outByte[i*2+24] = (lowByte(ch_out[i]));
    }

    outByte[31] = highByte(int(horizon*100));
    outByte[32] = lowByte(int(horizon*100));
    outByte[33] = highByte(int(vertical*100));
    outByte[34] = lowByte(int(vertical*100));
    outByte[35] = 0xff; // extra 1
    outByte[36] = 0xff; // Servo control mode

    time_now = millis();
    unsigned int temp02 = (unsigned int)(time_now/10);
    outByte[37] = highByte(temp02);
    outByte[38] = lowByte(temp02);
outByte[39] = 0xAA; // ender

void send_tf1(void)
{
    for (byte i=0; i<20; i++)
    {
        Serial.write(outByte[i]);
        delayMicroseconds(60);
    }
}

void send_tf2(void)
{
    for (byte i=20; i<40; i++)
    {
        Serial.write(outByte[i]);
        delayMicroseconds(60);
    }
}

void read_gcs(void)
{
    time_old3 = millis();
    time_now = millis();

    while ((Serial2.available()<20) && ((time_now-time_old3)<10))
    {
        time_now = millis();
        delayMicroseconds(200);
    }

    for (byte i=0; i<20; i++)
    {
        inByte[i] = Serial2.read();
    }

        inByte[19] == 170)
    {
        for (byte i=0; i<20; i++)
        {
            inByte2[i] = inByte[i];

            for (byte i=0; i<4; i++)
            {
                u3.b3[0] = inByte2[i*2+4];
                u3.b3[1] = inByte2[i*2+3];
                wing[i] = u3.uint3;
                u2.b2[0] = inByte2[i*2+12];
                u2.b2[1] = inByte2[i*2+11];
                gain[i] = u2.int2;
            }
        }
    }
} else {
    sizein2 = Serial2.available();
    Serial2.flush();
}

void send_gcs(void) {
    for (byte i=0; i<40; i++)
        Serial2.write (outByte[i]);
}
### B. COEFFICIENTS OF KANE’S EQUATION

\[
\begin{bmatrix}
C_{11} & C_{12} & C_{15} & C_{16} & C_{17} \\
C_{21} & C_{22} & C_{25} & C_{26} & C_{27} \\
C_{51} & C_{52} & C_{55} & C_{56} & C_{57} \\
C_{61} & C_{62} & C_{65} & C_{66} & C_{67} \\
C_{71} & C_{72} & C_{75} & C_{76} & C_{77}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\ddot{\theta}_3 \\
\ddot{\theta}_6 \\
\ddot{\theta}_7
\end{bmatrix}
=
\begin{bmatrix}
R_1 \\
R_2 \\
R_3 \\
R_6 \\
R_7
\end{bmatrix}
\]

\[
c_1 = \cos(\theta_1) \quad s_1 = \sin(\theta_1)
\]
\[
c_2 = \cos(\theta_2) \quad s_2 = \sin(\theta_2)
\]
\[
c_5 = \cos(\theta_5) \quad s_5 = \sin(\theta_5)
\]
\[
c_6 = \cos(\theta_6) \quad s_6 = \sin(\theta_6)
\]
\[
c_7 = \cos(\theta_7) \quad s_7 = \sin(\theta_7)
\]

