6-2014

Real-Time Hybrid Simulation with Online Model Updating

Adam Mueller
Western Michigan University, amueller1130@yahoo.com

Follow this and additional works at: https://scholarworks.wmich.edu/masters_theses

Part of the Civil Engineering Commons, Computational Engineering Commons, and the Construction Engineering and Management Commons

Recommended Citation
https://scholarworks.wmich.edu/masters_theses/506

This Masters Thesis—Open Access is brought to you for free and open access by the Graduate College at ScholarWorks at WMU. It has been accepted for inclusion in Master's Theses by an authorized administrator of ScholarWorks at WMU. For more information, please contact maira.bundza@wmich.edu.
REAL-TIME HYBRID SIMULATION WITH ONLINE MODEL UPDATING

by

Adam Mueller

A thesis submitted to the Graduate College
in partial fulfillment of the requirements
for the degree of Master of Science in Engineering (Civil)
Civil and Construction Engineering
Western Michigan University
June 2014

Thesis Committee:

Xiaoyun Shao, Ph.D., Chair
Upul Attanayake, Ph.D.
Haluk Aktan, Ph.D.
Hybrid simulations have shown great potential for economic and reliable assessment of structural seismic performance through a combination of physically tested components, called the experimental substructure, and numerically simulated components, called the numerical substructure. Current hybrid simulation practices often use a fixed numerical model without considering the possible availability of a more accurate model obtained during hybrid simulation through an online model updating technique. To address this limitation and improve the reliability of numerical models in hybrid simulations, this study describes the implementation of an online model updating method in real-time hybrid simulation (RTHS). The Unscented Kalman Filter (UKF) was selected as the model updating algorithm and applied to identify Bouc-Wen model parameters that define the hysteresis of the experimental substructure, and the identified parameters were therefore applied to the numerical substructures during the hybrid simulation. Firstly, a parametric study of the UKF system model parameters was carried out. Then the developed online model updating method was experimentally validated through RTHS. Finally, guidelines for implementing the UKF for online model updating in future RTHS are provided.
ACKNOWLEDGMENTS

Most importantly, I would like to express my gratitude to my advisor, Dr. Xiaoyun Shao, for all her guidance, assistance, and encouragement throughout my graduate studies. Additionally, I would like to thank the other members of my thesis committee, Dr. Upul Attanayake and Dr. Haluk Aktan.

I would also like to extend a special thanks to my predecessor Chelsea Griffith for her support and training, as well as Carlos Santana and Ramon Roberts-Perazza for their immense help in experimental setup.

Adam Mueller
# TABLE OF CONTENTS

ACKNOWLEDGMENTS ......................................................................................... ii

LIST OF TABLES ................................................................................................. vii

LIST OF FIGURES ............................................................................................... ix

CHAPTER

1. INTRODUCTION .............................................................................................. 1

   1.1. Conventional Seismic Testing Methods ..................................................... 1

       1.1.1. Shake Table Testing (STT) ................................................................. 2

       1.1.2. Quasi-Static Testing (QST) ............................................................... 2

   1.2. Hybrid Simulation ....................................................................................... 3

       1.2.1. Pseudodynamic (PSD) Hybrid Simulation ......................................... 4

       1.2.2. Online Model Updating in Hybrid Simulation ................................. 5

   1.3. Literature Review on Model Updating ....................................................... 7

       1.3.1. Numerical Studies ............................................................................. 9

       1.3.2. Experimental Studies .................................................................... 11

   1.4. Justification, Objectives, and Scope ........................................................ 13

   1.5. Thesis Outline ........................................................................................ 14

2. THEORETICAL BACKGROUND ....................................................................... 16

   2.1. Pseudodynamic (PSD) Hybrid Simulation Fundamentals ...................... 17

       2.1.1. Equation of Motion (EOM) ............................................................... 17

       2.1.2. Numerical Integration ..................................................................... 18
Table of Contents—Continued

CHAPTER

2.1.3. Explicit Newmark Algorithm ........................................ 20
2.1.4. PSD Hybrid Simulation Framework ................................. 23
2.2. Bouc-Wen Hysteresis Model ........................................... 25
2.3. Unscented Kalman Filter ................................................. 29
  2.3.1. Sigma Points ......................................................... 31
  2.3.2. State Prediction .................................................... 33
  2.3.3. Measurement Prediction ........................................... 33
  2.3.4. Kalman Gain ....................................................... 33
  2.3.5. State Update ....................................................... 34
2.4. Summary ........................................................................ 35

3. NUMERICAL SIMULATION AND PARAMETRIC STUDY ............ 36
  3.1. Bouc-Wen Model Parameter Identification using the UKF ....... 37
  3.2. Parametric Study of the UKF System Model ......................... 42
    3.2.1. Initial Covariance Matrix $P_{0|0}$ ................................ 46
    3.2.2. Process Noise Covariance Matrix $Q$ ............................ 64
    3.2.3. Measurement Noise Covariance Matrix $R$ ....................... 74
  3.3. Numerical Hybrid Simulation with Model Updating ............. 84
  3.4. Summary and Conclusions ............................................. 96

4. EXPERIMENTAL STUDY .................................................. 98
  4.1. RTHS Testing System .................................................. 99
## Table of Contents—Continued

### CHAPTER

4.1.1. Hybrid Testing System Components ............................................. 99  
4.1.2. Hardware Integration ................................................................. 102  
4.1.3. Software Integration ................................................................. 104  
4.2. Test Specimen ............................................................................... 104  
4.3. Quasi-Static Test and Offline Parameter Identification ................. 105  
4.4. Time Delay Compensation for RTHS ........................................... 113  
4.5. RTHS without Online Model Updating ......................................... 115  
4.6. RTHS with Online Model Updating ................................................ 119  
  4.6.1. RTHS 1 .................................................................................. 122  
  4.6.2. RTHS 2 .................................................................................. 123  
  4.6.3. RTHS 3 .................................................................................. 128  
  4.6.4. RTHS 4 .................................................................................. 129  
  4.6.5. RTHS 5 .................................................................................. 130  
  4.6.6. RTHS 6 .................................................................................. 132  
4.7. Implementation Issues ..................................................................... 136  
4.8. Summary and Conclusions ............................................................. 138  
5. CONCLUSIONS .................................................................................. 139  
  5.1. Summary of Findings ................................................................... 140  
    5.1.1. Parametric Study ................................................................. 140  
    5.1.2. Experimental Study ............................................................. 142
# Table of Contents—Continued

## CHAPTER

5.1.3. Implementation Guidelines .............................................. 143

5.2. Future Work ........................................................................ 144

REFERENCES ............................................................................. 147

APPENDIX .................................................................................. 152
LIST OF TABLES

1.1. Model updating in recent studies .............................................................. 8
2.1. Explicit Newmark formulation ................................................................. 22
3.1. Structural, ground motion, and integration parameters used in the parametric study ................................................................. 43
3.2. Initial state estimates .............................................................................. 45
3.3. Parametric study simulation summary ..................................................... 46
3.4. Error threshold of $P_{q\theta}(5,5)$ for all initial estimate sets ..................... 54
3.5. True and converged values of Bouc-Wen parameters when initial estimates are inaccurate .......................................................... 60
3.6. Adjustment of close initial estimates ....................................................... 62
3.7. Error threshold of $Q(1,1)$ for all initial estimate sets ............................. 71
3.8. Structural, ground motion, integration algorithm, and UKF system model parameters used in the numerical hybrid simulation .................................. 90
3.9. Exact and estimated Bouc-Wen parameters used in the numerical hybrid simulation .................................................................................. 90
4.1. Components and parameters of the hybrid testing system ...................... 100
4.2. Converged values of Bouc-Wen parameters from QST .......................... 112
4.3. Structural, ground motion, integration algorithm, and MFF compensation parameters used in the RTHS without updating .............................. 117
4.4. Structural, ground motion, integration algorithm, and MFF compensation parameters used in the RTHS with model updating .......................... 120
4.5. UKF system model parameters used for each RTHS ............................... 121
4.6. Converged values of Bouc-Wen parameters from QST and RTHS 2 .......... 124
4.7. Converged values of Bouc-Wen parameters from QST and RTHS 6 .......... 133
LIST OF FIGURES

1.1. Illustration of hybrid simulation of a multistory building with online model updating ................................................................. 7

2.1. Conceptual diagram of PSD hybrid simulation framework .................. 23

2.2. Bouc-Wen hysteresis example ............................................................ 26

2.3. Effects of decreasing and increasing Bouc-Wen parameters .............. 28

2.4. Unscented Kalman Filter block diagram ............................................ 31

3.1. SDOF system .................................................................................... 37

3.2. SDOF simulation block diagram ....................................................... 41

3.3. El Centro earthquake ground acceleration ........................................ 42

3.4. Time history response of SDOF system ............................................. 44

3.5. Hysteretic response of SDOF system .................................................. 44

3.6. Time histories of $k$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate .................................................................. 49

3.7. Time histories of $\beta$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate .............................................................. 51

3.8. Time histories of $\gamma$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate .............................................................. 52

3.9. Time histories of $n$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate .............................................................. 55

3.10. Time histories of $\alpha$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate .............................................................. 57

3.11. Time histories of all parameters when all initial estimates are (a) exact, (b) close, and (c) inaccurate ........................................... 59
3.12. Time histories of all parameters when initial estimates are close, and initial covariance elements corresponding to $\beta$ and $\gamma$ are equal to (a) 1 and (b) 0.1 ................................................................. 61

3.13. Time histories of all parameters when initial estimates are adjusted, and initial covariance elements corresponding to $\beta$ and $\gamma$ are equal to (a) 1 and (b) 0.1 ............................................................................. 63

3.14. Time histories when all initial estimates are exact for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ........................................................................................................... 66

3.15. Time histories when all initial estimates are close for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ........................................................................................................... 68

3.16. Time histories when all initial estimates are inaccurate for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ................................................................. 72

3.17. Time histories when all initial estimates are exact for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ........................................................................................................... 75

3.18. Time histories when all initial estimates are close for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ........................................................................................................... 77

3.19. Time histories when all initial estimates are inaccurate for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ........................................................................................................... 80

3.20. Time histories when all initial estimates are close and noise covariance is $10^{-5}$ for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$ ................................................................. 82

3.21. 3DOF system ........................................................................................................ 85

3.22. 3DOF simulation block diagram ........................................................................ 87

3.23. Restoring force function .................................................................................... 88

3.24. Absolute displacement time history response of three simulations for (a) first story, (b) second story, and (c) third story ........................................ 92

3.25. Zoomed-in absolute displacement time history response of three simulations for (a) first story, (b) second story, and (c) third story ............... 93

3.26. Hysteresis response of three simulations for (a) first story, (b) second story, and (c) third story ................................................................................. 94

3.27. Time histories of all parameters during “with updating” case ....................... 95
List of Figures - continued

4.1. Schematic diagram of the hybrid testing system ........................................ 103
4.2. External BNC connections .......................................................................... 104
4.3. Test specimen used in experimental study .................................................. 105
4.4. Cyclic loading pattern used for QST .......................................................... 107
4.5. Hysteretic response of specimen ................................................................. 107
4.6. Time histories for various $R$ values for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$  ................................................................................................................................. 110
4.7. Comparison of experimental and numerical hysteretic response ................ 112
4.8. Time delay estimation .................................................................................. 113
4.9. Command tracking performance of actuator with MFF compensation ....... 115
4.10. Hybrid simulation of 3DOF system ................................................................ 116
4.11. Absolute displacement time history results of the RTHS without updating .................................................................................................................. 118
4.12. Hysteretic results of RTHS without updating .............................................. 119
4.13. Time histories of all parameters for RTHS 1 .............................................. 122
4.15. Time histories of all parameters for RTHS 2 .............................................. 124
4.16. RTHS 2 absolute displacement time history results comparison for (a) first story, (b) second story, (c) third story .......................................................... 126
4.17. RTHS 2 hysteresis response comparison for (a) first story, (b) second story, (c) third story ........................................................................................................ 127
4.18. Time histories of all updated parameters for RTHS 3 .............................. 129
4.19. Time histories of all updated parameters for RTHS 4 .............................. 130
4.20. Time histories of all parameters for RTHS 5 ............................................. 131
4.21. Time histories of all parameters for RTHS 6 ............................................. 132
List of Figures - continued

4.22. RTHS 6 absolute displacement time history results comparison for
   (a) first story, (b) second story, and (c) third story ........................................... 134

4.23. RTHS 6 hysteresis response comparison for (a) first story, (b) second
   story, and (c) third story .................................................................................. 135

4.24. Hysteresis response of all RTHS with online model updating ................. 137
1. Introduction

Earthquakes are one of the most devastating and costly natural hazards as they have the potential to cause major damage to infrastructure. Earthquake engineering aims to design and retrofit structures so that they can achieve desired performance levels when subject to earthquake attacks, which requires an adequate understanding of structural responses due to seismic loading. However, structural seismic responses are usually highly unpredictable due to the many uncertainties associated with earthquake excitation intensities, material properties, construction method, etc. Therefore, physical experimentation is often conducted to better understand structural seismic response with the objective of calibrating numerical models that can be used to simulate structural seismic response for structural design.

There are several experimental methods that are commonly used in earthquake engineering research, including conventional methods such as shake table testing (STT) and quasi-static testing (QST). A newer method that combines physical testing and numerical simulation is known as hybrid simulation. This section briefly discusses these seismic experimental methods and introduces the concept of online model updating within the scope of hybrid simulation, which leads to a state-of-the-art advancement in hybrid simulation reliability.

1.1. Conventional Seismic Testing Methods

During conventional seismic testing, predefined loadings are applied to structural specimens. These conventional testing methods are open-loop type testing, implying that
feedback from the specimens is not required during the testing process. Both shake table testing (STT) and quasi-static testing (QST) are conventional seismic testing methods.

1.1.1. **Shake Table Testing (STT)**

During STT, the base of a structural specimen is fixed to a shake table which applies simulated seismic ground motion via hydraulic actuators. Inertial and damping effects are developed naturally within the specimen. With instrumentation attached/installled on the specimen, structural responses such as accelerations, displacements, and strains can be directly measured.

While STT may be the most intuitive form of earthquake engineering research, its versatility is limited. It usually requires the construction of an entire structural system as the specimen, which may not be economical. Moreover, the size and weight of the specimen is restricted by the size and load capacity of the shake table, respectively. This often entails the usage of small scale models, which may not accurately represent the behavior of a full scale structural system.

1.1.2. **Quasi-Static Testing (QST)**

During QST, hydraulic actuators are connected directly to a specimen and a slow predefined monotonic or cyclic loading history is applied. This allows for close investigation of hysteretic behavior and damage of a single structural member or small subassembly.

While QST is helpful in characterizing the post-yield structural performance of specimens, it is not a truly dynamic test, so inertial and damping effects cannot be captured, which is essential in earthquake engineering research. Moreover, interactions
between the specimen and the rest of the structural system cannot be accounted for in QST, so the specimen can only represent a standalone structure.

1.2. Hybrid Simulation

Hybrid simulation, as an advanced experimental method, is a combination of physical testing and numerical simulation. The most critical and/or complex components of a structural system that may be difficult to model numerically are physically tested using hydraulic actuators or shake tables. These components are known as the experimental substructure. Conversely, the rest of the structural system, which is generally simple to model and analyze, is numerically simulated and is thus called the numerical substructure. Hybrid simulations are executed using a time-stepping integration algorithm to solve an equation of motion which defines the responses of both the experimental and the numerical substructures. Consequently, the loadings imposed on the substructures are the same as they would be within the complete structural system. This allows overall system response to be established without testing the entire structural system. Since the response of the experimental substructure at each step is required for the integration algorithm to calculate loading commands of the next step, hybrid simulation is a closed-loop testing method. In other words, feedback from the specimen is required for the experiment to proceed during a hybrid simulation.

Hybrid simulation has advantages over pure numerical simulation as it addresses modeling uncertainties by replacing components that are difficult to model with physical specimens. Hybrid simulation also addresses many of the limitations associated with conventional testing methods such as STT. For example, since only a small portion of the structure needs to be physically constructed, it is much more economical than STT.
Furthermore, the size and weight restrictions present in STT are generally increased. In hybrid simulation, size is only limited by the amount of available space in the lab, and weight is only limited by the capacities of the strong floor and reaction frame. Hybrid simulation has advantages over QST as well, since inertial and damping effects can be captured, and interactions between the specimen and the rest of the structural system are accounted for, which is critical for evaluation of dynamic behavior of structural systems subjected to earthquake loading.

Hybrid simulation is divided into two broad categories: dynamic hybrid simulation and pseudodynamic hybrid simulation (Shao, et al., 2011). In dynamic hybrid simulation, inertial and damping effects are developed naturally within the experimental substructure. They can be executed using shake tables, dynamic actuators, or both. The loading commands sent to the experimental substructure take into account the ground motion input and the interaction with the numerical substructure. In pseudodynamic (PSD) hybrid simulation, inertial and damping effects of the experimental substructure are numerically simulated, allowing the test to be conducted at a slow rate or in real-time. PSD hybrid simulation is discussed in detail below.

1.2.1. Pseudodynamic (PSD) Hybrid Simulation

During a PSD hybrid simulation, the dynamic effects of the experimental substructure are numerically simulated and the specimen’s dynamic restoring force is captured experimentally. Since the dynamic effects are numerically simulated, PSD hybrid simulations can be conducted on an extended time scale (slow PSD) if high-velocity actuators are unavailable. However, the hysteretic response of some structures is rate-dependent, and thus conducting the PSD hybrid simulation at a slow rate would not
capture these effects. To address this issue, real-time PSD hybrid simulations have been continuously developed during the last two decades to resolve implementation challenges such as actuator time delay compensation and stability of numerical algorithms (Wallace, et al., 2005; Wu, et al., 2005; Stojadinovic, et al., 2006; Mercan & Ricles, 2007; Ahmadizadeh, et al., 2008; Chen & Ricles, 2009; Shao, et al., 2011).

1.2.2. Online Model Updating in Hybrid Simulation

Hybrid simulation is perhaps best suitable for structural systems in which the complex nonlinear behavior to be represented by the experimental substructure is concentrated in just a few areas of the structure. If the nonlinear behavior is instead distributed throughout the structure, it becomes uneconomical and infeasible to utilize numerous experimental substructures. Specimen construction would be very costly and there would likely be an insufficient number of hydraulic actuators in any given lab. Nevertheless, in many cases the numerical substructures in a hybrid simulation are intended to exhibit similar or even the same nonlinear behavior as the experimental substructure. In these instances, it is challenging to accurately model the numerical substructures to echo the behavior of the experimental substructure because nonlinear data from the experimental substructure is not available until the hybrid simulation has already begun. The nonlinear behavior of the numerical substructures can only be approximated, which could very likely lead to inconsistencies between the experimental and numerical substructures.

One potentially effective means of reducing these inconsistencies is online model updating of the numerical substructures. In this method, the most critical structural element that is expected to exhibit nonlinear hysteresis first is selected as the experimental substructure. The structural elements that are intended to have similar
properties as the experimental substructure are modeled as numerical substructures with adjustable parameters defining their hysteresis. As the hybrid simulation proceeds and the experimental substructure begins to exhibit hysteresis, the experimental data is used to identify a set of parameters that appropriately defines its hysteresis. These parameters are then applied to the numerical substructures during the hybrid simulation. In brief, the numerical substructures are updated to match the experimental substructure as data becomes available.

Online model updating applied to hybrid simulation has the potential to significantly increase the accuracy of numerical substructure modeling, and thus the overall accuracy of the structural system response. In addition, utilizing online model updating is economical, as the required number of experimental substructures decreases. For example, consider a traditional hybrid simulation of a multistory building without online model updating. To obtain the most accurate structural response, every story that exhibits nonlinear behavior would need to be an experimental substructure. However, if online model updating is implemented as illustrated in Figure 1.1, accurate results can be obtained by simply assigning the most critical story as the experimental substructure, since the numerical substructures inherit the properties of the experimental substructure throughout the hybrid simulation.
1.3. Literature Review on Model Updating

Recently, many researchers have begun to investigate model updating due to its perceived advantages in hybrid simulation and in other structural engineering applications. Table 1.1 contains a list of recent numerical and experimental studies that have used model updating. For the experimental studies, the table lists whether the tests were real-time or slow. In addition, the testing method, parameter identification algorithm, and hysteresis model is listed for each study.

