Numerical Study of Flow in Three-Dimensional Fiber Networks and Sedimentation of Curved Fiber by Lattice Boltzmann Method

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NUMERICAL STUDY OF FLOW IN THREE-DIMENSIONAL FIBER NETWORKS AND SEDIMENTATION OF CURVED FIBER BY LATTICE BOLTZMANN METHOD

by

Xiaoying Rong

A Dissertation
Submitted to the
Faculty of The Graduate College
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Department of Engineering, Chemical Engineering and Imaging
Dr. Dewei Qi, Advisor

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Fiber network is modeled as a three-dimensional random cylindrical fiber network. Nonlinear behavior of fluid flowing through the fiber network is numerically simulated by using the lattice Boltzmann (LB) method. Nonlinear relationship between the friction factor and the modified Reynolds number is simulated and analyzed by using the Fochheimer equation, which includes the quadratic term of velocity. There is a transition from linear to nonlinear region when the Reynolds numbers are sufficiently high, reflecting the inertial effect of the flows. The simulated permeability of such fiber network is compared to the experimental results and the finite element simulations.

Dynamic motions of fibers sedimenting and in shear flows are of great interest in the paper and pulp industry. Fibers are not just cylindrically shaped, but also curved. The sedimentation of curved fibers under gravity is studied. The aspect ratio, inertia of single curved fiber is found to affect the motion and orientation. Two and four fiber sedimentation under gravity are studied as well. The sedimentation of curved fiber is also compared with the sedimentation of cylindrical fiber. The curvature and position are found to affect the sedimentation of multiple fibers in Newtonain flow. The lattice Boltzmann method is employed in this study. The numerical simulation is
focused on migration and rotation of curved fibers and the interactions between fibers.
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Xiaoyong Rong
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INTRODUCTION

Flow in Porous Media

Permeability of porous media interests much research, such as in paper industries, petroleum industries, environmental, biological processes, and physiological systems. Permeability, as a property to understand the migration of fluid into porous media, has been studied theoretically and experimentally for many years. The fundamental information, which brought the most interest, is the relation of the applied pressure gradient and the resulting flow rate. For the flow at near zero Reynolds number, pressure gradient and flow rate have a linear relation, known as Darcy's law. For small but non-zero Reynolds number, pressure gradient is a non-linear function of the flow rate. The experimentation that showed this non-linear relation was carried out by Forchheimer (1930), who indicated that there exists a quadratic term of flow rate when the Reynolds number is sufficiently high. Modeling and simulating this non-linear relation and inertial effect of porous materials have brought more interest.

The inertial effect in periodic and random arrays have produced a large number of studies. Koch (1997) and Hill (2001) simulated the moderate Reynolds number flows through periodic and random arrays of aligned cylinders and spheres in two-dimension. These studies showed that the inertial term made a transition from linear to quadratic in random arrays. The inertial effect became smaller when the volume fraction of the array approached closing packing. Two-dimensional simulations considering inertia
were also studied by Andrade et al. (1999). The porous medium was created by using square plaquettes as obstacles for fluid flow. They showed that the departure from Darcy’s law in flow through high porosity percolation structures, and at sufficiently high Reynolds numbers inertia became relevant. Forchheimer equation was shown to be valid for low and also a limited range of higher Reynolds numbers. Clauge and Phillips (1997) investigated the hydrodynamic permeability in a three-dimensional array. The disordered fibrous medium was modeled as non-overlapped cylindrical fibers for pure collagen and proteoglycan fibers. They obtained a non-linear region at very dilute fiber volume fractions.

Because the flow in porous media and the permeability are directly related to pore geometry, the complex structures of porous media bring more difficulty for modeling and simulation. In most of studies, the porous medium is modeled either in two-dimension with random arranged cylinders or spheres (Martin 1998, Lee 1997, Ghaddar 1995), or in three-dimensional structure using ordered cubic, body centered cubic or face centered cubic lattices (Hingdon 1996). Koch and Hill (2001) reviewed recent research of inertial effects on porous media. The microstructure was found to be more important at finite Reynolds numbers than at zero Reynolds number. To enhance the understanding of inertial effects, the simulation must be considered in three-dimensions and close to the real pore structure. In numerical simulations, the lattice Boltzmann (LB) method has been employed to investigate flows in complex geometries, especially in three-dimensional modeling and simulation. Succi et al. (1989) and Cancelliere et al. (1990) used the LB method to simulate the flow through porous media at the pore scale.
and studied the microscopic behavior of flow. Koponen, et al. (1999) employed the LB method in a three-dimensional simulations of flow through fiber webs. The fibers were laid in either the $x$ direction or $y$ direction. They applied a gravitational body force to the fluid to simulate the creeping flow. Although the LB method is employed to study the flow in three-dimensional porous media, most of the simulations were focused on creeping flow, which eliminated the inertial effect. Recently, studies started to focus on the nonlinear relation in porous media using the LB method. For example, Inamuro et al. (1999) applied the LB method to simulate the isothermal flows in three-dimensional sphere packed porous media at a single porosity. The calculated pressure drops fit with Ergun equation for high Reynolds numbers.

Despite the numerous simulation studies, it seems that the simulation of flow through three-dimensional random fibrous porous media with medium porosity has not been well simulated by considering the quadratic term of velocity by the lattice Boltzmann method. In this study, the lattice Boltzmann method was employed to model and simulate fluid flows through a random fiber network at medium porosities.

**Particle Suspension**

Solid particles sediment in rivers and rain drops and dust sediments in air. Sedimentation is involved in chemical engineering, environment processes, fluidized beds, and many others. Sedimentation is also a useful method to separate different particles in industry. Many experiments and numerical simulations have been done to study the sedimentation of
regularly shaped particles, like spherical, elliptical particles, etc. In nature, particles are not all regularly shaped, for example, fibers used in paper manufacturing are not cylinders. During pulping and recycling processes, fibers are usually flexible; sometimes they may be curved permanently. The sedimentation of curved particles or fibers under gravity, the migration in pressure driven flows and the orientation in shear flow have not been paid enough attention, due to the complexity of modeling the flexibility or curved shapes. Fiber motion is directly related to flocculation and dispersion. In the pulp and paper industry, migration and aggregation of curved and flexible fibers are obviously important aspects.

Particle suspensions with zero or small Reynolds numbers have been studied experimentally and numerically. Spherical particles, benefited by their isotropic shape, have produced a large number of experiments and simulations. The off-center motions have been discovered by many experiments. Segre and Silberberg (1961) demonstrated that spherical particles in a pipe of Newtonian fluid migrate to the radial position about the middle of center and wall of the pipe. Karnis and Mason (1966) showed that a spherical particle migrates to the center of the pipe in a viscoelastic fluid, regardless of its initial position when inertia is negligible. When inertia is considered, a spherical particle exhibits off-center motion in a planar Poiseuille flow in the experiment done by Ho and Leal (1974). Yu et al. (2004) simulated the motion of spherical particles in Poiseuille flows within Newtonian fluids. It was shown that at low Reynolds numbers, particles move towards the center of the pipe, while at high Reynolds numbers they move towards the wall. Davis and Acrovos (1985) reviewed some
experimental and theoretical studies of the sedimentation of rigid spherical particles. It has been observed that, absent hydrodynamic interactions, individual particles sediment at the Stokes' velocity. As the concentration of particles increases, hydrodynamic interactions decrease the sedimenting speed, due to the effect of fluid backflow. Numerical studies can produce a precise description of the behavior of a small number of particles. Based on finite element and finite volume techniques, many-particle effects have been studied by Batchelor (1972) and Brady and Bossis (1988). Koch (1994) and Ladd (1992, 1996, 1997) simulated the fluctuation velocity of spherical particles in dilute random suspensions. These studies focused on the velocities of settling and the interactions between particles. The individual particle velocities fluctuate about the mean fall speed. There are increased numbers of paired particles, which leads to increasing the average sedimentation velocity.