---

\[
C_{11} = c_2*(c_2*m_1*l_1^2 + c_2*m_2*l_2^2 + I_{yy1}*c_2)
\]
\[
C_{12} = 0
\]
\[
C_{15} = c_2*(c_1*c_5*c_6*l_2*l_3*m_2 + c_6*l_2*l_3*m_2*s_1*s_5)
\]
\[
C_{16} = -c_2*(c_1*l_2*l_3*m_2*s_5*s_6 - c_5*l_2*l_3*m_2*s_1*s_6)
\]
\[
C_{17} = 0
\]
\[
C_{21} = 0
\]
\[
C_{22} = m_1*l_1^2 + m_2*l_2^2 + I_{yy1}
\]
\[
C_{25} = c_1*c_6*l_2*l_3*m_2*s_2*s_5 - c_5*c_6*l_2*l_3*m_2*s_1*s_2
\]
\[
C_{26} = c_2*c_6*l_2*l_3*m_2 + c_1*c_5*l_2*l_3*m_2*s_2*s_6 + l_2*l_3*m_2*s_1*s_2*s_5*s_6
\]
\[
C_{27} = 0
\]
\[
C_{51} = c_1*c_2*c_5*c_6*l_2*l_3*m_2 + c_2*c_6*l_2*l_3*m_2*s_1*s_5
\]
\[
C_{52} = c_1*c_6*l_2*l_3*m_2*s_2*s_5 - c_5*c_6*l_2*l_3*m_2*s_1*s_2
\]
\[
C_{55} = I_{xx2} - I_{xx2}*c_6^2 + I_{yy2}*c_6^2 + c_6^2*l_3^2*m_2
\]
\[
C_{56} = 0
\]
\[
C_{57} = I_{xx2}*s_6
\]
\[
C_{61} = c_2*c_5*l_2*l_3*m_2*s_1*s_6 - c_1*c_2*l_2*l_3*m_2*s_5*s_6
\]
C62 = c2*c6*12*13*m2 + c1*c5*12*13*m2*s2*s6 + 12*13*m2*s1*s2*s5*s6
C65 = 0
C66 = m2*13^2 + Iyy2
C67 = 0

C71 = 0
C72 = 0
C75 = Ixx2*s6
C76 = 0
C77 = Ixx2

R1 = (c2*c5*c6*12*13*m2*s1 - c1*c2*c6*12*13*m2*s5)*t5_d^2 + (- 2*c1*c2*c5*12*13*m2*s6 - 2*c2*12*13*m2*s1*s5*s6)*t5_d*t6_d + (c2*c5*c6*12*13*m2*s1 - c1*c2*c6*12*13*m2*s5)*t6_d^2 - t1_d*t2_d*(2*c2*m1*s2*l1^2 + 2*c2*m2*s2*l2^2 + 2*Iyy1*c2*s2) - Fz1*c1*c2*l1 - Fz2*c1*c2*l2 - Fx1*c2*l1*s1 - Fx2*c2*l1*s1

R2 = (c2*m1*s2*11^2 + c2*m2*s2*12^2 + Iyy1*c2*s2)*t1_d^2 + (c1*c5*c6*12*13*m2*s2 + c6*12*13*m2*s1*s2*s5)*t5_d^2 + (2*c5*12*13*m2*s1*s2*s6 - 2*c1*12*13*m2*s5)*t5_d*t6_d + (c1*c5*c6*12*13*m2*s2 - c2*12*13*m2*s6 + c6*12*13*m2*s1*s2*s5)*t6_d^2 + Fy1*c2*l1 + Fy2*c2*l2 - Fx1*c1*l1*s2 - Fx2*c1*l2*s2 + Fz1*11*l1*s1*s2 + Fz2*12*l1*s1*s2

R5 = (c1*c2*c6*12*13*m2*s5 - c2*c5*c6*12*13*m2*s1)*t1_d^2 + (- 2*c1*c5*c6*12*13*m2*s6 - 2*c6*12*13*m2*s1*s2*s5)*t1_d*t2_d + (c1*c2*c6*12*13*m2*s5 - c2*c5*c6*12*13*m2*s1)*t2_d^2 - My2 - t5_d*t6_d*(2*c6*m2*s6*13^2 - 2*Ixx2*c6*s6 + 2*Iyy2*c6*s6) - Fz2*c5*c6*13 - Fx2*c6*13*s5 + Ixx2*c6*t6_d*t7_d

