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How Secondary School Mathematics Teachers Construct an Understanding of "Appropriate Use" of Graphing Calculators in the Context of Collegial Inquiry

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HOW SECONDARY SCHOOL MATHEMATICS TEACHERS CONSTRUCT AN UNDERSTANDING OF "APPROPRIATE USE" OF GRAPHING CALCULATORS IN THE CONTEXT OF COLLEGIAL INQUIRY

by

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Marcia L. Weller Weinhold

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CHAPTER I: INTRODUCTION

Calculators Come to School

As soon as calculators and computers became affordable, some teachers began to use them in teaching mathematics. In 1974 the National Council of Teachers of Mathematics (NCTM) encouraged the use of calculators in mathematics classrooms (Hembree & Dessart, 1992, p. 23). The enthusiasm of some teachers and reluctance of others spawned what has become an ongoing debate, locally and nationally. A flurry of research studied the effects of calculator use in classrooms at all levels, and in 1986 Hembree & Dessart published a meta-analysis of 79 of these studies. Their results showed positive effects at all grade levels on tests of computation and problem solving when children were tested using calculators, and positive effects or no significant differences when students were tested without calculators, for all grade levels but grade four computation. In spite of these findings, falling calculator prices, and a renewed call for calculator use at all grade levels ("Calculators in the Mathematics Classroom," 1986), the curriculum remained largely unaffected.

Then, bracketing the release of NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), two important documents were produced by the Mathematical Sciences Education Board (MSEB) of the National Research Council: *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989), and *Reshaping School Mathematics: A Philosophy and Framework for*

Curriculum (1990). Recommendations for calculator use were made in all three documents, and NCTM included vignettes to show how calculators could be used effectively for learning mathematics with understanding. The controversy surrounding the use of this technology was not ignored, however. "Increased use of technology in mathematics education is inevitable, but wise use is not automatic" (Mathematical Sciences Education Board [MSEB], 1990, p. 18). Another cautionary note stressed that "Calculators and computers are not substitutes for hard work or precise thinking, but challenging tools to be used for productive ends" (MSEB, 1989, p. 84).

"Appropriate Use" Called For

Around the time of these landmark documents, supporters of mathematics education reform began to refer to "appropriate use" of calculators and computers in instruction (cf. "Board acts on Resolutions," 1988; NCTM, 1989,1991). More recently, *Principles and Standards for School Mathematics* (NCTM, 2000) includes a Technology Principle which asserts that "Students can learn more mathematics more deeply with the appropriate use of technology. ... In mathematics-instruction programs technology should be used widely and responsibly" (p. 25). Examples and vignettes have been available (Heid, Choate, Sheets, & Zbiek, 1995; NCTM, 1989; 1991) to help teachers get an idea of what "appropriate use" might look like, but few studies have been made of how teachers actually interpret this idea of "appropriate use" for their own classrooms.

How Prepared are Teachers to Use Graphing Calculators?

Recent surveys of technology use, such as the National Survey of Science and Mathematics Education for 2000 (Weiss, Banilower, McMahon, & Smith, 2001), often

raise more questions about calculator use in schools that they answer. For example, Weiss reports that although only 29% of responding high school mathematics teachers feel "very well qualified" to use "technology in the support of mathematics learning" (p. 23, 24), 80% of responding teachers had students using calculators in their most recent lessons (p. 74). When asked what specific things they felt well prepared to do with technology, teachers were not able to choose whether they were responding about calculator use or about computer use. However, two-thirds or more of responding high school teachers felt they could "use calculators/computers for drill and practice, ...to collect and analyze data, [and] to demonstrate mathematical principles" (p. 28). Also, over half felt prepared to use technology for "mathematical learning games" (p. 28). Only one of these most common choices seems to deal with calculators. Thus it seems reasonable to ask exactly what kind of uses students were making of calculators in those 80% of classrooms, and if and/or how their teachers decided these uses were "appropriate."

The MSEB prediction that use of technology is inevitable is supported by the National Survey of Science and Mathematics Education trend analysis (Weiss, Banilower, McMahon, & Smith, 2002). Surveyed teachers were asked to report specifically about one of their classes. The percent of these mathematics classes in which *graphing calculators* were used rose from 40% in 1993 to 77% in 2000, while the percent of mathematics classes for which teachers said graphing calculators were NOT needed dropped from 40% to 20% (p. 59). In Europe, where there has been lack of any official policy on calculator use, researchers have raised questions about what happens when students just begin using graphing or symbolic algebra calculators without any guidance

from teachers. Guin and Trouche (1999) point out that "while calculators are used by students, the French educational system has not properly acknowledged their use" locally (1999, p. 195). They agree with Artigue (1997) that when working with technology, student behavior that enhances understanding does not come naturally.

The last decade has witnessed increased numbers of professional development workshops and programs aimed at preparing teachers to use graphing calculators in school mathematics (Fleener, 1995b; Myers, 1998; Weiss et al., 2001). However, in spite of professional development, teachers who have become comfortable with technology, may not use it at all (Cuban, Kirkpatrick, & Peck, 2001; Norton, McRobbie, & Cooper, 2000; Windschitl & Sahl, 2002), or use it in a way that was not expected (Johnson, 1994; Zbiek, 1995).

Such findings lead us to ask whether there is something unusual about the teachers in studies that have shown effective teaching with calculators. Many of the researchers who have reported studies of teachers' or students' use of technology have worked with long-standing programs designed to improve instruction by using technology. Many use specially designed software and teachers trained or well-experienced with a particular technology (at times the teacher is the researcher). Some examples of these researchers and their projects are Heid, Blume, Zbiek, et al. (CAS-IM), Doerr (STELLA), Kaput (SimCalc), Dugdale (PLATO, Green Globs), Dick (Calculus Connections Curriculum Project), Schwarz et al. (CompuMath) and Yerushalmy (Geometric Supposer, Visual Math). In these types of studies, teacher decisions about technology use are not a focus. The researcher or project provides the materials used by the teachers. Only a few researchers, such as Simmt, have studied teachers that had no

particular preparation for using technology. Questions that remain open include what sort of preparation makes a difference, and how teacher knowledge of graphing calculators, for example, can be used in creating or selecting learning activities for their students. (See for example Zbiek, 1995.)

For teachers without special preparation beyond electing to attend a calculator workshop, their classroom use of calculators has been attributed to their beliefs and attitudes, not only about calculators, but also their beliefs about mathematics (Fleener, 1995b; Simmt, 1997; Tharp, Fitzsimmons, & Ayers, 1997). There are also issues surrounding the use of calculators that are not curricular issues. For example, using calculators changes the dynamics of the classroom in ways that cause disagreement among researchers about the benefits of such change. Ruthven saw great benefit in the increased use of technology in schools as a result of the calculator being seen as "personal technology" (1992). He notes particularly increased motivation to learn how to use a calculator if it is going to be available both at home and school. Yet Goos and colleagues found that the very personal nature of the calculator sometimes got in the way of students truly collaborating, and thus failed to provide a zone of proximal development for themselves in problem solving (Goos, Galbraith, & Renshaw, 2002). Little work has been done to determine how teachers who may have no formal preparation deal with the appearance of calculators in their classrooms, or how they develop the habit of mind that predisposes them to question what is "appropriate use?"

Studying the Complex System in Which Teachers Work

Because of the many factors that influence teacher decisions in the classroom, many researchers have developed combined methodologies which account for the social

environments in which teachers work, as well as the academic factors. Lave and Wenger's concept of *communities of practice*, more fully described in Wenger (1998), provides constructs that help describe the process of meaning-making to be expected in a group of teachers engaged in collegial inquiry. The community, for Wenger, is one of four components of a social theory of learning. The four components of learning in this view are Meaning – learning as experience; Practice – learning as doing; Community – learning as belonging; and Identity – learning as becoming (1998, p. 5). He asserts that the concept of practice should be understood at the level of negotiation of meaning (p. 72). Although he uses *community* and *practice* individually, he sees them both as abbreviations for *community of practice*, which he views as a unit. This is because practice is the source of coherence of the community which forms around it. Wenger's documentation of a community of practice among insurance claims processors provides the context in which he develops the language to characterize such communities. Wenger sees a community of practice as being defined by mutual engagement, joint enterprise and shared repertoire (p. 73). Most persons belong to several communities of practice at any given time, and move in and out of many such communities during their lifetimes. When they move between communities of practice, persons carry with them meanings learned in the communities in which they practice. When they relate their meanings to experiences in a new community, people act as *brokers,* perhaps opening possibilities for new meaning (p. 109).

In some cases, what passes between communities of practice are *artifacts*. These are physical objects containing *reifications* of a particular community in which the artifact has meaning. A *reification*, in turn, is the result of "the process of giving form to

our experiences by producing objects that congeal this experience into 'thingness'" (Wenger, 1998, p. 58). Artifacts must always be interpreted when they pass to another community. For an example of problems inherent in interpretation of state mathematics standards as artifacts, see Hill (2001). Researchers who have described communities of practice through which teachers experience professional development include Moore and Barab (2002), Franke and Kazemi (2001), Reynolds, Treahy et al. (2001), and Palincsar, Magnusson et al. (1998). This study seeks to describe how teachers act as brokers among their colleagues, and how they interpret artifacts during collegial inquiry.

Statement of the Research Questions

In order to begin to develop a sense of how teachers construct an understanding of "appropriate use" of graphing calculators, three questions will be investigated. Each will examine the collegial efforts of a group of secondary mathematics teachers as they create a shareable tool to aid in making decisions on "appropriate use" of graphing calculators in their classrooms.

1. What are the issues that teachers focus on when constructing an understanding of "appropriate use" of graphing calculators, and how do they negotiate those issues?
2. How do teachers in collegial discussions about "appropriate use" of graphing calculators incorporate their prior instructional experiences and artifacts that reify the ideas of others?
3. In collegial discussions about "appropriate use" of graphing calculators, what influence is exhibited by contextual factors such as the curriculum used and

student characteristics, and by teachers' beliefs and conceptions about graphing calculator use and about the nature of mathematics?

Overall Description of the Study

Participants

Participants for this study were initially recruited from high school mathematics teachers, defined as teaching at least one high school mathematics course during the time of the study. They were required to have access to graphing calculators for use in their classrooms. Recruitment at first concentrated on two large districts in southwestern Michigan, but the decision by one large high school faculty not to participate necessitated expanding recruitment to middle schools as well as other districts. Mathematics supervisors of surrounding districts were approached to seek permission to contact teachers, and to ask if professional development credit could be granted for participants. A letter of invitation was sent to individual teachers at their schools, including a description of the project, a tentative timeline, and whether or not they would receive professional development credit. A sample invitation packet can be found in Appendix A. The five participants who eventually agreed to take part were from the same large district, but three different schools, two high schools and one middle school.

Data Collection

The main work of the teacher participants in the study was development of a shareable tool to be used by teachers in making decisions about appropriate use of graphing calculators. This work took place in a series of four professional development

study group sessions. The discussions of these sessions were audiotaped and transcribed. At the end of each session, teachers wrote reflections on what was discussed and on their emerging understanding of appropriate use of graphing calculators to teach mathematics. Also, any written work on the development of the tool was archived for analysis.

Before and after these four sessions, teachers were individually interviewed. Part of the interview asked the teacher to sort 20 mathematical tasks according to whether calculators would "always," "sometimes," or "never" be appropriate to use with the tasks, or whether the teacher would just not use the task with students. For those tasks rated "sometimes," teachers were asked to explain the conditions under which calculators use would be appropriate.

Teachers also filled out a survey before and after the study group sessions. The survey began with several open-ended questions, then asked for background information, and finally asked for a degree of agreement, using a 5-point Likert-type scale, with statements about mathematics, calculators, and how students learn mathematics. In the final interview, only selected items of the survey were reviewed. For these items, teachers were asked if they would still answer the same, or, if not, why they would change their answers. The interviews were audiotaped and transcribed, and the survey and task sort responses were kept for analysis.

In each time interval between study group sessions, teachers gave permission for the researcher to observe one class and do a follow-up interview to the class. Field notes were made in each classroom observation, as well as the use of an observation checklist. Teacher interviews were audiotaped and transcribed. Some classroom activities which were not from the class textbook were collected as artifacts.

Analysis of Data

Analyses of the task sorts and surveys use descriptive statistics. Because the numbers are few, no generalizations are attempted of these results. However, comparison of teachers to each other and to themselves at the end of the sessions helps make inferences about whether and to what extent the work on the tool brought participants closer to a common understanding of appropriate use of graphing calculators to teach mathematics.

Analysis of the transcripts followed a system of coding the data with symbols relating to the three research questions. This allowed two types of analysis: looking at a chronology of statements by individual teachers to track their own trajectory toward a communal understanding of appropriate use of graphing calculators, and looking at statements made by different participants during the same time period to understand the negotiation that took place in working toward the common understanding.

Significance of the Study

In a recent review of published research on the use of graphing calculators at the secondary level, Burrill and colleagues (2002) identify gaps in the literature. Some of these gaps have been made more pronounced by recent research, and some are new areas of inquiry that have become more pressing. One of the questions framing the Burrill review is "How do teachers use handheld graphing technology and how is this use related to their knowledge and beliefs about technology, mathematics, and teaching mathematics?" (p. 13). In the section describing the gaps in the literature relating to that question, Burrill et al. assert that "Future research should seek to explore in greater depth

the relationships between the use of handheld graphing technology and the classroom norms that give meaning and purpose to those uses" (p. 16). The contention of the present study is that there is another relationship between the use of calculators and established norms that shapes individual teacher's use of handheld graphing technology. This other relationship is the teacher's relationship to the goals and norms of the community of practice (Bohl & Van Zoest, 2002; Wenger, 1998) in which he or she participates.

Another recommendation of the Burrill review is "Teachers should have opportunities to reflect upon and discuss their beliefs about mathematics, teaching, and learning in relationship to their knowledge and beliefs about the use of technology in the mathematics classroom" (Burrill et al., 2002, p. 19). The study group sessions of the present study provide an opportunity to investigate the benefits of collegial discussion in developing a common understanding of appropriate use of graphing calculators in the teaching of mathematics.

This study may also open the door to longitudinal studies that will test whether such communities formed to study appropriate use of calculators are robust, and can take in new members as well as continue the evolution of the understanding of "appropriate use" even as technology evolves to allow uses as yet undreamed of. Another possible significance of this study will be examined when a future community of practice tests whether the tool which is a product of this study, or the artifacts examined in the process, are sufficient to nurture change in beliefs, conceptions and use of calculators for its own teachers.

Even if the change observed in teachers is not impressive, this study will have given the five participants the opportunity to begin an inquiry they may continue in other

settings. It also provides understanding of the workings of a learning community formed to explore appropriate use of graphing calculators. This understanding will provide a basis for further development of processes that will encourage deeper exploration of appropriate use of graphing calculators in the secondary mathematics classroom among teachers who are willing to take up the inquiry.

CHAPTER II: LITERATURE REVIEW

The research questions assume that there will be use of graphing calculators in secondary mathematics classrooms, and also that an understanding of "appropriate use" is desirable among teachers working in the same school, and perhaps in broader communities of practice. The literature to be reviewed will provide further detail from documents cited in Chapter I, especially in application to professional development of teachers, and will also provide motivation for a concept of "appropriate use." The research questions also assume that teachers make decisions about how they use calculators themselves, and also about how students use calculators. Hence, what is known about the issues that influence teacher decisions about calculator use is also reviewed. This includes what students do with graphing calculators.

While conceding that "we are not sure how best to teach mathematics with [technology]" (1989, p. 62), MSEB listed the societal benefits of increasing the use of such tools in the teaching of mathematics:

> School mathematics can become more like the mathematics people actually use...Calculators in the classroom can help make higher mathematics more accessible. ...Mathematics learning can become more active and dynamic ... Students can explore mathematics on their own, ...[and] mathematics study can build long-lasting intuition and insight, not just short-lived strategies for calculation (1989, pp. 62-63).

In *Reshaping School Mathematics*, MSEB described a number of transitions that were already underway. "The teaching of mathematics is shifting from primary emphasis on paper-and-pencil calculations to full use of calculators and computers" (1990, p. 5). This shift was in part attributed to changes in the world of mathematics. Computers and

calculators "have affected not only what mathematics is important, but also how mathematics is done. ... [C]hanges in mathematics brought about by computers and calculators are so profound as to require readjustment in the balance and approach to virtually every topic in school mathematics" (p. 3). In a section on new priorities, the MSEB asserted that "Computers and calculators change what is feasible and what is important" (p. 20).

Trends in the Use of Technology

Research by Weiss et al. (2002) suggests that the MSEB's inclusion of both computers and calculators as "technology" to be used in teaching mathematics was insightful. For instance, it is noteworthy that the percent of mathematics classes whose teachers felt that *computers* were NOT needed rose from 29% to 35% from 1993 to 2000. Weiss and colleagues contend that this presumably reflects the sophistication of the available calculators. However, the percent of mathematics classes actually using *computers* rose from 44% to 60%. All of these changes were statistically significant (Weiss et al., 2002, p. 59). We see from this that, according to what teachers say, change is happening.

Cuban and colleagues (2001), however, report surprisingly little use of computer technology in schools that have high access. Their explanation for lack of computer use was based on the structure and climate of the school. They saw a difficulty in changing established patterns of time division and departmentalization of content studied in high schools. There was little opportunity for "cross-fertilization of ideas" (p.828). Also, when computers were used in class, it made teachers' jobs harder; often leading to exhaustion, and eventually to leaving the school for better-paying jobs and more advanced, more

reliable technology. These barriers to use of computer technology argues for a focus on graphing calculators.

Effects of Using Calculators in Mathematics Classes

We turn now to the literature that describes research on the use of calculators, and teachers' thinking about such use. Hembree and Dessart (1986; 1992) conducted two meta-analyses, one of studies of non-graphing calculator uses, and a second which included several studies of graphing calculator use. The preponderance of evidence indicates advantages gained on tests of computation and of problem solving by students who had studied mathematics using calculators. These gains were evident both on pencil-and-paper tests, and on tests for which students were allowed to use calculators. Positive change was also shown in students' attitudes toward mathematics. When Hembree and Dessart prepared their second study for the 1992 NCTM yearbook (Fey & Hirsch, 1992), no empirical studies were yet available on "how to integrate the calculator directly into the learning process," and little research had been reported on graphing calculators. (pp. 30-31) In that volume, however, many examples of uses of graphing calculators in schools were reported.

In Dunham's (1994, 2000) updated review of the literature reporting research on use of calculators and graphing calculators, she concluded that contradictions remain between what teachers believe and what research shows on topics such as student loss of basic computation skills as a result of calculator use. Her challenge to "design inservice and education programs that not only prepare teachers to teach with calculators but that also challenge their beliefs about mathematics and mathematics instruction" is heeded in the present study.

Some research also began to examine how curricula incorporating use of graphing calculators affects students' ability to perform on mathematical tests. Hirschhorn and Senk (1992) report that 7th and 8th grade students using scientific calculators with University of Chicago School Mathematics Project (UCSMP) teaching materials were able to out-perform students in comparison classes not using calculators or UCSMP materials on items dealing with percent, exponents, and exponential growth. Huntley and colleagues (2000) similarly report that a reform curriculum developed by the Core-Plus Mathematics Project (CPMP) was "more effective than conventional curricula in developing student ability to solve algebraic problems when those problems are presented in realistic contexts and when students are allowed to use graphing calculators" (p. 328).

Ruthven (1992) sees the fact that graphing calculators are *personal* devices as crucial to their affect on student thinking. Students may begin by imitating traditional solution methods, but the more familiar they become with the calculators, the more likely they are to use innovative solution methods. One such method uses the repeat calculations that calculators perform on previous answers to iteratively examine exponential growth. Another, which Ruthven calls "trial-and-improve," (p. 94) involves guessing and using the calculator to see how much improvement is needed in the guess. Ruthven notes that the initial guesses and methods of improvement become much more sophisticated and efficient as students have more exposure to the functions they are working with. He reports that students who have worked with graphing calculators are much more adept at describing symbolically the graph of which they are given a plot than are comparison groups that have not used graphing calculators. Among those using graphing calculators, there was also a lack of difference between males and females in

their ability to handle these questions, compared to a significant difference advantaging males in the non-calculator comparison groups.

A more recent survey of research on the use of graphing calculators in secondary mathematics classrooms (Burrill et al., 2002) reports gains for students using calculators over control groups not using calculators. Gains in conceptual understanding were particularly pronounced when the use of calculators included a change to a curriculum specifically designed for calculator use. Continuous use of graphing calculators was also tied to student ability to see functions as objects rather than operations (p. 38). Students using calculators also outperformed the comparison groups in solving multi-step problems and problems using applications and real data (p. 39). Burrill reported that the knowledge and skills students learned while using calculators were essentially those that they were taught, explicitly or implicitly (by teacher example, for instance). The biggest differences in how well they learned seemed to be related to how much access students had to calculators (p. 34). The studies reviewed by Burrill et al. that dealt with what students actually did with the calculators took place in interview situations outside classrooms. Many of the tasks given students in these studies directed them to use a graphing calculator, or to display or interpret a graph (p. 22). In one study designed to give students a spontaneous choice in calculator use, students chose to use a graphical approach rather than an algebraic approach for tasks such as "$x^3 - 3x = \ln x$" or "$\sin x + 2\cos x = 3/2$" (p. 23).

What Students Actually Do with Calculators

The National Survey of Science and Mathematics Education does provide some information about what teachers say they and their students are doing with computers and

calculators, but there is no way to separate what is done with which type of technology. Weiss et al. found that the most frequent use of calculators/computers at the high school level,was for taking tests or quizzes, followed closely by doing drill and practice. In roughly half of high school mathematics classes surveyed, calculators/computers were used to demonstrate mathematics principles on at least a weekly basis (2001, p. 73). Weiss and colleagues also constructed composite scores for teaching practice, one of which was "Use of Calculators/Computers for Investigation." Included in this composite are certain questions about how students used calculators/computers in classes – record, represent and/or analyze data; use calculators/computers as tools (e.g. spreadsheets, data analysis); do simulations; collect data using sensors or probes; retrieve or exchange data; solve problems using simulations (p. E-26). The mean score for classrooms of grades 9 – 12 for this composite was 31 out of a possible 100 (if each use occurred in each class period). This compares to a score of 34 for grades 5 – 8 classrooms (p. 75). This suggests that investigative use of calculators or computers decreases from middle to high school.

No distinction is made between scientific, four-function or graphing calculators in any questions of the Weiss survey, except one. This question asks, "For the following equipment, please indicate the extent to which each is available, whether or not each is needed, and the extent to which each is integrated in this mathematics class" (Weiss et al., 2001, Appendix B, Mathematics Questionnaire, p. 12). The report states only that graphing calculators were used "at some point" in 77% of grades 9 – 12 mathematics classes, compared to 26% of grades 5 – 8 classes (p. 88). Only 4% of high school teachers report that their students never "use calculators or computers to develop conceptual understanding," and 3% report that their students never "use calculators or

computers for learning or practicing skills" (p. 73). Thus we have some indication of activities for which students do and do not use calculators, but it is unclear what types of calculators are used and how the calculator use accomplishes the goal claimed.

Teachers were also asked (1) about the professional development they felt they needed, (2) about the emphasis of the professional development in which they chose to participate, and (3) about whether the professional development changed their teaching. The area which drew the highest percent of positive responses from high school teachers for all three of these questions was "learning how to use technology in mathematics instruction" (pp. 37, 43, 44). It is clear from the report of Weiss and colleagues that technology is being used more than ever in mathematics classrooms. Yet teachers express uncertainty in their preparedness to use technology, and desire professional development to enhance their use of technology. What the Weiss report leaves open is whether the reported changes made in teaching practice were limited to those categories in which these teachers felt well-qualified with technology.

Other researchers have gone into classrooms to investigate how students are actually using calculators. Dick (1992) reported a student's correct non-traditional solution to an integration problem, wholly based on his analysis of the calculator-drawn graph of the functions in question, rather than on his knowledge of the symbols. Ruthven (1992) also notes being surprised by students using graphical iterations rather than numerical iterations in a problem.

Doerr and Zangor (2000) classified students' strategies for using graphing calculators in five overlapping modes: (1) computational tool, (2) transformational tool, (3) data collection and analysis tool, (4) vizualization tool, and (5) checking tool. They do

not note the proportion of each of these uses, but do emphasize the role of the teachers' understanding of the calculator in promoting the rich diversity of calculator use. Several other studies in Burrill et al. (2002) reported that students used graphing calculators to draw graphs, but none showed a distinct preference for graphical solutions when a graph was not asked for (p. 24). Results were mixed, however, on whether students were using graphing calculators to check algebraic work (p. 25). More than one study concentrated on misunderstandings that were either caused or exacerbated by the graphing calculator's limitations, especially scaling of graphs, or accepting the first view of a graph as definitive (p. 25). Howere, Doerr and Zangor (2000) found that the knowledge of the teacher in their study was able to minimize such misunderstandings or to make their correction a focus of learning to use the calculator.

Zbiek (2002a) developed a framework for looking at students' work, but worked specifically to define categories that would transcend any particular mathematical concept and any mathematical tool (p. 14). Her framework does not take into consideration the role of teachers when students are using calculators. Earlier, Zbiek (2002b) had developed a two-tiered framework of categories for student use of technology that did not extend to representations. These categories describe various ways that students might use technology to check answers, get information, delegate work, get solutions, or improve presentation of their ideas to others. The richness of student work exhibited by these categories, as opposed to teachers' initial reactions to student use of calculators, influenced the researcher's decision to introduce these categories to teachers during the first study group session.

Guin and Trouche (1999) identify five types of student interaction with graphing calculators, and use these to study the various ways that students make the transition to symbolic calculators (the TI-92). These interactions were termed *random, mechanical, resourceful, rational,* and *theoretical.* They call a 'successful' transition the "instrumentation" of the calculator, after Vygotsky, as extended by Verillon and Rabardel (Verillon & Rabardel, 1995). They see "instrumentation" as a psychological construct involving the potentialities and constraints of an artifact (the calculator), together with the knowledge of a student, resulting in a reorganization of activity with and without the calculator. As a result of their study, Guin and Trouche see instrumentation of the symbolic calculator as a two-step process. The first step involves student discovery of the various commands of the calculator, their effects and organization. The second step is "characterised by a pruning attitude towards the first strategies"(1999, p. 214) which were essentially trial and error. This second step emerges while students are becoming familiar with the constraints and potential benefits of the tool, and are beginning to be able to mistrust its results.

Guin and Trouche contend that only those students with enough mathematical background (generally those with *resourceful, rational,* and *theoretical* interactions) to explain the discrepancies between the discrete nature of graph images and the mathematical object are willing to undertake "the additional work of adapting to the machine with positive effects." The others "often gave up any idea of understanding, copying the formula into their notebook without any interpretation" (Guin & Trouche, 1999). The successful negotiation of the second stage was characterized by combining all information sources (including paper-and-pencil work, peers, text, and calculator) in the

solution of problems. Guin and Trouche argue that this cannot be accomplished without "an explicit intervention at a conscious level" (p. 222). Specifically, they state that teachers often underestimate the amount of mathematical knowledge required for efficient instrumentation. Thus the selection of tasks that bring students to realize that calculator results must be judged mathematically is a crucial role of the teacher. They also stress that student work on calculators should be made visible through the use of an overhead calculator when students are sharing their ideas. Tasks should introduce a limited number of commands at once. Sufficient time should be spent in verifying all the types of representations available on a calculator. And finally, by means of class discussion, calculator techniques that are clearly understood and/or efficient should be "institutionalised" (p. 225). We might also say they should become part of the sociomathematical norms of the class (Hershkowitz & Schwarz, 1999). The process of "instrumentalization" described by Guin and Trouche agreed with the researcher's own classroom experience, and supported the decision to try to make the possibility of a second stage of student calculator use feasible to teachers.

Guin and Trouche gave detail about what students did, but only the barest information about how the teacher organized the students' work. The tasks used in the study were presumably provided by the researchers. There has been some study of what teachers do with calculators, and how that affects students. While studying student achievement and attitudes toward calculators in a college algebra class, Smith and Shotsberger (1997) found no differences in achievement or attitude toward mathematics between students using graphing calculators and those not using calculators. However, they did find a difference in the amount of time students reported using graphing

calculators when studying for each of two instructors. They attributed this to the different philosophies of the two instructors. The instructor whose students reported less calculator use in studying used the calculator only to check answers that were found in traditional ways. The other instructor used the calculator for demonstrations, for checking answers, and to replace other more traditional methods of solving problems. This finding on the limited ways calculators were used suggests that teachers' knowledge of how to use calculators is not sufficient for their creative use in teaching mathematics.

Overall, the studies reviewed in this section helped orient the researcher to a broader range of possibilities of student calculator use and teacher attitudes that could support or limit that use.

Effects of Teacher Beliefs and Attitudes on Classroom Use of Technology

Fleener (1995a; 1995b) adapted a survey to use with self-selected teachers concerning their use of graphing calculators. The 29-item Attitude Instrument for Mathematics and Applied Technology - Version II (AIM-ATII) removed the affective and experiential items of her earlier AIM-AT, and added more items on the types of calculator uses. In both studies, one using each of the surveys, she found that teachers could be classified by their response to a single item on whether students should master conceptual ideas before or after using calculators. When that classification was made, the two groups differed significantly on most of the other items on the survey. This indicates that the philosophical division affects many of their other decisions about the use of graphing calculators in the classroom. A similar, but less decisive division occurred when amount of experience using calculators was used to form two groups of respondents. Fleener's (1995a) study comparing experienced teachers and preservice

teachers indicated that there is a development continuum that should be taken into consideration when planning for work with teachers aimed at changing their use of graphing calculators.

Although Jeon (1999) did not survey her subject's beliefs as Fleener did, she noted a difference in the way calculators were used in two classes taught by the same teacher. The teacher Jeon studied emphasized not only the graphing approach, but basic algebraic skills as well as connecting algebraic and graphing approaches in her second-year algebra course. However, in her third-year algebra and trigonometry course, she did many mathematical experiments and tested the students' ideas with the help of the graphing calculator. Johnson (1994) had noted even more striking differences in two classes taught by the same teacher. Calculators were used in a class for students bound for 4-year college, but calculators were not used in a class for students intending to attend a technical college. The teacher in Johnson's study based his decisions on his understanding of the expectations of local colleges of each type, with which he made intentional contacts for the purpose of better preparing students for their intended future endeavors.

Similar to the previous section, the reports in this section prepared the researcher to recognize a variety of teacher attitudes and the sorts of influences they might have on teacher behavior with regard to graphing calculator use.

On Teachers' Concerns about Technology

One dissertation study (Myers, 1998) asked 21 teachers, self-selected by attending a workshop on the use of the computer algebra system (CAS) of the TI-92 graphing calculator, to express any concerns about using calculators with symbolic algebra

capabilities. Responses were recorded from interviews and surveys. It was evident that these teachers did not accept indiscriminant use of calculators. Their concerns included student dependence on the machine rather than understanding, loss of manipulation skills, poor preparation of teachers, standardized tests, and need to adjust classroom tests. The teachers seemed more ready to err on the side of caution, yet a third of them saw benefits out-weighing risks. Two teachers felt that the number of students helped would far exceed the number whose education might be harmed. In terms of the uses that would be made in the classroom, teachers were not specific (few had actually used the calculator yet by the time of the interviews), but seemed to be most concerned about whether to use the calculator before or after teaching pencil-and-paper skills.

Results in the Myers study provided the researcher with issues to include in the survey for the present study, as well as alerted her to the possibility that these issues of CAS use might not be the same as those for non-CAS graphing calculators.

How Teachers Make Decisions about Using Calculators

There is a paucity of research relating to how teachers actually make decisions about using graphing calculators in their classrooms. Simmt (1997) came closest to addressing the concerns the present study explores. She observed and interviewed six teachers as they used (or did not use) graphing calculators in a single unit of instruction. She examined how the teachers used the calculators and why they used them as they did. She also questioned teachers to uncover their philosophies of mathematics and mathematics education. Simmt concluded that teachers who already valued ways to vary their instruction welcomed the calculators as a new variation, whereas those who used direct instruction saw calculators as an unnecessary addition. The only indication of

change in teacher attitudes Simmt found in this one unit of calculator use was that one of the teachers who favored lectures saw the calculator as an occasional way to present content more efficiently. Simmt did not make any attempts at intervention, either to help teachers understand how to use calculators or why they were being mandated. Because the present study does intend to promote such understanding, Simmt's study lays out the challenge of the present study by indicating how unlikely it is that any short intervention will change attitudes or beliefs.

It is noteworthy that both Fleener and Simmt used self-selected participants who intended to use graphing calculators, and yet there were broad differences in their beliefs and attitudes. Even broader differences might be expected among faculty intentionally recruited from teachers who do not use calculators – either never have, or tried them and abandoned them, as well as from teachers who are known to use calculators. The researcher was forewarned that some beliefs would be deeply entrenched, and some misconceptions would be taken as fact.

Tharp, Fitzsimmons, and Brown Ayers (1997) found a similar division of teachers with their pre- and post survey of teachers taking a telecourse on the use of graphing calculators. Teachers who thought of mathematics as rule-based were more likely to see calculators as unhelpful, and perhaps detrimental to mathematics learning, whereas those who did not see mathematics as predominantly rule-based were more likely to welcome graphing calculators. The 'rule-based' teachers tended to use lectures as their instructional strategy, and when faced with student exploration of calculator uses they themselves were not familiar with, they "returned to the lecture in order to control the calculator use and avoid embarrassment" (p. 565). Like Simmt's teachers who did not value varying

their instruction, these teachers abandoned the calculators rather than change their instruction. In the case of teachers who did not think of mathematics as strictly rule-based, the telecourse was able to change the views that they held of graphing calculators by providing videos of a variety of classrooms in which calculators were used to help students understand mathematical concepts. When they saw these examples, teachers were more open to using calculators in their own classes because they could envision the ways in which their students would be helped. Although the present study would not use videotapes, the researcher was able to select specific lessons she observed that could be used as a basis of discussion of the innovation possible with graphing calculators.

Szombathelyi (2001) conducted a detailed survey examining personal factors that influence high school and college mathematics teachers' decisions to use calculators in their classrooms, interviewing a sample of them. She also asked teachers to classify themselves as novice, intermediate or expert users of calculators and to describe what that classification entailed. There was very close agreement on how the teachers described the categories, based on the intensity and depth of their use of calculators in their classes. Her observation sample, selected from volunteers of the surveryed group, included teachers from each of the three categories.

Szombathelyi (2001) found that the most important personal factors influencing teachers' use of graphing calcualtors were their beliefs, their familiarity with graphing calculators, their professional development and their mathematical and pedagogical knowledge. Their teaching experience and educational background did not play a determining role intheir decisions. She found that the differences in calculator use were that novices used calculators only for demonstration (even thought their students had

access to graphing calculators) and aswer checking, intermediate users would also use calculators for intorducing new material, but experienced users incorporate calculators every day, especially in problem solving and discovery learning.

Along with her findings, Szombathelyi (2001) reported on concerns that teachers raised. They were concerned about improper use of graphing calculators, or that calculator use would detract from understanding of concepts. Teachers were also concerned about the possible conflict that would arise when students had successive mathematics teachers who had different policies on calculator use. Standardized tests were also a concern.

Selection of Appropriate Tasks

One study which paid attention to the decisions teachers make about using calculators is reported by Branca, Breedlove and King (1992). They describe the actions of the teachers in one middle school who used calculators to answer the challenge they face in organizing instruction "so that it attracts, and develops the abilities of, the greatest number of students possible" (p. 13). These teachers felt that calculators must be seen as an important tool, not a panacea. Hence the decisions they make should be deliberate. The teachers produced a list of questions to be considered when making decisions about calculator use.

- Does the calculator allow the students to get closer to mathematical concepts being presented?
- Will the use of the calculator in mathematics activity increase student confidence and persistence?
- Could the concept be taught with an inductive approach?
- Would use of the calculator facilitate the study of real-life applications?
- Will using the calculator allow assessment to be focused on relevant educational objectives? (pp. 10 – 12)

These questions suggested to the researcher that teachers would be willing and able to put down in writing something that reified their deliberations on the topic of calculator use. These questions could also serve as a model for comparison if teachers became stymied over the structure of what they would write down about their own understandings. It did not become clear to the researcher until well into her analysis that these questions all deal with whether or not calculators should be used, and did not elaborate on why any specific use would be appropriate.

In order to attempt to bring into the study group sessions a discussion of specific uses of calculators that may or may not be appropriate, the researcher was guided by the notion of appropriate tasks for teaching mathematics. The *Professional Standards for Teaching Mathematics* (NCTM, 1991a) describe the "worthwhile mathematical tasks" that should be posed to students. Tasks should be based on "sound and significant mathematics; knowledge of students' understandings, interests and experiences; [and] knowledge of the range of ways that diverse students learn mathematics" (p. 25). In addition, such tasks should

- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics. (p. 25)

Thus, worthwhile mathematical tasks are those that satisfy these goals. They not only "help students to develop skills in the context of their usefulness," but they also

"illuminate mathematics as an intriguing and worthwhile domain of inquiry" (p. 24). Selecting such tasks is seen as a central responsibility of the mathematics teacher. The *Professional Standards* do not directly look at tasks to be used with various kinds of technology; rather, they look at calculators as one of the tools for enhancing discourse in the mathematics classroom. The calculator is not the center of instruction, but a means of clarifying discourse about the mathematics.

Following this intent of focusing on the mathematics even as we intentionally talked about using graphing calculators, the researcher looked for a framework within which to work that would allow the focus to be drawn back periodically to some specific mathematics content. For this purpose, the concept of cognitive demand of tasks, as applied to mathematical tasks seemed ideal.

Stein, Smith, Henningsen and Silver use the concept of *cognitive demand* (2000) to help teachers analyze cases based on classroom work. Such analyses are intended to be used in professional development preparing teachers to analyze their own practice. The term *cognitive demand* of tasks refers specifically to "the kinds of thinking needed to solve tasks" (p. 3). Tasks are broadly categorized as requiring low-level or high-level thinking. Low level cognitive demand tasks include memorization tasks and "procedures without connections" to meanings. Tasks with high level cognitive demand are "procedures with connections" to meanings and "doing mathematics" (p. 16).

Stein and colleagues do not limit such task analysis to tasks using technology. Rather, their examples assume that as students work on tasks, they "quietly get whatever paper, tools, or manipulatives they need to complete the task" (p. 2). The cognitive demand of a task may change during the time students are working on it (p. 3),

particularly depending on the teachers' interactions with working students. This is why, in professional development for which Stein et al. designed their task analysis guide, teachers analyze classroom cases rather than just the tasks themselves. In the proposed present study, by applying a cognitive demand analysis to students' use of graphing calculators, teachers can become aware of the idea that introduction of technology to tasks posed in traditional textbooks for the practice of algorithms often reduces the cognitive demand of such tasks. By comparing these more traditional tasks to tasks specifically designed for use with graphing calculators, is the researcher intended that teachers would be able to select tasks that satisfy the criteria of 'worthwhile mathematical tasks' no matter what tools students use to solve them. To some degree this comparison was intended to be instrumental in teachers' conclusions that they need to understand what calculators are capable of. If they are not familiar enough with the calculators their students are using, they may not be able to assess what happens to the cognitive demand of a task when students use the calculators.

Why Collegial Inquiry?

Up to this point, the studies examined have provided the researcher with insight into the concerns of teachers as they consider the use of mathematical tasks and graphing calculators to teach mathematics, and into the ways in which teachers and students have used graphing calculators. The final sections of this chapter will consider studies that influenced the researcher in designing the manner in which the present study engaged teachers in the topics of calculator use and mathematical tasks so that their construction of an understanding of appropriate use of graphing calculators could be examined.

In their discussion of a "consensus model" of professional development , (Hawley & Valli, 1999, p. 135) describe "inquiry" as one of the conventional approaches to professional development that is inadequate to the task of motivating lasting change that is effective in improving the education of students. They cite the definition of inquiry given by Sparks and Loucks-Horsley (1990), that requires teachers to identify an area of instruction to study, to collect data, and then to modify instruction on the basis of their interpretation of their data. Hawley and Valli agree that this approach resonates with the consensus model, but note that its effectiveness is vulnerable to varying abilities of teachers to identify areas of study and interpret data. They then identify eight "design principles for effective professional development" (pages 136-144). Principle Four provided the impetus for the researcher to incorporate collegial inquiry into the design of the present study. This principle called for collaborative problem solving, and one of the modes in which this is carried out is the study group (Hodges, 1996). Collaboration facilitates the identification of problems of common concern and a communal understanding of how to solve them. In the present study, the agreement on the problem of common concern was accomplished by self-selection when teachers chose to join the study group. The issues involved in addressing the problem, however, were a matter of intense negotiation within the group.

Coe (1990) provides one example of such collegial inquiry in his case study of a collaboration of two experienced teachers in what he calls "clinical supervision" but which in the U.S. might be termed 'peer evaluation of instruction.' Coe argues that although the goal of supervision is improvement of instruction, the collegiality of inquiry described in his case study was a worthwhile end in itself. Following Little (1987), Coe

describes the collegial relationship growing out of the process of collaborative inquiry as going beyond "social support that puts newcomers at ease," and more appropriately framed as "professional support that advances one's knowledge and practice of teaching" (from Little, 1987, p. 498; see also Little, 1993).

Ball and Cohen (1999) describe a broader vision of inquiry in their proposed "pedagogy of professional development" (p. 25). Besides engaging teachers in "tasks grounded in the activities of practice" (p. 25), this endeavor calls for "the development of a disposition of inquiry" (p. 27). Ball and Cohen include in this the skills of inquiry, such as "generation of multiple conjectures about an issue in practice," the "production of alternative explanations" and "efforts to weigh them rationally" (p. 27). They also give examples that indicate a *disposition* for inquiry, such as "learning to avoid leaping to definitive conclusions, cultivating the disposition to frame interpretations as conjectures, and ... how to identify and use appropriate evidence" (p. 27).

It is unlikely that such a disposition to inquiry could be internalized by a group of teachers in the four study group sessions of the present study. However, the researcher endeavored to model such a disposition to inquiry in her interactions with teachers, both in the study group sessions and interviews.

There remained the task of making as clear as possible to participants in the present study what the purpose of the collaboration was and how it would work. Because it was not known until days before the study began who the participants would be, the vision of the collaboration was probably more an ideal in the researcher's mind than a shared vision of the participants. It is important to understand what the researcher's vision was, because it influenced her decisions not only of the choice of artifacts and

mathematical content to introduce, but also of how she interacted with participants on the many levels at which they came into contact.

Community of Practice

The concept of community of practice, introduced by Lave and Wenger (1991) and further developed by Wenger (1998) provided anthropological language and constructs to describe and analyze the inextricable social aspects of teaching and learning mathematics. A more specific adaptation of the concept of community of practice to learning to teach mathematics has been made by Bohl and Van Zoest (2001). It is this adaptation that most strongly influenced the researcher's vision of what would take place in the study group sessions of the present study. Because the regime of competence by which a community of practice defines itself is not fully developed for the community of reform practice (which Bohl and Van Zoest define as those teachers endeavoring to follow the perspectives reified in the NCTM reform documents of 1989, 1991, 1995, and 2000), participation in this community includes defining what it means to be an expert participant (Bohl & Van Zoest, 2001, p. 14). In the same way, the researcher envisions participation in the study group sessions of the present study a first step in defining expert participation in a community of practice made up of teachers who appropriately use graphing calculators to teach mathematics. Other groups of teachers under other modes of inquiry have worked and are working on this definition also, but for the researcher and the participants in this study group, the definition of expert participation in the community of practice was open at the beginning of the present study.

The researcher was less concerned about whether the study group actually "became" a community of practice in the short amount of time they met, and was more

concerned about seeking to incorporate the characteristics by which such communities are defined and cohere, namely *joint enterprise*, *mutual engagement*, and a *shared repertoire* (Bohl & Van Zoest, 2001, p. 7). .

Other researchers who have used the concept of community of practice in their work have also concentrated on these three characteristics. In raising the question of whether this concept provides "added value" to research, Eraut (2002) draws attention to the varied meanings that the word "community" takes on. There is an ecological understanding relating to organisms in a geographical area, a political definition related to groups that are to be influenced, and an ideological definition implying that a characteristic such as learning is always part of any community. Eraut is critical of what he sees as Lave and Wenger's (1991) disallowing diversity in a community of practice because of the process of induction of newcomers and of the regime of competence. He prefers Engestrom's (1993) definition of community as comprising "multiple individuals and/or subgroups who share the same object" (p. 67), which would allow for diversity. Eraut then argues that activity theory starts from the practice end of community of practice while Lave and Wenger (1991) start from the community end of the same phenomenon, so that they complement each other. Eraut (2002) asserts that the theoretically driven definition of Lave and Wenger (1991) has "led to significant debates about the nature and context of learning, which have added value to our research community" (p. 5).

Palinscar and colleagues (1998) begin their research report with an acknowledgement that it is incongruous to write about the birth of a community of practice, because they are always in existence, and it is very difficult to mark their

formation, dissolving or evolving. They note that Lave and Wenger (1991) initiated their study for the purpose of capturing the negotiation of meaning within groups. Palinscar et al. also assert that there is no general agreement on what constitutes expert practice in education, and hence they suggest that in order to study communities of practice, they must first be built. The 18 participants in their study were recruited to take part in professional development in teaching science in grades K-5. The common enterprise was to engage in inquiry-based teaching of science. The researchers developed design principles to guide the building of their community of practice: (1) a common goal was "development of practice consonant with [the] orientation" of inquiry-based science teaching; (2) reliance on diverse expertise to contribute intellectual resources; (3) focus on the central work of teaching, including planning, enacting and reflecting on one's teaching. The activities around which the community grew included work with inquiry lessons as learners, implementing lessons in class while being observed or video-taped, sharing experiences with a small group, and general debriefing. The researchers claim that a community of practice is forming, based on examples of negotiated meaning, of shared ideas being greater than those of any one participant, and the draw of social as well as professional needs.

Moore and Barab (2002) and Reynolds et al. (2001) both report on the ILF, first called Internet Learning Forum (Reynolds et al., 2001), and then known as Inquiry Learning Forum (Moore & Barab) which better describes its mode of operation. These researchers use a definition of community of practice that differs from Lave and Wenger (1991). They see three components of the community of practice: (1) a shared cosmology, especially "shared goals, practices, belief systems and collective stories that

capture canonical practice" (Reynolds et al., 2001, p. 110); (2) more than one member; and (3) the community is constantly reproducing itself. The researchers assert that technological advances now allow development of a community of practice on the Internet suitable "for situating teacher professional development in classroom contexts" (Reynolds et al., 2001). The ILF provides video clips of classroom teaching to provide this classroom context, and a starting point for discussion.

Use of ILF begins with pre-service teachers as their professors build a culture of sharing and discussing teaching practices, and use the ILF to support that activity. A huge impetus for pre-service teachers to be interested in video is that the state of Indiana now requires a video of teaching as part of a new teacher's licensure portfolio. Video clips as well as transcripts are available via ILF. Since expertise is necessary for learning to take place in a community, ILF has also recruited practicing teachers to participate and allow video-tapes to be posted. Students in the taped classes are protected by HSIRB protocols and by password access to ILF (Moore & Barab, 2002). Thus ILF appears to be an extension of the university and school communities of practice, as encouraged by Bohl and Van Zoest (2001) to embrace students as they develop into mature teachers.

Moore and Barab (2002) report on further developments of the project, such as addition of student work, and provision for collaborative workspaces. The intended substantive discussion in connection with viewing of the video clips has not been at the level expected, so plans are being made to provide prompts that will inspire deeper thinking and critique.

Franke and Kazemi (2001) expand their previous understanding of "generative growth" by examining the communities that teachers create for themselves and their

students. Thus they begin by defining the community of practice as those exhibiting this "generative growth" four years after they had finished extensive experience with the Cognitively Guided Instruction (CGI) project. The researchers noted that these teachers structured their knowledge of teaching using the CGI ideas as a framework. They also regarded that knowledge their own, and altered their ways of expressing and applying it as needed. When incorporating the concept of community of practice, Franke and Kazemi (2001) include the reforming of identity as part of the learning that occurs in communities of practice. Of the other research reported in this section, only Bohl and Van Zoest put as much emphasis on that aspect of Wenger's conception.

Similar to most researchers whose work is reviewed here, Franke and Kazemi recognized that they could not "create" a community of practice, but rather could give teachers opportunities to come together and learn together about the teaching and learning of mathematics. Space was provided for teachers to share and make sense of their students' mathematical work, and the meetings were called "workgroups" (Franke & Kazemi, 2001, p. 57). The researchers included examples of the resulting reforming of identity on the part of one of the teachers. The growth of the researchers' own understanding as a result of interactions with participants in (and outside) the workgroups was also traced.

The work in this section influenced this researcher to disavow any claims of being able to "start" a community of practice, and also gave some suggestions of the types of activities which could evoke such processes as identity reformation and negotiation of meanings.

CHAPTER III: METHODOLOGY

Theoretical Framework

The theoretical framework for the design of this study, the research questions, and the analysis of transcripts of interviews and study group sessions draws on the structure of design experiments, described by Brown (1992) as an "attempt to engineer innovative educational environments and simultaneously conduct experimental studies of those innovations" (p. 141). The study also incorporates a framework for analyzing social construction of practice adapted from Wenger (1998) by Bohl and Van Zoest (2001). A design experiment is a method of studying complex systems. Such systems are described as (1) products of human construction rather than naturally occurring; (2) not capable of being isolated because such attempts to remove them from the holistic systems in which they are embedded completely alters their nature; (3) not observable except through their effects; and (4) not merely lying dormant waiting to be stimulated, but rather initiating action (Lesh, 2002). Complex systems tend to make the observer part of the system, so that there is no such thing as "immaculate perception" (Lesh & Clarke, 2000, p. 133). Schools are this sort of complex system, and teaching is a complex activity within this complex system. This study examines how a norm for "appropriate use" of graphing calculators is negotiated in the complex system of a community of practice.

A pilot study to gather feedback on the survey and other artifacts was conducted with a group of teachers who were experienced in using graphing calculators in their

mathematics classes. The difficulty of distilling, from the transcripts of sessions with the pilot teachers, language describing their common understanding of appropriate use of graphing calculators led the researcher to search for a way of involving the teachers themselves in the distillation. The design experiment provided a perfect vehicle. According to Zawojewski, the defining conditions of a design experiment, especially when used with a group of teachers, are (1) a product under design and (2) the experimentation (Zawojewski, under development). The product under design must be useful and shareable, its design process must result in thought-revealing documentation, and it must motivate iterative cycles of expressing, testing and revising teacher thinking. In this study, the product design took place in the context of collegial inquiry (Hawley & Valli, 1999) by a group of teachers as they participated in study and discussion of the concept of "appropriate use" of graphing calculators in their classrooms. The teachers were recruited for their desire to work at developing a tool to aid in making decisions about appropriate use of graphing calculators in secondary classrooms, and their willingness to share this tool with other teachers in their schools and districts. Hence, the product of this design experiment was a tool for teachers to use in making decisions about appropriate use of graphing calculators in high school and middle school mathematics classes. Henceforth this product of this design experiment will be referred to as the tool under development, the AUGC (Appropriate Use of Graphing Calculators) tool, or just the tool.

In a design experiment, the experimentation can take on many forms related to the product under design. In the present study the product under design was the AUGC tool, and the experimentation included teacher use of the emerging tool to analyze use of

calculators in their classroom instruction as students worked with the problems presented in their textbooks. Classes were observed, and in follow-up interviews teachers were asked to reflect both on the lesson and on the relevance of the emerging tool in planning for or analyzing the use of calculators by students during the lesson. The classroom observations were done between the sessions of collegial discussion. Each session included evaluation of the tool, based on teacher experiences. The group negotiated changes that would make the AUGC tool more useful or more easily shared. After changes were made, the revised tool was again tested by using it to guide and analyze classroom calculator use.

One of the assumptions of design experiments is that no one 'expert' or group of experts has exclusive insight on 'truth,' but rather, all stakeholders in a particular educational setting

> have ways of thinking that tend to evolve significantly if they are engaged in activities that repeatedly require them to express their views in forms that go through sequences of testing-and-revision cycles in which formative feedback and consensus building influence final conclusions that are reached (Lesh, 2002, p. 43).

Because at present there is no widely accepted criteria for "appropriate use" of graphing calculators, teachers engaged in the testing-and-revision cycles of this study compared the results of the alterations in their AUGC tool to an "end in view" (Zawojewski, p.6). That is, they were asked if the revised tool better fit their needs for identifying appropriate use of technology. As they considered this question teachers had to think about what the results of appropriate use should be for students, and whether the calculator use as guided by the emerging tool increased such results.

As seen through the lens of communities of practice, the researcher, and all of the participants at various times, served as brokers from other communities of practice in which graphing calculators were used. Teacher experiences and calculator activities, as well as frameworks for analysis of tasks and of calculator use, were artifacts used to convey meaning from one community of practice to another. Artifacts and activities brokered by the researcher were influenced by her experience with the teachers of the pilot study, and by her own understanding of appropriate use of graphing calculators constructed within the communities of practice in which she participated. See Appendix B for an attempt to put this understanding into words. The analysis of many sources of data was used to determine to what extent the brokers and artifacts were effective in supporting the construction of a collective meaning for the term "appropriate use" of graphing calculators.

The Research Study

All names in the remainder of this dissertation, including city and school names, have been replaced by pseudonyms.

Site

Five teachers from Riversmeet City Public Schools (RCPS) agreed to take part in a professional development study group to inquire into the question of appropriate use of graphing calculators in the teaching of secondary mathematics. Riversmeet is a mid-size city in the American midwest located on one of the major rivers in the geographical region. The city school district of Riversmeet, according to Standard & Poor's School Evaluation Services (http://www.ses.standardandpoors.com), had a population in 2001 of

just over 45,000. There were about 7,600 students enrolled in RCPS grades K – 12, which was about 85% of the school-aged children. About 18% of the children lived in households with only one parent. RCPS students attended 15 elementary schools, 3 middle schools, several special programs, and one high school with about 1200 students in grades 10-12. Over previous years, the city school population changed from 53% economically disadvantaged in 1997 to 57% economically disadvantaged in 2001. The ethnic make-up of the student population also shifted from 60% white in 1997 to 55% white in 2001. Of the non-white students in 2001 37% were Black, 6% Hispanic, 2% Native American, and less than 1% Asian/Pacific Islander. Gender balance among students has remained constant at about 48% female and 52% male.