Feng, Hu and Joseph (1994 a,b) investigated the motion of single circular and elliptical particles settling in a Newtonian fluid. It was found that the center of the channel is the equilibrium position regardless of the initial position of the particle. With inertia, the particle experiences five different regimes of motion: steady motion with and without overshoot, weak, strong and irregular oscillations. An elliptical particle always turns its long axis perpendicular to the fall and drifts to the centerline of the channel during the sedimentation. For two particles, which interact while settling, a steady staggered structure, a periodic wake-action regime and an active drafting-kissing-tumbling (DKT) scenario are realized at increasing Reynolds numbers. The DKT results when the trailing particle is sucked into the wake of the leading one, and drafted toward the leading particle with increasing
velocity. Two particles then almost touch (kissing). Because the configuration of the doublet, standing vertically is unstable, it tumbles and the top particle turns around the bottom particle to take the lead. The study also showed that, at high Reynolds numbers, the inertia of particles become strong enough and they tend to collide. Qi (1999) also numerically demonstrated the motion of drafting-kissing-tumbling by the lattice Boltzmann method.

Axisymmetric particles have been studied as nonspherical particles, mainly by many other numerical methods. Sedimentation and orientation of these particles are complicated because the particles rotate in flow. Many studies have been done to predict the motion of axisymmetric particles in dilute suspension at low Reynolds numbers. Davis (1991) calculated cylindrical rods and disks, prolate and oblate spheroids sedimentation in Stokes flow. He found that these particles experience the greatest drag when translating along their axes of symmetry and their symmetric axes aligned with the direction of flow. The thin disc-shaped spheroid has its thin side along the flow. When particle interaction exists, particles tend to be misaligned. When considering inertia, Huang et al (1997) simulated a circular cylinder in Coutte and Pois-suille flows. The simulation results numerically proved that the equilibrium position of neutrally buoyant particles migration is different in Newtonian fluids and viscoelastic fluids, which had been experimentally observed (Cox and Mason 1971). Huang et al (1998) reported that there are critical numbers, below which the settling turns the long side of the particle along the stream. It is also showed in the results that tilted off-center falls are unstable. In the dilute regime, particles migrating towards the center of the channel in viscoelastic fluids was numerically simulated. Three
dimensional rotation and translations of non-spherical particles including ellipsoids, cylinders and cubic-shaped particles at small aspect ratios were studied by Qi (1997a, 1997b, 1999) using the Lattice Boltzmann method. Qi (1999) found that at near zero Reynolds number, the long axis of an elliptical particle turns vertically, while it turns horizontally when the Reynolds number increases. Migration and orientation behavior of single and multiple elliptical particles in planar Poiseuille flow was studied by Qi et al. (2002). It was shown that at a steady state, an ellipse has higher probability to orient along the stream, rather than cross the stream. Multi-particles tend to locate between the wall and the center line of the channel. When the particle is slightly denser than the fluid, a single particle moves towards the wall. Only when the particle density is large enough, does it move towards the center.

Recently, with increasing the aspect ratio, the sedimentation of spheroid particles and rod-like particles were studied. The numerical simulation performed by Höfler (2000) showed that at a small but finite Reynolds number, an individual spheroidal particle of aspect ratio of 5 will rotate until the fiber becomes horizontal. This is different from sedimentation of a fiber in Stokes flow, in which the fiber will not change orientation during sedimentation. Experimental study of the sedimentation of dilute and semi-dilute suspensions of fibers given by Herzhaft and Guazzelli (1996, 1999) also showed that the individual fiber sediments without changing direction. At different aspect ratios, steady-state velocity and orientation distributions of fibers were measured (Herzhaft and Guazzelli 1999). The results showed that for dilute suspension, the fibers tend to align in the direction of gravity and clump together to form pockets. The experimental results also revealed that
the sedimentation of rod-like fibers is qualitatively different from the sedimentation of spheres. Mackaplow and Shaqfeh (1998) numerically studied the sedimentation of fibers at different aspect ratios. The results showed that fibers are aligned in the direction of gravity at low Reynolds numbers. The distribution of fibers in suspension becomes inhomogeneous as fibers cluster into long streamers in the direction of gravity. This drives a downward convective flow within the streamer, which serves to increase the mean fiber sedimentation speed. Another numerical simulation was performed by Butler and Shaqfeh (2002) to study the suspension of fibers at zero Reynolds number. The numerical results also confirmed that the sedimentation of non-spherical particles differ qualitatively from the sedimentation of spherical particles. The suspension of fibers tends to orient with gravity during the sedimentation process. The hydrodynamic interactions between the fibers cause the suspension to be heterogeneous during the sedimentation and unstable.

In previous studies, sedimentations of regular shaped particles were investigated. Without the influence of inertia, axisymmetric particles showed similar behavior in that the orientation of particles is along the stream. The hydrodynamic interactions between particles are the only influence of fiber sedimentation when inertia is negligible. Clusters of particles are generated under particle interactions. The orientation of a cluster is along gravity as presented in all the studies. At small but finite Reynolds number, the particle tends to rotate and became horizontal rather than vertical in the case of zero Reynolds number.

In pulping and papermaking processes, as well as biological processes,
many fibers are flexible rather than rigid. Experiment has shown that the rotation orbits of flexible fibers in shear flow in the dilute regime appeared springy, with "snake" shapes and S-turns (Forgacs and Mason 1965). Some theoretical studies have developed the simulation method to precisely reveal the motion of flexible fiber suspensions. Yamamoto and Matsuoka (1992, 1994, 1995) developed a method using a chain of spheres to represent a flexible fiber. Spheres are contacted and subjected to stretching, bending and torsion forces. Ross and Klingenberg (1997) demonstrated a method with chained spheroids of sockets and ball joints. The methods and results presented in the previous studies are at zero Reynolds number. In reality, fibers are normally suspended in higher Reynolds numbers during the process. Qi (2004) has studied flexible fibers in shear flow at finite Reynolds number. Flexible fibers as a chain of spheres with different stiffness were studied. It was observed that with decreasing stiffness, the rotation of a flexible fiber changes from rigid fiber rotation to springy, then the more flexible fiber shows an S-turn finally. When considering inertial effects, the rotation is slowed down due to streamline separation. This effect is larger for stiffer fibers.

It is evident that most of the particles and fibers are not regularly shaped rods and spheroids, which have at least two symmetry axes. Most of the particles and fibers are irregularly shaped, curved or flexible. To understand the suspension of fibers in pulping and papermaking processes, fibers with curved shape and flexibility need to be investigated further. As demonstrated in many studies, rods, prolate and oblate particles show similar migration properties in shear and pressure driven flows. The inertia effect,
aspect ratio and initial position have been proven to be important parameters in particle suspensions. In this study, curved fibers are shaped as an arc. The suspension of a curved fiber in shear and pressure driven flow is numerically simulated by the lattice Boltzmann method. Different initial positions, orientations and curvatures are studied to further understand the sedimentation characteristics of curved fibers. In addition to single fiber sedimentation, multi-fibers at certain concentrations will be studied as well to reveal the fiber interactions during sedimentation.

**LATTICE BOLTZMANN METHOD**

The method of lattice Boltzmann simulation has been successfully applied for simulating the interaction between fluid and solid particles (Aidun, 1998; Qi, 1999; Ladd 1994, 2001). In the lattice-Boltzmann (LB) method, fluid particles are divided into discrete nodes, every particle resides on the lattice nodes to simulate the nature of flow. The fluid particles move to their nearest neighbor along the link with unit spacing in each unit time step.