R6 = (c1*c2*c5*12*13*m2*s6 + c2*12*13*m2*s1*s5*s6)*t1_d^2 + (2*c1*12*13*m2*s2*s5*s6 - 2*c5*12*13*m2*s1*s2*s6)*t1_d*t2_d + (c1*c2*c5*12*13*m2*s6 - c6*12*13*m2*s2 + c2*12*13*m2*s1*s5*s6)*t2_d^2 + (c6*m2*s6*13^2 - Ixx2*c6*s6 + Iyy2*c6*s6)*t5_d^2 - Ixx2*c6*t7_d*t5_d - Mz2*c5 - Mx2*s5 + Fy2*c6*l3 - Fx2*c5*13*s6 + Fz2*13*s5*s6

R7 = Mz2*c6*s5 - Mx2*c5*c6 - My2*s6 + Ixx2*c6*t5_d*t6_d
APPENDIX

C. FIVE HOLE PROBE

The 5-hole probe is calibrated in the Advanced Design Wind Tunnel at Western Michigan University. The calibration range is from $-20^\circ$ to $+20^\circ$ angle of attack (AoA) and angle of side slip (AoS). Fig. 3.15 ~ 3.17 shows the collected data for calibration. The pressure sensor 1 measures the Pitot\textsubscript{measured}. The pressure sensor 2 measures the Pitch\textsubscript{measured}. The pressure sensor 3 measures the Yaw\textsubscript{measured}. These three measured data can be expressed as Eq. C.1 ~ C.3 using the collected data. Coefficients $c_1$–$c_7$ are defined by the measured calibration data. Q, $\alpha$, and $\beta$ values are functions of these equations. First, $A_1$ is defined as the ratio of Pitch\textsubscript{measured} to Yaw\textsubscript{measured} in Eq. C.4. By converting Eq. C.4 into Eq. C.5, it relates $\alpha$ and $\beta$. $A_2$ is defined as the ratio of Pitot\textsubscript{measured} to Yaw\textsubscript{measured} in Eq. C.8 by substituting $\alpha$ with $\beta$ using Eq. C.5. As a result, the 4th order $\beta$ function is obtained as Eq. C.9. By solving this equation using numerical method, $\beta$ value can be found. Next, $\alpha$ is calculated using Eq. C.5, and Q from Eq. C.1. Using Q, $\alpha$, and $\beta$ values, three axis wind vector components are obtained.

\[
\text{pitot}_\text{measured} = f_1(Q, \alpha, \beta) = Q \cdot (1-c_1 \cdot \alpha^2) \cdot (1-c_1 \cdot \beta^2) \tag{C.1}
\]

\[
\text{pitch}_\text{measured} = f_2(Q, \alpha, \beta) = Q \cdot (c_2 \cdot \alpha + c_3 \cdot \beta + c_4) \tag{C.2}
\]

\[
\text{yaw}_\text{measured} = f_3(Q, \alpha, \beta) = Q \cdot (c_5 \cdot \alpha + c_6 \cdot \beta + c_7) \tag{C.3}
\]
\[ A_1 = \frac{\text{pitch}_{\text{measured}}}{\text{yaw}_{\text{measured}}} = \frac{Q \cdot [(c_2 \cdot \alpha + c_3 \cdot \beta) + c_4]}{Q \cdot [(c_5 \cdot \alpha + c_6 \cdot \beta) + c_7]} \] (C.4)

\[ 0 = [(c_2 \cdot \alpha + c_3 \cdot \beta) + c_4] - [(c_5 \cdot \alpha + c_6 \cdot \beta) + c_7] \cdot A_1 \]
\[ \Rightarrow \alpha = \frac{[c_4 - c_3 \cdot A_1 + (c_3 - c_6 A_1) \cdot \beta]}{-c_2 + A_1 \cdot c_5} = c_4 \cdot \beta + c_9 \] (C.5)

\[ c_8 = \frac{c_3 - c_6 A_1}{-c_2 + A_1 \cdot c_5} \] (C.6)

\[ c_9 = \frac{c_4 - c_7 \cdot A_1}{-c_2 + A_1 \cdot c_5} \] (C.7)