Figure 1.1: Illustration of hybrid simulation of a multistory building with online model updating
<table>
<thead>
<tr>
<th>Author, Year</th>
<th>Experimental or Numerical</th>
<th>Real-Time or Slow</th>
<th>Testing Method</th>
<th>Parameter Identification Algorithm</th>
<th>Hysteresis Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yang, 2012</td>
<td>Numerical</td>
<td>N/A</td>
<td>Hybrid simulation</td>
<td>Simplex Method</td>
<td>Modified Giuffré-Menegotto-Pinto</td>
</tr>
<tr>
<td>Chen, 2013</td>
<td>Numerical</td>
<td>N/A</td>
<td>Hybrid simulation</td>
<td>N/A</td>
<td>Bouc-Wen</td>
</tr>
<tr>
<td>Hashemi, 2013</td>
<td>Experimental</td>
<td>Slow</td>
<td>Hybrid simulation</td>
<td>UKF</td>
<td>Bouc-Wen-Baber-Noori</td>
</tr>
<tr>
<td>Kwon, 2013</td>
<td>Both</td>
<td>Slow</td>
<td>Hybrid simulation</td>
<td>Objective function minimization</td>
<td>Numerical: Bouc-Wen-Baber-Noori Experimental: Giuffré-Menegotto-Pinto</td>
</tr>
<tr>
<td>Song, 2013</td>
<td>Both</td>
<td>Real-Time</td>
<td>QST and STT</td>
<td>UKF</td>
<td>Modified Bouc-Wen</td>
</tr>
<tr>
<td>Wang, 2013</td>
<td>Numerical</td>
<td>N/A</td>
<td>Hybrid simulation</td>
<td>Constrained UKF</td>
<td>Bouc-Wen</td>
</tr>
<tr>
<td>Elanwar, 2014</td>
<td>Numerical</td>
<td>N/A</td>
<td>Hybrid simulation</td>
<td>Genetic Algorithm</td>
<td>Nonlinear concrete and bilinear steel</td>
</tr>
</tbody>
</table>

Table 1.1: Model updating in recent studies
1.3.1. Numerical Studies

1.3.1.1. Unscented Kalman Filter (UKF)

Many researchers have employed the Unscented Kalman Filter (UKF), or variations thereof, as a tool for hysteresis parameter identification, which is a necessary component of model updating. For example, Song and Dyke (2013) conducted a numerical study to examine the performance of the UKF for parameter identification of a modified form of the Bouc-Wen model. It was concluded that the UKF can accurately capture the behavior of nonlinear models under low noise levels.

Wang and Wu (2013) indicated that the UKF does not consider the constraints of hysteresis parameters, which can cause large errors or possible divergent responses of parameters. A constrained UKF (CUKF) algorithm was proposed to alleviate this issue and was implemented with the Bouc-Wen model in a numerical hybrid simulation with model updating. It was concluded that the CUKF decreases parameter fluctuation and improves convergence speed compared to the UKF.

1.3.1.2. Optimization Techniques

Other researchers have studied the use of optimization techniques for parameter identification. For instance, Yang et al. (2012) proposed a model updating technique in which the Simplex Method is used to find a set of hysteresis parameters that minimize an error indicator, which is defined as the cumulative difference of strain energy between the experimental and numerical substructures. Since the method requires performing a nonlinear analysis for several sets of trial parameters, it is only suitable for slow hybrid simulations. A numerical hybrid simulation was performed on a two-pier bridge to verify
the method. The “experimental” pier was assigned a sophisticated fiber-sectioned element, while the numerical pier used a modified form of the Giuffré-Menegotto-Pinto model. It was concluded that the method is capable of improving the accuracy of hybrid simulation with multiple identical substructures.

Kwon and Kammula (2013) developed a novel model updating method in which the experimental substructure has several numerical counterparts with different sets of modeling parameters to represent the possible range of hysteresis of the experimental substructure. The numerical counterparts are all subjected to the same displacement as the experimental substructure, and a weighted average of the restoring forces of the counterparts is calculated. The weight factor for each counterpart is calibrated by minimizing the error between the weighted average and the restoring force of the experimental substructure. The proposed method was verified through a numerical hybrid simulation of an idealized two story structure, in which the first story was the “experimental” substructure and the second story was the numerical substructure. The Bouc-Wen-Baber-Noori (Baber & Noori, 1985) model was used for both substructures.

Elanwar and Elnashai (2014) take a different approach, in which the constitutive relationship of the numerical substructures is updated instead of the hysteretic behavior. A “shadow” finite element model is created and its constitutive relationship is compared to that of the experimental substructure at each step. If the error between them exceeds a given tolerance, a genetic algorithm is used to update the shadow. Two numerical hybrid simulations of a two-bay one-story frame were executed using finite element software. For both simulations, the parameters governing the constitutive relationship of the left column were identified and applied to the other two columns. A bilinear steel model was
used for the first simulation, whereas a nonlinear concrete model with bilinear steel reinforcement bars was used for the second one. It was concluded that updating the constitutive relationship of numerical substructures during a hybrid simulation can significantly reduce errors.

1.3.1.3. Reliability Assessment

Researchers have also looked into reliability assessment to quantify the cumulative effect of modeling errors for hybrid simulations with online model updating. Chen, et al. (2013) developed a reliability assessment tool called the model accuracy indicator (MAI). First, a numerical hybrid simulation of a single-degree-of-freedom (SDOF) system with one experimental spring and one numerical spring was carried out. Model updating was not employed to show that errors in the hysteresis parameters of numerical substructures could lead to inaccurate results. Subsequently, the MAI was formulated based on the synchronized subspace plot between the measured restoring force of the experimental substructure and the predicted restoring force of a corresponding numerical model subject to the same displacement response history. Further numerical simulations indicated that the MAI has great potential for reliability assessment.

1.3.2. Experimental Studies

1.3.2.1. Slow PSD Hybrid Simulation with Online Model Updating

So far there have been two instances of slow PSD hybrid simulation with online model updating reported in the literature. Hashemi, et al. (2013) performed a slow PSD hybrid simulation of a one-bay frame model consisting of nonlinear rotational springs at the base of each column and an elastic truss element connecting the top of both columns. The left
column and spring were represented by an experimental steel column bolted to a clevis at the base with sacrificial steel coupons, which provided the nonlinear behavior. The right spring acted as the adjustable part of the numerical substructure and was updated based on the measured response of the experimental substructure. The Bouc-Wen model with degrading behavior as implemented by Baber and Noori was used to model the right spring, and the UKF was used to identify its parameters. The UKF successfully delivered relatively accurate estimates of model parameters.

Kwon and Kammula (2013) utilized their previously described model updating technique in a slow PSD hybrid simulation as well. A single story structure with two slender braces was used. One brace was the experimental substructure whereas the other one was the numerical substructure modeled with the Giuffré-Menegotto-Pinto model. It was concluded that the proposed method shows potential for model updating in hybrid simulation but requires further theoretical investigation.

1.3.2.2. Real-Time Model Updating

No instances of real-time hybrid simulation (RTHS) with online model updating have yet been reported in the literature. However, Song and Dyke (2013) employed the UKF for real-time model updating in an experimental study. Two QSTs and a STT were conducted on a shear building structure, and it was concluded that the UKF is capable of model updating in real-time and can provide similar or better results than offline model updating. This demonstrates that the UKF shows promise for other real-time applications such as RTHS with online model updating.
1.4. Justification, Objectives, and Scope

As highlighted in section 1.2.1, performing PSD hybrid simulations in real-time is important so that rate-dependent behavior of experimental substructures can be captured to improve understanding of structural seismic response. In addition, section 1.2.2 emphasizes the significance of online model updating in hybrid simulation, as it can increase the accuracy of numerical substructure modeling in an economical manner. However, to the author’s knowledge, no researcher has yet attempted real-time hybrid simulation (RTHS) with online model updating. As shown in Table 1.1, Hashemi and Kwon performed slow hybrid simulations with model updating, and Song conducted real-time model updating, but not in the context of hybrid simulation.

Therefore, the primary objective of this study is to implement online model updating in real-time hybrid simulation (RTHS). The UKF was adopted in this study for parameter identification of the experimental substructure due to its demonstrated capability in previous studies. Since the Bouc-Wen model has been used alongside the UKF in previous studies (see Table 1.1), it was selected to model hysteresis in this study. To achieve the primary objective, an incremental approach was adopted, which began with numerical simulation of Bouc-Wen parameter identification. The hysteretic response of a single-degree-of-freedom (SDOF) system subjected to earthquake ground motion was simulated, and the UKF was used to identify the Bouc-Wen parameters defining the hysteresis. Then online model updating was simulated through a numerical hybrid simulation of a three-story shear-type building, and it was proven that the proposed online model updating technique can significantly improve the accuracy of the results. An experimental study was carried out next, which began with a QST of the test
specimen. Using the hysteretic response from the QST, the Bouc-Wen parameters defining the hysteresis of the specimen were identified offline using the UKF. Next, a stable RTHS without online model updating was realized, and the results demonstrated the need for online model updating. Lastly, six RTHS with online model updating were performed to verify the effectiveness of the proposed method.

The secondary objective of this study is to understand the effects of the UKF system model parameters when used with the Bouc-Wen model. Despite the abundance of literature on the use of the Bouc-Wen model in conjunction with the UKF, there appears to be a lack of information regarding how to select appropriate system model parameters for the UKF. To achieve the secondary objective, an in-depth parametric study of the UKF system model was conducted using the SDOF system mentioned previously. An assumed set of structural parameters was used for the SDOF system which allowed general observations and recommendations to be made regarding selection of the UKF system model parameters. However, the behavior of the UKF may vary depending on the specific structure under investigation, so the recommendations made in this parametric study may not be universal for all applications.

1.5. Thesis Outline

This thesis explores the implementation of online model updating in RTHS through numerical and experimental studies.

Chapter 2 presents the theoretical background behind PSD hybrid simulation and the model updating procedure developed in this study. Topics covered include numerical integration used in hybrid simulation, the Bouc-Wen hysteresis model, and the Unscented Kalman Filter (UKF).
Chapter 3 contains the numerical study, which first describes Bouc-Wen parameter identification of a SDOF system using the UKF. A parametric study of the UKF system model parameters is then presented using the SDOF system. Lastly, numerical hybrid simulation with model updating demonstrates the functionality of the model updating technique.

Chapter 4 comprises the experimental study that was conducted using the versatile hybrid testing system at the Laboratory of Earthquake and Structural Simulation (LESS). A discussion of the hardware and software at LESS is provided in addition to a description of the specimen used for the tests. Offline parameter identification using results from a QST of the specimen is presented. Then real-time compensation is addressed and verified through a RTHS without online model updating. Lastly, six proof-of-concept RTHS with online model updating are presented.

Chapter 5 summarizes the findings of this study and includes suggestions for future work involving RTHS with online model updating.
2. Theoretical Background

This chapter begins with a discussion of the fundamentals of pseudodynamic (PSD) hybrid simulation employed in this study. The equation of motion (EOM) is first introduced for elastic single-degree-of-freedom (SDOF) systems and is then extended for inelastic and multiple-degree-of-freedom (MDOF) systems. Next, numerical integration of the EOM is described in the context of PSD hybrid simulation, and the attributes and pros and cons of explicit and implicit algorithms are presented. The Explicit Newmark algorithm, which is selected for numerical integration in this study, is then formulated. Subsequently, the overall framework of PSD hybrid simulation is illustrated.

The Bouc-Wen model, which is used to model structural hysteresis in this study, is explained next. The governing equations are presented followed by descriptions of the parameters that define the shape of the hysteretic loop. The bounds on these parameters are established, and the effect of each parameter on the shape of the hysteretic loop is illustrated through an example.

Lastly, the Unscented Kalman Filter (UKF), which performs parameter identification for the model updating in this study, is outlined through detailed explanations of each of its five phases.
2.1. Pseudodynamic (PSD) Hybrid Simulation Fundamentals

2.1.1. Equation of Motion (EOM)

When an external dynamic force, such as earthquake excitation, is imposed on a structure, it is counteracted by an inertial force, an energy dissipation force, and a restoring force, according to structural dynamic equilibrium. By applying Newton’s second law, the equation governing the motion of a lumped mass in a single direction, or degree-of-freedom (DOF), can be determined. This equation is known as the equation of motion (EOM), and such a structure is called a single-degree-of-freedom (SDOF) system.

For an elastic SDOF system, the EOM is:

\[ m\ddot{u} + c\dot{u} + ku = p \]  

where \( m \), \( c \), and \( k \) are the mass, damping coefficient, and stiffness, respectively; \( \ddot{u} \), \( \dot{u} \), and \( u \) are the acceleration, velocity, and displacement responses, respectively; and \( p \) is the external dynamic force. When the dynamic force is due to ground motion, such as an earthquake excitation, it is defined as the opposite of the product of mass and the acceleration of the ground, and the EOM becomes:

\[ m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \]  

where \( \ddot{u}_g \) is the predefined ground acceleration history. If the structure’s response exceeds its elastic limit, the force-displacement relationship is no longer linear, and the restoring force cannot be defined simply in terms of a linear stiffness; rather, the restoring force is expressed as a function of the displacement and velocity of the system, and the EOM is:

\[ m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = -m\dddot{u}_g \]
where \( f_s \) is the restoring force function, which is related to the hysteretic response of the structure. There are different methods of numerically modeling \( f_s \), and one of the most common is the Bouc-Wen hysteresis model, which is detailed in Section 2.2.

For a multiple-degree-of-freedom (MDOF) system, the mass and damping properties are presented using matrices. The acceleration, velocity, displacement, restoring force, and external force become vectors. In addition, the external force vector necessitates an influence vector \( \mathbf{\iota} \), which represents the displacements of the masses resulting from static application of a unit ground displacement. The EOM for a MDOF system is:

\[
\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + f_s(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{p} = -\mathbf{m}\dddot{\mathbf{u}}_g
\]

where \( \mathbf{m} \) and \( \mathbf{c} \) are the mass and damping matrices, respectively; \( \mathbf{p} \) is the external force vector; and \( \dddot{\mathbf{u}} \), \( \dot{\mathbf{u}} \), and \( \mathbf{u} \) are the acceleration, velocity, and displacement vectors, respectively.

### 2.1.2. Numerical Integration

Since earthquake ground motions are random and the resulting modeled external forces in the EOM cannot be expressed using general functions, it is not possible to solve the EOM using classical differential equation solution methods. Instead, time-stepping integration algorithms are utilized. This allows solution of the EOM for each time step of the ground motion input. There are two broad categories of integration algorithms: explicit and implicit.

For explicit algorithms, solving the structural response of the next time step is straightforward, as it depends only on the structural response of the current and previous
steps, which have been computed. This makes explicit algorithms very simple, quick, and computationally efficient. However, explicit algorithms are conditionally stable, which means there is an upper limit on the step size that can be used. If a step size greater than the upper limit is used, the algorithm becomes unstable (Schellenberg, et al., 2009). The upper limit is proportional to the shortest period of the structure. For large, complicated structures with many DOFs, the shortest period tends to be very small, which can make the upper limit on the step size unreasonable.

On the contrary, for implicit algorithms, the structural response of the next time step depends on the response of the next time step as well as the computed response of the current and previous time steps. Because of this condition, algebraic formulae need to be solved to determine the solution at the end of each time step, requiring an iterative solution scheme. However, implicit algorithms are unconditionally stable, so larger step sizes can be used. This makes them ideal for highly refined structures with many DOFs. Naturally, implicit algorithms tend to be more computationally expensive due to their iterations.

When performing hybrid simulation, it is best to avoid or at least limit iterations. This is because commands sent to experimental substructures during the iteration process can overshoot the true displacements. This introduces false loading/unloading cycles, which can impair the desired behavior of the specimen. Nevertheless, implicit algorithms can be modified for use in hybrid simulation. For example, Shing and Vannan (1991) proposed a displacement reduction factor to reduce the chance of displacement overshooting and attain a more uniform convergence rate. Additionally, Dorka and
Heiland (1991) and Shing, et al. (1991) suggested fixing the number of iterations to a constant value.

The structure in this study is a shear-type building, so a simple and straightforward explicit algorithm was sought. According to Schellenberg, et al. (2009), the Explicit Newmark integration algorithm is the most straightforward among the explicit family. It does not require iterations, so displacement command overshoot is not a concern. Furthermore, no information on the stiffness of the structure is required within the algorithm, eliminating the need for tangent stiffness calculations, which are necessary when modified implicit algorithms are used for PSD hybrid simulation. On the other hand, the primary disadvantage of the Explicit Newmark algorithm is its inability to establish algorithmic damping to suppress modes with high frequencies. However, this is not a significant concern when dealing with simple structures with a small number of DOFs, like the shear-type building in this study. Hence, the Explicit Newmark algorithm was selected to solve the equation of motion in this study and is presented next.

2.1.3. Explicit Newmark Algorithm

Newmark integration algorithms utilize the following finite difference formulae for displacement and velocity:

\[
\mathbf{u}_{k+1} = \mathbf{u}_k + \Delta \mathbf{u} + \frac{\Delta t^2}{2} \left( (1-2\beta)\ddot{\mathbf{u}}_k + 2\beta \ddot{\mathbf{u}}_{k+1} \right) \tag{2.5}
\]

\[
\ddot{\mathbf{u}}_{k+1} = \ddot{\mathbf{u}}_k + \Delta t \left( (1-\gamma)\dddot{\mathbf{u}}_k + \gamma \dddot{\mathbf{u}}_{k+1} \right) \tag{2.6}
\]

where \( \dddot{\mathbf{u}}_k, \dddot{\mathbf{u}}_k, \) and \( \mathbf{u}_k \) are the acceleration, velocity, and displacement vectors, respectively, at time step \( k \); \( \Delta t \) is the step size of the integration; and \( \beta \) and \( \gamma \) are parameters that determine the variation of the accelerations over a time step.
For the Explicit Newmark algorithm, \( \beta \) is set to zero, and Equations 2.5 and 2.6 can be rewritten as:

\[
\begin{align*}
\mathbf{u}_{k+1} &= \mathbf{u}_k + \Delta t \mathbf{\dot{u}}_k + \frac{\Delta t^2}{2} \mathbf{\ddot{u}}_k = \mathbf{\ddot{u}}_{k+1} \\
\mathbf{\dot{u}}_{k+1} &= \mathbf{\dot{u}}_k + \Delta t \left( (1-\gamma) \mathbf{\dot{u}}_k + \gamma \mathbf{\dot{u}}_{k+1} \right) = \mathbf{\dot{u}}_{k+1} + \Delta t \gamma \mathbf{\ddot{u}}_{k+1}
\end{align*}
\]

where \( \mathbf{\ddot{u}}_{k+1} \) and \( \mathbf{\dot{u}}_{k+1} \) are henceforth referred to as the target displacement vector and target velocity vector. The following system of linear equations is obtained by substituting Equations 2.7 and 2.8 into the EOM in Equation 2.4:

\[
\mathbf{m}_{\text{eff}} \mathbf{\ddot{u}}_{k+1} = \mathbf{p}_{\text{eff}}
\]

where:

\[
\begin{align*}
\mathbf{m}_{\text{eff}} &= \mathbf{m} + \Delta t \gamma \mathbf{c} \\
\mathbf{p}_{\text{eff}} &= \mathbf{p}_{k+1} - \mathbf{f}(\mathbf{\ddot{u}}_{k+1}, \mathbf{\dot{u}}_{k+1}) - \mathbf{c} \mathbf{\dot{u}}_{k+1}
\end{align*}
\]

The effective mass matrix \( \mathbf{m}_{\text{eff}} \) is constant and is thus calculated prior to the start of the time-stepping calculations. The effective force vector \( \mathbf{p}_{\text{eff}} \) is evaluated at each time step using the target displacement and velocity vectors. After Equation 2.9 is solved for the acceleration vector of the next time step \( \mathbf{\ddot{u}}_{k+1} \), Equations 2.7 and 2.8 are utilized to update the target displacement and velocity vectors:

\[
\begin{align*}
\mathbf{u}_{k+1} &= \mathbf{\ddot{u}}_{k+1} \\
\mathbf{\dot{u}}_{k+1} &= \mathbf{\dot{u}}_{k+1} + \Delta t \gamma \mathbf{\ddot{u}}_{k+1}
\end{align*}
\]

The updated displacement and velocity vectors are in turn used in the next time step of the integration. This procedure is repeated for the entire duration of the external force. The steps of the Explicit Newmark algorithm are listed in Table 2.1.
If the parameter $\gamma$ is greater than 0.5, the algorithm introduces too much numerical damping, affecting the participation of lower modes to the solution. Therefore, $\gamma$ is usually set to 0.5, causing the Explicit Newmark algorithm to yield identical solutions to the central difference method (Schellenberg, et al., 2009). The initial conditions for displacement, velocity, and acceleration (i.e. $\mathbf{u}_0$, $\mathbf{u}_0$, and $\mathbf{u}_0$) must be defined before the start of the time-stepping calculations. For systems initially at rest, $\mathbf{u}_0$ and $\mathbf{u}_0$ are vectors of zeros, and $\mathbf{u}_0$ is calculated from the EOM. As previously stated, for explicit algorithms, the step size must meet a criterion for stability. For Explicit Newmark, the stability criterion is:

$$\Delta t \leq \frac{T_n}{\pi} \quad 2.14$$

where $T_n$ is the shortest period of the structure under investigation (Schellenberg, et al., 2009).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{m}_{\text{eff}} = \mathbf{m} + \Delta t \gamma \mathbf{c}$</td>
<td>Initialize: $\mathbf{u}_0$, $\mathbf{u}_0$, $\mathbf{u}_0$</td>
</tr>
<tr>
<td>$\mathbf{u}_{k+1} = \mathbf{u}_k + \Delta t \mathbf{u}_k + \frac{\Delta t^2}{2} \mathbf{u}_k$</td>
<td>Time-Stepping Calculations:</td>
</tr>
<tr>
<td>$\dot{\mathbf{u}}_{k+1} = \dot{\mathbf{u}}_k + \Delta t (1 - \gamma) \dot{\mathbf{u}}_k$</td>
<td>$\mathbf{u}<em>{k+1} = \mathbf{u}</em>{k+1} - f(z) (\dot{\mathbf{u}}<em>{k+1} \ddot{\mathbf{u}}</em>{k+1}) - c\mathbf{u}_{k+1}$</td>
</tr>
<tr>
<td>$\mathbf{p}<em>{\text{eff}} = \mathbf{p}</em>{k+1} - f(z) (\dot{\mathbf{u}}<em>{k+1} \ddot{\mathbf{u}}</em>{k+1}) - c\mathbf{u}_{k+1}$</td>
<td>$\mathbf{u}<em>{k+1} = \mathbf{u}</em>{k+1}$</td>
</tr>
<tr>
<td>$\ddot{\mathbf{u}}<em>{k+1} = \mathbf{m}</em>{\text{eff}}^{-1} \mathbf{p}_{\text{eff}}$</td>
<td>$\dot{\mathbf{u}}<em>{k+1} = \dot{\mathbf{u}}</em>{k+1} + \Delta t \gamma \ddot{\mathbf{u}}_{k+1}$</td>
</tr>
</tbody>
</table>

**Table 2.1: Explicit Newmark formulation**
2.1.4. PSD Hybrid Simulation Framework

Figure 2.1 shows a conceptual diagram of the PSD hybrid simulation framework. The three-story shear-type building shown is divided so that the first story is the experimental substructure, and the upper two stories are the numerical substructures. The EOM is solved at each time step using the Explicit Newmark algorithm described previously. For a shear-type building, the influence vector $\mathbf{i}$ in the EOM is a vector of ones. The rate-dependent terms (inertial forces $\mathbf{m}\ddot{\mathbf{u}}$ and damping forces $\mathbf{c}\dot{\mathbf{u}}$) and the external forces $-\mathbf{m}\ddot{\mathbf{u}}_g$ in the EOM are numerically simulated.

