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Changing Mathematical Discourse: A Case Study of a Secondary Mathematics Teacher

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CHANGING MATHEMATICAL DISCOURSE: A CASE STUDY OF A SECONDARY MATHEMATICS TEACHER

by

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CHAPTER I

INTRODUCTION

Calls for Reform

In the past, and to a great extent today, mathematics instruction has focused on procedural skills rather than conceptual understanding. However, current calls for reform (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000; National Research Council [NRC], 1989, 1990) encourage teachers to shift the emphasis of teaching to make conceptual understanding and sense-making paramount. The mathematics education community is beginning to understand the nature of teaching to be complex, challenging, and intellectually demanding for teachers intent on developing their students' conceptual knowledge of mathematics. The community's understanding has developed from the growing body of research on teacher education and professional development (e.g., Brown, Cooney, & Jones, 1990; Fennema & Franke, 1992; Koehler & Grouws, 1992; Silver & Stein, 1996; Thompson, 1992).

Education research has suggested that there is a connection between conceptual understanding and the discourse used by students in the classroom (Cohen, 1994; Prawat, 1989). Mathematics education research, specifically Hiebert and Carpenter (1992), indicates that conceptual understanding of mathematics is enhanced by students articulating their thinking, calling for a different learning environment with strikingly different discourse. The current calls for reform in mathematical discourse (NCTM, 1989, 1991, 2000; NRC, 1989, 1990) discuss creating classroom environments where students are encouraged to explore, conjecture, justify, and prove mathematical ideas (NCTM, 1989, 2000). In particular, the *Professional Teaching Standards for School Mathematics* (NCTM, 1991)

proposes that teachers concentrate on several aspects of their role in orchestrating discourse to assist students in the sense-making process:

> (a) posing questions and tasks that elicit, engage, and challenge each student's thinking; (b) listening carefully to students' ideas; (c) asking students to clarify and justify their ideas orally and in writing; (d) deciding what to pursue in depth from among the ideas that students bring up during a discussion; (e) deciding when and how to attach mathematical notation and language to students' ideas; (f) deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty; and (g) monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (p. 35)

These suggestions may seem obvious and rather ubiquitous, however, in light of the study by Stigler and Hiebert (1997) of a cross-section of American classrooms, there are definite discrepancies between what has recently been observed and the suggestions listed above. These findings make it clear that there remains a need to encourage and support mathematics teachers to alter existing practices to better serve their students.

Research Support for Calls

Ultimately, the goal for changing teachers' classroom practices is to improve student achievement. Through the use of discourse analyses of classroom interactions, two key aspects of classroom discussions have been shown to improve student achievement, namely, wait-time (Rowe, 1974, 1986; Tobin, 1980) and types of questions asked (Cazden, 1986; Redfield & Rousseau, 1981). Wait-time and types of questions were found to have a positive effect on other areas of discussions such as: (a) an increase in the length of student responses and the number of logical arguments they included; and (b) an increase in speculative thinking (DeTure & Miller, 1984; Rowe, 1986). Both of these results lead to a greater emphasis on sense-making as called for by the *Standards* (NCTM, 1991). It is important to note, however, that the questioning in these cases refers to moving from factual knowledge questions to those

that ask for explanations. It does not consider the line of mathematical questioning which requires a conceptual understanding of the mathematics by the teacher and is more difficult to achieve.

Some Obstacles

Although the "quick fixes" of wait-time and types of questioning previously mentioned are a start, there are more fundamental issues involved in changing the mathematical discussions in a classroom. Researchers have found that these other issues include the teacher's history of past practices (Smith, 1996; Lloyd & Wilson, 1998; Wilson & Lloyd, 2000), beliefs (Thompson, 1984, 1992), and lack of content knowledge (Fennema & Franke, 1992; Stein, Baxter, & Leinhardt, 1989) and pedagogical content knowledge (Carpenter, Fennema, Peterson, and Carey, 1988; Fennema & Franke, 1992). Additionally, as Lappan (1997) reports:

> The implications of new forms of discourse in the classroom pose a considerable challenge for teachers' professional development. Many teachers never experience learning mathematics in situations where value is placed on the quality of the thinking, the quality of explanation or argument, and the quality of decisions made based on the evidence. (p. 215)

Professional development has historically attempted to change how teachers teach, yet instruction remains relatively constant (Cuban, 1988, 1993; Goodlad & Klein, 1974). Traditionally professional development has taken place by means of external consultants providing brief one-day in-services (Little, 1993) on curriculum-independent issues without invoking long-term changes in teaching. In an effort to address the lack of success of this approach, professional development has shifted to include more long-term and intensive systemic initiatives (Loucks-Horsley, Hewson, Love, & Stiles, 1998; National Science Foundation, 2000) centered around innovative curricula.

A Solution

To assist teachers in reforming their practices, many school districts have adopted National Science Foundation (NSF)-funded curriculum materials based on the NCTM *Curriculum and Evaluation Standards* (1989). It is thought that by using these curriculum materials teachers would change their instruction and hence the classroom discourse. These curricula embody many aspects of the current reform, such as worthwhile tasks, suggestions for group problem-solving, and assessment. Teachers using these materials are expected to be facilitators of and participants in mathematical discussions rather than lecturers or merely askers of questions (Hirsch, Coxford, Fey, & Schoen, 1995; NCTM, 1991). For example, Schoen (2000) found that teachers using a particular NSF-funded curriculum used more whole-group discussions, rather than teacher presentation, and used open-ended questions during the discussions of the mathematical content. However, Schoen (2000) also found that the amount of professional development the teachers had participated in played a role in the level of success in their implementation of the curriculum materials.

Additional Obstacles

Although curricula can provide the framework and tasks, the success depends on the implementation by the teacher. Deborah Ball (2000) recollects from her own personal experience, "I was learning a central lesson of teaching: that the curriculum is made in class, in the enactment of tasks. I saw more vividly than ever how crucial was the teacher's role" (p. xi). Research as part of the Quantitative Understanding: Amplifying Student Reasoning and Achievement (QUASAR) project (c.f. Silver & Stein, 1996) supports this notion of the important role teachers play in the implementation of instructional tasks. Stein, Silver, and colleagues have concluded from their years of research that even mathematics instructional tasks that have the

potential for placing high cognitive demands on the students may be implemented in such a way as to circumvent student thinking.

It is evident that even the tasks provided in seemingly worthwhile curriculum materials are not a panacea (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998; Stein, Smith, Henningsen, & Silver, 2000). Although curricula may provide worthwhile tasks, implementation of the lesson remains teacher-dependent. It is likely that no curriculum is teacher-proof, nor is this what we should hope to achieve. As a result, it is imperative that research focus on the teacher's role to better understand the barriers and the supports necessary to assist teachers as they endeavor to create environments where student thinking is paramount.

Possible Solution

Research and contemporary theories (Cobb, Boufi, McClain, & Whitenack, 1997; Hart, Schultz, Najee-ullah, & Nash, 1992; Schon, 1983; Schon, 1987), indicate that teachers who are encouraged to consistently reflect on their teaching practices can then alter them as Shaw and Jakubowski (1991) advocate:

> If the many recommendations for change are to be realized in the classroom, we as teacher educators and researchers must look closely at individual teachers and be cognizant that [even though] teachers may appear to be enthused about changing, they may find it very difficult. A concerted effort is needed to perturb teachers in a worthwhile way and follow up the perturbations by assisting them to construct viable alternatives and encouraging and supporting them as they make changes to benefit their students' learning. (p. 20)

To accomplish this, professional development in teaching must move beyond the "closed-door" policy and isolationist perspective where once the classroom door falls shut, the teacher continues to teach in the same manner as before (Goodlad & Klein, 1974). A new paradigm for professional development is emerging and includes teacher assistance embedded in or directly related to the work of teaching (Ball & Cohen, 1999; Loucks-Horsley, Hewson, Love, & Stiles, 1998; Schifter & Fosnot,

1993; Stein, Smith, & Silver, 1999). This teacher assistance should be grounded in the content of teaching and learning (Stein, Smith, & Silver, 1999) and in collaboration with experts outside the teaching community. In other words, the professional development of mathematics teachers should be situated in the teacher's classroom focusing on the students' learning of the mathematical content in collaboration with mathematics education researchers.

One of these alternative paradigm models is the "teacher collaborative" (Little, 1993), examples of which are used in Local Systemic Change (LSC) projects funded by the NSF. These LSC projects are subject-specific collaborations between, typically, a university and group of school districts for a minimum period of three years. Through these collaborations teachers are provided opportunities to expand their knowledge of mathematics or science content, pedagogy, and learning theories in addition to opportunities for leadership training and networking with other teachers.

The goal of the LSC projects is to prepare teachers to implement reform curricular materials in ways consistent with the *Standards* (NCTM, 1989, 1991, 1995, 2000). One critical aspect of this implementation, as mentioned previously, is the teacher's role in the facilitation of classroom discourse. One approach to focusing on the teacher's role is to use videotaped lessons to promote deep thinking about the ways teachers elicit and engage with students' ideas. Segments of lessons, such as exchanges among teachers and students while they are at work, are used to encourage discussion about effective discourse practices (see Grant, Kline, and Van Zoest (2001), for specifics). Using videotape allows the teacher to reflect on a variety of aspects of the discourse that are not possible if one relies on recollection. For example, videotape allows the teacher to observe the wait-time, identify the exact

verbiage and phraseology of their comments and student utterances, and hear the tone of voice. The ability to stop and replay the videotape also allows the teacher more time to reflect on students' thinking and investigate missed opportunities. Viewing and reflecting on videotaped classroom episodes with researchers knowledgeable about the mathematics content can assist teachers in their quest to change the discourse in their classrooms.

The availability of this technology in conjunction with new paradigm models for professional development allow for investigations into the complexities of teacher change and ways to assist it. Enlightened by the new models, a teacher-researcher collaboration was established with a secondary mathematics teacher using an intervention based upon focused reflection on videotapes of his instruction. The purpose was to investigate the nature of change that occurs in both the teaching and reflection practices associated with the classroom mathematics discourse of a secondary school mathematics teacher using a reform curriculum. Given the complexity of teaching mathematics in this current environment of reform, mathematics teacher educators must improve their understanding of the process of changing teachers' reflective and discourse practices at the same time they are encouraging the teachers to meet the challenges of reform.

Research Question

In the process of the teacher-researcher collaboration, the researcher addressed the following question: In what ways does participation in a teacher-researcher collaborative intervention focused on mathematical discourse affect (a) the nature of the teacher's reflection about and vision of mathematical discourse, and (b) the mathematical discourse in the teacher's classroom?

Brief Overview

Chapter II includes a summary of the research literature related to aspects of meaningful mathematical discourse and methods of changing mathematics teaching practices. Chapter III describes the methodology used in this research study to answer the research question. Chapter IV elaborates on the data collected and the analysis procedures used by providing specific examples from the interviews and classroom episodes along with results of the analysis. Chapter V concludes the report by summarizing the findings and suggesting how this research could be applied and extended in future professional development and research on discourse.

CHAPTER II

LITERATURE REVIEW

The current study is guided by previous research regarding changing mathematics teachers' practices through professional development. This chapter reviews the research literature in three primary areas that guide the data collection and analysis of the study being reported here: (a) focus on mathematical content, (b) nature of meaningful mathematical discourse, and (c) methods of professional development.

Focusing on Mathematical Content

One body of literature concerning the professional development of mathematics teachers involves maintaining a focus on the importance of teachers' mathematical content. For example, Gearhart, Saxe, and colleagues (1999, 2001) assert the importance of providing teachers with professional development focused on the mathematical content students will be investigating and the ways in which students might be thinking about that content. They compared two interventions; one that provided "opportunities [for teachers] to build their knowledge of mathematics, their knowledge of students' understandings of concepts and problem-solving strategies in the same domains, and their expertise with assessment" (Gearhart et al., 1999, p. 289) and one that "met regularly to discuss strategies for implementing the curriculum" (Saxe, Gearhart, & Nasir, 2001, p. 55). They concluded that providing opportunities for teachers to focus on the mathematical content are significantly more effective than collegial support alone at implementing curricular materials as intended.

The results of a study by Jacobson and Lehrer (2000) also point to the importance of a focus on mathematical content. They discovered in their investigation

of four second-grade teachers, all of whom had participated in extended professional development on Cognitively Guided Instruction (CGI), that the teachers had strikingly different discourse and student achievement in their classrooms when investigating geometric versus arithmetic content. The difference between the four teachers was that two of them had continued to participate in a research program that investigated students' thinking about geometry. In these two teachers' classrooms, the teachers encouraged students to refine, elaborate, and extend their thinking about the geometric content. In the other two classrooms, teachers "simply elicited a series of unconnected student observations" (p. 85). The authors assert that teachers need professional development focused on mathematical content.

A third study that points to the need to make content salient in professional development is Quantitative Understanding: Amplifying Student Reasoning and Achievement (QUASAR), a 5-year professional development and research project designed to encourage teachers to implement reform-oriented instructional materials for the middle-school level. In the professional development component of this project, teachers were encouraged to reflect on the mathematical tasks aspect of a lesson and identify the mathematical goal of the task. Although teachers were provided with curriculum materials that consisted of meaningful mathematical tasks, when the enactment process did not provide a clear understanding of the mathematical goal of the task, these tasks often degenerated into less cognitively demanding and less meaningful tasks (Stein, Grover, & Henningsen, 1996). Similarly, Stein, Smith, Henningsen, and Silver (2000) found it was important for teachers to focus on the goal of the mathematical task to ensure that it is achieved in the implementation of a lesson.

This body of literature suggests that professional development for

mathematics teachers needs to focus on the mathematical content students will be investigating and learning. It is important that professional development encourages teachers to know and understand the goal of the mathematical task as well as focus on students' mathematical thinking.

Nature of Meaningful Mathematical Discussion

A second body of literature lays out the nature of what constitutes meaningful mathematical discussion. From research on elementary school classroom discussions, O'Connor and Michaels (1996) suggest that, "Success in this complex realm involves more than simply creating a friendly or non-threatening setting for discussion and problem-solving" (p. 65). Literature in this area notes that such discourse involves students being pressed to participate and to conjecture, justify, explore and reason (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Mendez, 1998; NCTM, 1989, 1991).

Although many teachers have moved away from purely lecturing, the typical classroom, in which students are more apt to practice memorized procedures than to engage in deep mathematical discussion (U.S. Department of Education, 1997; Stigler & Hiebert, 1997), is far from the suggestions in the *Standards* documents (NCTM, 1989, 1991, 1995, 2000). In an effort to help teachers change their practice, a number of mathematics education scholars have worked to identify the components of meaningful mathematics discussion (Yackel & Cobb, 1996; Mendez, 1998; Richards, 1991; Wood, 1998).

Cobb and his colleagues are among those who seek to contribute to the identification of meaningful mathematical discourse. Yackel and Cobb (1996) set out to encourage elementary mathematics teachers to establish learning environments based on student inquiry while at the same time developing tasks that promoted

mathematical understanding. While investigating these mathematical learning environments and collaborating with the teachers of second- and third-grade mathematics classrooms, they found that teachers often required students to explain their answers (social norm) but failed to discuss what constitutes an acceptable mathematical explanation (sociomathematical norm). For example, many teachers explicitly discussed the appropriateness of how to answer a question (e.g., "raise your hand after I've finished asking the questions") but did not explicitly discussed the acceptability of a mathematical justification. What typically occurred is that through the actions and verbal interactions of the teachers and students, sociomathematical norms are established. The importance of these discussions with students about the acceptable ways of talking about math became apparent in their year-long collaboration with one second-grade mathematics teacher (Cobb, Wood, & Yackel, 1993). As a result of this collaboration, they identified the need for a cycle of "talking about talking about mathematics" and "talking about and doing mathematics" (p. 99). Throughout this cycle, the students and teacher constantly renegotiate the classroom norms but, typically, "within a few weeks, most students routinely give conceptual explanations as the need arises and that they ask others clarifying questions that bear directly on their underlying task interpretations" (Cobb, p. 47, in Sfard, Nesher, Streefland, Cobb, and Mason, 1998). These studies suggest that meaningful mathematical discussion should involve the establishment of sociomathematical norms that include acceptable ways of talking about mathematics.

Mendez (1998) offers support for social and sociomathematical norms as important components of meaningful mathematical discussion. In her study, an eighth-grade mathematics teacher, who encouraged episodes of meaningful mathematical discussion within his classroom, explicitly discussed both the social and

sociomathematical norms in his classroom. For instance, during the second day of class, the class brainstormed about the role of student sharers and listeners. In the ensuing weeks, these norms were revisited and posters were hung around the room to remind students of their responsibilities as sharers or listeners. In addition, during the interactions in the classroom, the teacher would press students to justify or clarify their comments to make their arguments more mathematically sound.

Studies have shown that carefully established sociomathematical norms can contribute to increased inquiry-based instruction, higher order questioning, and cognitive activity of the students (Yackel & Cobb, 1996) and, as a result, greater understanding of mathematics (Cobb, Boufi, McClain, & Whitenack, 1997). What may seem an inconsequential difference between norms has been found to have a considerable effect on the quality and content of the mathematical classroom discourse (Cobb et al., 1997).

Another aspect of meaningful mathematical discussion that has been identified is the students' perception of where the mathematical authority lies in the classroom. In contrast to traditional classroom discourse, in which the teacher is constantly providing the evaluative feedback, meaningful mathematical discourse requires a shift in mathematical authority from the teacher to a shared authority between teacher and students.

Wilson and Lloyd (2000) studied authority as it relates to the mathematics classroom. They discovered, when working with three high-school mathematics teachers during their first year of implementing *Contemporary Mathematics in Context: A Unified Approach* (CMIC) (Coxford, Fey, Hirsch, Schoen, Burrill, Hart, Watkins, Messenger, & Ritsema, 1997), that sharing authority is a complex and difficult task for teachers trained to teach in traditional lecture-oriented ways. In each

case, the teacher was excited about the change and had volunteered to use the materials, but only one was able to establish a classroom where the teacher shared responsibility for sense-making with the students. This teacher allowed students to struggle in their small groups and stood in the back of the room during small-group presentations. In addition, when asked questions by a small group, he consciously decided not to share his thoughts on the answer, in some cases because he did not know it. In contrast, another teacher became fearful during the implementation that her students were not able to make the appropriate connections to encourage the conceptual understanding she desired, so she interrupted small groups and made the connections for them in the whole group. This undermined the students' ability to make sense of the mathematics on their own. The teacher's sharing of authority of the students in the classroom, then, is a critical component of meaningful mathematical discussion.

Some studies have made an effort to establish criteria for assessing meaningful mathematical discourse. Mendez (1998), for example, developed and refined a set of rating scales to be used in evaluating classroom mathematical discourse. Guided by the NCTM *Standards* (1989, 1991) and research on promoting student understanding, she proposed three components to assess the robustness of the mathematical dimension of the discussions: (a) justification, (b) multiple forms of representation, and (c) generalization.

Justifications provided by students indicate that the discussion has moved beyond the description and explanation of an answer into the level of logical argument, eventually leading to proof. Maher and Martino (1996), describe the results of following a student, from grades 2 through 5, who participated in mathematics classes that "actively engaged [her in the] construction of mathematical

ideas" (p. 198). She was engaged in opportunities that allowed her to seek multiple representations and was encouraged to explain and justify her thinking. Maher and Martino (1996) suggest that, as a result, the student was able to move to a level of reasoning that included invention of her own "proof-by-cases" argument.

Mendez (1998) found that teachers can assist students' justifications by scaffolding questions, revoicing student explanations, and praising students for referring to others' comments. Scaffolding questions are those that call on a speaker to elaborate on their meaning, such as, "And do you have anything else to say about it?" Revoicing is when the teacher uses students' words or vocabulary to elicit further responses along the same line. Giving praise is the explicit pronouncement and acknowledgment of students' listening to and building upon one another's responses.

Evidence of multiple forms of representation in the discussion indicates that students are communicating their mathematical ideas in a variety of ways. Much of the research on multiple representations focuses on algebraic understanding and, in particular, on the concept of functions. The idea is that when students are allowed to explore using multiple representations, they develop a far more coherent and conceptual understanding. For example, Yerushalmy (1991) found in a study in which students used a computer that the presence of simultaneous multiple representations allowed students to develop conceptual understanding that went beyond the understanding typically associated with algebraic manipulative skills. Encouraging students to communicate about the multiple representations appears to refine and enhance their conceptual understanding.

Generalizations heard in the classroom suggest that students are attempting to make sense of the mathematics by forming conclusions based on a series of facts. As part of a multiyear study on algebraic thinking in the middle grades, Masarik and

Nathan (2000) examined how students generalize from a verbal representation of a contextualized pattern of numbers to symbolic algebraic representation. As a result, they found that the classroom discourse, which promoted presenting, questioning, and defending ideas, provided opportunities for the development of a student's mathematical knowledge in the area of pattern generalization.

Although Mendez (1998) only includes three components in her criteria for assessing the mathematical dimension of meaningful mathematical discourse, a fourth component, evidence of connections, was appropriate to consider given the importance placed on connections within mathematics contexts and between real-world applications in the NSF-funded curricular materials used in this study. As a part of the design of the curriculum, connections are found both in real-life contexts of the mathematical investigations and the organization of the curriculum around fundamental ideas, such as functions (Hirsch, Coxford, Fey, & Schoen, 1995).

In summary, researchers have identified several components of meaningful mathematical discussions. These components include establishing sociomathematical norms that encourage acceptable mathematical talk, encouraging an environment of shared authority, and evidence of mathematical criteria. These mathematical criteria are justification, multiple forms of representation, generalization, and connections.

Methods of Professional Development

A third relevant body of literature focuses on the need for professional development for teachers as a key ingredient in improving U.S. schools (Sykes & Darling-Hammond, 1999). More important, the key lies in the type of professional development in which teachers engage. For example, the Eisenhower Professional Development Program (2001) conducted a 3-year study of teachers at all instructional levels of public school who participated in professional development supported with

Eisenhower funds. From this, a series of surveys of 287 teachers representing 30 schools in 10 districts in 5 states, the report's authors concluded:

> Professional development focused on specific, higher-order teaching strategies increases teachers' use of those strategies in the classroom. This effect is even stronger when the professional development activity is a reform type (e.g., teacher network or study group) rather than a traditional workshop or conference; provides opportunities for active learning; is coherent and consistent with teachers' goals and other activities; and involves the participation of teachers from the same subject, grade, or school. (Executive Summary, p. 1)

Similarly, Little (1993) suggests that "the most promising forms of professional development engage teachers in the pursuit of genuine questions, problems, and curiosities, over time, in ways that leave a mark on perspective, policy, and practice" (p. 133). Many studies of and literature about professional development suggest various ways in which effective professional development is implemented, including the use of teacher reflections, the use of videotapes, and the introduction of particular pedagogical techniques (Ball & Cohen, 1999; Loucks-Horsley, Hewson, Love, & Stiles, 1998; Schifter & Fosnot, 1993; Stein, Smith, Henningsen, & Silver, 2000). The following gives a brief summary of these ways.

Use of Teacher Reflections

A number of studies suggest that teacher reflection is a key component in changing teachers' practices. For example, in her seminal research with three middle-school mathematics classrooms, Alba Thompson (1984) identified teacher reflectiveness as the key for teachers to reconcile their inconsistencies between beliefs and practices. Thompson found that the teacher in the study with the fewest inconsistencies between her beliefs and practices and most highly integrated conceptual system of teaching tended to be the most reflective. This study indicates that teachers encouraged to reflect on their practices create opportunities for themselves to confront and reconcile the inconsistencies between their beliefs and

practices.

Shaw and Jakubowski (1991) describe how one of three elementary/middle school teachers in their study was able to enact changes in her classroom instruction through reflection. They propose a schema of six cognitive requisites for enacting teacher change (Tobin & Jakubowski, 1992) to help describe and discuss the reasons for change or lack of change in the teachers. The six cognitive requisites are: (a) perturbation or uneasiness with the way things are, (b) awareness that improvement requires change, (c) commitment to move into action, (d) vision of teaching, (e) visualization of the changes in the classroom and reflection as they are taking place, and (f) reflection on teachers' own practices and raising questions about their own actions in the classroom.

As a result of their analysis using this model, Tobin and Jakubowski (1992) conclude that because one teacher exhibited high level of commitment and collaborated with other teachers in her building, she was able to make the desired changes. In contrast, another teacher liked the idea of promoting an environment based on sense-making but lacked the commitment, while a third had high commitment but lacked access to alternative models to incorporate into her vision. Based on their research with these teachers and other elementary mathematics and science teachers, Tobin and Jakubowski (1992) suggest that "essential ingredients for teacher change were reflection in and on action and access to resources to provide foci for reflection" (p. 176).