\[ A_2 = \frac{\text{pitot}_{\text{measured}}}{\text{yaw}_{\text{measured}}} = \frac{Q \cdot [(1 - c_1 \cdot \alpha^2) \cdot (1 - c_1 \cdot \beta^2)]}{Q \cdot [(c_5 \cdot \alpha + c_6 \cdot \beta) + c_7]} \] (C.8)

\[ A_2 = \frac{[(1 - c_1 \cdot (c_8 \cdot \beta + c_9)^2) \cdot (1 - c_1 \cdot \beta^2)]}{[(c_5 \cdot (c_8 \cdot \beta + c_9) + c_6 \cdot \beta) + c_7]} \] (C.9)

\[ \Rightarrow 0 = [(1 - c_1 \cdot (c_8 \cdot \beta + c_9)^2) \cdot (1 - c_1 \cdot \beta^2)] - A_2 \cdot [(c_5 \cdot (c_8 \cdot \beta + c_9) + c_6 \cdot \beta) + c_7] \]
**APPENDIX**

**D. STATE-SPACE MODEL**

Linearized state space model at 500Pa

| A | x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| x1 | 0 0 0 0 0 0 1 0 0 0 |
| x2 | 0 0 0 0 0 0 1 0 0 0 |
| x3 | 0 0 0 0 0 0 0 1 0 0 |
| x4 | 0 0 0 0 0 0 0 0 1 0 |
| x5 | 0 0 0 0 0 0 0 0 0 1 |
| x6 | -26.79 0 10.95 0 0 -1.434 0 0.5883 0 0 |
| x7 | 0 -29.24 0 6.262 0 0 -1.484 0 0.6581 0 |
| x8 | 30.39 0 -88.98 0 0 1.793 0 -4.496 0 0 |
| x9 | 0 44.95 0 -42.7 0 0 2.005 0 -4.667 0 |
| x10 | 0 0 0 -6.913 0 0 0 0 0 -18.46 |

| B | u1 u2 u3 u4 u5 u6 |
|---|--------|--------|--------|--------|--------|--------|
| x1 | 0 0 0 0 0 0 |
| x2 | 0 0 0 0 0 0 |
| x3 | 0 0 0 0 0 0 |
| x4 | 0 0 0 0 0 0 |
| x5 | 0 0 0 0 0 0 |
| x6 | 0 0.0519 0 0 0 0.9735 |
| x7 | -0.046 0 0 0 -0.956 0 |
| x8 | 0 0.1632 0 0 0 0.6282 |
| x9 | -0.03 0 0 0 -0.703 0 |
| x10 | -0.005 0 0 0 0 0 |

| C | x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| y1 | 0 0 0.0428 0 0 0 0 0 0 0 |
| y2 | 0 -0.925 0 -0.116 0 0 0 0 0 0 |
| y3 | 0.925 0 0.1078 0 0 0 0 0 0 0 |

| D | u1 u2 u3 u4 u5 u6 |
|---|--------|--------|--------|--------|--------|--------|
| y1 | 0 0 0 0 0 0 |
| y2 | 0 0 0 0 0 0 |
| y3 | 0 0 0 0 0 0 |
### Linearized state space model at 600Pa

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<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
<th>x9</th>
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Linearized state space model at 900Pa

\[
\begin{array}{c|cccccccccc}
A & x1 & x2 & x3 & x4 & x5 & x6 & x7 & x8 & x9 & x10 \\
\hline
x1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
x2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
x3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
x4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
x5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
x6 & -41.84 & 0 & 18.38 & 0 & 0 & -1.535 & 0 & 0.6129 & 0 & 0 \\
x7 & 0 & -43.6 & 0 & 11.85 & 0 & 0 & -1.557 & 0 & 0.6438 & 0 \\
x8 & 56.76 & 0 & -136.7 & 0 & 0 & 1.858 & 0 & -4.6 & 0 & 0 \\
x9 & 0 & 67.03 & 0 & -80.81 & 0 & 0 & 1.952 & 0 & -4.677 & 0 \\
x10 & 0 & 0 & 0 & 1.655 & 0 & 0 & 0 & 0 & 0 & -18.46 \\
\end{array}
\]

