An Investigation into Relationships between Alternative Assessment and Pre-Service Elementary Teachers' Beliefs about Mathematics

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AN INVESTIGATION INTO RELATIONSHIPS BETWEEN ALTERNATIVE ASSESSMENT AND PRE-SERVICE ELEMENTARY TEACHERS' BELIEFS ABOUT MATHEMATICS

by

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The purpose of this study was to examine how including alternative assessments in a reform-based mathematics course affects pre-service teachers' mathematical beliefs. A single section of a mathematics course designed for elementary education majors that employed three different alternative assessments was the setting for the study.

A pre- and post-belief survey completed by students enrolled in the section under study represented the first level of data collection and analysis. The results of the pre-belief survey guided the selection of seven informants from the section and provided belief statements for the informants to verify during subsequent interviews. Based on the data from these interviews, it was determined that all but one of the informants believed mathematics to be a pre-existing set of facts and procedures that are passed along by some mathematical authority. Moreover, all the informants reported that they believed assessing in mathematics meant giving a paper-and-pencil test. Further data collection methods, including a card sort of assessment practices and perusing classroom vignettes, and subsequent analysis identified each informant's previous mathematical experiences as traditional, especially regarding assessment.

A comparison of the pre- and post-belief surveys indicated that the alternative assessments had indeed affected the mathematical beliefs of almost 50% of the students so that they now held more productive mathematical beliefs. During the final informant interviews, however, it became evident that apparent shifts in beliefs were not as profound as indicated by the comparison. Although all seven informants had become more aware that tests represent only one of many ways to assess students in mathematics, most of the informants clung to their original beliefs about the nature of mathematics. The two informants who exhibited more productive beliefs credited the alternative assessments as the most influential factor from the course under study.

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CHAPTER I

INTRODUCTION

Background

> I used to think that maths teachers were all teaching the same subject, some doing it better than others. I now believe that *there are two effectively different subjects being taught under the same name, 'mathematics'*. (Skemp, 1978, p. 11)

The 'two different subjects' are often evident in pre-service teachers' and reform-minded mathematics educators' differing views regarding the nature of mathematics. A review of the research in mathematics education (e.g., Brown & Borko, 1992; Dossey, 1992; Thompson, 1992) indicates that because of traditional educational experiences most pre-service teachers perceive mathematics to be a static discipline. Often referred to as absolutism (Ernest, 1991), this position focuses on content because mathematical knowledge is thought to be a set of unchanging truths that exists separate from human experience. From this standpoint, mathematics teaching typically relies on passing along the content from teacher to student through the use of demonstration lectures followed by structured practice (Schoenfeld, 1992). The National Council of Teachers of Mathematics (NCTM) calls for reforms in mathematics education that represent a dramatically different perspective of mathematics and mathematics instruction (McLeod, 1994; NCTM, 1989, 1991, 1995); a perspective that stresses mathematical process as well as mathematical

product (Schoenfeld, 1992). Courses taught from this perspective usually encourage students to learn mathematics by thinking and reasoning while engaging in non-routine problem solving situations. Students are expected to be active participants in constructing their own mathematical knowledge, an expectation many pre-service teachers find difficult to accept (Raymond & Santos, 1995). In order for these future teachers to effectively embrace the NCTM's vision, they must begin to view mathematics from a fallibilist position (Thompson, 1992). According to Ernest (1991), fallibilism holds that "mathematical truth is corrigible, and can never be regarded as being above revision and correction" (p. 3), which is in direct opposition to the absolutist view.

In an effort to challenge pre-service teachers' absolutist view, many mathematics educators are designing content and method courses to model a more fallibilist approach to mathematics. Unfortunately, research into the mathematical beliefs of pre-service teachers has shown that simply modeling change may not be sufficient since certain beliefs prove extremely resistant to reformers' modification efforts (McDiarmid, 1990; Pajares, 1992). The problem may be that to date much of this research has focused on addressing mathematical beliefs through changes to the content and instruction in mathematics education courses but not through changes in assessment practices (Bell, 1995).

Using traditional testing methods based on an absolutist view of mathematics conveys conflicting messages to students being taught from a fallibilist perspective (Emenacker, 1994). "As the curriculum changes, so must the tests. Tests also must

change because they are one way of communicating what is important for students to know… In this way tests can effect change" (NCTM, 1989, pp. 189-190). Assessments developed from a fallibilist perspective engage students in mathematical tasks that require them to construct and apply mathematical concepts in relatively new ways (NCTM, 1996). Such forms of assessment have the potential to challenge students to rethink their absolutist views towards mathematics in ways that shifts in content and instruction have not since their views are being challenged by the piece of the educational system many students value most – grades (Lester & Kroll, 1991; Oaks, 1987; L. Wilson, 1994a). Hence, the goal of this study is to examine how including alternative assessment techniques in a reform-based mathematics course affects pre-service teachers' mathematical views.

Significance of the Research

> Much is made of the absolutist-fallibilist distinction because, as is shown subsequently, the choice of which of these two perspectives is adopted is perhaps the most important epistemological factor underlying the teaching of mathematics. (Ernest, 1991, p. 3)

Because studies support the premise that teachers' mathematical beliefs influence their classroom instruction (Ball, 1990; Baroody & Ginsburg, 1990; Dossey, 1992; Middleton, 1999; Thompson, 1992), it is essential for mathematics educators to challenge pre-service teachers' absolutist view of mathematics. Otherwise the discontinuity between the absolutist view prevalent in today's schools and the fallibilist view supported by the NCTM will surely persist. Identifying experiences that can help pre-service teachers develop a more fallibilist approach to

mathematics is an important topic for research in mathematics education (Brown & Borko, 1992; McLeod, 1994; Thompson, 1992). Yet there is little research on the role assessment plays in the development and reinforcement of students' beliefs in mathematics (Bell, 1995).

There does exist, however, a great deal of speculation regarding assessments' role in the rise of an absolutist mathematical view. For example, Borasi (1990) suggests "[w]e should not be surprised at mathematics students' overwhelming concern with product and answers when the most important measure of academic success is given by the score received on standardized multiple-choice tests taken under considerable time pressures" (p. 177). While statements such as this may sound reasonable, it is important that studies like the one described here attempt to provide formal research into the complex relationships between assessments and beliefs in mathematics.

Research Questions

> Research on attitudes towards mathematics has looked at many aspects of the teaching process, but it falls short of investigating the impact assessment has upon these critical aspects of mathematical development (Bell, 1995, p. 11).

The central question addressed by this study is the following: What are the relationships between alternative assessment practices in a reform-based probability and statistics course for pre-service elementary teachers and their beliefs about what it means to learn and do mathematics?

Specific questions addressed in this research are:

1. What past experiences have pre-service elementary teachers had relative to learning and assessment in mathematics?

2. How do students explain the relationship between their experiences in learning mathematics, with a focus on assessments, and their beliefs about the nature of mathematics?

3. Prior to their completing a sequence of alternative assessments, what are pre-service elementary teachers' beliefs about what it means to assess a student's mathematical understanding?

4. Does experience with a sequence of alternative assessments affect students' perceptions about what it means to assess mathematical understanding? If so, how and why?

5. After completing a course using a sequence of alternative assessments, do students report any changes in their mathematical beliefs? If so, to what do they ascribe the changes?

Brief Overview of the Study's Methodology and the Dissertation's Organization

> Assessing beliefs is a difficult task since beliefs are often held without teachers being conscious of having explicit knowledge of their own beliefs. Any attempt to uncover how teachers think will necessarily rely on inferences from indirect tasks, and some may provide more meaningful data than others… (Jacobs, Yoshida, Stigler, & Fernandez, 1997, p. 8)

As the research questions suggest, the focus of this study was on understanding the relationships between assessment experiences in mathematics and

the development of mathematical beliefs from the individual's perspective. A variety of methods were used to gather data on students' mathematical beliefs and on any impact a course using non-traditional forms of assessment had on those beliefs. Data sources included a pre- and post-survey used to determine student mathematical self-perception and mathematical beliefs, interviews addressing student mathematical beliefs and assessment experiences in mathematics, and artifacts, including student work, from the course. As data accumulated, a procedure similar to Glaser and Strauss's (1967) Constant Comparative Method aided in uncovering similarities and differences among participants' responses. During this analysis, themes emerged that helped to direct the collection of subsequent information and in the development of theories grounded in the data.

Thus, the organization of the study reflects the organization of this dissertation. Previous research and theoretical essays addressing beliefs and assessment aided in the developing the framework for the data gathering and analysis. The ideas and results from this research literature are discussed in depth in the next chapter. Details of the methods used in gathering and analyzing the data from this study are provided in Chapter III. In Chapter IV, the findings from the study are supplied so that they might be scrutinized. Finally, conclusions from this research, the study's limitations, and the implications of the findings, including suggestions for further research, are discussed in Chapter V.

Closing Comments

More than a decade has passed since the publication of NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics.* Since then much has been made of the attempts to change the way mathematics is presented in schools so that it is more representative of the fallibilist perspective. The NCTM literature contains numerous examples of shifts in mathematics curriculum, instruction, and assessment that reflect the reform movement. While there is considerable formal research on how altering the curriculum and/or instruction affects students' beliefs, little is known about how changing assessment impacts those same beliefs. This study provides some insight into this often overlooked area of research in mathematics education.

CHAPTER II

LITERATURE REVIEW

The intent of this chapter is to establish a theoretical framework for the study and to review research findings relevant to the research questions. In the first section, the theoretical framework is developed through an examination of the philosophy of mathematics learning used in this study and a description of McEntire and Kitchens' (1984) theory on beliefs. The second section details the literature on beliefs pertinent to this study; a general definition of beliefs is provided along with an explanation of their structure. Research on students' beliefs about mathematics, the implications of these beliefs, and the results of attempts to change pre-service teachers' mathematical beliefs are also discussed in the second section. The third section focuses on mathematics assessment through a report on its history, current trends in mathematics assessment, and research on the impact of assessments on students' mathematical beliefs. The final section summarizes the results of the literature discussed in this chapter and explains how the present study contributes to the existing research.

Theoretical Framework

> (I)t is important that researchers interested in examining teachers' beliefs make explicit, to themselves and to others, the perspectives from which they are approaching their work. (Thompson, 1992, p. 137)

Philosophy of Mathematics Learning Employed in This Study

Dossey's (1992) review of the research in mathematics education identifies studies as reflecting either an external or an internal view of mathematics. Traditional teaching methods, which reflect an external or absolutist view, presume that a student enters the mathematics classroom as a blank slate, *tabula rosa*, to be filled with mathematical truths (Schoenfeld, 1987a). Because researchers holding an external view assume that mathematics consists of a fixed body of knowledge, they attempt to determine how well students absorb and accumulate information. Researchers studying mathematics education from an internal perspective maintain a more fallibilist approach toward the development of mathematical knowledge. Rather than studying how successful certain methods are at transmitting facts and procedures to a group of students, researchers maintaining an internal view concentrate on understanding the processes an individual uses in making sense of his or her mathematical experiences (Dossey, 1992). Because this approach assumes each one of us constructs unique interpretive frameworks of how the world works based on our distinct experiences, it is known as constructivism. This is the philosophical perspective employed in this study.

Piaget, whose ideas were a precursor to constructivism, saw intelligence and knowledge as biological functions whose development could be explained and mapped for each individual (von Glasersfeld, 1990; Schoenfeld, 1987a, 1992). Thus, constructivism stands in opposition to the absolutist presumption that knowledge is a

pre-existing set of truths. Instead, the constructivist views knowledge as the result of each individual making sense of his or her own personal experiences (Confrey, 1990; Jaworski, 1998; MSEB, 1994; Romberg, 1992; Steffe, 1990). Students do not simply receive knowledge as it is given, but interpret it based on mental frameworks constructed during prior experience (Romberg & Carpenter, 1986; Schoenfeld, 1992).

The interaction between existing ideas and new experiences is a continuous and complicated process, with learning occurring whenever a new experience forces adaptations to an individual's existing mental schema (Steffe, 1990; Teppo, 1998). The constructivism model, therefore, is a philosophical position of knowledge development, where learning is characterized as a cognitive cycle consisting of equilibrium, experience, disequilibrium, consideration, assimilation or rejection, and a return to equilibrium. Whether new experiences fit into the existing structure (assimilation) or not (rejection), the mind's final state after the experience is dependent on the individual's initial cognitive constructions (Janvier, 1992; Romberg & Wilson, 1995).

Learning is not a solely individual endeavor, however. Individuals also develop mathematical meanings through interaction and discourse with their teachers and peers. At times, the teachers must intervene and help learners to negotiate passage toward socially accepted knowledge. Constructivism does not deny the role social interaction plays in the construction of knowledge (Cobb & Yackel, 1996; von Glasersfeld 1992). Interactions with other people constitute a large portion of each person's experience, and these interactions influence which behaviors, concepts, and

theories are considered reasonable. The extent to which a student learns from another person during an activity, though, is still dependent on past experiences in similar social situations. "Consequently, individual learners construct unique and idiosyncratic personal knowledge even when exposed to identical stimuli" (Ernest, 1998, p. 29). Hence, social interaction does have an affect on what a child learns, but it is not the sole determining factor.

Along with knowledge, beliefs also appear to be the result of an intricate process involving both cognitive and social factors (Cooney, Shealy & Arvold, 1998; Pajares, 1992; Schoenfeld, 1988b, 1992). People are continuously constructing their own personal perceptions of the world surrounding them rooted in their experiences, including their interactions with other people. Sometimes their view matches the perceptions of others, but not always. In fact, the picture they see may be quite different – the result of a steady diet of alternative experiences (Schoenfeld, 1987a; Shaw, 1989). A succinct explanation of how experiences impact the development of an individual's beliefs and the implications of these beliefs is proposed in McEntire and Kitchens' (1984) Theory of Axioms.

McEntire and Kitchens' Theory of Axioms

As the literature on constructivism suggests, an intricate relationship exists between an individual's experiences and the mental frameworks a person constructs as he or she makes sense of these experiences (Schoenfeld, 1988a). According to McEntire and Kitchens' (1984) work, beliefs are also constructed from experiences.

Their theory provides a simplified snapshot of the construction process by concentrating on how experiences lead to the development of beliefs (axioms) that then affect behavior (Kitchens & Hollar, 1995).

McEntire and Kitchens describe axioms as assumptions about the world or the self that a person believes to be true. They identify two distinct types of axioms. The first type, WORLD axioms, defines the person's basic perception about what exists outside the individual. These form a person's views about his or her environment and other people. Beliefs that 'people only care about themselves' or that 'math is just a collection of boring facts' are examples of WORLD axioms. The I AM axioms, on the other hand, define the basic assumptions an individual has about self, such as 'I am not good at math' or 'I am a good dancer'.

A person's WORLD and I AM axioms are adopted when the mind is most open to new experiences. As a person interacts with other people and the environment, the individual develops a sense of the world and his or her place in it. Accepting what is seen or heard as the truth amounts to affirming the experience. Much like the constructivist position, this affirmation process is a personal act. As a person makes sense of his or her experiences, positive or negative, the individual develops beliefs about self and surroundings. Schoenfeld (1987a) offers an excellent scenario that demonstrates this phenomenon.

> Suppose that during your entire academic career, every mathematics problem that you were asked was in fact a straightforward exercise designed to test your mastery of a small piece of subject matter. You were expected to solve such problems in just a few minutes: If you did not, it meant that you had not understood the material and the material should be explained to you. Suppose in addition that this scheme was reinforced in class: Problems were expected

> to be solved rapidly, and teachers gave you the solutions if you did not produce the answers quickly. Having had this experience over and over again, you might eventually codify it as the following (implicit) rule: When you understand the subject matter, any problem can be solved in 5 minutes or less. (p. 27)

Hence, based on this experience the student may construct a counterproductive WORLD axiom – "If a math problem cannot be solved quickly, then give up."

Once fixed in the student's mind, this type of counterproductive belief can have disastrous results because beliefs are the primary force in influencing a person's behavior and have a dramatic affect on learning. The student who believes that mathematics problems should be solved quickly or abandoned acts in such a way as to demonstrate that this belief is true, thus reinforcing it (Pajares, 1992). In order for success to occur, more productive beliefs must replace the counterproductive ones.

Unfortunately, Pajares' (1992) review of educational research on beliefs found that long-held beliefs based on early experiences are among the most difficult to replace. "[T]he earlier a belief is incorporated into the belief structure, the more difficult it is to alter, for these beliefs subsequently affect perception and strongly influence the processing of new information" (p. 317). After years of viewing the world through certain beliefs, an individual will often ignore or distort contrary evidence in an effort to maintain their position (Wideen, Mayer-Smith, & Moon, 1998). Thus, beliefs can even affect an individual's actions when encountering experiences that might have the potential to alter the beliefs.

McEntire and Kitchens' theory attempts to explain how these early experiences are used in the development of a person's beliefs, and how beliefs affect

an individual's actions in future situations. The experience-belief-action chain is essential to this study and referred to throughout this dissertation. Beliefs, the central portion of this chain, are discussed in the next section.

Beliefs

> Once the student has sufficient evidence that he can't do math (from failed tests, negative chalk board experiences, peer success, teacher or parent judgement) the conscious mind induces the axiom, "I am not good at math" or I have no ability in math" (McEntire & Kitchens, 1984, pp. 141 - 142).

A student constructs beliefs about mathematics based on experiences with the subject (Cobb, Wood, & Yackel, 1990), and these mathematical beliefs, in turn, affect how the student acts during future encounters with mathematics (Pajares, 1992). The focus of this portion of the literature review is on the pivotal piece in this relationship – beliefs. A general discussion on beliefs and belief systems introduces this topic and is followed by an examination of the current research regarding mathematical beliefs, their implications, and recent attempts to alter them.

Beliefs and Belief Systems

Beliefs are sometimes incorrectly identified as knowledge or attitudes. The confusion probably results from the fact that beliefs fall somewhere between cognition and affect (Schoenfeld, 1985). Cirulis (1991) distinguishes between beliefs and attitudes by explaining that attitudes are more emotional in nature, while beliefs represent a more cognitive state of mind. Although beliefs and knowledge are both

cognitive functions, Thompson (1992) suggests that a distinctive trait of knowledge is that it

> must meet criteria involving cannons of evidence. Beliefs, on the other hand, are often held or justified for reasons that do not meet those criteria, and, thus, are characterized by a lack of agreement over how they are to be evaluated or judged. (p. 130)

Therefore, for the purpose of this study, beliefs are cognitive structures that: (a) result from a process of organizing and interpreting past experiences, (b) exist somewhere between feelings and knowledge, and (c) do not require a consensus for acceptance.

The complex structure depicting the way in which beliefs are held and how they relate to one another is referred to as a belief system. Green's (1971) description of this system is one of the most widely accepted (see Cirulis, 1991; Cooney et al., 1998; Emenaker, 1994; Thompson, 1992):

> First there is the quasi-logical relationship between beliefs. They are primary or derivative. Secondly, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. Each of these characteristics of belief systems has to do not with the content of our beliefs, but with the way we hold them. (Green, 1971, pp. 47-48)

The quasi-logical relationship asserts that some beliefs are connected in what resembles a causal relationship with the primary belief representing the cause and the derivative belief the effect (Thompson, 1992). For example, a student that holds the primary belief that all mathematics is a connected body of absolute facts may develop the derivative belief that doing mathematics means memorizing a collection of definitions, theorems and algorithms. Although a belief is never held in complete

isolation from other beliefs, the very nature of beliefs ensures that these relationships are unstable and unpredictable. Given a different set of circumstances the 'cause' in the above relationship most likely will result in a significantly altered 'effect'.

The second dimension in Green's description represents the conviction with which beliefs are held. Peripheral beliefs, as the name suggests, are near the edges of a belief system and are more susceptible to modification. The central beliefs, on the other hand, are at the core of a person's system of beliefs and therefore, are the toughest to examine and change (Pajares, 1992). Again, the context dictates the position of the belief in this relationship (Cooney et al., 1998). Using the example from before, the derivative belief about memorization in mathematics may be central to the student when facing the prospect of being tested, thus pushing to the periphery the belief that any connections exist among the mathematical concepts being assessed.

The final dimension of Green's beliefs system contends that while beliefs are never held in isolation from each other, it is possible for 'clusters' of beliefs to exist nearly independent of one another. The separation existing among belief clusters explains why a person's beliefs may appear inconsistent to an outside observer (Cirulis, 1991; Thompson, 1992). Shaw's (1989) study of three middle school teachers' mathematical beliefs provides a prime example of this clustering since these teachers held two distinct sets of beliefs regarding mathematics. One cluster contained their ideal vision of mathematics while the other cluster of beliefs addressed the actual way mathematics was taught in their classrooms. Two of the

three teachers exhibited a tremendous discrepancy between these two clusters of which they were completely unaware. While they ideally believed that mathematics teaching should stress deep understanding and connections, they taught for memorization of isolated skills. When this disparity was drawn to their attention, they expressed discomfort that their belief clusters were so different. Fortunately, "[t]his uneasiness not only caused more stress for them, but motivated them to try new ideas and approaches in teaching mathematics ... they had a potential to change" (p. 223). Thus, beliefs may be altered if the individual holding them can recognize the inconsistencies.

The revision of long held beliefs, however, is uncommon; this is another distinction between beliefs and knowledge. Although both are cognitive in nature and result from experience, knowledge tends to grow with subsequent experiences while beliefs are inclined to remain static (Pajares, 1992). As a result, mathematics educators' attempts to challenge pre-service teachers' counterproductive beliefs stemming from an absolutist view have met considerable resistance. The identification of some of the most damaging of these beliefs, their affect on how students interact with mathematics, and educators' attempts to replace these beliefs with productive beliefs reflecting a fallibilist view follow.