Use of Videotapes

Some literature in the area of professional development for mathematics teachers suggests that videotapes can function as a useful tool for prompting changes in teachers' behaviors. For example, the professional development project described

in Grant, Kline, and Van Zoest (2001) used videotaped lessons to promote deep thinking about the ways in which teachers elicit and engage with students' ideas. Segments of lessons, such as exchanges among teachers and students while they are working on problems, were used to encourage discussion about effective discourse practices.

Introducing Pedagogical Techniques

A third method suggested in the literature for professional development is introducing pedagogical techniques of various kinds. Three such techniques, in particular, have been identified as ones that encourage classroom discourse and were used in the design of the current study.

Wait-time

Wait-time is the period of time between two speakers' responses, generally between the teacher and student. Wait-time 1 is the time after a teacher poses a question and wait-time 2 is the time after a student has made a reply and before the teacher responds. A number of studies suggest that teaching the concept of wait-time to teachers enhances their effectiveness in the classroom. Rowe (1974, 1986) found that teachers without formal instruction on wait-time typically pause for less than one second between the time they ask a question and respond themselves (wait-time 1) and again wait less than one second to respond after a student answers (wait-time 2). Researchers found that if teachers' wait-time 1 and 2 were at least three seconds, their students' achievement increased (Rowe, 1974, 1986; Tobin, 1980). Furthermore, Rowe's (1986) work suggests that wait-time 2 is the most important type of wait-time.

The power of wait-time may be due to the influence it has on the nature of classroom discourse. Rowe (1986) found 10 results of increased wait-time related to

student talk: (a) student responses increased between 300-700%; (b) more inferences were supported by evidence and logical argument; (c) speculative thinking increased; (d) the number of questions asked by students increased as well as the number of experiments they proposed; (e) student-student exchanges increased as teacher-centered show and tell behavior decreased; (f) student failures to respond decreased; (g) disciplinary verbalizations decreased; (h) variety of students participating voluntarily in discussion and the number of unsolicited but appropriate contributions increased; (i) student confidence, as reflected in fewer inflected responses (those which had questions like "Is that what you wanted?"), increased; and (j) achievement improved on written measures where the items were cognitively complex. All of these results contribute to mathematical discussions that encourage autonomy and sense-making on the students' part. Pimm (1987), focusing on mathematics classrooms, likewise found that using what he calls "silences" encourages students to think. Silence, he cautioned, may sacrifice teachers' feeling of control over the conversation and may cause embarrassment.

Rowe (1986) summarized the findings with respect to the effects of increased wait-time on teachers. They are: (a) teachers exhibited fewer discourse errors and greater continuity in the development of ideas; (b) teachers asked fewer questions but at a cognitively higher level; and (c) teachers' expectations of certain students, especially minority students, increased.

Wait-time has been found to decrease when teachers engage in certain habits. DeTure and Miller (1984) found, in their investigation of a written protocol used as a training instrument with in-service elementary science teachers, that teachers who used techniques such as mimicry, compliance, rhetorical questions, or chaining questions had a negative affect on wait-time. Given the results that wait-time

influences both teaching and learning in the classroom, it is regarded as an important aspect to consider when changing classroom discourse practices in the classroom.

Questioning

Questioning techniques have also been identified as pedagogical techniques that could be shared with teachers in order to change the classroom discourse. In a meta-analysis of the research on the relationship between higher order questions and student achievement, Redfield and Rousseau (1981) found that the use of higher cognitively demanding questions improves student achievement. Pressley and his colleagues (1988) and Martin and Pressley (1991), working with adults and college students, investigated the relationship between remembering information (e.g., the hungry man got into his car) and being asked "why" questions. Asking "why" encouraged the participants to make sense of the situation. Results showed being asked a "why" question promotes learning, even when the individuals asked did not succeed in generating answers.

Studies also have shown that the types of questions asked have been found to affect the verbal discourse of the classroom. For example, King (1999), in her work with peer learning groups to mediate their own learning, found that increasing the complexity of questions increased the complexity of discourse. She categorizes questions according to the responses they elicit, such as factual, comprehension, or connection questions, with the latter two being considered more complex questions that stimulate greater cognitive activity on the part of the respondent.

Patterns of questioning are different from types of questions in that the pattern is how the interaction between questions and answers occurs in the classroom. The literature in this area suggests that patterns of questioning can be categorized into one of following types: (a) Initiation-Reply-Evaluation (IRE) or Initiation-Response-

Follow-up (IRF), (b) funneling, or (c) focusing. Sinclair and Coulthar (1975), in a pioneering analysis of discourse in classrooms, found that verbal interaction between the teacher and students generally follows an IRE pattern, with the teacher posing and initiating the questions and providing the evaluation or feedback. Richards (1991), in his work with high school mathematics classrooms, found that school math, which typically follows the IRE pattern, differs strikingly from other forms of mathematical talk (i.e., research math, inquiry math, and journal math). IRE involves the teacher leading students through a predetermined set of questions and answers, usually at a very surface level with little or no explanation from students. Students may be encouraged to ask questions, but they are not generally encouraged to make conjectures, share their reasoning, or pose alternative solution strategies, all expected behaviors in inquiry mathematics classrooms. In other research, Pimm (1987) found teachers using questions that he called "proof by intimidation." These are questions that suggest the answer because of the phraseology or tone of voice. For example, a question such as "When a coach, if you play sports, tells you to give 110%, is that ever possible?" elicits an immediate "no" response without requiring the listener to consider and think about the question.

A second pattern of questioning described by Wood (1998) is funneling. Funneling takes place when the teacher asks a series of questions that guide the students through a procedure or to a desired end. For example, Pimm (1987) identifies a technique he calls "clozed" questioning where teachers use a fill-in-the-blank type of question that the student completes. He proposes that this is a positive questioning technique because it allows the teacher to maintain control of the discourse and to highlight particular items along the way. In this case, though, the teacher is the only one engaged in higher-level cognitive activity, and the students are

merely answering questions to arrive at an answer, often without seeing the connection between questions. Cobb, Wood, and Yackel (1993), in their collaborative investigation into developing "instructional settings in second-grade mathematics' classrooms that are compatible with implications of the constructivist theory of knowledge" (p. 91), found that this sort of questioning often degenerates into a guessing game where the students try to guess at what they believe the teacher wants to hear.

A third questioning pattern found in the literature involves questions that focus rather than funnel students' thinking. A focusing question pattern requires the teacher to listen to students' responses and guide them based on what the student is thinking rather than how the teacher is thinking. Pimm (1987) found the technique of echoing to be helpful in encouraging this pattern of questioning. The teacher selectively repeats pupils' comments, a technique that returns the conversational impetus back to the group but at the same time focuses the conversation. This is mirrored in the research of O'Connor and Michaels (1996), in which they identified the idea of revoicing. Revoicing is a structure of talk in which a teacher uses student utterances either to align a student's comment with others or to provide an opportunity for the student to agree or disagree with the teacher's characterization. They found that revoicing can be used by the teacher to clarify, explain reasoning, introduce ideas, and redirect discussion.

Listening

Listening is another technique that encourages mathematical discourse and can be used in professional development for mathematics teachers. Davis (1997) found that when an eighth-grade mathematics teacher listened differently, the types of questions she asked changed. For example, when she listened evaluatively--focused

on listening for something--she would ask a question for which she already knew the answer or had a correct answer in mind. In contrast, listening interpretively prompted "information-seeking" versus "response-seeking" (Davis, 1997, p. 363) types of questions. In interpretive listening, the teacher does not necessarily anticipate what the student will answer. Instead, the teacher listens closely to the students to understand what they mean. The teacher also exhibited a third type of listening that Davis (1997) termed *hermeneutic,* which he defined as being listening that is oriented to sense-making. In addition, it is mutual sense-making by teacher and students that steers the course of the conversation. This type of listening goes beyond the interpretative in that how the student responds may change the trajectory of the discussion.

The three techniques of wait-time, questioning, and listening have been suggested as effective ways to enhance classroom discourse. Introducing such methods to teachers who wish to improve the discourse of their mathematics classroom can be a part of an effective professional development strategy.

Summary

Research in mathematics classrooms and work with mathematics teachers show that it is important for teachers to focus on mathematical content during professional development.

The nature of meaningful mathematical discourse includes teachers helping to establish sociomathematical norms that clearly delineate acceptable mathematical talk. The classroom culture should also include a sense of shared authority between the teacher and students. In addition, important components of mathematical discourse include evidence of students providing justifications, multiple forms of representation, generalizations, and connections.

The research literature also suggests introducing specific pedagogical techniques to encourage teachers to change their teaching practices. Suggested techniques include the use of reflections, the use of videotapes, and the introduction of pedagogical techniques, such as wait-time, questioning, and listening.

CHAPTER III

METHODOLOGY

This study uses a single-case case study design to investigate the nature of changes that occur in the teaching and reflection practices associated with mathematics discourse of a secondary school mathematics teacher using a reform curriculum. The following describes the design specifics, participant, setting, intervention, and data collection and analysis used this study.

Design of Study

By Yin's (1994) definition, the case study "is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p. 13). It was chosen for the design of this study due to the difficulty of teasing apart the phenomenon of teacher change from the context of classroom teaching. The study reflects the current direction of professional development (Stein, Smith, Henningsen, & Silver, 2000) by working collaboratively with a teacher in the context of his classroom. It also relies upon multiple sources of evidence where the data converges or is triangulated (Eisenhart, 1988), and takes into account prior research developments to guide data collection and analysis.

Setting

Jefferson High School, a pseudonym for this study, is the only high school in its district and is located in a small town adjacent to a racially diverse city of approximately 100,000 residents. The high school services approximately 450 students of primarily low to middle socio-economic status, where 30% of the students are eligible for free or reduced-price lunches. The student population is approximately 90% White, 5% African American, 3.5% Hispanic, 1% Asian or Pacific Islander, and

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0.5% American Indian or Alaskan Native.

This setting is of particular interest for two reasons: (1) the high school teachers chose to adopt textbook materials developed from a National Science Foundation (NSF)-funded curriculum project, and (2) the school district elected to participate in a NSF Local Systemic Change (LSC) professional development project.

A Reform Curriculum

The Jefferson High School teachers made the decision to adopt the curriculum produced by the Core-Plus Mathematics Project (CPMP) beginning the 1998-99 school year. CPMP was one of four projects funded by the NSF to develop high school mathematics curricula that would interpret and implement recommendations in the National Council of Teachers of Mathematics (NCTM) *Standards* (1989, 1991). Course 1 was field-tested in 36 high schools in 11 states beginning in the 1994-95 school year and subsequent courses were field-tested during the following years. The project materials were published under the title *Contemporary Mathematics in Context: A Unified Approach* (CMIC) (Coxford, Fey, Hirsch, Schoen, Burrill, Hart, Watkins, Messenger, & Ritsema, 1997, 1998, 1999; Coxford, Fey, Hirsch, Schoen, Hart, Keller, Watkins, Ritsema, and Walker, 2000).

The CMIC creators and developers grounded their four-year integrated and context-based curriculum in the *Standards* (NCTM, 1989), mathematical habits of mind (Cuoco, 1996), and themes derived from *On the Shoulders of Giants* (Steen, 1990). The intent was to produce a curriculum that integrates mathematical content within and across year-long courses instead of segmenting topics into single-subject courses, such as Algebra and Geometry. Throughout each course, strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics are interwoven to produce a mathematically connected curriculum. The

fundamental ideas that unify these strands include: symmetry, function, matrices, and data analysis and curve-fitting. In addition, they are unified by habits of mind such as "visual thinking, recursive thinking, searching for and describing patterns, making and checking conjectures, reasoning with multiple representations, inventing mathematics, and providing convincing arguments" (Coxford et al., 1999, p. xiv). Further unification comes from the "fundamental themes of data, representation, shape, and change" (Coxford et al., 1999, p. xiv).

The CMIC instructional materials are organized into connected units, comprised of multi-day lessons in which major ideas are developed through investigations of rich applied problems. Each lesson is divided into launch, explore, share and summarize, and apply phases. The launch phase (or Launch) piques the students' interest by introducing a problem situation related to the mathematical content of the lesson and provides an opportunity to assess the students' prior knowledge. The role of the teacher is to act as a director and moderator of the discussion (Hirsch, Coxford, Fey, & Schoen, 1995). When the teacher poses the question, he or she is to ask open-ended questions and allow for multiple students to provide suggestions; however, this is not the time to reach conclusions or a consensus. Then the teacher provides direction as needed to the students on the investigation that follows.

The explore phase (or Investigation) provides opportunities for students to work collaboratively in small groups to collect data, create models and meanings, and make and verify conjectures, often times using technology as a tool to aide in these processes. During this time, the role of the teacher is to facilitate discussion within the small groups (Hirsch et al., 1995). In this role, the teacher is meant to circulate from group to group providing guidance and support, clarifying or asking questions, giving

hints, providing encouragement, and drawing group members into the discussion to help groups work more cooperatively. As a facilitator it is important for teachers to listen carefully to students and encourage students to listen to and interact with others.

The share and summarize phase (or Checkpoint) provides closure to the lesson as the class members reconvene to discuss the ideas developed in small groups and arrive at a consensus and shared meaning of the mathematics under study. The teacher is again placed in the role of moderator (Hirsch et al., 1995), during which time he or she encourages student-student interaction to develop shared mathematical authority.

The last phase, apply, is intended for students to individually assess their understanding of a concept(s) and/or method(s) developed in the investigation. In this role as intellectual coach (Hirsch et al., 1995), the teacher again asks comprehension and connection questions and listens to the students as they describe their thinking.

The CMIC materials automatically provide teachers with many of the necessary conditions for reforming mathematics education, utilizing an instructional design that encourages rich discourse on contemporary mathematical content.

NSF Local Systemic Change Project

A second reason that Jefferson High School was chosen as the site for this study is the fact that the district participates in a NSF Local Systemic Change (LSC) project. This particular project is best described as one of Little's (1993) alternative models called the "teacher collaborative." It is a collaborative between a university and group of school districts, all of whom had chosen to adopt the CMIC materials. The mathematics teachers in the collaborative schools committed to 130 hours of professional development over four years, which included opportunities for them to expand their knowledge of mathematics content, pedagogy, and learning theories in

addition to leadership training and networking connections to other teachers.

Given the high school's choice of using a reform curriculum and participating in a LSC project, the experience of the participant in this study mirrors the experience of thousands of teachers across the nation. Although it may not be typical, it represents circumstances that are best thought to promote movement toward mathematics reform teaching practices.

Participant

Mr. Blume[1], the participant in this study, was a 6-year veteran teacher in the Jefferson High School mathematics department with several years prior experience teaching mathematics elsewhere. Mr. Blume is the lead teacher in his department and considered by his colleagues to be a "very good teacher." He was involved in the selection and adoption of the CMIC materials and has participated in CMIC workshops developed specifically for teachers implementing the curricular materials at all four of the year-long levels. Mr. Blume is a naturally reflective teacher who consistently strives to improve the teaching and learning in his classroom. Despite this, there was room for improving both the mechanics and mathematical content of his classroom discussions, and he was eager and willing to do so.

Description of Intervention

In a new paradigm for professional development, it is important to establish a collaborative and comfortable environment for the teacher's reflection to occur (Stein, Smith, & Silver, 1999). Mr. Blume was comfortable in his classroom, decorated with posters and quotes from recent movies interspersed with students' work, sitting at a four-person table where students sat moments before. For this reason, the intervention took place in Mr. Blume's classroom at Jefferson High

[1] This is not the participant's real name.

School approximately once a week for a 90-minute block of time during the 2000 Fall Semester. Because of school closings due to inclement weather there were a total of 12 meetings instead of the originally planned 14.

In addition to providing a collaborative and comfortable environment, Ball and Cohen (1999) suggest that the teachers' learning environment (i.e., professional development experiences) need to be situated in what is hoped to be produced and include "intellectual tools that could help them examine their own work with care and some detachment, to challenge their own thinking, and to draw reasonable conclusions from their inquiries, including generalizations that could help them to navigate future situations" (p. 11). To this end, experiences and intellectual tools were provided that were possible for the teacher to replicate and enact without the researcher.

What was paramount in all aspects of this intervention was what is called Focused Reflection. Focused Reflection involves deliberately calling to the conscious mind of the teacher aspects of teaching for him to cogitate and reflect upon. In this case, the teacher's reflection was centered around the salient features of mathematical discussions in order to improve the nature of his reflection and classroom practice in this area. The intervention elements related to these teacher-researcher collaboration sessions are described below and include a Discourse Reflection Tool (DRT) used in conjunction with the viewing of videotaped episodes, and selected readings from the research literature on classroom discourse. The descriptions are followed by a sample Focused Reflection Session (FRS).

Discourse Reflection Tool

The Discourse Reflection Tool (DRT) was developed by the researcher for

this study[2] to provide a model for the types of questions Mr. Blume could be asking himself daily as he reflects on his practice. See Appendix B and "Description of Discourse Reflection Tool" section, p. 42, for a complete description.

Implementation Procedure of DRT

The researcher videotaped Mr. Blume's class and recorded times, notes and questions regarding the discourse taking place. After the class was dismissed[3], she spoke briefly to Mr. Blume to obtain his immediate impressions and ask for his suggestion of a particular episode on which to focus the next FRS. The original intent was to encourage Mr. Blume to select the episodes to reflect upon, which would then allow additional opportunities to analyze the types and content of the episodes that were most salient to him.

At first, Mr. Blume was unwilling to identify episodes because he said he was not sure exactly what he should be looking for. After several weeks, he insisted that the researcher continue to select the episodes since "you do such a nice job choosing ones with lots to talk about, and I'm not sure I can think that hard at the end of the day anyway." Toward the end of the collaboration it was obvious that he was tired by this point in the day. Thus, each episode was selected by the researcher after consulting her written notes and reviewing the tapes the night of the observation in preparation for the next day's FRS using the DRT (see next section "Episode Selection Criteria" for the process of selecting the episodes for reflection and Appendix D for the list of videotaped episodes).

The following day, an episode of the videotaped observation was used in the

[2] This tool was revised after piloting it with two high school teachers as they viewed one of their own classroom videotapes. See Appendix C for the original instrument and description of the rationale for the changes.

[3] The videotaped class was Mr. Blume's fourth and final block class of the day.

FRS to help recall particular points in the lesson. Together, the teacher and researcher used the DRT to encourage reflection on both mathematical content and discussion mechanics.

Episode Selection Criteria

First it is important to define episode as it is being used in this study. In Sierpinska's (1997) words:

> For a mathematics educator, the primary unit of meaning is neither the word nor even the act of speech but *an episode of interaction*, characterized by a common and identifiable *theme* of conversation, relevant for the teaching of mathematics. Hence, what decides about the unity of an episode is its content and not its form. (p. 4)

Typically these episodes of interaction were 4-9 minute segments, possibly part of a greater mathematical discussion occurring within a lesson, that formed a cohesive unit based on their mathematical content.

Episodes were chosen to include: (a) aspects of worthwhile mathematical discussion, (b) missed opportunities to probe student thinking, and (c) extended discussion of one mathematical topic or problem. The first criteria was that episodes selected had to include at least one aspect of a worthwhile mathematical discussion, such as students providing explanations or building upon each other's thinking. The researcher decided it was important to have Mr. Blume reflect upon episodes that included these aspects to support his practice and encourage further improvement.

While it was important to point out aspects of worthwhile mathematical discussion, it was likewise important to choose episodes where Mr. Blume had missed opportunities to probe student thinking. Recognizing these instances provides excellent opportunities for teachers to reflect on why they missed them. It encourages teachers to think hard about their mathematical knowledge and rationalization for asking questions.

The third criteria relied on the length of time spent on a particular topic or problem. Episodes which stay focused on one problem or mathematical topic for a period of 5-10 minutes provide ample opportunity for a variety of aspects of worthwhile mathematical discussion to occur yet are manageable to remember and reflect upon.

The combination of these criteria helped the researcher determine the episodes to select for use during the following day's DRT discussion.

Research Literature/Readings

Relevant readings associated with the characteristics of effective mathematical discussions (e.g., justification and connections (NCTM, 1989, 2000), wait-time (Rowe, 1986), listening (Davis, 1997), and reflection (Etchberger & Shaw, 1992; Schifter & Fosnot, 1993)) were selected prior to the study to be provided for the teacher throughout the semester to provoke and encourage reflection. However, during the course of the collaboration, it was concluded that these were not useful to him for several reasons: (a) Mr. Blume did not think they were his type of reading,[4] (b) the researcher felt that Mr. Blume was so overwhelmed with the various happenings in his life that it would add an unnecessary burden (e.g., this was the first year of block scheduling and his first time in several years teaching only ninth-grade classes), and (c) the researcher had the impression that Mr. Blume was not ready to discuss that type of article. The intent was for the readings to be handed out and discussed at a subsequent meeting, giving the teacher time to read and think about the content. As it turned out, there were two discussion readings: wait-time (Rowe, 1986), and building mathematical discussions (Sherin, Louis, & Mendez, 2000).

[4] He explained in his 10/26/00 interview that he did not typically find education literature useful in his classroom teaching and instead tends to read books about personal philosophies or management (e.g., books about Zen religion, Phil Jackson).

These readings were discussed and incorporated into the FRS while viewing relevant videotaped episodes of his classroom.

The DRT provided a focus and structure for in-depth reflection that created opportunities for Mr. Blume to reflect on the mathematical discussions that occurred in his classroom. Coupled with the readings and his reflections on the readings, this intervention was expected to provide ample opportunity for the Focused Reflection necessary for change to occur during this study.

Sample Focused Reflection Session

The following is an excerpt from one of the pilot RFSs with a teacher who was using the same curricular materials in another school in the same LSC. In this excerpt, the DRT was used to discuss a videotape of her lesson. The purpose of the entire lesson was to develop the notion of periodicity and what it means for something to be periodic. The teacher had the students make a graph of some trend they would describe as periodic. The students drew a variety of graphs using different topics, usually depicted over time.

In the episode that we discussed using the DRT, the teacher was having the students share, in a whole-class setting, a description of the graphs they had created in small groups. In the episode, there were no instances of students conjecturing, providing justifications, making generalizations, or making connections, although during the collaboration session the teacher and researcher discussed the mathematics and pursued the idea of periodicity. Most of the talking during the classroom episode was done by the teacher.

The excerpt picks up during the discussion of the teacher's use of questioning during the episode.

(1) Researcher: So in terms of the questions, there weren't a whole lot of

questions?

(2) Teacher: No..no. I did ask the one about, I mean, it was almost rhetorical because I answered it myself, it um, can you predict.. but then I answered it myself. That one I should have let the students answer. Or ask somebody else to answer. The one about.. just some of them were just..okay you have [couldn't hear] The questions seemed to be, if there was a question, it was more clarifying so it would be a factual question..the one about "Is this reasonable?" I think it would be another comprehension question. It's more of a "do you understand this terminology?"

(3) R: So what do you think the purpose was of you asking these questions?

(4) T: I think it's..ah..what I generally try to do when I'm in class, and I usually try to make this clear to the class is that the questions that I ask are usually the questions which I think they should be asking themselves. So in a sense what I am doing, is that I'm thinking out loud about the way I am thinking about this whatever they are discussing. And I think that's sometimes why I answer the question myself. (Laugh) Because that is essentially because that's exactly what I am doing. Is.. I am asking the question that I think needs to be answered next in order to continue with this whatever it is that we're looking at. (Pause) So maybe that would be exactly what I should tell my students. You know, I am asking questions that I think needs to be answered next, and if you're not quick, I'm going to answer them myself. (Laugh) You either get right in there or .. but, see now, what I could do, what I should do, probably, is then instead of asking them that question, is that I could just ask a student, "okay, now, what's the next question?" (Pause) Because that would really get across to them that it's their job to come up with the questions. It's not my job to come up with the questions.

(5) R: Is there a particular "right" next question?

(6) T: No..not necessarily. Because the next question that you ask depends on where your own mind is. And maybe that would be even a better approach is, would be, to ask somebody okay, "what is YOUR next question about this situation?" And I also think that is a good way to get kids involved in

the lesson. Because then they don't feel they have to have an answer. They don't feel like they have to have it all figured out. All they have to know is "what's that next question?" And sometimes, and I do tell the students this, sometimes finding the next question is really the MOST important thing about solving this problem. It's not, "do I have an answer" because who knows what the question is. So once you can come up with that next question, so that might actually be, and..and, you know, certainly you can't see it in one small segment of a lesson, but I think um, but that is sort of my overall goal and maybe that's because I decided that that's the way that I learn. Is by figuring out, okay, what's the SMALL question I'm trying to answer here, which is the next step, and then does that lead me somewhere else. Does it always lead me in the right place? No. And that's also something that I think kids have to know is that sometimes their questions are a dead end, you know. And then ask which question got me off track. [Talks about related situation in her own studies] Now whether I actually do a good job of that in class or not, I don't know, but usually that's the way I decide what's the next question I'm going to ask. I think "if I were looking at this for the first time, what would be the next question?" that I would think would have to be asked. But I think it would be better for me probably to concentrate on "what's the next question YOU would think to ask in order to get closer to an understanding about whatever." In these periodic functions, "does it always have to start over at exactly the same point," maybe that's the question they are getting hung up on? Whereas somebody on the other side of the room might be getting hung up on the question "well does it always have to come up to the same height or could it be just getting smaller each time like a bouncing ball, would that be periodic or not?"