\[
\begin{array}{c|cccccc}
B & u1 & u2 & u3 & u4 & u5 & u6 \\
\hline
x1 & 0 & 0 & 0 & 0 & 0 & 0 \\
x2 & 0 & 0 & 0 & 0 & 0 & 0 \\
x3 & 0 & 0 & 0 & 0 & 0 & 0 \\
x4 & 0 & 0 & 0 & 0 & 0 & 0 \\
x5 & 0 & 0 & 0 & 0 & 0 & 0 \\
x6 & 0 & 0.05018 & 0 & 0 & 0 & 0.9641 \\
x7 & -0.1146 & 0 & 0 & 0 & -0.9561 & 0 \\
x8 & 0 & 0.1686 & 0 & 0 & 0 & 0.6689 \\
x9 & -0.0423 & 0 & 0 & 0 & -0.7027 & 0 \\
x10 & -0.0383 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
C & x1 & x2 & x3 & x4 & x5 & x6 & x7 & x8 \\
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y2 & 0 & -0.925 & 0 & -0.116 & 0 & 0 & 0 & 0 \\
y3 & 0.925 & 0 & 0.1124 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{c|cccccc}
D & u1 & u2 & u3 & u4 & u5 & u6 \\
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y1 & 0 & 0 & 0 & 0 & 0 & 0 \\
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### Linearized state space model at 1000Pa

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NOTATIONS

$\alpha$ : Angle of attack

$\beta$ : Angle of side slip

$D_{Droge_{outer}}$ : Drogue’s outside diameter

$D_{Droge_{inner}}$ : Drogue’s inside diameter

$D_{Bar}$ : Bar’s diameter

$S$ : Projection area of Drogue

$l_1$ : Bar length

$l_2$ : position of Bar’s C.G.

$l_3$ : Drogue length

$l_4$ : position of Drogue’s C.G.

$C_{L_{-Droge}}$ : Coefficient of lift of drogue

$C_{D_{-Droge}}$ : Coefficient of drag of drogue

$C_{M_{-Droge}}$ : Coefficient of moment of drogue

$Q$ : Dynamic pressure (Pa)

$m_{Bar}$ : Mass of rigid Bar($m_1$)

$m_{Droge}$ : Mass of Drogue($m_2$)

$I_{xx_{-Droge}}$ : Drogue’s Moment of inertia about X-axis
\( I_{yy\_Drogue} \) : Drogue’s Moment of inertia about Y-axis

\( I_{zz\_Drogue} \) : Drogue’s Moment of inertia about Z-axis

\( H_{G\_Drogue} \) : Angular moment of drogue

\( I_{yy\_Bar} \) : Bar’s Moment of inertia about Y-axis

\( I_{zz\_Bar} \) : Bar’s Moment of inertia about Z-axis

\( H_{G\_Bar} \) : Angular moment of bar

\( \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7 \) : Each joint angle

\( R_{\omega\_{Bar}} \) : Rigid-bar’s angular rate from reference frame

\( Bar_{\omega_{Drogue}} \) : Drogue’s angular rate from bar frame

\( R_{\alpha\_{Bar}} \) : Rigid-bar’s angular acceleration from reference frame

\( Bar_{\alpha_{Drogue}} \) : Drogue’s angular acceleration from bar frame

\( L_{ref\_bar} \) : Coordinate transformation matrix from reference to bar

\( L_{bar\_drogue} \) : Coordinate transformation matrix from bar to drogue

\( R_{G1}, R_{G2}, R_{p1}, R_{p2} \) : Position vector of each point

\( V_{G1}, V_{G2}, V_{p1}, V_{p2} \) : Velocity vector of each point

\( A_{G1}, A_{G2}, A_{p1}, A_{p2} \) : Acceleration vector of each point