The secondary school mathematics program offers two avenues for students to study mathematics. One four-year path uses the Core-Plus Mathematics Project (CPMP) materials, *Contemporary Mathematics in Context* (Coxford et al., 1998 - 2000). The courses in this pathway are called Integrated Mathematics 1, Integrated Mathematics 2, Integrated Mathematics 3, and Integrated Mathematics 4. When talking about these classes, however, teachers used such references as "Core 2" or "Core-Plus 1" or just "Course 1." The second pathway uses reform-oriented textbooks that maintain the titles Algebra 1, Geometry, Advanced Algebra and Precalculus, and the courses also use those names. The textbook used in the advanced algebra course is the only one the researcher had contact with, and that was *Advanced Algebra Through Data Exploration: A Graphing Calculator Approach* (Murdock, Kamischke, & Kamischke, 1998). Some students begin high school with Integrated Mathematics 2 or Geometry, but the block schedule allows students to accelerate themselves by taking two mathematics classes in

one year. After the third course of either sequence, students may choose Discrete Mathematics, Probability and Statistics, or AP Statistics. And after the fourth course of either sequence, calculus is an option. Standard & Poor's shows a student-teacher ratio of about 14 to one for the Riversmeet district, but most of the observed classes had 23 to 25 students.

Over the preceding decade, the RCPS had introduced new standards-based curricula at all three school levels. With the help of major grant funding, intensive professional development was conducted with elementary, middle, and secondary school teachers of mathematics. None of the teachers in the current study had participated in this grant-funded professional development, since they were all hired after the professional development was completed. The school district has experienced a nearly 25% turnover among secondary mathematics teachers each year, resulting in about two new mathematics teachers at the high school each year (SDInt, 276).

Participants

Classes taught by the teacher participants ranged from eighth-grade mathematics in a middle school, to freshman algebra and integrated mathematics classes in the high school, to advanced algebra in an alternative school. All of the classes have about the same ethnic make-up as the school in general. Each teacher was eligible for professional development credit for his or her participation in the present study, though two of them specifically stated that they already had the required number of credits and had joined the group because of the topic. The initial interviews, conducted before any group meetings took place, provided an opportunity to examine the teaching situation and educational

background of each teacher. This description is presented here as introduction to the group of participants.

At the time of the study, Lynn was a middle school mathematics and social studies teacher. She taught three classes of eighth-grade Connected Mathematics and one of American history. Lynn had been teaching in the district for three years, having previously taught three years in another district. The middle school schedule had six periods that met daily. Lynn was a white female and had attended a regional state university whose mathematics educators are national leaders in the use of technology in teaching mathematics, but Lynn did not finish all her coursework on that campus and had not taken any calculator-specific mathematics or methods classes.

Tess, a white female, was a high school mathematics teacher in her second full year of teaching in the district. She taught one year in another district after graduating from another regional state university, known for its use of technology in most mathematics classes, including those for teachers. Tess taught freshmen Integrated Mathematics 1 and Integrated Mathematics 2, and also a basic skills support class called Mathematics Investigations, which was an elective designed to be taken concurrently with another mathematics class for which students needed support.

Yvette, an African-American female, was new to the district at the time of the study. The classes Yvette taught were Integrated Mathematics1, Integrated Mathematics 2, and Algebra 1, but her students were all freshmen. She received her education as a teacher at a small midwestern private college and had no experience with using calculators in college classes, either for mathematics or for methods. She had decided not to stay in the district, but was very interested in appropriate use of calculators.

Karl, a white male, was a seasoned teacher, with 34 years of experience teaching mathematics and physical education, but he was in his second year of teaching in the district. Karl had received his teacher education before the age of technology, but had taught in another state in a district that prided itself in being up-to-date with technology. Karl had attended workshops for using calculators to teach mathematics, though he noted that most of the topics covered in the workshops were not applicable for the beginning algebra he was teaching at the time. He taught in a small school, a grades 6-12 academy that was set up in its own building. Karl taught all the high school level mathematics, for Integrated Mathematics 1 to Precalculus.

Rob, the fifth teacher, had just finished his teaching internship at Riversmeet HS as a science teacher when he was hired to replace a mathematics teacher who had left the district after only one month of school. Rob, a white male, received his teacher education at a large regional state university whose mathematics educators are well-known for curriculum development and large-scale professional development projects involving technology and standards-based curricula. Rob had completed most of an aeronautical engineering program before switching to education, and was also a member of the U.S. military reserves. Because the 2003 War in Iraq started just after this study began, Rob had other stresses to deal with as he taught freshmen Integrated Mathematics 1, Algebra 1, and Integrated Mathematics 2.

Role of the Researcher

The researcher was a participant in the study group sessions, questioning for clarification and sharing experiences and artifacts. Thus she was a broker like the rest of the study group members. She also took on the role of secretary for the on-going work of

developing the AUGC tool. However, she was also the facilitator of the group, planning the agenda on the basis of what was accomplished in previous sessions and on interactions with individual teachers during the observation interviews. In addition, the researcher was an observer of the group interactions during the development of the tool. The questions she asked were at least in part an attempt to clarify where ideas for the tool were coming from, and what influenced any seeming changes in opinions expressed by individuals. She recorded field notes after each encounter with participants, and specifically aimed at an objective analysis of what took place.

Research Design

By focusing the work of the study group on developing a tool to reflect collegial understanding of what "appropriate use" of graphing calculators is, the researcher provided a mutual engagement, one of the characteristics of a community of practice. Because the goal of the study group was a shareable tool, teachers' thinking processes and the social process by which the form and substance of the tool was negotiated were exposed to the researcher. Negotiating the production of the AUGC tool was the vehicle for construction of the collegial norm for appropriate use of graphing calculators. This construction was aided by the examination of artifacts such as activities, lessons, and frameworks for analysis that were brought to the discussion by participants or by the researcher as brokers from various communities of practice.

The process of negotiating an AUGC tool made observable the thoughts and concerns of teachers, based on their beliefs and conceptions about graphing calculators. Classroom observations over a period of time also revealed how the collegial inquiry was incorporated into classroom use of graphing calculators. But the main purpose of the

study was to understand the workings of the community and what sorts of issues and artifacts are important to the task of developing the tool.

Implementation Plan

Time frame	Participant activity	Researcher activity
February 2nd week	Complete Task sort Task sort Interview Complete survey	Pre-study Task Sort interview and Survey of beliefs and conceptions Analysis of surveys and interviews
February 4th week	Study group session Issue list for AUGC Tool Written reflection	First study group session – Student calculator use framework Continue analysis
March 1st week	Incorporate calculators, using Zbiek framework Post-observation Interview	Classroom observations & Interviews Analysis of observations and interviews Prepare for next study group session
March 2nd week	Study group session Negotiate AUGC Tool revision Written reflection	Second study group session – Levels of Cognitive Demand (LCD) Continue analysis
March 4th week	Incorporate calculators, using LCD task analysis Post-observation Interview	Classroom observations & Interviews Analysis of observations and interviews Prepare for next study group session
April 1st week	Study group session Negotiate AUGC Tool revision Written reflection	Third study group session – Branca –Teacher questions Continue analysis
April 3rd and 4th weeks	Incorporate calculators, using emerging Tool Post-observation Interview	Classroom observations & Interviews Analysis of observations and interviews Prepare for next study group session
April 4th week	Study group session Complete final draft of AUGC tool & evaluation Written reflection	Fourth study group session – consolidation of AUGC tool ideas & evaluation Continue analysis
May 2nd week June 2nd week	Complete Survey review Complete Task sort and Interview	Post-study Task Sort interview and survey review
May – July	Read analysis and react, if interested	Finish analysis, write up findings

Table 1: *Timetable for study*

Interviews and classroom observations were held at each teachers' place of work, and study group meetings were held after school hours at the high school where three of the teachers worked. The schedule that was followed is shown in Table 1.

Data Collection

At the beginning of the study, prior to group meetings, teachers were individually asked to do a task sort (Stein et al., 2000), describing for the researcher the conditions under which calculator use would be appropriate for certain tasks. This task sort provided a baseline of teacher thinking about appropriate use of calculators before the collegial inquiry began. The task sort interview was repeated at the end of the study, again with individual teachers. The tasks that were sorted, and instructions, can be found in Appendix C. All interviews were audio-taped and transcribed.

A preliminary survey provided baseline information on the teachers' beliefs and conceptions about using graphing calculators and about the nature of mathematics. It also gathered information on the teachers' educational backgrounds, their previous experience with calculator use as either a student or teacher, and the amount of professional development in which they had participated that focused on the use of graphing calculators in the learning and teaching of mathematics. The survey was completed between the time of the initial personal interview and the first group meeting. At the end of the study, teachers were asked to review and revise the responses they initially made on the survey, and to comment on any changes they made. The purpose of this repeat survey was to ascertain if beliefs and conceptions had changed. The surveys, which incorporate questions drawn from Simmt (1993), Fleener (1995b), Weiss (2001) and Ziebarth (2002) can be found in Appendix D.

The study group met four times to reflect on and share their ideas and their experiences with calculators in their classrooms, and to put their understanding of appropriate use of graphing calculators into an AUGC tool that would be shared with other teachers. Participants were using reform curriculum materials in their classrooms, which in some cases served as perturbations of the ideas of appropriate calculator use the teachers brought with them. All group sessions were audio-taped and transcribed.

To help with the process of examining calculator use in their classrooms between sessions, calculator use logs were provided as a checklist. A sample log page can be found in Appendix E.

At the end of each study group session, teachers were asked to write about how their own concept of "appropriate use" of graphing calculators had developed since they last wrote about it. A sample reflection page is given in Appendix F. Following the fourth session, final interviews with teachers were scheduled. During this interview, the final task sort was done, the survey was revisited in part, and teachers were invited to talk about the growth they felt they experienced in understanding the appropriate use of graphing calculators in teaching mathematics.

Observations of participants' classes were another way of gathering data about the teachers' conceptions of appropriate use of graphing calculators. An observation tool based on Zbiek's two-tiered categories (2002b) for student use of technology and on her MAGICAL categories for student use of representations (2002a) was developed for this study. It could not be used to full advantage by a non-participant observer, but served to focus the observer's attention on calculator use. This tool can be found in Appendix G.

Two tables showing the relationship of the data sources and the research questions they were used to answer can be found in Appendix H.

Data Analysis

Three levels of analysis were applied to most of the data gathered in this study. Each level of analysis involved all three of the research questions, so it is not possible to make neat divisions of this report by research question. Nor is it possible to divide the report entirely by individual participant, because the process of negotiating the AUGC tool is joint. Because showing development of thinking requires a presentation that is mostly chronological, that is the approach that will be used, with pauses to summarize development of the tool at different stages.

The first level of analysis was applied as soon as transcripts were available, which for the first two sessions was before the next session was held. This on-going analysis provided the basis, along with field notes, on which decisions were made by the researcher as facilitator of the study group sessions. This analysis can be compared to what a teacher does in making decisions about what to do in the next class period. It involved, for example, analysis of the initial interview, task sort and survey to learn how far apart participants were in their conceptualizations of mathematics and the teaching and learning of mathematics. It determined the topics and types of activities that would be used as common ground for interacting with artifacts in subsequent sessions. Informal assessment by conversation and questions about what happened in classroom observations also influenced these decisions. More detail on these sorts of decisions will be given in Chapter V.

The second level of analysis attempted to trace the trajectory of individual teachers from the beginning to the end of the study, from initial interview to final interview, examining not only the contributions they made to the group's work in developing an AUGC tool, but also how the contributions of others influenced the individual's thinking. More detail on the trajectories of the five participants is given in Chapters IV and VI.

The third level of analysis, and the most difficult to accomplish, directly addresses the research questions. It seeks to give a rich description of the process by which the group as a whole was able to produce a tool that they all agreed was shareable with their colleagues, and that would provide at least a catalyst for further discussion that would benefit the mathematics programs at each of their schools. The results of this analysis are found in Chapter V.

Answering the Research Questions

Transcripts of interviews and of study group sessions and the participants' reflections are used to answer the first research question: *What are the issues that teachers focus on when constructing an understanding of "appropriate use" of graphing calculators, and how do they negotiate those issues?* The first list of issues was gathered from the first task sort interviews, and was used by the researcher to prepare for the first session. A second list of issues that was used to test the validity of the first form of the AUGC tool was consolidated from the transcript of the first study group session.

The second research question, *How do teachers in collegial discussions about "appropriate use" of graphing calculators incorporate their prior instructional experiences and artifacts that reify the ideas of others?*, is answered by examination of

the session transcripts and the participant reflections. Some elucidating evidence is found in classroom observations and teacher interviews.

The third research question, *In collegial discussions about "appropriate use" of graphing calculators, what influence is exhibited by contextual factors such as the curriculum used and student characteristics, and by teachers' beliefs and conceptions about graphing calculator use and about the nature of mathematics?*, is answered primarily by evidence from study group transcripts. However, corroboration is found in the teacher survey, classroom observations, and particularly from the interviews following these observations, when teachers are speaking for themselves only. Comparison of the final task sort and interview to the pre-study sort and interview provided evidence of the extent to which collegial discussion has influenced the teacher's own thinking about the use of calculators with the sorted tasks. Overall effects on the teachers' beliefs or attitudes are reflected in the teachers' final review of the answers they gave to the initial survey. The reasons they gave for changes in responses further illuminate their thinking.

With all data sources, careful study of the sequence of teacher statements or actions was made to trace the development of teacher thinking. The analysis notes changes in conceptions, beliefs or actions based on introduction of new experiences. The interviews and discussions, as well as teacher writing also provide evidence of such effects. This analysis, using the findings from each of the three research questions, will provide whatever elucidation is possible of the overall question of the study, *How do teachers construct an understanding of "appropriate use" of graphing calculators in the context of collegial inquiry?*

Limitations

Limitations of the study are inherent in its short duration and the relatively small number of participants. Other limitations in the applicability of findings to other situations stem from the fragility of the community studied, since they were brought together under the limited conditions of this study, and at least one will not return to the school for the next year. The unique make-up of the group as having a preponderance of young teachers also limits the possible applicability of findings to planning for similar work with a group of experienced teachers. Because the definition of 'appropriate use' is locally determined, the definition reified in the AUGC tool will not necessarily directly transfer to any other community. Longer studies of how the construct develops over time and around future changes in technology, and comparisons to constructs developed in other communities would be necessary to determine any common themes that could be taken as a broader norm.

Organization of Following Chapters

The organization of the next three chapters might be compared and contrasted to the common concept of a set of "before and after" pictures. Like "before and after" pictures, a snapshot of the thinking of a group of teachers at two particular points of time will be presented. However, unlike "before and after" pictures, the major focus of this study is how teachers get from the "before" snapshot to the "after" snapshot. The "before" and "after" pictures verify that something actually occurred in between, so there is a reason to examine the process. Chapters IV and VI will present as clear a picture as possible of the thinking of this group of five teachers about the appropriate use of graphing calculators, using primarily three data sources to make that presentation:

interviews, task sorts, and surveys. Only part of the survey is revisited in the Chapter VI, because much of it gathered teacher background information. The final interview also differs from the initial interview, in that each final interview is designed for a specific teacher. Information on the district mathematics program was also enhanced by an interview of the district Mathematics and Science Coordinator, Sonia Day, who was a long-time mathematics teacher in the district before moving "across the street."

At the end of the Chapter IV, an introduction is made of the researcher, since she is also a member of the study group, as well as planner and facilitator of the study group sessions. Here the researcher looks at herself as facilitator in terms of the information, knowledge, past experience and beliefs that certainly influenced decisions made about the study group sessions. Descriptions of the decisions made will precede the report of each session. Extensive written reflections on each session plan, interview, observation, and study group session assist in this area of the analysis.

Chapter IV will also introduce data sources that were being developed at the same time as the study group sessions. These include the AUGC tool whose development was the focus of the study group sessions, and also the classroom observations and interviews that took place in the time between sessions. Chapter V, then, will weave together the different data sources to look at how this small community undertook the task of developing a tool that other teachers could use to make decisions about appropriate use of graphing calculators to teach mathematics. This chapter will also introduce the artifacts that were brought to the group by brokers who also participate in other communities of practice which use graphing calculators. Some of these artifacts are in the form of physical documents that reify the practice of using calculators, while others are stories or

bits of wisdom gathered from past experience or interactions. Finally, Chapter VI paints a picture of where teacher thinking was at the end of the study group sessions, and the teachers' relation to the final form of the tool that was developed. The metaphor of a painting rather than a snapshot is appropriate for the final picture, because the state of these teachers' thinking at the end is very much a work in progress, and the teachers are aware of that. They themselves will continue to paint on their canvas, and will interact with colleagues as they share their AUGC tool, helping to blend new colors. The reader should note that the unit of analysis in Chapter IV is the individual study group member, and in Chapter V the unit of analysis shifts to the group and to the tool being created by the group, and in Chapter VI, the focus returns to the individual study group members.

CHAPTER IV: CONTEXT

Participant Teachers Prior to Group Inquiry

In this chapter, the preexisting conceptions of individual teacher's understanding of the "appropriate use" of graphing calculators are charted as they enter the study group. First, each of the data sources is examined, and the information gained from each data source addressing each of the three research questions is reported, from the perspective of the individual teachers in the group. In Chapter V, analysis is organized chronologically, across all the data sources, from the perspective of the group. The research questions are restated here, and are henceforth referred to as RQ1, RQ2, and RQ3.

RQ 1: What are the issues that teachers focus on when constructing an understanding of "appropriate use" of graphing calculators, and how do they negotiate those issues?

RQ 2: How do teachers in collegial discussions about "appropriate use" of graphing calculators incorporate their prior instructional experiences and artifacts that reify the ideas of others?

RQ 3: In collegial discussions about "appropriate use" of graphing calculators, what influence is exhibited by contextual factors such as the curriculum used and student characteristics, and by teachers' beliefs and conceptions about graphing calculator use and about the nature of mathematics?

Initial Interviews

To provide a more complete understanding of how teachers were thinking about the appropriate use of graphing calculators to teach mathematics when the study began, an initial interview was done before any study group sessions were held. The protocol for the interview can be found in Appendix I. This interview probed the background of the

teachers, their experience with calculators, and their beliefs and concerns about using graphing calculators to teach mathematics. A task sort of 20 items adapted from a variety of high school mathematics textbooks with copyright dates ranging from 1968 to 1997 was used to provide opportunities for teachers to talk about calculator use with topics ranging from linear relationships to probability and statistics to rational functions. Teachers were asked to assign tasks they would use in teaching the topics to categories indicating whether use of calculators was always, sometimes, or never appropriate. They had the option of saying they would not use a task. The task sort items and instructions can be found in Appendix C, and the results of the initial sort are reported in the next section of this chapter.

Some of the results of this interview were reported in Chapter III, when participants' backgrounds were described. The results of the initial interviews were also used to begin to formulate the list of issues that needed to be addressed if the group was to be able to develop a tool that would be useful for teachers in deciding how to use graphing calculators appropriately in teaching mathematics. The remaining results of the interview are reported here as summaries of the conceptions the individual teachers expressed about appropriate calculator use. References in this section are to a document, PrI-TS1 (Pre-Interview, Task Sort 1), containing the transcripts of all initial interviews, divided into chronologically numbered segments that were then coded. References to this document and others include the initial of the teacher being interviewed, and the number of the segment quoted or referenced, in this form: (teacher initial if not included in title, document title, chronological segment number). Coding will be explained in a later section of this chapter.

Lynn, Teaching Eighth Grade Mathematics at Riversmeet MS

Lynn said she first fought against using calculators with her middle school students, but was using them now as directed by the curriculum she is using. When asked if she had been convinced or "beaten down," she replied,

> I think it was partly both. And there are sometimes that I don't let the kids use them, but it would be for things—one of my arguments was the order of operations. Important for the kids to know, especially in their later years as they're going through and working with algebra. There are sometimes that they need to understand that without a calculator, because the calculator does it for them. At least most of them [calculators] now, scientific. (L, PrI-TS1, 227)

Lynn described being shocked at how little some of her students understood about the meaning of the operations of addition and multiplication, and how many lacked basic computation skills. At the time of her initial interview, however, she justified using calculators "to get past some of that," saying "they would not be able to survive at this level of math without one." And also, "it's not in my curriculum or appropriate for me to go back and do that during class time. It is [appropriate] after school" (L, PrI-TS1, 230). She summed up by saying "So my views on calculators have changed, and I don't think that it's appropriate at most times to say it's never used, because some kids need it" (L, PrI-TS1, 230). Later in the interview, when she was discussing why she put tasks in certain categories in the task sort, Lynn expanded on her explanation of her thinking. One task asked which was greater, 15% or one-eighth. She said, "So, if I'm really just testing 'do they know the difference between these two numbers,' use of the calculator would be appropriate for some of them because I'm not looking at the task 'do you know how to do long division' " (L, PrI-TS1, 237).

Lynn was still particularly concerned with concepts her students need to understand, and felt that some must be learned before using calculators. The example she used from her seventh and eighth grade classes (she taught her present eighth graders when they were in seventh grade) was learning to graph equations. At first she had her seventh graders make the tables of points, that is, *x* and *y* values, by hand (using calculators for multiplying or adding, if needed), and then plotting the points on grid paper. She said, "they could not use calculators at that point – I expected them to plot the points by hand, show me the work" (L, PrI-TS1, 243). Now in eighth grade many of the students were figuring out how to use the calculator to graph their exponential functions. "I also allow that," Lynn said, "But it's not something that I have said, 'Oh look, you don't need to do it by hand' " (L, PrI-TS1, 243).

The whole matter of calculators in elementary schools is something Lynn felt strongly about. When she reported being shocked by the lack of computational skills of her students, she stated, "And it was because they were able to use the calculators...they could punch everything in and get the correct answer, but they didn't know why." (L, PrI-TS1, 254). She concluded, "I would not be for them at all in elementary schools. At all." (L, PrI-TS1, 255). However, once they get past those grades, "if they don't have that mastered, then we have to teach them ways to make up for, compensate for that lack of skill. Keeping a kid in the same level of math because they can't memorize their multiplication facts isn't the best for them either" (L, PrI-TS1, 255).

Lynn's struggle up to this point seems to be with whether or not students should be permitted to use calculators or not. She does not believe that there is any appropriate use of calculators in elementary grades, but has been led to accept that calculators can

make up for computational skills deficiencies that some students have when they enter middle school.

Karl, Teaching High School Math at Riversmeet Academy, RA

Karl stated early in his interview that calculators

> all have their place. I think there's times that you need to learn to do things without the use of a calculator before you apply the use of the calculator. And then there's other times where problems are so complicated and drawn-out that if you don't have the calculators to speed things along, you get bogged down. (K, PrI-TS1, 121)

Later, in describing the tasks that he put in the "sometimes appropriate to use calculators" category, Karl said, "In most of these, I would teach it without the calculator first, and use of the calculator after" (K, PrI-TS1, 130). He added, "I'd like to see them be able to do it by hand first to see whether their thinking processes are right" (K, PrI-TS1, 141). Karl also encouraged the use of mental math techniques rather than the graphing calculator. However, Karl also saw the importance of having students learn to use calculators correctly, and to be able to check their answers. He thought many students come to his class not knowing the advantages of using a calculator. But he also saw a problem in falling back on the calculator – he saw it in some students as a sign of insecurity in their own mathematical ability (K, PrI-TS1, 143).

Karl shared with Lynn the thought that for some students you had to admit that it was "almost a waste of time" (K, PrI-TS1, 152) to try to get them to learn their multiplication tables. The cause for this, Karl felt, was that "some teachers will just go ahead and let them use the calculator, not force them to know how to do certain things without the calculator first, just because it's easier" (K, PrI-TS1, 144). However, Karl

admitted that he preferred to avoid calculators himself, and learned from his students about how to do certain procedures on the calculator (K, PrI-TS1, 176).

At this time, Karl, like Lynn, was thinking in terms of whether students should be permitted to use calculators or not. He also went beyond that, perhaps because he taught a junior level class, and noted that students should learn to use calculators correctly and know the advantages of using them.

At the time of the study, the remaining teachers, Yvette, Rob and Tess, all taught in freshmen academies at Riversmeet High School (RHS). The academies were designed to give freshmen a smaller group of teachers and classmates to interact with so that they had a greater sense of community. The four core subject teachers (math, science, language arts and social studies) of each academy met together each week to discuss the academic and social progress of their group of about 125 students (SDInt, 30). The disadvantages of the academies, especially for beginning teachers, was that team meetings took up preparation time, and that interdisciplinary teaming cut down on the contact that academy mathematics teachers have with other mathematics teachers. In addition, the team mathematics teachers generally teach three different classes because academy membership is not assigned by math ability or any other academic characteristic (SDInt, 105). As a result, these participant mathematics teachers, each with less than two years of teaching experience, generally had more 'preps' than the other teachers in the school, because each academy has freshmen at all levels of mathematics achievement. New teachers tend to be assigned to academies because experienced teachers are allowed to choose their assignments first (SDInt, 105).

Tess, Teaching at Riversmeet HS in a Freshmen Academy

Tess loved to use technology. In one of her observed classes, Tess had downloaded an examination copy of an interactive geometry software program so that she could help her students experience how the software could perform the geometric transformations they had been sketching by hand. This was in preparation for having the students perform the same transformations by multiplying the matrix containing the coordinates of the vertices of a figure by an appropriate transformation matrix. Although the functionality of the examination copy was limited, and all the students took turns on the single computer she had in her classroom, Tess felt that the experience was very motivating for the next lessons her students would be studying.

However, this was with her 'advanced' (Integrated Mathematics 2) students. In her initial interview, Tess was very concerned that students should meet certain criteria before they use calculators. For example, in response to one of the tasks she sorted, Tess said, "I want them to see how to plot it and how to do the graphs. But if they already know, they did it by hand, they get the concepts, then I think the calculator is appropriate" (T, PrI-TS1, 19). In response to another task, she said, "sometimes it's easier for them to graph to make the conjecture of where it's going to, but they need to know how to graph it first" (T, PrI-TS1, 26). Tess recognized that one of the sort tasks came from an introductory "Think About This Situation" section of the Core-Plus Mathematics textbook she used. For that one, Tess said, "So this is the starting part of it, and then we go to the next phase. So once we've already introduced this, then I would take them to the calculator" (T, PrI-TS1, 37).

Another phrase that Tess used to describe her criteria of when to use calculators, had to do with "setting up" the problems. For one of the sort tasks she said, "Once you've set up these kind of problems with them and they're just going through and doing them, I think the calculator is appropriate" (T, PrI-TS1, 25). Perhaps this "setting up" referred to a mathematical procedure or a calculator procedure, because in another task students were asked to give the decimal equivalent of a fraction, and Tess said, "if they already know they divide the denominator into the numerator, I think it's appropriate" (T, PrI-TS1, 32).

Overall, Tess agreed that it was fair to say of her ideas that "you just think that by doing some of it by hand first, they'll have a better understanding of what they should expect when they go to the calculator, or, you're not taking away from their understanding if you let them use the calculators at that point" (T, PrI-TS1, 43). Tess's initial interview was cut short by an Academy team meeting.

Keeping in mind that the examples Tess had used were of plotting points for graphing and of knowing that dividing numerator by denominator will convert a fraction to a decimal, it appears that Tess did not think beyond the question of whether a student should be permitted to use a calculator or not. She had criteria for when students should be allowed to use calculators that seemed to depend on whether they could do the same thing without a calculator but just wanted to be faster. She worried about students' understanding of concepts, and whether using calculators would block that understanding. She felt responsible to be the gatekeeper of technology for her students, to be sure they understood concepts before they used calculators.

Yvette, Teaching at Riversmeet HS in a Freshmen Academy

Yvette had the least previous experience with calculators before coming to RHS. She had not used graphing calculators in any classes as a student, neither in mathematics classes nor in education classes. In fact, she was essentially introduced to graphing calculators, and to the idea that a calculator screen could be projected with an overhead projector (SD Inv, 336), at the Core-Plus Mathematics Course 1 implementation workshop she attended after she was hired by RCPS (Y, PrI-TS1, 278). She felt this introduction to calculators was good, because she was not the only one who knew very little about calculators, and there were many who were comfortable with them and could help those who needed help.

Yvette described her learning process on the graphing calculator with "okay, let's just show me and let me play with it" (Y, PrI-TS1, 294). Before this experience she felt "like you should know how to do this for yourself, versus plugging in" (Y, PrI-TS1, 296). Perhaps related to this was her response to a description of the purpose of the study. She said, "I guess my biggest fear is making handicapped, dependent children. I think they need to know how to do it, and then show them on the calculator as a quick way to do it, after you know how to do it" (Y, PrI-TS1, 304). This statement seems to agree with Tess's initial beliefs about doing procedures by hand first.

Yvette's comments on the appropriateness of calculators for the tasks she sorted seemed to be based on what would be easier or more accurate. For example, Yvette said, "they could graph it by hand, but graphing by hand is not – you can't see perfectly. Graphing it on the calculator is immediate, and then you can use the trace features and exactly see where your intercepts are" (Y, PrI-TS1, 333). In response to another task, she

said, "they could put the equation in, get a table. So much easier" (Y, PrI-TS1, 338). Yvette also seemed to be open to students using "guess and check" with the calculator. But on one of the tasks she rebelled: "Oh, honestly, you don't need a calculator .... I wouldn't have them do that with a calculator, because, honestly, the slope is given for you anyway, you don't even have to calculate the slope" (Y, PrI-TS1, 343-349). In fact, her explanation for most of the tasks that she put in the "never appropriate to use calculators" pile was that "they should be able to solve stuff like that [without calculators]" (Y, PrI-TS1, 365).

Yvette does not exhibit a conception of using calculators for anything other than what could be done by hand. She, in contrast to Tess, sees the calculator filling in for procedures students cannot do by hand, and thus she worries about making students dependent on calculators. Although she describes her own learning about calculators as wanting to play with them, the implication is that she can learn by playing because she knows what she would do by hand and now just needs to duplicate that on the calculator. Students do not have that ability in her understanding.

Rob, Teaching at Riversmeet HS in a Freshmen Academy

Rob's early college work in aerospace engineering had given him experience in using calculators in calculus classes, but mainly to produce graphs, not to circumvent algebraic manipulation. Rob had also used graphing calculators in his senior year in high school. He recalled hoping that they would help with the math computations, but said they were most useful in making graphs. It was not until his math methods course after he switched his college program to study education that he saw a calculator that would do algebraic manipulation.

In discussing the tasks that he sorted, Rob referred to many in his "never appropriate to use calculators" pile as "specific problems where I'd want them to be able to do them physically with paper and pencil" (R, PrI-TS1, 458). Others he thought should be done with mental math. In his "always appropriate to use calculators" pile Rob put "questions where we've already gotten to a certain point, I don't want them to spend a lot of time on manipulating .... do they really understand the problem?" (R, PrI-TS1, 467)

When the "sometimes appropriate" tasks were discussed, Rob's criteria seemed to depend on what he wanted students to be able to do before they used a calculator. For one task he said, "I want them to mentally visualize, or visualize this mentally before they go ahead and plot this. But then I would also take this equation and have them graph it, so they have both" (R, PrI-TS1, 471). Rob seemed to prefer that students be able to get a pretty good mental picture of what they expect to see and then use a calculator to check, "or maybe as an expanding of what they didn't know, or they didn't think they knew, or something they found that was curious that they didn't know before" (R, PrI-TS1, 472). Rob carried the visual theme into other comments, also wanting students to plot graphs by hand when starting out. Rob summed up his attitude toward calculator use this way: "The calculator's going to be a visual aid, it's going to be a checker, it's going to be something that we can use for simulations with random numbers. I guess I'm trying to say I'm big into visual math" (R, PrI-TS1, 481). Even the task that asks for the sine of 15° Rob would like to see approached by visualizing the sine curve (R, PrI-TS1, 502).

Rob's interest in visualization takes his thinking beyond the idea of duplication of by-hand calculation. He allows for the possibility that students might explore "what they didn't know, or ... that was curious..." (R, PrI-TS1, 472), but doesn't go as far as saying

that they should explore to help gain understanding. Rob does have his list of things that students should be able to do by hand, but he does not state how he expects students to learn to visualize before working on graphing calculators. In his "always appropriate" to use calculators pile, Rob placed the more complicated problems, with several steps or comparison of several functions. He implied that this meant he would not use such problems until students were experienced enough with the mathematics involved that his major concern was if they understood what the problem was asking, and how the mathematics they already knew could be used to solve it. For this the calculator would be appropriate in Rob's classroom.

All of the teachers, in the initial interview, expressed a desire to have students at some level of readiness before they used calculators. They expressed that condition in different ways: "understanding," "setting up," "visualizing," "right thinking processes." One of the tasks of the study groups would be to clarify just what background was necessary before calculators would be appropriate. Another worry expressed by at least two teachers was that "other teachers" were not using calculators appropriately, and three mentioned the implied result that students were unable to do what they should be able to do in secondary school mathematics. These worries match almost exactly the concerns expressed by teachers in Szombathelyi (2001).

During the initial interviews, all five teachers expressed an understanding of using calculators appropriately which involved answering the question, "Should students be allowed to use calculators or not?" The focus on this interpretation may have been driven by the "never," "sometimes," and "always" appropriate categories they were asked to use in the task sort. But in focusing on this question, these teachers were expressing the same

concerns as teachers did in the survey studies done by Fleener (1995) and Szombathelyi (2001), and the rule-based teachers in Tharp et al. (1997). In these studies also, the teacher responses were limited by the framing of the survey items to which they responded.

Initial Task Sort

Teachers were asked to place each of 20 tasks (see Appendix C) into one of four categories. The categories were assigned numbers for the sake of analysis, with higher values associated with greater attribution of appropriate use:

0 I would not use this task at all, with or without calculators
1 It would never be appropriate to use graphing calculators with this task
2 It would sometimes be appropriate to use graphing calculators with this task
3 It would always be appropriate to use graphing calculators with this task

The tasks were printed on separate small pieces of paper, so that they could be physically manipulated and categorized, and each teacher used his or her own set. At the end of the discussion of the sort, the papers were placed in four envelopes for later recording of sort results. The number associated with each category in the list above was then used to calculate a "mean use" index, both for each task (mean of all category values assigned to the task) and also for each teacher (mean of all category values assigned by each teacher). For example, Task G received the lowest mean use index of 1.2. It read "Give the decimal equivalent of $\frac{3}{4}$. Explain why your answer makes sense." A similar task, which was expected to receive a higher mean use index, was task H, which read, "Give the decimal equivalent of $\frac{156}{195}$. Explain why your answer makes sense." Task H received a mean use index of 2.6. Task I, which received the highest mean use index of 2.8, read,

Sales of Newspapers in the US (millions)

Year	A.M.	P.M.	Total
1982	33.2	29.3	62.5
1985	36.4	26.4	62.8
1988	40.5	22.2	62.7
1990	41.3	21.0	62.3

Fit regression lines for A.M., P.M. and Total data trends by year. How are the equations related? Why does that make sense?

Table 2 shows the results of each teacher's sort, along with teacher and task "mean use" values. Teachers have been placed in the table according to their mean use values, with lower levels of appropriate use first.

Task	Yvette	Tess	Rob	Karl	Lynn	Task Mean Use
A	2	2	2	3	2	2.2
B	2	2	2	2	3	2.2
C	1	1	2	3	3	2.0
D	2	2	2	3	3	2.4
E	1	2	2	3	2	2.0
F	2	2	1	2	3	2.0
G	1	1	1	1	2	1.2
H	3	2	3	3	2	2.6
I	3	3	3	3	2	2.8
J	1	2	1	3	3	2.0
K	2	1	2	2	2	1.8
L	2	1	1	1	2	1.4
M	2	2	3	2	3	2.4
N	2	2	2	3	3	2.4
O	1	1	1	1	3	1.4
P	1	1	1	2	2	1.4
Q	1	2	2	1	3	1.8
R	0	2	2	0	3	1.4
S	2	1	1	2	3	1.8
T	1	2	1	2	3	1.8
Teacher Mean Use	1.6	1.7	1.75	2.1	2.6	

Table 2: *Results of task sort by extent to which calculator use is deemed appropriate*

Since the assigned values are only ordinal, these means can give only an indication of the openness teachers have to using calculators relative to other teachers in the study. Likewise the "task mean use" indicates only the opinions of this group of teachers on the relative appropriateness of using a calculator on one task compared to

another task in the list. The sort category values were also used to compare the sort results for each pair of teachers.

To obtain a measure of the extent of agreement among the teachers on the categories to which tasks were assigned, a value of 1 was assigned if two teachers put a task into the same sort category, and 0 if they put a task into different categories. Results for all ten possible pairs of the five teachers appear in Table 3.

Task	T/R	Y/R	Y/T	Y/K	T/K	R/K	K/L	R/L	Y/L	T/L	Pair-Agree on Task
A	1	1	1	0	0	0	0	1	1	1	6
B	1	1	1	1	1	1	0	0	0	0	6
C	0	0	1	0	0	0	1	0	0	0	2
D	1	1	1	0	0	0	1	0	0	0	4
E	1	0	0	0	0	0	0	1	0	1	3
F	0	0	1	1	1	0	0	0	0	0	3
G	1	1	1	1	1	1	0	0	0	0	6
H	0	1	0	1	0	1	0	0	0	1	4
I	1	1	1	1	1	1	0	0	0	0	6
J	0	1	0	0	0	0	1	0	0	0	2
K	0	1	0	1	0	1	1	1	1	0	6
L	1	0	0	0	1	1	0	0	1	0	4
M	0	0	1	1	1	0	0	1	0	0	4
N	1	1	1	0	0	0	1	0	0	0	4
O	1	1	1	1	1	1	0	0	0	0	6
P	1	1	1	0	0	0	1	0	0	0	4
Q	1	0	0	1	0	0	0	0	0	0	2
R	1	0	0	1	0	0	0	0	0	0	2
S	1	0	0	1	0	0	0	0	0	0	2
T	0	1	0	0	1	0	0	0	0	0	2
Pair- agreement Totals	13	12	11	11	8	7	6	4	3	3	
								Least agreement			C, J, Q, R, S, T
								Most agreement			A, B, G, I, K, O

Table 3: *Pairwise agreement on task sort categories in initial task sort*

Summing the ones by task was then used as an indication of the amount of agreement among these five teachers on the categories of the tasks. It must be remembered that this pair-wise agreement does not necessarily imply overall agreement. In fact, there was no task that all five teachers put in the same category. The tasks that had the highest number (6) of pairs agreeing on category were tasks A, B, G, I, K, and O;

while the most disagreement occurred on tasks C, J, Q, R, S, and T. Summing the pair columns indicates that in the first task sort, Tess and Rob agreed on categories for 13 of the 20 tasks. There was nearly as close an agreement between Yvette and Rob. It is no surprise then that Yvette and Tess agreed next closely, but Yvette and Karl also agreed on the same number of task categories. The least agreement of any teacher with another on task categories was Lynn with Yvette, Tess, and Rob.

These results are not surprising given Rob's willingness to have students explore, and Yvette's decisions based on what is quickest or easiest, but it does not seem to agree with Tess's insistence on working by hand first, which seems to agree more with Lynn's position. One explanation is that Tess was thinking of her Integrated Mathematics 2 class, which she considered to be advanced in terms of understanding concepts, and thus more ready to use calculators. At this point, Karl seems to be a moderate user, perhaps comparable to Szombathelyi's (2001) intermediate users.

Initial Survey Results

The survey instrument (see Appendix D) was given to teachers at the initial interview, and they completed it individually prior to the first study group meeting. This was the first opportunity teachers had during the study to write on their thoughts about graphing calculators and their use in teaching. The survey consisted of two parts. Part A had three free-response items, and part B used a variety of limited choice responses. Tables 4 through 18 reproduce the survey items from Parts A and B, and provide responses of the five teachers prior to the first study group session. Teacher responses are identified using the teacher's pseudonym initial.

Results of Part A – Free Response

With respect to RQ1, the issue that is most evident in responses to A1 is that several teachers feel that students should "do problems first" (Lynn) before calculators are introduced, or "be able to do these problems without a calculator" (Tess). Rob's recognition that students used the calculator for quick calculations did not seem to be an issue at the time he wrote this response, nor did Yvette seem concerned with students using calculators to do arithmetic. The fact that these are pre-existing issues means they do not directly apply to the negotiations in the group referred to in RQ1, but they are most likely to be issues that will surface in the group sessions.

A1: Give a brief description of how you are presently using graphing calculators in teaching your mathematics classes.
K: *Integrated II and Algebra II are calculator courses [the textbooks call for use of graphing calculators].* L: *Use them daily in instruction and students use them for investigation. They need to show their work and do problems first without them – making tables and graphs, etc.* T: *Not using it in my Math Investigations class. I am teaching basic skills in this class. I want them to be able to do these problems w/o a calculator. [In] Core-Plus (Integrated I & II) depending on the topic, I usually introduce the topic first for understanding and then introduce the calculator.* R: *Generally, we use the calculators as a tool for understanding and generating graphs, tables, and data. Sometimes the students will use the calculator as a quick answer to simple computations.* Y: *I use graphing calculators to model equations and show similarities/differences between equations. I also allow students to check work and do arithmetic.*

Table 4: *Initial teacher responses to Survey Item 1 of Part A*

For input to RQ2, we look at Rob's response. He saw the calculator from what might have been a science teacher's perspective – " a tool for understanding and generating graphs, tables, and data." Karl and Tess address RQ3, indicating that their courses call for calculators, and indeed this was found to be the case, because Core-Plus Mathematics (*Contemporary Mathematics in Context*) (Coxford et al., 1998 - 2000),

which was also the text for Karl's "Integrated II," and *Advanced Algebra through Data Exploration* (Murdock et al., 1998) are textbooks that use the graphing calculator as a tool for student learning. Yvette also expressed her belief that the calculator is useful to teach certain concepts, such as comparison of graphs.

The responses to A2 in Table 5 provide some insight into these teachers' beliefs, and thus address RQ3. Three of the teachers use the word "first" or imply that something should happen before calculators are used. Two of them want to see "understanding" first, and the other wants "the fundamentals" learned first. Rob, on the other hand, does not see

A2. Please list the most important things you consider when you decide how and when to use (or <u>not</u> use) graphing calculators in your mathematics classes.
K: *Whether or not it's taking the place of learning the fundamentals first.* *Is it an enhancement to the course?* *Does speed count?* L: *They have to have basic understanding first then use as a tool not a crutch.* T: *I first look for understanding and then use the calculator.* R: *If I would like the students to use their mental math skills then the calculator is out of the question. If the calculator can be used as a visual aid, then I prefer the calculator.* Y: *concepts – of linear, parabolic, exponential models* *use of tables, regression models* *quicker* *visual tool*

Table 5: *Initial teacher responses to Survey Item 2 of Part A*

a predetermined sequence. Instead, he wants to be clear on what his purpose for the lesson is, and then whether the calculator will help fulfill the purpose. Yvette concentrates on which capabilities of the calculator she would use, and for what content. Karl makes some modifications to his "fundamentals first" beliefs. He seems to be open to "enhancement" of the course, and is also aware that speed [presumably of calculation] might make a difference in a lesson.

The purpose of A3 (see results in Table 6) was to guide the choice of activities to be used as examples in the study group sessions. Thus it was not intended to address any of the research questions. However, since A3 asks for concepts students have problems

A3. Please identify several of the mathematics concepts in your class that students have the most trouble understanding.
K: *Fractions, operations of signed numbers – especially subtractions, LCM* L: *fractions – number line place values; multiplication facts; basic difference between operations* T: *graphs – tables – equation; solving a linear system; simulation models; linear models* R: *fractions; exponents* Y: *negative numbers (adding/subtracting) fractions*

Table 6: *Initial teacher responses to Survey Item 3 of Part A*

"understanding," perhaps it gives some insight into the meaning intended in responses to A2. "Understanding" seems to also include certain memorized facts, such as "multiplication facts," and processes, such as "operations of signed numbers" and "basic difference between operations." Perhaps Rob's inclusion of "fractions" and "exponents" are also problems with the "basic difference between operations." At any rate, it is clear that all of these teachers, except perhaps Tess, are identifying as problematic concepts that students study long before they reach the secondary school classrooms in which these teachers work. This fact was one influence on the researcher's decision to select for group discussion mathematical examples on the level of linear relations.

Results of Part B – Limited Response Items

Part B of the survey used a variety of formats. In items 1 and 11, teachers were asked to select one of a series of choices that best described certain characteristics of their education and experience with calculators, providing background for RQ1 and RQ2. Note that Rob elected to best describe his background rather than checking only

1. Check one of the following that *best* describes your background in mathematics. __R____ I have learned much of my mathematics in science, business or engineering courses. __R____ I have an undergraduate minor in mathematics. K,T,Y,L_ I have an undergraduate major in mathematics. _______ I have taken some graduate level mathematics courses.
11. Circle YES [<u>R,T</u>] or NO [<u>K,Y,L</u>]: My college or graduate level methods class included use of calculators. Below, check one that *best* describes your experience using calculators (i.e. scientific or graphing calculators). _____ I haven't used calculators much except for basic operations (i.e. +, -, x, ÷). __Y__ I have used calculators in some math courses or workshops, but not much beyond basic operations and functions (i.e. powers, finding roots, trig functions, etc.). _K,L_ I have taken at least one math course or workshop that required significant calculator use including graphics capabilities (i.e. graph, tables, trace, programming, etc.), but little, if any, use of the statistics and probability capabilities. _R,T_ I am comfortable with most aspects of calculators including statistics and probability capabilities.

Table 7: *Initial teacher responses describing education and calculator background*

a single choice. Teacher responses are indicated in Table 7, using initials. We see that four of the teachers majored in mathematics, and the other studied science or engineering, and that two had methods classes that used calculators and are comfortable with calculators.

Results of Part B – Likert-scale Responses about Calculator Use

Other groups of survey items in Part B ask for responses on a five-point Likert scale, from Strongly Disagree (SD) to Strongly Agree (SA). The five-point scale allowed a neutral choice (N). Four such items further probed teachers' feelings of confidence for teaching mathematics, particularly with calculators. The responses shown in Table 8 indicate that all but one of the teachers were comfortable using calculators, and all were confident in their ability to teach mathematics. This addresses RQ2 by delineating a baseline of experience on which participants will draw during the study group sessions.

To further describe the experiences and artifacts addressed in RQ2, teacher knowledge of state and national standards for the teaching of mathematics was also probed in three items of the survey. The responses shown in Table 9 indicate consistent

Mark one "0" for each statement	SD	D	N	A	SA
14. I am confident in my ability to learn new mathematics concepts.	0	0 LY	0	RKT	
15. I am confident in my ability to formulate questions to guide students' understanding of mathematics.	0	0 LY	0	RKT	
37. I understand the fundamental ideas of mathematics.	0	0	0	RKLTY	0
50. I am confident in my ability to teach mathematics using calculators.	0	0	R	KLTY	0

Table 8: *Initial teacher comfort with mathematics and calculators*

familiarity with the Michigan *Mathematics Framework*, but less consistent knowledge of NCTM national standards, and even less knowledge about the Michigan state mathematics test for high school students.

Mark one "0" for each statement	SD	D	N	A	SA
34. I am well-informed about the National Council of Teachers of Mathematics (NCTM) *Principles and Standards* for the grades I teach.	0	K	R	LTY	0
35. I am well-informed about the Michigan *Mathematics Framework* standards and benchmarks appropriate for the courses I teach.	0	0	0	RKLTY	0
36. I am well-informed about the Michigan High School Proficiency Test for Mathematics (1998) **state mathematics test** for high school students.	0	KLT	R	Y	0

Table 9: *Initial teacher knowledge of standards and state test*

Following the lead of the *2000 National Survey of Science and Mathematics Education* (Weiss et al., 2001), the survey included six items about the school context in which these teachers were teaching. These items provide insights for answering RQ3. The responses shown in Table 10 indicate wide variation in perceived collegial support and shared vision. There is agreement that teachers have good access to calculators for teaching, but teachers did not agree on whether students are dependent on calculators.

Two of the teachers who agree that colleagues share ideas are not teaching in the same school as the two who disagree with the statement. The third teacher who agrees that colleagues share ideas was new to her building in the year of the study, and may have had an orientation period or mentor that involved some sharing of ideas. The fact that Rob is often neutral on these statements may be an indicator that Rob began teaching in his school only at the beginning of the semester during which the study group sessions were held. On the other hand, Tess may feel isolated from other mathematics teachers because she is the only mathematics teacher on her team, a group that teaches all the courses for a subgroup of the freshmen at this large high school.

Mark one "0" for each statement	SD	D	N	A	SA
13. I feel supported by colleagues to try new ideas in teaching mathematics.	0	L	R	TY	K
38. I have adequate access to graphing calculators for teaching mathematics.	0	0	0	RKLY	T
39. Mathematics teachers in my school have a shared vision of effective mathematics instruction.	0	T	R	KLY	0
40. Mathematics teachers in my school regularly share ideas and materials related to mathematics instruction.	0	RT	0	KLY	0
41. I have time during the regular school week to work with at least one other teacher on matters related to planning and teaching our courses.	0	RKTY	0	L	0
42. Students are dependent on calculators when they come to my class.	0	K	R	LTY	0

Table 10: *Initial teacher perception of teaching environment*

To shed further light on RQ3, specifically on how the teacher participants in this study think about mathematics, that is, their philosophical stance, four items were adapted from Simmt's study (1993) of how such beliefs affected use of calculators. The responses to these items, as seen in Table 11, show that the divisions in philosophical tenets about mathematics in this group of teachers focus on whether mathematics is hierarchical and cumulative, and on whether it would be possible to have a world with

different mathematical truths. Division is indicated by agreement of some teachers but disagreement of others to some of the statements, as in the responses to items 46 and 47. It is interesting to note that Rob is the teacher whose mathematics was studied in the context of science or engineering, and he is the only one who disagrees that mathematics is hierarchical and cumulative. This information was believed to be helpful in planning the study group activities, since Simmt (1993) showed that teachers' calculator use tended to reflect their philosophies of mathematics. The responses suggested that the activities used in the study groups did not have to appeal to teachers with vastly different philosophical convictions.

Mark one "0" for each statement	**SD**	**D**	**N**	**A**	**SA**
43. Mathematics is fixed and unchanging.	T	RKY	L	0	0
44. Mathematical ideas are constructed by human minds.	0	0	RY	KLT	0
46. Mathematics is essentially hierarchical and cumulative.	0	R	T	KLY	0
47. A world with different mathematical truths is impossible.	0	KL	RT	Y	0

Table 11: *Initial teacher conceptions of mathematics*

The two largest categories of items in the survey examined beliefs about calculator use, relating to RQ3, and about pedagogical ideas, relating to RQ2. These categories use several forms of items. The first to be considered will be the Likert-type items relating to RQ3, probing beliefs about calculator use in teaching mathematics. The sources of these items are indicated by capital letters after the items. The letter F indicates the source is Fleener (1995b), Z indicates Ziebarth (2002), and M indicates Myers (1998). Items that are not marked in any way were added as a result of feedback from a pilot study. Table 12 gives the teacher responses to the first group of these items.

Item 48 is Fleener's (1995b) "Mastery" item: "Students should not be allowed to use calculators until they have mastered concepts." Fleener (1995b) was able to characterize her survey respondents according to this item and have agreement within

each group on all other items for which there were significant differences between the groups. The other two items in Table 12 are those for which Fleener's groups showed

Mark one "0" for each statement	SD	D	N	A	SA
48. Students should not be allowed to use calculators until they have mastered concepts. (F, Z, M)	0	RLTY	0	K	0
32. Using calculators will cause students to lose basic computational skills. (F)	0	RT	L	KY	0
8. Continued use of calculators will cause a decrease in student estimation skills. (F)	0	KTY	R	L	0

Table 12: *Initial teacher responses to Fleener's 'mastery' item, and related items*

significant differences. It can be seen that the group of five teachers in this study does not follow Fleener's finding. One difference in study participants that may account for this is that Fleener's participants had just attended what was for most their first workshop on the Casio 7000 graphing calculator. The participants in the present study have all been using calculators in their classes for at least a year, and/or used calculators in classes they took during their teacher preparation. Karl's lack of calculator use in his teacher preparation coursework may explain why Karl agrees while the others disagree on items 48 and 32. Yvette also did not have calculator experience in her teacher preparation program, but did attend a curriculum implementation workshop that emphasized use of calculators. Why Karl then disagrees on 8, or why Lynn answers these three items so differently is not clear. In her first interview, Lynn related how she fought against the use of calculators by her middle school students. However, she finally came to accept that if she expected them all to do calculations by hand she would not have time to teach the concepts, such as graphing and exponential growth, that her curriculum included. This possibly explains her response to item 48.

On the eight items in Table 13 addressing RQ3 and RQ2, we see Rob and sometimes Yvette disagreeing with the others, or being neutral, while Tess is the only one

to be emphatic about her agreement. Comparing item 3 to item 48 above, Rob and Karl seem to be the only ones being consistent, or perhaps the others feel that procedures cannot be elucidated by using calculators, but concepts can be. We will examine other data for support of this conjecture about procedures and concepts. But if we use that interpretation, then the disagreement of Lynn, Tess, and Yvette with item 5 is not consistent. The more/less important items (9 and 10) were suggested by a summary of the impact of technology on the mathematics curriculum (NCTM, 1991b). Rob's science background may give him a different view of the utility of mathematics in item 9. When he reviewed this section of the survey during the final interview, Rob said that to him "math" included knowing what to do to solve a problem, regardless of the tool, and that none of that is less important.

Mark one "0" for each statement	**SD**	**D**	**N**	**A**	**SA**
3. Students need to demonstrate proficiency in using mathematical procedures before doing any similar work using calculators. (Z, M)	0	R	0	KLTY	0
6. More difficult mathematics problems can be done when students have access to calculators. (F)	0	R	0	KLY	T
7. Using calculators frees students to explore alternative solution strategies. (F)	0	R	0	KLY	T
9. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *less* important.	0	RY	0	KLT	0
10. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *more* important.	0	Y	R	KLT	0
49. Students should be allowed to use calculators on standardized tests. (F)	0	0	R	KLY	T
2. Incorporating calculators into teaching requires changing the types of problems assigned. (F)	0	KLTY	R	0	0
5. Students should be allowed to use calculators even before they understand the underlying concepts. (F)	K	LTY	R	0	0

Table 13: *Initial teacher responses regarding use of calculators*

The six items for which responses are shown in Table 14, although intended to address RQ3, may also be seen as affecting RQ1, since the statements themselves may

have raised issues that would be brought up later. Participant teachers for the most part either agreed or disagreed with these items as a group. In interpreting the agreement with item 4 in Table 14 below and the disagreement with item 2 in Table 13 above, it is important to know that all of these participating teachers are teaching from reform curricula that are designed to use graphing calculators, so "change" may be interpreted as doing other than these curricula suggest. The agreement with item 28 is consistent with this interpretation. Clearly these teachers consider it worthwhile to teach students to use calculators, and they use them for more than computation. At the beginning of the study only one teacher (Yvette) felt that the major value of calculators is to save time in performing computations.

