Polarization of Compton-Scattered Gamma Rays

James G. Hudson
Western Michigan University, Jim.Hudson@dri.edu

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Western Michigan University
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Intermediate energy photons (approximately 0.2 MeV to 10 MeV.) interact with matter of low atomic number principally through a process called the Compton effect. This process is an inelastic collision of a photon with a free electron at rest. Using the principles of conservation of energy and conservation of momentum the energy of the scattered photon can be found in terms of the incident photon energy and the angle through which it is scattered. The energy of this scattered photon is then given by

\[ E' = \frac{E_0}{1 + \alpha(1 - \cos \theta)} \] (1)

where \( E_0 \) is the energy of the incoming photon, \( \alpha = \frac{E_0}{m_0 c^2} \), and \( \theta \) is the angle through which the photon's direction is changed.

Theory

Since photons are quanta of the electromagnetic field they have electric and magnetic components oriented at right angles to each other both directed perpendicular to the direction of propagation. It is only necessary to look at the electric vector, \( \mathbf{E} \), since the magnetic vector is always perpendicular to it. A beam in which all electric vectors are parallel is referred to as linearly polarized. (Since we will be concerned only with linear polarization we will drop the word linear.) If the
electric vectors in a beam are randomly oriented then the beam is unpolarized. In this paper we are concerned with beams having some electric vector alignment or partially polarized beams. A well-known optical phenomenon is that partial polarization results when an unpolarized beam is scattered (sunlight on glass for instance). The same phenomenon occurs for gamma rays undergoing Compton scattering. Klein and Nishina have made theoretical calculations of the magnitude of this polarization. For polarized incident radiation this is

\[ d(e^\sigma) = \frac{r_0^2}{4} \, d\Omega \left( \frac{E'_f}{E_0} \right)^2 \left( \frac{E'_f + E_0}{E'_f - 2 + \cos^2 \theta} \right) \]  \hspace{1cm} (2)

which is the differential collision cross section. This is an absolute probability for a photon of energy \( E_0 \) passing through matter with a thickness such that there is one electron per square centimeter. It is the absolute probability that this particular photon will suffer a particular collision such that it emerges with an energy \( E'_f \), has its electric vector changed by an angle \( \theta \), and is directed within the solid angle \( d\Omega \). The other term \( r_0 \) is the classical electron radius \( e^2/m_0 c^2 \). This equation indicates that it is most probable that the photon come out of the collision with the smallest change in orientation of its electric vector. The dependence of \( d(e^\sigma) \) on the scattering angle \( \theta \), is implicit in the factor \( E'_f \).

The experiment reported in this paper was an attempt to compare the observed polarization with the value predicted by
the Klein-Nishina formula. Experiments have been done for scattering angles of 90° (Rodgers 19363 and Hamilton 19674) and agreement with the theory has been adequate. However this experiment was an attempt to measure the polarization of gamma rays from a 100 curie Cobolt60 source scattered 45° by an aluminum target. It is safe to consider the primary beam from the source to be unpolarized because there is no mechanism by which it could become polarized. However an unpolarized beam can be treated as two beams of equal intensity polarized at right angles to each other. Any two orientations of the electric vectors may be chosen as long as they are mutually perpendicular and also perpendicular to the direction of propagation. For convenience we will choose one direction of the electric vector in the plane of scattering and denote it by $E_0'$. The other electric vector will then be perpendicular to the scattering plane and be denoted as $E_0^\perp$. The beam scattered by the target should be partially polarized but we can treat this beam in a similar manner, that is, by breaking it into two orthogonally polarized beams. We will choose one of these beams to have its electric vector in the plane of scattering, denoted by $E_r'$ and the other beam with its electric vector perpendicular to the scattering plane and denoted by $E_0^\perp$. Because this total beam is partially polarized the intensities of $E_0''$ and $E_r'$ are not equal. If $J_0'$ and $J_r$ represent the intensities of the respective components of the secondary beam
then the degree of polarization, \( p \), of this beam can be defined as

\[
p = \frac{J_L - J_N}{J_N + J_L}
\]  

(3)

In this treatment of the two beams each component of the second beam must be made up of parts from each of the incident components. The relative amount of each component of the incident beam that is contained in each component of the secondary beam depends upon \( \bar{d}(\sigma) \) which was given earlier (equation 2). If we let \( I_N \) and \( I_L \) be the respective intensities of the primary beam (\( I_N = I_L \)) then we can express the intensities of the secondary beam quantitatively as

\[
J_N = \bar{d}(\sigma)(\Theta = \angle E_o'' E_L)I_N + \bar{d}(\sigma)(\Theta = \angle E_o'' E_L)I_L
\]  

