Commissioning of the Helical Orbit Spectrometer: A New Device for Measuring Nuclear Reactions in Inverse Kinematics

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COMMISSIONING OF THE HELICAL ORBIT SPECTROMETER:
A NEW DEVICE FOR MEASURING NUCLEAR REACTIONS
IN INVERSE KINEMATICS

by

Jonathan C. Lighthall

A Dissertation
Submitted to the
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Advisor: Alan H. Wuosmaa, Ph.D.

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The Helical Orbit Spectrometer (HELIOS) at Argonne National Laboratory is the first implementation of a radical new concept for measuring nuclear reactions. Direct nuclear transfer reactions are powerful tools for studying the properties of the atomic nucleus. A traditional example is the neutron-transfer reaction \((d,p)\), wherein an accelerated beam of deuterons \(d\) bombards a heavy target. The incoming deuteron transfers a neutron to the target nucleus and the outgoing proton \(p\) is detected to study the properties of the residual heavy nucleus. A new frontier of nuclear reaction studies involving short-lived exotic nuclei—which are unsuitable for use as targets—is being made available through the development of radioactive ion beam facilities.

In measurements made with radioactive beams, the role of the beam and target are reversed, with a heavy-ion beam bombarding a light target. In this regime of “inverse kinematics,” the center-of-mass system has a substantial velocity in the laboratory frame, the consequence of which is that the energy of the emitted light ion is highly angle-dependent. A successful measurement made in inverse kinematics thus requires a detector system with excellent resolution. HELIOS is a new approach to detecting the charged light ion reaction products which addresses the technical challenges of studying reactions in inverse kinematics.

The HELIOS spectrometer is based on a large-bore superconducting solenoid which uses a slender position-sensitive detector array to measure the energy and position of charged particles along the solenoid axis. This dissertation describes the advantages of the HELIOS concept as they relate to measurements made in inverse kinematics. The technical specifications of the spectrometer are described in detail. Three reaction measurements are discussed: the measurement used to commission HELIOS, the \(^{28}\text{Si}(d,p)^{29}\text{Si}\) reaction; the first experimental results from HELIOS, the \(^{12}\text{B}(d,p)^{13}\text{B}\) measurement; and an important measurement which is planned to be made with HELIOS, the \(^{132}\text{Sn}(d,p)^{133}\text{Sn}\) reaction. Each of these measurements is compared to past measurements made with other detector systems.
Acknowledgments

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As far back as I can remember, I have always been interested in physics. It was my grandfather, the late Dr. James Cockrell, that first showed me the allure that physics holds. This dissertation is dedicated to him.

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Part I

Scientific Motivation
Chapter 1

Theoretical Motivation

1.1 Nuclear Shell Model

One of the greatest achievements of nuclear physics was the development of the nuclear shell model by Goeppert-Mayer [1] and Jensen et al. [2] which ultimately earned them the Nobel Prize in 1963. In analogy to the noble gases (Group 18) in chemistry, certain configuration of protons and neutrons are more stable than others, suggesting the presence of “magic numbers.” Based on empirical data, it was apparent that nuclides comprised of 2, 8, 20, 28, 50, 82, or 126 protons or neutrons are particularly stable. This stability must arise from the nature of the nuclear potential. The presence of the magic numbers 2, 8, and 20 suggested that the nuclear potential must have a shape somewhere in between that of a harmonic oscillator and a square well. Fig. 1.1 shows that both of these potentials are able to reproduce the first three magic numbers. However, neither potential succeeds in predicting the positions of the larger magic numbers. Therefore, the simplified description of the nuclear potential must be missing something.

The shape of a potential that is between that of a harmonic oscillator and a square well is the Woods-Saxon potential

$$f(r, r_0, a) = \left[1 + \exp \left( \frac{r - r_0}{a} \right) \right]^{-1}$$  Woods-Saxon (1.1)

where $r$ is the dependent variable (the radius), $r_0 = RA^{1/3}$ is the nuclear radius; $R$ has a typical value of 1.25 fm and $a$, the diffuseness parameter, is on the order of 1 fm. The fundamental shape of this potential is shown in Fig. 1.1; a function plot of this potential is given in Fig. 1.2. The degeneracy of each level produced by the Woods-Saxon potential and the square well, both central potentials, is $2(2\ell + 1)$, where $\ell$ is the orbital angular momentum of the state. Although the Woods-Saxon potential shifts the position of the energy levels relative to the finite square well, it still does not reproduce the magic numbers observed in experiment.

The breakthrough of Goeppert-Mayer and Jensen et al. is the inclusion of the non-central nuclear spin-
Figure 1.1: Nuclear energy levels based on different potentials leading to the magic numbers. The oscillator number is given at left. Numbers in parentheses are the magic numbers predicted using the specified potential. The circled numbers in the right column corresponding to the Woods-Saxon potential with a spin-orbit interaction are the nuclear magic numbers close to stability. Figure modified from Ref. [3].
Figure 1.2: Functional plot of the Woods-Saxon potential. The volume potential \( f(r) \) (Eq. 1.1) and the surface-derivative potential \( g(r) \) (Eq. 2.6) are plotted. The depth of the well has been selected to be 50 MeV. The dashed line indicates the nuclear radius \( R = 6.6 \) fm, corresponding to an \( \Lambda = 28 \) nucleus.

The idea of spin-orbit coupling was already well-known in chemistry, arising from the electromagnetic interaction between a nucleus and an orbiting electron. The general form of the nuclear spin-orbit potential is the same as the atomic spin-orbit potential. However, empirically, the nuclear spin-orbit potential arises from the nucleon-nucleon interaction is of opposite sign and 20–30 times stronger than atomic spin-orbit potential \([4, 5]\). The nuclear potential is then rewritten as

\[
U(r) = -V f(r, R, a) + V_{SO} \frac{1}{r} \frac{d}{dr} f(r, R, a)(\vec{\ell} \cdot \vec{s}) \quad \text{Thomas form (1.2)}
\]

where \( V \) and \( V_{SO} \) are the depth of the volume and surface-peaked potentials, respectively. The depth of the volume potential \( V \) is typically on the order of 50 MeV. With this new potential, each level with \( \ell > 0 \) is split into two new levels. For a given value of \( \ell \), the \( j = \ell + \frac{1}{2} \) state has lower energy, and the \( j = \ell - \frac{1}{2} \) state has higher energy. The energy difference is proportional to \( \frac{1}{2}(2\ell + 1)\hbar^2 \). Starting with the \( n = 3 \) oscillator shell \( (2p1f) \), the highest angular momentum state is pushed down to energies comparable to the preceding oscillator shell (see Fig. 1.1). This is the interaction which correctly reproduces the observed magic numbers 28–126.

The stable nuclei with closed proton and neutron shells, such as \(^{40}\text{Ca} \) and \(^{208}\text{Pb} \), have been well understood within the context of the shell model for decades. However, only recently has it been possible to study experimentally exotic nuclei around the neutron-rich double shell closure of \( Z = 50, N = 82 \). As such, the doubly-magic \(^{132}\text{Sn} \) nucleus is the subject of increasing attention in the nuclear physics community. With the development of the Helical Orbit Spectrometer (HELIOS) and the Californium Rare Isotope Breeder Upgrade (CARIBU) at the ATLAS facility of Argonne National Laboratory, ground-breaking measurement opportunities
are on the horizon.

1.2 The Stellar $r$-process

1.2.1 Background

In 1957, the seminal work “Synthesis of the Elements in Stars” by Burbidge, Burbidge, Fowler, and Hoyle laid the foundation of understanding nucleosynthesis in the universe \[6\]. An important process in stellar nucleosynthesis is the rapid neutron capture process ($r$-process), which accounts for synthesis of about half of the chemical elements with atomic mass greater than $A = 50$ \[7\]. Nuclei involved in the stellar $r$-process are therefore of key importance in understanding the isotropic abundances found in the universe. The $r$-process occurs in environments of extreme neutron flux, where free neutron density is on the order of $n_n \geq 10^{21}$ cm$^{-3}$, and temperatures in the $T \geq 1$ GK range \[8\].

Based on these extreme environmental parameters, a number of exotic sites of the $r$-process have been suggested \[9\], however a likely candidate is the stellar atmosphere during a core-collapse supernova, leading to the creation of a neutron star. In such an environment, nuclei capture an increasing number of neutrons creating more and more neutron-rich isotopes. When the environmental temperatures decrease, reducing the rate of neutron capture, $\beta$-decay becomes dominant. The neutron-rich isotopes typically $\beta$-decay along a path of constant $A$ back towards stability. The transition from the neutron capture regime to the $\beta$-decay regime is called freeze-out.

One of the major influencing factors that dictates the path of the $r$-process is the competition between neutron capture ($n,\gamma$) and photodisintegration ($\gamma, n$). As a nucleus captures successive neutrons, each added neutron has a lower binding energy. This process continues until the neutron capture rate $\lambda_n$ is balanced by the photodisintegration rate $\lambda_\gamma$. When the two reactions are in thermal equilibrium, $(n, \gamma) \rightleftharpoons (\gamma, n)$, and can be described by Maxwell-Boltzmann statistics, the isotopic abundance is determined by the Saha equation

$$\frac{n(Z,A+1)}{n(Z,A)} = n_n \left( \frac{2\pi \hbar^2}{m_n kT} \right)^{3/2} \frac{(2j_{Z,A+1} + 1)}{(2j_{Z,A} + 1)(2j_n + 1)} \frac{G_{Z,A+1}^{\text{norm}}}{G_{Z,A}^{\text{norm}}} e^{Q_{\gamma}/kT} \quad \text{Saha equation}$$

(1.3)

where $n(Z,A)$ is the number density (abundance) of the nuclide $^A_ZX$, $k$ is the Boltzmann constant, $j_i$ and $G_i^{\text{norm}}$ are the spins and normalized partition functions of the individual particles, respectively; and $Q_{\gamma}$ is the neutron capture $Q$-value, or equivalently, the neutron separation energy $S_n$ of $^{A+1}_ZX$. The second line of the equation is a simplification to illustrate that the abundance of adjacent isotopes depends largely on the neutron
density $n_n$, the temperature $T$, and the $(n,\gamma)$ neutron capture $Q$-value. When the reaction rates are in equilibrium, in order for the rapid neutron capture to continue, the nucleus must undergo negative $\beta$-decay. It is said during this time that the $r$-process is “waiting” for the $\beta$ decay; this assumption is called the \textit{waiting-point approximation}.

Nuclei along the $r$-process path with longer $\beta$-decay half-lives have a substantial effect on the $r$-process. While the isotopic abundance is governed by the neutron capture $Q$-value, the elemental abundance is inversely proportional to the total $\beta$-decay rate of the isotopic chain (\textit{steady flow approximation}) \cite{8}. In particular, the relatively more-stable nuclei with magic neutron numbers cause a pause in the $r$-process. This pause leads to a greater abundance of these nuclide during the $r$-process, corresponding to greater solar abundance in the same mass region after $r$-process freeze-out. These so-called “waiting-point” nuclei at $N = 50, 82,$ and $126$ correspond to the three predominant peaks in the isotropic abundance spectrum shown in Fig. 1.3 said to be produced by the $r$-process \cite{10}. The effect of these waiting points is so substantial that the time it takes for a seed nucleus to travel the $r$-process path is largely determined by the sum of the half lives of nuclei near closed neutron shells \cite{7}. As a result, it is clear that knowledge of the shell gaps of exotic isotopes in the region of the magic numbers is essential to understanding stellar processes. Specifically, energy levels of isotopes near the doubly-magic $^{132}$Sn shell closure are vital to understanding the synthesis of isotopes in the $A = 130$ region (see Fig. 1.3).
CHAPTER 1. THEORETICAL MOTIVATION

Figure 1.4: Illustration of shell quenching in the Cd isotopes \((Z = 48)\). Measured isotope masses (full circles), short-range mass extrapolations (half-circles) and calculated masses (lines) are plotted relative to the finite-range liquid droplet model (FRDM). Mass trends below the FRDM line are a signature of shell quenching. Figure from Ref. [13, Fig. 3]

1.2.2 Theoretical Framework

Until recently, the only way to investigate the unstable neutron-rich nuclei of the \(r\)-process was through theoretical calculations. The first shell-model calculations were performed in the late 1960s and early 1970 with the advent of powerful computing systems. The following decades saw the development of advanced microscopic interaction models such as Skyrme and Gogny [11]. As the development of facilities for radioactive beams drew nearer in the early 1990s, the shell properties of heavy neutron-rich isotopes gained renewed interest. Many theorists sought to extrapolate models that described \(\beta\)-stable nuclei to exotic nuclei along the \(r\)-process path [12]. An illustrative example is the comparison of predictions of mass models. While the models tend to agree in the regions with experimental data, they diverge wildly when extrapolated to exotic nuclei as shown in Fig. 1.4 [13].

The lack of a shell model that globally fits experimental results thus invited theorists to apply a variety of techniques to try to predict the evolution of nuclear structure with increasing neutron excess towards the \(r\)-process nuclei (and beyond to the neutron drip line). For example, the Hartree-Fock-Bogoliubov (HFB) calculations using the Skyrme force SkP and the relativistic mean-field (RMF) approximation both show significantly different predictions for properties of exotic nuclei than for \(\beta\)-stable nuclei. While both models showed similar increased diffuseness in the neutron potential, they disagreed dramatically in their predictions of the spin-orbit splitting [14].

As discussed in § 1.1, the effect of spin-orbit splitting is a fundamental factor influencing the formation of
magic number shell gaps. More specifically, it is the presence of a non-central spin-orbit term in the nuclear potential that produces the shell gaps in the naive shell model [4]. This effect is shown in the rightmost column of Fig. 1.1. As shown in Eq. 1.2, the spin-orbit potential, and thus the spin-orbit splitting, is proportional to the gradient of the nuclear potential [15]. The diffuse nuclear surface and larger spacial extent of neutron-rich nuclei can produce a softening of the nuclear potential. The resultant weakening of the spin-orbit potential reduces, or otherwise alters, the shell gaps [16].

Another important aspect of shell spacing is the effect of the tensor force, the effect of which is not shown in Fig. 1.1. The tensor component of the nuclear potential connects the $^3S_1$ ($L = 0$) angular momentum state and the $^3D_1$-state ($L = 2$) in the deuteron to produce the measured magnetic dipole and electric quadrupole moments [17]. In a similar fashion, the tensor force connects single-particle orbitals above closed nuclear shells. The interaction between adjacent levels can be either attractive (deuteron-like) or repulsive and is responsible for trends in spacing of single particle levels [18].

If the shell gaps diminish or are not present, the shell effects are said to be "quenched." This was an early prediction of the HFB method and suggested that the $N = 82$ shell closure may be quenched along the $r$-process path. However, without making any assumptions about shell quenching in the model, the SkP force used in the HFB calculations intrinsically predicts smaller shell gaps than in other models [12]. Furthermore, when applied to shell gaps for which experimental data exists, not only does SkP underestimate the shell gaps, it does not fit the data as well as other calculations, such as relativistic Hartree-Bogoliubov (RHB) model.

That is not to say that shell quenching does not happen. Any reasonable force model shows decreasing shell gaps, i.e., lower as nuclei become more neutron-rich. This reduction of the shell gaps is due to the decreasing neutron separation energy as more neutrons are added to the nucleus, which is a key feature of the $r$-process. As the binding energy of each additional neutron becomes smaller and smaller, the shell gap eventually disappears. With enough neutron excess, all of the shell gaps quench at the terminus stratum of the neutron drip line.

### 1.2.3 Initial Measurements

In the absence of further experimental data, it would remain an open question where shell quenching occurred among the $N = 82$ isotones and to what extent the effect was present in $^{132}$Sn. In the meantime, the predictions from models explicitly involving shell quenching and experimental data from a suite of $\beta$-decay studies performed at the ISOLDE facility at CERN would seem to support a reduction of the shell gap above $N = 82$. The experimental evidence offered by the first of these studies was inconclusive due to isobaric contamination but suggested a relatively small $E(2^+)$ value of 957 keV for $^{130}$Cd [19]. The follow-up study utilized significantly more advanced background suppression, but showed inconsistent results for cadmium and tin [13]. These studies emphasized the difficulties of these measurements and the need for further measure-
Figure 1.5: Plot of the $N = 82$ shell gaps as a function of $Z$. The shell gaps are defined by the $S_{2n}$ energy and are determined experimentally from mass measurements. Also plotted are a number of theoretical calculations. The mass measurements show a peak at $Z = 50$, corresponding to the doubly-magic nucleus $^{132}$Sn, meaning that the $N = 82$ shell is not quenched in the tin isotopes. Figure from Ref. [21, Fig. 3].

ments while leaving the $N = 82$ shell gap on uncertain ground.

More recently, two experiments have brought about a “restoration” of the $N = 82$ shell gap in $^{132}$Sn. The first was a study done at GSI that measured $\gamma$-ray transitions in $^{130}$Cd, which at the time of publication was the most neutron-rich $N = 82$ isotope with observed $\gamma$-ray transitions [20]. The $^{130}$Cd isomers were produced in a knockout reaction and in a beam-fragmentation reaction. What made this study unique is that it measured $\gamma$-ray transitions in coincidence with ions unambiguously identified using a fragment separator. By measuring the transitions in coincidence with explicit ion identification, it was clear the $\gamma$-rays were being produced in isomeric decay of the isotone of interest. The study measured a larger $E(2^+)$ value than found in previous studies, strengthening the argument for the $N = 82$ shell gap.

Further reinforcement came from another experiment performed at CERN with ISOLDE. This most recent study was a mass measurement using the ISOLTRAP Penning trap mass spectrometer [21]. In order to circumvent problems with isobaric contamination, the $^{132}$Sn was made into a sulfide and the resultant $^{132}$Sn$^{34}$S$^+$ was then separated and analyzed. The subsequent mass measurement differed from the previously accepted value for $^{132}$Sn by 480 keV. This new mass value also increases the accepted value of the two-neutron separation energy $S_{2n}$ for $^{132}$Sn, giving it the largest shell gap of the $N = 82$ isotones (see Fig. 1.5).

1.2.4 Outlook

In order to further the understanding of the $r$-process and the evolution of nuclear structure towards the neutron drip line, additional experimental data are required. Even though $^{132}$Sn may appear to be less “exotic” than previously believed, its role as a doubly-magic nucleus along the $r$-process path cannot be overlooked or understated. Information about the single-neutron states above the $^{132}$Sn shell closure will be vital to forming a complete picture of nuclear structure in this region.
At present, the majority of published experimental data related to energy levels of nuclei in the vicinity of $^{132}\text{Sn}$ come from $\beta$-decay studies. The first study of the single-neutron states in $^{132}\text{Sn}$ was performed at ISOLDE [22]. Indium isotopes were produced in by beam-induced fission of uranium and the subsequent $\beta$-decays were studied. By measuring $\gamma$-rays in coincidence with the $\beta$-decays, the energy levels of the resultant $^{133}\text{Sn}$ isomers were deduced. Of the three energy levels proposed in this study, one has since been confirmed by measuring the prompt $\beta$-decay spectra following the spontaneous fission of $^{248}\text{Cm}$ [23]. By gating on known isomeric transitions in $^{112}\text{Pd}$, a fission partner of $^{133}\text{Sn}$, the 1561 keV transition, corresponding to the $h_{9/2}$ neutron level, has been confirmed.

Spins and parities can be tentatively assigned using $\beta$-decay selection rules and comparison to theoretical models, but no conclusive assignments can be made. An additional problem with the $\beta$-decay studies in the $^{132}\text{Sn}$ region is that they generally populate high-energy excited states [22]. The neutron separation energy of $^{133}\text{Sn}$ is only 2.45 MeV, so the higher energy states preferentially populated by $\beta$-decay are unbound against neutron emission, making them difficult to detect. In order to effectively study the structure of low lying states it is necessary to use another method of measurement. Direct nuclear reactions involving nucleon transfer provide a unique window to measure these quantities. The $^{132}\text{Sn}(d,p)$ measurement performed by Jones et al. [24] is an example of such a reaction study; this measurement is discussed in Chapt. 4.
Chapter 2

Nuclear transfer reactions

In terms of nuclear physics, a nuclear reaction occurs when two atomic nuclei collide and transform to produce new nuclei different from the reactants. Use of a particle accelerator is needed in order for the colliding particles to have sufficient kinetic energy to overcome the Coulomb repulsion due to the nuclear charge. A special class of nuclear reactions called “transfer reactions” involve the transfer of a small number of nucleons between the reactants during the process of the reaction. This chapter discusses the utility of studying such nuclear reactions and how they provides a powerful analytic tool for understanding nuclei. Transfer reactions are one of several methods that allow the study of the position of excited states in a nucleus. Other methods include $\beta$-decay $\gamma$-decay studies. What makes transfer reaction studies unique is the way in which they provide information on the quantum number associated with the spin and parity of nuclear energy levels. In addition, transfer reactions provide information on the single particle structure of a nucleus.

2.1 Direct Reactions

In the most basic form of a direct reaction, an incident beam particle has a single collision with a target nucleus, interacting with a single degree of freedom in the target nucleus. Direct transfer reactions are characterized by short interaction time $t \sim 2r_0/v_1$ where $r_0$ is the radius of the target nucleus and $v_1$ is the velocity of the incident ion. The amount of energy transferred in these reactions is small compared to the incident beam energy; thus these reactions are sometimes referred to as being “quasi-elastic” to differentiate them from deep-inelastic nuclear reactions.
2.1.1 Notation

In the prototypical example, a stationary target is bombarded by an accelerated beam of particles and a detector measures the scattered particles. The reaction may be written as

\[ A + a \rightarrow B + b \] (2.1)

where \( A \) is the heavy ion reactant; \( a \) is the light ion reactant; \( B \) is the heavy ion recoil; and \( b \) is the light ion recoil. In this notation, the reaction can be written in a more compact form as \( A(a,b)B \) with \( (a,b) \) identifying the type of reaction.

This notation convention was developed during a time with the reactions being studied typically involved a light ion beam and a heavy “ion” target—here the term ion is used loosely, as the target are not typically ionized. To make consistent use of this notation, the first reactant listed is defined as the target nucleus and the second reactant is defined as the beam particle or projectile. Therefore, under this convention, a reaction in which the heavy reactant is accelerated (“inverse kinematics”) would be written as \( a(A,b)B \), even though the reaction would still be referred to as an “\((a,b)\)” reaction. Similarly, the light ion ejectile \( b \) is traditionally the detected particle, or the particle of interest. Adopting the notation, a reaction in which the heavy recoil is the particle of interest or a measurement in which the heavy ion is detected can be written as \( A(a,B)b \).

In elastic scattering, the incoming particle \( a \) and the outgoing particle \( b \) are the same, leaving the ion species of the target nucleus unchanged. In a transfer reaction, \( a \) and \( b \) are different from each other. The form of this difference divides transfer reactions into three different classes, given names from the perspective of the light ion projectile. For example, in a “stripping” reaction, nucleons are stripped from the incident projectiles by the target nucleus. Examples of stripping reactions include \((d,p)\) and \((^3\text{He},d)\) which are neutron and proton stripping reactions, respectively. These reactions add nucleons to the target nucleus. The inverse
of this process is referred to as a “pick-up” reaction, where the incident projectile picks-up nucleons from the target. For example \((d, {}^3\text{He})\) and \((t, \alpha)\) are proton pick-up reactions.

In a typical nucleon transfer reaction, a heavy target is bombarded by an accelerated beam of stable light nuclei. A classic example of such a reaction is a deuteron beam striking a stable \(^{28}\text{Si}\) target to produce \(^{29}\text{Si}\), illustrated in Fig. 2.1. In such an experiment, the ejected proton is detected in order to study the properties of \(^{29}\text{Si}\). Traditionally, the ejected light ion is detected with silicon detectors through a range of fixed laboratory angles—other detectors may also be used, such as photographic plates, gas counters, etc. Charge collection in the detectors determines the ejected particle's energy and the position of the detector determines the laboratory angle. Detectors can be segmented or position-sensitive for enhanced angular resolution.

### 2.1.2 Single-particle States

What makes direct nuclear reactions unique is the wealth of information they provide and the relative ease with which they provide it. In a traditional experiment, involving an isotopically pure beam and a target which is either isotopically pure or of a known composition, there is little question as to the source of the measured results. In turn, this makes particle identification nearly unnecessary, eliminating the need for additional detectors and electronics.

Single particle states around closed-shell nuclei serve as a benchmark for testing nuclear structure theories. At present, there is no global theory describing nuclear properties across the chart of the nuclides. The properties of stable nuclei are well understood. However, the diffuse surfaces of neutron-rich nuclei may leads to changes in the nuclear potential and this effect is not completely understood \([14, 15]\). As a result, it is unclear how the ordering and spacing of single particle states evolve with increasing neutron excess.

One of the main goals of studying valence states around shell closures is to identify the ordering of the single-particle orbitals. To use \(^{133}\text{Sn}\) as an example, past studies populating states in \(^{133}\text{Sn}\) and measuring subsequent \(\gamma\)-ray transitions have shed some light on the energy levels above the \(^{132}\text{Sn}\) core \([22, 23]\). However, the value of transition energies gained in such studies yields no direct information on the ordering of the levels or the spin and parity of the states. The key to further progress in understanding the structure of \(^{133}\text{Sn}\) lies in nucleon-transfer reactions. One of the powerful aspects of direct transfer reactions is their selectivity. The reactions preferentially populate states that are well describe as target + nucleon system. In the \(^{132}\text{Sn}(D,p)\) reaction, one would expect states to be populated that correspond to neutron occupying the orbits in the \(2f 3p\) shell and the \(1h_{9/2}\) orbital.
2.2 Plane-wave Theory

Direct nuclear reactions, such as the nucleon transfer reaction \((d,p)\), are well suited to populate low energy, low angular momentum states. These low lying states tend to have single particle structure, particularly above closed-shell nuclei. It is the access to these single-particle levels provided by direct nuclear reactions which make them of specific interest in understanding both nuclear structure and astrophysical processes. It is not surprising then, that this technique has been well-established as an analytical tool for decades. The lowest-order theory describing direct reaction describes the incoming projection and outgoing ejectile as plane waves \((\text{first Born approximation})\) [26]. This method was first used by Butler [27] to describe the \((d,p)\) reaction.

2.2.1 Momentum Transfer

In a semi-classical description the incident plane wave will have a momentum vector \(p_i = \hbar k_i\). Due to the interaction of the projectile with the target, the change in momentum of the incident particle will be \(q = k_i - k_f\). Continuing from the assumption that the reaction occurs at the nuclear surface, the radius of the target nucleus may be written as \(r_0 = RA^{1/3}\) and the angular momentum transferred to the target nucleus is \(\ell = r_0 \times q\). The scattering angle connecting \(\ell\) and \(q\) is then given by

\[
\theta = \arccos \left( \frac{k_i^2 + f_i^2 - (\ell/r_0)^2}{2k_f k_i} \right). \tag{2.2}
\]

Fig. 2.2 shows the results of semi-classical calculations for momentum-matching for various proton-stripping reactions on \(^{118}\text{Sn}\). This naive interpretation of nuclear scattering actually provides a useful description of the reaction. Fig. 2.3 shows the excitation energy spectra of the same nucleus, \(^{118}\text{Sn}\), populated in two different proton-stripping reactions. The \(^{118}\text{Sn}(\alpha,t)^{119}\text{Sb}\) reaction, which has a large, negative \(Q\)-value of \(-14.7\text{ MeV}\), has enhanced yield to transitions corresponding to momentum transfers of \(\ell = 4\) and \(\ell = 5\). The yield to higher-spin states populated with the \(^{118}\text{Sn}(^{3}\text{He},d)^{119}\text{Sb}\) reaction is reduced.

Continuing further with a naive picture of nuclear scattering, if the effects of the Coulomb field are neglected, the incoming plane-wave can be modeled as scattering from a hard sphere. The Fraunhöfer diffraction equation

\[
f(\theta) = \frac{i k}{4\pi} (1 + \cos \theta) \int_S e^{i(q\cdot\vec{r})} dS \tag{2.3}
\]
gives the scattering amplitude as a function of angle and momentum transfer [5]. Rewritten in terms of a differential cross section, this relationship becomes

\[
\frac{d\sigma}{d\Omega} = k^2 r_0^2 \left( \frac{j_1(kr_0\theta)}{kr_0\theta} \right)^2 \tag{2.4}
\]
CHAPTER 2. NUCLEAR TRANSFER REACTIONS

Figure 2.2: Semi-classical calculations of momentum-matching for proton-stripping reactions on $^{118}$Sn. The calculated angular momentum transfer (in units of $\hbar$) is plotted as a function of beam energy.

Figure 2.3: Demonstration of momentum matching with two different proton-stripping reactions on $^{118}$Sn. (Top) The $^{118}$Sn($\alpha$,t)$^{119}$Sb reaction, which has a large, negative $Q$-value of -14.7 MeV, has enhanced yield to transitions corresponding to momentum transfers of $\ell = 4$ and $\ell = 5$. (Bottom) The yield to higher-spin states populated with the $^{118}$Sn($^3$He,d)$^{119}$Sb reaction is reduced and the low-spin states are enhanced. Figure by B. P. Kay from Ref. [28].
where \( j_\ell(kr_0\theta) \) spherical Bessel function of order \( \ell \). This relationship shows that measuring the angular variations in the intensity of the energy levels provides additional useful information. There is a direct relationship between the angular momentum \( \ell \) transferred to the heavy recoil and the angular distribution of the ejected particle. The resultant angular distribution is proportional to \( j_\ell(kr_0\theta) \). Fig. 2.4 shows an example of angular distributions from the \(^{28}\text{Si}(d,p)^{29}\text{Si} \) reaction which exhibit an interference pattern comparable to Eq. 2.4.

### 2.3 Optical Model

The phenomenological optical model was developed to improve upon the limitations of the plane wave theory of reactions. The optical model characterizes the reaction by a potential which *distorts* the incoming and outgoing plane waves.

#### 2.3.1 Nuclear Potentials

The optical model potential approximates the nuclear potential as

\[
U(r) = -Vf(r, r_0, a) - iWf(r, R', a') - iW'dg(r, r_0', a')
\]  

(2.5)
where $V$ depths of the real parts of the Woods-Saxon potential, $W$ is the depth of the imaginary (absorption) part of the Woods-Saxon potential and $W_D$ is the strength surface-peaked imaginary potential. The surface potential $g(r)$ is given by the derivative of the Woods-Saxon potential.

$$g(r, r'_0, a') = 4a \frac{d}{dr} f(r, r'_0, a')$$ (2.6)

In a similar fashion, the radial dependence of the spin-orbit term in Eq. 1.1 may be rewritten in terms of $g_{SO} = r^{-1}(d/d r)f(r, r_{SO}, a_{SO})$. The exact depths of the real potentials are varied to reproduce the previously-known nucleon separation energy for a given nucleus. The potential given in Eq. 2.5 includes imaginary terms in order to allow the removal of flux from the elastic scattering channel, i.e., for inelastic scattering.

### 2.3.2 Distorted Wave Born Approximation

Except for a number of special cases, the solutions to the Schrödinger equation for the optical model potential do not have a closed form and require computational solutions [26]. Calculating transition amplitudes for direct reactions in this way is called the distorted-wave Born approximation (DWBA). By comparing the measured shape of the angular distributions from an experiment to DWBA calculations, the angular momentum transfer $l$ can be determined and the degree to which the states are accurately described as a single-particle state. This provides a clear way of assigning the spin to the states populated in the heavy recoil. The relationship between the measured angular distribution and the calculated angular distribution is given by

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{meas}} = S \left( \frac{d\sigma}{d\Omega} \right)_{\text{DWBA}}$$ (2.7)

where $S$ is called the spectroscopic factor. For a state perfectly describe as a nucleon orbiting an closed core, that is, a single-particle state, the spectroscopic factor is identically equal to one. The spectroscopic factor provides a measure of the overlap between the final state and the initial state plus an added nucleon. Fig. 2.5 shows an example of calculations using the finite-range DWBA code PTOLEMY [29] using optical model parameters from Chapt. 13 for the $^{28}\text{Si}(d,p)$ reaction at a variety of bombarding energies.