Mathematical Beliefs

People's beliefs about the nature of mathematics are well documented in the research literature (Borasi, 1990; Frank, 1988; Garofalo, 1989; McLeod, 1992;

Schoenfeld, 1988a, 1992). These studies report that the mathematical beliefs held by a majority of students and teachers represent a narrow and static view of the subject, typically associated with absolutism and traditional school mathematics (Dossey, 1992; Raymond & Santos, 1995). Three of the most dominant mathematical beliefs found in the research literature concern perseverance, confidence, and predictability in mathematics. These three areas are the focus of this study.

The first belief, perseverance, relates to the amount of time and effort exerted when solving a mathematics problem. Many students argue that a problem in mathematics should be solved quickly or not at all (McLeod, 1994; NCTM, 1998; Spangler, 1992). "They believed that something was wrong either with themselves or with the problem itself if a problem took 'too long' [more than five to ten minutes] to solve" (Frank, 1988, p. 33). Thus, students are often willing to spend only a few minutes working on a problem before dismissing it as impossible, even when the problem might have been solved with a little more time (Schoenfeld, 1987b).

Students' unwillingness to put persistent effort into solving a problem relates directly to the second belief, confidence. In order for students to feel confident in their ability to do hard mathematics, they must become comfortable exploring different solution methods and validating their own answers (Ball, 1990). Oaks (1987), however, found that students did not believe thinking on their own could result in a mathematical solution. Typically, students believe that only extremely intelligent and creative people can discover and develop important mathematics

(Garofalo, 1989), and that ordinary people cannot expect to understand it, let alone create it (Schoenfeld, 1992).

Students' confidence is in their teachers, not in themselves. The research shows that many mathematics students, including pre-service teachers, believe the best way to learn mathematics is by passively absorbing the information as it is presented (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Frank, 1988; Garofalo, 1989; McDiarmid, 1990; Raymond & Santos, 1995). When middle school students were asked to select from six cartoons the one that best represented an ideal mathematics classroom, 65 of the 144 students were most comfortable with an image of the teacher as 'sage-on-the-stage' (Fleener, Dupree, & Craven, 1997). In a similar study, Fleener, Pourdavood, and Fry (1995) asked sixty-five pre-service elementary teachers to write a metaphor describing the role of a mathematics teacher. Almost half used a metaphor that implied a position of control and authority. A majority of the students and the pre-service teachers in these two studies believed that unless the teacher provided the proper facts, rules, and formulas for solving a problem, students would be incapable of finding the proper solution.

Students usually do not believe that the teacher is the ultimate authority, however. The teacher simply 'passes on' the pre-existing set of definitions and rules that makes mathematics predictable (Bell, 1995; Cooney & Shealy, 1995; Oaks, 1987; Raymond & Santos, 1995). According to Garofalo's (1989) findings, secondary students commonly believe that "[a]lmost all mathematics problems can be solved by the direct application of the facts, rules, formulas, and procedures shown by

the teacher or given in the textbook" (p. 502). From this primary belief comes the derivative belief that "[m]athematical thinking consists of being able to learn, remember, and apply facts, rules, formulas, and procedures" (p. 503). Mathematics is seen as predictable because solving a mathematics problem amounts to implementing a pre-existing procedure to obtain the correct answer (Kloosterman & Stage, 1992; McLeod, 1992).

These three categories of beliefs – perseverance, confidence, and predictability – were used extensively in the collection and analysis of data for this study. In each category, the beliefs of students participating in the research fell along a continuum from 'productive' to 'counterproductive'. Mathematics educators contend that many productive beliefs reflect a fallibilist nature of mathematics (McLeod, 1994; Thompson, 1992). If students are to be successful in true problem solving situations in mathematics, then they must hold beliefs that support perseverance, self-confidence, and flexibility in thinking (Schoenfeld, 1992). Consequently, the mathematical beliefs presented in this section are classified as counterproductive because they interfere with the learning and teaching of mathematics as suggested by the NCTM (Ball, 1990; Cirulis, 1991; McDiarmid, 1990).

Implications of Beliefs on Instruction and Learning

Research supports the McEntire and Kitchens' (1984) presumption that the best predictor of a person's behavior is his or her beliefs (Borko et al., 1992; Hersch,

1986; Pajares, 1992, 1993; Teppo, 1998). Both the teachers' and students' views of mathematics play significant roles in determining what happens in the mathematics classroom (Lubinski, 1994). Because most students and teachers hold counterproductive beliefs based on an absolutist view of mathematics (Dossey, 1992; Schoenfeld, 1992; Thompson, 1992), the effects of such beliefs are examined here.

These beliefs are counterproductive because mathematics education research reports several negative consequences that emerge from beliefs based on an absolutist view of mathematics. The first is that an absolutist view tends to ignore the mathematical process and focuses on the result (Borasi, 1990). As a result, students see it as a waste of time to delve into the details behind a problem because only the answer matters; they fail to appreciate the struggle and creativity involved in the achievement of certain mathematical results. A second consequence relates to students' belief that they learn mathematics best when they are absorbing the material as it is presented to them. In Fleener, Dupree and Craven's (1997) study, junior high students described students' role in the ideal mathematics classroom as, "listening and learning" (p. 43) while the teacher's role was, "To show what to do and how to do it" (p. 43). Consequently, these students fell short of constructing any personal meaning for the mathematics presented in class. A third consequence is that students with an absolutist view typically focus on the memorization of facts rather than the conceptual understanding of mathematical ideas (Oaks, 1987). Frank (1988) found that when students encountered a problem that could not be solved by an algorithm stored in their memory, they would either quit working on it or seek assistance from

the teacher. These students failed to develop any confidence in their own ability to do mathematics. Thus, these counterproductive beliefs resulting from the absolutist view of mathematics described above are often responsible for increased anxiety and limited success in school mathematics (Borasi, 1990; Oaks, 1987; Teppo, 1998).

Students' expectations of what it means to learn mathematics can also have a profound effect on how teachers present the material (Cooney, 1985; McLeod, 1994; Philipp, Flores, Sowder, & Schappelle, 1994; Raymond, 1997; Shaw, 1989). An excellent example of the power behind students' expectations is demonstrated in Foss's (1997) study of an instructor in a mathematics methods course for pre-service elementary teachers. While the methods instructor, Diane, felt she could model teaching in a constructivist manner, her students' expectations reflected a counterproductive "belief that they are not learning unless they are being told what to do" (p. 126) and caused her to rethink her methods. As Diane attempted to engage her students in group activities requiring discourse and reflection, she became aware of the lack of effort and interest displayed by a significant number of students. Endeavoring to catch the attention of these students, she shifted into a more authoritarian role. As Diane began to lecture with increasing regularity, her students became more attentive. After a particular lecture one student observed, "It was probably good for the class to get all that information because some have been saying that they are not learning anything here" (pp. 121-122). Ultimately, the counterproductive beliefs of the students won over Diane's attempts to teach from a

constructivist approach as she altered her methods to appease the pre-service teachers' expectations about how mathematics should be taught.

The mathematical beliefs held by the elementary education majors in Diane's methods course are not unique, and once they become teachers their instruction often cultivates an environment that proliferates their counterproductive beliefs (Ball, 1990). They see their students as repositories of knowledge and their role as teacher as presenting the information in the most efficient and clear manner possible (Arvold, 1997; Borko et al., 1992; Cooney et al., 1998; Shaw, 1989; Thompson, 1992). Every problem has an answer and a proper procedure for finding it (Ball, 1990; Brown & Borko, 1992; Jacobs et al., 1997). Students are only responsible for accepting the information and reproducing it on homework and tests as they demonstrate their mathematical ability (Schoenfeld, 1988a). As students experience this type of environment, the counterproductive beliefs are passed along, discouraging the students from constructing an accurate picture of what it means to learn and do mathematics. Therefore, it is imperative that mathematics educators attempt to modify pre-service teachers' beliefs to reflect a more fallibilist view of mathematics.

Attempts to Effect Change on Pre-service Teachers' Mathematical Beliefs

The literature on mathematical beliefs presented thus far has highlighted three important points. First, counterproductive mathematical beliefs permeate the belief systems of most pre-service teachers. They tend to view mathematics as "absolute, certain, and God-given" (Arvold, 1997, p. 466). Second, these beliefs influence how

future teachers will represent mathematics, which in turn impacts how their students will view mathematics. Unless pre-service teachers reconsider their counterproductive beliefs, they find it difficult to implement reform ideas in their teaching (Jacobs et al., 1997; Lubinski, 1994). Finally, because pre-service teachers' mathematical beliefs often develop early in life and are constantly reinforced throughout their schooling (Ball; 1990; Cirulis, 1991; Frank, 1988), these counterproductive beliefs are among the most central and deeply held beliefs regarding the nature of mathematics (Raymond, 1997). As a result, attempts to replace pre-service teachers' counterproductive beliefs with beliefs representing a more fallibilist approach have faced substantial resistance (Pajares, 1992, 1993; Vacc & Bright, 1999). Mathematics educators continue to try, however, as is evident in the following examples from the recent research literature.

One approach, which focuses primarily on pre-service elementary teachers, suggests increasing the number of mathematics courses elementary education majors need to graduate. The rationale is that with greater content knowledge their mathematical beliefs will become more enlightened. Researchers have concluded, however, that this strategy alone is ineffective in altering pre-service elementary teachers' counterproductive beliefs. Ball (1990) points out that although mathematics majors take significantly more mathematics than elementary majors, both tend to hold a similar, rule-based view of mathematics. In fact, if the additional courses are taught in the same fashion as those encountered previously, then the pre-service teachers' beliefs will only be reinforced rather than challenged (Borko et al., 1992).

Attempts to alter beliefs by simply talking about reform-minded methods or watching classrooms taught using these approaches have proven similarly unsuccessful. "These traditional programs which advocate progressive teaching practices seem to us to be a classic case of 'Do as I say, not as I do'" (Wideen et al., 1998, p. 160). Although students learned and could repeat the current reform rhetoric, they often failed to internalize it (Bright & Vacc, 1994; Cirulis, 1991). Even when pre-service teachers saw the methods successfully at work in the elementary classroom, many of them protected their original beliefs by saying that the teacher and students were exceptional in some way (McDiarmid, 1990). While the teacher in McDiarmid's study may have been remarkable compared to what these pre-service teachers had experienced as elementary mathematics students, the students were not atypical. These pre-service teachers' vested interest in maintaining their prior beliefs caused them to alter the experience to accommodate their own view of mathematics and its instruction (Pajares, 1992). This is a prime example of why belief modification can be so difficult to achieve.

Courses that directly engaged pre-service teachers with mathematics taught from a constructivist perspective did have success in affecting change in mathematical beliefs (Raymond & Santos, 1995), although some studies reported only marginal shifts. Those resulting in the most significant belief alterations shared several common factors: problem-solving activities, cooperative learning, reflective writing, and incorporating research on children's thinking and mathematical understanding (Bright & Vacc, 1994; Emenaker, 1995). The combination of

mathematics learning theory and actual reform methods in the mathematics education classroom resulted in experiences that fostered productive beliefs (Vacc & Bright, 1999). “Working in groups allowed the students to see, firsthand, people taking different approaches to the same problem” (Emenaker, 1994, p. 95), thus discouraging students’ previous view of mathematics as predictable (Geskus, 1994; Raymond & Santos, 1995). The group setting also encouraged students to become more confident in their own mathematical ability, with one student noting that “in this class, we have been forced to solve the problem on our own with a group. I realized that I don’t have to have the teacher tell me everything” (Raymond & Santos, 1995, p. 64). While this confidence seemed to improve at least one student’s willingness to put more time into solving difficult problems – “I also don’t give up on math problem so easily” (Raymond & Santos, 1995, p. 64), Emenaker (1994) found that upon completion of the course under his study, students still felt that mathematics problems should be solved quickly or abandoned as too hard. Therefore, the results regarding perseverance in these studies appear inconclusive.

Although a majority of these studies observed change in the counterproductive beliefs of pre-service teachers, the researchers were hesitant to describe the change as permanent or substantial (Bright & Vacc, 1994; Raymond & Santos, 1995; Vacc & Bright, 1999). Shifts in beliefs are sometimes the result of simply replacing one authority with another, which means that these changes cannot be expected to last (Oaks, 1987; Pajares, 1992). Furthermore, because of the clustering effect common in belief systems (Green, 1971), teachers’ beliefs about mathematics and the teaching

and learning of mathematics can be quite different from one another (Raymond, 1997; Shaw, 1989). For example, teachers may ideally espouse a fallibilist position toward mathematics, but demonstrate through their instruction an actual view supporting absolutism. This could explain why mathematics education programs seem to have only a minor impact on future teachers' instructional practice even when mathematical beliefs appear to have been altered (Lubinski, Otto, Rich, & Jaberg, 1995; Raymond, 1997; Wideen et al., 1998). The design of most of these programs limits them to addressing pre-service teachers' pedagogical beliefs, but not their central mathematical beliefs (Raymond, 1997).

Much of the research on the development of mathematical beliefs and mathematics educators' attempts to change pre-service teachers' counterproductive beliefs presented thus far has concentrated on curriculum and instruction, while leaving assessment practices relatively untouched. The fact that course assessments do not always reflect the otherwise fallibilist nature of a program does not escape the attention of study participants who frequently describe this discrepancy as confusing and hypocritical (Emenaker, 1994; MAA, 1994; Raymond, 1997). It is clear that students view assessment as a principal element in the educational system along with the curriculum and instruction. The role of assessments in the development and alteration of mathematical beliefs is the main focus of the next section.

Assessment

> Examinations tell [students] our real aims, at least so they believe. ...[W]e may completely sabotage our teaching by a final examination that asks for

> numbers to be put into memorized formulas. However loud our sermons, however intriguing the experiments, students will judge by that examination - and so will next year's students who hear about it. (Rogers, 1969, p. 956)

Depending on one's position, assessments can represent either end of the experiences-beliefs-actions chain (McEntire & Kitchens, 1984). Teachers' assessments represent actions resulting from their beliefs. For students, assessments are experiences that contribute to the development of their beliefs. The purpose of this section is to examine the research on this relationship between assessment and beliefs in mathematics from both standpoints – including mathematics educators' exiguous attempts to use alternative assessments to challenge pre-service teachers' absolutist beliefs. Before all this can be addressed, however, it is necessary to clarify how assessment and other related terms are used within this study.

Understanding Assessment

There is general agreement that assessment is a pivotal activity in the educational process (Crooks, 1988; Ernest, 1998; L. Wilson, 1993; Webb, 1992), yet it is not always clear what is meant by assessment since it can mean different things depending on use and context. Ordinarily, mathematics assessment refers to the collection of data used to evaluate the effectiveness of a curriculum, guide teachers in their instructional decision making, or determine students' abilities (MSEB, 1991; NCTM, 1995). Teachers choose from either informal or formal methods to accomplish these tasks. Informal assessments usually coincide with instructional evaluation (Clark, Clark & Lovitt, 1990) and have less influence on students' grades

(Clark, 1992; Long & Benson, 1998; L. Wilson, 1993). Formal assessment refers to a more organized, scored event (Clark et al., 1990), which in mathematics has commonly meant a paper-and-pencil test (Clarke, 1992; Long & Benson, 1998; L. Wilson, 1993, 1996).

Formal assessments can be further broken down into internal and external assessments. External assessments refer to mandated or externally developed tests used by school districts to evaluate mathematics programs and provide information about these programs to the public (MSEB, 1994). The Iowa Basic Skills Test, California Achievement Test, and the competency tests required by some states are all examples of externally developed tests (L. Wilson, 1993). Because teacher and district evaluations are sometimes tied to external testing, this form of assessment can encroach on teaching time and influence what and how teachers teach (Webb, 1992). It is not the chief influence, however, as students spend more classroom time engaged in internal assessments than in external testing (Crooks, 1988; MSEB, 1994). Internal assessments are the methods that individual teachers choose to use when determining the mathematical abilities of the students in their classroom. In an effort to evaluate students, a teacher may elect to create a new instrument or use a pre-existing one (such as a publisher's chapter test); in either case, the decision is usually very personal and represents the information the teacher believes is critical to assess.

Whether teachers realize it or not, students discern what mathematics is important to know and be able to do through these internal assessments (Clark et al., 1990; Garfield, 1994; MSEB, 1994; Raymond, 1996; Webb, 1992). Because grades

motivate many students, they come to value only those tasks they know will affect their grades (Cohen & Fowler, 1998; Garofalo, 1989; Lester & Kroll, 1991; Long & Benson, 1998; L. Wilson, 1993). In this way, students avoid all the assessment confusion by concentrating only on formal, internal assessments (L. Wilson, 1993). Therefore, for the purpose of this dissertation, assessment refers to student tasks developed or chosen by the teacher that aid in determining a student's grade for the course.

Teachers have traditionally employed paper-and-pencil tests when assessing students' ability in mathematics (Crooks, 1988), but some educators argue that these tests offer insufficient information about the students who take them (Garfield, 1993; Raymond, 1996; Webb, 1992, 1993a). "Tasks that provide insight into student's understanding of mathematics are often of a different form from ones typically found on tests" (Cooney, Badger, & Wilson, 1993, p. 245). These tasks are customarily referred to as alternative assessments, which include tests taken in a different format such as in groups, or tasks such as journal writing, projects, demonstrations, or portfolios (Cooney, Bell, Fischer-Cauble, & Sanchez, 1996; Lester & Kroll, 1991; MAA, 1994; MSEB, 1991; NCTM, 1996). Another form of alternative assessment is the use of authentic or 'performance-based' tasks. These realistic assignments are perhaps the assessments that best approximate *doing* mathematics since they require that students apply their learning in an authentic context (Garfield, 1994; Romberg & Wilson, 1995; L. Wilson, 1993). Although authentic assessment tasks were used extensively in my study, this is not to suggest that alternative assessment strategies

are commonplace in present mathematics classrooms, as the research examined in the next section demonstrates.

Current Trends in Mathematics Assessment – Assessment as Action

Recent research on mathematics assessments suggests that teachers still rely heavily on traditional, paper-and-pencil tests to evaluate their students' mathematical understanding (Dossey & Swafford, 1993; Raymond, 1996; Romberg, 1992; Senk, Beckmann, & Thompson, 1997; Webb, 1992, 1993b). While teachers at the elementary and middle school level are more likely to use diverse evaluation techniques than their secondary counterparts, they still tend to rely more heavily on tests when assessing mathematics than they do when assessing writing and speaking (Cooney et al., 1996; Webb, 1992). Since assessment represents an action from the teacher's standpoint, the assessment choices a teacher makes reflects his or her view of what constitutes knowing and doing mathematics (Dossey & Swafford, 1993; McEntire & Kitchens, 1984; Wheeler, 1993).

At all educational levels, mathematics tests share several common characteristics. A mathematics test is usually taken in a formal setting where a student works individually on a set of questions under some strict time limitation (Crook, 1988; L. Wilson, 1993). Although the types of questions on the tests may vary widely, rarely are they set in a realistic context (Cooney et al., 1993; Senk et al., 1997). Instead, the questions tend to reflect those done previously in class – perfunctory and algorithmic in nature and only meant to assess students' procedural

knowledge (Cooney et. al, 1996; Dossey & Swafford, 1993; Garfield, 1993; Lajoie, 1995; Lambdin, 1993). The results from students' tests are most often used to assign grades and rank-order the students (Cooney et al., 1993; Wilcox & Lanier, 1999).

Inasmuch as most traditional mathematics tests emphasize finding the percentage of answers a student got correct on problems that stress recognition and recall, these tests fail to identify students' strengths and weaknesses in areas that require higher-ordered thinking. Senk, Beckmann, and Thompson's (1997) research found that open-ended questions requiring students' rationale, justifications or explanations were significantly lacking from most of the mathematics tests they studied. For the most part, the questions on these tests required students to follow a prescribed algorithm in an effort to obtain a single correct answer. Cooney, Badger, and Wilson (1993) found this compatible with teachers' view of mathematics "as consisting primarily of a series of steps to be applied in isolated contexts" (p. 247). The teachers in this study believed that assessing mathematical complexity was simply a matter of developing a problem involving more and more steps, resulting in tests that represented mathematics as a large collection of separate skills and concepts. Thus, the effect of teachers' mathematical beliefs on their actions is evident in the types of test questions they choose to assess their students (M. Wilson, 1992).

It would be inaccurate, however, to assume that only teachers' mathematical beliefs determine their choice of assessment methods. Even though a teacher may hold a fallibilist view of mathematics, beliefs about teaching and learning

mathematics can also exert a significant amount of influence on assessment choices (Shaw, 1989). Some teachers believe that alternative assessments are more subjective than tests and therefore less likely to be accepted by society (Cooney et al., 1996; Cooney & Shealy, 1995; Crooks, 1988; Long & Benson, 1998; Wilcox & Lanier, 1999). Besides, they see assessments other than tests as too time consuming to evaluate and difficult to record (Cooney et al., 1993; Cooney et al., 1996; Cooney & Shealy, 1995; Lambdin, 1993; Senk et al., 1997). Many teachers are also uncomfortable developing alternative forms of assessment due to the fact that they feel ill-prepared to write good assessment instruments (Cooney et al., 1993; Crooks, 1988; Lambdin, 1993; Long & Benson, 1998; Senk et al., 1997).

One particularly strong teacher belief is that students will not be successful on any type of assessment unless they are given ample opportunities to practice what is being assessed (Crooks, 1988; Foss, 1997; Long & Benson, 1998). Typically, the questions on a traditional mathematics test are taken directly from problems presented during class or in the textbook allowing students to practice certain methods over and over again (Cooney et al., 1996; Lambdin, 1993; Senk et al., 1997; L. Wilson, 1993). The same 'rehearsal' strategy is commonly found when teachers attempt to use more alternative methods. Long and Benson (1998) suggested that

> [a]lthough this similarity might indicate alignment, the repetition led to a reduced level of mathematical reasoning required each time a similar example was solved... What started out as good thinking in instruction tended to be reduced to recall in evaluation. (p. 505)

This lack of confidence in students' ability to adapt to alternative assessment strategies is also cited by teachers as a reason for choosing to avoid altogether

incorporating alternative assessments in their mathematics classes (Cooney et al., 1996; Crooks, 1988) or only using it informally (L. Wilson, 1993).