The teacher continues the discussion about periodicity and definition and misconceptions students might have developed during the lesson.

(7) R: Getting back to this, I think we touched on the questions. Um. There weren't, as we talked about, there weren't a lot of questions, cause the students were..

(8) T: Right

(9) R: But who or what provided validation and reinforcement of correct answers?

(10) T: Yeah, that was, yeah I'm guilty. Shouldn't have been, but that was indeed the way it was. The one student who was saying, "yes, yes" to what I was saying, I should have been letting him say it. And in a sense he was validating what I was saying about his question, or about my question. It was actually my question.(Laugh)

(11) R: Right (Laugh).

(12) T: My question about his graph, which then I answered and then he validated it.

(13) R: So maybe then if you had couched it in the terms of "I understand this to be ..?"

(14) T: Right.

(15) R: "..is that the way you intended?"

(16) T: Uh hum. That actually might have been a nicer way to do that. Because then his validation would have had a little more meaning.

(17) R: Yeah.

(18) T: Uh hum. Which also would be, I think, showing a little more respect for the students' thinking.

In this pilot FRS, the DRT provided the focus and the original questions, however additional probing questions or suggestions are provided by the researcher. For example, the question in turn 3 was from the DRT, but the question in turn 5 was intended to make the teacher think harder about the pattern of questioning she was using with the students. It encouraged her to think about the reasons for her pattern and to consider other ways that she could have conducted the discussion to include greater student participation and engagement. Turn 9 is another example of a question

originating from the DRT. It was followed by a suggestion (turns 13 and 15) for another way to respond to the student, which the teacher then acknowledged could move the authority to be shared with the students and contribute to developing a classroom social norm of respect for students' thinking. The FRSs followed a similar form, with the teacher reflecting out loud about the mathematical discussion, prompted by questions or suggestions by the researcher, either from the DRT or naturally developing out of the conversation.

Data Collection and Analysis

Data was collected from two primary sources: classroom observations and teacher-researcher interactions. As depicted in Figure 1, data collected in order to answer the part of the research question related to the nature of Mr. Blume's reflection included the transcripts from 12 FRSs using the DRT and interviews positioned (a) before any sessions, (b) 8 weeks into the collaboration, and (c) after the final FRS. The researcher also completed weekly summaries of her reflections of the observations and FRSs.

Data collected in order to answer the part of the research question related to the nature of discourse in Mr. Blume's classroom included videotapes from the 14 observations. In addition, the researcher made notes during the observations detailing instances of worthwhile mathematical discussions as well as questions to ask Mr. Blume about reasons for his actions.

The data overlapped and were used to triangulate the findings and inform both parts of the research question. However, for ease of discussion, the following description of the data is divided in order to address the part of the question it most directly informed.

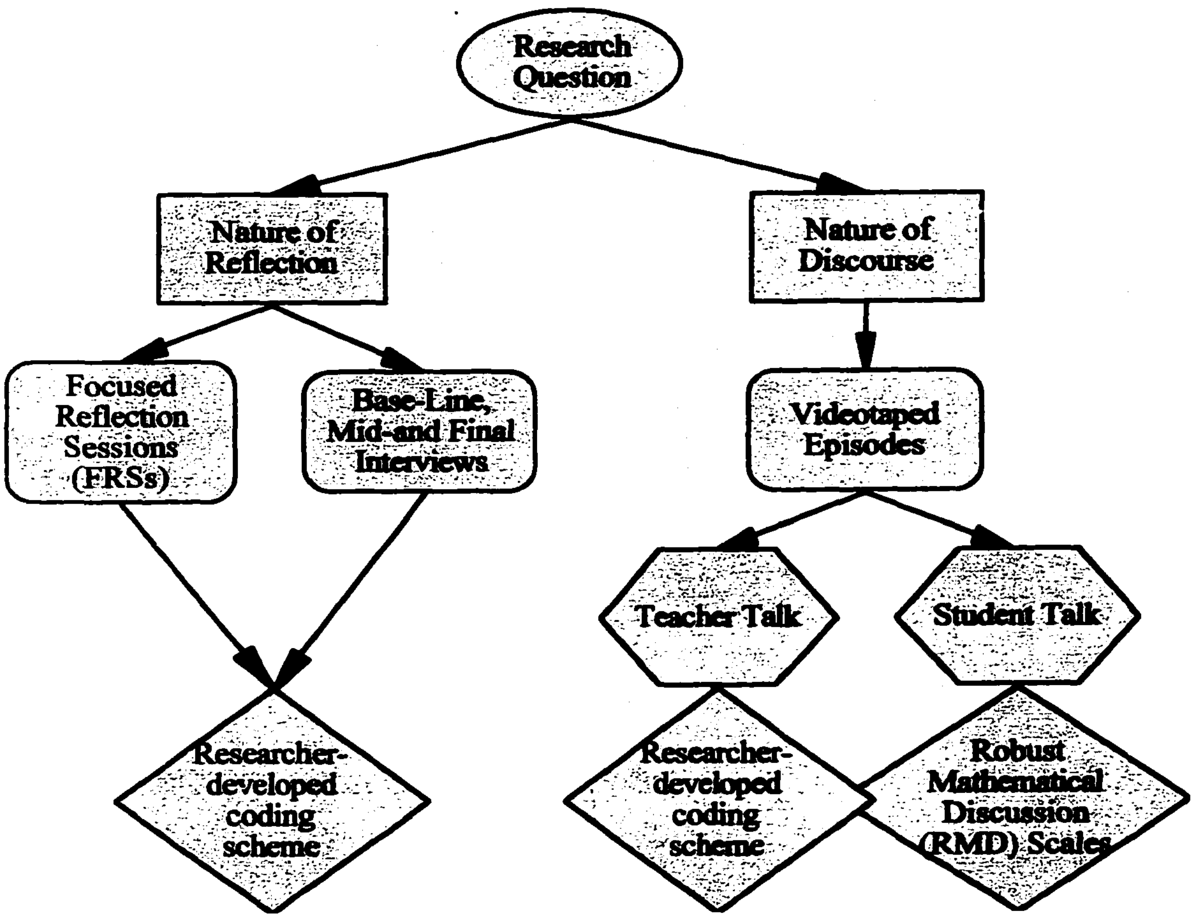

Figure 1. Data Collection and Analysis Schema.

<u>Nature of Reflection</u>

During the FRSs, the teacher and researcher met to identify and discuss episodes of mathematical discussions using the DRT and reflections on readings. Audiotapes of the sessions were transcribed and coded for the participant's nature of reflection on mathematical discussion. The following subsections describe the tools used to collect data from the interviews and FRSs. The coding of these interviews is described in the "Analysis of Nature of Reflection" section, p. 43.

<u>Interview Protocols</u>

There were three interviews, separate from the FRSs, which were intended to

provide additional specific information about Mr. Blume's vision of discourse. The protocols for these interviews are described in the following sections.

Baseline Interview. After the first observation (9/14/00), the Baseline Interview Protocol (see Appendix E) was used with Mr. Blume (9/21/00) to illuminate his perception of his discourse practices and provide a means to assess changes over the course of the collaboration. During the 90-minute interview, this protocol solicited information about Mr. Blume's vision of discourse and what he believed to be the current level of mathematical discussion in his classroom. The questions pertained to both discussion mechanics and mathematical content.

Mid-Interview. After a preliminary analysis of the collaboration sessions during the first half of the semester, a modified version of the Baseline Interview Protocol was developed. This Mid-Interview Protocol (see Appendix F) consisted of seven questions. The first three questions were identical to questions from the Baseline Interview and were asked in order to highlight changes in Mr. Blume's vision and in the nature of his reflection about discourse. From the preliminary analysis, it was evident that Mr. Blume's purpose for the discourse was different from that of the developers of the curriculum. Questions were added to solicit Mr. Blume's purpose for the discourse and the parts of a lesson. Students' thinking also became a focus of discussion during the collaboration, so a question about the role of students' thinking was added. A fourth question about which episode and FRS influenced his thinking was incorporated to investigate Mr. Blume's reflections and identify what was salient to him. The 120-minute Mid-Interview began on 11/2/00, but due to Mr. Blume's other obligations was completed the following day.

Final Interview. After a preliminary analysis of the collaboration sessions during the second half of the semester, the Final Interview Protocol was developed

(see Appendix G). The protocol was divided into the following parts: (a) similar questions, (b) key occurrences, and (c) verification. The similar questions portion placed quotations from Mr. Blume's Baseline and Mid-Interviews on index cards (see Appendix H) and asked Mr. Blume to agree or disagree with his earlier responses and provide further elaboration. These quotations were limited to statements made by Mr. Blume related to his vision of discourse and the purpose of discourse.

The second part was used to try to investigate possible salient moments from the discussions of the videotaped episodes. Mr. Blume was first asked to identify any episode that stood out in his mind as influencing his thinking about discourse. After this, a sequence of video clips that had been identified by the researcher were shown to Mr. Blume. The purpose of revisiting these episodes was to see if there were any changes in Mr. Blume's observations and reflections.

The third part consisted of a combination of questions and index cards (see Appendix I) chosen to check the validity of the researcher's thinking. Similar to Cooney's (1985) method of "clustering activity," quotations related to obstacles inhibiting his vision of discourse (e.g., comfort, time) from previous interviews were chosen and presented to Mr. Blume. He was then asked to categorize the quotations according to how closely they reflected his current thinking. Because of the anticipated length of the interview, the Final Interview Protocol was divided into two 90-minute interviews administered on consecutive days (12/19/00 and 12/20/00).

Description of Discourse Reflection Tool

The Discourse Reflection Tool (DRT) (see Appendix C) was used during the FRSs to promote reflection and discussion about the aspects of mathematical discourse investigated in this study. As a result, it provided an ongoing and consistent record of Mr. Blume's reflections during the collaboration. The DRT is divided into

three parts and provides prompts in the following areas: (a) background, (b) discussion mechanics, and (c) mathematical content. Each week, all aspects of the background and mathematical content were discussed, but given the limited time, the discussion mechanics rotated between wait-time, questioning, and listening.

Background. The background section collected data regarding the teacher's reflections about the goal and the perceived success of the lesson. The key areas of potential changes that this section assessed were (a) the way in which the teacher thinks about the goal for the lesson, (b) the differences in the number of students and which students participated in the episode, and (c) the nature of classroom interaction.

Discussion Mechanics. The questions in this section are directly related to the salient features of the mechanics of classroom discussion, namely: (a) wait-time, (b) questioning, and (c) listening. Generally, these questions require only brief answers, however Mr. Blume was pressed to provide rationale for his responses.

Mathematics Content. The largest portion of the DRT is reserved for questions regarding the mathematics content of the lesson. The questions in this portion assessed the teacher's reflections about (a) the verbal references to mathematical aspects of discussion, and (b) the unspoken aspects that affect the mathematical talk. The first type of questions encouraged the teacher to reflect on which conjectures, justifications, connections, or representations, suggested by the students, were pursued or not pursued. The second type of question encouraged the teacher to reflect on issues of authority and the establishing of sociomathematical norms (see "Nature of Meaningful Mathematical Discussion" section in Chapter II for more detailed descriptions of these issues).

Analysis of Nature of Reflection

The nature of reflection was analyzed by first coding transcripts from the 11

useable[5] audiotaped FRSs using the DRT and the three interviews. Statements made by Mr. Blume in these transcripts were imported into NUD*IST (Replee Pty. Ltd., 1993) and coded according to the preliminary scheme pictured in Figure 2.

The first level of the initial coding scheme was divided into three nodes: (a) reflective stage, (b) discussion mechanics, and (c) mathematics content. All statements coded to the Reflective Stage node were also coded to either the mathematical content or mechanics of discussion nodes. In instances where the content and mechanics of a statement could not be separated, the statement was coded to both nodes (see Appendix J for the nodes and definitions of the final coding scheme).

The reflective stage code was defined by and further divided into stages of reflective practice (Shaw & Jakubowski, 1992): (a) perturbation, (b) awareness, (c) commitment, (d) vision, (e) projection into vision, (f) reflective practitioner (see "Use of Teacher Reflections" section of Chapter II for more detailed descriptions of these categories).

The mechanics node was separated into wait-time, questioning, and listening. This initial coding scheme was elaborated on throughout the data collection and analysis process (see "Wait-time,""Questioning," and "Listening," sections of Chapter II for more detailed descriptions of these categories).

The mathematical content node was divided into four components: (a) justification, (b) generalization, (c) representation, and (d) connections. The statements at each of these nodes were coded as solicited or unsolicited by the researcher. In addition, the rationale provided by the teacher as to why he did or did

[5] The audio from teacher-researcher collaboration session 11/22/01 was not able to be transcribed due to poor sound quality.

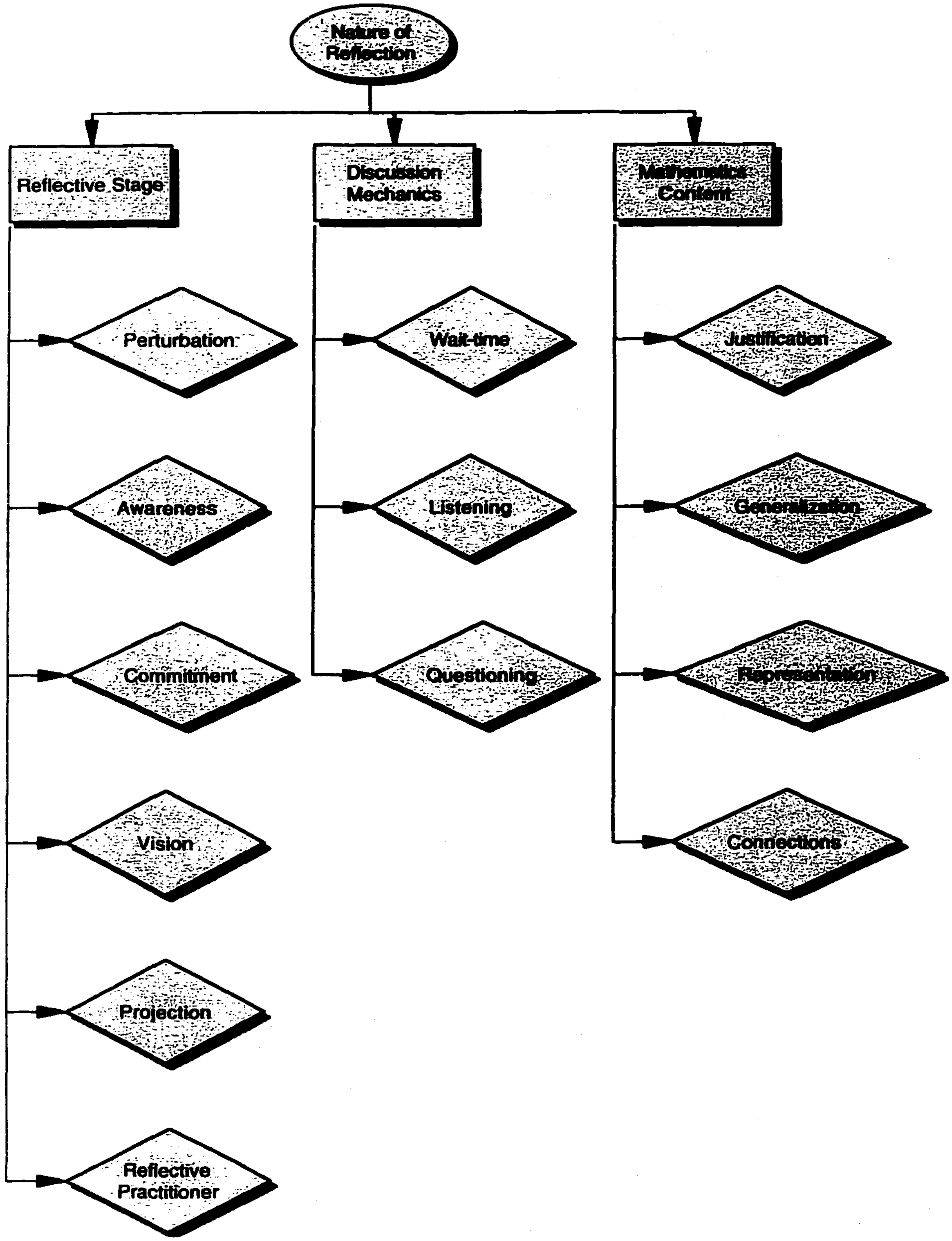

Figure 2. **Initial Coding Scheme for the Nature of Reflection.**

not pursue the mathematical content was coded (see "Nature of Meaningful Mathematical Discussion" section of Chapter II for more detailed descriptions of these components).

During the coding of Mr. Blume's perceptions, memos were made by the researcher on specific codes to identify instances where the mathematical content component perceived by the teacher contradicted what the researcher observed from the same episode. It was necessary to view Mr. Blume's comments about the mechanics and mathematics of his lessons with an understanding that his perceptions are filtered through his beliefs. Together, these data identified the nature of change in Mr. Blume's reflections on the mathematical discussions in his classroom.

Nature of Discourse

The first observations were performed during the early weeks of school, prior to the beginning of the teacher-researcher collaboration. It was assumed that the mathematical discussion may vary depending on the point in the lesson; therefore, observations of Mr. Blume included Launch, Investigation, and Checkpoint discussions (see "A Reform Curriculum" in this chapter for description of these parts of a lesson).

During these observations, Mr. Blume was videotaped with a microphone attached to him to record his verbal interactions with students. A total of nine videotaped episodes were chosen to be analyzed and all episodes were transcribed prior to any coding. They were then coded by (a) using the RMD Scales (see Appendix K) to analyze all utterances made by the students and (b) using a researcher-developed coding scheme to analyze all of the teacher utterances (see Appendix J for the final coding scheme).

The following sections describe the process of selecting the episodes to

analyze, the rating rubrics of the various components of the RMD Scales, and the procedure for analyzing the episodes using the data collected from the RMD Scales.

Selection of Episodes

Highest priority was given to those episodes that were discussed in the FRSs in order to triangulate the observation data with what Mr. Blume was thinking during the episode. Seven of the nine episodes were ones used in the FRSs, allowing additional opportunity to triangulate the data. The selection criteria of the episodes varied based on the point in the lesson. The following describes how the three Launch, Investigation, and Checkpoint episodes were chosen (see Appendix D for the list of videotapes and their characteristics).

No further selection criteria was needed for the Launch episodes because there were only three observations that included discussions of the Launch. This was due to the fact that Launches naturally occur infrequently and typically took place on a Monday when the researcher could not observe the classroom. These three Launch episodes were used in the analysis process; even though they were observed near the end of the semester they were spaced over a 5-week period.

During the majority of the observations students were working on an Investigation, so there were many such episodes from which to select. The following criteria were used to choose the three Investigation episodes: (a) space apart over the semester, (b) small-group settings, and (c) length of time with teacher interaction. The third criteria narrowed the options considerably, since it was often observed that the teacher would stand by or interact with a group for less than two minutes. The three episodes selected took place during the middle seven weeks with episodes showing students working in small groups while interacting with Mr. Blume for 4-9 minutes.

The Checkpoints, like the Launches, were less frequently observed. Of the 5

observations that included discussions about the Checkpoint questions[6], one was unsuitable because it was not complete and the second was not equally spaced from the others. The remaining three episodes spanned an 11-week time frame, occurred during whole class discussions, and ranged from 6-9 minutes in length.

Robust Mathematical Discussion Rating Scales

This collection of scales, an extension of the Robust Mathematical Discussion (RMD) Rating Scales developed by Mendez (1998) to evaluate an eighth-grade mathematics teacher's mathematical discussion, was used to investigate the nine selected videotaped and transcribed episodes (see Appendix J for the changes made for this study). The RMD Scales have two dimensions, each having several components: (1) Mathematics Rating, and (2) Discussion Rating. Numerical values for each component were assigned based on a rubric-type scoring system falling on a continuum from 0 to 1. The following provides a description of the dimensions, components, and rubric-type ratings.

Mathematics Rating Scales. The Mathematics Scale (see Appendix K) is divided into four components: (1) justification, (2) representation, (3) generalization, and (4) connections. The first three sections were modeled from Mendez's (1998) work, with the fourth section, connections, added to further evaluate the mathematical content in this study (see "Nature of Meaningful Mathematical Discussion" section of Chapter II for the justification for the inclusion of this component).

Justification described the level of reasoning that students used in the episode. The three levels of the justification scale are: (a) proof, (b) explain, and (c) none. An utterance was coded as "proof" and assigned one point if a "logical argument is given

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[6] On many occasions, the researcher was observing the classroom when a Checkpoint discussion could have taken place. However Mr. Blume skipped the questions because he did not think it was necessary or a good use of time to answer them at that point.

for being sure that an answer is correct or counterexamples provide a refutation." It was coded as "explain" and assigned a half-point if a "student explains why her answer holds and/or how she got her answer." It was coded as "none" and assigned no points if "no justification is given." Adding the total and dividing by the number of student turns (or utterances) determined the justification score.

Representation was scored based upon the number of representations used in a discussion by the students. A topic within an episode was coded as: (a) unpacked, (b) amplified, or (c) single. A topic was "unpacked" and assigned a 1 if "more than two representations" were suggested by the students. A topic was "amplified" and assigned 0.5 if "two representations" were suggested, and a topic was "compressed" with no points given if "only one representation" was supplied. To determine the score for this section, each topic within an episode was rated separately with the highest rating for an episode taken.

Generalization assessed the students' ability to extrapolate a concrete example or make predictions. This section was divided into either "generalization" or "concrete." One point was assigned to instances where there was "generalization beyond particular examples, categorization, recognition of a pattern, or broad prediction" and no points were assigned to instances where the discussion was "limited to one particular situation." The rating was determined by the number of points divided by the number of student turns.

Connections assessed students making connections within and outside of mathematics. This section is divided into two levels, "connected" and "none." One point was assigned to each student utterance when a student related the current mathematical topic of study to another mathematical or real-world topic. The rating was determined by taking the total number of points and dividing it by the student

turns.

Discussion Rating Scales. The Discussion Dimension (see Appendix K), as described in Mendez (1998), was divided into three components: (a) intensity, (b) engagement, and (c) building.

Intensity assessed how a student entered the discussion, and was rated on three levels: (a) volunteer, (b) elicited, and (c) none. An utterance was coded "volunteer" and given a value of 1, if "students voluntarily join the discussion without teacher mediation." It was coded "elicited" and assigned 0.5, if the "teacher nominates student speakers or asks questions of students." If an "off topic remark is given or no student enters the discussion" no points were assigned to the utterance. The rating was determined by totaling the points assigned and dividing by the number of student turns.

Engagement was difficult to measure, since it was likely that not all students who were intellectually engaged voiced an opinion. However, in mathematical discussions as defined in this study, it is important for students to verbally engage themselves in the discussion to further and deepen the understanding of all members of the community. For this reason the scale of engagement was calculated by counting "the number of student speakers as a proxy for number engaged" and dividing by the number of students present.

Building measures how much students were listening and building upon each other's responses and was divided into three levels (as opposed to two in Mendez (1998)): (a) build, (b) neutral, and (c) none. An utterance was coded as "build" and assigned a value of 1 if "responses build on earlier comments with new ideas and are integrated into the discussion." The following two categories were both coded as 0 in Mendez (1998), but in this study a statement was deemed "neutral" and

assigned 0.5 if it was a "repetition of earlier stated ideas by another." A student response was coded as "none" and given no points if it was a "first response or non-sequitur." The rating was calculated by totaling the number of points and dividing by the number of student turns.

Analysis of Nature of Discourse Data

The transcribed videotaped episodes were analyzed by (a) using the RMD Scale results to investigate the student talk, and (b) using a coding scheme to investigate the teacher talk. Student and teacher utterances were coded according to the following preliminary schemes, with new codes added as themes emerged (see Appendix J for the final coding scheme).

Student Talk. The analysis of the student talk consisted of using the RMD Scales to assess all of the components of the Mathematics Content and Discussion Rating Scales, as described above. Each utterance was assigned a value based on the rubric-scoring scale. In order to increase the study's validity, one episode (10/26/00) was randomly chosen for the researcher and another mathematics education researcher to individually code and then compare their findings. The purpose was for the researcher to work out the definitions to make a more systematic analysis of the remaining episodes. The two researchers were consistent in the categorization of all areas (over 95%), except the area of Building in which the second researcher had misinterpreted the scoring rubric. After a discussion, she agreed with the first researcher's interpretation and the results of the coding.