### 2.4 Conclusion

With few exceptions, most stable-beam stable-target combinations have been studied in terms of nucleon transfer reactions. In order to measure the properties of nuclei further away from stability, it is necessary to utilize either radioactive beams or radioactive targets. In principle, this technique would be useful to study exotic nuclei. In terms of the $r$-process, it would be particularly useful to study neutron-rich nuclei with the $(d,p)$ reaction which deposits a neutron on the target nucleus. However, the nuclei of interest are so unstable
that they are unsuitable for making targets. Until recently, this limitation has made transfer reactions inapplicable to the study of exotic nuclei. With the arrival of rare isotope beam capabilities at nuclear accelerator facilities, exotic nuclei can now be studied with this powerful technique.

The worldwide development of radioactive ion beam facilities is allowing for a new generation of nuclear reactions to be studied. Reactions utilizing such exotic beams are permitting measurements of nuclear properties further away from stability than previously possible. With radioactive beams, however, the kinematics of the measurement must be inverted, with a heavy-ion beam bombarding a light-ion target. The basic form of the reactions being studied is the same (nucleon stripping or knock-out), but the reaction kinematics are changed in a way which presents new measurement difficulties.
Part II

Experimental Techniques
Chapter 3

Two-body Kinematics

Direct nuclear transfer reactions, as discussed in the previous chapter, can be described in terms of two-body kinematics. The feature that makes the HELIOS spectrometer unique is the effect that the spectrometer's magnetic field has on the motion of the reaction products (solenoidal transport) and the way in which the products are measured (detector geometry). Therefore, in order to have a meaningful discussion of the benefits of HELIOS, it is necessary to introduce the key concepts of reaction kinematics. This chapter also discusses the specific challenges encountered when measuring reaction in inverse kinematics.

3.1 Basic Principles

Reaction kinematics form a subset of generalized two-body kinematics. In the typical setup, a beam of ions of mass $m_1$ are accelerated to a velocity of $v_1$ and bombard stationary target atoms of mass $m_2$. In “normal kinematics” a light ion beam strikes a heavy ion target with $m_1 < m_2$. An example of such a reaction is the $^{28}\text{Si}(d,p)$ as discussed in Ref [25], in which a deuteron beam strikes a silicon target. In “inverse kinematics” the masses are reversed—a heavy ion beam strikes a light ion target ($m_1 > m_2$). The same $(d,p)$ reaction on silicon is written in inverse kinematics as $d(^{28}\text{Si},p)$. This is the reaction presented in Ref. [30] and discussed in Chap. 13.

3.1.1 Energy

Before the reaction, given a stationary target and an accelerated ion beam, the total kinetic energy of the two-body system in the laboratory frame is equal to the beam energy $E_1$. The total kinetic energy in the center-of-mass system is then given by

$$T_{\text{cm}} = \frac{m_2}{m_1 + m_2} E_1$$  \hspace{1cm} (3.1)
where $E_1 = \frac{1}{2} m_1 v_1^2$ is the kinetic energy of the beam particle.

The system then gains or loses energy depending on the specific reaction energy, referred to as the $Q$-value. The $Q$-value will be defined here as the change in total energy of the system before and after the reaction in a ground-state transition. With this definition, the reaction $Q$-value may be calculated as the change in mass.

$$Q = [(m_1 + m_2) - (m + M)]c^2$$  (3.2)

where $Q$ is explicitly the energy gained in the reaction\(^*$*. Here the masses of the reaction products are written as $m$ and $M$, with $m < M$. In this notation, an endothermic reaction will have a negative $Q$-value. For transitions to states other than the ground-state, the excitation energy $E_x$ must also be included in calculating the resultant total kinetic energy in the center-of-mass frame.

$$E_{\text{total}} = T_{\text{cm}} + Q - E_x$$  (3.3)

With the energy $E_{\text{total}}$ defined above, the conservation of momentum gives the (non-relativistic) kinetic energy of the ejectile in the center-of-mass system as

$$E_{\text{cm}} = E_{\text{total}} \frac{M}{m + M} = \frac{1}{2} m v_0^2$$  (3.4)

where $v_0$ is the velocity of the ejectile (mass $m$) in the center-of-mass frame.

### 3.1.2 Velocity

The center-of-mass system is defined as the reference frame in which the reactants have equal and opposite momentum vectors. In the laboratory system, this center-of-mass has a velocity given by

$$V_{\text{cm}} = v_1 \frac{m_1}{m_1 + m_2}.$$  (3.5)

The velocity of the ejectile (mass $m$) in the center-of-mass frame $v_0$ is given in terms of the total kinetic energy in the center-of-mass $E_{\text{total}}$. Substituting Eq. 3.3 into Eq. 3.4 and solving for $v_0$ yields

$$v_0 = \sqrt{\frac{2M(T_{\text{cm}} + Q - E_x)}{m(m + M)}}.$$  (3.6)

The velocity magnitudes $V_{\text{cm}}$ and $v_0$ are constants of motion. Eq. 3.5 shows that for a given reaction, $V_{\text{cm}}$ is

---

\*Some texts define the $Q$-value as the total amount of energy lost in the reaction, in which case $Q' = -Q + E_x$. 


fixed by the bombarding energy of the beam particle. Similarly, Eq. 3.6 shows that $v_0$ is fixed by the reaction $Q$-value and transition energy $E_x$, in addition to the bombarding energy. These velocities are related to the laboratory velocity by $v_{\text{lab}} = V_{\text{cm}} + v_0$. The magnitude $v_{\text{lab}}$ varies as the emission angle $\theta_{\text{cm}}$ changes. With reference to Fig. 3.1, the magnitude $v_{\text{lab}}$ is readily obtained using the law of cosines

$$v_{\text{lab}}^2 = v_0^2 + V_{\text{cm}}^2 - 2v_0V_{\text{cm}}\cos(180^\circ - \theta_{\text{cm}}). \quad (3.7)$$

Rewriting in terms energy, one gets

$$E_{\text{lab}} = E_{\text{cm}} + \frac{1}{2} m V_{\text{cm}}^2 + m V_{\text{cm}} v_0 \cos(\theta_{\text{cm}}). \quad (3.8)$$

### 3.1.3 Angle

After the reaction, the ejectile is emitted at an angle in the center of mass $\theta_{\text{cm}}$ over the range of $0–180^\circ$. Due to the conservation of momentum, the ejectile and the heavy recoil are ejected back-to-back in the center-of-mass frame with $\theta_{\text{cm}} + \theta_{\text{cm}} = 180^\circ$. To be consistent with the convention used in most textbooks, the center-of-mass angle $\theta_{\text{cm}}$ is defined relative to $+V_{\text{cm}}$ direction, as illustrated in Fig. 3.1. This choice of coordinates is natural in normal kinematics where, in the rest frame of the heavy ion target, the beam is approaching at a velocity of $\approx v_1$. However, in inverse kinematics the target is approaching the heavy ion at a **negative** velocity.
CHAPTER 3. TWO-BODY KINEMATICS

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$Q$-value

$E_1/A_1$

$K_{g.s.}$

$\theta_{max}$

Ref.

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<th>Reaction</th>
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<th>(MeV/u)</th>
<th>(deg)</th>
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<td>4.78</td>
<td>0.01</td>
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<tr>
<td>$^{124}\text{Sn}(d,\text{He}^3)^{123}\text{In}$</td>
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<td>14.35</td>
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Table 3.1: Ground-state transition velocity ratios $K_{g.s.}$ for a number of reactions. For each reaction, the values are given for both normal and inverse kinematics. For reactions that have been performed, the beam energy has been selected to match the given reference.

of $-V_{cm}$. Therefore, in inverse kinematics, the center-of-mass angle is defined as the compliment of the angle shown in Fig. 3.1 ($180^\circ - \theta_{cm}$). Thus in both cases, $\theta_{cm} = 0^\circ$ refers to the light ion traveling in the same direction before and after the reaction from the standpoint of the heavy ion.

In the laboratory frame, the extent to which $\theta_{lab}$, and in turn $E_{lab}$, vary as a function of $\theta_{cm}$ depends on how $V_{cm}$ and $v_0$ compare to one another. The value of the ratio $K = V_{cm}/v_0$ divides the reactions into three classes. Table 3.1 gives the $K$-values for a number of reactions.

$K < 1$

For $K < 1$ the laboratory energy $E_{lab}$ is a single-valued function of the laboratory angle $\theta_{lab}$, with the ejectile being emitted over all laboratory angles. In the limit of normal kinematics, when the beam is much lighter than the target ($m_1 \ll m_2$), the center-of-mass of the reaction system is nearly at rest. Eq. 3.8 shows that when the velocity ratio $K \ll 1$—that is, when $V_{cm} \ll v_0$—the laboratory energy $E_{lab}$ begins to lose its dependence on the angle $\theta_{cm}$. This relationship is shown in Fig. 3.2 by the line $K = 0.05$ illustrating $\theta_{lab} \approx \theta_{cm}$.

As the ratio $K$ becomes comparable to 1, the reactions enter the realm of inverse kinematics. The laboratory energy $E_{lab}$ varies strongly with angle as the $\cos(\theta_{cm})$ term in Eq. 3.8 begins to dominate. When $V_{cm}$ and $v_0$ are more aligned, the energy in the laboratory $E_{lab}$ is large compared to the energy in the center-of-mass $E_{cm}$. This corresponds to forward angles in the laboratory; at rearward angles, the inverse is true.

$K = 1$

$K_{g.s.} = 1$ is the trivial case, arising in two ways. First, in elastic scattering, corresponding to $Q = E_x = 0$; and second, when $E_x \approx Q + E_1/A_1$. When no energy is transfered in the reaction, $V_{cm} = v_0$ and $\theta_{lab} = \frac{1}{2} \theta_{cm}$. The ejectile is emitted in the forward hemisphere only, with $\theta_{lab} \leq 90^\circ$. This relationship is shown in Fig. 3.2 by the line $K = 1$. 


Figure 3.2: Transformation of angles between the laboratory and the center-of-mass systems. For $K > 1$ the transformation is double-valued in the laboratory. The maximum laboratory angle for such solutions is plotted. Adapted from Michalowicz \cite[Fig. 9]{31}.

For $K > 1$, when $v_0 < V_{cm}$, the ejectile cannot be emitted in the rearward hemisphere. Thus, the laboratory energy $E_{lab}$ is a double-valued function of the laboratory angle $\theta_{lab}$ and the ejectile is emitted over a limited range of angles in the forward hemisphere. The ejectile has a maximum emission angle in the laboratory given by

$$\theta_{max} = \tan^{-1}\left(\frac{1}{\sqrt{K^2 - 1}}\right) = \sin^{-1}\left(\frac{1}{K}\right)$$ (3.9)

The shaded region of Fig. 3.2 illustrates the relationship between $K$ and $\theta_{max}$ and Table 3.1 gives $\theta_{max}$ for a specific reaction in inverse kinematics. In such cases, each angle $\theta_{lab}$ corresponds to a “high energy” solution and a “low energy” solution. The low energy solution corresponds to particles emitted at more forward angles (typically $\theta_{cm} \lesssim 30^\circ$) in the center-of-mass frame.

In the limiting case of $K \gg 1$, when $V_{cm} \gg v_0$, Eq. 3.8 again shows that the laboratory energy $E_{lab}$ will loose its dependence on the angle $\theta_{cm}$. However, in this case, the laboratory velocity is approximately the center-of-
mass velocity $v_{\text{lab}} \approx V_{\text{cm}}$. This is the typical situation for the heavy ion reaction product. Furthermore, in the limit of inverse kinematics ($m_1 \gg m_2$), the velocity of the heavy recoil is then also approximately the incident beam velocity $V_{\text{lab}} \approx V_{\text{cm}} \approx v_1$.

### 3.2 Inverse Kinematics

The utility of studying reactions in inverse kinematics is that it provides access to measurements involving heavy isotopes that are unstable against $\beta$-decay. However, as detailed in the previous section, the transition to inverse kinematics is not simply a reversal of target and beam. The rapidly changing laboratory quantities that are encountered have serious implications for measurements carried out in inverse kinematics. In addition, the difficulties associated with a radioactive ion beam must also be considered. This section discusses several of these challenges.

#### 3.2.1 Kinematic Compression

In inverse kinematics, the center-of-mass of the reaction has a substantial velocity in the laboratory frame. When the velocity of the light ion reaction product $v_0$ is comparable to the velocity of center-of-mass $V_{\text{cm}}$, the energies of the emitted light ions are highly angle-dependent. When these velocities are anti-aligned (forward center-of-mass angles), the laboratory energy is significantly smaller than in normal kinematics. This also has the effect of compressing the relative spacing between energy levels at a given $\theta_{\text{lab}}$.

**Compression Coefficient**

The separation between two energy levels in the laboratory frame is defined here as $\Delta E_{\text{lab}} = (E_{\text{lab}} - E'_{\text{lab}})$. Starting with Eq. 3.8 and holding $V_{\text{cm}}$ and $\theta_{\text{cm}}$ constant, the energy separation in the laboratory at a fixed emission angle is given by

$$\Delta E_{\text{lab}} = \Delta E_{\text{cm}} + m V_{\text{cm}} \Delta v_0 \cos(\theta_{\text{cm}}) \quad (3.10)$$

The energy separation in the center-of-mass frame is given by

$$\Delta E_{\text{cm}} = \frac{1}{2} m (v_0 + v'_0) \Delta v_0 \quad (3.11)$$
The ratio of the separation of the energy levels in the laboratory $\Delta E_{\text{lab}}$ to that in center-of-mass will be defined here as the “compression coefficient”

\[
\frac{\Delta E_{\text{lab}}}{\Delta E_{\text{cm}}} = 1 + \frac{2V_{\text{cm}} \cos(\theta_{\text{cm}})}{(v_0 + v'_0)(1 + v'_0/v_0)}
\]  

(3.12)

and is identically equal to 1 at $\theta_{\text{cm}} = 90^\circ$. For reactions in normal kinematics, with $K \ll 1$, this effect is suppressed and $\Delta E_{\text{lab}}/\Delta E_{\text{cm}} \approx 1$ for all angles. In both normal and inverse kinematics, at rearward emission angles ($\theta_{\text{cm}} > 90^\circ$) the effect causes an increase of the energy spacing ($\Delta E_{\text{lab}}/\Delta E_{\text{cm}} > 1$). However, in order to differentiate the angular distributions associated with different angular momentum transfers, it is necessary to measure the reactions at forward angles ($\theta_{\text{cm}} \lesssim 30^\circ$). This region of interest is where the effect of kinematic compression is the most dramatic. In cases when the $K$-value of the reaction is $> 1$, meaning the laboratory energy $E_{\text{lab}}$ is a double-valued function of the laboratory angle $\theta_{\text{lab}}$, the value of $\Delta E_{\text{lab}}/\Delta E_{\text{cm}}$ is negative for the low energy solution. In some cases, the low energy solution will also see an enhancement in the energy level spacing ($\Delta E_{\text{lab}}/\Delta E_{\text{cm}} < -1$) at the most forward laboratory angles.

Example

Fig. 3.3 shows calculated plots of energy $E_{\text{lab}}$ vs. angle $\theta_{\text{lab}}$ for the $(d, p)$ reaction on $^{132}\text{Sn}$ at 4.78 MeV/u. This bombarding energy is chosen to match the measurement of Jones et al. [24]. This reaction is a benchmark measurement for inverse kinematics, not only because of the importance of the physics associated with the measurement (see Chapt. 1.2), but also because of the difficulty of the measurement. In the figure, fictitious energy levels have been selected such that $\Delta E_x = 1.0 \text{ MeV}$ to illustrate kinematic compression.

$^{132}\text{Sn}$ is unstable against $\beta$-decay with a half-life of $T_{1/2} = 39.7 \text{ s}$. Therefore it is impossible to make a $^{132}\text{Sn}$ target for a reaction study in normal kinematics. However, if it were possible to perform this reaction in normal kinematics (left panel), the ground state transition would have a velocity ratio of $K_{\text{g.s.}} = 0.01$ and at $\theta_{\text{cm}} = 10^\circ$ the “compression” coefficient is 1.01. In inverse kinematics, the ground state transition has a velocity ratio of $K_{\text{g.s.}} = 0.70$ and at $\theta_{\text{cm}} = 10^\circ$ the compression coefficient is 0.30. Furthermore, in inverse kinematics, for excitation energies $E_x \gtrsim (Q + E_1/A_1)$, in this case $\approx 5 \text{ MeV}$, the energy solution is double-valued ($K > 1$).

3.2.2 Kinematic Broadening

The effect of kinematic compression emphasizes the need for high-resolution energy measurements in inverse kinematics. In addition, due to the strong dependence of the light recoil energy $E_{\text{lab}}$ on laboratory angle
Figure 3.3: Illustration of kinematic compression. In normal kinematics (left, \( K_{g.s.} = 0.01 \)), the energy level spacing approximately constant and equal to the spacing in the center-of-mass. In inverse kinematics (right, \( K_{g.s.} = 0.70 \)), the energy spacing is dramatically compressed at forward angles, corresponding to emission in the rearward hemisphere. The dashed lines indicate the approximate position of \( \theta_{cm} = 35^\circ \), with the arrows indicating forward angles in the center-of-mass. Note the laboratory energy \( E_{lab} \) becomes double-valued in inverse kinematics for \( E_x > 5 \text{ MeV} \).

\( \theta_{lab} \), it becomes essentially important to have precise angle measurement. Even in a system with perfect energy resolution (\( \delta E_{lab} = 0 \)), the range of angles covered by a finite detector element \( \delta \theta_{lab} \) corresponds to a range in measured energy; thus leading to a reduced measured energy resolution. This relationship is illustrated in Fig. 3.4. For example, in the \( d(132\text{Sn},p) \) reaction discussed above, at \( \theta_{cm} = 10^\circ \) (\( \theta_{lab} = 149^\circ \)) the rate-of-change (i.e. slope) of the ground state transition in the laboratory is \( d E_{lab}/d \theta_{lab} = 41 \text{ keV/deg} \); at \( \theta_{lab} = 90^\circ \) this value increases to 168 keV/deg. This unavoidable feature is referred to as the “kinematic broadening” of the energy resolution [38]. Put another way, due to the covariance between energy and angle, the energy resolution can be limited by angular resolution and vice-versa. This effect is discussed below and demonstrated in Fig. 3.5.

Resolution Propagation

The uncertainty of the measured quantity \( \theta_{lab} \) may lead to a kinematic broadening of the measured quantity \( E_{lab} \), as mentioned, but it also effects the derived quantities \( E_{cm} \) and \( \theta_{cm} \). To see exactly how the measurement uncertainties effect the derived results, it is necessary to perform an error analysis. Solving Eq. 3.8
for $E_{cm}$ and rewriting in terms of the measured quantities $E_{lab}$ and $\theta_{lab}$ (this substitution is derived in Eq. 5.7) yields

$$E_{cm} = E_{lab} - \frac{1}{2} m V_{cm}^2 - m V_{cm} V_0 \cos(\theta_{cm})$$

$$= E_{lab} + \frac{1}{2} m V_{cm}^2 - m V_{cm} V_{lab} \cos(\theta_{lab})$$

$$= E_{lab} + \frac{1}{2} m V_{cm}^2 - V_{cm}(\sqrt{2m E_{lab}}) \cos(\theta_{lab})$$ (3.13)

Given this relationship, the uncertainty in $E_{cm}$ due to the uncertainty in $E_{lab}$ and $\theta_{lab}$ can be calculated using the generalized error propagation equation [39, 40].

$$\left(\delta E_{cm}\right)^2 = \left(\frac{\partial E_{cm}}{\partial E_{lab}}\right)^2 \left(\delta E_{lab}\right)^2 + \left(\frac{\partial E_{cm}}{\partial \theta_{lab}}\right)^2 \left(\delta \theta_{lab}\right)^2 + 2 \left(\frac{\partial E_{cm}}{\partial E_{lab}}\right) \left(\frac{\partial E_{cm}}{\partial \theta_{lab}}\right) \delta E_{lab} \delta \theta_{lab}$$ (3.14)

The first two terms in this expression are the individual contributions of the two measured quantities, which are always positive. The third term is the contribution of the covariance between the two quantities and can (mathematically) take on positive and negative values. Each term is weighted by a partial derivative. The weight for the uncertainty in energy $\delta E_{lab}$ is given by

$$\frac{\partial E_{cm}}{\partial E_{lab}} = 1 - m V_{cm} \cos(\theta_{lab}) \sqrt{\frac{m}{2E_{lab}}}$$ (3.15)

and the weight for the uncertainty in angle $\delta \theta_{lab}$ is given by

$$\frac{\partial E_{cm}}{\partial \theta_{lab}} = V_{cm} \sqrt{2m E_{lab}} \sin(\theta_{lab})$$ (3.16)

Thus using this technique, the contribution of the individual uncertainties in the measured quantities can be calculated. Unless otherwise specified, the value of the uncertainties are characterized in terms of the
full-width at half maximum (FWHM) $\Gamma$, as opposed to the standard deviation $\sigma$; the two are related by $\Gamma = 2.35\sigma$. Table 3.2 shows the results of these calculations for a number of reactions in a manner similar to that presented by Winfield et al. [38]. Following Ref. [38], the contributions are calculated assuming measurement uncertainties of $\delta E_{\text{lab}} = 40$ keV FWHM, $\delta \theta_{\text{lab}} = 0.54^\circ$ FWHM and an uncertainty in the beam energy (discussed below) of 0.14%. However in Ref. [38], the individual contributions are added in quadrature, neglecting the effects of the covariance of the measured quantities. The two rightmost columns in Table 3.2 show the sum of the uncertainties added quadratically $\Sigma_{\text{quad}}$ and the sum including the covariant term $\Sigma_{\text{covar}}$. As expected, this contribution has little effect in normal kinematics where the covariance between $E_{\text{lab}}$ and $\theta_{\text{lab}}$ is negligible. However, in inverse kinematics the contribution of the covariance is significant. The last two rows of Table 3.2 show that in some cases (when $K > 1$), the contribution of covariance tends to improve the $Q$-value resolution.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_1/A_1$ (MeV/u)</th>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$\delta \theta_{\text{lab}}$</th>
<th>$\delta E_{\text{lab}}$</th>
<th>$\delta E_1$</th>
<th>$\Sigma_{\text{quad}}$</th>
<th>$\Sigma_{\text{covar}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{132}\text{Sn}(d,p)^{133}\text{Sn}$</td>
<td>4.78</td>
<td>9.9</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$^{124}\text{Sn}(d,\text{He}^3)^{123}\text{In}$</td>
<td>14.35</td>
<td>9.8</td>
<td>2</td>
<td>39</td>
<td>0</td>
<td>39</td>
<td>41</td>
</tr>
<tr>
<td>$^{28}\text{Si}(d,p)^{29}\text{Si}$</td>
<td>8.94</td>
<td>9.6</td>
<td>3</td>
<td>38</td>
<td>0</td>
<td>39</td>
<td>41</td>
</tr>
<tr>
<td>$^{12}\text{B}(d,p)^{13}\text{B}$</td>
<td>6.24</td>
<td>9.1</td>
<td>4</td>
<td>36</td>
<td>0</td>
<td>37</td>
<td>40</td>
</tr>
<tr>
<td>$d(^{28}\text{Si},p)^{29}\text{Si}$</td>
<td>6.02</td>
<td>157.9</td>
<td>31</td>
<td>85</td>
<td>8</td>
<td>91</td>
<td>116</td>
</tr>
<tr>
<td>$d(^{12}\text{B},p)^{13}\text{B}$</td>
<td>6.24</td>
<td>155.2</td>
<td>25</td>
<td>93</td>
<td>7</td>
<td>97</td>
<td>118</td>
</tr>
<tr>
<td>$d(^{132}\text{Sn},p)^{133}\text{Sn}$</td>
<td>4.78</td>
<td>149.0</td>
<td>22</td>
<td>111</td>
<td>7</td>
<td>113</td>
<td>133</td>
</tr>
<tr>
<td>$d(^{124}\text{Sn},3\text{He})^{123}\text{In}$</td>
<td>13.00</td>
<td>21.5</td>
<td>87</td>
<td>72</td>
<td>55</td>
<td>126</td>
<td>57</td>
</tr>
<tr>
<td>$p(^{77}\text{Kr},d)^{76}\text{Kr}$</td>
<td>30.00</td>
<td>15.1</td>
<td>118</td>
<td>54</td>
<td>85</td>
<td>156</td>
<td>106</td>
</tr>
</tbody>
</table>

Table 3.2: Calculated contributions to the uncertainty of $E_{\text{cm}}$ for a number of reactions. Values are calculated in keV FWHM for $\theta_{\text{cm}} = 10^\circ$, with $\delta E_{\text{lab}} = 40$ keV, $\delta \theta_{\text{lab}} = 0.54^\circ$ and $\delta E/E = 0.14\%$. The quadratic sum $\Sigma_{\text{quad}}$ and the sum including the covariant term $\Sigma_{\text{covar}}$ are given. Adapted from Ref. [38, Table 2].

### 3.2.3 Discussion

In normal kinematics, the dominant contribution to the uncertainty of the center-of-mass energy $\delta E_{\text{cm}}$ (i.e. the $Q$-value resolution) is the intrinsic energy resolution of the detectors. In addition, the energy separation between excited states in the laboratory is nearly the same as that in the center-of-mass frame. However, in inverse kinematics, the significant covariance between $\theta_{\text{lab}}$ and $E_{\text{lab}}$ due to the kinematics of the reaction tends to broaden the resolution of the measured (and derived) quantities. The net result of measuring reactions in inverse kinematics at forward center-of-mass angles is that separate energy levels are compressed together and are measured with broadened resolution, which tends to blur the states together. These effects are illustrated in Fig. 3.5. To address these problems, one approach is to implement a large detector array with excellent angle resolution. This method has the disadvantage that it requires a complicated assortment of detectors and electronics. However, the challenges encountered in conducting reactions in inverse kinematics...
Figure 3.5: Illustration of the combined effect of kinematic compression and kinematic broadening. Calculated energy spectra are plotted for a fixed laboratory angle corresponding to at $\theta_{\text{cm}} = 10^\circ$, consistent with Fig. 3.3 and Table 3.2. In normal kinematics (top, $\theta_{\text{lab}} = 9.9^\circ$), the energy level spacing approximately constant and equal to the spacing in the center-of-mass. These states are plotted assuming an energy resolution of 40 keV FWHM. In inverse kinematics (bottom, $\theta_{\text{lab}} = 149^\circ$), the energy spacing is dramatically compressed and the resolution is broadened to 133 keV FWHM. States above $E_x > 5\text{ MeV}$ are not emitted in the rearward hemisphere.

do not completely preclude their study. Chap. 4 describes two "traditional" approaches to measuring transfer reactions with exotic beams. Another approach is to measure the nuclear reactions in a new way that avoids or suppresses kinematic compression and kinematic broadening. The Helical Orbit Spectrometer (HELIOS) at Argonne National Laboratory, discussed in Chap. 5 and on, provides such an approach.

### 3.2.4 Other Contributions

#### Intrinsic Width

The discussion of resolution propagation presented above assumes that the states being measured have a natural width much smaller than the measurement uncertainty. The intrinsic shape of an excited state is a Lorenzian distribution with a width given by the Heisenburg uncertainty principle

$$\Delta E \Delta t \leq \hbar$$  \hspace{1cm} (3.17)
(here the standard symbol for uncertainty $\Delta$ has been used)\cite{5}. If $\tau$ is the lifetime of the excited state, the width $\Gamma$ is given by

$$
\Gamma \approx \hbar / \tau.
\quad (3.18)
$$

For example, the first-excited state in $^{29}\text{Si}$ has a lifetime of $\tau = 290 \text{ fs}$, which corresponds to a characteristic width of 2.3 meV (milli-electronvolts); this is a typical value for a bound nuclear excited state. Therefore, under realistic conditions, the shape and width of measured excited states is dominated by the intrinsic detector resolution and, depending on the reaction, kinematic effects. In addition to the intrinsic resolution properties of the detector system, an important contribution to the $Q$-value resolution is the quality of the incident beam and the thickness of the target.

**Beam Energy**

The beam energy $E_1$ enters into the calculation of $E_{\text{cm}}$ via the center-of-mass velocity $V_{\text{cm}}$. Rewriting $V_{\text{cm}}$ in terms of $E_1$ yields

$$
V_{\text{cm}} = \sqrt{\frac{2E_1}{m_1} \left( \frac{m_1}{m_1 + m_2} \right)}.
\quad (3.19)
$$

Substituting this expression into Eq. 3.13 and differentiating gives the contribution weight of the beam energy uncertainty $\delta E_1$ to the $Q$-value resolution.

$$
\frac{\partial E_{\text{cm}}}{\partial E_1} = \frac{m}{m_1} \left( \frac{m_1}{m_1 + m_2} \right)^2 - m(v_{\text{lab}} \cos(\theta_{\text{lab}})) \left( \frac{m_1}{m_1 + m_2} \right) \sqrt{\frac{1}{2m_1E_1}}.
\quad (3.20)
$$

The value of $\delta E_1$ depends on many factors, such as method of beam production. However, for an example with a stable beam, Fig. 3.6 shows several measurements of the beam energy $E_1$ and the uncertainty in the beam energy $\delta E_1$ over the first five days of the $d(^{28}\text{Si}, p)^{29}\text{Si}$ experiment in Ref. \cite{30}. The beam energy had a nominal value of 168 MeV (6 MeV/u) with a measured value of $E_1 = 168.53 \pm 0.24 \text{ MeV}$. This level of uncertainty corresponds to a relative uncertainty in the beam energy of $\delta E_1 / E_1 = 0.14\%$. The contribution of this uncertainty is shown in the sixth column of Table 3.2. For unstable beams, the uncertainty in the beam energy will typically be higher than that for stable beams, although to what degree this is so depends on the method of beam production.

**Beam Size**

Depending on the beam production technique, the heavy ion beam may have a substantial lateral extent—on the order of 1–5 mm—at the target plane. This transverse area is referred to as the beam spot size. An increase in the uncertainty of the transverse extent of the interaction area between the beam and the target...
\[ \delta x, \delta y \] leads to a smearing of the measured scattering angle. Fig. 3.7(a) shows the effect of a finite beam spot size. A similar effect, the effect of a misalignment of the beam, will also produce an error in determining the angle. The contribution of this effect is discussed in Chapt. 12.

**Target Effects**

The light ions classically available for accelerated beams are not necessarily readily available, or perhaps practical, for use as target. While most electrostatic accelerator facilities can produce pure beams of light ions, a solid cryogenic hydrogen target for high-resolution measurements, for instance, is impossible. Instead, hydrogen beams are replaced with targets made of polyethylene \((C_2H_4)_n\) or polypropylene \((C_3H_6)_n\). Similarly, the inverse kinematics analog of a deuteron beam could be a deuterated polyethylene \((C_2D_4)_n\) target.