Students are often unaware of the factors that influence teachers' assessment decisions though, which means that what a student comes to believe as a result of a heavy diet of paper-and-pencil tests may be quite different than what the teacher has intended. The aim here is not to suggest that paper-and-pencil tests are completely without merit and should be discontinued, only that the use of any single assessment style presents a limited view of the nature of mathematics. Because alternative forms of assessment are used relatively infrequently or have little impact on students' grades (Cooney et al., 1996; Senk et al., 1997; L. Wilson, 1993), one can safely assume that tests embody nearly the entirety of a student's assessment experiences in mathematics.

Taking Tests – Assessment as Experience

In a summary of the research on assessment, Crooks (1988) reports that "[a] substantial proportional of student time is involved in activities that are evaluated" (p. 440). Due to their frequency and perceived importance, tests are among students' most memorable experiences from mathematics classes (Clarke, 1992). Yet, teachers often either ignore or fail to realize the extent to which these testing experiences impact their students' view of mathematics (Crooks, 1988; Raymond, 1996). While teachers may intend for tests to define the mathematics skills they believe their students need to know, students have difficulty separating skills in mathematics from

beliefs about mathematics (Ma & Kishor, 1997). Hence, even when teachers try to pass along productive beliefs to their students, their message may unwittingly get lost in their choice of assessment.

The use of the traditional paper-and-pencil test to determine how well students know the material presented in class is based on a behaviorist theory of learning and an essentialist view of knowledge (Romberg, 1992; M. Wilson, 1992; L. Wilson, 1994b), thus encouraging students to embrace the counterproductive mathematical beliefs presented earlier. For instance, requiring students to identify and reproduce recently furnished information under a strict time restraint may reinforce inaccurate beliefs regarding perseverance, confidence, and predictability. An excellent example comes from Schoenfeld's (1988a) description of a unit test given in a class he observed which

> contained 25 problems – giving students an average of 2 minutes and 10 seconds to work on each problem. The teacher's advice to the students summed things up in a nutshell: "You'll have to know all your constructions cold so you don't spend a lot of time thinking about them." (p. 159)

The timed arithmetic tests that measure elementary students' computational ability offers another example of a commonly used assessment that rewards speed and accuracy in remembering pre-existing facts (Bell, 1995). The message students receive seems quite clear – mathematics means determining a quick, mechanical response from a memorized method developed by some mathematical authority (Bell, 1995; Clarke et al., 1990; Hancock & Kilpatrick, 1993). Thus, "there are dangers to the narrow assessments of competency that are currently employed" (Schoenfeld,

1988a, p. 146) because they fail to represent the true complexity of mathematics (Galbraith, 1993; Izard, 1993; Wheeler, 1993).

Instead, traditional mathematics tests reflect counterproductive beliefs by rewarding students for successfully memorizing and efficiently duplicating disconnected skills. Students believe that when they get the right answers then they have learned a great deal of important mathematics (Frank, 1988; Izard, 1993), when all they can truly claim is that their answers matched those on the key. Sometimes it is just a matter of luck, as when students haphazardly stumble upon a correct solution even though their methods are flawed (Schoenfeld, 1988b). A more likely scenario is that the students have memorized the steps for a particular procedure in order to pass a test yet lack the most basic understanding behind it (Bell, 1995; Borko et al, 1992; Clarke, 1992). Ball (1990) reports that although a group of pre-service elementary teachers knew the rule for dividing fractions well enough to sufficiently pass their mathematics tests, they had learned it without understanding how division of fractions is connected to the larger concept of division. "To allow them, and ourselves to believe that they understand the mathematics [in these cases] is deceptive and fraudulent" (Romberg, 1992, p. 46).

Attempts to implement alternative assessments in an effort to expose students to more diverse and authentic mathematical experiences and thereby challenge their absolutist views have proven generally unsuccessful. It appears that the primary reason behind this lack of success stems from the fact that the alternative tasks are commonly assessed informally due to the teachers' doubts described previously (L.

Wilson, 1993). Students believe that any mathematics material left ungraded is not worth knowing or doing (Crooks, 1988; Frank, 1988; Garofalo, 1989; Senk et al., 1997; L. Wilson, 1994a). Long and Benson's (1998) observation of a class found that "when graded assignments consistently did not require students to explain why a process worked or why the knowledge was useful, they tended to pay less attention to such discussions during instruction" (p. 506). These students were unwilling to put their energies into activities such as conjecturing and exploring which were not graded, and therefore, presumably not valued.

L. D. Wilson (1993) reports similar results in a study of a high school mathematics teacher attempting to implement alternative assessment methods such as reflective writing and group work. Many of the mathematics problems students were asked to write about were conceptual in nature, but few students did any of the writing since it was rarely collected and graded. Instead they concentrated on quizzes and tests that were comprised of almost entirely procedural questions yet had clear grades affixed. When Wilson asked seven students from the class if they were ever expected to do any sort of writing in the class, not one of them said yes. They did remember writing they had done in another mathematics course though, because that writing had been graded. The students were also unenthusiastic about the teacher's attempts to promote mathematical discourse through teaching pairs because these episodes involved informal assessments. Since all the student work that was graded was done individually, the students learned that the work done in pairs was fine, but less important than the tests they took individually. While this teacher's efforts to

present a more complete picture of mathematics were laudable, they proved ineffective as long as she tried to communicate the picture using informal assessments.

Once increased value is attached to assessment methods which engage students in alternative tasks that embrace problem solving, reasoning, communications, and making connections, then perhaps students will view mathematics “as a dynamic set of interconnected, humanly constructed ideas” (Romberg & Wilson, 1995, p. 4). To date, research related to the effect alternative assessments might have on mathematical beliefs is meager (Bell, 1995; Crooks, 1988; Webb, 1992). In particular, assessments have played only a minimal role in aiding mathematics educators’ attempts to challenge pre-service teachers’ counterproductive beliefs.

Alternative Assessment in the Mathematics Education Classroom

The research on alternative assessments used in the mathematics education classroom suggests that the inclusion of alternative assessments in a mathematics education course often results from a desire to address pre-service teachers’ beliefs about assessment, not mathematics. It has been determined that direct exposure of pre-service and in-service teachers to alternative assessment methods can alter their traditional view of mathematics assessment to the point that most of these teachers became more comfortable with using alternative forms of assessment (Barnes & Barnes, 1993; Cooney & Shealy, 1995, Raymond, 1996). Unfortunately, research on

using alternative assessments to affect change in pre-service teachers' mathematical beliefs is far less definitive.

Alternative assessment typically plays a supporting role in these studies – a single part in an all out effort to embody reform ideas in a mathematics education classroom. Research from Raymond and Santos (1995) provides a good example. They found that "a course emphasizing alternative assessment practices, problem solving, cooperative work, and student reflection" (p. 61) was capable of challenging pre-service teachers' counterproductive beliefs, but it was unclear to what extent the course's alternative assessments contributed to the challenge. Barnett (1996) suggests that it is important to understand exactly what aspects of a course encourages pre-service teachers to rethink their beliefs about the nature of mathematics because such knowledge can help in designing effective professional development experiences for future teachers.

Bell's (1995) research was one of the first to scrutinize the exact nature of the relationship between assessments and mathematical beliefs. Although her study focused primarily on how assessments impact students' attitude toward mathematics, she also discovered that "students who were required to take exams viewed the learning of mathematics more as a memorization process whereas the portfolio group was more prone to view the learning of mathematics through the understanding of the underlying concepts" (p. 32). With this result, evidence finally existed regarding assessments' impact on mathematical beliefs.

The research presented in this section has demonstrated that the majority of current assessment techniques reflect an absolutist vision of mathematics and identified the potential these tests have for shaping students' counterproductive mathematical beliefs. Unless mathematics assessments shift to represent a more fallibilist view of mathematics, the message sent by reform efforts in curriculum and instruction will be contradicted by the values traditional paper-and-pencil tests communicate (Graber, 1996; MSEB, 1994). If students believe that the results from an assessment are important, then their actions are guided by this perceived importance. Thus, it is critical to develop assessments that reward and encourage productive mathematical beliefs.

Summary

> The root of the problem is that we do not know very much about the origin of preservice elementary education students' beliefs, how they are supported through experiences, or how to convert existing beliefs to new beliefs. (Geskus, 1994, p. 14)

Currently, many teachers believe that mathematics is limited to a conglomeration of undeniable facts and formulas and, through their actions, pass along this view to their students. As a result, students encounter experiences (typically associated with the traditional mathematics classroom) which cultivate counterproductive mathematical beliefs that can impede their development in the subject. Thus far, mathematics educators' attempts to interrupt this absolutist cycle have met with varying levels of success. Their plan has been to challenge pre-service teachers' narrow mathematical views through a constructivist philosophy of learning

that provides experiences that represent the fallibilist nature of mathematics. This typically has meant shifts in the curriculum and in instruction. Although assessment has been called "an instrument of reform" (Cooney et al., p. 239), changes in how mathematics is being assessed are uncommon. Traditional paper-and-pencil tests that reflect the absolutist view remain the assessment of choice in mathematics, possibly sabotaging educators' otherwise reform-minded efforts. In this way, assessment may serve as an impediment rather than an instrument of reform.

As this chapter has shown, the role assessment plays in the development of students' mathematical beliefs is often written about, but without much formal research for support. The goal of this study is to address this deficiency by obtaining a better understanding of the relationship that exists between assessments and beliefs in mathematics. These results will aid mathematics educators as they choose the assessments they will use in courses designed to challenge pre-service elementary teachers' counterproductive mathematical beliefs. The methods used in this study to obtain data on this relationship are defined in the next chapter.

CHAPTER III

METHODOLOGY

During the1998 Fall semester, thirty-two students attending a large mid-western research university were offered the opportunity to participate in an investigative study into the relationships between mathematical experiences and beliefs. The students enrolled in a single section of an undergraduate mathematics course for pre-service elementary teachers without prior knowledge of the research. Seven students agreed to participate as informants for the research, which meant being interviewed on five separate occasions and sharing copies of all of their graded coursework. Their testimony would represent the majority of the data used in this research.

Figure 1 provides a visual representation of the context and methodology of the study. The figure also represents the framework of this chapter: a detailed account of the research setting, a description of the interventions employed in the course section under study, information regarding the study sample, and an explanation of the methods used in the gathering and analysis of data. Many of the decisions made when devising this research design were the result of findings from an earlier pilot study. Therefore, in order to put this design in perspective, the chapter begins with a brief examination of the relevant details from the pilot study.

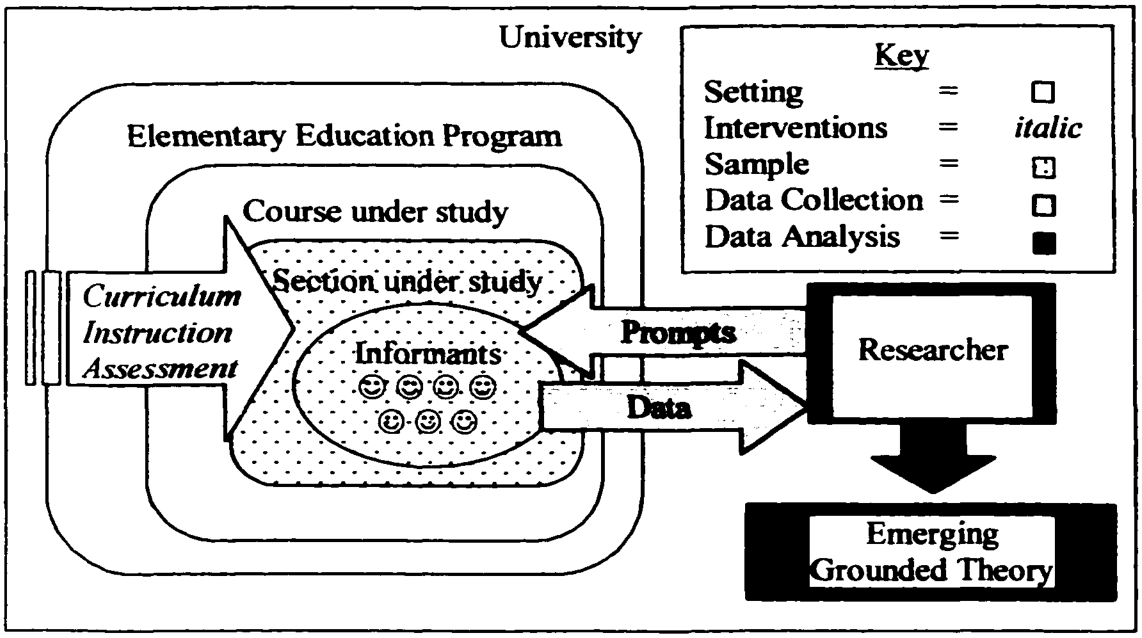

Figure 1. Context and Methodology of the Study.

Pilot Study

The pilot study began in the Fall 1997 semester, lasted a year, and was separated into four phases. Each phase built upon results from antecedent phases in an effort to determine appropriate methods for collecting data on the relationships between pre-service teachers' mathematical beliefs and assessment experiences. Only the pertinent details are presented here.

During the first phase, I observed a *Probability and Statistics* section led by the instructor teaching the class for the final study. Besides determining that the instructor's methods and assessments reflected the vision of the NCTM, this pilot also provided an opportunity to begin designing appropriate interview protocols and pilot a quantitative belief instrument. Background information collected during interviews

conducted with four students from this section provided the basis for a vignette activity and two card sorts used in the final study. Regarding the belief instrument, the original research plan was to administer a pre-existing, quantitative belief inventory (see Kloosterman & Stage, 1992) at the beginning and the end of the course to ascertain if there were any changes in students' mathematical beliefs. Unfortunately, this survey limited students to reacting to prewritten beliefs without being able to explain their position. The personal nature of beliefs required an instrument that allowed for varied responses and insight into students' rationale if the results were to be useful in addressing the research questions.

The second and third phases examined the effectiveness of the interview protocols at identifying informants' mathematical beliefs and determining their reactions to alternative mathematics assessments. These were conducted during the Winter and Spring semesters of 1998. Based on the results, it appeared that students' perception of ability and effort in mathematics might relate to their beliefs about the subject, which ultimately influenced the selection of students for the final study.

The pilot study's final phase took place during the 1998 Summer semester and involved testing a belief survey used by Cooney, Shealy, and Arvold (1998) and Gober (1997). Pajares (1992) suggests using metaphors and/or similes to aid in understanding pre-service teachers' beliefs and a portion of this survey included selecting the best and worst similes for teaching and learning mathematics and supporting each choice. I adapted the simile section to my research by adding metaphors for doing mathematics. When this modified version of the survey was

piloted on a group of elementary and secondary pre-service teachers and their mathematics instructors the results were promising. A few of the similes and metaphors proved too obvious, however, consistently resulting in rationale lacking depth. I replaced these with more neutral similes/metaphors suggested by members of the pilot group. With the survey completed, I was now prepared to begin researching the relationships between beliefs and experiences in mathematics.

The Setting

Geertz (1973) refers to contextual information as the 'thick description' of the study inasmuch as a research report must include a significant amount of background information if it is to furnish an accurate backdrop on which to examine the results. In support of this thick description, Lincoln and Guba (1985) wrote:

> [I]t is entirely reasonable to expect [a researcher] to provide sufficient information about the context in which an inquiry is carried out so that anyone else interested in transferability has a base of information appropriate to the judgement. (pp. 124-125)

The description provided in this section is separated into three increasingly smaller groups representing the study's setting – the university, the elementary education program, and a specific mathematics course for elementary education majors.

The University

The study was conducted at a public, four-year university located in a large mid-western city during the 1998 Fall semester. The university served approximately 27,000 students at the time of the study. Women represented 55% of the student

populace, and 11% of the 27,000 students belonged to a minority race. Roughly twenty-one thousand students, or 78%, were enrolled as undergraduates. These percentages have remained consistent over recent years.

The Elementary Education Program

During the 1998 Fall semester, slightly under 2,000 undergraduate students were identified as elementary education majors. As Table 1 demonstrates, this population was roughly homogeneous. The typical pre-service elementary teacher was white and female.

Table 1

1998 Elementary Education Majors Data

Total Students	Percent of Total
Men	17.4%
Women	82.6%
Minorities	7.5%

The university's elementary education program differs from many comparable programs in that, with a few exceptions, all pre-service elementary teachers are required to take four specific mathematics courses designed for elementary education majors to earn their degree. The first three courses stress mathematical content and each must be successfully completed with a grade of C or better before the fourth

class, *Methods in Elementary School Mathematics*, can be taken. The first course, *Number Concepts*, is a prerequisite for the other two content courses, *Geometry* and *Probability and Statistics*, which may be taken in either order. Equivalents to the *Number Concepts* and *Geometry* courses are offered at a local community college, though no such credit is available for the *Probability and Statistics* course. Therefore, the vast majority of all elementary education majors who graduate from the university must pass through this particular mathematics content course. Herein lies one of the reasons that the *Probability and Statistics* course was selected to be at the center of this study.

The Focus Course

Probability and Statistics is a four-credit course that usually meets for an hour and forty minutes twice a week. Six sections of the course are typically offered in the Fall semester in an attempt to accommodate the large number of students. In the fall of the study, 203 students completed this course, resulting in class sizes ranging from 32 to 36 students. (see Table 2 for complete statistics.)

Each section uses a text developed by mathematics educators from the university specifically for the course. The book contains information, activities, and exercises that attempt to reflect mathematics as problem solving, reasoning, communication, and connections – as envisioned in the *Curriculum and Evaluation Standards* (NCTM, 1989). Although some instructors use supplemental materials from outside resources – *Teaching Children Mathematics*, *Mathematics in the Middle*

Grades, *Mathematics Teacher*, the NCTM *Addenda* series, *Qualitative Literacy*, and the *Middle Grades Mathematics Project* – the required text provides a complete curricular framework for the course. It is separated into seven chapters with the first four addressing the study of descriptive statistics and the last three focusing on experimental and theoretical probability. From an instructional standpoint, the textbook incorporates the use of technology as it applies to probability and statistics. The text provides technical support and activities that employ the calculator in the exploration of concepts behind statistical functions, how data is represented in different graphs, and programs that examine simulations.

Table 2

Probability and Statistics Fall 1998 Enrollment Data

Section	Enrollment	Male	Female	White	Minority
A	36	6	30	31	5
B	35	5	30	31	4
C	34	6	28	31	3
D	33	6	27	27	6
E	33	6	27	29	4
Study	32	9	23	27	5
Total (%)	*203 (100)*	*38 (19)*	*165 (81)*	*176 (87)*	*27 (13)*

Another instructional technique shared by each section of the focus course is the participation of individuals in small groups. Students are customarily separated into groups of three or four in which they are expected to communicate their methods, solutions, and comprehension of the mathematics in the assignments and activities. Several sections attempt to incorporate this idea of cooperative learning into selected assessments as well. While this alternative assessment method is not common to all six sections, it was a portion of the intervention employed in the section under study [hereafter known as the study section].

The Intervention

The study section's attempts to intervene in the elementary education students' mathematical view are separated into three parts: Class Routine, Instructor, and Assessments. Class Routine begins by preparing a generalized picture of a typical day in the study section. Next, Instructor profiles the study section's teacher. Finally, Assessments describes the alternative assessments used in the study section.

Class Routine

The study section incorporated several significant pedagogical shifts, moving from traditional mathematics teaching toward alignment with the vision of the *Professional Standards for Teaching Mathematics* (NCTM, 1991). According to this document, good mathematics instruction requires the implementation of worthwhile mathematical tasks and increased involvement of students in classroom discourse.

Although a typical class period followed the traditional agenda of reviewing previously assigned work, introducing a new topic, and practicing this topic (Raymond & Santos, 1995; Schoenfeld, 1992), the similarities ended there. The class usually began with each group discussing amongst themselves problems that arose from the previous day's assignment. Next, a whole class discussion, led by one of the students, addressed any problems that remained unresolved. Students either volunteered or were selected to 'become the teacher' while the instructor took the student's place in his or her group. The 'teacher' was encouraged not to simply show how to get the answer, but to engage the entire class in determining any appropriate processes that would result in reasonable solutions. Once the problems from the previous assignment had been addressed, the new topic was introduced, usually through a small group exploration of mathematical ideas connected to some previous learning. Finally, time permitting, students worked collaboratively on a new set of problems. The instructor remained available to answer questions throughout the entire process, but regularly she would take any questions and open them up to the entire class. In this way, the teacher did not represent the study section's sole mathematical authority.

Instructor

The alignment of curriculum, instruction, and assessment in the mathematics classroom is a central goal of the NCTM (1989, 1991, 1995) *Standards* documents. In a constructivist classroom, the teacher plays an important role in this alignment,

especially in the creation of an environment where assessment and instruction are thoroughly integrated (L. Wilson, 1994b). Since this study was investigating the effect combining alternative forms of assessment with reform-based curriculum and instruction might have on beliefs, it was imperative to observe an instructor who was willing to embrace just such an atmosphere.

The study section's instructor, a professor of mathematics education at the university, met this requirement. First, she was a co-author of the course text and comfortable in its use. Second, as the description of the class routine suggests, she took a non-traditional, student-centered approach to instruction. The final piece in this curriculum-instruction-assessment alignment was her experience and enthusiasm in using alternative assessment methods. She taught the university's graduate course on mathematics assessment, where her knowledge and interest in alternative forms of assessment was evident.

As a result of the instructor, the study section was strikingly similar in structure to the courses described in the literature review that were found to be somewhat successful in challenging pre-service teachers' absolutist view of mathematics. In each course, students engaged in problem-solving activities in cooperative groups. They engaged in reflective writing and delved into the current research on students' mathematical thinking. What distinguished the study section from these previously researched courses was the instructor's commitment to not only align instruction and curriculum with reform views, but assessments as well.