To determine the mass density $\rho$ and momentum density, $\rho \vec{u}$, following equations are used,

\[
\rho = \sum_{\sigma} \sum_i f_{\sigma i} \quad \rho \vec{u} = \sum_{\sigma} \sum_i f_{\sigma i} \vec{e}_{\sigma}
\]  

(1)

The dynamics of these distribution functions in the single relaxation time (Bhatnager, Gross and Krook, 1954) scheme, is given by
\[ f_\sigma(x + \vec{v}_\sigma, t + 1) = f_\sigma(x, t) - \frac{1}{\tau} [f_\sigma(x, t) - f_\sigma^{eq}(\vec{x}, t)] \] (2)

where \( f_\sigma(x, t) \) is the fluid particle distribution function for particles with velocity \( \vec{v}_\sigma \) at position \( x \) and time \( t \), \( f_\sigma^{eq}(\vec{x}, t) \) is the equilibrium distribution function. In a widely used class of models (Qian et al., 1992), the kinematic viscosity is related to the relaxation time \( \tau \) for convergence by

\[ \nu = (2\tau - 1)/6 \] (3)

In this study, the simulation is performed using the D3Q15 model. It possesses a rest particle state, six links with the nearest neighbors, and eight links with the next nearest neighbors. Periodic boundary conditions in the flow direction with bounce back on the solid nodes were used. \( f_\sigma^{eq}(\vec{x}, t) \) are taken as

\[ f_\sigma^{eq}(\vec{x}, t) = A_\sigma + B_\sigma (\vec{v}_\sigma \cdot \vec{u}) + C_\sigma (\vec{v}_\sigma \cdot \vec{u})^2 + D_\sigma u^2, \] (4)

where \( \sigma = 1 \) represents the particles move to the nearest neighbors, \( \sigma = 2 \) represents the particles move to the second nearest neighbors, \( \sigma = 0 \) represents the particles rest at the nodes. Coefficients of \( A_\sigma, B_\sigma, C_\sigma \) and \( D_\sigma \) can be determined as (Qian 1990):
where \( \rho_f \) is the density of the fluid.

Collision Rule

When the particle is suspended in the fluid, the fluid particles and solid particles collide. Ladd (1994) presents a bounce-back boundary condition at the solid-fluid interface. Under moving boundary condition, if the solid is stationary, the incoming distributions are reflected back to the opposite direction that they came from. This is simply a bounce back rule. If a moving node, the distribution is proportion to the velocity of the node, which can be notated as \( \bar{U}_b \). The collision rules Ladd gave are as following:

\[
A_i = \frac{1}{6} \rho_f, \quad B_i = \frac{1}{3} \rho_f, \quad C_i = \frac{1}{2} \rho_f, \quad D_i = \frac{1}{6} \rho_f, \\
A_2 = \frac{1}{72} \rho_f, \quad B_2 = \frac{1}{24} \rho_f, \quad C_2 = \frac{1}{16} \rho_f, \quad D_2 = \frac{1}{48} \rho_f, \\
D_0 = \frac{1}{6} \rho_f
\]  

(5)

where \( \rho_f \) is the density of the fluid.
solid and fluid is conserved, as described in Ladd (1994). The hydrodynamic force exerted on boundary nodes is:

\[ F(\bar{x} + \frac{1}{2} \bar{e}_{\alpha}) = 2\dot{e}_{\alpha}(f_{\alpha}(\bar{x}, t) - B_{\alpha}(\bar{U}_{b} \cdot \dot{e}_{\alpha})) \]  

(7)

where \( \bar{U}_{b} \) is the velocity of the solid-fluid interface at the node, and it is determined by solid particle at its mass center, angular velocity \( \Omega \), and particle mass center \( \bar{R} \) can be calculated as:

\[ \bar{U}_{b} = \bar{U} + \Omega \times \left( \bar{x} + \frac{1}{2} \bar{e}_{\alpha} - \bar{R} \right) \]  

(8)

The total forces \( F_T \) of the particles can be obtained by summing the force term.

\[ F_T = \sum F\left( \bar{x} + \frac{1}{2} \bar{e}_{\alpha} \right) \]  

(9)

The total torques \( T_T \) of the particles is

\[ T_T = \sum \left( \bar{x} + \frac{1}{2} \bar{e}_{\alpha} - \bar{R} \right) \times F\left( \bar{x} + \frac{1}{2} \bar{e}_{\alpha} \right) \]  

(10)

PERMEABILITY OF POROUS MEDIA

Single-phase fluid flowing through microscopically disordered porous media at low Reynolds numbers is described by Darcy's law (Darcy 1856,
Bear 1972). The superficial flow rate \( \langle u \rangle \) of a viscous fluid through a porous medium of length \( L \) is proportional to the applied pressure difference \( \Delta P \) and inversely proportional to the dynamic viscosity \( \mu \).

\[
\langle u \rangle = \frac{k \Delta P}{\mu \, L} \quad (11)
\]

At low Reynolds numbers, where the flow is laminar, viscous forces are predominant and the linear Darcy's law is valid. \( k \) is used to describe the permeability with the unit of area (length square). However, as the Reynolds number increases, the inertial force has to be considered. With increasing Reynolds number, the inertial force describes the transition from viscous force predominated creeping flow to another inertial force governed laminar region, and gradually passes to turbulent flow.

In order to always satisfy Darcy's law in the creeping flow region and to correctly capture the influence of inertia at high Reynolds numbers, the well known Forchheimer equation (1984) is used. This equation consists a linear term of viscous component and a power term of inertial component.

\[
-\frac{\Delta P}{L} = \alpha \mu \langle u \rangle + \beta \rho \langle u \rangle^2 \quad (12)
\]

where \( \alpha \) is the viscous coefficient and \( \beta \) is the inertial coefficient. They are both resistance coefficients describing the physical properties of porous materials. At low Reynolds numbers, the quadric term of velocity is close to zero, therefore can be ignored, which turns Forchheimer equation to Darcy's law. \( \alpha^{-1} \) is defined as the permeability of porous media.
Forchheimer equation can be modified as a friction factor and Reynolds number correlation (Andrade 1999):

\[ f = \frac{1}{Re'} + 1 \]  \hspace{1cm} (13)

where,

\[ f = -\frac{\Delta P}{L \beta \rho \langle u \rangle^2} \]  \hspace{1cm} (14)

and

\[ Re' = \frac{\beta \rho \langle u \rangle}{\alpha u} \]  \hspace{1cm} (15)

The formula can be used for calculating the friction factor of a porous medium with various geometries and porosities. The universal factors give a good comparison of different porous materials and flow conditions.

**DYNAMICS OF A RIGID BODY**

The curved fiber studied here is considered as a rigid body, which is treated as a continuum distribution of mass with a fixed shape and a body-fixed reference frame that characterize the position and orientation of the body in the space.

There are nine components that are used to describe the rotation of a rigid body (Allen 1987). Generally, Euler angles \( \phi, \theta, \psi \) are a suitable convention. \( \phi \) is the angle that rotates along z axis, \( \theta \) is the angle rotates
along \( x' \) axis, finally \( \psi \) is the angle rotates along \( z'' \) axis. There is a rotation
matrix related body-fixed coordinates to space-fixed coordinates.

\[
A = \begin{pmatrix}
\cos \phi \cos \psi & -\sin \phi \cos \theta \sin \psi & \sin \phi \cos \psi + \cos \phi \cos \theta \sin \psi & \sin \theta \sin \psi \\
-\cos \phi \sin \psi & -\sin \phi \cos \theta \cos \psi & \sin \phi \sin \psi + \cos \phi \cos \theta \cos \psi & \sin \theta \cos \psi \\
\sin \phi \sin \theta & -\cos \phi \sin \theta & \cos \phi \cos \theta & 0 \\
\end{pmatrix}
\] (16)

In body-fixed coordinates, the angular velocity is \( \Omega^b_x \) in the \( x \) direction,
\( \Omega^b_y \) in the \( y \) direction and \( \Omega^b_z \) in the \( z \) direction. The angular acceleration is
given by:

\[
\dot{\Omega}^b_x = \frac{\tau^b_x}{I_{xx}} + \left( I_{yy} - \frac{I_{zz}}{I_{xx}} \right) \Omega^b_y \Omega^b_x \\
\dot{\Omega}^b_y = \frac{\tau^b_y}{I_{yy}} + \left( I_{zz} - \frac{I_{xx}}{I_{yy}} \right) \Omega^b_x \Omega^b_y \\
\dot{\Omega}^b_z = \frac{\tau^b_z}{I_{zz}} + \left( I_{xx} - \frac{I_{yy}}{I_{zz}} \right) \Omega^b_x \Omega^b_y
\] (17) (18) (19)

where \( \tau_x, \tau_y, \tau_z \) are the torques exerted on the solid particle in body-fixed
coordinates for \( x, y, z \) directions respectively. \( I_{xx}, I_{yy} \) and \( I_{zz} \) are the three
principal moments of inertia.