The relatively low intensity beams achievable with most radioactive isotopes can be compensated for, in part, by the use of thicker targets. However in a thicker target, the nuclear reactions can occur at a variety of depths. When the bombarding beam consists of heavy ions, the energy loss associated with passing through the target \(dE_1/dz\) (with \(+z\) the direction of the beam) can be substantial. For example, for a \(^{132}\text{Sn}\) beam incident on a \(200 \mu\text{m}\) thick \((\text{CD}_2)_n\) target will have an energy loss of about 15 MeV. In addition, as these effects...
Figure 3.7: Illustration of the effect of beam spot size and target thickness on the light ejectile. (a) The nuclear reaction occurs off the nominal beam axis, resulting in a disparity between the actual scattering angle $\theta_{1}^{\text{scat}}$ and the detected scattering angle $\theta_1$. (b) The nuclear reaction occurs near the front of the target and the light ejectile scatters again while passing through the target. Adapted from Ref. [38, Fig. 7]

The amount of energy loss that the beam experiences is not fixed, but has a finite distribution. The spread in energy of the beam after passing through the target is known as energy straggling. Finally, the collisions which lead to the energy loss also produce small angle scattering; this effect is known as multiple scattering. Fig. 3.7(b) illustrates the effect of a thick target on the light ion ejectile. The target thickness effects of energy loss and multiple scattering impact the kinematics of both the incoming beam particle and the outgoing reaction products. Due to the random nature of these effects, they are best treated using Monte Carlo simulation techniques. This method is discussed in Chapt. 12.
Chapter 4

Two Example Measurements

A successful measurement of a reaction in inverse kinematics requires a high-efficiency, high-resolution detector system with large acceptance and good background suppression. The traditional solution to these issues is to use a large detector array with fine angular resolution [41–44]. This chapter discusses two benchmark reaction measurements in inverse kinematics using the traditional—or non-solenoidal—detector approach wherein the laboratory energy $E_{\text{lab}}$ of the detected particles is measured as a function of the laboratory angle $\theta_{\text{lab}}$ to determine the center-of-mass quantities.

4.1 The $^{12}\text{B}(d,p)$ Measurement

4.1.1 Introduction

A measurement of the $^{12}\text{B}(d,p)$ reaction was carried out at Argonne National Laboratory to study the neutron-rich $N = 8$ nucleus $^{13}\text{B}$. The details of this measurement are described in Ref. [33]; this section summarizes those results. Originally, the main purpose of this experiment was a nuclear structure study. There is a pair of excited states in $^{13}\text{B}$ near 3.6 MeV, separated by 199 keV, which had (at the time of the experiment) unknown spins. The aim of this measurement was to resolve this doublet and through the analysis of the resulting angular distributions, determine the angular momentum transfer $\ell_n$ and assign spins and parities $J^\pi$ to the states.

4.1.2 Experimental Setup

The experiment was carried out in the scattering chamber upstream from the Enge Split-Pole Spectrograph in ATLAS Target Area III (SPSIII), referred to internally as Ludwig's Castle.
CHAPTER 4. TWO EXAMPLE MEASUREMENTS

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Figure 4.1: Gas cell used for in-flight beam production at ATLAS. The type of gas cell used to produce the radioactive $^{12}\text{B}$ beam in-flight are 3.7 cm long and 2.54 cm in diameter. Havar® foils 1.9 mg/cm² thick serve as windows on the entrance and exit of the gas cell.

Beam Production

$^{12}\text{B}$ is unstable against $\beta$-decay with a half-life of $T_{1/2} = 20.2$ ms. Therefore, a reaction involving $^{12}\text{B}$ must be performed in inverse kinematics. In this example, the $^{12}\text{B}$ beam is produced in-flight, following the method which is described in detail in Ref. [45]. A primary beam of stable $^{11}\text{B}$ ions at an energy of 81 MeV and an intensity of 100 pnA bombarded a cryogenic gas cell to produce the secondary radioactive beam. The production cell is shown in Fig. 4.1. The gas cell was filled with $\text{D}_2$ deuterium gas at a pressure of 1400 mbar and temperature of $-185$ °C to produce a target with an areal density of 1.6 mg/cm². The secondary $^{12}\text{B}$ beam was produced in the neutron transfer reaction $d (^{11}\text{B},^{12}\text{B}) p$. The resulting radioactive beam bombarded an 150 μg/cm² target of deuterated polyethylene ($\text{C}_2\text{D}_4)_n$ with an average beam intensity of $1.2 \times 10^5$ ions/s.

Detectors

The detector setup for measuring transfer reactions within Ludwig’s Castle is described in Ref. [46]. The same basic setup was used in this measurement. The detector array utilized in this experiment consists of three 500 μm thick double-sided silicon strip detectors (DSSD) of design S1 manufactured by Micron Semiconductor. The annular detectors have an inner radius of 24 mm and an outer radius of 48 mm, for a total active area of 53 cm². One side of each detector is segmented into 16 concentric rings of $\Delta r = 1.5$ mm, while the other side is segmented into 16 wedges, each covering $\Delta \phi = 22.5^\circ$; thus each detector requires 32 electronics channels. To suppress spurious counts, a detector signal is required in an element on both sides of a given detector in order to be included in the trigger logic. Heavy recoils are identified downstream from the target.
Figure 4.2: Detector setup for the $^{12}\text{B}(d,p)$ measurement in inverse kinematics (drawn to scale). Protons ejected in the rearward hemisphere ($\theta_{\text{lab}} > 90^\circ$) are detected by three annular detectors covering $114^\circ < \theta_{\text{lab}} < 162^\circ$.

\begin{table}[h]
\centering
\begin{tabular}{cccccc}
Det. & z & $\theta_{\text{lab}}$ & $\theta_{\text{cm}}$ & $\Delta \cos(\theta_{\text{cm}})$ & $\Delta \Omega$ \\
    & (mm) & $\theta_1$ & $\theta_2$ & $\theta_1$ & $\theta_2$ & (sr) \\
1 & -21.1 & 113.7 & 131.3 & 32.4 & 21.5 & 0.086 & 0.54 \\
2 & -39.9 & 121.9 & 141.2 & 27.0 & 16.4 & 0.068 & 0.43 \\
3 & -74.8 & 147.3 & 162.2 & 13.5 & 7.1 & 0.020 & 0.13 \\
Total & & & & 0.175 & & 1.10 \\
\end{tabular}
\caption{Detector positions and solid angle coverage for the $^{12}\text{B}(d,p)$ measurement. Protons ejected in the rearward hemisphere ($\theta_{\text{lab}} > 90^\circ$) are detected by three annular detectors covering $114^\circ < \theta_{\text{lab}} < 162^\circ$.}
\end{table}

in a $\Delta E\cdot E$ detector array discussed in Chapt. 10.2. In an adjacent scattering chamber downstream from the heavy recoil detectors, a surface barrier detector is placed in the beam path behind an attenuator to monitor the beam current.

Fig. 4.2 shows the physical relationship of the detectors to the target foil within the scattering chamber. The $^{12}\text{B}(d,p)$ reaction has a $K_{g.s.}$-value of 0.61, which means forward angles in the center-of-mass correspond to rearward angles in the laboratory ($\theta_{\text{lab}} > 90^\circ$); hence the detector array is position upstream from the target foil. Table 4.1 shows the solid-angle coverage for the detector array. The entire array covered a solid angle of 1.10 sr.

### 4.1.3 Results

As shown in Table 3.2, the expected energy resolution of the $^{12}\text{B}(d,p)$ reaction is on the order of 120 keV. This estimate neglects beam spot size ($\approx 3$ mm) and target thickness (150 $\mu$g/cm$^2$) effects. The beam spot size
Figure 4.3: Excitation energy spectra from the $^{11,12}$B($d,p$) reactions in inverse kinematics. The energy resolution is 250 keV. The black peaks in both spectra correspond states populated in the residual nucleus which lie below the neutron-decay threshold; 3.4 MeV for $^{12}$B and 4.9 MeV for $^{13}$B. The white (hatched) peaks correspond to states which are neutron-unbound. (a) In the $^{11}$B($d,p$) spectrum, the $\Delta E_x = 102$ keV doublet near 2.7 MeV is unresolved. (b) In the $^{12}$B($d,p$) spectrum, the $\Delta E_x = 199$ keV doublet near 3.6 MeV is unresolved. Figure from Ref. [33, Fig. 1].

will have the same effect on any measurement. The target thickness, however, has a pronounced effect on the $Q$-value resolution in inverse kinematics because the heavy ion experiences significant energy loss entering and exiting the target. Therefore, the reported $Q$-value resolution of 250 keV is not surprising. However, this resolution was insufficient to resolve the states at $E_x = 3.482$ and 3.681 MeV ($\Delta E_x = 199$ keV). Therefore the separate angular distributions of these states could not be analyzed, making a determination of the angular momentum transfer impossible. Fig. 4.3 shows the excitation energy spectra for both reactions. The measurement of this reaction was reattempted in order to separate these states using the HELIOS spectrometer as discussed in Chapt. 14.

Although the original aim of this experiment was not realized, the experiment did provide a new measurement that had astrophysical implications. Eq. 4.1 shows the $r$-process path through the light elements. Included in the reaction chain is neutron capture on $^{11}$B (indicated by the underbrace). The $^{11}$B($d,p$)$^{12}$B neutron-transfer reaction is an example of a measurement that can be used to study the $r$-process. Recoil tagging, using particle identification in the $\Delta E-E$ array (discussed in Chapt. 10.2), was used to measure the branching ratio of $^{12}$B decay. The neutron-unbound 3.389 MeV state in $^{12}$B is predominately populated in coincidence with the recoiling $^{12}$B nucleus, corresponding to $\gamma$-decay of $^{12}$B$^\ast$. However, a fraction of the events populating the 3.389 MeV state were measured in coincidence with a $^{11}$B, corresponding to in-flight $n$-decay. The ratio of the yield of these events is related to resonant neutron capture in $^{11}$B which contributes to both the overall rate of the $r$-process [9]. The details of this relationship are discussed in Ref. [33].

$$^{1}\text{H}(n,\gamma)^2\text{H}(n,\gamma)^3\text{H}(d, n)^4\text{He}(t, \gamma)^7\text{Li}(n, \gamma)^6\text{Li}(\alpha, n)^{11}\text{B}(n, \gamma)^{12}\text{B}(\beta^-)^{12}\text{C}(n, \gamma)^{13}\text{C}$$

(4.1)
CHAPTER 4. TWO EXAMPLE MEASUREMENTS

4.2 The $^{132}$Sn($d,p$) Measurement

4.2.1 Introduction

A sophisticated example of a large acceptance array with excellent resolution is the Oak Ridge Rutgers university Barrel Array (ORRUBA) in concert with the Silicon Detector Array (SIDAR) at the Holifield Radioactive Ion Beam Facility (HRIBF) at Oak Ridge National Laboratory. This detector array was used to study the ($d,p$) neutron transfer reaction on the neutron-rich, doubly-magic ($\textit{N} = 82, \textit{Z} = 50$) nucleus $^{132}$Sn. The results of this measurement are reported in Refs. [24, 36, 37]; this section summarizes those results.

4.2.2 Experimental Setup

A $^{132}$Sn beam was produced using the isotope separation online (ISOL) technique. The $^{132}$Sn ions were created as fission fragments from protons bombarding a uranium carbide target. The $^{132}$Sn fission fragments were re-accelerated with the HRIBF 25 MeV tandem Van de Graaff accelerator to an energy of 4.78 MeV/u, producing an essentially pure beam. A $100 \mu\text{g/cm}^2$ CD$_2$ target was used, rotated 60$^\circ$ to the beam axis for an effective thickness of 160 $\mu\text{g/cm}^2$. The target was rotated to allow particles emitted near $\theta_{\text{lab}} = 90^\circ$ to be detected.

The ORRUBA detector array, shown in Fig. 4.4, is specifically designed to meet the challenges of measuring inverse kinematics: it has a large spacial coverage and is capable of making high resolution measurements of both energy and angle. The details of the detector array construction are discussed in Ref. [35]. The ORRUBA detector array essentially consists of two rings of detectors positioned forwards and backwards of $\theta_{\text{lab}} = 90^\circ$. 
Laboratory Energy (MeV) vs. Laboratory Angle (deg) for the \( d^{(132}\text{Sn},p)^{133}\text{Sn} \) reaction at 4.78 MeV/\( u \) with ORRUBA. The simulation includes elastic scattering of protons, deuterons, and \( ^{12}\text{C} \). Analytical calculations have been plotted over the simulated results using the axes of the original figure. The calculations are color-coded to match Fig. 4.7. At \( \theta_{\text{lab}} = 120^\circ \left( \theta_{\text{cm}} = 22.9^\circ \right) \) the kinematic compression coefficient is \( \Delta E_{\text{lab}}/\Delta E_{\text{cm}} = 0.34 \). Annotated figure taken from Ref. \[35\].

For the \( d^{(132}\text{Sn},p)^{133}\text{Sn} \) measurement, the upstream ring of detectors consisted of single-layer position sensitive detectors; the downstream ring was made up of \( \Delta E-E \) telescopes, with the residual \( E \) detectors also being position sensitive. In this configuration, the detector array provides angular resolution of \(< 0.5^\circ \), position resolution of 0.5 mm FWHM, and (intrinsic) energy resolution of \(< 60 \text{ keV FWHM} \). Fig. 4.5 shows a simulated spectrum of the \( d^{(132}\text{Sn},p)^{133}\text{Sn} \) based on these parameters. The results of the simulation are in agreement with the analytic calculations of Fig. 3.3.

### 4.2.3 Results

The ground-state and three excited states at \( E_x = 0.845, 1.363, \) and 2005 MeV were identified in this measurement. The state at \( E_x = 1.363 \pm 0.031 \text{ MeV} \) was previously unobserved. The \( E_{\text{lab}} \) versus \( \theta_{\text{lab}} \) proton spectrum produced is shown in Fig. 4.6. The analytic calculations of Fig. 3.3 have been plotted over the data (with re-calculated excitation energies). Table 3.2 shows that, neglecting target thickness effects, the \( Q \)-value resolution should be, at best, 133 keV FWHM. Fig. 4.7 shows the measured \( Q \)-value spectrum which has an energy resolution of over 300 keV FWHM. Angular distributions were measured for the two lowest levels. The ground state showed \( \ell_n = 3 \) character, consistent with the \( 2f_{7/2} \) assignment; and the first-excited state at at \( E_x = 0.845 \) had an angular distribution characteristic of an \( \ell_n = 1 \) transfer, corresponding to the \( 3p_{3/2} \) orbital.

Other reports are available in the literature of neutron transfer reactions in the \( A = 130 \) region using ORRUBA. An early proof-of-concept experiment was carried out using the lampshade SIDAR array and a proto-
Figure 4.6: (color online) Measured $E_{\text{lab}}$ vs. $\theta_{\text{lab}}$ spectrum for the $d(^{132}\text{Sn},p)^{133}\text{Sn}$ reaction at 4.78 MeV/u with ORRUBA. Analytical calculations have been (roughly) plotted over the results, showing good agreement. The axes of the calculations plot are shown. Annotated figure taken from Ref. [36].

The typical form of the ORRUBA array to study the $^{124}\text{Sn}(d,p)$ reaction in inverse kinematics [47]. This measurement had a reported $Q$-value resolution of 200 keV FWHM. Additional $(d,p)$ studies have been carried out using the ORRUBA detector and other neutron-rich isotopes—$^{131}\text{Sn}$ [48] and $^{134}\text{Te}$ [37]—both have excitation energy spectra with resolution on the order of 200–300 keV FWHM. This collection of results provide a consistent description of the performance characteristics of this detector array. In order to improve on the results obtained with ORRUBA, a detector system is needed which can avoid or suppress the effects of resolution degradation due to the covariance of measured quantities (kinematic broadening).

Figure 4.7: (color online) $Q$-value spectrum from the $d(^{132}\text{Sn},p)^{133}\text{Sn}$ reaction at 4.78 MeV/u with ORRUBA. Measured at $\theta_{\text{cm}} = 54^\circ$. The $Q$-value resolution is 300 keV FWHM. Figure taken from Ref. [24].
Chapter 5

The HELIOS Concept

The HELical Orbit Spectrometer (HELIOS) offers a new way of studying reactions in inverse kinematics that has several advantages over the detection methods mentioned in Chapt. 4. The conceptual principle of HELIOS is introduced in Refs. [49, 50]; the proposed design and performance characteristics are laid out in Refs. [51, 52], and the technical realization and the details of the experimental commissioning are presented in Ref. [30]. The goal of this chapter is to summarize, derive, and expand on the key mathematical concepts presented in these references.

Schematically, HELIOS is based on a large-bore superconducting solenoid, as shown in an engineering model in Fig. 5.1. Accelerated heavy-ion beams enter the solenoid along the magnetic axis, passing through a hollow detector array. The beam then intercepts a “light-ion” target, also on the magnetic axis. In the configuration shown in the figure, charged reaction products ejected rearward in the laboratory frame—that is, $\theta_{\text{lab}} > 90^\circ$—move in helical orbits to the detector array. Beam-like recoils are kinematically focused forward in a narrow cone and intercepted by a detector array for identifying heavy ions.

5.1 Solenoid Kinematics

The solenoid used in HELIOS produces what is effectively an uniform axial magnetic field. The technical specifications of the solenoid as well as the features of the magnetic field are discussed in detail in Chapt. 6. When a reaction occurs within such a magnetic field, the trajectories of the reaction products are simplified due to the constraints of cyclotron motion. With the solenoid axis aligned colinearly with the beam axis, ions emitted at the target travel in helical orbits under the influence of the magnetic field and return to the magnetic field axis.
Figure 5.1: Cutaway schematic view of HELIOS in the “(d, p)” configuration. The accelerated beam enters from left. Shown are the light-ion silicon detector array suspended on the upstream alignment ring, rotating target fan, and heavy-recoil detector. Mechanical design by S. Heimsath. 3D rendering by B. J. DiGiovine. This figure also appears in Ref. [30].

5.1.1 Coordinates

The particle trajectories are defined by the orientation of the laboratory velocity $\vec{v}_{\text{lab}}$ relative to the solenoid axis. The symmetry of the solenoid defines a cylindrical coordinate system ($z, \rho, \phi$), with the beam traveling in the $+z$ direction and the azimuthal angle $\phi = 0$ defined relative to “beam right” (the $+x$ axis). Furthermore, the coordinate convention used with HELIOS defines the $+y$ axis as “up” yielding a left-handed coordinate system. With this convention in mind, the laboratory velocity may be written as

$$\vec{v}_{\text{lab}} = v_{\parallel} \hat{z} + v_{\perp} \hat{\rho}$$

with $v_{\parallel}$ and $v_{\perp}$ defined in terms of the polar angle as

$$v_{\perp} \equiv v_{\text{lab}} \sin(\theta_{\text{lab}})$$

$$= v_0 \sin(\theta_{\text{cm}})$$
CHAPTER 5. THE HELIOS CONCEPT

Figure 5.2: Vector diagram of relevant velocities of the mass $m$ ejectile. $v_{\text{lab}} = V_{\text{cm}} + v_0$ with $v_{\text{lab}}$ the ejectile velocity in the laboratory, $V_{\text{cm}}$ the velocity of the center-of-mass, and $v_0$ the ejectile velocity in the center-of-mass. The velocity projections $v_\perp$ and $v_\parallel$ are defined as illustrated.

and

$$v_\parallel \equiv v_{\text{lab}} \cos(\theta_{\text{lab}}) = V_{\text{cm}} + v_0 \cos(\theta_{\text{cm}})$$

as illustrated in Fig. 5.2. In these equations $v_0$ retains its definition from Eq. 3.6.

5.1.2 Cyclotron Motion

The velocity component perpendicular to the solenoid axis $v_\perp$ defines the radius of cyclotron motion

$$r = v_\perp \frac{m}{B q}$$

for a particle of mass $m$ and charge $q$ traveling in a field of strength $B$. A related quantity, the cyclotron period $T_{\text{cyc}}$, is fixed by the mass-to-charge ratio of the particle and the magnetic field strength.

$$T_{\text{cyc}} = \frac{2\pi r}{v_\perp} = \frac{2\pi}{B} \left(\frac{m}{q}\right)$$

The position at which particles return to the solenoid axis varies according to their velocity parallel to the magnetic field $v_\parallel$. As such, HELIOS disperses ions by $v_\parallel$. For a given value of $(m/q)$ and magnetic field, the axial path length (return distance) is given by

$$z_n = v_\parallel (n \times T_{\text{cyc}})$$
CHAPTER 5. THE HELIOS CONCEPT

after executing \(n\) number of orbits. For purposes of nomenclatural clarity, it is useful at this point to introduce the variable \(z_1\), the point at which a particle undergoing a single orbit intercepts the solenoid axis.

5.2 Determining the Center-of-mass Quantities

Particles are detected within HELIOS along the length of a hollow array of position sensitive silicon detectors (PSDs) suspended on the solenoid axis. This detector arrangement represents a fundamental departure from the tradition measurement approach wherein the scattering angle \(\theta_{\text{lab}}\) is measured. When a light ion intercepts an active portion of the silicon detector array, three quantities are measured: the energy \(E\), the time of flight \(t\), and the axial position \(z\). From these measured quantities, the center-of-mass quantities of energy \(E_{\text{cm}}\) and emission angle \(\theta_{\text{cm}}\) are derived.

5.2.1 Excitation Energy

The transformation of the energy of the ejectile in the laboratory frame to the energy in the center-of-mass frame can be reduced to a linear transformation. Substituting Eq. 5.3 into Eq. 3.8, the laboratory energy \(E_{\text{lab}}\) can be rewritten as

\[
E_{\text{lab}} = \frac{1}{2} m \left[ v_0^2 + V_{\text{cm}}^2 + 2 V_{\text{cm}} v_0 \cos(\theta_{\text{cm}}) \right]
\]

\[
= \frac{1}{2} m \left[ v_0^2 + 2 V_{\text{cm}} [v_0 \cos(\theta_{\text{cm}}) + v_0 - V_{\text{cm}}] + V_{\text{cm}}^2 \right]
\]

\[
= \frac{1}{2} m \left( v_0^2 + 2 V_{\text{cm}} v_\parallel - V_{\text{cm}}^2 \right). \tag{5.7}
\]

A given beam energy fixes the value of \(V_{\text{cm}}\) and, given a reaction and transition, \(v_0\) is fixed. Therefore, the measured energy \(E_{\text{lab}}\) depends linearly according to \(v_\parallel\), the laboratory velocity parallel to the beam axis. Assuming the particles are detected on the magnetic axis, this quantity has a value of \(v_\parallel = z_n/(n T_{\text{cyc}})\). Inserting this expression into Eq. 3.8 and rearranging to solve for the center-of-mass energy, defined as \(E_{\text{cm}} = \frac{1}{2} m (v_0)^2\), gives the center-of-mass energy as a linear offset from the laboratory energy.

\[
E_{\text{cm}} = E_{\text{lab}} + \frac{1}{2} m V_{\text{cm}}^2 - \frac{m V_{\text{cm}}}{n T_{\text{cyc}}} z_n \tag{5.8}
\]

The excitation energy \(E_x\) is then derived from the center-of-mass energy by correcting for the recoil mass of the residual nucleus. Substituting Eq. 3.4 into Eq. 3.3 and solving for \(E_x\) yields

\[
E_x = T_{\text{cm}} + Q - E_{\text{cm}} \frac{m + M}{M} \tag{5.9}
\]
5.2.2 Emission Angle

In order to study the angular distributions for transitions to different excited states, it is necessary to calculate the scattering angle in the center-of-mass $\theta_{\text{cm}}$. With reference to Fig. 5.2, the center-of-mass angle is readily obtained using the law of cosines.

Starting with the Eq. 3.7 and solving for $\theta_{\text{cm}}$ yields

$$\theta_{\text{cm}} = \arccos \left( \frac{\nu_0^2 - \nu_0^2 - V_{\text{cm}}^2}{2 \nu_0 V_{\text{cm}}} \right). \tag{5.10}$$

This is the familiar result of two-body kinematics from classical mechanics (cf. Goldstein et al. [53, Eq. 3.109]).

As alluded to earlier, $V_{\text{cm}}$ is fixed by the bombarding energy of the beam. Once the excitation energy is determined using Eq. 5.8, the value of $\nu_0$ is fixed. Thus, for a given transition, the center-of-mass angle is uniquely determined by $\nu_{\text{lab}}$, which is calculated from the measured laboratory energy $E_{\text{lab}}$.

5.3 Advantages

The principles laid out in this chapter describing the HELIOS concept provide a number of advantages over traditional detection techniques.

5.3.1 Particle Identification

The cyclotron period is an especially important quantity because it defines the time of flight for the detected particles and provides particle identification. The time-of-flight is approximately

$$T_{\text{cyc}} = 65.1 \text{ ns} \times \frac{A n}{q B_{\text{cal}}} \tag{5.11}$$

with $A$ the atomic mass number, $n$ is the number of orbits, $q$ in units of $e$, and $B_{\text{cal}}$ in Tesla. Table 5.3.1 gives the cyclotron periods ($n = 1$) for the H and He isotopes with $A \leq 4$ for a variety of field strengths. The minimum separation in ns of the cyclotron periods for these light ions is approximately $32.6/B$, where $B$ is in units of T. With a field strength of $B = 2.0$ T, this separation is 16.3 ns; timing resolution of this magnitude is readily achievable with typical silicon detectors. Thus, by means of measuring the time of flight, HELIOS solves the problem of particle identification of light-ion reaction products at low energy. The particle identification is valid up to ambiguities of $A n/q$. 
CHAPTER 5. THE HELIOS CONCEPT

<table>
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<tr>
<th>Ion</th>
<th>$T_{\text{cyc}}$ (ns)</th>
<th>$B/n = 1$T</th>
<th>2T</th>
<th>3T</th>
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<tr>
<td>$p$</td>
<td>65.6</td>
<td>32.8</td>
<td>21.9</td>
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</tr>
<tr>
<td>$^3\text{He}$</td>
<td>98.2</td>
<td>49.1</td>
<td>32.7</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>130.3</td>
<td>65.2</td>
<td>43.4</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>131.2</td>
<td>65.6</td>
<td>43.7</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>196.4</td>
<td>98.2</td>
<td>65.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Cyclotron periods $T_{\text{cyc}}$ for typical reaction products for a variety of field strengths.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_1/A_1$ (MeV/u)</th>
<th>$B$ (T)</th>
<th>$\theta_{\text{lab}}$ (deg)</th>
<th>$\delta z$</th>
<th>$\delta E_{\text{lab}}$</th>
<th>$\delta E_1$</th>
<th>$\Sigma_{\text{quad}}$</th>
<th>$\Sigma_{\text{covar}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d(^{28}\text{Si},p)^{29}\text{Si}$</td>
<td>6.02</td>
<td>2.00</td>
<td>157.9</td>
<td>11</td>
<td>40</td>
<td>13</td>
<td>43</td>
<td>32</td>
</tr>
<tr>
<td>$d(^{12}\text{B},p)^{13}\text{B}$</td>
<td>6.24</td>
<td>1.04</td>
<td>155.2</td>
<td>5</td>
<td>40</td>
<td>11</td>
<td>42</td>
<td>37</td>
</tr>
<tr>
<td>$d(^{132}\text{Sn},p)^{133}\text{Sn}$</td>
<td>4.78</td>
<td>2.00</td>
<td>149.0</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td>42</td>
<td>31</td>
</tr>
<tr>
<td>$d(^{124}\text{Sn},^3\text{He})^{123}\text{In}$</td>
<td>13.00</td>
<td>2.73</td>
<td>21.5</td>
<td>45</td>
<td>40</td>
<td>38</td>
<td>71</td>
<td>38</td>
</tr>
<tr>
<td>$p(^{77}\text{Kr},d)^{76}\text{Kr}$</td>
<td>30.00</td>
<td>2.00</td>
<td>15.1</td>
<td>24</td>
<td>40</td>
<td>51</td>
<td>70</td>
<td>54</td>
</tr>
</tbody>
</table>

Table 5.2: Calculated contributions to the uncertainty of $E_{\text{cm}}$ for measurements using HELIOS. For experiments that have already been preformed, the actual magnetic field value is used; for experiments that have not been run—$^{132}\text{Sn}$ and $^{77}\text{Kr}$—a 2.00 T field is assumed. Values are calculated in keV FWHM for $\theta_{\text{cm}} = 10^\circ$. The quadratic sum $\Sigma_{\text{quad}}$ and the sum including the covariant term $\Sigma_{\text{covar}}$ are given.

5.3.2 Q-value Resolution

The most significant feature of the HELIOS concept is that it avoids the problem of kinematic compression introduced in § 3.2.1. Instead of detecting ions at fixed laboratory angles, the ions transported by the magnetic field within HELIOS are detected at fixed axial positions. Eq. 5.8 gives a linear relationship between the measured quantities $E_{\text{lab}}$ and $z$ and the derived quantity $E_{\text{cm}}$. This linear relationship between the laboratory quantities and the center-of-mass system is the key to the enhanced $Q$-value resolution of HELIOS. Table 5.2 shows the calculated $Q$-value resolution for a number of reactions based on the HELIOS concept. Following the discussion in § 3.2.2, the contributions are calculated assuming energy measurement uncertainties of $\delta E_{\text{lab}} = 40$ keV FWHM, and an uncertainty in the beam energy of 0.14%. In addition, the contribution from the uncertainty of the position is based on $\delta z = 1.0$ mm FWHM. As can be seen in the table, the dominant contribution to the $Q$-value resolution is the intrinsic detector resolution, as was the case in normal kinematics.
Energy Separation

At a fixed z position, kinematic loci are separated by their laboratory energy \( \Delta \text{E}_{\text{lab}} = (E_{\text{lab}} - E'_{\text{lab}}) \). This quantity is related to the center-of-mass energy separation by

\[
\Delta \text{E}_{\text{lab}} = \left[ E_{\text{cm}} - \frac{1}{2} m V_{\text{cm}}^2 + \frac{m V_{\text{cm}}}{T_{\text{cyc}}} z_1 \right] - \left[ E'_{\text{cm}} - \frac{1}{2} m V_{\text{cm}}^2 + \frac{m V_{\text{cm}}}{T_{\text{cyc}}} z'_1 \right] \tag{5.12}
\]

in which the second term in the bracketed expressions is constant for a given bombarding energy and the last term is constant for fixed z. Taking the difference, these terms drop out and we are left with

\[
\Delta \text{E}_{\text{lab}} = \Delta \text{E}_{\text{cm}}. \tag{5.13}
\]

Comparing this expression to Eq 3.12, one sees the “compression” coefficient leading the \( \Delta \text{E}_{\text{cm}} \) term is identically equal to 1 and is independent of scattering angle. Hence, the separation between kinematic groups corresponding to different energy levels in the residual nucleus at a fixed z is the same as the energy spacing in the center-of-mass frame. Put another way, with measurements made with HELIOS, there is no kinematic compression. This result arises from the constant slope of the kinematic loci, given by

\[
\frac{\partial \text{E}_{\text{lab}}}{\partial z} = \frac{m V_{\text{cm}}}{n T_{\text{cyc}}} \tag{5.14}
\]

which is independent of z and the reaction Q-value and hence is free from kinematic compression. Substituting Eqs. 3.5 and 5.5 into this expression one also sees the slope is independent of the ejectile mass \( m \).

\[
\frac{\partial \text{E}_{\text{lab}}}{\partial z} = \frac{q B}{2 \pi n} \sqrt{\frac{2 E_1}{m_1 (m_1 + m_2)}} \left( \frac{m_1}{A_1} \frac{1}{A_1 + A_2} \right) \approx 2.21 \text{ keV/mm} \times \frac{q B}{n} \sqrt{\frac{E_1}{A_1 (A_1 + A_2)}} \tag{5.15}
\]

where \( A_1 \) and \( A_2 \) are the atomic mass numbers of the beam and the target nuclei, respectively; \( q \) in units of e, and \( B \) in Tesla. In the limit of inverse kinematics \( m_1 \gg m_2 \), the term in the parentheses is equal to one. Table 5.3.2 gives slopes for the H and He isotopes for a number of field strengths and bombarding energies.