**Blue:** Numerically simulated  
**Red:** For real-time PSD  
**Green:** For model updating

![Conceptual diagram of PSD hybrid simulation framework](image)

Figure 2.1: Conceptual diagram of PSD hybrid simulation framework
The restoring force vector \( \mathbf{f}_s(\mathbf{u}, \dot{\mathbf{u}}) \) is formed from the individual restoring forces of the substructures. The target displacement and velocity vectors (denoted by \( \dot{\mathbf{u}}_{k+1} \) and \( \dot{\mathbf{u}}_{k+1} \) in Explicit Newmark) are divided into a numerical component, which is sent to the numerical hysteresis models, and an experimental component, which is imposed on the specimen. For slow PSD, the displacements imposed on the specimen do not need to be sent at the actual target velocity, but for real-time PSD, it is required so that rate-dependent hysteretic effects can be captured. For the numerical substructures, the hysteresis models compute restoring forces, and for the experimental substructure, the restoring force is measured from the specimen using a load cell. The complete restoring force vector \( \mathbf{f}_s(\mathbf{u}, \dot{\mathbf{u}}) \) is then fed back into the numerical integration algorithm so that the target displacement and velocity vectors of the next step can be determined.

When adopting model updating in PSD hybrid simulation, responses measured from the experimental substructure are also utilized to identify hysteresis model parameters, which are then applied to numerical substructures that have the same structural configuration as the experimental substructure. In this study, the Unscented Kalman Filter (UKF) is used for parameter identification, and it is discussed in section 2.3.
2.2. Bouc-Wen Hysteresis Model

In this study, the Bouc-Wen hysteresis model is adopted. It can be used to describe the nonlinear relationship between the displacement, velocity, and restoring force of structures. It was introduced by Bouc (1967) and extended by Wen (1976). By varying the parameters that define the shape of the hysteretic loop, various hysteresis patterns can be created and fitted to experimental data, so a numerical model of a physical structural component can be extracted from experiments. The Bouc-Wen model and its variations are widely used within structural engineering to model the behavior of all types of materials.

In this study, the inputs of the Bouc-Wen model are structural displacement and velocity responses, and the output is restoring force. Two equations are used to define the relationship between the inputs and outputs:

\[
fs = \alpha ku + (1 - \alpha) kz 
\]
\[\text{Equation 2.23} \]

\[
\dot{z} = A\ddot{u} - \beta |\ddot{u}|z^{n-1} z - \gamma |\ddot{u}|z^n 
\]
\[\text{Equation 2.24} \]

where \(z\) is the hysteretic displacement and \(A, k, \beta, \gamma, n,\) and \(\alpha\) are the parameters that affect the shape of the hysteretic loop. All other variables are as previously defined. Equation 2.24 is a differential equation with initial condition \(z = 0\) at time zero. Once Equation 2.24 is solved, Equation 2.23 can be easily computed to obtain the restoring force, which consists of two terms: an elastic term \(\alpha ku\) and a hysteretic term \((1 - \alpha)kz\).

The bounds on \(n, \beta,\) and \(\gamma\) can be determined through a thermodynamic analysis (Ismail, et al., 2009) and are:

\[n > 0\]
\[\beta > 0\]
\[-\beta \leq \gamma \leq \beta\]
In addition, the bounds on $k$ and $\alpha$ are:

\[
\begin{align*}
  k &> 0 \\
  0 < \alpha < 1
\end{align*}
\]

The parameter $k$ defines the initial or pre-yield stiffness and has a unit of force per length. All other parameters are unitless. The parameter $\alpha$ is the ratio of the post-yield stiffness $k_f$ to the pre-yield stiffness. The parameter $n$ controls the sharpness of the yield. The parameters $A$, $\beta$, and $\gamma$ represent the size and shape of the hysteretic loop, but they are redundant. In other words, the same hysteretic loop can be created using different combinations of $A$, $\beta$, and $\gamma$. Therefore, it is common to set $A=1.0$ to ensure singularity (Ma, et al., 2004).

**Figure 2.2: Bouc-Wen hysteresis example**
Figure 2.2 shows an example hysteretic loop for the following parameters: $A = 1.0$, $k = 1.0$, $\beta = 1.5$, $\gamma = 0.0$, $n = 2.0$, and $\alpha = 0.3$. Figure 2.3 demonstrates the effects of decreasing and increasing each parameter individually while the other parameters are kept the same as above. Parts (a) and (b) show that changing $k$ simply scales the loop in the vertical direction. If $k$ decreases (a), the loop shrinks in the vertical direction, but if it increases (b), the loop is stretched in the vertical direction. Parts (c) and (d) show that $\beta$ controls the size and shape of the loop. If $\beta$ decreases (c), the loop broadens, but if it increases (d), the loop becomes narrower. A similar trend can be seen for $\gamma$ in parts (e) and (f). Part (g) shows that if $n$ decreases, the yield becomes more gentle and curved, but part (h) shows that if $n$ increases, the yield becomes sharper. As $n \to \infty$, the behavior becomes perfectly bilinear. Part (g) shows that if $\alpha$ decreases, the post-yield stiffness decreases. As $\alpha \to 0$, the behavior becomes perfectly elasto-plastic. Part (h) shows that if $\alpha$ increases, the post-yield stiffness increases, resulting in quasi-linearity. As $\alpha \to 1$, the behavior becomes perfectly linear.
Figure 2.3: Effects of decreasing and increasing Bouc-Wen parameters
The Unscented Kalman Filter (UKF) is an algorithm that estimates the state of a system based on a known system model and a series of noisy measurements. It is an extension of the ordinary Kalman filter, which is only applicable to linear systems. The UKF, on the other hand, can be applied to nonlinear systems, such as the Bouc-Wen hysteresis model presented in Section 2.2. In this study, the UKF is a key component for model updating. It is used for parameter identification of the experimental substructure (see Figure 2.1). Consider a nonlinear system defined by the following two equations:

\[
    n = 1.0 \\
    n = 10 \\
    \alpha = 0.1 \\
    \alpha = 0.9
\]
where $\mathbf{x}$ is a vector of the system state variables, $\mathbf{u}$ is the system input, $\mathbf{y}$ is a vector of the measurements, $\mathbf{v}$ and $\mathbf{w}$ are the process noise and measurement noise vectors respectively, and $k$ is the time step. The nonlinear function $f$ updates the state vector $\mathbf{x}$ for the next time step, and the nonlinear function $h$ predicts the measurement $\mathbf{y}$ based on the state vector $\mathbf{x}$. The process noise and measurement noise ($\mathbf{v}$ and $\mathbf{w}$) are assumed to be zero-mean Gaussian white noise with covariance matrices of $\mathbf{Q}$ and $\mathbf{R}$, respectively.

The UKF updates two variables each time step: $\hat{\mathbf{x}}_{k|k}$, which is an estimate of the system state at time step $k$, gets updated to $\hat{\mathbf{x}}_{k+1|k+1}$; and $\mathbf{P}_{k|k}$, which is the covariance matrix at time step $k$, gets updated to $\mathbf{P}_{k+1|k+1}$. Initial values $\hat{\mathbf{x}}_{0|0}$ and $\mathbf{P}_{0|0}$ must be set prior to performing time-stepping calculations. Figure 2.4 shows a block diagram of the UKF, which consists of five phases that are discussed in detail in the following sections.
2.3.1. Sigma Points

Sigma points are calculated differently in the standard UKF and the constrained UKF (CUKF). The standard UKF first selects a set of $2n+1$ sigma points, where $n$ is the number of elements in the state estimation vector $\hat{x}_{k|k}$, and places them into a single matrix $\chi_{k|k}$. In Equation 2.32, $\left(\sqrt{(n+\kappa)}P_{k|k}\right)_i$ represents the $i$th column of the matrix...
square root of \( (n + \kappa)P_{kj} \). Therefore, the dimensions of \( \chi_{klj} \) are \( n \times (2n+1) \). In addition, a weight \( W_i \) is defined for each sigma point. The weights are constant and do not depend on the time step \( k \). The parameter \( \kappa \) is for scaling and can be set to any real number.

\[
\chi_{klj} = \left[ \hat{x}_{klk} \right]_i + \left( \sqrt{(n + \kappa)P_{klj}} \right)_i \hat{x}_{klk} - \left( \sqrt{(n + \kappa)P_{klj}} \right)_i, \quad i = 1, 2, \ldots, n \tag{2.32}
\]

\[
W_0 = \frac{\kappa}{n + \kappa} \tag{2.33}
\]

\[
W_i = \frac{\kappa}{2(n + \kappa)}, \quad i = 1, 2, \ldots, 2n \tag{2.34}
\]

For the CUKF (Wang & Wu, 2013), an algorithmic parameter \( \theta_{k,i} \) denotes the sampling step size corresponding to the \( i \)th column of \( \sqrt{P_{klj}} \), and Equation 2.32 becomes:

\[
\chi_{klj} = \left[ \hat{x}_{klk} \right]_i + \theta_{k,i} \left( \sqrt{P_{klj}} \right)_i \hat{x}_{klk} - \theta_{k,i} \left( \sqrt{P_{klj}} \right)_i, \quad i = 1, 2, \ldots, n \tag{2.35}
\]

The sampling step sizes \( \theta_{k,i} \) are calculated according to the following equations:

\[
\theta_{k,i} = \theta_{k,n+i} = \min \left( \theta_{k,i}^C, \theta_{k,n+i}^C \right), \quad i = 1, 2, \ldots, n \tag{2.36}
\]

\[
\theta_{k,i}^C = \min \left( \theta_{k,i}^L, \theta_{k,i}^U \right), \quad i = 1, 2, \ldots, 2n \tag{2.37}
\]

\[
\theta_{k,i}^L = \min_{j=1}^{n} \left\{ \sqrt{n + \kappa} \left[ (x_L)_j - (\hat{x}_{klk})_j \right] / \left( \sqrt{P_{klj}} \right)_j \right\}, \quad i = 1, 2, \ldots, 2n \tag{2.38}
\]

\[
\theta_{k,i}^U = \min_{j=1}^{n} \left\{ \sqrt{n + \kappa} \left[ (x_U)_j - (\hat{x}_{klk})_j \right] / \left( \sqrt{P_{klj}} \right)_j \right\}, \quad i = 1, 2, \ldots, 2n \tag{2.39}
\]

where the subscript \( i \) is the column index, the subscript \( j \) is the row index, and the vectors \( x_L \) and \( x_U \) contain the lower bounds and upper bounds of the system state variables in \( \hat{x}_{klk} \), respectively.

The CUKF was not adopted for the numerical study in Chapter 3, but was found to be beneficial for the experimental study, and thus was implemented in Chapter 4.
2.3.2. State Prediction

Each sigma point is propagated through the $f$ function, and a weighted average is computed to obtain the predicted state estimate $\hat{x}_{k+1|k}$. The predicted covariance matrix $P_{k+1|k}$ is also computed.

$$
\left( \chi_{k+1|k} \right)_i = f \left( \left( \chi_{k|k} \right)_i , u_k \right), \quad i = 0,1,2, \ldots, 2n 
$$

2.40

$$
\hat{x}_{k+1|k} = \sum_{i=0}^{2n} W_i \left( \chi_{k+1|k} \right)_i
$$

2.41

$$
P_{k+1|k} = \sum_{i=0}^{2n} W_i \left[ \left( \chi_{k+1|k} \right)_i - \hat{x}_{k+1|k} \right] \left[ \left( \chi_{k+1|k} \right)_i - \hat{x}_{k+1|k} \right]^T + Q
$$

2.42

2.3.3. Measurement Prediction

Each updated sigma point from Equation 2.40 is propagated through the $h$ function, and a weighted average is computed to obtain the predicted measurement $\hat{y}_{k+1|k}$. It is important to note that for the $f$ function, the system input $u$ is at step $k+1$, but for the $h$ function, the system input $u$ is at step $k$. Thus, a unit delay is needed when feeding $u$ into the $f$ function in the state prediction phase. Lastly, the covariance matrix of the predicted measurement $P_{y_{k+1|k}^y}$ is computed.

$$
\left( Y_{k+1|k} \right)_i = h \left( \left( \chi_{k+1|k} \right)_i , u_{k+1} \right), \quad i = 0,1,2, \ldots, 2n
$$

2.43

$$
\hat{y}_{k+1|k} = \sum_{i=0}^{2n} W_i \left( Y_{k+1|k} \right)_i
$$

2.44

$$
P_{y_{k+1|k}^y} = \sum_{i=0}^{2n} W_i \left[ \left( Y_{k+1|k} \right)_i - \hat{y}_{k+1|k} \right] \left[ \left( Y_{k+1|k} \right)_i - \hat{y}_{k+1|k} \right]^T + R
$$

2.45

2.3.4. Kalman Gain

In this phase, the cross-covariance matrix $P_{y_{k+1|k}^y}$ is computed in order to calculate the Kalman gain $K_{k+1}$.
\[
P^{xy}_{k+1|k} = \sum_{i=0}^{2n} W_i \left[ (z_{k+1|k})_i - \hat{x}_{k+1|k} \right] \left[ (y_{k+1|k})_i - \hat{y}_{k+1|k} \right]^T \\
K_{k+1} = P^{xy}_{k+1|k} \left( P^{xy}_{k+1|k} \right)^{-1}
\]

2.3.5. State Update

The last phase of each time step updates the state estimate \( \hat{x}_{k+1|k+1} \). Here \( y_{k+1} \) is the measurement vector obtained from sensors. In addition, the covariance matrix \( P_{k+1|k+1} \) is updated.

\[
\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} (y_{k+1} - \hat{y}_{k+1|k}) \\
P_{k+1|k+1} = P_{k+1|k} - K_{k+1} P^{xy}_{k+1|k} K^T_{k+1}
\]
2.4. Summary

This chapter provided the theoretical foundation for the research presented in this thesis. First, the fundamentals of PSD hybrid simulation were reviewed and online model updating was introduced. The EOM typically used in earthquake engineering to solve structural seismic responses was presented. The Explicit Newmark algorithm was selected to solve the EOM due its straightforward formulation and fast calculation speed necessary for real-time hybrid simulation (RTHS). During a PSD hybrid simulation, the target displacement and velocity vectors are computed and sent to the experimental and numerical substructures, and the restoring force vector is obtained by combining the individual restoring forces of the substructures. When model updating is applied, parameters defining the experimental substructure’s response are identified and applied to the numerical substructures to enhance the hybrid simulation accuracy.

The Bouc-Wen hysteresis model was then discussed with a simple example to demonstrate the effects of the Bouc-Wen model parameters on the shape of the hysteretic loop.

Finally, the UKF, which is implemented for online parameter identification in hybrid simulation in this study, was introduced. The UKF computes an estimate of a system state based on a system model and a series of noisy measurements. Each of the five phases that occur within a time step of the UKF was discussed in detail.
3. Numerical Simulation and Parametric Study

This chapter consists of numerical simulations of Bouc-Wen parameter identification and hybrid simulation with online model updating. First, the Unscented Kalman Filter (UKF) was implemented to identify the Bouc-Wen parameters defining the hysteresis response of a single-degree-of-freedom (SDOF) system. To gain insight on the performance of the UKF when used for Bouc-Wen parameter identification and to determine optimal values of UKF system model parameters for the experimental study, an in-depth parametric study of the UKF parameters $\hat{x}_{00}$, $P_{00}$, $Q$, and $R$ was conducted using this SDOF system.

Lastly, to confirm that the model updating procedure was functioning properly before implementing experimentally, a numerical hybrid simulation with online model updating was conducted on a 3DOF shear-type building model whose three stories are assumed to have identical hysteresis responses. The UKF identifies the Bouc-Wen parameters of the first story and applies the identified parameters to the upper two stories, yielding more accurate results compared to hybrid simulation without model updating.
3.1. Bouc-Wen Model Parameter Identification using the UKF

Consider the SDOF system shown in Figure 3.1 which consists of a lumped mass $m$ supported by a column whose hysteresis is modeled using the Bouc-Wen parameters shown in the figure and a linear viscous damper with damping constant $c$.

![Figure 3.1: SDOF system](image)

The objective of this example is to numerically implement the UKF to identify the Bouc-Wen parameters of the column hysteresis response when the SDOF system is subjected to an earthquake ground motion $\ddot{u}_g$. First, the state vector $\mathbf{x}$ is populated with the Bouc-Wen parameters $k$, $\beta$, $\gamma$, $n$, and $\alpha$. The parameter $A$ does not need to be included due to redundancy (see Section 2.2) and is simply set to unity. In addition to the Bouc-Wen parameters, an intermediate variable $z$ must be included in the state vector to provide a link between the state vector $\mathbf{x}$ and the measurement vector $\mathbf{y}$ (Wang & Wu, 2013). The entire state vector is:
\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} z \\ k \\ \beta \\ \gamma \\ n \\ \alpha \end{bmatrix} \]

The system input \( u \) of the UKF consists of the Bouc-Wen model input, which is the displacement and velocity responses of the SDOF lumped mass:

\[ u = \begin{bmatrix} u \\ \dot{u} \end{bmatrix} \]

As stated in Section 2.3, the UKF function \( f \) updates the state vector \( x \) for the next time step. To establish the function \( f \), the derivative of \( x \) with respect to time is taken:

\[ \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} \dot{z} \\ \dot{k} \\ \dot{\beta} \\ \dot{\gamma} \\ \dot{n} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \dot{u} - \beta |\dot{u}|z|z|^{n-1} z - \gamma \dot{u} |z|^{n} \\ \dot{u} - x_3 |\dot{u}|x_1|^{n-1} x_1 - x_4 \dot{u} |x_1|^{n} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

where \( \dot{z} \) is defined in Equation 2.24. The derivatives of the Bouc-Wen parameters are all zero because they are assumed to be constant. The function \( f \) is then obtained by being presented in a discrete time form (Chatzi & Smyth, 2008):
\[
x_{k+1} = f\left(x_k, u_k\right) + v_k = \left(x_k + \Delta t x_k\right) + v_k
\]
\[
= \begin{bmatrix}
x_{1,k} + \Delta t \left(\hat{u}_{k} - x_{3,k} \left| \hat{u}_{k}\right| x_{1,k}^{5,k-1} - x_{4,k} \hat{u}_{k}\left| x_{1,k}\right|^{5,k} \right)
x_{2,k}
x_{3,k}
x_{4,k}
x_{5,k}
x_{6,k}
\end{bmatrix}
\] + v_k  

where \(\Delta t\) is the time step size and the subscript \(k\) represents the current time step. Note that every variable appearing in the function \(f\) is either part of \(x_k\) or part of \(u_k\), thus satisfying the input requirements of \(f\).

