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A COMPARISON OF THE HUTCHINGS' LOW-STRESS, FACTOR ANALYSIS, HAND-HELD CALCULATOR, AND CONVENTIONAL ADDITION ALGORITHMS FOR SPEED, ACCURACY, AND PREFERENCE WITH REGULAR EDUCATION STUDENTS

by

John C. Hampel

A Thesis
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Faculty of The Graduate College
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A COMPARISON OF THE HUTCHINGS' LOW-STRESS, FACTOR ANALYSIS, HAND-HELD CALCULATOR, AND CONVENTIONAL ADDITION ALGORITHMS FOR SPEED, ACCURACY, AND PREFERENCE WITH REGULAR EDUCATION STUDENTS

John C. Hampel, M.A.
Western Michigan University, 1993

Three repeated measures experiments were conducted to determine the efficiency with which medium and low math-achieving, fourth- and fifth-grade students computed two sizes of addition problems using each of four different algorithms. The experimenter selected the conventional algorithm and the hand-held calculator because they are currently in popular use. The experimenter selected the Hutchings’ low-stress and factor analysis algorithms because of their demonstrated performance enhancing characteristics and unique design features. The three written algorithms represented a design continuum that facilitated a powerful deductive analysis.

In all conditions the Hutchings’ low-stress algorithm produced incrementally superior performance consistent with its unique features. Seven of the eight ANOVA’s that compared the groups’ mean performances using the four algorithms were significant. Post hoc multiple comparisons showed addition performance using the Hutchings’ method to be consistently superior to the other methods.
ACKNOWLEDGEMENTS

What a veritable challenge this thesis has been. I almost do not believe it is complete. I would like to thank the people foremost involved in believing that I could (and would) complete it and in so doing help bring the project to fruition. To the members of my committee, Dr. Galen Alessi, Dr. Jack Michael, and Dr. Bill Redmon, I extend my sincere appreciation for their unwavering support and guidance. I say very special thanks to Jack for his continuing availability, wisdom, and statistical savvy, and to Galen for his immutable good humor, patience, and belief in environmental solutions.

I also owe a huge debt to Dr. Pat Steinert for taking time in a busy work schedule to help promote, organize and execute "Project Addition." I commend her willingness to extend herself on my behalf. I must also mention pertinent friends and family whose contributions have been timely and invaluable. They include: my friend Sheri for carrying on so diligently during the many weeks of data collection, bringing cookies for the kids, and support for me; my parents for helping out with the reliability scoring; my subjects, without whom this study would have been just a nice idea; and my best friend Kris for her unfailing encouragement.

Finally, I dedicate this thesis to Dr. Lloyd Barton Hutchings, the man who created the "low-stress" algorithm. From the stories Galen has shared with me I feel that you must be a very special man indeed. My hope is that someday the Hutchings'
low-stress algorithm will be more than an alternative, experimental commodity. I truly believe that, introduced proactively from the beginning, it has the power to streamline the teaching of computational mathematics, and thereby help ease our current educational crisis. This highly effective educational technology is available to us now, and it's free. Thank you Dr. Hutchings.

John C. Hampel
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A comparison of the Hutchings' low-stress, factor analysis, hand-held calculator, and conventional addition algorithms for speed, accuracy, and preference with regular education students

Hampel, John Carl, M.A.

Western Michigan University, 1993

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CHAPTER I

INTRODUCTION

Description of Research

This research study includes three experiments that examine the rate and accuracy of the performances of medium and low math-performing fourth- and fifth-grade students on addition problems using each of four different algorithms. A mathematical algorithm is generally defined as a "step-by-step written procedure for determining the result of an arithmetic operation (i.e., a sum, a difference, a product, a quotient)" (Ashlock, 1982, p. 2).

The first experiment involves 20 daily sessions and examines the children’s rate and accuracy of performance calculating addition problems that consist of 2x7 matrices (or arrays) of numbers using four different addition algorithms. The second experiment examines the same subjects, algorithms, and dependent variables, computing larger 5x7 matrix problems over a period of 10 daily sessions. The third experiment involves six daily sessions and examines the children’s preferences (and related performance) among the four addition methods for computing 5x7 array problems.
The Problem

The rationale and motivation for this research are the unsatisfactory results of current methods of instruction in arithmetic in the United States. The essential problem is that too many students graduate from our school systems and into adult society unable to perform basic addition computations. It has been suggested that computational methods different than those currently in use may improve many individual's addition performance. The question then becomes, which methods are most efficient. This is the essential question addressed by this study.

Historical Background

The debate over which method of addition is most efficient was resolved once over 1,000 years ago. It has only resurfaced within the last 30 years, with the development of the current mathematics crisis in education. To fully appreciate the problems we face and the options available to us we must know something of the past and how the debate over addition methods was once decided.

Relative to the history of man's use of numbers, both the Hindu-Arabic number system and our modern mathematical constructs that employ it represent recent developments (Gundlach, 1971). The current, diverse systems of mathematics that are, in fact, the driving wheel of most of the hard sciences, are the result of thousands of years of cultural and technical evolution. Gundlach (1971) describes this progression through time:

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Historically, concern with computation preceded by many centuries concern with properties of numbers. In turn, the concern with properties of numbers preceded concern with number systems by almost two thousand years. It is from this latter concern, however, that the powerful unifying patterns of modern mathematics are generated. Most historians ascribe the chief concern for computation to the Oriental peoples and the chief concern with numbers systems to those who more or less followed the predominantly Greek tradition. It appears, however, that the essence of the modern approach is due to a synthesis of both points of view—a synthesis that has been achieved rather recently. (pp. 26-27)

It is unknown when in history humankind first discovered (or created) cardinal numbers (Gundlach, 1971). The first numeration systems were very simple, and expressed quantitative events and rudimentary computations. These notation systems had limited power and flexibility, were almost always unique in form to their parent language, and were born of the needs of daily living (Benner, 1971; Davis, 1971). Davis (1971) writes,

The art of computation originated in the basic needs of human life and thus is found in the earliest records of the race. It was practiced by the Egyptians and the Babylonians. It appears in the oldest traditions in the Orient. The art has been discovered even in such an isolated culture as that of the Mayas. (p. 87)

Davis (1971) cites two primary needs that spurred the development of computation: (1) the need for accurate and consistent accounting of business transactions and spoils of war, involving such commodities as numbers of people, herds of animals, sacks of grain, tracts of land, and actual monies; and, (2) the need to keep track of the seasons, leading to the development of the calendar.

Originally, computations were carried out with the assistance of tables. In this fashion, the Babylonians accomplished multiplication as early as 2000 B.C.
The earliest known mechanical calculating device for computing simple arithmetic operations is the abacus (Davis, 1971). Developed by the Greeks (abacus is probably derived from the Greek words abax, or abakion, meaning a board strewn with dust or sand for counting or sketching figures), early versions of the abacus often used pebbles for calculation. The Greeks and Romans constructed more sophisticated designs that employed grooved tables with sliding counters and ruled lines to represent numbers. Eventually, a portable-frame version of the abacus evolved. It utilized a small metal or wooden frame holding a set of parallel wires or wooden rods on which moved counters or beads. Davis (1971) describes how the abacus functions:

The essential idea of any of the various forms of the abacus is that counters (pebbles, markers, or disks) on one line indicate units; on the next line, assuming that a base-ten system is in use, the counters indicate tens; on the next, hundreds; then thousands, and so forth. In its most rudimentary form this means that the moment ten counters appear on a line, they are replaced by a single counter on the proper adjacent line; similarly, one counter would be replaced by ten when moving in the opposite direction. This action is probably the basis of the terms "carry" and "borrow" used in arithmetic today. (p. 118)

The Chinese adopted the abacus during the sixth century A.D. (Davis, 1971). Their version, called the suan phan, is similar to the portable-frame type used by the Romans. The Japanese version, also similar to the Roman portable-frame model, called the soroban, was common by the seventeenth century. Until the introduction of written algorithms, employing the Hindu-Arabic number system, some type of abacus was the dominant method for performing number computations throughout
Europe and Asia. The abacus is still popular today in such countries as Japan, China, India, and Russia.

It is unfortunate that the exact details involved with the formation and development of the Hindu-Arabic system are not documented (Benner, 1971). Benner (1971) states, "Most historians have placed the final development of this system, with full systematic use of the zero and the principle of place value, probably sometime between the fourth century A.D. and the seventh century A.D." (p. 48). It is probable that the Hindus invented the system (hence, the primary name), while the Arabs transmitted it (Gundlach, 1971). The system was adopted by the Arabs when it was brought to Baghdad about 800 A.D. It was first explicated in print about 825 A.D. by the Moslem mathematician, astronomer, and geographer Mohammed ibn-Musa al-Khowarizmi. Al-Khowarizmi, who was easily the single greatest medieval influence in the area of mathematics, ascribed the system to the Hindus (Benner, 1971; Schrader, 1971). In his best-known book, Hisab al-jabr w' al-muqabala ("Science of Restoration [or reunion] and Opposition"), he demonstrates "what is easiest and most useful in arithmetic such as men constantly require in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealings with one another." (Al-Khowarizmi, 825/1915). The book describes how to perform computations using the Hindu-Arabic system in powerful ways that had never before been realized (Schrader, 1971). It laid the groundwork for Western mathematicians who would use it to create substantially faster and more
reliable methods for multiplication and division (Davis, 1971). (Imagine trying to multiply using Roman Numerals.)

Sadly, the original Arabic work by al-Khowarizmi was lost (Benner, 1971). Fortunately, a twelfth century Latin translation, likely done by Adelard of Bath (an English Monk) survived. Entitled Liber algoritmi de numero Indorum, and referred to as Liber algorismi, it dispatched al-Khowarizmi's book, his computational strategies, and the Hindu-Arabic system that made these strategies possible, to the western world. The system began to appear in Europe near the end of the thirteenth century (Davis, 1971). By the end of the fifteenth century arithmetic began to assume a somewhat modern form.

Al-Khowarizmi, himself, travelled to Europe to compete in mathematical contests where men would race one another to correctly solve a variety of computational problems (Alessi, 1991). The abacus had been generally superior to those written systems designed around Roman Numerals. However, the abacus was no match for the written algorithms produced by al-Khowarizmi using the Hindu-Arabic notation system. Davis (1971) notes, "One weakness of calculation by the abacus is that each step erases the preceding step, so that there is no way to check the answer except by recomputing" (p. 92).

The new written system was not only fast, but produced a permanent record (using sand tables) that could be analyzed for errors and, if need be, quickly corrected. When a competitor using the abacus failed to produce the correct answer it was necessary for him to restart his computations from the beginning. By
providing a written, correctable record of a computation the Hindu-Arabic number system permanently shifted the balance of relative "computing power" away from the abacus. The practical (and economic?) advantages of al-Khowarizmi's methods, and the fact they could only be employed with the Hindu-Arabic number system used by the Moslems, provided sufficient motivation for Western scholars (and eventually Western culture) to master and adopt the new methods (Schrader, 1971).

It is of interest to note that the opening words of the Latin translation (Liber algorismi), Algoritmi dixit ("Al-Khowarizmi says"), evolved into the term "algorithm" (or "algorism") meaning a type of computational process.

Repercussions of the Problem

The Educational Crisis

Sadly, over 1000 years later, even though we know efficient and (most) accurate ways for adults to add numbers, the problems with teaching children basic computational skills have not been resolved. The public education system in the United States has yet to create a system that can, more or less, guarantee success in teaching the basic math skills. Some children excel, whereas others fail repeatedly. Many become proficient and declare math as one of their favorite subjects, while an alarming percentage develop animosities and/or anxieties toward computational mathematics. Biggs (1965) indicates that learning mathematics seems to arouse anxiety more easily than any other subject. It is discouraging to realize that most
elementary school systems in the United States typically devote the largest percentage (amounting to many hundreds of hours) of mathematics classroom time to oral or written drill (Milgram, 1969).

To darken the picture even further one must also realize that the available classroom time for teaching these skills has been shrinking (Hutchings, 1976). We can no longer afford to devote an unlimited amount of time to the development of computational skills. One of the primary reasons for this time shortage appears to be the rate at which human knowledge is expanding. The volume of new conceptual mathematics that a person must master in order to be considered a competent worker or a competitive applicant is (constantly) increasing. Hence, children must learn substantially more conceptually in 1991 than in 1961, and in the same amount of classroom time. Concurrently, while conceptual demands increase, demands for computational competence have not decreased. Careers that demand quantitative expertise are growing in number, while existing careers are now demanding ever increasing levels of mathematical competence. Based on the increasing mathematical complexities introduced into and demanded by the high technology work-place, this trend is expected to continue.

Effects on the Individual Level

A second major contributing factor to the failure of students in the United States to master computational skills has to do with the nature of the instruction now used and the effects that it may have on students. Currently, computation skills are
taught over a period of years, involving thousands of repetitive (paper and pencil) drills and didactics. Students who chronically perform unsatisfactorily in computational arithmetic often find this repetition to be tedious, frustrating, unrewarding, even anxiety producing. Hutchings (1972) asserts that, for beginning students of mathematics, "the major portion of failure, boredom, and frustration experiences occur during the mastery of computational skills" (p. 3). Gordon (1972) and Alessi (1974) share the viewpoint that repetitive math exercises (that intermittently yet reliably lead to failure) may develop or condition negative attitudes toward the material being drilled. Failure to master (or frustration associated with) computational arithmetic can easily discourage interest in or appreciation for mathematics in general. As Yvon (1987) states, "Students who become mired in drill activities are often prevented from seeing the beauty and enjoying the fun of mathematics" (p. 16). Boyle (1975) suggests the possible negative effects of extensive drilling within the broader context of increased elementary classroom demands:

The repetitive drill on basic skills and the introduction of more conceptual material to be learned by children in their elementary school years may well contribute to a conditioning of negative emotional responses to the whole subject area of mathematics. (p. 2)

Published research reveals a reliable connection between mathematical performance and learning and anxiety (Callahan & Glennon, 1975). As Callahan and Glennon (1975) state, "anxiety appears to be more easily aroused in learning mathematics than it is in other subjects" (p. 82). Skemp (1971) describes a scenario
that suggests a typical way in which math-related anxiety might develop and intensify. The scenario begins as a student does not (completely) understand a teacher's explanation or lecture point. If the student feels anxious at their failure to comprehend, they may exert greater efforts to do so. However, their heightened anxiety level may actually diminish the efficacy of their efforts to understand. Hence, the student may continue to fail to understand and become even more anxious. This creates a vicious, self-defeating cycle. In describing the possible outcomes of this scenario, Callahan and Glennon (1975) report: "For mathematics lessons, at best the anxiety may be aroused in the single situation; at worst—and this is probably more common—there may be a spread of anxiety arousal to mathematical tasks in general" (p. 81). In a similar vein (from a different perspective) Skinner (1968) has observed that:

Few pupils ever reach the stage at which automatic reinforcements follow as the natural consequence of mathematical behavior. On the contrary, the figures and symbols of mathematics have become standard emotional stimuli. The glimpse of a column of figures, not to say an algebraic symbol or an integral sign, is likely to set off, not mathematical behavior, but a reaction of anxiety, guilt, or fear. (p. 18)

Skinner (1953) suggests that some of the covert stimuli, that result from human behaviors (e.g., performing number computations) that then lead to aversive consequences (such as confusion, failure, social ridicule or embarrassment), may be conditioned to elicit some of the same emotional or physical responses (e.g., sweaty palms, rapid heartbeat, dry mouth) as are elicited by the aversive consequences. By the same process this conditioning may also involve the concomitant environmental...
stimuli (both overt and covert), so that (for example) a classroom, desk, teacher, time of day, or thought about an upcoming arithmetic drill may each independently elicit some degree of aversive emotional arousal.

Such states of aversive emotional arousal also represent motivational states, or "establishing operations", wherein escape or avoidance of the aversive stimuli or behaviors becomes reinforcing and the individual's repertoire of behavior that has in the past been reinforced by such avoidance or escape will come to strength (cf. Michael, 1989). In other words, the child who fails (especially early on) in mathematics may quickly learn to avoid or escape any and or all math related tasks. Math and the things associated with it become aversive, undesirable, even punishing, while the behavior that avoids or escapes math related events (e.g., saying "I can't do this!" or watching television) becomes stronger and more prevalent.

Furthermore, the verbal behavior of the students' social community (including that of the student) that relates or refers to arithmetic events or performance may affect performance. A child overhearing another saying, "I hate these problems, they're too hard" may adopt the speaker's attitude (talking or acting like the speaker may be reinforcing) and subsequently not work as diligently (Skinner, 1957).

After examining the research on anxiety as it affects learning in mathematics, Biggs (1965) concluded that the impediments produced by anxiety seem to overwhelm any motivating effect for learning and performing mathematics. Aiken (1970a), on the other hand, believes that anxiety may effect mathematics performance either positively or negatively. The nature of the outcome depends on
the intensity of the anxiety, the specific mathematics task, and idiosyncrasies of the individual. Callahan and Glennon (1975) conclude that anxiety and mathematics performance are related, but this is a complex relationship. They indicate,

> It is difficult to make a simple generalization regarding the factors associated with anxiety in mathematics learning. Teachers can be quite confident, however, that high anxiety does have a debilitating effect on achievement in elementary school mathematics. From a selected set of studies, it would seem that association between achievement and anxiety will be affected by such factors in the instructional process as abstractness of the materials to be learned, familiarity with the material to be learned, grade level of the student, sex of the student, socioeconomic status of the student, as well as the type of cognitive processing required in the task. (Callahan & Glennon, 1975, p. 84)

If, for some students, mathematical events and stimuli become conditioned (over time) to elicit strong emotional responses such as anxiety, then one would expect to find these negative emotional responses mirrored in unfavorable or deteriorating attitudes toward mathematics. In fact, this appears to be the case. Callahan and Glennon (1975) indicate, "In general, studies would indicate a decline in favorable attitudes about mathematics as students continue in school" (p. 78).

The nature and stability of students' attitudes toward mathematics have been examined by a number of studies. However, it should be noted that measuring the "attitudes" of elementary school children appears to be problematic at best. Aiken (1970b) summarizes the major difficulties:

> A serious problem in drawing conclusions about the interaction between attitudes and achievement concerns the inadequacies of measures of attitudes themselves. The reliability of a Thurstone, Likert, or semantic-differential scale is usually fairly satisfactory at the high school and college levels, but reliable measures of attitudes
of elementary school pupils have yet to be devised. In fact, the shortcomings of all self-report inventories at the elementary school level are widely recognized; the limited reading abilities and experiences of pupils with the content of such inventories represent two sources of difficulty. (pp. 251-252)

Generally, the research suggests that most students hold strong feelings toward or attitudes about mathematics. That is, most students are not neutral regarding mathematics. Josephina (1959) surveyed 900 fifth-, sixth-, seventh-, and eighth-grade students asking them to rank order their most and least favorite school subjects. Arithmetic was ranked in the top three "best liked" and top three "least liked" school subjects for all grade levels. Rowland and Inskeep (1963) conducted a similar study surveying the attitudes of fourth-, fifth-, and sixth-grade students. They found arithmetic to be ranked first as the subject liked most across all grade levels, and fifth (out of ten subjects) as the subject liked least. Of all the subjects not mentioned by students as being liked or disliked, arithmetic ranked last. Faust (1963) surveyed more than 2500 upper elementary students from Iowa and found that, among the "skill" subjects, arithmetic ranked first (followed by reading, spelling, and language). In a survey of 366 eighth-grade students Callahan (1971) found that 62% of those surveyed tended to like math, 20% tended to dislike math, whereas only 18% expressed a neutral position toward mathematics. What the results of these studies suggest is that for most students there is no middle ground with mathematics. They either like it intensely or dislike it intensely.

Fedon (1958) found that some students form strong opinions (both negative and positive) toward mathematics as early as the third grade. However, there is
some disagreement as to whether the attitudes held by students in the early elementary years are generally lasting ones. Examining the attitudes of over 600 students, spanning a period of six years, beginning at the fifth or sixth grades and ending at the eleventh or twelfth grades, Anttonen (1969) found a (relatively low) positive correlation of .305 between early and late mathematics attitude scores. In a survey of third- through eighth-grade students Callahan (1971) found that the students believed that their attitudes toward mathematics developed and changed at each grade level. The students in this study also cited grades six and seven as being the most important for developing attitudes. Callahan and Glennon (1975) concur that the most important years in forming lasting attitudes toward mathematics are the upper elementary and middle school grades. These studies suggest that students' attitudes tend to shift over time (largely becoming less favorable).

On the other hand, there is also evidence suggesting that once a child has developed what is called a "bad attitude" toward math, it is most unlikely to be affected by the student's subsequent potential for improvement or the positive influences of teachers. Based on a long-term survey of students' changing attitudes toward math Poffenberger and Norton (1959) concluded:

The evidence from the study indicates that self-concepts in regard to mathematics ability are well established in the early years and it is very difficult for even the best teacher to change them in spite of the fact that potential ability is much in evidence. Students with an initially negative attitude toward mathematics may go into the classroom with a mental attitude set against the subject which may be maintained even when positive identification with the teacher is made. (p. 175)
Whereas it has been clearly shown that students' attitudes toward mathematics deteriorate over time, the effects of less favorable or deteriorating math attitudes on math performance is equivocal. Bassham, M. Murphy, and K. Murphy (1964) studied 159 sixth-grade pupils and found that 4 times as many students assessed as having a negative attitude toward mathematics were classified at .65 of a grade under there expected achievement as the number who overachieved that same amount. Similarly, almost 3 times as many students who were assessed as having positive attitudes toward math were classified at .65 of a grade above their expected achievement as the number who underachieved that same amount. Neale (1969) and Aiken (1970c) surveyed the research on the relationship between students' attitudes toward math and math achievement. Despite major differences in subject populations and assessment devices, most of the studies they examined found moderate, positive correlations between 0.20 and 0.40 (Callahan & Glennon, 1975).

It is possible (and believable) that the students' attitudes were pivotal in their performances. It is also possible that the reverse situation was true, that the students' performance levels determined their attitudes (Neale, 1969). Callahan (1971) reported that his subjects' most frequent reason for disliking mathematics was "not good in math, don't learn easily, not sure of myself" (p. 753). It also could have been that some third factor (or combination of factors e.g., parental attitudes, teaching methods, or the student's anxiety level) was responsible for both the students' performances and attitudes.
In summary, there is evidence supporting a moderate connection between math attitudes and math performance. However, given the difficulties of accurately measuring the attitudes of elementary school children, and the correlational nature of the supporting data, the effects of math attitudes on math performance among elementary school children remains uncertain.

**Weaknesses of the Traditional Algorithm and Drilling**

The traditional or conventional addition method may be ineffective with students because it requires too large a jump from previously learned abilities with numbers. This jump may explain the necessity for extensive drill, and the general ineffectiveness of this method. Brownell (1928) believes that children pass through a series of stages before they "attain the methods of apprehension which are characteristic of the adult" (p. 174). According to his position, children progress through a series of stages as they move from the most primitive or immature method of counting each item in a group (and perhaps using their fingers), to translating the objective representation into abstract symbols that they eventually learn to manipulate arithmetically as if they were concrete entities (and perhaps doing so covertly). The many steps between these two extremes of performance are characterized by increasing levels of abstractness. Brownell (1928) states,

> Children develop mature methods of apprehending concrete numbers by passing through a series of stages in the way they think about these numbers, stages which steadily tend to substitute a greater degree of abstractness for the concreteness of the external representation. (p. 174)
Our educational system does not acknowledge these learning processes by providing students with transitional mechanisms or experiences designed to move them from one level of abstractness to a higher one. Consequently, advancing students from the simple number counting stage to the basic math facts through drilling of these facts may be insufficient to develop a mature skill with their use. Rather than progress to an adult performance level they may rely on (and perfect) their existing (more immature) counting skills (such as counting on fingers or covertly, or breaking down the unknown math facts into known ones) to perform the basic math facts. The ability to add three numbers (what Brownell refers to as "second additions") using more mature methods is also achieved through a series of developmental steps. He indicates,

Pupils cannot, or at least do not, suddenly begin to make second additions rapidly, correctly, and with clear evidence of completely understanding the processes involved. On the contrary, they attain such skill by a series of steps or stages which are analogous to the series of steps by which they learned the additive combinations. (Brownell, 1928, p. 176)

The traditional method of addition may indeed represent too large of a conceptual jump for many students, even for those who have mastered the basic math facts. Brownell (1928) notes that, "Some pupils who seem to have a thorough understanding of the additive combinations (i.e., basic math facts) encounter difficulty in learning to perform the second additions in three-digit examples" (p. 178). This suggests that the traditional method itself may be poorly designed for the task it’s assigned, and that an algorithm requiring a lesser degree of abstraction may
be more appropriate for many students during some phase of their arithmetic education. It also follows that extensive repetition and drills with an algorithm requiring, as yet, unattained levels of abstraction will not overcome this barrier to learning, and may only exacerbate the problem. Brownell (1928) describes this process:

Prescription of drill (repetition) by no means insures that children will learn number facts by repeating them. Moreover, classroom drill in arithmetic, since it neglects the processes which are actually employed by children in learning arithmetic facts, cannot supply to children the type of assistance they need. Under conditions of drill, pupils who have not attained mature methods of dealing with numbers simply improve the efficiency with which they use their immature methods and fail to advance as they should to more mature, more effective methods of dealing with numbers. By its very nature, drill is almost powerless to help them. For these reasons drill represents a valuable method of teaching in the final stages of learning, not in the first stages (p. 200).

There is also notable research suggesting that children learn arithmetic skills better when less time is spent on drill and practice and more is spent on "meaningful developmental activities" (Callahan & Glennon, 1975, p. 95). In discussing the research literature Callahan and Glennon (1975) report, "There was general agreement among the studies that the classes which devoted 50% or more time to developmental activities performed better on achievement tests than those classes devoting 50% or more class time to drill and practice work" (p. 95). While drill should not be eliminated, it appears that as little as 25% of class time may be adequate. Summarizing the role of repetitive practice in the contemporary mathematics curriculum Callahan and Glennon (1975) echo Brownell's (1928)
conclusions: "it must be preceded by a thorough teaching program aimed at the building of meanings and understandings; or, stated otherwise, practice must follow understanding" (p. 126).

Using the traditional addition algorithm as an instructional device to establish a solid foundation of meanings and understandings, (presumably) prerequisite to a stage of productive repetition, appears problematic and counter-intuitive. In order to employ the traditional method with consistent success the user must first memorize, or learn to compute, not only the 55 simple additive combinations, or basic math facts (e.g., 1 + 2), but hundreds of complex math facts as well (e.g., 68 + 7). As Brownell (1928) implied, we appear to be teaching basic arithmetic functions (the meanings and understandings) using an addition method that incorporates advanced degrees of abstraction and requires relatively high levels of arithmetic understanding prior to its competent use. In effect, it is as if our students must practice before they understand. It is, therefore, not surprising that repetitive practice using the traditional addition algorithm frequently is ineffective for teaching competent addition skills.

Other evidence of inherent problems with the traditional addition algorithm come from Roberts (1968) who identified four categories of errors, or what he refers to as "failure strategies" that children, of a wide range of abilities, make in performing written computations. These categories include using the wrong operation, making obvious computation errors, applying an algorithm defectively or inconsistently (Roberts refers to an algorithm as "defective" if it does not always
produce correct results), and making random responses. His results indicate that "The largest number of errors was due to erroneous or incorrect algorithm techniques in all groups except the lowest quartile, which displayed more random responses" (p. 2). Even the highest quartile of the subjects in this study made substantial algorithm-related errors, fully 39% of their total mistakes.

In a follow-up study Engelhardt (1977) identified eight different classifications of computational errors. The three categories that involved defective or incorrect procedures (this includes algorithm-related errors) accounted for 61% of the total number of errors made by the children in the study. Commenting on these results Ashlock (1982) states, "Again, it is obvious that teachers dare not assume that errors in computation are caused by carelessness or by a child not knowing the basic facts. The actual procedures are likely to be wrong" (p. 3). Based on this research, the high degree of relative abstractness incorporated into the traditional algorithm (compared to the learning stages that precede it), the many years of repetition and drill required for most students to master it (compared to alternative algorithms and calculating devices) and the large percentage who never do, it appears that, in any number of ways, the traditional addition algorithm itself is inadequate as an instructional device.

What specifically about this algorithm is defective, overly abstract, or excessively demanding? Essentially, the problems appear to be twofold: (1) it requires that the operator perform most calculations covertly, and (2) it requires that the operator switch back and forth between two distinct types of operations,
computing basic math facts and carrying or regrouping. Further details as regards
these mechanisms are explicated (through comparisons to the Hutchings’ low-stress
algorithm) in the upcoming section "An Alternative Written Algorithm."

The Alternatives

The Japanese System and the Abacus

Some have suggested that we should revamp the American system along the
lines of the current Japanese system. In Japan, computational skills are not highly
stressed. The Japanese teach their children to compute basic arithmetic problems
effectively using the abacus (or soroban). This eliminates the necessity of
memorizing any math facts (basic or complex) and juggling abstract regrouping and
carrying operations covertly when the student is in the lower grades. This provides
immediate success for most Japanese students. The average Japanese student
masters the abacus with a minimum of instruction, and hence learns to compute
correct answers to arithmetic problems quickly and accurately without the years of
repetitive drilling required by the American system.

The Japanese system does teach the student how to use written methods of
computation. However, they are introduced years later than the American system
and well after the students have mastered many areas of conceptual mathematics.
The success of mathematics education in Japan suggests several ramifications:
(a) using a mechanical device like the abacus may enhance computational
performance by reducing the demand placed on what is loosely described as the "short-term memory" (Norman, 1969); (b) by facilitating early success it may increase interest in, and confidence with computational mathematics and mathematics in general; (c) eliminating the necessity of years of drilling and practice may reduce individual anxiety (especially as it pertains to long-term failure) and the avoidance of mathematics; (d) eliminating the hundreds of hours of valuable classroom time spent on drill and practice affords more opportunity for teaching higher level conceptual topics and real world problem solving that is desperately needed for our students to keep pace with the ever expanding human knowledge base; (e) waiting until the student is somewhat older and generally more capable of memorizing abstract information and executing abstract mathematical operations before teaching them the traditional written algorithms may enhance their long-term, adult performance with these algorithms.

It would be unfair, however, to suggest that the only significant differences between the two educational systems (and their students' relative performances) are computational tools and curriculum components. There are any number of non-algorithm, non-system differences that must be considered when evaluating the two systems overall performance. For example, whereas the average Japanese student spends 16 to 20 hours doing homework each week, their American counterparts today spend only 3 to 4 hours per week. This represents a dramatic change from the 1960's when American students spent an average of 16 to 17 hours per week studying and doing homework (Alessi, 1992a).
Regardless of the mechanisms at work, the Japanese system seems to produce results. In 1972, Japanese 13-year-olds ranked first in achievement among the 12 countries taking part in the International Study of Achievement in Mathematics. The American 13-year-olds ranked eleventh (Husen, 1967). These results should be interpreted cautiously, however. Callahan and Glennon (1975) remark,

> Although the study was never intended to be a contest, and was not undertaken as a head-to-head competition on achievement test performance between the participating countries, that was often the form reported in the news media in this country. ... It is extremely difficult to control the myriad of variables affecting mathematics achievement in order to get valid comparisons between and among different countries. (p. 46)

The Japanese system does appear to solve many of the problems inherent with the American system. One of the reasons for this may be the abacus. In light of Western history and culture, however, the abacus would seem to be a relative step backwards. This perception may be inaccurate (and unfortunate):

> The possibilities of the abacus as a calculating rather than a teaching aid are underestimated in the Western world...a few minutes practice each day for a week or so enables one to add columns of numbers at an impressive speed. (Gardner, Glenn, and Renton, 1973, p. 51)

**The Hand-held Calculator**

Regardless of whether or not the abacus is perceived as a viable alternative for western educational systems, the once resolved struggle between written algorithms and calculating devices for solving arithmetic computations is alive again. In search for a solution to the mounting educational crisis in the United States many
have argued that the now inexpensive, hand-held calculator is part of the solution and should be incorporated into the elementary school mathematics curriculum (Denenberg, 1983; Dick, 1988; Enright, 1992; Hutchings, 1976; Hembree, 1986; Hembree & Dessart, 1986; B. J. Reys & R. E. Reys, 1987; Suydam, 1987; Wiebe, 1987; Yvon, 1987). Callahan and Glennon (1975) quote this position statement adopted by the Board of Directors of the National Council of Teachers of Mathematics (NCTM):

Mathematics teachers should recognize the potential contribution of the calculator as a valuable instructional aid. In the classroom, the mini-calculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics. (p. 125)

In 1980 the NCTM recommended that "mathematics programs must take full advantage of the power of calculators and computers at all grade levels" (p. 2). In 1986 they reaffirmed their position in the following statement:

The National Council of Teachers of Mathematics recommends the integration of the calculator into the school mathematics program at all grade levels in classwork, homework, and evaluation. Although extensively used in society, calculators are used far less in schools, where they could free large amounts of the time that students currently spend practicing computation. The time gained should be spent helping students to understand mathematics, to develop reasoning and problem-solving strategies, and, in general, to use and apply mathematics. (NCTM, reprinted in Arithmetic Teacher, 1987, p. 61)

In regards to the current crisis in American education Denenberg (1983) suggests that the calculator could be a central part of a new, more efficient approach:

The recent report of the National Commission on Excellence in Education recommended that more time (that is, longer school days
and longer school years) is needed for education. What is actually needed is more efficient use of the time teachers and students now spend in the classroom. By having students use calculators, ten long division problems in a fifth grade class will take but five to ten minutes instead of 30 to 40. (p. 49-50)

Many have envisioned and or implemented uses for and benefits (both tangible and intangible) from the calculator far beyond its ostensible value as a portable, high power, easy to use computational device. The NCTM (1987) recommends that students use calculators to:

(a) concentrate on the problem-solving process rather than on the calculations associated with problems; (b) gain access to mathematics beyond the students' level of computational skills; (c) explore, develop, and reinforce concepts including estimation, computation, approximation, and properties; (d) experiment with mathematical ideas and discover patterns; (e) perform those tedious computations that arise when working with real data in problem-solving situations. (p. 61)

Hembree (1986) indicates that the calculator can expedite and augment algorithm instruction, reduce the need for memorization, motivate students, and encourage discovery, exploration, and creativity. Several have noted that using calculators can help build students' enthusiasm and confidence for solving problems and performing computations (B. J. Reys and R. E. Reys, 1987; Suydam, 1987; Yvon, 1987). Calculators can help teachers develop concepts about numbers and counting, e.g., what patterns or relationships emerge when you add 1, 3, or 10 repeatedly (Suydam, 1987). Calculators can assist in the instruction of basic arithmetic laws, e.g., does the commutative property apply to 475 x 598 as well as to 5 x 2 (Suydam, 1987). Calculators can help teachers understand decimals by relating the calculator's readout
to a familiar place-value concept such as money, e.g., one cent = 1/100 of a dollar = 0.01 (Yvon, 1987). Calculators can help foster students' abilities to estimate calculations and judge the reasonableness of results (Bobis, 1991; Suydam, 1987; Yvon, 1987). In regards to teaching the basic number facts Suydam (1987) reports: "Calculators actually seem to help students learn the basic facts: waiting until after the facts are fully mastered before using calculators does not appear to be necessary" (p. 22).

The calculator can be used successfully by both beginning and advanced users (Bitter, 1981). On one end of the continuum, it affords the novice easy access to accurate computational ability. At the other end it provides advanced capabilities to meet a host of future computational needs, e.g., for those students who are ready to advance into concepts and problems requiring mathematics that exceed their written computational abilities (Dick, 1988; Gardner, Glenn, & Renton, 1973; Hembree, 1986). Suydam (1987) believes that all students can profit from using calculators including those with physical and mental handicaps.