The remaining coding was performed by the researcher and took place during a 24-hour period of time, so that time would not distort the definitions of the codes in the researcher's mind. Since the purpose was to look for trends over the duration of the collaboration, the episodes were randomly ordered prior to analyzing so as to not

bias the results (see Appendix L for the order and raw data). Tables and graphs of the data were used to identify trends and look for patterns.

Teacher Talk. The same nine transcribed episodes were also analyzed by focusing on the teacher utterances. These transcriptions were imported into NUD*IST (Replee Pty. Ltd.,1993) and coded using an initial coding scheme developed by the researcher. This coding scheme was informed by previous research in discourse analysis, namely, Pimm (1987), Jacobson & Lehrer (2000), and Forman, Larreamendy-Joerns, Stein, & Brown (1998). As depicted in Figure 3, the initial coding scheme was separated into three nodes: (a) purpose, (b) revoicing, and (c) questioning.

Each utterance made by the teacher was coded according to its apparent purpose. Jacobson and Lehrer (2000) provided the initial classification based on their research about the function of the teacher's talk. The initial sub-nodes of purpose were: (a) elicit fact, (b) elicit explanation, (c) establish norms, (d) probe student thinking, (e) generate conjectures, and (f) confirm. Since each utterance of teacher talk was coded for its purpose it became necessary to add new codes to describe unanticipated purposes (see Appendix J for the complete list of codes and definitions). If appropriate, the utterance was also coded under revoicing and/or questioning.

Revoicing speaks to the ways in which a teacher uses students' utterances to extend the discussion. Prior research in revoicing found that there are four types of revoicing that teachers tend to use (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; O'Connor & Michaels, 1996) which formed the following sub-nodes: (a) clarify or explain, (b) explain reasoning, (c) introduce ideas, and (d) redirect discussion.

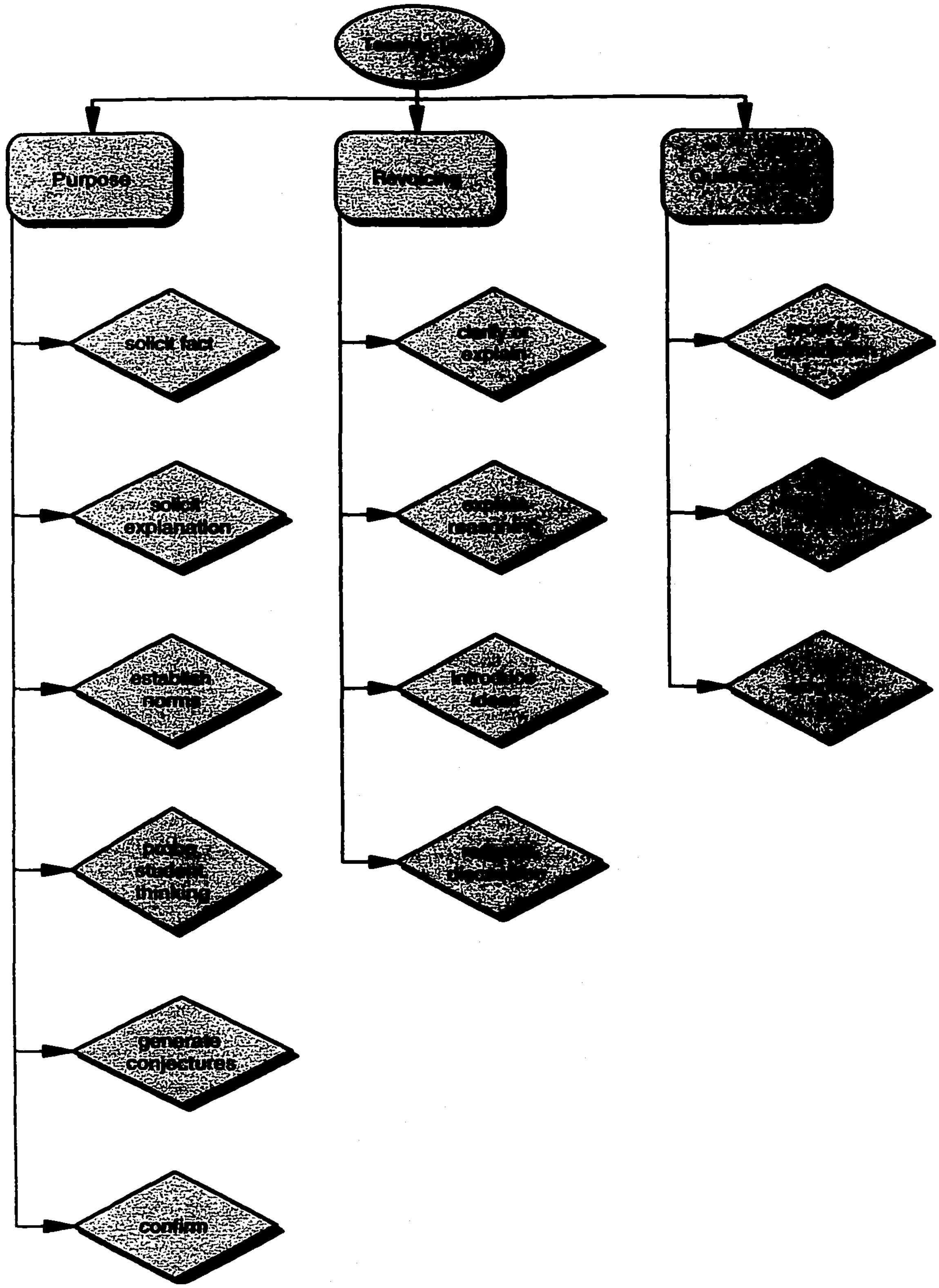

Figure 3. **Initial Coding Scheme for Teacher Talk in the Classroom Discourse.**

The questioning nodes and sub-nodes described the types of questioning techniques being used by the teacher. Pimm's (1987) research suggested that there were three types of questioning techniques that teachers could use to enhance the verbal interaction of the students in the classroom[7]. The three types formed the sub-codes for the questioning nodes: (a) proof by intimidation, (b) "clozed" questioning, and (c) echoing (see the "Questioning" section of Chapter II for a more complete description).

As the researcher coded the utterances, new nodes became necessary and were added to the existing nodes. After a preliminary coding of the nine episodes, the researcher read back through a report on each node to verify whether the utterance belonged. New sub-nodes were made to cluster together the data that showed similar properties and new nodes were established for the data that seemed out of place with the other data coded to that node. After this new categorization was in place, another round of coding was completed and the researcher again read back through a report for each node. Memos were made throughout the process to point out trends and patterns or salient features of a node.

Summary

This chapter outlined the methodology used in this study. It was a case study focusing on one experienced secondary school mathematics teacher who had prior professional development training regarding the CMIC (Coxford et al., 1997) curriculum. As part of this study, he participated in a teacher-researcher collaboration focused on mathematical discourse using videotaped episodes of his classroom. During Focused Reflection Sessions (FRSs), the participant was encouraged to

[7] The researcher acknowledges that these questioning techniques are used by teachers and agree that they would increase student involvement, but does not think that they are sufficient to encourage meaningful mathematical discussions.

discuss articles he had read and answer questions from the Discourse Reflection Tool (DRT) regarding the discourse captured in videotaped episodes of his classroom.

Data was collected to inform the nature of Mr. Blume's reflection and the nature of Mr. Blume's classroom discourse. To investigate the nature of reflection, data was collected during three interviews and 12 FRSs. The analysis consisted of coding and looking for patterns in the transcribed data. To investigate the nature of discourse, nine episodes were selected to analyze using (a) the Robust Mathematical Discussion (RMD) Scales to explore trends in the student talk and (b) a researcher-developed coding scheme to explore patterns in the teacher talk. Altogether, the analysis provided a picture of the struggles, challenges, and successes of Mr. Blume while engaged in this teacher-researcher collaboration. The results are introduced and discussed in the next chapter.

CHAPTER IV

RESULTS

This reporting of the results parallels the two components of the research question: the first about the change in Mr. Blume's nature of reflection, and the second about the observed change in the discourse in the classroom. The following subsections describe the results of the data analysis.

Change in the Nature of Reflection

"It is good to have an end to journey toward, but it is the journey that matters in the end."[8]

Three observations were made as a result of an analysis of the data: (1) Mr. Blume exhibited a shift from explaining away or defending his practices to openly reflecting and considering alternative ways of thinking; (2) Mr. Blume shifted from using general terms while talking about the "natural" development of the discourse to acknowledging, with specific strategies, that the teacher plays a significant role in the development of worthwhile discourse; and (3) Mr. Blume held as a consistent focus students' comfort over their learning of the mathematics.

Defending vs. Reflecting

"The first step to wisdom is silence, the second is listening."[9]

Over the course of the collaboration, Mr. Blume exhibited a shift in his reflection from defending what he did in class to openly sharing his thinking and considering alternative ways to facilitate the classroom discourse. In the beginning of the collaboration, Mr. Blume discussed why he asked a particular question or made a certain remark with certainty that what he was doing was the best way. For example,

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[8] Quotation written on whiteboard at the front of Mr. Blume's room, 10/10/00.

[9] Quotation written on whiteboard at the front of Mr. Blume's room, 10/26/00.

when he was asked why he always responded with "good" to any student's response, whether it was correct or not, he made the following comment:

> Positives I guess. Just it was "good" you responded, it is, I appreciate you doing that, it was a good answer. I try to make sure I don't say "good" if it wasn't good in some sense so they just don't stop listening to that word. But, yeah, I don't know if it is a sub-conscious thing or not, but I think along those lines, we just know it's a pleasant thing to hear and that engrains itself into them as well and helps the atmosphere in the room. (Focused Reflection Session (FRS) 9/22/00)

In another early instance, the researcher asked Mr. Blume if there was something he could have done to determine whether a particular silent student in a group understood what the rest of the group understood. He replied:

> I tried to–as I was asking questions back and giving a nudge here and there–I would try to look at her [the silent student] and Aaron just as much as Carol and Keith, just so they knew that, as far as I was concerned, they should be part of this, too. I didn't ask them directly. I could have done that. I guess I didn't feel that at the time, and I still don't, if that would have done it or not. I kind of wanted it to flow a little more naturally. I don't know any other way than somehow directly asking them to give a verbal response to some of this. (FRS 10/04/00)

In both of these excerpts, Mr. Blume was certain of what he did in the episode. Although he provided some justification for his actions, his tone was more one of explaining rather than reflecting. In the second excerpt, he did show some sign of considering other options when he said, "I could have done that," but quickly dismissed it.

Early in the collaboration, Mr. Blume made intermittent comments that showed less certainty about the appropriateness of and motivation for his actions in the classroom. For instance, in the following excerpt, the researcher had just asked Mr. Blume how he knew if the students understood what another student had said. In his response he admitted he did not have an answer:

> I don't know if they understand. I don't know if they pictured what she said in their own mind. Because it's always a good reminder for me when we do those discussions how they are seeing the same thing in so many different

> ways. That's a good question and I have no answer to it. [Laughs] I have no idea. (FRS 9/29/00)

Although Mr. Blume made other similar comments in his reflections at the beginning of the collaboration, they were the exception rather than the rule.

In what the researcher described in her notes as "a breakthrough," the session on October 18, 2000 was filled with instances of Mr. Blume reflecting on his classroom and considering the possibility of alternative ways to approach classroom discourse. In this very emotion-filled session, Mr. Blume admitted that he was uncertain and not completely sure how best to facilitate the classroom discourse.

Directly before the turning point in the session, Mr. Blume explained why he had used a questioning technique that funneled a student's thinking (as described in Wood, 1998), and impeded her ability to build on her own thoughts. The researcher pointed out in the episode that the student had shown she intuitively knew the answer before Mr. Blume led her through a series of questions. He began his response in his typical explanatory, certain mode:

> Right. I guess I didn't feel it was necessary to grab it all at once. Grab that one piece and you've got it, we'll get the rest. Maybe you'll get them as you think of it on your own and if you don't, maybe you'll get it when I tell you to do, maybe you already have it and if you don't have any of that, we'll get it at another point. We'll get those ideas of breaking down units into smaller increments to get more accurate readings. (FRS 10/18/00)

He and the researcher continued discussing what the student could have been thinking and why he asked particular questions. Suddenly, he paused and exclaimed, "It's something. The more we talk like this, the more I remind myself that I have no idea what I'm talking about" (FRS 10/18/00). When the researcher asked what he meant by the statement, he sadly responded, "I don't really have a clue to what they're thinking and how best to learn and how [voice trails off] all that. I don't have a clue" (FRS 10/18/00).

Later in the same discussion, Mr. Blume demonstrated that he was reflecting

on the episodes on his own time and was beginning to form his own conclusions, evidence of a change in his reflective practices:

> I'll think about what I've said sometimes, when we talk like this and think, "Boy, that was a stupid thing to say. Who do I think I am to say what somebody's thinking?" There's so much more involved in it, I guess, than just the subject matter. There are just so many other variables involved. To maximize all of them at once [voice trails off]. (FRS 10/18/00)

The entire tone of this FRS shifted from a teacher who sounded certain of what he was doing and who was not considering alternatives to one who was beginning to question himself and wonder about other possibilities. He began to realize that although he thought he knew what the students were thinking, for example, or why he was engaging in a particular pedagogical practice, he probably did not actually know and would need to change his questioning and thinking in the classroom to get the results he desired.

After this turning point, Mr. Blume continued to make statements that sounded explanatory, but they were less frequent. More often, the comments he made included questions and considerations of alternative ways to approach the class. The following exemplifies his greater awareness of alternatives:

> I see the strong personalities speaking up some, which leads me to think, "How can I get some of these other people involved?" I see myself having to prod a lot of things or thinking that I have to prod--not having the patience to wait to see if something just happens. Every time I see that, I think, "How, should I have just waited in this situation or how can I help get them to start?" (Mid-Interview 11/02/00)

Mr. Blume also evidenced a self-awareness of his own typical patterns of response to new ideas and alternatives in his later reflections. During the final interview, Mr. Blume was asked to categorize and comment on some of his prior statements placed on cards (see Appendix I). Card 1 stated:

> That's another problem I've had this year. The first time through it, my mind is pretty open to any response to it. Now the second time through, I'm starting to get locked into certain things and I don't adjust as well as I think I did that

> first year to alternative responses. I don't think it's a good thing [to get locked in]. (FRS 10/15/00)

His response to the card showed that he had shifted in his reflections to acknowledging that alternatives do exist and recognizing the value of reflecting on his practice:

> I had no set anything [the first time I taught CMIC]. The more years I teach, the more I get locked into certain things. I don't adjust well to alternatives. I said I don't think that's a good thing to happen and I strongly agree with that yet. But, I think now, and again through our discussions and your questions of me and just listening as I verbalize my thoughts, hear myself, it's helped me to be more aware of that and to really think about it and start to figure out ways to get through it. (Final Interview 12/20/00)

In the following excerpt, Mr. Blume was reflecting on a Launch episode where students had been asked to create a path to use to paint the lockers of their school and then determine which of the proposed paths was most efficient. As he contemplated the episode, he discussed how he was working hard to implement a new pedagogical practice:

> It was a Launch, and like we talked last time, I don't think the idea is to determine the right answer at this point so I try to keep my mind open and not veer toward something. When the first couple said they'd just paint this hallway and this hallway and this hallway and never going into which side they're painting first and what order, I had a hard time biting my tongue and waiting to see if that would come out and if others would pick up on it or not [voice trails off]. It did. It came out with Noah, but then it was lost again with others, and when it didn't come up again at the end of that, then I went up and asked again, "How are you going to do this?" I had to consciously make an effort not to do that four minutes earlier. (FRS 12/08/00)

In these two responses, Mr. Blume showed signs of becoming a reflective practitioner. He was reflecting during the episodes and was considering the consequences of his actions. In the final interview, Mr. Blume was asked to view three episodes from his classroom and reflect on what he saw. His tone had changed, and he appeared to be much more personally removed from the situation he was observing. He had almost adopted the role of a third party:

> It seemed to me, each succeeding episode, that I was pulling back from

> conducting the discussion. . . .The second was, I was sitting in the back. It was started by a question by a student. It wasn't started by me. Other students were responded by [*sic*] opinions that were given, some by my prompting and some without. There were more hands up. I think I was still, in a sense, leading it but not to the extent that I was in the first episode. I was listening to more of their answers. I wasn't necessarily looking to get straight to the solution that I thought was right. I was willing to take a 'round about way of getting there and listening to other people and seeing where that took us. (Final Interview 12/19/00)

Over the course of the collaboration, there were changes in the nature and frequency of Mr. Blume's reflective comments. In the beginning, Mr. Blume's comments tended to rationalize and defend what he was doing without considering other possibilities. His tone was one of certainty. After the session on October 18, the frequency of Mr. Blume's reflective comments increased, and he made efforts to reflect on his teaching practices by raising his own questions about his teaching and considering the alternatives.

Generalities vs. Specifics

"It's what you learn after you know it all that counts."-John Wooden[10]

Over the course of the collaboration, Mr. Blume shifted his reflection from talking in general terms about how the discourse of his classroom naturally developed to discussing specific strategies he used to proactively establish a discourse community in his classroom. In the beginning, Mr. Blume made little mention about how the teacher influences classroom discourse. Instead, many of his reflections spoke of how, over time, students improve the nature of their participation in classroom discussion, but with no clear description of how such improvements occur. For example, in the baseline interview, Mr. Blume was asked why he did not normally interrupt a whole class discussion to allow students to discuss a topic in small groups briefly reconvening the discussion. He responded:

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[10] Quotation written on whiteboard at the front of Mr. Blume's room, 9/21/00.

> My initial thought, I guess, would be that I expect us to get to the point where they can function at a high level whatever way we're doing it. They're in enough control of themselves and their minds, I guess, that no matter how we attack it, they can transition themselves to it and just do that well, regardless of what method it is. So, when we're into that, I feel that we can do that and if we're not doing a very good job, I feel that we need to stick through it 'til the end just so that we learn how to do this better. I guess, even if it's going poorly, I figure that we need practice at this way of doing this so we're going to take it to the end then we'll discuss it. We'll discuss how to make this more effective the next time. (Baseline Interview 9/20/00)

This showed Mr. Blume's focus on practice on the part of the students without a clear idea of how it occurred or the role of the teacher. When probed about how this discourse could be made more effective, Mr. Blume could not articulate any specific suggestions. Instead, he repeatedly asserted that good discourse takes a great deal of time to establish and requires much practice on the part of the students.

In the final interview, he had a different view of his role in facilitating discourse in his classroom:

> If I make it a point to help try and build these discussions and help them grow and work at it daily and persist at it, then it seems to happen that way. They seem to get better as the semester moves on. But, I guess I'm not sure that if a point wasn't made to help them grow, that they'd naturally...I don't think they'd naturally go as far. (Final Interview 12/19/00)

He attributed part of the difference in his thinking to participation in the collaborative:

> I think I thought much more this semester about what I was doing and what I was shooting for and how we were going about doing it a lot more than I have in the past. I think in the past, there was thought into it, a lot of it just came naturally and whatever happened is just how it naturally came out of me. So, that differed. That probably altered the way I did some things. (Final Interview 12/19/00)

This difference was apparent in Mr. Blume's reflections as he shifted from generalities to being specific about how he felt a teacher could encourage discourse, primarily by: (1) using specific pedagogical techniques in orchestrating the discussion, and (2) keeping an open mind. These developments are discussed in the following sections.

Orchestrating Discussion

Mr. Blume began to recognize and observe that some pedagogical techniques that the researcher suggested were successful in altering the discourse in his classroom. The three most frequently referenced techniques were utilizing wait-time after asking a question or after a student made a comment, asking different kinds of questions to increase student thinking, and both listening to students and modeling that listening for other students. Of the three, Mr. Blume found that he could encourage students to think and respond the most by not immediately answering after a student had spoken.

Over the course of the collaboration he made several references to the effects that increasing his wait-time was having on his classroom discourse. Prompted by the researcher to reflect on his wait-time during the 9/22/00 FRS, Mr. Blume claimed unprompted during the next meeting, "I was just going to tell you. I'm trying and I've been trying since last time to wait longer before speaking after they give an answer in many of the classes." He went on to describe what was happening as a result:

> They don't stop then. They'll pause and then they won't say anything and then, like with these students, all of a sudden they'll say something then like–I'm not sure what they're thinking but something else will come out kind of haltingly. Then if there is still no response [by me].[Laughs-voice trails off] (FRS 9/29/00)

The following week, he again talked about his role in the discourse when he was asked what he was thinking during a particular moment in an episode:

> I was probably thinking, like I do most of the time now, of trying to slow myself down and pause and just wait and see what happens. If nothing happens, then stepping back in but, taking a pause to allow something to happen just not expecting anything, just trying to provide an opportunity. (FRS 10/4/00)

In addition to trying some of the researcher's suggested pedagogical techniques, Mr. Blume developed one of his own and reflected on it with the

researcher. He called this pedagogical technique "playing dumb," and it was enacted in two ways--by lessening his own knowledge and by pretending he did not understand what students were saying. He decided approximately eight weeks into the collaboration to stop reading ahead in the teacher's text. The researcher recorded in her notes after the interview the following concern:

> I'm concerned about one of the approaches and changes he's made. He seems to think that if he doesn't know what's coming next (and tells the students that "there are no guarantees--because I haven't looked at the answers") (1:09) that they'll work together better and achieve better discourse. This concerns me, though, because it seems to be translating into him possibly not knowing what the purpose of the investigation or lesson is. He also mentioned that it might not even be necessary to answer the Checkpoint questions in class because from their discussions, he'll see that it would be redundant to do so. (Researcher Notes 11/3/00)

Mr. Blume thought that not reading ahead would enable him to answer the students' questions honestly with "I don't know" and encourage them to come up with their own answers. He also thought that this method would force the students to question what he said and to think for themselves:

> The day before yesterday, when we started this investigation, I told them that I haven't looked at it to plan ahead. I had not looked at it at all, and I'm not going to look at it. I'm just going to do it right along with them and, if they have questions, I will give them my impression of what I think is being asked, but I'm not basing it on anything else at this point, so I don't guarantee that what I'm telling them is right. I wanted to try to, that, to try and get them to rely more on each other again and don't just think that I've got all the answers and that I'm the one you always check with, and hopefully that would lead again, build into better class discourse and such and so, I don't know. (FRS 11/03/00)

The second way in which Mr. Blume began "playing dumb" was to act as he did not understand student speakers so they would have to describe their ideas in some other fashion. For example, in the following excerpt, he recounted his thoughts and actions during an episode:

> In that case yesterday, I knew what Heather was thinking, I think. I think, sometimes, to promote that conversation, though, you just kind of have to act as honestly as you can that you're unsure and that they can help you along

> with it. In doing so, they continue the conversation and that allows it to be easier for other students to join in because they're leading it, and I'm just the idiot up there trying to figure out what they're doing. Yesterday, in that case at least, I thought I had a pretty good idea what it was she was doing, but I wanted to keep going and keep going. (FRS 12/01/00)

When prompted by the researcher to suggest ways for a teacher to encourage the students to listen to and build upon one another's ideas the following week, Mr. Blume again described his idea of playing dumb:

> Like when Tina gave her answer and referred it to Trevor's then I went back and asked Trevor what he had said. At that moment, I personally knew full well what Trevor had said but, like we talked last time again, you play the fool and you ask them again. That just models what you just said. I'm listening. I'm trying to tie it into what other people said, and I can't remember, so I'm going to ask him again. (FRS 12/08/00)

Mr. Blume was not sure that his not looking ahead in the text or pretending not to understand were educationally sound ideas. He said that he would have to evaluate their effectiveness after he saw the results of the students' final semester test scores. He was fairly certain, though, that the changes he had made were leading to a better match between the actual discourse and his vision for the discourse in the classroom, in that the students were participating and interacting more during the classroom discussions.

Keeping an Open Mind

Mr. Blume's conviction that he needed to keep an open mind in order to promote his vision for discourse developed and deepened over the course of the collaboration. Early in the collaboration the concept of keeping an open mind was not defined or explicitly identified, but was more of a sense that he should not always be looking for a particular answer. For example, during one of the earlier FRSs, he made the following comment:

> Maybe I was a little harsh or I was a little, they did have a good response and it wasn't the one I was looking for. And I locked myself into that one response I was looking for so much that I didn't give enough credit to what was said. (FRS 9/22/00)

By the mid-point of the collaboration he was using the phrase "open mind" to describe the approach he wanted to take to his classroom discussions. He thought this would allow the students to share multiple perspectives and encourage more student interaction during the discussions. The following is an example of how he described what he should be doing, with similar statements reiterated numerous times over the second half of the collaboration:

> I think I could have a little more open mind. Just learn how to keep my mind open to many possibilities while still being aware of what I want, what needs to get discussed or needs to get spoken or needs to be understood. Have that in the back but keep, learn how to keep, an open mind so that we can come to that in many different ways where I'm not looking for it or to hear it in a particular fashion or set words being used. (Mid-Interview 11/02/00)

In another instance, Mr. Blume had asked during a class if some people had not arrived at an answer another student had posed. When a student responded that she did not get the same answer, but was not sure how she got hers, Mr. Blume did not ask her what her answer was. During the next day's FRS the researcher asked why he did not explore the alternative answer. He replied:

> Since I, just me, concentrate on one thing at a time, it seems. Since that was my focus [the right answer], the thought of even asking her what she really had didn't even…it wasn't even the faintest whisper in my mind. (FRS 11/03/00)

At this point in the collaboration Mr. Blume's focus was still on the answer in his mind and he was not yet considering other possible responses, let alone the importance of understanding students' thinking.