The measurement vector \(y\) consists of only one element for this SDOF system, i.e. the restoring force \(f_y\). The UKF function \(h\) can be derived based on Equation 2.23 and is presented in a discrete time form:

\[
y_{k+1} = h(x_{k+1}, u_{k+1}) + w_{k+1} = x_{6,k+1} x_{2,k+1} u_{k+1} + \left(1 - x_{6,k+1}\right) x_{2,k+1} x_{1,k+1} + w_{k+1}
\]

where \(x_1, x_2,\) and \(x_6\) are defined in Equation 3.1, \(u\) is the displacement response of the SDOF lumped mass, and \(w\) represents the restoring force measurement noise.

An implementation block diagram of the UKF applied to the SDOF example is shown in Figure 3.2. In the Explicit Newmark Part 1 box, Equations 2.17 and 2.18 are utilized to calculate the target displacement \(\tilde{u}_{k+1}\) and the target velocity \(\hat{u}_{k+1}\) of the next time step \(k+1\).

The variables \(\tilde{u}_{k+1}\) and \(\hat{u}_{k+1}\) are then fed into the Bouc-Wen Model box. The column’s response is numerically simulated (Equations 2.23 and 2.24) to compute the restoring force \(f_{s,k+1}\) for a set of specified parameters \(A, k, \beta, \gamma, n,\) and \(\alpha\). The
Forward Euler method is employed to approximate the solution of the differential equation (Equation 2.24). In the case of an experimental study (see Chapter 4), a physical specimen would be used in place of the Bouc-Wen model. The target displacement would be physically applied to the specimen using a hydraulic actuator, and the restoring force would be captured through a load cell.

Next, the target displacement $\ddot{u}_{k+1}$ and restoring force $f_{s,k+1}$ of the next time step $k+1$ and the target velocity $\dot{u}_k$ of the current time step $k$ are fed into the Unscented Kalman Filter box, which performs one time step of the block diagram in Figure 2.4. Note that $\ddot{u}_{k+1}$ and $\dot{u}_k$ are part of the UKF system input $u_{k+1}$, and $f_{s,k+1}$ is the measurement $y_{k+1}$. During this cycle, the state estimate $\hat{x}_{k|k}$ and the covariance matrix $P_{k|k}$ get updated to $\hat{x}_{k+1|k+1}$ and $P_{k+1|k+1}$, respectively.

Meanwhile, $\ddot{u}_{k+1}$, $\dot{u}_k$, and $f_{s,k+1}$ are fed into the Explicit Newmark Part 2 box, which calculates the displacement $u_{k+1}$, the velocity $\dot{u}_{k+1}$, and the acceleration $\ddot{u}_{k+1}$ of the next time step $k+1$ using Equations 2.19 through 2.22.
Figure 3.2: SDOF simulation block diagram
3.2. Parametric Study of the UKF System Model

To realize the secondary objective of this study, which is to understand the effects of the UKF system model parameters when used with the Bouc-Wen model, a parametric study was conducted using the SDOF system in Figure 3.1 with assumed values for the structural properties. The El Centro earthquake ground acceleration shown in Figure 3.3 was used as the excitation and was scaled down to ensure a more realistic excitation for the assumed structural properties. Table 3.1 lists the parameters used in this parametric study.

![Figure 3.3: El Centro earthquake ground acceleration](image-url)
The initial natural period of vibration (before yielding) of the SDOF system is calculated:

\[ T_n = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.05}{1}} = 1.405 \text{ sec} \]  

After yielding occurs, the SDOF system will exhibit softening, in which the tangent stiffness decreases with time. Since the stiffness \( k \) is present within the denominator in Equation 3.6, a decreasing stiffness will increase the natural period of vibration \( T_n \). In other words, the largest natural period that the structure will exhibit is 1.405 sec. The criterion for stability in Equation 2.14 is applied:

\[ \frac{1.405}{\pi} = 0.447 \text{ sec} > \Delta t = 0.01 \text{ sec} \]  

where \( \Delta t \) is the step size of the integration. The criterion for stability for the Explicit Newmark integration algorithm is met. The time history response and the hysteretic

<table>
<thead>
<tr>
<th>Structural Properties</th>
<th>Mass ((m))</th>
<th>0.05 kip\cdot\text{sec}^2/\text{in}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping Ratio ((\xi))</td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>Bouc-Wen Parameters</td>
<td>(A) 1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(k) 1.0 kip/in</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\beta) 2.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\gamma) 0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(n) 1.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\alpha) 0.2</td>
<td></td>
</tr>
<tr>
<td>Ground Motion</td>
<td>Earthquake El Centro</td>
<td></td>
</tr>
<tr>
<td>Earthquake Scale</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Integration Parameters</td>
<td>Time step ((\Delta t)) 0.01 sec</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\gamma) 0.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Structural, ground motion, and integration parameters used in the parametric study
response of the SDOF system subject to the scaled down El Centro ground acceleration are shown in Figure 3.4 and Figure 3.5, respectively.

**Figure 3.4: Time history response of SDOF system**

**Figure 3.5: Hysteretic response of SDOF system**
To study the effects of the accuracy of the initial state estimate $\hat{x}_{00}$ on the convergence performance of the Bouc-Wen parameters, three sets of initial estimates were created: exact, close, and inaccurate, as listed in Table 3.2. The initial estimate of $z$ is always zero to satisfy the initial condition of the differential equation in the Bouc-Wen model (Equation 2.24).

<table>
<thead>
<tr>
<th></th>
<th>Exact</th>
<th>Close</th>
<th>Inaccurate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
<td>1.2</td>
<td>5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2</td>
<td>2.5</td>
<td>6</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
<td>0.2</td>
<td>-2</td>
</tr>
<tr>
<td>$n$</td>
<td>1.5</td>
<td>1.8</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.2</td>
<td>0.3</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 3.2: Initial state estimates

The UKF system model parameters $P_{00}$, $Q$, and $R$ were each studied in conjunction with the three initial state estimates $\hat{x}_{00}$. Table 3.3 lists the simulations that were conducted for the parametric study and their corresponding figures and subfigures. For each simulation, the time histories of the Bouc-Wen parameters in the state estimate $\hat{x}_{kk}$ were examined to determine the sensitivity of their convergence performance to changes in the UKF system model parameters. Although the full El Centro earthquake record was used for each simulation, the time history plots presented henceforth only show the first several seconds required for convergence. The parametric study served as a tuning procedure for the UKF system model, leading to optimum parameter values that were utilized in the experimental study.
<table>
<thead>
<tr>
<th>UKF Parameter</th>
<th>Simulations</th>
<th>Figure</th>
<th>Subfigures</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀₀</td>
<td>k</td>
<td>3.6</td>
<td>(a) Exact $\hat{x}_{q₀}$</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>3.7</td>
<td>(b) Close $\hat{x}_{q₀}$</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>3.8</td>
<td>(c) Inaccurate $\hat{x}_{q₀}$</td>
</tr>
<tr>
<td></td>
<td>$n$</td>
<td>3.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>3.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>3.11</td>
<td></td>
</tr>
</tbody>
</table>
| | $\beta$ and $\gamma$ refinement | 3.12 and 3.13 | (a) $P_{q₀}(3,3) = P_{q₀}(4,4) = 1$  
(b) $P_{q₀}(3,3) = P_{q₀}(4,4) = 0.1$ |
| Q | Exact $\hat{x}_{q₀}$ | 3.14 | (a) k |
| | Close $\hat{x}_{q₀}$ | 3.15 | (b) $\beta$ |
| | Inaccurate $\hat{x}_{q₀}$ | 3.16 | |
| R | Exact $\hat{x}_{q₀}$ | 3.17 | (c) $\gamma$ |
| | Close $\hat{x}_{q₀}$ | 3.18 | (d) $n$ |
| | Inaccurate $\hat{x}_{q₀}$ | 3.19 | |
| | $R$ varies | 3.20 | (e) $\alpha$ |

Table 3.3: Parametric study simulation summary

3.2.1. Initial Covariance Matrix $P_{q₀}$

The initial covariance matrix $P_{q₀}$ is a square diagonal matrix with dimensions of $6 \times 6$, within which the diagonal elements represent the certainty level of the initial estimates. For example, the element in the $(2,2)$ position of $P_{q₀}$ represents the certainty level of the initial estimate of the second parameter in $\hat{x}_{q₀}$ (i.e. the stiffness $k$). In addition, the units of $P_{q₀}$ are the square of the units of $\hat{x}_{q₀}$, so the element in the $(2,2)$ position of $P_{q₀}$ has units of kip$^2$/in$^2$.

For the parametric study, each diagonal element of $P_{q₀}$ was investigated independently. For each simulation, the initial estimates of the parameters not under investigation were all set to their exact values, and the diagonal elements of $P_{q₀}$
corresponding to them were all set to $10^{-6}$. This helps restrict erratic fluctuations of parameters not under investigation and in turn ensures that the parameter under investigation is not influenced by them. For the parameter under investigation, the initial estimate and the corresponding diagonal element of $P_{q0}$ were varied to study the effects on its convergence performance.

For all $P_{q0}$ simulations, the process noise covariance matrix $Q$ was set to:

$$
Q = \begin{bmatrix}
10^{-6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Section 3.2.2 provides an explanation for this choice of $Q$ as well as a further refinement of its nonzero element.

Measurement noise was not considered in $P_{q0}$ simulations. Therefore, the restoring force calculated using the Bouc-Wen model was fed directly into the UKF without any simulated noise added. Correspondingly, the measurement noise covariance $R$ was set to zero.
3.2.1.1. Bouc-Wen Parameter $k$

Figure 3.6 shows the first three seconds of the time histories of $k$ for various values of its initial covariance element $P_{00}(2,2)$ when its initial estimate is exact, close, and inaccurate as listed in Table 3.2. As mentioned above, all other parameters have exact initial estimates and initial covariance values of $10^{-6}$ to limit the investigation strictly to $k$.

Figure 3.6 truly demonstrates the power of the UKF. The accuracy of the converged value is not at all affected by the initial estimate or the initial covariance. The estimate always converges to its true value of 1.0. Nevertheless, the speed of convergence is affected by the initial estimate and covariance. If the initial estimate is inaccurate as shown in Figure 3.6(c), a high initial covariance is beneficial as it increases convergence speed. However, if the initial estimate is accurate as shown in (a) and (b), a high initial covariance can cause excessive fluctuations prior to convergence. This is undesirable in the context of model updating because the state estimate is used to update the numerical substructures at every time step. If the state estimate is fluctuating severely, the numerical substructures will inherit this flawed behavior.

It is usually easy to get a very accurate initial estimate of $k$ through preliminary cyclic tests prior to conducting a hybrid simulation. This indicates that $P_{00}(2,2)$ should be relatively small since it represents how certain the initial estimate of $k$ is. A value of 0.1 is likely a safe choice.
Figure 3.6: Time histories of $k$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate
3.2.1.2. Bouc-Wen Parameter $\beta$

Figure 3.7 shows the first eight seconds of the time histories of $\beta$. It behaves similarly to $k$ in that the converged value is generally very accurate. Unlike $k$ however, $\beta$ does not exhibit excessive fluctuations. When the initial estimate is exact, there are some fluctuations, but they are small enough to be ignored for model updating.

Convergence speed does not appear to be affected by the initial estimate. For example, when $P_{q_0}(3,3) = 10$, $\beta$ converges at about 1.5 seconds regardless of its initial estimate. On the other hand, initial covariance does affect the convergence speed. The larger the value of $P_{q_0}(3,3)$, the sooner the estimate “realizes” it needs to change and the sooner it achieves its true value. For example, when $P_{q_0}(3,3) = 10$, the estimate starts shifting toward its true value at about 0.3 seconds and achieves it at about 1.5 seconds. However, when $P_{q_0}(3,3) = 0.1$, the estimate does not begin changing until almost 1 second and achieves its true value at about 2 seconds.

This demonstrates the advantage of using large values for $P_{q_0}(3,3)$, especially since there is no apparent risk of excessive fluctuations. Although this is true when $\beta$ is investigated independently, it does not hold true when all parameters are investigated together, as is demonstrated later in this section. For the moment, it is recommended to use $P_{q_0}(3,3) = 10$, but this value is further optimized when all parameters are investigated together.
Figure 3.7: Time histories of $\beta$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate.
3.2.1.3. Bouc-Wen Parameter $\gamma$

The first eight seconds of the time histories of $\gamma$ are shown in Figure 3.8, and similar trends to $\beta$ were observed. The converged value is always accurate, fluctuations are very small, and convergence is faster for larger values of $P_{00}(4,4)$. Based on these observations, it is also recommended that $P_{00}(4,4)=10$, but this value is also further optimized when all parameters are investigated together.

![Figure 3.8: Time histories of $\gamma$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate](image-url)
3.2.1.4. *Bouc-Wen Parameter n*

The first seven seconds of the time histories of $n$ are shown in Figure 3.9. Similar to $k$, $\beta$, and $\gamma$, the converged value is quite accurate. Also, convergence speed follows a pattern similar to the previous parameters, wherein larger values of $P_{00}(5,5)$ cause the estimate to start changing sooner and converge sooner than smaller values of $P_{00}(5,5)$.

There is one major difference between the behavior of $n$ and the behavior of the other parameters. Certain values of $P_{00}(5,5)$ cause the covariance matrix $P_{kk}$ to become non-positive definite. This causes an error in the UKF during the matrix square root operation in Equation 2.32. For example, when the initial estimate is exact or close (Figure 3.9(a) and (b)) and $P_{00}(5,5) = 0.1$, 1, or 10, the error occurs. Also, if the initial estimate is inaccurate as shown in (c), $P_{00}(5,5) = 10$ results in an error. The smallest
value of $P_{\alpha 0}(5,5)$ that causes the error will henceforward be called the error threshold of $P_{\alpha 0}(5,5)$.

There is a distinct relationship between the accuracy of the initial estimate and the error threshold of $P_{\alpha 0}(5,5)$ as shown in Table 3.4. If the initial estimate is inaccurate, a relatively large value of $P_{\alpha 0}(5,5)$ is acceptable and will not generate an error. However, if the initial estimate is close to the true value, a small value of $P_{\alpha 0}(5,5)$ is better for the UKF to run successfully. In other words, the more accurate the initial estimate, the smaller $P_{\alpha 0}(5,5)$ must be. Because of this relationship, it is recommended to conservatively select a small value for $P_{\alpha 0}(5,5)$. This may adversely affect the speed of convergence, but it reduces the likelihood of an error in the UKF. Therefore, it is recommended to use $P_{\alpha 0}(5,5) = 0.01$.

<table>
<thead>
<tr>
<th>Initial Estimate Set</th>
<th>Error Threshold of $P_{\alpha 0}(5,5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>0.032</td>
</tr>
<tr>
<td>Close</td>
<td>0.081</td>
</tr>
<tr>
<td>Inaccurate</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Table 3.4: Error threshold of $P_{\alpha 0}(5,5)$ for all initial estimate sets

The high risk of error demonstrates that $n$ is very sensitive to its initial covariance element $P_{\alpha 0}(5,5)$. This is because it takes the form of an exponent in the Bouc-Wen model, whereas the other parameters are coefficients.
Figure 3.9: Time histories of $n$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate
3.2.1.5. Bouc-Wen Parameter $\alpha$

The first four seconds of the time histories of $\alpha$ are shown in Figure 3.10. Like the other parameters, the converged value is very accurate. However, for $\alpha$, the convergence speed is not influenced very much by $P_{00}(6,6)$. For all cases, the estimate begins changing immediately after the start of the simulation. Also, the convergence time is about 1.5 seconds for all cases except when the initial estimate is inaccurate (Figure 3.10(c)) and $P_{00}(6,6) = 0.01$.

For all initial estimates, if $P_{00}(6,6) = 1$ or $P_{00}(6,6) = 10$, the UKF overshoots the true value of $\alpha$ and in some cases even goes beyond the lower bound of $\alpha$, which is zero from Equation 2.29. For all initial estimates, $P_{00}(6,6) = 0.1$ performs very well without overshooting. Therefore, it is recommended to use $P_{00}(6,6) = 0.1$. 
Figure 3.10: Time histories of $\alpha$ when the initial estimate is (a) exact, (b) close, and (c) inaccurate
3.2.1.6. All Bouc-Wen Parameters

After the optimal values for each diagonal element of $P_{00}$ were determined independently, the performance of the entire matrix $P_{00}$ with the optimal elements corresponding to each Bouc-Wen parameter was verified. The matrix $P_{00}$ was set to:

$$P_{00} = \begin{bmatrix}
10^{-6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 10 & 0 & 0 & 0 \\
0 & 0 & 0 & 10 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.01 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.1
\end{bmatrix}$$

(3.9)

The element in the (1,1) position, which corresponds to parameter $z$, is kept at $10^{-6}$. Theoretically, this value should be zero since the initial estimate of $z$ is entirely certain. However, $P_{00}$ is not positive definite if any of its diagonal values are zero, so a small value can be assumed for $P_{00}(1,1)$ instead.

The numerical simulation was conducted for each set of initial estimates $\hat{x}_{00}$ in Table 3.2, and the time histories of all parameters are shown in Figure 3.11.
Figure 3.11: Time histories of all parameters when all initial estimates are (a) exact, (b) close, and (c) inaccurate
If all initial estimates are exact or close as shown in Figure 3.11(a) and (b), the parameters converge quite well with the exception of some fluctuations and overshooting in \( \beta \) and \( \gamma \). However, these fluctuations were small when \( \beta \) and \( \gamma \) were being investigated individually (Figure 3.7 and Figure 3.8), which demonstrates that the parameters are all interrelated. The convergence of one parameter may be affected by the initial estimates and initial covariance elements of other parameters. In addition, if all the initial estimates are inaccurate as shown in (c), the parameters converge to completely incorrect values, further supporting the observation that the Bouc-Wen parameters are interrelated. Table 3.5 compares the true values and the incorrect converged values of the Bouc-Wen parameters for inaccurate initial estimates. It can also be observed that \( \alpha \) converges to a value below its lower bound of zero. Therefore, selecting initial estimates that are somewhat accurate plays a crucial role in parameter identification using the UKF.

<table>
<thead>
<tr>
<th>Bouc-Wen Parameter</th>
<th>True Value</th>
<th>Converged Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>1</td>
<td>0.879</td>
</tr>
<tr>
<td>( \beta )</td>
<td>2</td>
<td>0.071</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.5</td>
<td>0.020</td>
</tr>
<tr>
<td>( n )</td>
<td>1.5</td>
<td>3.45</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.2</td>
<td>-3.38</td>
</tr>
</tbody>
</table>

**Table 3.5: True and converged values of Bouc-Wen parameters when initial estimates are inaccurate**

Because of the fluctuations and overshooting in \( \beta \) and \( \gamma \) observed in Figure 3.11(a) and (b), further refinement of their initial covariance elements is necessary. Their values were decreased to both 1 and 0.1, and the time histories for both conditions are shown in Figure 3.12. The “close” set of initial estimates was used for both conditions.
When $\mathbf{P}_{q0}(3,3) = \mathbf{P}_{q0}(4,4) = 1$ (Figure 3.12(a)), the fluctuations and overshooting are still significant, but when $\mathbf{P}_{q0}(3,3) = \mathbf{P}_{q0}(4,4) = 0.1$ (Figure 3.12(b)), the fluctuations and overshooting are effectively eliminated with no detrimental effect on the speed or accuracy of convergence.

![Figure 3.12](image_url)

**Figure 3.12**: Time histories of all parameters when initial estimates are close, and initial covariance elements corresponding to $\beta$ and $\gamma$ are equal to (a) 1 and (b) 0.1
It appears as if \( P_{ii0}(3,3) = P_{ii0}(4,4) = 0.1 \) is the best option, but if the initial estimates are adjusted to different values that are still close to the true values (see Table 3.6), convergence problems arise. Figure 3.13 shows the time histories for the adjusted close initial estimates. If \( P_{ii0}(3,3) = P_{ii0}(4,4) = 0.1 \) (Figure 3.13(b)), it is evident that \( \beta \) cannot quite converge to its true value of 2.0. However, if \( P_{ii0}(3,3) = P_{ii0}(4,4) = 1 \) (Figure 3.13(a)), convergence performance improves drastically. Consequently, it is more prudent to use \( P_{ii0}(3,3) = P_{ii0}(4,4) = 1 \) to ensure good convergence. It may cause fluctuations and overshooting in some cases, but it is more important that the values converge accurately.