Yvon (1987) suggests that the calculator could eliminate much of the current drill required for mastering the traditional addition algorithm, by allowing students to do as many or as few examples as necessary to demonstrate their competence (with the calculator) and understanding of the underlying processes. He asks this pertinent question:

If students can demonstrate that they understand the reasons underlying the various operations, why should they not be allowed to
use the calculator to do them, especially if they know when and how to use each process? (Yvon, 1987, p. 17)

Utilizing calculators students can receive immediate results and don’t have to wait for teachers to grade their work. In this fashion, many students may progress independently (Yvon, 1987).

The calculator has become a common, inexpensive, time saving tool in the United States. Adults often use them for grocery shopping, balancing check books, and completing income tax forms. As preparation for entry into the modern world, teaching children to use them makes sense. Yet, despite its low cost, remarkable range of abilities, ubiquitous applications, the performance and educational potentials it promises, and despite the current crisis in education, the calculator has not gained a significant role in the teaching of mathematical skills (Wiebe, 1987). As Suydam (1983) reports: "You have probably heard it said that schools may be the last place where paper-and-pencil computation is used--while everyone else uses calculators!" (p. 20).

Wiebe (1987) notes that whereas the abundance of cheap, electronic calculators has virtually eliminated the need for paper-and-pencil computations, both at home and in the workplace, elementary mathematics instruction has changed little from the 1950s. As of 1982 in the United States, fewer than 20% of the elementary school teachers and 36% of the secondary school teachers had utilized calculators for purposes of instruction (Hembree, 1986). It appears that the NCTM’s official position on the use of calculators in the elementary classroom triggered a swell of
resistance, skepticism, and, in a few cases, sharp criticism. (Dick, 1988; Hembree, 1986). In a two part, two page advertisement in the February 1987 issue of the

*Arithmetic Teacher,* Saxon warned that:

Introducing calculators in elementary schools will convince many students that the calculator is a magic box that can be used as a substitute for understanding, and these students will resist the arduous mental effort that is required to develop a feel of numbers and the ability to estimate. If the presence of calculators will cause this reaction in only 5 to 10 percent of our elementary school students, then the loss to these students will far outweigh the temporary gains that a few other students might experience. ... these machines will cause great damage to a significant number of students if these machines are introduced while arithmetic is still being learned (1987a, 1987b. pp. 20-21)

Saxon (1986) published an article of very similar content on the editorial page of the

*Wall Street Journal.*

The primary concern of those opposed to the calculator has been that, if introduced too early in the elementary school curriculum, it would undermine the acquisition of basic arithmetic skills (Dick, 1988; B. J. Reys & R. E. Reys, 1987). In the ten years following the NCTM's initial pro-calculator position statement, over 200 research studies were conducted. Most of these studies focused on whether the use of calculators harmed mathematics achievement (B. J. Reys & R. E. Reys, 1987). The results consistently documented that the use of calculators in the classroom did not diminish achievement (providing that the basics had been taught first using paper-and-pencil methods) (Hembree, 1986; Hembree & Dessart, 1986; B. J. Reys & R. E. Reys, 1987). B. J. Reys and R. E. Reys (1987) conclude: "In
fact, not only are no measurable ill effects observed, but classroom research shows that the calculator is a powerful teaching and learning tool" (p. 12).

The most significant review and analysis of the research data on the use of the calculator in the classroom was done by Hembree and Dessart (1986). Using a meta-analysis technique they integrated and analyzed the results of 79 different studies that examined the effects of calculator usage on students' achievement and attitude in grades K-12. Hembree (1986) summarizes their findings:

1. In grades K-12 (except grade 4), students who use calculators in concert with traditional instruction maintain their skills with paper and pencil without apparent harm. Indeed, using calculators can improve the average student's basic skills with paper and pencil, both in working exercises and in problem solving.
2. Only in grade 4 does sustained calculator use appear to hinder the development of basic skills.
3. The use of calculators in testing produces much higher achievement scores than paper-and-pencil efforts, both in working exercises and in problem solving. This statement applies to all grade and ability levels. In particular, it applies to low- and high-ability students in problem solving. The better problem-solving performance is a result of improved computation and process selection.
4. Students using calculators have better attitudes toward mathematics and better self-concepts in mathematics than students not using calculators. This statement applies across all grade and ability levels. (p. 20)

Why does resistance to the calculator (seem to) persist? One possibility is that interest and resources channeled into microcomputer equipment for young students has shoved the average four function calculator to education's back burner (B. J. Reys & R. E. Reys, 1987). Dick (1988) believes that "the reasons for such continuing resistance have more to do with image than substance" (p. 37). It is ironic that the research that Saxon (1987a) cites as evidence that "these machines
will cause great damage to a significant number of students" (p. 42), is the Hembree and Dessart (1986) meta-analysis review. Saxon (1987a) appears to have isolated and grossly over-exaggerated one aspect of Hembree and Dessart's findings, failing to report that the bulk of the evidence supported calculator use in the classroom. Furthermore, Saxon (1987a) implicates that part of the reason that the Japanese education system is so successful is that they concentrate on teaching fundamentals and do not allow calculators into the elementary school classroom, while failing to note the central role of the abacus in the Japanese system.

However, there is justification for the critic's fears. Dick (1988) states, "calculators can be used in the classroom in a variety of ways, some bad, some good" (p. 38). Those on both sides of the calculator controversy seem to agree that the worst use of calculators involves students blindly (and perhaps mindlessly) punching buttons (as if into a "magic box") without employing estimation techniques or understanding the underlying mathematical concepts and relationships (Dick, 1988; B. J. Reys & R. E. Reys, 1987; Saxon, 1987b). It therefore seems ironic that one of the most common uses for the calculator in classrooms has been for double-checking students' written work, an activity that amounts to rote (possibly mindless) button-punching (B. J. Reys & R. E. Reys, 1987). In one study, Reys, Bestgen, Rybolt, and Wyatt (1980) found that 85% of the teachers interviewed indicated that calculators should be used for checking paper-and-pencil computations. The trend now is to discourage such checking. In 1985 the California State Board of Education (reprinted in Arithmetic Teacher, 1987) took the official position that
calculators "should not be used for busy work drill and practice or to check answers
explicated what they believe to be the limited educational value of checking written
work with calculators:

(a) Providing an easily accessible answer key is a more efficient use
of students' time than having them do the same exercise on a
calculator. (b) Checking paper-and-pencil work doesn't reflect the
real-world use of a calculator. No one does paper-and-pencil
computation and then uses a calculator. (c) Students wonder why
they should spend so much time doing the paper-and-pencil
calculation if a calculator is available. Such an approach reinforces
the notion that using calculators is "cheating" or not right, a feeling
that should not be evoked under any circumstances. (p. 13)

The (once) common practice of using calculators to check written work could explain
some of the continued resistance to the use of calculators in the classroom. The fact
that so many teachers have used the calculator in what appears to be an ineffective
(or counterproductive) way also points out the need for further research. As
Hembree (1986) notes: "Most of the research has focused on the possibility that the
use of calculators will harm basic skills. Little effort has been given to the
enhancement of students' achievement through a systematic use of calculators" (p.
19).

From the standpoint of the opponents of calculator usage in the classroom,
it is not unreasonable to fear that students will learn to rely on them as "magic
boxes" for performing computations, thereby failing to gain a thorough and working
understanding (or appreciation) of mathematical principles and techniques. It is not
inconceivable that, as a result, a student could become dependent upon the calculator

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as a computational crutch. The calculator might even come to function as a barrier to learning other mathematical principles and techniques. However, it is also reasonable to posit that the repeated, singular use of any algorithm (written or mechanical) could produce a similar condition of both a reliance on that algorithm (an ostensible computational ability) and a coexisting ignorance of (and even resistance to learning) the underlying principles (Dick, 1988). For instance, the repeated use of the traditional long division algorithm does not guarantee that the user will gain (or maintain) an understanding of the underlying concepts and mechanisms. A person may be able to perform long division competently, yet not understand the mathematical reason for moving the decimal point in both the dividend and the divisor (the same number of places to the right) until the divisor is a whole number (Dick, 1988). Conversely, it is also possible that a person could understand the mathematical concepts and mechanisms underlying a mathematical procedure (e.g., finding square roots), as well as that procedure’s importance or applications, without knowing how that procedure is executed using any specific algorithm (Dick, 1988). Clearly, as Wiebe (1987) suggests, comprehensive training in mathematics involves much more than the successful manipulation of any one procedural algorithm or the understanding of any single mathematical concept.

Used in isolation of a comprehensive mathematics curriculum (i.e., as a means to an end), no mathematical algorithm by itself can provide the learning foundation that a combination of written algorithms, manipulatives, and other didactic strategies can (Dick, 1988; Hembree, 1986; Wiebe, 1987). Mastering the
calculator as a computational device cannot replace learning the basic math facts or mathematical concepts (e.g., the commutative, associative, and distributive laws) necessary for written or mental computations (Denenberg, 1983; Hembree, 1986). Learning how to compute number problems quickly and accurately with a calculator will not effectively teach students how to employ the mental arithmetic techniques needed to first estimate the answer of a computation and subsequently judge the reasonableness of an answer (Dick, 1988; Wiebe, 1987). These abilities are seen as vital skills in mathematics, ones that, generally, most students lack and most school systems fail to teach adequately (Bobis, 1991; Dick, 1988; Wiebe, 1987). Mental estimation and judging skills appear to develop independently of any computational algorithm. As Dick (1988) points out:

mental estimation skills would not appear to be inherently tied to any particular computational device. If students do not employ estimation skills and number sense, then they will accept outrageous answers from a mistake in a written algorithm as readily as from a mistake in punching a button on a calculator" (p. 39).

Bobis (1991) suggests that the abilities to estimate, interpret, and judge the soundness of answers are skills that need to be deliberately taught and encouraged in students. Furthermore, these skills appear to be more crucial when using a calculator than paper-and-pencil methods (Bobis, 1991; Dick, 1988; Wiebe, 1987). The potential for making an error when remembering and entering numbers into a calculator, in conjunction with the tendency of many users to put great trust in the answers that it produces, increases the necessity of estimating and challenging the
reasonableness of any calculator-derived answer. Dick (1988) asserts that: "Since mental arithmetic and estimation are of such great importance, they should be incorporated into calculator use at all times" (p. 40).

Some have encompassed the skills of mental arithmetic, estimation and judging within a broader concept referred to as "number sense" (Bobis, 1991; Dick, 1988). Turkel and Newman (1988) characterize people who have a good number sense in this way:

They are comfortable and confident with numbers, know how they are used, know how to interpret them, and know when they make sense...[They] have a good understanding of the meaning of numbers. They are able to use numbers and understand how numbers are used in the world around them. Such people show good judgement about selecting an appropriate method of processing numbers; approximation, paper-and-pencil computation, mental estimation, or computation with a calculator. (p. 53)

Bobis (1991) explicates the specific approach, described as number sense, that should be employed during the three stages of calculation when using a calculator: "(1) Estimation--survey for numerical relationships; check for reasonableness; and estimate. (2) Calculation--are the steps I'm following reasonable? (3) Checking--Does the answer make sense? Or is the answer reasonable, bearing in mind initial numerical relationships?" (p. 42).

Dick (1988) believes that students need to be taught both the calculator's unique strengths and weaknesses. Students need to realize that, whereas the calculator is an excellent tool for mathematics, it is not a perfect one. Dick (1988) points out that,
Although calculators are not the villains portrayed by some, it would be just as erroneous to view them as a panacea for mathematics instruction. Regardless of whether calculators or paper and pencil are used, students learn by actively thinking about and reflecting on what they’re doing. (p. 41)

The calculator has a number of unique limitations of which students need to be aware. The most significant of these has to do with how the numbers are displayed. Most all calculators express fractions as decimal equivalents (Dick, 1988). This could cause individual students, whose work needed to begin and (or) end in fraction form, difficulty. This limitation could make the introduction of calculators in the early elementary grades problematic as most mathematics curriculata currently delay the presentation of decimals until after mastery of fractions (Wiebe, 1987). As a solution to this dilemma, Wiebe (1987) proposes a math curriculum that would introduce the concept of fractions (through such manipulatives as Fraction Pieces and Fraction Bars) in the first grade. Pupils would then be introduced to decimal equivalents of simple fractions, which, along with the basic number facts, they would have to memorize. Thus, by first converting the simple fractions into decimals the calculator could be integrated into math instruction from the beginning.

Another unique limitation of the calculator, also related to its display properties, is that it can only show a limited number of digits (usually eight to twelve). This weakness creates situations that Dick (1988) describes as "underflow" and "overflow" (or, obtaining answers that are too small or too big to be displayed on the calculator) (p. 40). Whereas situations of this type are unlikely to arise, students need to learn how to anticipate and circumvent them.
Other inherent weaknesses with the calculator (and possible solutions) include: They are not always available nor are they infallible (e.g., batteries can die). Solution—students should be taught written methods and employ estimation techniques so that unavailability or failures do not catch them unprepared. Some models may have buttons that are too small and or closely spaced for younger, less coordinated hands to operate without (accidentally) pressing more than more button at a time. Solution—use calculators with larger buttons specifically designed for smaller, less coordinated hands. Most hand-held calculators do not produce a permanent written record of the operator’s keystrokes that can be used by the teacher to diagnose and remediate systematic, procedural errors. Solution—employ printing calculators, especially during the acquisition phases of mathematics instruction.

The calculator appears to have many shortcomings. However, most of these appear to be of major consequence only if it is to be used as a number-crunching magic box, without supplemental written methods, manipulatives and other didactic strategies (e.g., estimation and judging techniques). Hutchings’ (1976) has suggested that the calculator and low-stress algorithms be used together. He states,

Low-stress algorithms and hand-held calculators complement each other and are a logical team. The calculator offers speed with complex operations; the algorithms offer independence from the machine, the power to check the machine easily or even to exceed the machine’s limitations, and a permanent, complete record of work. Facility in both and the option to use either as needed probably represents the ideal skills package, necessitating only a fraction of the time and stress required by conventional algorithms. (Hutchings’, 1976, p. 236)
Dick (1988) believes that the calculator and traditional algorithm are both necessary components in a comprehensive mathematics curriculum:

Which offers the better vehicle for mathematics instruction—paper-and-pencil algorithms or the use of the calculator? The answer must be "both" and "neither." The picture painted by opponents of students' mindlessly pushing calculator buttons is not a pretty one. But the picture of students mindlessly pushing pencils across paper is no less ugly. It seems obvious that the use of calculators per se does not preclude our students from thinking, but they must be presented with activities that require them to think! (p. 41)

In summary, the best ways of using the calculator (or any algorithm) in the classroom are those that facilitate a student's ability to solve mathematical problems and encourage the development of the student's understanding of the operations and principles underlying the computations. Rather than becoming a crutch and a barrier to learning, the calculator can be presented as a challenge for students, providing a powerful tool for investigating solutions to computationally complex problems. Suydam (1987) notes that when using calculators "Students appear to be able to take their attention off the computational aspects of the problem and focus on the relevant factors for solving the problem" (p. 22).

The calculator seems ideally suited to the job of reducing the time, effort, and frustration entailed with the mastery of computational skills. Accomplishing such a time reduction could facilitate the introduction of problem-solving strategies and techniques much earlier in the mathematics curriculum (similar to the function of the abacus in the Japanese system), and potentially reduce the frequency of anxiety evoking failures. With the exception of fourth-grade students, the research
affirms that, when used in tandem with paper-and-pencil methods, the calculator does not harm the development of students' basic math skills. Caution must be exercised when using the calculator with fourth-graders. Further research is needed to isolate the variables responsible for this performance discrepancy as well as how the calculator can best be utilized at all grade levels for maximizing mathematical learning and performance.

An Alternative Written Algorithm

In order to provide more classroom time to pursue the increasing volume of conceptual issues in mathematics, and to help students realize greater success with computational mathematics and thereby reduce operator anxiety, some have suggested that less stressful, more reliable, less training intensive methods (or algorithms) for computation be employed. Hutchings (1976) is one of the leaders in creating and advocating for, what are referred to as, low-stress (originally "low-fatigue") algorithms. He developed algorithms for all four of the basic math operations (addition, subtraction, multiplication, division) in the late 1960s while working in the arithmetic center at Syracuse University. Research on low-stress algorithms began at the Syracuse University Arithmetic Center in 1967. Work on alternative arithmetical algorithms also has been a major enterprise at the University of Maryland’s Arithmetic Center since 1971, where Hutchings relocated to work with John Wilson and Robert Ashlock.
The following explanation of the Hutchings' low-stress algorithm is adapted directly from "Low-Stress Algorithms" (Hutchings, 1976). Hutchings explicates the low-stress addition algorithm as follows:

The low-stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit.

These examples illustrate both conventional and half-space notation.

Conventional:

\[
\begin{array}{c}
7 \\
+ 8 \\
\hline \\
15
\end{array}
\]

In half-space notation, each of the conventional examples would look like this:

\[
\begin{array}{c}
7 \\
\; \; 8 \\
\end{array}
\]

Both the bar and the plus sign are omitted in the new basic-fact notation, so that a slower child is not tempted to repeat the bar or operation sign when doing column addition... Naturally, when the sum is less than 10, no tens portion is recorded at the lower left. So

\[
\begin{array}{ccc}
6 & 4 & 6 \\
+ 1 & + 1 & + 3 \\
\hline \\
7 & 5 & 9
\end{array}
\]
would be written like this:

\[
\begin{array}{ccc}
6 & 4 & 6 \\
17 & 15 & 30
\end{array}
\]

When half-space notation is applied to column addition, changes in the usual procedure are possible. As with all low-stress algorithms, a complete record of component operations is made, and different kinds of operations can be completed separately without step-by-step alternation. This means that every basic addition fact necessary to the algorithm is recorded. Rather than recalling another addition fact and regrouping again, the student can recall and record all the necessary addition facts in an uninterrupted sequence and then perform all the necessary regroupings. These regrouping operations, which are a major portion of the mental work of conventional algorithms, are drastically simplified by the new procedures. Thus large sums can be efficiently obtained through a series of basic addition facts.

Consider this exercise:

\[
\begin{array}{cccc}
6 & 8 & 9 & 4 \\
+ & 9 & \\
\end{array}
\]

Starting at the top, if we add the first two digits, 6 + 8, and record the sum in the new notation, we have

\[
\begin{array}{cccc}
6 & 8 & 9 & 4 \\
+ & 9 & \\
\end{array}
\]

It is after this first step that the low-stress procedure becomes very different from the usual procedure: the sum 14 is not used in the next step. Instead, only the 4, the ones portion of the sum, is used. So, the next step is 4 + 9 = 13, and the 13 is recorded in our new notation:
The rest of the work is done the same way. The complete sum of each two-digit addition is recorded in half-space notation, but only the ones portion of each sum is used in the next addition:

\[
\begin{array}{c}
6 \\
84 \\
93 \\
4 \\
+ 9 \\
\end{array}
\]

Work for the column is now complete. The ones portion of the column sum is always the same as the ones portion of the last two-digit sum—in this example, 6:

\[
\begin{array}{c}
6 \\
84 \\
93 \\
47 \\
+ 95 \\
\end{array}
\]

The tens portion of the column is always the same as the number of tens recorded at the left of the column. These are simply counted. Here there are three; so 3 is the tens portion of the column sum:

\[
\begin{array}{c}
6 \\
84 \\
93 \\
47 \\
+ 95 \\
\end{array}
\]

\[
36
\]
The low-stress procedures used for single-column addition can be applied without change to exercises involving more than one column. The advantage offered by low-stress procedures is increased in proportion to the length of the column. Columns should be spaced somewhat farther apart than with the usual procedure to allow space for the special notation. Carrying (or bridging, or regrouping, or whatever one might call it) can be done just as it is with the usual procedure.

\[
\begin{array}{cccc}
4 \\
5 & 7 \\
6 & 8 & 5 \\
7 & 6 & 1 \\
8 & 7 & 5 \\
9 & 9 & 7 \\
+ & 5 & 8 & 5 \\
\hline \\
5
\end{array}
\]

For a column in some multi-column exercise, then, the last step—that is, counting the tens at the left of the column—would be slightly changed. The counting itself is not changed in any way, but the answer, the total number of tens, is no longer written in the tens place of the first column’s sum but instead at the top of the next column at the left. This is, in fact, exactly the same as the regrouping operation of the traditional procedures. The carried number is the tens total from the preceding column...This value is used in the first two-digit addition of the column. This procedure could be extended to the addition of any multi-digit whole number. Consider the following example, which extends the previous one:

\[
\begin{array}{cccc}
4 & 6 & 8 & 6 \\
5 & 3 & 9 & 5 & 4 & 7 \\
6 & 3 & 1 & 0 & 6 & 7 & 8 & 5 \\
7 & 0 & 5 & 8 & 7 & 4 & 1 & 6 & 1 \\
8 & 0 & 7 & 2 & 4 & 8 & 2 & 7 & 8 \\
9 & 9 & 2 & 1 & 9 & 1 & 9 & 2 & 7 \\
3 & 2 & 5 & 5 & 8 & 1 & 8 & 3 \\
9 & 3 & 4 & 9 & 4 & 0 & 1 & 5 & 0 \\
8 & 0 & 5 & 4 & 8 & 3 & 6 & 8 \\
+ & 7 & 7 & 5 & 6 & 4 & 1 & 7 & 3 \\
\hline \\
5 & 7 & 4 & 3 & 3
\end{array}
\]
Work continues in this manner until the exercise is completed. Note, however, that the column sum for the last column in a multi-column example is recorded in exactly the same way as the sum of a single-column exercise. (pp. 220-223)

Hutchings (1976) cites three major effects on performance of his low-stress methods: (1) easy mastery after brief training, (2) greater computational power, especially with larger problems, and (3) sharp reductions in performance-related anxiety compared with conventional algorithms. These effects (and others) appear to be the by-product of the low-stress algorithms’ two basic mechanical characteristics. The Hutchings’ low-stress addition algorithm provides (1) a written, concise record of each and every binary operation performed, and (2) the ability to complete one distinct set of binary operations (or intermediate steps) without having to alternate between different kinds of operations or steps. In the case of the low-stress addition algorithm, the student, (a) records the partial sums for each binary addition operation (also referred to as a basic math or number "fact") within one entire column of numbers (in a problem consisting of any number of columns); and then, (b) proceeds to regroup or add the number of tens (hundreds, etc) to be carried to the next place or column.

In contrast with the low-stress method, the conventional addition algorithm (a) does not incorporate a complete written record, thereby requiring that the growing sum of a column of numbers be "remembered;" and, (b) demands that the student switch mentally between the addition of two single digit numbers (i.e., the number facts) and regrouping, or carrying numbers to the next place. Switching
between these two binary modes or intermediate steps (fact recall and regrouping) appears to demand competence well beyond knowledge of the basic math facts (Hutchings, 1976). As Brownell observed in 1928, most of his students who achieved proficiency with the basic math facts could not automatically perform the more complex ("secondary") addition operations required by the traditional algorithm (This is still the nature of the performance increment required by today’s mathematics programs). Brownell found that students needed intermediate learning steps to move from mastery of the number facts to mastery of complex addition operations.

Ashlock (1982) asserts that eliminating this switching "places fewer memory demands on the child who is computing" (p. 20). He sees this segregation of fact recall from regrouping as the advantage that low-stress algorithms hold over conventional algorithms. Alessi (1979) suggests that the low-stress procedures reduce operator attention load (less sustained concentration is needed). He points out that when using a low-stress procedure an operator can literally suspend a computation at any point during the process, and, because of the full written record feature, recommence at any time (e.g., after a distraction or competing event) (Alessi, 1992b).

Hutchings (1976) believes that the low-stress procedures produce two distinct cognitive characteristics or benefits for the low-stress algorithm operator, and that each correlates (roughly) with the mechanical features of the low-stress algorithms. The features and corresponding benefits are: (1) the complete record results in a
substantial decrease in the demands placed on memory (or, manipulations performed covertly); and, (2) eliminating the switching between binary modes (i.e., delimiting all addition operations to basic math facts) reduces (cognitive) dissonance that may serve to inhibit the student when performing computations. In summary, Hutchings (1976) states,

Together, these characteristics seem to constitute virtual elimination of the mental work necessary with conventional algorithms. ... The great reduction in mental work makes exercises involving large multidigit numbers relatively easy, and assignments can progress quickly to such exercises so that students appreciate the dimension of their new computational power. (p. 238)

Hutchings (1976) goes on to point out that the modest sizes of typical arithmetic problems used to teach the traditional method are more likely due to the limits of the algorithm, and not the actual computational demands made by today's highly quantitative workplace.

Ashlock (1982) points out the need to "teach computation procedures which make sense to a child" (p. 20). While it cannot be said with complete assurance that the low-stress addition algorithm makes more sense to children, there is experimental evidence suggesting that most children prefer using it. When given a choice between the Hutchings' low-stress and conventional algorithms, almost all children freely choose a low-stress or similar full-record technique (Buitendorp-Drew, 1980; Gillespie, 1976; VanHevel, 1981; Vaughan, 1982). Even under conditions requiring a 50% increase in response effort for choosing the low-stress method over the
conventional method, over 80% of the children in these studies preferred the Hutchings' technique.

Employing methods that "make sense to a child" may be even more vital when a child requires remedial instruction in computational arithmetic (Ashlock, 1982). By the time a child is identified as needing remedial instruction in addition, he or she has probably met with a great deal of failure. Such a child is unlikely to attend to remedial attempts that are essentially identical to past frustrating and/or unhappy experiences. For such a student getting the correct answer, experiencing immediate success, may be the most important need. (Behaviorally, it follows that any procedure that provides for this need may be a very powerfully reinforcing event). Towards this end, Ashlock (1982) suggests that, for slow-learning students having trouble with computational math, "there are times when a computational procedure which is fresh and new to the child can be taught as a machine to use whenever a result is needed" (p. 20). In this manner, introducing a novel and effective alternative may help to "circumvent the mind set of failure which beleaguers the child" (Ashlock, 1982, p. 19). On a similar note, Hutchings (1976) maintains that we should introduce the low-stress methods immediately for the low-functioning students and those with severe remedial needs in upper elementary and junior high schools.

Ashlock (1982) believes that, as professionals, our time is best spent, not scoring childrens' arithmetic work, but analyzing what children do and don't do correctly. In order to achieve this goal we must have methods that clearly ascertain
what a student is doing. That is, we must have methods that explicate the procedures children use to get the answers they get. Self-reports from children are neither reliable nor complete. Accordingly, Ashlock (1982) strongly endorses the use of arithmetic algorithms (e.g., Hutchings' low-stress algorithm) that, by their nature, produce a full record of the student's binary (single digit) calculations. Having a full record of the student's computation performance makes possible the subsequent analysis of what the child is and isn't doing correctly. In this fashion teachers may accurately diagnosis any number of error patterns. Ashlock (1982) states, "The diagnosis of errors in arithmetic is an essential part of evaluation in the mathematics program, and any such diagnosis must be accompanied by remedial or corrective instruction" (p. 1).

Pinpointing the precise nature of a child's performance deficits as they relate to procedural patterns also helps to unlock the mystery of what might otherwise appear to be debilitating, capricious, inner (cognitive) deficits or bad attitudes. It removes the problem from inside the child, where it cannot be solved, and places it the realm of the tools or methods that the child uses, which can easily be modified. Ashlock (1982) believes that (most of the time) the causes of poor performance are faulty methods, not faulty children. He states, "Children who learn patterns of error are capable of learning. They generally have what we might call a learned disability, not a learning disability" (p. 7). In other words, children learn methods that don't produce consistently accurate results in much the same fashion, and as well, as methods that do produce consistently accurate results. This is
consistent with Brownell's (1928) observations that requiring students to compute addition problems using methodology too abstract or advanced for their development will cause them to resort to (and in the process refine) their previously learned, more immature addition strategies.

If, as Roberts (1968) and Brownell (1928) have suggested, the algorithm currently in use is instructionally defective and overly demanding, it follows that students' performance will also be substandard. Identifying and employing more appropriate, efficacious methods may be the key to substantially improving the mathematics performance of the average elementary school student.

Other Alternative Algorithms

If we are to consider revamping the American educational system, then every viable avenue should be considered. There are other, lesser known or researched, alternative addition algorithms that may have potential for enhancing students' performance and educational experience.

Fulkerson (1963) developed an algorithm that is similar to the Hutchings' low-stress technique. Like the Hutchings' low-stress, the Fulkerson Tens requires that the operator perform only binary operations (i.e., only the basic math facts). Procedurally, however, this method differs from the Hutchings' in that only a partial record of the computation is made. The Fulkerson Tens involves recording the tens place value for each binary operation, but not the units value. The tens value is also recorded differently than with the Hutchings'. Using the Fulkerson Tens algorithm,
the operator draws a line through the last digit of an addend that results in the 
cumulative sum going over 10, as demonstrated here:

8 9
9 \ 
+ 6 \ 8

This is done within each column of numbers each time that the on-going sum 
exceeds 10. In effect, this records the tens place holder. One advantage to this 
difference is that it alleviates the special, broader spacing (between numbers) 
required to accommodate the half-step notation used with the Hutchings' low-stress 
technique. This would allow the Fulkerson's method to be used with existing work 
books and drill sheets.

Once a slash is made and the tens place holder is recorded the operator 
proceeds with the next binary addend operation until the next partial sum tops 10, 
or the operator reaches the end of the column. At the end of the column the 
operator adds the number of slashes made. This is the number to be carried to the 
next column, as demonstrated here:

2
8 9
9 \ 
+ 6 \ 8

4

As it provides half of the overt (written) feedback information potentially 
available, it should reduce the operator's memory load accordingly. However, if 
maximizing such reductions is both desirable and contingent on a complete written
record, then it would seem that the Fulkerson Tens addition algorithm does not afford the same performance advantages that the Hutchings' low-stress algorithm does.

O'Malley (1969) offers an alternative algorithm that is partially similar to the Hutchings' and Fulkerson's methods. With the O'Malley technique the operator immediately writes the tens portion of the addend pair (or whatever place the second digit of the addend pair represents) directly between the numbers of the adjacent column (up one space and to the left one space), as demonstrated here:

```
  5  7
 11
  8  4
  1
  6  8
 11
+  9  9
   3 0 8
```

This method does help to ensure that the operator carries the correct number to the next column to be added, and, like the Fulkerson's method, it reduces the memory and attention requirements on the operator. However, its partial record lacks the full advantages provided by the Hutchings' algorithm with its full written record (as previously discussed). Like the Hutchings' low-stress, the O'Malley method requires special spacing.

Sanders (1971) suggested a method that consists of the student silently computing the sum of the first two addends, then vocalizing the one's portion aloud while holding up fingers to represent the number in the "tens" place. Similar to the
Hutchings' method, the ones portion of each sum is used in the next addend operation. When the final pair of addends has been calculated, the ones portion of the number verbalized is written as the ones portion of the final answer (the column sum). Then, the number of fingers representing the tens portion of the column sum is recorded, as demonstrated here:

\[
\begin{align*}
8 & \quad 8+4=12 \text{ (student holds up one finger)} \\
4 & \quad 2+9=11 \text{ (student holds up a second finger)} \\
9 & \quad 1+7=8 \text{ (student continues to hold up two fingers)} \\
7 & \quad 8+5=13 \text{ (student holds up a third finger)} \\
+ & \quad 5 \\
\hline
3 & \quad 3 \text{ The sum of the units column is 3} \\
\text{} & \quad \text{The number of fingers held up is written} \\
\text{} & \quad \text{in the tens portion of the answer}
\end{align*}
\]

While this method does appear to reduce the operator's memory load, that is, the number of covert responses required, it does lack the complete written record (that facilitates analysis and diagnosis of performance) of the Hutchings' low-stress algorithm. The Sanders' requirement that calculation responses be verbalized might also have the disadvantage of creating excessive and distracting noise in a classroom setting.

VanHevel (1981) modified the Fulkerson Tens method so that it produces a full record of each binary operation. Coined the Fulkerson full-record method, it retains the use of slash marks to indicate the tens values, and uses half-step notation (like the Hutchings') to register the units portion of each binary operation, as demonstrated here:
The Fulkerson full-record appears to be nearly identical to the Hutchings' low-stress method. It seems reasonable to conclude that these two algorithms would afford very similar levels of arithmetic performance, offering similar advantages and disadvantages.

Bartarseh (1974) presented a method intended to reduce the number of errors made during the carrying process. Bartarseh recommended that, rather than carrying numbers to the next column, the numerals of an addend should be written partially under the adjacent column with the tens value of the binary product underlined. The underlined tens value should be aligned directly underneath the adjacent column, with the units value written in a position that is midway between the original and adjacent columns. The underlined digit is then added with the next column, as demonstrated here:

\[
\begin{align*}
1 & \quad 2 \\
9 & \quad 8 \\
7 & \quad 6 \\
+ & \quad 8 \quad 9 \\
\hline
1 & \quad 6 \quad 3
\end{align*}
\]

\[
\begin{align*}
8 & \quad 4 \\
6 & \quad 8 \\
+ & \quad 9 \quad 9 \\
\hline
2 & \quad 1 \\
2 & \quad 5 \\
\hline
2 & \quad 5 \quad 1
\end{align*}
\]
This method does not reduce the memory load on the operator, nor does it provide a full written record other than the answer itself. It does require special spacing.

Research on Alternative Addition Algorithms

Historically, increasing percent accuracy and rate of correct addition responding in arithmetic has been attempted (experimentally) by the manipulation of specific consequent events. Kirby and Shields (1972) employed an adjusting fixed ratio schedule of positive reinforcement (praise and feedback for correct answers) to increase the rate of correct responding (which they did successfully). They also observed an increase in the amount of time that subjects spent attending to problem solving. Copeland, Brown, and Hall (1974) reported that public praise from the school principal for both improving students and the highest performing students resulted in a percent accuracy increase in third graders' addition performance. McCarty, Griffin, Apolloni, and Shores (1977) used group contingencies in a token economy based on money to increase arithmetic problem solving rates.

Researchers have also manipulated time as an independent variable. Ayllon, Garber, and Pisor (1976) found that the rate of correct addition responding could be increased from 125% to 266% by systematically reducing the amount of time allowed for completion of the math assignment in a token economy.

Compared to research that has manipulated consequent events, little appears to have been done through altering antecedent events, other than changing the
algorithm itself. In one study (Lovitt & Curtiss, 1968) the subjects verbalized the problem prior to making a written response. This antecedent condition increased the rate of correct addition responding while decreasing the rate of incorrect responding.

The three most significant classes of antecedent events that affect the percent accuracy, correct rate, and incorrect rate of addition responding have been the alternative algorithm, the abacus, and the calculator. Alternative algorithm research has focused primarily on comparisons of the Hutchings' low-stress procedures with the conventional algorithm across various experimental conditions. Two unpublished studies (master theses) have contrasted one alternative algorithm with another. VanHevel (1981) compared the Hutchings' low-stress method with the Fulkerson full-record addition algorithm. Vaughn (1982) conducted a study that examined the differences between the Hutchings' low-stress and the Fulkerson Tens addition methods. Three unpublished studies have examined addition computation performance between the Hutchings' low-stress and conventional algorithms and the handheld calculator (Drew, 1981; Todd, 1980; Zoref, 1976). McGlinchey (1981) compared the Hutchings' low-stress subtraction algorithm, the conventional subtraction algorithm, and the pocket calculator. To date, no comparisons of the Sanders (1971), O'Malley (1969), or Batareseh (1974) addition methods have been reported in the literature (published or unpublished). Generally, published reports of alternative algorithms and their relative qualities have been of a descriptive nature.
The earliest alternative algorithm studies examined addition performance using the Hutchings' low-stress procedure in comparison with the conventional algorithm (Alessi, 1974; Boyle, 1975; Dashiell, 1974; Hutchings, 1972). Alessi (1974) compared the low-stress algorithm to the conventional addition method under varying conditions of token economy reinforcement and increasing problem difficulty using a 3x2x3 group factorial design. He found that, in the thirty minute experimental period, the low-stress method produced higher correct rates than the conventional method. He also reported, "As the test forms increased in difficulty, the extent of superiority for the Hutchings' algorithm increased in a consistent fashion" (Alessi, 1974, pp. 76-77).