A set of four quantities (quaternions) is used to calculate the
transformation matrix for convenience of computing (Evans 1977, Evans and
Murad 1977), which is \( Q = (q_0, q_1, q_2, q_3) \).

The quaternions are defined as:

\[
q_0 = \cos \frac{1}{2} \theta \cos \frac{1}{2} (\phi + \psi)
\] (20)
and the rotation matrix can be rewritten as

\[
A = \begin{pmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\
2(q_1q_2 - q_0q_3) & q_0^2 + q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\
2(q_1q_3 + q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2
\end{pmatrix}
\]

The quaternions for each solid particle satisfy the equation of motion.

\[
\dot{Q} = \begin{pmatrix}
\dot{q}_0 \\
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
q_0 & -q_1 & -q_2 & -q_3 \\
q_1 & q_0 & -q_3 & q_2 \\
q_2 & q_3 & q_0 & -q_1 \\
q_3 & -q_2 & q_1 & q_0
\end{pmatrix} \begin{pmatrix}
0 \\
\Omega_x^b \\
\Omega_y^b \\
\Omega_z^b
\end{pmatrix}
\]

These transform matrices are used to transform between body-fixed coordinates and space-fixed coordinates. The Gear predictor-corrector method (Gear 1966, 1971) is used to solve the system of first-order differential equations.

As in the translation case (Allen 1987), the modified leap-frog method is applied, which uses half time steps to update the positions and orientations. The algorithm is written as:
\[
\begin{align*}
V\left(t + \frac{1}{2} \delta t\right) &= V\left(t - \frac{1}{2} \delta t\right) + \delta \ddot{a}(t) \quad (26) \\
\tilde{r}(t + \delta t) &= \tilde{r}(t) + \delta \tilde{V}\left(t + \frac{1}{2} \delta t\right) \quad (27)
\end{align*}
\]

where, \(\tilde{V}\left(t - \frac{1}{2} \delta t\right)\) is the mid-step velocity of particle, \(\tilde{r}(t)\) is the current position of mass center of particle, \(\ddot{a}(t)\) is the acceleration of particle. The velocities leap over the coordinates to give the next mid-step values \(\tilde{V}\left(t + \frac{1}{2} \delta t\right)\). The current velocities may be calculated as

\[
\tilde{V}(t) = \frac{1}{2} \left[ \tilde{V}\left(t - \frac{1}{2} \delta t\right) + \tilde{V}\left(t - \frac{1}{2} \delta t\right) \right] \quad (28)
\]

A Verlet-equivalent algorithm (Verlet 1967, 1968) takes the form

\[
\begin{align*}
\tilde{r}(t + \delta t) &= \tilde{r}(t) + \delta \tilde{V}(t) + \frac{1}{2} \delta^2 \dddot{a}(t) \quad (29) \\
\tilde{V}(t + \delta t) &= \tilde{V}(t) + \frac{1}{2} \delta [\ddot{a}(t) + \dddot{a}(t + \delta t)] \quad (30)
\end{align*}
\]

The new positions at time \(t + \delta t\) are calculated by using equation (29), and the velocities at the mid-step are computed using

\[
\tilde{V}\left(t + \frac{1}{2} \delta t\right) = \tilde{V}(t) + \frac{1}{2} \delta \dddot{a}(t) \quad (31)
\]

The forces and accelerations at time \(t + \delta t\) are then computed, and the velocity move completed.
\[ \bar{V}(t + \delta t) = \bar{V}\left( t + \frac{1}{2} \delta t \right) + \frac{1}{2} \delta \ddot{\bar{a}}(t + \delta t) \]  

(32)

Positions and orientations of particle are updated by using quaternion \( \bar{Q} \). The method is described as follows:

\[ \bar{Q}(t + \delta t) = \bar{Q}(t) + \delta \dot{\bar{Q}} \left( t + \frac{1}{2} \delta t \right) \]  

(33)

\[ \bar{Q}\left( t + \frac{1}{2} \delta t \right) = \bar{Q}(t) + \frac{1}{2} \delta \ddot{\bar{Q}}(t) \]  

(34)

Now the positions and orientations are translated between two coordinates at a half time step. After that, the whole process is repeated.

STRUCTURE OF CURVED FIBER

A rigid curved fiber is shaped as an arc and its parameters are shown in Figure 1.

Three principal moments of inertia for this shape are calculated using following equations:

\[ L = R \cdot 2\theta , \ L \text{ is the contour length of the fiber} \]  

(35)

\[ X_G = \frac{R \sin \theta}{\theta} , \ X_G \text{ is the mass center} \]  

(36)

Moments of inertia related to the original point 0 can be calculated using the following equations. Then the plane motion theorem is applied to
transform the moments of inertia to its mass center.

\[ I_{xx} = \frac{m}{8} \left( 4R^2 + 5r^2 \right) \left( 1 - \frac{\sin 2\theta}{2\theta} \right) \]  
\[ I_{yy} = \frac{m}{4} \left( 4R^2 + 3r^2 \right) \]  
\[ I_{zz} = \frac{m}{8} \left( 4R^2 + 5r^2 \right) \left( 1 + \frac{\sin 2\theta}{2\theta} \right) \]  

When \( R \gg r \), equations 37 to 39 can be simplified as:

\[ I_{xx} = \frac{mR^2}{2} \left( 1 - \frac{\sin 2\theta}{2\theta} \right) \]  
\[ I_{yy} = mR^2 \]  

Figure 1: Arc shaped fiber and its parameters.
Physically, the level of bending can be changed by varying the angle \( \theta \), when \( r, L \) are fixed, the curl index is defined as \( C = \frac{L}{L'} - 1 \). \( L' \) is the end-to-end length of the curved fiber. The aspect ratio of the fiber is given by \( \frac{r}{d} \). \( d \) is the diameter of the fiber. The principal axes of the curved fiber are \( X-Z' \). \( X \) and \( Z \) are the coordinates of simulation channel.

The moments of inertia of a fiber in a simulation channel are more complicated in this case. The inertia can be obtained by the equations of an arc. By applying the parallel axes and rotation theorems to the fiber, the eigenvalues of the moment of inertia tensor can be calculated to obtain the moments of inertia of the principal axes.

When the parallel axes theorem is applied,

\[
I'_{xx} = I_{xx} + m\left( \Delta y^2 + \Delta z^2 \right) \\
I'_{yy} = I_{yy} + m\left( \Delta x^2 + \Delta y^2 \right) \\
I'_{zz} = I_{zz} + m\left( \Delta x^2 + \Delta y^2 \right)
\]  

By applying rotation theorem,

\[
I'_{xx} = I_{xx} - m\left( \Delta y \Delta z \right) \\
I'_{yy} = I_{yy} - m\left( \Delta x \Delta z \right) \\
I'_{zz} = I_{zz} - m\left( \Delta x \Delta y \right)
\]

where \( \Delta x, \Delta y, \Delta z \) are the displacements from body-fixed coordinates to space-
fixed coordinates.

The inertia of principal axes are the eigenvalues of the inertia of summary.

\[
\begin{pmatrix}
I_x \\
I_y \\
I_z
\end{pmatrix} = M \begin{pmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{pmatrix}
\] (45)

where \( M \) is a translation matrix.

To understand the migration and rotation of curved fibers and flexible fibers suspended in shear flow and pressure driven flow, a single fiber needs to be investigated first. Fibers with different curvatures and fixed aspect ratio will be investigated. Then, multi-fiber suspensions will be studied by considering collisions.