Assessments

Students' final grades were based on their aggregate score on quizzes, a comprehensive final exam, class participation, and three alternative assessment tasks. Initially, quizzes were simply unannounced homework quizzes meant to hold the students accountable for daily assignments. Later in the semester, the quizzes introduced the students to the type of problems found on the final exam so they would have some experience with the instructor's testing style. The final exam, worth approximately 30% of the total possible points available over the semester, was a comprehensive paper-and-pencil test consisting primarily of problems requiring students to demonstrate their understanding of probability and statistics. Students took the final exam in a traditional testing environment as each student worked on the test individually for a fixed period of time. Class participation scores depended on attendance, preparation, involvement in class activities and discussions, and students' willingness to lead a problem discussion. A written journal, collected on a weekly basis but not graded, also figured into the participation score. Journal entries were intended as a non-threatening method of communication between students and the instructor through which students would respond to some prompt or share their thoughts about the class activities, environment, or alternative assessments.

Although each alternative assessment addressed different topics within probability and statistics (see Figure 2), they were all similar in structure. All were authentic, which means the assessments engaged students in activities that teachers and researchers actually perform in their work. Students were also encouraged to

Assessment Task	Chapters	Topics Assessed
<u>Assessment Writing</u> Pairs of students will design assessment questions that could be used to evaluate their peer's understanding of the topics presented in class.	1	Interpreting Graphical Displays of Data* Interpreting Tabular Displays of Data* Real Graphs and Picture Graphs Bar Charts or Graphs Circle Graphs* Line Charts or Graphs Selecting Appropriate Graphical Displays
	2	Organizing Data Stem-and-Leaf Plots Frequency Tables and Histograms* Scatter Plots
<u>Evaluating Responses</u> In pairs, students will create a key for a pre-existing course exam. Individually, students will use their "pair" key to evaluate the answers from an exam completed by an imaginary student.	3	The Concept of Mean* The Mean of a Frequency Distribution The Median of a Data Set* Selecting a Measure of Central Tendency*
	4	Range and Interquartile Range* Box Plots* Mean Deviation Standard Deviation* Applications of Standard Deviation Normal Distributions
<u>Monte Carlo Task</u> Students will each receive a problem that can be solved using a simulation model. Each student team will design a model, carry out a simulation, and analyze the results.	1 – 4 Review	Designated by * above
	5	Likelihood of Events Randomness Probability Experiments Two Kinds of Probability Geometric Probability
	6	Problem Solving with Simulation Using a Random Number Table The Monte Carlo Procedure

Figure 2. The Study Section's Alternative Assessments.

undertake each of the three tasks in pairs in an attempt to align the assessment structure with the cooperative nature of the class, although some students did choose

to work individually. Finally, each assessment required students to self-assess their work, thereby reflecting on the activity and the quality of their product.

The first alternative assessment task, *Assessment Writing*, required students to write test questions that would evaluate their peers' understanding of the course topics presented in Figure 2. (Appendix A includes the actual description of each task and the accompanying rubric). Student pairs were given nearly a month to work on the task including portions of two class periods. While pairs' assessment items were not actually administered to their peers, the items and accompanying solutions did provide evidence of the students' depth of understanding of the material.

Evaluating Responses, the second assessment, began by directing new student pairs to create an answer key for a pre-existing *Probability and Statistics* exam covering topics in descriptive statistics (see Figure 2). Again, the students worked on their key during two class periods, but completed the majority of the task outside of class. Having developed an exam key, students worked individually during one class period on the evaluation of a fictional student's exam answers (see Appendix A). These answers represented typical student responses, right and wrong, found on past exams covering the same material. Not only did the student evaluators have to determine the correctness of each answer they also had to identify the thinking that might have resulted in a mistake and how a mistake ought to affect an answer's score. To accomplish the scoring, students relied on the same rubric that the instructor used to evaluate the students' keys. Thus, *Evaluating Responses* allowed the instructor an opportunity to assess students' procedural understanding through the evaluation of the

key and students' conceptual understanding via their explanations of how they evaluated the fictional student's exam.

The third alternative assessment was a *Monte Carlo Task* based on several simulation problems found in *The Art and Techniques of Simulation* (Gnanadesikan, Schaffer, & Swift, 1987). As seen in Figure 2, this assessment evaluated probability concepts along with previously assessed topics in statistics. Each student pair was assigned one of seven situations that they were to solve by developing and implementing a simulation model (see Appendix A for the situations). Using a programmable calculator, students designed a program that would accurately represent the model, carry out at least one hundred trials, and store the results of each trial in a data list. Having carried out the simulation, students created a report describing their solution plan, the results from carrying out their plan, and their interpretation of the descriptive statistics. Within this report, they also visually displayed their findings using graphical representations. Part of a class period was set aside for the pairs to work on this assessment, but the majority of the project was completed outside the class.

On any two of the three assessment tasks, students could correct their mistakes, thereby improving their understanding as well as their score. Although a small penalty, amounting to between 5% and 10% of the assessment grade, accompanied these second chance attempts, many students took advantage of the opportunity to resubmit their work or meet with the instructor to discuss their mistakes. Not only did all of these assessments emphasize that mathematics is

essentially a group endeavor, connected to real-life, and involves self-reflection, they also modeled that mathematics is a process not simply a product.

The alignment between curriculum, instruction, and assessment within the study section was complete. As a result, the pre-service teachers enrolled in this section would encounter mathematics from a fallibilist perspective with a consistency lacking in most earlier studies. The elementary education majors exposed to this combination of interventions represented the study's sample.

The Sample

The students enrolled in the study section, like those in the university's elementary education program as a whole, turned out to be a fairly homogeneous group in terms of gender and race (see Table 3). Inclusion of this information is not intended to suggest that the study section was representative of the university's population of elementary education majors, but to provide information that gives a complete picture of the sample from the study. Out of these thirty-two students, seven agreed to act as informants in this research.

My highest priority when selecting informants was that they would offer me the best opportunity to learn the most about the research questions (Patton, 1990; Stake, 1995). On the first day of class, I shared with the students in the study section the goals of this research in an effort to generate enthusiasm for their participation as informants (see Appendix C for the Oral Script). As an added incentive, I offered to tutor students for their *Probability and Statistics* final if they agreed to take part in the

study. The goal of this convenience sample was to recruit at least twelve students who were willing to discuss candidly their views regarding the relationships between assessments and beliefs in mathematics. From this pool of informants, a more purposeful sample could be selected.

Table 3

Study Section Enrollment Data

Total Students	32	Percent of Class	Percent of University Elementary Education Majors
Men	9	28.1%	17.4%
Women	23	71.9%	82.6%
Minorities	5	15.6%	7.5%

A further consideration in this study were results from the research (Oaks, 1987) suggesting that students' perception of ability and effort in mathematics are related to their beliefs about the subject. For this study, students identified themselves as fitting one of the following classifications: (a) students who did well in mathematics and felt it came easy to them [Type I], (b) students who did well in mathematics but really had to work at understanding it [Type II], and (c) students who had a hard time understanding mathematics and typically did poorly in the subject [Type III]. Consequently, it was decided that the purposeful sample should include six informants – two of each self-perception types.

Initially, only six students (all females) expressed interest in taking part in the study. Fortunately, this group of possible informants included each of the three types: one Type I, three Type IIs, and two Type IIIs. Having a single Type I informant involved in the study was a concern, however. Therefore, in an attempt to recruit another informant for this classification, I contacted students who identified themselves as someone who found mathematics easy to understand and earned good grades. Although many of these were males, they expressed disinterest or maintained that they did not have the time. Another female student who fit into this classification agreed to be an informant though, providing the study with at least two representatives from each type.

Further information on each informant is presented in Chapter IV. These detailed descriptions of each informant ought to assist the reader in linking informant characteristics with the research findings. The methods used in the collection of the data for this research are provided in the following section.

Data Collection

An important characteristic of research in mathematical beliefs is the use of a variety of concrete practices when gathering data (Borasi, 1990; Pajares, 1992). For this study, open-ended surveys, in-depth informant interviews, and artifacts from the study section (including journals) were all tools employed to uncover students' mathematical beliefs and related experiences. These instruments (surveys, interviews, and artifacts) are the focus of this section on data collection.

Surveys

Surveys provide an efficient method for obtaining large amounts of data from respondents (Krathwohl, 1993). This study employed two versions of the modified belief survey described earlier in the section on the pilot study. Students enrolled in the study section completed the initial survey on their first day in class and the exit survey during the final week of that same semester. These surveys were similar in that both asked students to select similes reflecting their ideal vision of mathematics teachers, students, and what it means to do mathematics. The most meaningful data, however, resulted as students justified their choices. Analysis of these explanations provided insight into the students' mathematical beliefs. Where the two surveys differed was in the remaining information each collected from the students.

The initial survey (see Appendix C) sought out data regarding students' high school and college mathematics course histories, where they attended high school and their year of graduation (used to help match students' initial and final surveys since the final surveys were completed anonymously), and how they viewed themselves in terms of their mathematical ability. Background questions required students to write the appropriate information on the spaces provided, but the self-perception portion was somewhat more complicated. Students reported their perception of their ability in mathematics on a two-dimensional coordinate system (see Figure 3).

Students placed a symbol (dot, asterisk, stick-person, etc.) on the coordinate system that corresponded to where they believed they were in regards to mathematical effort and grades. The labels for these self-perception types (described

in the previous section) reflected the corresponding quadrant in the coordinate system shown in Figure 3. Thus, the upper-right quadrant symbolized Type I students, the upper-left quadrant designated Type II students, and the lower-left quadrant stood for Type III students. It was not expected that any students would place themselves in the lower-right quadrant [Type IV], students who found mathematics easy to understand but typically earned poor grades, nor did any students in the sample do so. Any students who placed themselves directly on one axis or the other would be addressed on a case-by-case basis. The background data and mathematical self-perceptions from this initial survey communicated the various course histories and attitudes of possible interview informants.

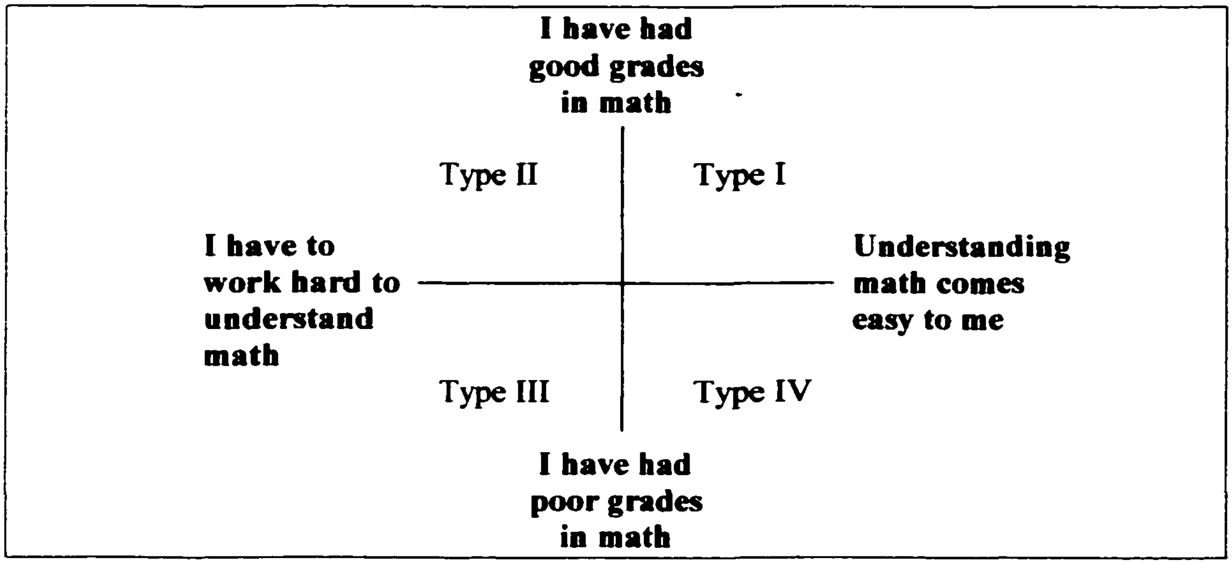

Figure 3. Self-perception in Mathematics Graph From Initial Survey.

In order to gather evidence of any changes to the pre-service elementary teachers' beliefs over the fifteen week semester and possible reasons behind the

changes, all of the study section students completed a second belief survey at the end of the semester (see Appendix C). On the exit survey the portion asking for the students' course history in mathematics was replaced by a pair of questions that attempted to ascertain what recent experiences the students felt might have shaped their current view of mathematics. Students selected from a list of experiences, both inside and outside the *Probability and Statistics* course, and were encouraged to add any influential experiences that they felt were missing. Once these experiences had been identified, students ordered from most to least significant the top six influential experiences. This information aided in determining students' perception of the impact alternative assessments had on their current mathematical beliefs.

Responses to the prompts from these belief surveys provided a starting point in the data collection process, but would not be sufficient in addressing the research questions. Consequently, it was necessary to conduct interviews with informants who could aid in the interpretation of this data and contribute further insights into the relationships between mathematical beliefs and experiences. This interview process is described next.

Interviews

Once an informant agreed to take part in the study, an interview schedule was developed. Each informant participated in five interviews lasting from forty-five to ninety minutes depending on the amount of information addressed. Spread throughout the semester, the interviews coincided with the study section's alternative

assessment schedule (see Figure 4). I conducted the interviews in my office at the university, with the exception of one set conducted at an informant's place of employment for her convenience. A structured plan developed for each interview ensured that all of the issues relevant to the research questions would be investigated (see Appendix C for each interview's protocol). These plans consisted of open-ended prompts meant to elicit detailed responses, while focusing the informants' attention on the objectives of the study.

Week of Course	Assessment	Interviews
1st		
2nd		Interview One
3rd		
4th		Interview Two
5th	Assessment Writing	
6th		Interview Three
7th		
8th	Evaluating Responses	
9th		Interview Four
10th		
11th		
12th		
13th		
14th	Monte Carlo Task	
15th	Final Exam	Interview Five

Figure 4. Assessment and Interview Schedule for Study Section (Fall 1998).

Interview One

The first round of interviews took place during the second and third weeks of the semester. The two main goals of this first interview were to investigate the informants' past experiences in mathematics classes, including assessment

experiences, and to begin examining their beliefs about mathematics upon entering the *Probability and Statistics* course. Data related to these objectives were gathered as the informants responded to direct questioning and engaged in three structured activities (see the Interview One Protocol in Appendix C).

The first activity required informants to read two vignettes describing a typical day in two very different mathematics classes (see Appendix C). One description provided a "traditional" view based on the research literature (Romberg, 1992) and findings from the pilot study. The other description was representative of the reform-mathematics classroom discussed in the NCTM (1989, 1991, 1995) *Standards* documents. As the informants read these vignettes they highlighted the portions that resembled their own experiences in yellow and those most foreign to their experiences in blue (color selection was arbitrary). Upon completion of this task, informants discussed any experiences that were missing, thereby presenting a comprehensive picture of each informant's experiences in mathematics.

The next activity focused on assessments in the mathematics classroom and involved sorting randomly arranged cards listing various assessment activities reported in previous research (the assessment list is provided in Appendix C). Informants separated the assessments into three piles labeled 'a lot', 'sometimes', and 'never' based on the frequency with which they had encountered each assessment during their mathematics experiences. Upon completing this sort, informants commented on the assessments in each set and discussed any assessment experiences missing from the 'sometimes' or 'a lot' piles. Hence, the resulting data represented

the entirety of assessment experiences that the informants recalled occurring during previous mathematics courses.

The final activity also required the students to sort through pre-shuffled cards, these containing various mathematical beliefs related to perseverance, confidence, and predictability (see the belief statements in Appendix C). Some of the phrasing of the belief statements came directly from students' rationales on the initial survey, while others related to prior research. Informants divided these statements into those they could agree with and those that they could not. Again, once the sort was finished the informant explained her reasoning behind the placement of each belief statement card, thereby clarifying her perspective concerning the nature of mathematics.

In responding to the first interview's prompts and activities, the informants described their experiences in mathematics and their view of the subject upon entering the *Probability and Statistics* course. These results provided information used as baseline data throughout this study, including the next interview when informants attempted to make connections between their beliefs and experiences.

Interview Two

After reviewing the transcripts from the first interviews, I adapted the plans for the second set of interviews to reflect the data already collected (see the Interview Two Protocol in Appendix C). The second interviews took place during the fourth week of the semester in an effort to complete them before the students encountered their first alternative assessment in the study section. This interview, and all the

subsequent interviews, began by asking each informant to clarify statements made during earlier interviews to ensure an accurate understanding of the data. The remainder of interview two examined informants' strongest beliefs about the nature of mathematics and the experiences that they perceived as relating to these beliefs.

First, informants returned to a subset of the belief cards they sorted during the first interview and chose the four they felt most strongly about either positively or negatively. (In an attempt to avoid the selection of seemingly obvious statements, I removed three belief cards that had been sorted identically by each informant – see the belief statements in Appendix C.) Once the informants had selected four of the remaining belief statements, they rewrote each so that they could strongly agree with all of them. During this exercise, they also rewrote a statement regarding their beliefs about their ability in mathematics – "I consider myself as {poor/average/good} in mathematics."

Upon the completion of this task, they learned of McEntire and Kitchen's (1984) theory that experiences lead to beliefs, which affect actions. They then shared how long they had held each of the five rewritten beliefs and tried to pinpoint possible experiences that might have contributed to the development of these beliefs. Most of the informants found it difficult to think of more than one experience for each belief so they took the rewritten belief cards with them and thought about other related experiences for the next interview.

Interview Three

The third interview was conducted after the class's first alternative assessment, during the sixth and seventh weeks of the semester. Besides examining any belief-related experiences they had thought of since the last interview, this interview focused on past and present assessment experiences in mathematics. Informants talked about the *Assessment Writing* task (the first alternative assessment) and compared it to past mathematics tests. Because tests figured so prominently in the informants' previous mathematics classes (a finding from the first interview), it was important to gain a clearer picture of how the informants viewed these past assessment experiences. The interview concluded with the assignment of another belief instrument that asked them to place Xs along several continuums that would represent what they believed to be true about mathematics and learning mathematics (see Raymond, 1993). This instrument aided in the examination of relationships between assessments and beliefs in the fourth interview.

Interview Four

After completing the second assessment task, *Evaluating Responses*, informants returned to report on their responses to Raymond's belief instrument; this fourth interview was scheduled for the ninth and tenth weeks of the semester. They also spent time comparing the study section's first two alternative assessments to each other and to the paper-and-pencil tests that they had taken in other mathematics courses. The central focus of the interview, however, involved an activity related to

McEntire and Kitchens' (1984) theory on the relationship between experiences, beliefs and actions.

In this activity, informants read a vignette describing a student preparing for and taking a mathematics test (see the Mathematics Test Vignette in Appendix C) and highlighted in yellow experiences that sounded familiar and blue those that were foreign. The scenario reflected details from earlier interviews and served two purposes: (1) confirmation that the informants' experiences taking a test in mathematics were clearly understood and (2) providing a context for the experience-belief-action activity. Next, the informants discussed how the teacher in the dramatization might view mathematics [beliefs] based on the characteristics of the teacher's test [action]. Finally, they went through the same process, but this time from the perspective of a student taking the test. In this case, the test represented an experience and the informants hypothesized about the beliefs a student might have developed in this situation.

Using these theoretical relationships as a basis, the informants went on to explored the role these types of testing experiences might have played in the development of the beliefs they had rewritten during their second interview. Consequently, this exploration resulted in a list of the possible causal relationships between assessments and beliefs in mathematics as seen by the informants. During a portion of the fifth interview, this list was used to determine if any consensus existed among the informants regarding such relationships.

Interview Five

All of the informants took the *Probability and Statistics* final exam before this last interview, except for one who scheduled her interview earlier on the same day as the final. Therefore, the informants had experienced all the study section's instructional sessions and alternative assessments prior to this fifth meeting. As in the previous interviews, the informants spent time describing their perceptions of recent experiences in the course, concentrating on those related to assessment. The main purpose for interview five, however, was to examine certain themes that emerged during previous interviews and determine to what extent the alternative assessments used in the study section affected students' mathematical beliefs.

The interview began by addressing the informant's beliefs regarding mathematics assessment in general. Each informant shared what she believed it meant to assess someone's ability in mathematics, how this view had changed over the semester, and how the alternative assessments related to any change. It was during this exchange that the informant discussed the study section's assessments in more detail, especially the last two – the *Monte Carlo Task* and the final exam.

Next, each informant documented any changes in her view of mathematics since the semester started and the factors contributing to these alterations. This involved three activities: (1) comparing rationale from the initial and exit belief surveys, (2) another sorting of the belief cards in order to identify any shift in the informant's beliefs, and (3) a review of the rewritten belief statements to ascertain if the informant still strongly agreed with every statement. In each activity, if there

were any differences between the informant's initial and exit beliefs, then she discussed her new beliefs and explained what experiences from the semester prompted such changes.

The final portion of the interview attempted to examine the validity of certain hypotheses suggested in the research literature and resulting from the informants' responses in earlier interviews. A list of nineteen statements regarding the relationships between assessments and beliefs in mathematics was developed (see the Relationship Confirmations in Appendix C). Informants chose to 'strongly disagree', 'disagree', 'agree', or 'strongly agree' with each statement based on the following criteria. 'Strongly disagree' meant, "I have never held that belief. Therefore, I cannot really answer this one because I have never felt that way." 'Disagree' implied, "Although I have held that belief, it arose from an experience other than the one suggested here." 'Agree' suggested, "I have held that belief and the experience makes sense but it is not the first one to come to mind." Finally, 'strongly agree' represented, "Yes, that connection between beliefs and experiences reflects my reality." Lincoln and Guba (1985) refer to this method as 'member checking' and state that it "is the most crucial technique for establishing credibility" (p. 314). Upon completion of this task, the informants explained the rationale for their position on each summative statement. The results allowed me to evaluate how accurately my interpretations of previous interview data reflected the perceptions of my informants, which represented the culminating piece of the interviewing process.