Boyle (1975) compared the low-stress addition algorithm to the conventional algorithm under three conditions: (1) antecedent algorithm instructions, (2) social reinforcement contingencies, and (3) simulated test and non-test conditions. The dependent variables in this study were the correct rate and percent correct (rate and accuracy). The design employed was a 2x2x2 group factorial configuration. This study reported that the Hutchings' low-stress algorithm produced significant increments in both dependent variables over the conventional algorithm, under all conditions of reinforcement and testing.

In the late 1970's and early 1980's a series of within subject studies were conducted at Western Michigan University (Buitendorp-Drew, 1980; Drew, 1981; Gillespie, 1976; Hadden, 1981; McCallum, 1981; McGlinchey, 1981; Rudolph, 1976; Todd, 1980; VanHevel, 1981; Vaughan, 1982; Zoref, 1976). Each of these
studies found performance under the low-stress algorithm to be generally superior to the conventional algorithm.

Zoref (1976) worked with six fourth-grade students (three males and three females), in a study that investigated the speed (correct rate and error rate) and accuracy (percent correct) of performing addition problems using the Hutchings' low-stress algorithm, the conventional algorithm, and the handheld calculator. Of the six subjects involved, three were identified as "low achievers" in math and three identified as "high achievers" in math. The subjects were tested using the three methods of addition on two levels of problem difficulty (2x7 arrays--matrices comprising two columns by seven rows; 5x7 arrays--matrices comprising five columns by seven rows). This study found that, overall, the Hutchings' method was the most stable, having the lowest standard deviations across all measures, regardless of level of student or problem difficulty. Using the Hutchings' low-stress algorithm, all six students performed with the lowest error rate compared with the other two procedures, regardless of problem difficulty. With the exception of low achievers working 2x7 problems using handheld calculators, the Hutchings’ method was also found to be the most accurate of the three methods.

In this study, using the Hutchings' low-stress algorithm, the high math achievers consistently scored higher correct rates and performed more accurately than the low math achievers, regardless of the size of the problem. This by itself is hardly a surprise. However, the error rates for the two groups using the Hutchings’ algorithm were essentially identical. Using the conventional algorithm,
a difference of 34 percentage points existed between the low and high performers (54% versus 88%). Using the Hutchings’ low-stress method, the range between the two groups was only 7.5 percentage points (87.5% versus 95.0%). These findings confirm the results of every other low-stress algorithm study (Alessi, 1974; Boyle, 1975; Buitendorp-Drew, 1980; Dashiell, 1974; Drew, 1981; Gillespie, 1976; Gordon, 1972; Hadden, 1981; Hutchings’, 1972; McCallum, 1981; McGlinchey, 1981; Rudolph, 1976; Todd, 1980; VanHevel, 1981; Vaughan, 1982; Zoref, 1976) that, for a wide range of students, the Hutchings’ low-stress algorithm is a more accurate calculation method than the conventional algorithm. That is, the Hutchings’ method appears to be highly effective for reducing calculation errors, despite the pre-existing math achievement level of the student.

Zoref (1976) also discovered that, among the three methods, the hand-held calculator appeared to produce the most variable performance over groups and level of difficulty. While the calculator produced the highest accuracy scores among all three methods for the low achievers on 2x7 arrays, it also resulted in the lowest accuracy scores of the three procedures for the high achievers on the 5x7 arrays. As the problems became more difficult a large decrease in accuracy was noted with the calculator for both high and low achievers, although the low achievers produced both a higher correct rate and a lower error rate on the larger arrays when compared with the high achievers.

Rudolph (1976) utilized the Hutchings’ low-stress and conventional addition algorithms to investigate two hypotheses. First, could easily distractible, emotionally
impaired students learn computational skills more readily using the Hutchings' low-stress addition algorithm? Second, how do distracting and non-distracting environments affect the performance of both normal and emotionally impaired students' using either the low stress or conventional algorithm? The subjects chosen for this study were four seventh-grade students, two from regular education classrooms and two from an emotionally impaired classroom. The results suggest that, compared with the conventional addition algorithm, both types of students performed addition problems more accurately when using the Hutchings' low-stress method, regardless of environmental distractions, or the order of algorithm presentation.

Gillespie (1976) investigated students' preferences for type of addition method by giving them a choice between the Hutchings' low stress addition algorithm and the conventional algorithm, using a Findley "min-max" choice procedure. These choices were further explored under conditions of differentially increasing response effort without reinforcement (experiment 1), and with reinforcement (experiment 2) using a reversal design. Six third-grade students, three males and three females, participated in this study. The two algorithms were presented and employed under three basic conditions, first, under conditions of equal response effort for both algorithms (subjects choosing either method were required to complete the same number of problems), then under two differentially increased response effort conditions of 50 percent and 100 percent for the preferred algorithm.
The dependent variables in this study were algorithm preference (percentage of students choosing one algorithm or another), rate of columns correct and incorrect (or error rate), and percent accuracy, all based on 4x5 problem arrays. In experiment 1 the subjects received three, 20-minute instructional sessions on the Hutchings' low-stress addition algorithm and one, 20-minute session on the conventional algorithm. After this, the pre-baseline phase began. During the four pre-baseline sessions the students were forced to alternate between the two methods. For all conditions that followed, each subject chose the method they preferred. A preference was established when the student used the same algorithm for six consecutive free choice sessions. In condition A (baseline), either algorithm chosen required an equal response effort (the same number of problems). In Condition B, subjects again had a free choice. However, in Condition B the algorithm preference established during Condition A required a 50 percent increase in problems completed over Condition A. Conditions C and B were the same except that a 100 percent increase in response effort (twice as many problems) over Condition A was required for the preferred algorithm. This process was then repeated beginning with Condition A followed by a second Condition B and a second (and final) return to Condition A (baseline). Experiment 2 was identical to experiment 1 except that tokens were made contingent on the accuracy of the subjects' answers in Conditions B and C. Also, there was only one return to baseline.

Gillespie (1976) reported (for experiment 1) that the Hutchings' low-stress algorithm was freely selected by students in 59 out of 61 (97%) equal response effort
conditions, in 36 out of 43 (84%) 50 percent increased response effort conditions, and in 28 out of 71 (39%) 100 percent increased response effort conditions. In other words, even when required to calculate twice as many problems as their baseline number, subjects' preferences for the Hutchings' algorithm diminished significantly only during the 100 percent increased response effort condition. Furthermore, the Hutchings' low-stress algorithm yielded consistently higher accuracy, greater mean session rates of columns correct, and lower mean session rates of columns incorrect, through all the experimental conditions than the conventional algorithm.

The results for experiment 2 were almost identical to experiment 1 during Conditions A and B. During Condition C, however, contingent reinforcement (tokens) for percent accuracy resulted in 50% of the students establishing a preference for their lesser accurate, conventional method. In other words, reinforcement did not maintain the students' preferences for the more accurate algorithm.

Buitendorp-Drew (1980) conducted a systematic replication of the Gillespie (1976) study. Algorithm preferences of 14 third grade students, 10 high accuracy and four low accuracy on a math facts test, for the Hutchings' low-stress and the conventional addition algorithms were investigated under increasing response efforts (greater number of problems), with and without reinforcement (representing experiments I and II). In experiment I, under equal response effort conditions, the low-stress algorithm was preferred by 100% of the subjects. Under the 50% more response effort condition the low-stress procedure was chosen by 83% of the
subjects. For the 100% more effort session 37.5% of the subjects preferred the Hutchings' method. Experiment 2 produced very similar data. For this experiment the results of Conditions A and B were identical to experiment 1. During the 100% increased effort condition 33% of the students preferred the Hutchings' low-stress method. Similar to the Gillespie (1976) study, Buitendorp-Drew (1980) found that, across both experiments and all conditions, the Hutchings' method produced more accurate arithmetic computations. However, unlike the earlier study, within the replication no subject ever established a preference for their lesser accurate method.

Todd (1980) compared the hand held calculator, Hutchings' low-stress, and conventional algorithms for subtraction, over two levels of problem difficulty (2x7 and 5x7 matrix problems). In this study the low-stress algorithm produced the lowest error rates, the conventional algorithm was the fastest, while the calculator was the slowest and least accurate at the lower level of problem difficulty (2x7 matrices). On the larger, more difficult problems (5x7 matrices), the Hutchings' method became the most accurate method as overall performance deteriorated significantly using the conventional algorithm or the calculator.

Drew (1980), conducted a systematic replication of the Zoref (1976) study. He examined the performances of high and low math-achieving students (a total of eight third-grade students) using the calculator, Hutchings' low-stress, and conventional addition algorithms over two levels of problem difficulty. The Hutchings' low-stress method proved to be superior to the conventional method regardless of subject math-achievement level or problem difficulty. Overall, the
calculator produced very similar results to the Hutchings' except for the more difficult problems (5x7 arrays) where it was less effective than the Hutchings' for the low achievers but still more effective than the conventional method.

Hadden (1981) studied the effects of Hutchings' low-stress addition and subtraction algorithms on the correct rate and percent accuracy of problem solving with low performing, fourth-grade math students. Multiple baseline designs were used (one for addition and one for subtraction). During baseline, subjects worked addition and subtraction problems using only the conventional algorithms. As each subjects' performance stabilized, a brief training program was introduced using Hutchings' low-stress addition and subtraction algorithms. Following this training, subjects performed addition and subtraction problems using the new algorithms. Before and after training assessment probes were given to measure knowledge of place value. A second probe was presented at the conclusion of the study to evaluate how well the subjects' abilities with the new algorithms had generalized to problems of various sizes and difficulty. This study reports that the accuracy and rate of problem solving improved for all subjects after training with the Hutchings' low-stress addition method, but only for one subject after being trained in the low-stress subtraction technique.

McCallum (1981) performed a component analysis of Hutchings' low-stress addition algorithm by comparing it to the conventional algorithm with a written record and the conventional algorithm without a written record. The results were rank ordered as follows: the Hutchings' algorithm produced the best results, as
measured by accuracy and speed of calculations, the conventional algorithm with a
written record was next, and the conventional algorithm without the written record
produced the lowest results.

Van Hevel (1981) compared the Hutchings' low-stress, Fulkerson full-
record, and conventional addition algorithms for speed, accuracy, and preference.
Van Hevel noted "that both the Hutchings' low-stress and Fulkerson full-record
algorithms were generally superior in producing stable, accurate, and efficient
calculations" (p. 28). The study did not show any differences between the low-stress
and full record algorithms on the 5x7 problem arrays (these algorithms are
essentially identical). A 17x12 problem array was used to test student preference.
Four chose the full record, three chose the low-stress, and two students chose the
conventional method.

Vaughan (1982) investigated the differential calculation power of the
Hutchings' low-stress, Fulkerson Tens, and conventional addition algorithms on 5x7
matrix addition problems. The dependent variables were the rate correct, the rate
incorrect, the percent accuracy, and the subjects' individual algorithm preference.
Six male first semester third-grade students (approximately eight and one half years
of age) were employed as subjects for this study. For the final three of the 22 total
daily sessions subjects were allowed to chose which of the three algorithms they
employed.

The results of this study showed the Hutchings' low-stress algorithm to be
superior to the other two methods for producing stable rates of accurate calculations
of 5x7 array addition problems. Using the Fulkerson Tens method the students performed less quickly and accurately than when using the Hutchings' method, and more accurately than when using the conventional algorithm, which was the least effective method overall. During the "free choice" sessions, four subjects chose the Hutchings' method (21 of 27 total choices), two students chose the conventional algorithm (6 of 27 total choices), and no students chose the Fulkerson Tens method. During this last phase the Hutchings' low-stress method produced a substantially higher mean correct rate, a lower mean incorrect rate, and a higher mean percent accuracy than the conventional algorithm.

The Purpose of This Study

In every similar algorithm study conducted to date (that the author was able to locate; n=14), with a wide variety of students and settings, the Hutchings' low-stress addition algorithm was found to produce more effective (faster and more accurate) columnar addition performance than the traditional method and in most cases the calculator. In most cases, and for most types of students, the Hutchings' method resulted in the lowest error rates of any of the methods compared. This would seem like major news for an education system beleaguered by problems of successfully teaching the basics of mathematical calculation. One wonders why something like the Hutchings' low-stress addition algorithm has not replaced (or at least supplemented) the traditional method. One possible reason for this may be that the existing research is almost entirely of the single subject variety and has not been
published. Most curriculum developers and educational systems planners may not know about alternative algorithms and their relative effectiveness to existing methods. Another possible barrier to adopting the Hutchings' algorithm (or any new computation method) may be the expense and trouble of modifying curriculums and textbooks.

This study was an attempt to explicate and compare the performance of four addition methods for computational speed and accuracy (as expressed in the composite "performance index" score) utilizing a repeated measures statistical design. This design increased the power of the (statistical) results in that individual performance differences (even large ones) did not effect the ANOVA F values, thus making the ANOVAs more sensitive to mean differences (Gravetter & Wallnau, 1985). This study was also an attempt to evaluate subjects' preferences among these methods. To date, no study has examined preferences between the calculator, the Hutchings' and traditional methods.

This study included the traditional method on the basis of its current use in education. This study included the calculator because it is currently a part of the curricula for many schools. Furthermore, despite its shortcomings, many believe that the hand-held calculator could be a significant part of the solution to the current crisis in teaching mathematics (Bitter, 1981; Denenberg, 1983; Enright, 1985; Hutchings, 1976).

This study included the Hutchings' low-stress algorithm on the basis of its design advantages and previous performance as documented by substantial single
subject research. This study included the factor analysis algorithm as an experimental contrast and control. The factor analysis method, which is identical in structure and purpose to McCallum's (1982) "conventional algorithm with a written record," served two basic purposes in this study:

1. It controlled for novelty. Like the Hutchings' low-stress algorithm, it was absolutely new for all subjects. If the novelty of the method was a determining factor in the children's performance then the results for the Hutchings' and factor analysis methods should be similar.

2. The factor analysis method, like the name it bears, may assist in identifying the crucial component(s) within each of the other two written algorithms (Hutchings' and traditional). The factor analysis algorithm comprises one main element of the traditional algorithm and one main element of the Hutchings' design. Like the traditional method it requires complex addition operations within columns. Therefore (as explained earlier), like the traditional method, the factor analysis algorithm may place more demand on a student's concentration and short-term memory load. If concentration and short-term memory load are crucial factors in determining the students' performance with an algorithm, then the traditional and factor analysis methods should produce similar results while the Hutchings' method, which reduces the demands placed on the student's short-term memory, should result in better performance. Similar to the Hutchings' low-stress algorithm, the factor analysis method requires the student to write a full record of all the computation operations performed within each column of the addition problem. If the full record
feature is crucial for improving performance then the Hutchings' and the factor analysis methods should produce similar results, while the traditional method should be less effective.

The three written algorithms represent a design continuum. The traditional method requires that all simple and complex number facts be reckoned covertly (in one's head). The factor analysis takes the traditional method and adds a full written record. The Hutchings' takes the traditional method, adds a full written record, and also limits all computations to basic addition operations (limited to the use of basic math facts). Together, the three written algorithms employed in this study will help facilitate the analysis of the components that they utilize.

Because (dramatic) differences in computational power of algorithms (often) appear at increasing levels of problem difficulty, two levels of problem difficulty were included in this study. Experiment 1 comprised smaller problems (2x7 arrays). Experiment 2 comprised larger problems (5x7 arrays). Experiment 3 continued with 5x7 array problems and attempted to measure the students' preferences by allowing them to choose the algorithm they wished to use.

Higher achievers may show different performance effects than lower achievers. Much of the history of educational research since the 1960's has been aimed at identifying techniques that differentially improve performance of low achievers over high achievers. Unfortunately, such efforts have been unsuccessful. In order to examine the differential performance between students of greater and lesser pre-study ability, two groups of children were recruited and selected for this
study. They were chosen from pre-existing ability groups labeled low and average functioning, and designated (for this study) as low and medium math achievers.

In summary, this study, comprised of three experiments, focused on ten questions. For experiments 1 and 2 the questions were:

1. For each of the eight discreet conditions in the first two experiments (two groups x two statistical perspectives x two levels of problem difficulty) is there a difference in performance between the four addition methods?

2. If there are differences in performance for any one of these eight conditions, to what degree do the differences appear to be functionally consistent with the features/components of the methods (novelty, written record, basic math facts, consistency of a calculating device)?

3. What underlying causal (operational) mechanisms may we infer from these results?

4. For each experiment and each statistical perspective is there a performance difference between low and medium achieving math groups?

5. For each of the two statistical perspectives is the relative performance between methods differential between the experiments (i.e., with problem difficulty)?

6. For perspective 2 for each group is there a performance differential between early and late sessions within experiments 1 and 2. (i.e., is there a practice effect)?

7. If there is a performance differential between early and late time periods, is this difference consistent across the four addition methods?
For experiment 3 the questions were:

8. Under conditions of choice and extensive experience with the methods, for what percentage of the total experiment 3 sessions will the medium and low achieving subjects select each of the four methods for adding 5x7 addition problems?

9. Under conditions of choice, extensive experience with all four methods, and no direct feedback concerning their performance, what proportion of the medium and low achievement subjects will select the method with which they are most proficient (based on their performance at the end of experiment 2)?

10. Will the medium and low achieving subjects' overall performances continue to improve on similar problems during experiment 3 over experiment 2, using the same methods (i.e., with extended practice)?
CHAPTER II

METHOD: EXPERIMENT 1

General Considerations

The design and execution of this study was in part determined by the following constraints: (a) the subjects were selected from regular education classrooms, (b) the sessions were held in the same room each day at the same time immediately after regular school hours (approximately 3:30pm), Monday through Friday; (c) all study-related expenses incurred were the responsibility of the experimenter, and (d) the results of the study were made available to the pertinent school personnel and the parents of the subjects.

Independent Variables

There were four independent variables in experiment 1. They were:

1. Addition methods, four levels: traditional, hand-held calculator, Hutchings' Low-stress algorithm, and factor-analysis.

2. Types of children, two levels: low functioning and average functioning with respect to pre-existing math skills. They were designated as low and medium functioning, respectively.
3. Time periods, two levels: an early sessions mean (the mean performance index of the first five sessions) and a late sessions mean (the mean performance index of the last five sessions).

4. Problem difficulty: addition problems comprised of two columns by seven rows of digits (2x7 arrays).

Data Collected/Dependent Variables

Data collected in experiment 1 included:

1. Rate of columns correct, defined as the number of columns correct in a five-minute test period.

2. Rate of columns incorrect, defined as the number of columns incorrect in a five-minute test period.

The dependent variable analyzed in this experiment was:

1. Performance index, defined as the rate of columns correct minus the rate of columns incorrect.

Subjects

Experiment 1 involved fourth- and fifth-grade children (nine boys and nine girls), ages 10 and 11. All subjects were students at the Later Elementary School in a small midwestern school district, and all were enrolled full-time in regular education classrooms. Half of the overall research group were identified as average achievers (designated medium functioning) in math while the other half were
identified as low achievers (designated low functioning) in math. The low and average ability groupings were based on the subjects' daily work and quarterly curriculum-based achievement tests. The school's instructional specialist chose the subjects on the basis of their pre-existing ability groups and the proximity of the child's home to the school. Children who normally walked home from school (as opposed to riding a bus) were selected first thereby minimizing transportation problems. After attrition there were 12 low achieving subjects and six medium achieving subjects in experiment 1.

Setting

Experiment 1 sessions were conducted immediately after school at 3:30 pm and lasted 40-45 minutes. Every afternoon at approximately 3:25 pm one of the school office staff announced the study over the school intercom. All sessions were held in one of the mathematics rooms in the Lower Elementary Building connected via an enclosed hallway to the Later Elementary complex. The children walked to this room with their books, coats, lunch boxes, etc from various locations in the Later Elementary building.

The room contained approximately 25 desks and chairs for the students, the teacher's desk and chair, various school-related posters, pictures, and storage cabinets (where pencils, calculators and other materials were kept) on the left and back walls (as one faced the front of the room), a bank of windows on the right wall, and blackboards on the front wall. The room was generally a quiet, non-
distracting environment except for the first few minutes of each session when the
school busses immediately outside the windows would board students and leave.

Procedure

Informed Consent

After the subjects were selected by the school education coordinator a letter of informed consent was mailed (on school letterhead) to each child's family. A
letter of assent for each subject was also included. The letter thoroughly explained
the nature of the research and requested the parents' written permission for their
child to participate in the study. The protocol for the study was approved by the
Western Michigan University Human Subjects Review Board, the school system
superintendent, and the school board.

Algorithm Training Procedures

All subjects from both groups received the initial training for each method
at the same time. The Hutchings' method was introduced and practiced first,
followed by the factor analysis method, then the traditional method, and finally the
calculator.

The instructional format used for teaching the Hutchings' method was taken
from the Hutchings' (1972) study (refer to appendix A). Instruction for the factor
analysis method was similar to the Hutchings', except that the Hutchings' combines
the unit's value of the first number with the next single digit number and records that sum, whereas the factor analysis procedure adds the entirety of the first number (the unit's and the ten's value) to the next single digit number and records that sum (refer to Appendix B). The review of the traditional method followed the format used in the VanHevel (1981) study (refer to Appendix C). The instructional format for the calculator was devised by the author (refer to Appendix D).

To assess the subjects' proficiency with the novel methods (the Hutchings' and factor analysis), and to reliably determine that the subjects were using the method prescribed, worksheets with practice problems were given to the subjects at various points of these trainings. The experimenter judged the subjects to be ready to employ these methods when they demonstrated accurate implementation on 100% of the practice problems. This did not mean that the subjects computed the problems correctly 100% of the time, or that they were so required. Rather, a subject was judged to be ready to begin the study when they demonstrated consistently the correct format for each method. This was easily assessed in that both the Hutchings' and factor analysis methods require written procedures that leave permanent evidence (response products) unique to each of them. In the case of the Hutchings' method, the student must add only the number in the digits place to the next single digit number (recording this result) while recording a "1" each time this sum is 10 or greater. Examining a problem computed using the Hutchings' algorithm reveals no written record (recorded within the matrix using half-space notation) exceeding 18. The written results of this method are easily distinguished from the factor analysis
method that requires the student to add the entirety of one addend to the next single digi digit number and subsequently record this entire result within the matrix (using half-space notation). A problem computed using the factor analysis method reveals a written record that increases progressively with each addition operation, and places no limits on the size of the recorded results. Lastly, in regards to identifying usage of the two alternative algorithms, there were no random methods (e.g., doodling) noted in any of the students' addition work that could successfully pass for either of the alternative algorithms.

Both the Hutchings' and factor analysis methods are easily contrasted with the traditional algorithm and calculator that leave no permanent response products on the problem matrix itself (and hence produce visibly similar results). Because addition problems computed using the hand-held calculator and traditional method usually appeared identical, reliably determining whether subjects used the calculator required the experimenter to observe the subjects' behavior during those training (and experimental) sessions that employed it. There were no occasions observed of a subject employing any other method than the calculator when this was the required method.

**Experimental Task**

For every session in experiment 1 the children computed 2x7 array addition problems using the four methods for a five-minute period each. The experimenter timed the periods using the stopwatch mode of a Casio W-26 alarm chronograph.
The subjects were told to do as many problems as they could. They received no qualitative feedback regarding their performance. Between five-minute periods just enough time was taken to briefly introduce the next method, reacquainting the students with the basic operations necessary. This experiment involved 20 daily sessions over a five week period.

In order to minimize any sequence or temporal order effects the succession of the four methods used was counterbalanced by random assignment. With four addition methods there were 24 discrete, sequential combinations (4! permutations). To predetermine the exact order in which the methods were presented during the sessions, a random selection procedure was used. Each discrete combination was written on a small piece of paper and then placed in a bag. The experimenter then drew the pieces of paper from the bag one at a time. Once used, a sequence was not available for reselection until all of the remaining sequences had been employed.

To help the children remember the methods (the differences in operations between the Hutchings' and the factor analysis were subtle), and to help the experimenter keep track of the changing daily sequences, each method was color coded as follows: traditional=white paper, calculator=pink paper, Hutchings'=green paper, and factor analysis=blue paper. For each session each subject was given a stapled set of four or eight worksheets, either one of each method (color) or two of each method (color) depending upon the individual subject's level of proficiency (for the slower subjects eight worksheets was a waste of four pieces of paper). The worksheets were stapled together in the order that the
methods were to be performed during that session. For experiment 1 each worksheet contained nine 2x7 array addition problems.

**Daily Sessions**

Prior to each daily session the worksheets (with the subjects’ names written on them), pencils, and calculators were placed on the desks in the classroom. The classroom was arranged so that the subjects sat facing the blackboard which was at the front of the room. The first students to arrive were offered the opportunity to help the experimenter with various session related tasks (erasing, passing out daily record sheets, passing out cookies).

When all the subjects had arrived for that day the experimenter presented a brief review of the first algorithm to be used. The subjects began each five-minute work period with a countdown from the number 5. Individual students were selected to conduct the countdown. Often, the entire class would join. When the experimenter’s stopwatch’s alarm beeped and the experimenter said "stop" the students were instructed to finish working, take a short break, and prepare for the next five-minute work period.

When all four work sessions were finished daily record cards were distributed. The record cards were colorful, fold out (commercially purchased) cards intended for tracking (and encouraging) a child’s performance or attendance. Each day that a child attended (beginning with day 1) they placed a small sticker in the appropriate box of the record card. The children were given a wide choice of
brightly colored stickers from which to select. The children were told that when the
.cards were full (35 stickers) that they would be finished with the study and would
receive a gift certificate to Toys-R-U's for $10.

The experimenter (and his assistant) also provided cookies and drinks
approximately three times per week. These were distributed after the record cards
were completed as the children were preparing to leave. Other attendance incentives
included small gifts, toys, and prizes (e.g., etch-a-sketch, bubble gum machines,
piggy banks, baseballs, frisbees, dolls, coloring books, Legos toys) that were
awarded after the four work sessions were finished and the record cards were
completed. Early in the study the prize winners were selected by drawing numbers
out of a hat. Later the experimenter created addition competitions wherein three
children would each add the same large number problem on the blackboard. All
three children would win something, however the first one to compute the correct
answer using the method specified could choose first from among the toys. Once
a child had competed he or she was not eligible again until all the others had had
their chance.

Materials

The worksheets (and the correct answers) were created using a Hypercard
program on a Macintosh II-Ci computer (see Appendix E for sample worksheets).
The program generated the problems from numbers selected randomly. However,
as recommended by Hutchings (1972), the identity element (zero) was not employed.
For experiment 1 there were 64 completely different worksheets. The worksheets were cycled so as not to be presented less than three sessions apart. This minimized the possibility of a subject memorizing the problem or the correct answers. The experimenter observed no behavior to suggest that this might have occurred.

The calculators were Sharp model EL-326S. These are small, solar-powered units. They were provided by the school system and are the same calculators used in the regular education classrooms. Pencils were provided primarily by the experimenter. Some children preferred to use their own. The experimenter provided the daily attendance cards, stickers, prizes, toys, and foods.

Scoring and Recording

The experimenter scored all of the problems (using computer produced answer sheets) away from the experiment site. For each child, for each session, the columns correct, columns incorrect, and percent accuracy were all recorded on the front face of each method's worksheet. These results were later transferred to a number of master data base worksheets and finally plotted using a computer graphing program, Harvard Graphics 3 (Jensen & Anderson, 1992). The performance indices were calculated and plotted using Harvard Graphics 3.

Experimental Design

For experiment 1 a within-subjects (repeated measures) design was employed; each subject came into contact with each level of the independent variable method
during each session. This technique produced a great deal of usable data in a short amount of time and allowed for a more powerful examination of the methods factor by partiailling out the variability due to between subjects differences.

Data Analysis

Within experiment 1 the performance of each group was statistically analyzed independently of the other group and from two different perspectives. Perspective 1 considers the main effect of the four methods. For this experiment the analysis averaged the subjects' average performance indices for each subject's first eight sessions using each method (i.e., each subject's first eight exposures to each of the four levels of the independent variable addition methods, regardless of the individual's actual session numbers). These mean values then formed the basis for a four level, one-way repeated measures ANOVA. This process was applied to both low and medium groups independently. This analysis yielded two, one-way ANOVAs.

Perspective 2 of the analysis requires two separate, two-factor (2x4), repeated measures ANOVAs. The first factor in each of these ANOVAs was the phase of the experiment, early verses late (two levels). It examined the main effects of changes in performance over time, comparing the subjects' average performance index early in the experiment (i.e., averaging the average of each subject's first five exposures across all four levels of the addition methods independent variable, regardless of the individual's actual session numbers) with their average performance index late in the
experiment (i.e., averaging the average of each subject's last five exposures across all four levels of the addition methods independent variable, regardless of the individual's actual session numbers).

The second factor in the perspective 2 analysis was the addition methods factor (with four levels, as in the first ANOVA). This factor differs from its counterpart in the perspective 1 analysis in that it uses a slightly different data base than the first ANOVA. Whereas the perspective 1 analysis averaged the average of each subject's first eight sessions, the perspective 2 analysis averaged the average of each subject's first and last five sessions (i.e., each subject's first and last five exposures to each level of the addition methods independent variable, regardless of the individual's actual session number). As in perspective 1, the perspective 2 process was applied to both low and medium functioning groups independently. For an overview of the these statistical analyses, refer to Table 1.

Once the ANOVAs were completed, a SAS computer program for multiple comparisons was run to test all pairwise comparisons for significance. This procedure was equivalent to performing a correlated $t$ test on each comparison. The critical alpha value for the multiple comparisons analysis was determined by dividing the standard critical alpha value of .05 by 6 (the total number of pairwise contrasts). (Huitema, 1993). It should be noted that this is a very conservative procedure. A substantial amount of power (ability to detect true differences) is lost due to the fact that all pairwise contrasts are being examined. To test for statistically significant mean differences between the performances of the two experimental groups in
Table 1

Data Analysis Summary Table: Experiment 1, Perspectives 1 and 2, Low and Medium Achievement Groups

<table>
<thead>
<tr>
<th></th>
<th>Experiment 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perspective 1</td>
</tr>
<tr>
<td>Low Group:</td>
<td>Low Group:</td>
</tr>
<tr>
<td>Methods (4)</td>
<td>Time Period (2) &amp; Methods (4)</td>
</tr>
<tr>
<td>First Eight</td>
<td>First and Last Five</td>
</tr>
<tr>
<td>ANOVA Configuration</td>
<td>1 x 4</td>
</tr>
<tr>
<td>Medium Group:</td>
<td>Medium Group:</td>
</tr>
<tr>
<td>Methods (4)</td>
<td>Time Period (2) &amp; Methods (4)</td>
</tr>
<tr>
<td>First Eight</td>
<td>First and Last Five</td>
</tr>
<tr>
<td>ANOVA Configuration</td>
<td>1 x 4</td>
</tr>
</tbody>
</table>
experiment 1, two sets of four independent $t$ tests were performed, one for each of the four addition methods within both statistical perspectives.

Reliability

Reliability data were collected and analyzed on the columns completed by the students. Because the calculations produced a permanent product (the written worksheets), it was possible for an independent grader to score the problems selected for the reliability analysis. For experiment 1, four worksheets, each from a different subject and method, were randomly selected from each daily session, making a total of 80 worksheets ($4 \times 20$; representing over 5% of the total worksheets).

A reliability coefficient was calculated based on the comparison of the independent grader's scores with the experimenter's scores. When the two columns scores were the same, an "agreement" was counted. When the two column scores differed a "disagreement" was counted. The reliability coefficient was calculated by dividing the number of agreements by the number of agreements plus disagreements.
CHAPTER III

RESULTS: EXPERIMENT 1

Reliability

For experiment 1, 80 worksheets were rescored. These worksheets made 1104 total columns. Among these columns eight disagreements and 1096 agreements were identified. This produced a reliability index of \( \frac{1096}{1104} = 0.99275 \).

It may be argued that a larger reliability sample should have been analyzed. However, given that the 5% sample selected was randomly chosen and evenly distributed across experimental sessions, and that this sample resulted in such a high reliability index, the experimenter was confident that these results would not drop below 95% with additional sampling.

Rationale for Data Analysis

Initially the data from experiment 1 was to be analyzed as one study: a split-plot group design with two "within subjects" (or repeated measures) factors and one "between subjects" (non-repeated measures) factor. The within subjects factors were the (two levels of) problem difficulty and the (four levels of) addition methods. The between group factor was the two groups, comprised of medium and low math functioning students. Although sound, this split-plot approach did not prove viable.
given the realities of working with 10- and 11-year-old school children. One ten-
year-old contracted chicken pox just days before the study was to commence. This
subject missed the first five days of data collection, after which attendance was
perfect. Another child’s family moved just prior to the study. After three weeks
another child’s mother decided to withdraw her child from the study. Numerous
others were occasionally sick for a day or two or would leave in the middle of a
session due to physical distress. One child dropped out because of a chronic illness.

The weather proved to be an additional complicating factor. About four
weeks into the study (the second week of data collection) the prevailing outdoor
weather conditions changed from mostly winter-like to almost summer-like. In that
the sessions were held every day immediately after school, from the time that the
weather improved until the end of the study attendance by many of the participants
was irregular despite incentives to attend, notes home, and direct contact with
parents.

These situational events resulted in many missing data points and imbalanced
group sizes (for attendance chart refer to Appendix F). After examining the plotted
raw data for individual subjects it appeared that, for most subjects in both groups,
even those that attended regularly, there was very little consistent growth over the
twenty initial sessions regardless of method or level of difficulty. Figures 1 through
6 show raw data plots for subjects 2, 3, 5, 7, 12, and 14 that display this type of
result for experiment 1 using various methods (The methods used are indicated on
the individual graphs). On the right ordinate these figures display the raw data,
Figure 1. Individual Daily Performance on Three Dependent Variables Using the Traditional Algorithm. Experiment #1, Subject #2.
Figure 2. Individual Daily Performance on Three Dependent Variables Using Hutchings' Low-stress Algorithm. Experiment #1, Subject #3.
Figure 3. Individual Daily Performance on Three Dependent Variables Using the Traditional Algorithm. Experiment #1, Subject #5.
Figure 4. Individual Daily Performance on Three Dependent Variables Using the Factor Analysis Algorithm. Experiment #1, Subject #7.
Figure 5. Individual Daily Performance on Three Dependent Variables Using the Hutchings' Low-stress Algorithm. Experiment #1, Subject #12.
Figure 6. Individual Daily Performance on Three Dependent Variables Using the Calculator. Experiment #1, Subject #14.
"columns correct" and "columns incorrect." On the left ordinate they display the percentage correct value derived from the raw data.

There were those subjects, such as 6, 9, 11, 17 and 19, using certain methods, whose performance showed consistent improvement over time. These results are presented in Figures 7 through 12 (with two samples from subject 19). This type of performance was the exception, not the rule. Additional examples of marked growth in performance were only seen from subject 3 (factor analysis method), subject 9 (factor analysis method), and subject 19 (calculator and traditional method). Only subject 19 (a medium achieving fifth grade student) showed consistent improvement using all four methods.