<table>
<thead>
<tr>
<th>True Values</th>
<th>Close Estimates</th>
<th>Adjusted Close Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>-0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>1.8</td>
<td>1.3</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Table 3.6: Adjustment of close initial estimates**

In conclusion, the optimum \( P_{ii0} \) matrix for the UKF implemented in this SDOF example was determined as:

\[
P_{ii0} = \begin{bmatrix}
10^{-6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.01 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.1
\end{bmatrix}
\]

\[3.10\]
3.2.1.7. Summary

The Bouc-Wen parameters were first investigated independently, and it was observed that their converged values are generally quite accurate regardless of the values within the $P_{00}$ matrix. High values of the elements within $P_{00}$ generally allow the Bouc-Wen parameters to converge faster, but they also can cause fluctuations and overshooting of the parameters, which should be restricted within reason when the UKF is used in the
context of model updating. Moreover, a non-positive definite $P_{00}$, which produces an error in the UKF, should be avoided by selecting a conservatively low value for $P_{00}(5,5)$.

When the Bouc-Wen parameters were investigated together, convergence performance changed in several ways, indicating that the Bouc-Wen parameters are interrelated. When the initial estimates in $\hat{x}_{00}$ were inaccurate, misleading convergence was observed, wherein the parameters converged to incorrect values. In addition, the fluctuations and overshooting of $\beta$ and $\gamma$ were much larger during the collective parameter simulations than they were during the independent parameter simulations, necessitating further refinement of their respective elements within $P_{00}$.

### 3.2.2. Process Noise Covariance Matrix Q

The process noise covariance matrix $Q$ in the UKF system model is a square diagonal matrix with dimensions of $6\times6$ for the SDOF example. Each diagonal element represents the process noise covariance associated with its corresponding parameter in the state estimate $\hat{x}_{kk}$. A simple conceptual explanation of the process noise is that if the true value of a parameter is constant, its corresponding process noise covariance element should be zero. Conversely, if the true value of a parameter is variable, process noise must be modeled with a nonzero value to reflect the level of parameter variation. For the Bouc-Wen model in this study, the true values of $k$, $\beta$, $\gamma$, $n$, and $\alpha$ are constant and thus have process noise covariance values of zero. However, the true value of the intermediate variable $z$ is dynamic as it is calculated every time step by solving the differential equation in Equation 2.24. This indicates that the $(1,1)$ element of $Q$ must be nonzero. If a zero value is used, the covariance matrix $P_{kk}$ becomes non-positive definite,
and the UKF cannot run. Also, the unit of \( Q(1,1) \) is the square of the unit of \( z \), which is \( \text{in}^2 \).

Throughout the numerical simulations of \( P_{00} \) in section 3.2.1, \( Q(1,1) \) was set to \( 10^{-6} \), but in this section, the effects of using other values of \( Q(1,1) \) are studied for each of the three sets of initial estimates \( \hat{x}_{00} \) in Table 3.2. For all simulations in this section, the optimum initial covariance matrix \( P_{00} \) established previously (see Equation 3.10) was used. Measurement noise was not considered, so the measurement noise covariance \( R \) was set to zero.

Figure 3.14 shows the first five seconds of the time histories of each parameter when initial estimates are exact. The value of \( Q(1,1) \) varies from \( 10^{-6} \) to \( 10^{-21} \). If \( Q(1,1) \) is smaller than \( 10^{-22} \), \( P_{kk} \) becomes non-positive definite after a few steps, and the UKF reports an error. Henceforward, this value will be referred to as the error threshold of \( Q(1,1) \).

The relationship between convergence performance and the value of \( Q(1,1) \) is not perfectly clear, but there is a trend that can be observed for all five parameters. In general, higher values of \( Q(1,1) \) decrease convergence speed and accuracy. This is especially true for \( Q(1,1) = 10^{-6} \). On the other hand, smaller values of \( Q(1,1) \) tend to introduce excessive fluctuations. Overall, \( Q(1,1) = 10^{-9} \) appears to be a reasonable choice as it produces good convergence and does not cause too much fluctuation in any of the parameters.
Figure 3.14: Time histories when all initial estimates are exact for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.14 cont’d: Time histories when all initial estimates are exact for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$

Figure 3.15 shows the first five seconds of the time histories of each parameter when initial estimates are close. The value of $Q(1,1)$ varies from $10^{-6}$ to $10^{-18}$, and the error threshold is $10^{-19}$, which is larger than it was for exact initial estimates.

As shown from Figure 3.15, convergence speed and accuracy is poor for $Q(1,1) = 10^{-6}$ but quite good for $Q(1,1) = 10^{-9}$ and $Q(1,1) = 10^{-12}$. When $Q(1,1) = 10^{-18}$, fluctuations are very large and convergence accuracy suffers greatly. This suggests that if $Q(1,1)$ is too close to the error threshold, convergence may be ruined as a result.
Therefore, it is crucial to select a moderate value for $Q(1,1)$. Convergence performance will be poor if the value is too high or too close to the error threshold. A value somewhere in between will likely provide sufficient convergence speed and accuracy.

Figure 3.15: Time histories when all initial estimates are close for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.15 cont’d: Time histories when all initial estimates are close for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.16 shows the first five seconds of the time histories of each parameter when initial estimates are inaccurate. The value of $Q(1,1)$ varies from $10^{-6}$ to $10^{-12}$, and the error threshold is $10^{-15}$, which is larger than it was for close initial estimates.

When $Q(1,1) = 10^{-6}$ and $Q(1,1) = 10^{-9}$, the parameters either converge to incorrect values or do not converge to anything at all. This problem was also observed during $P_{90}$ simulations in Figure 3.11(c). However, the problem is corrected when $Q(1,1) = 10^{-12}$ as all the parameters converge to their true values. This demonstrates the advantage of using a small value for $Q(1,1)$ for this study.

Table 3.7 compares the error threshold of $Q(1,1)$ for each set of initial estimates. The trend indicates that the more inaccurate the initial estimates are, the higher the error threshold is, and in turn, the more sensitive the UKF performance is to an error. If $Q(1,1)$ is too high or too close to the error threshold, convergence performance suffers as a consequence. On the contrary, a small value for $Q(1,1)$ can potentially resolve divergence or misleading convergence, wherein the parameters converge to incorrect values, but it is important that $Q(1,1)$ is not too close to the error threshold. In brief, the value selected for $Q(1,1)$ should be sufficiently small to obtain good convergence, but not so small that it is close to the error threshold. Based on these numerical simulation results, $Q(1,1) = 10^{-12}$ appears to be a reasonable choice:
\[ Q = \begin{bmatrix} 10^{-12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

3.11

<table>
<thead>
<tr>
<th>Initial Estimate Set</th>
<th>Error Threshold of Q(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>$10^{-22}$</td>
</tr>
<tr>
<td>Close</td>
<td>$10^{-19}$</td>
</tr>
<tr>
<td>Inaccurate</td>
<td>$10^{-15}$</td>
</tr>
</tbody>
</table>

Table 3.7: Error threshold of Q(1,1) for all initial estimate sets

Since the time history of the hysteretic displacement $z$ clearly depends on the parameters of the SDOF system, such as the structural properties and the earthquake ground motion, the optimal value of $Q(1,1)$ selected in this study may not be universal for all applications. Therefore, it is important that $Q(1,1)$ be tuned using a system that is similar to that which is being investigated. However, as can be seen from the parameter time history plots, there is generally a wide range of values of $Q(1,1)$ that produce suitable parameter convergence, demonstrating the robustness of the UKF.
Figure 3.16: Time histories when all initial estimates are inaccurate for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.16 cont’d: Time histories when all initial estimates are inaccurate for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$

3.2.2.1. **Summary**

The parametric study of $Q(1,1)$ indicated that low values generally result in faster parameter convergence but also may cause fluctuations of the parameters. In addition, it was observed that the more inaccurate the estimates in $\hat{x}_{00}$ are, the higher the error threshold of $Q(1,1)$ is. Convergence accuracy suffers if $Q(1,1)$ is too high or too close to the error threshold. Nevertheless, it was determined that small values of $Q(1,1)$ can
potentially correct misleading convergence resulting from inaccurate initial estimates in \( \hat{x}_{00} \). This observation suggests that the value of \( Q(1,1) \) should be small but not too close to the error threshold.

### 3.2.3. Measurement Noise Covariance Matrix \( R \)

The measurement noise covariance matrix \( R \) has dimensions of \( 1 \times 1 \) in this example, so it is in fact a scalar. The measurement noise covariance represents the level of noise in the restoring force measurement \( f_s \) and has units of kip\(^2\). In the experimental study (see Chapter 4), the actual covariance of the load cell noise was determined, but for this numerical study, various levels of noise were simulated using a Gaussian distributed random signal. The simulated noise was then added to the restoring force \( f_s \) to mimic realistic force readings experienced in hybrid simulation.

For each of the three sets of initial estimates in Table 3.2, different levels of simulated noise were added to \( f_s \). Initially, the \( R \) value was set exactly equal to the covariance of the simulated noise. Subsequently, the effects of an inaccurate estimation of the measurement noise covariance were considered by setting \( R \) to a value different than the true covariance of the simulated noise. The initial covariance matrix \( P_{00} \) and the process noise covariance matrix \( Q \) established previously (see Equations 3.10 and 3.11) were used for all the simulations in this section.

Figure 3.17 shows the time histories of each parameter when initial estimates are exact. The noise covariance varies from \( 10^{-3} \) to \( 10^{-5} \) and \( R \) is set equal to the noise covariance in all cases. Naturally, a lower noise covariance results in more accurate
convergence. Some parameters are less affected by noise than others. For example, $\alpha$ is particularly robust to excessive measurement noise.

Convergence speed trends are more difficult to discern because the time at which convergence is achieved is not easily distinguishable due to the simulated measurement noise. However, convergence speed does not seem to be affected much by the noise level. This is especially noticeable in the time histories of $\alpha$ wherein the estimate converges within about four seconds regardless of the value of the measurement noise covariance.

Figure 3.17: Time histories when all initial estimates are exact for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.17 cont’d: Time histories when all initial estimates are exact for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.18 shows the time histories of each parameter when initial estimates are close. Once again, $R$ was set equal to the noise covariance. Similarly to the simulations in which initial estimates were exact, the convergence speed does not appear to be affected much by the noise covariance. However, convergence becomes asymptotic for some parameters, particularly $\beta$ and $n$, wherein their estimates are always approaching their correct values but never actually reach them.

![Figure 3.18: Time histories when all initial estimates are close for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$](image-url)
Figure 3.18 cont’d: Time histories when all initial estimates are close for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 3.19 shows the time histories of each parameter when initial estimates are inaccurate. Once again, $R$ was set equal to the noise covariance. Here it becomes obvious that $k$ and $\alpha$ are very robust, but $\beta$, $\gamma$, and $n$ are not. This is likely due to the fact that $k$ and $\alpha$ appear directly in the equation for restoring force (Equation 2.23), whereas $\beta$, $\gamma$, and $n$ are only indirectly related to the restoring force through the intermediate variable $z$ (Equation 2.24). Furthermore, the time histories of $k$ indicate that convergence speed may actually be slightly affected by the noise covariance value. The estimate converges in about 0.25 seconds when the noise covariance is $10^{-5}$ but takes about 0.4 seconds when the noise covariance is $10^{-4}$.
Figure 3.19: Time histories when all initial estimates are inaccurate for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$. 
Lastly, the noise covariance was set to $10^{-5}$, but the value of $R$ was varied to observe the effects of inaccurate estimation of the measurement noise covariance. Figure 3.20 shows the time histories of each parameter using close initial estimates. Obviously, the best convergence is obtained when $R$ is exactly equal to the true noise covariance ($R = 10^{-5}$). If the noise covariance is overestimated ($R = 10^{-3}$), convergence quality suffers only slightly, but if the noise covariance is underestimated ($R = 10^{-7}$), convergence quality is significantly decreased. Therefore, it is recommended to
overestimate the measurement noise covariance if no accurate estimation is available, as the effects are less devastating.

Figure 3.20: Time histories when all initial estimates are close and noise covariance is $10^{-5}$ for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$. 
Figure 3.20 cont’d: Time histories when all initial estimates are close and noise covariance is $10^{-5}$ for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
3.2.3.1. Summary

The parametric study of the measurement noise covariance $R$ demonstrated that the Bouc-Wen parameters $k$ and $\alpha$ are robust to excessive measurement noise, but $\beta, \gamma,$ and $n$ exhibit asymptotic convergence, in which their estimates are always approaching their correct values but never actually reach them. The speed of convergence is largely unaffected by the level of measurement noise, but differences in speed were noticed in the time histories of $k$ when initial estimates were inaccurate. Lastly, it is recommended to overestimate the measurement noise covariance if no accurate estimation is available, as the effects are less devastating than underestimation of the measurement noise covariance.

3.3. Numerical Hybrid Simulation with Model Updating

This section describes a numerical hybrid simulation of a 3DOF system with model updating. The objective was to confirm that the model updating procedure was functioning properly before implementing it in an experimental hybrid simulation. A three-story shear-type building with lumped mass $m_i$ and damping constant $c_i$ at each story was used as the prototype structure (see Figure 3.21). The columns are assumed to have identical Bouc-Wen hysteresis responses defined by parameters $A, k, \beta, \gamma, n,$ and $\alpha$.

The first story acts as the experimental substructure, while the second and third stories are considered to be the numerical substructures. The Bouc-Wen parameters of the first story are assumed to be unknown constants that are identified by the UKF, while the Bouc-Wen parameters of the second and third stories are set in the numerical substructure model and are adjustable with given initial estimates. As the 3DOF system is subjected to
an earthquake ground motion ̇\( \ddot{u}_g \), the UKF uses the simulated response of the first story to identify its Bouc-Wen parameters in a manner similar to the SDOF simulation in Section 3.1. These parameters are applied to the second and third stories at each time step as the numerical simulation progresses.

![Figure 3.21: 3DOF system](image_url)
Figure 3.22 shows a block diagram of this 3DOF example. It is analogous to the SDOF example in Figure 3.2, except the displacement $u_k$, velocity $\dot{u}_k$, and acceleration $\ddot{u}_k$ are $3 \times 1$ vectors instead of scalar values.

The Bouc-Wen model of each story and the UKF are compressed into a single box called Restoring Force Function, which is shown in Figure 3.23. The Absolute $\rightarrow$ Relative box in Figure 3.23 converts the absolute displacement and velocity vectors $u_k$ and $\dot{u}_k$ (with respect to the ground) into relative displacements and velocities (with respect to each story).

The relative target displacement and velocity of the “experimental” first story ($\tilde{u}_{1,k+1}$ and $\tilde{\dot{u}}_{1,k+1}$) are fed into a Bouc-Wen model in order to obtain a relative restoring force $\tilde{f}_{1,k+1}$. As previously stated, the Bouc-Wen parameters of the first story are constant but unknown. In a true experimental study (see Chapter 4), a physical specimen would be used in place of the Bouc-Wen model for the experimental substructure in a hybrid simulation.
Restoring Force Function

Explicit Newmark Algorithm

Figure 3.22: 3DOF simulation block diagram
Figure 3.23: Restoring force function
Next, $\ddot{u}_{1,k+1}$, $\ddot{u}_{1,k}$, and $f_{1,k+1}$ are all fed into the Unscented Kalman Filter box, which updates the state estimate $\hat{x}_{k|k}$ and the covariance matrix $P_{k|k}$ to $\hat{x}_{k+1|k+1}$ and $P_{k+1|k+1}$, respectively. Then, $\hat{x}_{k+1|k+1}$ is sent to the Bouc-Wen models of the upper two stories along with the relative target displacements and velocities ($\ddot{u}_{2,k+1}$, $\ddot{u}_{2,k+1}$, $\ddot{u}_{3,k+1}$, and $\ddot{u}_{3,k+1}$). The relative restoring forces of the second and third stories ($f_{2,k+1}$ and $f_{3,k+1}$) are the output of the two numerical Bouc-Wen model boxes.

Lastly, the Relative → Absolute box converts the relative restoring forces of each story into an absolute restoring force vector used in the EOM (Equation 2.4).

Three cases were simulated: “exact”, “without updating”, and “with updating”. The simulation parameters that were used for all three cases are listed in Table 3.8. The same mass was used for all three stories, and the damping matrix was defined using Rayleigh damping with the damping ratio of the first two modes set to 0.05. The same $P_{0|0}$ and $Q$ matrices established in the parametric study were adopted, and no simulated noise was added, so $R$ was set to zero.

For the exact case, the exact Bouc-Wen parameters listed in Table 3.9 were utilized for all three stories. For the case without updating, the first story used the exact Bouc-Wen parameters in Table 3.9, and the upper two stories used the Bouc-Wen parameter estimates in Table 3.9 for the entire duration of the simulation. These estimates are the same as the close estimates from the parametric study. For the case with updating, the first story used the exact Bouc-Wen parameters from Table 3.9, and the second and third stories used the state estimate $\hat{x}_{k|k}$ from the UKF. The Bouc-Wen parameter estimates in Table 3.9 were used as the initial state estimate $\hat{x}_{0|0}$ in the UKF.
### Structural Properties

<table>
<thead>
<tr>
<th></th>
<th>Mass ((m_i))</th>
<th>0.05 kip-sec(^2)/in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping Ratio ((\xi))</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Natural Frequencies ((\omega_n))</td>
<td>1.99, 5.58, 8.06 rad/sec</td>
<td></td>
</tr>
</tbody>
</table>

### Ground Motion

<table>
<thead>
<tr>
<th></th>
<th>Earthquake</th>
<th>El Centro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake Scale</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

### Integration Parameters

<table>
<thead>
<tr>
<th></th>
<th>Time step ((\Delta t))</th>
<th>0.01 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

\[
P_{00} = \begin{bmatrix}
10^{-6} & 0 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0.01 & 0 \\
0 & 0 & 0 & 0 & 0.1 \\
\end{bmatrix}
\]

\[
Q = \begin{bmatrix}
10^{-12} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[R = 0\]

Table 3.8: Structural, ground motion, integration algorithm, and UKF system model parameters used in the numerical hybrid simulation

### Bouc-Wen Parameters

<table>
<thead>
<tr>
<th>Bouc-Wen Parameters</th>
<th>Exact</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k) (kip/in)</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>(\beta)</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>(n)</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3.9: Exact and estimated Bouc-Wen parameters used in the numerical hybrid simulation
Figure 3.24 compares the absolute displacement time history response of each simulation for all three stories, and Figure 3.25 presents a zoomed-in view from two to three seconds. The “exact” and the “without updating” cases show considerable discrepancies, but the “exact” and the “with updating” cases are nearly identical, demonstrating the effectiveness of the online model updating procedure implemented in hybrid simulation. The results are greatly improved even though the numerical substructure does not have accurate Bouc-Wen parameter estimates.

Figure 3.26 compares the relative displacement vs. relative force hysteresis response of each case for all three stories. Similarly, significant inconsistencies are observed between the “exact” and the “without updating” cases. However, when updating is implemented, the hysteresis results are almost identical to the results from the “exact” case.
Figure 3.24: Absolute displacement time history response of three simulations for (a) first story, (b) second story, and (c) third story
Figure 3.25: Zoomed-in absolute displacement time history response of three simulations for (a) first story, (b) second story, and (c) third story
Figure 3.26: Hysteresis response of three simulations for (a) first story, (b) second story, and (c) third story
Figure 3.27 shows the first five seconds of the time histories of all the Bouc-Wen parameters during the “with updating” case. They all converge to the exact values within about one second. As observed during the parametric study, the parameters $\beta$ and $\gamma$ exhibit some fluctuations prior to convergence, but these fluctuations do not appear to have significantly affected the accuracy of the results.

**Figure 3.27: Time histories of all parameters during “with updating” case**
3.4. Summary and Conclusions

In this chapter, Bouc-Wen parameter identification was numerically implemented using the UKF. A SDOF system having a nonlinear hysteretic response defined by the Bouc-Wen model was utilized for the parametric study of the UKF system model parameters including $\hat{x}_{00}$, $P_{00}$, $Q$, and $R$. The following observations and recommendations on how to determine the optimum UKF system model parameters are provided:

- **Initial covariance matrix $P_{00}$**: High values of the elements within $P_{00}$ generally allow the Bouc-Wen parameters to converge faster, but they also can cause fluctuations and overshooting of the parameters, which should be restricted within reason when the UKF is for online model updating. Also, if the initial estimates in $\hat{x}_{00}$ are inaccurate, misleading convergence may occur, in which the Bouc-Wen parameters converge to incorrect values.
  - **Bouc-Wen parameters $\beta$ and $\gamma$**: It appears to be more difficult to suppress the fluctuations and overshooting in $\beta$ and $\gamma$ while also maintaining convergence accuracy than it is for the other Bouc-Wen parameters.
  - **Bouc-Wen parameter $n$**: The more accurate the initial estimate of $n$, the lower the error threshold of $P_{00}(5,5)$. The value of $P_{00}(5,5)$ should be selected so that it is lower than the expected error threshold to avoid an error in the UKF.