RESULTS AND DISCUSSIONS

Permeability in Porous Media

The fiber network is modeled with equal sized and random distributed cylinders in three-dimension. Fibers can be over-lapped. This geometry is believed to be close to many fibrous materials, such as paper, filters and textiles. The correlation of pressure drop vs. velocity is studied to further prove the importance of the quadratic term of velocity in three-dimensional fibrous materials. The effect of inertia is focused at a porosity range from 48% to 72%. The simulated permeability is compared with the experimental results (Lindsay 1993).
In order to drive the flow, a pressure difference is imposed between the two faces normal to the axis of the superficial flow by applying a uniform body force to the fluid. The LB method is modified to account for the applied external body force which adds every time step a fixed amount of momentum (gravity $\vec{G}$) on the fluid points at every time step (Ladd 2001):

$$f_o(\vec{x} + \vec{e}_o \Delta t, t + \Delta t) = f_o(\vec{x}, t) - \frac{1}{\tau}[f_o(x, t) - f_o^*(x, t)] + \vec{G}$$

(C46)

Cylindrical fibers are used to simulate the random network structure of fibrous media. The structure is generated by randomly placing every fiber into the simulation box. With this grown method, the orientation of each fiber is random in the $x$-$y$ plane. In the $z$ direction, the fibers are laid with an angle less than ±15-degree. If a fiber meets the nodes occupied by the other fibers, these fibers occupy the same nodes. The friction of the fiber network or the porosity is calculated by dividing the number of nodes occupied by fibers, by the total number of lattices nodes. Fiber in this simulation is 25µm in diameter and 1mm in length. The fibrous web is simulated at 0.1mm in thickness or $z$ direction. Three different grid resolutions have been tested (64x64x64), (128x128x64), (160x160x80). The maximum errors are less than 4.9%, therefore size effects can be ignored. To reduce the computational load, the data reported are based on the simulation box with lattices at 128x128x64. The geometry is illustrated as in Figure 2.
Notice that porosity depends on the fiber length, the diameter of cylindrical fiber and the orientation of fibers. It is evident that this structure is close to the fibrous web, e.g. paper handsheet. In this geometry, to achieve a porosity of 72% needs 18 fibers in random arrangement and 29 fibers for a porosity of 63%.

Fluid flows through the fiber network in the $z$ direction in order to simulate the transversal permeability. The $x$ and $y$ directions of the simulation box are periodical. A non-slip boundary condition is used at the fluid and fiber interfaces. The flow is induced by applying a body force on fluid particles. For a given porosity, the geometry of the fiber network is the same. There is no change of fiber positions and orientations for every different velocity. As the pressure gradient increases, the velocity of flow increases.

At a given porosity, the simulation data were fit to equation (12), and
the coefficients $\alpha$ and $\beta$ were estimated thereafter. The modified Reynolds number $Re'$ and the friction factor $f$ are calculated by using equations (13) and (4).

The simulated pressure gradient vs. velocity is plotted in Figure 3. As shown in the figure, curves with quadratic term of velocity fit the simulation data very well.

![Figure 3: Quadric curves fit the simulation data of pressure gradient vs. superficial velocity.](image)

The curves obtained by using the LB method captured the expected tendency and the important transitions. The curve fitting parameters $\alpha$ and
Table 1: Curve-fit parameters obtained from equation (13) and (14) for random cylindrical fiber network.

As shown in the simulation results, pressure gradients vs. superficial velocity curves are nonlinear after the superficial velocity is higher than 25 cm/s at 72% and 63% porosities. The curve of 48% porosity is more linear in that range, which indicated that the inertial force has less effect in the fiber network with low porosity (Koch 1997).

By plotting the modified friction factor \( f \) vs. modified Reynolds number \( \text{Re}' \), it is observed the transition zone from linear to nonlinear in terms of modified \( f \) and \( \text{Re}' \). The curves showed in Figure 4 agree with experimental data (Bear 1972). It is clear that the linear to nonlinear transition starts at \( \text{Re}' \) around \( 10^1 \), which also agrees well with that by Andrade et al. (1999).

The calculated permeability \( \alpha^{-1} \) for random fiber network is compared with experimental results of transversal permeability for hardwood. The comparison is given in Figure 4. The simulation results showed good agreement with those obtained by Lindsay et al. (1993) at porosity over 60%. A slight discrepancy may reflect the geometrical
difference between the real paper and the simulated fiber network model.

Figure 4: Modified friction factor and Reynolds number show the transition zone from linear to nonlinear of random fiber network. Solid lines are the fit to the Forchheimer equation, dash line is the fit to Darcy's law at low Re'.

Further information from the data in Figure 5 is that the permeability calculated with the quadric velocity term has reasonable accuracy of predicting the permeability of paper fiber network. The simulated fiber network model by this method estimated the fluid transportation of hardwood sheets well.
Figure 5: Comparison of permeabilities of three-dimensional LB simulation and experimental data.

Sedimentation of Single Curved Fiber

An arc-shaped fiber with aspect ratio of 5 is studied. Refer to Figure 1, \( \theta \) is set as 60° as shown in Figure 6.

A single curved fiber is assumed to settle along the Z-direction in an infinitely long vertical channel. The channel is filled with a Newtonian fluid. Gravity is the only driving force for the settling. Once the initial position of the fiber is given, the flow is determined by several parameters: \( \rho_s, \rho_f, L, d, C \), where \( \rho_s \) is the density of the solid fiber, \( \rho_f \) is the density of fluid. The density ratio is defined as \( \frac{\rho_s}{\rho_f} \).

Two curved fibers with different curl indices and one straight fiber with flat ends are studied. These fibers are shown in Figure 7. All the fibers are set at same aspect ratio.
Figure 6: Curved fiber and its parameters used in simulation.

Figure 7: Two curved fibers and one straight fiber studied.

The Reynolds number is calculated by $Re = \frac{VL'}{V}$, where $V$ is the
average terminal velocity of the fiber and \( v \) is the kinematical viscosity of the fluid. For a straight fiber, \( L' = L \). When a fiber settles under gravity, the mean terminal velocity and Reynolds number are controlled by the density ratio.

A simulation box of \( 140 \times 60 \times 300 \) lattice units was used. In this scale, the length of the fiber is \( L = 40 \), and the diameter of the fiber is \( d = 8 \), which gives the fiber an aspect ratio of 5. The mass center of curved or straight fiber is placed at 0.3W in the X-direction. The fiber settles along the Z-direction. W=140 is the width of the channel. The Y-direction is a periodical boundary. Walls are placed at X-direction. Nodes 0 represent left wall. Nodes 140 represents left wall. The curl indices of curved fiber were set as 0.2 and 0.57. Two different density ratios are 1.01 and 1.05.

**Lateral Migration**

It has been shown in many studies that the centerline of the channel is an equilibrium position of settling. The curved and straight fibers will reach the center of the channel regardless of their initial position. However, the different initial positions of curved fibers may result in different trajectories of migration. The following figure demonstrates how a curved fiber changes its position during the sedimentation.

Figure 9 shows the lateral migration of curved and straight fibers at the density ratio of 1.05. The Reynolds number of the curved fiber is \( \text{Re} = 31.2 \), the Reynolds number of the straight fiber is \( \text{Re} = 37.7 \).
Figure 8: Positions of fiber during sedimentation.

Figure 9: Lateral migration of curved and straight fibers at a density ratio of 1.05. The diamond and square symbols represent the curved fibers with their concaves close to the wall. The triangle symbol represents the curved fiber with its convex close to the wall. The X-axis is the position of the fiber mass center in the Z-direction.
In the first case of C=0.2, the curved fiber with the density ratio of 1.05 is released from the initial position (42, 30, 80) with zero initial velocity. The initial position is not in the center of the channel. The concave of the fiber is close to the wall. The curved fiber shows a lateral migration towards the wall first, and it is repelled towards the center of the channel later. This motion can be clearly shown in the Figure 10 with flow streamlines. The curved fiber generates two vortices at the ends of the fiber, due to the curvature and rotation of the fiber. The left vortex is more intense. The pressure difference moves the fiber towards the wall. After the curved fiber turns itself perpendicular to gravity, the intensity of the left vortex, as shown in Figure 11, is reduced and the wall pushes the fiber towards the center of the channel.

It is necessary to determine whether the initial direction of the curved fiber has influence on migration and rotation. For this purpose, a curved fiber with the curl index of C=0.2 and the density ratio of 1.05 was placed at the same lateral position. The only difference is that the concave of the fiber is far away from the wall. The results show that this curved fiber has no initial movement towards the wall and the fiber moves towards the center of the channel monotonically. It is obvious that the initial direction of the curved fiber will influence the trajectory behavior even when they are placed at the same initial position.