Final Comments on Interviews

In these five interviews, the informants provided the bulk of the information used to address this study's research questions (see Figure 5). The informants' own words would paint the picture depicting how they perceived the relationships between their mathematics assessment experiences and their mathematical beliefs. Interviews were not the sole data source, however. As was mentioned earlier, surveys contributed a great deal of significant information and course artifacts, which will be discussed next, helped to complete the picture.

Artifacts

Artifacts represent non-interactive data sources that aid researchers in their attempts to make sense of a particular situation (Lincoln & Guba, 1985). In this study, the artifacts were documents collected from two different sources – the instructor and the informants.

The instructor contributed the *Probability and Statistics* textbook, copies of course handouts, information cards completed by the students, and a copy of the final exam. A description of the text is provided in an earlier section of this chapter (see The Focus Course section). Course handouts included descriptions of the tasks and the scoring rubrics (see Appendix A), daily agendas, and supplemental materials not covered in the text. The information cards, which students filled out the first day of class, contained personal information (phone number, address, and something unique about the person), a list of mathematics courses taken by the student and subsequent

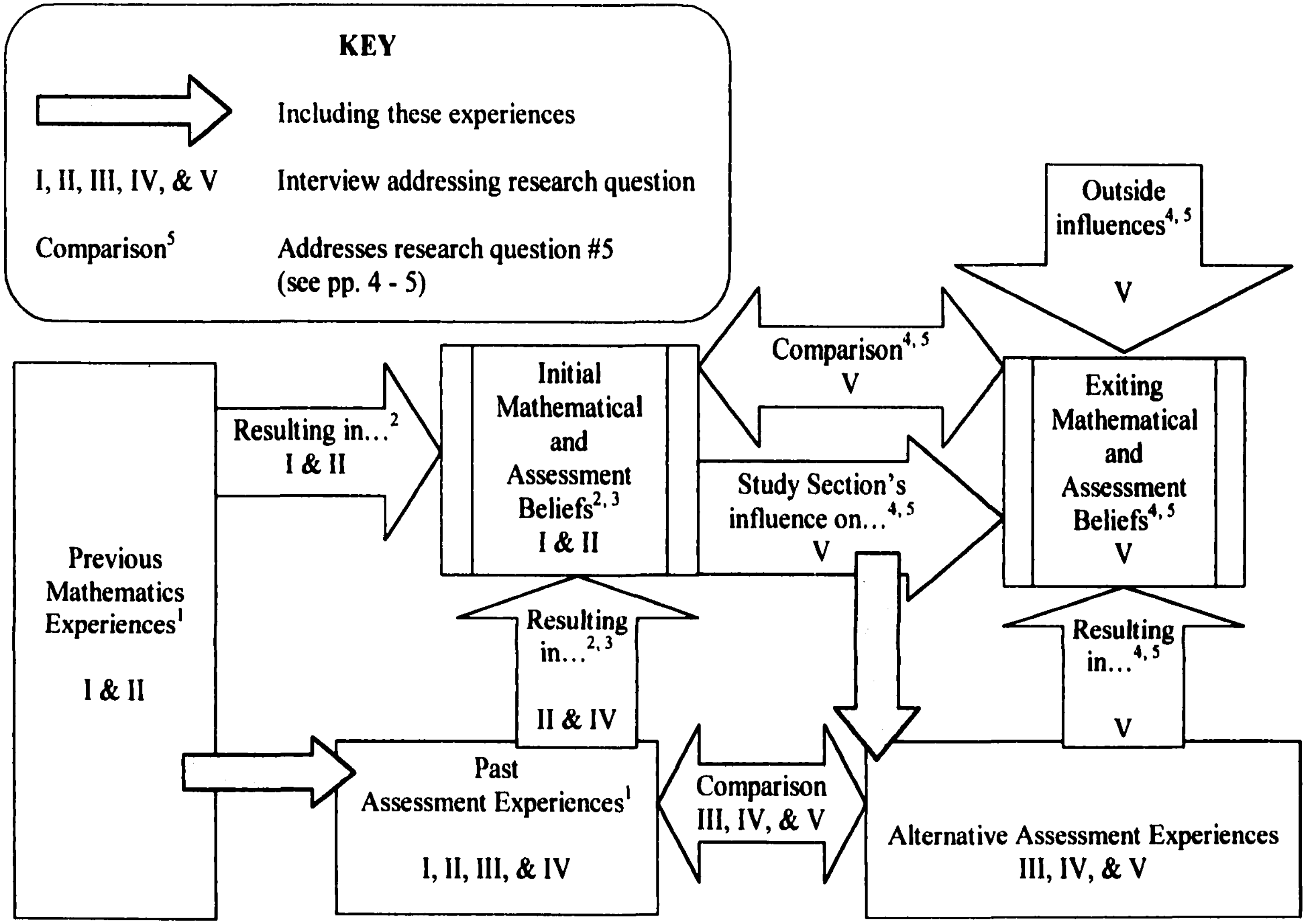

Figure 5. How Interviews Addressed the Study's Research Questions.

grades, and the student's attitude toward mathematics (drawn as a self-portrait). The final grade sheet was a printout from a spreadsheet program that showed the individual scores for each class assessment. While these documents did not contribute directly to the results of this research, they did aid in developing a backdrop for the data collected during the study.

Each of the informants provided copies of their completed assessment tasks (*Assessment Writing*, *Evaluating Responses*, and the *Monte Carlo Task*) and their course journal. Samples from the *Assessment Writing* project, in particular, assisted in identifying what the informants thought a mathematics test ought to look like and what they believed it meant to do mathematics. Journals, a weekly requirement for the course, were comprised of students' responses to the instructor's prompts, which usually involved questions related to issues raised in class, recent assignments, and group interactions. On occasion, the prompts addressed students' mathematical beliefs or their thoughts about the study section's alternative assessments. In those cases, the journal entries represented information that could clarify and support data gathered through other methods.

In this way, the artifacts provided evidence that either filled in the gaps or triangulated with the data gathered from the surveys and interviews. The researcher sifted through this amassed data in order to present a clearer picture of what a pre-service elementary teacher thinks it means to do mathematics and how these beliefs are related to assessment practices. Steps taken in the analysis of this data are described in the following section.

Data Analysis

In this research, data were analyzed using the constant comparative method (Strauss, 1987) in an effort to uncover common themes related to the relationships between assessment and beliefs in mathematics. In this method, data converges to represent themes that emerge during the classification of important results found in the data. Once identified, possible themes are shared with the informants in order to confirm their validity. The entire analysis process, which took place in two separate stages, followed this technique.

The first stage involved the initial collection of data. Upon completing a set of interviews, the interview transcripts were examined in preparation for subsequent interviews; this involved searching for commonalties across informants and interviews and highlighting statements requiring clarity. Whenever an informant's comments required clarification, I attempted to use her words verbatim during the next interview as a method of soliciting what she understood her statement to mean. Here is an example of just such an exchange:

> Interviewer: [W]e were talking about the fact that you've changed your perspective. And I asked why and you said, "I think just because of the way we were being forced to learn it here." How are you being forced to learn it here?
>
> Informant: They're not giving us the answers or telling us how to do things straightforward. They're making us go through the problem solving and thinking, sort of come up with the correct answers and we have to argue with each other in class, not looking to the teacher for a right or wrong response. (C2, 1)[1]

[1] (C2, 1) refers to informant Cynthia's second interview, the 1st page of the transcript.

This process allowed for greater comprehension of the ideas informants were trying to express through their responses without me guessing at their meanings.

As informants responded to the study's prompts and activities, certain statements seemed to imply possible responses to the research questions. Thorough analysis of these statements led to speculative themes related to relationships between assessment experiences and beliefs in mathematics. (Specific attention was given the counterproductive beliefs concerning perseverance, confidence, and predictability in mathematics described in Chapter II.) In order to test these assumptions, they were shared with all the participants using an informant's direct quotes whenever possible and reasonable modifications otherwise. Students' rationales from the initial belief survey incorporated into the belief statements used for Interview One's second card sort provide a prime example. In an attempt to explain why 'inventor' was the worst simile for an ideal mathematics student, one informant wrote, "students should not be inventing their own way of doing things" (WMSS, 1)[2]. The subsequent belief card read, "Students should not invent their own way of doing things in mathematics." Thus, it was possible to determine whether any of the other informants held this same belief.

While some themes emerged quickly, so that the informants could attest to their validity almost immediately (in the current or succeeding interview), other themes required more time to uncover. For example, during the first set of interviews

[2] (WMSS, 1) refers to the 1st student's entry for worst mathematics student simile on the initial belief interview (see Appendix D).

several informants noted that the methods used in their mathematics courses for elementary education majors were quite different from those they experienced in earlier mathematics classrooms. In the next interview, an informant reported that she strongly believed that:

> At a young level, memorization is used for basic math skills – addition, subtraction. As a student grows, how they solve the problem should be more important than memorizing the answer. They should be able to explain the problem and how they did it. (A2, 4)

She identified recent assessment experiences as the catalyst behind this belief.

> Well, in [*Number Concepts*] you had to write down all your stuff and that was more important in the grading process than if you got the right answer. How you explained it and the steps you took, you had to write those all down. (A2, 6)

Data from subsequent interviews suggested that this movement from memorizing answers to understanding the process paralleled many of the informants' experiences with assessments in mathematics.

> [I]t was, "this is how you do it, these are the correct answers, 4 x 3 is 12, that's the answer. 4 x 4 is 16. Now memorize them and we're gonna have a test on them on Friday." (C4, 15-16)

> So I would say that probably would be the difference in elementary/middle school and college level was more or less you already have the knowledge. They just want you to apply it, but in elementary school, they have to teach it to you first before you can apply it to something. (Ke3, 5)

> Usually in high school, it was more computation but now I think I'm learning more to write... (Lu3, 8)

Consequently, it appeared that some informants believed that students need to first memorize facts in mathematics and then later understand how the facts work because earlier timed tests required memorization while recent tests stressed understanding. During the final interview, informants confirmed or refuted this theme and several

other assumed connections developed over the course of the study. Based on their reactions to these conjectures, the second stage of the data analysis began.

While the analysis of the interview transcripts was an on-going process, an extensive analysis of all the data took place following the completion of the study. Because the final interview attempted to bring closure to the research, much of the data collected during these concluding conversations represented the most important information gathered over the semester. Therefore, informants' closing comments, including their confirmation of, or opposition to, the developed themes were scrutinized using all available data sources. For example, one informant disagreed with the statement: "Grades are very important in math class. Therefore, my assessment experiences have been some of the most influential in the creation of my beliefs about mathematics." An intensive examination of her journal entries and previous interview transcripts aided in determining whether this position was consistent throughout the study. Following this procedure for each informant allowed for an evaluation of the dependability of an informant's responses.

Having completed a thorough investigation into the validity of each theme, I conducted an exhaustive search of all the collected data for any possible themes overlooked in the initial phase of the analysis. Should new conjectures arise, informants granted me permission to contact them in order to check the validity of these new themes; this proved unnecessary however, because no new themes emerged.

Concluding Remarks on the Study's Methodology

Uncovering and identifying the mathematical beliefs of pre-service elementary teachers is by no means easy (Pajares, 1992). It requires detailed research plans that incorporate a deep understanding of how beliefs develop and manifest themselves. Plans that ignore the complex nature of an individual's belief system achieve limited success in obtaining worthwhile results. Due to extensive research on beliefs and effective methods used in revealing them, the methodology reported here provided significant data on how the pre-service teachers participating in this study viewed the relationships existing between mathematical beliefs and assessment experiences. These results are presented in the next chapter.

CHAPTER IV

ANALYSIS OF DATA

In this chapter, data collected using the methods described in Chapter III are presented and analyzed. Before addressing the results directly related to the focus of the study, however, I introduce the informants who supplied the bulk of the study's data. The remainder of the chapter examines the data in an attempt to explore the findings to the research questions found in Chapter I. Rewritten here so that they focus on the informants, this is the order in which the questions are discussed:

1. What were the informants' past experiences relative to learning and assessment in mathematics (question one)?

2. What were the informants' initial beliefs regarding what it means to assess a student's mathematical understanding (question three)?

3. What were the initial mathematical beliefs of the study section as a whole and how did the informants perceive the relationships between their experiences in mathematics and their own beliefs (question two)?

4. How and why did the mathematical beliefs of students in the study section change over the semester and what prompted any changes in the informants' beliefs in particular (question five)?

5. How did experiences with alternative assessments affect informants' perceptions about assessing mathematical understanding (question four)?

Answers to these questions arose as themes emerged during the data analysis.

The Informants

Seven students from the study section agreed to serve as informants for this investigation into the relationships between assessment experiences and mathematical beliefs. The data collected in this research provided insight into these relationships and about the informants themselves. Presented here, this information, along with my own impressions of each individual, forms a contextual basis for the data and subsequent analysis found throughout this chapter. The informants are introduced alphabetically, according to aliases of their own choosing, followed by my impressions of the group as a whole.

Anne

Anne, a 1995 graduate from a private high school in a large Midwestern city, comes from a family of teachers; a brother teaches English and her father is a mathematics teacher at a public high school. The father uses a reform-based textbook in his classroom and conducts demonstrations at other area high schools in an effort to convince them to use the textbook series as part of their curriculum. Yet, Anne did not view her father as having any impact on her view of mathematics. "I don't remember him talking about it, but I might have tuned it out" (A5, 14)[3]. When it came to identifying her own mathematics experiences, however, she admitted to

[3] (A5, 14) refers to Anne's fifth interview, the 14th page of the transcript.

getting confused because "I remember things he's had his kids do" (A1, 7). I attempted to remain cognizant of any influence her father's position might have on her responses throughout the five interviews.

Anne reported taking two mathematics courses in high school, algebra and geometry, and three collegiate mathematics courses before enrolling in the study section. The first college course stressed computational skills and Anne described it as "... basically a review. That was the easiest class I've ever taken in my life" (A1, 3). Next came an algebra course, which she also associated with the mathematics she had learned in high school, and finally the first mathematics course in her elementary education program, *Number Concepts*. She reported earning a B in this last course, although she was not completely sure.

On the initial belief survey, Anne identified herself as someone who had to work hard to get good grades [Type II]. She supported this in the first interview:

> To understand [mathematics] more I've had to work hard... My feelings on the whole going-to-school thing is their outcome, the grades. I've always had to work really hard to get, in my mind, good grades. And if I don't feel confident about something, then I won't get a good grade on my test, so I have to work hard enough to get confident. (A1, 1)

Regarding mathematics as a whole, Anne's position seemed fairly indifferent based on a picture she drew for the instructor showing her mathematical disposition. It showed an expressionless face, not happy or sad. She explained, "I never liked math, but ever since I came here I started liking it more and basically I'm a fair math student, I guess" (A1, 1).

Although my initial impression was that Anne was quite shy, she warmed to the interview process immediately and was both honest and reflective in her responses. I looked forward to our interviews because her forthright and introspective manner resulted in interesting, non-complicated data. Consequently, I often used Anne's answers as a means of checking the perspectives of the other informants. For example, many of the details in the assessment vignette came directly from her descriptions of past experiences with mathematics assessments.

Cynthia

Cynthia was the only informant that I actively recruited to join the study. When I contacted her, she confessed to being intrigued by the research, but hesitant to participate due to a particularly busy schedule. I explained that I needed another informant who, like herself, earned good grades in mathematics without working too hard [Type I]. She relented and agreed to try to set aside some times when we could meet. While I was concerned that she might not be able to commit to the entire study, without her help I might have had to conduct the research using a single Type I mathematics student.

The smiling face that Cynthia drew, with the word "HAPPY!" written next to it, succinctly describes her attitude toward mathematics. Partly, this has to do with the ease with which the subject seems to come to her:

> I enjoy math. I catch on pretty quick. If I do get confused, I always ask and it doesn't usually take too much to get me unconfused, you know? I seem to pick it up pretty easy, and I think that's why I like it is because I don't struggle with it like I do with history and English and stuff like that. (C1, 1)

When I asked how she prepared for mathematics tests, she provided further evidence to support that she was a Type I student:

> In fact, all through and until I hit college I very rarely did homework or did any studying of any kind. I just never really had to. Grades and stuff came pretty easy to me and I just never really had to study. (C3, 10)

Despite never studying, Cynthia experienced a great deal of success in mathematics. A 1994 graduate from a public high school in the Midwest, she completed four years of mathematics, including Advanced Placement Calculus. Even in her college courses, she reported earning either a BA or an A in *Number Concepts* and a BA in *Geometry*.

My fears that Cynthia would not follow through completely were unfounded as she took the study quite seriously. In fact, she proved to take everything seriously. While she had a very pleasant demeanor, her nature was very matter of fact and she typically wanted to get down to business. Her opinions were often presented quite forcefully and always accompanied by some supporting facts. As I expected from her high school mathematics background, she was intelligent and confident in her ability to do whatever mathematics came her way. She was a realist though, and recognized that there were circumstances, such as her busy schedule, that might make it necessary to accept less than a perfect score on certain assessments.

Janet

The only non-traditional student among the informants, Janet graduated from high school in 1977. She had attended a large Midwestern high school where she

recalled taking a pre-algebra and an algebra course. After high school, she "took a bunch of accounting classes in a technical college" (J1, 11) and kept the books for her family's small business. While this practical experience allowed her to see how mathematics applied to everyday life, it failed to ease the tension she felt toward the subject.

Janet's attitude towards mathematics, especially related to being assessed, can be summed up in one word – "panic" (J1, 4). Her attitude drawing reflected this negative view of mathematics as the face showed eyes looking down and a straight line across for a mouth, almost ambivalent. Given her less than positive disposition towards mathematics, Janet was chagrined to be in an elementary education program that listed mathematics as one of its minors:

> I just thought it was a big joke that there would be the possibility that someday I'd be taking math. That's just not something I've ever thought of myself as good at. (J1, 10)

Thus, Janet endeavored to get all of her mathematics courses out of the way as swiftly as possible by taking both *Geometry* and *Probability and Statistics* concurrently during the Fall 1998 semester.

Despite her anxiety, Janet reported earning good grades in mathematics, including an A in a course comparable to *Number Concepts* taken at a different college. When I asked for a description of herself as a mathematics student she responded, "One that works hard but things don't come automatically to" (J1, 1), which fits with her Type II identification on the initial belief survey. Further support included her explanation of why she did so much extra credit work in mathematics:

> I consider myself a hard worker in math, it's like I may not get them all right but if there's an opportunity to do something more, I'll do that in hopes that that will help me. (J1, 5)

I interpreted "help me" as meaning to help improve her grades, not her understanding of the subject.

As with Anne, my impression of Janet changed over the course of the study. Initially, many of Janet's responses were terse and flat, suggesting to me that she was somewhat reserved. When I probed more into the panic she felt towards mathematics, however, she began to open up both verbally and emotionally:

> I think when I first took it in high school; it was just no concept. Just did not get it. Never really clicked. So I took a pre-algebra in ninth grade. [Crying] I'm surprised I'm doing this... I struggled but I did okay. And then – but it scared me that I didn't know how to do it, and I guess I have a problem with failure. So I didn't like that I was looking at possibly not being able to do something. I'm kind of a goal-oriented person and liked to do things well. So I put off taking any more math for several years until I was actually a senior and then I took an Algebra 1 class and struggled with it quite a bit, so I guess [math] means failure. (J1, 10)

Throughout the remaining interviews Janet became more animated, although she continued to answer questions sparingly.

<u>Karen</u>

Karen reported graduating from a rural Midwestern high school in 1996 where she had taken a great deal of upper level mathematics including Advanced Placement Calculus. It was not surprising therefore, that she considered herself someone who gets good grades in mathematics and understands the material easily [Type I].

> I've always excelled in it. It's always been one of my best subjects. Well, up through probably sophomore year – so it was my best subject then. I loved it

> actually, my utmost favorite. I'm not doing too bad now. As it got harder, it was more work to put forth in it, but I've never really done terrible in it. It's come easy to me, real easy. (Ka1, 1)

She reflected this position by drawing a smiling face when depicting her disposition towards mathematics.

Yet, Karen acknowledged that, like many of the other informants, she suffered from test anxiety. "It does not fail. You put a test in front me and I go blank for the first couple minutes... but things will eventually start to click" (Ka1, 6). She explained that it happened in most subjects, but especially in mathematics.

> I would always get so stressed because of the fact that it was math. There was so much information compacted into one little test that I swore up and down I forgot half of what I learned, and I think that's part of the reason I would freeze. They'd give that test to me, I'd get my name and date on it, read the first question, and it'd be like, "Huh, where did that come from? I've never seen it before in my life." Then I'd have to kind of read through a couple of the questions, kind of answering the ones that were familiar to me. Then I'd have to go back through and by the time I got four or five questions answered, I was all right. I was like, "Okay, now I know what I'm doing." And go back into it. (Ka3, 15)

Unlike the other informants who suffered from test anxiety in mathematics, Karen had confidence that ultimately the answers would come, and this confidence enabled her to experience success in high school mathematics.

Karen reported completing *Number Concepts* and *Geometry* at a local community college, earning a B and a C respectively. She described these two course as "pretty much review for everybody" (Ka1, 4). The *Probability and Statistics* course was her first mathematics class in a year and she was convinced that finally she would be learning new material.

Like Cynthia, the other Type I student, Karen's busy schedule, which included school, working two jobs, and a long commute, made it difficult to find time to meet for interviews. At her request, our interviews took place at one of her jobs – an elementary school where she provided afterschool daycare. In fact, Karen was the sole informant who I interviewed outside of my office.

I never knew what to expect during these interviews. On occasion Karen demonstrated great intelligence and complexity, but she was just as likely to be scattered and inconsistent. While her manner of speaking belied an inner depth, her responses were sometimes conflicted. For example, at one point during the final interview, she stated that primarily due to her average scores on the *Probability and Statistics* assessments she felt she was only an average mathematics student. Later in the same interview, however, she disagreed with the statement, "The grades I got in math class determined my perception of my ability in math." These inconsistencies, along with her attempts to rationalize how these different positions were actually the same, made Karen's transcripts among the most difficult to analyze.