Later in the semester, Mr. Blume recognized that he was not keeping an open mind to what students were saying. In the final interview, while reflecting on his own previous comments, Mr. Blume acknowledged that probing student thinking was a necessary component of the whole class discussion:

> A student gave a response and you asked me why didn't I ask, why not follow it up with asking the student "what did you understand" or "what are you

> thinking now." I think with, if we do, the more full class discussions that we do, the more I'm going to have to do that. (Final Interview 12/20/00)

Although Mr. Blume had acknowledged this need to keep an open mind and suggested that it would be helpful for a teacher to do this in order to promote worthwhile mathematical discourse, he at the same time felt it was a difficult task. For example, during a FRS where the focus was on a Launch episode, he realized that he was not implementing the Launch in the way the curriculum developers had intended. He believed that he was side-tracked into a certain mind frame because he was frustrated that the students had not performed well on their quizzes the previous day. He described his frustration at directing the students' thinking:

> As I sit here and think now, and if I had to do it again in that Launch, I don't think I would...I don't know. I don't think I would always point that out. I think, more often than not, I would just let Irene's response go and let Chuck's go. And let them just be out there and say "just go into the Investigation." I think I was just, I had hoped, that they had gotten the previous lesson so well that they would all have done very well on the QT [quiz] and they didn't. It was, it seemed to me, it was these little things again. ...That related back in my mind to taking time and effort to pay attention to all the little details. Then, I was pretty much gone into that mode. I was frustrated. (FRS 11/15/00)

Later in the same FRS, prompted by the researcher's question about whether the textbook provided guidance about the length of time a teacher should spend on the Launch or the purpose of the Launch, Mr. Blume replied:

> From the trainings, it seems to be stuck in my mind that the Launches are just to get them thinking about the situation. Just to get them thinking and then going to the investigation. I don't think, I didn't do a good job of helping that happen yesterday when I got sidetracked on my crusade. (FRS 11/15/00)

Mr. Blume clearly had been introduced to the idea that during the Launch discussion the teacher should allow for a wide variety of responses and encourage the students to begin to think about the situation. He also acknowledged that it took concentration on his part to do this.

Mr. Blume's difficulty with open-mindedness seemed to stem from a fear of the unknown. This fear was related to his sense of efficacy and uncertainty about

whether these new teaching practices would guarantee students' mathematical understanding. As a result, he suggested that he tended to funnel students' thinking to assure himself that the student understood as he did:

> I think for me, for me sometimes, it's, I don't know if it's comfort or what. I would like all my students to understand these things really well, better than I understand them. I can...It's hard to know if they're really at that level or not. Sometimes I'll convince myself if I can hear them explain this in this way, which I know, to me, if it's explained to me in that way, then it means that you to understand it, well, it must mean that they understand it to the extent that I do. So, if I can hear it in those words, then I can say to myself "chances are they've got it." I can feel pretty good, I guess, because I think they've got it. Maybe it's just that I'm not used to hearing it in other ways, so, you don't even think of it coming from that direction or being spoken in that way. Sometimes, maybe, I like to think that I know it all! And, it's gotta' be done in this way because this is the way that I know it's gotta' be done! I can delude myself occasionally. (Mid-Interview 11/02/00)

He expanded on the sense of risk involved in keeping an open mind in the following quote:

> So it's hard, it's dangerous to keep an open mind. It's scary and it's unpredictable. You're just dangling, sometimes you feel like you're just dangling there. You don't know. Nothing is structured, nothing is set now and things could happen that are completely unplanned for, completely off what you need to get to...You're putting everything at risk. You're putting the learning at risk. You're putting the performance on the assessments at risk. You're putting the results at risk. You're putting your job at risk. I think there's lots of things that go into it at all different levels. (Mid-Interview 11/02/00)

Although Mr. Blume had identified and acknowledged that it would be beneficial to keep an open mind during classroom discussions, it was not an easy task for him. He often struggled between his knowledge of what he should do and his feeling of discomfort. He felt it was much safer, in terms of his students' performance, job security, and school test performance, when students responded in the way that he was thinking. In addition, funneling them into thinking his way allowed him to assess their knowledge compared to his knowledge and make the assumption that they understood in the same way and to the same level that he

understood.

Comfort vs. Mathematics

"You must do the thing you think you cannot do."-Eleanor Roosevelt[11]

As alluded to above, comfort was a major concern and source of tension for Mr. Blume. Throughout the collaboration, Mr. Blume consistently focused on the comfort level of the students in his classroom rather than the mathematics. Mr. Blume considered his primary responsibility and goal to be to encourage his students to understand the mathematics, but he often said that "it all came down to the people issues" and implied that mathematics learning would follow from a comfortable environment.

His belief throughout the collaboration was that establishing a comfortable environment for the students was paramount and took precedence over the mathematical content. This firmly held belief was shared during the baseline interview, when Mr. Blume was asked if there was anything unique to mathematics classroom discourse as compared to other classrooms. His response was, "What's talked about might be different, but what it takes to have a good discourse, I think that's a people issue. I don't think that really depends a whole lot on the topic" (Baseline Interview 9/20/00).

In his vision for good discourse, the most important element was comfort:

> They feel free enough to respond or ask, back and forth though. But it all works harmoniously together. Everything kind of melds smoothly together. Maybe there's disagreements but it's not where it's uncomfortable for people. There's always that comfort level where they can listen, then interject whenever they feel the need to and they'll do it intelligently on whatever we're talking about- just not throwing anything out. (Baseline Interview 9/20/00)

Similarly, in the Mid-Interview, he discussed the trade-off between comfort and

[11] Quotation written on whiteboard at the front of Mr. Blume's room, 11/2/00.

learning. He believed that a better learning environment is created if students are comfortable:

> If it's forced and it's not comfortable and it doesn't grow them, then, I think, in the long run, they end up learning less, even though they might be learning more right here and right now. So, I think you've got to look at both the present and present yet to come, keep them in mind when you're doing these things. (Mid-Interview 11/02/00)

Mr. Blume reiterated his notion of the importance of comfort in the final interview:

> I still thought that we need to take care of the main things first [voice trails off] The people things or whatever things stay consistent throughout all the years. When we finally got those in order in the classroom, then all these strategies that probably [pause] that could [pause] my poor vocabulary is limiting me here! But all the strategies that would magnify all the positives that are there, that could take care of the learning and boost it here or there beyond just if you took care of the people in the classroom. Then all the strategies, once you have that in place, then with the strategies on top of that, then they could have more of an effect, I guess. It seems we get so wrapped up into these strategies, these smaller issues, when we're leaving the big one [voice trails off]. (Final Interview 12/20/00)

Part of Mr. Blume's notion of content came from his own experiences. For example, he mentioned that when taking his undergraduate and graduate courses he did not tend to volunteer during class discussion and often felt uncomfortable when forced to participate. His concern for the students' well-being and comfort stemmed largely from Mr. Blume's religious and philosophical beliefs, as exemplified by the following:

> I have my own beliefs, my own belief system and, you know, and religion plays a factor in that, although it's not spoken outrightly here, it plays a factor in how I view people and learning and everything. (FRS 10/18/00)

In addition, Mr. Blume's beliefs were influenced by his interests outside of the education realm. He shared information about books he liked to read that indicated his interests during a conversation after the 10/26/00 FRS that was not audiotaped but was recorded in the researcher's notes:

> Reads Zen, team management types books--doesn't read and go to conferences on teaching because whenever he does it is nothing that he hasn't

> already thought about. So he's come to the conclusion that the more you know about people and how people interrelate, the better you can teach and base your teaching on those principles. (Researcher Notes 10/26/00)

One particular book he referenced was a book on sports team management written by Phil Jackson. He felt that these books and his Christian beliefs provided guidance on establishing environments conducive to learning mathematics.

Tensions did arise within Mr. Blume as he reflected on the balancing of comfort and mathematics. Mathematics and the teaching of mathematics were important to him, and he shared that he wanted the students to understand the mathematics conceptually. There were, however, several instances where the students had provided incorrect or questionable answers, and Mr. Blume chose not to pursue the mathematics. When these situations were pointed out to him, he responded in this way:

> Yeah, I wasn't, either [sure if she was correct], but I didn't want to shoot her down, and [pause] it was a Checkpoint and just asking for thoughts and I didn't feel it had to be just chiseled in stone at this point. Again, I guess I thought it would be more important not to shoot her down at that point than to correct it. I thought it was good that she at least [was] concerned about having the variables on the right axis. (FRS 10/26/00)

In another instance, Mr. Blume was confronted with an episode where it was unclear in what way a student was understanding the mathematics. When Mr. Blume was introduced to the possibility of the situation's richness, he agreed that is was a missed opportunity--not because of the mathematical issues, but because "it would have been a good opportunity to point out the use of language, so when students write it in their homework, that the ways they present it are accurate" (Researcher Notes 9/22/00).

Mr. Blume's concern with comfort extended to his own comfort. He expressed the need to be comfortable with his students and the situation before he could implement changes in his classroom. During the final interview, after viewing a

sequence of three video clips taken from his classroom over the course of the semester, Mr. Blume felt the last one most closely resembled his vision of classroom discourse. When asked if this episode would be possible to achieve at the beginning of the year, Mr. Blume replied, "I don't know if I could. At least with a group of students who it's the first time I've had them" (Final Interview 12/19/00). When prompted to explain why, he replied:

> I guess for no other reason, I'm not sure if my comfort level would be at a good enough level to do that. I don't think that all the factors in the classroom very often would allow that to happen anyways at the beginning of the semester, at least with freshman students in high school. All the things that go into that good discourse, not just giving an answer but listening, thinking about what you're saying and thinking about what others are saying, I'm not sure, I guess I don't feel that many of those students come into high school with that in this setting. (Final Interview 12/19/00)

Later in the same interview, unprompted, Mr. Blume reiterated his need to feel comfortable with the students before he can encourage them to develop the type of discourse community he envisions:

> Whether it's right or wrong, I think it takes time for it to come about and, for my own comfort level, I seem to need those things, I need to feel those things or at least believe those things are there. I can't step right into that at the beginning of the semester unless I've had the students before. That might be a personal shortcoming, where some of this could take place sooner but that seems to be who I am at this point. (Final Interview 12/19/00)

Summary

By the end of the semester, the form of Mr. Blume's reflections had shifted from defending his practices to discussing what he saw on the videotapes as an outside observer. He suggested that although the discourse naturally evolves, teachers influence the trajectory through their actions and words. The content of Mr. Blume's reflections also included talk about pedagogical techniques. Some reflections focused on techniques suggested by the researcher, while others were created and implemented by Mr. Blume himself. He continued to view comfort as the most

important factor in establishing good discourse.

Classroom Mathematical Discourse

"If a task is once begun, never leave it till it's done, be the labor great or small do it well or not at all."[12]

Three salient observations emerged from an analysis of the data related to the mathematical discourse in Mr. Blume's classroom: (a) A shift occurred in the dominant speaking role from teacher to student; (b) A shift occurred in the purpose of Mr. Blume's questioning from facts to probing student thinking; and (c) A tendency not to pursue talking about meaningful mathematical content persisted. These results are discussed in the following sections. (See Appendix K for a tabular representation of the complete results from the Robust Mathematical Discussion (RMD) Scales applied to the nine video episodes.)

Shift in Dominant Speaking Role

"I hear and I forget; I see and I remember; I do and I understand."-Chinese Proverb[13]

Over the course of the collaboration, who was doing the speaking and how it was carried out shifted. The first noticeable change was in Mr. Blume's physical position in the classroom from front and center to less conspicuous locations. Table 1 shows Mr. Blume's location in the room and his movement during whole-class discussions.

At the beginning of the collaboration, Mr. Blume was typically the focal point of the class discussion. Regardless of whether he was standing at the front of the classroom or walking around the room, he did most of the talking and explaining. As noted in Table 1, during the collaboration, Mr. Blume became less conspicuous in his location in the room and his participation in the discussions. There was a dramatic

[12] Quotation written on whiteboard at the front of Mr. Blume's room, 10/17/00.

[13] Quotation written on whiteboard at the front of Mr. Blume's room, 10/3/00.

Table 1

Teacher Location and Movement During Observations

Observation	Location/Position		Movement
9/14/00	Front	Stand/Sit	Around overhead
9/21/00	Front/ Back	Stand	Walking around
9/28/00	Front	Sit	Gets up to point to board
10/03/00	Front	Sit	Around overhead
10/10/00	Front/ Back	Stand/Sit	Recording student thoughts on board, sits momentarily at back while students share solutions
10/12/00	Front	Stand	Around overhead
10/17/00	No whole group discussion		
10/26/00	Back	Sit	None
11/2/00	Back	Sit	None
11/9/00	Back	Sit	None
11/14/00	Back	Sit	None
11/30/00	Front/ Side	Stand/ Sit	Recording student ideas, then sits when students share
12/7/00	Front/ Back	Stand	Moves to the back while students draw on board
12/19/00	Front/ Back	Sit/ Stand	Moves to the back while students draw on board

shift in his location beginning on October 26[14] as he sat at the back or at side tables and asked students to go to the whiteboard to record their ideas. This shift was so noticeable that during the next day's FRS, the researcher asked Mr. Blume why he chose to conduct the discussion in this way. He responded:

> Change! Where I was in the back and there were students up in the front, trying to slowly start to turn it over to them more and more, where they'll more and more take it and run with it, feed off of each other, not wait for me or depend on me or even count on me to necessarily make things work right,

[14] Also note that this was the next observed class session after Mr. Blume's "turning point" FRS reported earlier.

> where they start doing all that and taking it on themselves a little bit more. Working towards that, I guess. That was it. So, I sat in the back and let someone else go up in the front and keep asking questions to keep explaining this and that, hoping someone else will jump in at some point and say "what about this" or [voice trails off]. (FRS 10/26/00)

Mr. Blume continued this trend in decentering his location until the last two observations. During these, he sat or stood at the front to begin the discussion, then moved to the back while the students recorded solutions on the board. Once the discussions resumed, he moved back to the front to point to the board while students explained. Mr. Blume had altered his position in the classroom to encourage the students to take a greater part in the discussions. By moving himself out of the focal point of the students, he thought he could promote greater interaction among the students.

Other noticeable differences included a change in the percentage of types of teacher talk coupled with the fact that students began building on one another's comments. Mr. Blume was trying to shift the discourse toward a conversation, where students would interject, listen, and build on one another's responses more freely. This shift was most apparent in the Checkpoint discussions and, for this reason, they will be the focus of the following discussion.Over the course of the three Checkpoint episodes, which covered a period of two months, Mr. Blume's facilitating comments decreased from 13% (20/159[15]) of the talk to 7% (13/191). Facilitating questions functioned to keep the discussion going, and Mr. Blume frequently used them for calling on or prompting students to speak. As an example, in the following excerpt, Mr. Blume used facilitating questions when he asked the students to "buy it" or "sell it," meaning whether they agreed or disagreed with Anna's equation:

Mr. Blume: Check what you're getting with what's on the board.

[15] These ratios are based on length of the teacher utterances reflecting facilitating comments compared to the length of the episode.

[Anna writes y= 8x, pauses, then continues y= 8x –12]

Mr. Blume: Okay. Buy it? Any buyers?

Student: Buy it.

Mr. Blume: One buyer.

Multiple students: Buy it.

Mr. Blume: Two?

[several hands are raised]

Vinny: I didn't get that, but I don't think mine's right, so I'm buying that.

Mr. Blume: Sellers? [student mumbling] 'Kay, Trevor, what do you think went, what do you think it should be and explain why.

Trevor: 12 + 8x

[several students moan in disagreement]

Mr. Blume: Write your equation up there, Trevor. Let's see what you're talking about.

(Video Transcript 11/30/00, Unit 3 Lesson 3 Investigation 4)

The Building Rating Rubric of the RMD assessed the depth of a discussion using the ratings of "building," "restating," and "none." The building rating was assigned when student responses that built on earlier comments with new ideas were integrated into the discussion. The restating rating reflected student responses that repeated or reiterated another student's response. Table 2 shows that there was an increase in both categories of the Building Ratings Rubric for the Checkpoints. In the first Checkpoint episode, there was only one instance of restating, where a student acknowledged another student's explanation and responded:

> I know that much, but I don't get how variables deal with this (pointing to the book), how it's asking in the question. I know exactly what she (gesturing in Irene's direction) said, I just don't know how it would fit in the question. (Video Transcript 9/28/00, Unit 2 Lesson 1 Investigation 1)

Table 2

Building Ratings by Rating and Episode

Episode	Building	Restating
Checkpoint-9/28/00	0	0.05
Checkpoint-10/26/00	0	0.11
Checkpoint-11/30/00	0.07	0.17

In the second Checkpoint, there was evidence that students were thinking about and listening to others, but only two students were involved:

> Mr. Blume: Is that why you like that better than a data table because in the data table you gotta look through all the numbers to find the one, but in the equation you can just [voice trails off]
>
> Ralph: Yeah, you can find it without looking at it.
>
> Irene: So, basically it's the same thing--"easy to read" [pointing to datatable] or easy to, aah?
>
> Cindy: When you need to find the answer.
>
> (Video Transcript 10/26/00, Unit 3 Lesson 1 Investigation 1)

In this episode, Mr. Blume chose Irene to record other students' comments on the board, and because of this, she was required to listen to and interpret the students' comments. Ralph previously had provided his reason for selecting the equation as the best representation, and then Mr. Blume tried to help him say it more succinctly. As Irene was trying to figure out what Ralph was saying to write it on the board, another student, Cindy, jumped in to assist her.

In the third Checkpoint episode, more students participated in the restating and building process. For example, in this episode, two students had just simplified the equation $y=15x-(12+7x)$. Anna had written $y=8x-12$, and Trevor had written $y=12+8x$ as their answers. The students were then discussing which was correct and why in the following excerpt:

Anna: I don't know, I just took 15x and I minused 7x from it and then I just, and then, and then, okay now I don't get it, Oh, yeah, I had 15x and I took 7x away from it and then it was just that. I still don't get it now.

Mr. Blume: Do you know why you did minus 12?

Anna: No.

Mr. Blume: 'Kay, Eva?

Eva: Because, um, she minused 12 because, 15 minus 7 and it's 15 minus 12 also [waves finger in air] also.

Mr. Blume: Because the minus sign [voice trails off]

Eva: [making hand gestures with curved hands in and out]

Mr. Blume: Did you hear this Trevor?

Trevor: Yeah, but I thought you said you're not supposed to do that?

(Video Transcript 11/30/00, Unit 3 Lesson 3 Investigation 4)

In this excerpt, Eva was listening and was willing to explain what another student had done. Trevor was also listening but was prompted by Mr. Blume to make a statement, in which he questioned something that Mr. Blume had said in their group interaction the previous day. There was evidence in these Checkpoint episodes that students were listening to one another and were beginning to respond to and affirm their classmates' ideas.

In the beginning of the collaboration, when the percentage of teacher talk was the greatest, the order of utterances followed a teacher-student-teacher-student pattern. Mr. Blume would ask a question, to which a student would provide a brief response without much elaboration. Prompted again by Mr. Blume, the student provided further explanation. However, by the end of the collaboration, the pattern of talk had shifted to include instances of students responding to students without teacher interaction.

The shift in pattern of talk was most evident in the sequence of Launch

episodes. In this sequence, the students moved from restating one another's answers to responding without prompting to one another. In the first Launch episode, students were prompted by Mr. Blume to agree or disagree or were pressed by Mr. Blume with questions such as, "Is that what you were just saying, Heather?" In the second and third Launch episodes, students responded and agreed with other students' comments, unsolicited and without teacher facilitation, as seen in the following two excerpts. In the first excerpt, students were discussing how they would determine a path for painting the lockers lining the hallways in their school:

> Irene: Okay, I would start in the 100 hallway and get the smallest job out of the way, so that you have more time to concentrate on the larger areas. So I'd go from 100 to 200 to 300.
>
> Mr. Blume: 'Kay, Vinny?
>
> Vinny: I'd do the 100 because it's the shortest and then the 300 because it's the second shortest and 200 because it's the longest.
>
> Heather: That's what I would do.
>
> Another: Right on.
>
> (Video Transcript 12/7/00, Unit 4 Lesson 1 Investigation 1)

In this excerpt, Irene provided her opinion and Vinny followed it with a slight modification. This showed that he was listening to Irene and provided further elaboration. Unprompted, Heather and the other student agreed with and confirmed what Vinny had said. In the next excerpt, Donald prompted students to think about ways in which the Federal Communications Commission (FCC) could assign frequencies to radio stations to initiate an investigation of graph coloring. It again suggested that students were listening to one another:

> Heather: Um, the one that we have like, it like, there's like a couple of, there's like, it's like 90, okay it's like 97 point or whatever, and there's one that follows and then one that follows. It's the same radio station on like three different, like three different.

Vinny: Yeah, like 103.4 is the same thing as, well 103.6 is the same as 103.4.

Rob: It just doesn't come in as clear.

Mr. Blume: And why is that, do you think?

Irene: Radio stations don't [unclear]

Sarah: I think it's because the frequency [unclear]

Irene: There are so many frequencies.

Sarah: Yeah.

(Video Transcript 12/19/00, Unit 4, Lesson 2, Investigation 1)

In this second excerpt, Heather had a difficult time articulating the fact that a radio station may be heard through multiple frequencies but Vinny and Rob were listening to and understood what she was trying to say. When Mr. Blume asked why, instead of Rob responding, Irene and Sarah jumped in and tried to explain. These types of interactions were not found in the earlier episodes and suggest that students were beginning to listen to and build upon one another's ideas.

Overall, there was a shift in who was doing the majority of the talking and in the pattern of speech. Mr. Blume had physically placed himself out of the center of attention and the students were beginning to respond to and elaborate on the comments of other students.

Shift in the Purpose of Questioning

"There are no shortcuts to anyplace worth going."-Beverly Sills[16]

Analysis of transcripts of Mr. Blume's classroom discourse revealed that his talk had shifted from focusing on eliciting facts to probing students' thinking. During the first half of the collaboration, especially when working with individual groups, Mr. Blume tended to funnel the students' thinking by asking questions that would

[16] Quotation on whiteboard at the front of Mr. Blume's room, 11/9/00.

lead the students to arrive at his predetermined conclusion. This was reinforced by his practice during whole-group discussion of encouraging students to agree or disagree with a student speaker and then saying himself "I agree" or "I think you've got it [pause] you're seeing it the same way I'm thinking of it" (Video Transcript 11/2/00) rather than asking for justification.

In the following excerpt mid-way through the collaboration, a student was frustrated because she did not know how to determine the time when a batted ball would hit the ground. She had entered the equation that modeled the height of the ball's path into her graphics calculator and was confused by what she observed in the table[17]:

> Ellen: [reads the question in book] What? I don't know when the ball's going to hit the ground. [Mr. Blume walks around to her side] I don't understand how to get there.
>
> Mr. Blume: What's the height when the ball hits the ground?
>
> Ellen: I don't know, I don't know [voice trails off]
>
> Mr. Blume: Well [Laughs]
>
> Ellen: I don't know what the [pause] What the how [pause] Like where would it hit the ground? Like what would the number be? Cause it can't be zero, because it goes down to negatives?
>
> Mr. Blume: Well, yeah, the calculator will keep on going down into negatives because the calculator doesn't understand about a ball and the ground and stuff [voice trails off]
>
> Ellen: Where, where will the ball hit the ground?
>
> Mr. Blume: Okay, let's look here. In the height one [Heather begins paying attention], what's the one for?
>
> Ellen: I don't know.

---

[17] Zero, representing the ball's height when on the ground, did not appear in the table. To determine the time, the student needed to change the interval for the independent variable instead of looking merely at the integer values.

Mr. Blume: Any idea?

Ellen: No.

Mr. Blume: In the velocity, what was the 12 for?

Ellen: [Both students speak, unclear]

Mr. Blume: Okay, that was the initial velocity, right? 12 meters per second. So in the height equation, the 1 in front [student says something] is the initial height. When you throw the ball, the ball doesn't start on the ground, it starts from your hand. So the 1 means, when the ball left your hand, it was 1 meter above the ground. Does that make sense?