Referring to Figures 13 through 16 it can be seen that when the performances of all subjects within each group were averaged over daily sessions, these averaged results appeared much closer to the trend established by the first group of subjects, showing little change over the course of this experiment. These figures are based on the same raw data calculations as the individual Figures 1 through 11. Based on this lack of significant overall growth over time it was suggested that the raw data be averaged, or "collapsed." The resulting "collapsed" values would then be used for data analysis. The collapsing was done in two ways based on the two perspectives of data analysis. For perspective 1, which analyzed the four levels of the methods factor, the performance indices for the first eight sessions were averaged. Eight sessions were utilized because a large performance sample was desired and eight corresponded with the minimum number of sessions that any one
Figure 7. Individual Daily Performance on Three Dependent Variables Using the Hutchings' Low-Stress Algorithm. Experiment #1, Subject #6.
Figure 8. Individual Daily Performance on Three Dependent Variables Using the Hutchings' Low-Stress Algorithm. Experiment #1, Subject #9.
Figure 9. Individual Daily Performance on Three Dependent Variables Using the Hutchings’ Low-Stress Algorithm. Experiment #1, Subject #11.
Figure 10. Individual Daily Performance on Three Dependent Variables Using the Calculator. Experiment #1, Subject #17.
Figure 11. Individual Daily Performance on Three Dependent Variables Using the Hutchings' Low-stress Algorithm. Experiment #1, Subject #19.
Figure 12. Individual Daily Performance on Three Dependent Variables Using the Factor Analysis Algorithm. Experiment #1, Subject #19.
Figure 13. Four Addition Methods Compared on Dependent Variable Columns Correct. Experiment #1, Low Achieving Group.
Figure 14. Four Addition Methods Compared on Percent Correct Daily Means. Experiment #1, Low Achieving Group.
Figure 15. Four Addition Methods Compared on Dependent Variable Columns Correct. Experiment #1, Medium Achieving Group.
Figure 16. Four Addition Methods Compared on Percent Correct Daily Means. Experiment #1, Medium Achieving Group.
subject completed for experiment 1. For perspective 2 (the time period factor) the first and last five sessions were collapsed. The first and last five sessions were utilized because large performance samples were desired and all but one subject completed ten sessions or more for experiment 1. Collapsing the data served two purposes: (1) averaging actual sessions (i.e., ignoring missing days) alleviated the missing data problem; and (2) condensing 20 data points down to one and two greatly simplified the statistical analysis.

With attrition also came the problem of imbalanced group sizes. For the first 20 sessions the low group comprised 12 subjects and medium group 6 subjects. For the next ten sessions the groups dropped to 11 and 5 respectively. Common sense and another look at the raw data resolved these issues. While differential performance between groups is of interest, it is not the primary focus of this study. In fact, it will surprise no one that, on average, the medium group did (what appears to be) significantly better than the low. This lends some validity to the predetermined classifications. However, it is not necessary to evaluate this factor within the overall ANOVA analysis. Hence, breaking off the between subjects factor from the split-plot design so as to create two separate ANOVA analyses where before there was one did not compromise the primary goal of this study which was to examine differential performance among the four addition methods. Furthermore, between group inferences were drawn from both visual/graphic analyses and independent \( t \) tests. Graphing the performance of the two groups across the four methods on a single graph allowed a visual inspection and contrast of the mean
differences (where they existed). These mean differences between the groups' performances on the four methods were then analyzed for statistical significance using a simple, independent $t$ test for each of the four methods.

Organization of Dependent Data

The results of experiment 1 are presented in the order of the seven experimental questions (refer to pages 68 and 69 of this text). Where appropriate to each question, the two perspectives of data analysis were adopted. When both perspectives of analysis applied to a question, the perspective 1 data analysis is addressed for both groups, then the perspective 2 analysis is performed for both groups. Questions 2 and 3 are qualitative and inferential in nature and are addressed in the discussion section. Question 5, which addresses between experiment differences, is addressed in the experiment 2 results. Summary data for these comparisons are presented in 6 tables and 12 figures that accompany and support the text.

Dependent Data

**Question 1-A. Utilizing perspective 1 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the low achieving group?**

Figure 17 indicates that, for the low achievers working $2\times7$ addition problems, no one method resulted in performance significantly above or below the
Figure 17. Group Mean Performance Indices, Low Achieving Group, Experiment 1, Perspective 1. (Based on Mean Index of Each Subject's First Eight Sessions.)
rest. Also, the incremental differences between any two adjacent means is about the same. On average, for perspective 1 of this analysis, the low achieving group performed best (on the performance index) using the Hutchings' low-stress algorithm ($M=12.59$). Next was the calculator ($M=10.65$), followed by the traditional method ($M=8.83$), and lastly the factor analysis method ($M=7.41$). The difference between the largest and smallest mean effects (Hutchings minus factor analysis: 12.59-7.41) was 5.18.

The Hutchings' method produced the least intersubject variance ($s=5.83$), the factor analysis method followed second ($s=6.29$), the calculator was third ($s=7.54$) while the traditional method produced the most variance among the four methods ($s=9.37$). Additional descriptive data for the low achieving group working 2x7 problems (including range and median) are presented in Table 2.

Figure 18 displays both group and individual means for the four methods along the abscissa (group means are connected via dotted lines and individual means are connected via solid lines). As can be seen, even though the Hutchings' method produced the best overall mean performance, there was a fair amount of intersubject variance (or "noise", as seen in the standard deviations) when comparing all subjects' performance between any two methods. Overall, seven subjects performed best using the Hutchings' method, three subjects did their best using the traditional method, and two delivered their best addition performance using the calculator.

In pairwise comparisons between the Hutchings' and the other three methods, eight subjects performed better with the Hutchings' while four performed better...
Table 2

Experiment 1, Perspective 1: Descriptive Statistics of Low Math Achievement Group

<table>
<thead>
<tr>
<th>Performance Index Means</th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings' Analysis</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>-12.88 to 22.13</td>
<td>-4.13 to 25.88</td>
<td>2.63 to 22</td>
<td>-4.25 to 19.38</td>
</tr>
<tr>
<td>Mean</td>
<td>8.83</td>
<td>10.65</td>
<td>12.59</td>
<td>7.41</td>
</tr>
<tr>
<td>Median</td>
<td>8.76</td>
<td>11</td>
<td>12.32</td>
<td>7.32</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9.37</td>
<td>7.54</td>
<td>5.83</td>
<td>6.29</td>
</tr>
</tbody>
</table>
Figure 18. Group and Individual Mean Performance Indices, Low Achieving Group, Experiment 1, Perspective 1. (Based on Mean Index of Each Subject's First Eight Sessions.)
using the calculator; eight subjects performed better with the Hutchings' and three performed better with the traditional algorithm (one subject performed about the same); eleven subjects performed better with the Hutchings' and one performed about the same using the factor analysis method. The inconsistency of these results indicates that caution needs to be exercised when applying any of these methods to similar populations.

In summary, using the perspective 1 data, for the low achieving students computing 2x7 array addition problems, the Hutchings' low-stress algorithm produced the highest mean performance index of all the methods, although the relative differences were small and there was a substantial amount of variation and overlap between and within subjects across the four methods.

**Question 1-B.** Utilizing perspective 1 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the low achieving group?

A one-way repeated measures analysis of variance (ANOVA) compared the mean performance indices of subjects using the four addition methods. The differences among the means approached significance ($F[3,33]=2.81; p < .055$). This indicates that, given the null hypothesis is true, the probability of obtaining differences among the four means this large by chance is .055. With alpha equal to .050 this analysis indicated that there were no significant differences among the performance index means of the four methods.
Question 1-C. Utilizing perspective 2 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the low achieving group?

Based on the averages of the first and last five sessions for each subject, the low achieving subjects performed best using the Hutchings' low-stress algorithm ($M=13.69$). The calculator was the next most effective method ($M=11.07$), the traditional method was third ($M=9.14$), while the factor analysis algorithm was the least effective computing method with the low achieving group on $2\times7$ math problems ($M=8.09$).

Referring to Figure 19 it can be seen that the order of performance among the methods was the same and the spacing of the performance among the methods was very similar to the perspective 1 methods analysis for this group in this experiment. Perspective 2 performance increased slightly over perspective 1 for all four methods. The difference between the largest and smallest means also increased. For perspective 2 this was 5.6 (Hutchings minus factor analysis: 13.69-8.09) compared with 5.18 for the perspective 1 analysis.

The amount of variability produced between subjects for each method of the perspective 2 analysis was also very similar to the perspective 1 analysis. The Hutchings' produced the least variance ($s=5.75$), the factor analysis method was next ($s=6.81$), followed by the calculator ($s=8.53$), while the traditional method produced the most variability ($s=9.23$). For additional descriptive perspective 2 data for the low group in experiment 1, refer to Table 3.
Figure 19. Group Mean Performance Indices. Low Achieving Group, Experiment 1, Perspectives 1 and 2 Compared. (Perspective 1 is Based on the Mean Index of Each Subject's First Eight Sessions; Perspective 2 is Based on the Mean Index of Each Subject's First and Last Five Sessions)
Table 3

Experiment 1, Perspective 2: Descriptive Statistics of Low Math Achievement Group

<table>
<thead>
<tr>
<th>Performance Index Means</th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>-12.88</td>
<td>-4.13</td>
<td>5.13</td>
<td>-4.25</td>
</tr>
<tr>
<td></td>
<td>to 22.6</td>
<td>to 29.6</td>
<td>to 24.1</td>
<td>to 21.3</td>
</tr>
<tr>
<td>Mean</td>
<td>9.14</td>
<td>11.06</td>
<td>13.69</td>
<td>8.09</td>
</tr>
<tr>
<td>Median</td>
<td>9.15</td>
<td>11.45</td>
<td>12.84</td>
<td>9.15</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9.23</td>
<td>8.53</td>
<td>5.75</td>
<td>6.81</td>
</tr>
</tbody>
</table>
Figure 20 displays this group’s means for the four methods across the abscissa and includes individual performances between methods. As was the case for the methods analysis in perspective 1, the Hutchings’ method produced the best overall group mean performance. However, as with the first perspective, the second perspective also shows a fair amount of intersubject variance, or inconsistency, when comparing all subjects’ performance among all the methods. Overall, seven of the twelve subjects did their best work with the Hutchings’ method, three with the calculator, and two delivered their best performance with the traditional method.

Comparing the Hutchings’ and factor analysis methods, eleven subjects performed better with the Hutchings’. Nine subjects did better with the Hutchings’ method compared to the calculator, and nine subjects performed better with the Hutchings’ compared to the traditional method. The inconsistency of these results indicates that caution needs to be exercised when applying any of these methods to similar populations.

In summary, utilizing perspective 2 data, for the low achieving students computing 2x7 array addition problems, the Hutchings’ low-stress algorithm produced the highest mean performance index of all the methods, although (as was the case for the perspective 1 analysis) there was a substantial amount of variation and overlap between and within subjects across the four methods.
Figure 20. Group and Individual Mean Performance Indices, Low Achieving Group, Experiment 1, Perspective 2. (Based on Mean Index of Each Subject's First and Last Five Sessions.)
Question 1-D. Utilizing perspective 2 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the low achieving group?

A 2x4 (time period x method) repeated measures ANOVA compared the means of the averaged early and late performance indices. The differences among the four means were significant ($F[3,33]=3.54; p<.025$). This indicates that the probability of obtaining mean differences of this magnitude (among all four means) by chance is less than .025 if the null hypothesis is true. This finding is inconsistent with the perspective 1 analysis that did not indicate statistical significance in the differences among the four addition methods.

Post hoc multiple comparison analyses indicated one significant contrast. The low achieving subjects' performance index was significantly higher ($F_{critical}=0.00833; F[1,11]=13.91; p<.0033$) using the Hutchings' method than with the factor analysis method. The multiple comparison procedure revealed no other significant pairwise contrasts.

Question 1-E. Utilizing perspective 1 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the medium achieving group?

Figure 21 indicates that, for the medium achievers working 2x7 addition problems, the Hutchings' low-stress algorithm shows moderate superiority over the other three methods that appear to be grouped together at a lower level. Using the Hutchings' low-stress algorithm this group recorded a mean performance index of
Figure 21. Group Mean Performance Indices, Medium Achieving Group, Experiment 1, Perspective 1. (Based on Mean Index of Each Subject's First Eight Sessions.)
25.83. Their next best performance was with the calculator \((M=19.32)\), followed by the factor analysis method \((M=16.82)\), with the traditional method being just slightly less effective than the factor analysis \((M=16.80)\). The difference between the most and least effective methods (Hutchings' minus factor analysis: 25.83-16.80) was 9.03.

The calculator produced the least average variability between subjects \((s=5.96)\), the Hutchings' method was second \((s=6.31)\), the traditional method was third \((s=8.43)\), while the factor analysis method produced the most mean variance between subjects \((s=9.19)\). For additional descriptive data for this group in this experiment refer to Table 4.

Even though the overall mean differences between the methods are not dramatic, the Hutchings' method resulted in uniformly superior effectiveness within subjects. As can be seen in Figure 22, with the exception of one subject who did as well on the factor analysis method as with the Hutchings', every subject performed best with the Hutchings' algorithm. The performances for the other methods were not as uniform. Four subjects performed better with the second place calculator than with the factor analysis and traditional methods, while performance between the factor analysis method and the last place traditional method was evenly split.

In summary, using perspective 1 data, for the medium achieving subjects calculating 2x7 array addition problems, the Hutchings' low-stress algorithm produced the highest performance index mean. The Hutchings' method also showed
Table 4

Experiment 1, Perspective 1: Descriptive Statistics of Medium Math Achievement Group

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>7.88</td>
<td>10.88</td>
<td>13.75</td>
<td>4.88</td>
</tr>
<tr>
<td></td>
<td>to 32.38</td>
<td>to 29.38</td>
<td>to 33.5</td>
<td>to 33.88</td>
</tr>
<tr>
<td>Mean</td>
<td>16.79</td>
<td>19.32</td>
<td>25.83</td>
<td>16.82</td>
</tr>
<tr>
<td>Median</td>
<td>15.07</td>
<td>18.07</td>
<td>26.75</td>
<td>13.75</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>8.43</td>
<td>5.96</td>
<td>6.31</td>
<td>9.19</td>
</tr>
</tbody>
</table>
Figure 22. Group and Individual Mean Performance Indices. Medium Achieving Group, Experiment 1, Perspective 1. (Based on Mean Index of Each Subject's First Eight Sessions.)
consistent results within the subjects of this group, as every subject did best using
the Hutchings’ method.

**Question 1-F.** Utilizing perspective 1 data, does the statistical analysis of the main
effects of the methods factor (and, when appropriate, post hoc pairwise comparisons)
suggest that there are significant differences in mean and individual performances
between the four addition methods for the medium achieving group?

A one-way repeated measures analysis of variance (ANOVA) compared the
mean performance indices of subjects using the four addition methods. The
differences among the means were significant \( F[3,15]=9.58; \ p<.001 \). This
indicates that the probability of obtaining mean differences of this magnitude (among
all four means) by chance is less than .001 if the null hypothesis is true. Post Hoc
multiple comparison tests indicated that there were no significant pairwise contrasts.

**Question 1-G.** Utilizing perspective 2 data, do the descriptive statistics and a visual
(graphic) summary suggest that there are differences in mean and individual
performances between the four addition methods for the medium achieving group?

For this perspective of the analysis the medium achieving subjects performed
best using the Hutchings' low-stress algorithm \( M=27.92 \). The calculator was the
next most effect \( M=19.50 \), the factor analysis algorithm was third \( M=18.32 \),
while the traditional computing method was the least effective with the medium
achieving group on 2x7 problems \( M=15.65 \).

As can be seen in Figure 23, the order of performance among the methods
for both perspectives one and two was the same (Hutchings’--Calculator--Factor
Figure 23. Group Mean Performance Indices. Medium Achieving Group, Experiment 1, Perspectives 1 and 2 Compared. (Perspective 1 is Based on the Mean Index of Each Subject’s First Eight Sessions; Perspective 2 is Based on the Mean Index of Each Subject’s First and Last Five Sessions.)
analysis--Traditional) however, the spacing of the performance was somewhat different. When averaging early and late sessions' performance the Hutchings' increased slightly (27.92 > 25.83), the factor analysis increased slightly (18.32 > 16.82), while the other two methods remained about the same. In the perspective 2 analysis the difference between the largest and smallest means (Hutchings'-Traditional) was 14.27 compared to 9.03 for the perspective 1 analysis.

The amount of variability produced between subjects for each method of the perspective 2 analysis changed somewhat compared to the perspective 1 analysis. As before, the calculator produced the least variance (up from \( s=5.96 \) to \( s=7.08 \)), the Hutchings' method was next (up from \( s=6.31 \) to \( s=8.88 \)), followed by the factor analysis method (up from \( s=9.19 \) to \( s=10.11 \)), while the traditional method produced the most variability (up from \( s=8.43 \) to \( s=11.97 \)). Additional perspective 2 descriptive data for the medium group computing 2x7 array problems can be found in Table 5.

Figure 24 displays this group's means for the four methods across the abscissa and includes individual performances between methods. As was the case for the methods analysis in perspective 1, the Hutchings' method does produce the best overall mean performance. And, even though, as in the previous case, the overall mean differences are not dramatic, the Hutchings’ method produced uniformly superior effectiveness between subjects. As can be seen in Figure 24, with the exception of one subject who did as well on the traditional method as with the Hutchings', every subject performed best with the Hutchings' algorithm. The
Table 5

Experiment 1, Perspective 2: Descriptive Statistics of Medium Math Achievement Group

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>2.3 to 40.2</td>
<td>10 to 32.1</td>
<td>13.6 to 40.5</td>
<td>4.1 to 37.7</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>15.65</td>
<td>19.5</td>
<td>27.9</td>
<td>18.32</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>11.25</td>
<td>19.45</td>
<td>26.7</td>
<td>16.45</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>11.97</td>
<td>7.08</td>
<td>8.88</td>
<td>10.11</td>
</tr>
</tbody>
</table>
Figure 24. Group and Individual Mean Performance Indices. Medium Achieving Group, Experiment 1, Perspective 2. (Based on Mean Index of Each Subject's First and Last Five Sessions.)
performances for the other methods were not as clearly defined. Four subjects performed better with the second place calculator, while two performed better with the third place factor analysis method. Five subjects performed better using the factor analysis method compared with one who performed better using the last place traditional method.

In summary, using perspective 2 data, for the medium achieving subjects calculating 2x7 array addition problems the Hutchings’ low-stress algorithm produced the highest performance index mean. The differences between the Hutchings’ method, the traditional method, and calculator were even greater using the perspective 2 data than the perspective 1. For the perspective 2 analysis, the Hutchings’ method also showed consistent results between the subjects of this group, as every subject did best using the Hutchings’ method.

**Question 1-H.** Utilizing perspective 2 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the medium achieving group?

The methods main effects within the perspective 2 statistical analysis utilized a two-way repeated measures ANOVA that compared the means of the averaged early and late performance indices. The differences among the means were significant \( F[3,15]=8.19; p<.01 \). This indicates that the probability of obtaining mean differences of this magnitude (among all four means) by chance is less than...
.01 if the null hypothesis is true. This finding is consistent with the perspective 1 repeated measures ANOVA analysis.

Post hoc multiple comparison analysis revealed no statistically significant pairwise contrasts. This result is consistent with the results from the perspective 1 post hoc analysis.

**Question 4-A. Utilizing perspective 1 data, do the descriptive statistics, a visual (graphic) summary, and a simple statistical analysis suggest that there are significant performance differences between low and medium achieving groups for experiment 1?**

Figure 25 clearly indicates that, on average, the medium achieving subjects did consistently better across all four addition methods than the low achieving subjects. As can be seen, the order and degree of increase in performance for all four methods between the two groups was very similar. The medium achieving group's degree of increase over the low group for the calculator and factor analysis methods appeared almost identical. The medium group's degree of increase for the Hutchings' method was somewhat greater than for the calculator and factor analysis methods, while their degree of performance increase using the traditional method was slightly less than the calculator and factor analysis methods.

The two groups' performance on the four methods compared as follows (presented in rank order of performance for the low group): Using the Hutchings' low-stress algorithm the mean performance index for the medium group was 25.83 and for the low group 12.59. Using the calculator the medium group mean was
Figure 25. Group Mean Performance Indices. Experiment 1, Perspective 1, Low and Medium Achieving Groups Compared. (Based on Mean Index of Each Subject's First Eight Sessions.)
19.32 and the low group mean was 10.65. Using the traditional method the medium group mean was 16.8 and the low group mean was 8.83. Using the factor analysis method the medium group mean was 16.82 and the low group mean was 7.41. Refer to Table 6 for a summary of descriptive data comparisons between groups.

Independent measures $t$ tests were run on these differences. Setting the individual alpha levels for each $t$ statistic at .01, the overall (experiment-wise) alpha level becomes .04. For experiment 1 only the Hutchings' low-stress algorithm produced a statistically significant difference between the two groups ($t$-critical = 2.92; $t_{[16]}$ = 4.43).

The variance in performance using the four methods produced by the respective groups compared as follows (presented in order of increasing variability for the low group): Using the Hutchings' algorithm the low group standard deviation was 5.83 and the medium group standard deviation was 6.31; using the factor analysis method the low group standard deviation was 6.29 and the medium group standard deviation was 9.19; using the calculator the low group standard deviation was 7.54 and the medium group standard deviation was 5.96; using the traditional method the low group standard deviation was 9.37 and the medium group standard deviation was 8.43. The low group's performance was less variable than the medium for the factor analysis and Hutchings' methods, while the medium group's performance was less variable than the low using the traditional method and the calculator.
Table 6
Experiment 1, Perspective 1: Descriptive Statistics
of Low and Medium Math Achievement Groups

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Range</td>
<td>-12.88</td>
<td>7.88</td>
<td>-4.13</td>
<td>10.88</td>
</tr>
<tr>
<td></td>
<td>to 22.13</td>
<td>to 32.38</td>
<td>to 25.88</td>
<td>to 29.38</td>
</tr>
<tr>
<td>Mean</td>
<td>8.83</td>
<td>16.79</td>
<td>10.65</td>
<td>19.32</td>
</tr>
<tr>
<td>Median</td>
<td>8.76</td>
<td>15.07</td>
<td>11</td>
<td>18.07</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9.37</td>
<td>8.43</td>
<td>7.54</td>
<td>5.96</td>
</tr>
</tbody>
</table>
Comparison of the ranges of performance index values between the medium and low groups indicated that there was a substantial amount of overlap between the medium and low groups' subjects' performances. Using the Hutchings' method the low group's range of mean performance indices was 2.63 to 22 and the medium group's range was 13.75 to 33.5. Using the calculator the low group's range of mean performance indices was -4.13 to 25.88 and the medium group's range of mean performance indices was 10.88 to 29.38. Using the factor analysis method the low group's range of mean performance indices was -4.25 to 19.38 and the medium group's range was 4.88 to 33.88. Using the traditional method the low group's range of mean performance indices was -12.88 to 22.13 and the medium group's range values was 7.88 to 32.38.

It is clear from these results that the "low" and "medium" achievement distinction yields groups that are not entirely different. Despite what seem like substantial mean differences between the two groups' performances, there was a substantial amount of overlap in individual performances between the two groups. This overlap in individual performances is not surprising considering that the low and medium group subjects were selected from adjacent, pre-existing ability groups (low and average). Greater separation in individual performances would have been expected had the groups been selected from low and high ability groups.

The overlap in individual performance was explicated by comparing the individuals' means from each group with the other group's mean (and a one standard deviation confidence interval) for the respective method. The results were (presented
in order of increasing overlap: Using the Hutchings' low-stress algorithm one subject’s performance index mean from the low group fell within one standard deviation of the medium group overall performance index mean, and one medium group subject’s mean score fell within one standard deviation of the low group’s overall performance index mean. Using the calculator four low group subjects’ performance index means were within one standard deviation of the medium group’s overall performance index mean (one low group subject scored higher than the medium group overall mean), and three medium group subjects’ performance index means fell within one standard deviation of the low group’s overall performance index mean (one medium group subject scored below the low group’s overall mean). Using the factor analysis method six low group subjects’ performance index means were within one standard deviation of the medium group’s overall performance index mean (one low group subject scored higher than the medium group overall mean), and two medium group subjects’ performance index means fell within one standard deviation of the low group’s overall performance index mean (one medium group subject scored below the low group’s overall mean). Using the traditional method six low group subjects’ performance index means were within one standard deviation of the medium group’s overall performance index mean (four scored higher than the medium group mean), and four of the medium group subjects’ performance index means were within one standard deviation of the low group’s overall performance index mean (one scored below the low group mean).
In summary, based on the visual analysis there appears to be a difference between low and medium group performances for all four methods. Only the Hutchings’ method resulted in statistically significant differences in performance between groups. There was a substantial amount of variability within the performance of both groups for all methods and, hence, increasing amounts of overlap among the individual performances between the two groups as their respective overall performance index means converged. This overlap diminishes the interpretability of the differences between group means. Among the four methods in experiment 1 the Hutchings’ displayed the greatest differential increase in performance between the two groups and, hence, the least amount of overlap between subjects. In terms of generalizability, anyone wishing to apply these results to other populations should consider the individual differences that exist within such group classifications before making judgements about the applicability of any of these addition methods with any specific individual or group.

**Question 4-B. Utilizing perspective 2 data, do the descriptive statistics, a visual (graphic) summary, and a simple statistical analysis suggest that there are significant performance differences between low and medium achieving groups for experiment 1?**

Figure 26 indicates that, as was the case for the perspective 1 data analysis, on average, based on the perspective 2 analysis, the medium achieving subjects did consistently better across all four addition methods than the low achieving subjects. As can be seen, the order and degree of increase in performance between the two
Figure 26. Group Mean Performance Indices. Experiment 1, Perspective 2, Low and Medium Achieving Groups Compared. (Based on Mean Index of Each Subject’s First and Last Five Sessions.)
groups for all four methods was very similar. This overall pattern is also similar to the perspective 1 results.

The two groups' performances on the four methods compared as follows (presented in rank order of performance for the low group; the corresponding values for perspective 1 are in parentheses): Using the Hutchings' low-stress algorithm the mean performance index for the medium group was 27.9 (25.83) and for the low group 13.69 (12.59). Using the calculator the medium group mean was 19.5 (19.32) and the low group mean was 11.06 (10.65). Using the traditional method the medium group mean was 15.65 (16.8) and the low group mean was 9.14 (8.83). Using the factor analysis method the medium group mean was 18.32 (16.82) and the low group mean was 8.09 (7.41). Refer to Table 7 for a summary of descriptive data comparisons between groups.

Independent measures \( t \) tests were run on these differences. Setting the individual alpha levels for each \( t \) statistic at .01, the overall (experiment-wise) alpha level becomes .04. For the perspective two analysis of experiment 1 only the Hutchings' low-stress algorithm produced a statistically significant difference between the two groups (\( t \)-critical = 2.92; \( t \) [16] = 4.13). This result is consistent with the perspective 1 findings.

The variance in performance using the four methods produced by the respective groups compared as follows (presented in order of increasing variability for the low group; perspective 1 values are in parentheses): Using the Hutchings' algorithm the low group standard deviation was 5.75 (5.83) and the medium group
Table 7

Experiment 1, Perspective 2: Descriptive Statistics of Low and Medium Math Achievement Groups

<table>
<thead>
<tr>
<th>Performance Index Means</th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Range</td>
<td>-12.88 to 22.6</td>
<td>2.3 to 40.2</td>
<td>-4.13 to 29.6</td>
<td>10 to 32.1</td>
</tr>
<tr>
<td>Mean</td>
<td>9.14</td>
<td>15.65</td>
<td>11.06</td>
<td>19.5</td>
</tr>
<tr>
<td>Median</td>
<td>9.15</td>
<td>11.25</td>
<td>11.45</td>
<td>19.45</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9.23</td>
<td>11.97</td>
<td>8.53</td>
<td>7.08</td>
</tr>
</tbody>
</table>
standard deviation was 8.88 (6.31); using the factor analysis method the low group standard deviation was 6.81 (6.29) and the medium group standard deviation was 10.11 (9.19); using the calculator the low group standard deviation was 8.53 (7.54) and the medium group standard deviation was 7.08 (5.96); using the traditional method the low group standard deviation was 9.14 (9.37) and the medium group standard deviation was 11.97 (8.43). The low group’s (perspective 2) performance was less variable than the medium for the traditional, factor analysis, and Hutchings’ methods, while the medium group’s performance was less variable than the low using the calculator. In comparing the perspective 1 and 2 variances for each group, the medium achieving group’s (perspective 2) performance for each of the methods became more variable, whereas for the low achieving group in perspective 2 the only method that resulted in greater performance variance was the calculator.

As was the case for the perspective 1 analysis, comparing the ranges of performance index values between the medium and low groups using the perspective 2 data indicated a substantial amount of overlap between the medium and low groups’ subjects’ performances (the perspective 1 range values are in parenthesis). Using the Hutchings’ method the low group’s range of mean performance indices was 5.125 to 24.1 (2.63 to 22), and medium group’s range of mean performance indices was 13.6 to 40.5 (13.75 to 33.5). Using the calculator the low group’s range of mean performance indices was -4.125 to 29.6 (-4.13 to 25.88), and the medium group’s range of mean performance indices was 10 to 32.1 (10.88 to 29.38). Using the factor analysis method the low group’s range of mean performance indices was -
4.25 to 21.3 (-4.25 to 19.38), and the medium group's range of mean performance indices was 4.1 to 37.7 (4.88 to 33.88). Using the traditional method the low group's range of mean performance indices was -12.875 to 22.6 (-12.88 to 22.13), and the medium group's range of mean performance indices was 2.3 to 40.2 (7.88 to 32.38).

As was the case for the perspective 1 between groups analysis, it is clear from these results that the low and medium achievement distinction yields groups that are not entirely different. To explicate the degree of overlap in performance between the two groups for each method, the individuals' performance index means from each group were compared with the other group's overall performance index mean (and a one standard deviation confidence interval). The results were (Presented in order of increasing overlap): Using the Hutchings' low-stress algorithm three subjects' mean performance indices from the low group fell within one standard deviation of the medium group's overall performance index mean, and one of the medium group subject's performance index means fell within one standard deviation of the low group's overall performance index mean (this same medium group subject scored below the low group overall mean). Using the calculator six low group subjects' performance index means fell within one standard deviation of the medium group's overall performance index mean (one low group subject scored higher than the medium group overall mean), and three medium group subjects' performance index means fell within one standard deviation of the low group's overall performance index mean (one scored below the low group overall mean).
Using the factor analysis method seven low group subjects' performance index means fell within one standard deviation of the medium group's overall performance index mean (one low group subject scored higher than the medium group overall mean), and two medium group subjects' performance index means fell within one standard deviation of the low group's overall performance index mean (one scored below the low group overall mean). Using the traditional method ten low group subjects' performance index means fell within one standard deviation of the medium group's overall performance index mean (three scored higher than the medium group overall mean), and four of the medium group subjects' performance index means were within one standard deviation of the low group's overall performance index mean (one scored below the low group overall mean).

These results are inconsistent with the perspective 1 findings. The number of medium achievement group subjects whose performance index means fell within one standard deviation of the low group's overall performance index mean remained the same between perspective 1 and 2. However, the number of low achievement group subjects whose performance index means fell within one standard deviation of the medium achievement group's overall performance index mean increased for each method (Hutchings', from one to three subjects; calculator, from four to six subjects; factor analysis method, from six to seven subjects; traditional method, from six to ten subjects). These changes in the degree of overlap between the two groups appear to be mostly the result of increases in the medium achievement group's
performance variability (the standard deviations) for each of the methods, rather than notable changes in mean performance index values.

In summary, there appears to be substantial differences between low and medium group performances for all four methods, although there is a substantial amount of overlap between the individual performances between the two groups. This overlap is greater employing the perspective 2 data than perspective 1. Among the four methods in experiment 1, for both perspectives one and two, the Hutchings' method displayed the greatest differential increase in performance between the two groups and the least amount of overlap. In terms of generalizability, anyone wishing to apply these results to other (similarly labeled) populations should consider the individual differences that exist within such group classifications before making judgments about the applicability of any of these addition methods with any specific individual or group.

Question 6-A. Utilizing perspective 2 data do descriptive statistics suggest that there are differences in overall performance between the early and late time periods (i.e., is there a practice effect) for the low achieving group?

The average of the four methods' means for the early sessions in experiment 1 was 9.89. The average for the late sessions was 11.1. This small difference suggests that there is no notable difference in performance across the four methods between the early and late sessions. That is, there appears to be a minimal overall practice effect for the low achieving group working 2x7 addition problems.
Question 6-B. Utilizing perspective 2 data, does a statistical analysis indicate that there are significant differences in overall performance between the early and late time periods across the methods (i.e., is there a practice effect) for the low achieving group?

A 2x4 (time period x method) repeated measures ANOVA compared the means of the averaged early and late performance indices. This analysis was non-significant ($F[1,11] = 1.18; p > .05$) indicating that for this group in experiment 1 there was no significant difference between early and late performance indices averaged across all four methods.

Question 6-C. Utilizing perspective 2 data do descriptive statistics suggest that there are differences in overall performance between the early and late time periods (i.e., is there a practice effect) for the medium achieving group?

The average performance index of the four methods' means for the early sessions in experiment 1 was 17.82. The average performance index for the late sessions was 22.88. This moderate difference suggests that there is some difference in performance across the four methods between the early and late sessions. That is, there does appear to be some overall practice effect for the medium achieving group working 2x7 addition problems.
Question 6-D. Utilizing perspective 2 data, does a statistical analysis indicate that there are significant differences in overall performance between the early and late time periods across the methods (i.e., is there a practice effect) for the medium achieving group?

A 2x4 (time period x method) repeated measures ANOVA compared the means of the averaged early and late performance indices. This analysis was non-significant ($F[1,5]=3.47; p > .05$) indicating that for this group in experiment 1 there was no significant difference between early and late performance indices averaged across all four methods.

Question 7-A. Utilizing perspective 2 data, does a visual (graphic) summary and statistical analysis suggest that there is an interaction between the methods and time periods factors (i.e., is the performance differential between early and late time periods consistent across all four methods) for the low achieving group?

Referring to Figure 27, it can be seen that there was virtually no difference between the early and late session means for three of the four methods. Only the Hutchings' method produced a slight increase in mean performance (from 11.98 to 15.4) between early and late sessions for the low achieving math group computing 2x7 array problems. Given this increase only the Hutchings' method produced results suggesting an interaction between the methods and time period factors. It also suggests that, using the Hutchings' method, the low achievers' performance improved with extended practice. However, a 2x4 (time period x method) repeated measures ANOVA determined that there was no statistically significant interaction between the two factors ($F[3,33]=.85; p > .05$).
Figure 27. Group Mean Performance Indices. Low Achieving Group, Experiment 1, Early and Late Sessions Compared. (Early Sessions Based on Mean Index of First Five Sessions; Late Sessions Based on Mean Index of Last Five Sessions.)
Question 7-B. Utilizing perspective 2 data, does a visual (graphic) summary and statistical analysis suggest that there is an interaction between the methods and time periods factors (i.e., is the performance differential between early and late time periods consistent across all four methods) for the medium achieving group?

Figure 28 indicates that there were small increases between early and late sessions means for the traditional method (up from a performance index mean of 14.43 to 16.87) and the calculator (up from 17.97 to 21.03), and somewhat greater degrees of improvement for both the Hutchings’ and factor analysis methods. The Hutchings’ method resulted in a mean performance index increase from 24.1 to 31.73; the factor analysis method resulted in a mean performance index increase from 14.77 to 21.87. These results suggest that there is a greater practice effect when the medium achieving subjects use either the Hutchings’ or factor analysis methods, as compared to the calculator or traditional methods, for computing 2x7 addition problems. However, a 2x4 (time period x method) repeated measures ANOVA determined that there was no statistically significant interaction between the two factors ($F [3,15]=1.66; \ p > .05$).
Figure 28. Group Mean Performance Indices. Medium Achieving Group, Experiment 1, Early and Late Sessions Compared. (Early Sessions Based on Mean Index of First Five Sessions; Late Sessions Based on Mean Index of Last Five Sessions.)
CHAPTER IV

METHOD: EXPERIMENT 2

General Considerations

The same as experiment 1.

Independent Variables

There were four independent variables in experiment 2 that included the first three from experiment 1 and added:

4. Problem difficulty: addition problems comprised of five columns by seven rows of digits (5x7 arrays).

Data Collected/Dependent Variables

Data collected in experiment 2 was the same as in experiment 1. The dependent variable analyzed in experiment 2 is the same as in experiment 1.

Subjects

The subjects for experiment 2 were the same as for experiment 1 except that subject 20 from the low achieving group and subject 13 from the medium achieving group dropped out at the end of experiment 1.

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Setting

The setting for experiment 2 was identical to experiment 1.

Procedure

The procedures for experiment 2 were identical to experiment 1 with the exception of algorithm training and the experimental task. For experiment 2 no training was required in that this had been done prior to the beginning of the sessions for experiment 1. For experiment 2 the subjects added 5x7 array addition problems whereas experiment 1 involved 2x7 arrays.