To compare the influence of curvature to migration and rotation, a curved fiber with the curl index of C=0.57 and the density ratio of 1.05 was studied (Figure 9). In this case, the curved fiber was placed with its concave is close to the wall. With a higher curl index, the curved fiber shows less movement towards the left wall before it starts to move towards the center.
of channel. On the contrary, the straight fiber at the same density ratio and aspect ratio moves towards the center of the channel without moving towards the wall first.

Figure 10: Flow streamlines of curved fiber with curl index of C=0.2 and density ratio of 1.05 at time step of 3,000.
Figure 11: Flow streamlines of curved fiber with curl index of $C=0.2$ and density ratio of 1.05 at the time step of 8,000.

Rotation

Studies showed that, without the influence of inertia, a non-spherical particle would fall along the stream under gravity with its long axis aligned vertically. When the inertia effect is considered, a non-spherical particle will rotate itself until its long axis becomes horizontal. In this study, the curved and straight fibers sediment under gravity and the inertia effect is considered. The rotation angle is defined as the absolute value of the angle between the Z-axis of the fiber in body-fixed coordinates and the Z-axis of the simulation.
box in a space-fixed coordinates.

The results show that the curved fiber always turns itself perpendicular to gravity at the studied Reynolds numbers. At the density ratio of 1.05, the Reynolds number of the curved fiber reaches Re=31.2. The curved fiber turns quickly to be perpendicular to gravity while the straight fiber takes a longer time (Figure 12). The curved fibers with different curl indices show the same tendency of rotation. With a larger curl index, the fiber rotates faster than the one with a smaller curl index. This can be explained by the different curvatures that associate with different moments of inertia. The curved fiber with the larger curl index induces a lower moment of inertia about its rotation axis than the straight one.

![Graph](image)

**Figure 12:** Rotations of curved and straight fibers at the density ratio of 1.05. In this case, the concave of the fiber faces to the left wall.
Figure 13: Angular velocities of curved fiber and straight fiber at the density ratio of 1.05. In this case, the triangle symbol represents the curved fiber with the curl index of $C=0.2$, and with its convex close to the wall. The diamond symbol represents the curved fiber with the curl index of $C=0.2$, but with its concave close to the wall.

The angular velocities shown in Figure 13 also demonstrate that the initial direction and curvature of fiber result in different rotations. When the concave of curved fiber is close to the wall, the fiber turns clockwise to be perpendicular to gravity. When the convex of curved fiber is close to the wall, the fiber turns counter-clockwise to be perpendicular to gravity. The straight fiber turns counter-clockwise as well. The curvature and initial direction of the
curved fiber have a significant influence on rotation.

The sedimentation of non-spherical particles differs quantitatively from the sedimentation of spherical particles. As shown in the results, the sedimentation of a curved fiber differs from that of a straight fiber.

At the same density ratio, the terminal settling velocities of curved fibers at different curl indices are different. Figure 14 shows the settling velocities of curved fibers at the density ratio of 1.01. The settling velocity increases rapidly when the fiber is just released. The fiber with a higher curl index can reach a higher maximum settling speed. The settling velocity becomes stable when the fiber is horizontal and close to the center of the channel. The fiber with a larger curl index has smaller overshooting speed as shown in Figure 14. The terminal velocity of the fiber with a larger curl index is higher than the one with smaller curl index. This may be explained by the drag force on different shapes of fiber.

Sedimentation of Two Curved Fibers

Symmetrical particles sedimenting in finite Reynolds number flows was discussed in several documents (Joseph 1987, Feng 1994a,b, Qi 1999). An important feature of the sedimentation is called drafting, kissing and tumbling (DKT). Drafting is the stage that when particles sediment under gravity, the trailing particle approaches the leading particle due to the low pressure in the wake of the leading particle. Kissing is the stage that the trailing particle is sucked into the wake of the leading particle and two particles contact. Tumbling is the stage that the trailing particle becomes the leading one, then the two particles part.
Figure 14: The terminal velocity of curved fibers at the density ratio of 1.01. The Reynolds number of curved fiber at $C=0.57$ is $Re=11.7$. The Reynolds number of curved fiber at $C=0.2$ is $Re=9.8$. The concave of fiber close to the wall.

Two curved fibers with curl index of $C=0.2$ were placed in an infinitely long channel. The channel is filled with Newtonian fluid. The simulation box is set at $80 \times 80 \times 300$. Fiber length is $L=40$. Fiber diameter is $d=8$. Aspect ratio is 5. The initial positions of curved fiber mass center are $(30, 30, 40)$ and $(30, 30, 80)$ respectively. The fibers settle along Z-direction. Y-direction of the channel is set as periodical boundary. In X-direction, nodes 0 represent the left wall, and
nodes 80 represent the right wall. Each fiber is placed as its concave side facing the left wall. The density ratio is set as 1.10. The curved fibers are allowed to sediment under gravity.

The following figure shows the displacement of two curved fibers in the X-direction.

![Figure 15: Displacements of two curved fibers in the X-direction. Fiber 1 is the trailing particle. Fiber 2 is the leading particle. Re=16.3.](image)

As shown in Figure 15, two curved fibers move slightly towards the left wall at the beginning of sedimentation. Then they are pushed far away towards the right wall. Two curved fibers finally swing back and become stable in the center of the channel. This confirmed that the center is the stable
position during sedimentation under gravity.

Displacement of two curved fibers is different compare to the displacement of a single curved fiber. In the case of a single curved fiber, the curved fiber is sucked toward the left wall first. Then the curved fiber is pushed towards the center of the channel. The curved fiber oscillates around the centerline until it is stable. In the case of two curved fibers, the fibers are pushed far away to the right wall and then become stable at the center of the settling channel without strong oscillation.

The following figure shows the streamline of flow at a time step of 10,000.

![Streamline of two curved fibers sedimenting under gravity at a time step of 10,000.](image)

Figure 16: Streamline of two curved fibers sedimenting under gravity at a time step of 10,000.
When two curved fibers sediment under gravity, they generate vortices on their sides. It does not show that the vortex appears at the back of the fiber. Two curved fibers keep their distance in Z-direction during the sedimentation. The following figure shows the displacements of two curved fibers in the Z-direction.

![Displacements in the Z-direction of two curved fibers during sedimentation.](image)

Figure 17: Displacements in the Z-direction of two curved fibers during sedimentation. Fiber 1 is the trailing particle. Fiber 2 is the leading particle. Re=16.3.

Two curved fibers turned themselves 90 degrees when they become stable in the center of the channel. The following figure shows the rotation angles of two curved fibers during sedimentation.
Two curved fibers show strong oscillation during sedimentation, especially when the two curved fibers are pushed far away towards the right wall. The rotation angles around time step of 50,000 show that the vigorous change when the two curved fibers are at the nearest position to the right wall. When the two curved fibers are pushed back towards the centerline of the channel, the rotation becomes smoother. Until two curved fibers reach the centerline of the channel, they turn themselves, becoming perpendicular to gravity. A fiber rotates itself, becoming perpendicular to the flow is observed in single fiber sedimentation. This shows that in finite Reynolds
number flow, the particle always turns itself to become perpendicular to the flow and becomes stable.

In this case, two curved fibers are placed far from each other. The wake effect of the leading particle is not clearly observed. This may tell that the drafting, kissing and tumbling only happens when two particles are close enough to each other, so that the wake generated by the leading particle is strong enough to trap the trailing particle.

When two curved fibers are placed closer to each other, a drafting-kissing-tumbling scenario is observed. In this case, a simulation box is set to be the same as 80×80×300. Two curved fibers with curl index of C=0.2 were placed at (30,30,40) and (35,30,75) respectively. Each fiber is placed as its concave side facing the left wall. The density ratio is set as 1.20. The curved fibers are allowed to sediment under gravity. The following figures show how two curved fibers sediment in an infinitely long channel filled with a Newtonian fluid. When two curved fibers are placed in such positions, the trailing fiber does not catch the leading fiber as fast as the trailing circular particle (Qi 1999).