The effect of kinematic compression as exhibited by HELIOS is illustrated in Fig. 5.3. In practice, this effect is utilized by correcting for the slope of the kinematic curves by means of Eq. 5.8 and projecting onto the energy axis. The intercept of these lines corresponds to the energy at \( z = 0 \), or equivalently \( \theta_{\text{lab}} = 90^\circ \), and is given by

\[
\text{E}_{\text{lab}} \bigg|_{z=0} = E_{\text{cm}} - \frac{1}{2} m V_{\text{cm}}^2, \tag{5.16}
\]
Table 5.3: Typical slopes of $E_{lab}$ vs. $z$ with HELIOS. Values are given in keV/mm for various bombarding energies and parameter values $q = 1, 2$ e; $\beta = 1, 2, 3$ T; and $n = 1, 2$.

This value is independent of the number of orbits and the resultant spectrum is then independent of $z$.

**Position Dispersion**

As mentioned in §3.2.2, kinematic broadening arises from the finite resolution of any realistic detector system when measuring covariant coordinates. The degree of covariance between the measured quantities determines the contribution of the individual resolutions to the final $Q$-value resolution. The effect of covariance is intimately related to the effect of kinematic compression, which alters the laboratory spacing of center-of-mass energy levels. The combined effect of covariance and kinematic compression can be gauged by the relative spacing of the individual covariant quantities—that is, the laboratory spacing divided by the resolution.

An alternate approach to determining the center-of-mass quantities is based on the separation in $z$ of kinematic loci at a fixed energy $E_{lab}$. In a manner similar to the procedure describe above, the kinematic groups can be rotated by a linear transformation and projected onto the position axis to produce a spectrum independent of $E_{lab}$. Rewriting Eq. 5.8 to solve for $z_1$ yields

$$z_1 = \frac{T_{cyc}}{m V_{cm}} \left( E_{lab} - E_{cm} + \frac{1}{2} m V_{cm}^2 \right).$$ (5.17)

The position of the $z$-intercept of these lines, corresponding to $E_{lab} = 0$, is then given by

$$z \bigg|_{E_{lab}=0} = \frac{-m V_{cm}}{T_{cyc}} \left( E_{cm} - \frac{1}{2} m V_{cm}^2 \right).$$ (5.18)

and the separation between states at a fixed $E_{lab}$ is then given by

$$\Delta z_1 = \frac{T_{cyc}}{m V_{cm}} \Delta E_{cm}$$

$$= \frac{2\pi}{\beta q V_{cm}} \Delta E_{cm}.$$ (5.19)
Here the “compression coefficient” $\Delta z/\Delta E_{cm}$ has units of mm/MeV, and is more accurately described as the dispersion in $z$. So whereas the dispersion in $E_{lab}$ is fixed, the dispersion in $z$ is set by the parameters of the experiment, namely the field strength $\mathcal{B}$ and the bombarding energy of the beam. For example, this coefficient has a value of 98.7 mm/MeV for the $d^{(28}\text{Si},p)^{29}\text{Si}$ reaction at 6.02 MeV/u and a field strength of 2.00 T.

In order to assess the possible advantage of using the $z$ dispersion to derive a $Q$-value spectrum, the relative spacing in $z$ must be compared to the relative spacing in $E$. The uncertainty in $z$, i.e. the width of the lines projected onto the $z$ axis, is calculated by adding in quadrature the intrinsic position resolution and the contribution of the energy resolution, based on the slope, as follows

$$
(\delta z_{cm})^2 = \left( \frac{\partial z_{cm}}{\partial z} \right)^2 (\delta z)^2 + \left( \frac{\partial z_{cm}}{\partial E_{lab}} \right)^2 (\delta E_{lab})^2
$$

$$
= (\delta z)^2 + \left( \frac{2\pi}{\mathcal{B} q V_{cm}} \right)^2 (\delta E_{lab})^2
$$

(5.20)

where $z_{cm}$ is given in analogy to $E_{cm}$ in Eq. 3.14. The relative resolution, or resolving power, based on the

---

Figure 5.3: Calculated kinematic groups from the $d^{(28}\text{Si},p)$ reaction at 6.0 MeV/u. The seven strongest transitions populated in the reaction are plotted. The measured pair $E$ and $\theta_{lab}$ (left) exhibit kinematic compression, while the pair $E$ and $z$ measured in a $\mathcal{B} = 2.00 \text{T}$ field do not. Lines of constant laboratory angle are plotted in gray every 15°.
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50

Reaction $E_i/A_i$ $\beta$ $\Delta z/\Delta E_{cm}$ $\delta z_{cm}$ $\Delta z/\delta z_{cm}$ $\Delta E_{cm}/\delta E$

<table>
<thead>
<tr>
<th>Reaction</th>
<th>(MeV/amu)</th>
<th>(T)</th>
<th>(mm/MeV)</th>
<th>(mm)</th>
<th>—</th>
<th>—</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d(^{28}\text{Si},p) ^{29}\text{Si}$</td>
<td>6.02</td>
<td>2.00</td>
<td>98.7</td>
<td>4.1</td>
<td>24.2</td>
<td>24.2</td>
</tr>
<tr>
<td>$d(^{12}\text{B},p) ^{13}\text{B}$</td>
<td>6.24</td>
<td>1.04</td>
<td>202.8</td>
<td>8.2</td>
<td>24.8</td>
<td>24.8</td>
</tr>
<tr>
<td>$d(^{132}\text{Sn},p) ^{133}\text{Sn}$</td>
<td>4.78</td>
<td>2.00</td>
<td>105.1</td>
<td>4.3</td>
<td>24.3</td>
<td>24.3</td>
</tr>
<tr>
<td>$d(^{124}\text{Sn},^3\text{He}) ^{123}\text{In}$</td>
<td>13.00</td>
<td>2.73</td>
<td>23.3</td>
<td>1.4</td>
<td>17.1</td>
<td>16.6</td>
</tr>
<tr>
<td>$p(^{77}\text{Kr},d) ^{76}\text{Kr}$</td>
<td>30.00</td>
<td>2.00</td>
<td>41.8</td>
<td>1.9</td>
<td>21.5</td>
<td>21.3</td>
</tr>
</tbody>
</table>

Table 5.4: Compression coefficients based on dispersion in $z$ using HELIOS. The compression in $z$ is given as the inverse-slopess of the kinematic loci, $\Delta z/\Delta E_{cm}$. The two rightmost columns give the relative resolving power, where a higher number corresponds to higher resolving power.

Position and the energy are related by

$$\frac{\Delta z}{\delta z_{cm}} = \frac{2\pi}{\beta q V_{cm}} \left( \frac{\Delta E_{cm}}{\delta E} \right).$$  \hspace{1cm} (5.21)

Table 5.4 shows the $z$-dispersion compression coefficients and resolving power for a number of different reactions performed with HELIOS, using intrinsic resolution values $\delta z = 1.0$ mm FWHM and $\delta E_{lab} = 40$ keV FWHM. The relative resolution is calculated for states assuming $\Delta E_{cm} = 1$ MeV. As Table 5.4 shows, any possible advantage gained by the $z$-dispersion tends to be canceled out by the line width $\delta z_{cm}$ and there is no improvement in the resolving power. Utilizing the variable dispersion may allow one to compensate for kinematic broadening under certain circumstances. However, this technique has limited applicability because the field strengths at which the technique is beneficial ($\beta < 1$ T), reduces the acceptance of the spectrometer.

5.3.3 The Acceptance

In a traditional detector scheme, the acceptance or solid angle coverage is defined by the range of angles subtended by the detector $(\theta, \phi)$—in other words, the fraction of a sphere surrounding the target covered by detectors. In the HELIOS detector scheme, the solenoid transports the ions in such a way as to ostensibly redefine the acceptance as the fraction of area covered on a cylinder on the solenoid axis $(z, \phi)$.

Calculating the Solid Angle

The solid angle coverage for an individual detector element in HELIOS is given by

$$\Omega = \int \int \sin(\theta) d\theta d\phi = \Delta \phi \left[ \Delta \cos(\theta) \right] \hspace{1cm} (5.22)$$

$$= [2 \arctan(u_0/2p_0)] \left[ \Delta \cos(\theta) \right]$$
where $w_0$ is the width of the detector and $\rho_0$ is the radius of the detector array.

Given the relation $z_1 = v_0 T_{\text{cyc}} = [v_0 \cos(\theta_{\text{cm}}) + V_{\text{cm}}] T_{\text{cyc}}$, each detector subtends the same range of $\cos(\theta_{\text{cm}})$.

Here $\cos(\theta)$ is given by

$$
\cos(\theta_{\text{cm}}) = \frac{z/t - V_{\text{cm}}}{v_0}
$$

where $t$ is the time of flight, given in Eq. 5.31. The actual range of angles covered in the center-of-mass frame depends on the position of the array. As shown in Eq. 5.19, the dispersion and thus the solid-angle acceptance also depends on the magnetic field and the bombarding energy studied. An increase in the magnetic field decreases the dispersion and thus increases the coverage in center-of-mass angles for a given detector position. Similarly for the bombarding energy.

**Example**

For the ground-state transition in the $d^{(28}\text{Si}, p)^{29}\text{Si}$ reaction at 6 MeV/u with a central magnetic field of 2.0 T, each detector subtends between 2–5° in the center-of-mass frame, depending on its distance from the target. As seen from Fig. 5.5, the range of center-of-mass angles covered for the entire array is 21–42° given the interval covered by the array is between −680 and −340 mm from the target. With these settings, each detector covers an interval of $\Delta \cos(\theta_{\text{cm}}) = 0.028$ and covers an azimuthal range of $\Delta \phi = 0.24 \pi$, giving a solid angle of 0.021 sr per element, and a total solid angle coverage of 0.50 sr for the silicon array in the center-of-mass frame.

**Radial Acceptance**

For particle orbits originating on the solenoid axis, as is the case in HELIOS, the radial excursion goes as $\rho = r[1 - \cos(\varphi)]$ with a maximum of $\rho = 2r$. This radial extreme is related to the laboratory energy by

$$
v_{\text{lab}} \sin(\theta_{\text{lab}}) = \frac{2\pi r}{T_{\text{cyc}}}$$

$$
v_{\text{lab}} = \frac{rq \theta}{m \sin(\theta_{\text{lab}})}$$

$$
E_{\text{lab}} = \frac{1}{2m} \left(\frac{r}{\rho/2q \theta}\right)^2
$$

When $\rho$ is the radius of the solenoid bore, Eq. 5.24 gives the high-energy acceptance cutoff imposed by the size of the solenoid. This limit is plotted with a wide-dashed line in various single-orbit energy versus position spectra (e.g., Figs. 5.5 and 14.1). When $\rho = \rho_0$, the radius of the detector array, the limit imposed by the radial extent of the array is given. However, this theoretical limit is not typically the effective limit. The minimum-radius orbit acceptance of the array is not typically determined by the radial excursion of the orbit, but by the emission angle. For a given target-to-detector separation $\Delta z$, the minimum angle is given by $\theta_{\text{lab}} = \arctan(\rho_0/\Delta z)$. This limit is plotted in various $E_{\text{lab}}$ vs. $z$ histograms, represented by a narrow-dashed
CHAPTER 5. THE HELIOS CONCEPT

Eq. 5.24 may be rewritten in terms of the momentum \( p \) of the ejectile using the relation \( E = p^2/(2m) \). Using this formulation, the axial acceptance can be written in terms of magnetic rigidity\( ^* B r \).

\[
\begin{align*}
    p_{\text{lab}} &= \frac{(\rho/2)q B}{\sin(\theta_{\text{lab}})} \\
    p_{\text{lab}} &= \frac{rq B}{\sin(\theta_{\text{lab}})} \\
    p_{\perp} &= \frac{rq B}{r} \\
    B r &= \frac{p_{\perp}}{q}
\end{align*}
\]

(5.25)

Axial Acceptance

Ref. [52] includes an expression similar to Eq. 5.24 for determining the \( z \)-acceptance of the array.\(^†\) Whereas Eq. 5.24 is based on \( v_{\perp} \), the \( z \)-acceptance is based on \( v_{\parallel} \) as follows

\[
\begin{align*}
    v_{\text{lab}} \cos(\theta_{\text{lab}}) &= \frac{z_1}{T_{\text{cyc}}} \\
    v_{\text{lab}} &= \frac{z_1 q B}{2\pi m \cos(\theta_{\text{lab}})} \\
    E_{\text{lab}} &= \frac{1}{2m} \left( \frac{z_1 q B}{2\pi \cos(\theta_{\text{lab}})} \right)^2
\end{align*}
\]

(5.26)

The maximum axial excursion—which is included here as part of the discussion of the acceptance—requires the consideration of the effect of a finite detector array, which is discussed in the following section. The maximum axial excursion is found by taking the partial derivative of Eq. 5.30 (on page 54). The extremum occurs when the condition given in Eq. 5.27 is satisfied, which assumes \( \rho_0/2r \ll 1 \). This point occurs at approximately a fixed value of \( \theta_{\text{cm}} \) for the energy levels populated in a given reaction. For example, in the \( d^{(28}\text{Si},p)^{29}\text{Si} \) reaction illustrated in Fig 5.5, the maximum longitudinal excursion occurs at approximately 8° in the center-of-mass frame.

\[
\begin{align*}
    \frac{\partial}{\partial \theta_{\text{cm}}} z &= -v_0 \sin(\theta_{\text{cm}}) T_{\text{cyc}} + \frac{\rho_0}{v_0 \sin^2(\theta_{\text{cm}})} [V_{\text{cm}} \cos(\theta_{\text{cm}}) + v_0] \\
    \frac{\partial}{\partial \theta_{\text{cm}}} z &= 0 \quad \text{critical point condition} \\
    v_0 \sin(\theta_{\text{cm}}) T_{\text{cyc}} &= \frac{\rho_0}{v_0 \sin^2(\theta_{\text{cm}})} [V_{\text{cm}} \cos(\theta_{\text{cm}}) + v_0]
\end{align*}
\]

(5.27)

Note that when \( \rho_0 = 0 \), \( z \) reaches a maximum at the expected value of \( z_1 = -v_0 \sin(\theta_{\text{cm}}) T_{\text{cyc}} \).

\(^*\)The magnetic rigidity is typically written as \( B \rho \) ("bee-rho") where \( \rho \) is the radius of curvature of the particle orbit, defined here as \( r \).

\(^†\)The equation given in Ref. [52, Eq. 11] contains a typographical error. The "cos" terms are meant to be "sin."
5.4 Considerations

5.4.1 The Effect of a Finite Detector

So far in this chapter, the equations are valid for any charged particle moving in a homogeneous magnetic field—trajectories begin and end on the magnetic field axis and the time of flight of the orbits is equal to $T_{\text{cyc}}$. However, in order to be detected, particles must be intercepted by the detector array before returning to the solenoid axis, thereby truncating the trajectory of the particle. This process is illustrated in Fig. 5.4. The axis intercept $z_1$ is slightly greater in magnitude than the position where the particle is detected $z$ due to the finite transverse extent of any given detector array. Similarly, the time of flight $t$ is reduced from $T_{\text{cyc}}$.

Formulation

The effect of a finite detector array may be formulated in terms of its effect on the path length of a particle orbit. The transverse path length of a particle orbit may be written as $l_0 = \varphi r$ where $\varphi$ is the angle of rotation about the orbit’s center. For a detector of “radius” $\rho_0$, the path length of a single orbit $l_0$ is given by

$$l_0 = r(2\pi - \varphi_0)$$
$$= r \left[ 2\pi - 2\arcsin \left( \frac{\rho_0}{2r} \right) \right]$$
$$\approx 2\pi r - \rho_0 \quad (5.28)$$

where $\varphi_0$ is the angle of the cyclotron orbit excluded by the detector array. For a detector array with a non-circular (polygonal) cross section, the radius $\rho_0$ is a function of the azimuthal coordinate $\phi$. In such case, a fixed $z$-position corresponds to a finite range of radii. This leads to a variation in energy for a given transition.
at a fixed \( z \), which is an example of kinematic broadening. Thus, the detected position is given by

\[
\begin{align*}
    z &= \nu_0 t \\
    &= \nu_0 \left( \frac{\rho_0}{\nu_\perp} \right) \\
    &= (v_0 \cos(\theta_{cm}) + V_{cm}) \frac{2\pi r - 2 \arcsin \left( \frac{\rho_0}{2r} \right)}{v_0 \sin(\theta_{cm})}.
\end{align*}
\]  

(5.29)

As shown in Eq. 5.28, the transverse path length of the cyclotron orbit is reduced from \( 2\pi r \) by approximately \( \rho_0 \), the radius of the detector array. This approximation is valid when the quantity \( \rho_0/2r \) is sufficiently small (\( \ll 1 \)). Due to the acceptance limit imposed by a realistic detector array (see § 5.3.3), this condition is always true for detected particles. Given this approximation, the relationship between the detected position of the ions \( z \) and the axis intercept \( z_n \) is given by

\[
\begin{align*}
    z &\approx (v_0 \cos(\theta_{cm}) + V_{cm}) \frac{2\pi r - \rho_0}{v_0 \sin(\theta_{cm})} \\
    &= v_{lab} \cos(\theta_{lab}) \left( T_{cyc} - \frac{\rho_0}{v_{lab} \sin(\theta_{lab})} \right) \\
    &= z_n - \frac{\rho_0}{\tan(\theta_{lab})}.
\end{align*}
\]  

(5.30)

Therefore, the deviation from the zero-radius detector limit is exaggerated for shallow emission angles with respect to the magnetic field axis, corresponding to smaller helical orbit radii. The time of flight \( t \) is reduced from the cyclotron period \( T_{cyc} \) in a similar fashion

\[
\begin{align*}
    t &= T_{cyc} - \frac{2r}{\nu_\perp} \arcsin \left( \frac{\rho_0/2r}{r} \right) \\
    &\approx T_{cyc} - \frac{\rho_0}{v_0 \sin(\theta_{cm})}.
\end{align*}
\]  

(5.31)

In the rearward hemisphere (\( \theta_{lab} > 90^\circ \)), the effect of a finite detector array manifests itself in the appearance of “knees” in the kinematic loci. Fig. 5.5(a) shows an analytic calculation of proton energies and center-of-mass angles vs. position for the \( d^{(28}\text{Si},p)^{29}\text{Si} \) reaction. The calculation assumes a cylindrical detector array of radius \( \rho_0 = 11.4 \) mm and an ideal solenoid with a uniform field of 2.00 T. The effect of a non-zero radius detector array is illustrated by comparing the dotted lines in Fig. 5.5 with the solid lines corresponding to the ground-state transition.

In the forward hemisphere (\( \theta_{lab} < 90^\circ \)), the effect of a finite detector array is the same, insofar as the detected position \( z \) is reduced in magnitude from the axis intercept \( z_n \) as given in Eq. 5.30. However, the velocity \( \nu_0 \) and the slope \( \partial E_{lab}/\partial z \) have opposite sign, leading to a single-valued function of \( E_{lab} \) vs. \( z \) at low energy. In
CHAPTER 5. THE HELIOS CONCEPT

Table 5.5: Calculated contribution of $\delta t$ to the uncertainty in $E_{cm}$. Contributions are given in keV FWHM and are tabulated for several values of $\delta t$, holding $\delta E_{lab}$ and $\delta \theta_{lab}$ fixed. The values are calculated at $\theta_{cm} = 10^\circ$.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_1/A_1$ (MeV/u)</th>
<th>$\mathcal{B}$ (T)</th>
<th>$\delta t$ (ns)</th>
<th>$\delta z$</th>
<th>$\delta E_{lab}$</th>
<th>$\delta t$</th>
<th>$\delta E_{cm}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d^{(28}\text{Si},p)^{29}\text{Si}$</td>
<td>6.02</td>
<td>2.00</td>
<td>5.00</td>
<td>11</td>
<td>40</td>
<td>1,280</td>
<td>1,280</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.00</td>
<td>11</td>
<td>40</td>
<td>511</td>
<td>512</td>
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<td></td>
<td></td>
<td>1.00</td>
<td>11</td>
<td>40</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.50</td>
<td>11</td>
<td>40</td>
<td>128</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td>11</td>
<td>40</td>
<td>64</td>
<td>70</td>
</tr>
</tbody>
</table>

The excitation energy can be determined in the region of the knees with a sufficiently fine time resolution. Table 5.5 shows the contribution of the time resolution to the overall $Q$-value resolution. However, with the prototype array discussed in § 7.1.2 having a time resolution of $\delta t = 3.87$ ns, this correction has a prohibitively deleterious effect on the final $Q$-value resolution.
Figure 5.5: Calculated ejectile energies for the $d(^{28}\text{Si},p)^{29}\text{Si}$ and $d(^{124}\text{Sn},^3\text{He})^{123}\text{In}$ reactions. Panels (a) and (b) correspond to $d(^{28}\text{Si},p)^{29}\text{Si}$, and panels (c) and (d) correspond to $d(^{124}\text{Sn},^3\text{He})^{123}\text{In}$. Panels (a) and (c) show the ejectile energy $E_{\text{lab}}$ vs. detected axial position $z$ (relative to the target) for protons ejected in the rearward hemisphere and helions ejected in the forward hemisphere, respectively. In panels (b) and (d), the emission angle of the ejectile in the center-of-mass $\theta_{\text{cm}}$ is plotted against the detected position $z$. Excitation energies were selected such that $\Delta E_{\text{cm}} = 1 \text{ MeV}$. The dotted line in each plot corresponds to the ground-state transition as measured by a detector array of zero radius, illustrating the difference between the axis intercept $z_1$ and the detected position $z$. The upper dashed line running through panels (a) and (c) represents $\theta_{\text{lab}} = 0^\circ$. The upper dashed line in panel (a) corresponds to the limit imposed by the solenoid bore. This limit occurs above the axis range in (c). Both reaction calculations assume an ideal magnetic field of 2.00 T.
5.4.2 Corrections for Relativity

So far in this chapter, the effects of relativity have been ignored. However, with heavy-ion reactions such as $^{132}$Sn$(d,p)$ planned to be measured with HELIOS at beam energies in the GeV-range, it is important to ensure that this premise is valid. Stating the problem in terms of velocities and inertial frames, $v_{\text{lab}}$ is the velocity of the projectile in the (stationary) laboratory frame; the center-of-mass frame is moving with a velocity of $V_{\text{cm}}$ relative to the laboratory frame in the $+z$ direction; and $v_{0}$ is the velocity of the ejectile in the center-of-mass frame.

Transformation Factors

Here, we have a choice of how best to keep the notation both brief and consistent. The velocities $v_{\text{lab}}, v_{0},$ and $V_{\text{cm}}$ have been clearly defined; therefore, the speed parameters may be written as $\beta, \beta_{0}, B_{\text{cm}}$ and the Lorentz factors as $\gamma, \gamma_{0}, \Gamma_{\text{cm}}$. The velocity of the center-of-mass is defined in terms of the speed parameter $B_{\text{cm}} = \frac{V_{\text{cm}}}{c}$ where $c$ is the speed of light and $B_{\text{cm}}$ is given by

$$B_{\text{cm}} = \frac{p_{1}c}{\gamma_{1}m_{1}c^{2} + m_{2}c^{2}} = \sqrt{\frac{E_{1}^{2} + 2m_{1}c^{2}E_{1}}{E_{1} + (m_{1} + m_{2})c^{2}}}.$$ (5.34)

Here the subscripts on $m_{1}$ and $m_{2}$ refer to the masses of the incident beam particle and the target particle, respectively. The momentum $p_{1}$ and total energy $\gamma_{1}m_{1}c^{2}$ of the beam particle are rewritten in terms of the total beam energy $E_{1}$ (cf. Goldstein et al. [53, Eq. 7.98]). The Lorentz factor for the center-of-mass frame is then written as $\Gamma_{\text{cm}} = 1/\sqrt{1 - B_{\text{cm}}^{2}}$.

Energy

With the total energy in the laboratory frame given by $W_{t} = E_{1} + (m_{1} + m_{2})c^{2}$, the total energy in the center-of-mass frame is $W'_{t} = W_{t}/\Gamma_{\text{cm}}$. Here, prime notation refers to quantities in the center-of-mass system. With $W'_{t}$ defined, the total energy of the ejectile in the center-of-mass is given by

$$W' = \frac{(W'_{t})^{2} + (m - M)c^{2}}{2W'_{t}}$$ (5.35)

where $m$ and $M$ are the mass of the ejectile and heavy recoil, respectively. The excitation energy enters Eq. 5.35 implicitly via the heavy ion mass, given $M = M_{0} + E_{x}$. Similarly, the reaction $Q$-value is represented in the particle masses (cf. Eq. 3.2). Finally, the kinetic energy of the ejectile is given as a function of scattering angle.
as

\[ E = W - mc^2 \]
\[ = W' \Gamma_{\text{cm}}(1 + B_{\text{cm}}' \beta_0 \cos(\theta_{\text{cm}})) - mc^2 \]  

(5.36)

**Velocities**

The trajectories within HELIOS are defined by the velocity projections. Using the standard velocity-addition formula based on the Lorentz transformation, the axial velocity may be written as

\[
v_\parallel = \frac{v'_\parallel + V_{\text{cm}}}{1 + v'_\parallel V_{\text{cm}}/c^2} = \frac{v_0 \cos(\theta_{\text{cm}}) + V_{\text{cm}}}{1 + [v_0 \cos(\theta_{\text{cm}})] V_{\text{cm}}/c^2} = \frac{v_0 \cos(\theta_{\text{cm}}) + V_{\text{cm}}}{1 + B_{\text{cm}}' \beta_0 \cos(\theta_{\text{cm}})}
\]  

(5.37)

where \( v'_\parallel \) is the z-projection of the ejectile velocity in the center-of-mass frame. Similarly, the radial velocity may be written as

\[
v_\bot = \frac{v'_\bot}{\Gamma_{\text{cm}}(1 + v'_\parallel V_{\text{cm}}/c^2)} = \frac{v_0 \sin(\theta_{\text{cm}})}{1 \mp B_{\text{cm}}' \beta_0 \cos(\theta_{\text{cm}})}
\]  

(5.38)

With these velocity transformations, the scattering angle in the laboratory \( \theta_{\text{lab}} \) is derived in the usual way

\[
\tan(\theta_{\text{lab}}) = \frac{v_\bot}{v_\parallel} = \frac{v_0 \sin(\theta_{\text{cm}})}{\Gamma_{\text{cm}}[v_0 \cos(\theta_{\text{cm}}) + V_{\text{cm}}]} = \frac{1}{\Gamma_{\text{cm}} \cos(\theta_{\text{cm}}) + K}
\]

(5.39)

Here \( K \) retains its definition as \( V_{\text{cm}}/v_0 \), but may be rewritten as \( B_{\text{cm}}/\beta_0 \) (cf. Goldstein et al. [53, Eq. 7.112], Michalowicz [31, Eq. 3.2]).

**Cyclotron Orbit**

The cyclotron motion is defined by the velocity perpendicular to the magnetic field \( v_\bot \). As such, the relevant Lorentz factor is defined as \( \gamma_\bot = 1/\sqrt{1 - (v_\bot/c)^2} \). The equations of the cyclotron orbit may then be rewritten. Eq. 5.4 becomes

\[
r = v_\bot \frac{\gamma_\bot m}{\beta q}
\]

(5.40)
and Eq. 5.5 becomes

\[ T_{\text{cyc}} = \frac{2\pi}{\mathcal{B}} \left( \frac{\gamma \sqrt{m}}{q} \right) \]  

(5.41)

**Example**

To use a representative example, consider the $^{124}\text{Sn}(d,^3\text{He})$ reaction at 13 MeV/u. At the time of this writing, this reaction has the highest kinetic energy of any approved HELIOS experiment. At the quoted energy, the $^{124}\text{Sn}$ beam ion has a kinetic energy of $T = 1.61$ GeV and a rest mass of $m_0c^2 = 115$ GeV. Even at this comparatively high energy, the collision is still in the classical energy regime of $T \ll m_0c^2$, with $\gamma_0 = 1.01$.

With $K = 1.42$, the energy solution is double-valued in the laboratory with a maximum ejection angle of $\theta_{\text{lab}} = 44.6^\circ$. Taking relativistic kinematics into consideration, the maximum correction to the classically-calculated energy for a given $\theta_{\text{cm}}$ scattering angle is 0.49%. For the high-energy solution, neglecting this correction corresponds to a calculation error of less than 500 keV. However, in the range of energies relevant to the prototype silicon detector array, which has a sensitivity of about 1–12 MeV, this correction has an RMS value of 13 kEV. Therefore, for the purposes of calculating—and simulating—particle trajectories within HELIOS in order to determine an optimal experimental setup, the effects of relativity are negligible.

**5.4.3 Technical Requirements**

It is useful here to briefly review the technical requirements to realize the HELIOS concept; these requirements are discussed in Ref. [52] and summarized in Table 5.6. The solenoid should have a radial parameter $r$ on the order of 1.25 T·m and a length parameter $L$ on the order of 3.5 T·m. Standard “3 T” Magnetic Resonance Imaging (MRI) medical scanners match these requirements. For particle identification, a detector timing resolution of approximately 10 ns is needed to separate particle groups with different values of $A/q$. In order to straighten the knees of kinematic groups, timing resolution below 0.50 ns is required; however, this feature is not necessarily required for successful operation of the spectrometer. In order to take advantage of the $Q$-value resolution provided by the HELIOS measurement approach, certain limits need to be placed on the position and energy resolution of the detector array. Position resolution on the order of 0.5–1.0 mm FWHM is required. Energy resolution on the order of 25–50 keV FWHM is required. The following chapters discuss how the actual performance characteristics of the HELIOS spectrometer compare to these requirements.
### Table 5.6: Nominal performance specifications required to realize the HELIOS concept. These requirements are as outlined in this chapter and in Ref. [52]. The value of $\delta t$ is calculated based on the minimum separation of kinematic groups at $B=3$ T. $\Delta \Omega$ is calculated for one target-to-detector setting for the simulated $^{132}\text{Sn}(d,p)$ reaction discussed in Ref. [52].

<table>
<thead>
<tr>
<th>Solenoid</th>
<th>Detectors</th>
<th>Derived</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B R$</td>
<td>$B L$</td>
<td>$\delta z$</td>
</tr>
<tr>
<td>(T·m)</td>
<td>(T·m)</td>
<td>(mm)</td>
</tr>
<tr>
<td>1.25</td>
<td>3.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Part III

Technical Description
Chapter 6

The Solenoid

The magnet used in HELIOS is a superconducting solenoid from a decommissioned Siemens model OR63 Magnetic Resonance Imaging (MRI) scanner. The solenoid had previously been used as a research MRI at the Max Planck Institute for Biological Cybernetics in Tübingen, Germany. The OR63 solenoid is a prototype model similar to the OR64 production model, going under the trade name 3T MAGNETOM Trio, a whole-body medical diagnostic scanner. When used as medical diagnostic, the MRI scanner produces an RF perturbation to the otherwise homogeneous magnetic field of the solenoid. However, for the purposes of HELIOS, the solenoid field is kept at a fixed value and left unperturbed. The HELIOS solenoid was delivered to Argonne and installed in the then-named General Purpose Area in December, 2006.

Figure 6.1: The “Patient End” of the HELIOS solenoid before and after conversion to a spectrometer; (left) showing MRI scanning hardware, and (right) showing the large adapter flange and alignment “spider”. Photo (left) by B. B. Back, Nov. 2006
CHAPTER 6. THE SOLENOID

6.1 Vacuum Chamber

For use as a nuclear spectrometer, the hardware associated with MRI scanning had to be removed (see Fig. 6.1). Stripped of scanning hardware, the interior diameter of the solenoid bore is 92.5 cm and 234.7 cm in length. Based on sonic measurements, the solenoid wall was determined to be 5.3 mm thick. With the interior hardware removed, the bore-surface of the solenoid was polished to make it suitable as a vacuum chamber. The ends of the solenoid featured square-faced mounting faces for annular field shims. These mounting faces permitted the entire solenoid volume to be converted to a vacuum vessel by sealing the ends of the bore with large-opening aluminum adapter flanges.