Mark one "0" for each statement	**SD**	**D**	**N**	**A**	**SA**
4. It is not necessary to change what is taught in order to effectively use calculators. (F)	0	0	0	RLTY	K
28. The calculator can be used to explore mathematical concepts. (F)	0	0	0	RKLY	T
22. The use of calculators is fine as enrichment for better students, but not as part of "mainstream" mathematics for all students. (Z)	LT	RKY	0	0	0
25. Use of calculators will eventually replace all paper and pencil work in mathematics.	L	RKTY	0	0	0
26. It takes too long to teach students which buttons to push on a calculator.	LT	RKY	0	0	0
51. The major value of calculators in mathematics classes is to save time from performing computations. (F)	0	RKLT	0	Y	0

Table 14: *Initial teacher responses showing substantial agreement on calculator use*

Responses to the last three Likert-scale items dealing with RQ3 are shown in Table 15. These are items that, along with items 8, 32 and 48, show definite disagreement among the participating teachers. This information influenced the choice of artifacts which the researcher brought to study group sessions, and the questions asked in sessions and in interviews. The initial interview also provided insight into the disagreement on

item 33. Those who agreed with the statement were thinking particularly about elementary school mathematics classes. Even though Tess disagreed with item 33, she stated more than once her opinion that elementary teachers were using calculators as replacement for teaching, particularly when they did not understand the mathematics. For items 53 and 54, disagreement may indicate teachers thinking of particular uses of calculators that would not produce the stated results.

Mark one "0" for each statement	SD	D	N	A	SA
33. It is not appropriate for calculators to be used in some mathematics classes. (F)	0	RT	0	KLY	0
53. Using calculators in the teaching of mathematics results in greater student understanding of concepts. (Z)	0	T	RY	KL	0
54. Using calculators in the teaching of mathematics encourages a more active, conjecturing approach to the learning of mathematics. (Z)	0	L	0	RKTY	0

Table 15: *Initial teacher responses showing disagreement on use of calculators*

Results of Part B – Likert-scale Responses about Pedagogy

The final set of survey items were intended to provide baseline information for RQ2, by probing teachers' pedagogical beliefs and conceptions. The sources of the items are indicated by initials, as above.

The first group of these items uses the same Likert scale described previously. The items reported in Table 16, with which all teachers agree or are neutral, indicate that teachers think calculator skills are important, as is paying attention to how students think and learn. On the other hand, the participant teachers all agree that a number of pedagogical ideas are not acceptable, namely, that teachers should dictate methods that students use, that students learn by mimicking, that it takes too long to teach calculator use, that students understand better by using paper and pencil, and that it is unreasonable to expect students to be creative in mathematics. These responses led the researcher to

believe that there would be little disagreement among the participants around these issues.

There was disagreement, however, in the responses to some of the items dealing with pedagogical issues, particularly those dealing with how students learn. Two of the few items for which participants chose broadly differing responses were items 20 and 27, as shown in Table 17. Item 20 claims that "students learn mathematics by the personal building of mathematical understanding." The teacher with the most

Mark one "0" for each statement	SD	D	N	A	SA
16. It is important for mathematics teachers to be aware of how students learn mathematics. (Z)	0	0	0	RK	LTY
17. It is important for teachers to ask how students are thinking when studying mathematics even if it limits the amount of material covered. (Z)	0	0	0	RKY	LT
29. Calculator skills are as important as paper and pencil computational skills. (F, M)	0	0	0	RKLY	T
30. Students can gain understanding of computational procedures by using calculators. (F)	0	0	R	KLTY	0
18. Students should use procedures taught by a teacher instead of ones they develop on their own. (S)	T	RKLY	0	0	0
19. Students learn mathematics by studying examples and practicing mathematical concepts and skills. (S, Z)	T	RKY	L	0	0
25. Use of calculators will eventually replace all paper and pencil work in mathematics.	L	RKTY	0	0	0
26. It takes too long to teach students which buttons to push on a calculator.	LT	RKY	0	0	0
45. Expecting students to be creative in mathematics is unreasonable.(S)	0	RKLTY	0	0	0
52. Students understand math better if they solve problems using only paper and pencil. (F)	0	RKLTY	0	0	0

Table 16: *Initial teacher responses showing agreement on certain pedagogical ideas*

experience (Karl) disagreed, the teacher with least experience and a science background was neutral, and the other three agreed. Item 27 proposed that in order to understand the mathematical concepts underlying an algorithm, one has to be able to perform the

algorithm accurately. In this case, the most experienced teacher is neutral and the middle school teacher and the teacher with the least calculator experience agree.

Responses to the last five items in Table 17 remove us from the familiar patterns of Rob or Karl being the one who does not agree with the others, and we see that Karl, Tess, Rob, Yvette, and Lynn can all play that role. Karl, for example, does not see a connection between understanding and accurate performance of procedures. Tess says that students needn't show calculator work on paper. As we have seen before, Yvette is the only one who thinks the major value of using calculators is to speed computation. Rob alone disagrees that teachers should decide when calculators are used, and Lynn alone agrees that accurate performance of an algorithm is necessary for understanding underlying concepts. These, then, are some of the issues on which the researcher prepared to hear disagreement.

Mark one "0" for each statement	**SD**	**D**	**N**	**A**	**SA**
20. Students learn mathematics by the personal building of mathematical understanding. (S)	0	K	R	L	TY
27. Being able to accurately perform an algorithm is necessary for understanding the underlying mathematical concepts. (S)	T	R	K	LY	0
21. If a student understands a mathematical concept, then that student will be more able to accurately perform procedures associated with the concept. (S)	0	K	0	RLT	Y
24. When solving problems with calculators, students don't need to show their work on paper. (F)	L	RKY	0	T	0
31. The teacher should decide when it is appropriate for students to use calculators. (F, M)	0	R	0	KLTY	0
51. The major value of calculators in mathematics classes is to save time from performing computations. (F)	0	RKLT	0	Y	0
53. Using calculators in the teaching of mathematics results in greater student understanding of concepts. (Z)	0	T	RY	KL	0

Table 17: *Initial teacher responses showing disagreement on certain pedagogical ideas*

Results of Part B – Other Descriptive Responses about Pedagogy

The final item left to examine on the survey actually deals with how prepared teachers feel they are to teach using certain strategies. This item borrows its form and content from the 2000 National Science Foundation Local Systemic Change teacher survey (Horizon, Inc., 2000), but was modified following feedback from teachers in the pilot study.

Teacher responses, given in Table 18, show that most participants considered all the strategies except "using graphing calculators" fairly or very important. Two of the five teachers thought "using graphing calculators" was only somewhat important. Again, Rob was the consistent stand-out, thinking all strategies were only fairly important, but he was joined at times by Karl or by Yvette, who felt that conceptual understanding and hands-on activities were only somewhat important. In terms of how well prepared

12. For each item below, please								
use the *left section* to rate its **importance** for effective mathematics instruction.					Use the *right section* to indicate how **prepared** you feel to do each one.			
NI = not important **SI** = somewhat important		**FI** = fairly important **VI** = very important			**NAd** = not adequately **S** = somewhat		**FW** = fairly well **VW** = very well	
importance				Please mark one "0"	**prepared**			
NI	SI	FI	VI	←in each section for each item →	NAd	S	FW	VW
0	0	RY	KLT	Provide concrete experiences before abstract concepts.	0	R	TY	KL
0	Y	R	KLT	Develop students' conceptual understanding of mathematics.	0	R	TY	KL
0	Y	RK	LT	Have students participate in appropriate hands-on activities.	0	RY	K	LT
0	0	RK	LTY	Engage students in investigative activities.	0	RK	Y	LT
0	KL	RY	T	Use graphing calculators.	0	K	RY	LT
0	0	RY	KLT	Engage students in applications of mathematics in a variety of contexts.	0	RY	KT	L

Table 18: *Initial teacher responses on importance of strategies and preparedness to use them*

teachers felt they were to teach with certain strategies, Rob felt only somewhat prepared in all strategies except using graphing calculators, for which he felt fairly well prepared. The other teachers felt fairly well or well prepared for all the strategies, with the exception of Karl who felt only somewhat prepared to use graphing calculators and investigative activities, and of Yvette, who felt only somewhat prepared to use hands-on activities and a variety of contexts.

In summary, results of the survey instrument give a thorough description at the beginning of the study of how the five participant teachers were thinking about the use of graphing calculators in teaching mathematics. All five feel confident in their knowledge of mathematics and in their ability to teach mathematics. They are all fairly well acquainted with national and state mathematics standards, but seem to feel less knowledgeable about the state mathematics test. All but Rob feel at least fairly well prepared to use teaching strategies that they think are effective. Most of the teachers are aware of the collegial context in which they teach, as well as the student context. The three schools represented in the group do not have the same levels of collegial support or shared vision.

For the most part, these teachers do not see mathematics as fixed and unchanging. They believe that mathematical ideas are constructed by human minds, and that mathematics is hierarchical and cumulative. Contrary to Fleener's findings (1995b), at the beginning of the study these teachers do not neatly divide over the question of mastery. Perhaps because they have had more experience with students using calculators, they have different views on the effect of calculator use on computational skills and estimation. With the exception of Rob, these teachers at the beginning of the study agreed

on many positive uses of calculators. Again, with the exception of Rob, who was neutral, they all believe that using calculators does not require changing the types of problems assigned, and that calculators should not be used before underlying concepts are understood.

Similarly, at the beginning of the study, all five teachers had the same view of the consequences of teaching with calculators, such as more exploration is possible but calculators will not replace paper and pencil work entirely. They also did not see a great need for change in the content studied, though some content may become less important. However, they were divided on whether the use of calculators would inspire better understanding or more conjecturing.

Also, at the beginning of the study all of the teachers were in agreement on many pedagogical issues, such as the importance of knowing how students were thinking, and letting students develop their own solution methods. However, there was distinct disagreement on the relationship between using calculators and understanding concepts, and this disagreement seems to be deeper than deciding which comes first. The initial survey showed that teachers agreed that calculator skills were important and should be taught, but they disagree on whether using calculators results in better understanding of concepts and on whether students must develop their own understanding.

Researcher Decisions about Study Group Sessions, Based on Initial Interview, Task Sort and Survey

In the spirit of full disclosure of possible sources of bias, and because the researcher is also a member of the study group, a background of the researcher is now given, similar to that given above for other participants. The researcher received an

undergraduate degree in physics and mathematics, then spent three years working with teachers in the Philippines as a U.S. Peace Corps Volunteer before returning to graduate school for teaching credentials. She taught physical science for two years, and then taught mathematics at two other schools – including computer programming, geometry, advanced algebra, statistics, pre-calculus, calculus, and a hybrid course starting with abstract algebra and leading to the design and building of computer circuits. She had no experience with calculators in college, but was introduced to computers in her second teaching assignment, and to graphing calculators in her next teaching position. She attended conferences and professional development whenever possible, and worked hard to keep up with new technology. Her master's degree was earned in Educational Computing. A full statement of the researcher's understanding of appropriate use of graphing calculators is found in Appendix B.

Following the first reading of the transcripts of the initial interviews, and coding of the interviews using the initial codes found in Appendix J, the researcher wrote a reflection on what she had learned about the group of teachers she would be working with, and their thinking about using calculators to teach mathematics. This reflection helps set the stage for understanding decisions made about the study group sessions. In part, this reflection says

> Some of what these teachers say strikes me as "ideally, this is what I would do"-type thinking. The teachers seem to have an ideal of where students should be when and if they use calculators. Only L and K have been in the classroom long enough to consider how they have had to adjust their ideals. They would like for students to have operational fluency with arithmetic, but they have to face the fact that some don't. They recognize what is the mathematics in their curriculum, and what weaknesses in fluency have to be compensated for. Both assert that students who are using calculators for number crunching can and do make progress in mathematical thinking. However, lack of certain fluency

> makes it difficult for students to recognize patterns that allow them to make models of situations.
>
> One common issue is that teachers do not want students to be dependent on calculators. They want them to be able to do the math "on their own," with paper and pencil, by visualizing, or with mental math. Most of them see this as coming before the use of the calculators, so I don't think there is much concept of using the calculator as a tool of exploration. Yet, several have said that they are not against discovery, or doing things in different ways, but, at least in one case, it is a matter of control. The teacher needs to be aware of what the students are doing, and that they are working on math. This is not always evident when they are using calculators – they play, even when they don't have games. (Reflection – InitInterviews, ¶ 3-4)

In planning for the specific examples and artifacts to use in the first study group session, the researcher reflected on the difference between these teachers and the experienced teachers who had participated in her pilot study. While the pilot teachers had brought many examples from their own classrooms into the discussion, this new study group had much less classroom calculator experience to draw on. However, one of the advantages of working with the new group was that they were all teaching the same content if not exactly the same courses, and that was linear relationships and graphs. The pilot study group had carried out a particularly productive discussion about a problem which focussed on the concept of linearity in the context of a teenager wanting to earn money, so it was decided to use that problem again. This problem did not emphasize use of a calculator, but the pilot teachers had used theirs with interesting results. A contrasting problem would also be used, because the researcher intended to lay groundwork for the discussion of tasks in a later session. The contrasting problem specified the use of a calculator in a purely abstract setting, and was seen by the researcher as typical of the activities added to standard textbooks when calculator use was encouraged by the National Council of Teachers of Mathematics (NCTM) in its

Curriculum and Evaluation Standards for School Mathematics(1989). The contrasting task has also been included in the Task Sort, as was a problem similar to the linearity problem. The two problems and their sources are included in Appendix K.

Originally, the researcher had planned for the first session a discussion based on contrasting these two tasks and examining differences in tasks and how using calculators might change the cognitive demand of tasks. However, the limited experience of using calculators with students that was exhibited in the initial interviews suggested a change of plan. It was decided that a look at these two tasks through the categories of student calculator use developed by Zbiek (2002b) would be a way to introduce a common vocabulary to enhance future discussion. Optimistically, the researcher also readied copies of Zbiek's paper on the MAGICAL framework (2002a) developed to describe student uses of different representations.

Chapter V will report on what happened during the study group sessions as these teachers began to negotiate their understandings of appropriate use of graphing calculators. But first, the next two sections will introduce concurrent activities that also affected the discussions in the study groups, and the decisions made by the researcher as each new session was planned.

Evolution of a Tool to Share with Other Teachers

The design experiment methodology has as its central focus the design, development and testing of some kind of shareable tool. For this study, the focus was on developing and testing a tool that was to be useful to other teachers in making decisions about appropriate use of graphing calculators to teach mathematics.

Initial Issues

From the initial interviews it was clear that there was no one participant who seemed eager to take on the job of being the scribe to write down and keep track of ideas about the tool and how it should be put together. Searching for a way to take on this work for the group without becoming a dictator of what the tool should look like, the researcher consolidated from her coded initial interviews, a list of 99 issues that had been raised related to using graphing calculators to teach mathematics. Before the first session, these were loosely organized into 11 subgroups. The list and subgroups can be found in Appendix L. Following Palmer's (1998, p. xvi) method of jump-starting open discussion by asking for extremes,[1] the researcher asked the teachers to introduce themselves to each other at the first study group session by responding to two questions: What do you like best about teaching with graphing calculators? and What do you like least about teaching with graphing calculators? Naturally, more issues arose.

Narrowing the Focus

The issues raised at the first session were also consolidated, and there were 58 of them. In Session 2 participants were asked to go through a list of issues and mark those that they felt the AUGC tool the group was going to develop could actually address. From those chosen, the researcher made a further consolidation, and a "first attempt" at formulation of the tool, using suggested wording from Session 2. Teachers were then asked to begin using the draft tool to decide what they would watch for to record in their calculator use log. Feedback from teachers on how the tool worked in filling out the log,

[1] Palmer began a Courage to Teach session by asking teachers to introduce themselves by telling their greatest joy and greatest sorrow in teaching. Once the group had cried together, anything could be said.

both from observation interviews and from the next session, helped the researcher to prepare the next draft of the tool for the following session. How this evaluation and revision of the tool proceeded in the study group sessions will be part of Chapter V.

Classroom Observation and Testing of the Tool

The researcher's extensive classroom experience taught her that talking about ideas and implementing them in the classroom are very different things. Partly for this reason, and partly to provide further common experience with each of the participants, teachers were asked during the recruitment process to agree to be observed for one class period a total of three times – once during each inter-session time period – with a brief interview following.

Classroom Calculator Use Observed

Teachers chose the class that would be observed, within the constraints of three different school schedules. RHS was on an alternating block schedule, with one everyday class in the academies; RMS followed a six period daily schedule; and RAA started its seven period day later and ended later than RHS. Calculator use was not made a requirement for the observed class, partly because these teachers were not experienced enough with the curriculum to change the order of lessons. Also, the Connected Mathematics and Core-Plus Mathematics curricula assume the use of calculators and capitalize on them, so there was always the possibility that someone would be using calculators.

Unavoidable Contextual Factors

Presence of the researcher in the teachers' classrooms also allowed observation of some of the contextual factors that affected the way these teachers were able to use calculators in class. For example, in one observed class, a teacher offered to pay $5 for each classroom calculator that was returned – a personal outlay of cash. It was also mentioned that one student had already collected on two calculators. Another example was the fact that every observed class was interrupted at least once by the public address system, as was every interview and session held at school, with the exception of the one held on a snow day after roads were cleared.

Some of the students in several observed classes seemed unable to work on their own or with their peers. They were demanding of the teacher's attention. Others refused to work at all, some seeming to blame the teacher for all that was wrong in their lives, and some just ignoring the teacher. Teachers were dealing with this uncooperative behavior in a number of ways. Some moved quickly from group to group as students were working, aiming to head off distraction and keep people on track. Others gave their attention to students who were working on mathematics and ignored those who were not until they became too disruptive of the class as a whole.

In other classrooms, the lesson proceeded according to a well-established routine and teachers were able to visit students as they worked and ask questions to probe their understanding. The researcher was not surprised that the newer teachers craved feedback, or at least they were accustomed to politely asking classroom visitors for suggestions. She tried to explain that she was not there to criticize or correct, but when pressed, encouraged teachers to talk about what they had tried in the past and who else was

helping them adjust to teaching in this school. Some replied with stories of students who were living in cars or with a different relative every week. Motivating students clearly was an ongoing concern of these teachers. Several teachers approached this by creating new lessons in the spirit of the curriculum they used, but using simpler words and contexts they thought the students would more easily relate to.

Only one mathematical error in notation was observed, which was taken to mean that this group of math majors had learned their content well. Another observation was perhaps more relevant to this study. In two classrooms in which teachers were working with the Core-Plus Mathematics curriculum for the first time, teachers and students were becoming frustrated because students were not able to "solve" an inequality such as $3x + 2 > 2x - 1$. Students did not know how to respond to words such as "like terms" or "undoing" operations. Yet, for specific numbers they knew whether the statement was true or not, once they recalled what "the alligator," >, meant. In each case, after class, when the frustration was voiced, the researcher asked what the intent of the lesson was. Reading more closely, one of the teachers immediately saw that the students were being asked to solve the inequalities by using tables or graphs, not by manipulation of the symbols. This teacher was greatly relieved and thanked the researcher for providing a clearer view of what the curriculum was intending. The other teacher reacted somewhat differently, not having noted any more frustration than usual, and feeling that this was something students had done before. Both teachers felt there was much to learn about how mathematics was taught in the middle schools of the district, and were happy that they were able to hear some of that in the study group sessions.

Classroom Calculator Use Logged

The intent of the Calculator Use Log (see Appendix E) was to give teachers a way to provide evidence of the usefulness of the AUGC tool or other concepts developed in study group sessions. The researcher asked about the log at each interview and each session. At the end of each study group session teachers were asked to choose four aspects of using calculators that they would consciously watch for in their classrooms. The presence of these aspects would be indicated on the Calculator Use Log and reported in the next study group session. In Session 1 teachers chose four of Zbiek's (2002b) categories (see Appendix M) to look for. They all expected to see calculators used as an arithmetic aide (AA), and some wondered if they would observe some other way that students delegate work (DW) to calculators, get information (GI), or check answers(CA). In the second session, teachers decided use their logs to categorized their lesson tasks in terms of the cognitive demand levels of Stein, et al. (2000). At the third session teachers named their own categories. Karl, who was the most complete log-keeper, named his categories for whether the calculator was "absolutely necessary to do the lesson," "not necessary but helpful," or whether it "didn't use" calculators. Yvette missed that session, Lynn didn't use the log at that time because she was out of school for a week, Tess did not use the log, and Rob found his use of the log to be too mechanical and so intended to invent new categories.

Overall use of the log varied widely.

- Tess made only one entry in her entire log, but did not date it or say what topic was being worked on. When she turned it in she apologized, but wrote a general statement that in her Integrated Mathematics 2 class, for which she

made particular observations, "All students used calculators as a tool to help them think through the mathematics" (TLog, p.1).

- Yvette missed two of the sessions at which we chose what to look for, but after the first session Yvette observed her Integrated Mathematics 2 class (without writing dates or content) and noted that her students used the calculator as an "arithmetic aide" and "answer checker" in every class, but on about half the days they also used it to "get information" (such as square roots or sine values) or to "enhance presentation" (to help explain their answers) (YLog, p. 1).
- Lynn's log shows that she tried to also say how much her students used calculators in each way. Using the Zbiek (2002b) categories, there was "always some" use as arithmetic aide, and "some" use to check answers until a test day when there was "lots" of answer checking. On two days of this inter-session period, Lynn's students used the calculator to "get information, reference chart," perhaps to look at lists. After Session 2, Lynn found that all the Connected Mathematics tasks that she had her students work on rated at the "procedures with connections" (Stein et al., 2000) level of cognitive demand (LLog, pp. 1-2).
- Karl's intern teacher saw that, according to the Zbiek (2002b) categories, students used calculators as "arithmetic aides" every day, and as ways to "get information" on one day (KLog, p. 1). When Karl analyzed the cognitive demand (Stein et al., 2000) of lessons of his algebra 2 class he found that the Murdock et al. (1998) tasks were about 70% "doing mathematics" and 30%

"procedures with connections" (KLog, p. 2). The categories for Karl's third page are given above. He noted that half of the lessons of that period absolutely needed calculators and the other half didn't require them but they were helpful. (KLog, p. 3).

- The log Rob kept was completely filled out, including topic studied and whether the AUGC tool seemed useful up until the time that Rob didn't know from one day to the next if his military unit would be called up. Rob used the Zbiek categories for the two time-periods that he kept the log, and added "assessing" for one period. He found nearly daily use for arithmetic aide, checking answers, manipulation aide, and motivation provider. Lesser use was noted in the areas of example generator and assessing.

In summary, only a few of the teachers consistently recorded their observations of calculator use in the pages of the log books. What is recorded provides corroboration for the verbal comments made during the study group sessions, as will be reported in the following chapter.

CHAPTER V: PROCESS

The mere formation of the study group to inquire into "appropriate use" of graphing calculators, aside from the artifacts studied, was itself a possible source of change in teacher understanding. Change may have resulted from hearing what other teachers said in the sessions, or from the fact that a teacher may not have had occasion to talk or think about appropriate use of graphing calculators before. It is not possible to identify causes of change, but the tool under development served to elucidate results of the same interactions that might also bring about change in individual teachers' understanding of appropriate use of graphing calculators in teaching mathematics.

It is also expected that teachers' discussions about graphing calculator use will continue outside the study group sessions, so there is no claim that effects can be separately attributed to study group discussions, artifacts, interviews, or teacher reflection. All are parts of the complex system in which the AUGC tool was developed. Indeed, one goal of forming the study group was to educe a consciousness of the community of practice in which these teachers are working, and to facilitate recognition of the processes by which teachers negotiate meaning and norms for their community.

In this chapter, the analysis shifts from individual teachers to what they do together in community. In order to use calculators at all, each teacher must come to some kind of understanding of what is appropriate. But asking a number of teachers individually what each thinks is appropriate invites difficulty in assigning meaning to the words each uses to express himself or herself. The trustworthiness of comparisons of the

ideas would be suspect. By having teachers talk together, with the researcher as a participant, it is expected that at least a common vocabulary will emerge, if not a common understanding of appropriate use of calculators. By adding the focus of designing a tool that will be useful to other teachers, an incentive has been added to clarify language and to speak to as wide an audience as possible.

The self-selected group that agreed to take part in the study group have taken on some of the outward appearance of a community of practice, for, by Wenger's description, "Communities of practice are groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis" (2002, p. 4). What remains to be seen is whether they will develop a "shared repertoire" (Bohl & Van Zoest, 2001, p. 7), or shared "practices, belief systems and collective stories that capture canonical practice" (Reynolds et al., 2001, p. 110). The duration of this study is too short to verify the formation of a community of practice, but as the interactions are examined, Wenger's language will be used, identifying as *artifacts* and *boundary objects* the ideas and stories brought to the group. The persons who bring these artifacts are *brokers*, people who also belong to another (or many) community of practice in which calculators are used, and who share the understanding developed in the other community. Once ideas or artifacts are presented, the new community negotiates the understandings attached to them and whether they are useful for the goal of the community.

It is noteworthy that none of the five teachers in the study belonged to the same 'other' local community of practice, because even though three taught in the same school, they worked in different academies and so didn't interact professionally on a daily basis.

The researcher belonged to an 'other' local community of practice at the university, and was also active in the community of practice at her former school, and also in the Michigan Council of Teachers of Mathematics. Several of the teachers had concurrent activity in professional development communities, and most had recent university experience. Thus there were many opportunities for brokering.

Table 19 gives a chronological grid of the types of data collected at each time period surrounding the four study group sessions. Time moves down the table, and the data sources used before and after the series of sessions are summarized in the first and final rows of the table, in the order gathered. The data sources provide evidence for tracking the brokering and negotiating, as well as the construction of the AUGC tool. The reference to "artifacts" as data sources refers to both the artifacts that teachers bring to the sessions and their interactions with artifacts brought by others. For each "X" under class observations and observation interviews there are 5 sets of transcripts, calculator use checklists, and field notes. Missing from the table are the study group session transcripts,

Prior to Study Group Sessions – Interview, Task Sort, Survey						
	Tool Work	Artifacts	Session Reflections	Class Observations	Observation Interviews	Logs
Study Group Session 1	X	X	X			X
	X			X	X	X
Study Group Session 2	X	X	X			X
	X			X	X	X
Study Group Session 3	X	X	X			X
	X			X	X	few
Study Group Session 4	X	X	X			
After Study Group Sessions – Revisit Parts of Survey, Task Sort, Interview						

Table 19: *Sources of data and their chronology*

which are the major data sources examined in this chapter, and are also the source of much of the "tool work" and "artifact" data. Some lists and drafts were generated as part of tool work, and one of the artifact interactions has two transcripts of its own. The logs and reflections are written documents produced by teacher participants.

Because the intent is to trace the negotiation of teacher understanding of appropriate use of graphing calculators to teach mathematics, there is also a chronological flow to this section. To provide for this, each study group session is examined as a slice across all research questions and their related data sources at the time of the study group meeting. Figure 1 provides a schematic that indicates the cascade effect of the results of each round of data collection on the planning and execution of the next round. There is also a milder influence of the initial interview, survey and task sort on all of the rounds of data collection. This figure deals almost exclusively with the study group sessions, classroom observations and interviews, and development of the AUGC tool. Other influences, such as the session reflections and log entries are omitted. In some cases, the arrows indicate effects felt by teachers only, such as the arrow from "5 Observations" to each session icon. Other arrows represent effects on researcher and teachers, such as the ones from "5 Observations" and "5 interviews." Arrows indicating effect only on the researcher are any arrows terminating at "Researcher decisions."

It should be noted that the number of arrows in Figure 1 do not represent the amount of influence. For all sessions, the researcher had a pre-existing plan, but considered all prior experiences with group participants before implementing the plan. The main focus is at the end of each chronological iterative row, where the sessions produce the next iteration of the tool intended to reify the group's understanding of

appropriate use of graphing calculators. This figure is intended only to represent the slices that provide structure to this section of the report. Later figures will represent influences evident within a typical group meeting.

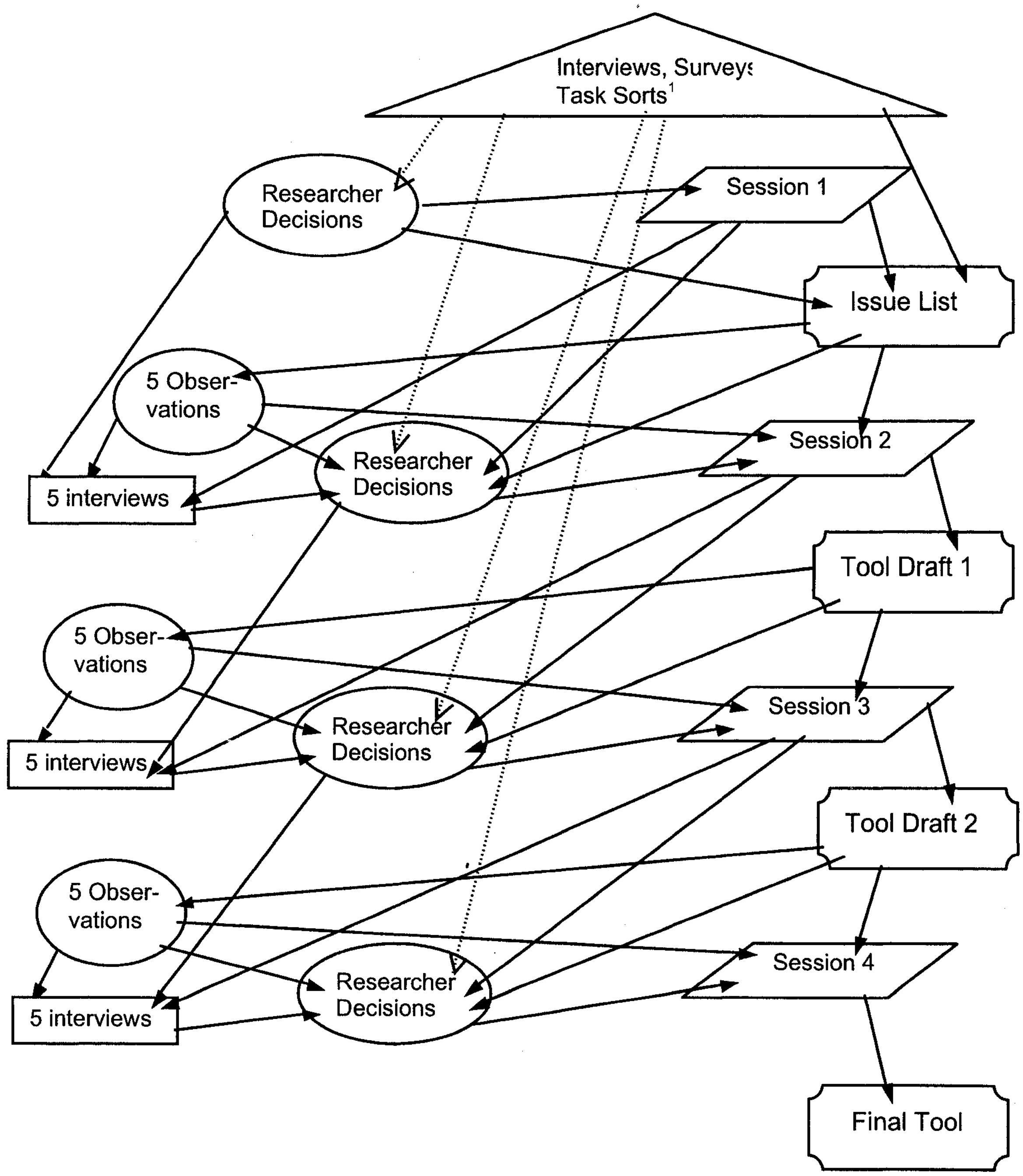


Figure 1: *Schematic of how each round of data collection affects the next*
[1]Includes 5 Surveys, 5 Task Sorts, and 5 Initial Interviews

Preliminary to the results of each slice of data for each study group session will be an analysis of the decisions made by the facilitator (researcher) as she planned for that particular session. This analysis has already been presented for Session 1, but there will also be prior observations and interviews. What has not been reported is that overall the study group sessions had 80% attendance, and two had to be rescheduled at short notice. The teachers were enormously generous with each other and with the researcher, trying to make things work out for everyone. This was taken as an indication of how important the topic was to these teachers. Email communication saved a great deal of time in making needed changes. Some time of each session was taken up with passing out papers, handing papers in, scheduling observations, and pouring out the latest classroom turmoil or administrative frustration.

Session 1

Although all five teachers taught in the same district, the only time they all met together was at district mathematics meetings, at which there were many other teachers also present. One potentially influential pre-existing relationship was explained at the first session, amid laughter. Karl had been Tess's high school geometry teacher years ago in another district. They related the story of what happened when they met at a new teacher's gathering a year and a half before. By the time of this session, Tess was accustomed to calling Karl by his first name, and Karl seemed to have adapted to learning from Tess's ideas. Overall, the introduction of each teacher was helpful, for although the researcher had heard all teachers' views on teaching with calculators, they had not all heard the views of the others before this time. For this reason, however, the issues that were brought up resulted in more discussion time than had been planned for. Not all the

issues brought up related directly to using calculators, so part of the task in designing the AUGC tool would be identifying which issues could be dealt with by our tool. For Session 1, most issues are reported to give a sense of how broad a range of issues teachers saw in teaching mathematics.

Issues from Session 1

The second teacher to respond in the introductions turned the question, "What do you like least ..." into "What is your greatest *fear* about teaching with calculators?" The teachers generally agreed that what they feared most about teaching with calculators was that students came to be dependent on calculators. Yvette said "But my greatest fear ... is that they become handicapped, dependent. That's the thing I don't like about the calculators" (SG1, Y, 12). Lynn was even more emphatic, and brokered an example from her own experience.

> My fear is the same thing, and it's not even a fear, it's true—they don't have a basic understanding of what the calculator's doing. So when it gives them a totally off-the-wall answer, that makes no sense, they have no clue that it's wrong because they're so dependent on everything that it says that they can't even make guesses as to if it's correct or not (SG1, L, 14).

Tess then served as broker, supporting Lynn with a specific case in which her students gave a "totally off-the-wall" response, but adding how she dealt with it. She reported the following interchange with students:

> "Mrs. Ernest, that's right," and I'll look at them and go "when in the world did it change that negative two squared is negative four, because that's not what I learned, and I know math doesn't change like that!"
> They'll go, "But Mrs. Ernest!"
> And I'll go, "Two negatives..."
> "Oh, yeah, that should—Mrs. Ernest, but the calculator says it's negative four" (SB1, P, 34)

Karl, on the other hand, saw the same problem, but was also concerned about his own lack of calculator knowledge. Not only was he willing to expose his own uncertainty to the group, but he also subtly introduced the need to teach students to use the machine correctly. In this act, Karl seemed to be brokering more general knowledge from long years of teaching. He said,

> My biggest fear – actually, there's two of them. Number one is, I'm not that great a user of the calculator, so I have to do a lot of home study on how to punch the buttons and make things work before I try to show the kids. In a lot of cases, I'm learning right along with the kids.
>
> And the other greatest fear I have is that all it takes is just one little thing that you leave out and the kids will say, 'this is what the calculator says, so that must be right.' And you know what the answer is, but you got it out of the answer book, so you know it's [theirs is] not right, but I don't know if I know how to figure it out myself [on the calculator]. (SG1, K, 21-22)

Karl continued to call on his own teaching experience in negotiating an understanding of dealing with student dependence. He was very diplomatic in phrasing his comments. "I agree with her ... what it boils down to is that we as teachers have to teach estimation before we teach anything else. So that ... I can say to the kids, all right, does your answer make sense?" (SG1, K, 28) He closed this comment by saying, "The kids won't do that! They don't want to—they just want to get to the problem and get an answer. And they don't want to reason" (SG1, K, 29). In this way Karl mitigated what might have been taken as a criticism of fellow teachers (needing to make students estimate), but he also introduces discussion of a contextual issue (RQ3a), that of the students they teach.

In response, Lynn suggested that calculator dependence is a matter of habit that students picked up in elementary school, but Tess saw more than that. She said,

> I don't think it's that, I just think they don't 'get' math. And they rely on the calculator because they don't get it. And they've learned how to push buttons. They haven't learned how to do the mathematics. They haven't learned why two plus two equals four, or why two times three equals six, but they know how to push it on a calculator. (SG1, T, 39-40).

This assertion was met with agreement by Lynn and Yvette. Karl was quiet while Tess continued, comparing what happened to learning an algorithm but not understanding why it worked. Then Karl interjected, "The sad part of it all is that these were really only meant for higher level activities. These only replace the slide rule.... And we didn't use slide rules down in the lower level courses" (SG1, K, 46-47). Some thought that was a good point, but the researcher protested that she had never used her slide rule to balance her checkbook. Karl replied, "Well, that's the thing, see, it was meant for up here, and it's filtered its way back down because of laziness" (SG1, K, 49). Yvette agreed that 'laziness' was a key word. In the course of this episode we see the negotiating moving from using evidence based in concrete teaching experiences (RQ2) to the use of beliefs about calculator use and about the nature of students (RQ3).

Other issues introduced in the first session often came with an anecdote, and the contextual issues of students' previous education and curriculum became part of the discussion. Tess related an experience that suggested an elementary school connection:

> I go and talk to those elementary teachers that are teaching my [own] kids mathematics – and they're doing our basics, they're doing our foundation – and they looked at me, they had no idea who I was, and they said, "You know what? I hate math – that's the worst subject." I looked at them, and I go, "Here, my name is Tess and my son's coming to your school and I'm a math teacher" (SG1, P, 56)

But Lynn countered with a defense, noting that in middle school math she was "starting exponents, it is the first time these kids have ever seen exponents. Eighth grade—first time I think that it's ever introduced. The book introduces it with a calculator. And I just

totally disagree with that" (SG1, L, 59). Tess agrees that some is curriculum-based, saying, that the Core-Plus curriculum she uses sometimes calls for calculator use before she thinks they should be used, so she waits until she thinks students are ready for the calculators (SG1, T, 63). The researcher then asked how Tess made that decision. Her reply was,

> Well, my first year, I never got the calculators out until I knew if they had the concept or not. I'd introduce it without the calculators, and maybe the warm up that day would be—here's this equation, tell me what the slope is and *y*-intercept (SG1, T, 69).

Here Tess seems to indicate that she doesn't want students to use calculators on linear relation problems until students can look at a linear equation and identify the slope. She later contradicts this notion by claiming that she really thinks the Core-Plus development of slope shows appropriate use of graphing calculators. Tess's comment greatly interested the eighth-grade teacher, Lynn; and Yvette was also ready to give Lynn information on how much the ninth graders remembered from the previous year. The following discussion seemed a perfect segue into the two activities on slope which the researcher had planned for the session. In the interchange below, Lynn is responding to Tess's question, "tell me what the slope is and *y*-intercept." I is the investigator.

L: Just by looking at it?
T: Right.
L: Can they do that?
Y: Yeah. Most of them. If it's in a *y*-equals [form].
L: Okay.
Y: If they have to solve…
T: Because we go through, and we go through, and we go through that.
Y: Yeah, I had to go over it and over it and over it again…
T: But when you first introduced it, they didn't get it
Y: Oh, no.
L: And that is a huge thing. So when they leave eighth grade, probably 80% of them do know it. If I give them an equation, and

they have it, I would say right now 80% of my kids today, they do it on their openers almost every day—different forms, looking at tables, looking at graphs, looking at the equation—tell me the y-intercept, tell me the slope.

Y: After the summer then, they lose it.

I: So what is it that they lose? Do they lose the name of the...is that what they've lost, when you ask for slope?

L: We use the same names, though.

T: I don't think it's the name, because our book, the Core-Plus material, doesn't even introduce it as slope. It introduces it as the rate of change.

L: So does ours. And we started telling them it's slope very early because then they go through so many different things—and like, it's the same thing, why didn't you just say that in the beginning? So we'll say it's the rate of change, it's also called the slope later on...

T: But I think though, that Core-Plus does a really good job. That's one of the units that I really like, because I think they do a really good job. They do it as a rate of change first and they introduce delta—delta-y over delta-x and all that kind of stuff. And then they go back and they say okay, this is what the y-intercept is, and then this is slope—slope is the rate of change. So I think they've got it now, but to me it's like the concept—someplace we're missing out on the concept. And I don't think that's the teacher's fault, I don't think it's the calculator's fault. For some reason I think it's—we need to show them why it's important. They don't see the importance of it (SG 1, 70-87).

At a later point, spurred by a calculator answer of .5 and a value of 1/2 to which the calculator answer was to be compared, the teachers launched into a discussion of how their students hated fractions and didn't know how to deal with them. In elementary classes they had calculators that worked with fractions, and when they moved to a new calculator in seventh grade they wanted to know where the fraction key was on the calculators there.

T: I would be so frustrated, though, if I had to teach fractions.

L: I don't [in 8^{th} grade]. They do in sixth grade and they do in elementary. That's how they learn them in elementary is the fraction key. That floored me as a seventh-grade teacher—you're seriously coming in asking me for a fraction key? No, you can't use that.

I: I'm thinking about what Karl said, way back. That one of the things that a calculator is appropriate for is to check answers. ...

T: I don't care if my kids did their math problem and then they're going back and they're going to check to make sure it's correct, that's okay, that's the way to learn the calculator—the proper way to use a calculator.

I: Because otherwise, is there a concept of checking the calculator? You've already said, no, they don't have that concept. Can we get them that concept somehow?

L: I think if you do it by hand first, or at least have an estimation, like Karl said, or have some idea.... But it *is* a calculator issue, because that's how they were taught to learn it. And I understand fractions are difficult, but...

Y: It's a calculator issue because that's how they were taught. And there again, that's where they're being handicapped. (SG1, 198-207)

Karl had been quiet for a long time at this point, but now he saw his chance. Again, Karl seemed to be trying to be diplomatic about bringing in differing ideas.

> Well, I could play the devil's advocate for a minute here, and say, the way technology is going these days...why don't we just use the calculator to begin with? Really, how many engineers are there today, how many statisticians, compared to the everyday person? (SG1, K, 221)

This brought out quite a few explanations about needing the foundation, about being able to check the technology, about not being taken advantage of, and the ubiquitous example of the folks at McDonald's who can't make change. Karl responded, and there was a sense that he may have had specific students in mind when he said,

> Well, there can be a point, where you do not use a calculator and you get to a point where you identify the students that ... are actually going to need that understanding, so they can develop those things. But the rest of us, who have no clue, could care less to have a clue or whatever, why do we need to be—continue to be forced to not use the calculator? (SG1, K, 233)

Then the issue arose that each of them had made it through most of college without using calculators, but they had to admit that calculus would have been a lot easier with a TI-83. (Rob, who had used calculators in calculus, was absent for this session.)

The final issue that arose was who should teach students how to use a calculator. But there didn't seem to be any consensus on how long that should take, or whether all students would learn the use of calculators. Here is part of the discussion:

> T: No, see, the point was, I didn't have a calculator until I was in college, and I've adapted just fine.
> L: That's what I'm saying, so why do they need them two years?
> T: The ninth graders can get a calculator when they're in ninth grade, and they will adapt fine.
> L: Exactly. But why do they need it in seventh grade for two years to adapt to it, they're going to get it—how long does it take to introduce it?
> I: Well, we do have to remember that we're probably not the norm, as mathematicians.
> K: Well, it all boils down to is that the teachers don't want to teach how to use the calculator, they want it already known how to use it so they can go ahead and do their things. (SG1, 245-250)

Artifacts which Reify the Ideas of Others

After the interchange on students missing the concept of slope, teachers were asked to work on two mathematical tasks dealing with slope (Appendix K), chosen as a focus for discussion of Zbiek's (2002b) categories of uses of technology. Although they were asked to work on the tasks together, teachers worked independently, using calculators even with no specific instructions to do so.

Two Contrasting Problems

One problem explored linearity by using a context of doing yard work at different hourly rates to earn money to purchase something (henceforth referred to as the yard work problem). The other problem used the calculator to explore the definition of slope by tracing the graph of a linear function (henceforth the tracing problem). The researcher asked what these problems dealt with, and Lynn replied that she didn't think students would see the \$3.75 in the yard work problem as a slope. When asked if the students

could make sense of what they should do to fill in a table of earnings for hours of work, the teachers generally agreed that they could, if they had a calculator to help them come up with the correct numbers. The teachers enumerated several ways students might approach the table by adding or multiplying. Tess noted, "They wouldn't do it like we did it, because I notice that some of us went to our table and put the table increment [of 3.75] on the calculator [with an equation of $y = x$], and we just wrote things down" (SG1, T, 99). The next part of the problem generated the following interchange:

I: How about that next set, then, making the rough sketch?
T: I think that some of the kids would get it, I think 50% of my kids could do it, 50% of my kids couldn't.
Y: I would agree with that estimate.
I: What's the hard part about it?
T: They don't get the *x* and *y*. They don't get the times *x* and chart a plot per the dollars. I mean, they don't...
I: But it says right there the number of dollars...
T: But you know what? They don't read! It's not that they can't read, they don't take the time to read the problem and to look at it.
L: Yeah, I would say, after having them for two years [teaching the same students for 7th and 8th grades], that they should know that. If I took this over to my class on Monday, and gave this to the kids, 75% of them would get it without a problem, today. Because we're working on it, we just finished it . When they come back in the fall, obviously that's not true! But they would be able to sketch this, they should be able to have a good sketch (SG1, 113-120).

Given a specific problem, teachers were able to make statements about what proportion of their students might react a certain way. Earlier statements about students may have been interpreted to apply to all their students. Again, there is a mix of statements based on mathematical activities and statements based on beliefs about student characteristics. The discrepancy in proportions of students who could do the problems is not so great when one notes that of the 75% of eighth graders that Lynn said could do the problem today, some of them would go into Integrated 2 in ninth grade, and they would not be

studying the linear problems of Integrated 1 in the classes that Tess and Yvette say only 50% could do the problem.

After a lengthy digression on inequality symbols and teaching mathematics as language, the discussion turned to the tracing problem. It was not clear to teachers what was intended by the authors of the task, as the following exchange shows.

> T: To me, some of this stuff right here, I wouldn't want my kids to be able to use a calculator on … the slope of a line, because all the information's right there.
> Y: Yeah, you don't really need it.
> I: In fact, she [Y] said [as they were working], "don't I already know that the slope is one-half?" So think about when might you actually do something like this. …
> T: Maybe an introduction, something new, …and then that … question: "How was your result related to the *x*-coefficient in the linear equation you graphed?" They're trying to say that, well, that coefficient is slope. That's what they're trying to say….
> Y: No, they never said it was the slope.
> T: No, it just says slope of the line at the top, they never brought slope into any of the…
> L: Do you think that they wanted them just to see that? So if they're asking for the quotient, …, okay, the quotient's also the same …
> T: I think they were doing a lot of tie-ins. They're showing that the difference of *y*'s and the difference of *x*'s and when you take that quotient, that gives you the same thing. But if they did anything like I did, I used .5 … hmm, that's [that coefficient is] a half, are those the same? (SG1, 163-175)

Tess ends wondering if her students would recognize that the result of calculating the quotient (.5) was the same as the coefficient of x (1/2).

Zbiek – Categories of Student Uses of Technology

As time ran short in the two-hour session, the researcher gave a brief introduction to the summary table from Zbiek's paper, *A Two-tiered Category Perspective to Describe Purposes of Mathematics Technology Use*, which had been intended as the major artifact to help thinking about the AUGC tool in this session. The entire paper was also given to

the teachers so that they would have the larger context to pursue it further. A very brief introduction was given to the categories of student uses of technology. A summary of the categories is given below.

CA – Checking Answers
- Answer Checker – use one method
- Alternative Checker – two methods

GI – Getting Information
- Reference Chart – trig/log values, formula lists
- Information Conveyer – to show as an illustration to students
- Puzzler – results or appearance puzzles students

DW – Delegating Work
- Arithmetic Aide – computations
- Example Generator – multiple instances from which to reason
- Representation Generator – table, graph, line through data
- Algorithm Executor – regression line, intersection, max, min
  (usually not yet capable of doing without technology)

GS – Getting Solutions
- Answer Giver – direct command, e.g. Solver
- Dual Processor – use two or more methods for same problem

IP – Improving Presentation
- Work Replication – duplicate another's work, or recall earlier work
- Report Helper – communication or illustration of ideas
- Motivation Provider – demonstration of effect of parameter
- Attention Helper – calling attention to particular characteristic

Zbiek's complete table (reproduced by permission) and the transcript of its introduction can be found in Appendix M.

Teachers had questions. At first they thought the categories were a sample guideline like the one the group would develop. The researcher explained that this was empirical research, the result of actually observing students working with technology. She explained that their Calculator Use Logs that had been handed out in colored folders would allow them to do the same sort of observation of what their students were doing. They would have a choice of what to observe, but Zbiek's categories might suggest things to look for. They were to pick just one of their classes to observe, during each class time

until the next study group session. On the Calculator Use Log pages (see Appendix E), there were four columns to fill in as they wished. The way teachers actually used the logs has already been reported. The discussion following the introduction of Zbiek's table began this way:

> T: So you want us to look to see if our students are doing stuff? Good or bad stuff, or…
> I: Whatever way you think—what do you think would be useful for other teachers to think about, because remember, we're trying to make a tool that's useful for us, but that we also would like to share with other teachers.
> T: Well, one of our biggest pet peeves right now is when the students can't add two plus three in their head.
> L: I was going to say, maybe when they're using it as a crutch instead of a tool, if they're using it because they don't know—I don't know how you would observe that.
> K: Using the graphing calculator as a scientific calculator, or just as a general operation.
> I: So maybe that category….
> Y: Getting the answer, versus…
> I: …arithmetic aid.
> Y: Okay.
> L: Yeah, that could be.
> T: So we put on there arithmetic aid, DW 1 AA? (SG1, 291-301)

Lynn's mention of what couldn't be observed (student's reasons) perhaps went unnoticed by the others, but was nonetheless insightful. There followed a very brief effort to apply the Zbiek categories to the activities teachers had worked on and how they had used calculators. This interchange also reveals their thinking about prerequisites.

> I: Thinking about the difference between these two activities—the one didn't necessarily say to use calculators, but most of you did, and I think if we would think about how we used calculators on that, we could probably go down this list and say oh yeah, we were doing—getting information…
> L: Checking the answers—I know I used mine for checking answers, and going through—I did it by hand and then I graphed it out just to see what the kids would do.
> T: I didn't do anything by hand! (laughs) If I couldn't do it in my head, I went to table set. …I did the equation, and then I went to table set

and did it that way. But it was because I had the prior knowledge to know what an equation is. And my kids wouldn't know that.

L: Well, you also did sort of, used it as a check, because you knew if you typed it in wrong, that it was going to not give you the correct answers. If you got off the wall answers, you knew—because you knew what you expected to get.

T: But I wasn't doing it for a check, though, I was *doing* it....

L: Doing it to get answers.

I: Doing it to get it done. But she knew how to predict that what she got was the right thing.

Y: She knew her basics (SG1, 317-326).

As teachers filled out their log's four columns, they were talking with each other about why they would pick certain categories to use with their students. They were again thinking of their individual situations, and they were immediately seeing ways to apply Zbiek's categories to what they expected to see their students doing..

L: I'm wondering about 'getting information,' because I think that would be appropriate for mine right now because they're just starting, like I said—although it's not going to be lasting long, because they're finishing that book next week, but—just getting information, can they graph it on there, are they using the exponential graphs to come up with information, but when we start a new book, that could be totally different. I don't know....

K: But if I use algebra II, then they have to use it to gather information, you know, with the problems all like that, so they....

L: Maybe I would be doing the reference chart, then. They're just going to...

I: If they're just going to get some value or they're finding out what five raised to the sixth power is, that's getting information.

L: That would just be the reference chart.

T: In Integrated I right now, we're working on the example generator, because today they had to graph four different equations to find out what quadrant they're in....

Y: You're farther than me (SG1, 358-406).

Whatever they were discussing, it didn't take long before teachers were exchanging information about their day-to-day work – "where are you in ...?" It was time for teachers to write down reflections on what they did in Session 1, and time for the researcher to pack up the snacks, go home, reflect, and work on planning for classroom

observations and the next session. The researcher purposely did not look at the teacher reflections at the end of each session. They were intended to be personal records of teacher growth. They were reviewed by the researcher before the fourth session in considering a change in form for reflections on the fourth session. Tables of all four session's reflections are in Appendix N.

Researcher Plans for Study Group Session 2

The first classroom observations followed on the next school days after the first study group session. Two of those observations, involving solving inequalities were reported in the earlier section on observations. Those incidents with inequalities, and the two differing reactions of the teachers, had considerable influence on the thinking of the researcher as she finalized plans for the second meeting of the study group.

Prior Observations and Interviews

The question of just what was meant by knowing "concepts" before students used calculators had been in the researcher's mind as she set out on the first set of observations. Perhaps this is what called attention, in the two classes already mentioned, to the symbolic approach to solving inequalities. In another class, the calculator was used as the curriculum called for, to produce graphs and tables for comparison of the functions $y = x^3$, $y = 3x$, and $y = 3^x$. Students were able to predict what the graphs would look like with another number instead of 3, and they were able to check with their calculators. Another class was using Geometer's Sketchpad to perform reflections and rotations of triangles on a grid, and then reading coordinates from the grid to derive the symbolic representation of the transformation, for example, $(x, y) \rightarrow (x, -y)$. This was not the technology called for in the curriculum, but the exploration was in keeping with the spirit

of the lesson, which had called for the same conclusions from by-hand sketches. The final observed class was examining two salary offers to determine which was best over 6 years – a constant increase per year or a percent increase per year. The students were able to write equations, make graphs, compare tables, and explain their reasoning.

The interviews following class observations were short. The class that was working on the salary problem was reviewing for a test, and the teacher noted that these students were comfortable using calculators for this topic. The parts they had problems with were writing the equation once they had a table. The conversation with Karl was directed more at the curriculum than the lesson that had been observed. Karl felt that the teaching materials he was using in his classes (except geometry) were designed to be used with graphing calculators. He said, "Well, I just know that kids could not do it without the calculator. The kids are doing stuff in this class, probably, that back before the graphing calculator, they wouldn't have had this early in their math career" (ObsInt 1, K,17). And this seemed appropriate to him, for "It doesn't make sense that you would be using a graphing calculator to do the same thing that you did 20 years ago, that you could do by hand" (ObsInt 1, K,51). Another benefit Karl saw with the calculators he expressed this way, "So I guess what it does—the calculator allows you to stretch your thinking. Because now you are able to use numbers that physically you may not have been able to use before" (ObsInt 1, K,57). So Karl saw the calculators as making mathematics possible that wasn't available to high school students when he was that age. Particularly for the observed lesson,

> with the power models there, you can set up your initial equations with *x*-squared or *x*-cubed and you can quickly with a calculator, go back and just change one or two numbers. Then punch the button—you regraph the

whole thing, where by hand, that would be insurmountable (ObsInt 1, K, 59).