Materials

The materials for experiment 2 were identical to experiment 1 with the exception that the worksheets for experiment 2 contained six 5x7 array problems as compared to nine 2x7 array problems in experiment 1.

Scoring and Recording

The scoring and recording techniques used for experiment 2 were identical to those used in experiment 1.

Experimental Design

The experimental design for experiment 2 was identical to experiment 1.
Data Analysis

The data analysis for experiment 2 was identical to experiment 1 with these exceptions: Whereas the experiment 1, perspective 1 data analysis averaged the subjects’ mean performance indices for each subject’s first eight sessions using each method (regardless of the individual’s actual session numbers), the experiment 2, perspective 2 data analysis averaged the subjects’ mean performance indices for each subject’s first five sessions (regardless of the individual’s actual session number). Whereas the experiment 1, perspective 2 data analysis averaged the subjects’ mean performance indices for each subject’s first and last three sessions using each method (regardless of the individual’s actual session numbers), the experiment 2, perspective 2 data analysis averaged the subjects’ mean performance indices for each subject’s first and last three sessions (regardless of the individual’s actual session numbers).

Reliability

The strategy for determining the reliability coefficient for experiment 2 sessions was the same as experiment 1 except that a total of 40 worksheets (4 x 20; representing just over 5% of the total worksheets) were selected for rescoring.
CHAPTER V

RESULTS: EXPERIMENT 2

Reliability

For experiment 2 the 40 worksheets made 461 total columns with 2 disagreements and 459 agreements. This produced a reliability index of (459/461) .9957.

It may be argued that a larger reliability sample should have been analyzed. However, given that the 5% sample selected was randomly chosen and evenly distributed across experimental sessions, and that this sample resulted in such a high reliability index, the experimenter was confident that these results would not drop below 95% with additional sampling.

Rationale for Data Analysis

The strategies employed for analyzing the data in experiment 2 were the same as those used for experiment 1 with this exception: Whereas experiment 1 collapsed the subjects' performance indices across the first eight sessions for perspective 1 and the first and last five sessions for perspective 2, experiment 2 collapsed the subjects' first five sessions for perspective 1 and the first and last three sessions for perspective 2. The underlying rationale for selecting the number of sessions to
collapse in experiment 2 was the same as experiment 1: A large performance sample was desired, and six corresponded with the minimum number of sessions that any one subject completed for experiment 2.

Organization of Dependent Data

The results for experiment 2 are presented in the same fashion as they were for experiment 1 with the exception that the discussion of experimental question 5 (between experiments differences) is included in the experiment 2 results. Also, the "E-2" designation is added to each experiment 2 question.

Summary data for experiment 2 comparisons are presented in 6 tables and 16 figures that accompany the text.

Dependent Data

Question 1-A-E2. Utilizing perspective 1 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the low achieving group?

On average, for perspective 1 of this analysis the low achieving math group performed 5x7 array addition problems best using the Hutchings' low-stress algorithm ($M=9.32$). Their next best performance was with the traditional algorithm ($M=5.93$), followed by the factor analysis method ($M=5.89$), with the calculator being the least effective method ($M=2.85$). The difference between the largest and smallest means (Hutchings'-Calculator) was 6.47.
Referring to Figure 29 the Hutchings' low-stress algorithm shows moderate superiority over the calculator and modest superiority over the other two methods. The traditional and factor analysis methods produced nearly identical performance that was about half way between the Hutchings' and the calculator.

The factor analysis method produced the least intersubject variance ($s=4.49$), the Hutchings' method was next ($s=4.83$), the traditional method was third ($s=6.59$), while the calculator produced the most variance among the four methods ($s=7.04$). Additional descriptive data for the low achieving group working 5x7 problems (including range and median) are presented in Table 8.

While visual examination of the results indicates that for most subjects the Hutchings' was their most effective method (refer to Figure 30), there was some intersubject variance (or "noise", as seen in the standard deviations) suggesting that caution should be exercised when generalizing these results to similar populations. Six subjects performed best using the Hutchings', two using the traditional method, one using the factor analysis method, one using the calculator, and one did equally well using either the traditional method or the calculator.

In pairwise contrasts eight subjects did better with the Hutchings' method compared to three who did better with the traditional. Nine subjects performed better with the Hutchings' method while two performed better with the factor analysis algorithm. Ten subjects performed better using the Hutchings' compared to one who performed better using the calculator.
Figure 29. Group Mean Performance Indices. Low Achieving Group, Experiment 2, Perspective 1. (Based on Mean Index of Each Subject's First Five Sessions.)
### Table 8

Experiment 2, Perspective 1: Descriptive Statistics of Low Math Achievement Group

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>-7 to 19</td>
<td>-4 to 17.2</td>
<td>3.6 to 21.2</td>
<td>-1.6 to 15.6</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>5.93</td>
<td>2.85</td>
<td>9.32</td>
<td>5.89</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>5.8</td>
<td>.4</td>
<td>9.8</td>
<td>6.6</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>6.59</td>
<td>7.04</td>
<td>4.83</td>
<td>4.49</td>
</tr>
</tbody>
</table>
Figure 30. Group and Individual Mean Performance Indices. Low Achieving Group, Experiment 2, Perspective 1. (Based on Mean Index of Each Subject's First Five Sessions.)
In summary, the low-achieving group performed 5x7 array addition problems best using the Hutchings' algorithm, although there was a fair amount of intrasubject variability across methods and intersubject variability within methods. There was a moderate mean difference between the Hutchings' and both the factor analysis and traditional methods, and a more dramatic mean difference between the Hutchings' and the calculator.

Question 1-B-E2. Utilizing perspective 1 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the low achieving group?

A one-way repeated measures analysis of variance (ANOVA) compared the mean performance indices of subjects using the four addition methods. These differences were significant ($F[3, 30] = 6.18; p < .002$). This indicates that the probability of obtaining mean differences of this magnitude (among all four means) by chance is less than .002 if the null hypothesis is true.

Post hoc multiple comparison analysis found that the Hutchings' algorithm produced significantly better addition performance than the calculator ($F$-critical = .00833; $F[1, 10] = 20.89; p < .001$) and the factor analysis method ($F$-critical = .00833; $F[1, 10] = 12.79; p < .005$). The multiple comparison procedure revealed no other significant pairwise contrasts.

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Question 1-C-E2. Utilizing perspective 2 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the low achieving group?

For this perspective of the analysis the low achieving subjects performed 5x7 array addition problems best using the Hutchings' low-stress algorithm ($M=8.72$). The factor analysis was the next most effect ($M=6.22$), the traditional algorithm was third ($M=5.96$), while the calculator proved to be the least effective ($M=3.34$). The complete summary of descriptive statistics are presented in Table 9.

As can be seen in Figure 31 the order and spacing of performance among the methods for both perspectives one and two was similar with mean performance on the Hutchings' dropping off slightly ($8.72 < 9.32$), the factor analysis method gaining slightly ($6.22 > 5.89$), the traditional method gaining very slightly ($5.96 > 5.93$), and the calculator performance improving slightly ($3.34 > 2.85$). In the perspective 2 analysis the difference between the largest and smallest means (Hutchings'-Calculator) was 5.38 compared to 6.47 for the perspective 1 analysis.

The variability in performance among the subjects for each method of the perspective 2 analysis was also very similar to the perspective 1 analysis. The order of the methods from least to most variability was the same with each method for both perspectives. Comparison of perspective 2 with perspective 1 data showed that variability from the factor analysis method increased from 4.49 to 4.57, increased from 4.83 to 5.46 for the Hutchings' algorithm, decreased from 6.59 to 6.15 for the...
Table 9

Experiment 2, Perspective 2: Descriptive Statistics
of Low Math Achievement Group

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>-6 to 17.84</td>
<td>-4.34 to 26.67</td>
<td>2.92 to 22.5</td>
<td>.335 to 18</td>
</tr>
<tr>
<td>Mean</td>
<td>5.96</td>
<td>3.33</td>
<td>8.72</td>
<td>6.22</td>
</tr>
<tr>
<td>Median</td>
<td>5</td>
<td>.335</td>
<td>6.47</td>
<td>6.17</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.15</td>
<td>8.46</td>
<td>5.46</td>
<td>4.57</td>
</tr>
</tbody>
</table>
Figure 31. Group Mean Performance Indices. Low Achieving Group, Experiment 2, Perspectives 1 and 2 Compared. (Perspective 1 is Based on the Mean Index of Each Subject's First Five Sessions; Perspective 2 is Based on the Mean Index of Each Subject's First and Last Three Sessions.)
traditional method, while the calculator produced the greatest intersubject variance, increasing from 7.04 to 8.46.

Figure 32 displays the low achieving group’s means for the four methods across the abscissa and includes individual performances between methods. As was the case for the methods analysis in perspective 1, the Hutchings’ method produced the best overall group mean performance. However, as with the first perspective, the second perspective also shows relatively small differences between the method performance index means and a large amount of intersubject variance, or inconsistency, when comparing the subjects’ performance within any one method. Overall, four of the subjects did best using the Hutchings’ method, four with the traditional method, two with the calculator, and one with the factor analysis method. The inconsistency of these results indicates that caution needs to be exercised when applying any of these methods to similar populations.

**Question 1-D-E2.** Utilizing perspective 2 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the low achieving group?

The methods main effects within the perspective 2 statistical analysis utilized a two-way repeated measures ANOVA that compared the means of the averaged early and late performance indices. These differences were significant ($F_{[3,30]}=3.90; p<.025$). This indicates that the probability of obtaining mean differences of this magnitude (among all four means) by chance is less than .025 if
Figure 32. Group and Individual Mean Performance Indices. Low Achieving Group, Experiment 2, Perspective 2. (Based on Mean Index of Each Subject's First and Last Three Sessions.)
the null hypothesis is true. This finding is consistent with the perspective 1 statistical analysis.

Post hoc multiple comparison analysis revealed that the subjects’ performance index was significantly higher when Hutchings’ method was used than when the calculator was used \((F\text{-critical}=.00833; F [1,10]=12.22; p<.005)\). This result is consistent with the findings of the perspective 1 analysis. However, unlike perspective 1, the perspective 2 analysis did not reveal a significant difference between the Hutchings’ and the factor analysis methods. The multiple comparison procedure for perspective 2 revealed no other significant pairwise contrasts.

**Question 1-E-E2. Utilizing perspective 1 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the medium achieving group?**

On average, for perspective 1 of this analysis, the medium achieving math group performed 5x7 array addition problems best using the Hutchings’ low-stress algorithm \((M=24.28)\). Their next best performance was with the factor analysis method \((M=18.28)\), followed by the traditional method \((M=11.08)\), with the calculator being the least effective method \((M=2.32)\).

The factor analysis method produced the least average variability between subjects \((s=8.04)\), the Hutchings’ method was second \((s=8.32)\), the calculator was third \((s=9.13)\), while the traditional method produced the greatest amount of variability between subjects \((s=9.89)\). For additional descriptive data for this group in experiment 2 refer to Table 10.
Table 10

Experiment 2, Perspective 1: Descriptive Statistics of Medium Math Achievement Group

<table>
<thead>
<tr>
<th>Performance Index Means</th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>1.2 to 28.4</td>
<td>-8.8 to 12.4</td>
<td>11.4 to 34.8</td>
<td>7.4 to 32.4</td>
</tr>
<tr>
<td>Mean</td>
<td>11.08</td>
<td>2.32</td>
<td>24.28</td>
<td>18.28</td>
</tr>
<tr>
<td>Median</td>
<td>10.6</td>
<td>4.8</td>
<td>25.2</td>
<td>17.6</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9.89</td>
<td>9.13</td>
<td>8.32</td>
<td>8.04</td>
</tr>
</tbody>
</table>
Referring to Figure 33 the Hutchings' low-stress algorithm shows moderate superiority over the factor analysis method, substantial superiority over the traditional method, and dramatic superiority over the calculator. These differences are quite uniform between subjects across all four methods. Figure 34 indicates that each subject in the medium achieving group performed best using the Hutchings' method. Four of the five subjects in this group performed second best using the factor analysis method. For one subject the traditional method was the second most effective. Four of the five subjects performed third best using the traditional method, while all subjects recorded their lowest scores using the calculator.

In summary, utilizing perspective 1 data, for the low achieving students computing 5x7 array addition problems, the Hutchings' low-stress algorithm produced the highest mean performance index of all the methods, showing moderate to substantial superiority over the other three methods. Furthermore, the individual performance differences between methods were quite uniform.

Question 1-F-E2. Utilizing perspective 1 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the medium achieving group?

A one-way repeated measures analysis of variance (ANOVA) was used to compare mean performance indices for the four addition methods. The differences among the four methods were significant ($F[3,12]=16.68; p<.0001$). This
Figure 33. Group Mean Performance Indices. Medium Achieving Group, Experiment 2, Perspective 1. (Based on Mean Index of Each Subject's First Five Sessions.)
Figure 34. Group and Individual Mean Performance Indices. Medium Achieving Group, Experiment 2, Perspective 1. (Based on Mean Index of Each Subject’s First Five Sessions.)
indicates that the probability of obtaining mean differences of this magnitude (among all four means) by chance is less than .0001 if the null hypothesis is true.

Post hoc multiple comparison analyses revealed that the factor analysis method produced significantly better results than the calculator ($F$-critical = .00833; $F_{[1,4]} = 26.29; p < .0069$). The multiple comparison procedure revealed no other significant pairwise contrasts.

Question 1-G-E2. Utilizing perspective 2 data, do the descriptive statistics and a visual (graphic) summary suggest that there are differences in mean and individual performances between the four addition methods for the medium achieving group?

For this perspective of the analysis the medium achieving subjects performed $5 \times 7$ array addition problems best using the Hutchings' low-stress algorithm ($M = 24.80$). The factor analysis was the next most effective ($M = 19.06$), the traditional algorithm was third ($M = 10.00$), while the calculator proved to be the least effective ($M = 4.00$).

As seen in Figure 33, the order and spacing of performance among the methods for both perspectives one and two were very similar; perspective 2 mean performance on three of the four methods improved slightly over perspective 1 (Hutchings': 24.8 > 24.28; Factor Analysis: 19.06 > 18.28; Calculator: 4.0 > 2.32) while performance with the traditional method decreased slightly (10.0 < 11.08). The difference between the largest and smallest means was 20.8 for the perspective 2 analysis and 21.96 for the perspective 1 analysis.
Figure 35. Group Mean Performance Indices. Medium Achieving Group, Experiment 2, Perspectives 1 and 2 Compared. (Perspective 1 is Based on the Mean Index of Each Subject's First Five Sessions; Perspective 2 is Based on the Mean Index of Each Subject's First and Last Three Sessions.)
The amount of variability produced between subjects for each method of the perspective 2 analysis changed somewhat compared to the perspective 1 analysis. Whereas before the factor analysis method produced the least variability, now the calculator resulted in the least variance (down from 9.13 to 7.17), the Hutchings' method was next (up from 8.32 to 8.43), the factor analysis method was next (up from 8.04 to 9.60), while the traditional method produced the most intersubject variance of the four methods (up from 9.89 to 12.39). For additional descriptive data refer to Table 11.

Figure 36 displays the medium achieving group's means for the four methods across the abscissa and includes individual performances between methods. As was the case for the methods analysis in perspective 1, the Hutchings' method produces the best overall mean performance showing significant superiority over the calculator and traditional methods. Figure 36 displays a parallel performance picture that looks very similar to the perspective 1 equivalent. Four of the five subjects performed best using the Hutchings' method while the fifth performed equally well on the Hutchings' as the factor analysis method. The factor analysis method resulted in the next best performance for four of the five subjects. The traditional method was next for four of the five subjects, while four of the five medium achieving subjects recorded their weakest addition performance with 5x7 array problems using the calculator.

In summary, utilizing perspective 2 data, for the medium achieving subjects ciphering 5x7 array addition problems the Hutchings' low-stress algorithm produced
Table 11
Experiment 2, Perspective 2: Descriptive Statistics
of Medium Math Achievement Group

<table>
<thead>
<tr>
<th>Performance Index Means</th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>-4 to 32</td>
<td>-5.67 to 15</td>
<td>11.67 to 36.17</td>
<td>7.17 to 36.34</td>
</tr>
<tr>
<td>Mean</td>
<td>10</td>
<td>4</td>
<td>24.8</td>
<td>19.07</td>
</tr>
<tr>
<td>Median</td>
<td>10.5</td>
<td>5.34</td>
<td>26</td>
<td>18.34</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>12.39</td>
<td>7.17</td>
<td>8.43</td>
<td>9.60</td>
</tr>
</tbody>
</table>
Figure 36. Group and Individual Mean Performance Indices. Medium Achieving Group, Experiment 2, Perspective 2. (Based on Mean Index of Each Subject's First and Last Three Sessions.)
the highest performance index mean. The differences between the Hutchings' method and the other three methods was very similar between the two perspectives. The Hutchings' method also showed consistent results between the subjects of the group, as every subject did best using the Hutchings' algorithm.

Question 1-H-E2. Utilizing perspective 2 data, does the statistical analysis of the main effects of the methods factor (and, when appropriate, post hoc pairwise comparisons) suggest that there are significant differences in mean and individual performances between the four addition methods for the medium achieving group?

The methods main effects within the "perspective 2" statistical analysis utilized a two-way repeated measures ANOVA that compared the means of the averaged early and late performance indices. The differences among the methods were significant \( F[3,12]=16.37; p<.0009 \). This indicates that the probability of obtaining mean differences of this magnitude (among all four means) by chance is less than .0009 if the null hypothesis is true. These results are consistent with the perspective 1 statistical findings.

Post hoc multiple comparison analysis found that the subjects' mean performance index was significantly higher using the Hutchings' low-stress method compared to the calculator \( (F_{\text{critical}}=.00833; F[1,4]=44.43; p<.0026) \). The factor analysis method was also found to produce significantly better results than the calculator \( (F_{\text{critical}}=.00833; F[1,4]=73.95; p<.001) \). These findings are not consistent with the perspective 1 results, wherein the mean difference between the
Hutchings' and the calculator was not found to be statistically significant. The multiple comparison procedure revealed no other significant pairwise contrasts.

**Question 4-A-E2. Utilizing perspective 1 data, do the descriptive statistics, a visual (graphic) summary, and a simple statistical analysis suggest that there are significant performance differences between low and medium achieving groups for experiment 2?**

Figure 37 indicates that, on average, using the Hutchings' and factor analysis methods the medium achieving group's degree of increase in performance over the low achieving group was similarly substantial. Using the traditional method the medium achieving group performed better than the low, and the degree of betterment of the low group was not as substantial as the with the Hutchings' and factor analysis methods. Using the calculator the low achieving group actually performed slightly better than the medium achieving group.

The two groups' performance on the four methods compared as follows (presented in rank order of decreasing performance for the low group): Using the Hutchings' method the mean performance index for the medium group was 24.28 and for the low group 9.32. Using the traditional method the medium group performance index mean was 11.08 and for the low group 5.93. Using the factor analysis method the medium group performance index mean was 18.28 and the low group 5.89. Using the calculator the medium group performance index mean was 2.32 and the low group 2.85. Refer to Table 12 for a summary of descriptive statistics between the groups.
Figure 37. Group Mean Performance Indices. Experiment 2, Perspective 1, Low and Medium Achieving Groups Compared. (Based on Mean Index of Each Subject's First Five Sessions.)
Table 12
Experiment 2, Perspective 1: Descriptive Statistics of Low and Medium Math Achievement Groups

Performance Index Means

<table>
<thead>
<tr>
<th>Performance Index Means</th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings'</th>
<th>Factor Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Range</td>
<td>-7</td>
<td>1.2</td>
<td>-4</td>
<td>-8.8</td>
</tr>
<tr>
<td></td>
<td>to</td>
<td>to</td>
<td>to</td>
<td>to</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>28.4</td>
<td>17.2</td>
<td>12.4</td>
</tr>
<tr>
<td>Mean</td>
<td>5.93</td>
<td>11.08</td>
<td>2.85</td>
<td>2.32</td>
</tr>
<tr>
<td>Median</td>
<td>5.8</td>
<td>10.6</td>
<td>.4</td>
<td>4.8</td>
</tr>
</tbody>
</table>
Independent measures (between groups) $t$ tests were run on these performance differences between the groups. Setting the individual alpha levels for each $t$ statistic at .01, the overall (experiment-wise) alpha level becomes .04. For experiment 2 the Hutchings' low-stress algorithm produced a statistically significant difference ($t$-critical = 2.98; $t$ [14] = 4.59) between the groups as did the factor analysis method ($t$-critical = 2.98; $t$ [14] = 4.01).

The variance in performance using the four methods produced by the respective groups compared as follows (presented in order of increasing variance for the low achievement group): Using the factor analysis method the low group standard deviation was 4.49 and the medium was 8.04; using the Hutchings' low-stress algorithm the low group standard deviation was 4.83 and the medium was 8.32; using the traditional method the low group standard deviation was 6.59 and the medium group was 9.89; using the calculator the low group standard deviation was 7.04 and the medium group was 9.13. The low achieving group's performance was less variable then the medium for all four methods.

Comparing the ranges of performance index values between the medium and low groups indicated that there was a substantial amount of overlap between the groups. The results of these comparisons were: Using the Hutchings' low-stress algorithm the low group's range of performance index means was 3.6 to 21.2, and for the medium group the range was 11.4 to 34.8; using the factor analysis method the low group's range of performance index means was -1.6 to 15.6, and for the medium group the range was 7.4 to 32.4. Using the traditional method the low
group's range of performance index means was -7 to 19, and for the medium group the range was 1.2 to 28.4; using the calculator the low group's range of performance index means was -4 to 17.2, and for the medium group the range was -8.8 to 12.4.

These results indicate that, even for the traditional, Hutchings', and factor analysis methods that resulted in the apparently superior performance of the medium group over the low group, there was a substantial amount of overlap in performance between the individuals in the two groups (for any one method). This was explicated by comparing the individuals' means from each group with the other group's mean (and a one standard deviation confidence interval) for the respective method. The results of these comparisons were (presented in order of increasing overlap): For both the factor analysis and Hutchings' methods one of the low achievement group subjects' performance index means was within one standard deviation of the medium group's respective overall performance index mean (for both methods this was low group subject 17; none of the low group subjects exceeded the medium group's overall mean), and one of the medium achievement group subjects' performance index means was within one standard deviation of the low group's respective overall performance index mean (for both methods this was medium group subject 14; none of the medium group subjects scored below the low group's overall mean). Using the traditional method nine low group subjects' performance index means were within one standard deviation of the medium group's overall performance index mean (two scored higher than the medium group mean), and three of the medium group subjects' performance index means were within one
standard deviation of the low group's overall performance index mean (two scored below the low group mean).

In summary, a visual analysis indicates that the medium achieving group did much better than the low achieving group using three of the four methods, and slightly worse using the calculator. The Hutchings' and factor analysis methods resulted in statistically significant differences in performance between the two groups. For all four methods the medium group's performance showed more variability between subjects than the low group's performance. There was substantial variance in performance within both groups using any of the methods and hence an increasing degree of overlap among the individual performances between the two groups as their respective overall performance index means converged. This overlap diminishes the interpretability of differences between group means. Among the four methods in experiment two utilizing perspective 1 data, the Hutchings' low-stress algorithm displayed the greatest differential increase in performance between the two groups (the factor analysis method was next), and hence, both of these methods resulted in the least amount of overlap between subjects. In terms of generalizability, anyone wishing to apply these results to other populations should consider the individual differences that exist within such group classifications as "low" and "medium" before making judgements about the applicability of any of these addition methods to any specific individual or group.
Question 4-B-E2. Utilizing perspective 2 data, do the descriptive statistics, a visual (graphic) summary, and a simple statistical analysis suggest that there are significant performance differences between low and medium achieving groups for experiment 2?

Figure 38 indicates that, as was the case for the perspective 1 data analysis, on average, the medium achieving subjects did consistently better than the low achieving subjects using the traditional, Hutchings', and factor analysis methods, and about the same using the calculator. Comparing perspective 1 and two data, the overall pattern of performance between the two groups was very similar.

The two groups' performances on the four methods compared as follows (presented in rank order of performance for the low group; the corresponding values for perspective 1 are in parentheses): Using the Hutchings' low-stress algorithm the mean performance index for the low group was 8.72 (9.32), and for the medium group 24.8 (24.28). Using the factor analysis method the low group mean was 6.22 (5.89), and the medium group was 19.07 (18.28). Using the traditional method the low group mean was 5.96 (5.93), and the medium group was 10.00 (11.08). Using the calculator the low group mean was 3.33 (2.85), and the medium group was 4.00 (2.32). Considering the similarities between the groups' performances for perspective 1 and 2 data, no statistical analysis was conducted to examine between group performance differences utilizing perspective 2 data. For a summary of descriptive data between the two groups refer to Table 13.

Independent measures (between groups) t tests were run on these performance differences between the groups. Setting the individual alpha levels for each t statistic
Figure 38. Group Mean Performance Indices. Experiment 2, Perspective 2, Low and Medium Achieving Groups Compared. (Based on Mean Index of Each Subject's First and Last Three Sessions.)
Table 13

Experiment 2, Perspective 2: Descriptive Statistics of Low and Medium Math Achievement Groups

<table>
<thead>
<tr>
<th>Performance Index Means</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>Calculator</th>
<th>Hutchings' Analysis</th>
</tr>
</thead>
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<td>Low Medium</td>
<td>Low Medium</td>
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<tr>
<td>Range</td>
<td>-6 to 17.84</td>
<td>-4 to 32</td>
<td>-4.34 to -5.67</td>
</tr>
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<td></td>
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<td>2.92 to 11.67</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>.335 to 7.17</td>
</tr>
<tr>
<td>Mean</td>
<td>5.96, 10</td>
<td>3.33, 4</td>
<td>8.72, 24.8</td>
</tr>
<tr>
<td>Median</td>
<td>5, 10.5</td>
<td>.335, 5.34</td>
<td>6.47, 26</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.15, 12.39</td>
<td>8.46, 7.17</td>
<td>5.46, 8.43</td>
</tr>
</tbody>
</table>
at .01, the overall (experiment-wise) alpha level becomes .04. For the perspective 2 analysis of experiment 2 the Hutchings’ low-stress algorithm produced a statistically significant difference ($t$-critical=2.98; $t_{[14]}$=4.60) between the groups as did the factor analysis method ($t$-critical=2.98; $t_{[14]}$=3.65). These results are consistent with perspective 1 findings.

The variance in performance using the four methods produced by the respective groups compared as follows (presented in order of increasing variance for the low group; perspective 1 values are in parentheses): Using the factor analysis method the low group standard deviation was 4.57 (4.49), and the medium group was 9.60 (8.04); using the Hutchings’ method the low group standard deviation was 5.46 (4.83), and the medium group was 8.43 (8.32); using the traditional method the low group standard deviation was 6.15 (6.59), and the medium group was 12.39 (9.89); using the calculator the low group standard deviation was 8.46 (7.04), and the medium group was 7.17 (9.13). The low group’s performance was less variable than the medium group for the traditional, Hutchings’ and factor analysis methods, while the medium group’s performance was less variable than the low for the calculator. Comparing the variance in performance between perspectives one and two, the perspective 2 variability in performance between subjects decreased in all cases except for the low achieving group using the traditional method and the medium achieving group using the calculator.

As was the case for the perspective 1 analysis, comparing the ranges of performance index values between the medium and low achieving groups using the
perspective 2 data did indicate that there was a substantial amount of overlap between the medium and low groups’ subjects’ performances (the perspective 1 ranges are in parentheses). Using the Hutchings’ algorithm the low group’s range of performance indices was 2.9 to 22.5 (3.6 to 21.2), and the medium group’s was 11.7 to 36.2 (11.4 to 34.8). Using the factor analysis method the low group’s range of performance indices was .3 to 18 (-1.6 to 15.6), and the medium group’s was 7.2 to 36.3 (7.4 to 32.4). Using the traditional method the low group’s range of mean performance indices was -6 to 17.8 (-7 to 19), and the medium group’s was -4 to 32 (1.2 to 28.4). Using the calculator the low group’s range of mean performance indices was -4.3 to 26.7 (-4 to 17.2), and the medium group’s was -5.7 to 15 (-8.8 to 12.4).

As was the case for the perspective 1 between groups analysis, it is clear from these results that the low and medium achievement distinction yields groups that are not entirely heterogeneous. There was substantial overlap in individual performances between the two groups, even for the three methods that yielded larger mean differences between the two groups. This overlap was explicated by comparing the individuals’ means from each group with the other group’s mean (and a one standard deviation confidence interval) for each method. The results were (presented in order of increasing overlap): Using the Hutchings’ algorithm one low group subject’s performance index mean fell within one standard deviation of the medium group’s overall performance index mean, and one medium group subject’s performance index mean fell within one standard deviation of the low group’s overall
performance index mean. Using the factor analysis method two low group subjects' performance index means were within one standard deviation of the medium group's overall performance index mean (one scored higher than the medium group overall mean), and one medium group subject's performance index mean fell within one standard deviation of the low group's overall performance index mean. Using the traditional method nine low group subjects' performance index means fell within one standard deviation of the medium group's overall performance index mean (two scored higher than the medium group's overall mean), and four medium group subjects' performance index means fell within one standard deviation of the low group's overall performance index mean (two scored lower than the low group's overall mean). These results are very similar to the perspective 1 findings.

In summary, a visual analysis indicates that the medium achieving group did much better than the low achieving group using the traditional, Hutchings', and factor analysis methods, and about the same using the calculator. For three of the four methods the medium group's performance showed more variability between subjects than the low group's performance, while for the calculator the medium group's performance exhibited less variability. There was substantial variance in performance within both groups using any of the methods, and hence increasing amounts of overlap among the individual performances between the two groups as their respective overall performance index means converged. This overlap diminishes the interpretability of differences between group means. Among the four methods in experiment two utilizing perspective 2 data, the Hutchings' low-stress
algorithm displayed the greatest differential increase in performance between the two groups, and hence, the least amount of overlap between subjects. In terms of generalizability, anyone wishing to apply these results to other populations should consider the individual differences that exist within such group classifications as "low" and "medium" before making judgements about the applicability of any of these addition methods to any specific individual or group.

**Question 5-A. Utilizing Perspective 1 data, do descriptive statistics and a visual summary suggest that there are differences in performance among the methods between experiments 1 and 2 for the low achieving group?**

Examining Figure 39 it is clear that when the addition problems became more difficult (going from 2x7 to 5x7 matrices) the low achieving group's performance diminished for each of the four methods. In terms of raw score differences the factor analysis showed the least performance decrement between experiments (7.41-5.89=1.52), the traditional method was next (8.83-5.96=2.87), followed by the Hutchings' method (12.59-9.32=3.27), with the calculator losing the most between the experiments (10.65-2.85=7.8). When these raw score differences are expressed as percentages of the experiment 1 performance index averages, the factor analysis showed the lowest degree of relative loss (1.52/7.41=21%), the Hutchings's was next (3.27/12.59=26%), the traditional method was third (2.87/8.83=33%), while the calculator showed the greatest degree of relative decrement between the experiments (7.8/10.65=73%).

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Figure 39. Group Mean Performance Indices. Low Achieving Group, Perspective 1, Experiments 1 and 2 Compared. (Experiment 1 is Based on Mean Index of Each Subject’s First Eight Sessions; Experiment 2 is Based on Mean Index of Each Subject’s First Five Sessions.)
The variability in performance from experiment 1 to experiment 2 dropped for each method as follows: for the traditional method the standard deviation fell from 9.37 to 6.59; for the calculator it fell from 7.54 to 7.04; for the Hutchings' method it fell from 5.83 to 4.83; for the factor analysis method it fell from 6.29 to 4.49.

In summary, for the low achieving subjects, when the problems become more difficult, there appears to be a moderate amount of performance decrement using the factor analysis, traditional, and Hutchings’ methods, and a substantial loss in performance when using the calculator. Between the two experiments the variability in performance dropped for each method.

Question 5-B. Utilizing Perspective 2 data, do descriptive statistics and a visual (graphic) summary suggest that there are differences in performance among the methods between experiments 1 and 2 for the low achieving group?

Figure 40 clearly shows that, utilizing perspective 2 data when the addition problems became more difficult (going from 2x7 to 5x7 matrices) the low achieving group’s performance diminished for each of the four methods. In terms of raw score differences the factor analysis showed the least performance decrement between the experiments (8.09-6.02=2.07), the traditional method was next (9.14-5.96=3.18), followed by the Hutchings’ method (13.68-8.72=4.96), with the calculator losing the most between the experiments (11.06-3.33=7.73). When these raw score differences are expressed as percentages of the experiment 1 performance index averages, the factor analysis showed the lowest degree of relative loss
Figure 40. Group Mean Performance Indices. Low Achieving Group, Perspective 2, Experiments 1 and 2 Compared. (Experiment 1 is Based on Mean Index of Each Subject's First and Last Five Sessions; Experiment 2 is Based on Mean Index of Each Subject's First and Last Three Sessions.)
(2.07/8.09=26%), the traditional method was next (3.18/9.14=35%), the Hutchings's was third (4.96/13.68=36%), while the calculator showed the greatest degree of relative decrement between the experiments (7.73/11.06=70%). Furthermore, there were no notable differences between the contrast of experiments 1 and 2 within perspective 1 and the contrast of experiments 1 and 2 within perspective 2.

The variability in performance between the two experiments decreased for each of the methods as follows: For the traditional method the standard deviation dropped from 9.23 to 6.15; for the calculator it dropped from 8.53 to 8.46; for the Hutchings' method it dropped from 5.75 to 5.46; for the factor analysis method it dropped from 6.81 to 4.57.

In summary, for the low achieving subjects, utilizing perspective 2 data, when the problems become more difficult, there appears to be a moderate amount of performance decrement using the factor analysis, traditional, and Hutchings' methods, and a substantial loss in performance when using the calculator. With each of the four methods the variability in performance declined between experiments 1 and 2.

**Question 5-C.** Utilizing Perspective 1 data, do descriptive statistics and a visual (graphic) summary suggest that there are differences in performance among the methods between experiments 1 and 2 for the medium achieving group?

Figure 41 indicates that when the addition problems became more difficult the medium achieving group's performance increased slightly when using the factor
Figure 41. Group Mean Performance Indices. Medium Achieving Group, Perspective 1, Experiments 1 and 2 Compared. (Experiment 1 is Based on Mean Index of Each Subject's First Eight Sessions; Experiment 2 is Based on Mean Index of Each Subject's First Five Sessions.)
analysis method, and diminished in varying degrees for the other three methods. In terms of raw score differences the factor analysis method resulted in the best overall performance, increasing from a performance index mean of 16.82 to 18.28. The Hutchings' method was next losing 1.55 points between the experiments (25.83-24.28=1.55); the traditional method was next and showed a loss of 5.72 points (16.8-11.08=5.72); between experiments the calculator showed a performance decrement of 17 points (19.32-2.32=17). When these raw score differences are expressed as percentages of the experiment 1 performance index averages, the factor analysis method showed a relative gain of 9% (1.46/16.82=9%), the Hutchings's resulted in a relative loss of 6% (1.55/25.83=6%); the traditional method resulted in a relative loss of 34% (5.72/16.8=34%); at 88% (17/19.32=88%) the calculator showed the greatest degree of relative decrement between the experiments.

The variability in performance from experiment 1 to experiment 2 changed as follows: Using the traditional method the standard deviation increased from 8.43 to 9.89; using the calculator it increased from 5.96 to 9.13; using the Hutchings' method it increased from 6.31 to 8.32; using the factor analysis method it decreased from 9.19 to 8.04.

In summary, for the medium achieving group, when the problems become more difficult, overall performance increases with the factor analysis method, and decreases for the other three methods. The Hutchings' method showed the least performance decrement between experiments while the calculator resulted in the greatest. Performance variability decreased between the experiments when the
medium achieving subjects used the factor analysis method and increased for each of the other three methods.

Question 5-D. Utilizing Perspective 2 data, do descriptive statistics and a visual (graphic) summary suggest that there are differences in performance among the methods between experiments 1 and 2 for the medium achieving group?