Each adapter flange has an opening diameter of 71.12 cm and features twelve 4.45 cm diameter feedthroughs. These feedthroughs are used for a variety of functions as described throughout the following chapters, including detector signals. An additional removable flange reduces the opening to mate with an 20 cm diameter beam-line pipe. This flange features four 16.51 cm diameter feedthroughs. A 20 cm beam pipe is used for HELIOS instead of the ATLAS-standard 10 cm beam pipe to aid in vacuum pumping. All of the hardware in the immediate vicinity of the magnet is constructed from non-magnetic materials—primarily aluminum alloys and 316 stainless steel. A photograph of the HELIOS solenoid, fully converted for use as a spectrometer, appears in Fig. 6.2.
6.2 Field Map

Particle trajectories calculated analytically, such as those presented in Chap. 5, assume a purely axial, uniform magnetic field. Such a field is produced by an “ideal” solenoid of infinite length. However, a realistic solenoid of finite length will produce a magnetic field with systematic non-uniformities. To account for the effects of such non-uniformities, the non-uniformities can be modeled, as in Ref. [51], or the field map of an existing magnet can be used, as in Ref. [52]. In order to assess and characterize the HELIOS solenoid, a map of the magnetic field was made in October, 2007.

6.2.1 Equipment

The components of the magnetic field were measured using an F. W. Bell Model ZOA73-3208 three-axis Hall probe. The probe was read out via an F. W. Bell Series 9900 Gaussmeter. The probe consisted of an aluminum rod, 20 cm long and 0.79 cm in diameter, mounted in a plastic handle with the Hall probe sensors near the tip of the rod (see Fig. 6.3). The Hall plates within the probe had a stated mutual perpendicularity of $\pm 2^\circ$. Both pieces of hardware were used in a previous application.

Measurements were read out passively with the gaussmeter in “master” mode by a purpose-written program using LabVIEW™. Throughout the field measurements the gaussmeter was kept in a region of low magnetic flux ($< 500 \mu T$) while extension cables allowed the probe to be moved throughout the solenoid volume. The gaussmeter had a stated DC resolution accuracy of $\pm 0.035\%$. For example, $0.01 \mu T$ (1 mG) resolution at a field of $300 \mu T$ (3 G) and $100 \mu T$ (10 G) resolution at 3.0 T. The probe was calibrated for probe and circuit offset errors by placing the probe in the gaussmeter’s 80 dB attenuation shielded “zero gauss” chamber.

After considering a variety of probe jig designs, such as an articulating arm on a cart, it was decided to mount a rotating probe jig to the flange faces of the solenoid. This approach was chosen to provide reproducibility in measurement position by having the probe jig mounted to the solenoid and to take advantage of the cylindrical symmetry of the solenoid. In this configuration, the three measured components corresponded to the axial $(\vec{B} \cdot \hat{z})$, radial $(\vec{B} \cdot \hat{\rho})$, and azimuthal $(\vec{B} \cdot \hat{\phi})$ components of the magnetic field.

6.2.2 Alignment

Two parallel rails ran the length of the solenoid axis, allowing the probe to move on linear bearings over a range of $-175.85 \leq z \leq +114.15$ cm, relative to the mechanical center of the solenoid (the solenoid itself covering a range of $\pm 117.35$ cm). The axial rails of the probe jig straddled the mechanical axis of the solenoid such that the probe could travel along a radial path, thus sampling the field along the solenoid axis ($\rho = 0$). Due to the great length of the axial rails, the probe jig had to be structurally reinforced. Trusses were added to the axial rails to increase their rigidity and minimize sagging.
The field probe was oriented with the body of the probe aligned radially, with the tip of the probe pointing outward. With the trusses in place, the probe was aligned to rotate in a circular path, concentric with the rim of the flange faces to within 0.5 mm. The radial and axial rails were graduated every 5 cm ±0.5 mm for linear measurement reproducibility. Each downstream end of the magnet (shown in Fig. 6.1) had screws protruding from the solenoid bore every 10°. These screws were used as reference points (and anchors) for measurements made at different rotation angles.

The design iteration shown in Figs. 6.1 and 6.3 featured two linear bearings connecting the central carriage of the jig to the axial arms. Measurements made with this configuration showed 100+ G variations and discontinuities through central region of the solenoid where the field is expected to be uniform. It was determined that these variation were due to the radial arm of the probe jig pivoting in place. This design was later upgraded to six anchoring points on the axial rails, keeping the radial arm of the jig rigidly perpendicular to the axial rails. The reinforcement of the radial arm eliminated the apparent variations in the axial field.

Within the tip of the probe, the Hall plate sensors had to be aligned to the mechanical axes of the solenoid. The method by which the probe was aligned within the jig is illustrated in Fig. 6.4. The tip of the probe was placed within 1 cm of center of the solenoid. Then the probe was then oriented within the jig to maximize the reading on the “axial” Hall probe.
Figure 6.4: Illustration of the method of alignment of the 3-axis Hall probe. The magnetic field is directed right to left in both figures. The probe is aligned by a) rotating the probe along its radial axis to minimize the reading in the “azimuthal” Hall plate. Next, b) the probe is tilted in the $\rho - z$ plane to minimize the reading in the “radial” Hall plate.

### 6.2.3 Method

Measurements were carried out by rotating the jig with the probe at fixed $z$ and $\rho$ positions, taking a measurement every $10^\circ$. At each $z$ position, this process was repeated at radii every 5 cm between 0–45 cm, inclusive. Thus, each cross section contains 360 data points comprised of 10 concentric circles. A total of 59 such field cross sections were measured every 5 cm over a range of nearly 3 m. The result is a magnetic field map of 21,240 points inside and outside the solenoid volume with a corresponding average measurement lattice spacing of 4.4 cm. An averaged and interpolated version of the field map made up of 410 points appears in Appendix A.

### 6.3 Field Analysis

Fig. 6.5 shows the results of the field mapping. For each combination of $z$ and $\rho$, the points have been averaged over the 36 angular measurements. Every other radius set up to $\rho = 40$ cm has been plotted for clarity. The deviations from uniformity of the magnetic field take the form of systematic variations in the axial and radial field components of the field.
Figure 6.5: Field map of the HELIOS solenoid. The axial and radial field components are plotted with a spline fit to the measured values as a function of axial position for five different radii. The vertical dashed lines indicate the fiducial cylindrical volume.
6.3.1 Gross Structure

Axial

The structure of the magnetic field is consistent with that produced by a 6-coil superconducting solenoid with a 2-coil active shield [54]. This is the structure which is presented schematically in the solenoid service manual. The effect of these coils is evinced by the 6 peaks and 2 prominent troughs in the $\rho = 45$ cm axial field map in Fig. 6.5. The axial field is symmetric to within the precision of the measurements about a point offset -0.5 cm from the mechanical center of the magnet. This symmetry is demonstrated in Fig. 6.6. The “kink” in the axial field at about $z = \pm 95$ cm from the center is consistent with two concentric solenoids with opposite current [54].

On the solenoid axis, the axial field falls to 10% of its central value at approximately $z = \pm 150$ cm from the center of the magnet and to 0.1% at approximately $z = \pm 230$ cm from center. At distances greater than 175 cm from the center of the magnet, the fringe field is well approximated by an inverse-cubic function ($\mathcal{B}_z \approx 1/z^3$). The south pole of the average axial magnetic field corresponds to the “Patient End” of the solenoid; as shown in Fig. 6.1, this end is the downstream end of the solenoid.

Radial

Throughout the solenoid, the radial field component is well approximated by a third-order polynomial function of the cylindrical radius $\rho$.

$$\mathcal{B}_\rho = A + B\rho + C\rho^2 + D\rho^3$$ (6.1)
The radial field reaches a maximum absolute value of 63% of the central field approximately $z = \pm 95$ cm from the center of the magnet at a cylindrical radius of $\rho = 45$ cm, corresponding to the location of the fringe field reducing coils.

**Tangential**

The “tangential” or azimuthal component of the magnetic field ($\vec{B} \cdot \hat{\phi}$) was measured to be less than 1.8% of the total magnetic field and had an average value of 0.32% of the total magnetic field. Fig. 6.7 shows a representative example of the tangential field, measured at $\rho = 15$ cm. Included in the plot are fitted projections of both the axial and radial fields. The results of such fits at all radii indicate that the “azimuthal” Hall plate sensor was $0.07^\circ \pm 0.02^\circ$ away from perpendicular, relative to the “axial” sensor, and $1.31^\circ \pm 0.14^\circ$ away from perpendicular, relative to the “radial” sensor. The residual azimuthal component is systematic and structureless, consistent with zero.

**Uniformity**

The magnetic field is homogeneous and purely axial in a spherical field region about the geometric center of the solenoid, referred to as the Diameter Spherical Volume (DSV). For medical purposes, similar magnets are guaranteed to have a homogeneous region (“Field of View”) 40 cm in diameter. In this region, the measured variation in the magnetic field is less than the stated measurement uncertainty (0.035%) and is consistent with zero. In a 90 cm DSV, which extends to the solenoid bore, the mean absolute field non-uniformity is slightly
A detailed analysis of the field map revealed systematic azimuthal variations in the measured radial field components. An example such variations is shown in Fig. 6.8. Variations of this kind are consistent with an offset between the rotational axis of the field mapping jig and the magnetic field axis. Fig. 6.9 illustrates how these variations may arise. If the probe is rotated along a circular path which is offset from the magnetic field axis, a range of magnetic equipotentials are sampled. The result would be a sinusoidal variation in the measured magnetic field as shown in Fig. 6.8.

Using the relationship illustrated in Fig. 6.9, the radius relative to the magnetic axis $\rho$ can be written in terms of the measured radius $\rho'$ as

$$
\rho = \sqrt{[(\rho' + \delta \rho)\cos(\phi) + x_0]^2 + [(\rho' + \delta \rho)\sin(\phi) + y_0]^2}
$$

where $x_0$ and $y_0$ are the horizontal and vertical alignment offsets, respectively, and $\delta \rho'$ is a linear offset in $\rho'$. 

higher at 0.05%. The absolute field non-uniformity increases to a maximum of 3.1% within a central cylindrical volume 1 m long and 40 cm in radius. Section 12 describes how these characteristics define the fiducial volume of the spectrometer.

### 6.3.2 Fine Structure

Figure 6.8: Measured azimuthal variation in the radial field component at $z = 94.15$ cm, $\rho = 45$ cm. ±0.035% error bars are shown. The data have a fluctuation of ±63 G about the mean. The line is a sine cure fit to the data ($\chi^2$/dof = 1.98).
The linear offset parameter is included because the exact position of the Hall plate sensor within the tip of the probe had to be estimated from non-technical schematics. All of the readings in the “axial” sensor for the measurements at $z = \pm 95$ cm corresponded to a positive radial flux, except for those at $\rho' = 0$, which measured a negative flux. Therefore, those measurements were actually sampling the field at a radius of $-\delta \rho'$. Also, since this feature was absent from the measurements at $\rho' \geq 1$ cm, the value of $\delta \rho'$ must be $< 1$ cm.

When the HELIOS solenoid was installed on the ATLAS beam line, it was aligned such that the beam path was collinear with the mechanical axis of the solenoid. Therefore, an offset between the magnetic field axis and the mechanical axis of the solenoid could have an impact on experiments conducted with HELIOS. To assess any possible offset, the radial field was analyzed at $z = \pm 95$ cm, where the radial field is strongest. In order to have a more statistically significant sample, the field cross sections at those points included an additional 576 measurements, made at radius intervals of 1 cm in the range $0 < \rho < 10$ cm.

To fit the data, the measured radius was related to the sample-point radius by Eq. 6.2. Then, the sample-point radius $\rho$ was fit with a cubic function. For measurements made at $\phi \geq 180^\circ$, the measured radius and field were reflected about the origin to ensure a symmetric fit function. To determine a $\chi^2$ fit value, the average value of the tangential field component (for all measurements) was used as the value of the uncertainty in the magnetic field measurement $\delta \mathcal{B} = 15.4$ G. The results of fitting the radial field at the given cross sections is shown Table 6.3.2 and Fig. 6.10.
Table 6.1: Offset parameters for two different cubic fits to the radial field at $z = \pm 95$ cm. Values are given in mm. For both fit ranges $x_0$ is consistent with zero and $\delta \rho'$ is consistent with the required range of 0–10 mm.

<table>
<thead>
<tr>
<th>$z$</th>
<th>$\rho' \leq 30$ cm</th>
<th>$\rho' \leq 45$ cm</th>
<th>$\rho' \leq 30$ cm</th>
<th>$\rho' \leq 45$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_0$</td>
<td>$y_0$</td>
<td>$\delta \rho'$</td>
<td>$\chi^2$/dof</td>
</tr>
<tr>
<td>$-95.85$</td>
<td>0.26</td>
<td>4.30</td>
<td>-0.08</td>
<td>5.19</td>
</tr>
<tr>
<td>$+94.15$</td>
<td>-0.22</td>
<td>-1.59</td>
<td>0.26</td>
<td>1.38</td>
</tr>
<tr>
<td>Average</td>
<td>0.02</td>
<td>2.71</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.10: Radial field data for the $z = 94.15$ cm field cross section measurement with two cubic fits.
The fits indicate that $x_0 \approx 0$ mm and $y_0 \approx 3$ mm. This may represent an actual offset in the mechanical structure of the solenoid. However, the magnitude of the fitted value of $y_0$ is greater for the measurement at $z = -95$ cm. This measurement point was nearly 1 m away from the point where the probe jig was suspended. It is possible that despite the reinforcement from the trusses, the axial rails of the probe jig were sagging and this is the cause of the vertical offset. The effect of a genuine mechanical offset is discussed in Chapt. 12. Finally, the linear offset is in the range $0 < \delta \rho' < 1$ cm, as required by the flux inversion present only in the measurements at $\rho' = 0$.

Accounting for the eccentric rotation of the rotation of the probe jig effectively eliminates the azimuthal dependence of the radial field component for smaller radii ($\rho' \leq 30$ cm). For larger radii, this correction still leaves a residual variation. However, the residual azimuthal variations in the radial field components are on the order of 0.5%, and scales roughly with the axial field component, which is consistent with a trivial non-perpendicularity of the Hall plate sensors.

### 6.3.3 Determining the Absolute Field

The field mapping was conducted with a current of 363.00 A in the solenoid coils. This value was chosen to achieve a 2.00 T field based on the field-to-current ratio given in the solenoid service manual as shown in Table 6.3.3. A field of 2 T (20,000 G) was selected in order to comply with safety guidelines, specifically, the whole body exposure ceiling limit value (TLV-C) as suggested by the American Conference of Governmental Industrial Hygienists (ACGIH). However, the initial result of the field mapping indicated a central field of 1.9159 T. This measured value corresponds to a different field-to-current ratio, as shown in Table 6.3.3.

This discrepancy introduces a dilemma: either the solenoid’s field response is nonlinear or the field mapping measurements are systematically inaccurate. To address this issue, the magnetic field was measured by a one-dimensional Hall probe at a fixed representative point on the flange face during the process of energizing the solenoid. The results of this measurement are shown in Fig. 6.11. It is clear from this figure that the field response is indeed linear with current. However, it is also clear from the self-consistent nature of the field mapping data that whatever systematic error is present affects all of the data points equally. The conclusion is then that the measured field map quantities are 95.8% of the actual value.
This conclusion is further supported by an analysis of the data from the first reaction measured with HELIOS. For a given reaction and bombarding energy, Eq. 5.15 gives the expected slope of the kinematic loci based on the (uniform) magnetic field. Fig. 6.12 shows data from three different position settings that have been shifted to line up with calculations for the two possible field strengths. The calculations for $B_0 = 1.91$ T clearly do not line up with the data, while the calculations for $B_0 = 2.00$ T are in good agreement.

An additional implication of understanding the field-to-current ratio of the HELIOS solenoid is the maximum field setting. The HELIOS solenoid is rated as a 3.0 T magnet; this field value is achieved with a solenoid current of 543.96 A, as shown in Table 6.3.3. However, the Siemens Model 3600 Magnet Power Supply used to energize the HELIOS solenoid will only output 518.00 A to the solenoid. This corresponds to a maximum possible central field value for the HELIOS solenoid of 2.8568 T.\footnote{The value reported in Ref. [34] (2.7 T) is off by a factor of 95.8%, which is the value based on the field map.} This maximum field value corresponds to a length parameter of $B \rho = 1.32$ T·m and a radial parameter of $B L = 6.70$ T·m, exceeding the requirements put forth in Ref. [52].
Figure 6.12: Determination of the absolute field by slope analysis. Calculated proton energies (fine dashed lines) are overlaid on two analyses of the same data set. The data have been shifted in $z$ to best fit the calculations for $\mathcal{B}_0 = 1.91$ T (top) and $\mathcal{B}_0 = 2.00$ T (bottom). In the top figure, note that the calculations overestimate $E_{\text{lab}}$ at low energy and underestimate $E_{\text{lab}}$ at high energy, indicating a mis-match in slope ($\mathcal{B}$-field). In the lower figure, the data and calculations are aligned, supporting the conclusion that the field strength was indeed $\mathcal{B}_0 = 2.00$ T. In each panel, pairs of vertical lines indicate detector positions and bold dashed curves indicate the (calculated) acceptance limits. The upper limit occurs above the axis limit in the lower figure.
Chapter 7

The Silicon Detector Array

In order to take advantage of the HELIOS concept, the light ion reaction products transported through the HELIOS solenoid must be detected in a slender array close to the beam axis. The radial extent of the array should be on the order of 1 cm. For rearward hemisphere operation, the array must also be hollow to permit the transmission of the beam to the target foil. The silicon detectors which make up the array must be position sensitive along their length with a position resolution on the order of $\delta z = 0.5–1.0$ mm FWHM. In addition, the intrinsic detector energy resolution should be on the order of $\delta E_{\text{lab}} = 25–50$ keV FWHM. This chapter describes characterization and construction of the silicon detector array as they relate to these design requirements.

7.1 The Detectors

7.1.1 Specifications

The prototype array utilizes detectors that were manufactured by Canberra Industries. The detectors were designed for and used in a previous application as a nuclear reaction calorimeter [57, pp. 100–104]. The detectors are position-sensitive passivated implanted planar silicon detectors. The detectors are $12 \text{ mm} \times 56 \text{ mm}$ silicon wafers, $700 \pm 15 \mu\text{m}$ thick$^*$ with active areas $9 \text{ mm} \times 50.5 \text{ mm}$. The particle energy which this detector thickness is capable of stopping is shown in Table 7.1.1 for a number of light ions.

The construction of the detectors follows the standard fabrication method of passivated implanted planar silicon detectors [58]. First the edges of the silicon wafers are passivated by thermal oxidation which, after etching, produces an insulating boundary of SiO. Next the front of each detector (the side shown in Fig. 7.1) is implanted with boron ions to form a $p^+ n$ junction and the back of the detectors is implanted with arsenic to form an $n$-type junction. Then after annealing, aluminum contacts, discussed below, are patterned on either

$^*$Thickness quoted in private communication from the manufacturer.
Table 7.1: Stopping power calculations for the HELIOS silicon detectors. Approximate light ion energies in MeV with stopping ranges of 700 μm for two different incident angles $\theta$ are given. The calculations were carried out using the Monte Carlo simulation program SRIM (Stopping and Range of Ions in Matter), which has reported accuracy of 4.3% [59].

<table>
<thead>
<tr>
<th>Nucleon</th>
<th>Incidence Angle</th>
<th>$\theta = 0^\circ$</th>
<th>$\theta = 45^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>9.9</td>
<td>12.1</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>13.2</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>15.7</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>$^3$He</td>
<td>35.1</td>
<td>42.9</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>39.6</td>
<td>48.7</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.1: Detail of a silicon detector mounted on a modular section of the HELIOS silicon detector array. Dual bonding wires connect each end of the detector to the PC board. On the detector shown, the guard ring is connected to the left-hand side bonding pad. The target-end of the PC board (right-hand side in the figure) features a temperature sensor. Photo by A. H. Wuosmaa, Feb. 2008. This figure also appears in Ref. [30].

side of the detector for electrical connections. The thin layer of boron provides position sensitivity along the length of the detector by resistive division. The dead layer introduced by the ohmic contact on the back of the detector—typically the entrance window—is equivalent to a silicon thickness of < 50 nm. The dead layer on the junction side is < 100 nm silicon equivalent. A typical detector is fully depleted at a reverse-bias voltage of 190 V producing nominal leakage current of 350 nA.

Each detector has three aluminum signal contacts; one covering the back of the detector and one at each end of the front of the detector. The implanted resistive layer provides about 17 kΩ of resistance between the two front contacts. One of the contacts is also connected to a guard ring that separates the active area of the detector from the oxide passivated edges of the detector. This guard ring helps define the electric field along the edges of the detector. The position signals may be read out from the bonding pads at either end of the detector as shown in Fig. 7.1. In addition, the total energy of the detected ions is measured from the contact at back of the detector. With the position signals identified as $X_{\text{far}}$ and $X_{\text{near}}$, with “far” and “near” relative to say, the target, the position on the detector is defined as
\[ X = \frac{L}{2} \left[ 1 + \frac{(X_{\text{far}} - X_{\text{near}})}{E} \right] \]  
(7.1)

with \( X = 0 \) corresponding to an event at the \( X_{\text{near}} \) end of the detector and \( X = L \) corresponding to an event at the \( X_{\text{far}} \) end, where \( L = 50.5 \) mm, the length of the detector.

However, all three signals need not be measured in order determine the position of a detected ion. When the detectors were used in the nuclear calorimeter, only two channels were read out for each detector. The bonding pad connected to the guard ring was grounded and the position was signal was read out only from the contact at the other end of the detector. The ratio of the position signal to the energy signal was then used to determine the position. The position can be determined using any two of the available measured quantities.

\[ X = \frac{L}{2} \left[ 1 + \frac{(X_{\text{far}} - X_{\text{near}})}{(X_{\text{far}} + X_{\text{near}})} \right] \]  
(without \( E \)) \hspace{1cm} (7.2a)
\[ X = L \frac{X_{\text{far}}}{E} \]  
(without \( X_{\text{near}} \)) \hspace{1cm} (7.2b)
\[ X = L \left[ 1 - \frac{X_{\text{near}}}{E} \right] \]  
(without \( X_{\text{far}} \)) \hspace{1cm} (7.2c)

Measuring all three quantities leaves the system over-determined and and has a number of advantages. In the HELIOS configuration, reading out both position signals provides a redundant energy measurement. And as will be explained in Chapt. 9, reading out all three signals also allows for noise rejection.

### 7.1.2 Characterization

The HELIOS PSD detectors were characterized at Western Michigan University using both an \(^{241}\)Am radioactive decay source and the Physics Department’s model EN 6.0 MeV tandem Van de Graaff accelerator. The results of the characterization experiments are reported briefly in Ref. [30]; this section expands on that discussion and details additional measurements. The accelerator tests were carried out over a time period between October, 2006 and March, 2007. Fig. 7.2 shows the testing mount that was used, including the scattering mask that was used to assess the position resolution of the detectors.

#### Position & Energy

The position and energy resolutions were measured by elastic scattering of a proton beam at four different energies from a carbon foil into an individual, masked detector. The detector was positioned at a radius of 105 mm from the beam axis with a proton scattering angle of \( \theta_{\text{lab}} = 60^\circ \) corresponding to normal incidence at
the center of the detector. The experimental setup is shown in Fig. 7.3. The results of these runs are shown in Fig. 7.4. The symmetric U-shaped energy cutoff at low energy is due to an electronic threshold of 112 keV on all three signals, as indicated by the dashed curve in the figure.

Both the position and energy resolutions depend on energy. Fig. 7.5 shows a projection of Fig. 7.4 onto the position axis for the kinematic groups corresponding to \( E_1 = 2 \) and 5 MeV. The position resolution of the detector for these two sets varies between 0.5 and 1.2 mm FWHM. For a given incident particle energy, the position resolution also varies weakly (±6%) along the active length of the detector. As shown in Fig. 7.6, the position resolution at the center of the detector is poorest. Fig. 7.7 is a projection of Fig. 7.4 onto the energy axis for the kinematic group corresponding to the slit located at 30 mm (\( \theta_{\text{lab}} = 60^\circ \)). The detector has a measured energy resolution between 27 and 53 keV FWHM, decreasing with lower energy. Both the position and energy resolutions of the PSDs are consistent with the design requirements suggested in Ref. [52].

**Time**

Additional measurements using \((p,p)\) scattering were made in March 2007 to evaluate the timing response of the detectors. One of the HELIOS silicon detectors was installed at a scattering angle of \( \theta_{\text{lab}} = 45^\circ \) relative to the beam line. A scintillator with a photomultiplier tube was also installed 45° relative to the beam axis, such that the opening angle between the detectors was 90°. The experimental setup is shown in Fig. 7.3. A polyvinyl
CHAPTER 7. THE SILICON DETECTOR ARRAY

Figure 7.3: Scattering chamber setup for the PSD characterization experiments. (left) For the position, energy, and length measurements, a masked detector was placed at $\theta_{\text{lab}} = 60^\circ$ to measure elastic $^{12}\text{C}(p,p)$ scattering. (right) For the timing measurements a scintillator and a silicon detector were placed 90$^\circ$ apart at $\theta_{\text{lab}} = \pm 45^\circ$ to measure elastic $^{1}\text{H}(p,p)$ scattering. Figure from Ref. [60].

Figure 7.4: Characteristic energy versus position spectrum of a HELIOS PSD. Protons were elastically scattered from a $^{12}$C foil into an individual detector at four different beam energies $E_1 = 2, 3, 4, 5$ MeV. The detector was covered by a mask with 0.50 mm wide slits at 10 mm and every 5 mm starting at 20 mm. Elastic scattering from $^{1}\text{H}$ (box) and $^{16}\text{O}$ is also present due to water in the target. The series of points at 5.5 MeV, indicated by the arrow, correspond to $\alpha$ particles from a $^{241}\text{Am}$ calibration source. The dashed curve corresponds to a threshold of 112 keV required for all three signals. This figure appears similar form in Ref. [30].
Figure 7.5: Position resolution of protons from \((p,p)\) at two energies in a HELIOS PSD. Protons are elastically scattered from \(^{12}\text{C}\). (a) For \(E_1 = 5.0\ \text{MeV}\), the average position resolution is 0.532 mm FWHM. (b) For \(E_1 = 2.0\ \text{MeV}\), the position resolution is 1.17 mm FWHM. This figure appears similar form in Ref. [30].
Figure 7.6: Detector resolution as a function of position for scattered protons at 5 MeV. Peak widths measured from the spectrum shown in Fig. 7.5(a). The position resolution varies ±6% over the length of the detector.

Figure 7.7: Energy resolution spectrum of a HELIOS PSD for the slit located at 30 mm. This position corresponds to a proton scattering angle of 60° in the laboratory frame. The peaks near 1 MeV correspond to the protons from ${}^1\text{H}(p,p)$ with an energy resolution of 52.8 keV FWHM. The peaks in the range of 2-5 MeV correspond to protons from ${}^{12}\text{C}(p,p)$ with an energy resolution of 35.5 keV FWHM. The peak near 4.7 MeV (indicated with an arrow) corresponds to protons from ${}^{16}\text{O}(p,p)$. The peak near 5.5 MeV corresponds to $\alpha$ particles from a $^{241}\text{Am}$ calibration source with an energy resolution of 27.1 keV FWHM. This figure appears similar in Ref. [30].
Figure 7.8: Characteristic timing spectrum for a HELIOS PSD. Energy versus TAC signal for protons from \((p,p)\) scattering at four different beam energies \(E_1 = 5, 7.5, 9, 10\text{ MeV}\). The width of the timing locus near 64 ns (arrow) corresponding to coincidence events broadens from 1 ns FWHM to over 3 ns FWHM with decreasing energy.

formal (Formvar®) foil was used for its hydrogen content for \((p,p)\) scattering. With the detector arranged in such a way, coincident elastic scattering events may be detected. A time to amplitude converter (TAC) was used to measure the difference in time-of-flight between protons detected in the scintillator and those in the silicon detector. Fig. 7.8 is a composite of the energy measured in the silicon detector at four different proton beam energies plotted as a function of the TAC signal.

As discussed in Ref. [61], the timing response of a position-sensitive detector varies with both energy and position. The slower rise times of the energy signal both at lower energies and towards the center of the detector lead to a broadening of the timing signal. This effect is present in the data shown in Fig. 7.8. The timing locus near 64 ns (indicated by the arrow) broadens with decreasing energy. Near 5.0 MeV the detectors have a time resolution of approximately 1.11 ns FWHM. The width of the timing signal increases steadily to 3.28 ns FWHM near 2.5 MeV. The timing resolution required for particle identification is only on the order of 10 ns, which these detectors easily meet. The position dependence of the timing response was not assessed in this series of measurements, however, this feature is discussed in Chapt. 9 which covers the calibration of the
Length

A longer PSD was also characterized to assess the possibility of reducing the number of detectors making up the array. Tests were conducted with a Design X2 PSD manufactured by Micron Semiconductor which had an active area $22.2 \times 94.8 \text{ mm}^2$. As with the previous characterization measurements, protons were elastically scattered from a $^{12}\text{C}$ foil at a variety of beam energies. The detector was located at a radius 125 mm away from the beam axis such that protons scattered at $\theta_{\text{lab}} = 60^\circ$ were normally incident on the detector. A mask covered the detector which had 0.5 mm wide slits spaced every 5.0 mm. Fig. 7.9 shows the results of these tests. Whereas the 50.5 mm-long detector has a position resolution of $0.532 \pm 0.031 \text{ mm FWHM at } E_1 = 5 \text{ MeV}$, the 94.8 mm-long detector has a position resolution of $1.370 \pm 0.065 \text{ mm FWHM}$. Furthermore, at energies below about 1.6 MeV the position resolution of the longer detector degrades below 5 mm—which is consistent with the manufacturer’s stated resolution of 5650 $\mu\text{m}$—making it unsuitable for many HELIOS applications. However, the 50.5 mm detectors are suitable for HELIOS, meeting the necessary design requirements.

7.2 The Array

The detector array discussed in this section is a working prototype which was developed to demonstrate the feasibility of the HELIOS concept. Two ostensibly opposing design requirements had to be reconciled in the design of the HELIOS detector array. The first is that the silicon detector array must have a small outer radius such that particles are detected as close as possible to the solenoid axis. A small radius of detection $\rho_0$ reduces the effects discussed in §5.4.1, such as the degree by which particle flight times are reduced from the cyclotron period. The minimum diameter of the detector array would be on the order of the width of the detector, in this case 12 mm.

The second design requirement of the detector array is a large inner diameter. This feature is necessary to permit transmission of the beam through the array to the target. The inner opening of the array must also be large enough to allow for small misalignments in the beam which are encountered during the process of tuning the accelerator. With this consideration in mind, the inner diameter of the detector array should be on the order of a typical tuning collimator; about 10 mm. Combining both of these design features suggests a thin-walled structure with a regular-polygonal cross section.

7.2.1 Construction

Extruded aluminum profiles are well-suited to this application because they are light-weight, rigid, and can be made in a variety of lengths and cross sections. The core of the HELIOS silicon detector array uses a
Figure 7.9: Characteristic energy versus scattering angle plot for a 94.8 mm-long detector. Protons were elastically scattered from a $^{12}$C foil at three different beam energies $E_1 = 1, 4, 5$ MeV. The detector was covered by a mask with 0.50 mm wide slits every 5 mm. Elastic scattering from $^1$H (box) is present due to water in the target (faint loci corresponding to $^{16}$O($p,p$) can also be seen). The series of points at 5.5 MeV (arrow) correspond to $\alpha$ particles from a $^{241}$Am calibration source. At low energy, the position resolution is less than the slit separation; thus the structure due to the slits is not present in the locus near 1 MeV.
portion of an extruded aluminum 80/20 brand T-slotted profile. The T-slotted profiles consist of a hollow core, square in cross section, and four T-slot flanges for mounting hardware (see Fig. 7.10). In the original design of the silicon detector array, the T-slot flanges were to be utilized as the mounting point for the detector array. These flanges were ultimately removed along the entire length of the extrusion.