As shown in Figure 23, the trailing fiber gets closer to the leading fiber at time step around 10,000, and then they part. Until time step over 26,000, the trailing fiber and the leading fiber exchange their positions, the leading fiber becomes the trailing one, and the trailing fiber becomes the leading one. Also shown in Figure 22, at time step of 26,000, two curved fibers contact each other at the ends; this is the kissing stage of drafting-kissing-tumbling scenario.
Figure 19: Two curved fibers sediment in an infinitely long channel at time steps of (left to right) 0, 3,000, 4,000 and 5,600. "T" represents the trailing fiber, and "L" represents the leading fiber.
Figure 20: Two curved fibers sediment in an infinitely long channel at time steps of (left to right) 6,400, 7,800, 10,000 and 12,200. "T" represents the trailing fiber, and "L" represents the leading fiber.
Figure 21: Two curved fibers sediment in an infinitely long channel at time steps of (left to right) 14,000, 19,200, 21,200 and 26,000. "T" represents the trailing fiber, and "L" represents the leading fiber.
Figure 22: The displacements in the X-direction for two curved fibers. Fiber 1 is the trailing fiber, and fiber 2 is the leading fiber. Re=45.35.

Figure 23: The displacements in Z-direction for two curved fibers. Fiber 1 is the trailing fiber, and fiber 2 is the leading fiber. Re=45.35.
Figure 24: The rotation angles of two curved fibers. Rotation angle is the absolute value of the angle between Z-axis of the fiber in body-fixed coordinates and Z-axis of the simulation box in a space-fixed coordinates. Fiber 1 is the trailing fiber. Fiber 2 is the leading fiber. Re=45.35.

Figure 25: The settling velocities in Z-direction of two curved fibers which are placed closer to each other.
Figure 24 shows that the rotation of two curved fibers is different compared to the rotation of single fiber sedimentation. When single fiber settles along the channel, it rotates to become perpendicular to the stream and oscillates around that position until it is stabilized. In the case of two curved fibers sedimentation, two fibers both turned 180-degree before they exchange positions. The leading fiber turns faster than the trailing fiber. At the time two curved fibers contact, the trailing fiber turn 180-degree and the leading fiber is at the same angle as it starts to settle. After this point, two fibers turn close to 90-degrees, which is the position close to perpendicular to the stream. At the same time, two fibers are apart again, and the leading fiber changes to be trailing one.

As shown in Figure 25, the trailing fiber accelerated during sedimentation to reach the leading one. Interesting enough, the trailing fiber accelerated twice during sedimentation. One was at the beginning of settling, also shown in Figure 22 and 23, two fibers are closer to each other in Z-direction, but they are pushed away from each other in X-direction. During the second acceleration, two fibers are close to each other again and exchange positions in Z-direction, at the same time, they pushed each other away in X-direction again.

It is clearly shown in Figures 26-28, that during the settling, the vortices generated by the leading particle are at its two sides. At the same time, the trailing fiber generates vortices at its sides as well, which pushes the leading fiber to turn faster than the trailing one. Because of this rotation, two fibers push each other away in the X-direction. Then they are pushed back by the fixed wall and rotate again. This rotation again sucks two fibers together,
and two fibers contact at the ends, then exchange positions.

Figure 26: Streamlines of two curved fibers settling under gravity at time step of 3,000. The upper fiber is the trailing fiber. The lower fiber is the leading fiber.

For comparison, two straight cylindrical fibers with same aspect ratio were placed in the same channel. The simulation box is 80×80×300. The length of the straight fiber is \(L=40\), and the diameter is \(d=8\). The aspect ratio is 5. The positions of each straight fiber are (40,40,45) and (40,40,90). Each straight fiber is placed with its long axis along \(Z\)-direction. The density ratio is set at 1.20. Again, in the \(X\)-direction of the channel, node 0 represent the right wall, and
node 80 represent the left wall. Gravity is the only driving force during sedimentation.

Figure 27: Streamlines of two curved fibers settling under gravity at time step of 10,000. The left fiber is leading one, and the right fiber is the trailing one.

Compared to the X-direction displacements in the case of curved fibers sedimentation (Figure 22), the displacements of two straight fibers are smoother (Figure 29). Two straight fibers are separated in opposite directions and then they pass each other and part again.
Figure 28: Streamlines of two curved fibers settling under gravity at time step of 20,000. The left fiber is leading one, and the right fiber is the trailing one.

The drafting, kissing and tumbling scenario was observed as well. As shown in Figure 30, two straight fibers keep their distance in Z-direction at the beginning of the sedimentation. Then two straight fibers move close to each other and exchange positions. The trailing fiber passes over the leading one at time step of 10,400. Comparing to two curved fiber sedimentation (Figure 23), two curved fibers exchange their positions at time step of 26,000. The straight fibers reach DKT scenario much faster than curved fibers at the same
aspect ratio and density ratio.

Figure 29: The displacements in X-direction for two straight cylindrical fibers. Fiber 1 is the tailing fiber, and fiber 2 is the leading fiber. Re=38.43.

Figure 30: The displacements in Z-direction for two straight cylindrical fibers. Fiber 1 is the tailing fiber, and fiber 2 is the leading fiber. Re=38.43.
The rotations of straight fibers (Figure 31) are also different from the rotations of curved fibers (Figure 24). In the case of two curved fibers sedimentation, two curved fibers both turn 180-degree during the settling. While in the case of two straight fiber sedimentation, two straight fibers turn a little bit over 90-degree and oscillate around 90-degree.

Figure 31: The rotation angles of two straight cylindrical fibers. Rotation angle is the absolute value of the angle between Z-axis of the fiber in body-fixed coordinates and Z-axis of the simulation box in a space-fixed coordinates. Fiber 1 is the trailing fiber, and fiber 2 is the leading fiber. Re=38.43.
In Figure 32, it shows that the trailing straight fiber only accelerated once during the sedimentation. Unlike in the case of two curved fiber sedimentation (Figure 25), the trailing curved fiber accelerated twice to reach the leading one. Similar to what happened in two curved fibers case, during the acceleration, two straight fibers are close to each other and exchange positions in Z-direction. But two straight fibers move away from each other in X-direction during the acceleration.

The following figures show the positions of two straight cylindrical fibers sedimenting in an infinitely long channel. The gradient of each fiber represents the two-ends of the fiber.
Figure 33: Two straight cylindrical fibers sediment in an infinitely long channel at time steps of (left to right) 0, 3,000, 4,000 and 5,000. "T" represents the trailing fiber, and "L" represents the leading fiber.
Figure 34: Two straight cylindrical fibers sediment in an infinitely long channel at time steps of (left to right) 6,000, 7,000, 8,000 and 9,000. "T" represents the trailing fiber, and "L" represents the leading fiber.
Figure 35: Two straight cylindrical fibers sediment in an infinitely long channel at time steps of (left to right) 10,000, 12,000, 14,000 and 16,000. "T" represents the trailing fiber, and "L" represents the leading fiber.

It is shown that two straight fibers kept their distance in Z-direction before the leading fiber turned over 45-degrees. As the leading fiber turns more, the trailing fiber gets closer to the leading one. Two straight fibers
contact at the ends. After two straight fibers contact with each other, they part and exchange positions. Streamlines of two straight fibers at time step of 5,000 and 10,000 are shown in Figure 36 and 37. When two straight cylindrical fibers start to sediment, two large vortices appeared at the two sides of the fibers along the long axis (Figure 36). The leading straight fiber starts to tile and generate a low pressure area between the leading fiber and the trailing fiber (Figure 37). This low pressure vortex attracts the trailing fiber to tumble, and the ends of the fibers to contact. After the contact, the two fibers are apart again. Like the sedimentation of two curved fibers, two straight fibers contact at the ends, then exchange positions. Unlike the sedimentation of two curved fibers, two straight fibers exchange positions in X-directions after they exchanged position in Z-direction. Two curved fibers exchanged positions in X-direction before they exchange positions in Z-direction.