(4)

\[
J_L = \bar{d}(\sigma)(\Theta = \angle E_o' E_L)I_N + \bar{d}(\sigma)(\Theta = \angle E_o' E_L)I_L
\]  

(5)

(see fig. 1)

Since \( E_o \) and \( E_L \) are both perpendicular to the scattering plane they are parallel to each other and \( \Theta = 0^\circ \). \( E_o \) is perpendicular to \( E_L ^{''} \) and \( E_o'' \) is perpendicular to \( E_L \) because in each case one vector is in the scattering plane and the other is perpendicular to it, hence \( \Theta = 90^\circ \) in these cases. \( E_o'' \) and \( E_L'' \) are both in the same plane with the two beams and perpendicular to the two beams respectively. Hence they make the same angle with each other as the scattering angle, \( \Theta \), so that in this case \( \Theta = \Theta \). These considerations yield

\[
p = \frac{\sin^2 \Theta}{\left( \frac{E_o}{E_o} + \frac{E_L}{E_L} \right)}
\]  

(6)
For our arrangement we calculate $p = 0.279$.

**Experiment**

To experimentally measure $p$ we used a gamma ray polarimeter which uses NaI(Tl) crystals to analyse the secondary beam for polarization. Sodium iodide crystals detect gamma rays by converting high energy photons into visible light which is used to produce electrons in a photomultiplier adjacent to the crystal. The number of electrons produced is proportional to the amount of energy deposited in the crystal by a gamma ray. However this number of electrons must be multiplied by successive reproductions of these electrons as they are accelerated across dynodes. This multiplication is not always the same and a Gaussian distribution of pulse heights results after the pulse leaves the photomultiplier. A cathode follower (pre-amplifier with gain≈1) must be used to match the impedance of the photomultiplier to that of a cable so that the pulse can be passed to a detector amplifier, with a large enough gain so that the pulse can be analysed. The pulse can be sent into a multi-channel analyser where the pulses are sorted into channels according to their amplitude. A memory bank then records the number of pulses which fall into each channel and the number of pulses per channel can be displayed on an oscilloscope or typed out on paper.

One way a NaI crystal captures energy from a gamma ray is the Compton effect discussed previously. Sometimes the energy of the gamma ray is completely absorbed by the crystal but often
it loses only part of its energy to the crystal and moves off at a different angle. These are the events we are interested in because by examining these gamma rays Compton-scattered from a crystal we can determine the polarization of a beam.

Our gamma polarimeter had a central cylindrical crystal, crystal A, one and one-half inches in diameter and one inch high which was placed in the beam, to be analysed (see fig. 2). It was calculated that about one-half of the gamma rays incident on this crystal would be absorbed or scattered by it. The number and angle of those which were Compton-scattered depended upon $d(\sigma)$ as shown previously. If the beam to be analysed were unpolarized then $d(\sigma)$ would depend only upon $E_0$ and $\Theta$. However since the secondary beam was partially polarized $d(\sigma)$ was not homogeneous even for constant $E_0$ and constant $\Theta$. If we now place two other NaI crystals near the first crystal and at identical angles from the secondary beam we should not detect the same number of Compton-scattered gamma rays from the first crystal if the beam being analysed is partially polarized. In our polarimeter a three inch diameter by three inch high NaI crystal, crystal B, was placed 90° from the secondary beam and four inches from the first crystal. It was moved from a position in the scattering plane to a position perpendicular to the scattering plane to determine a relative number of counts.

Since we were only interested in recording events in which
photons were scattered from the first crystal to the second it was necessary to use a large amount of lead to shield both crystals from direct radiation from the very intense source. The second crystal also had to be protected from radiation scattered off the target. Because of backscattering from the walls and other objects in the room it was even necessary to protect the crystals with lead on all sides.

With all this shielding there were still many counts recorded by the detectors but since we were only interested in photons scattered by crystal A into crystal B we could bias out most undesirable counts by requiring that only events which happen simultaneously in both crystals be recorded. Since photons travel at the speed of light \(3 \times 10^8\) meters/second a true scattering event would be virtually simultaneous and not biased out by this requirement. An Ortec coincidence unit was used so that we could record only these simultaneous events. This device puts out a simple logic pulse only if a pulse, regardless of amplitude, enters the unit from both crystals within a controlled length of time (a few nanoseconds = \(10^{-9}\) seconds).