From a single observation in each classroom, it was certainly not possible to determine the order in which certain work has been done on a regular basis. One class was reviewing, another was just beginning a new section, and a third was working on a calculator-based activity, so it was not possible to see what the "usual" use of calculators was. Only in the classes dealing with inequalities did the teachers seem to assure that students worked problems by hand. It was not clear whether this by-hand work was instead of calculator work, or even whether it was before calculator work, because calculators were used in parts of these classes. However, the frustration of both teachers and students seemed to be connected to solving inequalities by using symbols. Because of this apparent connection, the subject of inequalities was subsequently used in a later session.

Researcher Decisions about Session 2

The researcher wrote in a reflection on planning for Session 2, "Task analysis, both for intended learning and for calculator use, is definitely important" (S2 Plan, ¶ 2). Even with very good mathematics backgrounds, and a week-long implementation workshop for one of them, two teachers had been unable to ascertain the intentions of the authors of the Core-Plus Mathematics materials in working with inequalities. In one case the suggestion that symbolic solving might not be the intention was met with relief that the teacher could instead work with what students did understand. In the other case, the reaction might have been interpreted as disagreement with, or difficulty in interpreting, the curriculum. The researcher considered another possibility in a reflection written shortly after the observation, "[the teacher] wants them to be able to do it by hand. It is

not clear whether that is because [the teacher] thinks that's better math, or because [the teacher] doesn't know how to do it on a calculator..." (Obs 1, [initial], ¶ 12). In either case, the analysis of the cognitive demand of tasks, according to the guide developed by Stein et al.(2000), still seemed a worthwhile activity for Session 2.

The intention was to then apply the task analysis to a subset of the tasks from the task sort which had begun the study, and to the two tasks that had been used in Session 1. However, in the interest of actually getting something written down for the group's AUGC tool, and in the light of the huge list of issues that had emerged in the initial interviews, the researcher prepared a reduced list of issues that needed teachers' attention at the beginning of Session 2. Thus, the question of what was meant by knowing "concepts" before using calculators was left for future sessions.

A series of schematics presented on the following pages shows the interactions the researcher saw at play in all of the study group sessions.

Figure 2 shows the influences of the various communities of practice of which participants are members. The work on the AUGC tool is affected when teachers draw on their understandings from these outside communities.

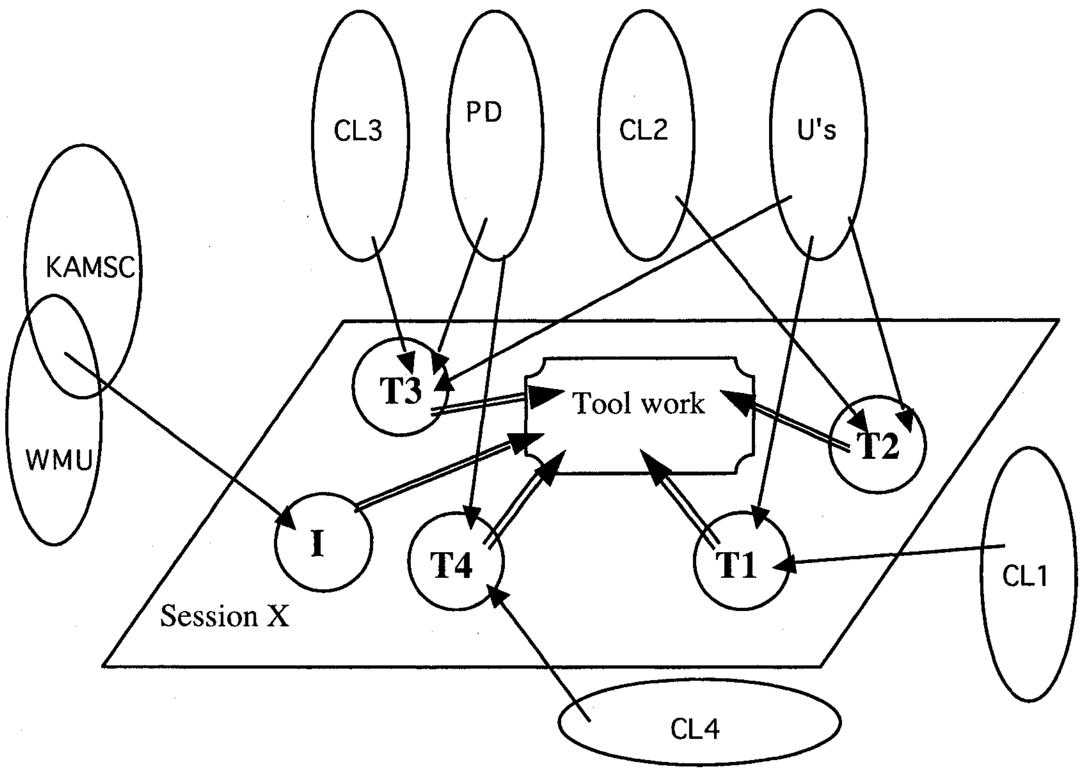


Figure 2: *Other communities of practice affect the tool work during sessions*
Note: The two print styles represent different time references. **Bold** represents persons and influences concurrent with the duration of the session, while normal weight denotes influences called up from past experience. "Tool work" represents not only the developing AUGC tool, but also the 'space' within which the work is done. Four teachers are represented because only four attended each sesseion. Notations include:
represents a community of practice; CL = classroom, academy, school; PD = professional development; U's = various universities; I = investigator; T = teacher.

Figure 3 shows the added influence of the classroom observations and calculator use logs on the work of the next study group session. When teachers bring ideas to the session that came up in the discussion of the interview, or that using the Calculator Use Log brought to mind, work on the AUGC tool is affected.

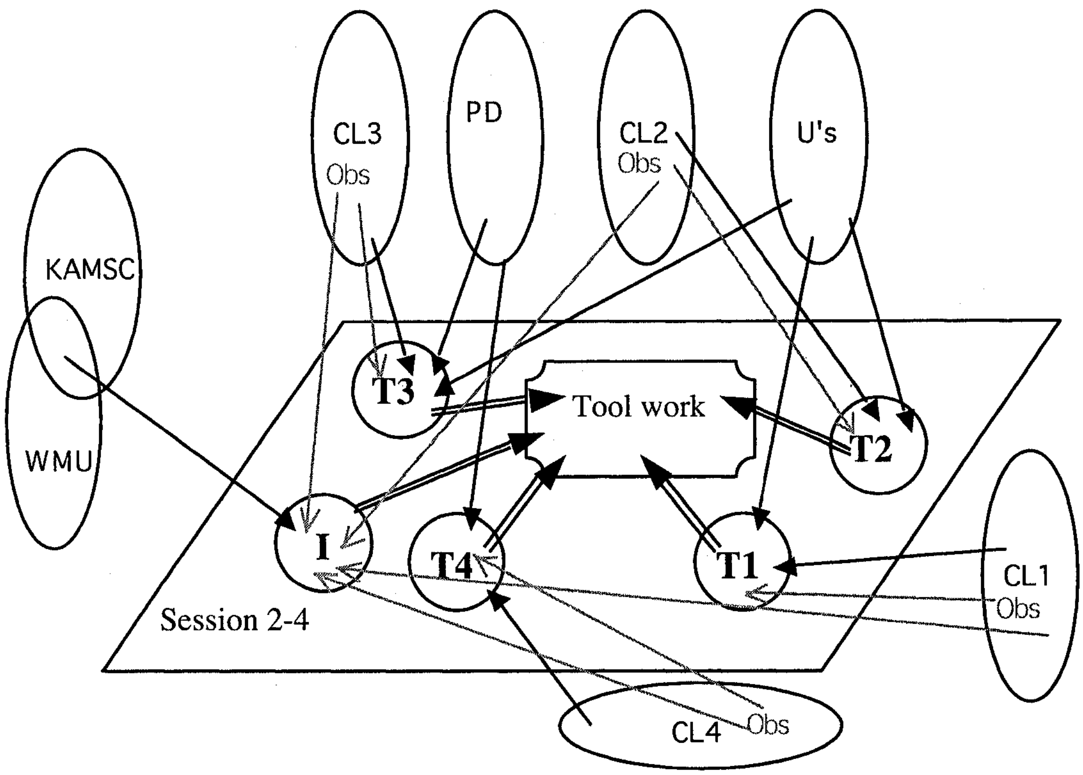


Figure 3: *Earlier classroom observations and Use Logs affect tool work during sessions*
Note: For further details, see Note on Figure 2.

Finally, Figure 4 depicts the reciprocal effect of the tool work on the teacher, as well as the influence of the artifacts brought into the session, including their experience with reform curricula. Teachers' interactions with the artifacts as they negotiate the meaning of that artifact in their community provides language and understanding that might be transferred to constructing their own tool. The reciprocal effect of the tool work allows influence to also be felt in teacher's classroom.

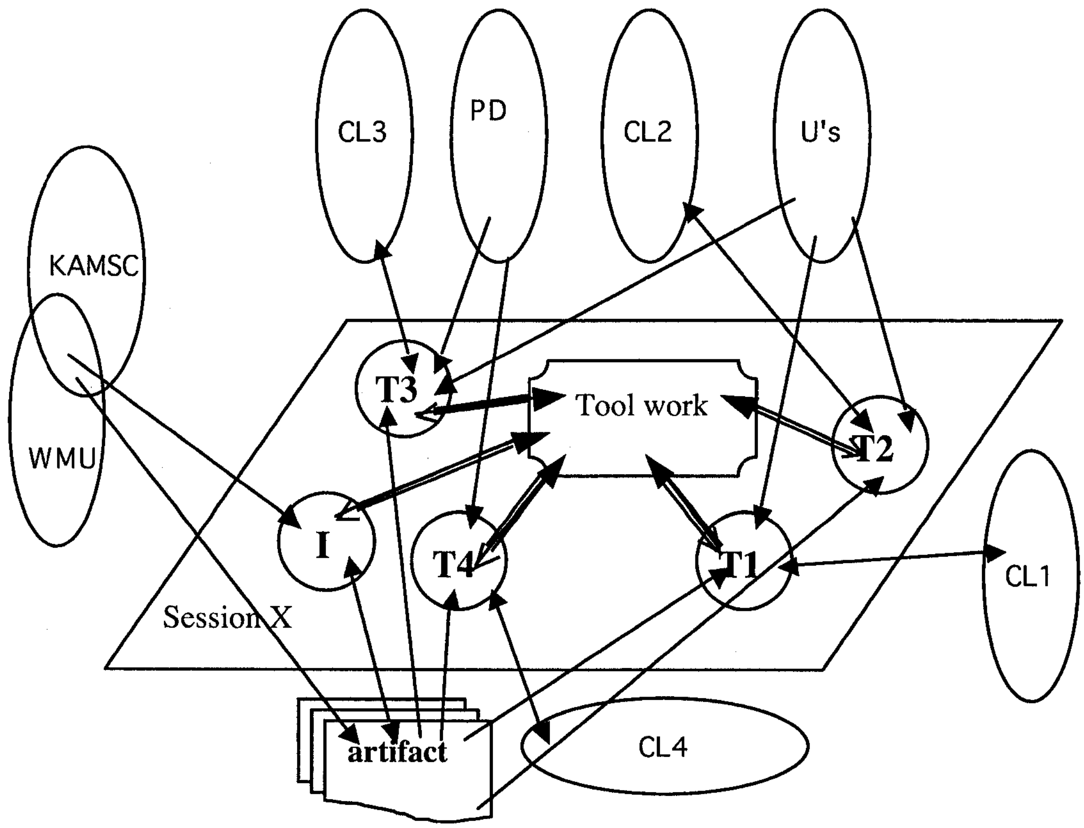


Figure 4: *Artifacts affect tool work during sessions, and work affects teachers*
Note: For further details, see Note on Figure 1.

Session 2

Work began on the AUGC tool in the opening minutes of Session 2. First, teachers were asked about how they used their logs and the Zbiek categories to concentrate on what their students did with calculators. Rob, who had missed the first session but had received the materials, was the first to speak. "It was amazing, it was striking, because I've never looked at that before, ... even thought about that—about what kids are doing with the calculators" (SG2, R, 3). Rob was amazed that they used calculators to do "calculations that they could do simply by thinking about it. I actually took the calculator away from a couple of kids because I wanted them to think about that" (SG2, R, 2). When asked for an example, he responded, "something as ridiculous as 2 times 3—it was just automatic for them to punch it in" (SG2, R, 7).

Lynn commented that the same issue had been raised in Session 1, but she went on to recount what she had observed her students doing. "We've been doing exponential functions—we just finished with—and we did use the calculators ... 3 days out of the week [the calculator] was actually used as a tool to do the work" (SG2, L, 9). The work they did with the calculators was graphing, comparing exponential growth and decay, and comparing equations. Students also had to deal with window settings to look at the graphs. Lynn's use of the phrase "as a tool" was in contrast to "as a crutch," which had earlier been used pejoratively. It also may indicate that she did not find Zbiek's categories general enough for an overall expression of what she observed. Rob also shared a positive experience related to Zbiek's 'technology as a puzzle" category:

> I've been finding – we're into matrices right now with my Integrated 2 class, and that's who I'm focusing on – I've been finding that the kids know more about the calculators and how to use the matrices than I do.

> And I've been allowing them to share with me what they've found. So in an investigatory way, they've been manipulating the numbers and manipulating matrices to really learn this themselves. I thought that was very interesting (SG2, R, 15).

Rob found that the calculators gave him a way to learn from his students, to allow them to be more competent than he was. He did not hide from them what was happening, but said, "okay, show me how you're doing this before I go to a book and try to figure it out myself – show me what you've found, and do you think that's right?" (SG2, R, 15) Then to his fellow teachers he added, "And 9 times out of 10, they've led themselves down the right path. And I thought that was very interesting" (SG2, R, 15). Rob did not find this embarrassing, but "very interesting."

The researcher did not follow up on Rob's idea of learning from students or invite others to share similar experiences. It was a lost opportunity. Then Karl brokered his "technology as a puzzle" experience from an algebra 2 class.

> Generally speaking, the kids use the calculator... and they think they have the answer. Then they look in the back of the book and see what the answer to the problem is. If they get that answer, then they're overjoyed and they think they've done everything correct. If they don't get that answer, then they come to me—what's wrong with this, why didn't we get this? So we have to go back to look if they entered the information correctly (SG2, K, 23).

This was another lost opportunity, because the researcher did not ask the others to relate their experiences with textbooks that don't have answers in the back (Core-Plus Mathematics does not put answers in student books).

Then Tess referred back to Rob's contribution. "I noticed what Rob noticed, [that] instead of thinking about what's in front of them, they grab the calculator and let the calculator do the thinking for them" (SG2, T, 25). Picking up on what seemed to be of interest to the group, Rob then related an incident of students taking time to enter several

4 x 4 matrices into a calculator to add them. But he also reported staging a 'race' to show them how much quicker it was to add those particular matrices by hand. While Rob was relating what he and his students did, Tess told more about her interpretation of what her students did. She continued, "They think that the calculator's more exact, more right-on. The calculator can be trusted more than their mind can be trusted" (SG2, T, 31). Lynn also reported her interpretation. "I think they're more confident that if their calculator gives me the answer, it's right. If I just add them by myself, I could make a mistake" (SG2, L, 34).

At this point, the researcher hoped to explore other explanations of why students might "grab the calculator." She asked Tess what her students had been working on at the time when they were grabbing the calculator inappropriately. Tess replied,

> They were solving for *x*. And it was very hard for them, because ... what they do first is they learn to do it with a table. Putting the equation in, and then solving it with a table. Well all the kids wanted to do it that way, and I told them "no." I said, "If I see you're using your calculator to solve it with your table, I'll take them away" (SG2, T, 37).

This action was in line with Tess' agreement with survey item 31, which stated that teachers should decide when it was appropriate for students to use calculators.

Focusing on the AUGC Tool

A more direct approach was then used to bring discussion back to the Zbiek categories, and the goal of developing an AUGC tool. The researcher said,

> Is there something that you think, just from this short experience that you've had with those categories, that we want to put down for other teachers to think about? ... maybe we can think in terms of, you just mentioned, Rob, that just the fact that you were paying attention brought some things to mind... something as simple as "be aware of what your students are actually doing with the calculators"? (SG2, I, 42)

The interchange that followed was used to put the first pieces of the AUGC tool on paper. The phrases and language used appeared in the "first attempt" tool.

> R: Well, I think – you can't just say "watch your students and what they're doing with the calculator," because normally calculator use in my classroom is not my number one priority. But keeping a list like this can certainly help.
>
> L: I also look at some of the other kids that are maybe special ed, actually labeled special ed, or the slow-rate learners, that really don't know their multiplication, and they need the calculator in order to move on as – it is a crutch at that point, they don't know how to do it, they have to come up with alternatives to move past that stage....
>
> I: And they might not be capable of it.
>
> L: Right. So I think there's also a difference in the type of student you're working with on whether you're going to let them use the calculator, and when you're not. And working with ... 10-15% are either identified as slow-rate learners, or have been labeled in the last year. So a lot of their IEPs will say "for all tests, for all things, they need to have the use of the calculator." ... Some of them have been doing a lot more with the use of the calculator, being able to look at the exponential functions and understand what's going on, and they wouldn't be able to do that without.
>
> I: ... I put down "Things that we might consider" and what I'll do is put this together and then we can talk about it more in depth next time as we're adding things along. I put down "Be aware of what students are doing with calculators" and "Be aware of which students legitimately need them."...
>
> T I think the other thing that's important, ...we're not just teaching them how to push the buttons. That when they get the answer out, they know what to do with that answer.... But then the other hard thing is, that they get confused with the stat functions and the linear functions. Whenever they go to a table, they think they can go into a linear table to put information....
>
> I We sometimes forget how complicated even the 83's are....I'm going to put down "types of calculators and complexity" I'll think of something to say. (SG2, 43-58).

A short digression about the number of students not passing any middle school mathematics but still going on to high school ended with Lynn saying "We have no control over that. That's not our decision" (SG2, L, 71). This thought provided the segue the researcher was waiting for. She passed out the list of issues shown in Table 20, and

said, "I want you to go through here and pick out the...issues...our tool... [can] actually address" (SG2, I, 72).

After teachers had marked their list, the researcher asked, "Is there something you want to say about any of these issues?" (SG2, I, 73) Tess said,

> I think that our tool can address the problem that kids believe the calculator over themselves. If we just put some questions up on the board, that we know if the kids put it into the calculator the wrong way, they get the wrong answer, but make it simple enough where the kids can think it through, and then say, "that's how come I said the kids believe the calculator." ... I think it's important that we teach them the calculator is only as good as the person that's pushing the buttons (SG2, T, 74).

Issues from Session 1 (58 in all)	Which can our tool address?
I - always believe calculators	I - kids believe calculator
I - benefit for students	I - kids don't read problems
I - better to do by hand, but no time	I - kids just want answer
I - book introduces new concept with calc	I - know what technology is doing
I – calculators intended only for "higher level"	I - lack of math skill irritating
I - calculator faster, ease of comparisons	I – laziness
I - calculator smarter than me	I – meaningful to kids
I - check answers	I – misuse
I - checking the calculator	I - more problems in same time
I - common term	I - no slide rules in elementary
I - concepts first	I - only look at numbers
I - curric says use calculators -have to follow	I – operations
I - dependent	I – pacing
I - do they know the concept?	I – reading
I - don't know difference between 2x3 and 2^3	I - reading math
I - don't know operations	I - retention
I - elementary teachers do not know math	I - see another way
I - estimate	I - sometimes don't use calculators
I - faster by hand	I - student placement
I - foundations	I - syntax errors hard to find
I - fraction calculators	I - teachers don't want to teach use of calc
I - fraction key	I - teaching can be fun
I - fractions	I - teaching math reading
I - garbage in, garbage out	I - technology here to stay
I - go deeper	I - technology makes some math obsolete
I - group work	I - we do teach how to use
I - I didn't use a calculator until college ...	I - what do they lose?
I - I'm doing just fine	I - who learns what on the calculator?
I - if I disagree with curric, I don't use calc	I - work to learn calculator so I can show

Table 20: *Issue list given to teachers at Session 2*

When the researcher asked for other comments, Tess had more. She said "We have to teach them to think even though they have a calculator" (SG2, T, 78). Lynn supported that by brokering a comment from one of her middle school colleagues. He said "I can't believe, as soon as they get that calculator in their hands, it's like they forget everything they just learned" (SG2, L, 79). Rob had been quiet for awhile, but showed that he was still thinking of action. He interjected, "Maybe a questionnaire for the students and their calculator use in the classroom. How do they feel about it? I think I might do that" (SG2, R, 81). Tess then carried her idea one step further. "We need to teach them to make choices. Or make decisions on when you should pick up a calculator" (SG2, T, 83). Not only did she see the need to help students learn to think *with* a calculator, but also to make the choice to think *without* a calculator sometimes. So, although she had threatened to take away her students' calculators, Tess had the ideal that students should make their own decisions. Lynn may have been wondering if that would be best, because she related an incident of balancing a checkbook with a calculator because it was easier. She added, "I think they [students] perceive it sometimes the same way—it seems easier...just to get the calculator, punch the buttons, and see what it tells me, than actually having to go through the process of thinking about it" (SG2, L, 87).

A long discussion ensued about the attitude of society toward mathematics, including parents who were happy with their children's D's in mathematics classes. Lynn closed that discussion by saying, "But until we make it meaningful for them, that's where I think their hangup is, a lot of that meaning comes from the use of the calculator. Once they're hooked on that, there is no meaning" (SG2, L, 97). She seemed to say that the foreignness of the calculator was an excuse to not have to think about the math –' just let

the black box do it for me.' The researcher responded by gleaning one more point for the AUGC tool, "So thinking is actually one of the things that we want to aim at" (SG2, I, 98).

Artifacts that Reify the Ideas of Others

Attention was then directed to the agenda, which is found in Appendix O.

> Okay, now I would like to turn to this sheet that I gave you for Session 2. We've already done the first two, so we can check those off. There's a little note there that all of the stuff from there down comes from this book, ...*Implementing Standard-Based Mathematics Instruction: A Casebook for Professional Development* (Stein et al., 2000)(SG2, I, 99).

Again, for the sake of being open about the abbreviated introduction this group of teachers received of the ideas in the book, the entire introduction is found in Appendix O, along with The Task Analysis Guide being introduced, which is reproduced by permission. The guide describes four levels of cognitive demand of tasks, paired as high-level or low-level tasks. The low-level tasks are (1) memorization, (2) procedures without connections; and the high-level tasks are (3) procedures with connections, and (4) doing math. Participants worked in pairs to apply the Task Analysis Guide to the tasks that were provided in Stein's casebook (2000), and separate audiotapes were made of each pair's negotiations.

Levels of Cognitive Demand

After the pairs finished their work ranking tasks, the group came together again and the researcher asked how they defined the four levels of cognitive demand for themselves. The following language was used to describe their understandings.

For the level of memorization tasks, teachers thought of "Rote memorization, mindless," " Like television watching," " Something like simple addition or

multiplication," or "You already know this, write it down." Procedures without connections brought to mind, "Spitting it back out," or something "A little bit higher than memorization, but it just wasn't making real connections to actually what the mathematics was about." When asked what the difference was, then, for procedures with connections, Rob said, "The big word, I think, is cognitive effort." Tess added that "You have to think about it a little bit. Something that they have to take it a step further." The researcher was surprised that they didn't explain what the "connections" were, but asked "what, in addition, do we have in the doing mathematics task?" Here Rob was a bit more specific. He said, "The question that goes further by asking the student to reflect on what they were doing, explain what they were doing, what do you think would be next, or how could you add to this question, … would be a mathematical task." To which Lynn added, "And the analyzing part. Can you think of – I kind of think of a metacognition – they have to think about what they're thinking" (SG2, 121-137).

It was evident that these teachers "spoke the same language," at least superficially. Rob would henceforth call tasks in the category "doing mathematics," by his preferred "mathematical tasks." Toward the end of this discussion they began quoting their students – for examples of what the opposite of "doing math" was. Phrases such as "Just tell me what to do," and "Can I just do the problem?" or "Am I right or wrong?" were familiar to all of them. The majority of the rest of the session was taken up in negotiating the classification of eight tasks into these four categories.

Applying the Task Analysis Guide

The tasks are not all reproduced here, because it was the application of the task analysis guide, and not the tasks themselves, that were of interest to the researcher.

However, the results of the classification activity by the two pairs of teachers are given in Table 21, along with the classification given by Stein et al. (2000, p. 21) as a comparison.

Task	Stein Rating	Pair R - L	Pair T - K	Non-consensus	Group
A	Do Math	P with C	P with C		P with C
B	P with C	Do Math	P with C		P with C
C	Do Math	P with C	P w/o C		P w/o C
D	P w/o C	P with C	P w/o C		P with C
E	P with C	P with C	P with C	Do Math	Do Math
F	P w/o C	Mem	Mem		Mem
G	P with C	Mem	Mem		Mem
H	Mem	P w/o C	Mem		Mem

Table 21: *Pair and Group classification of tasks using Task Analysis Guide*

Note: The categories are represented by abbreviations:

Mem = Memorization
P w/o C = Procedures without connections
P with C = Procedures with connections
Do Math = Doing mathematics

The fifth column of the table records the rating of Karl for the one task on which he declined to reach a compromise with Tess in their pair work. This became significant in the negotiation of the group consensus. The final column of Table 10 lists the classifications assigned the eight tasks by group consensus.

Before examining the negotiation of the group consensus, Table 22 gives another view of the classifications of the eight tasks, following the suggestion of Silver (2003) to compare classifications of tasks by using only the high-level or low-level designations. By this method, we see that the two pairs of teachers disagreed on the distinction between high and low level cognitive demand on only two tasks. As they worked on consensus on these two, they resolved both disagreements in a manner contrary to the Stein rating. Only one of the task ratings on which the two pairs agreed did not agree with the "official" rank, so they ended up with a 62% agreement overall. The goal of analyzing the tasks, according to Stein et al. (2000), was "not to achieve complete agreement but rather to provide teachers with a shared language for discussing tasks and their characteristics

and to raise the level of discussion among teachers" (p. 20). Only slight evidence of this shared language can be seen in the brief exposure of the present study.

Task	R - L Pair	T - K Pair	Compared to Pair/Stein	**Stein** H/L rating	Group Consensus	Compared to Stein
A	H	H	agree	**H**	H	Agree
B	H	H	agree	**H**	H	Agree
C	H	L	disagree/H	**H**	L	
D	H	L	disagree/L	**L**	H	
E	H	H	agree	**H**	H	Agree
F	L	L	agree	**L**	L	Agree
G	L	L	agree/'wrong'	**H**	L	
H	L	L	agree	**L**	L	Agree

Table 22: *Comparison of ratings of high or low cognitive demand of tasks*

In examining an example of the negotiations of the consensus ratings of the tasks, the focus is on how thinking about calculator use influenced judgement of the cognitive demand of a task. This is practical because the description of each task included what tools were available for student use. Task C was a task on which the two pairs disagreed, and it indicated a calculator as a tool. It read,

> Your school's science club has decided to do a special project on nature photography. They decided to take a little over 300 outdoor photos in a variety of natural settings and in all different types of weather. Eventually they want to organize some of the best photos into a display and enter the State nature photography contest. The club was thinking of buying a 35mm camera, but someone in the club suggested that it might be better to buy disposable cameras instead. The regular camera with autofocus and automatic light meter would cost about $40.00 and film would cost $3.98 for 24 exposures and $5.95 for 36 exposures. The disposable cameras could be purchased in packs of three for $20.00 with two of the three taking 24 pictures and the third one taking 27 pictures. Single disposables could be purchased for $8.95. The club officers have to decide which would be the best option and they have to justify their decisions to the club advisor. Do you think they should purchase the regular camera or the disposable cameras? Write a justification that clearly explains your reasoning. (Stein et al., 2000, p. 19)

Tess and Karl used the numbers 1 through 4 to indicate the levels from Memorization (1) to Doing Math (4). They were easily dissuaded from the guide's

requirement for explanation by the particular situation, because the justification only required "saying one is cheaper than the other" (T-K, 39). Karl said he didn't "see any really high level of thinking ability that has to be done with task C" (T-K, 40). They finished the discussion noting the presence of the calculator as allowing them to agree.

> T: Especially with the calculator. All they're going to do is push buttons.
> K: Well, that's the thing. I forgot about the calculator being in there.
> T: Yes, I agree with you, I think it's procedures without connections tasks because all they're going to do is push buttons. (T-K, 41-43)

In spite of her earlier plea for students to need to know what to do with the answers they get, Tess did not see the requirement of writing a justification (which may well include such things as how many club members could take pictures at once) as going beyond the pushing of buttons.

The negotiation of consensus began with Rob asserting that Task C was "Mathematical tasks" (SG2, 157). After clarifying that their designation corresponded to what she and Karl had called "level 4," Tess called on Karl to explain their position for a level 2 rating. Karl said, "They used calculators, all you have to do is punch in the buttons" (SG2, 165). Rob noted difficulty reading the problem, and predicted students would have difficulty figuring that out and *then* would have to "justify the reasoning" (SG2, 169). Karl replied, "I didn't think there was a whole lot of justification in it. Once you punch these buttons..., one's bigger than the other" (SG2, 172).

Karl had been unusually quiet for the earlier discussion on the calculator use logs, but in this part of the session he seemed to want to have his way. His diplomatic tendencies from Session 1 were gone. When Lynn and Rob talked more about the specific complexities of the task for middle school students, Karl seemed to abandon

mathematical discussion and instead effectively stopped the discussion by saying, "I figured my wife could figure it out, so it couldn't be too bad" (SG2, 176).

After a general outcry, Rob said, "I'm not going to argue it" (SG2, 178), followed by Lynn's "I'm not either" (SG2, 179). However, Rob does not let the departure from mathematical argument go unacknowledged. He specifically stated that he would not join that part of the discussion, but he then brought the conversation back to the task, conceding that the cognitive effort needed was perhaps not so high as level 4. Rob tried humor to even things out again, and Karl conceded that a case could be made in several ways. He then made a more reasonable statement of his case for the calculator making the problem much easier, "if you give somebody a task to go to the store and say which is the better buy, ... you do this calculation, you do that calculation, here it is" (SG2, 187). The researcher's attempt to bring the discussion back to the complexities of the problem by getting consensus that the task was not memorization, brought only half-hearted support for a higher level rating from Lynn. It must be noted that Tess was at this point out of the room, and could not be appealed to. Lynn finally said that she could agree with Karl's assessment of the task.

The next task discussed, the other one on which the two pairs did not agree, won a higher rating because a calculator was not allowed. Ironically, Stein et al. had rated the task lower, so the study group teachers missed agreeing with the "official" rank again. At the end of the session teachers responded to the same reflection prompts as they did after the first session. Their responses are in Appendix N. Evidently, their thinking about appropriate calculator use was still varied. Lynn's response to the final prompt may have mirrored her frustration with trying to negotiate the task levels with Karl, because she

wrote only "??" as a reply to "something that was discussed today that I do not agree with was ...".

At the end of Session 2, the apparent gap in agreement between what students should be able to do without calculators and what they should be encouraged to do with calculators had not narrowed. The concern for knowing concepts first was still not clarified, and the calculator still had the reputation for being a way to avoid understanding.

Researcher Plans for Study Group Session 3

Plans to apply the Task Analysis Guide to some of the Task Sort items were not realized, nor was the hope of introducing other thought-revealing tasks (Lesh, Hoover, Hole, Kelly, & Post, 2000). However, it was clear that participants were now fully engaged in thinking about how calculator use affected what was happening in their classrooms. The researcher determined to get as much interaction as possible on the emerging AUGC tool in Session 3 before introducing the Branca et al. (1992) questions for teachers using calculators.

Focusing on the AUGC Tool

The researcher had taken on the role of trying to put into words the concerns group members thought the AUGC tool should address. First, a narrowed list was made of the issues that had been selected in Session 2 as ones that could be addressed by the AUGC tool. The issues were arranged in categories, one of which contained those the researcher questioned on grounds of addressability. The entire document used in Session 3 is found in Appendix P, and the categories are presented in Table 23 in the section for Session 3.

Comparing this list of issues with that presented at Session 2 (see Table 20), it may be significant that those dealing with disagreement with curriculum were not chosen. Perhaps these teachers do not feel that curriculum can be questioned – or at least not in a document to be shared. By not pursuing the omission of these, the researcher missed an opportunity to pursue the question of whether it is important that teachers understand the intent of the use of calculators in their district's curriculum.

The researcher then prepared, based on the conversation of Session 2, what she called a "First Attempt at a Tool." The word "draft" was intentionally not used because that might have implied too much finality. In Session 3, teachers would be asked to judge if the AUGC tool dealt with the issues that had been raised. The main content of that first attempt is given in Table 24, which is presented in the section for Session 3, and the full document used in the third session is in Appendix P.

Hoping to get feedback so that a second version of the first attempt could be prepared for Session 3, the researcher emailed copies of the documents in Appendix P to teacher participants almost two weeks before Session 3. However, only one teacher took time to write comments, and she did not know how to email them back, so the researcher first received feedback at Session 3.

Prior Observations and Interviews

Yvette's observed class was working on distances on a coordinate grid. She had done some preparation for the students' calculator use as a reference. Yvette did not use a calculator herself during class. She drew diagrams on the board and brought individuals who had questions to the board to work with the diagrams while the other students were working in groups. The numbers in the problems were such that students could reason

from the diagrams. Only a few of the students actually went to get calculators. When the text asked them to generalize what they learned from the problems they did with numbers, Yvette called them together and they worked through it together, using the Pythagorean Theorem. When Yvette talked about her class, she was more concerned with their lack of motivation than their use of calculators.

Karl, on the other hand, had only two students in attendance. Perhaps for the observer's benefit, Karl did a fairly traditional presentation of the two forms of exponential functions – recursive and explicit, doing examples on his overhead calculator. The students then worked on their assignment, conferring once in awhile. When asked what level tasks he thought the students were working on, he said, "Basically 3 [procedures with connections tasks]. Because they're learning through the examples and applying them to their homework problems" (K2ObsInt, 2).

Tess's observed class was solving systems of equations by graphing. She was still pulling in other technology when she could. Tess had used Fathom to demonstrate the concept of solving a system by finding intersections, and in this class they were learning to do the same thing on the calculator graphics screen. When Tess asked how they could check their work, they knew they could use the tables. She also gave them another system whose solution was not integral, perhaps to encourage using the intersect option rather than *only* the tables.

Lynn's observed class was working in one of their most difficult units. They were using an area model to look at the multiplication of binomial factors. They had also done by-hand generation of tables of values for x, x^2, $x + 4$, $x - 4$, and

$(x + 4)(x - 4)$ and had plotted the values by hand to see the shape of the graphs. During class they were able to completely go over one problem after students had solved it. But then Lynn had to instruct them to work on two other problems for homework, except the calculator part, which they would do the next day in class. This was because most of the students did not have calculators to use at home. The students Lynn called on seemed to be able to explain how graphs, tables and equations were related, but in the group the researcher sat near, there was at least one student who merely wrote down what his "helpful" group members told him, and did not seek explanation.

Rob's observed class again showed creativity. For the first 45 minutes the class took a quiz, and the room was absolutely silent. After the quiz, groups put finishing touches on projects they had worked on for several days. They had chosen a product to sell, determined start-up costs, including securing a patent, set a price, predicted sales, and used a graph to find their break-even point. Calculators were not evident, because they were presenting the projects.

After class, talking about the study group, Rob said,

> It certainly makes me think about what I'm using calculators in the classroom for. I'm starting to lock my calculators up, and only keeping them out at certain points where I think they're necessary....I'm really gauging, I'm looking at my lessons now, and saying ...how am I going to use the technology in the classroom ... to enhance lesson, or to allow them to think without jumping into calculator exercises (R2ObsInt, 2).

He said he had at this point not considered the cognitive demand of the projects, so the researcher asked about that. The project task had the students making connections between all the mathematics of the unit, and since they also created the situations themselves, the projects rated as "doing math." Rob gave his explanation of level 4 (doing math) this way, "Well, ...I think you have to question the students at a level where

you're going to get the students to think about, not just react, but to think about what they're doing" (R2ObsInt, 28). He contrasted this to what his students think about math, "They wait for someone to tell them, okay, you have to multiply these two numbers together, what do you get—and they think that's math" (R2ObsInt, 34).

Rob then explained what he still needed to do in his work with the study group.

> You know, I haven't correlated the cognitive thinking with the calculators. That's something—we've talked about that 2 weeks ago, and I never really threw that into the—I was thinking more of just intellectual questioning when we were thinking about the intellectual problems that they could do, but now that we talk about it, how could I cross that over with calculators? How can I use the calculators to help them get to that level 4? (R2ObsInt, 38)

Researcher Decisions about Session 3

Not having enough time for everything in Session 2 caused some ideas to be carried over into Session 3, and others to be reluctantly abandoned. Session 3 would certainly have to begin with work on the AUGC tool – evaluating the First Attempt document, and judging whether it responded to the issues. Then the group would spend a small amount of time on cognitive demand of tasks, applying the Task Analysis Guide to two tasks on the topic of inequalities. In a reflection written just before Session 3, she explains,

> I will be watching for this glimmer of understanding about cognitive demand, and also for clarification of what is meant by "understanding concepts before using calculators." I have chosen inequalities as the focus of content today because I observed two classes in which they were part of the lesson, and got the impression in both classes that teachers were pushing symbolic manipulation, when the curriculum was really calling for examination of graphs. What concept did the teachers have in mind? I am choosing to compare [a revised version of the observed CPMP inequalities lesson] with a more traditional approach (UCSMP) because to me, the cognitive demand of the activities is obviously different. I want to see if teachers can see it too (ReflPre3, ¶ 2, 3)

Focus would be on parts of the two lessons to avoid worse time problems.

And finally, the questions reported by Branca et al. (1992) for use with middle school teachers using calculators would be presented to the group for comparison to our 'first attempt' tool.

Session 3

The emphasis of Session 3 was on the substance and format of the emerging AUGC tool, with some attention given to the application of the Task Analysis Guide. The researcher kept a timer going to enforce moving on to the next topic, partly because it was the last day before Spring Break and minds would wander.

Focusing on the AUGC Tool

The session started with an invitation to examine the categories for the issues chosen in Session 2, as presented in Table 23. The researcher introduced it thus:

> What I did was I took the issues that we had looked at last time, and you … marked which ones you thought the tool might be able to address, and I categorized them in five categories. And the ones at the bottom, I'm just not sure that we can actually address those, but if you can convince me, or give me a little more detail in that, we might want to incorporate those, too (SG3, I, 2).

Lynn had looked at these when they were sent by email and commented about the issues in the last category.

> I think they need to be addressed, but I don't think in the tool that we're supposed to be coming up with, because…there are so many other complicated issues that deal with these, that I don't think a tool for calculator use is going to solve any of them (SG3, L, 3).

The wording of how to talk about the issue of elementary teachers' math knowledge was a question for Tess. She felt we had to express something about how calculators should be used at that level, because

> That's how come they use the calculator, because they don't feel comfortable enough teaching the mathematics, they fall back on the tool, and that's what we don't want the tool used for! We want it to be a tool, not an … I don't know what to call it—do you know what I mean? (SG3, T, 7)

1. How students benefit from using calculators		
I - benefit for students	4	
I - calculator faster, ease of comparisons	2	
I - go deeper	2	
I - technology here to stay	3	
I - meaningful to kids	2	
I - teaching can be fun	1	
2. Which students benefit from using calculators		
I – calculators intended only for "higher level"	2	[of math or of students?]
3. Student misconceptions about calculators		
I - always believe calculators	1	
I - calculator smarter than me	1	
I - checking the calculator	1	
I - kids believe calculator	2	
I - kids just want answer	2	
I - know what technology is doing	2	[they often don't]
4. Teachers need to think about		
I - concepts first	1	
I - do they know the concept?	2	
I – estimate	1	
I - kids don't read problems	1	
I - teachers don't want to teach use of calculator	1	
5. Drawbacks from calculator use?		
I - don't know difference between 2x3 and 2^3	1	
I - don't know operations	1	
I - what do they lose?	2	[originally: what do they lose over summer?]
I – operations	1	
**Can the tool deal with these?		
I - elementary teachers do not know math	1	
I - lack of math skill irritating	1	
I – reading	1	
I – retention	1	

Table 23: *Issues categories presented at Session 3*

Note: Each issue, preceded by " I –," is followed by the number selecting it in Session 2

The others joined in the following interchange, which shed light on what Tess thought about what came before calculator use.

> I: I think so, but let's talk that out a little more, so we can get it down on tape.
> K: A facilitator.
> T: Exactly.
> I: Okay, a facilitator of what, though?
> K: Of whatever you're doing, you know?
> T: If you're adding four plus three, we want the kids to know that it's four objects and three objects, and we're adding them together, and be able to visualize it, just not push a button on a calculator....
> L: No, if they could visualize addition, subtraction, multiplication, and division, if they knew those four operations, and what they meant, I think it would be a huge step in the elementary years. If the kids could come to the middle school at least having
> T: I think it's the old concept that we used to teach math. Do this algorithm, do this, you don't have to understand why we do it, just do it. And I think that's – the elementary hasn't made that transition that we've tried to make because they don't understand (SG3, 8-20).

It was not procedures that Tess wanted before calculators, in fact here she argued against that. What she wanted was conceptual understanding of what operations do to numbers, and what numbers mean. The others agreed, though Rob was quiet at this moment. What these teachers may not have realized at the time was that their district had spent five or more years in teacher professional development and implementation of a curriculum that would specifically encourage this type of learning for elementary students. The researcher had observed elementary children in that district who could do the reasoning these teachers were hoping for. Had their limited time in the district not been enough to learn what was happening in elementary schools? Or had they been told the name of the elementary curriculum by someone who assumed they would know the philosophy behind it. The researcher did not try to explain, since the group had other work to do.

Karl stated that he had contributed the issue in the second category, and that he meant it for level of mathematics. He explained,

> So if you're teaching a certain course, then it might require the use of the calculators.... I never thought about the level of the students, but I don't know how you can allow one student in class to use a calculator, and not another person (SG3, K, 34).

This led to further discussion of which students should use calculators, and when. Karl didn't see much promise in trying to set a some do/some don't guideline, and calculators were already required by the Individual Education Plan (IEP) of some students.

> K: It makes me think that—the analogy is the telephone. Well, I'm sorry, but you can't use the push button telephone until you know how to use the dial telephone. ... how relevant is some of this to make the kids not use the...
>
> T: But this is the ordeal that I see exactly, because Sonia [the district Mathematics Coordinator] and I have gone back and forth about this because I'm teaching the Math Investigations class. And we had this big discussion about fractions and why I'm not letting the kids use the calculator to do fractions (SG3, 42-43).

The issue of students not understanding variables on the calculator came up, and Rob thought he could add something. The way he negotiated a chance to speak, aided by the researcher, is illustrated in the following interchange:

> I: They don't understand that that's what the calculator's doing. So I'm wondering if...
>
> R: Are we talking about misconceptions?
>
> T: We're talking about which benefits students.
>
> R: Oh, I see.
>
> I: But I think that really is a misconception, if they don't understand that the calculator is taking some value for x. If they don't understand that the calculator really can't do x-squared for them, except for a specific value, then...
>
> R: I actually took my kids this week, because they kept trying to put equations into the calculator and expecting the calculator to solve it for them, and give them some number for x, when in actuality, the calculator – and I started them off with, okay, put x in there, and what you should see, and I had it on the overhead, and they would do it and get something like negative 2.05 something, just the wrong

> number. And I said, well, that's [what's on the overhead] what you should be getting, and the whole class just erupted, because nobody had that. They didn't understand – and I said, no, your calculators are wrong, or you must be wrong, because your calculator is always right – you put it in your calculator wrong. We started this huge debate on it, and I think they finally figured it out, but I did that with each of my classes this week... (SG3, 53-58).

What Rob brokered here was an example of how a teacher might deal with a difficulty in understanding as a calculator problem, rather than as a student problem (what students don't understand). The researcher was able to support him with a similar example, where the value in variable x was used to the student's advantage to operate on the home screen with a traced value from the graphics screen.

Be aware of what students are doing with calculators. Are they being used to (as) Check answers? Arithmetic aid? Reference sheet? Answer finder? Presentation helper? Visualizer?
Be aware of which students legitimately need calculators, due to disability.
Beyond teaching students how to use the calculator, ask them to interpret answers once they get them.
Be aware of the complexity of the calculator students are using. Is it more complex than is needed for the task?
Ask students to make connections between what they do with the calculator and the problem they are trying to solve.

Table 24: *First Attempt at an AUGC tool, based on discussion of Session 2*

At that point, the timer signaled that it was time to discuss the 'first attempt,' so the researcher called teachers' attention to the statements in Table 23. The researcher's decision to enforce movement on the agenda in order to finish work on the AUGC tool in the four sessions teachers had agreed to attend certainly affected the depth of discussion

in some cases, such as this. But it also helped reduce the amount of time spent on "venting." She introduced their new focus this way,

> These issues [Table 23] were the issues that we brought up about calculators. And then I asked you to mark which ones might our tool that we're trying to create, address. Then I put those in the categories. Now, the last time we talked about what things we might actually write down for the tool, and I just made notes of those in my notebook, and now I've just typed them up [Table 24]. So now, the question is, do these things that we put down, actually address these issues that we said should be... [addressed]? (SG3, I, 70)

Again, Lynn was ready with a suggestion. "I added one question, and I didn't know how to word it, but—that we need to be aware of students' ability if they have mastered the concept or not" (SG3, L, 76). It took the group awhile to focus on what Lynn was trying to say. The researcher began,

> I: Okay, so you're thinking of this one question, or a couple questions in number 4 [category 4 on the issues list]. About the concepts.
> L: Yes. I don't know how you'd want to word that, but...
> I: ... how do you think that might be—you said their "ability to master the concept"?
> T: Assessing students' learning with concepts, or something like that?
> I: Well, and what might we want to say about that in terms of the calculators? We've reworded it, but then what? (SG3, 80-84)

Later in the conversation, they come back to this topic again, worrying that many students entering high school do not have the concrete understanding needed to succeed in high school math. The discussion proceeded,

> R: Maybe it's the [middle school] curriculum.
> L: And a lot of it is the curriculum, especially with this book, with the quadratic functions. A lot of it's just way over their heads. It's so far above them. Instead of going back and building a solid foundation...
> T: You have to have the basics before you go into the upper.
> R: I think it's good, though, that they're showing the calculator use with that. And maybe they're just using it as kind of a picture, a technological picture, and that's it, but...
> I: What sort of concrete understanding would you rather they have?

> L: I would rather be able to have them make the graphs, understand what the calculator's doing, that these—kind of what we talked about—that these *x* values in the equation are being replaced over and over and over, and they're giving you back a different y value for each *x* that you put in. And have a good understanding of that, and understand, I would say even spend more time on linear equations. Understand how to write the equations, how to make graphs, how to interpret information, and solving basic proportion problems (SG3, 110-115).

Here Lynn was again expressing the desire to have conceptual understanding at least along with calculator use, if not before it. And a little later, she seemed to think they had reached an understanding of what her addition to the AUGC tool should be.

> L: That's really the only thing I see ... missing, would be however we would word that about assessment, do they have a mastery or an idea about this concept.
>
> I: Okay. Assessing conceptual understanding. Does anybody else want to add some points to that? (SG3, 130-131)

Tess commented that she had done hers the opposite way – placing every item in the 'first attempt' into one of the categories of the issues list, so she did not see a need for another category. As the timer signaled another shift in focus, Rob suggested that he wanted to have until the next session to formulate additional items. Karl had no other comments.

Two Contrasting Tasks

The researcher now asked teachers to talk briefly about how they had used their logs to record what they found about the cognitive demand of the lessons they taught. Tess had not written in her log, but was ready to share her thoughts.

> With my Integrated 2 kids, we're starting the power models unit, and that's the unit where we talk about *x* going to a certain power, and what happened. And I figured – I really thought that that was a good cognitive thinking – the calculator was just used as a tool, it [the task] told them what to graph, it told them what to put in there, and then they had to be able to read, interpret the tables, interpret the graph, and see what happened when they added that coefficient on to the *x*-squared, the *x*-

> cubed, and then it took them to the next step. What happens if it's x to the fourth, or x to the fifth? So there the calculator was the perfect tool for them, because if we would've had to do that all by hand, it would have taken forever. It was the perfect tool for them to use because they could see it. Now some of them did stupid things and put them all in the calculator at once, so they didn't know which was which, but the kids that got it, could see what was going on. And it took them to the next level. And finally, they were saying, hey, Mrs. Ernest, it's like if they're odd they're this, and if they're even, they're this. And it's like, you got it! So there I thought the calculator was the perfect tool, and it was used correctly (SG3, T, 159).

Not only did Tess give the task a rating of "good cognitive thinking," which seemed to be her equivalent of "doing math," but she also acknowledged that it demonstrated appropriate calculator use, "as a tool." Others who were teaching the same lessons agreed.

The group then went on to apply the Task Analysis Guide to two contrasting tasks. The first was taken from a recent revision of the Core-Plus Mathematics (Coxford, et al. 2003) materials for Course 1. The context was different from what had been taught in the two observed classes, but the concepts of understanding inequality by looking at graphs and tables were the same. After some digression over the alligator analogy for inequality – Rob said, "I spent a whole freaking day on alligators and what do they eat! (SG3, R, 188) – and over the distributive property, Tess gets everyone back to the task.

> T: Do you want to talk about cognitive stuff here?
>
> I: Yes, I want to talk about what's the cognitive demand of this little piece here, where they're asking them to solve these equations and inequalities by finding the value or range of x that satisfies the given conditions. ... But what are they actually asking them to do here?
>
> L: Well, they're asking them to, I assumed, solve for x.
>
> R: Oh, no.
>
> I: [reading] Explain how the solutions can be found in tables and graphs of these two functions (SG3, 194-206).

The task under discussion here used linear models to describe data of the percent of male and female doctors in the United States since 1960. The percent of male doctors is modeled by the equation $Y_1 = 98 - 0.54X$, and the percent of female doctors is modeled by the equation $Y_2 = 2 + 0.54X$. The task analysis is being applied to part 3, quoted here:

> 3. Solve each of these equations and inequalities by finding the value or range of values of X that satisfy the given conditions. Then explain what each solution tells about prospects for male and female percents of all U.S. medical doctors. Explain how the solutions can be found (or at least estimated) in tables and graphs of $Y_1 = 98 - 0.54X$ and $Y_2 = 2 + 0.54X$.
> $70 = 2 + 0.54X$
> $98 - 0.54X = 2 + 0.54X$
> $98 - 0.54X > 80$
> $98 - 0.54X = 65$
> $2 + 0.54X < 40$
> $98 - 0.54X = 4(2 + 0.54X)$ [Hint: Consider $Y_3 = 4\ Y_2$.]
> $98 - 0.54X = 1.5(2 + 0.54X)$ [Hint: Consider $Y_4 = 1.5\ Y_2$.]
> (Coxford et al., 2003, p. 213)

Tess had taught the original lesson more than once, and Rob was interested because, as he said, "I've just given up on inequalities, it's horrid" (SG3, R,191). But Tess also showed, in the exchange below, that she understood how to maintain the cognitive demand of the task as she helped students use calculators.

T: They know that the 65 and the 70 is Y. They know that. So they know how to use their tables, and they can find Y for 70 and then that's their *x*. ... So then, we talked...
R: You do a table or something like that?
T: The table. And then we sit there and we talk about what that means. And then we go to the inequality. ... And I go, but let's look at what happens at 80, where is it less than, or where is it greater than? But, I mean, they can solve it, but the cognitive part—it's hard to get them there....
T: The way that I teach this lesson, is we do 1 and 2 together...because it introduces...all of the aspects. So I get the little overhead calculator thing out, and we go over it and we talk about it. But then when they get over here, when they have to think on their own, I have to ask them more questions because they lose it.
I: Because we are asking them to think on their own.

T: But they can push the buttons to get the answer, but when the cognitive part comes in, to talk about what that answer means—and those ones where they're equal to, oh my word, "I can't put that in the calculator, Mrs. Ernest!" "Yes you can. What does that stand for? In the other problem, what did that 70 represent?" "Y." "Well can't one of those be Y?" "Well then what do I do with the other one?" "Can't you make the other side Y, too?"

I: That really is a difficult concept... And I think that's why they specifically restate that these are the two graphs that we're using, here's Y_1 and here's Y_2. And if you look at the hints that they give, what would you expect a student who's catching on to do with that hint?

T: To make it 4 times whatever the Y is. That's what they did.

I: So, it says here, Y_3, that means make another graph, or make another table. And that one is 4 times Y_2.

R: I do like the fact that they use those two graphs over and over in a repetitive manner so they understand that it's—we're just trying to solve everything, they don't just give you different equations and say, what about this one, or what about this one, because I think you would be very confused by that. But they use the same two equations....

T: They have a hard time, though, about that substitution thing. In this, you have to sub that whole equation for *y*. (SG3, 209-220).

The teachers have not yet given the task a rating of cognitive level, but they have brought out connections between the problem situation and the mathematics, and also between the representations. In the following interchange, Tess acknowledged that it was not easier to teach using calculators.

T: That's one thing that really bugs me about being a teacher and using the calculator. ...[students] don't know how to do the thinking after the calculator gives them the answer.

I: So in our tool, we really have to—we really want to emphasize that. Right? Make sure that the thinking is still going on.

T: The cognitive has got to be there, because what good is the calculator if the cognitive ability isn't there?

I: So the interpretation of the answer? ...

R: ...I see a lot of benefits, too, where I can have the students switch back from a graph to a table to an equation, interchange them, and if they don't get one understanding, they can throw it in a table, and they can go, oh, now I see how those relate to each other, or boom, I can switch to a graph ....

T: ... we're saying that our kids struggle with the greater cognitive thinking that goes along with the usage of the calculator, ... And the kids, a lot of them, just like Rob said, we teach them how to solve it on a table, a lot of the kids can't solve these by hand, but they get how to push the button on the calculator, and they know that ...they can go to the table and find out what x solves that (SG3, 226-244).

The researcher tried to bring them once more back to the analysis of the task, pointing out that the connection to the context is part of what makes this "doing math." Rob ends up looking at it from a student's point of view, rather than how hard it is for the teacher.

R: They don't know how to read math, that's the problem.
I: But isn't that the big idea here? That we're trying to show them that the math actually does connect up with the context somehow, and does allow us to do something with it?
R: It's a lot easier doing it this way than, I think, [the way] I learned. Connecting the reading and the math together.
T: How many times do you have a kid in Integrated I ask you when am I ever going to use this stuff? In algebra, I used to hear that at least six or seven times in a day.
L: That's a very good point, that's true (SG3, 254-258).