Keona

Keona represented the lone minority student (African-American) in this study. Before graduating from a large urban high school in the Midwest in 1995, she took four years of mathematics through Algebra 2. Since entering college, she reported taking a general algebra course and two mathematics courses for elementary education majors, earning a C in *Number Concepts* and a DC in *Geometry*.

The DC may explain why Keona placed herself in the quadrant representing a student who tries hard to understand mathematics yet earns poor grades [Type III].

> I would describe myself as average math student. Usually, I mean, I have difficulty with math so I always need some type of help with it, and I'm really not good with story problems at all. (Ke1, 1)

When I asked her what made her feel she was average, she responded, "My grades have led me to believe that I'm just an average student in math" (Ke1, 2). Keona reported having to put a great deal of effort into her mathematics classes in order to earn passing grades. The frowning face she drew to reflect her attitude toward mathematics showed her frustration with the subject, which seemed consistent with the other self-reported data.

Despite her struggles in mathematics, Keona was always very pleasant and forthcoming with information during her interviews. She seemed very comfortable explaining her responses and often went out of her way to do so. My intention was to keep each interview under an hour, but on occasion Keona's interviews took longer as she went into detail about some experience or shared an opinion about an assessment. Even as she was expressing displeasure about some aspect of the class, I still got the impression from Keona that she was in an overall positive frame of mind.

Lucia

Lucia graduated from a Midwestern high school in 1995, where she took Algebra 1, Geometry, and Algebra 2. She failed high school Geometry though, and "had to take that over in summer school" (Lu1, 3). Ironically, she also had to take the

Geometry course for elementary teachers twice before earning an acceptable grade – a B. Her grade in *Number Concepts* was a CB and, overall, she saw herself as an average student who had to work hard to get good grades [Type II].

When she described her relationship with mathematics during the first interview, however, she did not seem quite so confident:

Lucia:	It's always been hard for me ever since I was in elementary school. I don't know, each year I think I'm understanding – I'm getting more confident with it.
Interviewer:	Describe what you mean by it was hard.
Lucia:	Well, it's just frustrating for me because in third grade I was put in a remedial class… And then I was always in the remedial math. In sixth grade I had a great teacher, absolutely loved him, and I got an A in his class, and that's when I started getting better and better and better because of my confidence level. Maybe he did it for me. And then I came here and last semester I had [*Geometry*] and I had to take it over because I did absolutely terrible… and now I don't know. (Lu1, 1)

The mixed results Lucia experienced in mathematics left her confused about her ability in the subject. What she seemed sure of was the impact being placed in the remedial class had on her; "…it totally brings your confidence down and it's like you have no hope. I already had it in my head so I just thought I would never do well" (Lu1, 2). Hence, her drawing of her attitude toward mathematics showed a face with a straight-line mouth and angled eyes suggesting a look of frustration.

In the course of our interviews, Lucia exhibited some behaviors that might explain why she struggled in a traditional mathematics setting. She interpreted the instructions of certain interview activities in unexpected ways, which led to unique results. For instance, instead of highlighting sentence by sentence the mathematics classroom vignettes as the other informants did, Lucia looked at the task more

holistically. If most of the details from a paragraph were familiar to her then she put a yellow dot next to it, even when foreign aspects were present. When I attempted to gain further insight by asking her to explain her responses another problem arose; Lucia often experienced difficulties trying to express clearly what she was thinking. Therefore, it became necessary for me to restate what I understood her to be saying, and then she either agreed or disagreed with my evaluation. Although Lucia continually assured me that I was not putting words in her mouth, I remained concerned that her interview responses were more a reflection of me than they were of her.

Lynn

Lynn graduated from a large, suburban, Midwestern high school in 1994. As a junior, she had gotten through Algebra 2, but struggled enough that she decided to take Consumer Math her senior year. She described that class as "an easy math class. You know, how to balance a checkbook, all that stuff. Stuff that in a way I needed to know" (Ly1, 2). Her struggles continued in college where she had to repeat three mathematics courses. These included *Number Concepts*, which she finally passed on the third try with a C, and *Geometry*, which she was currently retaking because of an earlier DC grade. The study section represented her first attempt at *Probability and Statistics*.

Understandably, Lynn saw herself as a Type III student. She explained:

> I mean I'm dedicated to doing the work but the outcome of my work is, I guess, just horrible. I mean, I've had to retake math classes, had to retake

> math tests. It's like I go in feeling that I know it and once the professor's talking or the teacher's talking and we're given an assignment, everything I just learned is gone. It's like it's all vanished. I'm looking at this trying to figure out, what the heck am I supposed to do. (Ly1, 1)

Confusion was also evident in her drawing representing her attitude toward mathematics, which showed a person with squiggly eyebrows and a squiggly mouth. Next to her drawing she had written, "worried, stressed, & confused".

Like Keona, Lynn was very comfortable sharing and her interviews often took longer than the time allotted. On occasion, the transcription of her response to a single question was more than two and a half pages in length. Not only were these responses thorough, but they also tended to get off into other subjects. Whenever Lynn took a breath, I found myself trying to steer the interview back on course. Analyzing her interviews required sifting through a tremendous amount of extraneous information.

Informants as a Whole

Over the course of the study, these seven students distinguished themselves as worthwhile informants. They exemplified characteristics of pre-service elementary teachers deemed important to this research: representative of each of the three important student types (see Figure 6) and a varied mathematical background (see Table 4). Furthermore, each informant felt safe discussing details of their lives, even when the details might be considered embarrassing. Therefore, a tremendous amount of beneficial data on their mathematical experiences, their beliefs, and the relationships that exist between them, were gathered and analyzed.

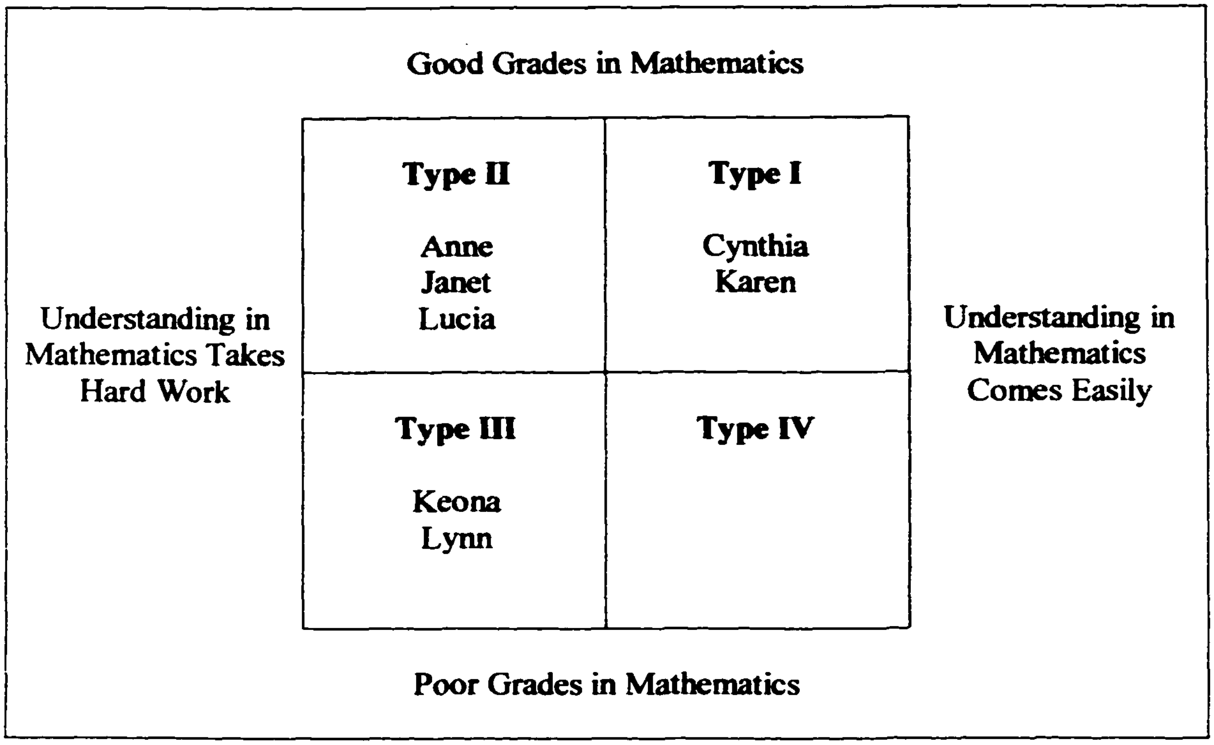

Figure 6. Informants Reported Self-perception as a Mathematics Student.

Table 4

Informant's Reported High School Mathematics Courses

Anne	Cynthia	Janet	Karen	Keona	Lucia	Lynn
Algebra	Algebra 2	Pre-algebra	Algebra 2	Basic Algebra	Algebra 1	Algebra 1
Geometry	Geometry	Algebra	Geometry	Algebra 1	Geometry	Geometry
	Pre-calculus		Pre-calculus	Geometry	Algebra 2	Algebra 2
	A.P. Calculus		A. P. Calculus	Algebra 2		Consumer Math

The Informants' Experiences in Mathematics

In order to identify experiences that might contribute to pre-service teachers' mathematical beliefs, the informants compared their experiences to two classroom vignettes (see Appendix C). With a few exceptions, the informants' reactions to these scenarios suggested that they had all participated in comparable mathematical experiences. Therefore, this section examines the informants' combined results.

Past Mathematical Experiences – Benchmark Data

Recall that in this activity, informants read two vignettes; one described traditional mathematics instruction while the other presented methods based on recent research and reform in mathematics education. As they read the vignettes, the informants highlighted experiences that were consistent with their own in yellow and any that were foreign using blue. We then discussed the highlighted sections so that I could gain a clearer picture of each informant's experiences in mathematics.

When choosing typical and atypical experiences from the traditional classroom vignette, the informants consistently reported familiarity with most of the mathematical experiences described there. Lucia and Cynthia highlighted nearly the entire description in yellow without using any blue, suggesting that the scene was reminiscent of their previous mathematics classes. While the remaining informants found isolated elements of the traditional vignette foreign, they still identified the traditional classroom as the primary source of their memories of mathematics. Janet

referred to these mathematics experiences as predictable. When I asked her, "How is it predictable?" she responded:

> That basically you did the same thing every day in class. That there wasn't much difference. Like on the [traditional side] where there's a lot of yellow. It was that you came into class, you sat down, you opened up your book, you worked on what you did the day before, you worked on some new stuff, and then you are assigned homework for take home. And that was every single day. (J1, 3)

Anne and Karen expressed strikingly similar memories from their mathematics classes:

> We were always on the [traditional] side, ... our desks were always arranged in rows all facing the front blackboard, and then usually [the teacher] would begin by asking if there were any questions on the homework.... And then I remember the teacher would go over whatever we were gonna do that day and show us examples from the book, and then usually do some by ourselves... (A1, 2)

> I've had a couple where you've been arranged in rows and kind of stayed that way except maybe one or two times a year... The teacher just read the text and demonstrated the examples and maybe a few of his own but not really anything too in depth... Similar practice problems are assigned. I think just about every class students went to the board to show how they did the problem. (Ka1, 2)

Doing problems on the board proved to be a quite powerful experience for Lynn.

> And sometimes they would have the students go up and have them redo the problem, and I'd feel sorry for them because the teacher would say, "Ah, what about that," you know? I mean, there was a couple times where I was up there and it was like, "You need to put the one on top to show that you were carrying over." I mean, I must have gotten at least three lectures from one teacher about carrying over the one, and that was in algebra... it felt intimidating. That's one reason why I barely rarely raised my hand. (Ly1, 3-5)

Other informants also expressed animosity toward, and frustration with, traditional mathematics teaching.

> Just sitting there watching the teacher. Pretty boring. (Lu1, 2-3)
>
> I didn't care too much for those [experiences]. (Ka1, 2)
>
> I didn't learn a lot from just being directed at all the time. (J1, 2)

The informants were unanimous in their consternation for these common mathematics experiences.

On the surface, data from the research-based, reform classroom vignette appeared to lack the consistency found in the traditional vignette data. Anne and Janet found nothing familiar in the reform scenario, Cynthia found only one piece similar to her own experience, and the remainder – Karen, Keona, Lucia, and Lynn – reported encountering mathematics lessons that shared many of the same qualities as those described in the reform vignette. During the discussions that ensued, however, it became apparent that most of the reform experiences occurred in recent mathematics courses for elementary education majors. The statement, 'Students seated in small groups…', provided one example of this. Keona highlighted this statement yellow and then explained

> Usually elementary, middle school, high school, we usually always worked individually. I don't remember any time until I got to college where we worked in groups. It wasn't until I got to [*Number Concepts*] that we started working in groups… It's like every math class now we always are in a group. (Ke3, 15-16)

Lynn's comments were similar, and pointed out that some of the teacher's questions from the reform vignette that she highlighted yellow were also the result of more recent mathematics experiences.

> Lynn: … students sitting in single seats in a row made me think of grades first through pretty much high school, my senior year,

and a few of my college classes. But the one on the other side reminded me of [*Number Concepts*] and also [*Geometry*] and [*Probability and Statistics*]... I mean, even some of the questions like the one I highlighted, "How is this like other problems you've done?" I've heard that.

Interviewer: That's something that's been asked post-high school?

Lynn: Yeah, in college. (Ly1, 3)

Only Karen recalled experiencing anything close to the situations described in the reform vignette prior to taking any of the mathematics courses for elementary education majors.

> Yeah I had one teacher that, when I took calculus [in high school], it was really a lot of small group work activities for that, with all the thinking stuff, and a couple of my other classes were that way, too. (Ka1, 3)

Otherwise, the informants agreed that most of the experiences related to the reform vignette were a recent phenomenon.

A majority of the informants found the reform-based approach to teaching mathematics much more to their liking.

> I love small groups because if you know it well you can get to explain to my peers about how to do it and then it makes me understand it better. And then when I don't understand that, I have like three, four people that can help me. (Lu1, 4)

> ...we didn't have any of the manipulatives unlike my college classes where like just yesterday we were working with the geoboards. I really like those too... That's one thing I'm enjoying about my math classes, the amount of manipulatives. (Ly1, 3-5)

> I had a ball in the first math class here when we were figuring out all that stuff in the base ten and all that... I don't remember learning any of that [before] and a light bulb went on as soon as I learned all that stuff here. It made everything else make sense... [Before, it] was all, "This is how it's done," and "That's how you get the right answer," and "That's just how it is." (C1, 2)

> ... wouldn't it have been nice to have had the [reform] side, had been taught like the [reform] side and how different it was, and [my experience] was much

> more rigid and people sitting in classrooms. And there wasn't a lot of interaction going on the math classes I was in. (J1, 2)

Only Anne acknowledged an initial struggle with accepting these new methods. *Number Concepts* was her first mathematics class that required that she explain how she got an answer and involved a significant amount of group work. Her previous collegiate mathematics courses were more like those in high school, "because we didn't do group work then. And we never really talked about how you got the answer" (A1, 3). Thus, she had trouble adapting to this new mathematical environment.

> I'd never thought like that before... I mean, I've always just had to do the problem, get the answer, well the right answer. "Good job, let's go on." But in [*Number Concepts*] you had to think about how you would do it and ... that was just too hard for me... And he – yeah, a lot of group work and I was not used to that. I had a hard time with it. (A2, 1-3)

Anne had become accustomed to the changes though, and admitted, "[Probability and Statistics], I like that class," (A1, 4).

Upon asking the informants if there were any memorable experiences not illustrated in the vignettes, most mentioned taking notes. Cynthia explained that, "It was all notes on the board that we copied into a notebook" (C1, 4). Keona and Anne made similar comments and mentioned that this was another difference between early mathematics experiences and recent ones.

> A lot in the past I would take a lot of notes in math but now it's like I really don't take notes at all. (Ke3, 13)

> [Teachers] explained to us how to do it and we took notes, so we'd have an understanding. And then if you had a problem with homework, you had your notes to look at... And that's always weird to me now because you don't really take notes... in [*Probability and Statistics*] I haven't taken notes... If

> there's something I'm not sure about then I just write it down to remind myself how to do it, or an idea that she gives us for when we're teachers I'll write that down, but it's not like in high school where you have five pages of notes. (A1, 4)

Prior to the study section, note taking represented a considerable piece of their mathematical memories. Therefore, the informants agreed that note taking ought to be included in their list of previous experiences in mathematics. With this addition, the seven students reported having nothing else to add and considered the list complete.

Conclusions From the Data on Informants' Mathematical Experiences

Two themes emerged from the informants' descriptions of their mathematical experiences. First, all but Karen remembered encountering almost entirely traditional mathematics instruction prior to taking *Number Concepts*, and even Karen's non-traditional experiences occurred in her last two years of high school. The informants verified that those experiences from the reform vignette that they found familiar typically occurred exclusively in the *Number Concepts* and *Geometry* courses, while the traditional scenario reflected the methods used in previous classes. Cynthia thought the changes found in the mathematics courses for pre-service elementary teachers (hereafter referred to as the mathematics education courses) were intentional.

> I think that's the point of [the mathematics education courses]. I think they know that most of us were probably taught in this memorization manner, you know? And I think they're trying to change it so we don't teach that way. (C1, 4)

The second emergent theme suggested that the informants' felt the reform vignette reflected a better way of teaching mathematics than the traditional scenario. As each informant shared their collective distaste for the traditional dissemination model of teaching and their appreciation for the reform model, most agreed that their attitudes toward mathematics were more positive because of recent experiences. Potentially, the combination of improved attitudes and a reform environment may already have translated into informants' holding a more fallibilist view toward mathematics upon enrolling in the study section. The design of study anticipated this possibility (see question two), and I examine it after addressing the informants' thoughts about mathematics assessments.

Assessments in Mathematics

Obtaining information related to the informants' mathematical assessment experiences and their beliefs about how to assess mathematical understanding involved using a variety of data collection methods. These methods included a card sort of assessment experiences with subsequent explanations, interview prompts regarding assessment beliefs, and a vignette activity focusing on testing experiences in mathematics. The results from the three data collection methods shared several common characteristics that are reported here along with my conclusions.

Assessment Experiences in Mathematics

Results from the card sort on mathematics assessment experiences mirrored the findings regarding the informants' overall mathematical experiences. The methods of assessments that the informants recalled encountering most often in their mathematics classes were also mostly traditional in nature. These included quizzes, assignments, cumulative finals and certain types of tests (see Table 5). In particular, Karen, Cynthia, and Lucia described tests that assessed their recall of basic facts.

> Those were in middle school for like fractions and things, converting them all to percents and decimals. We had to memorize them but yet know them to apply them to other things. We were always timed on those to see how many we could get done in this time period. Those were pretty popular. Oral tests in elementary with basic addition and counting and multiplication in third or fourth grade. (Ka1, 9)

> And I remember in fourth grade doing just rote memorization for times tables, and we'd have to write them out like ten times and then we'd have them like spelling tests. We'd have multiplication tests. (C1, 2)

> I remember we had to do orally, like multiplication tables. Like the teacher would say and we would have to just like say them all to her. She would time us and it was really scary… it took me so long to memorize them all and she'd like – kids would be standing around, so that was something. (Lu1, 10-11)

The pattern of experiencing traditional mathematics followed by alternative practices in *Number Concepts* and *Geometry* held true for assessments as well. "I think the only test I've ever taken with a group was in [*Number Concepts*]" (A3, 12). "Yeah, the first time I started written work was here…" (Lu1, 8). Finally, many of the informants continued to view all things traditional, instruction or assessment, as less appealing than non-traditional methods. Lucia explained, "[Q]uizzes in school… were frustrating. I didn't like them much" (Lu1, 10). On the other hand, Janet said,

"Group tests sound good, don't they? … Because if you struggle with a concept, you're able to work it out together" (J1, 5).

Table 5

Informants' Reported Assessment Experiences

Assessment Type	Never	Sometimes	Always
Cumulative Final	0	1	6
Extra Credit	0	7	0
Group Tests	2	5	0
Homework Assignments	0	0	7
Math Projects	2	5	0
Oral Reports	6	1	0
Participation	0	3	4
Portfolios	6	1	0
Quizzes	0	0	7
Take-home Tests	3	4	0
Test (no partial credit)	1	3	3
Test (credit for work)	0	1	6
Test (with cheat sheet)	3	4	0
Written Work	3	4	0

Not everyone was as enthusiastic, however. Just as Anne was reluctant in embracing non-traditional instructional methods, lack of experience with alternative assessments made some informants wary of these new methods. They had become comfortable with the traditional assessment approach and had experienced success with it. Karen reflected, "I love [tests where you got partial credit], I really do… I can show you exactly what I know, but if I forget something at least I'm getting something…" (Ka1, 8-9). The informants' familiarity with traditional testing continued to manifest itself throughout our discussions on assessment in mathematics.

<u>Informants' Initial Beliefs Regarding Assessing Mathematical Ability</u>

During the first interview, I asked the informants what they would expect if I told them I was about to assess their ability in mathematics. Typically, they either described problems that might be on a test:

> Probably just computations kinds of things at whatever level you were trying to assess… (C1, 4)
>
> story problems … (Ly1, 10)
>
> word problems, algebra problems, and different things like that. (Ke1, 4-5)

or came right out and said:

> Give me a test. (A1, 5)
>
> The first thing that comes to mind is a test. (Ka1, 5)
>
> Some type of a written test. (J1, 4)
>
> Just giving me a test and work out problems. (Lu1, 6-7)

The informants' preoccupation with mathematics tests was of interest because they had reported encountering a variety of assessment practices in mathematics, especially in recent mathematics education courses.

Therefore, in the final interviews I asked why they had initially felt that the way to assess someone's mathematical ability was through a test. Again, their responses were nearly identical.