Figure 42 indicates that when the addition problems became more difficult, utilizing perspective 2 data, the medium achieving group’s performance increased very slightly when using the factor analysis method, and diminished in varying degrees for the other three methods. In terms of raw score differences the factor analysis method resulted in the best overall performance, increasing from a performance index mean of 18.32 to 19.07. The Hutchings’ method was next losing 3.1 points between the experiments (27.9-24.8=3.1); the traditional method was next and showed a loss of 5.7 points (15.7-10.0=5.7); between experiments the calculator showed a performance decrement of 15.5 points (19.5-4.0=15.5). When these raw score differences are expressed as percentages of the experiment 1 performance index averages, the factor analysis method showed a relative gain of 4% (.75/18.32=4%), the Hutchings’s resulted in a relative loss of 11% (3.1/27.9=11%); the traditional method resulted in a relative loss of 36% (5.7/15.7=36%); at 79% (15.5/19.5=79%) the calculator showed the greatest degree of relative decrement between the experiments. Furthermore, there were no notable differences between the performance differential between experiments 1 and
Figure 42. Group Mean Performance Indices. Medium Achieving Group, Perspective 2, Experiments 1 and 2 Compared. (Experiment 1 is Based on Mean Index of Each Subject’s First and Last Five Sessions; Experiment 2 is Based on Mean Index of Each Subject’s First and Last Three Sessions.)
2 within perspective 1 and the performance differential between experiments 1 and 2 within perspective 2.

The variability in performance between the two experiments changed as follows: Using the traditional method the standard deviation increased from 11.97 to 12.39; using the calculator it increased from 7.08 to 7.17; using the Hutchings' method it decreased from 8.88 to 8.43; using the factor analysis method it decreased from 10.11 to 9.6.

In summary, for the medium achieving group, utilizing perspective 2 data, when problems become more difficult, overall performance increases slightly with the factor analysis method, and decreases for the other three methods. The Hutchings' method showed the least performance decrement between experiments while the calculator resulted in the greatest. For the traditional method and calculator variability increased slightly between experiments 1 and 2, and for the Hutchings' and factor analysis methods it decreased slightly.

Question 6-A-E2. Utilizing perspective 2 data, do descriptive statistics suggest that there are differences in overall performance between the early and late time periods (i.e., is there a practice effect) for the low achieving group?

The low achieving group's average of the four methods' means for the early sessions in experiment 2 was 5.2. The average for the late sessions was 6.92. This small difference suggests that there is no notable change in performance across the four methods between the early and late sessions. That is, there appears to be
minimal overall practice effect across methods for the low achieving group working 5x7 array addition problems.

Question 6-B-E2. Utilizing perspective 2 data, does a statistical analysis indicate that there are significant differences in overall performance between the early and late time periods across the methods (i.e., is there a practice effect) for the low achieving group working 5x7 array addition problems?

A 2x4 (time period x method) repeated measures ANOVA compared the means of the averaged early and late performance indices. This difference was non-significant ($F[1,10]= 3.26; p > .05$) indicating that for this group in experiment 2 there was no significant difference between early and late performance indices averaged across all four methods.

Question 6-C-E2. Utilizing perspective 2 data, do descriptive statistics suggest that there are differences in overall performance between the early and late time periods (i.e., is there a practice effect) for the medium achieving group?

The average of the four methods' performance index means for the early sessions in experiment 2 was 13.08. The average performance index for the late sessions was 15.85. This modest difference suggests that, for the medium achieving group ciphering 5x7 addition problems, there is no notable change in performance between the early and late sessions. That is, there appears to be minimal overall practice effect across methods for the medium achieving group working 5x7 array addition problems.
Question 6-D-E2. Utilizing perspective 2 data, does a statistical analysis indicate that there are significant differences in overall performance between the early and late time periods across the methods (i.e., is there a practice effect) for the medium achieving group working 5x7 array addition problems?

A 2x4 (time period x method) repeated measures ANOVA compared the means of the averaged early and late performance indices. This difference was non-significant ($F[1,4] = 4.26; p > .05$) indicating that for this group in experiment 2 there was no significant difference between early and late performance indices averaged across all four methods.

Question 7-A-E2. Utilizing perspective 2 data, does a visual (graphic) summary and statistical analysis suggest that there is an interaction between the methods and time periods factors (i.e., is the performance differential between early and late time periods consistent across all four methods) for the low achieving group?

Figure 43 indicates that there was virtually no difference between the early and late session means for the traditional and factor analysis methods, and very slight increases in performance between the early and late sessions using the calculator (from 2.52 to 4.15) and the Hutchings' method (from 7.06 to 10.37). These increases, however slight, suggest that using these methods the low achiever's performance improved with extended practice. A 2x4 (time period x method) repeated measures ANOVA determined that there was no statistically significant interaction between the two factors ($F[3,30] = .32; p > .05$).
Figure 43. Group Mean Performance Indices. Low Achieving Group, Experiment 2, Early and Late Sessions Compared. (Early Sessions Based on Mean Index of First Three Sessions; Late Sessions Based on Mean Index of Last Three Sessions.)
Question 7-B-E2. Utilizing perspective 2 data, does a visual (graphic) summary and statistical analysis suggest that there is an interaction between the methods and time periods factors (i.e., is the performance differential between early and late time periods consistent across all four methods) for the medium achieving group?

Figure 44 indicates that for the medium achieving group there were moderate mean increases between early and late sessions means using the calculator (up from a mean performance index of 1.6 to 6.4) and the Hutchings’ method (up from a mean performance index of 22.4 to 27.2). Using the traditional and factor analysis methods there was virtually no change in mean performance between the early sessions and the late sessions. These results suggest that, for the medium achieving group ciphering 5x7 addition problems using either the calculator or the Hutchings’ low-stress algorithm, there is a notable practice effect. However, a 2x4 (time period x method) repeated measures ANOVA determined that there was no statistically significant interaction between the two factors ($F [3,12] = .49; p > .05$).
Figure 44. Group Mean Performance Indices. Medium Achieving Group, Experiment 2, Early and Late Sessions Compared. (Early Sessions Based on Mean Index of First Three Sessions; Late Sessions Based on Mean Index of Last Three Sessions.)
CHAPTER VI

METHOD: EXPERIMENT 3

General Considerations

The same as experiments 1 and 2.

Independent Variables

There were three independent variables in experiment 3. These included the first two from experiments 1 and 2 and the fourth from experiment 2.

Data Collected/Dependent Variables

Data collected in experiment 3 was the same as in experiments 1 & 2. The first dependent variable analyzed in this experiment was the same as experiments 1 and 2. Experiment 3 also analyzed the following dependent variable:

2. Daily-session, a measure of a subject’s algorithm preference; defined as the choice of method made by each subject prior to each of the five daily sessions; each subject’s daily choice is counted as one "daily-session."
Subjects

The subjects for experiment 3 were the same as for experiment 2 except that subject 18 from the low achieving group dropped out at the end of experiment 2.

Setting

The setting for experiment 3 was identical to experiments 1 & 2.

Procedure

The procedure for experiment 3 was identical to experiment 2 with the following exceptions: Whereas for experiments 1 and 2 the subjects used each of the four methods during each experimental session in an order predetermined by the experimenter, for experiment 3 each of the 15 subjects (5 medium achieving and 10 low achieving) who participated were required to choose an addition method (from the four) at the beginning of each daily session. As a result the color of the worksheets no longer indicated (or were correlated with) the addition method to be used, as had been the case in experiments 1 and 2. The subjects were instructed to use the method that they chose for all four of the 5 minute work periods.

Materials

The materials for experiment 3 were identical to experiment 2 with the exception that the worksheets used were chosen irrespective of color.
Scoring and Recording

The scoring and recording techniques used for experiment 3 were identical to those used in experiments 1 & 2.

Experimental Design

Experiment 3 of this study employed a within-subjects (repeated measures) design similar to experiments 1 and 2. However, rather than employing all four methods within each daily session, each subject came into contact with the same method (of their choosing) for each of the four successive work periods.

Data Analysis

The data analysis for experiment 3 was very different than for experiments 1 and 2. For experiment 3, three descriptive analyses were performed. First, the number of daily-sessions for which each method was selected was calculated for the whole group, as well as for both medium and low achieving groups. These values were also expressed as percentages of the total number of daily-sessions for each group.

Next, each subject's experiment 3 performance was compared with their performance at the close of experiment 2 in order to determine how many subjects chose the addition method with which they were most proficient. For each group
the number of subjects selecting their "best" method was also expressed as a percentage of the total group.

Finally, experiment 3 performance was compared with performance at the close of experiment 2 in order to determine how the subjects' performance had changed with continued practice. These changes were described for all subjects individually and, where each group member chose the same method, within groups.

Reliability

The strategy for determining the reliability coefficient for experiment 3 sessions was the same as experiments 1 and 2 except that three worksheets from each daily session (from three different subjects) were randomly selected and rescored. For this experiment a total of 15 worksheets were selected for the reliability check (3 x 5; representing just over 5% of the total worksheets).
CHAPTER VII

RESULTS: EXPERIMENT 3

Reliability

For experiment 3, the 15 worksheets made 465 total columns with one disagreement and 464 agreements. This produced a reliability index of \( \frac{464}{465} \times 0.9978 \).

Organization of Dependent Data

The results for experiment 3 are presented in the order of the three experimental questions (8-10). The designation "E-3" is added to each.

Dependent Data

Question 8-E3. Under conditions of choice and extensive experience with the methods, for what percentage of the total daily-sessions was each of the four methods for adding 5x7 addition problems selected by the medium and low math achieving groups?

Over the five days of experiment 3 the (15 total) subjects completed a total of 60 daily-sessions. 20 of these represented medium achieving subjects, and 40 low achieving subjects. Of the 60 total daily-sessions, the subjects chose the Hutchings' method for 39 daily sessions (equalling 65% of the total), the traditional method for
13 (equalling 21.7% of the total), and the calculator for 8 daily sessions (equalling 13.3% of the total). No subjects chose the factor analysis method.

Examining preferences of the subjects according to their group, of the 20 daily-sessions completed by medium achieving subjects, all 20 employed the Hutchings' low-stress algorithm. Of the 40 total low achievement daily-sessions, 19 employed the Hutchings' method (equalling 47.5% of the total), 13 employed the traditional method (equalling 32.5% of the total), and 8 used the hand-held calculator (equalling 20% of the total). It should be noted that, because the 15 subjects did not attend an equal number of sessions during experiment 3, these results are unequally weighted by the preferences of those subjects who actually participated.

In summary, it appears that, once familiar with these four addition methods, given a choice, a clear majority of all the subjects chose the Hutchings' low-stress algorithm. When this analysis is broken down by group, the medium achieving subjects chose the Hutchings' algorithm 100% of the time, while the low achieving subjects chose the Hutchings' method 47.5% of the time.

Question 9-E3. Under conditions of choice, extensive experience with all four methods, and no direct feedback concerning their performance, how many (what percentage of) subjects selected the method with which they are most proficient adding 5x7 addition problems?

Each subject's choices of most effective methods were established by examining the results from the last sessions of experiment 2 (i.e., the averages of
last three days of the experiment). Each of the five medium achievement group
subjects (subjects 2, 3, 9, 14, & 19) chose the method that had been most effective
for them during this period, this was the Hutchings' low-stress algorithm (in
experiments 1 and 2 subject 19 did almost as well using the factor analysis method
as the Hutchings').

In the low achievement group subject 1 chose their most effective method
(Hutchings') two sessions out of three, subject 4 chose their most effective
(Hutchings') method one session out of five, subject 5 chose their most effective
method (Hutchings') three sessions out of three, subject 6 chose their most effective
method (Hutchings') two sessions out of five, subject 7 chose their most effective
method (traditional) three sessions out of four, subject 11 chose their most effective
method (Hutchings') four time out of five, subject 12 chose their most effective
method (traditional) one session out of three, subject 15 chose their most effective
method (Hutchings') five times out of five, subject 16 chose their most effective
method (traditional) three sessions out of three, and subject 17 chose their most
effective method (calculator) four times out of four. Of the 40 total low achievement
group daily-sessions, for 28 of them, or 70% or the sessions, the individual chose
their most effective method. For a summary of this individual data, refer to Table
14.

In summary, the subjects in the medium achievement group chose their most
effective addition method 100% of the time, while the subjects in the low
achievement group chose their most effective addition method 70% of the time.
### Table 14

**Experiment 3: Subjects' Most Effective Method Choices, and Subjects' Performance Changes from Experiment 2 to Experiment 3**

<table>
<thead>
<tr>
<th>Subject Number and Group: M or L</th>
<th>Proportion of Total Daily-sessions that Subject Selected Most Effective Method</th>
<th>Which Methods Continued to Improve in Experiment 3 over Experiment 2?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-L</td>
<td>2/3 (Hutchings')</td>
<td>Hutchings': No</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Traditional: No</td>
</tr>
<tr>
<td>2-M</td>
<td>4/4 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td>3-M</td>
<td>3/3 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td>4-L</td>
<td>1/5 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Traditional: Yes</td>
</tr>
<tr>
<td>5-L</td>
<td>3/3 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td>6-L</td>
<td>2/5 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculator: No</td>
</tr>
<tr>
<td>7-L</td>
<td>3/4 (Traditional)</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Traditional: Yes</td>
</tr>
</tbody>
</table>
### Table 14--Continued

<table>
<thead>
<tr>
<th>Subject Number and Group: M or L</th>
<th>Proportion of Total Daily-sessions that Subject Selected Most Effective Method</th>
<th>Which Methods Continued to Improve in Experiment 3 over Experiment 2?</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-M</td>
<td>4/4 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td>11-L</td>
<td>4/5 (Hutchings')</td>
<td>Hutchings': Yes, Traditional: No</td>
</tr>
<tr>
<td>12-L</td>
<td>1/3 (Traditional)</td>
<td>Hutchings': Yes, Traditional: No</td>
</tr>
<tr>
<td>14-M</td>
<td>5/5 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
<tr>
<td>15-L</td>
<td>5/5 (Hutchings')</td>
<td>Hutchings': No</td>
</tr>
<tr>
<td>16-L</td>
<td>3/3 (Traditional)</td>
<td>Traditional: No</td>
</tr>
<tr>
<td>17-L</td>
<td>4/4 (Calculator)</td>
<td>Calculator: No</td>
</tr>
<tr>
<td>19-M</td>
<td>4/4 (Hutchings')</td>
<td>Hutchings': Yes</td>
</tr>
</tbody>
</table>
Question 10-E3. Do descriptive statistics suggest that the subjects' overall performance continued to improve during experiment 3 over experiment 2 (i.e., is there evidence of a continued practice effect)?

The performance index means from last three sessions of experiment 2 (i.e., the last portion of the perspective 2 data) were used as the baseline for comparing experiment 3 results. The medium achievement group's mean performance index using the Hutchings' low-stress algorithm for the last three days of experiment 2 was 27.2. The medium group's mean performance index using the Hutchings' method for experiment 3 was 32.13. This continued improvement during experiment 3 suggests that during experiment 3 there was a practice effect for the medium group using the Hutchings' method to cipher 5x7 addition problems.

The low achievement group cannot be evaluated as a group because not all of the subjects chose to use the same methods. However, individually their relative performances changed as follows (the first mean performance index values are from experiment 2 and the second are from experiment 3): Subject 1's performance using the Hutchings' method diminished from 2.67 to 0, and their performance using the traditional method fell from -7.33 to -11.25. Subject 4's performance using the Hutchings' method increased from 6.33 to 10.25, and their performance using the traditional method increased from 1 to 4.75. Subject 5's performance using the Hutchings' method rose from 9.5 to 15.92. Subject 6's performance using the Hutchings' rose from 16.67 to 17.38, and their performance using the calculator dropped from 11.33 to 8.08. Subject 7's performance using the Hutchings' method
rose from 6.67 to 10.0, and their performance using the traditional method rose from 11.67 to 17.25. Subject 11’s performance using the Hutchings’ method rose from 5 to 7.8, and their performance using the traditional method fell from 2.67 to 0.00. Subject 12’s performance using the Hutchings’ method rose from 6.6 to 11.25, their performance using the traditional method fell from 11 to 5, and their performance using the calculator rose from 2.67 to 11. Subject 15’s performance using the Hutchings’ method fell from 15.33 to 14.2. Subject 16’s performance using the traditional method fell from 16.33 to 8.2. Subject 17’s performance using the calculator fell from 41.33 to 34.9. For a summary of this individual data, refer to Table 14 (pp. 205-206).

Six of the eight low achievement subjects who did choose the Hutchings’ algorithm improved during the course of experiment 3. The two subjects whose performances diminished did not decrease a significant amount. Of the six subjects who chose to use the traditional method for some of the sessions in experiment 3, four did notably worse while two improved. Of the three subjects who chose to use the calculator for some of the sessions in experiment 3, two did somewhat worse while one improved.

In summary, for experiment 3, the medium achievement group continued to improve using the Hutchings’ algorithm (the only method these subjects chose), while six of the eight low achievement group subjects who chose to use the Hutchings’ method continued to improve.
CHAPTER VIII

DISCUSSION

Methodological and Design Considerations

I designed this study to measure addition computation abilities without the provision of immediate feedback regarding performance. In this way I intended to simulate more "real world" conditions where students seldom receive timely information about how well they're doing. I realized, after the study had been conducted, that this could have been a strategic planning error. Not providing the subjects with (relatively immediate) information about their performance (so that they could correct computational error patterns) may have rendered those aspects of the results that pertain to changes in performance over time, somewhat less convincing, potent, and interpretable. Generally speaking, it is difficult to evaluate an individual’s potential for accomplishing a task when, during the learning process for that task, they do not receive feedback about what is and what is not an adequate performance.

A second post-hoc methodological consideration concerns controlling for what appeared to be changing or ineffective motivative conditions during the study. Three types of observations suggest a lack of consistent (or consistently effective) motivative conditions: (1) large, session to session variances in the performances
indexes of some individuals (especially the low group) were recorded throughout the study; (2) there were no significant differences in performance between early and late sessions for any group or any method. Furthermore, many subjects' performance declined sharply during the experiment; (3) the speed and intensity with which some subjects worked was also observed to vacillate over time, suggesting changing motivation (recall that this study was run immediately after school and the improvements in the weather that occurred about half way through experiment 1).

In designing this research it was my hope to produce results that would be applicable to everyday classroom situations. In this sense this experiment was more naturalistic (rather than experimental). The current study's motivative conditions were similar to the typical classroom in that: (a) parents, teachers, and the school administrative staff supported this project and the childrens' continued participation; (b) the study was held in a school classroom and attempted to implement similar rules of conduct and discipline; (c) the study offered generalized, conditioned incentives for attendance and good performance, e.g., regular verbal and written praise (on test forms and notes to parents) from the experimenter. Most of the children seemed motivated to please the experimenter: (d) many children enjoyed the competitive nature of the test sessions; (e) many children seemed to enjoy the group itself and the comradery of their friends; (f) many children seemed to gain a sense of importance from being involved. They were told that they were unique and special to be a part of "Project Addition."
However, it may be that the motivative conditions of this study were not as constant (or powerful) as those typical for an average, upper elementary classroom. Unlike school: (a) participation in the study was (literally) after school, and therefore optional, carrying no school-related consequences; (b) the current study did not provide any direct information about or consequences for accurate or inaccurate performance.

At this point I realize, based on what appears to be variability in the current study's motivative conditions, that some form of incentive system for good performance may have helped to produce more consistent and dramatic results. However, it also seems that employing an incentive system could have compounded the interpretation of the efficacy of the methods as students use them in the natural environment. Whether or not implementing such measures would have created an educational environment more functionally equivalent to a typical classroom remains an issue for further empirical analysis. Research is needed to replicate these results in "actual" classrooms, and to extend the generality of these findings to other student populations (both older and younger). Other, more generic issues for future research include: (a) determining techniques for creating "in-class" equivalent environments. How can we study the elements of the grade school classroom without being in it (and hence disturbing it)? and (b) examining how children's performances using these methods change over time under more consistently "optimal," experimental conditions. Such investigations might address how best to engineer a grade school classroom so as to optimize specific performance variables.
Although the results of the current study that relate to changes in performance over time may be somewhat unclear, this study does provide a reasonably adequate record of the addition performance of 10- and 11-year-old-grade school children when they are given comprehensive, pre-study instruction using the four methods of addition. This study is also unique in that the dependent variable chosen for analysis is a much more straightforward, unambiguous measure of computational performance. In all similar studies to date that the author was able to locate the dependent variables of interest were the rate of columns (or problems) correct, the rate of columns (or problems) incorrect, and the percentage of columns (or problems) correct. Each of these three standard dependent variables for describing computational performance is, in some way, incomplete and therefore, by itself, misleading. For example, on the surface it appears that a subject who scored an average of 25 columns correct over a series of sessions performed well until one discovers that this subject also computed 25 additional columns incorrectly. In this case the number of columns correct is very misleading. Consider another example: the subject computes each problem perfectly and receives a percentage of 100% for the accuracy of their work. This sounds like success until you realize that the subject computed only three columns (one 2x7 problem) of numbers. Consider yet another example: The subject fails to compute any columns correctly and receives 0% for their work. At first you might suspect that this subject was sloughing off until you find that they incorrectly computed 35 columns of numbers. This subject appears to be trying hard, yet is making some costly errors. It would be a mistake
to focus on this subject’s overall percentage score as this fails to convey the true nature of the performance.

To eliminate the ambiguity inherent with the dependent variables columns correct, columns incorrect, and percentage of columns correct, and to present the essential information in a simplified form, the dependent variable "performance index" was created. By subtracting columns incorrect from columns correct the performance index value embodies the essential information describing computational performance. It is a more complete measure as it describes overall efficiency.

Discussion of Results

In all eight discrete conditions (2 experiments x 2 perspectives x 2 groups) designed to examine the methods factor, the Hutchings’ low-stress algorithm proved to be the most effective of the four methods, resulting in the highest mean performance indices. Furthermore, in all conditions, except for the experiment 1, perspective 1, low achievement group condition, the overall repeated measures ANOVA $F$ was significant (for a summary of the statistical results refer to Appendix G).

In all three experiments every subject in the medium achieving group delivered their best performance using the Hutchings’ method. For the low achieving subjects, in three of the four conditions, the majority of the group performed best using the Hutchings’ algorithm. In the experiment 2, perspective 2 condition the plurality of low group subjects performed best with the Hutchings’
method. In experiment 3, where the subjects had to choose which method they employed, all five of the medium achieving subjects chose the Hutchings' method (their most effective) for 100% of their daily-sessions. The performance of all five continued to improve over experiment 2 levels. In contrast, the low achieving subjects selected their most effective method for 70% of the experiment 3 daily-sessions. Eight of the ten low achieving group subjects chose to use the Hutchings' method for at least one session (for six of the ten low achieving subjects it was their most effective method), amounting to 47.5% of the low group's total daily-sessions. Of the eight who chose to use the Hutchings' for at least one session, six continued to improve their performance over experiment 2 levels.

What real world value do these performance increases have? That is, do the statistically significant differences in performance revealed by this study lead to or reflect some worthwhile, practical differences in the classroom? Unfortunately, the answer to this question is beyond the intended scope of this study. Longer-term research is needed to assess the impact of high performance computational algorithms similar to the Hutchings' on such (relatively distal) parameters as class grades, problem-solving abilities, achievement scores, tendency to use the Hutchings' algorithm in the classroom subsequent to the research study, and attitudes toward mathematics. It would be surprising if the types of improvements in performance witnessed during this study would have no impact on classroom or academic functioning. In many cases the performance differences between the Hutchings' method and the traditional method and calculator were dramatic. The
fact that 100% of the average students who participated in this study chose the Hutchings' method, which they had just learned for purposes of this study, over the traditional method, which they had known and practiced many times longer, suggests that, for average students, the Hutchings' represents a major improvement in efficiency and user friendliness (over the traditional method) that could lead to greater success and enjoyment with mathematics. These issues need to be validated empirically.

Given that the medium and low achieving groups represent adjacent (predetermined) skill level classifications, it is likely that the performance differences (between groups classified as "lower" and "higher") would be much more dramatic were an actual "high achieving" group contrasted with a low achieving group. Given their adjacent classification status, it is notable that in experiments 1 and 2 the medium math achieving group scored better than the low for all three written methods, and in experiment 1, with the calculator. In experiment 1 the Hutchings' method produced a significant performance difference between the groups. In experiment 2 both the Hutchings' and factor analysis methods produced significant performance differences between the groups. Somewhat more predictably, in experiment 2, using the calculator, the low and medium groups performed at about the same (exceedingly poor) level.

There was a substantial amount of variance in performance across all methods, both within and between groups. This has a number of ramifications. In conjunction with the small group sizes (for experiment 1: medium group = 6, low
group = 12) the high variance levels contributed to a failure to achieve significance in many of the statistical tests, especially the Post Hoc multiple comparison procedures. The importance of the size of the mean difference between groups and not just the statistical results has been cited as important when analyzing the effectiveness, or real-world impact of an experimental intervention (Huitema, 1986). Research employing larger groups sizes and some measure of pre-test ability (to function as a covariate) may produce more dramatic statistical results.

In experiment 1 there did not appear to be a pattern to the variance in performance between methods or groups. In experiment 2, however, the medium achievement group's performance became substantially more variable (than experiment 1) making the low achieving group's performance substantially less variable (or more uniform) than the medium group for all four of the methods. The greater uniformity of the low group's performance (when the problem difficulty increased) suggests that the low achieving (sample) group may represent a more homogeneous population than the medium group as their skills appear to be more consistent across the four addition methods than the medium group. There were very large differences between the medium achieving group subjects' performances, especially for the traditional method and calculator. Using these two methods medium group subjects 9 and 14 routinely performed on par with low group means. By comparison (based on mean performance indices), medium group subject 19 performed over three times as well as subject 14 using the Hutchings' algorithm, over four times as well using the factor analysis method, over five times as well
using the calculator, and over fifteen times as well using the traditional method. Subject 19 performed over 23 times better than subject 9 using the traditional method. It should be point out that there were also many dramatic contrasts in performance within the low group. For example, subject 17 consistently performed almost 5 times better than subject 4 using the Hutchings' method, over 11 times better than subject 18 using the traditional method, and over 25 times better than subject 1 using the calculator. Using the calculator, subject 17 of the low achieving group scored higher than all but one of the medium group subjects.

This study found that group divisions or classifications based on past academic performance (and teacher appraisal) appear (at the group level) to be valid. For example, the performance differences between the medium and low groups using the Hutchings' method were statistically significant (in favor of the medium group) for both perspectives and both experiments. However, as the previous paragraph clearly explicates, this study also found substantial intersubject variance within methods and intrasubject variance between methods for both groups. The large intersubject variances in performance and the resulting overlap between groups suggests that children labeled by teachers or educators as "medium (or average) achieving" and "low achieving" will not necessarily produce vastly different performances. That is, groups labelled as medium and low math achievers are not necessarily heterogeneous. The large intrasubject variance in performance carries a number of implications: (a) no one addition method appears to produce the best (or worst) performance for all subjects (of either low or medium classification); (b)
students who do poorly using one method may perform very well with another (and vice versa); (c) performance using one method cannot be used to predict performance on any other method; and (d) a student's (historically determined) classification is not necessarily an accurate indicator of how well that student will perform with any specific addition method.

In summary, although the medium achieving group generally performed better than the low achieving group, the considerable fluctuations in performance (both within and between subjects) found in this study suggest that caution needs to be exercised when estimating an addition method's appropriateness (i.e., when generalizing to similar groups or individuals). Regardless of academic history or achievement classification, individual performance needs and abilities should be assessed before prescribing a change in a student's individual academic program.

The Written Algorithms

**Question 2.** If there are differences in performance within any one of these eight conditions, to what degree do the differences appear to be functionally consistent with the features of the methods (novelty, written record, basic math facts, consistency of calculating device)?

**Question 3.** What underlying causal (operational) mechanisms may we infer from these results?

A pattern in the performance differences between the two groups over the three experiments suggests that, for fourth- and fifth-graders, substantial performance gains can be realized by delimiting addition computations to the basic
math facts. Examining Figure 25 (page 127) it can be seen that, for both low and medium achieving groups in experiment 1, utilizing perspective 1 data, the Hutchings' method delivers performance that is superior to the other three methods. If either the novelty of the Hutchings' method or its incorporation of a complete written record were the key, functional components responsible for this superior performance, then the factor analysis method should have produced similar results in that it shares these features with the Hutchings' method. The fact that the traditional and factor analysis methods produced similarly inferior performance, compared to the Hutchings' algorithm, suggests that these methods share common functional elements. The traditional and factor analysis methods both require that the student perform complex addition operations. Therefore, the superiority of the Hutchings' method may be due to its major dissimilarity with the other methods, the fact that it delimits all computations to the basic math facts.

This deduction is not surprising given that basic math fact skills are learned earlier (usually first grade) and hence are older and more polished than complex addition skills that are learned later (usually second grade). That the medium achieving group appears to make even better use of the Hutchings' features only points out that they probably are more fluent in the basic math facts and that, in general, they are likely to learn and adopt different procedures more readily than the lower achieving group. However, this is a general, group-level explanation that may not apply equally to all individuals in the group. In this study substantial variance was observed in the performance of the subjects in both groups across all four
methods. The large variance in performance using the Hutchings' method may be due, in part, to the subjects' differential histories of success with the basic math facts.

Referring to Figures 27 (page 142) and 28 (page 144), it can be seen that, for both groups, in experiment 1 utilizing perspective 2 data, the Hutchings' method again produced the best overall results and the greatest improvement between early and late sessions' means. It appears that the low group only benefitted from practice using the Hutchings' method, whereas the medium group experienced similar rates of growth for the Hutchings' and factor analysis methods, and a lower rate of growth for the calculator and traditional method. As a result, the medium group's performance with the factor analysis method improved so as to be somewhat higher than their performance with the traditional method. This begins to suggest that, for the medium math achieving group, some element other than delimiting computations to the basic math facts may be positively affecting performance. Given the similarity between the growth rate of the two alternative algorithms, this element may be common to both of them. The two components that the factor analysis method shares with the Hutchings' are the novelty of the method and the use of a full written record. If novelty is a functional factor affecting performance, then two things might be expected: (1) performance with the factor analysis method should be more in line with the Hutchings' from the beginning, and (2) performance with both of these methods should drop off over time (as the novelty wears off). Neither of these hypothetical expectations was realized in this study, which suggests that the full
written record feature of both the factor analysis and Hutchings’ methods may be another functional device for enhancing addition computation performance in grade school children.

The fact that the experiment 1, perspective 1 results did not indicate that the written record feature positively affected addition performance suggests that, for many subjects, utilizing the full written record feature takes more time and practice. This seems especially reasonable given that, prior to this study, the children were conditioned to ciphering numbers with one primary method. These experiments required them to master four different addition methods (involving subtle and gross methodological differences). At the outset of the experiment this demand for new learning could have easily overwhelmed and confused many of the subjects.

Referring to Figure 26 (page 133), the experiment 1, perspective 2 data analysis placed the low group’s performance using the Hutchings’ algorithm on par with the medium group’s performance using the traditional method. For educators wishing to bridge the gap between low and medium functioning children, introducing the Hutchings’ method when a child has clear difficulty mastering the traditional method might be part of the answer.

This phenomenon also points out that, in order to be valid, the labels affixed to children (based on some aspect of their academic performance) need to acknowledge all variables that influence performance. As this study has demonstrated, in their effects on performance, the inherent abilities of the individual to do the job are sometimes secondary to the tools provided (the addition methods).
For a comprehensive understanding of the variables affecting human performance and strategies for evaluating their relative impact, consult Gilbert’s *Human Competence* (1978, chap. 3).

When the problems became more difficult (experiment 2, 5x7 arrays) the resulting performance patterns became more pronounced. Referring to Figure 37 (page 172) it can be seen that, again, the Hutchings’ algorithm results in the best performance for both groups. The overall performance for the low group shifted downwards from experiment 1. In this first perspective of experiment 2 the traditional and factor analysis methods produced nearly identical results. For the low achievers then, as was the case in experiment 1, the full written record feature alone adds little or nothing to their performance. Their overall superior performance using the Hutchings’ algorithm indicates that they gain some advantage by having only basic math facts to compute. As was the case in experiment 1, using the Hutchings’ method equates the low group’s performance with that of the medium group using the traditional method.

For the medium achievers in experiment 2, performance using three of the four methods moved downwards from experiment 1. The factor analysis method was the only method (for either group) on which improvement was evidenced from experiment 1 to experiment 2 (As explained in the "Purpose of This Study" section (p. 64) of this text, a method’s computational power can be affected dramatically by the level of problem difficulty). Whereas it may be surprising that either group, using any method, could improve when the problems became more difficult, it
should be noted that, functionally, based on the difficulty of adding one column of numbers taken from either size problem using any of the three written algorithms, there is no difference between computing the 2x7 and 5x7 array problems. Each of these problems has seven rows. Attaching three additional columns of the same length to each problem may make them look harder. It does not actually make the task of adding any one column, or carrying its results to the next, any more or less complex. To this extent it is somewhat puzzling why, for the medium achieving group, the Hutchings' and traditional algorithms did not produce results in experiment 2 similar to experiment 1 as did the factor analysis method. The traditional method produced substantial losses between experiments, while the Hutchings' produced minor decrements in performance. As a group the medium achievers did not regain the level of performance using the Hutchings' method that they had reached at the end of experiment 1 until the conclusion of experiment 3. This is an aspect of the groups' performance that may have been significantly affected by feedback and incentives.

I would like to offer two explanations for the reduction in overall performance when the problems became "harder" (i.e., larger) that are not associated with specific algorithm characteristics. The first is what might be termed "array shock" (Alessi, 1992c). Similar to the new car buyer's "sticker shock" phenomenon, a student sees the larger size of the addition problem and is immediately intimidated or overwhelmed. This type of reaction could be the result of direct experience or from observing others' reactions when those others were
unable to successfully compute large problems. Now, when exposed to a larger math problem they might experience inhibitory emotional states such as anxiety or fear, and, or tell themselves, "I can’t do this," and, or try to avoid or escape the computational task. As Skinner (1968) pointed out, "The glimpse of a column of figures, not to say an algebraic symbol or an integral sign, is likely to set off, not mathematical behavior, but a reaction of anxiety, guilt, or fear" (p. 18).

A second explanation examines relative rates of reinforcement and perceived levels of difficulty associated with problem size. This analysis first presumes that completing an addition problem is (automatically) conditionally reinforcing for a student (based on positive outcomes for having completed problems in the past). Consequently, even though completing one column in a 5x7 problem is no harder than for a 2x7 problem, the overall number of problems completed will be lower. If the current rate of computing behavior is to some degree the result of past rates of reinforcement for that behavior, then, when the frequency of (automatic) reinforcement diminishes (i.e., the subject completes fewer problems per five minute period), the rate of responding also diminishes. Since 5x7 array addition problems are over twice as large as 2x7 problems, a subject ciphering both size problems at the same rate of speed will complete less than half as many of the larger than the smaller. Based on this logic, to maintain the same rate of reinforcement ciphering 5x7 problems as 2x7 problems, a student would have to work over twice as fast. This suggests that, from the student’s perspective, the response effort required to attain the same relative reward (i.e., to finish the same number of problems)
ciphering 5x7 arrays is twice that required for 2x7 arrays. Such a perceived increase in effort could help explain why, after the problems increased in size (from 2x7 to 5x7 arrays), it took over ten sessions, even for the medium achieving subjects, to reattain experiment 1 performance levels.

It should be noted that a student's past history of relative success or failure may determine how much of an effect a decrease in the rate of reinforcement will have. The rate of computing behavior of those students who have met with consistent failure may not diminish when the problems become larger because completing addition problems (possibly of any size) hasn't become automatically reinforcing for them. As Skinner (1968) (somewhat pessimistically) noted, "Few pupils ever reach the stage at which automatic reinforcements follow as the natural consequence of mathematical behavior" (p. 18). The fact that this study did not provide qualitative feedback about performance may have exacerbated this phenomenon, as those students whose experience with addition is basically unsuccessful may have simply assumed that their answers were wrong. The failure to provide feedback about answers may have denied students the opportunity to experience more successful mathematical outcomes (especially using the Hutchings' method) and condition more positive feelings.