After this the discussion centers on how to deal with the variety of students in a classroom, and the various things districts have done to deal with this in the light of state test pressure. Teachers pull in ideas from everywhere – from their previous experience to courses they are taking now. They again return to how their district is trying to cope, and Tess talks about her Math Investigations class that is supposed to support those having trouble in math. She is disillusioned by the difference between the intent of the class and what is actually happening in it. Worry for each teacher comes back to his or her own students, and Tess says, "But I don't have time to go back, because we have to get ready for the MEAP [the state test], they have to be ready to go to Integrated 2" (SG3, T, 278). The following interchange shows the differing ways the teachers dealt with this outside pressure. Karl seemed to try to ignore it by labeling it, "See, that's the problem. There's

too many agendas out there that we have to book. Theoretically, we can't get the stuff done that we want to get done" (SG3, K, 279). Tess seemed to have, at this point, a rising sense of panic.

> But the other thing is, too, I don't know if you've seen the new benchmarks for the State of Michigan, but if those go through, the kids will be starting in high school with Integrated 2. And I laugh, I looked at Sonia, and I go, I can't believe you're saying that to me. Because something's got to give, because our kids are going to fail left and right (SG3, T, 280).

Lynn seemed to have gotten past that panic stage, "It's not that they're going to, they already are. I mean, they're already failing. It's not that it's a big new surprise that they're just starting now... and we're rushing too fast and we don't have choices" (SG3, L, 281). She went on to share ideas from a course she is taking on differentiated instruction, something reminiscent of the researcher's early experiences with 'individualized instruction.' Lynn had faced the facts and was willing to keep trying to make things better.

Finally, attention was centered on the second task, or pair of tasks, to be analyzed, also dealing with inequalities. These tasks were from *Algebra* (McConnell et al., 1990) lesson 6-7. The introduction, which was distributed to teachers, showed the solution of the linear inequality, $13x + 18 > 10x + 12$. First, $-10x$ was added to both sides to give $3x + 18 > 12$. Then -18 was added to both sides, leaving $3x > -6$. Multiplying by one-third then left a solution of $x > -2$. This was then graphed on a number line and checked using a two-step check. Two further examples were given, one of which was in a real-life context. The two tasks that teachers were asked to analyze were written on the board. They were one of the "Covering the Reading" tasks, $-48 + 10a < -8 + 20a$ (p. 292), and the Exploration, which said, "There are certain numbers which are less than their squares.

(a) Find one such number. (b) Find all such numbers" (McConnell et al., 1990, p. 295). The following discussion gives a taste of the analysis of cognitive demand that the teachers made of these two tasks. Some of the discussion deals with figuring out what the exploration is asking. First Lynn comments, "I'm thinking the cognitive demands are less,... this is more memorization on how to solve equations and the steps that they have to memorize to do that" (SG3, 286). Tess then pointed out, "It's not related to a real situation. There's nothing there...you're just plugging numbers and then you're graphing it with an open circle or a closed circle and whichever way the arrow goes....(SG3, 287-94). Lynn concurs, "If they memorize those steps, then that's like level one ...." (SG3, 295)

Tess then moves to the Exploration problem, asserting that it has a higher cognitive demand.

> T: Now, the next one, I think, is cognitive.
> L: Yeah.
> T: Wait a minute. Aren't all numbers less than their squares? It says there are certain numbers that are less than their squares. Aren't all numbers less than their squares?
> I: How about one-half?
> T: Oh, okay. I'm just thinking integers, sorry....
> K: That's the same thing that happens in the classroom with the kids, that's all they think about too....
> T: That has nothing to do with what this [earlier problem] is, though. To me that's totally cognitive. (SG3, 296-306).

There was some discussion about whether the solution of $x < x^2$ answers the given question. The researcher then asked,

> I: Now how do you think about that, just as a mathematician yourself, how do you think about that problem? ...
> T: Who cares? ...
> R: I don't know, I could think of all integers, and then it's mindless to me.
> I: I picture the two graphs, $y = x$ and $y = x^2$....

R: Right, it would be....That's wonderful, I never even thought about using the two graphs, but I think that's wonderful, because there'll be certain portions between zero and 1 that will be less than...
L: That's here, right here [looking at her calculator].
I: And I think students who have worked with calculators will do it like that (snaps). They just have a whole different view.
T: Or they'd look at tables. They'd look at the graphs or they'd look at the table. My kids will look at tables, they love those tables.
R: Now I see that as a really cool question!
L: Me too, to think of graphs or a table, I didn't...
R: Yeah, I didn't at first... (SG3, 316-332).

The teachers now see the Exploration in a completely different light. So the researcher brings up the connection she is hoping they have made. Lynn had experience teaching with the *Algebra* book.

I: Now what I want us to think about is, in these two contexts with these two various kinds of cognitive load, what happens when we introduce calculators to the problem? Does this [-48 + 10a < -8 + 20a as it was presented in lesson 6-7] look like it was designed to be solved by calculators?
R: No.
T: No.
I: No. What happens when kids bring calculators into the picture?
L: ...Well, I think it just totally takes away. I think their objective here is to teach the skills to solve the equation. Once you let the calculator do that [-48 + 10a < -8 + 20a], your whole objective of the lesson is done. They don't have to know anything except how to punch a button. Here, [the male and female doctors] ... the objective is totally different, the calculator use has to be there, or the graphs and tables, have to be there in order to solve them. These, they don't. They're two totally different objectives.
R: Well, this right here [*Algebra*], you're going to have to set up your curriculum to fit the calculators. There's no way, you have to make up extra problems...
L: See, and I taught this and we didn't use calculators. We had graphing calculators and I got them out one or two days a year.
R: Yeah, but now think about what she said about using the graphing calculator. How many people would actually saw that, the two graphs, and understood that, and said, wow?
L: They didn't, you're right.
T: But I don't see how to get the kids from that to that [lesson 6-7 introduction to Exploration], or this to this.

R: I'm sure there's more steps here, than just that [a copy machine malfunction had given them three copies of the introduction rather than the whole lesson].
L: There's not [speaking from experience].
T: No, there's not.
L: This is what I'm going to tell you happens in the classroom—we don't do those [the Explorations].
I: Didn't you have one or two kids that needed to be challenged? And you wouldn't suggest they do that?
L: No, because the one or two kids that needed to be challenged were in the higher class, and they still didn't do those. (SG3, 348-363)

There ensued some emotional discussion about teaching loads and who teaches the "higher" classes. Then the researcher returned to what happens when calculators are introduced, and finally to what that meant we should say in our AUGC tool.

I: Okay. But let's think about what we want to say to teachers who are going to use calculators in terms of the tasks that they ask students to do.
L: I think they need to understand what are your objectives. And I think that was addressed in one of these lessons....
R: No, but how can I use a calculator and enhance my understanding? That's what I'm saying.
L: That's better, to enhance my understanding.
T: But if they don't know how to use it basically, they can't get to that exploration process. There's got to be a point where, if you teach this whole lesson and you don't use a calculator, and then all of a sudden you bring out this exploration question, the kids aren't going to think about tables and graphs. They're not going to have an idea. And they're going to think about—do the same thing I did—and think about just the integers. They're not even going to think about negatives, they're just going to think about whole numbers...(SG3, 399-411).

The researcher pulled this together by saying, "Okay, so the implications for our AUGC tool, are that we need to add that stuff about asking the question, 'When does the calculator enhance understanding or is it even necessary?'" (SG3, I, 427)

Comparing with Another Tool

The researcher then handed out the last item for discussion in this session, an article by Branca, Breedlove and King (1992), reporting on a list of questions that a group of middle school teachers had developed to help themselves and others think about using four-function calculators in their mathematics classes. She also handed out a list of the questions along with one elucidating quote. This document is found in Appendix P. The questions themselves are listed here for reference.

1. Does the calculator allow the students to get closer to mathematical concepts being presented?
2. Will the use of the calculator in a mathematics activity increase student confidence and persistence?
3. Could the concept be taught with an inductive approach?
4. Would the use of the calculator facilitate the study of real-life applications?
5. Will using the calculator allow assessment to be focused on relevant instructional objectives?

After some time to read the questions, Lynn commented that our 'first attempt' did not contain anything similar to question 4, about real-life applications. The researcher suggested that might be because of the tasks the group studied, but Tess asserted that it was because the curricular materials they teach from always used real-life situations. For whatever reason, because other teachers might not use such curricula, it was suggested that a shareable tool might need such a consideration. Then Lynn said, "The other one is five. And I think that's maybe what we were trying to word—will using the calculator allow assessment to be focused on relevant educational objectives. And I think that's maybe what I was trying to word" (SG3, L, 453). To check her understanding of Lynn's comment, the researcher ties question 5 to the earlier discussion, "So in other words,

'What is it that we're trying to see if they understand here, is it exponential functions, or is it can they multiply 8 times 8?'" (SG3, I, 454)

The continuing discussion again returned to concerns for the students they taught.

T: ... the whole thing is, we have to meet our students where they are to a certain point, and take them where they are, and some of our kids are low.
L: That just needs to be addressed all over the place. That needs to be addressed.
T: This is, number two, I think, and we've talked about number 2, I think that is so important. Will the use of the calculator in the mathematics activity increase student confidence and persistence? They don't think they can do math without a calculator, what are we teaching our kids? They can do it. We've stopped them from doing it, because we've given them the calculator wrongly.
I: Do you think so?
T: I'm sorry, Marcia, but when my second grader has a calculator in his hand, and he still doesn't know how to do some stuff, that's wrong.... I don't care if he's going to have a calculator when he gets older. I want him to get a high school diploma. And with a high school diploma, he has to know his basics, without that calculator in his hand (SG3, 466-472).

Tess's impassioned reply seemed to indicate that although she could value the use of calculators in teaching the mathematics she taught on the high school level, Tess still could not imagine how calculators could possibly do anything but harm in an elementary classroom. The passion of her reply reminded the researcher of Yvette's accusation that her students had been disabled by using calculators in elementary grades. She wished Yvette was there to respond to Tess. Lynn reminded the group that we had already discussed the problem of elementary teachers not understanding enough mathematics, so things calmed down and the researcher was able to make a suggestion for the AUGC tool,

I: So, on any level, again, we have to think about what is the objective of the lesson. If the objective is just to get answers, then why are we bothering?
T: Exactly.

I: And maybe we should state that we're specifically thinking about middle school, high school, as we write this up, and somebody else might want to adapt it for elementary. I mean, we realize that there are different things that they might do. So that might be something that we need to put... But even for our purposes, for the objectives that we have on the secondary level, there are times when we don't want them to use calculators.

T: Well, I think they need to be able to think about mathematics. And I think that the cognitive part of thinking about mathematics comes from thinking through some problems, without just pushing the buttons (SG3, 474-477).

Tess was able to agree, and admitted that she needed to get home because she was stressed out by the fight that had occurred in her room just at the end of the day. So the Calculator Use Logs were pulled out to decide what to consciously watch for until the final session. Rob indicated that his use of the log had gotten too mechanical. Lynn reported that she found that her lessons since Session 2 had actually involved at least procedures with connections.

R: As I'm going through this, I'm like, all right, yeah, we do this, we do this, we do this, we don't do this, all right, big deal, but what are some of the other categories that I can get in here, that maybe I should be focusing on? ...

I: Okay. So now stop those, and pick out ... something that you really think you need to think hard about.

T: How about the assessment issue? We really hit that a lot.

L: I was thinking that too.

T: And really look about and seeing if the kids are using it the way we want them to use. I mean, are we really getting the objectives out of our lessons that we want to, using the calculator?

R: Can I pick my own categories? Is that a problem? I think I'm going to do that....

L: I looked at, and I just wrote some of the things down, looking at the four steps, like the memorizing the task, without procedures, with, and doing math. I went through, and most of it is kind of procedures with connections. It's just—I looked each day at the lesson and how the kids were using it, and I'm like, you know, some of them really were making connections with it, and actually probably some of them were even in this category

I: Doing math, yes.

L: I didn't see with the quadratic functions that they were working with, none of them, because of the context of the curriculum, were mindless tasks. They actually had to use it. So that's kind of all I did.
I: That's fine! So that's what I wanted, an indication of what you're seeing in the requirements of the task that you're asking the kids to do.
T: So it's all right for us to see what we want to focus on in our own classroom, then pick that.
I: Yes. But I would like you to either do stuff from our tool, or from these questions, so that we can ... in the next session, ... do sort of an evaluation of it (SG3, 505-521).

The researcher closed with a preview of what the final study group session would be planned around. Teacher reflections on this session are given in Appendix N. It is evident that one part of the discussion that made an impact on teacher thinking was that elementary and secondary might have different appropriate uses for calculators. It is also clear that these teachers were able to recognize the difference in cognitive demand between tasks, and that different objectives required different tasks. Some teachers were beginning to understand that calculator use need not be mindless, but elementary use was still a sore point, at least for Tess. Teachers had also concluded quite forcefully that the curriculum they were teaching from had "built in" the real-world problems that others saw as benefits of using calculators. Teachers did not have to make up cognitively demanding problems to use the calculators with because their curriculum materials provided them.

Researcher Plans for Study Group Session 4

The researcher was forced to be content that at least this group of secondary school teachers had conceded that maybe teaching with calculators would be different in the elementary school, so elementary teachers would need to create their own tool to

judge appropriate use. Session 4 would be given over to honing our own barely emerging AUGC tool.

Prior Observations and Interviews

The final round of classroom observations took place just before the final study group session because of a school break in the district. Yvette's class was working on understanding reflections and transformations and how they are represented in the coordinate plane. She began class by checking the students' understanding of reflection by having one of them come to the front and be her reflection. After she showed the class where the imaginary mirror was based on the floor, she asked them questions about where her 'reflection' should be – how far from her, how far from the mirror – and how he should move as she moved (she had selected the tallest volunteer, so the students were amused and interested). They seemed to understand the concept of reflection, but when the 'mirror' was placed on the coordinate plane, especially when it was not on an axis, the students had difficulty. Yvette said it was a problem with negative numbers, and that was also the reason they used calculators in this lesson. After class she talked about the use of calculators to work with negative numbers, and the students' difficulty with new vocabulary in spite of her many schemes to help them. More than once during class students had asked what the book was saying. Yvette said, "The book is not hard to understand, they just don't comprehend what it's saying" (Y3ObsInt, 28).

The researcher asked why she had chosen to work on parts 1 and 2 of the investigation together, and she said, "Because … if they can do 1 and 2, they can do 4 through 8" (Y3ObsInt, 56). At the end of the interview Yvette asked what she had missed in Session 3 because of her emergency with her daughter. The researcher did a quick

summary of the session, finally coming to the Branca questions. As she explained the group's noticing that question 4 did not match what we had in our AUGC tool, Yvette interrupted, "Because it was in the book already, it was built in" (Y3ObsInt, 86). She also agreed that the curriculum they were using had the real-life applications built in.

Tess, on the other hand, had her students working entirely in groups once their warm-up was finished. For a warm-up she used the Checkpoint of a previous lesson which served as a reminder of what they would be investigating in their groups. The warm-up dealt with the quadratic model for the height of an object as a function of time under the influence of gravity. In the new lesson they would explore the effect of the individual parameters a, b, and c, in the equation $y = ax^2 + bx + c$ on the graph of the equation. Because Tess decided to collect the warm-up instead of discuss it, there was little to be observed of the students' understanding of the quadratic model for height as a function of time. Likewise, perhaps because she was not feeling well, Tess decided to have students work until the end of the period rather than share what they had learned about the effects of the parameters.

After class, Tess emphasized that this lesson used the calculators appropriately – "as a tool" (T3ObsInt, 2). The students already knew about parabolas from the gravity model, so now they had the need to graph many parabolas quickly so that they could observe patterns. This was what the calculator was good for. Overall, Tess thought the tasks they use in Integrated Mathematics 2 have high cognitive demand. She says,

> I do feel that when I use calculators with this class… —remember we had those categories? For the most part we're at three or four. We're on the higher level thinking order, I think, with the Integrated 2. I love teaching Integrated 2 (T3ObsInt, 32).

Again, Tess expressed her concern with what she saw as a "district attitude" toward using calculators instead of mastering the basics. She thought it conflicted with the requirement for three years of high school math. The researcher protested that she was not sure that was really the overall attitude, but Tess said,

> When I teach Math Investigations, which is our elective math class, I am told to teach them how to do it on the calculator. Well, that doesn't help them, because then when we do stuff in Integrated 1 and Integrated 2, and Integrated 3—because we expect these kids to go all the way up to Integrated 3—we expect them to be able to do this stuff, like fractions, work with fractions (T3ObsInt, 44).

The reason she gave for not wanting to just have them use calculators is that they have no way to check themselves to see if they pushed the wrong buttons. This seemed to be a back-down from what she had expressed in Session 3 about students being taught with the calculator so that they would have an idea about how to explore a problem such as "which numbers are less than their squares" with graphs. Or perhaps it was a return to a well-ingrained belief. She had not stopped thinking about it, however. She added,

> You know, with the calculator, there's times when I sit there and I think about it—I think the calculator belongs in a classroom, I think it belongs in the classroom more than it ever has before, but it's to take the students to deeper thinking. We still have to get back to basics, we still have to teach them how to think for themselves. And I think that the calculator can help them think for themselves, if they're given the right instructions way back when (T3ObsInt, 52).

Later Tess reiterated the same point when the researcher asked if she had anything to add.

> I think that's the whole thing, the calculator should be used as a tool, not as a crutch. And if it's something that we expect them to know in the fourth or fifth or sixth or seventh grade, they shouldn't be using the calculator for it. Mathematically, we don't need those calculators. We need those calculators for something we did like today in my group, in my last class. About taking it deeper....So what happens when you don't have a "b" or a "c"? They can graph all those equations and they can see what

> happens to that function, and then it's like oh, wow! That's what I see the use of a calculator for. The calculator's not a crutch. (T3ObsInt, 86-88).

After listening to Tess worry about students who skip Integrated Mathematics 1 not having a background in quadratics, the researcher observed Lynn's eighth-grade class working on quadratics of various types. Most of the class was spent examining the equation, table and graph of a ball thrown straight up. First they built the table, then looking at the table, described what the graph will look like. After graphing the equation, they trace it to compare to the table. They even looked at second differences to judge if the graph was quadratic. It was hard to believe that these might be students that Tess thought had no background in quadratics! They were able to answer questions using both table and graph. Then they compared to another equation that had a constant term.

Lynn's main worry for her students was that they do not have access to graphing calculators at home, so homework had to be spread out and done in class. She said, "I don't ask them to do that much at home, because then they'll just get further behind" (L3ObsInt, 12). Asked if this problem has given her any thoughts about the AUGC tool the group was working on, she said,

> I don't see that the tool would really be able to address that, because, even looking back, I know the problem is time. They want to move them forward at a faster pace, and in order to do that, they have to use the graphing calculator. But even if we had this tool in place, ... I'm going to look at that one that says "do the students have the basic understanding of the material before they get the calculator out," and my answer is no. And there's nothing I can do about it. I don't have the time to go back and reteach it, and it's not the purpose of this lesson. It would be nice if they had the background...(L3ObsInt, 14)

With all the work that was observed, Lynn felt that her students "still don't know what a quadratic is" (L3ObsInt, 16). When the researcher protested that they had been working with them for quite awhile, the following interchange ensued.

> L: And today was really the first day that we used the graphs, looked at tracing for a quadratic, so actually, they'll spend three or four days on this. So the more that they get better and better—they'll get there, yes, and they'll understand that. But my frustration is they don't understand what that quadratic is doing.... Does that make sense? What that equation is doing? If they started out and said $2x^2 - 3$, they would see that much easier.
>
> I: Yeah, but when they're using those simple ones, then they're just using the area model, and they're not doing the graphs of those?
>
> L: No, they're not doing the graphs of those, they're just doing the area model. Yes.
>
> I: So it's almost as if they're two totally different things.
>
> L: Exactly how they see that. Two totally separate things. (L3ObsInt, 18-24).

Not unlike her colleagues in the study group, Lynn seemed to be having disagreements with the curriculum materials she was using, or she did not understand the intent of the lessons on quadratics. It was not clear what Lynn meant by "what the equation is doing," but it seemed an important part of her own understanding of quadratics.

Karl's class was taking a quiz, so the conversation afterwards centered on the content they had been studying, and the quiz. One of the quiz problems, dealing with exponents had been designed so it was calculator neutral, asking students to rewrite an expression with a rational exponent in as many ways as possible. A calculator might have served as a check, but students had to come up with the different ways. Karl was aware that there was a difference in what students can do with the calculators, even if they own one. He noted,

> the calculator won't necessarily give you a fraction, for example, it gives you a decimal. And I say, well, you know, that's the same as the fraction—if you know what the decimal equivalent is to the fraction, so

> you want to go into your calculator and push the button that tells you to change it from a decimal to a fraction, and then you'll see that it's a fraction. But here again, the knowledge of what the calculator will do for you is important. So if you don't know certain things are in there, that you are able to use, then you're out of luck (K3ObsInt, 36).

This led to further discussion about how teacher and students learn about calculators.

> K: As a teacher – and I'm not in this category, necessarily – you have to be very knowledgeable of what the calculator will do for you, because there are certain things that you want the kids to do on their own without using the calculator, and some kids are so smart, are so keen, they'll go to the catalog, and then ha, ha! (laughs)
> I: They'll find something that you don't know!
> K: And you say, no, no, you're not supposed to use the catalog button, this is what I want you to know! (laughs) So it's imperative in these calculator courses that the teacher be up on what's available, and what you're going to select out of there to use for the kids to actually learn what they're supposed to learn.
> I: Right. So do you sometimes sort of actually eavesdrop on them to learn things that they know about calculators that you don't know?
> K: Oh, I'm telling you, truthfully, that's how I learned a lot of it, yes, I went and took a calculator course, and then I come back and I'd be explaining away, on the overhead—this is how you do it, I learned this in the course, you know. But Mr. Yancey, if you do this, it's a lot quicker! Okay, your turn. Show me what you're doing (K3ObsInt, 38-43).

Overall, Karl was also concerned about his students. He wanted to find the best ways to reach them. The conversation concluded this way,

> K: Yeah, I think that's important, that's good, because like we say, some kids are visual learners, some kids have to think and hear it and know it to do it. That's one thing I try to do, is try to say, well, kid doesn't get it one way, what can I do to explain it to him differently to get him to understand it? I always try to do that.
> I: Yeah, and sometimes the calculators allow you to take a different route with some students that you couldn't have done when you had to sketch everything by hand.
> K: That's right. Technology does—let's face it—it does add to your ability to reach those kids, if they want to be reached! (K3ObsInt, 60-62).

Rob's class was not using calculators at all. They were working with three-dimensional geometrical shapes and how they are named. Rob kept the class moving, and they were busy right up until the bell rang. They did top, front, right drawings, and some of them learned that three drawings was not always enough to show everything. He had decided not to have students build the objects, but rather to use figures built by another class. The researcher asked him about that decision. He said it was partly about time and partly about not helping the students understand the mathematics any better. Rob also said he made decisions about calculators that way. He said,

> When does the calculator become a tool to use effectively, and when does it become a hindrance to the overall lesson itself? If I can explain something visually or computationally without using the calculator, where a calculator would just bog us down into mindless calculations, that's when you have to decide that the calculator's not beneficial. It could be used as another tool maybe visually, but it's going to take too much time or too much effort to get the students somewhere, where they could get so easily somewhere else. I don't know of any other way to say that (R3ObsInt, 46).

All five of these teachers had concerns about their students and how they learned. Some had concerns about the district policies, state test, and the curricular materials they were using. Some of these concerns were disagreements, and some due to inexperience, but all had roots in the work they were doing with students and whether they felt successful in that work. In these concerns they agreed with the teachers of all levels in Szombathelyi's (2001) study.

Researcher Decisions about Session 4

There was no decision remaining about the goal of the final session. Obviously the work on the AUGC tool was uppermost. The group needed to make final changes and

also to make a commitment that would show ownership of their work. The decision to be made was how that could be done.

In her notes on planning Session 4, the researcher returned to two ideas that had been postponed earlier. One was what was meant by "concepts first" and the other a desire to make connections back to the tasks of Session 1, or to the Task Sort tasks. She wrote of priorities for Session 4:

> First Priorities – to finalize the Tool
> – to evaluate its usefulness
> Second priorities – to make connections to the sort tasks,
> or to tasks from Session 1
> – to clear up the meaning of "concepts first"
> Perhaps "concepts first" can be approached through one of the sort tasks? That may deal with both second priorities, and the evaluation. I should probably select one of the tasks on which there was least agreement – C, J, Q, R, S, T (PlS4, ¶ 2, 3, 5).

Task S was chosen, partly because it dealt with a linear relationship, but also because it had been sorted by different teachers as "never," "sometimes," and "always" appropriate for calculator use. It would provide a base for a broad discussion. Task S is included in the agenda for the session, in Appendix Q, and will be included in the discussion of Session 4.

The researcher decided that perhaps the definitive sign of ownership would be if these teachers, all relatively new to the district, would be willing to have their AUGC tool shared with all the district's secondary and middle school mathematics teachers. So that was planned as the 'ultimate question.'

A new 'second attempt' version of the AUGC tool was then prepared, incorporating the changes that had been suggested in Session 3. It was emailed to teachers, but this time they were all on break, so no feedback was received. The full

document can be found in Appendix Q, and the working form in Table 24 in the section on Session 4.

Session 4

The final two-hour meeting of the study group had four teachers in attendance. Yvette was back, but Tess could not attend, and Lynn had to leave early. The first order of business was to review changes in the AUGC tool, and to make final adjustments. The researcher introduced it this way:

> What I've done since last time is, I tried to incorporate the things that were mentioned while we were talking in the group looking at the first draft, and then I changed the wording somewhat. And so all of the changes that I made, I put in italics, ... things that I added I put in italics. So what I want you to think about is, is there something missing, is there too much here, could we collapse any of this together? And then I put that note at the top, because we had that discussion about it might be actually different when you're thinking about it in terms of elementary students. So we'll just make a disclaimer, that we're doing this for middle and high school (S4, 1).

Evaluating the AUGC Tool

The document being examined, also called the 'second attempt,' is found in Table 25. Numbers have been added to the items to simplify references. Lynn registered her support for question 5 by brokering an example that really puzzled her. Her students often were assuming that they could use their calculators to multiply x by x, but she was puzzled by the fact that their calculators often had the value 10 for x. She knew that she had to get her students to understand that the calculator was giving the value of $x*x$ for only a specific value of x, but why was it always 10? A number of suggestions were given, not including what was probably the actual reason, that when the calculator graphs

a function in a window with $-10 \le x \le 10$, the last value of x is 10. Teachers did demonstrate to each other various ways to leave values in x, for example, by tracing. But this was definitely a limitation of the calculator that students and teachers needed to understand!

Note: This tool is intended for middle school and high school teachers. Elementary teachers may need to consider other questions.

1. *What are students doing with calculators*? Are *calculators* used to (as)
   Check answers?
   Arithmetic aid?
   Reference *to look up information, such as* $\sqrt{7}$?
   Answer finder, *such as finding intersections or intercepts*?
   Presentation helper, *to help explain ideas to others*?
   Visualizer?

2. *Which* students legitimately need calculators, due to *their IEPs*?

3. Beyond *learning to push calculator buttons*, are students asked to interpret answers once they get them? *Is conceptual understanding assessed with and without calculators?*

4. *Is the calculator necessary for the lesson's objectives? Are students asked for a deeper understanding – more than numerical answers?*

5. *Do students understand the limitations of the calculator?* Is *the calculator students are using* more complex than is needed for the task?

6. Are students making connections between what they do with the calculator and the problem they are trying to solve. *This opportunity is a major benefit of using calculators with real-life problems.*

Table 25: *Second Attempt at an AUGC tool, prepared for Session 4*

Rob supported the use of IEP in question 2 rather than "disability" as the earlier version said. Several teachers voiced the opinion that IEPs were sometimes not based on a real understanding of what calculators can and cannot do to help children learn mathematics. However, our AUGC tool could not address that. But then Rob brought up

another idea that sparked interest in the others, and resulted in another question for the final AUGC tool:

R: Something that I've always—maybe because of my ignorance or lack of knowledge for the calculators—do the teachers really know what things the calculators are used for?
L: No.
Y: No.
R: If I'm doing a specific lesson, and because of my ignorance of the calculators, where I could use the calculator as a tool, I wonder what I'm missing out.
L: I'll use myself as an example. We don't have 83s but we have 82s, and there's a lot of stuff it does that I don't know. But I also don't teach any of that, so I haven't really played around with it enough to know. But some of my kids, that are more advanced in computer programming and different things like that, play around with it and do things. So there probably is a lack or ignorance for teachers as far as the calculator and different things.
R: I wonder if my limitation on the calculator is affecting the students. If I could be better serving the students—and I'm trying to make a question out of this. I'm wondering if...
I: Do I have enough knowledge to...
R: ...use it effectively in a lesson? Or use the calculators effectively in a lesson. Or background in calculators, or enough knowledge over the content.
L: Maybe content. Do I have enough knowledge over this content to see if the calculator's beneficial?...
R: Oh, no, I want knowledge of the calculator.
I: Okay. Do I have enough knowledge of the calculator to make it useful for the content?
L: I think that's a good question to add.
Y: Yeah. Because these things do an awful lot (S4, 42-61).

Karl continued the discussion by pointing out that teachers seldom get enough professional development with new materials or equipment. This seemed to hit a sore spot with the group, and after some discussion the researcher suggested, "So what do you think might be a good way to ... I'm going to add this, 'how might I get more information about the calculators?' as kind of a follow-up on the questions we're going to put down"

(S4, 76). This concern with professional development for use of calculators also concurred with the concern of teachers in Szombathelyi (2001).

Even a course on using technology to teach secondary school mathematics that the researcher had taught at the University took a hit because "that really never touched on a lot of these calculators that we use here in school. I had to buy a [TI-] 92, and that's all I used, was 92, Geometer's Sketchpad..." (S4, R, 81). After clarifying what the TI-92 was, Lynn's question, referring back to the Tool's questions 4 and 5, was "is this necessary, though, at this level?" (S4, 88) Then Lynn made a connection to her students who couldn't deal with negative numbers. In some ways, that made calculators necessary. She said,

> If we're looking at first and second differences as our goal for our lesson, it's not to teach you how to subtract negative numbers, so for the lesson, you need to use your calculator so you can understand what the first and second difference is and see if it's linear or quadratic. And then after school, or on your half-sheet, you can practice negative-number part of it (S4, 132).

After this, the interest of talking across schools about mathematics curriculum and calculator use was again expressed.

Testing the AUGC Tool on a Familiar Task

The researcher then moved the group on to the second priority – testing the AUGC tool by applying it to a task or content, and at the same time, tying back to one of the Task Sort tasks. She asked them to work individually on Task S, thinking specifically about the questions that were asked above it. The task and questions are quoted here from Appendix Q.

> Please work on the following task on your own for about 10 minutes. Think especially about the second part of the question, and about these questions:

- What concepts need to be understood before students use calculators for this task?
- What level task is this? Why?
- How might students approach this task? Is any way the "right" way?
- Does our Tool help decide whether calculators are appropriate for this task?

Task S
Solve for x : $5(x-2)-3(2x+1)=2+5x$ What does this tell you about the intercepts of the graph of $y = -15-6x$? Why?

After teachers worked for awhile, they were asked about prerequisite concepts. Yvette began (Lynn had left the meeting):

Y: I would think they need to know how to solve for x, without the use of the calculator. And the only thing would probably be the division, basically to check arithmetic in that portion of it.
I: So you're thinking they might want to use the calculator just doing the arithmetic.
Y: Yeah, the division, instead of having to do it longhand. But that very first part, I think they should know how to do the basic algebra first.
R: Starts with the distributive property, divided by a term.
Y: Yes.
K: Exactly. As a matter of fact, that's the quickest way, as far as I'm concerned, to do it. I wouldn't have even thought about trying to solve it...
I: Certainly for anybody who knows about algebra, it should be the easiest way.
K: For me. That's the point that I'm making, is that if you are well enough grounded in that, the calculator doesn't even come to mind as a tool to find the answer.
Y: And actually, only after I finished did I go back and graph it, because I said, oh, well, she said graph it, so I went back and graphed it, but you already knew what it should be, before you graphed it.
I: You knew what the x should be.
Y: And the y, because of the form—if you have a linear model, so you should also know what a linear model—what the numbers...
R: How to find your x and y-intercept.
I: Well, except that the first question only asks you to solve for x.
Y: Okay, but then you asked about intercepts (S4, 143-156).

Rob then gave a sort of stream-of-consciousness summary of trying to answer the question about the relationship of the two functions. He said, in part,

> Yeah, I was looking at these and saying, how do I correlate between the two? If I can solve, instead of just solving for x, but that was—the point was to solve for x, ... but if I left the x and the integer on one side, then it looks like an equation. Do I have an x-intercept at that point, if I can stop right there and say, okay, if that's equal to zero, and that was a graph, or a linear model, already I'd have an x-intercept. ... I was trying to correlate the two, and if I was a student, how would I go through that. And if I was a student asked this question, would I get the correlation between the two? (S4, 157)

Karl sees no correlation between the two, but Yvette says the correlation Rob found still depends on knowing the distributive property and combining like terms. The following rather lengthy discussion about which concepts are prerequisite must be allowed to speak for itself.

> I: Okay, now which comes first?
> Y: What do you mean?
> I: Before you were saying, okay, first of all they would have to know how to solve for x. Could they come at it from the other direction, understanding linear relationships?
> R: Do they need to understand linear relationships before they understand the distributive property, combining like terms, is that what you're asking? If we're going to get back to...
> I: Or just taking that particular thing, which would use the distributive property and combining like terms to solve...
> Y: They need to know the distributive property and combining like terms before they can solve for *x*.
> I: They do?
> K: Not necessarily.
> Y: Not necessarily, but that just makes sense.
> K: Not if they know the other.
> R: Not if they know how to solve...
> I: For example, what?
> K: If I know to make two equations out of that, and put it in y-equals, and I can graphically see my answer, or I know that I can look it up in the table. Maybe I was never taught the distributive property and combining like terms.
> I: Or, I never caught on to it.

K: That's right. But I do understand the other.
Y: Oh, you're saying graphing first?
I: I'm putting the left side [Y_1 = 5(x-2)-3(2x+1)] into the calculator, just the way it is, and the right side [Y_2 = 2+5x] into the calculator, just the way it is. Okay, I've got an intersection here, what about that intersection is going to tell me what *x* is?
K: Hm?
I: Here are the two equations, they intersect right there [pointing]. How am I going to interpret that as a solution to this equation?
K: Well, I would have to—you have two equations, you have to know that the point of intersection solves both equations. So that the *x*-value has to be able to work...
Y: So you're solving a system of equations, that's basically what that is.
I: Right.
K: Yes.
I: But will that allow me to write down what is the solution?
Y: Yes, it will allow you to get there.
K: If you can get the solution exactly off the graph itself, the intersection, you don't necessarily—without going to the table.
I: And this was not a very good choice—negative 15 over 6 is not going to be easy to find on the table.
K: I worked it down and put it into a decimal, so I could work with it easier. I wanted to know—do I have an even decimal to work with, or...
I: And then, by going into the table, you mentioned going into the table, and looking for where the two functions have the same value—and that actually happens at what, negative 2.5?
K: But really I was just verifying what I already figured out, without using that.
I: Sure, but how do we think about it? We think about it exactly what [Yvette] said, that you first have to know about the distributive property, and you have to know all this stuff, because that's the way we've already learned to do it (S4, 162-192).

At this point Karl made an impromptu oral report on the Branca (1992) article that was handed out in Session 3. He suggested that rather than insisting on paper and pencil first, maybe we should teach paper and pencil only for so long, and then introduce calculator methods for those who aren't catching on, which will keep the large middle group of students together much longer in mathematics. But Rob replied,

> Yes, but I don't understand why—if that's the logic, then why can't you just start with the calculator, and get everybody up to speed at the same

> time, if you're just using the calculator. ... why can't we use the calculators as conceptual tools, like for graphing, to do something like this, where you have x and y-intercepts? I think that's what we're talking about, why can't you go the other way around, start with calculators, and use that to build that basis, build that understanding, if they can go on there and tinker with it themselves to build that understanding without us specifically saying okay, this is this (S4, 201-203).

But the gifted or talented, Karl thought, would need to be treated differently because they could handle both ways. Yvette and Rob weren't sure they agreed:

> K: I think that they're [the gifted and talented are] going to jump past the mainstream of everything, so we should be using the calculator with them on a higher level.
> Y: I don't know. I don't think anybody in algebra would see this [Task S] to do it the way you just did it.
> R: I would have never thought of it this way.
> Y: I would have never thought to do it that way. But I know that yeah, you can do it, but I don't know of a kid that would think that way.
> I: So, it might have to be suggested (S4, 206-210).

Trying to get back to the questions, the researcher asked, "So that's why I asked this question, is there any right way? And who decides what the right way is?" (S4, 214) Yvette responded, "There's no particular right way. I think you decide for yourself which way is easiest for you and best for you" (S4, 215). The researcher continued,

> What concept would they have to understand before they could take this and make a system of equations out of it, do you think? Would they have to understand the idea of a linear graph, and how it might be represented? Or do you think it would work just as well if one side of this was a quadratic and the other side was a linear? Is that the same concept? (S4, 218)

Again, Yvette replied, "That's the same, I think. All you're talking about is the intersection point being the solution. They're going to see linear models intersecting quadratics, and know that those are the solutions" (S4, 219).

Trying to press on to deeper analysis, the researcher suggested,

> Some students, when they get used to doing things on the calculator, will do something else with that. They'll say, okay, all I really need is the x. If I just want to know where the x is, if I can make one function out of it, I can get that intersection on the x-axis [where the value is much easier to find in the tables] (S4, 233).

Yvette noted that was essentially what we did in the comparison function, $y = -5x - 15$. But students might do it in another form. Suppose in Y_4 we enter $Y_1 - Y_2$? Karl quickly saw that was the same as moving the $2 + 5x$ to the other side of the equation and graphing. How can we know that is legitimate? What line is it? All questions that would allow connections between the various methods of solving. Yvette was troubled, and said,

> Y: ...do you think you're taking away some of their knowledge? Like having to know the distributive property, having to know to combine like terms?
> I: I don't know that I'm taking it away, because I don't think they have it yet.
> Y: I guess I'm just entrenched in old school.
> I: No, but now...
> Y: I see exactly what you're saying. I understand everything.
> I: It can be solved that way.
> Y: It can be solved exactly how you said, but if you do it your way, you're taking away, or they didn't have to use the distributive property or combining like terms.
> K: Well, then are we saying that we're teaching something—a different concept here? By using this task, we're teaching a different concept, other than usual.
> I: I think what I'm trying to say is that there may be students who would do it this way, rather than use the distributive property, even if they knew the distributive property. And I think that your discussion is, would I want to teach them that? Would I want to teach them how to do it that way, because then they're avoiding doing this other stuff? (S4, 242-250)

Further discussion ran the gamut of students' self-image, real-life problems as motivation, and showing work so teachers can assess whether students are making small computation errors or have misconceptions. The solution seemed to be to have some kind of balance

and to keep in mind the objectives of the lesson. Although Szombathelyi (2001) reported that her teachers had similar concerns about students missing some things, she did not report any examples in detail.

Did the AUGC Tool Help?

The researcher then asked about the AUGC tool, sparking the following exchange:

I: Okay, was this tool that we're creating here, helpful in trying to say whether or not calculators are appropriate for this problem?

K: Well, are we getting down to topics and where we put little asterisks after the copy "may use calculator" or "may not use calculator"? Is that where we're going with this?

I: I don't know, do you think that's where we should be going?

R: I thought we were just getting teachers, or people who were going to be looking at this, to just think about their own decision making for use of these questions. Because I think we're all going to interpret these questions a little bit differently.

I: Okay, so maybe this question, is the calculator appropriate for the lesson's objective...

R: Maybe it's not so much of interpreting the question, but they're going to interpret, hopefully, I think, what we're trying to do is get the teacher or the mentor to look at the questions that they're giving their students and discuss—I guess it's not for us to decide what is appropriate and what is not appropriate. Because they'll know their students better than we do. But I think the idea is for them just to take a look and think about what they're getting the calculators for. That's my understanding. Is that right?

I: That would be the way I would think about it. So I'm thinking, this question that we have about the lesson's objective—if the objective of this task is that they should be practicing the distributive property, then perhaps you don't want them to be using the calculator on this, because you're aware that they might do it this other way.

Y: But if you want them to think outside the box, then the calculator is necessary.

R: Or if the objective is more of finding x and y intercepts, or understanding x and y intercepts, then I think the calculator would be a great tool.

I: Right. So this question makes the teacher think about that at least, I guess, is what I'm trying to say. Is there any other question that you think might be applicable?

K: Okay, so we're thinking about whether the content that's being taught, the questions are guided towards the teacher on whether or not to use the calculator in that regard. Based on the content.
I: Right. And what's your goal with this content? ...
K: I think this may be the number one question—but under that, then we have to—there's these underlying factors of is the teacher himself able to teach that, or even have the knowledge that he was talking about to know it even exists, that technology exists. So how would I know how to answer that question if I did not know some of this other? (S4, 277-289)

In this last session teachers had been confronted with another way to look at a "standard" algebra problem. At least one was concerned about 'losing' knowledge, but others seemed more interested in having students understand how to solve a problem, even if they don't use "standard" methods. Again, the crucial matter is the objectives of the lesson, so that the task can be designed to meet them. Karl's discussion of the Branca article (1992) led Rob to question whether the logic behind "do it by hand first" was flawed. He began to argue for a more pragmatic approach – use whatever helped students understand.

The Ultimate Question

A short discussion resulted in the combining of questions 3 and 6, with a promise to email final copies to each teacher for final approval. The session ended with the researcher asking "the ultimate question."

I: Okay, there's just one more thing I want to ask. I hope you understand that I am going to be sharing what we've come up with here in my dissertation. But what I want to ask you is, if you would be willing to share this also with your colleagues that you teach with. And what would be a preferable way for you to do that? Would you like to be the ones who stand up and say, we did this, and we need your comments, or we'd like to get you started talking about this, or an option would be, I could say to Sonia, this is what we've produced, and she could present it anonymously and ask people to talk about it. What do you think about that?

R: I wouldn't mind presenting it. It doesn't really matter.
K: We're presenting it basically to our department, right?
I: Yeah. And I don't know how you might want to set that up, whether you'll each want to do that in your schools, or ...
K: I would just as soon put it on...
R: Yeah, I would too.
I: Rather than have somebody else try to interpret it (S4, 308-318).

The researcher took this response to be a sign of ownership and commitment to continue growing in this process. The real validation will be to get confirmation that the presentations actually took place. If Sonia would approve it as a professional development project for the district, it would have a better chance. Understandably, she wants to know what the teachers are going to say. It is hoped that she will meet with them to find out, since the researcher will have left the area.

Reflections on Session 4

In preparing for Session 4, the researcher had reviewed the teacher reflections from earlier sessions. She decided that new prompts that provided more opportunity for summarizing the experience would be appropriate for the final session. Those who either did not attend the final session, or left early, were asked to write their reflections as part of the final interview. The form given to the teachers is provided in Appendix R, and the responses of teachers are in Appendix N.

Researcher Plans for Final Interviews

Based on a reading of all available transcripts and of all field notes, the researcher prepared protocols for each teacher's final interview. Each protocol asked the teacher to review his or her own answers to some parts of the initial survey (see Appendix D). All responses of Part A of the survey were reviewed by all teachers, to see if they still agreed

with what they had said originally. Background information on the survey was not reviewed. The survey responses reviewed by all teachers included items 2 – 10, 20, 27, 32, 36, 42 – 54. In addition to these, each teacher might have been asked to clarify responses that the others were not asked about.

In the second stage of the final interview, teachers were asked to again sort the 20 tasks they had sorted prior to the study. The categories were again "I would not use this task," or, if they would use it, would calculators be appropriate for the task "never," "always," or "sometimes?" Teachers would be asked if they preferred to talk about the tasks as they sorted, or to sort first and talk later. If he or she did not talk about all tasks, each teacher was asked at least to explain the circumstances under which calculators would be appropriate for tasks in the "sometimes" category.

In the final part of the interview, each teacher was asked to clarify ideas expressed in previous interviews or sessions, and to give final concurrence to the final form of the group's AUGC tool. The questions planned for individual protocols for the final interviews are in a researcher's planning document provided in Appendix S. The final form of the tool will be presented in the last section of Chapter VI.

CHAPTER VI: PRODUCT

This chapter again looks at individual teachers, describing the broad strokes of the painting referred to in Chapter III, and in some cases identifying the brush that added a stroke. The chapter closes with a final look at the AUGC tool, the artifact which, at the time of the last study group session, reified the understanding of the group in regard to appropriate use of graphing calculators in teaching secondary mathematics.

End of Session Teacher Reflections

At the end of each session, teachers were asked to complete a short reflection sheet. After the first three sessions the prompts were the same, asking them for concerns about "appropriate use" of graphing calculators and for their intentions when they plan for such calculator use. They were prompted to say what, if anything, had been something they hadn't thought of before, and they were also given the opportunity to privately express disagreement with anything discussed in the session. Prompts for the fourth session reflection asked for more summary about the entire project and thoughts about the AUGC tool. The reflections for each teacher were kept together in a folder, so that each teacher could review what they had previously written before writing more.

The intent of the reflections was to give teachers a chance to slow down and think about how they were thinking about "appropriate use" of graphing calculators in mathematics teaching. The teachers were also aware, however, that their written reflections would be part of the data of this dissertation. The complete set of responses is

in Appendix N. Quotes in the following analysis are taken from the responses there, and S1, S2, S3, and S4 are used to denote references to the sessions, and teacher initials are used followed by prompt number to identify the quotes. For example, (Y2, S3) would be a quote from Yvette's response to prompt 2 following Session 3.

By making a chronological analysis of these teacher reflections, it was possible to patterns that indicated development of each teacher's understanding of appropriate use of graphing calculators. The most abrupt change noted was for Yvette. That is because Yvette missed Sessions 2 and 3, so her comments are separated by three classroom observations, discussions with her colleagues at school, and Session 4. Following Session 1, Yvette was most concerned with dependence of students on calculators – that they would use a calculator as a crutch. Her planning for calculator use would intend to "further" or "enhance student knowledge and provide quicker solutions after skills were learned" (Y2, S1). At the end of Session 4, Yvette says the most important thing she learned from the sessions was the "connections between paper and pencil concepts and calculator use" (Y1, S4). Rather than seeing the "use or not use" dichotomy prevalent in the early sessions, or her own 'reward' system for learning skills, Yvette had been influenced by Karl's argument for presenting both calculator methods and paper and pencil methods as complements of each other to maximize student understanding. Yvette said that as she planned next year, she would remember the first question of the AUGC tool, which provides a rich list of uses students make of calculators, only one of which is arithmetic aide. Yvette is ready to be open to the many ways her students could use calculators appropriately.

Karl's reflections did not show much change in his concerns. After Session 1 he was concerned that calculators not be used for arithmetic, but for "the intended purpose" (K1, S1). After Session 2 he indicated that he did not want calculators used for "non-problem use" (K1, S2), and following Session 3 Karl was concerned that students should only use calculators "when they can do it otherwise" (K1, S3). In response to the prompt asking for their intentions when planning for calculator use, however, Karl shows development from "enhance what they are learning" (K2, S1), to "using it in achieving the answer I couldn't get without it" (K2, S2), to "use them in addition to (as a help) the concepts I'm teaching" (K2, S3). It is interesting that Karl seems to contradict what he said in (K2, S2) by the response he made in (K1, S3).

Yet there is a marked change in the tone of what Karl said in (K2, S3) when he recognized calculators as a help in teaching concepts. Why that change took place might be related to his response to the prompt asking if anything was discussed that he had never thought of before. Karl said, "using them [calculators] to express something several different ways to allow the student to grasp the concept the best way" (K3, S3). Session 3 featured, among other things, a boring problem that became "wonderful" when viewed graphically. Is that what motivated his comment? Karl made excellent use of this new view allowing students to "grasp the concept the best way," in Session 4, when he was the one who suggested that students did not need to know the distributive rule if they knew how to graph both sides of the equation and turn it into a system of equations. Looking at Karl's reflections for Session 4, his claim is that he learned to ask himself whether calculator use is appropriate, to ask what purposes should be achieved, and to guard against students using calculators "under false pretenses" (K1, S4). The researcher

missed the chance to probe those "false pretenses," but it is clear that Karl will at least think about appropriate use as connected to the purposes one is intending to achieve. This is surely a more solid plan than "enhance what they are learning" (K2, S1)! In fact, in response to the prompt asking what he would remember next fall, Karl said, "Do I need to investigate the uses of the calculator to help me in my presentation of the material?" (K5, S4). For a man who started out admitting he avoided calculators, this is a huge step! Karl sees the possibility. Will he seek to form a community with others who want to pursue that possibility?

Lynn reports that her concerns are "they are using [calculators] too soon ...before they have the concept mastered ..." (L1, S1), "Do they understand what the calculator is actually doing?" (L1, S2), and "...Can the calculator enhance the learning?" (L1, S3). Note the shift in (L1, S3) from what the students are doing or not doing to the question of what the calculator can do. Not that Lynn believed that the calculator would single-handedly help students understand. Implicit in this change was her own acceptance of responsibility to plan use of the calculator that will help students understand. She also repeated in (L1, S3) her concern that students need to understand what the calculator is doing, so coupled with the quote above, the implication is that she was ready to accept the challenge and knew she was justified in continuing to demand that students understand, rather than letting the calculator 'do their thinking.'

The same shift in focus from student-centered concerns to calculator/mathematics concerns was evident in Lynn's responses to prompt two. She moved from an intent to save time or follow the curriculum (L2, S1) to plans for student mastering of skills (L2, S2), to finally to asking "What are the lesson's objectives?" (L2, S3). She received the

message in Session 3 that calculator use needed to be tied to the lesson objectives. Perhaps this arose from applying cognitive demand analysis to tasks from familiar curricula, or two contrasting curricula. Since the lesson objectives are her domain as teacher, again Lynn appears to be accepting responsibility to plan for appropriate calculator use. In reflecting on what she learned in the discussions, Lynn said she was comforted that other teachers struggle with questions of calculator use, and expressed the desire to follow "best practices" for calculator use in her classroom (L1, S4). Will this motivate Lynn to meet with other teachers and continue her inquiry into the appropriate use of graphing calculators?

In writing, Tess had much less to say than she did in speaking! Following Session 1 she said her concerns were "using calculators for the wrong reasons" (T1, S1). In following reflections, she said her concerns "have not changed" (T1, S2; T1, S3). In her response to prompt two, however, there is a similar shift seen in the other teachers. She begins somewhat vaguely wanting to "take students further" (T2, S1), or "to a higher level" (T2, S2), but after Session 3, Tess said, "make sure the calculator is being used to get to my objectives" (T2, S3). Very similarly to Lynn, Tess accepted responsibility to plan for calculator use by tying it to her objectives.

Unlike Karl, for whom the clue to his change lay in his response to prompt three, Lynn and Tess do not mention anything about objectives in response to prompt three. However, both of them mentioned levels of tasks/thinking in reply to prompt three after Session 2. Tess, in fact, said something she had never thought about before was "levels of tasks and how the calculator can change that; how are they/what are they getting out of my questioning and the calculator use?" (T3, S2). This may be the clue to the change in

Tess's focus from what students do (or should be allowed to do) with calculators) to what she does to plan for appropriate calculator use. In reflecting on the entire project, Tess said she learned that "middle/high school teachers want the same thing from a calculator. They want students to use it as a tool and not a math 'Bible'" (T1, S4). Tess also said that for next year she will remember that "I want to make sure that I'm using the calculator as a tool. I want to make sure that they are thinking mathematically" (T5, S4). "Using the calculator as a tool" was Tess's way of saying it should not be used as a black box with no understanding. She is still talking like a technology gatekeeper, but Tess is no longer talking about criteria for what they have to know first.

Rob did not attend Session 1, but following Session 2 he expressed concern for knowing what operations students were using calculators for, and that the teacher needed to decide (R1, S2). After Session 3 Rob's concern is for "assessment – when are calculators enhancing cognitive development of students?" (R1, S3). In responding to prompt two after Session 2, Rob was already concerned about what the student could get out of calculator use (R2, S2), and he adds to that after Session 3, "What can they use the calculator for? When is it inappropriate to use calculators?" (R2, S3). Rob seems to begin where the others ended, looking at how calculators can help students. However, he does not get stuck there. He is interested in the breadth of what students can do with calculators, and then he turns the question around – "When is it inappropriate to use calculators?" – indicating that he thinks that might be the smaller category. Under things he hadn't thought of before Rob noted that adding calculators to a curriculum added work for the teacher (R3, S3). When he was asked about this later, he mentioned both learning to use the calculator effectively and taking time to plan for appropriate use. In his

reflection on the whole program, Rob said he learned to ask "Are we randomly using calculators, or can we strengthen our students' understanding, cognitively, with calculator usage?" (R1, S4).

Final Interviews

Each of the five teachers seemed at least more aware of the ways students use calculators. They all seemed more open to the possibility that further understanding of their students could be gained by consciously planning calculator use. They also acknowledged the importance of examining the objectives of curricular tasks, especially those designed to use calculators. In all cases their teaching materials called for the use of calculators, and their district colleagues supported that, but some of these teachers either did not completely understand the goals of such calculator use, or they feared that others did not implement it properly. On the other hand, the session discussions had brought to their attention some of the benefits of continuity in teaching with calculators, for example, that students develop useful habits of mind with regard to using calculators as tools to solve problems.

When they began, these teachers had been having a hard time figuring out how to teach mathematics to the students in their classes. Lynn had fought using calculators because there was so much calculation that her students could not do. Tess at first seemed to put a wholesale ban on calculators for any work on 'basic math.' Yvette didn't want to handicap students by making them calculator dependent. Even Rob, who seemed to relish the opportunities for exploration their calculators warranted, was frustrated that his students were not able to deal with distribution and like terms. Karl at the beginning had

been more compliant, letting the curriculum dictate that he was teaching a "calculator course."

Also, most of them had a sense that students needed some prerequisite before it made sense to use calculators. In the final interviews, the researcher tried hard to get clarification of what that prerequisite was for each teacher. In the final session it had been concluded that even the prerequisites depended on the lesson objectives, as in the work done on Task S. Tess, however, who missed Session 4, commented in her final interview, "Task S, ... I'd expect them to do the first part of solving for the *x* by hand because they can't do that on a calculator" (FinInt T, 184). What Tess had missed was that the prerequisite of "solving for *x*" depended on whether you intended students to practice the distributive property and like terms, or if you wanted them to be able to find a value for *x* that satisfied the linear equation.