> Because I think that's the majority of the way teachers assess students, especially in the past – when the seven of us were in high school. (A5, 1)
>
> Because that's what we've always had. That's the only way I've really ever been assessed in school. (C5, 1)
>
> Because that's the way it's always been. Very seldom was your grade dependent on anything besides the test. (J5, 1)
>
> It's the most popular way. (Ka5, 1)
>
> I guess because that's usually the way we were always evaluated on our math ability or any type of ability we usually are given a test so we just figured that it was gonna be a test. (Ke5, 1)
>
> Because that's how we had been tested. That's all that we've experienced. (Ly5, 1)
>
> That's the way I've been taught my whole entire life... That's the most common. (Lu5, 1)

Because testing in mathematics was so common place, the informants attached great importance to this particular assessment practice. Thus, it was necessary to determine what the informants remembered about their mathematics testing experiences.

A Mathematics Test Vignette

A portion of the fourth interview involved an activity asking the informants to read and react to a vignette describing a student preparing for and taking a mathematics test (see Mathematics Test Vignette in Appendix C). Anne's response was typical. "I could have highlighted pretty much the whole page" (A4, 4), meaning that the vignette was an accurate representation of her own test taking experiences. Janet agreed, "It sounds pretty realistic to true life – that this is the kind of things you do to study for a test. And you're kind of off in your own world and kind of boring" (J4, 11). Their descriptions presented a traditional testing scenario with a few exceptions.

One of these exceptions was the statement, "If she still doesn't get it she might call some other students from the class and ask for their assistance." The intent of such statements was to broaden the choices available to the informants by also including actions representing productive mathematical behaviors. More than any other, the informants identified the above statement as being foreign to their assessment experiences. They explained:

> I guess it just never really occurred to me to do it. (Ka4, 6)
>
> I probably should, but I only find that happening like right before the test in class, with the rush to make sure you understand something. (Ly4, 8)
>
> If I'm that lost … they can't explain it so I can understand it. (J4, 9)
>
> I'd just wait and ask the teacher like in the morning before school. I very rarely called other kids. (C4, 11)

The idea that students cannot help each other to understand mathematics because only the teacher can, is one of the most common counterproductive mathematical beliefs.

Cynthia pointed out two other discrepancies with the vignette that confirmed the traditional nature of her testing experiences in mathematics. Specifically, "connect[ing] math to the real world" and "using their calculators" were foreign to her. She explained:

> Well, I guess a few story problems here and there that kind of do that, but those were more I think not to connect things to the real world but – well, maybe they were – but I think the focus of those more is to see if you can pull out the correct information out of the thing and figure out what you're supposed to do with the information once you get the numbers out of there and figure out what you're supposed to do with them. (C4, 11-12)
>
> I don't remember being allowed to use calculators until probably I hit pre-calculus or AP calculus, things like that… they wanted to know if we knew how to do it, not the calculator knew how to do it. (C4, 12)

These experiences may explain why Cynthia described the evaluation of an individual's understanding in mathematics as involving "probably not a lot of thought questions or problem-solving kinds of questions. More just systematic kinds of questions, things you do a certain way to get the correct answer" (C1, 5)

Concluding Comments on Mathematics Assessments

Cynthia's assessment experiences in mathematics and subsequent beliefs about mathematics assessments were typical of all the informants, thereby representing commonalties across the data. The informants all reported being taught and assessed from an absolutist view. The teacher lectured on the mathematics content as the students took notes making the teacher the mathematical authority in

the classroom. A few weeks later, the teacher tested the students to determine whether they could repeat back the information. Only recently, since taking some mathematics education courses had they encountered alternatives to traditional teaching and testing. Still, the informants' view of assessing students in mathematics reflected the absolutist perspective of their earlier assessment experiences. Whether these experiences also influenced their central beliefs regarding the nature of mathematics is forthcoming.

Relationships Between Mathematical Experiences and Beliefs

A majority of the data collected during this study related to the relationship between an informant's beliefs about the nature of mathematics and her experiences in mathematics. The personal nature of this data makes it necessary to examine each informant on an individual basis. Within these examinations, I report on the informant's initial beliefs about the nature of mathematics and the experiences that she feels contributed to her strongest beliefs. Themes common among the seven informants' individual data conclude this portion of the results. The section opens with the findings from the initial belief survey administered to the entire study section and what these results imply about the mathematical beliefs of these students as a whole.

Survey Results

Thirty-one students, present at the study section's first meeting, completed the initial belief survey described in Chapter Three and included in Appendix C. A portion of this survey required students to select from predetermined lists a best and a worst simile in three categories –mathematics teacher, mathematics student, and doing mathematics – along with explaining the reasons behind each choice. While simile selections occasionally offered some information regarding a student's mathematical beliefs, the explanations typically provided more insightful data. The explanations of two students could support opposing mathematical viewpoints even if they had chosen identical similes.

Some explanations proved more helpful than others did in ascertaining whether students held productive or counterproductive mathematical beliefs. A few students misunderstood the directions for selecting best and worst similes; for example, a common error was choosing a simile that reflected their own experiences in mathematics rather than one to represent the ideal mathematical experience. Another problem arose when a student selected a simile but offered no rationale for his or her choice or simply failed to complete a section. The latter was a particular problem with the very last prompt, "Choose the simile from the list that you believe does the worst job of describing doing mathematics and explain your choice." Due to lack of time or waning interest, six students left this section blank. Still, the remaining usable data presented a picture of a class of students holding, to varying

degrees, typical counterproductive beliefs associated with an absolutist view of mathematics (see Appendix D for a complete set of this data).

Twenty-seven of the thirty-one students who completed the belief survey wrote at least one explanation containing an obvious reference to predictability or lack of confidence, two of the counterproductive beliefs discussed in Chapter Two. No explanations related directly to the third counterproductive belief that a mathematics problem ought to be solved quickly or abandoned [perseverance], although three statements suggested that "after you do it enough it becomes very easy to you" (BDMS, 25)[4]. Students' rationale for their simile selections for mathematics teachers and students commonly reflected a reliance on an outside authority [confidence]. One student admitted, "I need to have someone teach me…" (WMTS, 19). Typically, "someone" referred to the teacher because students "soak up as much information as possible from their instructor" (BMSS, 32). Explanations suggesting that students viewed mathematics as a set of unquestionable truths that exist separate from experience [predictability] were most prevalent in the reasoning associated with simile choices for doing mathematics. Some focused on one right answer – "you should come out with the same finished product" (BDMS, 22). Others stressed using a predetermined method to arrive at the answers – "I think math is more analytical; a step-by-step process as opposed to a fluid group of movements" (WDMS, 12). On first glance these statements appear to describe a class filled with students holding

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[4] (BDMS, 25) refers to the 25th student's entry for the best doing mathematics simile on the initial belief survey; WDMS is the worst doing mathematics simile; BMTS and WMTS are entries for the best and worst mathematics teacher similes.

counterproductive mathematical beliefs, but remember that these are isolated references.

When examined as a whole, the explanations of most students suggested they held both counterproductive and productive beliefs. One such student wrote, "A mathematics student is like an explorer, lots of roads [productive] will get you to the same spot [counterproductive], but how you get there is the interesting thing" (BMSS, 9). Another student, who also chose explorer as the best simile for a mathematics student, demonstrated the same combination of beliefs – "taking different routes to arrive at the same destination" (BMSS, 22). Further evidence of this second student's belief in the predictability of mathematics was found in her rationales for the best and worst similes for doing mathematics – "you should come out with the same finished product" (BDMS, 22) and "while conducting an experiment your results are not always the same" (WDMS, 22). While these students had seemingly embraced productive beliefs associated with fallibilism and the reform movement, the fact that they continued to hold onto certain counterproductive beliefs calls into question the true extent of their acceptance of fallibilist thinking.

For six students, however, there was no question; their rationales exhibited strictly counterproductive beliefs (see the statements of students 1, 4, 19, 27, 28, and 31 in Appendix D). An examination of their choices and explanations revealed that three of these six selected inventor as the worst simile for a mathematics student. They explained this choice thusly:

> I choose inventor because students should not be inventing their own way of doing things. (WMSS, 1)

> Students are not like inventors because they are taught, they don't teach themselves. (WMSS, 4)
>
> I think math is already set up for you, you just need to do it. (WMSS, 19)

Given these statements and the fact that the inventor simile mirrors the constructivist process (e.g., Romberg & Carpenter, 1986; Wilson, 1992), it seems reasonable to suggest that simply selecting inventor as the worst simile indicated a lack of confidence in students' ability to invent methods for solving problems in mathematics. There is no evidence that the opposite is true, selecting inventor as the best mathematics student simile indicates acceptance of constructivism, but this raises further questions about the mathematical beliefs of the students in the study section taken as a whole.

The fact that none of the thirty-one students selected inventor as the best simile does not justify labeling their mathematical beliefs as counterproductive. Considering the previous data related to many of the students' split beliefs, however, it does create serious doubts as to whether the productive beliefs were central to their mathematical belief systems. It is possible that they viewed the many solution methods that a mathematics student can 'explore' as representing a fixed body of rules that cannot be invented. If such is the case, then most of the students entered the study section viewing mathematics as set of unquestionable truths. Thus, their willingness to allow students to explore mathematics represented a pedagogical belief, not a mathematical belief.

An analysis of the survey data found four students who did not demonstrate any counterproductive beliefs in their rationale writing; this does not mean that they

held productive beliefs though. Out of the four, one wrote such ambiguous statements that they could not be classified as either counterproductive or productive (see the rationales for student 18 in Appendix D) and another failed to provide any explanations for three of the six simile selections (see the rationales for student 30). The two remaining students did exhibit productive beliefs in their writing, but it was limited to:

> Learning how math works using trial/error. (BMSS, 6)
>
> I believe a mathematics student is like an explorer. Through learning about math, students can explore math and use it in any kind of situation. (BMSS, 29)

As is the case with each student's responses, further data was necessary to identify more accurately these students' beliefs regarding the nature of mathematics.

Interviewing all the students in the class was unrealistic, however, due to various constraints. Therefore, I focused on the seven students who volunteered to become informants using these results from the initial belief survey as a starting point. Each informant allowed me to discern a clearer picture of her mathematical beliefs and reported on the early experiences in mathematics, especially assessment experiences, which had influenced these beliefs.

Anne's Entering Mathematical Beliefs and Related Experiences

Based on her simile choices and corresponding rationale from the initial belief survey (see Appendix D, student 26), Anne seemingly embraced the productive beliefs associated with the fallibilist nature of mathematics, especially regarding

predictability. Her explanation for why 'doing a puzzle' was the best simile for doing mathematics provided a prime example: "There are many ways to do math as there are a puzzle, but the answer should end up similar." Apparently, for Anne mathematics was not about using one method to find a single correct answer. As I mentioned in the previous section, however, the survey data alone represented an incomplete picture of a person's mathematical beliefs, and in this case was somewhat misleading. During the first two interviews, it became apparent that Anne's productive beliefs were not as strong or central as originally assumed.

In the first interview, Anne sorted the mathematical belief cards as shown in Figure 7; these results reflected productive beliefs concerning perseverance, but not always in confidence or predictability. Her struggle with confidence was most evident in her disagreement with the statement, "Learning math is often the result of a lot of trial and error." She explained that she 'disagreed' with this statement because, "I think you should learn – you should understand it more than, 'Oh, that's not the answer. Let's try a different way'" (A1, 10). Clearly, Anne failed to accept the constructivist view that understanding results from students engaging in just such behavior. Regarding predictability in mathematics, the results of the card sort were mixed. Although Anne remained adamant that solving mathematics problems could involve using a variety of methods, she was not as sure that the answers might be different.

> I guess there's always a single solution. I don't know about that… See, I don't know because I guess the answer should always be the same. There's always the answer, but I think there's always ways – different ways – you can go about getting that answer. (A1, 13)

Therefore, the results of the first belief card sort confirmed that Anne did hold various productive beliefs toward mathematics, but she was unable to accept students' ability to construct understanding through experimentation and struggled with the concept of more than one correct answer to a mathematics problem.

Disagree
After you do math long enough it becomes very easy.
Before attempting a math problem, a person needs to know how to get the answer.
Learning math is often the result of a lot of trial and error.
Mathematics is made up of a lot of separate categories and unrelated rules.
Memorization is the most important tool to use when doing mathematics.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should not invent their own way of doing things in mathematics.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
In order to do mathematics well, students must absorb as much information as possible.
Math is a step by step process where there is always a single solution to be found.
Students should make discoveries in mathematics on their own, without it always being shown to them.
There are many ways to do a math problem, but the answers should be the same.
There are some basic rules in mathematics, but there is always room for interpretation.

Figure 7. Results From Anne's Card Sorts of Mathematical Beliefs (Shaded Statements Represent the Informant's Choices for the Second Card Sort).

The second interview presented further indicators that Anne's mathematical beliefs might not be what they seemed. Recall that as part of this interview, informants selected the four belief statements about which they felt strongest and then rewrote the statements to reflect their beliefs more accurately. (These four choices are

shaded in Figure 7.) Notice that Anne did not select to rewrite either of the statements concerning single correct answers, suggesting that she remained unsure about the possibility of multiple answers. Furthermore, the 'trial and error' statement, "one of those I didn't really know" (A1, 10) from the first card sort, was chosen as one of the four to rewrite as being most representative of her view of mathematics. Surprisingly, this time around Anne chose to agree with the statement, suggesting that her beliefs surrounding this idea were unstable.

Anne rewrote the 'trial and error' belief as follows, "Students have different ways of understanding a problem, so trial and error would result in the way a student is most comfortable doing the problem" (A2, 5). The reason for her inconsistent view of the use of 'trial and error' seemed to depend on whether it involved learning a concept [disagreement] or solving a problem [agreement]. She reported that her rewritten statement was a relatively recent belief, related to three experiences in her *Number Concepts* course.

> I have seen a problem done in my group worked different ways and I've tried to do it the way this one girl did it or the other girl did it and for me I have to pick which way I'm most comfortable with... And also the teacher, I think, showed that – he showed it too – that there's different ways to do it to get the same answer. And then on the test, sometimes he would say you can do it using the three methods. Pick one method and do the problem. And I think that's good because you may not be comfortable doing one of those methods and if he just asked for that one, then if it's me I might freeze up and get the whole thing wrong. (A2, 8)

As Anne described the experiences that supported her belief, it became apparent that by 'trial and error' she meant choosing from a variety of pre-existing methods the one

the student understood best. Thus, predictability was at the core of what might otherwise be considered a productive belief.

Additional evidence of Anne's conflicting views was apparent in her rewrite of the statement concerning the mathematics teacher's role in learning mathematics. She contended that the teacher should not always show how to solve problems in a smooth fashion, which was consistent with the rationale provided with her choice for worst simile for a mathematics teacher. While this might imply that Anne did not view the teacher as a mathematical authority, her rewritten belief suggested otherwise.

> Math teachers should explain each step to the problem. More than one way is possible because all students learn differently. There should be lots of time for questions while going through the problem. (A2, 4)

Experiences appeared to be a significant influence here, as Anne's pre-college mathematics teachers had always explained each step as they went along.

> I think that they've explained each step to the problem. I don't know if I've seen it more than one way. I mean, it might have been the way they were teaching it. So I guess that would be different. I don't really think I've ever had that one. Maybe in college... (A2, 7)

In fact, the data accompanying the rewritten 'trial and error' belief supported the fact that her *Number Concepts* instructor had indeed shown several approaches to solving a problem. Herein lies the conflict. Anne no longer believed that it was the teacher's responsibility to demonstrate clearly the single best procedure for solving a problem. Recent experiences established that the instructor ought to provide instead a variety of methods, regardless of how messy, and it was up to the students to determine which

was best for them. Again, the theme of a productive belief (a variety of methods) surrounding a counterproductive belief (the teacher dispenses the methods) emerged.

Regarding 'strong math students,' Anne maintained her position that they would not necessarily work problems quicker than weak students. Instead, she wrote:

> Strong math students may not work quickly on a problem but may understand how to do the problem better than poorer math students. (A2, 4)

Anne based this on a high school experience involving a good friend:

> My best friend from high school was a very strong math student and she could take forever to do a problem because she would want to make sure she understood everything and she was doing it exactly right rather than, "Oh, I think I know how to do it" and just get it done. And then I could do the same problem and probably get it done quicker but it wouldn't be the same answer. Or it might be the same answer, but I couldn't tell you how to do it and she would be able to explain it. (A2, 7-8)

While she did not equate speed with ability and recognized that some mathematics problems required perseverance if one was to be successful, Anne again referred to the necessity that the answers to a mathematics problem be the same.

Finally, although Anne did not believe memorization to be the most important tool when doing mathematics, she admitted in the first interview that

> I used to think that until I came here... [W]hen you have to explain something, how to do it, you can't just memorize the answer. I think that the whole 1 +1, 2 + 2 thing, that's more memorization. Everybody does that. But I really think that now it's not because if I do a problem that's gonna be on a test, I can't just memorize the answer. I have to figure out how I'm gonna work it, how I'm gonna explain how I worked it. (A1, 12)

Predictably, her rewritten statement reflected both productive and counterproductive mathematical beliefs.

> At a young level, memorization is used for basic math skills – addition, subtraction. As a student grows, how they solve the problem should be more

> important than memorizing the answer. They should be able to explain the problem and how they did it. (A2, 4)

Anne identified the assessment experiences from her mathematics education courses as being significant contributors in the development of this belief.

> I think the classes that I've taken the math [education courses]. It doesn't matter if you get the right answer. You need to be able to explain how you got it... Well, in [*Number Concepts*] you had to write down all your stuff and that was more important in the grading process than if you got the right answer. How you explained it and the steps you took, you had to write those all down. (A2, 6)

The first assessment in the *Probability and Statistics* course supported for Anne the importance of understanding over memorization.

> [W]e haven't been there that long, but it seems that if you can explain it to yourself, then it's better than oh, I memorized the answer, I got it. Because we have to write those tests, and how can you write a test if you don't understand how to do what you're writing about? How to make a pie chart. If you don't understand how that all comes together, then you can't do it. (A2, 7)

Because of recent mathematical assessments, Anne deemed it necessary to understand the mathematics she was learning, but this was only after she had memorized the basics. This raises the question of whether Anne truly understood the mathematical concepts she was learning or if she was memorizing the reasons behind rules and calling it understanding.

Anne exhibited a combination of productive and counterproductive characteristics as she discussed her beliefs about mathematics and possible experiences that had influenced them. She gave the impression that she agreed with many principles from the reform mathematics movement, yet was reluctant to let go of certain counterproductive beliefs completely. This reluctance, which seemed to

mirror her initial hesitancy to accept alternative instructional methods as preferable to her traditional experiences, emerged as a theme throughout Anne's data. In essence, her productive beliefs concerning mathematics masked a belief system containing a counterproductive core.

Cynthia's Entering Mathematical Beliefs and Related Experiences

The analysis of Cynthia's responses to the initial belief survey determined that she held mostly productive beliefs about mathematics (see Appendix D, student 14). Statements suggesting that mathematics students ought to experiment, interpret, and use various methods in determining "what makes the most sense" were found throughout her explanations. Cynthia alluded to the "rules of math" in a couple of her rationales, however, which raised the possibility that she viewed mathematics as being predictable. Comments made during the interview process provided evidence that Cynthia had only recently begun making the shift toward fallibilist thinking in mathematics and that she continued to find it difficult to release certain aspects of her counterproductive beliefs.

When asked to describe the experiences that had contributed to her changing perspective, Cynthia focused on her *Number Concepts* and *Geometry* courses.

> I think just because of the way we're being forced to learn it here and how we're going to be teaching it when we get out there it makes a lot more sense. (C1, 3)

> Yeah, if somebody stands up there and says this is how you do it and you carry the one and all that kind of stuff – that's what happened to me, and you don't know what – you don't know why it works. You just know that it works and besides, like I said, in [*Number Concepts*] a lot of light bulbs went off in

> my head thinking. I know how to do this stuff but, holy cow, now it makes sense. (C1, 10)
>
> [My mathematics education instructors are] not giving us the answers or telling us how to do things straightforward. They're making us go through the problem solving and thinking. Sort of, come up with the correct answers and we have to argue with each other in class, not looking to the teacher for a right or wrong response. (C2, 1)

Although Cynthia had experienced success in the traditional mathematics classroom [Type I], she appreciated that her mathematics education courses modeled a more effective instructional approach that she hoped to employ in her own teaching.

While Cynthia's sort of the belief statements reflected her desire to do more than simply pass along facts to her future students, there were indicators that she might also maintain counterproductive beliefs regarding predictability, perseverance and confidence (see Figure 8). For example, she both agreed and disagreed with statements addressing multiple methods of solutions, but a single correct answer. Using a recent example, she explained why she had agreed that "There are many ways to do a math problem, but the answers should be the same":

> This is one I had to go back and forth and pick a category so I put it in here because I think the answer's the same. We had one today in [*Probability and Statistics*]. The answer was – it was asking for a fraction and $^2/_8$ was the fraction, but you could have said $^1/_4$ or you could have said 25 percent, and all the answers are the same but there were different ways of getting those answers but in the long run they are all the same. (C1, 10)

Cynthia recognized that by disagreeing with "Math is a step-by-step process where there is always a single solution to be found" she appeared to be inconsistent.

> This one kind of contradicts the one I put in the other one, but there's not – when it says single solution I thought of like a single way to do it and there's not always a single way to do things. You're probably gonna end up with the

> same solution in the end in one form or another but it doesn't mean everybody uses the same route to get there. (C1, 11)

Thus, there was no contradiction in this case, but questions remained concerning Cynthia's view of a predictable answer. Her position seemed similar to Anne's, that different methods were acceptable when solving a problem, but the results ought to be the same. Regarding perseverance, she did not think that speed in solving

Disagree
Before attempting a math problem, a person needs to know how to get the answer.
Math is a step by step process where there is always a single solution to be found.
Mathematics is made up of a lot of separate categories and unrelated rules.
Memorization is the most important tool to use when doing mathematics.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should not invent their own way of doing things in mathematics.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
After you do math long enough it becomes very easy.
In order to do mathematics well, students must absorb as much information as possible.
Learning math is often the result of a lot of trial and error.
Students should make discoveries in mathematics on their own, without it always being shown to them.
There are many ways to do a math problem, but the answers should be the same.
There are some basic rules in mathematics, but there is always room for interpretation.