Ellen: No, not really.

Mr. Blume: Like if you throw a ball, when it leaves your hand, it's above the ground already, right? [student nodding head, "Um, hum"]. So the 1 means, when it left your hand the ball was 1 meter above the ground. So when the height's on the ground, what's the height going to be?

Ellen: I don't know.

Mr. Blume: The ground is going to be the height of [patting his hand on tabletop]? Height of? When you're on the ground [pause] What was the maximum height here? What'd you put for this? [pointing to question in the book]

Ellen: The maximum height was 5.4 meters.

Mr. Blume: 5.4 meters. 5.4 meters from what?

Ellen: From the point where you threw it.

Mr. Blume: It's not the, it's height off the?

Ellen: Ground.

Mr. Blume: Ground. So when it hits the ground, it comes back down and hits the ground, the height's going to be?

Ellen: Zero.

Mr. Blume: Zero.

Ellen: But it can't be because it's going into the negatives.

(Video Transcript 10/17/00, Unit 2 Lesson 4 Investigation 1)

By the end of the excerpt, Mr. Blume and the student seemed to be back at the

beginning again, with the student having no more clearer understanding of what the equation was doing or why the negatives and positives appeared on the screen without a zero. Mr. Blume used funneling-type questions through the majority of the excerpt and, toward the end, used "clozed" questions (Pimm, 1987), where he was not finishing his sentences purposely so that the student would fill in the blanks. This, however, did not help the student or Mr. Blume investigate her thinking about the situation.

Later in the collaboration, an episode began with Mr. Blume interested in and open to hearing students' thoughts, but it quickly changed in tone. In this episode, a student who was taking a quiz in the hallway interrupted the discussion to ask Mr. Blume a question about the quiz. Prior to the interruption, Mr. Blume was probing student thinking by eliciting explanations and clarifying what was being understood. After the interruption the focus and atmosphere changed to eliciting factual information. The following excerpt occurred when the students were discussing a graph of the percent of television viewers watching network versus cable programming and shows this shift toward eliciting factual information:

> Irene: Um, I think that the major networks would actually increase because there's a lot more shows on ABC and um CBS and stuff like that and then cable, I think, would just keep on increasing because people just watch cable. So.
>
> Mr. Blume: So you think they'd both go up, then?
>
> Irene (and others): Yeah.
>
> Mr. Blume: Same speed? Same rate of change?
>
> Irene: [shakes head] Maybe, I mean, maybe major networks would go up a little slower and cable would go up pretty much the same rate, I think. [Chuck's hand goes up.]
>
> Mr. Blume: Chuck?
>
> Chuck: Um, I think she's wrong.

Mr. Blume: What do you think?

Chuck: Because if more cable, if more people that want cable are buying it, then the major networks will start going down because no one's gonna want, ah, regular old, they're gonna want something new.

In this situation, Mr. Blume was asking questions to encourage the students to elaborate on their thoughts and was not pressing them into following his thinking about the situation. At this point, two other students entered the conversation and agreed with Irene's thought that the two would continue to increase. Mr. Blume had continued to probe what the students were thinking and their justifications when the student taking the quiz interrupted. After the interruption, his approach shifted:

Mr. Blume: Now, ah, everybody, everybody, now. We are missing something here.

Irene: What are we missing? [not sure it was heard]

Mr. Blume: We are. The li. The little things, the little things that make the difference that you're looking for between, between consistently doing excellent work and doing sometimes good, sometimes not and not always understanding why it's not also going well. These little things. What is a little thing everyone is missing here because no one's mentioned it yet? There's a reason, Irene, that your solution, I don't think, can happen. [Irene raises hand] Do you see what it is?

Irene: Yes.

Mr. Blume: What is it?

Irene: Um, because the major networks, it was kind of going up and down but then it got more steady and it was still decreasing as it was..is that what you mean?

Mr. Blume: No, well, you said you think that's going to start going up again because you think more people will start watching them because they got better shows again, right?

Irene: [Looks at him and nods]

Mr. Blume: Yeah, which makes sense, now there's something in this graph, though, that doesn't allow that line of thought, at least as I look at it.

Irene: Well, then, I have no clue.

Mr. Blume: Look at it. Look at the graph. Everybody, look at the graph. Not

just look at it, but think about what it's saying, each part of it. Don't wait to be told what it is. Can you figure it out? The day you can figure these things out before you're told is the day you've got it. And when that day comes, you can rest assured that you've got it. But if it's not there yet, then you're not where you want to be, probably. [student raises hand] Heather?

Heather: Okay, I think it's because of the thing you run or whatever that line is it's, it's after the last play or whatever it's still decreasing.

Mr. Blume: 'Kay, but Irene's, it's still decreasing, but she thinks at some point it's going to be increasing, and she explained why, and it, it sounded kinda reasonable, except, that's good, I guess, but there's something else, something else. [students hand raised] Vinny?

Vinny: Is it like yesterday when we were talking about the quadrants that will keep going on and going on?

(Video Transcript 11/14/00, Unit 3 Lesson 3 Investigation 1)

The questioning had degenerated into a "guessing game" (Cobb, Wood, & Yackel, 1993); Mr. Blume had something in mind, and he wanted the students to figure it out. In particular, Mr. Blume suggested there may be some merit to Irene's, Heather's, and Vinny's responses, but that he was looking for something else. The amount of speaking time by students and teacher in the two excepts confirms the switch in type of interaction. Before the interruption, the students' responses were longer than Mr. Blume's, but after the interruption, Mr. Blume was doing the majority of the talking and, apparently, thinking. Focusing on student thinking did not seem to come naturally to Mr. Blume. It took great effort on his part and was sometimes abandoned when he thought the class was not going well, felt pressed for time, or was distracted.

There were other instances in which Mr. Blume encouraged students to investigate their own thinking and allowed them to share what and how they were thinking. For example, in the following excerpt from a Launch where students were going to investigate paths and circuits, students were sharing their strategies about how they would go about painting the school's lockers. Mr. Blume began by calling

on people and allowing for a variety of responses without funneling their thoughts:

> Rob: I'll start in this hall [pointing to diagram] because it's the longest. Then this hall [and puts x's at the ends of each hallway] because it's the second longest and then this. I would go down [tracing the path on the board] and then move here. Or I could do it here, here, here, it doesn't really matter. [He shows painting hallway 1, 2, 3 or the reverse order.]
>
> Mr. Blume: Okay. Trevor?
>
> Trevor: I would do it the same order, but I think it's because you wouldn't have to like, ah, I would start at the back of the building and go around to the front, but I don't know why, I just would.
>
> Mr. Blume: Okay, Noah?
>
> Noah: I think I'd start in the 100's hall, go down the 100 hall, come down and go around [tracing with finger in the air] the 200 up and back down the 200.
>
> Mr. Blume: Okay, can you diagram it just with or finger or so, I think I know what you're talking about, but [student getting up to go to the board.] 'Kay, take a look at this one [student is at the board], you go [voice tails off] Shh.
>
> Noah: You go down there like that and like that. [shows the path on the board]

Here Mr. Blume tried to encourage the students to listen to one another's ideas by asking Trevor to reiterate his suggestion:

> Tina: I'd like, do what Trevor did, and I'd do the 300 and get the longest out of the way and then the 200 and 100.
>
> Mr. Blume: 'Kay, Trevor, what did you do again?
>
> Trevor: I just said start at the back and go to the front.
>
> Mr. Blume: Where's the back?
>
> Trevor (and many others): The back of the building! [Pointing at the board- several students say "The 300 hall"].

At this point Mr. Blume refocused the discussion back to developing a pathway by asking a focusing question. Several students exclaimed and seemed to understand and then go on to describe the actual pathways they would use to paint the hallways:

> Vinny: I'd do the 100 because it's the shortest and then the 300 because it's

the second shortest and 200 because it's the longest.

Heather: [turns around to face Mr. Blume] That's what I would do.

Another: Right on.

Mr. Blume: Okay, now again, Vinny, when you went down the 300, and Trevor and others. When you, when you're gonna go down these, how are you going to do these lockers? Are you going to (with arm motions acting it out) hold brushes on both sides?

Trevor: Oh!

[several other ohs!]

(Video Transcript 12/7/00, Unit 4 Lesson 1 Investigation 1)

Mr. Blume's classroom performance shifted over the course of the collaboration in terms of the nature of the interaction he had with students. Early in the collaboration, he focused on getting the students to arrive at the same answers he had. Later, he began to allow a variety of thinking and to ask probing questions to try to understand better what his students were thinking.

Lack of Talk about Mathematical Content

"He who is afraid of doing too much always does too little."-German Proverb[18]

Over the course of the collaboration, the majority of the classroom talk did not focus on meaningful mathematical content. Table 3 shows how Mr. Blume's classroom scored consistently and substantially lower on the Mathematics Content Dimension of the Robust Mathematical Discussion (RMD) Scales than on the Discussion Dimension.

An analysis of the data revealed two primary ways in which Mr. Blume inhibited students from talking about meaningful mathematics. The first was by steering the discussions toward a procedural knowledge of mathematics rather than

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[18] This was the last quotation written on the whiteboard at the front of Mr. Blume's room, 11/14/00. Mr. Blume discontinued putting the phrases on the board because as the pressures of the semester increased he felt he did not have the time to worry about these sorts of things anymore.

sense-making or conceptual knowledge. The second way was by a tendency to probe student thinking about opinions rather than mathematical statements. Related to this

Table 3

Comparison of Mathematics Content and Discussion Dimension Ratings by Episode

Episode	Mathematics Content	Discussion
Launch- 11/14/00	0.17	0.43
Launch- 12/7/00	0.02	0.48
Launch- 12/19/00	0.03	0.55
Investigation- 10/3/00	0.02	0.50
Investigation- 10/17/00	0.16	0.37
Investigation- 11/14/00	0.26	0.60
Checkpoint-9/28/00	0.31	0.40
Checkpoint-10/26/00	0.18	0.37
Checkpoint-11/30/00	0.17	0.42

was Mr. Blume's focus on procedural objectives for his lessons as opposed to mathematical goals and his failure to establish norms in his classroom for talking about mathematics.

An example of inhibiting student discussion about meaningful mathematics occurred in an episode that began with the textbook question, "What reasoning with the symbolic forms alone would confirm the equivalence of the expressions?" (Coxford et al., 1996, p. 237). This question, as written, encouraged students to make a generalization, but during Mr. Blume's implementation, it quickly degenerated into the exercise of simplifying an equation:

> Mr. Blume: Yeah, just with the equations. The symbolic form means the equation. Just with the equations, how could you see if they're really equal to each other or not?
>
> Chuck: Um, simplify, I guess.

Mr. Blume: Ooh, simplify. All right, so which one do we have to simplify?

(Video Transcript 11/30/00, Unit 3 Lesson 3 Investigation 4)

The class then spent the rest of the 6-minute episode simplifying two previously given equations to determine if they were equivalent and never returned to answering the question explicitly.

Making connections or generalizations is one way in which students make sense of the mathematics they are studying. The results from the RMD Scales analysis of Mr. Blume's class showed very few instances of students making connections or generalizations. In fact, of the 256 student utterances over the nine episodes, only 3 were recognized as Generalizations and 13 as Connections. An example of each of these can be seen in the same episode from the same student, Trevor. In this excerpt, a later portion of the discussion above, Trevor was trying to make sense of the fact that he was supposed to have simplified $y=15x - (12 + 7x)$ into $y = 8x - 12$,[19] when it seemed to conflict with what he had been told the previous day:

Trevor: Yeah, but I thought you said you're not supposed to do that?

Mr. Blume: The minus sign goes [pause] you minus the 12 and you minus the [drawing arcs on the board between - and 12 and - and 7].

Mr. Blume was trying to show visually how the distributive property with the subtraction sign would work. Trevor was confused because during the previous day, there had been an emphasis on combining only like terms:

Trevor: I thought you couldn't do that without a number and an x. I thought you couldn't do that.

Student: That's why you put it [voice trails off, speaker not acknowledged]

Mr. Blume: Okay, Trevor, when you do that, you've got 15x minus 12 minus 7x [writing on the board]. Subtracting both those, distribute it through the parenthesis. Okay, okay with this?

---

[19] Trevor had simplified the equation into $y=12 + 8x$.

Trevor: Yep, I was just saying, you said that earlier you couldn't do that but that. 'Member, you said when you walked over here?

Mr. Blume: Yeah?

Trevor: We were asking you. It's because ah, adding ...on like ah c and d, you said that [interrupted by Mr. Blume]

At this point, Trevor made a verbal connection with the problems he was assigned the previous day,[20] wondering how they were different and why the same mathematics did not apply in this situation. Mr. Blume, rather than acknowledging Trevor's thinking or sense-making process, focused on the procedural aspect of the mathematics:

Mr. Blume: Once you get this, how can you simplify that?

Trevor: 15 take away the 7 and then 12 take away that.

Mr. Blume: And just minus 12. Can you subtract these two [pointing to the 8x and 12 of the equation y=8x-12 written on board]?

Trevor: No.

Mr. Blume: No.

Trevor: Okay.

Mr. Blume: Okay, this that one's right. Good. Good.

(Video Transcript 11/30/00 Unit 3 Lesson 3 Investigation 4)

The episode ended with Mr. Blume satisfied that Trevor understood why the answer was $y=8x-12$ and Trevor accepting that fact, leaving the question of whether Trevor had made sense of the mathematics unanswered.

Although Mr. Blume agreed that learning mathematics should include students' making sense of it, there were numerous times that sense-making was not pursued. For example, on the same day, while students were working on the Investigations in small groups, students asked Mr. Blume for help with "simplifying"

[20] Trevor was referring to simplifying the following expressions: $8x + 5 - 3x$ and $5 + 3x + 12 + 7x$.

problems like 8(5 + 2x). In the course of the conversation, he did not make use of context or sense-making to help them understand--such as looking at the repeated addition aspect of multiplication and making connections with the distributive property--but instead relied solely on the rules.

Mr. Blume also missed opportunities to pursue students' mathematical comments. In one early episode, Haley suggested that the way to determine whether a line was the y=x line was if the points below the line had x-values values that were greater than the y-values. Mr. Blume responded very quickly, "Would that always be true?" His manner implied that the only right answer was "no." Mr. Blume glossed over her response and turned to other students to get the answer for which he was looking--that the x and y coordinates of the points on the line all would be the same. Although what Haley meant by her statement was unclear, it might have provided a rich opportunity to investigate the mathematics involved in the situation.

A second way Mr. Blume showed a tendency to avoid mathematical content was by inconsistently probing student thinking. In particular, Mr. Blume tended to encourage justifications in situations where students were providing opinion as opposed to mathematical arguments. This tendency became particularly evident during analysis of the justification component of the RMD Scales. For example, the Checkpoint discussion on October 26, 2000 included two questions. The first question, "Which representation do you think is the easiest to use and most accurate for making predictions? Give reasons for your choice" (Coxford et al., 1996, p. 161), could be answered with student opinion. There were six responses to this question, and in each case the students either provided an explanation or Mr. Blume asked why they thought that way. The second question was, "What factors could cause inaccurate predictions from a linear model of (projector distance, enlargement factor)

data?" (Coxford et al., 1996, p. 161). There were only two students who responded to the second question, and there was no prompting for justification or explanation by Mr. Blume.

When Mr. Blume was asked to identify the mathematical goals of his lessons, he did not focus on the mathematical concepts that students should understand. In a few episodes, Mr. Blume was uncertain about the mathematical goal of the lesson; in one case, he exclaimed "I wasn't quite sure and I guess I'm still not" (FRS 10/26/00). In most cases, however, he could share his ideas, but they tended to focus on what students should be able to do instead of what they should mathematically understand as a result of the lesson. In some cases, the statement of his goals was just telling what the students would do during the lesson. For example, "They'd be modeling the situations with the equations again, that they made and then they would be simplifying those equations. Combining them then simplifying them" (FRS 12/01/00). In other cases, when Mr. Blume talked about the mathematics students needed to think about, he would talk very specifically in the context of the lesson, but not about how the mathematical goal was connected to a larger mathematical concept. For instance, when he was asked about the mathematical goal of a launch, he replied, "Just to get them thinking about the different ways they could go about painting those lockers, with that scene, I guess" (FRS 12/08/00). It was apparent that Mr. Blume did not have a clear picture in his mind about what mathematical concept students should understand as a result of participating in the lesson.

It is interesting to note that in Mr. Blume's 222 transcribed utterances during the nine episodes there were no examples that addressed the issues of appropriate mathematical talk, such as what constituted an acceptable justification. There was also only one example of Mr. Blume making an explicit comment that contributed to

establishing the social norms in the classroom. At the end of a discussion about the definition of a variable, Mr. Blume concluded by saying:

> Now just what you did there, it took us about, about 10 different people to share their thoughts to work our way through and we end up at, we ended up at the place that we needed to and everyone listened. It looked like, at least you were polite enough to keep quiet, and that's the way that we do it, that's good. (Video Transcript 9/28/00, Unit 2 Lesson 1 Investigation 1)

As a result of the lack of explicit talk about discussion norms in Mr. Blume's classroom, the norms were established primarily through his example.

Summary

Many of the aspects of discourse in Mr. Blume's classroom shifted from a teacher-centered to a student-centered focus. Mr. Blume removed himself physically from the center of the room to encourage students to talk to one another. The students restated and built upon one another's thoughts to a greater extent over the course of the collaboration. Mr. Blume also began to focus more on students' thinking by using fewer funneling-type questions and more questions that honestly and overtly probed what the students were thinking. Most of these shifts, however, did not encourage the talk to focus on meaningful mathematical content.

The next chapter concludes this report by highlighting salient aspects of the study and providing implications for professional development and suggestions for further study.

CHAPTER V

CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to investigate the nature of change that occurred in both the teaching and reflection practices associated with the classroom mathematics discourse of a secondary school mathematics teacher using a reform curriculum. The first chapter briefly summarized the calls for reform, the obstacles often experienced when teachers try to enact these reforms, and suggestions for overcoming these obstacles. The second chapter provided a review of research literature related to aspects of meaningful mathematical discourse and methods of changing mathematics teaching practices. The third chapter described the research design, intervention, and data analysis. The fourth chapter outlined the results of the analysis of transcripts from videotaped episodes, interviews, and collaboration sessions. This chapter provides a brief summary of the study, answers the research question posed in the first chapter, suggests implications of the results, and provides recommendations for future research.

Summary of the Research Study

The study focused on Mr. Blume, an experienced secondary school mathematics teacher chosen for his past experience in curriculum workshops grounded in the National Council of Teacher's of Mathematics (NCTM) *Standards* (1989, 1991, 1995), as he participated in a teacher-researcher collaboration. This collaboration, based on a new paradigm for professional development (Loucks-Horsely, Hewson, Love, & Stiles, 1998; Stein, Smith, & Silver, 1999), focused on the mathematical discourse in Mr. Blume's classroom.

The teacher and researcher met for 12 Focused Reflection Sessions (FRSs) over the course of the Fall 2000 semester. The collaboration elements related to these

FRSs included a researcher-developed set of questions, Discourse Reflection Tool (DRT), used in conjunction with the viewing of videotaped episodes, and selected readings from the research literature on classroom discourse. These elements were used to encourage Mr. Blume to reflect deeply on his classroom mathematical discourse practices.

As part of the data collection and analysis procedure, data was collected to inform the nature of Mr. Blume's reflection and the nature of Mr. Blume's classroom discourse. To investigate the nature of reflection, data was collected during three interviews and 12 FRSs. The analysis consisted of coding and looking for patterns in the transcribed data. To investigate the nature of discourse, episodes were selected to analyze using (a) using the Robust Mathematical Discussion (RMD) Scales to explore trends in the student talk and (b) using a researcher-developed coding scheme to explore patterns in the teacher talk. Analysis of transcripts from interviews, FRSs, and selected episodes from the videotaped classroom observations led to a series of findings.

The research question that guided the data collection and methodology of this study was: In what ways does participation in a teacher-researcher collaborative intervention focused on mathematical discourse affect: (a) the nature of the teacher's reflection about and vision of mathematical discourse, and (b) the mathematical discourse in the teacher's classroom?

Conclusions

The following sections draw upon the results of this study to describe conclusions related to the two parts of the research question. The first two sections answer the two parts of the research question directly. The third section looks at the relationship between the nature of Mr. Blume's reflection and the discourse in his

classroom.

Nature of Reflection

The nature of reflection portion of the study focused on answering the question: In what ways does participation in a teacher-researcher collaborative intervention focused on mathematical discourse affect the nature of the teacher's reflection about and vision of mathematical discourse?

The analysis gave rise to three observations that answer this question. First, Mr. Blume exhibited a *shift* from explaining away or defending his practices to openly reflecting and considering alternative ways of thinking. Second, Mr. Blume shifted from using general terms while talking about the "natural" development of the discourse to more clearly identifying and articulating the role the teacher plays in developing meaningful mathematical discussions. Third, Mr. Blume held as a consistent focus students' comfort as opposed to learning the mathematics. The following two sections discuss the affective phases that heralded Mr. Blume's reflective shift and possible explanations for this shift.

Affective Phases

Mr. Blume's shift toward becoming more reflective was heralded by a series of three affective phases. These phases are labeled *perturbed*, *frustrated*, and *contented reflection*.

In the first phase, Mr. Blume was perturbed and unsettled by what he witnessed on the videotapes and made comments about how unhappy he was with the classroom discourse. He felt his practices were problematic as a result of perturbations prompted by the researcher's questions and viewings of his teaching episodes. He appeared to be most bothered by the instances when the researcher had identified and confronted him with missed opportunities for probing his students'

mathematical thinking where he had not even considered asking a question. This first phase is consistent with other researchers' claims that teachers need to see a need for change before they are willing to invest their time and energy into changing their practices (Etchberger, & Shaw, 1992; Loucks-Horsley, Hewson, Love, & Stiles, 1998; Stein, Smith, Henningsen, & Silver, 2000).

As a result of the perturbations he experienced, Mr. Blume entered a second phase best described as confusion and frustration. He wanted to change his ways, but he struggled with various obstacles that made improving his performance difficult. These obstacles included his frustration with his inability to keep an open mind in the face of restrictions such as time. In particular, Mr. Blume struggled with his feeling that the discourse developed over time and the school's change to block scheduling. In the case of block scheduling, a traditional year's worth of content is to be completed in a semester and Mr. Blume felt he did not have time for the discourse to develop over a year. Another restriction was a focus on the procedural content he felt needed to be discussed often as a result of poor test performance. Mr. Blume's restrictions seemed to be rooted in his lack of comfort and fear of the unknown. In other words, Mr. Blume was uncomfortable and hesitant because he was not convinced that changing his practices would have a positive impact on students' learning. Teachers who have experienced, as a student or teacher, the success of a learning environment where students discuss meaningful mathematics, and are expected to pose questions, conjecture, investigate, and justify, may not have this fear and lack of comfort. Mr. Blume, however, had not experienced this type of learning environment and could not be certain of its effectiveness.

From an outsider's perspective, this can be seen as a vicious cycle of not wanting to change because one has not personally experienced the change's success,

yet without changing, never being able to experience that success. This cycle contributed to Mr. Blume's confusion and frustration. Mr. Blume reconciled this by making small attempts to change, such as trying different techniques. By doing so, he began to feel successful in creating a discourse environment more conducive to his vision, and, for the most part, entered a third phase. This study suggests the importance of breaking the never-experiencing-never-trying cycle by encouraging teachers to enact changes and reflect on the effects so that eventually teachers will feel comfortable making more significant changes.

Once he entered the third phase of contented reflection, Mr. Blume was less agitated and more thoughtful about what was happening in the classroom. He was no longer defensive and viewed the classroom episodes almost as an outside observer. There were, however, still instances of frustration caused by his feeling of uncertainty about the best course of action and the long-term effectiveness of his changes. It was during this phase that he began to be more thoughtful about the changes he tried in the classroom, both during the classroom enactment and reflection sessions, and began to reflect on the episodes as a more critical and outside observer. Although Mr. Blume had not reached the point where the researcher was convinced that he would continue to reflect as deeply on his own, there were instances that showed that he could carry on the practice if he made it a priority.

Possible Explanations

The following discussion considers possible explanations for Mr. Blume's shift in his reflections. The first explanation attributes the shift to an increased familiarity and level of comfort with the researcher. It is possible that Mr. Blume had the same depth of reflection at the beginning of the collaboration, but was uncomfortable or hesitant to share his reflections with the researcher. Based on this

explanation, one would expect linear growth between his comfort and his willingness to share his thoughts and feelings. Stein, Smith, and Silver (1999) do emphasize the need to provide a comfortable environment for the collaboration, both in terms of physical environment and with the collaborators. In this case, however, it seemed that Mr. Blume became more comfortable with the process of thinking deeply, rather than with the researcher.