As a result of the final study group session, the tool for teachers to use in making decisions about appropriate use of graphing calculators was revised once more, emailed to participants, and then verified by each teacher in his or her final interview. At no time was this seen as a "finished" product. As evidenced by Rob's comment in the final session, "what we're trying to do is get the teacher or the mentor to look at the questions that they're giving their students and discuss—I guess it's not for us to decide what is appropriate and what is not appropriate. Because they'll know their students better than we do. But I think the idea is for them just to take a look and think about what they're getting the calculators for" (SG4, 282).

Similarly, when the group talked about sharing their tool with the rest of the mathematics teachers in the district, they wanted to present it themselves rather than have

someone else interpret it. The goal is discussion, not dictating a set of topics that must be or must not be taught with calculators. The key to understanding when it is appropriate to use calculators is two-fold according to these teachers: (1) understanding the objective of your lesson and the effect of calculators on the cognitive demand of mathematical tasks, and (2) understanding the thinking of the students you are working with.

By struggling with the issues the tool could or could not address, by struggling with wording for such things as assessment and sufficient knowledge about calculators, and by finally agreeing that this tool could not address what was done with calculators in elementary schools, this group of teachers not only put into writing something that they themselves saw as useful, but that they also stated they would like to actively engage in presenting to others rather than merely letting it be read. It is hoped that the researcher's presentation of their tool matches their expectations that it serve as a catalyst for further discussion.

The following sections look at changes in individual teacher thinking as expressed in their repeated task sort, and the changes they acknowledged when reviewing their survey responses. The final section of this chapter reviews the development of the tool and argues for its future utility.

Revisiting the Task Sort

Teachers showed considerable change in how they thought about appropriate calculator use for the specific tasks of the Task Sort between the beginning and end of the study. In Table 26, the task ratings for each teacher are sorted, and the means, medians and modes are calculated. A visual comparison can easily be made of changes in teacher thinking about the appropriateness of calculator use with these particular tasks. The table

also shows enhanced comparison by shading of the ratings that are modal values. The shading shows that although the mean use indices increased, the

Note that each set of results below is ordered by Teacher Use Mean

		PRE Sort, Sorted by Use levels					POST Sort, sorted by Use levels				
task count		Y	T	R	K	L	K	R	L	T	Y
1		0	1	1	0	2	0	1	2	1	2
2		1	1	1	1	2	0	1	2	1	2
3		1	1	1	1	2	1	1	2	1	2
4		1	1	1	1	2	1	1	2	2	2
5		1	1	1	1	2	1	1	2	2	2
6		1	1	1	2	2	1	2	2	2	2
7		1	1	1	2	2	1	2	2	2	2
8		1	2	1	2	2	1	2	2	2	2
9		1	2	2	2	3	2	2	2	2	3
10	Median	2	2	2	2	3	2	2	2	2	3
11	Line	2	2	2	2	3	2	2	2	3	3
12		2	2	2	2	3	2	2	2	3	3
13		2	2	2	3	3	2	2	2	3	3
14		2	2	2	3	3	3	2	2	3	3
15		2	2	2	3	3	3	3	2	3	3
16		2	2	2	3	3	3	3	2	3	3
17		2	2	2	3	3	3	3	2	3	3
18		2	2	3	3	3	3	3	3	3	3
19		3	2	3	3	3	3	3	3	3	3
20		3	3	3	3	3	3	3	3	3	3
Mode =		Y	T	R	K	L	K	R	L	T	Y
	Median	2	2	2	2	3	2	2	2	2.5	3
	Mode	2	2	2	3	3	3	2	2	3	3
	Mean	1.6	1.7	1.75	2.1	2.6	1.85	2.05	2.15	2.35	2.6
		Pre-Study Sort					Post-Study Sort				

Table 26: *Pre- and post- use levels for tasks sorted by teacher, teachers ordered by mean use, shaded modal values*

mode of the ratings moved toward the "sometimes" part of the spectrum, perhaps indicating more thoughtfulness in considering the possible contexts of calculator use. It is clear by comparing the range of teachers' use means for the post-study sort to the corresponding range for the pre-study sort that the overall sense of appropriateness of calculators for the tasks rose. Care must be taken in using this table to compare individual teachers, however, since the order of teachers is determined by their mean use index, it is not the same for both sorts.

Keeping this in mind, we can see that three teachers changed their thinking in the direction of more use, and two in the direction of less use. Yvette, for example, went from being least likely to find calculator use appropriate (mean use = 1.6), to being most likely to find calculator use appropriate (mean use = 2.6) at the end of the study. Both the median and the mode of her ratings increased. Karl, on the other hand, went from mean use of 2.1 to 1.85 – from the second most likely to the least likely to find calculator use appropriate. However, neither Karl's median rating value, nor his mode rating value changed. Tess, like Yvette, changed in the post-study sort to being much more likely to consider calculator use appropriate for the tasks that were sorted, and both her median and mode rating values increased. Rob joined those (all but Karl) whose number of "never appropriate" ratings fell, and his mean use increased due to more "always" ratings. Lynn, however, changed the rating for many tasks from "always" to "sometimes," commenting, "why I'm doing a lot of 'sometimes' is because it would make a difference if it's the first time they saw it or if it's something that you're looking for 'do they understand mastery of that?'" (FinInt L, 206).

Taking another view of changes for individual teachers, Table 27 compares the pre-study and post-study ratings, by teacher, for each task, indicating whether the post-study sort showed the teacher thinking the calculator appropriateness was the same, more than, or less than they indicated in the pre-study sort. This view easily shows that in the post-study task sort, Karl and Rob rated over half of the tasks as they had rated them in the pre-study sort, while Lynn changed over half of her task ratings to levels of less appropriateness of calculator use. Tess and Yvette both changed over half of their task ratings to levels of more appropriateness of calculator use. The direction of Lynn's

Karl 34 yr Pre	Post	calculator appropriate S	M	L	Lynn 6 yr Pre	Post	calculator appropriate S	M	L	Task	Tess 3 yr Pre	Post	calculator appropriate S	M	L	Yvette 1.5 yr Pre	Post	calculator appropriate S	M	L	Rob 6 wk Pre	Post	calculator appropriate S	M	L
3	3	x			3	2			x	A	2	3		x		2	2	x			2	3		x	
2	1			x	3	2			x	B	2	3		x		2	3		x		2	2	x		
3	3	x			3	3	x			C	1	2		x		1	3		x		2	2	x		
3	3	x			3	3	x			D	2	3		x		2	3		x		2	2	x		
3	3	x			2	2	x			E	2	3		x		1	2		x		2	3		x	
2	2	x			3	2			x	F	2	2	x			2	2	x			1	2		x	
1	1	x			2	2	x			G	1	1	x			1	2		x		1	1	x		
3	3	x			2	2	x			H	2	2	x			3	3	x			3	1			x
3	3	x			2	3		x		I	3	3	x			3	3	x			3	3	x		
3	1			x	3	2			x	J	2	3		x		1	3		x		1	2		x	
2	2	x			2	2	x			K	1	1	x			2	2	x			2	2	x		
1	2		x		2	2	x			L	1	2		x		2	3		x		1	1	x		
2	2	x			3	2			x	M	2	3		x		2	3		x		3	3	x		
3	0			x	3	2			x	N	2	3		x		2	3		x		2	3		x	
1	1	x			3	2			x	O	1	2		x		1	2		x		1	1	x		
2	2	x			2	2	x			P	1	1	x			1	3		x		1	1	x		
1	1	x			3	2			x	Q	2	2	x			1	2		x		2	2	x		
0	0	x			3	2			x	R	2	3		x		0	2		x		2	2	x		
2	1			x	3	2			x	S	1	2		x		2	3		x		1	2		x	
2	3		x		3	2			x	T	2	3		x		1	3		x		1	3		x	
mean use		70	10	20	mean use		40	5	55		mean use		35	65	0	mean use		25	75	0	mean use		60	35	5
2.1	1.9	%	%	%	2.7	2.2	%	%	%		1.7	2.4	%	%	%	1.6	2.6	%	%	%	1.8	2.1	%	%	%
*2.2	*2.1	S	M	L			S	M	L				S	M	L	*1.7		S	M	L			S	M	L

Table 27 : *Comparison of direction of changes from pre-study to post-study task sorts*

Note: S = task rated Same on both sorts, M = post-study sort found calculator use more appropriate for task than did pre-study sort, L = post-study sort found calculator use less appropriate for task than did pre-study sort. Teachers ordered by total teaching experience.
Mean use is average of ratings in each sort: 0 = not use; 1 = calculator never appropriate; 2 = calculator sometimes appropriate; 3 = calculator always appropriate. *denotes "mean use" if tasks rated "0" are left out of the average. Percent is calculated out of 20 tasks rated.

changes were contrary to the others' because of the way she thought about the tasks each time. In the pre-study sort, she summed up her thinking by saying, "I don't think that it's appropriate at most times to say it's never used, because some kids need it" (L, PrI-TS1, 230). When looking at the tasks the first time, Lynn was thinking of that group of students. In the final task sort, Lynn was instead focusing on the different purposes the tasks might have, in either introducing a concept or in assessing whether students have mastered it. For this reason many of her "always appropriate [for some students]" ratings were redefined as "sometimes appropriate [for all students]" ratings.

But perhaps the strongest indicator that the teachers had come closer together in their thinking about the appropriateness of calculator use for the tasks of the task sort comes from comparing the pair-wise agreements from the pre- and post-study sorts. In Table 28 we see that the total number of pair agreements increased from pre- to post-study sorts, and the variance across pairs decreased considerably. This phenomenon of reduced variance is best illustrated in the comparison of the box plots in Figure 5.

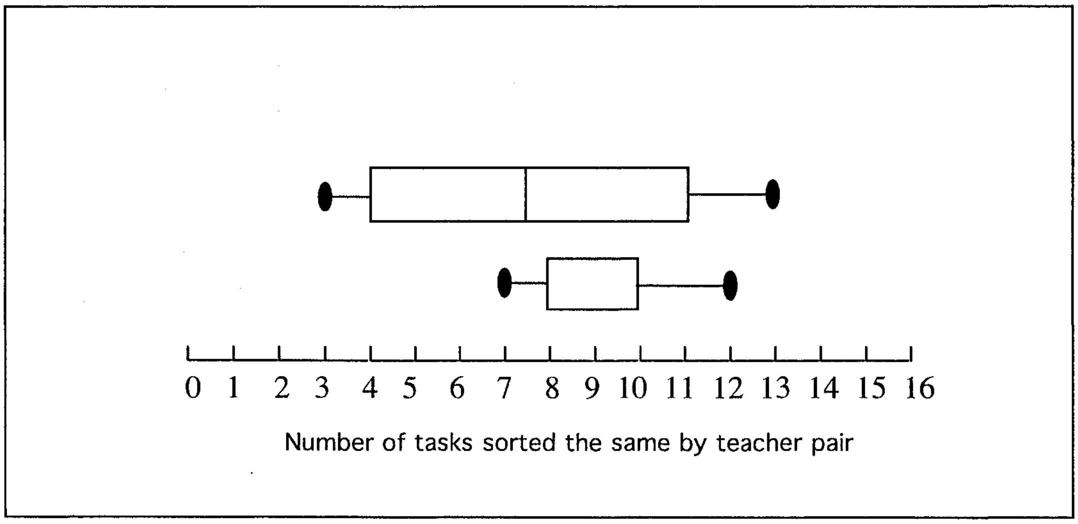


Figure 5 : *Boxplots of teacher pair agreement totals for pre-study task sort (top) and post-study task sort (bottom).*

Pre-Study Sort Agreements, by Task											Post-Study Sort Agreements, by Task									
R	R	K	T	K	K	K	L	L	L		R	L	T	K	L	L	K	K	R	K
T	Y	Y	Y	T	R	L	R	Y	T	**Task**	T	Y	Y	T	T	R	R	L	Y	Y
1	1	0	1	0	0	0	1	1	1	A	1	1	0	1	0	0	1	0	0	0
1	1	1	1	1	1	0	0	0	0	B	0	0	1	0	0	1	0	0	0	0
0	0	0	1	0	0	1	0	0	0	C	1	1	0	0	0	0	0	1	0	1
1	1	0	1	0	0	1	0	0	0	D	0	1	1	1	1	0	0	1	0	1
1	0	0	0	0	0	0	1	0	1	E	1	1	0	1	0	0	1	0	0	0
0	0	1	1	1	0	0	0	0	0	F	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	0	0	0	0	G	1	1	0	1	0	0	1	0	0	0
0	1	1	0	0	1	0	0	0	1	H	0	0	0	0	1	0	0	0	0	1
1	1	1	1	1	1	0	0	0	0	I	1	1	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	0	0	0	J	0	0	1	0	0	1	0	0	0	0
0	1	1	0	0	1	1	1	1	0	K	0	1	0	0	0	1	1	1	1	1
1	0	0	0	1	1	0	0	1	0	L	0	0	0	1	1	0	0	1	0	0
0	0	1	1	1	0	0	1	0	0	M	1	0	1	0	0	0	0	1	1	0
1	1	0	1	0	0	1	0	0	0	N	1	0	1	0	0	0	0	0	1	0
1	1	1	1	1	1	0	0	0	0	O	0	1	1	0	1	0	1	0	0	0
1	1	0	1	0	0	1	0	0	0	P	1	0	0	0	0	0	0	1	0	0
1	0	1	0	0	0	0	0	0	0	Q	1	1	1	0	1	1	0	0	1	0
1	0	1	0	0	0	0	0	0	0	R	0	1	0	0	0	1	0	0	1	0
1	0	1	0	0	0	0	0	0	0	S	1	0	0	0	1	1	0	0	0	0
0	1	0	0	1	0	0	0	0	0	T	1	0	1	1	0	0	1	0	1	1
13	12	11	11	8	7	6	4	3	3	Agreement totals	12	11	10	8	8	8	8	8	8	7

X
Z represents a pair of teachers

1 means the pair agrees on sort category of task
0 means pair disagrees on sort category of task

Table 28 : *Agreement of teacher pairs on sort categories for tasks in Pre-study and Post-study Task Sorts*

lc

Comparing the pre-study task sort with the post-study task sort also allows some insight into the tasks themselves. Table 29 gives the total number of teachers, for each task, who in the final sort felt that calculators would be more, less, or equally appropriate than they indicated in the initial task sort. The topic of each task is also given. We can see that only Task K did not inspire any change of thinking in the teachers. This task had students build or visualize a series of triangles made of toothpicks. Most teachers said they did not see how a calculator could be useful for such a problem, but thought students might need an arithmetic aide to help find the pattern. Only Tess said a calculator would

Task topic	Task	Total Same approp	Total More approp	Total Less approp
Graphing quadratic equation	A	2	2	1
Graphing linear equation	B	1	2	2
sin 30°	C	3	2	0
sin 15°	D	3	2	0
$x^2 + b$ exploration	E	2	3	0
Cookies - optimization	F	3	1	1
Decimal equivalent of 3/4	G	4	1	0
Decimal equivalent of 156/195	H	4	0	1
Linear regression	I	4	1	0
Slope by tracing points	J	0	3	2
Toothpick triangles -pattern	K	5	0	0
Comparing % and fraction	L	2	3	0
Copy machine – best deal	M	2	2	1
Population - exponential	N	0	3	2
Division of 29 by 7	O	2	2	1
Arithmetic order of operations	P	4	1	0
Probability	Q	3	1	1
Rational Function zeroes	R	2	2	1
Algebra order of operations	S	0	3	2
Predicting from linear graph	T	0	4	1

Table 29 : *Number of teachers rating each task more, less, or the same level of appropriateness for use with calculators in the post-study sort compared to pre-study*

never be appropriate for Task K, and she did not change her mind, saying, "because it's the toothpick thing and I don't know how you do that on a calculator so I never do that on a calculator" (FinInt T, 183). It is not clear whether Tess was referring to her own solving of the problem of finding a pattern, or to how she would expect students to approach it, or whether she would not allow calculators even as arithmetic aide. Generalizations cannot be made from the information in Table 29, because it is not always possible to say why teachers changed their ratings. For instance, in all but one of the cases in which only one teacher changed a rating from that of the initial task sort, the teacher who changed picked a rating that said calculators were more appropriate for the task than they had indicated in the initial sort. But Rob is the one teacher who went from 'always appropriate' to 'never appropriate' for finding the decimal equivalent of 156/195. He commented as he rated it

in the final interview, that he would like to see how his students might "take that apart" (FinIntFNR, ¶ 28). This seemed to be a matter of momentary curiosity rather than changing his mind about using calculators on any such tasks.

This comment lead the researcher to conjecture that when teachers look at the tasks, they imagine them being used with a particular set of students, and they respond according to their expectations for that group. This conjecture is supported by other teacher comments. For example, when giving reasons for putting tasks in the "always appropriate" pile, Yvette included "if the students had difficulty" (FinInt Y, 84). Tess also commented as she was giving her final agreement on the tool, "if I look at it a month or two from now, it might be totally different" (FinInt T, 217), implying that students' needs and teacher experiences can change teachers' views over time. Some of the change could possibly be attributed to greater familiarity with the tasks, particularly those that were also used in session work. To try to assess this possibility, the researcher asked some of the teachers during the second observation interviews if the activities done in class were similar to any of the tasks they had sorted at the beginning of the study. Each of the teachers asked this question replied that they could not remember any of the sorted tasks. However, in the final sort, the three teachers who had been present when Task S was discussed all remembered it.

Another relationship to notice in Table 29 is that, of the four tasks on which all teachers changed their minds, two (J and S) had been used for activities during study group sessions. And for those, more than half of the teachers in the final sort rated them as more appropriate for calculator use than they had indicated for those tasks in the initial sort.

Reviewing Selected Survey Responses

By asking teachers to review during the final interview the responses they made to the initial survey, the researcher was able to ascertain which responses, if any, teachers changed because their thinking had changed, and which items they were merely reading differently the second time through. Appendix T gives pre-study and post-study responses for the items that were reviewed. Here a closer examination of each teacher's changes in response is made, while noting which response changes teachers said specifically indicated a change in thinking.

In Part B, Karl (K) made four changes in his survey responses, shown in Table 30. When explaining his change on item 3, Karl reiterated something he said in Session 4. He said, "At some point we must introduce the calculators….No, I don't think

Mark one "0" for each statement		SD	D	N	A	SA
*3. Students need to demonstrate proficiency in using mathematical procedures before doing any similar work using calculators.	Pre	0	R	0	KLTY	0
	Post	0	KRY	0	LT	0
20. Students learn mathematics by the personal building of mathematical understanding.	Pre	0	K	R	L	TY
	Post	0	0	0	RKL	TY
*42. Students are dependent on calculators when they come to my class.	Pre	0	K	R	LTY	0
	Post	0	0	RK	LTY	0
*48. Students should not be allowed to use calculators until they have mastered concepts.	Pre	0	RLTY	0	K	0
	Post	0	RLTYK	0	0	0

Table 30: *Karl's changed responses to some survey items*
Note: * denotes items on which teacher acknowledges change in thinking

there's any set principle that always says, 'Well, I must do 1 through 10 before I can do 11'" (FinIntK, 37). Instead, Karl now saw the need for balance, working with calculators and without calculators for different parts of a lesson – "bring them hand-in-hand together" (FinIntK, 37). On item 5, which is similar to 3 but deals with concepts rather than procedures, Karl still strongly disagreed. He said, "They've got to be taught it

without the calculator first.... Show them the big picture" (FinIntK, 51). The researcher called Karl's attention back to an earlier discussion of calculator use to solve systems of equations in his algebra 2 class. Karl said that use was appropriate because they already had understood the concept of intersections of graphs being the common solution. As the researcher tried to press Karl on how they might have learned that concept before they used calculators, he abruptly went on, "I wonder what I was thinking of on #9?" (FinIntK, 74).

Karl changed his response to item 20, but this was a case that he didn't really consider a change, only a more careful reading of the statement. Karl wavered on possibly changing his response to item 32, which asserted that students who use calculators lose their basic computational skills. He said, "I really think that if a student has learned their skills, they don't lose them," but also, "If they're not reinforced on a regular basis they don't maintain their efficiency" (FinIntK, 92-94). So he decided to remain in agreement. On item 48 Karl acknowledged that he had to change his response to be consistent with what he said earlier about doing calculators together with non-calculator activities. And finally he nudged his response on item 42 from disagree to neutral, saying, "the more I work with these kids, the more I think they were weaned on calculators" (FinIntK, 134).

Recalling that item 48 was Fleener's 'mastery' item, it is reasonable to conclude that Karl's admitted need to change his response to item 48 indicates a significant change in his thinking. In Fleener's study (1995b), this would have indicated a change in Karl's willingness to use calculators in the classroom. However, Karl's situation seems rather to be one in which he used the calculators because his curriculum called for them, he

learned about the calculators from his students, he joined a study group doing inquiry on the appropriate use of calculators, and now he has come to the decision that his beliefs have to change in order to be consistent with what he is doing. His impromptu report on the Branca article in Session 4 also supports this interpretation. This support is not diminished by Karl's explanation of why he spoke up when he did in the sessions, "Well, see, I like to be the devil's advocate, that's me. Anything to stir the pot, see. Whether I agree with it or not, I throw it out there" (FinIntK, 321). The researcher asked this question in the final interview because in the first session Karl seemed very diplomatic when bringing up something that seemed to disagree with what the others said, but in later sessions he seemed to be more abrupt with such statements. His response in the final interview seems to indicate an explanation of style rather than of actual disagreement with what he was saying.

Lynn (L) changed only three of her responses in Part B of the survey. These are shown in Table 31. Two of them she changed because she hadn't understood the statement the first time through. But on item 32, why she changed is clear in her statement, "I think I really should say that I would agree with that... once they don't have a full understanding of the computations that, just by using the calculator, they are losing those" (FinInt L, 170). It seems that Lynn was speaking of a lost opportunity

Mark one "0" for each statement		SD	D	N	A	SA
*32. Using calculators will cause students to lose basic computational skills.	Pre	0	RT	L	KY	0
	Post	0	RT	0	KYL	0
43. Mathematics is fixed and unchanging.	Pre	T	RKY	L	0	0
	Post	T	RKYL	0	0	0
47. A world with different mathematical truths is impossible.	Pre	0	KL	RT	Y	0
	Post	0	KR	T	YL	0

Table 31: *Lynn's changed responses to some survey items*
Note: * denotes items on which teacher acknowledges change in thinking

rather than loss of actual skills. A distinction was pointed out in Session 4 after Lynn left, when Yvette questioned whether solving the linear equation by graphing a system of equations didn't "take away" their knowledge of distribution and like terms. The distinction was between knowledge that students had not yet been exposed to or that they had been exposed to and not understood, and knowledge that they had actually possessed and then lost or had "taken away" (SG4, 242-250).

Lynn also spoke during the final interview about knowledge prerequisite to using calculators. In reviewing her response to item 48, she said, "I do not think they have to have it [concepts] mastered ... but I think they need to have some idea of what's happening ... some understanding" (FinInt L, 183). Lynn gave two examples of what sort of understanding she meant. The first had to do with graphing. She reasoned that if students had not had some experience with making a grid and putting numbers on it and then using the numbers to plot points that had two numbers as coordinates, they would see graphing on the calculator as some kind of magic rather than something they could understand (FinInt L, 66). She also argued that students need basic understanding in order to know when a calculator is giving them wrong answers. But she does not tie this "understanding" that she is looking for to "basic facts." In fact, she specifically noted that some of her students "don't have any comprehension of multiplication facts .. but they could... get some information on how to graph things, how to read the x and y intercepts and put the equation in, change the window" (FinInt L, 22). She saw this as getting them beyond what they would have been able to do with by-hand calculations.

Tess (T) registered seven items with new responses. These are shown in Table 32. On item 2 Tess specifically noted that she meant that changes were needed for what she

called "strand math" – the single-subject courses such as algebra or geometry – as opposed to the integrated curriculum that she teaches. She saw strand math as being too

Mark one "0" for each statement		SD	D	N	A	SA
*2. Incorporating calculators into teaching requires changing the types of problems assigned.	Pre	0	KLTY	C	0	0
	Post	0	KL	0	RY	T
*5. Students should be allowed to use calculators even before they understand the underlying concepts.	Pre	K	LTY	R	0	0
	Post	K	L	R	Y	T
9. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *less* important.	Pre	0	RY	0	KLT	0
	Post	0	RYT	0	KL	0
36. I am well-informed about the Michigan High School Proficiency Test for Mathematics (1998) **state mathematics test** for high school students.	Pre	0	KLT	R	Y	0
	Post	0	KL	R	Y	T
46. Mathematics is essentially hierarchical and cumulative.	Pre	0	R	T	KLY	0
	Post	0	R	0	KLYT	0
53. Using calculators in the teaching of mathematics results in greater student understanding of concepts.	Pre	0	T	RY	KL	0
	Post	0	0	RY	KLT	0
*54. Using calculators in the teaching of mathematics encourages a more active, conjecturing approach to the learning of mathematics.	Pre	0	L	0	RKTY	0
	Post	0	L	0	RKY	T

Table 32: *Tess's changed responses to some survey items*
Note: * denotes items on which teacher acknowledges change in thinking

algorithmic or "button pushing," with too little connection to real-world problems (FinInt T, 42). She did not change her response to item 3, but in talking about it she clarified a little more what she sees as prerequisite to calculator use. "They have to know the terminology. They have to know what we're asking. And I think that's a definition type of thing more than maybe the mathematical stuff" (FinInt T, 46). Tess also still agreed that you don't have to change *what* is taught with calculators, but rather the *way* it is taught.

On item 5, Tess jumped from not agreeing to strongly agreeing, "because ... the calculator being used properly as a tool can help them understand" (FinInt T, 52). No explanation was given for the change in item 9, but it might have been like Karl's "What was I thinking…?" Tess's item 36 change was explained by attendance at a professional development session on the MEAP. Like Lynn, Tess had not understood item 46. When

given the idea that "cumulative" meant it built on itself, she changed away from neutral. Finally, on items 53 and 54, Tess thought she had not read 53 correctly, and felt that she wanted to be stronger in her agreement on item 54.

Table 33 shows the changes Yvette (Y) made as she reviewed her responses to Part B of the survey. Her final interview was the shortest, but the changes she acknowledged were perhaps the most dramatic. In item 2 she gave an example of using a calculator to generate data rather than just giving students a list. She seems to be speaking more of possibilities than requirement of change, however. Pressed a little further, Yvette brought up the example of Task S from Session 4, and the many different ways we "solved" the problem, when she had only been taught to do it algebraically. When she came to Task S in the Task Sort, Yvette said, "This is the one that was the eye-opener for me...I'm serious" (FinInt Y, 98-100). In changing her

Mark one "0" for each statement		**SD**	**D**	**N**	**A**	**SA**
*2. Incorporating calculators into teaching requires changing the types of problems assigned.	Pre	0	KLTY	R	0	0
	Post	0	KL	0	RY	T
*3. Students need to demonstrate proficiency in using mathematical procedures before doing any similar work using calculators.	Pre	0	R	0	KLTY	0
	Post	0	KRY	0	LT	0
*5. Students should be allowed to use calculators even before they understand the underlying concepts.	Pre	K	LTY	R	0	0
	Post	K	L	R	Y	T
10. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *more* important.	Pre	0	Y	R	KLT	0
	Post	0	0	R	KLTY	0
44. Mathematical ideas are constructed by human minds.	Pre	0	0	RY	KLT	0
	Post	0	Y	R	KLT	0
51. The major value of calculators in mathematics classes is to save time from performing computations.	Pre	0	RKLT	0	Y	0
	Post	0	RKLTY	0	0	0

Table 33: *Yvette's changed responses to some survey items*
Note: * denotes items on which teacher acknowledges change in thinking

response to item 3, Yvette used the same term Lynn had used in reviewing item 48. She said, "they would not need to demonstrate proficiency but just some type of

understanding" (FinInt Y, 34). When she acknowledged her need to change her response to item 5, Yvette said, "Now I'm going to say agree because as stated earlier, in the 4th session we looked at solving an equation for x and you can utilize systems of equations solving for x without even knowing that you're doing it" (FinInt Y, 36). The researcher clarified that she meant without knowing how to do the manipulative method. In explaining her change in item 10, Yvette mentioned mathematics such as computer graphics, transformations, and others that have become even more interesting because of computer games and simulations (FinInt Y, 38).

For item 44, Yvette had a hard time expressing why she changed. Finally she decided it was the word "constructed" that she didn't like, "I think that they're discovered" (FinInt Y, 66). And finally, she said she had misread item 51, saying, "I would say the major value of calculators is being able to interpret data differently or solve problems differently. It gives students variety in learning styles" (FinInt Y, 68).

Table 34 gives Rob's few response changes. Note that in each case, Rob was moving from a neutral stance to taking a stand. Rob had been specifically asked about his seeming neutrality. He indicated that it was not so much neutrality as being able to see both sides to almost any situation (FinIntFN R, ¶2). To justify his change on item 2, Rob seemed to be looking at possibilities of change rather than requirements, because he gave an example from his classroom that had been discussed before – the student exploration of matrices that occurred when his class was studying systems of equations.

For item 20, Rob made the change away from neutrality acceptable to himself by stipulating that he interpreted "mathematical understanding" as "foundational" (FinIntFN R, ¶12). On item 47, Rob asked for clarification of meaning of "another world." Since

the discussion had already included a Poincaré disk, the researcher suggested that a person living on a Poincaré disk might be considered to be living in another world. With

Mark one "0" for each statement		SD	D	N	A	SA
*2. Incorporating calculators into teaching requires changing the types of problems assigned.	Pre	0	KLTY	R	0	0
	Post	0	KL	0	RY	T
20. Students learn mathematics by the personal building of mathematical understanding.	Pre	0	K	R	L	TY
	Post	0	0	0	RKL	TY
47. A world with different mathematical truths is impossible.	Pre	0	KL	RT	Y	0
	Post	0	KR	T	YL	0
*50. I am confident in my ability to teach mathematics using calculators.	Pre	0	0	R	KLTY	0
	Post	0	0	0	KLTYR	0

Table 34: *Rob's changed responses to some survey items*
Note: * denotes items on which teacher acknowledges change in thinking

that explanation Rob changed his neutral response to 'disagree' (FinIntFN R, ¶17). And finally, Rob's change in item 50 reflected his gain in confidence not only of being able to identify what his students were doing with calculators, but also that he could plan the calculator use he intended (FinIntFN R, ¶20). Rob saw his neutrality as a good thing – seeing both sides of everything meant he was better able to understand the various ways his students approached problems.

At the end of the study, the final interviews portray a group of teachers who have shown some ability to change. Lynn now sees herself as cautiously using calculators to allow students to do math beyond what their by-hand skills would allow. Tess still bans calculators for most work on 'basic math,' but the clarification is that she wants students to have enough understanding of the operations the calculator is taking over so that they can interpret what the calculator tells them is the answer. Lynn shares this concern of Tess's. On the other hand, Tess does recognize the advantages of appropriate calculator use, and understands how to keep the cognitive demand of a task at a level that makes students think.

Yvette is just beginning to see calculators as a possible way to help students overcome the handicap of lack of by-hand skills. She is cautious though, and shares the concern for "basic understanding" of Lynn and Tess. She also worries about still assuring skills of algebraic manipulation, whose lack she sees as a handicap for higher mathematics. Rob continues to relish the opportunities for exploration the calculators warrant, and is more confident in his ability to make appropriate choices for calculator use. Karl's early compliance has become a realization that teaching a "calculator course" has brought him to modify his beliefs about the prerequisites for calculator use. He does express concerns similar to Tess, Lynn and Yvette, wanting students to see the "big picture" first, but he also said, "there might be one or two people in our group that [are] not flexible at all" (FinInt K, 323). Rob's way of dealing with this concern seems to follow the logic he pointed out in Session 4. "Why can't you go the other way around, start with calculators, and use that to build ... understanding, if they can go on there and tinker with it themselves to build that understanding without us specifically saying okay, this is this" (SG4, 203). This way of arguing may well be brokered from the very course in technology-based methods that Rob had criticized earlier for not doing more with TI-83 calculators. In that course Rob had taken part in a class-wide debate that required students to argue both sides of the technology question, and now with his teaching colleagues he is showing his ability to see both sides.

What Will You Remember Next Year?

In the final interviews several teachers were asked what about the AUGC tool they would remember as they were planning lessons next school year. All that were asked said they would particularly remember the very first question – being aware of how their

students were using calculators. Does that mean that all the other parts of the tool were a waste of time? The researcher contends that this is not the case, and will argue this claim by reviewing the evolution of the tool.

The impetus for the title of this section, and the metaphor for the argument comes from the early discussion in the study group sessions of the problem of retention from eighth grade to ninth grade of students' knowledge of linear functions. This was a very passionate discussion, fraught with anxiety, but carefully worded to avoid any personal implications. Lynn went over many of the things they studied in eighth grade, and Yvette and Tess reported how students responded in ninth grade, and it didn't seem to have any other explanation than the students 'forgot everything.' In another study, what might be given more attention is what connections students were making in eighth grade between their understanding of the mathematical concepts they were learning and the terminology that was used to assess their knowledge in both eighth and ninth grades. In response to the researcher's question implying that it might be the *name* that was forgotten, both grade level teachers conceded that their textbooks used "rate of change" to introduce the concept, but that they told the students up front that "rate of change" was the same as slope, and then used the term "slope." This opens up another possibility besides "forgetting." To which term did the students attach whatever they learned from the mathematical activities? Or did they have no name for their understanding? And if so, how would a teacher assess the understanding? Tess insisted that students be able to recognize "slope" in a linear equation before they could use calculators. How did that connect with the understanding the students had gained by using calculators in eighth grade?

The difficulty the participants had in formulating a response to the question about what they will remember about the tool as they plan lessons next year may have a similar root. At this point they have some ideas to work with, but these are not firmly fixed to any terminology (recall the many ways of invoking the highest level of the Stein task analysis guide). What their reference to the first question calls to mind for teachers may be the fact of a long and rich list of things that students do with calculators, and recalling that length and richness may be enough for them to continue their growth. A review of the evolution of the final tool will elucidate this idea.

The beginning point is the final form of the tool, as verified in final interviews. This is found in Table 35, with numbers added for easy reference. As teachers plan their lessons (the context about which the question was addressed), in most cases they will be looking at the next sections of their textbook – perhaps the teacher's guide, if one is handy. They may be consulting the district curriculum guide to see which sections they should be teaching next and if they are on schedule. As they look at the textbook, at least some attention will be given to the tasks that students will be asked to work on. If the researcher's earlier conjecture is correct, when they look at the tasks, they will be thinking about a particular group of students. At the beginning of the year, that group may be a previous class, or for a new teacher, it may be either an ideal class they have always expected or a worst case class from their student teacher experiences. And as they think about that class, it seems highly appropriate for them to consider what these students will do with calculators.

This does not mean that they will not consider the other questions. Recall that in the development of the AUGC tool, the researcher decided to start with Zbiek's

categories precisely because the teachers did not have enough common prior experience to begin with cognitive demand of tasks. The researcher also did not have enough

Appropriate Use of Graphing Calculators: Questions to Consider
Note: This tool is intended for middle school and high school teachers. Elementary teachers may need to consider other questions. 1. What are students doing with calculators? Are calculators used to (as) Check answers? Arithmetic aid? Reference to look up information, such as $\sqrt{7}$? Answer finder, such as finding intersections or intercepts? Presentation helper, to help explain ideas to others? Visualizer? To make connections between representations, such as graphs and tables? 2. Which students legitimately need calculators, due to their IEPs? 3. Does the calculator help focus on the lesson's objectives rather than on computation? 4. Are students asked for a deeper understanding – more than numerical answers? Is conceptual understanding assessed with and without calculators? 5. Are students making connections between what they do with the calculator and the problem they are trying to solve? Are students asked to interpret answers once they get them? This opportunity is a major benefit of using calculators with real-life problems. 6. Do students understand the limitations of the calculator? Is the calculator students are using more complex than is needed for the task? 7. Do I have enough knowledge of the calculator to make it useful for the content I am teaching? How might I get more information about using the calculator with this content?

Table 35: *Final form of AUGC tool developed by study group*

common experience of teachers' classrooms. This is analogous to any teacher at the beginning of the school year. Thus, the availability of the long rich list of possibilities is an appropriate starting place from which the need for other questions will arise, as it did

in the study group sessions. Teachers' eye-opening experiences with Zbiek's list led to formulation of the first question.

One of the immediate issues that arose was "who should be using calculators, and for what?" Our response was to incorporate that concern into the second question, which reminds teachers that there is no decision about use or no use to be made in some cases. That lack of decision, however, does not absolve the teacher from considering Question 1 for those students.

Question 3 was not worded in final form until Session 3, when Lynn recognized that one of the Branca questions said what she had been trying to put into words for the group. The group concurred. It earned third position in the list by being the next step in considering the original issue, "who should be using calculators, and for what?" If a student did not have blanket permission to use a calculator, what is another criteria to allow it for some students? Lynn had brought up this idea quite early, in explaining why she felt justified in using calculators when students did not have the by-hand skills to proceed in daily work otherwise. Lynn noted that these students do learn the new material. Tess supported that idea with her own report of students doing well in Integrated Mathematics 2 in spite of poor by-hand skills.

Up to this point the underlying question has been the one that predominated the initial interviews with every teacher – should we allow students to use calculators or not? The impetus to go beyond these first three questions of the AUGC tool when planning for lessons next year will depend on the teacher's desire to actually plan for calculator use, rather than letting it happen (or not happen). The group developing the tool was compelled to go beyond the first three questions by trying to address issues that arose in

Session 1, some of which recurred at subsequent sessions. It was at this juncture that the group began to consider the mathematical activities that students could (and did) engage in, rather than focusing primarily on the deficiencies of the students themselves.

One discussion surfaced in every session, and raised emotions every time. That was the perception of some of these teachers that calculators had not been used properly in earlier grades. As the discussion developed, it was discovered that the perceived problem was not so much that the students coming into middle school did not have basic facts memorized (though that was a problem), but that many students did not understand what the operations of addition, subtraction, multiplication and division meant, or how to represent the operations. Two additions to the tool stemmed from this discussion. First, a recognition that secondary school teachers could not write a tool for elementary school teachers, which resulted in the disclaimer in the note at the top. Second, the first question under number 4 was added, "Are students asked for a deeper understanding – more than numerical answers?" to bring to the front the idea that calculators must not be predominantly treated as black boxes. Note that the wording "asked for a deeper understanding" developed after teachers had experience analyzing tasks using Stein's (2000) guide.

The issue of prerequisite knowledge for calculator use was another that was hotly discussed in the group. Much of the development of that discussion was given in the summary of teacher positions at the end of the study. The issue was not settled in terms of particular knowledge, but enough examples were brokered from classroom experiences to give a general conceptualization. The concern is for understanding of "what the calculator is doing," by which they do not mean what is taking place in the silicon chip, or even in

the hard-wire programming, but rather, "what mathematical principle is the calculator allowing us to apply to this problem situation?" The examples of plotting points and of interpreting intersections of graphs as solutions of systems of equations were apt, and helped the group reach a consensus on the second question under number 4, "Is conceptual understanding assessed with and without calculators?"

The questions under number 5 were a consolidation of two questions in the "second attempt." The second question, "Are students asked to interpret answers once they get them?" has roots in the above discussion of avoiding the "black box" image. The first question, "Are students making connections between what they do with the calculator and the problem they are trying to solve?" and the last statement, "This opportunity is a major benefit of using calculators with real-life problems," were the result of several long discussions about "reading mathematics." This began when teachers were sharing frustrations about their teaching context, and specifically about the difficulty students had in reading their mathematics textbooks (the debate over ability to read versus motivation to read was not settled). Reading was seen as a very important tool, by which students understand the tasks on which they work, and by which they interpret what others have done to solve problems. "Reading the mathematics" came to mean more than reading just the words. It combined the words with symbolic expressions and facilitated the translation from one to the other. A powerful example was drawn from the Core-Plus inequalities lesson, because teachers noticed that students were asked to interpret in words the meanings of various mathematical models of the problem situation.

The reading issue arose again in the context of applying the Task Analysis Guide to tasks from Stein's casebook (2000). The influence of that context on Question 5 is seen

in the use of the word "connections," but that word was also a major influence on the decision of the researcher to hand out copies of Zbiek's MAGICAL framework at the final session. This framework provides teachers with an enhanced list rich in connections by which to analyze student (and teacher) use of technology in learning (and teaching) mathematics. The final statement in number 5 was seen as tying the reading issue more closely to the question.

Questions 6, "Do students understand the limitations of the calculator? Is the calculator students are using more complex than is needed for the task?" and 7, "Do I have enough knowledge of the calculator to make it useful for the content I am teaching? How might I get more information about using the calculator with this content?" are more directed to the teacher's knowledge of the calculator and how transparent the teacher's struggles are made to the students. In her study, Szombathelyi (2001) found that teacher knowledge of the calculator was one of the major factors in determining calculator use. Rob brokered a very helpful example of admitting what he didn't know and learning from his students. Karl said the same in interviews, and Lynn used herself as an example of a teacher not knowing why something was happening on students' calculators – why do the students' calculators always have $x = 10$? In fact, Question 7 was added partly to provide impetus for teachers to deal with Question 6. The questions were not combined specifically because Question 7 goes beyond understanding the cybernetic problems of the calculator, or the misconceptions students have about the calculator's abilities to do such things as compute $x*x$. Question 7 is intended to spur the teacher to seek all possible approaches that students might use, or that should be suggested to students. The

formulation of the question followed the work done in Session 4 on the multiple ways to solve Task S.

Thus, beginning with Question 1, teachers planning lessons will unavoidably be drawn to think about Questions 2 and 3 as they think about their students. If they take time to plan specifically for calculator use, which after all is the intent of the AUGC tool, they will consider at least aspects of Questions 4 and 5. And finally, what takes place in most classrooms will bring up Question 6, whose solution may lie in Question 7.

True to her own conjecture of a teacher looking at a task with certain students in mind, the researcher has made this argument that the development of Questions 2 through 7 were not a waste of time by considering the five participants in the present study. As she reflects on the group of more experienced teachers who were participants in a pilot study that did not include the construction of a tool, she notes that there is no concern in the present AUGC tool that was not discussed by the pilot group. Had a tool been developed by the pilot group, it would probably have taken a different form, and would most assuredly have been influenced by different artifacts, but many of the issues to be addressed would have been the same. This conjecture is supported by the similarity between the issues Szombathelyi (2001) found and those raised in this study. It is left to future research to examine similarities and differences of such tools created by different groups of teachers.

In closing, the researcher provides one final figure, Figure 6, representing the possible future of the AUGC tool that has been developed in the present study. This represents only local use of this particular tool, not the extended use that will ensue if the tool is used in other local study groups. The district in which the tool was developed is

not far from the University at which this work is being done, and some teachers are doing/will do graduate work at the University, hence a possible influence extends from

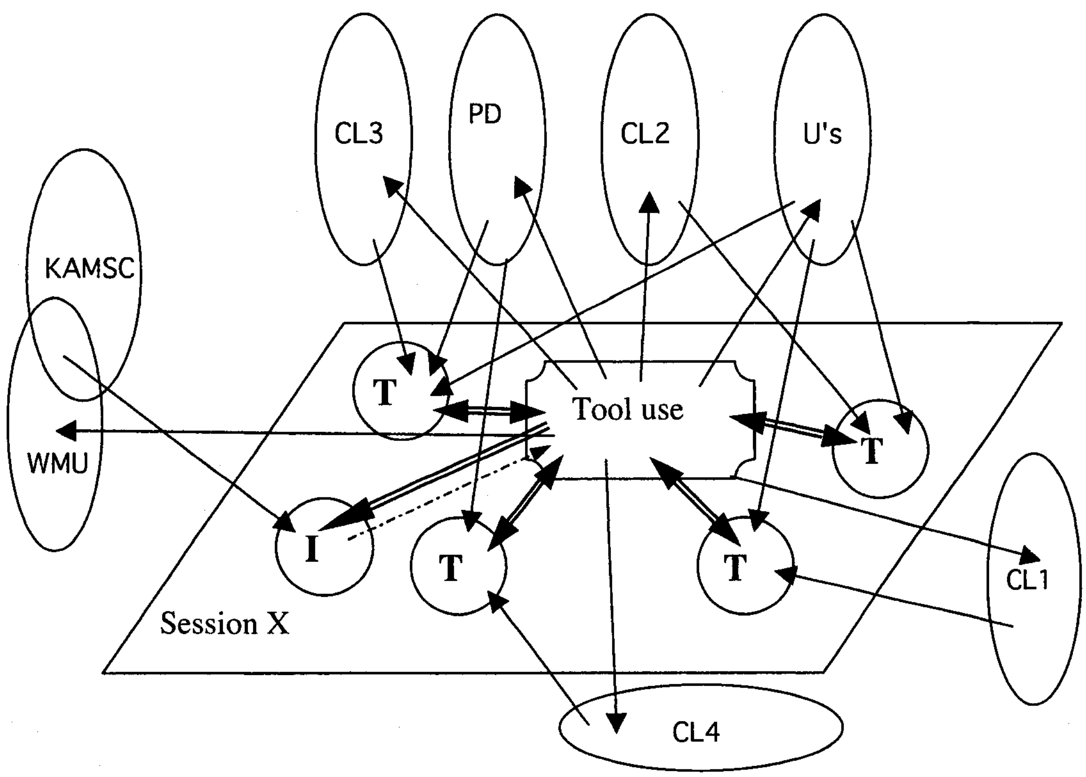


Figure 6: *The future influence of the AUGC tool as a focus of further study*
Note: The two print styles represent different time references. **Bold** represents persons and influences concurrent with the duration of a particular session, while normal weight denotes influences called up from past experience, and arenas in which future effects will be realized. "Tool work" represents not only the developing tool, but also the 'space' within which the work is done. Notations include:
represents a community of practice; CL = classroom, academy, school; PD = professional development; U's = various universities; I = investigator; T = teacher.

use of the tool to WMU and to the other universities teachers may attend. The AUGC tool may also affect professional development sessions the teachers attend, and it is hoped that the tool will affect the classrooms of the teachers who use it. The reciprocal arrows from tool use to teachers (who are now generic users, but may include original

participants of the development), indicate the hope that the tool will receive updates and modifications from teachers who use it. And finally, the investigator (I) will have only slight possible influence on the use of this particular AUGC tool, but this tool will have a major affect on structuring the future research of this investigator.

CHAPTER VII: DISCUSSION

This chapter elaborates how the analyses of results in Chapters IV, V, and VI have supported the formulation of answers to the three research questions. A synthesis of those findings is used to summarize the group's understanding of appropriate use of graphing calculators in the teaching of mathematics. Implications and limitations of the study are examined, and finally, suggestions for further research are offered.

Answering the Research Questions

Because of the nature of the study, and the nature of the analyses, the discussion of how the research questions are answered is, itself, a summary based on chronology. It is structured somewhat differently from the summary at the end of Chapter VI, because rather than focusing on the product of the inquiry, the research questions concern themselves with the influences that shaped the product. Thus, this section will look at the issues that provided the impetus to formulate a tool, the understandings of mathematics and of calculators that each teacher brought to the work, and the specific experiences and artifacts that entered into the negotiation that brought the group to agree on a tool that reified their collective understanding as it stood at the time of the final session.

Question 1: Issues

The first research question was: What are the issues that teachers focus on when constructing an understanding of "appropriate use" of graphing calculators, and how do they negotiate those issues?

The initial interviews with participant teachers and the first study group session produced a long list of issues. These issues surfaced repeatedly whenever participants discussed classroom use of graphing calculators. The full lists are found in Appendix L and Appendix O. The purpose of the study group was to design and construct a product that could be shared with other teachers to help in making decisions about appropriate use of graphing calculators in teaching and learning mathematics. This purpose provided focus for the group as they determined which issues the product under development could address. The winnowing of the list of issues took place in the second session, based on a discussion of issues in the first session, so that by the third session a reduced list of issues was used as a check for the "first attempt" at a product, which was henceforth called the "AUGC tool," or just the "tool."

The issues on which teachers focused in producing the AUGC tool were the issues that were finally incorporated into the tool. Many of these issues are the same as those found by Myers (1998) and Szombathelyi (2001). The final section of Chapter VI gives the record of this incorporation and how the evolution of the wording of the issues was negotiated. The main issues are listed below, first in a form based on what was heard in initial interviews or in Session 1, and then in an expression more akin to the wording that was used in the tool:

- "The kids are always grabbing a calculator, even if they don't know what to do with it!" ...BECAME ... What is the range of activities that students can do with graphing calculators?
- "You can't ban calculators, because some kids have them in their IEPs." ...BECAME ... Which students have blanket permission to use graphing calculators, and what use is helpful for them?
- "They have no business using a calculator for 2 times 3!" ...BECAME ... How can graphing calculators be used to benefit students who lack by-hand skills without handicapping them?

- "They have no idea what the answer means after they've pushed the buttons!" ...BECAME ... How do we keep graphing calculators from being magical boxes that don't promote student understanding of mathematics?
- "I just won't let them use calculators until they can ..." ...BECAME ... How do we assure that students have the appropriate prior knowledge to allow them to interpret information they get from graphing calculators?
- "They just won't read the problem!" ...BECAME ... How can graphing calculators help students learn to "read mathematics?"
- "They think they can use the calculator to solve for *x*!" ...BECAME ... What limitations of graphing calculators do teachers have to make clear to students, and how is that accomplished?
- "I have no idea how to do that on the calculator..." ...BECAME ... How do teachers improve their own knowledge of ways to teach mathematics using graphing calculators?

Just how the transition from the first expressions of the issue to the wording incorporated in the tool was accomplished is elucidated by looking at the general tone of the initial common complaints and that of the tool-like language. With the exception of the last issue listed above, the early statements of the issues all focus on characteristics of the students, so they might be called "student-centered complaints." The general tone of these complaints is that they seem to call into question the very use of calculators, and to make the decision seem very 'black or white.' So a teacher might decide, "If students are unable to make a table of values for a function y of x for various values of x, and to make a plot of those points on grid paper, then those students should not use calculators to make the plots." And indeed, many of the early statements of these teachers sounded similar to this, and similar to Simmt's (1997) participant who felt that calculator answers were not mathematics. The first factor that led to the reinterpretation of those issues was the focus on creating a tool to guide the appropriate *use* of graphing calculators – not to guide whether or not to use them.

This focus caused participants to begin to view their student-centered complaints as being preceded by a conditional. Then issues began to take a form such as, "*If* we are

going to use graphing calculators with students who ...'grab them,' 'have no idea...,' 'have no business...,' 'just won't read...', or who 'think they can solve for *x*,' then how can we use them *appropriately*?" Just that small change, partly made possible by the very district policy that some questioned – that calculators would be used – and their own belief that district policy could not be challenged in a public document, created an openness to Zbiek's (2002b) long and rich list of *activities* students do with calculators. As teachers watched their own students and saw evidence of activities other than "arithmetic aide," the focus on *activity* opened participants' interest to ask what type of tasks would encourage calculator use other than "arithmetic aide."

Refocusing on the second form of the statement of each of the major issues that was incorporated into the tool, we see evidence of this focus on *activity*. Besides the direct references to some kind of calculator use, such as "what use is helpful for them?" and "How can graphing calculators be used to benefit students...," there are also references to interpretation of answers, "reading mathematics," and other characteristics of tasks with high levels of cognitive demand.

The interpretation of these issues, and of others that were not included in the developing tool, was negotiated by the teachers through seeing them as issues of activity and task selection rather than as issues of student characteristics. Recall the example Rob gave of asking all his students to find the value of *x* on their calculators, and then telling them that they were wrong if they didn't get what his calculator said. Not only did this give an example of how a teacher might deal with student misunderstanding of calculator representation of variables, but the fact that he brokered that example required his colleagues to think about that issue as a calculator problem rather than a student problem.

The activity he proposed also focused student attention on the problem of the calculators, rather than on how each of them was right or wrong (although that was how he got their attention).

Also through this reinterpretation of issues as focused on calculator activities rather than student characteristics, teachers decided what would be in the tool and how it would be worded. When Tess reported how she taught the inequalities lesson, Rob was able to see how she used activity with calculator-generated tables to help her students reason about the difference between "equals 80" and "is less than 80." The AUGC tool's suggested wording "making connections between what they do with the calculator and the problem they are trying to solve" would not have had meaning for the teachers without examples such as Tess's.

One method of negotiation that was evident in the discussions were brokering of experiences from their own classrooms, such as that done by Rob and Tess. Brokering the ideas gained from the artifacts will be discussed under research question 2. There was one apparent attempt to sabotage the discussion in order to influence decisions. When this happened, a member of the group was able to get the discussion back on track. Other specific experiences and artifacts that were brokered come under the other research questions.

Question 2: Experiences and Artifacts

The second research question was: How do teachers in collegial discussions about "appropriate use" of graphing calculators incorporate their prior instructional experiences and artifacts that reify the ideas of others?

Classroom experiences were often recounted. Some were incorporated directly into work on the AUGC tool, and others into individual teacher's understanding expressed in interviews. Several of the activities worked on in study group sessions were also referred to as examples in negotiations. The inequalities tasks from Integrated Mathematics 1 course (see Coxford et al., 2003, p. 213) became the defining example of helping students learn to "read mathematics." The varied calculator methods to solve Task S from the Task Sort (see Appendix C) were an "eye-opener" for Yvette, and a graphical approach changed a "who cares?" problem ($x < x^2$) into a "wonderful" one. Karl's use of calculators because he taught a "calculator course" brought him to the realization that he had to change his thinking about what was prerequisite to using calculators.

The teachers' recollections of use or nonuse of calculators in their own education ranged from the somewhat unrealistic "we didn't use calculators but we adapted to using them in college" (so why should our students use them before high school?) to the somewhat wistful "It's a lot easier doing it this way than ... when I learned." These recollections allowed them to consider the fact that the reform curricula they were teaching from used tasks that were much different from those through which they learned mathematics, and that their objectives now go beyond computation. The third question of the tool came from that branch of reasoning, as did the second question of the tool's item 6.

The three major artifacts that were studied and interpreted by the group were all incorporated into the work of developing the AUGC tool in various ways. Zbiek's (2002b) two-tiered categories (see Appendix M) for analyzing how students use

technology in studying mathematics were first used as a list of descriptions that teachers set out to give meaning to by observing their own students. Many terms were adopted into the vocabulary of the group, and then into the wording of the tool, especially in Item 1. The most often used terms include "delegating work," "arithmetic aide," "reference table," and "technology as a puzzle."