Sedimentation of Four Curved Fibers

Four curved fibers were placed in an infinitely long channel and allowed to sediment under gravity. The curl index of the individual fibers is $C=0.2$. The length of each curved fiber is $L=40$. The diameter of each fiber is $d=8$. The aspect ratio of the curved fiber is 5. The dimension of the channel is $100\times80\times300$. The Y-direction is set as periodical boundary. Walls of the channel are set at X-direction. Nodes 0 represent the left wall. Nodes 100 represent the left wall. Curved fibers sediment along the Z-direction. The positions of fibers are as following: fiber 1 is at $(40,40,40)$, fiber 2 is at $(40,40,80)$, fiber 3 is at $(65,40,40)$, and fiber 4 is at $(65,40,80)$. Each fiber is placed with its concave side facing the left wall. The density ratio is set to be 1.20. The
positions of four curved fibers during sedimentation are shown in the following figure.

Figure 36: Streamlines of two straight cylindrical fibers settling under gravity at time step of 5,000. The lower fiber is leading one, and the upper fiber is the trailing one.

Fibers move towards the wall first, as shown in Figure 38. Two curved fibers that are placed close to the left wall are pushed away from the center of the channel. Two curved fibers that are placed close to the right wall move towards the right wall. Because of wall effects, two curved fibers, which have
moved towards the left wall, are pushed back by the wall, thus, moving towards the center of the channel. Two curved fibers that moved towards the right wall are pushed by the right wall as well. During the sedimentation, a cluster of fibers is observed. The fiber cluster settled faster than the single fiber.

Figure 37: Streamlines of two curved fibers settling under gravity at time step of 10,000. The right fiber is leading one, and the left fiber is the trailing one.
Figure 38: Four curved fibers sediment under gravity in an infinity long channel. From left to right, time step of 0, 5,000, 10,000, 15,000 and 20,000.

No fiber turns itself to be perpendicular to the stream in the figures shown above. Because of the fibers are not symmetric, the rotation is similar to the two fiber sedimentation. Fibers oscillate more than in the single fiber sedimentation.

When four fibers are placed a little further from each other, the displacements of fibers in X-direction are very different. In this case, fiber 1 is placed at (35,40,40), fiber 2 is placed at (35,40,80), fiber 3 is placed at (75,40,40), and fiber 4 is placed at (75,40,80). As shown in the following figure, fibers that are placed at the left side of the channel move towards the left wall. At the same
time, fibers that are placed at the right side of the channel move towards the right wall. They never moved towards the center of the channel.

Figure 39: The displacements for four curved fibers sediment in an infinitely long channel under gravity. The density ratio of fiber to fluid is 1.20. Re=3.08.

Figure 40: The rotation angles of four curved fibers sediment in an infinitely long channel under gravity. The density ratio of fiber to fluid is 1.20. Re=3.08.

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Figure 41: The displacements in X-direction for four curved fibers sediment in an infinitely long channel under gravity. The density ratio of fiber to fluid is 1.20. Re=3.08.

Figure 42: The rotations of four curved fibers sediment in an infinitely long channel under gravity. The density ratio of fiber to fluid is 1.20. Re=3.08.
Figure 42 also shows that when four fibers are placed far from each other, the wake effect is not strong enough to affect each individual fiber. Two fibers that are placed at the left side of the channel rotate the same way, so do the fibers placed at the right side of the channel. This is very similar to the case that two curved fibers are placed far away from one another.

CONCLUSIONS

The nonlinear behavior of fluid flowing through a three-dimensional random fiber network in the porosity range from 48% to 72% was numerically simulated by the lattice Boltzmann method. It was found that the curves of pressure gradient vs. superficial velocity are nonlinear after the superficial velocity is higher than 25 cm/s at 72% and 63% porosities. The curve at 48% porosity is more linear. It is shown that the inertial effect is important at relatively high Reynolds numbers. The relation between the modified Reynolds number and the friction factor are in excellent agreement with the Forchheimer equation. The results of permeability in the fiber network have good agreement with experimental data.

Curved fiber sediments in an infinitely long channel under gravity were studied. The results suggest that sedimentation of a curved fiber is very different from the sedimentation of axisymmetric particles. When multiple fibers sediment in the channel, the vortices generated by the curved nature of the fibers make the rotation and displacement of fibers more vigorous than for axisymmetric particles.

In the case of single fiber sedimentation, the migration and rotation of
the fiber are sensitive to curvature. The curved fiber always turns its long body perpendicular to the direction of gravity at all the Reynolds numbers tested. A fiber with a smaller curl index reaches the terminal velocity slower than the fiber with larger curl index. However, the terminal velocity of the fiber with larger curl index is higher than the fiber with smaller curl index. The fiber with higher curl index also shows smaller overshooting during settling.

It was demonstrated that the initial direction of the fiber affects the migration trajectory. When the concave side of the fiber is close to the wall, the fiber initially moves towards the wall due to the pressure difference. After the vortex intensity is reduced, the wall effect pushes the fiber to the center of the channel. The curved fiber turns clockwise to be horizontal under this condition. When the concave side of the fiber faces to the channel center, the fiber moves towards the center monotonically and turns itself counterclockwise to be perpendicular to gravity.

In the case of two curved fibers sedimentation, the position of each fiber affects the migration and rotation. The drafting-kissing-tumbling scenario is observed when two fibers are placed close to each other. During sedimentation, the wake generated by the leading fiber is not right behind the fiber. The vortices generated on the concave and convex sides of the individual fiber have different intensity. The vortices turn each fiber 180 degree during the sedimentation. The trailing fiber does not fall into the wake right away like what has been observed in circular particles sedimentation. It takes much longer for two curved fibers to reach each other. At the moment that two curved fibers contact, they tumble and exchange positions. The
leading fiber becomes the trailing one, and the trailing fiber becomes the leading one. The rotation of each fiber in the two fiber sedimentation is more vigorous than for single fiber sedimentation. The leading fiber turns much faster than the trailing one. Then two fibers turn close to be horizontal and close to the center of the channel.

For comparison, two straight cylindrical fibers with the same aspect ratio and density ratio were placed in the same channel. The sedimentation of curved fibers is significantly different to the sedimentation of straight fibers. The leading fiber turns faster than the trailing fiber. But no straight fiber turned 180 degree. The drafting-kissing-tumbling scenario occurred much faster than the sedimentation of curved fibers. The same as what happened in curved fibers case, two straight fibers contact at the ends of the fibers and tumble.

During two fibers sedimentation, the trailing fiber accelerates to reach the leading one. In the case of two curved fibers sedimentation, the trailing covered fiber accelerates twice during settling. When the trailing fiber accelerates, two curved fibers move far away from each other. In the case of two straight fibers sedimentation, the trailing straight fiber accelerates only once to reach the leading one and exchange positions. Similarly, during the acceleration, two straight fibers move away from each other.

When two curved fibers are placed far away from each other, the wake effect is not strong enough to affect the fibers. Then, the sedimentation of two fibers is similar to the migration and rotation observed in single fiber sedimentation. Two fibers move to the right wall and are pushed away by the wall, then move towards the center of the channel. When two fibers
become stable, they stay in the center of the channel and turn themselves to be perpendicular to gravity.

When four curved fibers are placed in the settling channel, the initial positions also have effects on the migration and rotation. When four fibers are placed far from each other, two fibers that are placed at the left side of the channel move towards the left wall, two fibers that are placed at the right side of the channel move towards the right wall. The center is not the final position of four fibers. Fibers do not turn to be perpendicular to the stream, because of the effect from the wall and the effect from other fibers.

Four curved fibers become unstable when they are placed close to each other. Two fibers that are placed closed to the left wall move towards the left wall. Two fibers that are placed close to the right wall move towards the right wall. Then the wall effect pushes the fibers towards the center again. One of the fibers is left behind. The other three fibers create a cluster during the sedimentation. The fiber cluster settles faster than the single fiber. The rotation of the fibers is very unstable as well. No fiber turns to be perpendicular to the stream in the case studied.

Sedimentation of curved fiber is different compared to axisymmetric particles. Because of the curvature, the migration and rotation are affected by not only the shape of the particle but also the position of the particle.
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