Furthermore, increased comfort with the researcher does not explain the shifts in Mr. Blume's reflections about the role of the teacher in the discourse, nor does it account for his emotion-filled breakthrough. It is more likely that the shift in Mr. Blume's reflections was a result of the constant probing about his instructional decisions while being confronted by episodes of his classroom teaching. These experiences raised his consciousness by allowing him to reflect on aspects of discourse and his facilitation of it that he had not considered before. In addition, the act of watching himself on videotape allowed for a more critical and objective assessment of what occurred.

Mr. Blume's travel from using general to more specific or explicit language can be seen as his way of defining and refining his philosophy. An appropriate parallel to this situation of collaborator and teacher is that of the teacher and students in a classroom. Like a student, Mr. Blume was provided activities and tasks that allowed him to talk about his teaching and to investigate its nuances. He also was provided with suggested pedagogical skills that were meant to provide options, which he tried and investigated throughout the semester. In the same way he conjectured about some self-developed techniques. Just as students make conjectures and develop algorithms, some of which may be faulty, Mr. Blume tried his idea of "playing dumb" to investigate its capacity to provide meaningful mathematical discussion. Facilitating

the teaching of mathematics using a constructivist paradigm requires allowing for student conjecturing and exploration that includes exposing the students to counterexamples for their faulty algorithm. The collaboration provided Mr. Blume the space to conjecture and investigate some of his own ideas about establishing meaningful mathematical discussion. The Focused Reflection Sessions provided the opportunity for him to notice the inconsistencies in his new techniques. The results of this study mirror the sentiments of Cobb, Wood, and Yackel (1990) when they realized that they were not providing the same constructivist learning environment for the teacher they were collaborating with as they were encouraging her to provide for her students. It was important for Mr. Blume's growth that he observe patterns, make conjectures, and test the pedagogical hypotheses he was developing in a way that made sense to him.

Continuing with this parallel, teachers model behaviors and questioning to situate students in the learning environment they hope to produce. In the same vein, the researcher asked Mr. Blume a series of questions as a model. Over time he internalized these questions and they became a basis for his thinking. This was evident as Mr. Blume moved toward more objectively looking at his teaching practices and questioning himself. This is corroborated by Ball and Bass (2000) when they outlined the necessity of providing "intellectual tools" and situating teachers in the actions the professional development is hoped to produce. A critical factor in assisting the development Mr. Blume's reflections was modeling for him questions he could raise himself as he reflected on his classroom.

Nature of Discourse

The nature of discourse portion of the study focused on answering the question: In what ways does participation in a teacher-researcher collaborative

intervention focused on mathematical discourse affect the mathematical discourse in the teacher's classroom?

Out of the analysis emerged three observations that answers this part of the research question. First, an analysis of the data showed a shift in dominant speaking role from teacher to student. Second, there was a shift in the teacher's talk from eliciting merely answers to probing student thinking. Finally, a tendency not to talk about meaningful mathematical content persisted.

One of the most important shifts was an increased emphasis on student thinking, exemplified by a change in the types of questions asked and increased student participation. However, as observed in the results, Mr. Blume tended to press student thinking in instances that involved students' opinions rather than students' mathematical understanding of mathematics. These are easier and safer for a teacher to accomplish in his or her classroom, and can be seen as an important first step. Once the teacher has developed a culture of sharing thinking in his or her classroom, further reflection on the necessity of understanding students' mathematical thinking could lead to the teacher truly probing all students' thinking.

There was no indication that the discourse in Mr. Blume's classroom had reached a level of meaningful mathematical discussion that encouraged conjecturing, exploring, and conceptual understanding. These results are not unlike other researchers' and professional developers' experiences with teachers trying to implement changes in their classrooms (Ball, 1990; Clarke, 1997; Cohen, 1990; Wiemers, 1990; Wilson & Goldenberg, 1998). The following discusses the possible explanations for why some changes took place and others did not.

The primary reason for Mr. Blume's new focus on student thinking might be due to Mr. Blume's realization, while watching the videotaped episodes, that he often

did not know the content or reasons behind particular student comments. As a result, he could not help the students produce the kind of discourse that he envisioned. These perturbations translated into Mr. Blume becoming more aware of the need to allow multiple student responses, but without necessarily realizing the importance of understanding students' mathematical thinking. This raises the question of whether introducing videotapes of another teacher's classroom where this level of discourse had been achieved might have been helpful in perturbing him. It is possible that providing exemplars might have provided alternative ideas for achieving this discourse, as implied by Shaw and Jakubowski (1991). Of course, a longer time for the collaboration time might also have achieved that goal.

Mr. Blume was hindered by several factors in his ability to create meaningful mathematical discussions. The factors identified are interconnected, but for ease of discussion will be artificially divided into the following categories: (a) goals, (b) sociomathematical norms, and (c) inconsistencies.

The first of these factors was often Mr. Blume's lack of a clear mathematical goal for a lesson. Whether it was because he was "not reading ahead" or he had not fully developed in his own mind what he wanted the students to understand rather than what he wanted them to do, he did not have a good sense of the desired outcomes. This lack of a clear picture limited his ability to focus the students toward discussing meaningful mathematics and led to students sharing their ideas and opinions in no cohesive fashion. This corroborates Silver and Smith's (1996) research that emphasized the importance of reform teachers needing clear goals in order to enact cognitively demanding tasks. In addition, this study recognizes that greater cognitive demands are placed on the teacher as they need to artfully respond and incorporate students' comments and questions in order to facilitate the discussion

toward the predetermined mathematical goal.

The second factor is related to the establishment of the classroom's socio-mathematical norms. Previous research has documented that the actions of the teacher and students in the classroom lead to the establishment and re-establishment of norms throughout the course of the year (Yackel & Cobb, 1996). The socio-mathematical norms in Mr. Blume's classroom were not established through any explicit talk by Mr. Blume. For example, Mr. Blume did not discuss with students what constituted an acceptable mathematical explanation. Instead, all socio-mathematical norms in his classroom were established based upon experiences in the whole-class discussion. As shown in the results, Mr. Blume had a tendency not to push students to justify their responses with mathematical explanations, nor did he encourage or model conjecturing and generalizing. This is unlike the teacher in Mendez (1998), successful at creating meaningful mathematical discussions, who reminded himself daily of the importance of explicitly talking about talking about the mathematics and identifying acceptable mathematical explanations.

In addition to not explicitly discussing acceptable mathematical talk, Mr. Blume did not hold high expectations for what he thought the students could mathematically think about and discuss. Often, he missed opportunities to pose questions that would encourage a mathematical discussion due to his perception that students had not had the proper training to engage in this type of discussion or that they did not have the mathematical tools. Mr. Blume suggested repeatedly that it was not possible to conduct his vision of classroom discourse until late in the semester because he did not feel that he or the students would be ready. This, however, is contrary to other researchers' findings. For example, Cobb states in Sfard, Nesher, Streefland, Cobb, and Mason (1998) that "We have found that, within a few weeks,

most students routinely give conceptual explanations as the need arises and that they ask others clarifying questions that bear directly on their underlying task interpretations" (p. 47). Mr. Blume's lack of experiences with this kind of teaching and learning limited his ability to achieve his discourse goals as he was not confident that they could be met.

The third factor was associated with a series of inconsistencies in Mr. Blume's talk or actions when facilitating the classroom discussions. For example, throughout the semester Mr. Blume introduced and incorporated new ideas, such as his "buy it" and "sell it" technique for students to indicate agreement or disagreement with their peers' comments. Each time he introduced a technique, the students were required to renegotiate the classroom norms. When Mr. Blume did not consistently use these ideas, the students were unsure about what was expected of them and their participation in the classroom discourse was constrained.

A second inconsistency appeared in Mr. Blume's constant tension between the goals of students comfort and students thinking. He was concerned about the students' comfort to the point that it would deter him from following-up on student thinking. Mr. Blume typically did not sustain pressure for the explanation and meaning of a students' response, especially when it dealt with a mathematical question as opposed to an opinion. As suggested by Stein, Grover, and Henningsen (1996), this did not allow the tasks to be implemented in the fashion they were intended and instead they deteriorated into tasks that were less cognitively demanding.

Another type of inconsistency involved mixed messages Mr. Blume sent concerning the role of authority. Although sometimes his words and actions expressed that he wanted the students to validate one another's responses, other times

his words or actions undermined this goal. A prime example of this was when he listened to student responses and then ended the discussion by identifying the response that matched his thinking. Actions such as these ended any further discussion and at the same time re-established Mr. Blume as the only mathematical authority in the classroom. In these situations, Mr. Blume was undermining the shift in authority from him to the students both by his direct actions and the confusion they created in his students' understanding of his expectiations of them. As Wilson and Lloyd (2000) reported, this shift in authority can inhibit reform.

Mr. Blume also sent mixed signals about students sharing answers to seemingly open-ended questions. Even though, most open-ended questions have a correct answer, the point is that they can be arrived at from different ways. In an effort to reach closure or meet some time constraint, Mr. Blume had a tendency to shift the focus to the correct answer. Stein, Grover, and Henningsen (1996) list this as one of the six ways that worthwhile tasks can decline to become non-problems. Mr. Blume acknowledged his focus on the correct answer and it appeared to have motivated his decision to not look at the answer key and "play dumb" when students asked questions.

Relationship Between Reflection and Discourse

As would be expected, there was a clear connection between Mr. Blume's reflections and classroom discourse in his classroom. Areas in which interaction could be seen included comfort, openness, experimentation, and efficacy.

Comfort, both his and his students', played a primary role in Mr. Blume's reflections and, consequently, his classroom practice. A major concern for Mr. Blume was providing an environment that was comfortable for his students. He did not press students to elaborate on their responses when he felt they might become

uncomfortable. Mr. Blume's focus on his students' comfort frequently contributed to his hesitancy to follow-up on students' mathematical thinking. Mr. Blume also felt he needed time to establish rapport with the students before he felt comfortable probing their mathematical thinking. In addition, he was uncomfortable making changes whose long-term outcome he felt was unknown to him. These feelings of discomfort contributed to the cycle of non-action discussed earlier and also led to the undermining the promising practices. This was particularly evident when he unintentionally modified the curriculum by transforming questions that asked for high-level mathematical thinking (e.g., generalization type questions) into an exercise of symbol manipulation.

A second interaction involved openness. After Mr. Blume moved beyond defending and explaining away his practices, he began to openly reflect. This allowed Mr. Blume to approach the classroom discourse differently, which in turn altered the outcome in the classroom. These new shifts then provided more to discuss in the ensuing collaboration sessions. By the end of the collaboration Mr. Blume was talking so freely and found so much to reflect on that it was difficult to discuss the entire Discourse Reflection Tool (DRT) during a 90-minute meeting. This idea of encouraging the change process by situating the professional development within the classroom to assist teachers in the interplay between reflection and action has been recognized as being effective in other studies (i.e., Cobb, Boufi, McClain, & Whitenack, 1997; Cobb, Wood, & Yackel, 1990; Yackel & Cobb, 1990). The current study supports the importance of these connections.

The experimentation that occurred when Mr. Blume was in the frustration and confusion stage provides an explanation for the randomness in many of the ratings scales. During this period, Mr. Blume was inconsistently trying out many different

techniques to create a better discourse community. In doing so, he was sending mixed messages to the students, which in turn led to mixed outcomes. Reflecting on these episodes sometimes increased Mr. Blume's level of frustration because of his lack of knowing what to do next and confusion about why he was not getting the results he expected. On the other hand, his consistent of lack of focus on mathematics in his reflections showed up with similar consistency in the observational data.

The area that ties all of these interactions together is efficacy. Mr. Blume knew and understood well what he had experienced, and it had served him well, along with countless numbers of former students. He was truly concerned about students' conceptual understanding of the mathematics, but is equally concerned with the long-term effects of how the mathematics is learned in his classroom. This would suggest that in order for Mr. Blume to continue to change, his sense of efficacy would need to change as well. This has been similarly suggested by Smith (1996), "For the current reform to generate deep and lasting changes, teachers must find new foundations for building durable efficacy beliefs that are consistent with reform-based teaching practices." Given the experience with Mr. Blume, it seems that building a different set of beliefs is possible, and likely, through longer interaction and reflection where he is consistently urged to try out his new ideas and reflect on them. Once he sees the effectiveness of these practices, like he did with the use of wait-time, using them would modify his sense of efficacy and no longer threaten it.

Summary of Implications

It is important to remember that this study is one example of one teacher-researcher collaboration; with other participants in other situations there may be different results. Having said that, this study can help to inform future professional development intended to assist teachers in changing their classroom discourse toward

a mathematical learning community that encourages conjecturing, exploring, justifying, and generalizing. The paradigm of professional development described in this study has as prerequisite that (1) the collaborator is working in conjunction with the teacher to change the teacher's instructional practices; (2) the collaboration is an opportunity for the teacher to conjecture, investigate, and make generalizations about ways that best would promote meaningful mathematical discussion; (3) the collaboration is a peer relationship based on mutual respect; and (4) the teacher is seen as knowledgeable and motivated, and his or her perspective is valued. Given these prerequisites, six characteristics of professional development suggested by this study are: (1) awareness of the teacher's sense of efficacy, (2) guidance without evaluation, (3) reflections on videotaped episodes, (4) focus on students' mathematical thinking, (5) awareness of the three affective phases, and (6) focus on inconsistencies. The first four support previously made recommendations and the last two emerged from this study.

Awareness of the Teacher's Sense of Efficacy

Collaborators need to be aware of the fact that reform efforts may make teachers uncomfortable and cause them to struggle with reconciling their feelings of efficacy. Helping teachers to identify and make known their feelings of discomfort with the unknown is important. Telling teachers that the reforms will be effective does not alleviate their discomfort with their sense of efficacy; they must experience the effectiveness themselves and may need to incorporate changes in small ways first. Currently, teachers are often taking risks as they implement reforms that are not accepted as valid by all the constituencies that a teacher must deal with, often on a day-to-day basis. Collaborators must be aware of this and find ways to help teachers navigate this difficult terrain.

Guidance Without Evaluation

Teachers need an appreciative audience to listen to and hear them, as they verbalize their thoughts. Collaborators must be prepared to withhold evaluation as they guide teachers on their journey. That is collaborators need to listen to and probe teachers' thinking to understand and assess, but not to evaluate either the teachers' reflections or classroom discourse. This requires patience and an understanding that the collaborators' role is to assist teachers as they move on their individual trajectories toward reform teaching.

Reflections on Videotaped Episodes

Videotaped episodes of teachers' classroom practices are effective means to perturb and probe teachers' thinking to encourage them to investigate their teaching. Videotape allowed Mr. Blume to observe and investigate aspects of his teaching that were not possible through reliving the experiences through memories only. In addition, it allows teachers to reflect closely on their mannerisms and speech to parse out any inconsistencies in their practices.

Focus on Students' Mathematical Thinking

Students' mathematical thinking needs to be paramount, without losing sight of the mathematical goal of the lesson. As was true in this case, teachers in the process of reforming their mathematical discourse practice often focus only on the mechanics (e.g. asking questions to encourage greater participation) resulting in classroom discourse which is often absent of meaningful mathematics. Thus it is important to help teachers focus not only on the social norms and orchestration of the classroom discourse, but also on the sociomathematical norms and sense-making processes. First, collaborators can work with teachers to identify the mathematical goals of their lesson. During implementation of the lessons, collaborators can help

teachers analyze the types of norms being established. Finally, collaborators can encourage teachers to concentrate on making explicit comments during the classroom discussions regarding what constitutes acceptable mathematical talk.

Awareness of the Three Affective Phases

It is important to be conscious of the affective phases that teachers pass through will enable collaborators to empathize with and encourage teachers in different ways as they travel this emotion-filled path on their way to becoming reflective practitioners. For example, when working with teachers who are not familiar with reform teaching philosophies and practices, it would be beneficial to use activities like videotape comparisons to perturb teacher thinking. Whereas, when teachers are already perturbed and aware of their incongruous teaching practices, but frustrated or confused, activities that provide guidance without evaluation would be more appropriate. For instance, when Mr. Blume began to feel frustrated about not knowing the next best course of action the researcher asked guiding questions to (a) help Mr. Blume think about alternative practices, (b) help Mr. Blume investigate the ramifications of the different choices, and (c) model the types of reflective questions that Mr. Blume could be asking himself. These questions are not evaluative and allow teachers to step back and view their teaching more constructively so that they can eventually reach the point of regularly reflecting in order to improve their teaching. Once a teacher reaches the contented reflection phase they are much more able to direct their own learning and continue a constant focus of improving their instruction.

Focus on Inconsistencies

Collaborators need to encourage teachers, while observing episodes of their teaching, to reflect closely on their mannerisms and speech to parse out any

inconsistencies or mixed signals they might be sending to their students. Although a teacher may want the authority to be shared within the classroom community, this is not likely to occur if the teacher has inadvertently set the stage as the judge. In addition, if the teacher always provides a judgment to each student answer (e.g., "good"), students do not have a need to share their thoughts and the class misses out on possibly rich opportunities to talk about the mathematics.

Limitations of the Research Study

This study looked at change in a 4-month period of time. It is possible that different changes might have appeared in the participant's classroom discourse given a longer study. In addition, the focus of this study was on the nature of change; considerations of the sustainability of the change were beyond its scope.

As with all case studies, this study does not intend to imply that what is found can be generalized to all teachers. The teacher in this study was in a specific situation with unique circumstances. In this case, the participant had several years of teaching and professional development experience, in addition to using NSF-funded curricular materials. It is possible that the same study done with different teachers, or even with the same teacher at various stages in his teaching career, would lead to different results.

Another limitation is that the researcher played the role of both the collaborator and researcher. Recognition that the participant might want to leave the study if pressed too hard by the researcher/collaborator may have led to hesitation and tentativeness on the part of the researcher. Studies in which different individuals assume the two roles might produce more detailed insight into how much a collaborator could push teachers' thinking without sacrificing their comfort.

Finally, the two instruments used in this study may have limited the results.

The DRT was developed by the researcher for this study, and although it was used with two other teachers prior to the study, with further refinement it might have encouraged more movement. As mentioned previously, the RMD Scales (Mendez, 1998) were developed based on the NCTM *Standards* (1989, 1991) and then revised and tested with one teacher in a middle school classroom. They may not consider and assess accurately the mathematical discourse in a high-school classroom using NSF-funded curricular materials. Further refinement of the instruments and their use in similar studies in the future may produce more detailed and discriminatory findings.

Suggestions for Future Research

This study provided an in-depth look at one teacher involved in a teacher-researcher collaboration using focused reflection to investigate the discourse in his classroom. Further research is needed to more fully understand the conditions necessary to assist teachers in changing their classroom practices in such a way as to promote meaningful mathematical discussions. The results of this study indicate the following possible directions for future research.

(1) Future studies need to be conducted that replicate this study. This research investigates the nature of change in just one teacher's reflections and classroom practice. Replication would test the proposed model of affective changes (i.e., perturbation, confusion, reflective practitioner) and perhaps refine it by identifying other relationships between diverse teachers and their affective changes.

(2) Studies that combine videotaped episodes of the teachers' classrooms and exemplary episodes of meaningful mathematical discussion are recommended. This study only used videotaped episodes from this one teacher's classroom. The addition of a set of exemplary videotapes in future studies might provide alternative visions for teachers and better enable them to become a reflective practitioner.

(3) The fact that the teacher-researcher collaboration in this study spanned only a 4–month period and showed some movement in the participant's reflections and classroom discourse is encouraging. Future studies should focus on a longer, sustained collaboration. Such studies might encourage teachers to demonstrate repeated episodes of meaningful mathematical discussions and more consistently show evidence of becoming a reflective practitioner.

(4) Studies should be done in which the research design includes reflection within a small collaborative group. In this study, the teacher reflected individually, prompted by the researcher. Opportunities for teachers to reflect in a group setting may encourage greater reflection and more insightful reflection as group members prompt and build on one another's thinking.

(5) A future line of research is needed concerning how teachers' comfort affects their ability to change their classroom practices. The results of this study suggest that sense of comfort plays a significant role in one's ability to change one's practice. Therefore, the proposed research should include opportunities for teachers to identify and reconcile inconsistencies between their beliefs and practices associated with their level of comfort. Additionally, this research may help to identify ways to confront and reconcile teachers' feelings of discomfort that inhibit them from implementing reform curricula in the way it was envisioned.

Final Remarks

There are skeptics who wonder if the vision of the mathematical discourse community, as suggested in this study, is possible. For instance Clarke (1997) concludes:

> As I reflected on the challenges for a teacher in creating a discourse community of the kind touted in reform documents and in research reports, it

became clear that the creation of such a learning environment required considerable knowledge (of mathematics content, pedagogy, and student learning) and placed such great demands on a teacher in terms of energy level, that perhaps the emphasis on this aspect of reform has been too great. My experience in this study raised a question for me as to how many true "discourse communities" actually exist, and whether the creation of a discourse community is reasonable, attainable goal for most teachers. (p. 291)

Although, like the teacher in Clarke's study, the teacher in this study did not achieve the desired level of discourse community, there are examples of teachers achieving such environments (Ball, 1996; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Mendez, 1998). Even though worthwhile mathematical discussions may not occur daily, it is possible that teachers armed with knowledge of and an interest in understanding student thinking can achieve this level of mathematical discourse over time. The current study merely suggests the complexity and difficulty in determining what it takes to help teachers achieve this goal.

In closing, Mr. Blume, a wonderful and caring teacher, reiterated the possibility and difficulties in achieving this vision of discourse:

> I think it would take a very secure person who really understands what's going on with learning and adolescents and all that, to let it go and not worry about how it fits into the structured assessments and grades and such. Trust that if good learning is going on, exceptional learning is going on, then at some point, the cream is going to rise to the top and it's going to show itself. When it does, it's just going to be stupendous! But, until that point, until you get to that point, it could be really rough. (Mid-Interview 11/02/01)

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Appendix A

Clearance From Human Subjects Institutional Review Board

Kalamazoo, Michigan

WESTERN MICHIGAN UNIVERSITY

Date: 29 August 2000

To: Laura Van Zoest, Principal Investigator
M. Lynn Breyfogle, Student Investigator for dissertation.

From: Sylvia Culp, Chair Sylvia Culp

Re: HSIRB Project Number: 00-07-05

This letter will serve as confirmation that your research project entitled "The Immediate and Ripple Effect of a Teacher-Researcher Collaboration" has been approved under the expedited category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note that you may only conduct this research exactly in the form it was approved. You must seek specific board approval for any changes in this project. You must also seek reapproval if the project extends beyond the termination date noted below. In addition if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

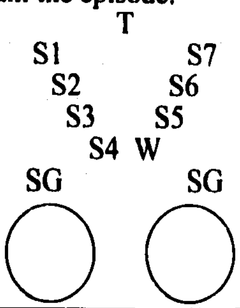
The Board wishes you success in the pursuit of your research goals.

Approval Termination: 29 August 2001

Appendix B

Discourse Reflection Tool

Pseudonym__________Interview # __________ Unit __________ Lesson__________ Investigation __________

Discourse Reflection Tool			
Background	Mechanics	Mathematical Content	
What was the mathematical goal of the lesson? What did you expect the students to understand as a result of the lesson?	What was the approximate wait-time between exchanges? [ ] less than 1 second [ ] more than 1 but less than 3 [ ] over 3 seconds	What student mathematical utterances were you surprised by or did not expect? Why?	Who or what provided validation and reinforcement of correct answers?
To what extent was the goal achieved?	What types of questions were asked during the whole episode? [ ] factual [ ] comprehension [ ] connection	What mathematics did you decide to pursue? Why? Were they related to students' justification or representations? How?	Were there places in the episode where students could have generalized?
Who were the students participating in the episode?	What was the purpose of the questions you asked?	What mathematics did you decided not to pursue? Why? Were they related to students' justification or representations? How?	What mathematics do you think the students understand as a result of this episode?
Diagram the episode.  	Were you listening: [ ] for expected answers [ ] to understand thinking [ ] to guide the student in their thinking	Are there examples during the episode where students are making connections with other mathematical, applications, or real-world topics? What other connections might have been made that were not?	In what ways did this episode encourage students to think about the mathematics?

Appendix C

Discourse Reflection Tool Pilot Study

Name________________Date______________

Discourse Reflection Tool

What was the mathematical goal of the lesson? What did you expect the students to understand as a result of the lesson?	Wait-time between 1st exchange? [ ] less than 1 second [ ] 1-2 seconds [ ] 2-3 second [ ] over 3 seconds	What type of questions were asked during the whole exchange? [ ] process [ ] factual [ ] product [ ] comprehension [ ] connection	What mathematics do you think the students understand as a result of this exchange? What mathematics did you pursue? Why? What mathematics did you not pursue? Why?
To what extent was the goal achieved?	Wait-time after 1st response? [ ] less than 1 second [ ] 1-2 seconds [ ] 2-3 second [ ] over 3 seconds	What was the purpose of the questions the teacher asked?	In what ways did this exchange encourage students to think about the mathematics?
What started the exchange?	Diagram the exchange. T S1 S7 S2 S6 S3 S5 S4 W	How did the questions contribute to the goal for the lesson?	Would you consider this a mathematical discussion? [ ] yes [ ] no On what basis did you make this conclusion?
What was the 1st response?	Who were the students participating in the exchange?	Who or what provided validation and reinforcement of correct answers?	What were you thinking about during the exchange? How did it contribute to the goal for the lesson?