A second artifact that provided meaningful ideas to the group, but without inspiring a consistent adoption of vocabulary, was the Task Analysis Guide (see Appendix O) provided by Stein et al. (2000). The ideas of high-level and low-level cognitive demand easily gained meaning as the teachers used the guide to analyze tasks, first those provided by Stein et al. (2000), and later, tasks in their own textbooks. Generally teachers classified tasks as "high- or low-level tasks," but some used words reminiscent of Bloom's taxonomy, or simply, "this task is purely cognitive!" The incorporation of these ideas into the AUGC tool are not immediately evident when looking at the tool because the language is not recognizable. However, "Are students making connections..." and "Are students asked for a deeper understanding..." are both phrases whose meaning is tied to the understanding of the cognitive demand of tasks.

The third artifact that was studied was a list of questions that Branca et al. (1992) developed for use by middle school teachers who were using four-function calculators. This was seen as an opportunity to compare the group's emerging AUGC tool with a similar tool. The comparison led to an interesting discovery. The Branca questions referred to facilitating 'real-life' applications, which the group's tool did not mention. The reason was immediately apparent. Real-world contexts were used for all of the mathematics studied in the curriculum materials these teachers were using, and hence

they were "built in" and did not need to be facilitated! The wording of Item 3 of the AUGC tool, however, was modeled closely on the wording of Question 5 of Branča et al. (1992; see Appendix P), because “focused on relevant instructional objectives” convincingly addressed an issue that the group had been trying to find words for. Other site-specific influences on evolution of the tool are discussed under Question 3.

Question 3: Context and Beliefs

The third research question was: In collegial discussions about "appropriate use" of graphing calculators, what influence is exhibited by contextual factors such as the curriculum used and student characteristics, and by teachers' beliefs and conceptions about graphing calculator use and about the nature of mathematics?

One interesting influence of curriculum and perhaps of lack of experience was the realization that in spite of undergraduate methods class experience with calculators, and the benefit of an implementation workshop for one teacher, the new teachers of Integrated Mathematics 1 and 2 classes were not grasping the intent of the calculator use in the reform curricula they were implementing. Sometimes they were supplementing to make up for the skills of Algebra 1 that were not evident in Integrated Mathematics 1, or were making assumptions about concepts that were not actually taught in eighth-grade integrated mathematics. This had an indirect effect on the AUGC tool, because it led the researcher to choose to use the inequalities lesson that became the model of the concept of “reading mathematics.”

In other cases, curricular modifications were made by each of the Integrated Mathematics 2 teachers that showed mature understanding both of the curriculum and of the students with whom the teachers were working. Yvette did “live reflections;” Tess

designed a discrete math project around Michigan lighthouses; and Rob had his students prepare a small business plan for a product that interested them. Each of these modifications drew from teachers' prior experience, and then provided a context from which to view the emerging AUGC tool.

Contrary to the findings of both Jeon (1999) and Johnson (1994), the present study did not find glaring differences in the way calculators were used with most students of different grade levels or course levels. Calculators were used, in accordance with the district curriculum, for exploration, for graphing, for generating tables, and for comparison of graphs in all classes from eighth grade to advanced algebra. The exception was the Mathematics Investigations class, a non-credit elective that Tess taught, in which she did not allow students to use calculators to work with fractions because she wanted them to understand the concept of fractions first. The importance of the district's policy of calculator use and its adopted curriculum cannot be overemphasized in this finding. Especially among the young teachers, district policy is extremely important. Policy is the standard against which they are evaluated for tenure. According to Sonia Day, the district mathematics coordinator, the ability to use technology and reform curricula is also an important consideration in the hiring process.

The seeming contradiction between this summary and the earlier emphasis on, for example, Tess's insistence that students identify the slope from an equation before using calculators has a two-fold explanation. First, what the teachers said did not limit what they did. Initially, these young untenured teachers did not know what the intentions of the researcher were in presenting this professional development opportunity under the auspices of the district administration. Would she be evaluating them? They erred on the

side of caution when they spoke about calculators, but followed the prescribed curriculum when they taught classes that were observed. Thus, in actual observation the researcher did not see any denial of calculator use when the curriculum called for it. The lack of enhancing beyond-the-curriculum calculator use noted was taken to be the result of inexperience of the teacher with the calculator or with the curriculum. Second, the early focus of the researcher was on issues that may have influenced development of the AUGC tool, thus the claim that calculator use was sometimes denied received attention that may have been greater than it deserved.

One contextual factor that affected the discussions of the study group was that all the participants had taught fewer than three years in the district. This put some limits on the successful classroom experiences that they could draw on, because some were still struggling with classroom management. This may have contributed to the initial focus of issues on the characteristics of students, because individual students were sapping teachers' energy in negative ways. The two teachers with more total classroom experience, Lynn and Karl, were more likely to suggest that the calculator was part of the solution rather than the source of mathematical difficulties of individual students.

However, the limited district experience also meant that two of the teachers had experienced undergraduate mathematics methods courses that used graphing technology, and all had attended at least workshops dealing with graphing calculators. This experience, and the fact that all the teachers were actually using calculators in their classes, provided context and basic vocabulary about calculator use that promoted understanding among participants. No one was confused when Tess said her students "solved in the tables" when she threatened to take their calculators away. Nor did anyone

wonder why students might ask, "What do I put in Y =?" Without this common vocabulary the study group would have had very different discussions.

Also, the fact that three of the teachers were new in the district meant that they were teaching in freshman academies, and thus were teaching most of the same lessons during the duration of the study. As a result, some examples used in the study group sessions were particularly meaningful, and could be examined from the classroom perspective of several teachers. The presence of one eighth-grade teacher in the group also allowed cross-grade and cross-school discussions that would not have been possible without her participation. AUGC tool development was influenced by these circumstances, particularly in the discussion about the habits of mind of using calculators that would develop if students consistently used them. This made teachers conscious of how important it was to know what teachers were doing at other levels, and may well result in contact with elementary school teachers in the future.

Some district issues arose that were certainly context related. Items 4 and 5 of the AUGC tool were at least partly reactions to what these new teachers perceived was happening with mathematics in district elementary schools. These new teachers were also overwhelmed with an unusually (for the district) heavy teaching load because of the structure of the academies. Three courses to prepare for probably contributed to the lack of familiarity with the intent of the curriculum in any one course at a given time.

Contextual factors relating to students included poor motivation, poor by-hand calculation skills, and poor reading ability of a significant number of students. These affected the discussion and negotiation of what went into the AUGC tool. For instance, Item 5 of the tool came from an extended discussion of "reading mathematics" which

began with concern about the low reading skills of some students. The concern for conceptual understanding and its assessment without calculators was partly motivated by a perception that many of the students these teachers worked with had been handicapped by inappropriate use of calculators in early grades. At the secondary school level, then, this concern focused on trying to be sure that students understood what they were asking the calculators to do and why. Connecting all that to the mathematical tasks students were asked to do was a daunting task for this group of young teachers. One teacher seemed relieved when he learned that the curriculum did not expect algebraic solutions in an Integrated Mathematics 1 lesson, but rather graphically determined solutions.

The one teacher belief that continually found its way into the discussion was that there was some kind of prerequisite that students must have before using calculators. This belief exhibited itself usually in what they perceived other teachers to be doing, for example, elementary school teachers. But it also led to a seeming anxiety about using calculators with students whose by-hand skills were not what was expected, as if the by-hand skills were an indication of ability to think mathematically. One expression of this anxiety was that teachers believed that others (i.e. college mathematics professors) would not allow students to use calculators.

In spite of the fact that all five teachers answered Fleener's (1995b) 'mastery' question the same way that in Fleener's study would have indicated a greater tendency to use calculators, most (except Karl, who unabashedly stated that he had to change his belief) hedged when they were asked to review their response to that item. They would not require 'mastery,' but there had to be "some understanding" or "proficiency" before it would be appropriate to use graphing calculators. Much discussion finally led to a sense

that those prerequisite concepts were not always the "traditional" ones such as algebraic manipulation, and they could often be taught hand-in-hand with calculator methods of solution.

The comparison with Fleener's (1995b) participants may not be fair, because the circumstances of Fleener's study make the responses of her participants seem solid. The difference is that those participants just responded to the survey and had no way to indicate a hedging in understanding of the word "mastery." The teachers in the present study, on the other hand, had the opportunity to discuss what was meant by "mastery" and what prerequisite knowledge was needed for a student to gain understanding by using a graphing calculator. Although some of their reticence may be related to beliefs about calculators, they have by the end of the study discussed many experiences of students' misconceptions of the calculator. Therefore, they have gained a healthy skepticism of the calculator as a panacea. Thus the hedging does not indicate a change in belief on the "mastery" question, but merely a clarification of what that means (or doesn't mean).

Appropriate Use of Graphing Calculators

The understanding of appropriate use of graphing calculators that this group of teachers held, however tenuously, at the end of the four study group sessions seemed to have two facets, as was claimed earlier. In addition to familiarity with the calculators being used, the prerequisite knowledge for teachers who wanted to use them appropriately included: (1) understanding the objective of the lesson and the effect of calculators on the cognitive demand of the mathematical tasks involved, and (2) understanding the thinking of the students as they work with calculators.

Part (1) of the teachers' understanding concerns lesson objectives and tasks. While planning for calculator use, understanding the objective of a lesson would allow teachers to avoid having the calculator sabotage students' achievement of the objective. For example, if the objective of the lesson is to practice the distributive law with algebraic symbols, then using a graphing calculator to solve linear equations by graphing a system of equations would undermine that objective. However, if the objective of the lesson is to solve a linear equation as part of the task of understanding the relationship between two linear models, then the method of solving by graphing a system of equations does not interfere with the objective of the lesson. Similarly, if the mathematical task asks students to compare two models of a real-life situation for various values of a variable in the models, then graphing on a calculator, or looking at calculator-generated tables are explorations that not only can assist students in "finding answers," but also in understanding what the task is asking in the first place. The cognitive demand of this task is not lessened by using calculators. Rather, the calculator may actually provide enough understanding of the situation so that students who may not have understood the task as it was stated in words can begin to use their visual and numerical understandings to proceed with finding solutions.

Part (2) of the teachers' understanding responds to their worry about what must be understood by the students before they use calculators to solve problems. After much discussion, this concern was clarified to focus on whether students understood the mathematical principle they were asking the calculator to apply to the situation for them. Again using the example of graphing a system of equations to solve a linear equation such as $3(x + 2) - 4(x - 1) = 2x - 5$, students did not need to understand the rules for

distribution and combining like terms. However, they must first understand the principle that the values of the expression $3(x + 2) - 4(x - 1)$ can be represented for all values of x by graphing the line $Y_1 = 3(x + 2) - 4(x - 1)$, and likewise for $Y_2 = 2x - 5$. They must also understand that when the two y-values are the same for a given x-value, the two lines intersect, and thus the intersection gives the x value for which the original equation is true, i.e., the solution. These teachers recognize that this is not a trivial set of understandings. Nevertheless, for many students this visualization of the problem can be more readily understood than can the rules for distribution and combining like terms. Thus teachers must be aware of what students are thinking as they use calculators. The newer teachers were not yet familiar enough with their curriculum to know when the rules for distribution and combining like terms would be taught, so they worried about that. Karl recognized that his advanced algebra text was teaching algebraic and graphical methods hand-in-hand.

Part of what was lacking for the newer teachers in the study group, but which Karl and also Lynn exhibited, was a comfort with the realities of their students. It was not a comfort that left them resigned to failing students, but, as Lynn so clearly demonstrated, a dogged facing of the facts that they would be working with and a determination to learn to use whatever tools could help improve things. Some of these teachers were still concerned that calculators might make matters worse, or that they might be more trouble (calculators were always disappearing from classrooms – or the batteries were) than the possible benefits were worth. The study group provided one community with whom they could share questions and ideas, and perhaps a desire to form new groups in their own schools to further their understanding.

The "ultimate question" did demonstrate that if the AUGC tool is to be shared, they want a part in it. The indications are that this will not be a possessive or didactic sharing, but rather to help explain the evolution of the wording, and to help other teachers interpret the meaning that is reified in this product of their effort.

Limitations and Implications

The form of the final tool and the understanding it reifies is peculiar to the six persons involved in the study group. The dynamics of the group were strongly influenced by the fact that three of the five teachers were relatively inexperienced, and perhaps more open to the ideas of others. The use of reform curricula by the teachers was a benefit to the group, but also a limitation because so many factors that might have been discussed in another setting were "built in." The small size of the study precludes generalizations of any kind, yet the process can be duplicated, given a facilitator with time to observe classes and experience to capitalize on activities that will be meaningful to the study group participants. The use of the three artifacts seems to have been fruitful with this group of teachers, but another group may need more time with one or the other of them.

The short duration of the present study was surely another limitation. It is impossible to know how robust this newly-constructed understanding of appropriate use of graphing calculators will be. Given another year in the same position, teaching the same lessons from the same curriculum as during the development of the AUGC tool, these teachers may be able to consolidate their understanding and truly test the usefulness of the tool. However, it is not clear whether the deeper understandings of the development of the tool would survive transfer to another district, another grade level or another curriculum. Under conditions of such change, would teachers be able to focus on

the activities students can do with calculators to improve understanding of mathematics, rather than on the possible mathematical shortcomings of the students themselves? Will these teachers continue to seek collegial discussion on the appropriate use of graphing calculators?

A limitation of the task sort was that it asked teachers to consider too many tasks, so that they looked at them only superficially in terms of finding a numerical result. They did not consider the questions such as, "Why does this make sense?" In using such a sort it may be preferable to use four to six problems that teachers actually work out. One drawback of this alternative is that teachers may find that more intimidating.

Implications stemming from the research questions are important for those working with teachers in district and school settings, and also for teacher educators. Related to Research Question 1, this study indicates that there is a strong tendency in some teachers, even those with experience using calculators as students in college courses, to suspect that what other teachers (especially those teaching different grade levels) do with calculators in their classes damages the students' understanding of mathematics. This suggests that opportunities must be created for teachers on different levels to work together on a tool such as the one created by this group. Focus on specific content, as was the case in the present study, would not be feasible, but examining the development of a strand across grades might provide a suitable context.

Another finding related to Research Question 1 is that, to reach agreement, negotiation of issues often involved examples from teachers' classrooms, and also replaced an interpretation of the issue focused on student deficiencies with an interpretation focused on calculator activity by the students. An implication of this

finding for professional development aimed at helping teachers learn to use calculators with students, would be to carefully select or elicit relevant examples, and steer discussion of issues away from student characteristics toward calculator activity for students. Exposure to Zbiek's (2002b) categories or video tapes of classes using calculators in a variety of ways could provide an early focus on student activity.

A finding related to Research Question 2 is that teachers do feel limited by what they themselves know about using a calculator to "do mathematics." This is in agreement with Szombathelyi (2001), and suggests a need for more opportunities for teachers to “play” with calculators and with their own curriculum. The AUGC tool shared in the present study may be useful in helping teachers ask questions of their curriculum, such as, "What would be the effect on the cognitive demand of a given task if students used calculators? How might the task be altered?"

In relation to the artifacts studied, it was found that the ideas of Zbiek (2002b) and Stein et al. (2000) were easily applied to work on the AUGC tool by this group of teachers. The questions of Branca et al. (1992) led to the conclusion that many of the same issues were addressed by the two tools developed over a decade apart, with the exception of facilitation of real-life applications. The implication is that these same artifacts may assist the development of a similar tool in another setting, but that differences may be expected when curriculum materials do not incorporate the use of calculators or of real-world contexts in which to study mathematics. It should also be noted that teachers in the present study found that nearly all the tasks that students were asked to do in the reform curricula had high cognitive demand, and were designed to be used with calculators. Thus they were not faced with the problem of demand levels being

reduced by the introduction of calculators. Care must be taken to provide such examples for teachers or students whose classroom materials do not regularly incorporate calculators.

A finding related to the beliefs of teachers targeted in Research Question 3 is that, with much discussion, and brokering of examples, teachers are able to recognize alternatives to their initial thinking about prerequisites to calculator use. This has implications for continuing work with teachers resistant to using calculators. The softening of the "no calculators until you understand the concept" stance to one of "let's see if we can develop a better understanding of this concept if we look at it using different representations" is a significant change, and requires examples from the teachers' own context. For this reason, it would be a great challenge to find a context powerful enough to affect the thinking of pre-service teachers enough to make their understanding of appropriate calculator use resilient in whatever context they begin their careers.

And finally, a finding outside the focus of the research questions, but made possible by the particular group of participants in this study, is that teachers using reform curricula for the first time do not necessarily discern the intent of calculator use in the curriculum, even though they themselves may have used calculators in college classes. Implications of this finding are similar to those for teachers who feel the limitations of their knowledge of calculators. Both pre- and inservice teachers should have opportunities to engage in activities that illustrate the interactions of calculators and mathematics curricula. This might be a particularly appropriate activity for a mathematics methods course, or a technology course for future teachers of mathematics. Analysis of

tasks for cognitive demand and careful observation of what students do with calculators as they solve tasks can provide context and examples for comparison to the AUGC tool constructed in the present study, thus providing a basis on which to judge the appropriateness of some of the tasks, and a more conscious effort to use calculators in that appropriate manner.

Suggestions for Future Research

Certainly there is no guarantee that this group of five teachers will ever meet again, or that they will actually present their tool to their colleagues. There are compelling reasons to want to know what happens. Can they make better use of calculators next year – or at least smarter use? Can they assimilate the three new mathematics teachers the district will hire for next year, and help them interpret the tool for their own use? Will using the tool make anything easier for those new teachers? If any of these questions could be researched, information would be gained on the extended effects of the process on the teachers who participated and on the group's possible evolution into a community of practice by amassing those "collective stories that capture canonical practice" (Reynolds et al., 2001, p. 110).

Another question that might fit only for this particular group is what happens when a group of new teachers presents such a tool to the veterans of the district? That question would not necessarily relate to calculator use, but surely would relate to the concept of a district community of practice and how it reacts to those peripheral participants who have something to share. But even more difficult might be the intended interaction among elementary, middle and high school teachers around the use of

calculators. Could there be any agreement on appropriate use of calculators? And what benefits might come from agreement?

The presence of a reform curricula perhaps made the present study possible. What would be the dynamics of a group consisting of users of traditional and reform curricula talking about calculator use? Would they be able to agree on a tool? On the other hand, what would the resulting tool look like if all the teachers were using traditional curricula? What would the common elements be in tools that were developed in different settings?

The present study included the Branca (1992) questions as a comparison. What would be the effect of bringing the AUGC tool of the present study, as a comparison, to another group developing a similar tool? Would the effect be the same if the tool came as an artifact to be modified?

In summary, although the phrase "appropriate use" of graphing calculators is commonplace in literature supporting mathematics reform, this study shows that the phrase is not necessarily well understood by teachers using calculators in mathematics classrooms. Collegial inquiry can help teachers reach a common understanding of appropriate use of graphing calculators, especially when focussed on producing a tool to be used by other teachers in making decisions about calculator use.

APPENDICES

Appendix A

Sample Invitation Packet

Mathematics Department Letterhead

Dear «Title» «Last»,

My name is Marcia Weinhold. For 17 years I taught mathematics in Kalamazoo and Wisconsin, but am currently working on my Ph.D. dissertation in mathematics education at Western Michigan University. I am sending you this information because I need your help. Worthwhile education research cannot be done without the participation of teachers like you. If you agree to participate in this dissertation study, you will not only make *it* possible, but you will earn 5 professional development credits for the work you do. Besides that, we will produce a tool that will be helpful to other teachers.

Many of you have watched calculators come to school with mixed emotions. Others have grown up with calculators. What are the issues that must be considered when graphing calculators are used to teach mathematics? My interest is in how teachers make decisions about what is appropriate use for calculators in their classrooms. No matter what curriculum materials you use, you often make adjustments.

Over four 2-hour study group meetings from February to May, I will ask participants to work together to develop a tool for secondary mathematics teachers to use in making decisions about appropriate graphing calculator use for their classes. This tool may take the form of a list of questions to ask yourself, or a checklist, or a chart with sample tasks – whatever makes it useful for other teachers. The process of designing this tool will require your professional judgment, your years of experience as student and teacher, and your ability to communicate. I believe it will be a good professional experience for all of us, and that the tool developed will provide a starting point for discussion in many other schools and districts as the research is shared.

My interest as a researcher is in the process through which this tool is developed, and the ideas that are incorporated into the final design. The discussions of the group will be audio taped to help me trace this development. I will also bring some ideas from recent research to the discussion; specifically, ideas about analyzing the cognitive demand of tasks, and of analyzing students' use of graphing calculators. You will be free to use these ideas or not in constructing a tool you feel will be useful for other teachers.

I will also ask for four to six volunteers from the study group to test the tool in considering calculator use in their own classrooms, to allow me to observe one of their classes once after each meeting, and to share their experience of using the tool with the whole group. There will also be an initial and a closing survey and interview for all participants. These will be audio taped and scheduled at your convenience before the first study group meeting and after the final meeting.

I am including in this mailing a tentative timeline for the study and an information form to fill out to register to participate. I would be greatly honored to have you as a colleague in this project.

Sincerely, marcia.weinhold@wmich.edu Work: 387-xxx Home 383-xxxx

Intent to Participate in
"Appropriate Use of Graphing Calculators Study Group"

Name ______________________ School ________________________

Best WAY to contact me at school: phone ________________ ext. __________

email ___________________________

Best TIME to contact me at school: _____________________

Best WAY to contact me at home or 'abroad': phone _______________

cell phone _________________ email ___________________________

Best TIME to contact me at home: _________________________

If you will need to pay for childcare during the 2 hr. time, please estimate total (8hr) cost ________

(I will personally reimburse childcare cost.) **Preferred time** for 2-hour study group meetings:

Mark each time as OK 1, OK 2, ... etc. to show best (OK 1) time and other acceptable times. Mark as CAN'T those times that will not work. Use the "Other" blank to propose an alternate time. Give as many possibilities as you can, so we can make a match with 4 to 6 people from several districts.

Day				Which weeks of month?
Monday	_______ 3:30 – 5:30	_______ 5 – 7	_______ 6 – 8 p.m.	1 2 3 4 5
Tuesday	_______ 3:30 – 5:30	_______ 5 – 7	_______ 6 – 8 p.m.	1 2 3 4 5
Wednesday	_______ 3:30 – 5:30	_______ 5 – 7	_______ 6 – 8 p.m.	1 2 3 4 5
Thursday	_______ 3:30 – 5:30	_______ 5 – 7	_______ 6 – 8 p.m.	1 2 3 4 5
Friday	_______ 3:30 – 5:30	_______ 5 – 7	_______ 6 – 8 p.m.	1 2 3 4 5

Saturday_______ 8 – 10 a.m. _______ 10 – noon

Other: Suggest day and time: ______________________________________

Please return your "Intent to Participate" to Sonia Day at your District headquarters to register for professional development credit. She will forward copies to me so I can contact you about times and place that coordinate with other participants. Thanks!!

Proposed Schedule for "Appropriate Use of GC Study Group"

Probable time commitment follows each activity, and total is at bottom:

Proposed Timeline	Participant activity	Agenda
January	Complete survey (15-20 min.) Complete Task sort (20-30 min) Task sort Interview (10 min.)	Pre-study Task Sort interview and Survey of beliefs and conceptions
Early February	Study group session (1:52 hr) Beginning the Tool Written reflection (8 min)	First study group session - Levels of Cognitive Demand (LCD)
February	Incorporate calculators, using tentative Tool (1 class sess) Post-observation Interview (5-10 min.)	Classroom observations Interviews
March - week 1 or 2	Study group session (1:52 hr) Complete second draft of Tool Written reflection (8 min)	Second study group sessions- MAGICAL framework
March	Incorporate calculators, using revised Tool (1 class sess) Post-observation Interview (5-10 min.)	Classroom observations Interviews
March - week 4 or 5	Study group session (1:52 hr) Complete third draft of Tool Written reflection (8 min)	Third study group session - Branca -Teacher questions
March - April	Incorporate calculators, using revised Tool (1 class sess) Post-observation Interview (5-10 min.)	Classroom observations Interviews
April - after spring break	Final Study group session (1:52 hr) Complete final form of Tool & evaluation Written reflection (8 min)	Fourth study group session - consolidation of Tool ideas & evaluation
April - May	Complete Task sort and Interview (30-40 min.) Complete Survey review (15-20 min.)	Post-study Task Sort interview and survey review

Total time commitment per participant: 10 to 10.5 hours

If you are interested in participating, but want more information, email me, marcia.weinhold@wmich.edu and I will email you the consent forms you will be asked to sign if you participate. They are a requirement of the University for all research. I will also send them to you if you reply to Sonia, since you will need to read and sign them before the initial interview if you wish to participate.

Appendix B

Researcher Position on Appropriate Use of Graphing Calculators

My own ideas of "appropriate use" of graphing calculators
Note: Items 1 & 2 were written in February 2003, before sessions began. Items 3 – 5 were written in June 2003, when I realized the exercise had not been completed.

Before I begin to observe this development in other teachers, where am *I*?
The following statements attempt to say what I think is appropriate use of graphing calculators:

1. **Calculators of any kind should be used to enhance the thinking of the student, not to replace such thinking.** [I also don't think it is possible for a machine to replace student thinking, but that's another discussion.] This takes different forms at different times. For example, if the question "What is 7 + 5?" (one of my un-favorites) is asked in early elementary school, we intend for the child to reason about the relative sizes of the two numbers, how they relate to 10 or some other landmark number, and how they might figure out the answer. Eventually, that reply should be automatic. But for some people, it never gets there. [8 + 7 is another of my un-favorites.] At some point we want students to work on other problems, of which 7 + 5 is only part. For example, how many grumbies can Dick and Jane buy together at $2.97 if Dick has $5 and Jane has $7? Here the intent of the problem is much larger than 7 + 5, yet it is true that if students can't find that sum all their other thinking will be hard to evaluate. This is precisely why teachers always want to see students' work! At some point, we want a child to be able to recognize his/her own weaknesses and either *work at strengthening* them *or use a machine to compensate* for them. In both cases, the child is learning how to deal with mathematics.

2. **Calculators, and especially graphing calculators, can provide a variety of representations to help students understand mathematics more deeply.** For example, using a calculator to count up from 7 using the recursive capability of calculators, might help students get a visual (or even tactile) image (to corroborate other images used) of how "far" it is from 7 to 10, as well as "how big" 5 is. Because the calculator is doing the adding, the student can concentrate on the patterns, from which the automaticity might be built. The same is true for graphing calculators. If graphing many linear functions is helpful to develop an automatic sense of what the graph of a linear function looks like, then some students might be helped by letting a machine plot the points so they can concentrate on pattern instead of finding points. This only helps the student, however, if he/she has some understanding of the cartesian plane, and locating points on a grid according to ordered pairs. Then, the step can be made to placing an entire graph on the grid using a rule such as an equation. The progression from point plotting to equation plotting is similar to the progression from counting by ones to counting by other multiples. The calculator can assist in making both automatic. But we have to be sure that students see that as the goal! This is done by testing or quizzing the concepts without calculators, but allowing calculators when the goal is a more complex application.

3. **Calculators can help students deal with the limitations of working memory.** Theorists who explain how people enhance their mental capabilities by 'chunking' use the idea of a 'working area' of memory that can hold only a given amount of information that can be mentally worked on at the same time. Chunking allows more information to be held and worked on, or easily recalled when needed. Students who have not developed chunking abilities, or for whom arithmetic has not yet become automatic, have difficulty mentally handling all the calculations needed to graph a relationship expressed by an equation, even when they understand the plotting of points. Letting a calculator handle the numbers so that students have correct points to plot takes some of the stress off so they can concentrate on the graph. But if the goal is to understand the relationship between parameters in an equation and the *y*-intercept of a graph, then it makes sense to let a machine handle the plotting of many graphs so that students can use their mental capacity to work out the relationship.

4. **Using calculators will not make teaching easier, but it will be different.** Teachers do need to be aware of the limitations of calculators so they can caution students about errors that will arise. They also need to emphasize estimation so that students will know if a calculator's answer makes sense. They need to require students to justify answers even if they come from a machine. A teacher using technology must be prepared for many different approaches to problems, and must be ready to challenge those that are not mathematically sound, as well as marvel over those that are ingenious. Teachers must also keep the "big ideas" in mind – those things that we want students to understand even without calculators, but that calculators might help uncover for some students. And, yes, there will be some things that students will not be good at because they keep letting the calculator do it, but think of what else they might use their minds for – and then challenge them with it!

5. **Using calculators will change students' thinking, but need not damage it.** I doubt that any engineer or physicist, who no longer had to extract square roots by hand, lacked for productive thinking to do instead. Some mathematicians whose work is built on techniques that are being 'taken over' by machines might worry that future students won't know the basic techniques that will allow their work to proceed. Perhaps other work will develop instead. But surely there will be those who doubt the machine, or who want to know how it does certain calculations, or want to build faster machines by finding better algorithms. They will be among those who study 'pure' mathematics. And I'd be surprised if their numbers or abilities turn out to be less than those who now succeed at higher mathematics. But they will have come to it by a somewhat different route, one that did not lose so many others along the way.

Appendix C

Task Sort and Instructions

Task Sort instructions

The purpose of this sort is to understand the types of tasks for which you think calculators can be appropriately used. A broad range of tasks is included. Assume that the content is within the curriculum covered by your students.

Please read each task, and decide first of all whether you would use the task with your students or not. You should consider it "used" if students might do the indicated calculation as part of solving another task.

If you would not use the task with students at all, place it in the envelope labeled NOT USE and write your name in the space on the label. Don't seal the envelope in case you change your mind while working on the others.

For the remaining tasks, decide whether calculator use would ALWAYS, NEVER, or SOMETIMES be appropriate for each task. Sort the tasks into these three categories.

Place the ALWAYS tasks into the envelope so labeled, and put your name on the label (don't seal it in case you change you mind on another).

Place the NEVER tasks into their envelope, and put your name on the label (don't seal it in case you change you mind on another).

Before placing the SOMETIMES tasks into their envelope, make a brief statement on the back of each task of the CONDITIONS under which calculator use WOULD be appropriate. When all these tasks are in the envelope, put your name on the label.

Return the four envelopes to the researcher. As part of your interview, she may ask some questions about the tasks you placed in various categories.

Task A

Graph $y = (x + 1.5)^2 - 6.25$
for values of x from -10 to 10.

Task B

Graph $y = 3x - 2$ for values of x from -15 to 15.

Task C

Give the value of the sine of 30°.

Task D

Give the value of the sine of 15°.

Task E

Graph $y = x^2 + 3$
Graph $y = x^2 - 2$
Graph $y = x^2$
Make a conjecture about the role of b in the graph of the equation $y = x^2 + b$. Test your conjecture for b = -1. Revise your conjecture if necessary.

Task F

Maggie's Bakery sells giant cookies for 85¢ each but includes a $2 delivery charge for orders less than a dozen cookies. The math club likes to have cookies delivered when they meet. If 5 members attend, how much will each have to pay to have 5 cookies delivered? If 6 attend? What is the smallest number of members needed for each to save the most on their order? Explain.

Task G

Give the decimal equivalent of $\frac{3}{4}$.

Explain why your answer makes sense.

Task H

Give the decimal equivalent of $\frac{156}{195}$.

Explain why your answer makes sense.

Task I[1]

Sales of Newspapers in the US (millions)

Year	A.M.	P.M.	Total
1982	33.2	29.3	62.5
1985	36.4	26.4	62.8
1988	40.5	22.2	62.7
1990	41.3	21.0	62.3

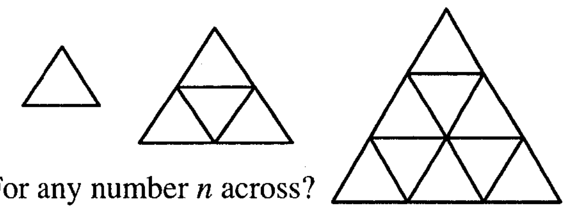
Fit regression lines for A.M., P.M. and Total data trends by year. How are the equations related? Why does that make sense?

Task J[2]

Graph the equation $y = \frac{2}{3}x + 1$. Trace any two points on the line and write the coordinates (x_1, y_1) and (x_2, y_2). Then calculate the quotient $\frac{(y_1 - y_2)}{(x_1 - x_2)}$. Compare values with others in your class. What can you conjecture about the value of this quotient along any part of the line? This quotient is called the *slope* of the line.

Task K[3]

The following figures are made of toothpicks all with the same length. How many toothpicks are needed to make a similar figure with 10 toothpicks across the bottom?



For any number *n* across?

Task L

Determine which is larger, 15% or $\frac{1}{8}$.

How do you justify your answer?

Task M[3]

An office manager must rent a copy machine. Ace Copiers charges \$50 per week plus 2.1¢ per copy. Speedy Print charges \$180 per week and 0.5¢ per copy. Make a graph showing charges for each company for up to 20,000 copies per week. Would the manager's decision change if each company lowered its weekly charge by \$50? How would the graph change?

Task N[4]

In 1992 the US population was changing each year in the following ways:

- Births equaled 1.6% of the population.
- Deaths equaled 0.9% of the population.
- Immigrants numbered 0.9 million.

If the population in 1990 was 248 million, and those trends continued each year, what would be a reasonable estimate for population in 2000?

Task O

Solve for x.

$$7x = 29$$

Task P[5]

Find the value of the expression

$30.5 \div (-1/4) \div [2 - 7(10 - 1)]$.

Task Q[6]	Task R[7]
A batch of 40 widgets produced on November 15 contained 5 defective widgets. If a sample of four widgets was drawn, what is the probability that none of the sample widgets was defective?	Find the zeros of the given function. $f(x) = x^2 - \frac{16}{x^2}$ Explain why these zeros make sense by writing the function in factored form.
Task S[5]	Task T[4]
Solve for x: $5(x-2) - 3(2x+1) = 2 + 5x$ What does this tell you about the intercepts of the graph of $y = -15 - 6x$? Why?	A survey of amusement park customers predicts the following number of bungee jumpers at certain prices. Price charged \$20 \$30 \$40 \$50 \$60 Jumpers daily 100 70 40 20 10 Predict the number of jumpers daily if the price is set at \$25, at \$45, at \$100. Explain your reasoning for these predictions.

[1]Task adapted from *Advanced Algebra*, UCSMP, second edition, Scott Foresman, 1996.
[2]Task adapted from *Advanced Algebra*, Holt, Rinehart, Winston, 1996.
[3]Task adapted from *Algebra in a Technological World*, NCTM, 1995.
[4]Task adapted form *Contemporary Mathematics in Context*, CPMP, Everyday Learning, 1997.
[5]Task adapted from *Algebra 2 and Trigonometry*, Dolciani, et al., Houghton Mifflin, 1968.
[6]Task adapted from *Foundations of Advanced Mathematics*, Kline et al., Heath, 1975.
[7]Task adapted from *Functions and Graphs*, Pownall, Prentice Hall, 1983.

Appendix D

Surveys

Teacher Survey, Part A Teacher Name ____________________

1. Please give a brief description of how you are presently using graphing calculators in teaching your mathematics classes. If you are not using graphing calculators in your classes (or in any one class), please give a brief statement of your reason(s) for not using them.

2. Please list the most important things you consider when you decide how and when to use (or <u>not</u> use) graphing calculators in your mathematics classes.

3. Please identify several of the mathematics concepts in your class that students have the most trouble <u>understanding</u>.

Teacher Survey, Part B Teacher Name ____________________

For all questions on this survey, take words such as *mathematics, algebra, geometry, probability* and *statistics* to mean your broad understanding of these subjects.

1. Check one of the following that *best* describes your background in mathematics.
___ I have learned much of my mathematics in science, business or engineering courses.
___ I have an undergraduate minor in mathematics.
___ I have an undergraduate major in mathematics.
___ I have taken some graduate level mathematics courses.

For each statement, please mark the 0 under the response that most accurately represents your feelings.
SD = strongly disagree, **D** = disagree, **N** = neutral or don't know, **A** = agree, **SA** = strongly agree

Mark one "0" for each statement	SD	D	N	A	SA
2. Incorporating calculators into teaching requires changing the types of problems assigned.	0	0	0	0	0
3. Students need to demonstrate proficiency in using mathematical procedures before doing any similar work using calculators.	0	0	0	0	0
4. It is not necessary to change what is taught in order to effectively use calculators.	0	0	0	0	0
5. Students should be allowed to use calculators even before they understand the underlying concepts.	0	0	0	0	0
6. More difficult mathematics problems can be done when students have access to calculators.	0	0	0	0	0
7. Using calculators frees students to explore alternative solution strategies.	0	0	0	0	0
8. Continued use of calculators will cause a decrease in student estimation skills.	0	0	0	0	0
9. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *less* important.	0	0	0	0	0
10. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *more* important.	0	0	0	0	0

11. Circle YES or NO: My college or graduate level methods class included use of calculators.
Below, check one that *best* describes your experience using calculators (i.e. scientific or graphing calculators).
___ I haven't used calculators much except for basic operations (i.e. +, -, x, ÷).
___ I have used calculators in some math courses or workshops, but not much beyond basic operations and functions (i.e. powers, finding roots, trig functions, etc.).
___ I have taken at least one math course or workshop that required significant calculator use including graphics capabilities (i.e. graph, tables, trace, programming, etc.), but little, if any, use of the statistics and probability capabilities.
___ I am comfortable with most aspects of calculators including statistics and probability capabilities.

12. For each item below, please use the *left section* to rate its **importance** for effective mathematics instruction.

Use the *right section* to indicate how **prepared** you feel to do each one.

NI = not important **FI** = fairly important
SI = somewhat important **VI** = very important

NAd = not adequately **FW** = fairly well
S = somewhat **VW** = very well

importance				Please mark one "0"	prepared			
NI	SI	FI	VI	← in each section for each item →	NAd	S	FW	VW
0	0	0	0	Provide concrete experiences before abstract concepts.	0	0	0	0
0	0	0	0	Develop students' conceptual understanding of mathematics.	0	0	0	0
0	0	0	0	Have students participate in appropriate hands-on activities.	0	0	0	0
0	0	0	0	Engage students in investigative activities.	0	0	0	0
0	0	0	0	Use graphing calculators.	0	0	0	0
0	0	0	0	Engage students in applications of mathematics in a variety of contexts.	0	0	0	0

For each statement, please mark the 0 under the response that most accurately represents your feelings.
SD = strongly disagree, **D** = disagree, **N** = neutral, **A** = agree, **SA** = strongly agree

Mark one "0" for each statement	SD	D	N	A	SA
13. I feel supported by colleagues to try new ideas in teaching mathematics.	0	0	0	0	0
14. I am confident in my ability to learn new mathematics concepts.	0	0	0	0	0
15. I am confident in my ability to formulate questions to guide students' understanding of mathematics.	0	0	0	0	0
16. It is important for mathematics teachers to be aware of how students learn mathematics.	0	0	0	0	0
17. It is important for teachers to ask how students are thinking when studying mathematics even if it limits the amount of material covered.	0	0	0	0	0
18. Students should use procedures taught by a teacher instead of ones they develop on their own.	0	0	0	0	0
19. Students learn mathematics by studying examples and practicing mathematical concepts and skills.	0	0	0	0	0
20. Students learn mathematics by the personal building of mathematical understanding.	0	0	0	0	0
21. If a student understands a mathematical concept, then that student will be more able to accurately perform procedures associated with the concept.	0	0	0	0	0
22. The use of calculators is fine as enrichment for better students, but not as part of "mainstream" mathematics for all students.	0	0	0	0	0

23. For each topic below, in the **IT** section, mark "0" if you teach the topic.
IT = I do teach this topic
If you do teach it, in the **IU NC** section, indicate whether or not you use graphing calculators.
IU = I use graphing calculators to teach this topic
NC = I do NOT use graphing calculators to teacher this topic
If you do NOT teach it, in the **CH NH** section, give your opinion of whether graphing calculators would be helpful in teaching the topic. **CH** = I would use graphing calculators to teach this topic
NH = Graphing calculators would NOT help teach this topic

Please mark all that apply.	IT	IU	NC	CH	NH
Estimation	0	0	0	0	0
Measurement	0	0	0	0	0
Algebra and symbol sense	0	0	0	0	0
Patterns and relations	0	0	0	0	0
Geometry and spatial sense	0	0	0	0	0
Geometric transformations	0	0	0	0	0
Functions (including trigonometric functions)	0	0	0	0	0
Pre-calculus concepts (e.g. rates of change, area under a curve)	0	0	0	0	0
Data analysis (including curve fitting and regression)	0	0	0	0	0
Probability	0	0	0	0	0
Graphical displays, descriptive statistics and hypothesis tests	0	0	0	0	0
Topics from discrete mathematics (e.g. combinatorics, vertex-edge graphs, recursion)	0	0	0	0	0
Mathematical structures (e.g. matrices, complex numbers, fractals)	0	0	0	0	0
Calculus	0	0	0	0	0

For each statement, please mark the 0 under the response that most accurately represents your feelings.
SD = strongly disagree, **D** = disagree, **N** = neutral, **A** = agree, **SA** = strongly agree

Mark one "0" for each statement	SD	D	N	A	SA
24. When solving problems with calculators, students don't need to show their work on paper.	0	0	0	0	0
25. Use of calculators will eventually replace all paper and pencil work in mathematics.	0	0	0	0	0
26. It takes too long to teach students which buttons to push on a calculator.	0	0	0	0	0
27. Being able to accurately perform an algorithm is necessary for understanding the underlying mathematical concepts.	0	0	0	0	0
28. The calculator can be used to explore mathematical concepts.	0	0	0	0	0
29. Calculator skills are as important as paper and pencil computational skills.	0	0	0	0	0
30. Students can gain understanding of computational procedures by using calculators.	0	0	0	0	0

For each statement, please mark the 0 under the response that most accurately represents your feelings.
SD = strongly disagree, **D** = disagree, **N** = neutral, **A** = agree, **SA** = strongly agree

Mark one "0" for each statement	SD	D	N	A	SA
31. The teacher should decide when it is appropriate for students to use calculators.	0	0	0	0	0
32. Using calculators will cause students to lose basic computational skills.	0	0	0	0	0
33. It is not appropriate for calculators to be used in some mathematics classes.	0	0	0	0	0
34. I am well-informed about the National Council of Teachers of Mathematics (NCTM) *Principles and Standards* for the grades I teach.	0	0	0	0	0
35. I am well-informed about the Michigan *Mathematics Framework* standards and benchmarks appropriate for the courses I teach.	0	0	0	0	0
36. I am well-informed about the Michigan High School Proficiency Test for Mathematics **state mathematics test** for high school students.	0	0	0	0	0
37. I understand the fundamental ideas of mathematics.	0	0	0	0	0
38. I have adequate access to graphing calculators for teaching mathematics.	0	0	0	0	0
39. Mathematics teachers in my school have a shared vision of effective mathematics instruction.	0	0	0	0	0
40. Mathematics teachers in my school regularly share ideas and materials related to mathematics instruction.	0	0	0	0	0
41. I have time during the regular school week to work with at least one other teacher on matters related to planning and teaching our courses.	0	0	0	0	0
42. Students are dependent on calculators when they come to my class.	0	0	0	0	0
43. Mathematics is fixed and unchanging.	0	0	0	0	0
44. Mathematical ideas are constructed by human minds.	0	0	0	0	0
45. Expecting students to be creative in mathematics is unreasonable.	0	0	0	0	0
46. Mathematics is essentially hierarchical and cumulative.	0	0	0	0	0
47. A world with different mathematical truths is impossible.	0	0	0	0	0
48. Students should not be allowed to use calculators until they have mastered concepts.	0	0	0	0	0
49. Students should be allowed to use calculators on standardized tests.	0	0	0	0	0
50. I am confident in my ability to teach mathematics using calculators.	0	0	0	0	0
51. The major value of calculators in mathematics classes is to save time from performing computations.	0	0	0	0	0
52. Students understand math better if they solve problems using only paper and pencil.	0	0	0	0	0
53. Using calculators in the teaching of mathematics results in greater student understanding of concepts.	0	0	0	0	0
54. Using calculators in the teaching of mathematics encourages a more active, conjecturing approach to the learning of mathematics.	0	0	0	0	0

Appendix E

Calculator Use Log

Calculator Use Log　　　　Name: ______________________

Class monitored ______________________

For each study group session, fill in agreed upon tool items to check for, then in ONE class per day, check off which items were used or considered for calculator use.

Comments on back →

Check for → (Session) Class Date						Tool Useful? Yes or No

Bring logs to each study group session - they are your evidence for the usefulness of the tool, and of further issues that need to be considered in revision.

Appendix F

End of Session Teacher Reflections Form

Reflecting on Session 1 2 3 Name ______________________

When I think of "appropriate use of graphing calculators," my chief concerns are --

When I plan for "appropriate use of graphing calculators," my main intention is --

Something that was discussed today that I had not thought of before was --

Something that was discussed today that I do not agree with was --

Appendix G

Observation Checklist

Observation Checklist[1]

Classroom use of Technology for Teaching Mathematics

Teacher ____________________________ Date _____________

School: ____________________________ Grade level _________

	Teach/ **S**tudent		- - + +	Technology Use (Zbiek, 2002)					Representation Use S=same (Zbiek, 2002) D=Diff						
Start time of epi-sode	Instigator	User	User Comfort	CA Check Answer	GI Get Information	DW Delegate	GS Get Solution	IP Improve Presentation	M Manipulate	Au Augment	G Generate	I Interpret	C Connect/ Cf	As Ascribe	L Link

Note: If end time is not beginning of next use, draw arrow to appropriate end time.

[1]This form is based on Zbiek (2002a, 2002b), which give descriptions of each category. The codes are designed for use with transcripts of classroom activity, but will be used for observation.

Appendix H

Tables Relating Research Questions to Data Sources

Research Questions in Relation to Sort Tasks, Interview Questions and Survey			
Research Question	Sort Task	Interview question	Survey question
1a. What are the issues ...?	Explanation of 'sometimes' tasks	1, 3, 4, 5	Part A: 1, 2 Part B: 2, 3, 9, 13, 24, 26, 28, 29, 30, 32, 34, 35, 36, 42
1b. ...how do they negotiate those issues?	Explanation of 'sometimes' tasks	1, 3, 4, 5	
2a. How do teachers ... incorporate their prior instructional experiences ...?	All tasks	1, 2, 3, 4, 5	Part A: 1, 2 Part B: 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 44, 45, 50
2b. How do teachers ... incorporate ... artifacts that reify the ideas of others?	All tasks	1, 2, 3, 4, 5	Part A: 1, 2 Part B: 7, 10, 11
3a. ... what influence is exhibited by contextual factors ...	Explanation of 'sometimes' tasks	2, 4, 5	Part A: 1 Part B: 13, 24, 28, 38, 39, 40, 41, 42
3b. ... what influence is exhibited ... by teachers' beliefs and conceptions about graphing calculator use	All tasks	1, 2, 3, 4, 5	Part A: 2, Part B: 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 42, 48, 49, 51, 53, 54
3c. ... what influence is exhibited ... by teachers' beliefs and conceptions about the nature of mathematics?	All tasks	1, 2, 3, 4, 5	Part A: 2, 3 Part B: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 33, 37, 39, 43, 44, 46, 47, 52

Relation of Data Sources other than Session Transcripts to Research Questions						
Research Subquestions	Tool Work	Artifacts	Reflect-ions	Observa-tions	Observ.ation Interviews	Logs
1a. What are the issues ...?	X	X	X		X	
1b. ...how do they negotiate those issues?	X		X	X	X	
2a. How do teachers ... incorporate their prior instructional experiences ...?	X		X		X	
2b. How do teachers ... incorporate ... artifacts that reify the ideas of others?	X	X	X	X	X	X
3a. ... what influence is exhibited by contextual factors ...	X	X	X	X	X	X
3b. ... what influence is exhibited ... by teachers' beliefs and conceptions about graphing calculator use	X	X	X	X	X	
3c. ... what influence is exhibited ... by teachers' beliefs and conceptions about the nature of mathematics?	X		X	X	X	

Appendix I

Initial Interview Protocol

Protocol for introductory interview, before the task sort:

1. Ask if teacher has read the consent forms, if not, have them read them both.

2. Do you have any questions about the consent forms, or about the study?

3. Are you willing to participate, assuming times can be worked out?

4. If so, please sign the consent forms.

----

5. Do you need the 5 hours of PD credit? Would you be willing to start study groups at 4:30 instead of 3:30 so that someone else could join and receive PD credit?

----

6. How many years have you been teaching? How many years at RCPS?

7. Where did you receive your teaching degree? Were you a math major? Minor?

8. Tell me about any classes or workshops you have had that prepared you in some way to use calculators in teaching mathematics. Could you say some more about __________?

9. Did you use calculators at all in your high school classes? Talk a little about that experience.

Appendix J

Original Codes for Transcripts

Starting Codes		
Research Question	Initial Codes	explanation
1. What are the issues . . . ?	I	Teacher identification of Issues, topics, concepts seen as controversial
1b. . . . how do they negotiate those issues?	IRT , IRP, IRS, IRA ClEx	Responses (as reported by teachers) to issues by Teacher, Parents, Students, Administration; Collegial explanations
2a. How do teachers . . . incorporate their prior instructional experiences . . ?	TU TExp TBr	Teacher reference to own classroom uses or their experiences with technology in other venues Teacher as Broker – bringing new ideas
(2a) Teacher Pedagogical ideas, background	TPI – lrn - or.ct - p.u	Teacher pedagogical ideas – how students learn, order of content, procedure – understanding relations
2b. How do teachers . . . incorporate . . . artifacts that reify the ideas of others?	TA - cd TA - thr SU - cha - ginfo - delw - gsol - impp - M - Au - G - I - C - A - L RBr	Teacher references to Task Analysis, for cognitive demand or thought-revealing qualities Reports of Student Uses incorporating Zbiek's categories: cha = check answer; ginfo = get information; delw = delegate work; gsol = get solution; impp = improve presentation (of explanations); MAuGICAsL refer particularly to student use of representations -- M = manipulate; Au = augment; G = generate; I = interpret; C = connect; As = ascribe; L = link Researcher as Broker
3a. . . . . what influence is exhibited by contextual factors . . .	CX - tm (curr) - st - col	Teacher references to their teaching context, such as teaching materials or curriculum, student characteristics and exploration, colleagues
3b. . . . what influence is exhibited . . . by teachers' beliefs and conceptions about graphing calculator use	TBC - ppf - st.dp - expl - st.th - c.err	Teacher beliefs about calculators; e.g. Paper & pencil first; student dependence; exploration uses; student thinking effects; calculator errors
3c. . . . what influence is exhibited . . . by teachers' beliefs and conceptions about the nature of mathematics?	TBM - st.c - rl.b - liv - fal	Teacher beliefs about mathematics; e.g. static, unchanging; rule-based; living, changing; fallible

Appendix K

Tasks for Session 1

Problems for Session 1

I. Exploring Linearity – Adapted from *Algebra in a Technological World* (Heid, 1995, p. 73-74).

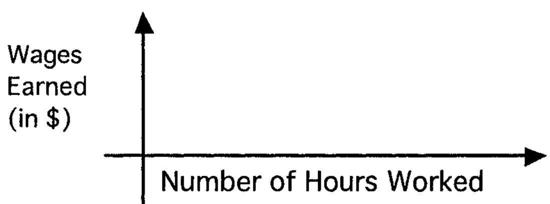
You plan to charge \$3.75 an hour for weekend yard work and want to calculate –

- the charge for time spend on the yard work – 2 hours, 3 hours, 4.5 hours, 10 hours, and so on;
- the amount of yard-work time needed to research some earning goal -- \$129 for a portable CD player or \$9.95 for a new CD.

1. Complete the following table showing the charges for 1 through 6 hours of yard work at \$3.75 and hour.

Time (in hours)	1	2	3	4	5	6
Charge (in dollars)						

2. Make a rough sketch of the graph of the relation between the number of hours worked and the wages earned on the unscaled axes below.



3. Write a function rule describing wages earned as a function *w* of the number of hours *h* worked.
4. Using your rule from question 3, write an equation or inequality whose solution provides the answer to the following question: How long must you work to earn at least \$25? Find the answer by producing a table of values or a graph.
5. Because it is sometimes difficult to get yard workers during the school year, hourly wages are higher then. Suppose that during the school year you are hired by a yard-work company at the rate of \$4.35 an hour with a \$50 bonus if you agree to work steadily.
   (a) Write a function rule that describes the relation between the number of hours *h* worked and the amount *w* you will be paid.
   (b) Use your rule to predict how much you will be paid for 45 hours of work. ...

8. Sometimes knowing rates of increase can help in comparing the growth in function values. Assume that you worked for \$3.75 an hour and your cousin worked for \$6.75 an hour in another state, but you both worked steadily for the same number of hours. She earned \$54 more than you. Discuss different ways to use the hourly rates to determine how many hours you both worked.

Problems for Session 1

II. Exploring Slope – adapted from *Advanced Algebra*, Holt, Rinehart, Winston, 1996, p.16 .

Graph the equation $y = \frac{2}{3}x + 1$. Trace any two points on the line and write the coordinates (x_1, y_1) and (x_2, y_2). Then calculate the quotient $\frac{(y_1 - y_2)}{(x_1 - x_2)}$. Compare values with others in your class. What can you conjecture about the value of this quotient along any part of the line? This quotient is called the *slope* of the line.