From Figure 33 (page 163) it can be seen that, for the medium group in experiment 2, performance using the three written algorithms was very evenly spaced. This corresponds nicely with the features of these methods and their corresponding performance advantages. The traditional method, which requires
simple and complex computations and requires that they all be done covertly, delivered the lowest performance. The factor analysis method, which utilizes a full written record and overtizes some of the work done covertly with the traditional method, delivered the next highest performance. The Hutchings' low-stress algorithm, which utilizes both the full written record feature and delimits all computations to basic math facts, delivered the highest performance.

The overall results from experiment 2, perspective 2 appear very similar to the perspective 1 results. Referring to Figures 43 (page 195) and 44 (page 197), it can be seen that, in experiment 2, utilizing perspective 2 data, the Hutchings' method again produced the best overall results with both groups as well as the greatest improvement between early and late sessions' means (for the written methods). It appears that the low group benefitted from practice using the Hutchings' method, and to a lesser extent the calculator, whereas the medium group's performance improved to a similar degree using these same methods.

The full written record feature (by itself) of the factor analysis method, did not appear to benefit either the low or medium group through the course of experiment 2. This could indicate that the subjects reached their maximum performance capabilities using this algorithm. It could also be indicative of a lack of feedback or effective incentives for improving performance.

In summary, systematic analysis of the data through all three experiments indicates that the enhanced performance using the alternative algorithms is functionally consistent with their unique designs (i.e., the tools they provide). The
Hutchings’ low-stress algorithm appears to produce the best addition performance for both groups, and most of the subjects, primarily because it delimits all addition operations to the basic math facts. However, it is not possible to state that this factor alone produced these results because no "basic math facts" without the "full written record" condition was included in this experiment (cf Fulkerson Tens method, Fulkerson, 1963). However, research has been done that suggests that the performance gains seen using the Hutchings’ algorithm are due to the combination of its features. Vaughn (1982) compared the Hutchings’ low-stress and Fulkerson Tens methods. The Fulkerson Tens algorithm is like the Hutchings’ in that it delimits all computations to the basic math facts, however, it does not incorporate a full written record. The Fulkerson Tens methods is the alternative algorithm closest to a just "basic math facts" design. Using the Fulkerson Tens method only the tens portion of an addend is written (a slash is made when an addend is 10 or larger), while the ones portion is held in memory to be added to the next digit in the problem (For additional information and demonstration of the Fulkerson Tens algorithm, consult pp. 48-50 of this text). Vaughn (1982) found that, ciphering 5x7 array problems, the Hutchings’ algorithm produced the highest rate and accuracy of computations, followed by the Fulkerson Tens method, with the traditional method producing the lowest rate and accuracy of computations. These results indicate a hierarchy of performance, consistent with that found in this study. The difference is, whereas this study examined the performance difference of an algorithm incorporating the basic math facts and full written record (the Hutchings’) with one
incorporating only the full written record (factor analysis), Vaughn (1982) compared an algorithm incorporating the basic math facts and full written record (the Hutchings') with an algorithm incorporating the basic math facts and a partial written record (the Fulkerson Tens). The fact that the Hutchings’ method produced superior performance compared to both the factor analysis and Fulkerson Tens methods suggests that using the basic math facts and full written record features together is essential for maximizing computational performance.

Utilizing the "basic math facts" and "full written record" features of the Hutchings' low-stress algorithm produced superior computational performance for both groups from the beginning of experiment 1. These combined features continued to enhance the performance of both groups throughout all three experiments. By the conclusion of experiment 2 the medium achieving group’s performance index mean using the Hutchings’ low-stress algorithm was over 2 1/2 times greater than their performance index mean using the traditional method.

Also important to enhanced performance, but seemingly less so for the low achieving group, is the full written record feature. Utilizing this feature of the factor analysis algorithm did not immediately produce results superior to the traditional method for either group. The low achieving group improved very slowly with this method. Over the first 20 sessions their mean performance index went from 7.53 to 8.66. Their mean performance index declined in the first phase of experiment 2 to 5.68. By the end of this experiment it reached 5.89. Conversely, the medium achieving group’s performance with this algorithm improved considerably during the
first 20 sessions. At the conclusion of experiment 2 the medium group's performance index mean using the factor analysis method was almost twice that using the traditional method.

These results indicate that, even under less than favorable conditions (i.e., no feedback or incentives), the factor analysis and Hutchings' algorithms offer medium achieving (average) students the tools with which to greatly enhance performance, especially given extended practice. For the large majority of both medium and low achieving students, the Hutchings' low-stress algorithm appears to be the most effective, i.e., less "defective" algorithm of the four methods tested. Most of the subjects in this study learned to use it effectively within two to six (45 minute) instructional sessions.

In that it emphasizes basic math and number skills, and eliminates the need for more complex math facts and abstract number abilities (e.g., regrouping), the Hutchings' low-stress algorithm demands a smaller incremental advancement in math skills for mastery than the traditional algorithm. The consistently superior performance of the Hutchings' low-stress algorithm suggests that it does demand less mental effort (i.e., "memory", "attention") than the traditional method. And, unlike the traditional method that requires all computations be performed covertly where they cannot be examined or reviewed, the Hutchings' and factor analysis methods incorporate a full written record of all computations. This feature provides instructionally valuable feedback for the identification and strategic remediation of computational error patterns.
The Calculator

The calculator, which showed promise as a viable method in experiment 1 (2x7 arrays), produced exceedingly poor results when the problems became larger (5x7 arrays), and for both medium and low groups (see Figure 37 on page 172). Given that correctly entering the numbers of an addition problem into a calculator (and properly manipulating the function buttons) will produce the correct answer every time, it seems reasonable to assume that the subjects encountered problems with doing this consistently. Based on direct observations, it seemed as if many of the subjects had difficulty physically discriminating the different buttons on the calculators (I frequently heard a subject repeat the right number to be entered, then press the wrong key, or inadvertently press two keys at once). This seemed to create frustration and confusion.

The experimenter also observed the subjects to make procedural errors. Often times a subject would fail to clear their previous answer before beginning a new problem. Others had difficulty remembering which numbers they had already entered, entering some twice while omitting others. Many subjects seemed to lack (or failed to employ) estimation or answer judgement skills. Often, subjects' answers were either too large or small by a full decimal place. It also seemed as if some subjects experienced difficulty remembering the proper order of the numbers to be entered (saying to themselves, "34597", then pressing 35497).
The calculators' solar-powered design also created problems. On cloudy days the majority of the available light came from the overhead florescent light fixtures. When a subject would lean over his or her desk and inadvertently block the light, the calculator would often cease functioning. Furthermore, even though the calculators used in this study were the very ones used by the school, the subjects had widely differing levels of experience with the units (due to individual teacher and grade differences). One low achieving fifth grade subject (17) was very proficient with the calculator beginning with session one (it was also her most effective method overall). Her mean performance index during the last portion of experiment 2 was almost three times larger than the highest performing medium group subject. This subject was one of the two subjects to voice positive feelings for the calculator, and to choose it during experiment 3. The majority of the low functioning group, however, frequently complained about having to use calculators (only one of the medium functioning group, subject 14, ever complained openly), and often asked for direct help from the experimenter when they would arrive at unexpected results. The experimenter encouraged these subjects to keep trying as they would often seem overwhelmed by frustration and give up. For many subjects, the session to session results using the calculator vacillated between extremes of mostly correct and mostly incorrect.

It seems as if some of the reason for the calculator's poor showing may be inherent with the device and the demands that it places on the user. However, it is far from clear that correlated, and to some degree controllable, variables were not
in some large part responsible for these poor results. The combination of slightly uncoordinated young hands operating calculators with small, hard to discriminate buttons, intermittent low light performance, and differing levels of experience with calculators in general may have resulted in the very poor showing for this method. Using larger, battery operated calculators with big buttons designed for less sophisticated users, providing instruction in the systematic application of estimation and judgement techniques, and controlling for experience may make a big difference in the results.

**Individual Preferences and Idiosyncrasies**

The strength of the Hutchings' and factor analysis methods was also reflected in individual preferences and performances. Each of the medium functioning subjects, especially subjects 3, 9, and 19, expressed strong preferences for and performance with the Hutchings' method, and generally seemed to enjoy the factor analysis method. In the last part of experiment 2, subject 9's mean performance index using the factor analysis algorithm was more than 15 times what it was using either the traditional method or calculator. Subject 9's mean performance index using the Hutchings' was more than 30 times what it was using either the traditional method or calculator. Subject 3 performed over twice as well using the Hutchings' compared with the traditional method, and over three times as well compared with the calculator. In the last part of experiment 2, subject 14 performed about 10 times as well using the Hutchings' compared with the traditional method, and about 15
times as well compared with the calculator. Each of the five medium functioning subjects delivered their best performances in experiments 1 and 2 using the Hutchings' algorithm. In experiment 3, all of the medium achieving subjects chose the Hutchings' (their most effective method) exclusively for use in experiment 3. Each of the medium group subjects improved upon their performance in experiment 3 over what it had been at the conclusion of experiment 2.

For the lower achieving subjects as a group the Hutchings' was the method of choice. At the individual level, however, there was less consistency with and preference for the Hutchings' method than demonstrated by the medium achieving group. In experiment 1, seven of the twelve low achieving subjects delivered their best performance using the Hutchings' method. Based on performance indices at the end of experiment 2, seven of the eleven low achieving subjects delivered their best performance using the Hutchings' method. In experiment 3, eight of the ten low group subjects chose the Hutchings' algorithm for at least one session, equivalent to 47.5% of the total daily-sessions. Of those eight, six improved their performance over what it had been at the end of experiment 2.

Comparisons of the low achieving subjects' performance using the Hutchings' with other methods yielded fewer, less dramatic contrasts as similar comparisons for individuals in the medium group. There were notable (low group) individual examples of increases in performance using the Hutchings'. At the end of experiment 2, subject 15 performed at a rate five times greater using the Hutchings' than with the traditional method. Subject 1's mean performance index using the
Hutchings’ was about five times greater than the traditional method. Subject 4’s performance was similar. Subject 5 performed about three times better using the Hutchings’ than the traditional method, and about nine times better than using the calculator. As impressive as these comparisons are, they represent only six of the ten low achieving group subjects. During this same period, subject 16 performed about 30% better using the traditional method than the Hutchings’. Subjects 7 and 12 performed about twice as well using the traditional algorithm as the Hutchings’. In contrast, all five of the medium group subjects performed impressively using the Hutchings’ method (compared to their performance using the other methods).

The relative differences between the two groups’ mean performance indices for the Hutchings’ and traditional methods were also striking. There was a statistically significant difference between the groups’ mean performance indices using the Hutchings’ method. During the last part of experiment 2, the Hutchings’ performance index for the low achieving group was about 10, and, for the medium group, about 28. This relative difference is much smaller (and not significant) using the traditional method. For the low achieving group the traditional method performance index was about 6, compared to about 10 for the medium achieving group.

These substantial, sometimes dramatic, differences between members of the low and medium groups, when contrasting performance using the Hutchings’ with performance using the traditional algorithm, raise a number of issues. If the low-stress algorithm is easier and more efficient because it only requires a knowledge of
simple math facts and utilizes a full written record, then why did only seven of the eleven low achieving subjects deliver their best performances (during the last portion of experiment 2) using the Hutchings’ method? Why did three subjects from this group cipher most effectively using the traditional method? Why wasn’t the low achieving subjects’ performance using the Hutchings’ algorithm greater than their performance using the traditional algorithm in proportions similar to the medium achieving group?

Based on observations during the work periods the experimenter noted that many low achieving subjects had consistent trouble discriminating between the Hutchings’ and factor analysis methods. Subjects 11 and 12 would routinely seem confused. Even after many weeks of sessions (and ostensible mastery of the basic procedures), they would routinely ask for direct assistance from the experimenter; often immediately following a review of the method that had been presented to the group. Typically, these subjects could not remember which method involved adding the entirety of the previous sum to the next number in the column (factor analysis), and which one involved adding just the unit’s portion of the previous sum to the next number (Hutchings’). On those days when they seemed to understand the format, each of these subjects did very well using the Hutchings’ method. Their weaker overall results using the Hutchings’ method may not reflect their inability to work more quickly using just the basic facts, rather it may be that, in the context of having to perform four addition methods, each method became more difficult.
In the past many of the low achieving students have had difficulty using one method well. They are not accustomed to success with math. They have routinely experienced failure, and demands to learn new material may evoke anxiety (that could easily increase confusion) and/or avoidance responses. During the actual work periods the experimenter observed many stereotypic avoidance behaviors, such as working slowly, staring out the window, drawing on the worksheets, interacting with and sometimes disrupting other subjects verbally or physically, becoming angry or obdurate when disciplined and redirected to do their work, and displaying frustration or displeasure at having to perform certain methods. Most of them voiced strong dislike for the calculator and the factor analysis method; e.g., in a whiny voice, "why do we have to do this one?" As a result, many of the low math achieving subjects (especially subjects 1, 4, 15, 16, & 18) frequently became "behavior problems." By contrast, the medium achieving group subjects displayed almost none of these types of behaviors.

Subjects 1 and 4 would often refuse to work unless they were seated next to one another. Subject 18 would often talk at inappropriate moments, and sometimes physically disrupt other students. Subject 16 initially appeared to be have moderate success with the traditional algorithm for the first 11 sessions (refer to Appendix H). Then, for the next 4 sessions they experienced great difficulty using the traditional method and complained loudly that they could not do the 2x7 problems. Subsequently, during the free time just prior to session 18, the experimenter observed this subject working at the blackboard ciphering a large (9x8 array)
addition problem. They were using the traditional method quickly and accurately. The experimenter gently confronted the subject saying, "I thought you told me that you couldn’t do this, it looks to me like you can do this pretty well." Beginning with that day this subject’s performance with the traditional method improved dramatically. In experiment 2 this subject’s mean performance index using the traditional method exceeded the medium group mean.

This raises a number of possibilities in regards to the low math achieving students. It may be that they exhibit poor in-class behavior because they are generally low achieving. It may also be that they are low achieving in part because of non-productive, in-class behaviors. It may be that because of early failures in math they developed ways to cope, such as avoiding difficult or confusing tasks as much as possible (e.g., learning two new, alternative addition algorithms). Unfortunately, such avoidance leads to inadequate learning and more failure. The poor, in-class behaviors become bad habits that are not necessarily indicative of inability, but patterns of behavior maintained by their immediate consequences. As such they may be both a cause and an effect of poor academic performance.

These considerations suggest that a between groups research strategy may be preferable (to a repeated measures design as used in this study) for determining performance differences between addition methods. A between groups design would assign one group of subjects for each method investigated, as compared with each subject using all methods. In a between groups study, each subject would learn and use only one method. This would require more subjects (than an equivalent repeated
measures study), however, it would reduce the learning demands placed on subjects, and eliminate any confounding effects of carry-over or confusion between the methods. It seems likely that a between groups study would better assess the abilities of a specific group of students using a single addition method.

Many of the low achieving students were observed to count out their answers to themselves, and/or tap out the answers using a finger or pencil employing a counting scheme taught to them by the school system. (Each single digit number has as many points on it as its value, e.g., an 8 as eight points; the student learns where the points are on the written number, so that when adding two numbers together they begin by saying the first number then tapping on the second number as many points as there are while counting upwards.) The whispering and tapping noises that this created sometimes disrupted some subjects. It also indicated that some of the subjects did not know their basic math facts by rote. This system of ciphering (which circumvents the need for rote math fact ability) could help explain why three of the low achieving subjects did better with the traditional method and failed to excel more consistently using the Hutchings' method. The tapping method appeared to be, at best, unreliable, and at worst, it seemed to give many subjects a false sense of competence. Furthermore, in as much as the school assured our research team that all of the children who participated knew the basic math facts, the tapping method (and methods like it, e.g., counting on fingers) may lead teachers and educators to inaccurately assess their children's computational abilities. To evaluate the true performance limits of the Hutchings' low-stress algorithm, future research
(as all previous Western Michigan University research) should thoroughly pretest subjects to assess that they know the basic math facts by rote memory.

The results of experiment 3 raise another concern: Why did the subjects in the low achieving group chose their most effective method for only 70% of the total daily-sessions? That is, why did some subjects choose lesser effective methods? Based on the frequent stereotypic avoidance behaviors that many of the low achievers exhibited, it is not unreasonable to suggest that many of them found computational arithmetic and its related processes to be mildly to moderately aversive. If this were the case, it follows that some of the low achievers were not sensitive to the positive differences that varying the methods had on their performance, because the consequences of any mathematical behavior are aversive, even those that lead to ostensibly positive outcomes. Unlike the medium achieving subjects, for whom the consequences of mathematical behavior appear to be more automatically reinforcing, many (perhaps four or five) of the low achievers did not seem to care about how they did. It was as if they had assumed that their answers would be wrong. It was as if they had given up before they had started.

Some of low achieving subjects may not have been as astute in selecting their most effective addition method in experiment 3 because they were not as proficient at self-checking and self-evaluation as the medium achieving subjects. Lower achieving subjects may not be as aware of the quality of their own performance as higher achievers, and consequently, may fail to choose a more effective method. Variation in self-checking abilities may be due to differences in specific learning
histories (some students were reinforced for or taught self-evaluation and self-
checking skills where others were not), and it may be that more efficient self-
checking is encouraged as the result of positive mathematical experiences (i.e.,
there's less aversiveness to avoid). Regardless of the conditioning history, failure
to accurately evaluate performance in general could explain why some subjects failed
to choose their most effective addition method in experiment 3.

Other variables potentially biasing both groups' method preferences involve
social pressure, influence, and modeling. The experimenter presented the
Hutchings' method as a powerful, accurate, scientific alternative to the traditional
method. The experimenter encouraged and reinforced the subjects for learning,
using, and succeeding with the Hutchings' algorithm. Consequently, some subjects
may have selected the Hutchings' for use in experiment 3 for reasons other than the
efficacy of the methods themselves. Some subjects may have selected the Hutchings'
because they wished to please or imitate the experimenter (similar to how they may
have pleased or imitated teachers in the past), or wished to gain the social approval
of peers who expressed a preference for the Hutchings' method. Conversely, some
subjects may have resisted using the Hutchings', not because of its ineffectiveness,
but because not selecting it was somehow reinforcing. Following routine classroom
disciplinary measures with subjects 1, 4, and 16 (e.g., not letting them sit together
or by the window), these subjects often would seem sullen or angry with the
experimenter. It is conceivable that these, or other subjects, may have chosen to use
the calculator or traditional method (instead of the Hutchings') out of what might be called belligerence or defiance.

It is also conceivable that some subjects did not use the Hutchings' algorithm, when it was, in fact, their most effective method, for reasons of potential stigmatization or other negative perceptions. They may have told themselves, "I don't want to be different", or, "I should be able to do the same method that my regular classmates use; if I can't I must be stupid; there's something wrong with me if I have to use this easier method."

These potentially confounding social variables suggest the need to reduce the influence that the experimenter has on subjects. To reduce these influences, future research on computational algorithms should present and teach the different algorithms as impartially and equally as possible. Perhaps someone (less invested in the results) other than the experimenter should conduct the instruction phase of the study. The primary experimenter is likely to be (at least minimally) biased. The subjects (children) may detect and react to these subtle prejudices, altering the nature of the results in ways not reflecting the attributes of the methods.

It is also necessary to recall that while this study emulated real classroom conditions, it was not a real class. The consequences for misbehavior or poor performance during the study were relatively slight. The very worst that could happen was that a subject would be dismissed and sent home (as was the case with subject 10). The subjects' performance and behavior in the study did not affect their standing in school in any way. As such, while the medium achieving group was
generally very well behaved, many of the low achievers needed frequent verbal redirection and discipline. It may be that many of the low achieving subjects' mathematical behavior was not as it might have been under more strict (i.e., realistic) classroom conditions.

In summary, for the medium achieving subjects, individually and as a group, the Hutchings' low-stress algorithm produced dramatic increases in performance over the traditional method and calculator, especially with harder problems. These dramatic increases appear to have been achieved by giving students an addition method that is designed to tap the operations they know best and eliminate those that are typically problematic. No artificial or elaborate motivative or feedback component was necessary to attain these results, merely a combination of simple math fact proficiency and an effective addition method that capitalizes on that proficiency. Such dramatic performance gains may also be testimony to the (automatic?) reinforcing nature of performing (well understood, basic) addition for some students. These results clearly illustrate the value of assigning the right tool for the workers' abilities.

For the low achieving subjects using the Hutchings' method the results were not nearly as dramatic. The Hutchings' method produced the best overall group performances, and there was a much larger amount of within group variation. In regards to the low achieving subjects' potential using the Hutchings', it may be that, given basic math facts proficiency at the rote memory level, and when used exclusively for a longer period of time in a classroom setting designed to manage
more difficult students, the Hutchings' method may produce significantly better results for the lower math achieving student.

Comparisons With Previous Research

Because this is the first study of its kind to employ the "performance index" dependent variable, no exact comparisons may be made to previous research outcomes. However, the three dependent variables typically used in previous studies may be interpreted together to produce a rough equivalent to the performance index.

There are two (unpublished) single subject studies (Zoref, 1976; Drew, 1980) that investigated the rate and accuracy of computational performance using the calculator, the traditional and Hutchings' methods, used identical problem sizes (2x7 and 5x7), and employed similar groups of children (Zoref worked with three low and three medium achieving fourth graders; Drew worked with four low and four medium achieving third graders). Zoref (1976) found that, using the Hutchings' low-stress algorithm, all six subjects performed with the lowest error rate (columns incorrect) of any of the methods over both levels of difficulty. For the low achieving group ciphering 2x7 array problems the calculator produced the highest columns correct rate (and therefore the highest accuracy). With this exception, for both levels of problem difficulty, all six subjects produced the highest columns correct and accuracy rates using the Hutchings' low-stress algorithm. These results are consistent with the current study. For the low achieving group in experiment 1 (2x7 problems) of the current study, the calculator was a close second to the
Hutchings' algorithm. Furthermore, it could be that, given the Hutchings' lower error rate, if the Zoref (1976) study's dependent variables were converted into performance index values, the Hutchings' method would show superior results in all conditions.

Consistent with the current study, Zoref (1976) also found that, for both medium and low achieving subjects, performance using the calculator diminished drastically when the problem size increased from 2x7 to 5x7. Furthermore, in both studies, computing the larger 5x7 problems using the calculator, both medium and low achieving subjects performed equally poorly.

Drew (1980) replicated the Zoref (1976) research and found similar results, i.e., that the Hutchings' low-stress method was superior to the traditional methods regardless of problem difficulty or achievement level of the subjects. Drew did find that, overall, performance using the calculator was somewhat greater than in the study he replicated. Unlike the current study, with 2x7 problems, the calculator produced results similar to the Hutchings' for both groups. Drew (1980) also found that, with the harder 5x7 problems, performance with the calculator diminished, however not as dramatically as in the current study. Drew found that, for both groups calculating the harder problems, performance with the calculator fell to a position in between that of the Hutchings' and traditional methods.

McCallum (1981) conducted a study that analyzed the components of the Hutchings' low-stress addition algorithm in the same way that the current study did. McCallum compared performance using the Hutchings' method with the conventional
McCallum compared performance using the Hutchings' method with the conventional algorithm utilizing a written record and the conventional algorithm without a written record (his conventional algorithm with a written record is identical to the factor analysis method). The rank ordering of performance from that study is very similar to the medium achievement group results from the current study (refer to Figure 33 on page 163). In the McCallum study (1981), the Hutchings' low-stress algorithm produced the best results, the conventional algorithm with a full written record was next, and the conventional algorithm without the written record produced the lowest results. These results are consistent with the analysis of this study and the algorithms' features. They add further weight to the supposition that the Hutchings' algorithm produces superior results to the traditional and factor analysis algorithms because it delimits all computations to the basic math facts.

The results of the current study are also consistent with all previous research (known to the experimenter) that have examined performance using the Hutchings' algorithm under various conditions (Alessi, 1974; Boyle, 1975; Dashiell, 1974; Hutchings, 1972; Rudolph, 1976; Gillespie, 1976; Buitendorp-Drew, 1980; Todd, 1980; Hadden, 1981; McGlinchey, 1981; VanHevel, 1981; and Vaughan, 1982). In general, it has been found that for a wide variety of subjects, conditions, levels of problem difficulty, and motivative states, the Hutchings' low-stress algorithm produces faster, more accurate addition computation than the traditional algorithm, modified traditional algorithms that use a written record (e.g., the factor analysis method), and the calculator. The Hutchings' method has been shown to be as
Fulkerson's "full record" which is functionally identical to the Hutchings'). In every study examining subject preferences the Hutchings' method has been chosen over all other methods, and often by substantial margins.

Conclusions

Given the apparent inefficiency of the algorithms currently in use in this country, the accelerating pace of new conceptual mathematical skills that students are required to know (and competent workers must utilize), increasing competition in the world marketplace, and the limited financial resources of our educational system, our society should no longer accept the status quo timeline for training computational skills. As educators we do a grave disservice when we graduate a math-incompetent and/or math-anxious individual. The system expends valuable, limited resources. The individual devotes essential developmental years not learning or advancing. The culture does not receive the benefits of a math-proficient worker, and so may become less competitive compared with other cultures. The individual may be emotionally tainted against mathematics and education in general and, as a result, fail to reach their potential. The individual may be bitter and non-supportive toward education (consider how often millage elections fail), and the individual will probably have a very difficult time remediating either their negative feelings toward math or their inadequate math skills. Toward finding viable solutions to the problems of teaching computational mathematics, this study has demonstrated that there are superior alternative addition methods currently available that embody fewer
defects, and hence produce consistently better results.
Appendix A

Hutchings’ Addition Algorithm, Lesson
(Adapted from Hutchings, 1972)
I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

\[
\begin{array}{c}
7 \\
+8 \\
15
\end{array}
\]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

\[
\begin{array}{c}
7 \\
\_8,5
\end{array}
\]

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? \(7 7\) (Point to the little 15.) \(\_8,5\)

Let's look at another one. I can write "9 plus 5 is 14" like this: Or, like this: \(9 9\) \(\_5,4\)

Both of these say "9 plus 5 is 14."

Tell me what these say: (Call on students.)

\[
\begin{array}{cccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 6 & 6 \\
\_8,7 & \_7,3 & \_5,9 & \_6,1 & \_6,2 & \_2,7
\end{array}
\]

The little number on the right is understood to be in the one's place, as are the big 7, the big 3, the big 9, the big 2 and the big 7. (Point to all the big numbers in the one's place.)

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The little number on the left is understood to be in the ten's place. In other words, this is the same as this (point from "big 7" to "little 7").

\[
\begin{array}{c}
9 \\
+8 \\
\hline \\
17
\end{array}
\]

And this is the same as this. (Point from "big 1" to "little 1" in the example above.)

Now watch me write the following facts both ways.

\[
\begin{array}{cccccc}
6 & 6 & 8 & 8 & 4 & 4 \\
+5 & 5 & +5 & 5 & +3 & /3 \\
\hline \\
11 & 13 & 7
\end{array}
\]

Look at the last pair. Are they different from the others? Note that there is no ten's place number and there is no "little one" on the left. (Do not draw the box in the empty area until saying this)

Let's look at some other examples. (In all cases do not draw the box (/\) until asking the question.

a) Is there any ten's number here?

\[
\begin{array}{c}
7 \\
+2 \\
\hline \\
/9 \\
\end{array}
\]

NO!

So will there be a little number here?

\[
\begin{array}{c}
7 \\
2 \\
\hline \\
/9 \\
\end{array}
\]

NO!
b) Is there any ten's number here?

\[
\begin{array}{r}
6 \\
+ 2 \\
\hline
\_ / 8
\end{array}
\]

NO!

So will there be a little number here?

\[
\begin{array}{r}
6 \\
\_ 2 \\
\hline
\_ / 8
\end{array}
\]

NO!

c) Is there any ten’s number here?

\[
\begin{array}{r}
1 \\
+ 8 \\
\hline
\_ / 9
\end{array}
\]

NO!

So will there be a little number here?

\[
\begin{array}{r}
1 \\
\_ 8 \\
\hline
\_ / 9
\end{array}
\]

NO!

d) Is there any ten’s number here?

\[
\begin{array}{r}
3 \\
+ 2 \\
\hline
\_ / 5
\end{array}
\]

NO!

So will there be a little number here?

\[
\begin{array}{r}
3 \\
\_ 2 \\
\hline
\_ / 5
\end{array}
\]

NO!

Again, if there is no ten’s place number there is no "little number" on the left.

Now watch me write the rest of these.
Notice:

\[
\begin{array}{c}
3 \\
\hline
14 \\
\end{array}
\]

no ten's number here, so no "little number" here

but

\[
\begin{array}{c}
9 \\
\hline
176 \\
\end{array}
\]

there is a ten's number here, so there is "little number" here

Again, notice:

\[
\begin{array}{c}
2 \\
\hline
57 \\
\end{array}
\]

no ten's number here, so no "little number" here

but

\[
\begin{array}{c}
8 \\
\hline
164 \\
\end{array}
\]

there is a ten's number here, so there is "little number" here

(Repeat this point using the following examples.)

\[
\begin{array}{cccccccc}
5 & 5 & 1 & 1 & 4 & 4 & 6 & 6 \\
+5 & 5_0 & +7 & 7_8 & +9 & 9_3 & +9 & 9_5 \\
\hline
10 & 8 & 13 & 15 \\
\end{array}
\]

Now I am going to show you a special way of adding that uses only those "little numbers" on the right.
I’ll say that again. (Repeat previous statement.)

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

8 First, do you see that this example is just number facts
5 piled one atop the other? (Point to various facts in the
7 column.)
9
8 OK! Here we go, starting at the top, writing facts as you
6 learned and using only numbers on the right for addition.
8
+ 7

8 a) The first fact we do may look a bit dif-
15,3 ferent because we do not have any little numbers
7 yet. (Point.)
9
8 b) This is the only time we will use two big
6 numbers. In the rest of the example we use one
8 little number and one big one.
+ 7

c) Now, eight plus five is thirteen.

d) (Write the thirteen: i.e. ,53 )

8 a) We’ve written the thirteen but we’ll use
15,3 only the three.
9
b) (Draw arrow between these numbers: 73.)
8
8 c) Three plus seven is ten.
+ 7
d) (Write the 10; i.e. 70 in the example.)
a) We’ve written the ten but we’ll use only the 0.

b) (Draw arrow between these numbers: 90.)

c) Zero plus nine is nine.

d) (Write the 9; i.e., 9, in the example.)

---

a) We’ve written the nine and look that’s all we have this time because zero and nine is just nine. But that’s okay because we only use the right-hand number anyway.

b) (Draw arrow between these numbers: 89.)

c) Nine plus eight is seventeen.

d) (Write the seventeen; i.e., 18, in the example.)

---

a) Say, We’ve written the seventeen but but we’ll use only the seven.

b) (Draw arrow between these numbers: 67.)

c) Seven plus six is thirteen.

d) (Write the thirteen; i.e., 63, in the example.)
255

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer. Watch!

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of them, so the first half of the answer is five.
Now, watch me do another. Remember, we use only the right side "little numbers." We will not bother to write the arrows anymore, just say with me,

\[
\begin{align*}
6 + 8 & = 14 \\
7 + 1 & = 11 \\
6 & = 6 \\
9 + 6 & = 16 \\
5 & = 5 \\
8 & = 8 \\
3 & = 3 \\
\hline
+ \, 2 & = 2 \\
\end{align*}
\]

Now the last number on the right is a 2, so the right half of the answer is a 2. We get the left half of the answer by counting the little numbers on the left that we didn’t use. One, two, three, four, five. There are five of them so the left hand of the answer is 5. The answer is 52.

Now say the work for these with me as I do them at the board. (Children do not copy this.)

\[
\begin{align*}
8 + 9 & = 17 \\
5 & = 5 \\
6 & = 6 \\
9 & = 9 \\
\hline
+ \, 2 & = 2 \\
\end{align*}
\]

Now copy these examples and do them by yourself. If you have any questions, ask me.
(After most have finished.) Check your work with mine as I do them at the board.

(After doing the examples.) Now let’s review.

I’ll write the work for another one on the board. I want someone to raise their hand and tell me what the answer is.

\[
\begin{array}{cccc}
6 & 8 & 5 & 9 \\
5 & 2 & 4 & 8 \\
9 & 7 & 9 & 3 \\
8 & 6 & 8 & 2 \\
5 & 9 & 7 & 7 \\
6 & 8 & 9 & 6 \\
\end{array}
\]

\[\pm 9\] \[\pm 5\] \[\pm 8\] \[\pm 9\]

(After most have finished.) Check your work with mine as I do them at the board.

(After doing the examples.) Now let’s review.

I’ll write the work for another one on the board. I want someone to raise their hand and tell me what the answer is.

\[
\begin{array}{c}
6 \\
8_4 \\
9_3 \\
5_8 \\
7_5 \\
5_0 \\
9_9 \\
3_2 \\
\end{array}
\]

6 plus 8 is 14
4 plus 9 is 13
3 plus 5 is 8
8 plus 7 is 15
5 plus 5 is 10
0 plus 9 is 9
9 plus 3 is 12

I’ll point to the box. Who will tell me what the right side of the answer is and how they got it?

(Locate correct response.) Good! That’s correct. The last little number on the right becomes the right side of the answer.

Now, who will tell me what the left side of the answer is and how they got it.

(Locate correct response.) Good! That’s correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you’re adding. Like this:
Can we write our left-hand answer number at the bottom if there is more than one column? (Point to the 1 in the tens place of the answer) No, we can't.

When there's more than one column, each column can have only one number at the bottom, except for the very last column which does have the usual two.

So the single number that we put at the bottom is always the right-hand number.

What can we do with the left-hand number? Would it make sense to throw it way? No, because it's part of the problem. Can we leave it down here (point to the "1" in the tens place of the answer) as part of the answer? No, because it's part of what we need to carry to the next column. (Draw the box above the second column.)

(Erase the 1 in the tens place of the answer.) * Instead of putting this little number on the left into the answer, what we need to do is add it to the other little numbers on the left. Add these with me. (Point consecutively at each little "1" and count.) One, two, three. There are three little numbers on the left of this first column, so we write a 3 at the top of the next column. (Write a 3 in the box above the left column.)
Now, when I start adding the second column I will start with the 3 first.

This is called carrying, some of you already understand it. Good. Carrying is very easy. And carrying is very important. You must never forget to carry.

Let's be sure you understand. (Repeat from the *)

Now let's review from the beginning. Look at these examples and tell me what to write. Okay, what do we do first? (Proceed with all the "basic" computations in the first column, locating and recording the answers in the appropriate half-space notation.) Good! Now that we have our last two little numbers, what do we do with the one on the right? (Locate the correct answer and write it in the digit's position.) Good! Now tell me what to write at the top of the left-hand column. (Point to the box above that column; locate the correct answer and write it in the box; proceed with the computations for the second column.) Good!

Now what is the next number in our answer? (Point to the tens place of the answer; locate the correct answer and write it in.) Now, what do we do with the little numbers on the left of the second column? (Point to the little numbers on the left of the tens column.) For the last column only, the left-hand total of the little numbers becomes the left-hand number in the answer. You treat the last column as though it were a single column, if there were just one column in the problem. Count with me. (Count the little numbers on the left and write down the answer in the hundred's place of the answer.) Good job, who would like to come up to the board and try one?

(Have volunteers from class work similar problems at the board.)
Now, copy these examples on the paper I gave you and do them with me.

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<tr>
<td>+5</td>
<td>+9</td>
<td>+7</td>
<td>+8</td>
<td></td>
</tr>
</tbody>
</table>

Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help.

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) I do this except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good. Are there any questions?
Be sure to make and place your numbers neatly! (Allow time needed for most to finish.)