- **Process noise covariance matrix $Q$**: Low values of $Q(1,1)$ generally result in faster parameter convergence but may also cause fluctuations of the parameters. The more
inaccurate the initial estimates in $\hat{x}_{\text{ij0}}$ are, the higher the error threshold of $Q(1,1)$ is, and convergence accuracy suffers if $Q(1,1)$ is too high or too close to the error threshold. Nevertheless, small values of $Q(1,1)$ can potentially correct misleading convergence resulting from inaccurate initial estimates in $\hat{x}_{\text{ij0}}$. The value of $Q(1,1)$ should be relatively small but not too close to the expected error threshold.

- **Measurement noise covariance $R$**: Lower levels of measurement noise result in more accurate convergence, but convergence speed is generally not affected much by the level of measurement noise. If no accurate estimation of the measurement noise covariance is available, it is recommended to overestimate the value than to underestimate it, as the effects on convergence performance are less devastating.
  
  o **Bouc-Wen parameters $k$ and $\alpha$**: The parameters $k$ and $\alpha$ are robust to measurement noise, as their converged values are accurate regardless of the noise level.
  
  o **Bouc-Wen parameters $\beta$, $\gamma$, and $n$**: The parameters $\beta$, $\gamma$, and $n$ are not as robust and exhibit asymptotic convergence, in which their estimates are always approaching their correct values but never actually reach them.

A numerical hybrid simulation with model updating was also conducted using a 3DOF shear-type building model. It was concluded that the UKF, when used for model updating, can effectively improve the accuracy of hybrid simulation results even if the Bouc-Wen parameter estimates of the numerical substructures are not accurate.
4. Experimental Study

To implement online model updating in real-time hybrid simulation (RTHS) and experimentally validate its effectiveness on improving accuracy of RTHS results, a series of experiments were conducted at the Laboratory of Earthquake and Structural Simulation (LESS) at Western Michigan University (WMU). This chapter begins with an introduction to the hardware and software components utilized in the experimental work with explanations of their integration for RTHS.

Next, the test specimen (i.e. experimental substructure in RTHS) is devised, and its suitability for implementing RTHS with model updating was verified. A quasi-static test (QST) was carried out on the test specimen, and the Bouc-Wen parameters defining its hysteretic response were identified offline using the Unscented Kalman Filter (UKF).

To conduct stable and reliable RTHS, a time delay compensator named the modified feedforward (MFF) scheme was adopted and implemented, and its functionality was verified through a RTHS of a 3DOF system without model updating.

Lastly, six proof-of-concept RTHS with online model updating were conducted using the same 3DOF system. For the first two, the UKF was used to update all the Bouc-Wen parameters; for the second two, the UKF system model was modified to keep the nonlinear parameter $n$ constant; and for the last two, the constrained UKF (CUKF) was implemented. Lastly, issues related to the implementation of the UKF and CUKF are discussed.
4.1. RTHS Testing System

The Laboratory of Earthquake and Structural Simulation (LESS) at Western Michigan University (WMU) consists of a versatile hybrid testing system that can be used to continuously improve hybrid simulation techniques. Many large-scale laboratories that are capable of hybrid simulation are usually occupied with various research projects. In addition, large-scale laboratories have expensive operating costs, and specimen and instrumentation installation can be very time consuming. These hindrances make development of hybrid simulation techniques nearly impossible at large scale laboratories. Alternatively, benchmark scale testing systems, such as the one developed at LESS, can facilitate economic development of hybrid simulation techniques, including the model updating for RTHS in this study (Shao & Enyart, 2012).

4.1.1. Hybrid Testing System Components

The major components of the hybrid testing system and their key parameters are listed in Table 4.1. They are described in detail throughout this section.

4.1.1.1. Shake Table

The uniaxial shake table can impose ground motion during a shake table test (STT), but the tests in this study are pseudodynamic (PSD) and thus do not necessitate a shake table. Instead, the shake table is utilized as a strong floor to fix the test specimen at its base using a bolt hole pattern. The hydraulic actuator that controls the shake table (i.e. table actuator) is held at a constant position during the RTHS in this study.
<table>
<thead>
<tr>
<th>Components</th>
<th>Performance/Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hardware</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Uniaxial shake table (built in house) | Table size
Maximum specimen mass
Frequency of operation
Maximum acceleration
Maximum displacement | 3 ft x 3 ft (915 mm x 915 mm)
500 lbf (228 kg)
0 - 20 Hz
4g (with 500 lbf specimen)
±3 in (±76.2 mm) |
| Hydraulic actuators (Shore Western model #910D-1.08-6(0)-4-1348) | Force
Stroke
Swivel base and end rod
Servo valve | ±3240 lb (±13.3 kN) at 3000 psi
6 in (± 2.5 in plus ±0.5 in cushions)
±90° swivel, ±7° tilt |
| Steel reaction frame | Height
Maximum capacity | 15 ft (4.6 m)
15 lb-ft (20.3 N-m) overturning moment |
| Hydraulic power supply (Shore Western 110.11S model) | 20 horsepower TEFC motor
380 - 480 V, 3 phase, 50/60 Hz power
Local and remote control, 24 VDC control voltage
High/low pressure controls |
| Embedded (real-time) processor (NI PXI-8108) | 2.53 GHz Intel Core 2 Duo T9400 dual-core processor
1 GB 800 MHz DDR2 RAM standard
10/100/1000BASE-TX (gigabit) Ethernet
Integrated hard drive, GPIB, serial, and peripheral I/O |
| Data acquisition (PXI-6229, PXI-6221) | Integrated chassis with four signal conditioning module slots
16-bit, 48 analog inputs, 6 analog outputs, 72 digital I/O |
| **Software** |                          |
| Generic hydraulic controller (Shore Western SC6000) | Two servo amplifiers
Two servo valve drivers
Two internal function generators
Either voltage or amperage valve drive signals
Four transducer amplifiers per controller card
Integrated data acquisition |
| Hybrid testing controller | Hybrid testing model: programmed in MATLAB/Simulink
Deployed using NI VeriStand |

Table 4.1: Components and parameters of the hybrid testing system
4.1.1.2. Hydraulic Actuator and Reaction Frame

The hydraulic actuator used in the RTHS (i.e. structure actuator) imposes displacements directly to the test specimen. It is attached to a steel reaction frame at the appropriate specimen height via a bolt hole pattern on the frame. The structure actuator contains a built-in linear variable differential transducer (LVDT) and load cell, allowing displacement and force to be monitored and utilized as feedback in the RTHS.

4.1.1.3. Hydraulic Controller

The hydraulic controller is the SC6000 manufactured by Shore Western. It is connected to the LVDT, load cell, and servo valve of the actuator, as well as the hydraulic power supply (HPS). This provides the user with low-level control of the actuator via proportional-integrator-derivative (PID) control as well as monitoring of displacements and forces.

The hydraulic controller is capable of conducting open-loop tests, in which feedback from the LVDT and load cell is not necessary to calculate displacement commands. It also contains an external input for the actuator, which allows displacement commands to be sent from an external source (i.e. real-time processor). This is required for the RTHS in this study since they are closed-loop tests.

4.1.1.4. Real-Time Processor and Data Acquisition

The National Instruments (NI) PXI-8108 real-time processor runs the RTHS models in this study. It analyzes the numerical substructures based on the LVDT and load cell feedback from the experimental substructure, which it obtains from the analog input channels on the National Instruments data acquisition (DAQ) cards (PXI-6229/6221). Furthermore, it calculates displacements commands for the experimental substructure and
sends them to the analog output channels on the DAQ cards. Both the real-time processor and the DAQ cards are enclosed in a single chassis, which facilitates real-time transfer of data between them.

4.1.1.5. Hybrid Testing Controller

The hybrid testing controller is a PC dedicated to development and deployment of the RTHS models. The models are developed in MATLAB/Simulink and deployed to the real-time processor using the software environment NI VeriStand. Section 4.1.3 provides a detailed description of the integration of MATLAB/Simulink and NI VeriStand.

4.1.2. Hardware Integration

Figure 4.1 illustrates the integration of the hardware components schematically. The HPS, hydraulic controller, and actuator are all connected via cables provided by the manufacturer. This forms the internal hydraulic control connection shown in red arrows which facilitates open-loop testing, such as shake table testing (STT) or quasi-static testing (QST).

The DAQ cards housed with the real-time processor are connected to the hydraulic controller through external Bayonet Neill-Concelman (BNC) connectors (see Figure 4.2), providing the critical link between the RTHS model running in the real-time processor and the experimental equipment. The LVDT and load cell readings of the actuator are fed into the DAQ cards so that the real-time processor can use them in the RTHS model to update the Bouc-Wen parameters and calculate the target displacements and velocities. Meanwhile, the target displacements of the experimental substructure are sent to the hydraulic controller through the external command connection, where they are then executed.
The connection between the hybrid testing controller and the real-time processor is wireless. The RTHS model created in the hybrid testing controller is deployed to the real-time processor via the internet using NI VeriStand, as shown by the dotted arrow in Figure 4.1. This connection and the external BNC connections in Figure 4.2 together form the external hybrid testing connection shown in blue arrows in Figure 4.1. Without this connection, closed-loop testing, and therefore RTHS, would not be possible.

![Figure 4.1: Schematic diagram of the hybrid testing system](image-url)
4.1.3. Software Integration

MATLAB/Simulink and NI VeriStand are both used in the hybrid testing controller to conduct RTHS. Models are created using Simulink, a block diagram environment integrated with MATLAB, and contain special blocks that correspond to software channels in NI VeriStand. Before deploying the model to the real-time processor, the user maps the software channels created in the Simulink model to the hardware channels on the DAQ cards and vice versa. This allows displacement commands to be sent to the actuator and feedback from the LVDT and load cell to be used in the RTHS model. Then the model is wirelessly deployed to the real-time processor, which runs it when the test is started. The test is monitored in NI VeriStand using the graphs and indicators available in the customizable workspace. In addition, the workspace has functions for calibration of hardware channels and data logging.

4.2. Test Specimen

Figure 4.3 shows the specimen devised for the implementation of online model updating in RTHS. It consists of a lumped mass attached to the structure actuator, a base fixed to the immobile shake table, and four replaceable columns. At the bottom, each column is placed between the base and an inside plate, and at the top, each column is placed
between the lumped mass and an outside plate. At the top and bottom of each column, two bolts are tightened on each side, providing the friction force necessary to clamp the columns.

The specimen emulates typical steel frame behavior, and the columns are oriented so that their weak axes are in bending to ensure that they yield and exhibit hysteresis during the tests, which is necessary for this study. The setup allows for easy replacement of the columns in between each test.

4.3. Quasi-Static Test and Offline Parameter Identification

Before conducting RTHS, a quasi-static (QST) reversed cyclic test was carried out on the test specimen, and its Bouc-Wen parameters were identified offline using the results. By investigating the performance of the UKF in offline parameter identification, it could be
determined whether the specimen was suitable for RTHS with model updating in this study.

The cyclic loading pattern used for the QST is shown in Figure 4.4. The specimen was loaded quasi-statically, and the displacement amplitude increased with each cycle up to a peak of ±2.8 inches. The QST utilized almost the full stroke of the actuator for two reasons: 1) so the specimen would exhibit the maximum achievable hysteresis, which is desirable for the UKF to accurately identify Bouc-Wen parameters, and 2) so the specimen would achieve its maximum possible peak restoring force, hence increasing the signal-to-noise ratio of the load cell, which is also beneficial for the UKF.

The hysteretic response of the specimen obtained from the QST is shown in Figure 4.5. Upon inspection, the response appears quite similar to those which the Bouc-Wen model is capable of exhibiting, but it is actually asymmetric. When the specimen is subjected to negative displacements, the corresponding restoring force is greater in magnitude than the respective positive displacements. For example, the peak positive displacement of 2.8 inches corresponds to a restoring force of about 0.17 kips, but the peak negative displacement of -2.8 inches corresponds to a restoring force of about -0.20 kips. However, the Bouc-Wen model does not take asymmetric hysteresis into account, so obtaining an accurate fit for the experimental data may be challenging. The asymmetry may be due to the single-ended piston rod in the actuator, which has a greater effective piston area in compression, whereas a double-ended piston has equal areas on both sides for balanced performance.
Figure 4.4: Cyclic loading pattern used for QST

Figure 4.5: Hysteretic response of specimen
In spite of the specimen’s asymmetric hysteresis, parameter identification was conducted offline using the displacement and force data from the QST. The Bouc-Wen parameters $k$ and $\alpha$ were roughly approximated from the hysteretic response in Figure 4.5.

$$k \approx \frac{(0.07 \text{ kips}) - (-0.07 \text{ kips})}{(0.5 \text{ in}) - (-0.5 \text{ in})} = 0.14 \frac{\text{kips}}{\text{in}}$$  \hspace{1cm} 4.1

$$k_f \approx \frac{(0.15 \text{ kips}) - (0.14 \text{ kips})}{(2.0 \text{ in}) - (1.5 \text{ in})} = 0.02 \frac{\text{kips}}{\text{in}}$$  \hspace{1cm} 4.2

$$\alpha = \frac{k_f}{k} = \frac{0.02}{0.14} = 0.14$$  \hspace{1cm} 4.3

The approximations of $k$ and $\alpha$ in Equations 4.1 and 4.3 were used in the initial state estimate $\hat{x}_{00}$. The parameters $\beta$, $\gamma$, and $n$ are difficult to approximate by inspecting the hysteresis response, so they were assigned reasonable initial estimates. The initial estimate of $z$ was set to zero, as explained previously in section 3.2. Altogether, the initial state estimate was set to:

$$\hat{x}_{00} = \begin{bmatrix} 0 \\ 0.14 \\ 1.00 \\ 0.50 \\ 2.50 \\ 0.14 \end{bmatrix}$$  \hspace{1cm} 4.4

The initial covariance matrix $P_{00}$ and the process noise covariance matrix $Q$ established in sections 3.2.1 and 3.2.2 were utilized again for the parameter identification of the specimen. They are:
The measurement noise covariance $R$ is a scalar value and represents the load cell noise covariance in this study. As discussed in section 3.2.3, overestimation of the measurement noise covariance does not severely degrade convergence performance, but underestimation does. To determine the optimal value for the structure actuator load cell at LESS, the UKF was run with three $R$ values: $10^{-5}$, $10^{-6}$, and $10^{-7}$. Figure 4.6 shows the time histories of the Bouc-Wen parameters for the three $R$ values. When $R=10^{-7}$, the time histories of all the parameters are erratic and unreasonable, indicating underestimation of the measurement noise covariance. When $R=10^{-5}$ and $R=10^{-6}$, the time histories are much more realistic, demonstrating that either of these values is acceptable. Since $R=10^{-5}$ is likely an overestimation and thus too conservative, $R=10^{-6}$ is selected as the optimal value and is used for both offline and online model updating.
Figure 4.6: Time histories for various R values for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$
Figure 4.6 cont’d: Time histories for various $R$ values for (a) $k$, (b) $\beta$, (c) $\gamma$, (d) $n$, and (e) $\alpha$

The converged values of the Bouc-Wen parameters from the offline parameter identification of the QST data when $R = 10^{-6}$ are listed in Table 4.2. A numerical simulation of the QST was performed on the Bouc-Wen model using these parameters. The model was subjected to the same loading pattern shown in Figure 4.4. A comparison of the hysteretic response from the QST and the numerical hysteretic response is shown in Figure 4.7. The two responses match quite well, with the exception of the peak negative restoring force. As expected, the Bouc-Wen model is not able to capture the
asymmetric hysteresis of the specimen. Nevertheless, the offline UKF was able to identify Bouc-Wen parameters that provide a good fit for the specimen used in this study. Hence, it was concluded that the specimen is suitable for proof-of-concept tests of RTHS with online model updating, and it was used as the experimental substructure in the tests discussed next.

\[
\begin{array}{c|c}
 k & 0.14 \\
 \beta & 0.60 \\
 \gamma & 0.44 \\
 n & 1.60 \\
 \alpha & 0.15 \\
\end{array}
\]

**Table 4.2: Converged values of Bouc-Wen parameters from QST**

![Figure 4.7: Comparison of experimental and numerical hysteretic response](image.png)
4.4. Time Delay Compensation for RTHS

Before online model updating can be implemented in RTHS, compensation for the inherent time delay in the actuator while ensuring stability of the test needs to be addressed.

The time delay value $\tau$ is the amount of time between when a displacement command is sent to the actuator and when the displacement is actually achieved. To determine the $\tau$ value of the structure actuator at LESS, an open-loop test was conducted with the actuator attached to the specimen, in which the peak displacement and velocity of the actuator were similar in magnitude to those that would be expected to occur during RTHS. A zoomed view of the displacement command and LVDT time history from the open-loop test is shown in Figure 4.8, and the value of $\tau$ was estimated to be about 0.012 seconds.

![Figure 4.8: Time delay estimation](image_url)
In closed-loop testing, feedback from the specimen (i.e. restoring force) is required to control the test, as discussed in Chapter 0. When closed-loop tests are conducted in real-time, actuator time delay can introduce effective negative damping, which causes vibration amplitude to increase over time if the effective negative damping is greater than the total damping of the system. This will cause the test to become unstable. The higher the time delay value, the lower the natural frequency of the structure must be to achieve stability (Ahmadizadeh, et al., 2008).

The modified feedforward (MFF) compensation scheme was adopted in the RTHS in this study since it has proven to be effective in previous studies at LESS (Sanchez, 2013). It is characterized by the following equation:

\[
u(k\Delta t + \tau) = \left(1 + \frac{\eta^2}{2}\right)u_k - \left(\eta + \frac{\eta^2}{2}\right)u_{k-1} + \left(\frac{\eta^2}{2}\right)u_{k-2}\]

where \( k \) is the pseudo-time index, \( \Delta t \) is the control time step, \( \tau \) is the control time delay compensation value, \( u_k \) is the displacement command at pseudo-time \( k \), and \( \eta = \frac{\tau}{\Delta t} \) is a dimensionless control parameter (Dion, et al., 2011).

A closed-loop RTHS without model updating was performed with the MFF compensation scheme to verify its functionality prior to implementing online model updating. Figure 4.9 presents a zoomed view of the actuator displacement command, the measured displacement (LVDT), and the desired displacement. The figure clearly shows that the MFF is sufficiently compensating for the actuator time delay since the measured displacement matches the desired displacement very closely. Moreover, the time delay between the command and the measured displacement is still about 0.012 seconds, indicating that the previous estimation of \( \tau \) is satisfactory.
Figure 4.9: Command tracking performance of actuator with MFF compensation

4.5. RTHS without Online Model Updating

The prototype structure used for the RTHS without online model updating was the same as the 3DOF system used in the numerical hybrid simulation in section 3.3 with different structural properties (i.e. mass, damping, etc.). The first story was the experimental substructure (see Figure 4.10), and the upper two stories were the numerical substructures whose responses were simulated using identical Bouc-Wen parameters.
Table 4.3 lists all the parameters used in the RTHS without model updating. Using the known initial stiffness of the specimen, a value for the masses $m_i$ was determined that resulted in reasonable natural frequencies. The Bouc-Wen parameters of the numerical substructures, i.e. the upper two stories, were set to the parameters identified from the QST results in section 4.3. The scale applied to the El Centro ground motion was set accordingly so that the peak displacement of the experimental first story would have a magnitude of about 2.5 inches to take advantage of most of the actuator stroke without too much risk of exceeding it. The integration time step $\Delta t$ was set to 0.001 seconds, as this value is typically used for RTHS and has demonstrated success in previous RTHS conducted at LESS (Sanchez, 2013). Lastly, the time delay value $\tau$ was set to 0.012 seconds, as estimated from section 4.4.
<table>
<thead>
<tr>
<th>Structural Properties</th>
<th>Mass ( (m_i) )</th>
<th>0.003 kip-sec^2/in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping Ratio ( (\xi) )</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Bouc-Wen Parameters of Numerical Substructures</td>
<td>( A )</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>( k )</td>
<td>0.14 kip/in</td>
</tr>
<tr>
<td></td>
<td>( \beta )</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>( \gamma )</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>( n )</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>( \alpha )</td>
<td>0.15</td>
</tr>
<tr>
<td>Natural Frequencies ( (\omega_n) )</td>
<td>3.04, 8.52, 12.31 rad/sec</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ground Motion</th>
<th>Earthquake</th>
<th>El Centro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake Scale</td>
<td>0.85</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Integration Parameters</th>
<th>Time step ( (\Delta t) )</th>
<th>0.001 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

| MFF Compensation | Time Delay Value \( (\tau) \) | 0.012 sec |

**Table 4.3:** Structural, ground motion, integration algorithm, and MFF compensation parameters used in the RTHS without updating

The time history results of absolute displacement (displacement with respect to the ground) for each story are shown in Figure 4.11. The displacement of the experimental first story is the measured displacement from the LVDT. The magnitude of the peak displacement of the experimental first story is about 2.3 inches, which is slightly less than the intended 2.5 inches.
Figure 4.11: Absolute displacement time history results of the RTHS without updating

Figure 4.12 shows the hysteretic response of each story. Even though the first story only attained a peak displacement of 2.3 inches, its hysteretic response shows that the columns exhibited ample nonlinear behavior as expected. Also, the hysteretic response of the first story appears to be have a different initial stiffness $k$, post-yield stiffness $k_f$, and overall shape than the hysteretic responses of the numerical stories, which were determined based on the Bouc-Wen parameters identified from the QST results. This implies that the hysteretic behavior of the specimen changed between the QST and this RTHS, which may be attributed to the drastic difference in the loading rate between the QST and the RTHS and/or variations in the installation of the columns. Consequently, offline parameter identification cannot fully predict the hysteretic response of a different test when testing conditions are slightly changed. This illustrates the importance of online parameter identification during hybrid simulation (i.e. model updating).
RTHS with online model updating was carried out using the same 3DOF structure shown in Figure 4.10. The first story was the experimental substructure, and the upper two stories were the numerical substructures. The implementation of the online model updating used in these proof-of-concept tests was based on the numerical hybrid simulation with model updating described in section 3.3. During the RTHS, the 3DOF system was subjected to an earthquake ground motion $\ddot{u}_g$, and the UKF used the restoring force feedback from the experimental first story to identify its Bouc-Wen parameters. These parameters were applied to the numerical model of the second and third stories at each time step as the RTHS progressed.
Six RTHS with online model updating were conducted, and Table 4.4 lists the parameters that were used for all of them. These parameters are the same as those used in the RTHS without updating. For RTHS 1 and 2, all of the Bouc-Wen parameters were updated, and for RTHS 3 and 4, \( n \) was excluded from the UKF and kept constant in an attempt to improve the convergence of the other parameters. The constrained UKF (CUKF) was implemented for RTHS 5 and 6 to prevent the parameters from exceeding their bounds as stated in Equations 2.25 through 2.29.