The desired pulses coming from each crystal represent discrete energies because they must be scattered through only certain angles in order to follow the path from the source, to the target, to crystal A, then to crystal B. The Compton equation then specifies what these energies can be to correspond with the scattered angles.
The energy of the pulses of interest in the second crystal can be calculated applying the Compton equation twice. For $E_0 = 1.25$ mev., the average energy from the Cobalt$^{60}$ source, a 300 kev. gamma ray is expected in crystal B due to scattering of 45° from the aluminum target and 90° from crystal A. In the first crystal we are interested in the energy deposited by the Compton-scattered gamma ray which scatters into the second crystal. This is obtained by subtracting the energy carried on to the second crystal, about 300 kev., from the full energy peak of the first crystal, about 720 Kev., to obtain 420 kev. We could then reduce the chances of counting random coincidences by biasing out pulses which were not approximately 420 Kev from the first crystal and 300 Kev. from the second crystal. This was done by placing single channel analyzers between the individual amplifiers and the coincidence unit. With the single channel analyzer specific "window"widths" could be selected allowing only pulses of desired energies to enter the coincidence unit. (see fig.3)

The energies of interest deposited in each crystal were not exactly 420 kev. and 300 kev. but for several reasons were spread over a certain range. In the first place the Co$^{60}$ source really gives off two gamma rays of closely spaced energies 1.17 mev. and 1.33 mev. Second the finite size of the target and first crystal allow a range of scattering angles at the aluminum target so that instead of measuring the polarization of 1.25
mev. gamma rays Compton-scattered at $45^\circ$ we were actually measuring the polarization of 1.17 mev. and 1.33 mev. Compton-scattered gamma rays at $43.8^\circ$ to $46.5^\circ$ (see fig. 4). Third and most important there is another angular spread for the photons scattered by crystal A into crystal B. In other words the second scattering does not occur at $90^\circ$ but may range from $76^\circ$ to $114^\circ$ (see fig. 5).

Fourth the multiplication and amplification of the pulse, as stated previously, gives a statistical distribution to even a monoenergetic photon. With all these factors considered we must decide how to gate out undesirable pulses without hampering the true counting rate. A range of energies was decided upon for each detector and with the aid of a multi-channel analyser the gates were set with the single channel analysers. Various other gates were experimented with but smaller gates drastically changed count ratios indicating that many desirable pulses had been left out. Wider windows only added unnecessary counts outside of the energy range of interest which were not counted anyway.

In order that a spectrum could be observed the pulses from each detector amplifier were fed into the multi-channel analyser. So that we could observe a full energy peak the pulses from each detector amplifier were added by a circuit which simply adds simultaneous pulse heights. The pulse from the adder circuit was fed into the multi-channel analyser to be sorted, displayed,
and read out. In order that we would only record in the multi-channel analyser those pulses which represented true scattering events it was necessary to use the logic pulse from the coincidence unit as a trigger which would allow the multi-channel analyser to accept only those pulses from the adder which occurred simultaneously with the logic pulse. In this manner only true coincidence pulses would be analysed by the multi-channel analyser and only these would be displayed or read out. By having the pulse height of the coincident pulses displayed we could further discard pulses which did not correspond to desired energies.

It was necessary to have equal delays in the detector analysers so that true coincidences could be detected by the fast coincidence unit, but this was fairly easy to do using an oscilloscope.

The resolving time of the fast coincidence unit was set by using a positron annihilation source and setting the time for maximum counting rate.

There was an internal delay of about two microseconds in the multi-channel analyser which necessitated delaying the logic pulse from going into the multi-channel analyser. This delay had to be adjusted for maximum counting rate.

An additional correction had to be made because there was a considerable amount of primary scattering done by the air between the source and crystal A. However this was easily
discounted for by making background subtraction runs with the target removed from the beam.

Each data gathering run consisted of four equal time periods. Two sets of runs with crystal B in each position, parallel and perpendicular to the scattering plane, After each run with the target in place a run of equal time was made with the target removed and the counts subtracted from the corresponding first run. Various time periods of 10, 20, and 30 minutes were used and nine runs were used in the final analysis.

The data gathered consisted of merely the raw number of counts recorded in each crystal position. From this information had to be deduced the degree of polarization of the scattered beam. In order to do this we use the same Klein-Nishina formula used in the first scattering at the target. Essentially we are using the polarization of Compton-scattered gamma rays at 90° to measure the polarization of Compton-scattered gamma rays at 45°. This is justified by previous experiments referred to earlier in this paper and 4.

Results and Interpretations

In the analysis of this second scattering we are not concerned with the polarization of the scattered beam so we can integrate \( d(\epsilon, \sigma) \) over all possible orientations of the second scattered electric vector yielding

\[
d(\epsilon, \sigma) = \frac{r_0}{2} \left( \frac{E'}{E_0} \right)^2 \left( \frac{E'}{E_0} + \frac{E_0}{E'} - 2 \cos^2 \xi \right)
\]  (7)
where all terms are the same as before and $\theta$ is the angle between the incident electric vector and the direction of the scattered photon. All photon beams leaving the first crystal must be made up of components of the two mutually perpendicular beams of the first scattered beam. The proportion of each component will depend upon its intensity and differential cross section.