Using the central core of a T-slotted profile as the mounting surface of the silicon detector array required that the T-slot flanges to be removed. The T-slotted profile was machined down on a lathe to a diameter of 20.32 mm, as shown in Fig. 7.10 (right). The resulting structure is a 680 mm long, 15.96 mm square aluminum rod with a 10 mm diameter central bore which forms the central support of the HELIOS detector array. It was intended from the onset of the array’s design that it would eventually be replaced. Therefore, some of the components of the prototype array were designed to be compatible with an updated array. In this instance, the overall length of the array was designed to ultimately accommodate 10.5 mm-long detectors for the proposed updated array.

The prototype array has six detectors on each side of the array. The detectors are installed on the central support of the array in a modular fashion with each side of the array constituting an individual module. The detectors reside on four printed circuit (PC) boards, each with dimensions of 388 mm × 18 mm. Each detector is affixed to an electrical contact on the PC board with conductive epoxy. The PC boards are assembled with the detectors aligned end-to-end, separated by bonding pads, with an average gap between wafers of 3.0 mm\(^\dagger\). Table 7.2 shows the precise positions of each detector. During the construction of the array, a precision mounting jig was used to aid in the placement of the detectors on the PC boards and to align them to

\(^\dagger\)The value reported in Ref. [30] (2.4 mm) is off by a factor of 4/5, based on a schematic that unintentionally omitted one of the gaps.
### Chapter 7. The Silicon Detector Array

<table>
<thead>
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<td>292.05</td>
<td>294.8</td>
<td>348.05</td>
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Table 7.2: Positions of the silicon detectors as mounted on the silicon detector array. Positions are given in mm relative to the active area of the target-end of Detector 6. These values are based on the engineering schematic of the PC board and have been measured to be accurate to within 200 μm.

better than 200 μm [60]. Each position contact on the detectors are electrically connected to the contacts on the PC board by two aluminum bonding wires. The three signals from the detectors are carried along traces in the PC board to where they are read out from one end of the board using two 34-pin connectors. Each of these connectors attaches to a ribbon cable.

Each of the modular PC boards described above is epoxied to aluminum L-brackets 573 mm long. The L-brackets are mounted to each side of the array support and are held in place with several screws. The target-end of the array support is capped with an attachment for a removable 4-jaw slit which is used in beam tuning. The signals from the slits run along the array in the space provided by the rounded edges of the array support (see Fig. 7.11). On the far end of the array, about 100 mm of the central support is exposed as a clamping surface to hold the array in place. The fully assembled HELIOS detector array is shown in Fig. 7.11. The array has a square cross section 23 mm on a side and is 710 mm long with the active length covering 345 mm.

The aluminum end of the array is held inside a liquid-cooled copper block, shown in Fig. 7.12. The mechanical contact between the array and the cooling block is made with thermoelectric coolers (“Peltier coolers”), which can supply additional cooling. The electrical connections for the Peltier coolers and the cooling lines both occupy their own 4.45 cm feedthrough. Neither system was utilized during the commissioning experiment. However, circulating a chilled ethyl glycol solution allows the array to be cooled to temperatures below 0°C. The temperature of the array is monitored by four small temperature sensors, one at the target-end of each PC board making up the detector array (see Fig. 7.1). Further cooling from the Peltier coolers is possible, but as of this writing, the system needs to be redesigned to utilize the additional cooling.

#### 7.2.2 Alignment

The cooling block is seated inside an insulating fiberglass frame which is fitted to the end of a support tube (see Fig. 7.13). The support tube is part of a linear bearing which allows the detector array to be translated axially within the solenoid volume over a range of about 400 mm. The recent installation of a chain drive on
In order for the transmission of the beam through the array, the array must be aligned with respect to the beam axis to better than 2.5 mm, or 2 mrad, along its length. The collimator at the target-end of the array is used to align front of the array while a removable centering jig is installed at the end of the support tube furthest away from the detector array. These two reference points are aligned to the beam line optically using a surveying telescope. The linear bearing system—and thus the array—are mounted to the alignment ring via a translation stage which provides motion perpendicular to the solenoid axis. The entire alignment ring is suspended by a pivot at the top of the solenoid chamber, occupying one of the 4.45 cm feedthroughs. Two mechanical feedthroughs occupy the positions 120° from vertical (at 4 o’clock and 8 o’clock) allowing the alignment ring to be tilted about the pivot in order to align the array angularly. Iterating these two alignment motions allows the array to be optically aligned to the beam axis with a precision of < 1 mm. With the addition of the array chain drive, the array alignment hardware use 4 of the 12 4.45 cm feedthroughs on the end of the solenoid holding the array.
CHAPTER 7. THE SILICON DETECTOR ARRAY

Figure 7.12: One half of the copper cooling block which clamps the silicon array. As shown, two of the four Peltier coolers are installed, held in place by a fiberglass bracket. The inlet and outlet of the liquid cooling channel are shown at right.

Figure 7.13: Enlargement of Fig. 5.1 detailing the alignment structures.
Chapter 8

Electronics

The pulse signals produced by the silicon detectors in HELIOS are the characteristic charge-collection
pulses produced by semiconductor detectors; a low-amplitude pulse with a fast rise-time and a slow decay.
The data acquisition system requires that the detector pulses be digitized in some way; specifically, the am-
plitude of the pulse is converted into a number stored by the computer system. In order to be digitized, the
detector pulses must be processed. Each detector signal passed through a linear amplifier (preamplifier) be-
fore a shaping amplifier is used to produce a pulse shape suited to digitizing. The signal processing performed
by the shaping amplifier also produces a timing signal which is used to create a logic trigger. Both of these
processes are described in this chapter. The use of position-sensitive detectors in HELIOS—as apposed to seg-
mented detectors—reduces the number of electronics channels that are read out from the array. The reduced
amount of hardware required allows for a streamlined electronics setup described below.

8.1 Patch Boards

Each of the four PC boards which make up the array is connected to an electronics feedthrough by two
94.1 cm-long ribbon cables (see Fig. 7.11). This is the shortest length of cable that allows a full range of motion
for the array. The cables are terminated by 34-pin micro IDC header sockets. This configuration was used
with the prototype array to accommodate, and thus provide compatibility with, 30 signals per board. The
electronics feedthroughs, shown in Fig. 8.1, pass through a slot in a modified feedthrough cap and are held in
place with sealing epoxy.

On the atmosphere side of the feedthrough, stock aluminum card guides are used for mechanical support.
The electronics feedthrough serves as a patch board to distribute the detector signals. Each board has three
rows of 10 LEMO sockets, with each row corresponding to a detector signal: $E$, $X_{far}$, and $X_{far}$. The energy-signal
connectors are mounted on the back side of the patch board so that the LEMO plugs can be inserted and
removed by hand (without using tools). An additional two LEMO connectors are present to read and power the temperature sensor at the target-end of each array PC board.

### 8.2 Signal Processing

#### 8.2.1 Preamplifiers

The detector array signals from the feedthrough patch boards are first processed with a Mesytec MSI-8p preamplifier. Each preamplifier has eight input channels and each PC board has 18 output channels, so each patch board is connected to three preamplifiers, nine in total. Fig. 8.1 shows a photograph of the preamplifier as they are installed and Fig. 8.2 is a connection diagram. The preamplifiers are mounted to the front flange of the solenoid to bring them as close as possible to the feedthrough patch boards. The supports for the preamplifiers are largely made out of material repurposed from the field mapping jig. The connection between patch boards and preamplifiers is made with 40 cm coaxial cables constructed with specialized low-loss transmission RG-174 cables and terminated with LEMO connectors. Such care is taken with shielding the cables and minimizing the cable length in order to preserve shape of the un-amplified signals from the detectors.

The detectors are reverse-biased through the energy contact with the position contacts terminated into 50 Ω. For the commissioning experiment, the detectors were biased using a collection of Ortec Model 210 Detector Control Unit 4-channel high voltage bias supplies. Subsequent experiments utilized an Iseg High Voltage Mpod-mini computer controlled multi-channel bias supply. Both the bias supply and the power supply for the preamplifiers are positioned in a region of lower magnetic field at a distance of about 1.5 m from the flange face of the solenoid. As a result of this significant separation, specialized shielded DB9 connectors had to be used.

*Belden 7805 25 AWG solid-core coaxial wire with tinned copper braided shielding and aluminum foil-polyester tape shielding.*
8.2.2 Shaping Amplifiers

The 8-channel preamplifiers are connected in pairs to 16-channel Mesytec MSCF-16 shaper amplifiers, five in all. The shaping amplifier is of type $CR - (RC)^5$, meaning a single stage of $CR$ differentiation followed by 5 stages of $RC$ integration to produce a nearly Gaussian output waveform. The amplifiers provide gain and shaping time adjustment in 4-channel blocks, while threshold and pole-zero cancellation adjustments are made for each channel. Pole-zero cancellation is an automatic process while setting the threshold of each channel is done manually. Thresholds are set to minimize or eliminate trigger on baseline noise.

In terms of bulk signal processing, the amplifier modules have one input and two outputs, each consisting of a 34-pin header. The incoming and outgoing energy signals are carried on shielded cables (RG-174) to reduce signal noise and because the shape of the signals is important. The shaper output is connected to a peak-sensing analog-to-digital converter (ADC). The timing signal, which is a logic pulse, is output over a
8.3 Triggering

The Mesytec MSCF-16 shaper amplifiers contain timing filter amplifier (TFA) and constant fraction discriminator (CFD) circuitry to produce timing signals. When the discriminator threshold of an individual detector is exceeded, a timing signal, or trigger, is produced. The output signals are of the form of emitter-coupled logic (ECL) signals. The 16-channel ECL timing output from each shaping amplifier module is connected to a level translator. The level translators—typically a LeCroy 4616 or a Phillips 726—are used to convert the ECL input signal to a nuclear instrumentation module (NIM) output signal. Fig. 8.3 shows a typical connection diagram used for producing the event trigger.

The level translators are used as a switchboard to select which signals are included in the event trigger. The individual outputs from each level translator (up to 16 signals) are fed into a logic fan-in/fan-out module,
which acts to produce the logical 'OR' of the timing pulses from each shaping amplifier module. The outputs from each of the five logic fan-in/fan-out modules are ORed together again to produce "Array OR" trigger signal.

The time of flight measurement is generated based on the Array OR trigger. The master RF signal used to drive the accelerator resonators (a sine wave) is put through a level discriminator to generate narrow, square pulses with a period equal to 82 ns. This RF timing signal serves as the start signal on a time-to-amplitude converter (TAC). A delayed version of the Array OR signal is then used as the stop signal for the TAC.

In the commissioning experiment, only the energy signals—and only those with low noise—were used to establish an event trigger. For measurements involving additional detectors, for instance a recoil detector, the trigger signals from those additional detectors would also enter into the event trigger; either in two-fold coincidence with the array trigger or in fan-out ("singles") mode. Once the event trigger is generated, it is used to start the Scarlet data acquisition system, discussed below. During the acquisition process, the Scarlet system issues an “busy” signal to inhibit the generation of new triggers while acquisition is taking place. For this reason, the event trigger is generated using a coincidence module which requires the absence of an inhibit signal to produces coincidences.

### 8.4 Acquisition

The energy signals from the Mesytec shaper amplifiers modules are each connected to a dedicated Phillips 7166H ("H" for “header”) peak-sensing ADC. The event trigger is used to start the acquisition, gate the ADCs, and is also counted in a scaler to monitor the event rate. Using a gate and delay generator, the event trigger is delayed and broadened to produce a logic signal which overlaps in time with the shaped pulses from the Mesytec shaper amplifiers. The amplitude of the pulse within the gate is is converted by the ADC into a 12-bit number.

For each event trigger, the Scarlet acquisition system issues a series of readout commands to a Weiner CC32 CAMAC controller which communicates with the individual CAMAC modules, such as the ADCs. Each ADC corresponding the detector array is interrogated for the number of channels with successful conversions, or “hits.” Only those channels with hits are read out. Additional ADCs, which may be used to read out signals are read out sequentially with each channel being read. As this process occurs, the Scarlet system inhibits further events triggers until the current event has been processed. Once complete, the busy signal is cleared and acquisition continues.
Chapter 9

Calibration

The shaper amplifiers used to process the detector signals provide only a rough gain adjustment. Each of the 72 signals that are read out from the detector array must be calibrated using analysis software. In addition, each detector has a timing signal associated with it which also needs to be calibrated. In all, 96 different signals need to be matched with one another. The visualization and analysis of the data is carried out with the program ROOT, which is based on the programming language C++ [62]. The method by which the data are calibrated takes the form of C++ macros which are used to generate functional fits to the data. The calibration constants determined by the fits are stored in calibration files which are read in and applied during the sorting of the data.

9.1 Position

The measured position as formulated in Eq. 7.1 assumes that the two position signals $X_{\text{far}}$ and $X_{\text{near}}$ are gain-matched. This relation further assumes that the sum of the two position signals are in turn gain-matched to the measured energy $E$. The latter consideration may be circumvented by use of Eq. 7.2a.

9.1.1 Gain-matching

In an un-calibrated detector, the following relation holds true.

$$a(E) = b(X_{\text{near}}) + c(X_{\text{far}})$$

(9.1)

The most fundamental level of calibration of the detector array is the determination of the constants $a$, $b$, and $c$ for each given detector. The first step of this process is the relative, mutual calibration, or gain-matching, of the two position signals. If the position signals are not gain matched, then the derived position is distorted. In
CHAPTER 9. CALIBRATION

Figure 9.1: Polynomial fit to a profile of a histogram of \( X_{\text{far}} \) vs. \( X_{\text{near}} \) for an individual detector. In this example, the slope of the linear fit to the data (dashed line) is -0.951, meaning gain of the \( X_{\text{far}} \) signal is 95.1% that of the \( X_{\text{near}} \) signal.

In the case when the calculation of the position includes all three signals (Eq. 7.1), the uncalibrated position will be compressed by a factor of \((b + c)/2\). If instead, only two signals are used to calculate the position (Eq. 7.2), the uncalibrated position will be compressed and skewed in a non-linear fashion. Fig. 9.1 shows the method of determining the constants \( b \) and \( c \), discussed below.

When properly gain-matched, the position signals are negatively correlated with a correlation coefficient of \(-1\). The first step of the calibration procedure is to measure the correlation coefficient of the uncalibrated detector. This coefficient is determined by plotting the position signals in a two-dimensional histogram with one position as the abscissa and the other as the ordinate. Measuring the slope of this histogram at a fixed energy gives the correlation coefficient. This can be accomplished by measuring a spectrum of known, fixed energies, for instance the \( \alpha \) particles from the decay of a radioactive decay source.

The fits shown in this chapter are made using the TProfile class available in ROOT [62]. A “profile” of a two-dimensional histogram (scatter-plot) is made by plotting the mean value of the \( Y \)-coordinate at each \( X \)-position. Fig. 9.1 shows the profile of the \( X_{\text{far}} \) vs. \( X_{\text{near}} \) histogram for a specific detector. For a clear demonstration of this technique, an example has been selected where only one energy level is present, that of the 3.27 MeV \( \alpha \)-decay of \(^{148}\text{Gd}\). The figure shows a quadratic fit to the distribution of counts with an additional linear fit connecting the intercepts of the quadratic fit. Higher-order polynomial terms do not improve the quality of the fit. The relationship between the two detector signals is clearly not best described by a linear function. This is due to an effect known as the “ballistic deficit,” discussed below.
Using this dual-fit technique, the slope of the linear fit provides the gain-matching coefficient very accurately. The slope parameter is applied to the position signals such that the dispersion of either $X_{\text{far}}$ or $X_{\text{near}}$ is increased while the other is left unscaled. This corresponds to setting either $b$ or $c$ in Eq.9.1 equal to 1 and assigning the other a value $>1$. For example, the slope of the linear fit in Fig. 9.1 is $-0.951$, which means the $X_{\text{near}}$ signal is unscaled ($b = 1$) and the $X_{\text{far}}$ signal is scaled by $1/0.951 = 1.051 = c$.

### 9.2 Noise Rejection

Once the two position signals have been gain-matched, the energy signal can be gain-matched to the position signals by fitting a profile of a plot of $E$ vs. $(X_{\text{far}} + X_{\text{near}})$. As was the case with the gain-matching of the positions signals, this step of the calibration is based on a linear fit of the above-mentioned plot. Fig. 9.2(a) shows such a plot after the gain-matching parameter has been applied. It is interesting to note most of the points lie along a sharply-defined locus corresponding to $E = (X_{\text{far}} + X_{\text{near}})$. This pronounced correlation between all three detector signals is accomplished simply by gain-matching all three signals—no higher-order correction have been applied. However, not all of the points exhibit proper correlation. The loci of points in the region indicated by the arrow in Fig. 9.2(a) do not lie along the same line.

Fig. 9.2(b) shows a projection of Fig. 9.2(a) along the line $E = (X_{\text{far}} + X_{\text{near}})$. From the area under the peaks, 91.5% of the points lie along the line $E = (X_{\text{far}} + X_{\text{near}})$. The remaining 8.5% of the points are spurious. The width of the main peak corresponds to the intrinsic detector resolution. This peak has been aligned with zero using a new calibration constant $d$, which is applied as follows $E - (X_{\text{far}} + X_{\text{near}}) + d$. Aligning the $E - (X_{\text{far}} + X_{\text{near}})$ spectra this way allows the entire array to be gated with a single parameter. Fig. 9.2(c) shows a plot of $E$ vs. $X$ which includes all points. The spurious points form a ridge at the edge of the detector, indicated by the arrow. Fig. 9.2(d) shows a plot of $E$ vs. $X$ for the same data set which has been gated on Fig. 9.2(b) to exclude points outside of the range $|E - (X_{\text{far}} + X_{\text{near}})| < 30$. Comparing Fig. 9.2(c) to Fig. 9.2(d) shows that the points along the edge of the detector have been eliminated.

### 9.3 Energy

The energy of the detector is calibrated by measuring a known energy spectrum. The peaks in the spectrum are found using the T Spectrum class within ROOT [63]. In the example shown in Fig. 9.3, 7 known peaks are identified in the $\alpha$-decay chain of $^{228}\text{Th}$. The T Spectrum class provides the channel numbers of the peaks, which are then mapped to their corresponding energy values with a linear fit. In this example, the ADC channel number is converted to energy in MeV with the relation $E = (E_0 + 1386)/523$, where $E_0$ is the uncalibrated energy. To preserve the relation $E = (X_{\text{far}} + X_{\text{near}})$, these same calibration constants can be applied to the
Figure 9.2: Method of rejection spurious counts by gating $E - (X_{\text{far}} + X_{\text{near}})$. (a) Plot of $E$ vs. $(X_{\text{far}} + X_{\text{near}})$ for an individual detector. The loci of counts that do not lie on the line $E = (X_{\text{far}} + X_{\text{near}})$, indicated by the arrow, are spurious. (b) Projection of panel (a) along the line $E = (X_{\text{far}} + X_{\text{near}})$. The peak corresponding to the “good” events have been aligned with zero. The width of the peak is equal to the intrinsic resolution of the detector. (c) $E$ vs. $X$ spectrum with no points excluded. The loci of points in the region indicated by the arrow correspond to the smaller peak in panel (c). (d) $E$ vs. $X$ spectrum requiring $E = (X_{\text{far}} + X_{\text{near}})$ which no longer has pathological points at the detector edges.
position signals as $X = (X_0 + f)/g$, where $f$ and $g$ are the offset and slope, respectively, of the linear fit.

In all, the basic calibration of the three individual detector signals can be accounted for with five calibration constants per detector: two parameters to gain-match the position and energy signals; one parameter to align the projection of $E - (X_{far} + X_{near})$; and two parameters to calibrate the energy. In practice, these calibration constants are stored in a single calibration file, 24-lines long with 5 elements per line (plus an indexing element). The varying subtleties of each detector (such as measured background, noise, and resolution) preclude the automated generation of these constants and mandate that this process be carried out manually. Such a calibration file can be generated for a set of data in a few hours.

### 9.4 Ballistic Deficit

Given a finite pulse-shaping time constant in a shaping amplifier, a slower rise time of a preamplifier signal will lead to an effective attenuation of the resulting shaped-pulse signals. In other words, the signals with slower rise times are measured to have a lower energy. The amount by which the amplitude of the shaped signal is reduced from what would be attained with an infinite shaping time is known as the ballistic deficit [64]. If the rise time of the preamplifier signal is the same for all signals, this effect is uniform for all pulses and can be accounted for with the linear energy calibration technique discussed above. However, if there is a variation in rise time of the preamplifier signals, the amplitude of the shaped energy signal will be dependent on rise time.
Figure 9.4: Charge collection times for a typical PSD calculated at a variety of detection positions. The charge collection times—and therefore the preamplifier signal rise times—are greatest at the center of the detector. Adapted from Ref. [65, Fig. 4]

### 9.4.1 Formalism

Kalbitzer and Melzer [65] show that the charge collection at the energy contact of a PSD is described by the equation

\[
Q_E(x, t) = (-2Q_0/\pi) \sum_{n=1}^{\infty} n^{-1} \sin[n\pi(x/L)] \times [1 - \cos(n\pi)] \left[1 - e^{-n^2\pi^2 t/(RC)}\right]
\]  

(9.2)

where \(Q_0\) is the charge of the incident particle; \(R\) is the resistance of the implanted resistive strip; and \(C\) is the capacitance of the detector junction. Fig. 9.4 shows a plot of this equation, specifically \(Q/Q_0\) vs. \(\pi^2 t/(RC)\), for a variety of detection positions. Eq. 9.2 depends explicitly on \(x/L\) and this dependence is shown clearly in Fig. 9.4. For example, the charge collection time at the energy contact to go from 10\%\(Q_0\) to 90\%\(Q_0\) is 1.6\times longer at \(x/L = 0.5\) compared with \(x/L = 0.1\). This position dependence of the charge collection time translates to a position dependence of the rise time of the signal from the preamplifier. The rise times of the position signals can be described in an equivalent fashion.

The effect of the ballistic deficit can be clearly seen in Figs. 9.1 and 9.5 by deviation between the measured data and the dashed lines corresponding the no ballistic deficit. The detector characteristics shown in these plots are very similar to those shown in Ref. [66, Fig. 2], which presents computational modeling of ballistic
deficit. To first order, the effect of ballistic deficit can be ignored. For example, close examination of Figs. 13.4 and 13.5, as reported in Lighthall et al. [30], reveals that this effect has not been corrected for. However, neglecting this effect reduces the effective detector energy resolution. If an energy level is averaged over the length of a detector element, the position-dependence of the energy will lead to a decrease in the resolution of the energy. In addition, given the relationship in Eq. 9.1, the position signals also exhibit ballistic deficit. The result is that the measured position is artificially compressed about the central region of the detector.

### 9.4.2 Compensation

There are a number of approaches for correcting for the ballistic deficit causing the position dependence of the energy. Perhaps the most straightforward approach begins with an even-order polynomial fit of the X-projection of the energy. Fig. 9.5 shows such a for an individual detector. With a fit in hand, the ballistic deficit can be corrected using the equation

\[
E = E_0 - \sum_{i=1}^{2n} a_i x^i
\]

where \( E_0 \) is the uncorrected energy. Given Eq. 9.1, once a detector has been gain-matched, the same fit may be used to correct the ballistic deficit in both the position signals and the energy. The position signals are scaled by a factor of \( E/E_0 \). However, it is important to note that the effect of ballistic deficit is also energy-dependent. Although this feature is not represented in the simplified expression of Eq. 9.2, it is empirically true. Therefore caution must be used when applying this technique; it is suitable for optimizing a particular energy level, but a full correction must utilize energy dependence.

### 9.5 Time-of-flight

Two energy-dependent effects contribute to the timing signal being skewed. The first arises from kinematics; shallow orbits intercept the array at earlier times; this effect is sharply energy-dependent and decreases the measured flight times on the order of about 2 ns. However, the contribution of this effect is generally less than the time resolution of the detector at the energy which it occurs. The second effect is related to the ballistic deficit and is on the order of the time resolution at low energy.

Fig. 9.6 shows an uncalibrated energy versus time plot for a typical detector. The structural details of this histogram are effectively identical to those reported in Ref. [61, Fig. 9], which discusses the timing response of PSDs in general. The apparent skewing towards longer flight times at low energy arises from the use of a constant fraction discriminator to produce the timing signals. The timing signals produced by a CFD are susceptible to rise time walk. As discussed in the previous section, the pulse shapes (rise times) are position
CHAPTER 9. CALIBRATION

Figure 9.5: Polynomial fit to the projection of a $E$ vs. $X$ histogram for a detector exhibiting ballistic deficit. The dashed line corresponds to an ideal detector with no ballistic deficit. Compare to Fig. 9.1.

As can be seen in Fig. 9.6, the time walk feature only occurs at lower energy; thus a fit is needed that is only applied at low energy. The solution employed in calibrating the HELIOS data uses piecewise quadratic fit to the low-energy timing structure. The fit is generated by an iterating macro which takes as its input the rough coordinates of an individual timing peak in $E$-$T$ space. For example, in Fig. 9.6 the single-orbit proton peak is roughly located in a box 400–1000 channels in $T$ and 400–3000 channels in $E$. The macro takes the profile of this box and fits the profile with a quadratic function. Then macro then iterates the endpoint of the fit until the critical point of the quadratic function corresponds to the endpoint of the fit. The result is shown in Fig. 9.7.

Once the piecewise fit has been calculated, the time walk is corrected with the following

$$
\forall T_0 < \left( \frac{-p_1}{2p_2} \right), \quad T = T_0 - p_2 \times \left( E + \frac{p_1}{2p_2} \right)^2
$$

(9.4)

where $p_n$ is the $x^n$ coefficients of the quadratic fit and $T_0$ is the uncorrected time. The resulting time spectrum is independent of energy, to within a possible residual linear offset.

Finally, the timing response of the detectors is also position dependent. The form of which is nearly identical to ballistic deficit and can be corrected for using the same method described above for correcting the position-dependence of the energy. Fig. 9.8 shows the time versus position spectra for an individual detector before and after calibration. In the uncalibrated spectrum, the timing response is slower in the center of the detector (corresponding to lower channel numbers). In the calibrated spectrum, an even-order polynomial fit
has been used to remove the position dependence and the channel number has been calibrated to time in ns.

Figure 9.6: Uncalibrated timing spectrum for a typical detector showing characteristic time walk. Increasing times correspond to decreasing channel number with a dispersion of approximately $-18$ ns per channel number. The prominent locus near channel 700 corresponds to single-orbit protons and the locus near channel 100 corresponds to double orbits.
Figure 9.7: Piecewise quadratic fit to a time profile to correct time walk. The Y-profile of the histogram in Fig.9.6 is plotted. The endpoint of the fit (channel 1422) corresponds to the critical point of the fit. The distribution beyond the critical point ($E > 1422$) is roughly constant, i.e., linear.

Figure 9.8: Time vs. Position spectrum before (left) and after (right) calibration for an individual detector. An even-order polynomial fit has been used to correct the position dependence. The range of the y-axis in both panels corresponds to 82 ns, but is reversed on the left due to the dispersion of $-18$ ns per channel.
Chapter 10

Heavy Ion Detection

10.1 Kinematics

In HELIOS, light ions are detected at essentially a fixed radius—\( \rho_0 \) the radius of the detector array—over a range of \( z \). However, this detection scheme is not typically suitable for the heavy ion reaction products which are emitted over a narrow range of angles and due to their rigidity, only go through a fraction of the helical orbit. Instead, a detector is placed at a fixed \( z \), measuring heavy ion reaction products over a range of \( \rho \). The kinematics of the light ion reaction product as they relate to the silicon detector array have been discussed in Chapt. 5. This chapter discusses the relevant kinematics of the heavy ion reaction product and a number of heavy ion detector approaches. The acceptance of a given detector system is related to the radial excursion and angle of rotation of the heavy ion products, discussed below.

10.1.1 Excursion

The radial excursion \( \rho \) of a particle orbit is expressed in term of the angle of rotation through the orbit of the particle. Here let us define two angles of rotation, as illustrated in Fig. 10.1. The position of the particle in the transverse plane of the solenoid, relative to the fixed solenoid axis, can be written as \( (x, y) \) or \( (\rho, \phi) \), where \( \phi \) is the angle of rotation. These are the same coordinates defined in § 5.1.1. For a given particle orbit, the angle of rotation about the orbit's center is written as \( \varphi \). In this notation, the path length of an orbit is written as \( l_0 = \varphi r \) and the radial excursion is given as a function of orbit-centric rotation angle.
Figure 10.1: Illustration of the relationship between the orbit-centric coordinates \((r, \varphi)\) and the array-centric coordinates \((\rho, \phi)\) or \((x, y)\), as viewed in the transverse plane of the solenoid.

\[
\rho = \sqrt{y^2 + x^2} \\
= \sqrt{r^2 \sin^2(\varphi) + (r - r \cos(\varphi))} \\
= \sqrt{r^2 \sin^2(\varphi) + r^2 + r^2 \cos^2(\varphi) - 2r^2 \cos(\varphi)} \\
= \sqrt{2r^2 - 2r^2 \cos(\varphi)} \\
= r \sqrt{2 - 2 \cos(\varphi)}
\]  

(10.1)

With the definition given in Eq. 10.1, \(\rho(0) = 0\) corresponds to the beginning of the orbit and the maximum excursion is given by \(\rho(\pi) = 2r\). This equation is plotted as a function of \(z\) in Figs. 11.4 and 12.4.

10.1.2 Rotation

The angle \(\varphi\) can be written as a function of the axial position \(z\)

\[
\varphi = \frac{t}{T_{\text{cyc}}} 2\pi \\
= t \omega_c \\
= \frac{z}{v_0} \omega_c
\]  

(10.2)
where $\omega_c$ is the cyclotron frequency of the ion. This expression can be rewritten as a function of the emission angle by using Eq. 5.3.

With reference to Fig. 10.1, the angle of rotation relative to the solenoid axis $\phi$ can be written in terms of the orbit-centric rotation angle $\varphi$ and orbit radius $r$. In formulating this relationship, one encounters a half-angle identity which simplifies the relationship as follows:

$$
\phi = \arctan \left( \frac{y}{x} \right)
$$

$$
\tan(\phi) = \frac{r \sin(\varphi)}{r - r \cos(\varphi)} \quad \text{half-angle identity}
$$

(10.3)

Substituting Eq. 10.2 into Eqs. 10.1 and 10.3 allows the calculation of the radial excursion $\rho$ and rotation angle $\phi$ as a function of the axial position.

### 10.2 Silicon Array

The heavy ion detector array used in Refs. [34, 67] is shown in Fig. 10.2. This is also the same heavy recoil array used in non-solenoidal (traditional) detector schemes as in Ref. [33]. The array consists of four sets of two quadrant detectors arrayed in a $\Delta E$-$E$ configuration, discussed below. The silicon detectors are Design QQQ1 single area pad detectors manufactured by Micron Semiconductor. The typical thickness of the $\Delta E$ detector is in the range of 75–500 $\mu$m, depending on the reaction being studied, and the thickness of the $E$ detector is typically in the range of 500–1500 $\mu$m. The PC board on which the detectors are mounted covers 90°, while the silicon detectors themselves cover 82°. The active area of the silicon detectors has an inner radius of 9.0 mm and an outer radius of 50.0 mm, for a total active area of 1731.0 mm$^2$ per detector. For the measurements reported in Refs [34, 67], the heavy recoil array was mounted to the center of the downstream alignment ring, at a fixed position of $z = +1,032$ mm.