The UKF system model parameters \( (\hat{x}_{0|0}, \hat{P}_{0|0}, Q, R) \) used for each RTHS are listed in Table 4.5. They were determined based on the parametric study discussed in Chapter 3 and modified for the various RTHS as described throughout this section.

<table>
<thead>
<tr>
<th>Structural Properties</th>
<th>Mass ((m_i))</th>
<th>0.003 kip-sec(^2)/in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Damping Ratio ((\xi_i))</td>
<td>0.05</td>
</tr>
<tr>
<td>Ground Motion</td>
<td>Earthquake</td>
<td>El Centro</td>
</tr>
<tr>
<td></td>
<td>Earthquake Scale</td>
<td>0.85</td>
</tr>
<tr>
<td>Integration Parameters</td>
<td>Time step ((\Delta t))</td>
<td>0.001 sec</td>
</tr>
<tr>
<td></td>
<td>(\gamma)</td>
<td>0.5</td>
</tr>
<tr>
<td>MFF Compensation</td>
<td>Time Delay Value ((\tau))</td>
<td>0.012 sec</td>
</tr>
</tbody>
</table>

Table 4.4: Structural, ground motion, integration algorithm, and MFF compensation parameters used in the RTHS with model updating
<table>
<thead>
<tr>
<th></th>
<th>RTHS 1</th>
<th>RTHS 2</th>
<th>RTHS 3</th>
<th>RTHS 4</th>
<th>RTHS 5</th>
<th>RTHS 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>**( \hat{x}_{0</td>
<td>0} )**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( 0.14 )</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>( 0.60 )</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>( 0.44 )</td>
<td>0.44</td>
<td>0.44</td>
<td>1.00</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>( 1.60 )</td>
<td>2.50</td>
<td>0.15</td>
<td>0.60</td>
<td>2.50</td>
<td>0.15</td>
<td>0.60</td>
</tr>
<tr>
<td>( 0.15 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.60</td>
<td>1.74</td>
</tr>
</tbody>
</table>

|                  |        |        |        |        |        |        |
| **\( P_{0|0} \)** |        |        |        |        |        |        |
| \( 10^{-6} \)    | 0      | 0      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 1      | 0      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 1      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 0      | 1      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 0      | 0      | 0.01   | 0      | 0      |
| \( 0 \)          | 0      | 0      | 0      | 0      | 0.1    | 0      |

|                  |        |        |        |        |        |        |
| **\( Q \)**      |        |        |        |        |        |        |
| \( 10^{-12} \)   | 0      | 0      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 0      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 0      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 0      | 0      | 0      | 0      | 0      |
| \( 0 \)          | 0      | 0      | 0      | 0      | 0      | 0      |

|                  |        |        |        |        |        |        |
| **\( R \)**      |        |        |        |        |        |        |
| \( 10^{-6} \)    |        |        |        |        |        |        |

Table 4.5: UKF system model parameters used for each RTHS
4.6.1. RTHS 1

For the first RTHS, the initial estimate vector $\hat{x}_{00}$ was populated with the converged values of the Bouc-Wen parameters from the QST offline parameter identification, and $P_{00}$, $Q$, and $R$ were kept the same as they were in the QST offline parameter identification. Figure 4.13 shows the time histories of all the Bouc-Wen parameters during RTHS 1. While $k$, $\beta$, and $\gamma$ appear to converge, $n$ and $\alpha$ diverge and take on values that exceed their bounds as stated in Equations 2.25 and 2.29. The numerical substructures inherited these flawed $n$ and $\alpha$ values, causing them to output unrealistically large restoring forces, which in turn caused large displacements to be generated for the experimental substructure. This triggered the safety limit of the actuator, effectively ending the test (see Figure 4.14).

Figure 4.13: Time histories of all parameters for RTHS 1
4.6.2. RTHS 2

The failure of the first RTHS demonstrates the decreased robustness of the UKF when implemented for RTHS as compared to the numerical hybrid simulation. As mentioned in the parametric study in Chapter 3, much of the UKF’s sensitivity in this study originates from the nonlinear Bouc-Wen parameter $n$, which takes the form of an exponent in Equation 2.24. In an attempt to prevent $n$ from becoming too low and dropping below its bound of zero, the initial estimate of $n$ was increased to 2.50 for RTHS 2. All other initial estimates in $\hat{x}_{0|0}$ and the other UKF parameters ($P_{0|0}$, $Q$, and $R$) were kept the same.

As demonstrated from the time histories of the parameters in Figure 4.15, the increased initial estimate of $n$ resolved the divergent behavior for both $n$ and $\alpha$. The converged values of the Bouc-Wen parameters are listed in Table 4.6 along with the converged values from the QST offline parameter identification. They are slightly
different, which further confirms the concept that the Bouc-Wen parameters defining the hysteresis of a physical specimen change from test to test due to factors such as the loading rate and the column installation.

<table>
<thead>
<tr>
<th></th>
<th>QST</th>
<th>RTHS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.60</td>
<td>0.54</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.44</td>
<td>0.30</td>
</tr>
<tr>
<td>$n$</td>
<td>1.60</td>
<td>1.74</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.15</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 4.6: Converged values of Bouc-Wen parameters from QST and RTHS 2

Figure 4.15: Time histories of all parameters for RTHS 2
To demonstrate the effectiveness of online model updating, two numerical simulations of the 3DOF system were conducted. For the first one, the hysteresis of all three stories was modeled using the Bouc-Wen parameters identified offline from the QST (first column of Table 4.6). For the second one, the hysteresis of all three stories was modeled using the Bouc-Wen parameters identified online from the second RTHS (second column of Table 4.6). The second simulation represents the “exact” case, in which the true hysteresis parameters of all three stories are known. The time history results of absolute displacement for both numerical simulations and the RTHS with online model updating are compared in Figure 4.16 for each story. The hysteresis responses are also compared in Figure 4.17.
Figure 4.16: RTHS 2 absolute displacement time history results comparison for (a) first story, (b) second story, and (c) third story.
Figure 4.17: RTHS 2 hysteresis response comparison for (a) first story, (b) second story, and (c) third story
In general, the RTHS results with online model updating match the numerical simulation results using the parameters identified online better than the numerical simulation results using the parameters identified offline. This illustrates the importance of online model updating in RTHS. Nevertheless, there are still some major discrepancies between the RTHS results with online model updating and the numerical simulation results using the parameters identified online. The most likely cause of these discrepancies is the specimen’s asymmetric hysteresis response, which cannot be captured by the Bouc-Wen model as mentioned earlier. In addition, the excessive fluctuation of the Bouc-Wen parameters in the first several seconds of the RTHS (see Figure 4.15) likely caused unrealistic hysteresis response in the upper two stories during that time, which could certainly skew the results of the RTHS.

4.6.3. RTHS 3
In an attempt to eliminate the nonlinearity caused by the Bouc-Wen parameter $n$, the UKF system model was adjusted by removing $n$ from the state estimate vector $\hat{\mathbf{x}}_{k|k}$ and setting it to a constant value of 1.74. This value was selected because it was the converged value of $n$ from RTHS 2 (see Table 4.6). In addition, the matrices $\mathbf{P}_{0|0}$ and $\mathbf{Q}$ were reduced to $5 \times 5$ by removing the column and row associated with $n$.

For RTHS 3, the initial estimates of the updated parameters ($k$, $\beta$, $\gamma$, and $\alpha$) were kept the same as they were in the first two RTHS. Figure 4.18 shows the time histories of all the updated parameters during RTHS 3. For the majority of the test, $\alpha$ is far below its bound of zero, although it eventually converges to a value just above zero. Furthermore, the converged value of $\gamma$ is greater than that of $\beta$, violating the bound in
Equation 2.27. This indicates that, even though they all converged, the UKF identified an impossible set of Bouc-Wen parameters.

Figure 4.18: Time histories of all updated parameters for RTHS 3

4.6.4. RTHS 4

RTHS 4 used the same $5 \times 5$ $P_{00}$ and $Q$ matrices that were used in RTHS 3, and the value of $n$ was again kept constant at 1.74. The initial estimates of $\beta$ and $\gamma$ were set further apart (1.00 and 0, respectively) in an attempt to prevent them from intersecting and in turn violating the bound in Equation 2.27. The initial estimate of $\alpha$ was increased to 0.6 in an attempt to suppress the large negative values it was exhibiting in RTHS 3.

Figure 4.19 shows the time histories of all the updated parameters during RTHS 4. The increased initial estimate of $\alpha$ appears to have improved its convergence performance, but the converged value of $\gamma$ is again larger than that of $\beta$. Therefore, the UKF again identified an impossible set of Bouc-Wen parameters.
4.6.5. RTHS 5

To resolve the parameter bound violations that occurred in RTHS 1, 3, and 4, the constrained UKF (CUKF) proposed by Wang and Wu (2013) was implemented (see Equations 2.35 through 2.39) for RTHS 5 and 6. The CUKF constrains the sigma points $\chi_{\text{kk}}$ to stay within the bounds of the Bouc-Wen parameters, effectively preventing the parameters in the state estimate $\hat{x}_{\text{kk}}$ from exceeding the bounds as well. In addition, for RTHS 5 and 6, the numerical substructures were not updated until two seconds into the simulation to prevent them from inheriting the oscillatory behavior of the Bouc-Wen parameters prior to convergence.

RTHS 5 used the same initial state estimate $\hat{x}_{00}$ that was used in RTHS 1, and the time histories of the Bouc-Wen parameters are shown in Figure 4.20. The parameters $n$ and $\alpha$ still tend to diverge slightly but not nearly as severely as they did in RTHS 1. This improvement caused stable results for RTHS 5, whereas the standard UKF utilized in
RTHS 1 produced unstable results. Therefore, the CUKF has the benefit of helping to stabilize the results of RTHS compared to the standard UKF.

Another advantage of the CUKF is the improved convergence speed. When the standard UKF is used, convergence can be extremely slow in some cases, such as $\alpha$ in RTHS 3 which required more than 15 seconds to converge. However, all the parameters appear to converge within about six seconds when the CUKF is utilized. Furthermore, the CUKF appears to significantly decrease the fluctuations that occur prior to convergence compared to the standard UKF.

Figure 4.20 clearly demonstrates that the CUKF is constraining the Bouc-Wen parameters successfully as $\alpha$ is prevented from exceeding its upper bound of one at about two seconds. In addition, $\gamma$ converges to the same value as $\beta$, indicating the enforcement of its upper bound. However, this suggests that the converged value of $\gamma$ may not accurately represent its true value. Nevertheless, the CUKF is capable of providing stability, which is crucial for RTHS.

![Figure 4.20: Time histories of all parameters for RTHS 5](image-url)
4.6.6. RTHS 6

The initial state estimate $\hat{x}_{0|0}$ for RTHS 6 was set to the converged values observed in the successful RTHS 2. The time histories of the Bouc-Wen parameters during RTHS 6 are shown in Figure 4.21. Again, the CUKF improved the convergence speed and decreased the fluctuations compared to the standard UKF. However, $\gamma$ again converges to the same value as $\beta$, and $\alpha$ converges to its lower bound of zero. This indicates that $\gamma$ and $\alpha$ are attempting to violate their bounds, but the CUKF is preventing this by holding their values at their bounds. Consequently, the converged values of $\gamma$ and $\alpha$ likely do not represent the optimum values for modeling the hysteresis of the experimental substructure.

![Figure 4.21: Time histories of all parameters for RHTS 6](image)
The converged values of the Bouc-Wen parameters from RTHS 6 are listed in Table 4.7 along with the converged values from the QST offline parameter identification. Similarly to RTHS 2, two numerical simulations of the 3DOF system were conducted: one using the Bouc-Wen parameters identified offline from the QST, and one using the Bouc-Wen parameters identified online from RTHS 6. The results of these numerical simulations were then compared to the results of RTHS 6. The time history results comparison is shown in Figure 4.22 for each story, and the hysteresis responses are also compared in Figure 4.23.

<table>
<thead>
<tr>
<th></th>
<th>QST</th>
<th>RTHS 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.60</td>
<td>0.26</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.44</td>
<td>0.26</td>
</tr>
<tr>
<td>$n$</td>
<td>1.60</td>
<td>1.83</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.15</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.7: Converged values of Bouc-Wen parameters from QST and RTHS 6
Figure 4.22: RTHS 6 absolute displacement time history results comparison for (a) first story, (b) second story, and (c) third story.
Figure 4.23: RTHS 6 hysteresis response comparison for (a) first story, (b) second story, and (c) third story
Similar to RTHS 2, the results of RTHS 6 match the numerical simulation results using the parameters identified online better than the numerical simulation results using the parameters identified offline. However, the agreement is not quite as close as it was in RTHS 2. This demonstrates that when Bouc-Wen parameters converge to their bounds (RTHS 6), the results are not as accurate as when parameters converge to values within their bounds (RTHS 2). Nevertheless, the CUKF is able to provide a reasonable approximation of the parameters defining the hysteresis of the experimental substructure.

4.7. Implementation Issues

Even though the results of RTHS 2 demonstrate that the online model updating method proposed in this study can function successfully, the results of the other RTHS with online model updating largely indicate the need for an increase in robustness. The CUKF helps to stabilize the results of RTHS, but $\gamma$ and $\alpha$ tend to converge to their bounds, which are likely not their optimum values.

Wang and Wu (2013) state that modeling errors are a possible reason for parameter bound violation in the UKF. Since the Bouc-Wen model does not account for asymmetry, modeling errors were introduced into the UKF in this experimental study. Song and Der Kiureghian (2006) proposed a generalized version of the Bouc-Wen model that is capable of modeling highly asymmetric hysteresis, and implementation of this model could reduce modeling errors and in turn improve the performance of the UKF and/or CUKF.

In addition, the hysteresis of the test specimen in this study was not very repeatable as its Bouc-Wen parameters tend to differ from test to test. Figure 4.24 compares the hysteresis response observed in each stable RTHS (i.e. 2 through 6), and it
can be seen that the degree of asymmetry varies. The fluctuations in the force measurement observed in some of the RTHS were a result of the displacement commands momentarily exceeding the safety limit of the actuator.

Both the standard UKF and the CUKF appear to be very sensitive to the initial state estimate $\hat{x}_{00}$ in this experimental study, as parameter convergence is heavily influenced by the selection of $\hat{x}_{00}$. Further research is needed to better understand the effects of $\hat{x}_{00}$ on the convergence performance so that optimum initial estimates can be selected.

Figure 4.24: Hysteresis response of all RTHS with online model updating
4.8. Summary and Conclusions

This chapter began by discussing the hardware and software components and the test specimen utilized in the experimental study. A QST was carried out on the test specimen, and the Bouc-Wen parameters defining its hysteretic response were identified offline using the UKF. Despite asymmetry in the specimen’s response, the UKF was able to identify Bouc-Wen parameters that provide a good fit for the specimen.

A RTHS without online model updating was conducted next. The Bouc-Wen parameters identified offline from the QST were used for the numerical substructures, and the modified feedforward (MFF) scheme was implemented to compensate for time delay. The results implied that the hysteretic behavior of the specimen can change from test to test, demonstrating the need for online model updating.

Lastly, six proof-of-concept RTHS with online model updating were conducted, and it was observed that the convergence of the Bouc-Wen parameters is very sensitive to the initial state estimate \( \hat{x}_{0|0} \). When the standard UKF is used, the parameters tend to exceed their bounds whether or not \( n \) is held constant. This may cause instability of the test as observed in RTHS 1. However, if the parameters converge correctly, the accuracy of the results are improved as seen in RTHS 2.

Implementation of the CUKF is helpful in stabilizing the results of RTHS as it prevents the Bouc-Wen parameters from exceeding their bounds. Moreover, the CUKF improves convergence speed and significantly reduces fluctuations prior to convergence. Nevertheless, the parameters tend to simply converge to their bounds when the CUKF is utilized, which implies that the CUKF may not be identifying the optimum parameter values for modeling the hysteresis of the experimental substructure.
5. Conclusions

A literature review comprising the most recent numerical and experimental studies on model updating in hybrid simulation suggested that no researcher has yet attempted online model updating in real-time hybrid simulation (RTHS), to the author’s knowledge. Therefore, the work presented in this thesis focused on development and implementation of an online model updating method for RTHS.

Chapter 2 discussed the theoretical background for this study. The Explicit Newmark algorithm was selected for numerical integration of the equation of motion (EOM), and the framework for pseudodynamic (PSD) hybrid simulation was presented. The Bouc-Wen model was designated to model hysteresis in this study, and the Unscented Kalman Filter (UKF) was employed for parameter identification.

Because of the lack of literature regarding selection of appropriate system model parameters for the UKF when used in conjunction with the Bouc-Wen model, a parametric study was carried out in Chapter 3. Through the parametric study, a thorough understanding of the behavior of the UKF was acquired so that an experimental study could be conducted with proper UKF system model parameters. The findings of the parametric study are summarized in section 5.

The experimental validation of the developed model updating method, which utilized the hybrid testing system at the Laboratory of Earthquake and Structural Simulation (LESS), was presented in Chapter 4. Offline parameter identification using results from a quasi-static test (QST) was conducted, and the results of a RTHS without online model updating demonstrated the need for online model updating. Six proof-of-
concept RTHS with online model updating were carried out. The findings of the experimental study are summarized in section 5.1.2.

5.1. Summary of Findings

5.1.1. Parametric Study

In Chapter 3, a parametric study of the UKF system model parameters was conducted using a SDOF system. The initial state estimate vector $\hat{x}_{00}$, the initial covariance matrix $P_{00}$, the process noise covariance matrix $Q$, and the measurement noise covariance $R$ were each investigated.

Several general trends regarding convergence accuracy and speed of the Bouc-Wen parameters were observed through the parametric study. High values of the elements within $P_{00}$ and a small value of $Q(1,1)$ generally cause the parameters to converge faster, but also introduce fluctuations and overshooting, which is detrimental for online model updating. Also, if the initial estimates in $\hat{x}_{00}$ are inaccurate, misleading convergence may occur, in which the Bouc-Wen parameters converge to incorrect values. In addition, the parametric study indicated that the Bouc-Wen parameters are interrelated, in that the convergence of one parameter may be affected by the initial estimates and initial covariance elements of other parameters.

Because the UKF employs a matrix square root operation, the $P_{00}$ matrix must be positive definite at every step, and a non-positive definite $P_{00}$ results in an error. The parametric study revealed important trends regarding positive definiteness which are helpful in avoiding errors in the UKF. The more accurate the initial estimate of $n$ is, the
lower the error threshold of its corresponding initial covariance element. Therefore, selecting a conservatively low value for $P_{(5,5)}$, the initial covariance element corresponding to $n$ in the Bouc-Wen model, reduces the chances of an error in the UKF. Furthermore, the more inaccurate the estimates in $\hat{x}_{(5,5)}$ are, the higher the error threshold of $Q_{(1,1)}$ is, so it is important that the value of $Q_{(1,1)}$ not be too low so as to avoid an error in the UKF. Additionally, high values of $Q_{(1,1)}$ can worsen convergence speed and accuracy, so $Q_{(1,1)}$ should be sufficiently low to obtain good convergence but should not be too close to the error threshold.