Discourse Reflection Tool Pilot Study

To pilot the Discourse Reflection Tool (DRT), the researcher met with two teachers for approximately one hour each. The researcher had the opportunity to preview the videotape of the first teacher prior to the scheduled meeting to select a particular episode. This episode was chosen because it was the only episode in which the teacher and students interacted in a mathematical discussion (which could be heard by the microphone). During this meeting, the teacher and researcher focused on this particular episode of the lesson, and the teacher answered the questions from the DRT. For the second teacher, the researcher asked the teacher to review the videotape and look for episodes of mathematical discussions for us to focus on during our meeting. As a result of the meetings with these two teachers, the following changes were made on the DRT.

The two questions regarding what began the episode and the first response were eliminated, for they did not seem to encourage reflection. Several other questions were altered to make them more user friendly. For example, the wait-time questions were combined and made less specific, as it was found that it wasn't necessary for the teachers to time each time lapse for them to reflect on how little time they gave students to respond. One pilot teacher found that she had never waited over 1 second and in several cases completed students' statements for them. She went on to reflect about why this was so. She decided that she was asking just rhetorical questions and didn't expect the students to answer and provided the following reasons: (a) she felt the need to cover the material in the hour, and (b) she thought it was important to model her thinking process and the questions she asked herself. In general, the two pilot teachers found the idea of the DRT and most of the questions to be useful in encouraging them to reflect. In fact, one of the teachers after the

interview wished she were the teacher being studied and said, "the teacher you'll be working with is pretty lucky."

Although there were not many changes to the DRT, a substantive shift to emphasize mathematical content has been made on the new version and a reorganization occurred to highlight this shift. Of the four columns of questions, the last two focus entirely on the mathematics content or questions related to it.

The first column asks the teacher to provide the context of the lesson and the intended mathematical goal(s) of the lesson as well as their reflections on how well this was accomplished. Studies have shown that teachers who reflect on the goal of lessons and to what extent they feel they have been achieved are more effective teachers (Onosko, 1989). The other two cells in the first column call into mind which students and how often they participated. In the pilot, one teacher noticed after several viewings that although some students did participate, they were primarily the ones in the front of the room and not whom she expected. The diagramming cell has been changed to include the possibility that the teacher, during whole group discussion might speak to a small group, in which they may then talk amongst themselves before reporting back to the class. Many other diagrammatic possibilities may occur, and in such cases, an appropriate diagram can be represented on the back of the DRT.

The second column was changed to include the salient aspects of the mechanics of mathematical discussions. Due to the lack of time, and the importance of discussing the mathematical content, only one aspect of the mechanics will be discussed, on a rotating schedule, each session. For example, during the first week of the intervention, we will discuss all of the background column, only the "wait-time" cell of the mechanics column and all of the mathematical content squares. Then the

following week, we will not discuss the wait-time aspect and move onto the two questioning cells. This will allow for greater time spent on the mathematical content, but yet provide an opportunity to address some of the characteristics of mathematical discussion that could affect the content discussed.

The last two columns focus on the mathematical content by asking the teacher to reflect on what mathematics was discussed, as well as what was not discussed and why not. This will provide the opportunity to discuss the mathematical concepts in order to deepen the teacher's understanding as well as highlight what mathematical concepts were or were not explored during the lesson. During the pilot study with one teacher, this prompted a very interesting and enlightening discussion of periodic waves. She came to the realization that she had not challenged the students' understanding of periodicity, and they may have come away with an incomplete notion and possibly misconceptions about the topic.

Appendix D

Observation Videotapes

Observation Videotapes

Date	Unit/ Lesson/ Investigation			Whole Group	Small Group	Launch	Invest	Check-point	Extension	Episode Times
9/14	1	3	2/3	XX	X		X		X	
9/21	1	4	2	X*	X				X*	1:23-1:28
9/28	2	1	1	X*	X		X	**X***		1:43-1:49
10/3	2	2	1	X	XX*		**X***	X		1:07,1:09-1:12
10/10	2	3	1	X	XX		X			
10/12	2	3	2	X*	X		X*			1:18-1:26
10/17	2	4	1		X*		**X***			2:19-2:22
10/26	3	1	1	X*	X			**X***		1:26-1:35
11/2	3	2	1	X*	X		X*			1:10-1:19
11/9	3	2	2	X*	XX		X*			1:03-1:10
11/14	3	3	1	X*	X	**X***	**X**			1:49-1:58
11/30	3	3	3/4	X*				**X**	X*	1:23-1:31
12/7	4	1	1	X*		**X***	X			2:23-2:28
12/19	4	2	1	X*	X	**X***	X	X		2:15-2:18

Bold denotes the episodes coded and analyzed using the Robust Mathematical Discussion Scales.

* denotes the episodes discussed during the Focused Reflection Sessions using the Discourse Reflection Tool.

Appendix E

Baseline Interview Protocol

BASELINE INTERVIEW PROTOCOL

Which of the following best describes the discourse in your classroom: a tennis match, football game, or symphony (or something else)? Why?

At what points in a lesson do you generally engage in whole-group discussions during investigations? How about during small-group discussions? How would you describe your role in each of these?

How would you characterize good discourse? What are the students doing? What is the teacher doing?

In what ways do you think your discourse should be different? What makes you think it should be this way?

When students make a conjecture, how do you respond? How do you decide whether to pursue the mathematics?

Appendix F

Mid-Interview Protocol

MID- INTERVIEW PROTOCOL

How would you characterize good discourse? What are the students doing? What is the teacher doing?

How is the discourse similar in small versus whole group settings? How do you see it as different? How would you describe your role in each of these?

In what ways do you think your discourse should be different? What makes you think it should be this way?

What episode stands out in your mind as influencing your thinking about discourse? How about an episode which altered your teaching?

What role do you think understanding students' thinking plays in discourse?

What do you see as the purpose of discourse in your classroom? What are you hoping to achieve as a result of student discussion?

What do you see as the purpose of the launch? the investigation? the checkpoint?

Appendix G

Final Interview Protocol

FINAL INTERVIEW PROTOCOL

Day 1

Part 1: Agreeing or disagreeing with responses on index cards (see Appendix H):

(1) What does the discourse in the classroom look like?

(2) What are the students doing and how are they contributing?

(3) What does the teacher say and do?

(4) How does a teacher establish and maintain this type of environment?

Part 2: Looking at sequence of video clips for progression:

What episode stands out in your mind as influencing your thinking about discourse? How about an episode which altered your teaching?

Part 3: Questions to verify my thinking:

(1) You have probably noticed that some people in the reform movement want to encourage students to think of mathematics in a different way. What do you think about what students think mathematics is and how people would like to change this thinking?

(2) What role do you think a teacher's understanding of students' thinking plays in discourse?

Day 2

Categorize quotes on index cards into things like: Ambivalent, Accurately describes, or Really gets at what I mean (see Appendix I)

Other Questions:

(1) How does the type of lesson you're teaching affect the discourse in your classroom? (If it's review in nature do you use different questioning techniques?)

(2) You have often mentioned that you didn't want to press the students to answer because they had already provided one right answer and you didn't think they would be comfortable if you pressed them for more. Is this accurate? Have you thought about ways to encourage students to feel comfortable but yet still encourage them to provide justifications?

(3) In many of the interview you suggested there are several areas of tension you feel pulling you in different directions and keeping you from implementing what you think might be the best discourse practices.

(4) Have you had any thoughts on how you were going to continue reflecting on

your teaching when I'm not here?

(5) Why did you agree to meet with me and talk about your classes? Did you get out of it what you were hoping?

Appendix H

Index Card Quotations: Similar Questions

INDEX CARD QUOTATIONS: SIMILAR QUESTIONS

Vision of discourse:

(9/21/00)

Where there is that back and forth, back and forth, between anybody in here, including myself. They feel free enough to respond or ask, back and forth though. But it all works harmoniously together. Everything kind of melds smoothly together. Maybe there's disagreements but it's not where it's uncomfortable for people. There's always that comfort level where they can listen, then interject whenever they feel the need to and they'll do it intelligently on whatever we're talking about – just not throwing anything out.

(9/21/00)

Well, the people involved in it know why they're there. That's what they're focusing on. They have open minds. Fairly confident in their abilities and what they know but they have an open enough mind to listen to other people. Confident enough to share their ideas. Maybe that confidence is also in being comfortable with anyone else involved in that discourse. So they've got that, too. They're comfortable with everybody enough to do that, at least.

(11/3/00)

Again, if the discourse, if it gets better, those things will come out more naturally without any prodding from me. I would hope they would say "Well, that's not what I got. I got this. Even if I can't explain why I got this, this is what I got." … "If you can explain it to me, maybe it's probably better than mine since I can't explain mine at all but prove something to me first." I'd like them to start thinking along those lines.

Purpose of discourse:

(11/2/00)

Because, if done right, I feel it will enhance the learning. Enhance the learning, broaden it, deepen it and that's why we're here. That's what I see as the purpose.

(11/2/00)

I'm hoping that their understanding of whatever topics getting discussed is full. They have a full understanding of whatever's…I suppose they can at the time…have an understanding of whatever we're talking about. There again, being breadth and depth

and hearing different viewpoints. It helps the classroom environment. Comfort level helps with the learning in days after that as well. I think it helps both immediate learning and it helps enhance learning that is to come yet. It just makes a better environment and a better place to learn.

Appendix I

Index Card Quotations: Verification

INDEX CARD QUOTATIONS: VERIFICATION

Feelings vs. intellect

(10/15/00)

That's another problem I've had this year. The first time through it, my mind is pretty open to any response to it. Now the second time through, I'm starting to get locked into certain things and I don't adjust as well as I think I did that first year to alternative responses. I don't think it's a good thing.

(10/13/00)

Yea, because I always like things to go very smoothly, just tremendous learning going on at the same time, them excelling in their learning and grade-wise. And, of course that's not...Learning is a messy business, it seems. If it's going smooth, I'm not sure how much they're learning but I feel better some days. But if it's not going smooth then I know if we get it, then they'll have learned something. But it's frustrating.

Time & Content

(10/18/00)

I would think I was little more direct with my responses yesterday than I would be, again, if I wasn't allowing myself to be time pressured. I think I wouldn't have been as direct or would have backed off more if we would have just went to the check point. If the teams worked on their own on that investigation, we would have gotten much more varied responses...I think it's a give and a take. In each way, you're gaining and you're losing. I guess, the first year, we did it the other way, where the teams were working, pretty much, do the whole investigation on their own and then we'd come together. This time I'm doing it a little bit different – see how that goes and make adjustments in future years.

(10/13/00)

I guess they just frustrated me because I wasn't feeling well and I wanted them to do well. If they don't understand them, I know they won't do as well as they should be doing, in my eyes. So I was frustrated that we still hadn't gotten it. What did I not do, or what could I have done or what should I have done where they could have understood this better (can't hear him). Now maybe they're not supposed to. I've been told that this core-plus training session that they're not supposed to get it all completely right away. If they don't all have it, move on. It'll come back and that seems to be how the (?) has been built into the program. But with myself and who I am and with the mastery of learning that's at the school here, they have to get some of it at a certain point. And, so, I'm still looking for a balance between the two, I guess.

(10/3/00)
They might not get quite as far but I think they'll understand more of what their doing and then, again, I think the inter-personal skills they're learning will improve their chances to learn more in the future, too. Lots of things. I think the pluses far outweigh the negatives, which might be not covering quite as much material as quickly and, I guess, a loss of some efficiency maybe by not having everything directed.

Comfort & Questioning:
(10/26/00)
I didn't want to shoot her down and...It was a checkpoint and just asking for thoughts and I didn't feel it had to be just chiseled in stone at this point. Again, I guess I thought it would be more important not to shoot her down at that point then to correct it. I thought it was good that she, at least, concerned about having the variables on the right axis. So, whether, at this point, whether they understood that it would cause a difference or not, I didn't feel that was as important, again, as shooting down her answer or changing the focus from making sure you have your variables on the right axis on your graphs to it doesn't really matter. ...Again, I didn't think that was worth it.

(10/18/00)
Researcher: So, you didn't follow it up with asking her what she got or "so, what did you understand" or "what are you thinking now" and you didn't do that because...?

MR. BLUME: I didn't do that because – one reason is I don't want to become too overbearing on them where they will hesitate or avoid asking my help because they know once they do, I'm not going to leave until they can explain everything to my satisfaction. I have done that in the past and I don't...you know, that was good. Another reason was that I thought that she probably did get it. She understood the zero part – when it hits the ground it was going to be zero. That's what she should be looking for, a value very close to zero. I don't know for sure, but I thought she probably got it. So that weighed in with not wanting to be overbearing with her and not wanting to break that moment by possibly confusing her again as I tried to ask questions that would prove to me that she understood what I hope she understood, whereas, if I did that and I asked them the wrong way and she gets confused again and frustrated again then I think sometimes we've lost whatever gained in that moment of "ah-ha".

(10/13/00)
I'm pretty direct with a lot of things. I want to really know something so I just ask that person the question about what I'm wondering, if they know it. ... I guess that's how I interact with people so I decided that's how I'm going to interact with the people in my classroom, too. When I'm in that situation, I won't beat around the bush, I won't try and sneak it out of them without them knowing it. If I want to know if you know it, I'm gonna' ask you. And we'll find out, I guess.

(10/3/00)
There are times when I will ask one [question] after another after another to the same person and not just...trying to drive home some kind of point. I asked a few times but I didn't do that here. I could have done that with Kim or Shawn. If I thought that they were completely out of the loop, there are times when I think that they'll be able to handle it ok when I ask them one and they answer it. Which leads a step closer so I'll ask them another question and then again and then again. It kind of puts them on the spot a little. If they can handle it, sometimes they talk themselves through it and make that connection. Sometimes I feel that I need to try and force the issue. So, when I just asked a few times here, I guess it just didn't seem like a whole lot to me. There's many times when I do a lot more than that – good or bad.

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Appendix J

List of NUD*IST Nodes

Node	Name	Definition
1	Reflective Stage	Exhibits any indication of being related to a reflective stage.
1 1	Reflective Stage/Perturbation	Exhibited a dissatisfaction or uneasiness with how things went.
1 2	Reflective Stage/Awareness	Exhibited a realization that something may need to be changed in order to improve.
1 3	Reflective Stage/Commitment	Exhibited moving beyond awareness to thinking about action.
1 4	Reflective Stage/Vision	Description of image of what discourse should look like.
1 5	Reflective Stage/Projection	Exhibited a visualization of the changes to occur and reflection on them as they take place.
1 6	Reflective Stage/Practitioner	Exhibited reflecting on own practice and raises questions about own practice.
2	Mechanics	References made about the mechanics of the discussion.
2 1	Mechanics/Wait-time	References to the use of wait-time pedagogical technique or its effectiveness.
2 2	Mechanics/Questioning	References made about the use of questioning.
2 2 1	Mechanics/Questioning/Types	Refers to the types of questions, like higher-order, or comprehension.
2 2 2	Mechanics/Questioning/Form	Refers to the form of the questioning, how it transpires, not the questions being asked.
2 3	Mechanics/Listening	References to the use of listening as a pedagogical technique or its effectiveness.
2 3 1	Mechanics/Listening/ Evaluative	References and reflections on listening for a particular answer.
2 3 2	Mechanics/Listening/ Interpretive	References and reflections on listening closely to the students to understand what they mean.
2 3 3	Mechanics/Listening/ Hermenuetic	References and reflections on listening that is oriented to sense-making.

2 4	Mechanics/Norms	References made regarding establishment of social or sociomathematical norms
3	Mathematics	References made regarding mathematical aspects of the discussion.
3 1	Mathematics/Justification	References made to justifications provided by students or its purpose.
3 2	Mathematics/Generalization	References made to generalizations provided by the students or its purpose.
3 3	Mathematics/Representation	References made to multiple forms of representation in the discussion or purpose.
3 4	Mathematics/Connections	References made to connections provided by the students or its purpose.
3 5	Mathematics/Conjectures	References made to conjectures provided by the students or its purpose.
3 6	Mathematics/Surprises	References to instances where he was surprised by students' mathematical talk.
3 7	Mathematics/Expectations	References made about his mathematical expectations of the students.
4	Whole-class discussion	References made to the purpose and role of whole-class discussions
5	Small-group discussion	References made to the purpose and role of small-group discussions
6 1	Teacher Talk/Revoicing	Instances of teacher talk that revoices students' talk.
6 1 1	Teacher Talk/Revoicing/ Clarify	Teacher talk that clarifies or amplifies what a student has just said.
6 1 2	Teacher Talk/Revoicing/ Explain Reasoning	Teacher talk that uses students' words to explain reasoning.
6 1 3	Teacher Talk/Revoicing/ Introduce Ideas	Teacher talk that uses students' words intending to introduce new ideas into the discussion.
6 1 4	Teacher Talk/Revoicing/ Redirect	Teacher talk that uses students' words but redirects the conversation.
6 2 1	Teacher Talk/Purpose/ Elicit Fact	Teacher talk that asks a questions that elicits factual or previously discussed information

		(e.g., drill-type questions).
6 2 2	Teacher Talk/Purpose/ Elicit Explanation	Teacher talk that elicits an explanation from a student for their answer procedural in nature.
6 2 3	Teacher Talk/Purpose/ Establishing norms	Teacher talk that explicitly made to establish either social or sociomathematical norms.
6 2 4	Teacher Talk/Purpose/ Probe student thinking	Teacher talk that is intended to understand student thinking. These go beyond the "explain your procedure" and ask questions about "why".
6 2 5	Teacher Talk/Purpose/ Generate Conjectures	Teacher talk that encourages students to make conjectures.
6 2 6	Teacher Talk/Purpose/Judging	Teacher talk that explicitly agrees with or refutes a statement made by a student.
6 2 7	Teacher Talk/Purpose/ Textbook question	Teacher talk that is reading aloud from the textbook.
6 2 8	Teacher Talk/Purpose/ Directive-Non-Mathematical	Teacher talk is statements or questions that are giving non-mathematical direction to students.
6 2 9	Teacher Talk/Purpose/ Telling	Teacher talk that is telling students how to do something mathematically.
6 2 10	Teacher Talk/Purpose/ Facilitating	Teacher talk that focuses on keeping the mathematical discussion going. Often student prompts.
6 2 11	Teacher Talk/Purpose/ Classroom Management	Teacher talk that are related to general classroom management issues.
6 2 12	Teacher Talk/Purpose/ Verify student thinking	Teacher talk that compares to and verifies the students' thoughts.
6 2 13	Teacher Talk/Purpose/ Natural Responses	Teacher talk that serves the purpose of acknowledging the speaker.
6 3	Teacher Talk/Questioning	Teacher talk that uses one of the questioning techniques identified by Pimm (1987)
6 3 1	Teacher Talk/Questioning/ Proof by intimidation	Teacher talk that makes the answer to a question obvious by the verbiage or tone of voice.
6 3 2	Teacher Talk/Questioning/	Teacher talk where the utterance intentionally

	Clozed Questioning	ends so that the student can fill-in-the-blank.
6 3 3	Teacher Talk/Questioning/ Echoing	Teacher talk where the question echoes back what the student had previously responded.
F 1	Description of Class Discourse	References made to how the classroom discourse played out in the classroom.
F 2	Comfort Level	References made about the importance of students (or teacher) being comfortable in the classroom.
F 3	Time	References made to the lack of time in the school day, semester, or etc.
F 4	Disagree	Comments made by the teacher in the interview referring to his perception of what occurred in the classroom, which the researcher saw differently.
F 5	Philosophy	References made related to his own philosophy.
F 5 1	Philosophy/Personal	References made to his personal philosophy about life in general.
F 5 2	Philosophy/Teaching	References made about how one should set up their classroom environment, carry out instruction, or educational policy.
F 5 2 1	Philosophy/Teaching/ Basic Needs	References made about the basic needs of the classroom teaching environment.
F 5 3	Philosophy/Mathematics	References made to his beliefs about the nature of mathematics.
F 5 4	Philosophy/Discourse	References made about his vision of discourse.
F 6	Tensions	References made to frustrations or conflicts that arise between thoughts and feelings.
F 6 1	Tensions/Efficacy	References made that exemplify the tension between his sense for what he should do for the students' benefit.
F 7	Noted Changes	References made that describe his perceived changes in the classroom.
F 8	Authority	References that describe his perception of authority in the classroom.

F 9	Student Thinking	References made to students' thinking; usually about what he thought students were thinking.
F 10	Goals	Description of the purpose of the lesson.
F 10 1	Goals/Launch	References made regarding the purpose of launches in general.
F 10 2	Goals/Investigation	References made regarding the purpose of investigations in general.
F 10 3	Goals/Checkpoints	References made regarding the purpose of checkpoints in general.
F 11	Open mind	References made about keeping an open mind during a classroom discussion either on part of teacher or student.

Appendix K

Robust Mathematical Discussion Rating Scales

Robust Mathematical Discussion (RMD) Rating Scales

Mathematics Content Dimension

Justification*

Proof (1)	Logical argument is given for being sure that an answer is correct or counterexample provides a refutation	[1(#proof) + 0.5(#explain)] ÷ # student turns
Explain (0.5)	Student explains why her answer holds and/or how she got her answer	
None (0)	No justification given, but may describe the answer	

Representations

Unpacked (1)	Topic is unpacked with more than two representations	Rate each separate topic; take the highest rating for the episode
Amplified (0.5)	Topic is amplified with two representations	
Single (0)	Compressed with only one representation	

Generalization*

Generalization(1)	Generalization beyond particular examples, categorization, recognition of a pattern, or broad prediction	# generalize ÷ # student turns
Concrete (0)	Limited to one particular situation	

Connections*

Connections(1)	Examples or statements in which student makes connection with different mathematical or real-world topic	# connections ÷ # student turns
None (0)	No connections made with other topics	

Average the scores on the individual scales to obtain the ranking on the Mathematics Dimension. Calculate to the nearest tenth and represent the ranking geometrically on the Mathematics Continuum.

Simple ⟶ Complex

*Note: These sections of the instrument remained unchanged from the Mendez's (1998) Robust Mathematical Discussions scales.

Robust Discussion Dimension

Intensity*

Volunteer (1)	Students voluntarily join the discussion without teacher mediation	[1(#volunteered) + 0.5(#elicited)] ÷ # student turns
Elicited(0.5)	Teacher nominates student speakers or asks questions of students	
None (0)	Off topic remarks or no student enter the discussion	

Engagement*

Many engaged		# speakers in contiguous discussion ÷ # students present
Several engaged	Count the number of student speakers as a proxy for number engaged	
Few engaged		

Building

Build(1)	Responses build on earlier comments with new ideas and are integrated into the discussion	[1(# build) + 0.5(#neutral)] ÷ # student turns
Neutral (0.5)	Repetition of earlier stated ideas by another	
None (0)	First response or non-sequitur	

Average the scores on the individual scales to obtain the ranking on the Discussion Dimension. Calculate to the nearest tenth and represent the ranking geometrically on the Discussion Continuum.

Monologic ⟶ Dialogic

*Note: These sections of the instrument remained unchanged from the Mendez's (1998) Robust Mathematical Discussions scales.

Appendix L

RMD Tabular Results

						Discussion Scales				Mathematics Content Scales				
Type	Date	Coding order	Unit	Lesson.	Investigation	Intensity	Engagement	Building	Overall	Justification	Representation	Generalization	Connection	Overall
Launch	11/14	8	3	3	1	0.62	0.6	0.09	0.43	0.09	0.5	0	0.10	0.17
	12/07	2	4	1	1	0.60	0.6	0.25	0.48	0.09	0	0	0	0.02
	12/19	6	4	2	1	0.86	0.57	0.22	0.55	0.13	0	0	0	0.03
Investigation	10/03	3	2	2	1	0.74	0.75	0.03	0.50	0.05	0	0.03	0	0.02
	10/17	9	2	4	1	0.61	0.5	0	0.37	0.04	0.5	0	0.09	0.16
	11/14	4	3	3	1	0.69	1	0.13	0.60	0.01	1	0	0.02	0.26
Checkpoint	09/28	7	2	1	1	0.80	0.40	0.02	0.40	0.15	0.5	0	0.1	0.31
	10/26	1	3	1	1	0.55	0.5	0.05	0.37	0.17	0.5	0	0.08	0.18
	11/30	5	3	3	3	0.55	0.57	0.15	0.42	0.08	0.5	0.06	0.03	0.17