#### 10.2.1 Principle

The heavy ion is identified by an energy loss measurement. The front detectors (visible in the figure) are selected to be thin compared to the stopping range of the incident particles. With this configuration, the incident particles do not deposit their total energy in the $\Delta E$ detector. Instead, in incident particles are transmitted through the $\Delta E$ detector, losing a fraction of their total energy. The residual energy of the incident particle is then deposited in the $E$ detector which is selected to be thick enough to stop the incident particles. The expression that describes the energy loss is known as the Bethe formula; at low energy (non-relativistic velocities), the
equation takes the form

\[- \frac{dE}{dz} = 4\pi e^4 n \frac{Z^2}{mv^2} \left( \ln \frac{2mv^2}{I} \right) \]

Bethe formula \hspace{1cm} (10.4)

where \( n \) is the electron density of the absorber; \( m, v, \) and \( Z \) are the mass, velocity, and atomic number of the incident particle, respectively; and \( I \) is the average ionization potential of the absorber—in this case, the silicon detector. When \( I \) is given as a function of \( Z \), the atomic number of the absorber material, Eq. 10.4 is referred to as the Bethe-Bloch formula. The parameter \( I \) is typically determined experimentally for each element or detector type. The Bethe formula may be rewritten without a loss of generality as

\[ \frac{dE}{dz} = C_1 \frac{mZ^2}{E} \ln \frac{E}{I} \]

where the parameters relating to the detector material have been replace by the constants \( C_1 \) and \( C_2 \) \cite{64}.

Due to the quasi-elastic nature of direction reactions, the heavy ion reaction products corresponding to different transitions have nearly the same energy. However, for a narrow range of incident particle energy,
Eq. 10.5 shows that the energy loss in fixed thickness of the transmission detector ($\Delta E$) scales roughly with $mq^2$, which allows for the separation of reaction products. Fig. 10.3 the characteristic heavy ion particle identification spectrum from Ref. [33] which plots the energy loss in the $\Delta E$ detector versus the residual energy in the $E$ detector. The figure shows that the reaction products from $d(^{12}B,p)^{13}B$ have been separated, with the dashed ellipse indicating the loci corresponding to $^{12}B$ and the solid ellipse indicating $^{13}B$. The presence of $^{12}B$ in the spectrum is due to the in-flight $n$-decay of the recoiling $^{13}B$ nucleus. Although this spectrum was obtained in a “traditional” detector setup, the detection principle is the same and the resulting spectrum is the same as those obtained with HELIOS [34].

### 10.2.2 Alignment

Each of the recoil detectors covers an azimuthal range of $\Delta \phi = 0.46 \pi$, while the range covered by each detector in the silicon detector array is $\Delta \phi = 0.24 \pi$. A corollary of this configuration is that each detector array has rotationally symmetric dead areas separated by $90^\circ$; this dead area is $0.26 \pi$ wide for the silicon detector array and $0.14 \pi$ wide for the heavy recoil detector array. This relationship is displayed in Fig 10.4 which shows the simulated detector yields for both arrays. In this figure, the arrays have been aligned such that active area of each array is centered relative to one another. However, in this configuration it is not guaranteed that a particle detected in the active area of one array will have its corresponding reaction product intercept an active area of the other array. Therefore, the detector arrays must be aligned azimuthally so that coincident events may be detected.

The description of particle orbits discussed in this chapter applies to both the light ion ejectile and the
Figure 10.4: (color online) Simulated detector array yields illustrating the azimuthal coverage of each array. The silicon detector array (black) is in its normal orientation with the top of the detector array centered at 90° (vertical). The heavy recoil detector (gray/red) has been rotated so that the center of each detector is aligned with the corresponding detector element on the silicon detector array to illustrate the azimuthal coverage of each array.

heavy ion recoil. In an experimental setup, the heavy recoil array would be rotated azimuthally (see Fig. 10.2) to maximize its measured yield and thus maximize the measured coincidences between the arrays. This procedure of determining the optimal phase-alignment of the detector arrays can be accomplished with simulations or analytic calculation. Take as an example the $^{12}\text{B}(d,p)$ reaction at a bombarding energy of 6.24 MeV/u, a central field of 1.04 T in HELIOS, and a target-to-detector separation of $\Delta z = 369$. The light ion ejectiles—protons, in this case—corresponding to transitions to the 3.48 MeV state in $^{13}\text{B}$ rotate through an average angle of $\Delta \phi = 348°$ ($\Delta \phi = 186°$) before being detected by the silicon detector array. For the purposes of alignment, this angle of rotation can be taken as intercepting the center of a given detector on the silicon detector array. Therefore, the corresponding heavy ion recoil should intercept the center of a detector element (41° from the active edge) on the heavy recoil array for optimal alignment.

If the light ion ejection is taken as being emitted at an angle of $\phi_0$, the heavy recoil is ejected at an angle of $\phi_0 + 180°$ and is detected at an angle of $\phi_0 + 180° - \phi/2$. In this example, the recoiling $^{13}\text{B}$ ions rotate through an angle of $\Delta \phi = 77.1°$ before hitting the heavy recoil detector located at $z = +1032$ mm. This angle of rotation is equivalent to a difference in axial rotation (phase difference) of 41.8°. Therefore, the heavy recoil should be rotated $-0.8° \pm 21°$. For the actual experiment, the array was rotated 15°, which is within this range. The optimal position has such a large range because of the heavy recoil array has about twice the azimuthal coverage of the silicon detector array.
10.3 Ionization Chambers

As the mass of the incident heavy recoil increases, an increasingly thin $\Delta E$ detector is needed in order to allow the transmission for the incident particle. Also, as the thickness of the $\Delta E$ detector decreases, the uniformity of the detector thickness becomes increasingly important. Furthermore, the radiation damage in a silicon detector increases dramatically for heavy ions in the mass range of typical fission fragments [64]. An alternate approach to using semiconductor detectors which is suitable for heavy ion measurements is the use of gas-filled ionization chambers. As of this writing, a new heavy recoil detector array is being characterized for use in HELIOS.

The new heavy recoil detector array consisting of two gas-filled ionization detector components. The arrangement of the new detectors is shown in Fig. 10.5. The first component of the array is a large area parallel-plate avalanche counter (PPAC). This type of detector is based on two closely-spaced parallel electrode plates which are held at high voltage. The gap between the plates is filled with an ionizing gas and the voltage between the plates is selected so that the gas is near breakdown, i.e. the avalanche regime. This detector is ideal for providing timing information and a grid of anode wires provides position information. Since the incident heavy ions fully pass through the PPAC detector, this is the gas-filled analogue of the $\Delta E$ detector. However, in avalanche mode, the energy resolution is only about $\delta E/E = 20\%$.

The second component of the heavy ion detector is a Bragg curve detector which was used in a previous application [68]. Fig. 10.5 shows a photograph of this detector being characterized next to the HELIOS spectrometer. The Bragg curve detector is a ionization chamber which is filled with gas at a sufficient pressure ($\approx 50$ mbar) to stop the incoming heavy ions. The detector has an active area of 24 cm long and 14.4 cm in diameter. The residual energy $E$ of the incident ions is measured as well as the position profile of the energy loss. The energy loss profile is known as the Bragg curve (hence the name of the detector) and position of the peak of the curve allows for $q$-identification of the incident particles.

![Figure 10.5: Gas-filled recoil detectors for HELIOS. (Left) Schematic drawing of the redesigned downstream HELIOS door including the Bragg curve detector (exterior) and a PPAC detector (interior). Drawing by A. Smith. (Right) Magnetic field testing of a Bragg curve detector during an energizing procedure of the HELIOS solenoid.](image-url)
Chapter 11

The Target Fan

The target system within HELIOS consists of three main parts. First, a mounting rail runs the length of the solenoid at the bottom of the vacuum chamber (see Fig. 5.1 on pg. 42). This rail is an 80/20-brand 3.81 cm square extruded aluminum T-slotted profile. The mounting rail is attached to a mechanical feedthrough at the downstream end of the solenoid which allows the rail to rotate under vacuum. The second component of the target system is a support arm connected to the mounting rail by a linear bearing which allows for longitudinal motion of the assembly. The final component of the target system is the modular target fan.

11.1 Targets

The original target fan design, shown on the left in Fig. 11.1, was milled out of a 6.35 mm thick aluminum block. Nine target positions are located along the outer edge of the fan at a radius of 43.50 cm with each position separated by 4°. The spacing of the target positions is fixed by the width of a typical target frame. With this geometry, the restricted rotation of a 3.81 cm square support rail mounted at the bottom of the vacuum chamber limits the number of possible target positions to nine.

In a typical experimental setup, four or five of the target positions are occupied by target foils. For example, in a \((d,p)\) reaction, a number of deuterated polyethylene \((\text{C}_2\text{D}_4)_n\) targets will be loaded into the target fan along with a carbon (foil for background measurements). Various polyethylene foils may be installed for a range of target thicknesses or in case of breakage or damage.

11.1.1 Diagnostics

In addition to target foils, the target fan can also hold a variety of diagnostic equipment including a radioactive decay source (typically \(^{228}\text{Th}\) or \(^{148}\text{Gd}\)) for detector calibration; a Faraday cup and collimator to measure the on-target beam current during tuning; and a silicon detector telescope in a \(\Delta E-E\) configura-
11.1 Acceptance

One face of the target fan has 2.54 mm recesses for mounting target frames—this side of the target frame is shown on the left-hand side of Fig. 11.1; the other face of the target fan has a 1.59 cm aperture countersunk with a 90° chamfer at each target position. This design is implicitly optimized for reactions with $K < 1$ when the light ion ejectiles of interest are emitted in the rearward hemisphere. With the target-foil side of the target fan facing the beam, ejectiles can be emitted at angles which are greater than 9.1° from vertical. The side of the target fan with countersunk apertures is intended to face downstream where the heavy ion reaction products are emitted in a narrow cone much smaller than the aperture chamfer. However, if this side were to face the beam, the edge of the chamfer blocks orbits emitted at angles less than about 22° from vertical. This is consistent with the acceptance limit reported in Ref. [30] and shown in Figs. 13.4 and 13.5.

To address the limited acceptance imposed by the original target fan, an updated design was made from a piece of 1.5 mm sheet metal. This design also features nine “target” positions, separated by 4°. Two of the positions are dedicated to holding Faraday cup and a calibration source (see Fig. 11.1). On the target-foil side of the updated fan, the acceptance limit has been lowered to 1.7° from vertical.
11.2 Positioning

To align different target positions on the target fan with the beam axis, the target fan assembly is rotated about the axis of the support rail. The longitudinal position of the target fan is adjusted by sliding the target assembly parallel to the beam axis by means of a linear bearing. Both of these adjustments can be made while the spectrometer is under vacuum.

11.2.1 Translation

The target-to-array separation distance can be changed by moving the target fan as well as the detector array. The linear bearing supporting the target fan moves along the mounting rail by means of a plastic chain drive which is controlled via a mechanical feedthrough. For the commissioning experiments, the position of the target fan was determined using a 2 m aluminum ruler, measuring the distance between the support arm of the target fan and the flange face of the solenoid bore. The support arm is milled out of a 1.27 cm thick block of aluminum. The interior surface of the support arm is milled down to a thickness of 0.32 cm (see Fig.11.1). This recessed face of the support arm is the reference face for measuring the position of the target fan. A measurement of 1175 mm was defined as the center of the magnet ($z = 0$).

Subsequent measurements utilized a AccuRange model AR1000 Laser Distance Sensor. The laser range finder uses standard RS232 serial interface to communicate with the computer. A simple LabVIEW™ “virtual instrument” (VI) was created to monitor the output from the laser. Initially, the range finder was mounted inside the vacuum chamber on the rail supporting the target fan. However, in this configuration, the laser system fails to operate at a solenoid current of 82 A, corresponding to a central field of 0.45 T. The laser was then moved to the beamline stand upstream of the solenoid, at a distance of 2.5 m from the solenoid where the on-axis fringe field is less than 130 G. A small anti-reflective glass port was installed on one of the 16.51 cm diameter feedthroughs.

Range measurements are made by rotating the target fan into the path of the laser, so that the laser hits the reference plane of the target fan support (now, on the upstream side of the target fan). The position of the target fan was calibrated by zeroing the range finder reading with the target fan at the $z = 0$ position described above. The range finder has a stated accuracy of ±2.5 mm. A reflecting arm has also been attached to the detector array support which can be rotated into the path of the laser beam for distance measurements.

11.2.2 Rotation

The target fan can rotate over a range of about ±18° from vertical. The rotation of the target fan is controlled by a Danaher Motion series CTP3 stepper motor. The motor is connected to the mechanical feedthrough of the target fan mounting rail by a worm gear which is driven by a 1 m drive shaft. The target drive motor is located
The stepper motor is controlled with a National Instruments model PCI-7330 stepper motor controller and an auxiliary manual control interface. The software package NI-Motion™ which accompanied the stepper motor controller provided programing modules which allows basic control of the stepper motor such as move and stop. A high-level user interface, shown in Fig. 11.3, is built upon these commands. The electronic noise produced by the stepper motor power supply interferes with the detector array preamplifiers. Therefore, the stepper motor is de-energized during data acquisition.

The stepper motor has a rear-end drift shaft which is connected to a BEI Industrial Encoders model HMT25 encoder to monitor the position of the target fan. Three bytes are read out from the encoder. First, a 12-bit integer (output over two bytes) indicates the position of the encoder within a given turn, yielding 4096 digits per turn. This is followed by a 4-bit integer (output over one byte), giving the turn number (0 to 15). This corresponds to an encoding resolution of 4096 bits/revolution or 0.088° (1.5 mrad) over 16 rotations.

The stepper motor is capable of moving in increments of 1.8° (200 steps/revolution). Therefore, one step of the stepper motor—an interval of 1.8°—corresponds to 20 or 21 (20.48) encoder digits. As a result, the stepper motor must be operated using live feedback from the encoder. For example, given a specific position that
CHAPTER 11. THE TARGET FAN

Figure 11.3: Screenshots of the program using LabVIEW™ which controls the rotation of the target drive (left). The Manual Control VI displays the output of the encoder and allows simple control of the stepper motor. (right) The Load and Select Targets VI allows the user to store and recall target positions based on the encoder readings. Once position settings are loaded, the user may move the target fan to a specific target by selecting the desired target within the program and issuing the move command. The target drive program then automatically moves the target fan to the correct position using the encoder readings for feedback and adjusting for backlash, if necessary. The program provides a reproducibility (precision) of < 0.5 mm.

needs to be reproduced, the stepper motor is adjusted until the encoder reads the desired position within ±10 encoder digits. Furthermore, to account for the finite spacing between the teeth of the worm and the worm gear (“backlash”), all aligning motions must be performed with the desired position approached from the same direction. The target drive program, shown in Fig. 11.3, accounts for both backlash and the precision of the stepper motor automatically.

The a worm gear which transfers the rotation of the stepper motor to the mounting rail has a ratio of 1:100. With the target foils rotating on a radius of 43.50 cm, a rotation increment of 1.8° at the stepper motor corresponds to a target rotation of 0.018° and a (theoretical) arc length precision of 0.14 mm. However, the effective arc length precision is about 0.5 mm. Each target foil is “sighted in” using an alignment telescope and the manual control interface. The encoder positions of target foil are then saved in the automated target drive controller program (see Fig. 11.3). Once a reference set of target foil position has been stored—since the spacing of the target foils is fixed—the position of the entire target fan assembly can be calibrated by sighting in a single target.
11.3 Further Designs

11.3.1 Luminosity Monitor

In order to make absolute cross section measurements, a precise determination of both the beam current and the target thickness must be made. Due to the complex nature of the accelerator system, fluctuations in beam current are a normal part of operation. In addition, any given target foil will have an inherent uncertainty in its thickness as well as possible variations in thickness. Furthermore, depending on the beam intensity, the isotopic content of the foil may degrade significantly during the course of an experiment.

To monitor the isotopic content of the target foil and the incident beam current, the “luminosity monitor” illustrated in Fig. 11.4 is used [70]. In the downstream configuration shown in the figure, the beam passes through the silicon detector array and intercepts the target foil in the normal fashion. The beam and beam-like heavy recoils are stopped in a Faraday cup which is at the end of a tube to block backscattered particles from being detected by the array. The Faraday cup monitors the beam current which can be used to determine the differential cross section. The luminosity $L$ is related to the differential cross section $d\sigma/d\Omega$ by the equation

$$
\frac{d\sigma}{d\Omega} = \frac{1}{L} \frac{d^2N}{d\Omega dt}
$$

(11.1)

where $N$ is the number of interactions, determined by the count rate in the detector array; and $d\Omega$ is the differential solid angle (determined from geometry, see §5.3.3).

Past the beam stop, a surface barrier silicon detector measures elastically scattered target ions. For ex-
ample, with a deuterated polyethylene target, this detector is used to monitor deuterons from \((d,d)\). In the arrangement shown in Fig. 11.4, the detector is placed 360 mm upstream from the target to measure 3.5 MeV elastically scattered deuterons, corresponding to a scattering angle of \(\theta_{\text{lab}} = 72.5^\circ\). In the Rutherford energy regime, the count rate in the monitor detector can be compared to previously-determined Rutherford scattering cross sections. The ratio of these cross sections can be used to scale the differential cross section measured in the detector array to calculate an absolute cross section. At energies above the threshold for Rutherford scattering, the rate in the surface barrier detector can be used to monitor the isotopic content of the target.

Although not specifically a target, the luminosity monitor was originally installed on the rotating support rail of the target fan. In this configuration, the luminosity monitor rotated with the target fan and could only be used in conjunction with one target position. Later designs utilized auxiliary support rails running the length of the solenoid. With the luminosity monitor mounted independently of the target fan, various target positions could be utilized.

### 11.3.2 Gas Cell

As of this writing, construction is under way of a cryogenic gas cell target which can be put in place of the HELIOS target fan. The cryogenic target fan features one gas cell and six positions for solid target foils and diagnostics. An engineering schematic is shown in Fig. 11.5. The length (thickness) of the gas cell is about 1.5 mm and has an opening angle of 144\(^\circ\). The gas cell is a redesign of a similar gas cell which was used in a previous measurement at Argonne [71]. As of this writing, 2–4 days of beam time have been approved to test the gas cell in HELIOS with the \(^{25}\text{Al}^{(3}\text{He},d)^{28}\text{Si}\) reaction.

![Figure 11.5: HELIOS cryogenic gas cell target fan. Designed and rendering by B. J. DiGiovine.](image)
Chapter 12

Simulations

As mentioned in Chapt. 3 there are important processes that contribute to the overall $Q$-value resolution that are random in nature. These random contributions, which include target thickness effects and detector resolution, are important when assessing the feasibility of using the HELIOS spectrometer to measure a given reaction. Monte Carlo simulations are performed to characterize the realistic response of HELIOS; these simulations appear in Refs. [52] and [30]. The latter incorporates tracking of particles through the actual measured field map of the HELIOS solenoid, and a detector array with dimensions of the actual array. The simulations appearing in these references both use the method described below.

12.1 Method

Three separate programs, all based on C++ code, are used to generate the simulations which model the performance characteristics of HELIOS. The first program is a kinematics program which takes as an input the reaction being studied, the bombarding energy of the beam and the excitation states of interest. Also input are the range of angles over which the reaction product are emitted and the thickness of the target. Using the principles of two-body kinematics as described in Chapt. 3, this program generates initial trajectories of the reaction products. For each trajectory, the program randomly generates the emission angle $\theta_{\text{lab}}$ (within the specified range) and calculates the energies of the reaction products. The results of this step of the simulation are equivalent to the analytic calculations shown in Fig. 3.3. Part of the output of this program is a randomly generated azimuthal emission angle $\phi$ and interaction depth within the target.

The initial trajectories produced by the first program serve as the input for the second program which tracks the particle trajectories through HELIOS. The features that distinguish the particle tracking simulation from analytical calculations are the inclusion of target thickness effects, beam spot size, and realistic detector resolution. Each of these contributions have a random component and are thus best calculated in a Monte Carlo simulation.
Carlo simulation. In addition, tracking the particles through the real magnetic field map, and using realistic detector geometry lend themselves to iterative calculation. To run a simulation, a list of parameters is used to specify the physical dimensions and characteristics of the solenoid and the detector arrays. Most of these parameters remain constant; the ones that are typically changed for each simulation are the magnetic field strength $B_0$ and the position of the detector arrays.

Particle tracking begins in the target. The simulation uses the interaction depth with the target to calculate energy loss and multiple scattering. Then a position within the beam spot is randomly generated and the new starting trajectory—a position and velocity vector—is passed to the tracking simulation. At each point in tracking the particle trajectory $\vec{r}_n$, the value of the magnetic field is interpolated from the field map $B(\vec{r}_n)$, where $n$ is the iteration step. Interpolation algorithms of varying sophistication can be used; a linear interpolation based on the values listed in Appendix A is used in the simulations appearing in this document. At each iteration step in the particle tracking, the components of the radius and velocity vectors are calculated as

$$ (v_i)_n = (\vec{v}_{n-1} \times \mathcal{B}(\vec{r}_{n-1})) \frac{q}{m} dt $$

$$ (r_i)_n = (v_i)_n dt $$

where the quantity in the square brackets is the acceleration due to the magnetic field and $i$ can be $x$, $y$, or $z$. A typical value of the time step used in the simulation is 0.02 ns, corresponding to about 1,500 tracking iterations before a successful particle detection. Tracking terminates if the particle hits the solenoid bore, intercepts the detector array, or exits the end of the solenoid.

Finally, the output of the tracking simulations are histogramed using ROOT [62]. For the simulated spectra appearing in this document (and in Refs. [30, 52]), the histograms are produced in a manner very similar to those made with experimental data. That is, the simulated energy and position signals from each simulated detector are used to generate the histograms.

### 12.2 Results

Fig. 12.1 shows a simulated spectrum of proton energy versus position for the $d^{(28}\text{Si}, p)^{29}\text{Si}$ reaction, calculated at three different target-detector separations. The simulations assume uniform angular distributions, a CD$_2$ target thickness of 84 $\mu$g/cm$^2$, and an energy resolution of 50 keV FWHM. The simulation does not include detector threshold effects. The vertical gaps in the spectrum are due to the spacing between detectors on the array. The simulated response of the spectrometer shown in Fig. 12.1 is ostensibly identical to the analytic calculations shown in Fig. 5.5. For this set of simulations, the particle trajectories probe the central region.
Figure 12.1: Simulated $E_{\text{lab}}$ vs. $z$ spectrum from the $d^{(28}\text{Si},p)^{29}\text{Si}$ reaction with HELIOS. Seven states in $^{29}\text{Si}$ are plotted using the field map of the actual HELIOS solenoid. Pairs of vertical lines indicate the length of the array coverage with leading edges at $-94$ mm, $-342$ mm, and $-492$ mm. The narrow-dashed line indicating the low-energy cutoff is due to particles intercepting the front of the detector array. This figure also appears in Ref. [30].

of the solenoid where the magnetic field is most uniform. The effect of field non-uniformities on peripheral trajectories is discussed below. As previously mentioned, an important difference between the analytic calculations and the simulations is the ability to study the realistic excitation energy resolution. Fig. 12.2 shows the effect that the realistic magnetic field has on the excitation energy resolution as compared to a simulation assuming an ideal (uniform) magnetic field. For this pair of simulations, the target is located at the center of the magnet ($z=0$) and the silicon detector array is positioned upstream at $\Delta z = 94$ mm. With this experimental setup, the target, the entire array, and all of the particle orbits are confined to the 90 cm DSV of greatest field uniformity. Both simulations assume 50 keV detector resolution, 1 mm$^2$ beam spot size, and the target effects from a 84 $\mu$g/cm$^2$ thick CD$_2$ target.

Following the method illustrated in Fig. 12.2, several pairs of simulations were run, changing one simulation parameter in each pair. The results of the $Q$-value analysis of these simulations are shown Table 12.1. Neglecting the beam spot size, target thickness effects, and the non-uniformity of the magnetic field, Table 5.2
Figure 12.2: (color online) Simulated $Q$-value resolution of the ground state of $^{29}\text{Si}$ using realistic parameters. The simulation producing the black histogram assumes an ideal magnetic field; the width of the peak is 89.6 keV FWHM. The gray (red) histogram is produced using the measured field map; the width of the peak is 100.0 keV FWHM. Based on these and other simulations, the average contribution of the realistic magnetic field 48.7 keV (which gets added in quadrature to the other contributions).

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_1/A_1$ (MeV/u)</th>
<th>$\beta$ (T)</th>
<th>Origin of contribution</th>
<th>$\delta E_{cm}$</th>
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</thead>
<tbody>
<tr>
<td>$d(^{28}\text{Si},p)^{29}\text{Si}$</td>
<td>6.02</td>
<td>2.00</td>
<td>PSD beam target field</td>
<td>$\Sigma_{quad}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50.0 65.4 35.3 48.7 102.0</td>
<td></td>
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</tbody>
</table>

Table 12.1: Derived contributions to the uncertainty of the center-of-mass energy $E_{cm}$ based on Monte-Carlo simulations. Values are determined based on a 50 keV FWHM detector resolution (PSD), 1 mm$^2$ beam spot size (beam), a CD$_2$ target 84 $\mu$g/cm$^2$ thick (target), and the measured field map (field). The quadratic sum $\Sigma_{quad}$ is equal to the simulated $Q$-value resolution.

shows that the $Q$-value resolution should be dominated by the intrinsic detector resolution; as is the case in normal kinematics. However, in inverse kinematics, due to the heavy ion reactant being the particle which passes through the target foil, Table 12.1 shows that even for a relatively thin target, target thickness effects are significant—on the order of 35 keV. The simulations also show that careful alignment of the silicon detector array and a well-defined beam are also important. A 1 mm$^2$ beam spot size, or equivalently, a ±0.5 mm beam misalignment contributes over 60 keV to the $Q$-value resolution; this contribution is greatest in the region of the knees. Finally, the contribution of the measured magnetic field in the central region of the solenoid is on the order of the intrinsic detector resolution, about 49 keV. All of these contributions get added in quadrature, according to the law of error propagation [39, 40].

Fig. 12.3 shows the simulated proton spectrum for the $d(^{28}\text{Si},p)$ reaction with the silicon detector array placed as far upstream as it will move, with the leading edge (target-end) of the active area of the silicon detectors placed at $z = -396$ mm. Two target-to-detector settings are shown, $\Delta z = 100$ mm and $\Delta z = 450$ mm to
mimic the detector coverage of the $d^{(28}\text{Si},p)$ measurement. Analytic calculations of the locations of the protons groups have been plotted over the simulations for comparison. It is clear that simulated protons detected at the $\Delta z = 100$ mm setting have had their orbits altered by the inhomogeneities of the magnetic field.

Fig. 12.4 shows calculated proton trajectories for the two target-to-array settings used to produce the simulation. Trajectories have been calculated for the ground state transition to $^{28}\text{Si}$. Taking $T_{\text{cy}}$, $v_\parallel$, and $r$ as inputs, Eqs. 10.2 and 10.1 are used to plot the radial excursion $p$ as a function of $z$. At each target position six trajectories are plotted, each intercepting the center of a detector element. Setting I ($\Delta z = 100$ mm) corresponds to emission angles of $\theta_{\text{lab}} = 95^\circ$–$110^\circ$ and orbit excursions of $\rho = 37$–$46$ cm. Setting II ($\Delta z = 450$ mm) corresponds to emission angles of $\theta_{\text{lab}} = 114^\circ$–$158^\circ$ and orbit excursions of $\rho = 10$–$34$ cm. Inspection of Figs. 12.3 and 12.4 shows that the distortions due to the magnetic field are greatest for orbits with the greatest radial excursion. Comparing Figs. 12.1 and 12.3 shows that this negative effect is absent for orbits in the center of the solenoid.

Despite the deformation of the spectrum, two important features should be recognized. First, by inspection, the spacing of the kinematic groups remains constant, even for particles whose orbits probe the regions of more non-uniform magnetic field. The dependence of energy on position is not precisely linear, however the separation in energy between different kinematic groups is retained. Therefore, the performance of the spectrometer as defined by $Q$-value resolution is not adversely effected by the field non-uniformities. The second important feature to note is that the simulated protons groups measured at the $\Delta z = 100$ mm setting—corresponding to orbits with smaller radii ($r < 17.5$ cm)—exhibit almost no deformity. The conclusion to be drawn from this observation is that the fiducial volume for small orbit radii ($r < 17.5$ cm) extends over the entire length solenoid which is within the acceptance of the array ($|z| < 74$ cm). In this region, the mean absolute field non-uniformity is 0.67%. It is also interesting to note that the $z = -396$ mm silicon detector array setting is nearly the same position used in the $^{11,12}\text{B}(d,p)$ experiment which exhibits no field-induced deformities in the shape of the kinematic loci (see Fig. 14.1).
Figure 12.3: Simulated proton spectrum showing distortions due to field inhomogeneities. The $d^{(28}\text{Si},p)$ reaction is simulated with the detector array at $z = -396$ mm (furthest possible upstream position). Two target-to-detector settings are shown, $\Delta z = 100$ mm and $\Delta z = 450$ mm. Distortions due to field inhomogeneities are more pronounced for orbits with larger radii ($\Delta z = 100$ mm setting). The apparent discontinuity between the data sets is due protons probing different regions of the magnetic field for the two different target-to-detector settings. Refer to Fig. 12.4 for an illustration of the particle trajectories.

Figure 12.4: Calculated proton trajectories from $d^{(28}\text{Si},p)$ for two different target-to-detector settings. Separations of $\Delta z = 100$ mm and $\Delta z = 450$ mm are shown, with the silicon detector array positioned at $z = -396$ mm. Drawn to scale.
Part IV

Commissioning Experiments
Chapter 13

The $^{28}\text{Si}(d,p)$ Measurement

13.1 Introduction

To demonstrate the properties of the solenoidal spectrometer technique, HELIOS was commissioned with a study of the $^{28}\text{Si}(d,p)^{29}\text{Si}$ reaction in inverse kinematics. As the HELIOS concept was previously untested, the commissioning experiment served as a proof of principle to verify the HELIOS performance characteristics. The results of the commissioning experiment are reported in Ref. [30]; this chapter expands on the discussion of those results. The $d^{(28}\text{Si},p)$ reaction was chosen because it has a number of advantages for this study. The $(d,p)$ reaction on $^{28}\text{Si}$ has been measured several times over the last six decades and the reaction mechanisms are well understood [25, 72–74]. The reaction has a large, positive $Q$-value of 6.249 MeV and typical cross sections at $\theta_{cm} = 0^\circ$ are of the order of 0.5–10 mb/sr. There are 8 states in $^{29}\text{Si}$ that are strongly populated by the $(d,p)$ reaction with excitation energies between $E_x = 0$ MeV and $E_x = 7$ MeV. These states are separated by an average interval of 0.91 MeV. Fig. 13.1 shows a representative excitation energy spectrum for the $^{28}\text{Si}(d,p)$ reaction performed in normal kinematics by Mermaz et al. [25]. The measurement has a reported $Q$-value resolution of 60 keV and serves as a benchmark for comparison for transfer reactions in normal and inverse kinematics.

In $^{28}\text{Si}$ there exists a pair of states near 6.2 MeV that are separated by separated by 187 keV. Resolving these states in inverse kinematics is a challenge. Table 3.2 shows that under ideal conditions, the $Q$-value resolution of this reaction would be, at best, on the order of 116 keV for a measurement made using a standard detection technique. However, as stated in Ref. [52] and shown in Table 5.2, the same measurement made with HELIOS should have a $Q$-value resolution on the order of the intrinsic detector resolution (neglecting beam and target effects). Therefore, the resolution of the members of this doublet would clearly demonstrate the $Q$-value resolution achievable with this device.
Figure 13.1: Proton spectrum from the $^{28}\text{Si}(d,p)^{29}\text{Si}$ reaction in normal kinematics. The spectrum was measured at $\theta_{\text{lab}} = 45^\circ$, with a bombarding energy of 9.0 MeV/u. The peaks are labeled with the corresponding residual nucleus and the spin, parity, and excitation energy of the state populated in that nucleus. States in $^{17}\text{O}$ are populated due to the use of a silicon oxide target and states in $^{12}\text{C}$ are present because of carbon contamination in the target. States labeled with $(d_0)$ are elastically scattered deuteron. Figure from Mermaz et al. [25, Fig. 1].
CHAPTER 13. THE $^{28}$Si($d,p$) MEASUREMENT

Table 13.1: Detector positions and solid angle coverage for the $^{28}$Si($d,p$) measurement. The target was positioned at three different distances from the detector array during the experiment. The total ground-state solid angle coverage of 1.18 sr is less than the sum of the individual ranges because the detector positions overlapped. The solid angle coverage for setting III was reduced because the detectors in position 1 (furthest from the target) were at a distance greater than the maximum axial range of the protons from the reaction. The figures given in parentheses are the solid angle coverage with poorly-performing detectors excluded (see § 13.3.3).