Investigation of the measurement noise covariance $R$ also provided some notable trends. Lower levels of measurement noise result in more accurate convergence, but convergence speed is generally not affected much by the level of measurement noise. The Bouc-Wen parameters $k$ and $\alpha$ are robust to excessive measurement noise, but $\beta$, $\gamma$, and $n$ may exhibit asymptotic convergence, in which their estimates are always approaching their correct values but never actually reach them. Also, if no accurate estimation of the measurement noise covariance is available, it is recommended to overestimate the value, as underestimation can lead to poor convergence performance.

The values within the $P_{(5,5)}$ and $Q$ matrices that were selected in the parametric study may not be universally applicable because the parametric study was conducted using a SDOF system with an assumed set of parameters. This is especially true for $Q_{(1,1)}$ because the time history of the hysteretic displacement $z$ clearly depends on the structural properties, earthquake ground motion, etc. However, the parametric study indicated that the acceptable range of $P_{(5,5)}$ and $Q$ is reasonably broad as convergence
accuracy is rarely affected by their values. This signifies that the UKF is robust to the selections of $P_{00}$ and $Q$. Nevertheless, it is important that $P_{00}$ and $Q$ be tuned using a system that is similar to that which is being investigated.

5.1.2. Experimental Study

After the QST was conducted, it was observed that the hysteretic response of the specimen is asymmetric, which the Bouc-Wen model is incapable of capturing. Nevertheless, the UKF was able to identify Bouc-Wen parameters offline whose converged values provided a good fit for the specimen. In addition, the RTHS without online model updating suggested that the hysteretic behavior of the specimen can change from test to test, which demonstrates the benefit of online model updating.

The RTHS with online model updating indicated that the convergence of the Bouc-Wen parameters is very sensitive to the initial state estimate $\hat{x}_{00}$. When the standard UKF is used, the parameters tend to exceed their bounds, which may cause the test to become unstable. However, accurate structural responses can be obtained from RTHS with online model updating if the Bouc-Wen parameters converge properly.

Stability of RTHS results can be improved through implementation of the CUKF, which prevents bound violations of the Bouc-Wen parameters. The CUKF also improves convergence speed and reduces fluctuations prior to convergence. However, in this study the parameters tend to simply converge to their bounds when the CUKF is utilized, which implies that the CUKF may not be identifying the optimum parameter values.

The asymmetry of the specimen introduces modeling errors into the UKF since the Bouc-Wen model is not capable of capturing it. This demonstrates the importance of selecting a hysteresis model that is suitable for the test specimen being used. In addition,
the specimen used in this study was very flexible, causing the signal-to-noise ratio of the load cell to be quite low. For RTHS with online model updating, the load cell should be selected such that its measurement noise covariance provides an acceptable signal-to-noise ratio based on the peak signal that is expected to occur.

5.1.3. Implementation Guidelines

The following guidelines for implementation of the UKF in online model updating were extracted from the numerical and experimental studies presented in this thesis:

- It is recommended to perform numerical simulation prior to implementing the UKF experimentally so reasonable values for the UKF system model parameters can be established. The model used for numerical simulation should be similar to the structural system which is being investigated experimentally.
- A reasonable estimation of the hysteretic behavior of the test specimen should be well-established prior to conducting RTHS with online model updating so that modeling errors can be minimized by selecting the most suitable hysteresis model.
- Preliminary offline parameter identification using experimental hysteresis data may prove to be helpful in fine tuning the UKF system model parameters, particularly the measurement noise covariance $R$.
- If parameter bound violations are observed when using the standard UKF, the CUKF may be implemented, which improves RTHS stability, increases convergence speed, and reduces fluctuations prior to convergence.
5.2. Future Work

Several additional studies can be conducted to further improve the online model updating scheme proposed in this thesis:

- **Parametric study on the UKF/CKF for structural model updating:**
  
  It is highly desirable to obtain a better understanding of the sensitivity of the initial state estimate $\hat{x}_{0|0}$ for RTHS with online model updating. Additional parametric studies using a wide range of structural system models and realistic experimental data may prove to be helpful in this regard.

- **Expand the use of the UKF/CKF into other structural systems and models:**
  
  The UKF can be used with other hysteresis models, such as the Bouc-Wen-Baber-Noori model (Baber & Noori, 1985). Furthermore, RTHS with online model updating can be performed using other types of materials and structural elements. For example, if a wood shear wall is used as the experimental substructure, the CASHEW model (Folz & Filiatrault, 2001) could be implemented with the UKF for parameter identification. Additionally, for preliminary numerical hybrid simulations with model updating, finite element modeling of the experimental substructure could be employed, which could potentially provide a more realistic representation of experimental RTHS.

- **Explore other parameter identification algorithms for RTHS with online model updating:**
  
  As discussed in the literature review, optimization techniques have also been used for hybrid simulation with online model updating. However, Yang et al. (2012) and Kwon and Kammula (2013) have both indicated that their model updating techniques
involving optimization may not suitable for real-time applications due to the large computation time associated with nonlinear analysis of several sets of trial parameters. Further research is needed to assess the suitability of optimization techniques for online model updating in RTHS.

In addition, other state observers besides the UKF have been used to facilitate model updating in real-time. For example, Song (2011) implemented the Extended Kalman Filter (EKF) and the high gain (HG) observer in addition to the UKF for real-time model updating. This demonstrates the potential for the EKF and HG to be used for online model updating in RTHS.

- **Further experimental investigation:**

One potential reason for the bound violations observed in the RTHS with online model updating in this study is modeling errors in the UKF, and additional research should be carried out to minimize these errors. This can be achieved by using a test specimen and a hysteresis model that are better correlated than those used in this study.

The Explicit Newmark integration algorithm employed in this study has the limitation of being conditionally stable, making it particularly unsuitable for large, complicated structures with many DOFs. To address this deficiency, unconditionally stable algorithms can be implemented for RTHS with online model updating. Significant development work on unconditionally stable algorithms for RTHS using the hybrid testing system at LESS has recently been carried out (Santana, 2014).

As mentioned throughout this thesis, fluctuations of parameter estimates prior to convergence can deteriorate the accuracy of RTHS results with online model
updating. For the last two RTHS presented in Chapter 4, the numerical substructures were not updated until two seconds into the simulation. However, if fluctuations are still present after the specified time at which updating begins, errors may still be introduced. This suggests the need for strictly defined criteria for when to begin updating numerical substructures. For example, an algorithm could be developed that is able to detect the time at which convergence is approximately achieved for each parameter. Implementation of such an algorithm could reduce modeling errors in the numerical substructures.

When the CUKF is implemented, the Bouc-Wen parameters tend to converge to their bounds, indicating that the identified parameters likely do not accurately represent the optimum values. The upper and lower bounds of the parameters in the CUKF can be more strictly set so that they encompass a reasonable range expected to occur in the specimen rather than the entire possible range. This may allow the CUKF to identify more realistic values for the parameters, which in turn would improve the accuracy of the RTHS results.

- **Develop implementation guidelines for broad application:**

  The scope of this study was limited to the UKF, Bouc-Wen model, Explicit Newmark algorithm, etc. so the implementation guidelines listed in section 5.1.3 may not be universal for all applications. As online model updating is further investigated and more tools and techniques are put into practice, it is desirable to develop implementation guidelines for broad application of online model updating.
References


Appendix

MATLAB/Simulink Files
Parametric Study

Initialization File:

clc; clear all; close all;

% The following script is an initialization file for numerical and experimental pseudodynamic (PSD) simulations.

%% Begin - Input Variables

g = 386.089;

%% Seismic mass and damping
% Inertial properties are modeled analytically in PSD simulation.
m = 0.05; % Mass
zeta = 0.05; % Damping ratio

%% Ground Motion
ga_scale = 0.5; % Scale for ground motion
load 'elcentro.mat'
pga = max(abs(ga))*g*ga_scale; % Peak ground acceleration

%% True Bouc-Wen parameters
A = 1;
k = 1;
beta = 2;
gamma = 0.5;
n = 1.5;
alpha = 0.2;

%% Natural frequency and period
Wn = sqrt(k/m); % Natural frequency
Tn = 2*pi/Wn; % Natural period

%% Damping coefficient
% Damping constant

c = 2*zeta*sqrt(k*m);

%% Explicit newmark parameters
% Time step
sd = 0.01;
gammaN = 0.5;
meff = m+dt*gammaN*c;

%% UKF parameters
Q = 0*eye(6);
Q(1,1) = 1.e-6;
R = 0;
kappa = 2;

%% Preprocessing
% Scaling ground motion
scale = pga/max(abs(ga));
ga = scale.*ga;
num = length(ga);

% Interpolate ground acceleration and calculate new time step, dt
t1 = linspace(0,(num-1)*dtga,num);
i = floor((num-1)*dtga/dt + 1);
t = linspace(0,(ni-1)*dt,ni);
ga = interp1(t1,ga,t);

t = t(:);
ga = ga(:);
Ag = ga(:);
tga = [t,ga];

Simulink Model:
Restoring Force Model:
UKF Algorithm:
Numerical Hybrid Simulation with Model Updating

Initialization File:

clc; clear all; close all;

% The following script is an initialization file for numerical and experimental pseudodynamic (PSD) MDOF simulations with model updating

%% Begin - Input Variables
g = 386.089;

%% Seismic mass and damping
% Inertial properties are modeled analytically in PSD simulation.
M = zeros(3);
for i=1:3
    M(i,i)=0.05; % Mass matrix
end

Pcoeff = -M*ones(3,1);
zeta = 0.05; % Damping ratio of first two modes

%% Ground Motion
% Scale for ground motion
load 'elcentro.mat'
pga = max(abs(ga))*g*ga_scale; % Peak ground acceleration

%% Bouc-Wen parameters of experimental substructure
A = 1;
k = 1;
beta = 2;
gamma = 0.5;
n = 1.5;
alpha = 0.2;

%% Stiffness matrix
K = [2*k,-k,0;-k,2*k,-k;0,-k,k];

%% Natural frequency and period
[v,D]=eig(K,M); % Evaluate natural frequencies and modes
Wn=zeros(1,3);
Tn=zeros(1,3);
for i=1:3
    Wn(i)=sqrt(D(i,i)); % Extract natural frequencies
    Tn(i)=2*pi/Wn(i); % Extract natural periods
end

%% Damping matrix
a0=zeta*2*Wn(1)*Wn(2)/(Wn(1)+Wn(2));
a1=zeta*2/(Wn(1)+Wn(2));
C=a0*M+a1*K; % Damping matrix
%% Explicit Newmark parameters
dt = 0.01; % Time step
sdt = 0.01;
gammaN = 0.5;
Meff = M+dt*gammaN*C;

%% UKF parameters
Q = 0*eye(6); % Process noise covariance matrix
Q(1,1) = 1.0e-12;
R = 0; % Measurement noise covariance matrix
kappa = 2;

%% Preprocessing

% Scaling ground motion
scale = pga/max(abs(ga));
ga = scale.*ga;
um = length(ga);

% Interpolate ground acceleration and calculate new time step, dt
t1 = linspace(0,(num-1)*dtga,num);
ni = floor((num-1)*dtga/dt + 1);
t = linspace(0,(ni-1)*dt,ni);
ga = interp1(t1,ga,t);
t = t(:);
ga = ga(:);
Ag = ga(:);
tga = [t,ga];
Simulink Model:

- **State Estimation Vector**
- **Covariance Matrix**

### Variables
- \( x \) = State Estimation Vector
- \( P \) = Covariance Matrix
- \( D \) = Displacement
- \( V \) = Velocity
- \( A \) = Acceleration

### Equations

- **Restoring Force Model**
  \[ F = \gamma_N \]  

- **Explicit Newmark Part 1**
  \[ \text{Part 1} \]

- **Explicit Newmark Part 2**
  \[ \text{Part 2} \]

- **Ground Acceleration**
  \[ g \]

- **Covariance Matrix**
  \[ P_{\text{coeff}} \]

- **Displacement**
  \[ D \]

- **Velocity**
  \[ V \]

- **Acceleration**
  \[ A \]

- **Time Step**
  \[ \Delta t \]

- **Gamma N**
  \[ \gamma_N \]
Restoring Force Model:
1<sup>st</sup> Story:

2<sup>nd</sup> and 3<sup>rd</sup> Stories:
RTHS with Online Model Updating

Initialization File:

clc; clear all; close all;

% The following script is an initialization file for numerical and
% experimental pseudodynamic (PSD) MDOF simulations with model updating

%% Begin - Input Variables
g = 386.089;

%% Seismic mass and damping
% Inertial properties are modeled analytically in PSD simulation.
M = zeros(3);
for i=1:3
    M(i,i)=0.003; % Mass matrix
end

Pcoeff = -M*ones(3,1);

zeta = 0.05; % Damping ratio of first two modes

%% Ground Motion
% Scale for ground motion
load 'elcentro.mat'
ppga = max(abs(ga))*g*ga_scale; % Peak ground acceleration

%% Stiffness matrix
A = 1;
k = 0.14;
K = [2*k,-k,0;-k,2*k,-k;0,-k,k];

%% Natural frequency and period
[v,D]=eig(K,M); % Evaluate natural frequencies and modes

Wn=zeros(1,3);
Tn=zeros(1,3);
for i=1:3
    Wn(i)=sqrt(D(i,i)); % Extract natural frequencies
    Tn(i)=2*pi/Wn(i); % Extract natural periods
end

%% Damping matrix
a0=zeta*2*Wn(1)*Wn(2)/(Wn(1)+Wn(2));
a1=zeta*2/(Wn(1)+Wn(2));
C=a0*M+a1*K; % Damping matrix

%% Explicit Newmark parameters
dt = 0.001; % Time step
sdt = 0.001;
gammaN = 0.5;
Meff = M+dt*gammaN*C;

%% UKF parameters
Q = 0*eye(6); % Process noise covariance matrix
Q(1,1) = 1.0e-12;
R = 1.0e-6; % Measurement noise covariance matrix
kappa = 2;

%% Preprocessing

% Scaling ground motion
scale = pga/max(abs(ga));
ga = scale.*ga;
um = length(ga);

% Interpolate ground acceleration and calculate new time step, dt
t1 = linspace(0,(num-1)*dtga,num);
ni = floor((num-1)*dtga/dt + 1);
t = linspace(0,(ni-1)*dt,ni);
ga = interp1(t1,ga,t);
t = t(:);
ga = ga(:);
Ag = ga(:);
tga = [t,ga];

%% Feedforward compensation
Tau = 0.012;
Eta = Tau/dt;

%% Modified Feedforward compensation
MFF1 = 1+Eta+Eta^2/2;
MFF2 = -(Eta+Eta^2);
MFF3 = Eta^2/2;
Simulink Model:

State Estimation Vector

Covariance Matrix

Displacement Velocity Acceleration

Displacement

Velocity

Force

Restoring Force Model

Covariance Matrix

Ground Acceleration

Explicit Newmark Part 1

Explicit Newmark Part 2
Restoring Force Model:
Modified Feedforward:
Embedded MATLAB functions

Explicit Newmark Part 1:

```matlab
function [Dp,Vp] = Part1(D,V,A,dt,gammaN)
Dp = D+dt*V+dt^2/2*A;
Vp = V+dt*(1-gammaN)*A;
```

Explicit Newmark Part 2:

```matlab
A=Meff\(P-F-C*Vp);
V=Vp+dt*gammaN*A;
```

Bouc-Wen Part 1:

```matlab
function z_dot=z_dot(A,z,beta,gamma,n,V)
z_dot=A*V-beta*abs(V)*abs(z)^(n-1)*z-gamma*V*abs(z)^n;
```

Bouc-Wen Part 2:

```matlab
function F=F(alpha,k,D,z)
F=alpha*k*D+(1-alpha)*k*z;
```

Sigma Points (Standard UKF):

```matlab
function [X,W]=SigmaPoints(x,P,kappa)
n=numel(x);
X=zeros(n,2*n+1);
W=zeros(1,2*n+1);
X(:,1)=x;
W(1)=kappa/(n+kappa);
U=chol((n+kappa)*P);
for k=1:n
    X(:,k+1)=x+U(k,:)';
    W(k+1)=1/(2*(n+kappa));
end
for k=1:n
```
\[ X(:,n+k+1)=x-U(k,:)'; \]
\[ W(n+k+1)=1/(2*(n+kappa)); \]

\section*{Sigma Points (CUKF):}

\begin{verbatim}
function [Xi,W]=SigmaPoints(x,P,kappa)
    n=numel(x);
    Xi=zeros(n,2*n+1);
    W=zeros(1,2*n+1);
    thetaL=zeros(1,2*n);
    thetaU=zeros(1,2*n);
    thetaC=zeros(1,2*n);
    theta=zeros(1,2*n);
    Xi(:,1)=x;
    W(1)=kappa/(n+kappa);
    s=[chol(P)',-
    chol(P)'];
    for k=1:2*n
        thetaL(k)=min([sqrt(n+kappa),(0-x(2))/
        abs(s(2,k)),(0-x(3))/
        abs(s(3,k)),(-x(3)-x(4))/
        abs(s(4,k)),(1-x(5))/
        abs(s(5,k)),(0-
        x(6))/abs(s(6,k))]);
        thetaU(k)=min([sqrt(n+kappa),(x(3)-
        x(4))/abs(s(4,k)),(1-
        x(6))/abs(s(6,k))]);
        thetaC(k)=min([thetaL(k),thetaU(k)]);
    end
    for k=1:n
        theta(k)=min(thetaC(k),thetaC(n+k));
        theta(n+k)=min(thetaC(k),thetaC(n+k));
    end
    for k=1:2*n
        Xi(:,k+1)=x+theta(k)*s(:,k);
        W(k+1)=1/(2*(n+kappa));
    end
end
\end{verbatim}

\section*{f and h Functions \(n\) updated:}

\begin{verbatim}
function [fX,hX]=f_and_h(X,D,V,dt)
    n=size(X,1);
    fX=zeros(n,2*n+1);
    hX=zeros(1,2*n+1);
    for k=1:2*n+1
        X(:,n+k+1)=x-U(k,:)';
        W(n+k+1)=1/(2*(n+kappa));
    end
end
\end{verbatim}
\[
fX(1,k) = X(1,k) + dt \cdot (V - X(3,k) \cdot \text{abs}(V) \cdot \text{abs}(X(1,k))^{n-1} \cdot X(1,k) - X(4,k) \cdot V \cdot \text{abs}(X(1,k))^n) \\
fX(2:6,k) = X(2:6,k) \\
hX(1,k) = fX(6,k) \cdot fX(2,k) \cdot D + (1-fX(6,k)) \cdot fX(2,k) \cdot fX(1,k) \\
\]

**f and h Functions (n constant):**

```matlab
function [fXi,hXi]=f_and_h(Xi,d,v,dt,n)
num=size(Xi,1);
fXi=zeros(num,2*num+1);
hXi=zeros(1,2*num+1);
for k=1:2*num+1
    fXi(1,k)=Xi(1,k)+dt*(v-Xi(3,k)*abs(v)*abs(Xi(1,k))^(n-1)*Xi(1,k)-Xi(4,k)*v*abs(Xi(1,k))^n);
    fXi(2:5,k)=Xi(2:5,k);
    hXi(1,k)=fXi(5,k)*fXi(2,k)*d+(1-fXi(5,k))*fXi(2,k)*fXi(1,k);
end
```

**State Prediction:**

```matlab
function [xp,Pp]=StatePred(fX,W,Q)
[n,kmax]=size(fX);
xp=zeros(n,1);
for k=1:kmax
    xp=xp+W(k)*fX(:,k);
end
Pp=zeros(n,n);
for k=1:kmax
    Pp=Pp+W(k)*(fX(:,k)-xp)*(fX(:,k)-xp)';
end
Pp=Pp+Q;
```

**Measurement Prediction:**

```matlab
function [yp,Pyy]=MeasPred(hX,W,R)
[n,kmax]=size(hX);
yp=zeros(n,1);
for k=1:kmax
    yp=yp+W(k)*hX(:,k);
end
```
Pyy=zeros(n,n);
for k=1:kmax
    Pyy=Pyy+W(k)*(hX(:,k)-yp)* (hX(:,k)-yp)';
end

Pyy=Pyy+R;

Kalman Gain:

function K=KalmanGain(fX,xp,hX,yp,Pyy,W)

n=numel(xp);
m=numel(yp);

Pxy=zeros(n,m);
for k=1:2*n+1
    Pxy=Pxy+W(k)*(fX(:,k)-xp)* (hX(:,k)-yp)';
end
K=Pxy/Pyy;

State Update:

function [x,P]=StateUpdate(xp,Pp,K,yp,Pyy,y)

x=xp+K*(y-yp);
P=Pp-K*Pyy*K';