Appendix L

Issues List from Initial Interviews

Issues from Pre-study Interviews

Perceived Conditions
I -- if they don't know their 'tables'?
I -- "should be able to -"
I -- depends on their age
I -- If you set it up with them...
I -- calculator for computation only
I -- if they already know what to do
I -- If they've set it up already

Misuses of calculators
I -- not appropriate if learning order of operations

I -- children becoming dependent
I -- simple operations
I -- kids don't do long division
I -- playing
I -- students playing with calc
I -- some teachers just let them use calculators instead of learning how

Calculators may not help -
I -- students don't recognize proportions
I -- students lack facility with fractions
I -- domain and range
I -- proportions tested on MEAP
I -- know what they're looking at

Non-use better
I -- divide, fract for remain
I -- do in head
I -- fought against use of calc
I -- memorize
I -- more errors on calculator
I -- some things should be memorized
I -- some students faster w/out
I -- memory
I -- need to know some things without calculators

I -- teacher does not want to deal with this topic
I -- still work on skills, but outside class time; but kids that need it don't show up

Reasons to Use?
I -- how much the kids don't know
I -- students have different levels of proficiency

I -- why make kids do what I wouldn't do?
I -- Also need to learn to use calc
I -- learn from the kids
I -- should know, but don't
I -- student insecurity

Do First before Calculator Use
I -- Concepts first
I -- plot by hand at first
I -- by hand first, then calculator to check
I -- discuss first
I -- discuss first; visualize
I -- estimate first
I -- graph by hand first
I -- first time with problem different
I -- first understand problem
I -- expect some before using calculators; paper & pencil first
I -- know concept first
I -- know intercept first
I -- visualize first
I -- without a calculator first
I -- write tables first
I -- students should visualize before they graph on calculator
I --summary - do some by hand first

"By hand" not in "first"
I -- can't understand regress by hand
I -- hands on is necessary
I -- hands-on
I -- need to know by hand
I -- plot by hand, then calc
I -- plot by hand
I -- should be able to do by hand
I -- hands-on; do it yourself, not plug in
I --"plug and chug" by hand
I -- graph without calculator

Positive Uses
I -- faster with calc
I -- For making conjectures
I -- at some point they have to learn to compensate for the things they don't know
I -- easier with calculator
I -- break the equation down, what makes sense?
I -- fun
I -- good playing
I -- predicting from graph
I -- calculators help them not get bogged down in the things they can't do
I -- allows guess & check, trying
I -- easier to change graph
I -- plot on calculator
I -- calculator can be like a tutor
I -- calculators are included in some IPs

I -- if you haven't used it, you don't know what advantages it offers
I -- patterns are hard to recognize if you don't know multiplication
I -- memory is tough
I -- no exploration?
I -- some students cannot do basic arithmetic

Possible Misconceptions
I -- put equation in (but equation not given)
I -- slope is given in equation
Unknown implications
I -- I don't teach that
I -- context
I -- skills vs. concepts

I -- calculator use different later
I -- changing mind

I -- at some point we want to stop spending a lot of time on number crunching
I -- deliberate use of calculators comes with experience
I -- deliberate use of calculators can be guided
I -- 'exact' intercepts

Calculator types, access
I -- 82 vs 83+
I -- 83 vs 89
I -- 83 vs 90
I -- access to calc
I -- need for some kind of uniformity

Appendix M

Agenda, Artifacts, and Introduction for Session 1

Session One Agenda Feb. 17, 2003

1. Collect Surveys, Introductions: What do you like most, least about calculator use?
2. Fill out class observation schedule, with class time to visit (short time to talk after)

3. Revisit the Task Sort (10 min.)
   Rationale: I may have not been consistent in stressing the word "appropriate" in giving verbal instructions to each interviewee. Also, this will help recall the variety of tasks, and to tie together the tasks of the task sort and the tasks used as examples for Zbiek's framework.

4. Spend some time doing specific activities (Exploring Slope, Exploring Linearity -- reduced), thinking particularly about what students might do with them (30 min.)

5. Look at Zbiek's definitions and examples for <u>Purposes of Calculator Use</u> (10 min.)
   - CA – Checking Answers
     - Answer Checker – use one method
     - Alternative Checker – two methods
   - GI – Getting Information
     - Reference Chart – trig/log values, formula lists
     - Information Conveyer – to show as an illustration to students
     - Puzzler – results or appearance puzzles students
   - DW – Delegating Work
     - Arithmetic Aide – computations
     - Example Generator – multiple instances from which to reason
     - Representation Generator – table, graph, line through data
     - Algorithm Executor – regression line, intersection, max, min (usually not yet capable of doing without technology)
   - GS – Getting Solutions
     - Answer Giver – direct command, e.g. Solver
     - Dual Processor – use two or more methods for same problem
   - IP – Improving Presentation
     - Work Replication – duplicate another's work, or recall earlier work
     - Report Helper – communication or illustration of ideas
     - Motivation Provider – demonstration of effect of parameter
     - Attention Helper – calling attention to particular characteristic

6. Apply Zbiek's categories to the uses we made of calculators, and to what we thought students would do (35 min.)

*7. Decide what to put into our Tool for a beginning (20 min.)

8. Fill in Teaching Log items to check (5 min.)
9. Write Reflections on Session (10 min.)

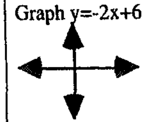
In this introduction to Zbiek's categories of student technology use, the researcher is speaking, while pointing to the document they all have (see following pages).

> Okay, we better, before we run out of time entirely, get at least to the topic that I wanted to introduce to you, and this is just the summary, this is actually the table of summarizing. ...When Rose Zbiek watches students, and she does sometimes high school, and sometimes college students, and sometimes graphing calculators, and sometimes computers, so she's trying to put it all in the same framework, to look at how they use...What I want you to look at are her categories that she developed.... You'll notice that there are heavy bars across here—the heavy bars divide up the big categories, so she really only has I think five categories. The first category, the CA, that stands for checking answers, but you notice that she breaks it down into three different ways, and I don't necessarily want us to have to get into that nitty-gritty, but that idea of checking.... The checking answers is one that we've already brought up, that that is a use that students make of calculators.
>
> The second big one, the GI, that's getting information. You can treat it as a reference chart—you're just looking up the value of pi, or whatever. An information conveyor, so that you can actually as a teacher use it to give information to your students, if you ever demonstrate a table or a graph or something like that. And then the last one, technology as a puzzle, this is what we're trying to get students to do is to question their calculator, "Why in the world did you give me that answer?" So that would be an example of technology as a puzzle. Sometimes the teacher will pose something that they know is going to give them—like when you did the negative 2-squared, and you wanted them to get that cognitive knock in the head, and some of them did and some of them didn't—you had to almost wring it out of them. So that's another use that the calculator may have.
>
> Then that next big long section, the DW, that's delegating work. There are lots of different ways—the arithmetic aid, the manipulation aid ...she's also including the ones that actually do algebraic manipulation, so the higher level calculators, so that would be included in this. Construction aid—that would be a computer thing, like the Geometer's Sketch Pad. Example generator—L, you were giving an example of that, when you graph a couple of graphs to compare them, you can do it quickly if you use a calculator. Representation generator—would be if you do tables or graphs or all of the different ways of representing. And the last one, on the next page, the algorithm executor—if you have a program that solves the quadratic equation or something like that, or actually, there are solve keys on these things, but I don't know if your students have discovered them yet. ...
>
> Okay, there's two more categories. The getting solutions—answer giver—and this is again, where you ask the calculator, for example, on a

> graph, when you use that intersect choice, "Find me the intersection." That would be getting the solution. I'm not just trying to check an answer, I'm not trying to just get information, I want the answer—I want nothing else, just give me the answer. And then dual processor is using two different ways to get the answer. So kind of using the calculator as a check on itself....
>
> Then the very last part, ...IP? ... I was going to say, it's enhancing your presentation. It must not be enhance. It can't be "I" if it's enhance! Anyway, it's using it to help make something clear to somebody else. So when the students are working together in groups, they might say, oh, look here at the table, this is how I can tell that this is the correct answer, or something like that. I'm also going to give these [Zbiek article on Two-tiered framework] to you, these are optional, but they give you way more detail than you might want to ask. (SG1, I, 255-282)

Zbiek's original table is reproduced here by permission of the author, as it was given to the participants.

Table 1. Technology purpose categories with descriptions and examples [* denotes category retained from 1998 version]

	Category	*Description*	*CAS/GC Example*	*Geometry Tool Environment*
CA 0	Unknown UK	Technology addressed or touched with an unknown attempt.	Student is asked to match 11 expressions with 6 graphs. Student reaches for calculator; interviewer/curriculum asks student to answer the question first without using the calculator	I could use the Sketchpad to do this. (followed by no Sketchpad use and no follow-up on what the use may have been)
CA 1	Answer Checker* AC	Task is completed by hand; the same task is then done by the tool; answers are compared	Graph y=-2x+6 on these axes:  Use a calculator to verify your work.	Given degree measures of two angles of triangle, add and subtract their sum from 180 degrees. Use tool to construct triangle with these measures and measure third angle.
CA 2	Alternative Checker* ALC	Complete a task by hand with one method; calculator is used with a related task or different method; students compare results to determine if first answer is valid	Simplify $\dfrac{x^4-4x^3-8x+32}{x^2-6x+8}$ To check your answer with tool, create a table that includes columns for both the original expression and your simplified expression.	Reason that changing constant in CAS-IM logistic function rule will move graph up and down. Use slider graph with tool to check this.
GI 1	Reference Chart* RC	Obtain factual information using tool like trig/log tables, formula lists, dictionaries, etc.	Approximate π to the nearest hundred thousandth.	Approximate e to the nearest hundred thousandth.
GI 2	Information Conveyer* IC	Used as handout or overhead to provide illustrations/information to students.	Shown in a [-5, 5, 1, -1.5, 1.5, .5] window are graphs of y = sin x and y = cos x. Students compare the two graphs, recalling complementary angles, to note that sin a = cos b if a + b = 0.5π.	Two triangles appear on screen and students determine if one can be the image of the other under a reflection.
GI 3	Technology as Puzzle TP	Tool results or process creates a result or appearance that puzzles students.	"Steps" appear on tool graph for step function family.	Dragging A of ∠ABC causes degree angle measure to increase through 170s then turn negative.
DW 1	Arithmetic Aide* AA	Tool used for numerical computations as part of a larger process that itself may or may not involve technology use Notion of delegating work to the tool is essential aspect of this category.	Enter $\boxed{2}\boxed{.}\boxed{7}\boxed{x}\boxed{8}\boxed{\wedge}\boxed{2}\boxed{=}$ while sketching by-hand graph of $y = 2.7x^2$.	Computing 360÷16 to determine the central angle measure when constructing a regular 16-gon using rotations.
DW 2	Manipulation Aide MA	Tool used for manipulation as part of a larger process that itself may or may not involve technology use. Notion of delegating work to the tool is essential aspect of this category.	Student solves quadratic equation with CAS in process of determining where graph of rational function with quadratic expression crosses the horizontal axis.	[Example may not exist]
DW 3	Construction Aide CA	Tool used for construction as part of a larger process that itself may or may not involve technology use. Notion of delegating work to the tool is essential aspect of this category.	[Example may not exist]	Student constructs horizontal line from point on slider graph to vertical axis to ascertain constant value of y for horizontal asymptote.
DW 4	Example Generator* EG	Quickly generate new instance or multiple instances from which to reason. Reasoning might involve observation and statement of patterns, followed by informal or formal explanation of why the patterns make sense.	Graph the following [using a graphics calculator]: $y = 3x^3$, $y = 7x^3$, $y = -2x^3$, and $y = -4x^3$. What effect does the sign of a have on graphs of equations of the form $y = ax^3$?	Dragging vertex of triangle to see that medians intersect over multiple examples of triangles.
DW 5	Representation Generator RG	Use tool to generate representation; representation may then be used in subsequent technology or non-technology work. Includes generation of an intentional case.	Use curve fitter to generate fitted function in mathematical-modeling problem-solving task.	Use script (with natural number and center point as parameters) to generate regular n-gon from which to reason about central angle measures.

DW 6	Algorithm Executor* AE	Performs tasks not (yet) doable by the user but necessary or useful in solving a problem or developing a concept	Fit various curves to data points to develop function model. What characteristics of the fitted functions (e.g., intercepts, monotonicity) match or conflict with real-world situation?	Determine image of point under circle inversion when not knowing what circle inversion is.
GS 1	Answer Giver* AG	Apply direct command to solve the stated problem	Solve $2x^2$-8x+9=17 using direct solve command	Given three points construct the circle passing through the three points.
GS 2	Dual Processor* DP	Use two tool-based methods to solve problem or explore concept	Solve a quadratic equation with the direct solve command, by zooming with graphs, and by successive tables	Show two rectangles are congruent by measuring corresponding parts and by generating image of one under product of isometries.
IP 1	Work Replication RW	Use the tool to replicate, to reproduce, or to recover the same tool work as previously done	Reproduce a series of examples while explaining a conclusion in order to show the examples that were considered in generating the conclusion	Using a script to record a construction for the purpose of playing it back to the class
IP 2	Report Helper* RH	Aids in communicating results of investigation Includes writing summary or conclusion.	Use text editor to write up observations about a family of functions	Add text box with problem statement. Use perpendicular congruent segments with common midpoint as axes.
IP 3	Motivation Provider* MP	Use the tool to produce images or other material to illustrate or inspire a concept, process, problem, lecture or lab	Recall of graphs of $f(x) = ax^3$, for a = -5, -3, -1, 1, 3, and 5 in quick succession to illustrate the effect on the graph of increasing the value of a in $f(x) = ax^3$	Show animation of dynamap to motivate study of rate of change.
IP 4	Attention Helper AH	Use the tool to draw the draw the attention of another person to some component of the display in conveying mathematics	Moving cursor to origin while saying " This lowest point matches where the dynamap hits the 0."	Clicks on the hypotenuse of a right triangle to show "the long side" that goes with c in $a^2+b^2=c^2$.

Appendix N

Teacher Responses to Reflection Prompts

Sessions 1 - 4

Session 1	End of session teacher reflections
Prompt 1:	When I think of "appropriate use of graphing calculators," my chief concerns are -
K	*Is the calculator being used for the intended purpose, not for arithmetic operations*
Y	*Dependence; Crutch*
R	R did not attend session 1
L	*They are using graphing calculators before they have the concept mastered or understood. They are using them too soon.*
T	*Using calculators for the wrong reasons*
Prompt 2:	When I plan for "appropriate use of graphing calculators," my main intention is --
K	*enhance what they are learning*
Y	*Enhancing student knowledge; furthering student knowledge; quicker solutions after learned skills*
L	*Time; the curriculum*
T	*to take the students further*
Prompt 3:	Something that was discussed today that I had not thought of before was --
K	*use of calculators in elementary school*
Y	*retention levels*
L	*How much do they lose over summer?*
T	*retention*
Prompt 4:	Something that was discussed today that I do not agree with was --
K	*starting the use of calculators at too early of age*
Y	*none*
L	*???*
T	*none*

Session 2	End of session teacher reflections
Prompt 1:	When I think of "appropriate use of graphing calculators," my chief concerns are --
K	*using the calculator for non-problem use*
Y	Yvette did not attend session 2
R	*what operations are they using them for; [student's] need basis; Teacher needs to decide*
L	*Do they understand what the calculator is actually doing?*
T	*They [concerns?] have not changed.*
Prompt 2:	When I plan for "appropriate use of graphing calculators," my main intention is --
K	*is to use it in achieving the answer I couldn't get without it*
R	*What can they get out of it?*
L	*Do they have this skill mastered? Will they get anything out of it?*
T	*to take the students to a higher level*
Prompt 3:	Something that was discussed today that I had not thought of before was --
K	*none*
R	*levels of problem-solving or tasking?*
L	*The task analysis guide and the levels of thinking*
T	*levels of tasks and how the calculator can change that; how are they/what are they getting out of my questioning and the calculator use?*
Prompt 4:	Something that was discussed today that I do not agree with was --
K	none
R	none
L	??
T	nothing

Session 3	End of session teacher reflections
Prompt 1:	When I think of "appropriate use of graphing calculators," my chief concerns are --
K	*only using them when you can do it otherwise*
Y	Y did not attend session 3
R	*assessment; when are calculators enhancing cognitive development of students*
L	*Do they have an understanding of what it is doing; Can the calculator enhance the learning?*
T	*have not changed*
Prompt 2:	When I plan for "appropriate use of graphing calculators," my main intention is --
K	*is to use them in addition to (as a help) the concepts I'm teaching*
R	*what can they use the calculator for? when is it inappropriate to use calculators?*
L	*What are the lesson's objectives? What do I want the students to learn?*
T	*to make sure the calculator is being used to get to my objectives*
Prompt 3:	Something that was discussed today that I had not thought of before was --
K	*Using them to express something several different ways to allow the student to grasp the concept the best way*
R	*appropriate use for elementary/ secondary; calculators added to a curriculum/ added work!*
L	*Are we discussing this only in terms of secondary? What about elementary?*
T	*secondary vs. elementary*
Prompt 4:	Something that was discussed today that I do not agree with was --
K	NA
R	*No disagreements except the complete use of the unabridged Core-Plus curriculum without dissolving complete use of calculator [without getting rid of use of calculator -- would like to see happy merger between Core-Plus concepts]*
L	??
T	

Session 4	End of session teacher reflections
Prompt 1:	What is the most important thing you learned from the discussions we have been having about appropriate use of graphing calculators? Why is it important?
K	*When is it appropriate (ask yourself) to use them? What purposes are you trying to achieve? Are the students using the calculators under false pretenses?*
Y	*Connections between paper and pencil concepts and calculator use concept*
R	*Proactivity (if you will) in deciding the necessity of calculators within certain lessons. Are we randomly using calculators, or can we strengthen our students' understanding, cognitively, with calculator usage?*
L [left session early]	*That all teachers or most teachers have concerns about the technology and calculator use. Important to know because I struggle with the idea and what is the best practices for calculator use in my room.*
T [missed session]	*Middle/High School teachers want the same thing from a calculator. They want students to use it as a tool and not a math "Bible." It is important to know this because we are all headed in the same direction.*
Prompt 2:	Do you think this group [you may exclude the researcher] agrees on the use of graphing calculators in teaching math?
K	*Yes*
Y	*Yes*
R	*Mostly, I want to continue a nontraditional route of math/exercising calculator use.*
L	*No*
T	*Yes*
Prompt 3:	If not, what is the most significant point of disagreement?
K	[left blank]
Y	[left blank]
R	*In math introduction --traditional/nontraditional use of calculators?*
L	*How often to use them and who uses them -- low or high students.*
T	[left blank]

Prompt 4:	If any, what might be the consequences of this disagreement?
K	[left blank]
Y	[left blank]
R	*A better understanding of where and when calculators can be introduced into the curriculum and/or lesson.*
L	*Expectations in one class may be lower or higher depending on philosophy.*
T	[left blank]
Prompt 5:	Next year, when you are planning lessons, what about our Tool for Teachers is most likely to enter into your thinking as you plan?
K	*Do I need to investigate the uses of the calculator to help me in my presentation of the material?*
Y	*The first question of the Tool [on how students are using calculators]*
R	*How can I use calculators to not only strengthen the lesson, but to also introduce, keep out, or randomly use for the lesson.*
L	*Just being aware of who is using them and for what reason.*
T	*I want to make sure that I'm using the calculator as a tool. I want ot make sure that they are thinking mathematically.*
Prompt 6:	Please tell me anything that would have made these sessions more worthwhile in your exploration of appropriate use of graphing calculators.
K	*Maybe more time for discussions and having everyone here at each session.*
Y	N/A
R	[left blank]
L	*What students thought.*
T	*I don't know.*

Appendix O

Agenda, Artifacts, and Introduction for Session 2

Cognitive Demand of Tasks Session Two March 13, 2003

What was useful about the Calculator Uses categories and Log? Write something for our Tool.

To help focus: Issues from Session 1 – which can we solve by our calculator use decisions?

All ideas and illustrations below are taken from *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development,* by M. K. Stein, M. S. Smith, M. A. Henningsen, and E. A. Silver (New York: Teachers College Press, 2000).

- Not all tasks are created equal – different tasks require different levels and kinds of student thinking. We refer to the kinds of thinking needed to solve tasks as their *cognitive demands.*
- The cognitive demands of tasks can change during a lesson.

THE MATHEMATICAL TASKS FRAMEWORK

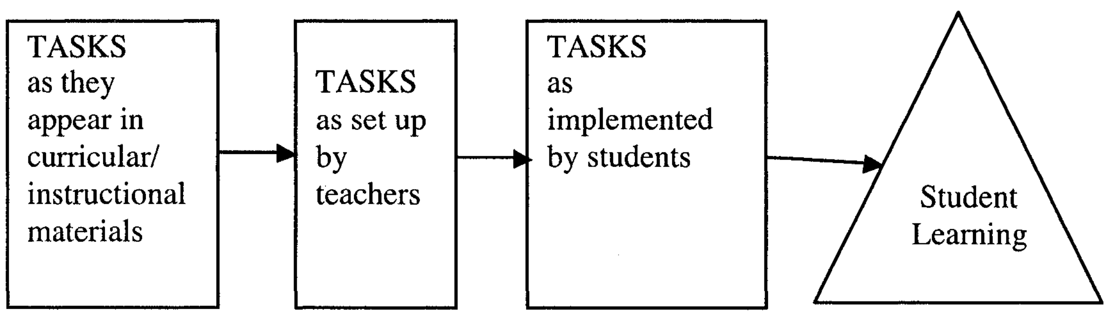


Figure from *Mathematics Teaching in the Middle School*, by Stein and Smith, copyrighted 1998 by NCTM.

Matching Tasks with Goals for Student Learning
Not all tasks provide the same opportunity for learning.

Differentiating Levels of Cognitive Demand
The Task Analysis Guide (p. 16)

Gaining Experience in Analyzing Cognitive Demand
Using the Task Analysis Guide on Stein et al. sample activities (p. 19)
A collaborative sorting activity – in pairs

Applying the Task Analysis Guide to Session 1 Activities
If time, Model-eliciting activities

What to watch for until April 3 – put these in your Calc Use Log

Write reflection

Issues from Session One
(58 in all)

I - always believe calculators
I - benefit for students
I - better to do by hand, but no time
I - book introduces new concept with calculators
I - calculators intended only for "higher level"

I - calculator faster, ease of comparisons
I - calculator smarter than me
I - check answers
I - checking the calculator

I - common term
I - concepts first
I - curriculum says use calculators -have to follow

I - dependent

I - do they know the concept?
I - don't know difference between 2x3 and 2^3
I - don't know operations
I - elementary teachers do not know math
I - estimate
I - faster by hand
I - foundations
I - fraction calculators
I - fraction key
I - fractions
I - garbage in, garbage out
I - go deeper
I - group work
I - I didn't use a calculator until college ...
I - I'm doing just fine
I - if I disagree with curriculum, I don't use calc
I - kids believe calculator
I - kids don't read problems
I - kids just want answer
I - know what technology is doing
I - lack of math skill irritating
I - laziness
I - meaningful to kids
I - misuse
I - more problems in same time
I - no slide rules in elementary
I - only look at numbers
I - operations
I - pacing
I - reading
I - reading math
I - retention
I - see another way
I - sometimes don't use calculators
I - student placement
I - syntax errors hard to find
I - teachers don't want to teach use of calculator
I - teaching can be fun
I - teaching math reading
I - technology here to stay
I - technology makes some math obsolete
I - we do teach how to use
I - what do they lose?
I - who learns what on the calculator?
I - work to learn calculator so I can show

Which can our tool address?

Transcript of the Session 2 introduction to the Task Analysis Guide (Stein, 2000).

Guide is reproduced on the next page by permission.

Two of the points that they make in bringing up, "why do we want to look at tasks?" are numbers 1 and 2 that are given here: not all tasks are created equal; different tasks require different levels and kinds of student thinking. We refer to the kinds of thinking needed to solve tasks as their cognitive demands. So that's the framework that they're using, they're looking at tasks and trying to focus on what kind of thinking do students have to do in order to solve this task. And then the second point, that brought them to develop this book, is that the cognitive demands of tasks can change during a lesson. And I think you've probably seen that happen. You are planning your lesson, and you pick out this activity and you think this is what they're going to do with it, and you give it to them, and it amazes you what they do with it. And that's an example of the cognitive demands changing, just because of what the students do with it. Sometimes it's what the teacher does with it. I think what I would like us to concentrate on is what does a calculator do to certain kinds of tasks.

This first diagram just illustrates that question #2, and really, this is as far as we're going to go with that part of it. You start out with the tasks as they appear and then the teacher picks it out and says good, sets it up for the kids, then the kids take it and they do something. It's kind of like—those of you who have been to school recently would have heard about—how do they put that—the intended curriculum, the implemented curriculum, and then what the kids actually learn. It's sort of that same idea. Really, the student learning depends on how the students themselves implement the tasks, which depends on...we go all the way back. Let's go a little bit further now, past the figure—what they emphasize then is that it's important to match the tasks with the goals that we have for student learning. So if our goal is that students should memorize the multiplication tables, then probably calculators are not the best way to do that. But there would be tasks that allow that to happen more efficiently.

But if at a different time, our goal is to get those kids thinking about something, then probably this memorization task is not the best way to get them thinking about something. So the selection of the task is an important thing, and that's another point that they make in here. They wanted to be able to somehow—and this is similar to what I gave you last week, which was a researcher trying to get at describing what kids do with calculators, so they set up these categories. That's the same thing that they did here. They observed teachers teaching tasks, and observed what happened in classrooms where various tasks were used, and decided that there were four categories that they wanted to use, and that's what I would like us to take a look at today, is what these four categories are. And then

in order to do that, we're going to look at the task analysis guide—I have it here—then we're going to look at some sample tasks that they used.

That's where I want you to split up into pairs and then I'll get both tape recorders going. First you need to take a look at this, and this is straight out of the book except I retyped it.... You notice that they have four types of tasks, but they also have two columns. The one column is the low level demands, and this has to do with the cognitive activity of the students—how hard are they thinking—and then the high level demands of the task.

Memorization tasks have these [pointing] certain characteristics. These two, the last one in the lower level and the first one in the upper level, both are about procedures. But the difference between the lower level and the higher level is how those procedures are connected. In the lower level it's procedures without connections, and in the higher level it's procedures with connections. The last category is the one they call "doing mathematics." I think that's actually fun to do with kids so they get a sense that there's really more to doing mathematics than regurgitating what you've memorized, or following some procedure that you learned how to do. So getting to the "doing the mathematics" tasks is one of the goals.

I know that you're going to have to take some time to read through these, but I want to give you the tasks that I would like you to look at. These are also from this book. And this is actually from a research project that was done over a period of four or five years with middle school teachers and students. So the tasks will be more on the middle school level, but I figure you can probably figure out what that means. These are some samples of tasks that they had teachers work with, along with these characteristics to try to decide which of these fall in those categories. I'd like you to pair up and talk it through. You can talk first about the definitions if you'd like to, and then look at the individual tasks (SG2, I, 100-105)

The Task Analysis Guide

Lower-Level Demands	*Higher-Level Demands*
Memorization Tasks –	Procedures With Connections Tasks –
▪ involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. ▪ cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. ▪ are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. ▪ have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	▪ focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. ▪ suggest pathways to follow (expicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. ▪ usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. ▪ require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks –	Doing Mathematics Tasks –
▪ Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. ▪ Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. ▪ Have no connection to the concepts or meaning that underlie the procedure being used. ▪ Are focused on producing correct answers rather than developing mathematical understanding. ▪ Require no explanations, or explanations that focus solely on describing the procedure that was used.	▪ Require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). ▪ Require students to explore and understand the nature of mathematical concepts, processes, or relationships. ▪ Demand self-monitoring or self-regulation of one's own cognitive processes. ▪ Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. ▪ Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. ▪ Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

from *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development,* by M. K. Stein, M. S. Smith, M. A. Henningsen, and E. A. Silver (New York: Teachers College Press, 2000), p. 16. Reprinted with permission from "Mathematical Tasks as a Framework for Reflection: From Research to Practice," *Mathematics Teaching in the Middle School*, Jan. 1998, 268-275, copyright 1998 by the National Council of Teachers of Mathematics. All rights reserved.

Appendix P

Agenda and Artifacts for Session 3

Session Three April 3, 2003

3:30 Look at the Issues – can our tool address these? What about the last ones?
Are any issues missing?

3:45 Look at the tentative Tool
Does this address any of the issues?
How might we address more?

4:00 Calculator Use Log – did you think at all about the cognitive demand of
the lessons you used in class?

4:10 One more look at cognitive demand of lessons – a Core-Plus activity, an algebra activity
CPMP 1A p. 211 – 215 (inequalities); UCSMP Algebra p. 4 – 8 (variables, inequalities)
What is the intent of the lesson?
Do students need to solve symbolically?
What are the 'big ideas' of the lesson?
If time, examples of model-eliciting problems – Big Foot

4:45 The Branca questions – How some teachers guided their own thinking about calculator use
What do you think of these questions?
Do they address our issues?
Is the question format useful? Could we use it for our issues?

5:00 Revising our tool

5:15 What to look for until April 24 – Calculator Use Log

5:20 Set classroom observations – April 15 – 23

5:22 Write reflections

For Feedback

The number following each issue is the number of you who indicated the tool could deal with the issue.

I have placed the issues that at least one person thought could be addressed into a number of categories. I would like to know if you disagree with any of the issues' placement, or have a disagreement (or support) you want to express for any of these.
Note that I question the last category, and would especially like feedback on that one. Note that I bracketed comments that I added.

Possible categories for the tool to deal with [keep in mind that we are trying to help DECISIONS]:

How students benefit from using calculators

I - benefit for students 4
I - calculator faster, ease of comparisons 2
I - go deeper 2
I - technology here to stay 3
I - meaningful to kids 2
I - teaching can be fun 1

Which students benefit from using calculators

I - calculators intended only for "higher level" 2 [of math or of students?]

Student misconceptions about calculators

I - always believe calculators 1
I - calculator smarter than me 1
I - checking the calculator 1
I - kids believe calculator 2
I - kids just want answer 2
I - know what technology is doing 2 [they often don't]

Teachers need to think about

I - concepts first 1
I - do they know the concept? 2
I - estimate 1
I - kids don't read problems 1
I - teachers don't want to teach use of calculator 1

Drawbacks from calculator use?

I - don't know difference between 2x3 and 2^3 1
I - don't know operations 1
I - what do they lose? 2 [originally: what do they lose over summer?]
I - operations 1

**Can the tool deal with these?

I - elementary teachers do not know math 1
I - lack of math skill irritating 1
I - reading 1
I - retention 1
I - student placement 1

First Attempt at Tool March 18, 2003
for making decisions about appropriate use of graphing calculators in teaching mathematics

From notes made at the meeting, March 13, 2003:

Be aware of what students are doing with calculators. Are they being used to (as)
- Check answers?
- Arithmetic aid?
- Reference sheet?
- Answer finder?
- Presentation helper?
- Visualizer?

Be aware of which students legitimately need calculators, due to disability.

Beyond teaching students how to use the calculator, ask them to interpret answers once they get them.

Be aware of the complexity of the calculator students are using. Is it more complex than is needed for the task?

Ask students to make connections between what they do with the calculator and the problem they are trying to solve.

Obviously, I added the possible uses – would you add others that you think should be thought about?

Does this wording make it clear that we are trying to help teachers MAKE DECISIONS?

Do you have suggestions for better wording? "Be aware" might be difficult for teachers to relate to.

Branca, N. A., B. A. Breedlove, et al. (1992). Calculators in Middle Grades: Access to Rich Mathematics. Calculators in Mathematics Education. J. T. Fey and C. R. Hirsch. Reston, VA, National Council of Teachers of Mathematics. **1992:** 9-13.

The authors use their students' voices to show the potential of calculator use. They devised questions to guide teachers in calculator use.

Questions for Teachers:

1. Does the calculator allow the students to get closer to mathematical concepts being presented?

"In fact, manipulating calculator keys becomes the gateway for engaging some students in thinking about mathematical problems and the concepts underlying those problems." p. 11

2. Will the use of the calculator in a mathematics activity increase student confidence and persistence?

3. Could the concept be taught with an inductive approach?

4. Would the use of the calculator facilitate the study of real-life applications?

5. Will using the calculator allow assessment to be focused on relevant instructional objectives?

Appendix Q

Agenda and Artifacts for Session 4

Agenda for Session 4 April 23, 2003

This is the last session!

First Priorities – to finalize the Tool – discuss "second attempt" 15 min.
– to evaluate its usefulness – see task below

Please work on the following task on your own for about 10 minutes. Think especially about the second part of the question, and about these questions:

- What concepts need to be understood before students use calculators for this task?
- What level task is this? Why?
- How might students approach this task? Is any way the "right" way?
- Does our Tool help decide whether calculators are appropriate for this task?

Task S

Solve for x:

$5(x-2)-3(2x+1)=2+5x$

What does this tell you about the intercepts of the graph of $y = -15 - 6x$? Why?

Discussion of the task, and the usefulness of the tool 30 min.

Finally, to identify whatever has been overlooked
(e.g. connections, a la MAuGICAsL). 30 min.

Final adjustments to our Tool – agree on wording 15 min.
Would you be willing to share this with your colleagues? With Teresa?
Do you want to read what I write?
Would you be willing to read parts that I feel need verification?
Summer contact Choosing pseudonym?

Writing the last reflection 10 min.

THANK YOU SO MUCH!!

Second Attempt at Tool April 23, 2003

for making decisions about appropriate use of graphing calculators in teaching mathematics

From notes made at the meeting, April 3, 2003, with changes in italics:

Note: This tool is intended for middle school and high school teachers. Elementary teachers may need to consider other questions.

What are students doing with calculators? Are *calculators* used to (as)

Check answers?
Arithmetic aid?
Reference *to look up information, such as* $\sqrt{7}$?
Answer finder, *such as finding intersections or intercepts*?
Presentation helper, *to help explain ideas to others*?
Visualizer?

Which students legitimately need calculators, due to *their IEPs*?

Beyond *learning to push calculator buttons,* are students asked to interpret answers once they get them? *Is conceptual understanding assessed with and without calculators?*

Is the calculator necessary for the lesson's objectives? Are students asked for a deeper understanding – more than numerical answers?

Do students understand the limitations of the calculator? Is *the calculator students are using* more complex than is needed for the task?

Are students making connections between what they do with the calculator and the problem they are trying to solve. *This opportunity is a major benefit of using calculators with real-life problems.*

Appendix R

Session 4 Teacher Reflections Form

Reflections on Final Session 4-24-03

What is the most important thing you learned from the discussions we have been having about appropriate use of graphing calculators? Why is it important?

Do you think this group [you may exclude the researcher] agrees on the use of graphing calculators in teaching math?

If not, what is the most significant point of disagreement?

If any, what might be the consequences of this disagreement?

Next year, when you are planning lessons, what about our Tool for Teachers is most likely to enter into your thinking as you plan?

Please tell me anything that you think would have made these sessions (or future sessions) more worthwhile in your exploration of appropriate use of graphing calculators.

Appendix S

Planning Document for Final Interviews

Final Interview plans and Planning Notes

Note: After the first final interview (with K) I decided that it took too long to read through the entire survey, even if they commented only on the items they decided to change. So for succeeding final interviews, I asked teachers to look only at questions they had not answered the first time through, or at questions on which the earlier survey results showed broadly diverse answers. Specifically, I asked that they look at questions 2 – 10, 20, 27, 32, 36, 42 – 54. Note that in some cases I also asked that they look at surrounding questions, so that there would not be a tendency to change just because I asked them to look at one or two questions. When they spoke about why they changed, it didn't seem to be just because they were asked to look at them again.

Most of the questions designed for individual teachers are drawn from re-reading the first interview and task sort, the early survey responses, and my general impressions of these teachers during the sessions. Not all questions were asked in the actual interviews – mostly due to time, but sometimes because of the direction the conversation took.

Survey A

T, L, K – Understanding before calculators – understanding of what? Concepts? Such as – for graphing linear equations, understanding of slope? Definition?

Survey B

Y, L – Academic major – Survey B1

R – as you review Part B of the survey, you are "Neutral" on many of the questions about calculator use – esp. questions 2 – 10. Comments?

R, L – Methods classes that used calculators – Survey B11

All – Q B36 – clarify the knowledge of the state high school mathematics test – did I call it by the wrong name?? Did that affect your response to the question?

Task Sort – talk as you sort, or sort first then talk?

All – Did you have any district or regionally-based professional development specifically dealing with the mathematics curriculum you are teaching?

R – you mentioned "traditional/ non-traditional" use of calculators. Please expand on what you mean by that. . . . And you said you would like to continue "nontraditional" use, is that correct?

R – are you the one who mentioned "stages" of calculator use? I think you thought of that as a way of working with students? Do you think that might also be true with teachers?

K – In our first interview, you said several times that "I would teach it without the calculator first, and use the calculator after." Do you think there is ever a time that using calculators first would be legitimate? Can you think of an example? [Does this ever happen in your alg 2 book?]

K – You also said you would like students to "do it by hand first to see whether their thinking processes are right." Do you think it is easier to see their thinking processes when they are working by hand? Why?

K – Talk about the times you decided to speak up in the sessions – what motivated you to speak up? [It seemed to me that K often took the other side of things, and often it had to do with new thoughts he had, or ideas he gleaned from readings.]

L – Ask about elective calculator class – at [university]. What was your reaction to that class? So basically, did you have any guidance in how to use calculators to teach? Do you have any idea why you were so against calculators in your first two years?

L – You mentioned why some of your ideas against calculators changed as you were teaching. Have any of our discussions either confirmed your thinking or suggested other changes?

L – Talk about the strengths and weaknesses your students have as they go over to the high school. What percent do you have the most concern about? Has calculator use in middle school hurt them, do you think? Now think of the rest of the students, as a whole. Has using calculators helped or hindered them?

T – Did anything in your education specifically help you think about using calculators for teaching mathematics?

T – You have been pretty definite about not wanting elementary students to use calculators, and about being careful to do things by hand first. Can you think of any situation for which it might be appropriate to introduce a topic by using a calculator? Why would that be appropriate? [Or why would nothing be appropriate? How about inverse functions?]

Y – As I re-read our first interview, you seemed to be very concerned that calculators should be allowed to make children "dependent." I think you even used the word "handicapped." Did you have that worry about calculators before you started teaching at RCPS?

Y – In what ways do you think our tool (give copy) will help combat the causes of that dependency? Which part of the tool do you think is most useful?

Y – Are there any parts of the sessions that you attended that stand out in your mind – either as controversial, or as particularly helpful to you? If so, why? If not, what had you hoped for?

Appendix T

Pre- and Post-Study Teacher Responses

to Selected Survey Items

During the final interview, teachers were asked to specifically review their responses to a number of the survey questions, and to talk about whether they would answer any of the questions differently as a result of the sessions. Questions reviewed were 2 –10, 20, 27, 32, 36, 42 – 54.

Teacher Survey, Part B   Response Review   Teacher Name L, Y, T, K, R

For each statement, please mark the 0 under the response that most accurately represents your feelings. **SD** = strongly disagree, **D** = disagree, **N** = neutral or don't know, **A** = agree, **SA** = strongly agree

Mark one "0" for each statement		**SD**	**D**	**N**	**A**	**SA**
2. Incorporating calculators into teaching requires changing the types of problems assigned.	Pre	0	KLTY	R	0	0
	Post	0	KL	0	RY	T
3. Students need to demonstrate proficiency in using mathematical procedures before doing any similar work using calculators.	Pre	0	R	0	KLTY	0
	Post	0	KRY	0	LT	0
4. It is not necessary to change what is taught in order to effectively use calculators.	Pre	0	0	0	RLTY	K
	Post	0	0	0	RLTY	K
5. Students should be allowed to use calculators even before they understand the underlying concepts.	Pre	K	LTY	R	0	0
	Post	K	L	R	Y	T
6. More difficult mathematics problems can be done when students have access to calculators.	Pre	0	R	0	KLY	T
	Post	0	R	0	KLY	T
7. Using calculators frees students to explore alternative solution strategies.	Pre	0	R	0	KLY	T
	Post	0	R	0	KLY	T
8. Continued use of calculators will cause a decrease in student estimation skills.	Pre	0	KTY	R	L	0
	Post	0	KTY	R	L	0
9. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *less* important.	Pre	0	RY	0	KLT	0
	Post	0	RYT	0	KL	0
10. Presence of calculators in classrooms, and outside of school, makes some mathematics topics *more* important.	Pre	0	Y	R	KLT	0
	Post	0	0	R	KLTY	0
20. Students learn mathematics by the personal building of mathematical understanding.	Pre	0	K	R	L	TY
	Post	0	0	0	RKL	TY

For each statement, please mark the 0 under the response that most accurately represents your feelings. **SD** = strongly disagree, **D** = disagree, **N** = neutral, **A** = agree, **SA** = strongly agree

Mark one "0" for each statement		SD	D	N	A	SA
27. Being able to accurately perform an algorithm is necessary for understanding the underlying mathematical concepts.	Pre	T	R	K	LY	0
	Post	T	R	K	LY	0
32. Using calculators will cause students to lose basic computational skills.	Pre	0	RT	L	KY	0
	Post	0	RT	0	KYL	0
36. I am well-informed about the Michigan High School Proficiency Test for Mathematics (1998) **state mathematics test** for high school students.	Pre	0	KLT	R	Y	0
	Post	0	KL	R	Y	T
42. Students are dependent on calculators when they come to my class.	Pre	0	K	R	LTY	0
	Post	0	0	RK	LTY	0
43. Mathematics is fixed and unchanging.	Pre	T	RKY	L	0	0
	Post	T	RKYL	0	0	0
44. Mathematical ideas are constructed by human minds.	Pre	0	0	RY	KLT	0
	Post	0	Y	R	KLT	0
45. Expecting students to be creative in mathematics is unreasonable.	Pre	0	RKLTY	0	0	0
	Post	0	RKLTY	0	0	0
46. Mathematics is essentially hierarchical and cumulative.	Pre	0	R	T	KLY	0
	Post	0	R	0	KLYT	0
47. A world with different mathematical truths is impossible.	Pre	0	KL	RT	Y	0
	Post	0	KR	T	YL	0
48. Students should not be allowed to use calculators until they have mastered concepts.	Pre	0	RLTY	0	K	0
	Post	0	RLTYK	0	0	0
49. Students should be allowed to use calculators on standardized tests.	Pre	0	0	R	KLY	T
	Post	0	0	R	KLY	T
50. I am confident in my ability to teach mathematics using calculators.	Pre	0	0	R	KLTY	0
	Post	0	0	0	KLTYR	0
51. The major value of calculators in mathematics classes is to save time from performing computations.	Pre	0	RKLT	0	Y	0
	Post	0	RKLTY	0	0	0
52. Students understand math better if they solve problems using only paper and pencil.	Pre	0	RKLTY	0	0	0
	Post	0	RKLTY	0	0	0
53. Using calculators in the teaching of mathematics results in greater student understanding of concepts.	Pre	0	T	RY	KL	0
	Post	0	0	RY	KLT	0
54. Using calculators in the teaching of mathematics encourages a more active, conjecturing approach to the learning of mathematics.	Pre	0	L	0	RKTY	0
	Post	0	L	0	RKY	T

Appendix U

Human Subjects Institutional Review Board Approval

Human Subjects Institutional Review Board

Centennial
1903-2003 Celebration

Date: January 23, 2003

To: Christian Hirsch, Principal Investigator
Marcia Weller Weinhold, Student Investigator for dissertation

From: Mary Lagerwey, Chair Mary Lagerwey

Re: HSIRB Project Number 03-01-17

This letter will serve as confirmation that your research project entitled "How Teachers Construct an Understanding of "Appropriate Use" of Graphing Calculators in the Context of Collegial Enquiry" has been **approved** under the **exempt** category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note that you may **only** conduct this research exactly in the form it was approved. You must seek specific board approval for any changes in this project. You must also seek reapproval if the project extends beyond the termination date noted below. In addition if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: January 23, 2004

Walwood Hall, Kalamazoo, MI 49008-5456
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BIBLIOGRAPHY

Artigue, M. (1997). *Computer environments and learning theories in mathematics education.* Paper presented at the International Derive and TI-92 Conference, Bonn.

Board acts on delegate assembly resolutions. (January 1988). *NCTM News Bulletin*, 3.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice based theory of professional development. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco, CA: Jossey-Bass.

Bohl, J. V., & Van Zoest, L. R. (2001). *Becoming a mathematics teacher: A framework for understanding development across communities in the context of reform.* Paper presented at the 25th annual meeting of the International Group for the Psychology of Mathematics Education, Utrecht, Netherlands.

Branca, N. A., Breedlove, B. A., & King, B. W. (1992). Calculators in middle grades: Access to rich mathematics. In J. T. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education* (Vol. 1992 Yearbook, pp. 9-13). Reston, VA: National Council of Teachers of Mathematics.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences, 2*(2), 141-178.

Burrill, G., Allison, J., Breaux, G., Kastberg, S., Leatham, K., & Sanchez, W. (2002). *Handheld graphing technology in secondary mathematics: Research findings and implications for classroom practice.* Texas Instruments.

Calculators in the mathematics classroom. (May 1986). *NCTM News Bulletin,* 6.

Coe, D. E. (1990, April 18, 1990). *Toward collegial inquiry: is there more to clinical supervision than the improvement of practice?* Paper presented at the Annual Meeting of The American Educational Research Association, Boston, MA.

Coxford, A. F., Fey, J. T., Hirsch, C. R., Schoen, H. L., Burrill, G., Hart, E. W., & Watkins, A. E. (1998-2000). *Contemporary Mathematics in Context: A unified approach* (1st ed.). Chicago: Everyday Learning.

Coxford, A. F., Fey, J. T., Hirsch, C. R., Schoen, H. L., Burrill, G., Hart, E. W., & Watkins, A. E. (2003). *Contemporary Mathematics in Context: A unified approach.* Columbus, OH: Glencoe McGraw-Hill.

Cuban, L., Kirkpatrick, H., & Peck, C. (2001). High access and low use of technologies in high school classrooms: explaining an apparent paradox. *American Educational Research Journal, 38*(4), 813-834.

Dick, T. P. (1992). Super calculators: Implications for the calculus curriculum, instruction, and assessment. In J. T. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education* (Vol. 1992, pp. 145-157). Reston, VA: National Council of Teachers of Mathematics.

Doerr, H. M., & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics, 41*, 143-163.

Dunham, P. H. (2000). Hand-held calculators in mathematics education: a research perspective (pp. 39-47). Columbus, OH: The Ohio State University.

Dunham, P. H., & Dick, T. P. (1994). Research on graphing calculators. *The Mathematics Teacher, 87*(6), 440-445.

Engestrom, Y. (1993). Developmental studies of work as a testbench of activity theory: The case of primary care medical practice. In S. Chaiklin & J. Lave (Eds.), *Understanding practice: Perspectives on activity and context* (pp. 64-103). Cambridge: Cambridge University Press.

Eraut, M. (2002, April 1-5, 2002). *Conceptual analysis and research questions: do the concepts of "learning community" and "community of practice" provide added value?* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Fey, J. T., & Hirsch, C. R. (Eds.). (1992). *Calculators in mathematics education.* Reston, VA: National Council of Teachers of Mathematics.

Fleener, M. J. (1995a). The relationship between experience and philosophical orientation: A comparison of preservice and practicing teachers' beliefs about calculators. *The Journal of Computers in Mathematics and Science Teaching, 14*(3), 359-376.

Fleener, M. J. (1995b). A survey of mathematics teachers' attitudes about calculators: The impact of philosophical orientation. *The Journal of Computers in Mathematics and Science Teaching, 14*(4), 481-498.

Franke, M. L., & Kazemi, E. (2001). Teaching as learning within a community of practice. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: teaching elementary school mathematics* (pp. 47-74). Mahwah, NJ: Lawrence Erlbaum Associates.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49*, 193-223.

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: the case of calculators. *International Journal of Computers for Mathematical Learning, 3*, 195-227.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* . San Francisco: Jossey-Bass.

Heid, M. K., Choate, J., Sheets, C., & Zbiek, R. M. (1995). *Algebra in a technological world*. Reston, VA: National Council of Teachers of Mathematics.

Hembree, R., & Dessart, D. J. (1986). Effects of hand-held calculators in precollege mathematics education: A meta-analysis. *Journal for Research in Mathematics Education, 17*, 83-99.

Hembree, R., & Dessart, D. J. (1992). Research on calculators in mathematics education. In J. T. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education* (Vol. 1992, pp. 23-32). Reston, VA: National Council of Teachers of Mathematics.

Hershkowitz, R., & Schwarz, B. (1999). The emergent perspective in rich learning environments: Some roles of tools and activities in the construction of sociomathematical norms. *Educational Studies in Mathematics, 39*, 149-166.

Hill, H. C. (2001). Policy is not enough: Language and the interpretation of state standards. *American Educational Research Journal, 38*(2), 289-318.

Hirschhorn, D. B., & Senk, S. (1992). Calculators in the UCSMP curriculum for grades 7 and 8. In J. T. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education* (Vol. 1992, pp. 79-90). Reston, VA: National Council of Teachers of Mathematics.

Hodges, H. L. B. (1996). Using research to inform practice in urban schools: Ten key strategies for success. *Educational Policy, 10*(2), 223-252.

Horizon, Inc. (2000). National Science Foundation Local Systemic Change teacher survey . Reston, VA.

Jeon, K. (1999). *Teacher in transition: A mathematics teacher's understanding of teaching with the graphing calculator*. Unpublished Ph.D. dissertation, University of Georgia, Athens.

Johnson, T. M. (1994). *A teacher's roles and calculator tasks in two twelfth-grade mathematics courses*. Unpublished Ph.D. dissertation, University of Wisconsin, Madison.

Lesh, R. (2002). Research design in mathematics education: Focusing on design experiments. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 27-50). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., Carmona, G., & Post, T. (2002, October 26-29). *Models and modeling*. Paper presented at the North American Chapter of the International Group for the Psychology of Mathematics Education, Athens, GA.

Lesh, R., & Clarke, D. (2000). Formulating operational definitions of desired outcomes of instruction in mathematics and science education. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 113-150). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-645). Mahwah, NJ: Lawrence Erlbaum Associates.

Little, J. W., & McLaughlin, M. W. (1993). Introduction: Perspectives on Cultures and Contexts of Teaching. In J. W. Little & M. W. McLaughlin (Eds.), *Teachers' Work: Individuals, Colleagues, and Contexts* (pp. 1- 8). New York: Teachers College Press.

Little, J. (1987). Teachers as colleagues. In V. Richardson-Koehler (Ed.), *Educator's handbook: A research perspective* (pp. 491-518). New York: Longman.

MSEB. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

MSEB. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum.* Washington, DC: National Academy Press.

Moore, J., & Barab, S. (2002). The inquiry learning forum: a community of practice approach to online professional development. *Tech Trends, 46*(3), 44-49.

Murdock, J., Kamischke, E., & Kamischke, E. (1998). *Advanced algebra through data exploration: A graphing calculator approach.* Berkeley, CA: Key Curriculum Press.

Myers, K. A. (1998). *The use of graphing/symbolic calculators in high school mathematics: A study of teacher knowledge, beliefs, and practices.* Unpublished Ed.D. dissertation, University of Cincinnati, Cincinnati, OH.

NCTM. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s.* Reston, VA: Author.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

NCTM. (1991a). *Professional standards for teaching mathematics.* Reston, VA: Author.

NCTM. (1991b). Standards blackline masters and presentation guide, *Impact of technology on the mathematics curriculum* (Transparency 34). Reston, VA: National Council of Teachers of Mathematics.

NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Norton, S., McRobbie, C. J., & Cooper, T. J. (2000). Exploring secondary mathematics teachers' reasons for not using computers in their teaching: Five case studies. *Journal of Research on Computing in Education, 33*(1), 87-109.

Palincsar, A. S., Magnusson, S. J., Marano, N., Ford, D., & Brown, N. (1998). Designing a community of practice: Principles and practices of the GIsML community. *Teaching and Teacher Education, 14*(1), 5-19.

Palmer, P. J. (1998). *The courage to teach.* San Francisco: Jossey-Bass.

Reynolds, E., Treahy, D., Chao, C., & Barab, S. (2001). The internet learning forum: Developing a community prototype for teachers in the 21st century. *Computers in the Schools, 17*(3/4), 107-125.

Ruthven, K. (1992). Personal technology and classroom change: A British perspective. In J. T. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education* (Vol. 1992, pp. 91-100). Reston, VA: National Council of Teachers of Mathematics.

Silver, E. A. (2003, April 7-9). *Looking at teachers' self-selected samples of their best instruction: What do they reveal? What can we learn?* Paper presented at the 35th Annual Conference of the National Council of Supervisors of Mathematics, San Antonio, TX.

Simmt, E. (1993). *Living one's philosophy of mathematics: Graphing calculators in high school mathematics.* Unpublished M.Ed. thesis, University of Alberta, Edmonton, Alberta.

Simmt, E. (1997). Graphing calculators in high school mathematics. *The Journal of Computers in Mathematics and Science Teaching, 16*(2-3), 269-289.

Smith, K. B., & Shotsberger, P. G. (1997). Assessing the use of graphing calculators in college algebra: Reflecting on dimensions of teaching and learning. *School Science and Mathematics, 97*(7), 368-376.

Sparks, D., & Loucks-Horsley, S. (1990). Models of staff development. In W. R. Houston (Ed.), *Handbook of Research on Teacher Education* . New York: Macmillan.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teachers College Press.

Szombathelyi, A. (2001). *Personal factors that influence teachers' decisions about graphing calculator use and a descriptive model of teachers' operational levels for using the graphing calculator in mathematices instruction.* Unpublished Ph.D., University of South Carolina.

Tharp, M. L., Fitzsimmons, J. A., & Ayers, R. L. B. (1997). Negotiating a technological shift: Teacher perceptions of the implementation of graphing calculators. *The Journal of Computers in Mathematics and Science Teaching, 16*(4), 551-575.

Thomas, G., Wineburg, S., Grossman, P., Myhre, O., & Woolworth, S. (1998). In the company of colleagues: An interim report on the development of a community of teacher learners. *Teaching and Teacher Education, 14*(1), 21-32.

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology in Education, 9*(3), 77-101.

Weiss, I. R., Banilower, E. R., McMahon, K. C., & Smith, P. S. (2001). *Report of the 2000 National Survey of Science and Mathematics Education* . Chapel Hill, NC: Horizon Research, Inc.

Weiss, I. R., Banilower, E. R., McMahon, K. C., & Smith, P. S. (2002). *The National Survey of Science and Mathematics Education: Trends from 1977 to 2000* . Chapel Hill, NC: Horizon Research, Inc.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* New York: Cambridge University Press.

Wenger, E., McDermott, R., & Snyder, W. M. (2002). *Cultivating communities of practice*. Boston: Harvard Business School Press.

Windschitl, M., & Sahl, K. (2002). Teachers' use of technology in a laptop computer school: The interplay of teacher beliefs, social dynamics, and institutional culture. *American Educational Research Journal, 39*(1), 165-205.

Zawojewski, J., Chamberlin, M., Hjalmarson, M., & Lewis, C. (under development). The role of design experiments in teacher professional development. In E. Kelly & R. Lesh (Eds.) *Design research in mathematics, science and technology education.* Mahweh, NJ: Erlbaum.

Zbiek, R. M. (1995). *Her math, their math: An in-service teacher's growing understanding of mathematics and technology and her secondary students' algebra experience*. Paper presented at the 17th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH.

Zbiek, R. M. (2002a, May 30). *MAGICAL framework describing the nature of students' use of representations*. Paper presented at the Research Presession of the 80th Annual Meeting of the National Council of Teachers of Mathematics, Las Vegas, NV.

Zbiek, R. M. (2002b, April 21). *A two-tiered category perspective to describe purposes of mathematics technology use*. Paper presented at the Research Presession of the 80th Annual Meeting of the National Council of Teachers of Mathematics, Las Vegas, NV.

Ziebarth, S. (2002). Survey Questions adapted from evaluation instruments used in numerous NSF-funded projects. For further information on projects and intruments, contact steven.ziebarth@wmich.edu.