<table>
<thead>
<tr>
<th>Set</th>
<th>Target (mm)</th>
<th>Array (mm)</th>
<th>$\Delta z$ (mm)</th>
<th>$\theta_{lab}$ ($^\circ$)</th>
<th>$\Delta \cos(\theta_{cm})$ (sr)</th>
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<td>−250.0</td>
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<td>93.7</td>
<td>111.4</td>
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<td>−250.0</td>
<td>491.60</td>
<td>115.1</td>
<td>173.1</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
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</table>

13.2 Experimental Setup

The commissioning experiment was carried out using the Argonne Tandem Linear Accelerator System (ATLAS) at Argonne National Laboratory in August 2008, less than two years after the arrival of the solenoid. A beam $^{28}$Si ions was accelerated from the ECR ion source in charge-state +11 to an energy of 168.52 ± 0.12 MeV (6.024 MeV/u). The measured energy of the beam throughout the experiment is plotted in Fig. 3.6 (on pg. 32). The beam was bunched with the characteristic 82 ns packet spacing of ATLAS with a timing resolution of 0.471 ns FWHM. For each of the data sets presented below, the beam was incident on an 84 $\mu$g/cm$^2$ target of deuterated polyethylene (C$_2$D$_4$)$_n$ with an average beam intensity of 25 ppA. This target thickness was used in order to minimize energy straggling in the target and to emphasize the resolution properties of the spectrometer. The magnetic field of the solenoid was set to a central value of $B_0 = 2.00$ T.

The leading edge of the active region of the detector array was positioned at −250 mm with respect to the center of the magnet. The detector array remained in this position throughout the course of the experiment. To cover different center-of-mass angle ranges, the target was placed at three different positions during the experiment. The position settings are summarized in Table 13.1. For the ground-state transition in $^{28}$Si($d,p$), the array covered a total axial range of 743 mm, corresponding to forward proton center-of-mass angle range of 49.4$^\circ$.

13.3 Results

13.3.1 Particle Identification

Particle identification was achieved by measuring the flight time of the emitted particles relative to the 82 ns RF period of the ATLAS beam. Following the method described in Chapt. 9, the timing response of each detector was corrected for position and energy dependence. The resulting time resolution combining all sili-
con detectors at all energies was 9.1 ns FWHM, which was sufficient to isolate the reaction products of interest. Fig. 13.2 shows a representative time of flight spectrum from the experiment, which has two main features. The larger peak near 32.8 ns represents protons that intercept the detector array at the end of one cyclotron orbit. The smaller peak in the timing spectrum near 65.6 ns corresponds to particles with a mass-to-charge ratio of \( \frac{A}{q} = 2 \) (\( \alpha \) particles and deuterons), and to protons executing two cyclotron orbits. However, in this experiment reaction products with an \( \frac{A}{q} = 2 \) are outside of the acceptance of the array. Since the timing resolution is energy-dependent, a two-dimensional timing cut as shown in Fig. 13.3 was used to gate the data. Spectra gated on both flight times are shown below.

### 13.3.2 Energy Resolution

Fig. 13.4 shows the measured spectrum of proton energy \( E \) versus axial position \( z \) for data taken with the target-to-detector separation of \( \Delta z = 340 \text{ mm} \) (setting II). An energy-dependent time gate similar to the one shown in Fig. 13.3 was used, requiring a time-of-flight consistent with single-orbit protons. The six vertical bands of counts correspond to the six silicon-detector positions within the array. The diagonal loci in the data correspond to transitions to different excited states in the residual \( ^{29}\text{Si} \) nucleus. The slope of the kinematic groups is 10.1 kev/mm (by inspection, the slope is roughly 1 MeV/100 mm). The kinematic loci in the plot are overlaid with the analytically calculated position of these states and are labeled accordingly. The experimental
CHAPTER 13. THE $^{28}\text{Si}(d,p)$ MEASUREMENT

Figure 13.3: Energy vs. time spectrum for all detectors at $\Delta z = 94$ mm (setting I). A typical two-dimensional timing gate selecting flight times corresponding to single-orbit protons has been plotted. The bands corresponding to the $\alpha$-decay source are seen to have no time dependence (random coincidence). The centroid of the locus near 66 ns is energy-independent; however the low-energy time walk correction leads to the rounded cutoff starting near 3 MeV (cf. Fig. 9.6).

The backgrounds in Fig. 13.4 arise from two sources. The horizontal lines in the spectrum (fixed $E$ as a function of $z$) are correspond to $\alpha$ particles from $^{224}\text{Ra}$ contamination in the vacuum chamber from a $^{228}\text{Th}$ calibration source. The $\alpha$ particles are in random time coincidence with the accelerator RF (see Fig. 13.3) and cannot be completely eliminated using a time-of-flight selection. Also present in the data is a smooth background of protons produced in fusion-evaporation reactions of $^{28}\text{Si} + ^{12}\text{C}$ in the CD$_2$ target. Protons from these reactions have the same time-of-flight as those from the reaction of interest, and hence can also not be eliminated by a time gate alone. In subsequent measurements, such events are distinguished using a recoil detector to detect and identify the forward-moving heavy recoils [34, 67]; this detector was not implemented for the $^{28}\text{Si}$ measurement.

Only a small fraction of protons can execute two cyclotron orbits without first hitting the silicon array. Fig. 13.5 shows the energy versus position spectrum at the furthest position, $\Delta z = 492$ mm (setting III), gated
Figure 13.4: Energy vs. position spectrum gated on single-orbit protons at $\Delta z = 342$ mm. The thin dashed lines indicate the analytically calculated position of the kinematic groups with excitation energies of (a) 0.00, (b) 1.27, (c) 2.03, (d) 3.07, (e) 3.62, and (f) 4.90 MeV. The thick dashed curves are the acceptance cutoffs discussed in the text. The pair of vertical lines indicate the range of the array coverage for this setting. This figure also appears in Ref. [30].

on a time-of-flight consistent with 65.6 ns. Despite the lower statistics, diagonal loci are still present with a slope that is half that of the single-turn data, 5.1 keV/mm, representing transitions to states in $^{28}$Si. Although the acceptance for such events is limited, as Fig. 13.5 shows, it is possible to extend the center-of-mass angle range through the use of multi-turn orbits in certain situations. The acceptance limits plotted in the figure are the same as those in Fig 13.4; however, the lines are stretched in $z$ by a factor of 2 for double orbits because of the longer flight time.

Fig. 13.6 shows a composite spectrum of proton energy versus the axial position for all three target positions. The spectrum is gated on single-orbit protons. The dashed curve (A) in Fig. 13.6 indicates the acceptance limit imposed by the array. Comparing the measured spectrum in Fig. 13.6 to simulated (calculated) spectrum in Fig. 12 shows the relevant features predicted to be present in the spectrum are clearly reproduced.

Extracting the relevant center-of-mass quantities from the measured laboratory quantities is straightforward, following the prescriptions of Chapt. 5. The timing resolution of the HELIOS detectors in insufficient...
to straighten the knees in the spectrum; therefore the center-of-mass energy $E_{cm}$ is simply obtained by projecting the laboratory energy along the slope of the kinematic loci using Eq. 5.8. The excitation energy is then obtained from the center-of-mass energy using Eq. 5.9 by accounting for the reaction $Q$-value and the recoil mass. Fig. 13.7 shows the resultant excitation energy spectra for $^{28}$Si. A smooth background from fusion evaporation has been subtracted using the TSpecrum class [63]. This background subtraction is shown explicitly in Fig. 13.7(c). The $Q$-value resolution for all detectors at all positions was 127 keV FWHM, with the best detector having a resolution of 74 keV FWHM. The measured $Q$-value resolution is within a few keV of the simulated resolution representing the ideal case including target thickness effects (cf Table 12.1). This result is an emphatic demonstration of the validity of the HELIOS concept.
13.3.3 Efficiency

In addition to testing HELIOS as a concept, one of the goals of this commissioning experiment was to characterize HELIOS as an instrument. Thus, not all its systems were fully optimized. Due to a problem with signal shielding on the electronics feedthrough patch boards, 9 of the 24 detectors in the array were excluded from the trigger due to noise. This left 2–3 active detectors at each of the six positions along the detector array, corresponding to an azimuthal coverage of 24% or 36%, respectively. Additional shielding of the detector cables and patch boards has since eliminated this problem. Of the remaining detectors, some can be clearly seen to have reduced efficiency towards the center of the detector.

Fig. 13.8 shows the energy versus position spectrum for \( \Delta z = 94 \) mm (setting I) with the effect of the software energy threshold plotted as well as the regions of greatest efficiency loss. The position- and energy-dependence of the regions of reduced efficiency is characteristic of the effect of ballistic deficit. This is due to the fact that the shaping times and electronic thresholds were not optimized for the slower rise times of the en-
ergy signals produced at these positions. This is effect is reduced by lowering the electronic threshold on the detectors, increasing the shaping time, and by including the position signals in the trigger.

Figure 13.7: Excitation energy spectrum for $^{29}\text{Si}$ measured with HELIOS. (a) Spectrum with smooth background subtraction for an individual detector covering 386–437 mm from the target (furthest detector position from the target with the leading edge of the array 94 mm from the target). The average energy resolution is 103 keV FWHM. (b) Composite spectrum for one detector per position again with the leading edge of the array 94 mm. (c) Composite spectrum for all detectors at all three positions showing background subtraction generated by TSpec$\text{turm}$ [63]. The average energy resolution is 127 keV FWHM. Identified energy levels are labeled by their energy in keV. The two lowest levels are also labeled with their identified spins. This figure appears similar form in Ref. [30].
Figure 13.8: Example of the position- and energy-dependent variations in detector efficiency. The $E_{\text{lab}}$ vs. $z$ spectrum for $\Delta z = 94$ mm is plotted. The dashed U-shaped curves corresponds to a (software) threshold of 210 keV required for all three signals on each detector. The dashed $\Lambda$-shaped curves highlight the regions where variations in energy signal rise times lead to a dramatic loss of counts. Finally, the curve at the bottom of the figure corresponds to $\theta_{\text{lab}} = 0^\circ$.

13.3.4 Angular Distributions

The laboratory energies of the protons from the ground and first-excited state transitions were sufficiently large so that the effects of ballistic deficit were negligible. For these states the relative detector efficiencies were normalized to the $\alpha$ spectrum of the $^{228}$Th calibration source. For each detector the $\alpha$ yield was used to determine the detector efficiency as a function of $X$, the detector position. This function was then used to scale the counts for each state. Fig. 13.9 shows the angular distributions extracted for these two transitions using data from the $\Delta z = 490$ mm position setting. Each point includes data from approximately half of one silicon detector which corresponds to a range of $\Delta \cos(\theta_{\text{cm}}) = 0.014$, or $\Delta \Omega = 10.6$ msr. The ground-state transition covers detectors 2–6 (10 points) and the first-excited state covers detectors 3–6 (8 points). Each distribution includes one additional point forward of $\theta_{\text{cm}} = 10^\circ$, corresponding to the region beyond the bend of the knees where protons with very shallow trajectories are detected well before completing their full cyclotron orbit ($\Delta \varphi = 325^\circ$); the solid-angle acceptance for these orbits is very sensitive to the relative alignment of the
silicon array. Due to this uncertainty in the solid-angle acceptance, the forward-most data point was omitted from the ground-state angular distribution published in Ref. [30]. Also, in this commissioning experiment, the beam current was not measured, and so the cross-section scale is arbitrary.

The curves correspond to distorted-wave Born approximation calculations using the code PTOLEMY [29]. The calculations for this reaction assume a deuteron bombarding energy of 12.09 MeV using the optical-model parameters from listed in Table 13.3. The angular distribution for the $^{28}$Si ground state shows a shape characteristic of an angular momentum transfer of $\ell_n = 0$, with a strong maximum near $\theta_{cm} = 0^\circ$, and minimum near $\theta_{cm} = 23^\circ$. The transition to the ground-state is in fact an $\ell_n = 0$ angular momentum transfer, corresponding to a ground-state spin assignment of $j_n = \ell_n + \frac{1}{2} = \frac{1}{2}$, with positive parity [72]. The angular distribution for the $^32^+$ first-excited state shows a much weaker dependence on scattering angle, as expected for an $\ell_n = 2$ transition. The transition to the first excited state corresponding to the assignment $j_n = \ell_n - \frac{1}{2} = \frac{3}{2}$. The shapes of these angular distributions are similar to those observed by Mermaz et al. [25] in normal kinematics at a deuteron bombarding energy of 18 MeV. The relative cross sections for the ground- and first-excited state transitions also agree well with those obtained by Mermaz et al. which demonstrates the ability of HELIOS to provide spin assignments.

### 13.3.5 Discussion

The HELIOS spectrometer was successfully commissioned by measuring the $^{28}$Si(d,p)$^{29}$Si reaction in inverse kinematics. All of the suggested design features discussed in Ref. [52] (summarized in Table 5.6) were demonstrated in this experiment. The time resolution of 9.1 ns was sufficient to isolate single-orbit protons from double-orbit protons. The measurement had a large acceptance with the detector array subtending a total of 1.18 sr over three detector array settings. The overall $Q$-value resolution was on the order of 100 keV FWHM, representing a substantial improvement over “traditional” detector schemes (cf. Chapt. 4). A $Q$-value resolution at this level is approaching that achievable in normal kinematics where the overall resolution is dominated by the intrinsic detector resolution. Finally, given an excitation energy and bombarding energy, the center-of-mass angles were derived to generate angular distributions. The angular distributions provided a signature of the angular momentum transferred in the reaction and allowed for the spin assignment of the ground and first-excited states in the residual $^{29}$Si nucleus. All of these results validate the HELIOS concept, making the commissioning experiment a resounding success.
CHAPTER 13. THE $^{28}$Si($d,p$) MEASUREMENT

<table>
<thead>
<tr>
<th>Solenoid</th>
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Table 13.2: Nominal performance specifications of the HELIOS spectrometer. The value of $\mathcal{B} R$ and $\mathcal{B} L$ are calculated for the maximum field (2.86 T) based on the solenoid bore (462 mm) and the maximum range of the array ($|z| < 741$ mm), respectively. $\Delta \Omega$ is calculated for one target-to-detector setting for the $^{28}$Si($d,p$) reaction. This value is a factor of two smaller than the one suggested in Table 5.6 because of the azimuthal acceptance of the prototype array. All other values are reported as measured.

<table>
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<th>$r_0'$</th>
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Table 13.3: Optical model parameters for the $^{28}$Si($d,p$)$^{29}$Si reaction from various sources. Ref. [25] uses a parameter ($\lambda = 25.0$) to scale the spin-orbit strength based on the real part of the Woods-Saxon potential $V$; hence, no additional parameters are specified.

Figure 13.9: Proton angular distribution from the $^{28}$Si($d,p$)$^{29}$Si reaction. Transitions to the ground (left) and first-excited (right) states of $^{29}$Si are plotted. The curves correspond to DWBA calculations for this reaction at a deuteron bombarding energy of 12 MeV using optical-model parameters from Ref. [25] (solid), Ref. [73] (dotted), and Ref. [74] (dashed). The angular distributions from Mermaz et al. have been digitized and plotted (triangles) for comparison. The cross section of the Mermaz data have been scaled to fit the plotted DWBA calculations, and the angles in both plots have been scaled by the same factor ($\approx 23^\circ/18^\circ$, see Fig. 2.5) to align the first minimum in the $\ell_n = 0$ distribution.
Chapter 14

The $^{12}$B$(d,p)$ Measurement

14.1 Introduction

The commissioning experiment described in the previous chapter served to demonstrate the validity of the HELIOS concept. The first groundbreaking physics measurement made with the HELIOS spectrometer was the study of the $d(^{12}$B,$p)^{13}$B reaction conducted in March, 2009. The details of this experiment are reported in Ref. [34]; this chapter summarizes those results. This experiment was a repeat measurement of the reaction discussed in Chapt. 4. The measurement needed to be repeated because the previous experiment, which was carried out using conventional detector geometry, was unable to resolve a pair of states in $^{13}$B separated by $\Delta E_x = 199$ keV. Separation of this doublet would provide a strict test of the resolution performance of the spectrometer. This experiment was the first reaction study using HELIOS with a radioactive beam and is considered the first “experiment” with HELIOS.

14.2 Experimental Setup

The method of beam production used in this experiment is identical to those described in Chapt. 4 [33]. Briefly, a 81 MeV primary beam of $^{11}$B bombarded a cryogenic gas cell filled with deuterium gas to produce in-flight a 75 MeV $^{12}$B secondary beam. The primary beam had an intensity of 50 pnA ($3.1 \times 10^{11}$ ions/s) and the secondary beam had an intensity of $6 \times 10^5$ ions/s on target. A 73 $\mu$g/cm$^2$ thick CD$_2$ target was used, placed at the center of the magnet at $z = 0$ mm. For the duration of the measurement the solenoid was set to a central field value of $B_0 = 1.04$ T and the array was placed upstream of the target at a target-to-detector separation of $\Delta z = 368.7$ mm. Table 14.1 shows the corresponding detector coverage for both reactions. For recoil detection, a 4-quadrant $\Delta E\cdot E$ telescope array was placed 1,032 mm downstream from the target (as shown in Fig. 5.1), covering $\theta_{lab} = 0.5^\circ$–$2.9^\circ$. The $\Delta E$ energy loss detectors were nominally 80 $\mu$m thick (the actual thicknesses
CHAPTER 14. THE $^{12}$B$(d,p)$ MEASUREMENT

Table 14.1: Detector positions and solid angle coverage for the $^{11,12}$B$(d,p)$ measurement. Since the ground-state transition was outside of the acceptance for both data sets, the coverage of a state which spans the array is given; the 2.62 MeV state in $^{12}$B and the 3.48 MeV state in $^{13}$B. The target and array remained at the same position for both measurements.

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<td>154.4</td>
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<td>0.120</td>
</tr>
</tbody>
</table>

ranged from 79–81 $\mu$m) and the $E$ residual energy detectors were nominally 500 $\mu$m thick (493–496 $\mu$m). The method of recoil detection is described in Chapt. 10.2. By gating the proton spectra on both the 63.0 ns time-of-flight and the detection of recoiling heavy ion, the measured backgrounds are largely suppressed.

14.3 Results

14.3.1 Energy Resolution

Fig. 14.1 shows the characteristic energy versus position spectrum for the $^{11}$B$(d,p)^{12}$B stable beam calibration measurement. The spectrum is gated on two event criteria. First the flight times consistent with single-orbit protons (63 ns). As was shown with the data from the $^{28}$Si$(d,p)$ reaction in Fig. 13.4, gating on time alone is insufficient to eliminate the background due to fusion-evaporation reactions of involving the $^{12}$C in the CD$_2$ target. The second gating criterion in Fig. 14.1 is the requirement of a coincidence measurement of the recoiling $^{12}$B ion in the heavy recoil detector. This added step dramatically reduces the background (cf. Fig. 13.4).

The ground state of $^{11}$B has a spin and parity of $J^\pi = 3/2^-$. The $\ell_n = 0$ transition populates states corresponding to $J = 3/2 \pm 1/2$, which are the second- and third-excited states. Both of these states were populated in this reaction; the $J^\pi = 2^- \text{ state}$ has an excitation energy of $E_x = 1.67 \text{ MeV}$ (b in Fig. 14.1) and the $J^\pi = 1^- \text{ state}$ has an excitation energy of $E_x = 2.62 \text{ MeV}$ (c). Fig. 14.3 shows the excitation energy spectrum from this measurement. The $Q$-value resolution was about 100 keV. The $\Delta E_x = 102 \text{ keV}$ doublet near 2.7 MeV is unresolved. The angular distributions of the 2.62 and 3.39 MeV states are shown in Fig. 14.4.

Fig. 14.2 shows a simulated energy versus position spectrum for the $^{12}$B$(d,p)$ reaction, requiring coincidence with the recoiling $^{13}$B nucleus. As the simulation shows, using similar resolution parameters as those discussed in Chapt. 12, the $\Delta E_x = 199 \text{ keV}$ doublet near 3.6 MeV should be able to be resolved in this measurement; and indeed it was. Fig. 14.3 shows the excitation energy spectrum from this measurement. Comparing this figure to Fig. 4.3 is a direct, practical demonstration of the enhanced $Q$-value resolution provided by the HELIOS technique. This measurement made with HELIOS had $3 \times$ better resolution than the previous mea-
CHAPTER 14. THE $^{12}$B($d,p$) MEASUREMENT

Figure 14.1: Energy vs. position spectrum for protons from the $^{11}$B($d,p$) reaction. Events are gated on a time-of-flight consistent with single-orbit protons and on coincidence with the recoiling $^{12}$B identified in the heavy ion recoil detector. The thin dashed lines indicate the analytically calculated positions of the kinematic groups; the excitation states are labeled (a) 0.95 MeV ($J^\pi = 2^+$), (b) 1.62 MeV ($J^\pi = 2^-$), (c) 2.62 MeV ($J^\pi = 1^-$), (d) 3.39 MeV ($J^\pi = 3^-$). The bold wide-dashed line corresponds to the acceptance limit imposed by the solenoid bore. This data appear in a similar form in Ref. [34, Fig. 2].

14.3.2 Angular Distributions

The two states with the most statistics from the $^{11}$B($d,p$)$^{12}$B reaction—the 2.62 MeV and 3.39 MeV excited states—were used to to calibrate the efficiency of the detector array. The 3.39 MeV excited state was populated with over $4 \times$ the statistics of any other state; this state was used to calibrate most of the array. However, examination of Fig. 14.1 reveals that the 3.39 MeV state (labeled b in the figure) does not extend onto detector position 1. This position was calibrated with the 2.62 MeV excited state (c in the figure). The array is binned into equal ranges of $\Delta z$ with each bin corresponding to one detector. The equal ranges of $\Delta z$ are equivalent to equal ranges of $\Delta \cos(\theta_{cm})$, in this case $\Delta \cos(\theta_{cm}) = 0.019$ or 14.5 msr.

The angular distributions from the $^{11}$B($d,p$) reaction were fitted to the results of DWBA calculations carried...
out using three different parameter sets that had previously been used to reproduced the angular distributions for this reaction [75–77]. For each solid angle bin (one per detector position), an efficiency factor equal to the ratio of DWBA/data was produced. These fixed efficiency factors were then applied uniformly to all of the data. The results of this procedure are shown in Fig. 14.4. However, even without applying this technique, the 3.48 MeV state displays the characteristic behavior of an $\ell_n = 0$. Fig. 14.5 shows the excitation energy spectrum of the 3.6 MeV doublet for two different angle ranges. In the top panel of the figure, counts from the entire array are used. In the bottom panel of the figure, counts in the range of $\theta_{\text{cm}} = 5^\circ$–$17^\circ$ have been excluded (half the array). The dramatic reduction of counts in the 3.48 MeV peak when excluding forward angles is an illustration of the forward-peaked nature of the 3.48 MeV state, which is consistent with a $\ell_n = 0$ transfer.
Figure 14.3: Excitation energy spectra from the (top) $^{11}$B($d,p$) and (bottom) $^{12}$B($d,p$) reactions, as reported in Ref. [34]. Comparing this figure to Fig. 4.3 (on page 37) is a direct, practical demonstration of the enhanced $Q$-value resolution provided by the HELIOS technique. Figure from Ref. [78].
CHAPTER 14. THE $^{12}$B($d,p$) MEASUREMENT

The characteristic shape of the $\ell_n = 2$ angular distribution of the 3.39 MeV in $^{12}$B was used to calibrate the efficiency of the detector array. Applying this calibration to the $^{13}$B data reveals the clear $\ell_n = 0$ character of the 3.48 MeV state and the $\ell_n = 2$ character of the 3.68 MeV state. Figure taken from Ref. [34, Fig. 3].

Figure 14.4: Angular distributions from the (left) $^{11}$B($d,p$) and (right) $^{12}$B($d,p$) reactions as reported in Ref. [34].

Figure 14.5: Excitation energy spectrum of the 3.6 MeV doublet for two different angular ranges. Panel (a) includes counts from the entire array (detector positions 1–6), corresponding to a center-of-mass range of $\theta_{cm} = 5^\circ - 29^\circ$. Panel (b) is includes counts from only detectors 3–6, corresponding to a center-of-mass range of $\theta_{cm} = 17^\circ - 29^\circ$. The dramatic reduction of counts in the 3.48 MeV peak when excluding forward angles is an illustration of the forward-peaked nature of the 3.48 MeV state, which is consistent with a $\ell_n = 0$ transfer. Figure from Ref. [78].
Chapter 15

Conclusions

15.1 Summary of Results

The HELIOS spectrometer was successfully commissioned using the \((d,p)\) neutron-transfer reaction on \(^{28}\text{Si}\) in inverse kinematics at 6.0 MeV/\(u\). The purpose of the commissioning experiment was to verify the resolution, acceptance, and transport properties suggested by the design of this novel spectrometer. Analytical calculations and Monte Carlo simulations were carried out to model the performance of the spectrometer and the experimental results were in excellent agreement, as demonstrated in Figs. 13.6 and 14.1. This experiment demonstrated that a \(Q\)-value resolution of less than 80 keV FWHM in inverse kinematics is attainable with HELIOS. This remarkable result approaches those of measurements made in normal kinematics.

The large acceptance of HELIOS was established with this measurement covering nearly 50° in the center-of-mass frame (\(\Delta \Omega = 1.18\ \text{sr}\)), and proton energies from 210 keV to over 9 MeV. The performance properties unique to solenoidal transport provided by HELIOS were also confirmed. Namely, the time-of-flight of the detected ions was approximately the cyclotron period \(T_{\text{cyc}}\), providing \(A/q\) particle identification and the measured kinematic groups were separated by their excitation energy \(\Delta E_{\text{lab}} = \Delta E_{\text{cm}}\).

The enhanced resolution of the HELIOS spectrometer is due to the linear correlation between the measured quantities \(z\) and \(E_{\text{lab}}\). This relationship eliminates kinematic compression and dramatically suppresses resolution broadening due to the kinematic covariation of the measured quantities. Comparing excitation energy spectra produced with HELIOS (Figs. 13.7 and 14.3) to state-of-the-art measurements made in conventional detector geometry (Figs. 4.3 and 4.7), it is clear that HELIOS has substantially superior center-of-mass energy resolution (\(\approx 3\times\)). Based on the advantages of the HELIOS spectrometer, it is clear that HELIOS is the ideal instrument for measuring the \(d(^{132}\text{Sn},p)^{133}\text{Sn}\) reaction. Its unique geometry provides the high-efficiency, high-resolution measurements required by inverse kinematics.
15.2 Outlook

15.2.1 The $^{132}\text{Sn}(d,p)$ Measurement

The future of HELIOS measurements utilizing rare isotope beams far from stability is dependent on the full completion of the Californium Rare Isotope Beam Upgrade (CARIBU) radioactive ion source, currently undergoing the final stages of development. As of this writing, the $^{132}\text{Sn}(d,p)$ reaction has not yet been measured with HELIOS. However, the opportunity to carry out this measurement is on the near horizon. In March, 2011, the CARIBU ion source was successfully commissioned. A beam of $^{143}\text{Ba}$ ions ($T_{1/2} = 14.5$ s) produced by CARIBU was detected at the ATLAS High-Energy Diagnostics region, identified by the characteristic $\gamma$-decay spectrum. While a $^{132}\text{Sn}$ beam has not yet been produced, HELIOS has been used to measure an $N = 82$ heavy ion reaction, specifically $(d,p)$ on $^{136}\text{Xe}$.

Fig. 15.1 shows the characteristic $E$ vs. $z$ spectrum $d(^{136}\text{Xe},p)^{137}\text{Xe}$ reaction at 10 MeV/u and $B_0 = 2$ T for two target-array positions. These data are gated on events corresponding to a cyclotron period of 32.8 ns—the cyclotron period for protons at 2 T. The background is protons from fusion-evaporation of target and beam. Fig. 15.1 also shows the excitation energy spectrum measured in this experiment; 11 states above $E_x = 1.5$ MeV had quantities identified or derived for the first time in this measurement. The $Q$-value resolution of this measurement is $\Delta E_{\text{cm}} = 100$ keV FWHM, which is consistent with the lighter mass studies discussed in Chapt. 13. In addition, angular distributions were measured for several states. This measurement demonstrates that the HELIOS spectrometer is poised for an effective and groundbreaking measurement of the $^{132}\text{Sn}(d,p)$ reaction.

Part of a successful measurement of $^{132}\text{Sn}(d,p)$ with HELIOS would possibly involve the first-ever measurement of angular distributions of states in $^{133}\text{Sn}$ above $E_x = 1$ MeV.

15.2.2 Instrumental Improvements

While the results of the commissioning experiment are very encouraging, the present implementation of HELIOS is still that of a demonstration prototype. A number of improvements to the apparatus are planned, including new silicon detectors in an arrangement that will provide twice the solid-angle coverage and improved overall resolution, as well as greater center-of-mass-angle acceptance. The new array will be hexagonal in cross section, modular in 5-detector lengths. Also planned is the addition of a gas parallel-plate avalanche counter and ionization chamber for the detection and identification of recoiling heavy nuclei that cannot be so identified using silicon $\Delta E - E$ techniques. While the present spectrometer configuration is tailored to the detection of particles emitted with laboratory angles greater than $\theta_{\text{lab}} = 90^\circ$, the advantages of simplified kinematics and improved resolution also apply to reactions in inverse kinematics that emit light ions forward of $\theta_{\text{lab}} = 90^\circ$, such as $(d,^3\text{He})$. As of this writing, the silicon detector array as been moved to the downstream position and additional work is currently underway to accommodate such $K > 1$ reactions. With these features
in mind, it is clear that HELIOS is a powerful tool for measuring nuclear reactions in inverse kinematics.

Figure 15.1: (color online) (Top) Energy vs. position spectrum for the $d(^{136}Xe,p)^{137}Xe$ reaction at 10 MeV/u and $B_0 = 2$ T for two target-array positions; $\Delta z = 50$ mm (blue) and $\Delta z = 350$ mm (red), as indicated by the horizontal lines. (Bottom) Excitation energy spectrum for $^{137}Xe$. Peaks are labeled with their energy, and where known, their $\ell$-value, spin, and parity. States marked with a $\triangle$ symbol are those with energy, $\ell$-value, or both, deduced for the first time in this work. Spins and parities in parentheses are to be regarded as tentative. A smooth background (evaporated protons from $^{136}Xe+^{12}C$ fusion) has been subtracted. Figures by B. P. Kay [79]
## Appendix A

### Field Map

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Table A.1: Average axial field strength $B_z$ of the HELIOS solenoid in gauss.
The field values presented in this appendix represent the interpolated field map matrix used in the simulations. It is based on the 21,240 point field map discussed in Chapt. 6.2. For each combination of \((z, \rho)\) in the original field map, the azimuthal variations in the measured field have been averaged. The value of the magnetic field at fixed intervals of \(\Delta z = 5\) cm are interpolated from the azimuthally-averaged values in a manner similar to the fit shown in Fig. 6.5.
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