Therefore
\[
N_H = J_H \delta(\phi) (F = \zeta_H, \gamma_H) + J_H \delta(\phi) (F = \zeta_H, \gamma_H^*) \tag{8}
\]
\[
N_L = J_H \delta(\phi) (F = \zeta_L, \gamma_L''') + J_L \delta(\phi) (F = \zeta_L, \gamma_L''') \tag{9}
\]
Since the geometry of the two positions of crystal B was symmetric about crystal A we can conclude that
\[
(F = \zeta_H, \gamma_H) = (F = \zeta_L, \gamma_L) \tag{10}
\]
\[
(F = \zeta_L, \gamma_L'') = (F = \zeta_H, \gamma_L') \tag{11}
\]

We now let
\[
R = \frac{\delta(\phi) (F = \zeta_H, \gamma_H)}{\delta(\phi) (F = \zeta_L, \gamma_L)} \tag{12}
\]
then we can write $p = \frac{J_H - J_L}{J_H + J_L}$

as
\[
p = \frac{(R+1)}{(R-1)} \frac{N_H - N_L}{N_H} \tag{13}
\]

Under ideal geometry, that is no angular spread in any of the scattering but exactly 45° and 90°, we would have
\[
R = \frac{E' + E''}{E'' + E'''} \tag{14}
\]

$R$ in this case would be 3.38.
As has been mentioned before we did not have ideal geometry so \( \xi \) is not simply either 0° or 90° but it can vary somewhere near these values. Because we must consider all possible angles that the photon could take from the first crystal to the second it is impossible to integrate over such a complicated geometry. It was necessary to break each crystal into a number of zones and calculate \( d(\xi) \) for all combinations of scattering from the centers of each zone of crystal A to the centers of each zone of crystal B. The zones of each crystal were made of equal volume so that a simple average could be taken. However it was necessary to weight the zones of the first crystal because there was a large amount of self attenuation in crystal A which made the side of the crystal closer to crystal B much more effective. (see figs. 6 and 7) After 196 calculations for \( R \) it was found to be \( R = 3.2 \pm 0.4 \) so that

\[
\frac{R+1}{R-1} = 1.9 \pm 0.2
\]

Experimentally

\[
\frac{N_u - N_x}{N_{t+} + N_x} = 0.132 \pm 0.009
\]

Therefore

\[
p = 0.25 \pm 0.03
\]

There were many sources of error in this experiment some of which could be alleviated to some extent. The correction for finite geometry was by no means exact and was substantially an approximation. A much more ideal geometry could have been
obtained if the distances between the crystals had been made larger. This would reduce the count rates to a large extent but the counts that would be recorded would be those of least ideal geometry leaving a more ideal geometric system which would require much less calculating for R. Count rates were high enough (15,000 to 20,000 per ten minutes) that the distances could have been greatly extended while still retaining enough counts to obtain reasonable statistics.

The geometry factor could also have been helped if the crystals used were smaller. The size of crystals A could be reduced by about one-half without even losing counts since only about half of this crystal was actually effective in scattering gamma rays into crystal B. This is because gamma rays scattered from the side of crystal A away from crystal B were to a great extent absorbed by the rest of crystal A before they ever reached crystal B.

When crystal B is in the parallel position the effective primary scattering angle is increased because scattering which occurs in the side of crystal A away from crystal B is biased out by self absorption and balance between 43.8° and 46.5° scattering is lost and the larger angle scattering prevails. The same effect applies when crystal B is in the perpendicular position but here the crystal is split into zones of equal scattering. This overall effect would tend to raise N/ε as
opposed to $N_L$ and thus raise $p$.

The value of $p$ would have been more precise and easy to measure if we had used a monoenergetic source of gamma rays but Co$^{60}$ was the only large source available.

The theoretical value of $p$ for the arrangement in this experiment, 0.279, lies just inside of the value found by the experiment 0.25 ± 0.03. Considering the crudeness of the geometry this is a fairly good result and it can be said that the Klein-Nishina formula holds up well. It would be desirable to perform the experiment again under a more ideal geometry but time did not permit another trial.

I wish to acknowledge my advisor Dr. Gerald Hardie who suggested the problem and helped me carry it through.
References

2. Evans, p.678
4. Hamilton, M.C., "Measurement of Polarization of Gamma Rays
   Inelastically Scattered by Electrons in Lead."
5. Evans, p. 680
Breaking each beam into two orthogonal polarized components.

figure 1
Experimental arrangement.

figure 2
Block diagram of the electronics.

figure 3
Angular Spreads

angles exaggerated

figure 4

figure 5
Crystals divided into zones

Crystal A

Right side disregarded

Bottom same as top

Crystal B

Figure 6

Figure 7