Now, I will do them. Check your work against mine. (Do examples on board. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly; demonstrate the latter while doing the work. State that the carry number is always written at the top of the column to which it is carried.)
Appendix B

Factor Analysis Algorithm, Lesson
(Adapted from Hutchings, 1972)
I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

\[
\begin{align*}
7 \\
\pm 8 \\
15
\end{align*}
\]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

\[
\begin{align*}
7 \\
\pm 8_s \\
15
\end{align*}
\]

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? 7 7
(Point to the little 15.) \(\pm 8_s \)
15

Let's look at another one. I can write "9 plus 5 is 14" like this: Or, like this:

\[
\begin{align*}
9 \\
\pm 5 \\
14
\end{align*}
\]

Both of these say "9 plus 5 is 14."

Tell me what these say: (Call on students.)

\[
\begin{array}{ccccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 6 & 6 & 5 & 5 \\
\pm 8 & 18_s & \pm 7 & 17_s & \pm 5 & 5_s & \pm 6 & 16_s & \pm 2 & 2_s \\
17 & 13 & 9 & 12 & 9 & 7
\end{array}
\]

The little number on the right is understood to be in the one's place, as are the big 7, the big 3, the big 9, the big 2 and the big 7. (Point to all the big numbers in the one's place.)
The little number on the left is understood to be in the ten's place.
In other words, this is the same as this (point from "big 7" to "little 7").

\[
\begin{array}{cc}
9 & 9 \\
+8 & 8 \\
\hline
1 & 7
\end{array}
\]

And this is the same as this. (Point from "big 1" to "little 1" in the example above.)

Now watch me write the following facts both ways.

\[
\begin{array}{cccccc}
6 & 6 & 8 & 8 & 4 & 4 \\
+5 & 5 & +5 & 5 & +3 & 3 \\
\hline
1 & 1 & 1 & 3 & 7
\end{array}
\]

Look at the last pair. Are they different from the others?
Note that there is no ten's place number and there is no "little one" on the left.
(Do not draw the box in the empty area until saying this)

Let's look at some other examples. (In all cases do not draw the box (/\) until asking the question.

a) Is there any ten's number here?

\[
\begin{array}{c}
7 \\
+2 \\
\hline
9
\end{array}
\]

/\ NO!

So will there be a little number here?

\[
\begin{array}{c}
7 \\
2 \\
\hline
9
\end{array}
\]

/\ NO!
b) Is there any ten's number here?

\[
\begin{array}{c}
6 \\
+2 \\
\hline
\end{array}
\]

\[\underline{\_ / 8}\]

NO!

So will there be a little number here?

\[
\begin{array}{c}
6 \\
+2 \\
\hline
\end{array}
\]

\[\underline{\_ / 8}\]

NO!

c) Is there any ten's number here?

\[
\begin{array}{c}
1 \\
+8 \\
\hline
\end{array}
\]

\[\underline{\_ / 9}\]

NO!

So will there be a little number here?

\[
\begin{array}{c}
1 \\
+8 \\
\hline
\end{array}
\]

\[\underline{\_ / 9}\]

NO!

d) Is there any ten's number here?

\[
\begin{array}{c}
3 \\
+2 \\
\hline
\end{array}
\]

\[\underline{\_ / 5}\]

NO!

So will there be a little number here?

\[
\begin{array}{c}
3 \\
+2 \\
\hline
\end{array}
\]

\[\underline{\_ / 5}\]

NO!

Again, if there is no ten's place number there is no "little number" on the left.

Now watch me write the rest of these.
Notice:

\[
\begin{array}{c}
\begin{array}{c}
3 \\
+1 \\
\hline
4 \\
\hline
\end{array} \\
& 3 \\
& 1_4
\end{array}
\]

no ten's number here, so no "little number" here

but

\[
\begin{array}{c}
\begin{array}{c}
9 \\
+7 \\
\hline
16 \\
\hline
\end{array} \\
& 9 \\
& 17_6
\end{array}
\]

there is a ten's number here, so there is "little number" here

Again, notice:

\[
\begin{array}{c}
\begin{array}{c}
2 \\
+5 \\
\hline
7 \\
\hline
\end{array} \\
& 2 \\
& 5_7
\end{array}
\]

no ten's number here, so no "little number" here

but

\[
\begin{array}{c}
\begin{array}{c}
8 \\
+6 \\
\hline
14 \\
\hline
\end{array} \\
& 8 \\
& 16_4
\end{array}
\]

there is a ten's number here, so there is "little number" here

(Repeat this point using the following examples.)

\[
\begin{array}{cccccccc}
5 & 5 & 1 & 1 & 4 & 4 & 6 & 6 \\
+5 & 15_0 & +7 & 7_8 & +9 & 19_3 & +9 & 19_3 \\
\hline
10 & 8 & 13 & 15
\end{array}
\]

Now I am going to show you another special way of adding a column of numbers that uses both "little numbers" not just the little number on the right, like I showed you before. In other words, we will add both little numbers to the next big number. Actually, this method is just like the traditional method that you've
always used except that you write down all of your computations.

I’ll say that again. (Repeat previous paragraph.)

This method should make your addition much easier and more accurate. Watch.

8
5
7
9
8
6
8
+ 7

OK! Here we go, starting at the top, writing down the result after we perform each addition operation.

8
5
7
9
8
6
8
+ 7

a) Like the Hutchings’ method the first fact we do may look a bit different because we do not have any little numbers yet. (Point.)

b) This is the only time we will use two big numbers. In the rest of the example we use two little numbers (if there are two) and one big one.

c) Now, eight plus five is thirteen.

d) (Write the thirteen: i.e. 17.)

8
5
7
9
8
6
8
+ 7

a) We’ve written the thirteen and we use the entire thirteen.

b) (Draw a circle around these numbers: 17.)

c) Thirteen plus seven is twenty.

d) (Write the 20; i.e. 27 in the example.)
8
15
27
29
8
6
+ 7

a) We've written the twenty and we'll use the entire 20 for our next addition.

b) (Draw a circle around these numbers: 29.)

c) Twenty plus nine is twenty-nine.

d) (Write the 29; i.e., 29 in the example.)

8
15
27
29
38
6
8
+ 7

a) We've written the twenty-nine and we'll use the entire twenty-nine for our next addition. Notice how this method is just like the traditional method except that, instead of keeping these numbers in your head, you write them down.

b) (Draw a circle around these numbers: 289.

c) Twenty-nine plus eight is thirty-seven.

d) (Write the thirty-seven; i.e., 37 in the example.)

8
15
27
29
38
46
8
+ 7

a) We've written the thirty-seven, and we'll use both little numbers, we'll use the thirty-seven.

b) (Draw a circle around these numbers: 367.)

c) Thirty-seven plus six is forty-three.

d) (Write the forty-three; i.e., 43 in the example.)
8  
153  
270  
299  
387  
465  
581  
+7  
---

a) We've written the forty-three and we'll use the entire forty-three. Notice how this method is different than the Hutchings' method. With the Hutchings' method we only use the little numbers on the right. With this method we use both little numbers.

b) (Draw a circle around these numbers: 483.)

c) Forty-three plus eight is fifty-one.

d) (Write the fifty-one; i.e., 581 in the example.)

---

8  
153  
270  
299  
387  
465  
581  
+7  
---

a) We've written the fifty-one and we'll use both little numbers, we use the entire fifty-one.

b) (Draw a circle around these numbers: 571.)

c) Fifty-one plus seven is fifty-eight.

d) (Write the eight; i.e., 578 in the example.)

---

Now our work adding this column of numbers is basically finished. Each time we added another number we wrote down the entire result, so now that we've reached the end, we have our entire answer. All we need to do is write down the result from our last addition in big numbers below the bar, and that's our answer.

8  
153  
270  
299  
387  
465  
581  
+7  
---

The last pair of little numbers is the entire answer.

---

58
Let's work through another example. Watch and listen as we go.

\[
\begin{align*}
6 & \quad 6 \text{ plus } 8 \text{ is } 14 \\
4 & \quad 14 \text{ plus } 7 \text{ is } 21 \\
2 & \quad 21 \text{ plus } 6 \text{ is } 27 \\
9 & \quad 27 \text{ plus } 9 \text{ is } 36 \\
5 & \quad 36 \text{ plus } 5 \text{ is } 41 \\
8 & \quad 41 \text{ plus } 8 \text{ is } 49 \\
+3 & \quad 49 \text{ plus } 3 \text{ is } 52 \\
\hline
5 & \quad 2
\end{align*}
\]

Now the last number pair of little numbers is our answer, so our answer is 52.

Now say the work for these with me as I do them at the board. (Children do not copy this.)

\[
\begin{align*}
8 & \quad 9 & \quad 4 \\
5 & \quad 5 & \quad 8 \\
6 & \quad 3 & \quad 3 \\
7 & \quad 2 & \quad 6 \\
9 & \quad 6 & \quad 3 \\
8 & \quad 8 & \quad 8 \\
3 & \quad 7 & \quad 7 \\
+1 & \quad 9 & \quad 6 \\
\hline
5 & \quad 2 & \quad 4 & \quad 4 & \quad 3
\end{align*}
\]

Now copy these examples and do them by yourself. If you have any questions, ask me.

\[
\begin{align*}
6 & \quad 8 & \quad 5 & \quad 9 \\
5 & \quad 2 & \quad 4 & \quad 8 \\
9 & \quad 7 & \quad 9 & \quad 3 \\
8 & \quad 6 & \quad 8 & \quad 2 \\
5 & \quad 9 & \quad 7 & \quad 7 \\
6 & \quad 8 & \quad 9 & \quad 6 \\
+9 & \quad +5 & \quad +8 & \quad +9
\end{align*}
\]
(After most of the students have finished.) Check your work with mine as I do them at the board.

(After doing the examples.) Now let's review.

I'll write the work for another one on the board. I want someone to raise their hand and tell me what the answer is.

\[
\begin{array}{c}
6 \\
8_4 \\
9_8 \\
5_8 \\
7_5 \\
5_0 \\
9_9 \\
3_2 \\
\hline
\end{array}
\]

6 plus 8 is 14
4 plus 9 is 13
3 plus 5 is 8
8 plus 7 is 15
5 plus 5 is 10
0 plus 9 is 9
9 plus 3 is 12

(Point to box.) Who will tell me what the right side of the answer is and how they got it?

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Now, who will tell me what the left side of the answer is and how they got it.

(Locate correct response.) Good! That's correct. The last little number on the left becomes the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this:

\[
\begin{array}{c}
4 \ 6 \\
7 \ 8_4 \\
6 \ 7_1 \\
8 \ 6_7 \\
7 \ 8_3 \\
\hline
3 \ 5 \\
\end{array}
\]
Can we write our left-hand little number at the bottom if there is more than one column? (Point to the 3 in the tens place of the answer) No, we can’t.

When there’s more than one column, each column can have only one number at the bottom, except for the very last column which does have the usual two.

So the single number that we put at the bottom is always the last right-hand little number.

* What can we do with the left-hand little number? Would it make sense to throw it way? No, because it’s part of the problem. Can we leave it down here (point to the "3" in the tens place of the answer) as part of the answer? No, because it’s what we need to carry to the next column. (Draw the box above the second column and erase the 3 in the tens place of the answer.)

\[
\[
\begin{array}{c}
1 \\
4 \\
6 \\
7
\end{array}
\]
\[
\begin{array}{c}
\downarrow \\
8_4 \\
6_7_1 \\
8_6_7 \\
7_8_5
\end{array}
\]
\[
\hline
\end{array}
\]
\[
5
\]

The ten’s place value from this first column is 3, so we write a 3 at the top of the next column. (Write a 3 in the box above the left column.)

\[
\[
\begin{array}{c}
\bar{3} \\
4 \\
6 \\
7
\end{array}
\]
\[
\begin{array}{c}
\downarrow \\
8_4 \\
6_7_1 \\
8_6_7 \\
7_8_5
\end{array}
\]
\[
\hline
\end{array}
\]
\[
5
\]

Now, when I start adding the second column I will start with the 3 first.
This is called carrying, some of you already understand it. Good. Carrying is very easy. And carrying is very important. You must never forget to carry. Let's be sure you understand. (Repeat from the *)

Now let's review from the beginning. Look at this example and tell me what to write. (Use a problem similar to the one above.) Okay, what do we do first? (Proceed with all the computations in the first column, locating and recording the answers in the appropriate half-space notation.) Good! Now that we have our last two little numbers, what do we do with the one on the right? (Locate the correct answer and write it in the digit's position.) Good! Now tell me what to write at the top of the left-hand column. In other words, what number do we carry to the next column? (Point to the box above that column; locate the correct answer and write it in the box; proceed with the computations for the second column.) Good! Now what is the next number in our answer? (Point to the tens place of the answer; locate the correct answer and write it in.) Now, what do we do with the last little number on the left of the second column? (Point to the last little number on the left of the tens column.) For the last column only, the last left-hand little number becomes the left-hand number in the answer. You treat the last column as though it were a single column, if there were just one column in the problem. Good job, who would like to come up to the board and try one?

(Have volunteers from class work similar problems at the board.)

\[
\begin{array}{cccc}
\bar{1}/ & \bar{1}/ & \bar{1}/ & \bar{1}/ \\
6 & 8 & 9 & 4 \\
2 & 9 & 8 & 5 \\
7 & 9 & 7 & 8 \\
5 & 3 & 7 & 2 \\
8 & 6 & 9 & 7 \\
7 & 7 & 8 & 5 \\
+5 & 9 & +9 & 6 \\
\end{array}
\]

\[
\begin{array}{cccc}
\bar{1}/ & \bar{1}/ & \bar{1}/ & \bar{1}/ \\
3 & 8 & 4 & 2 \\
9 & 6 & 5 & 7 \\
6 & 3 & 5 & 9 \\
+7 & 9 & +8 & 5 \\
\end{array}
\]

(When the students at the board have finished, work through the problems with the class and make any corrections necessary.)

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Now, copy these examples on the paper I gave you and do them with me.

\[
\begin{array}{cccc}
7 & 6 & 7 & 9 \\
5 & 9 & 8 & 7 \\
8 & 7 & 6 & 8 \\
6 & 9 & 4 & 7 \\
8 & 3 & 6 & 9 \\
+ & 9 & 8 & 3 \\
\hline
+ & 4 & 2 & 2
\end{array}
\]

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) I do this except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good. Are there any questions?

Now take these xeroxed examples and do them by yourselves. If you have trouble, ask me for help.

\[
\begin{array}{cccc}
7 & 6 & 7 & 9 \\
6 & 8 & 4 & 7 \\
8 & 6 & 9 & 5 \\
9 & 0 & 6 & 3 \\
4 & 3 & 9 \\
6 & 9 & 1 \\
+ & 8 & 5 & 8 \\
\hline
+ & 3 & 4 & 2
\end{array}
\]
Be sure to make and place your numbers neatly! (Allow time needed for most to finish.)

Now, I will do them. Check your work against mine. (Do examples on board. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly; demonstrate the latter while doing the work. State that the carry number is always written at the top of the column to which it is carried.)
Appendix C

Traditional Algorithm, Lesson
(Taken from VanHevel, 1981)
Since you all know how to write the number facts, I thought I would spend a few minutes reviewing how to do column addition problems.

When you write number facts you add the two numbers and write down your answer. For instance, when you write six plus seven, all you do is write thirteen at the bottom, like this:

\[
\begin{align*}
6 \\
+7 \\
\hline
13
\end{align*}
\]

The three is said to be in the ones place and the one in the tens place.

When you add more than two numbers, like in the problem nine plus seven plus eight, you add nine plus seven which equals sixteen. Then you add sixteen to eight to reach your answer of twenty-four.

\[
\begin{align*}
9 \\
7 \\
+8 \\
\hline
24
\end{align*}
\]

Does everybody know how to do this problem?

Good.

Okay, not I want you to write this problem down on the paper I gave you: Six plus seven, plus eight, plus five. Let's see you do this problem.

Okay everybody is finished. the answer is twenty-six.

Let's see how you reached your answer. You added six plus seven and it equaled thirteen. Next you added thirteen plus eight. It equaled twenty-one. Then you added twenty-one plus five and it equaled twenty-six. This is the answer. You wrote twenty-six at the bottom. Very good.
Now let me review how to add when there is more than one column.

Look at this problem:

\[
\begin{array}{c}
6 & 8 \\
8 & 7 \\
4 & 5 \\
\hline
+5 & 9 \\
\hline
\end{array}
\]

The sum of the first column is twenty-nine. Can we write the twenty-nine under that column? No, we can’t.

When there is more than one column, each column can have only one number at the bottom, except for the last column.

So the number we put at the bottom is always the right hand number of the units.

What do we do with the left hand or ten’s number? We carry it to the top of the next column and circle it. Watch me:

\[
\begin{array}{c}
6 & 8 \\
8 & 7 \\
4 & 5 \\
\hline
\hspace{1cm}+5 & 9 \\
\hline
\hspace{1cm}9 \\
\end{array}
\]

Now when we begin to add the second column, we always begin with the number we carried. In this example we carried the two.

To finish this problem we add:

\[
\begin{array}{c}
2 \text{ plus } 6 = 8 \hspace{2cm} 6 & 8 \\
8 \text{ plus } 8 = 16 \hspace{2cm} 8 & 7 \\
16 \text{ plus } 4 = 20 \hspace{2cm} 4 & 5 \\
20 \text{ plus } 5 = 25 \hspace{2cm} +5 & 9 \\
\hline
\hspace{1cm}9 \\
\end{array}
\]
On the last column we write both the left and right sides of the answer. Watch me.

$$\begin{array}{cccc}
\hline
127 \\
6 & 8 \\
8 & 7 \\
4 & 5 \\
+ & 5 & 9 \\
\hline
\end{array}$$

Your answer should be two hundred and fifty nine . . . . . . . . 259

Remember, if there is a column to the left, you always carry the tens or left hand answer to the top of the next column and circle it.

Now I want you to do some practice problems. (Hand out practice sheets.)
Appendix D

Hand-held Calculator, Lesson
The calculators we will be using are the same ones that you use here in school. Since some of you may be more familiar than others with using these calculators, I thought I would spend a few minutes reviewing how to use them. Okay.

To turn the calculator on, you must do two things. Since these are solar-powered calculators, you must make sure that they are getting enough light. So be sure that you don’t block the solar cells on the calculator. The cells are right here, along the top of the machine. The next thing you need to do to turn it on is press the button marked ON. Do you all see this button? Good. If the calculator is getting enough light, you should see a black 0 appear in the display window. Right now, then everybody turn on their calculators. Is everyone’s calculator showing a black 0 in the display? Good. Now, let’s try a simple problem, a basic number fact. Let’s add 7 + 8 with the calculator. First, press 7. This enters the seven into the calculator. Then, press the plus (+) button. Now press 8 which enters the eight into the calculator, and finally press the equals (=) button. This gives you the answer. What answer did you get? 15, yes. Did anyone not get 15? Very good.

Now, let’s try a slightly harder one. Let’s add 15 to 37. First, press the one button then the five. Do you see the 15 on the display? Good. Now press the plus (+) button. Next, press the three then the seven. Finally, press the equals (=) button to get your answer. What answer did you get? 52, right! Good job.

Now I want you to do these problems that I have on the board. Copy them on a piece of paper and use your calculator to get the answers.

<table>
<thead>
<tr>
<th>Row #1</th>
<th></th>
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</thead>
<tbody>
<tr>
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<td>68</td>
<td>87</td>
</tr>
<tr>
<td>78</td>
<td>75</td>
<td>39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Row #2</th>
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<tr>
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<td>57</td>
</tr>
<tr>
<td>86</td>
<td>54</td>
<td>85</td>
</tr>
<tr>
<td>43</td>
<td>87</td>
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<table>
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<td>42</td>
</tr>
<tr>
<td>87</td>
<td>29</td>
<td>54</td>
</tr>
</tbody>
</table>
Now, let's review your answers. I am going to write the correct answers on the board and I want you to mark your own work. (In order the answers are: 137, 143, 126, 192, 180, 239, 523, 421, 453). Was anyone not able to get at least one problem in row #1 correct? Good. Was anyone not able to get at least one problem in row #2 correct? Good. Did everyone get at least one problem in row #3 correct? Good, most of you got at least one of these correct.

Now, let's get some more practice with the calculators. I want you to do these problems that I am handing out (Sheets with 2x7 matrix problems similar to row #3 were distributed).
Appendix E

Addition Worksheets
\[
\begin{array}{ccc}
4 & 2 & 4 \quad 2 \quad 7 \quad 1 \\
3 & 7 & 2 \quad 2 \quad 6 \quad 1 \\
9 & 1 & 6 \quad 9 \quad 3 \quad 5 \\
2 & 2 & 5 \quad 9 \quad 3 \quad 2 \\
3 & 9 & 7 \quad 8 \quad 6 \quad 5 \\
4 & 7 & 9 \quad 5 \quad 1 \quad 8 \\
\hline
+ \quad 5 & 2 & \hline
& + \quad 2 & 8 & \hline
& \hline
& + \quad 6 & 5 & \hline
\end{array}
\]

\[
\begin{array}{ccc}
3 & 8 & 8 \quad 9 \quad 3 \quad 2 \\
4 & 3 & 4 \quad 3 \quad 4 \quad 1 \\
9 & 1 & 3 \quad 1 \quad 7 \quad 5 \\
9 & 5 & 9 \quad 2 \quad 6 \quad 3 \\
8 & 3 & 5 \quad 1 \quad 9 \quad 9 \\
5 & 6 & 5 \quad 7 \quad 9 \quad 7 \\
\hline
+ \quad 8 & 6 & \hline
& + \quad 6 & 9 & \hline
& \hline
& + \quad 2 & 7 & \hline
\end{array}
\]

\[
\begin{array}{ccc}
1 & 4 & 5 \quad 8 \quad 2 \quad 9 \\
4 & 7 & 6 \quad 1 \quad 9 \quad 6 \\
4 & 1 & 9 \quad 4 \quad 7 \quad 8 \\
5 & 4 & 6 \quad 6 \quad 4 \quad 5 \\
4 & 5 & 6 \quad 1 \quad 9 \quad 4 \\
1 & 5 & 3 \quad 5 \quad 2 \quad 2 \\
\hline
+ \quad 9 & 6 & \hline
& + \quad 1 & 9 & \hline
& \hline
& + \quad 5 & 2 & \hline
\end{array}
\]

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Appendix F

Attendance Summary Tables 1 and 2
### Attendance Summary, Table 1

**Experimental Sessions 1-19: X = Attendance**

<table>
<thead>
<tr>
<th>Session</th>
<th>1</th>
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<th>3</th>
<th>4</th>
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<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject No. &amp; Group: M or L</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
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Appendix G

Inferential Statistics Summary Table
Inferential Statistics Summary Table: Methods Factor (Question 1), Experiment 1, Perspectives 1 and 2, Low and Medium Achievement Groups

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<th>Methods Factor</th>
<th>Critical ANOVA F value</th>
<th>Obtained ANOVA F value</th>
<th>p value</th>
<th>S = Significant</th>
<th>NS = Non-Significant</th>
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<td>p &lt; .055</td>
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Experiment 1

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Inferential Statistics Summary Table: Methods Factor Multiple Comparisons (Question 1), Experiment 1, Perspective 2, Low and Medium Achievement Groups

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Experiment 1, Perspectives 1 and 2

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### Inferential Statistics Summary Table: Methods Factor (Question 1), Experiment 2, Perspectives 1 and 2, Low and Medium Achievement Groups

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<td>$F_{\alpha=.01}$=5.95</td>
<td>$F_{\alpha=.25}$=3.59</td>
<td>$F_{\alpha=.01}$=5.95</td>
<td></td>
</tr>
<tr>
<td>Obtained ANOVA F value</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{[3,30]}=6.18$</td>
<td>$F_{[3,12]}=16.68$</td>
<td>$F_{[3,30]}=3.90$</td>
<td>$F_{[3,12]}=16.37$</td>
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<tr>
<td>$p$ value</td>
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<tr>
<td>$p&lt;.002$</td>
<td>$p&lt;.0001$</td>
<td>$p&lt;.025$</td>
<td>$p&lt;.0009$</td>
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</tr>
</tbody>
</table>
| $S$=Significant | $S$ | $S$ | $S$ | $S$ | $S$
| $NS$=Non-significant | | | | |

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Inferential Statistics Summary Table: Methods Factor Multiple Comparisons (Question 1), Experiment 2, Perspective 1, Low and Medium Achievement Groups

<table>
<thead>
<tr>
<th>Methods Factor Multiple Comparisons</th>
<th>Experiment 2, Perspective 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Group: Question 1-B-E2</td>
</tr>
<tr>
<td>Statistically Significant Comparison</td>
<td>Hutchings’ &gt; Factor Analysis</td>
</tr>
<tr>
<td>Critical alpha value</td>
<td>.0083</td>
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<td>Obtained ANOVA F value</td>
<td>$F_{[1,10]} = 12.79$</td>
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<tr>
<td>p value</td>
<td>$p &lt; .005$</td>
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Inferential Statistics Summary Table: Methods Factor Multiple Comparisons (Question 1), Experiment 2, Perspective 2, Low and Medium Achievement Groups

<table>
<thead>
<tr>
<th>Methods Factor Multiple Comparisons</th>
<th>Experiment 2, Perspective 2</th>
<th>Low Group: Question 1-D-E2</th>
<th>Medium Group: Question 1-H-E2</th>
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<tbody>
<tr>
<td>Statistically Significant Comparison</td>
<td>Hutchings’ &gt; Calculator</td>
<td>Hutchings’ &gt; Calculator</td>
<td>Factor Analysis &gt; Calculator</td>
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<td>Critical alpha value</td>
<td>.0083</td>
<td>.0083</td>
<td>.0083</td>
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<tr>
<td>Obtained ANOVA F value</td>
<td>$F[1,10]=12.22$</td>
<td>$F[1,4]=44.43$</td>
<td>$F[1,4]=73.95$</td>
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<td>$p$ value</td>
<td>$p&lt;.006$</td>
<td>$p&lt;.0026$</td>
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Inferential Statistics Summary Table: Between Groups $t$ tests (Questions 4-A-E2 and 4-B-E2), Experiment 2, Perspectives 1 and 2

<table>
<thead>
<tr>
<th>Methods that Yielded Significant Results</th>
<th>Experiment 2, Perspective 2, Question 4-A-E2</th>
<th>Experiment 2, Perspective 2, Question 4-B-E2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings'</td>
<td>Hutchings'</td>
<td></td>
</tr>
<tr>
<td>$t_{\alpha=.01} = 2.98$</td>
<td>$t_{\alpha=.01} = 2.98$</td>
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<tr>
<td>Critical $t$ value</td>
<td>$t_{[14]} = 4.59$</td>
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</tr>
<tr>
<td>Obtained $t$ value</td>
<td>$t_{[14]} = 4.01$</td>
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<tr>
<td>Factor Analysis</td>
<td>Factor Analysis</td>
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<tr>
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<td>$t_{\alpha=.01} = 2.98$</td>
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<td></td>
<td>$t_{[14]} = 4.60$</td>
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<tr>
<td></td>
<td>$t_{[14]} = 3.65$</td>
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Inferential Statistics Summary Table: Time Periods Factor (Question 6), Experiments 1 and 2, Low and Medium Achievement Groups

<table>
<thead>
<tr>
<th>Time Periods Factor</th>
<th>Critical ANOVA F value</th>
<th>Obtained ANOVA F value</th>
<th>p value</th>
<th>S=Significant</th>
<th>NS=Non-significant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment 1</td>
<td>Experiment 2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Low Group: Question 6-B</td>
<td>Medium Group: Question 6-D</td>
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<td>$F[\alpha=0.05] = 4.84$</td>
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<td>S</td>
<td>NS</td>
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<td>Obtained ANOVA F value</td>
<td>Obtained ANOVA F value</td>
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<td></td>
<td>$F[1,11] = 1.18$</td>
<td>$F[1,5] = 3.47$</td>
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<td>p &gt; .05</td>
<td>NS</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Low Group: Question 6-B-E2</td>
<td>Medium Group: Question 6-D-E2</td>
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<td></td>
<td>$F[\alpha=0.05] = 4.96$</td>
<td>$F[\alpha=0.05] = 7.71$</td>
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<td>S</td>
<td>NS</td>
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<td></td>
<td>Obtained ANOVA F value</td>
<td>Obtained ANOVA F value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F[1,10] = 3.26$</td>
<td>$F[1,4] = 4.26$</td>
<td></td>
<td>p &gt; .05</td>
<td>NS</td>
</tr>
<tr>
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</tr>
</tbody>
</table>

$F = $ Value, $\alpha $ = Significance Level, $S$ = Significant, $NS$ = Non-significant
Inferential Statistics Summary Table: Method by Time Period Interaction (Question 7), Experiments 1 and 2, Low and Medium Achievement Groups

<table>
<thead>
<tr>
<th>Method by Time Period Interaction</th>
<th>Critical ANOVA F value</th>
<th>Obtained ANOVA F value</th>
<th>p value</th>
<th>S=Significant</th>
<th>NS=Non-significant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F[\alpha=.05]=2.90$</td>
<td>$F[\alpha=.05]=3.29$</td>
<td>$p&gt;.05$</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>Low Group: Question 7-A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium Group: Question 7-B</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Low Group: Question 7-A-E2</td>
<td>$F[\alpha=.05]=2.92$</td>
<td>$F[\alpha=.05]=3.49$</td>
<td>$p&gt;.05$</td>
<td>NS</td>
<td>NS</td>
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<tr>
<td>Medium Group: Question 7-B-E2</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$F = $ Significant
$NS = $ Non-significant

Experiment 1

Experiment 2

Low Group: Question 7-A

Medium Group: Question 7-B

Low Group: Question 7-A-E2

Medium Group: Question 7-B-E2

$F[3,33] = .85$

$F[3,15] = 1.66$

$F[3,30] = .32$

$F[3,12] = .49$
Appendix H

Experiment 1, Daily Sessions, Subject 16, Traditional Method
Appendix I

HSIRB Approval Letter, Letters to Parents, Letters to Subjects
Date: January 21, 1991
To: John Hampel
From: Mary Anne Bunda, Chair
Re: HSIRB Project Number 91-01-05

This letter will serve as confirmation that your research protocol, "A Comparison of the Hutchings 'Low-Stress,' Fulkerson tens method, hand-held calculator, and conventional addition algorithms for speed, accuracy, and preference with regular education students," has been approved after expedited review by the HSIRB. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the approval application.

You must seek reapproval for any change in this design. You must also seek reapproval if the project extends beyond the termination date.

The Board wishes you success in the pursuit of your research goals.

xc: Galen Alessi, Psychology

Approval Termination: January 21, 1992
Parent(s) of student  
Small Town, MI, zip code

Dear Parents:

You and your child are cordially invited to participate in a research study that will take place at your child's elementary school.

Did you know that there are dozens of ways to add a column of numbers together? This study will investigate the effectiveness of four different methods of performing addition problems. These four different methods (or algorithms as they are called) are: The traditional algorithm, the hand-held calculator, the Fulkerson tens algorithm, and the Hutching's "Low-stress" algorithm. The traditional method is the one with which most people are familiar. The hand-held calculator is also well known and widely used. The other two methods for adding numbers are novel approaches designed to improve both the speed and accuracy of performance.

The study sessions will be scheduled every weekday right after school for six to eight weeks. The sessions will last for 30 minutes each day, between 3:30 pm and 4:00 pm. During this time, your son or daughter will learn to compute addition problems using each of the four different addition methods.

In return for your child's participation he or she will receive training that will lead to mastering the basic addition facts at 100% accuracy. Your child will learn that math problems can be solved by using more than one method. Also, your child will benefit by knowing that he or she can accurately compute answers to large addition problems using methods that are much faster than the traditional method. Also, your child will be offered small incentives for attendance and participation during the study, and for completing the study (for instance, a small gift certificate to "Toys-R-Us"). In addition, your child will be offered the opportunity to learn how and why research of this nature is done. After the study is finished, the participants will be invited to attend two or three learning sessions. During these sessions we will show them, in an understandable and entertaining way, how and why research of this kind is done.

In return for your participation, we will provide you with a written summary of the overall results from this study as well as a verbal or written account of
your own child's performance. This personal information will not be released to your child's school, nor will we have access to data from your child's school except the assessment test that we ask they take. Also, no information about your child will be gathered until this form is signed and you have given your consent for your child's participation.

All information obtained in this study will be held in confidence. Only the researchers involved in this project will have access to this information. Your child's name will not be used. Instead, your child will be assigned a code number which will be used on all scoring forms and reports.

Your participation in this experiment is voluntary; your decision will not in any way affect your relations with your child's school or with Western Michigan University. It is recommended that you and your child participate in the entire project. However, if at any time you wish to stop, you are free to do so. Withdrawal from the study will not carry any penalties.

Questions or complaints regarding this research may be directed to John Hampel, 343-9466, Dr. Galen Alessi, 387-4470, or your school principal

YOUR SIGNATURE BELOW INDICATES THAT YOU UNDERSTAND THE ABOVE INFORMATION AND HAVE DECIDED TO PARTICIPATE IN THIS STUDY. You will receive a copy of this form.

______________________________  _______________________
signature of parent or guardian       date

______________________________  _______________________
investigator signature           date
Please mail this consent form and your child's assent form to:

Supervisor of Instruction
School's address
Small Town USA

Thank you for your time and consideration.

Sincerely,

John C. Hampel, Investigator
630 Minor St.
Kalamazoo, MI 49008
343-9466 (Home) 343-6109 (Office)

Galen Alessi, Ph.D.
Professor
Department of Psychology
Western Michigan University
Kalamazoo, MI 49008
385-3305 (Home) 387-4470 (Office)
387-4456 (Pediatrics Lab)
344-3355 (24 Hr Answering Service)
February 5, 1991

Dear Student:

You are invited to be a part of a research study. This study will see how well children add numbers. If you decide you would like to sign up you will stay after school each Monday through Thursday afternoon for about 30 minutes. This study will last several weeks.

If you sign up you will probably become very good at adding. To make it even more fun, you will be able to earn small gifts and prizes. You will also be given the chance to learn about research. We will explain how and why it is done in a way that you will understand and enjoy.

If you agree to be in this study we hope that you will stay in it for the entire time. However, if at any time you wish to stop, you are free to do so.

YOUR SIGNATURE BELOW SHOWS THAT YOU HAVE DECIDED TO BE A PART OF THIS STUDY. If you have any questions you should ask your parents.

__________________________________________  _________________
signature of student                     date

__________________________________________  _________________
investigator signature                     date

Sincerely,

John C. Hampel, Investigator
630 Minor St.
Kalamazoo, MI. 49008
343-9466 (Home)  343-6109 (Office)
April 30, 1993

The Graduate College
Western Michigan University
Kalamazoo, MI. 49008-5121

To whom it may concern:

I, Lloyd Barton Hutchings, do hereby give my permission to John C. Hampel and UMI to reproduce and publish (within John C. Hampel's masters thesis) a three page segment from my paper entitled: *Low-Stress Algorithms*, which appeared in the National Council of Teachers of Mathematics 1976 Yearbook, *Measurement in School Mathematics*. The section reproduced and quoted began on page 220, at the heading Notation for Low-Stress Addition, and concluded on page 223 with the line: "Note, however, that the column sum for the last column in a multi-column example is recorded in exactly the same way as the sum of a single-column exercise." I give my permission so that UMI may supply copies on demand.

Cordially,

Dr. Lloyd B. Hutchings, Ph.D
School of Education
Francis Marion University
Florence, South Carolina, 29051


Alessi, G. J. (April, 1991). Personal communication.

Alessi, G. J. (May, 1992a). Personal communication.

Alessi, G. J. (June, 1992b). Personal communication.

Alessi, G. J. (November, 1992c). Personal communication.


Bitter, Gary (1981). Five, six--math is kicks, when you seven, eight--calculate. *Instructor, 21*(2), 130-133.


Buitendorp-Drew, P. G. (1980). Preferences of students for the Hutchings' "low-stress" compared to the conventional algorithm under conditions of differentially increasing the number of problems with and without reinforcement. Unpublished specialist in education project, Western Michigan University, Kalamazoo.