Figure 8. Results From Cynthia's Card Sorts of Mathematical Beliefs.

mathematics problems corresponded to ability, but did agree with the statement, "After you do math long enough it becomes easy." As she explained her reasoning, it was apparent that she related this belief to prolonged exposure to certain types of

problems and not to the amount of perseverance required. "If I have questions or if I don't understand, I try to look deeper until I do understand. I don't just blow it off and say it doesn't matter" (C2, 5). Cynthia expressed confidence in her ability to solve problems in mathematics. The results of the belief card sort, however, presented conflicting views about the ability of students in general to solve problems without the assistance of a mathematical authority. On the one hand, she felt that students "need to be encouraged to invent their own ways and invent their own rules and then do things to prove those rules, right or wrong… instead of just saying well just carry the one here" (C1, 12). Yet, she also believed that "In order to do mathematics well, students must absorb as much information as possible." I interpreted this absorption to be a passive act, but Cynthia explained it much differently.

> I think if they sit back and they just zone out in class and they don't pay attention and participate and they don't take in everything that's being said and process it in their head, they're not gonna form their own opinions and they're not gonna know what works and doesn't work. So I think they need to be in tune and take in everybody's aspect and figure out, sort of, in their heads what they agree with and what they don't. (C1, 10)

Therefore, with the exception of predictable answers, the findings from Cynthia's card sort of mathematical belief statements supported earlier results from the belief survey that she held mostly productive mathematical beliefs.

Cynthia selected "In order to do mathematics well, students, must absorb as much information as possible" as one of the statements she felt strongly about and wished to rewrite (see shaded statements in Figure 8), which provided further opportunity to clarify her position. The rewrite, "Listening to and processing a lot of

different information is the key to understanding math," suggested active involvement and supported her earlier statements, but left open the possibility that the teacher was providing all of this information. As she described the experiences related to this belief, however, she discussed learning mathematics as a student-centered process.

> [W]e used to bounce ideas off each other and talk through things and stuff all through high school in the classes I was in, but I didn't really form an opinion about that until I'd gotten here at [this university] and how effective that can be and how helpful that can be to have the students thinking it through in their head and processing, "Well does that make sense or doesn't that make sense" and "Do I have another opinion of that." … before we'd throw things off of each other, but then the teacher would always have still one way of doing it and here they don't – it's not the answer that they're concerned about here. It's the process of getting to the answer and the learning process of it and we never really focused on that at all in high school and stuff. There was always still one correct way of doing things. (C2, 7-8)

Recent experiences in her mathematics education courses had fostered the belief that students ought to be actively engaged in understanding the mathematics they were doing. Although this belief was relatively new, it emerged as a theme in two other rewritten beliefs.

Cynthia altered the 'trial and error' statement to read "In order to truly learn and understand math, a student must be allowed to experiment with different options." Therefore, it was not surprising when she disagreed that "Students should not invent their own way of doing mathematics" and wrote instead, "The best way for a student to really get a sense of mathematics is for them to experiment and make up their own rules regarding the way a problem should be solved." Both beliefs were recent developments related to experiences in *Number Concepts* and *Geometry*,

although she remembered some isolated early experiences that upon reflection also supported this perception of mathematics.

> I can remember one time in eighth grade. I guess it was Algebra I then. I had this teacher and he was amazing... And it seems like he used to teach us that way. He wouldn't tell us how to do things right off the bat. He'd let us talk amongst ourselves and try to figure out about some ideas around about things we had done in the past and what make sense to work and what might not. And then we'd talk about it as a class. But I think that was the only time that really ever happened, so I didn't really think much more about it until I got into college and that's how they teach us here and that's really where I've seen the benefit of it more than I ever did before. (C2, 6)

> I mean that was the whole point of basically [*Number Concepts*] was okay, how many ways can you get to the same answer? How many different – that was where part of the questions – how many different ways can you get to this. So I guess that's something here and something I've believed since I've got here, since I've been in these classes. Not really before because it was never an option to me before. We never – that part of it was – I didn't even know about that part of it... It was just one way of doing things. (C2, 7)

These experiences had challenged the traditional notion that the teacher, as the mathematical authority, held the only acceptable method for solving the problems encountered in the mathematics classroom. A possible counterproductive belief, "the same answer" to a mathematics problem, remained in these descriptions though. Certainly, some problems do result in a single correct answer, but such is not always the case.

Predictability was also a possible factor in Cynthia's belief that "Math is a conglomeration of rules and categories that are all intertwined." While it is encouraging that she saw mathematics as interconnected, one might also interpret this statement to suggest that she viewed mathematics as a set of unquestionable truths.

Coincidentally, she identified this as the longest held belief of the four – since at least middle school.

> [Y]ou had to know what you were doing in pre-algebra in order to be able to do what you had to do in Algebra I. And the same for Algebra II. It just all built right on, and it – even going back farther than that, I mean all that's built on knowing how to add and subtract. That's the basis of everything and knowing how to multiply and things like that. (C2, 7)

Perhaps a belief in the incontrovertible rules of mathematics had survived Cynthia's mathematics education experiences while at the university.

The results from the initial belief survey and two card sorts leave little doubt that Cynthia had begun to assimilate certain aspects of fallibilism within her mathematical belief system. She disagreed with counterproductive beliefs related to lack of perseverance and lack of confidence and dismissed traditional methods as ineffective. Only in terms of the predictable nature of mathematics were her views unclear. Certainly, Cynthia embraced the idea of employing various methods to solve a problem, but her ability to acknowledge that some mathematics problems might have more that one acceptable answer was vague. Based on the data collected during interviews with other informants, it is highly likely that before this study all the mathematics problems Cynthia remembered encountering had a single correct answer. If this was the case, then the alternative assessments from the study section might address this counterproductive belief.

Janet's Entering Mathematical Beliefs and Related Experiences

The rationale accompanying Janet's simile selections on the initial belief survey indicated that she viewed mathematics as authority driven and predictable (see Appendix D, student 16). She believed that students ought to "soak up everything the instructor has to teach" including "procedures for winning or getting the correct information." Although the teacher who simply lectured to students was unappealing to her, all she wanted was a teacher who made "time to discuss questions." Hence, Janet exhibited little confidence in herself or others to find the predetermined answer to a mathematics problem without the teacher's help.

Results from Janet's first belief card sort reinforced this initial impression of an individual holding mostly counterproductive mathematical beliefs (see Figure 9). Janet was one of the few informants who still seemed to believe that being good at mathematics meant solving problems quickly, but admitted that as a Type II student she was willing to persevere. In regards to predictability and confidence, she also appeared to hold conflicting views. For example, as was the case with Cynthia, she both agreed and disagreed with statements concerning multiple methods but one acceptable answer. The confusion again was the result of different interpretations of the term 'solution' in the statement, "Math is a step-by-step process where there is always a single solution to be found." While I envisioned 'solution' to mean 'answer,' apparently Janet's definition matched Cynthia's because she explained that "Sometimes there's different ways to do it..." (J1, 12). Apparently, she accepted the

productive belief that solving problems in mathematics could involve various methods. Otherwise, her reasoning in both belief card sorts was counterproductive.

> Even though I think that they should figure it out on their own, I still think they're coming up with an answer that's predetermined. If they can't come up with – if it's X is 4, they can't come up with X is 3. (J1, 15)

> [I]t's nice when teachers give you different options on how to figure it out. (J2, 5)

Although her rationale suggested that students could determine appropriate solution methods on their own, Janet appreciated the teacher's aid in finding them and believed that any of these methods would result in a predetermined answer.

Disagree
Before attempting a math problem, a person needs to know how to get the answer.
Math is a step by step process where there is always a single solution to be found.
Memorization is the most important tool to use when doing mathematics.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
There are some basic rules in mathematics, but there is always room for interpretation.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
After you do math long enough it becomes very easy.
In order to do mathematics well, students must absorb as much information as possible.
Learning math is often the result of a lot of trial and error.
Mathematics is made up of a lot of separate categories and unrelated rules.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should make discoveries in mathematics on their own, without it always being shown to them.
Students should not invent their own way of doing things in mathematics.
There are many ways to do a math problem, but the answers should be the same.

Figure 9. Results From Janet's Card Sorts of Mathematical Beliefs.

Despite the fact that the interval between the first and second interviews represented a relatively short time period, Janet reported that certain experiences in her *Geometry* and *Probability and Statistics* courses had already affected some of her mathematical beliefs. She explained:

> [T]he classes that I'm taking this semester have been taught much differently than any classes I've taken before. And so it seems as though there are different approaches to getting the answer, and so there's a little more freedom in the process where before it only seemed like there was only one way to do it. (J2, 3)

Perhaps this explains why Janet selected "There are some basic rules in mathematics, but there is always room for interpretation" as a statement she strongly agreed with, although she had originally placed it in the disagree pile. Janet rewrote this statement to read, "It's good if math can be interpreted differently. No two [people] think exactly the same;" a belief she had only recently begun to develop.

> I just think that some of the teaching that I've had in the last six months in math class allows for a little bit more creative thinking than my prior math classes... For instance, the group working. I think that's just a totally new concept to me. And I think when you work in groups, you have that option to see how people work differently versus just assuming that when you're sitting in a single seat in rows of 30 that each person is doing it exactly the same way. You really have no idea. (J2, 8-9)

Earlier findings verified Janet's awareness that numerous methods were available when solving most mathematics problems, but exposure to group-work had shown her that students themselves might think differently about these methods.

Further support of this position was found in Janet's strong agreement with the statement "There are many ways to do a math problem, but the answers should be

the same." This time, however, she focused on the answer adding, "It seems like there's always one right answer." Since elementary school,

> ... it was pretty explicit in the fact that you turned in a paper and if you didn't have the answer on that paper, it was wrong. It was marked with a big red X or check... Every math test. If X is not equal to 2, it's wrong. You can show your work, although some teachers do give partial credit for your work and your thinking, so if you write it all out and explain your process, then you get some credit. So I guess that wouldn't go along with the thinking that there's always one right answer. Well, it's still one right answer but you can still get partial credit for thinking. (J2, 9)

As with so many of her peers, Janet was able to accept that in the process of doing mathematics a variety of methods might be used, but the final result was a single, predictable answer. It seemed as though this belief was often the result of long term exposure to problems with only one acceptable answer.

In contrast, the belief that "Learning math is often the result of a lot of trial and error" was relatively new to Janet and resulted from a recent encounter with a classmate at her previous college.

> I had this great tutor that wasn't a tutor. It was a fellow student that told me one time – it was the best thing I was ever told about math. She said, "You know, you're not supposed to just sit down and be able to – actually this is a little different – you're not supposed to be able to sit down and just be able to do any problem. The whole process of math is to allow yourself to fail and sit down and look at a problem and you have to be able to work it a few times and math is the process of working it through. It's not just sitting down and knowing the answer." So that was the first time that anyone ever told me that you were allowed to do that. (J1, 13)

Previously, Janet had "thought that [in] math you're supposed to already know how to do it when you sit down and look at it based on your reading the chapter and listening in class" (J2, 9). Because a friend shared with her a different perspective that made

more sense, Janet now wrote that she believed, "Learning math is the process of trying to solve the problem."

For Janet, this process involved absorbing information. She wrote, "You must be able to absorb the information or you can't learn the concepts." Not surprisingly, she reported believing this since at least junior high school.

> [W]hen we started learning pre-algebra and stuff and I started feeling like I wasn't getting that information, so if I wasn't getting the information, then I couldn't do the work. If I didn't absorb the information. You can absorb some of it, but if you don't get it all or get most of it, you're going to be in trouble...Not be able to do the work. (J2, 10)

Success in mathematics, being able to do the work, required students to soak up all the necessary information. Consequently, strong mathematics students were those who could assimilate all the material they encountered while solving a mathematics problem. "Stronger math students or possibly students with more experience do math easier." Janet started believing this once she started to struggle in mathematics and recent events had reinforced it.

> Just last week, I had a student in my geometry class and we were measuring angles and we were trying to figure something out and I looked at her and she did it really easy and I said, how did you know that? And she goes, oh well that was from my high school geometry class. So well darn, I guess I should have taken that geometry class in high school then. So I think that it was easy for her because of her prior experience. (J2, 10)

While she had been encouraged by a classmate who suggested that learning mathematics involved struggling with a problem, Janet remained uncomfortable with the idea of not knowing how to solve a problem as it was presented.

Ultimately, her friend's comment had not altered her previous beliefs, only created a new disconnected one. Janet desperately wanted to believe that doing

mathematics involved experimentation and learning from one's mistakes, but her central beliefs held fast to the ideas of predetermined answers and outside authorities. Given that Janet's K-12 mathematics experiences predated 1977 and that she had taken her college mathematics courses elsewhere, it was not surprising that she entered the study section with mostly counterproductive mathematical beliefs.

Karen's Entering Mathematical Beliefs and Related Experiences

As was mentioned when introducing Karen earlier in this chapter, the data from her interviews often seemed inconsistent, which made it difficult to analyze the results. Her responses on the initial belief survey contributed to this confusion. Not that her explanations for her simile choices were inconsistent. Karen was one of the three students who selected inventor as the worst simile for a mathematics student and her explanations exemplified counterproductive beliefs regarding confidence and predictability (see Appendix D, student 1). In fact, her rationale that "students should not be inventing their own ways of doing things" became one of the statements included in the first belief card sort. Other statements from her survey supported this sentiment, such as, "if everyone did math differently there would be no need for this course." Therefore, it was surprising that Karen presented a quite different position during the first interview.

The most striking change was that Karen disagreed with her written comments about how students learn mathematics (see Figure 10). Although she wrote, "I chose sponge because ideally a math student should be absorbing every bit of information"

(BMSS, 1) she placed the belief card espousing absorption in the disagree pile. "Because you don't have to take in as much as possible. You can almost know hardly anything at all and still be able to do it well" (Ka1, 12). Similarly, Karen reversed her position on students inventing their own way of doing things.

> They're always gonna come up with a different way of doing things. It may not be the way that they were shown, but it's going to be a way that can get them to what they need to get to. It may not be the way the teacher told them to do it, but it works for them and it may be easier for them to understand that way. (Ka1, 13)

Disagree
After you do math long enough it becomes very easy.
Before attempting a math problem, a person needs to know how to get the answer.
In order to do mathematics well, students must absorb as much information as possible.
Math is a step by step process where there is always a single solution to be found.
Memorization is the most important tool to use when doing mathematics.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should not invent their own way of doing things in mathematics.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
Learning math is often the result of a lot of trial and error.
Mathematics is made up of a lot of separate categories and unrelated rules.
Students should make discoveries in mathematics on their own, without it always being shown to them.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
There are many ways to do a math problem, but the answers should be the same.
There are some basic rules in mathematics, but there is always room for interpretation.

Figure 10. Results From Karen's Card Sorts of Mathematical Beliefs.

Perhaps, like Anne, Karen was simply looking at these statements from two different perspectives. Her comments suggest that in one case she might be addressing the

ideal and in the other her perception of reality. Thus, further evidence was necessary in order to identify Karen's beliefs concerning perseverance, confidence, and predictability in mathematics.

Perseverance was not an issue with Karen, as she admitted to having to "study for hours to make sure [she] knew what [she] was doing" (Ka1, 2), but in the areas of confidence and predictability her comments implied that she held counterproductive beliefs.

> I kind of prefer that so that [teachers are] giving their input on how you're getting it done and it's a different way but it's still working. It's not really correct, but it's kind of a bit more acceptable. (Ka1, 4)

> [I]f [teachers are] coming around and they're always telling you, oh that looks good, that looks good, or it's kind of that extra reassurance that you are getting it, you are doing what you're supposed to. You're not completely on the wrong track. (Ka1, 5)

> [T]here could be a couple possible right answers, but which one is more correct. (Ka1, 5)

> There's many ways to a solution normally for each problem, three, four, many different ways. Nine times out of ten there's going to be only one solution for it, but there's many ways to get there. (Ka1, 11)

Karen's use of "nine times out of ten" is another example supporting the theme that she believed most mathematics answers were predictable, yet there are various methods of arriving at the answer. Furthermore, Karen described the mathematics teacher as a gentler type of authority, but an authority just the same. Rather than telling students how to solve problems, the teacher serves as the critic of the students' methods. Without the teacher's reassurances, she had little confidence that the students would remain on the right track.

When it came time to rewrite the beliefs about which she felt strongest, Karen again seemed to present a productive belief system (see the shaded statements in Figure 10), but her comments hinted at a counterproductive core. She continued to disagree with her own 'inventing' statement, which she rewrote as, "Students need to find a way to do mathematics to best suit their needs and understanding." Karen related this belief to three different experiences.

> [M]y seventh/eighth grade year, getting more into algebra where there's a couple different ways to do a problem and it just depends on what type of person you are with learning, if you need shortcuts, if you need the longer version, if you can do it almost in your head type of thing, but everybody has to do it differently. I think that's kind of why they make a couple different ways to do things so that there's not just one set way, that everybody has to follow one thing because everybody learns differently. (Ka2, 5)
>
> They'll say well, here's a shorter way of doing it after we've learned the long way. Just kind of to reiterate the fact that there are a couple different ways to do things. (Ka2, 6)
>
> The only other possibility I can probably think of is like when I know I've helped other people, in the way I've explained it of the way I do it, it doesn't really work for them... so you've got to try to find like different ways of explaining things, so it kind of almost turns out to be a different way of doing it for them when in actuality it's the same way of doing it but it's a twist on the words so it's more understandable for them. (Ka2, 6)

In the course of describing the experiences behind this belief, Karen acknowledges that students learn differently [productive pedagogical belief], but they still depend on some expert to pass along to them the prescribed methods for solving problem in mathematics [counterproductive].

When Karen chose to rewrite the 'trial and error' belief as, "Students learn best by trying what they believe is the correct way and fixing any mistakes along the way or afterwards," she again represented both perspective. Through her own

experience with mathematics and working with students having trouble in mathematics, she indicated that she saw value in learning from one's own mistakes.

> Again back to like elementary school, it's just your basic addition. They'll tell you – I mean, they'll show you how to do it but then they'll give you 10, 15 problems and say, here you go... I know, I've worked in a classroom to where I've had to go around and correct them and you circle them and they have to go back through and try to correct them themselves, you know? And a lot of times they'll catch their own mistakes, saying that doesn't look right. I'm getting the same answer as I have for this one and the numbers aren't the same. So they can go back and correct what they've already done, so they're catching their own mistakes at times. (Ka2, 9)

Yet, Karen's description characterizes the teacher as disseminating solution methods before allowing time for practice and being responsible for judging the correctness of students' work. This represents a rather traditional view of teaching that often results in counterproductive beliefs about mathematics.

The remaining rewritten beliefs – "All students have their strong points. Some just have more than others" and "Math can be a very challenging subject for some students, even after many years of doing it" – emphasized that Karen recognized that doing mathematics sometimes required perseverance.

> This one I think is just of kind of again like a basic knowledge type of thing where you could – any classroom you walk into you can see the students that have, you know, more strong points than others that are constant participation people, and then you have your other students that every once in a while you get the ones that will click automatically. That will say, "Oh yeah, wow." And every once in a while you'll see them, but then they have a harder time grasping the concept of what is what. (Ka2, 6-7)

> [I]n elementary it's your basic math of adding, subtracting, multiplication, division, a few fractions here and there. But in middle school you kind of get to see the kids that you've always kind of thought weren't too bad in it, you know, with all the basics. But then after see them kind of struggling after having five, six years of math, you kind of get the point that not everybody is going to be quite as strong. Even though they understood the basics, there's

> those special parts that they just don't seem to get, the concepts that they don't seem to get. (Ka2, 8)

It is possible that Karen's own struggles in high school mathematics contributed to this view. Recall that in the first interview she reported how she had "always excelled in it… up through probably sophomore year… As it got harder, it was more work to put forth in it but I've never really done terrible in it" (Ka1, 1).

Perhaps Karen's success in mathematics explains why all the experiences she identified as affecting her mathematical beliefs were outside her *Number Concepts* and *Geometry* courses. She had mostly fond memories of early mathematics classes and therefore had no reason to embrace new experiences. Furthermore, her mathematics education courses were taken at a community college and not the university, which might explain why Karen's strongest held mathematical beliefs resulted from earlier experiences. Whatever the reasons, Karen clearly entered the study section with counterproductive beliefs related to confidence and predictability; her productive beliefs related to teaching mathematics, not the subject itself.

<u>Keona's Entering Mathematical Beliefs and Related Experiences</u>

The rationale Keona provided for her selections on the initial belief survey related more to her attitudes toward mathematics than her mathematical beliefs (see Appendix D, student 28). When explaining her choices for worst teacher simile and worst simile for doing mathematics, it was evident that she would not consider mathematics among her favorite subjects. "Gardening seems to be relaxing – math is not." "Dancing is fun, and math is not fun for me." A single statement relating to

Keona's mathematical beliefs suggested that she relied on the teacher as the supreme authority when learning mathematics: "They give the orders & you do it." The results from the two belief card sorts offered a clearer idea of Keona's overall view of the nature of mathematics.

Based solely on her sorting of the belief cards during the first interview, Keona appeared to agree with the productive belief statements and disagree with most of the counterproductive ones (see Figure 11). Only her agreement with statements regarding 'teacher showing' and 'memorization' implied otherwise. As Keona began to provide her thinking behind each decision though, it became obvious that her beliefs about mathematics were mostly counterproductive. Perseverance represented the lone exception; she disagreed that mathematics should be done quickly or not at all. In fact, "a strong math person might think again about the problem. They might just keep going over it to make sure that it is right and they did do step by step correctly" (Ke1, 11). Even this statement, however, hints at the fact that Keona saw mathematics as somewhat predictable.

Of course, Keona's experiences had reinforced the predictable nature of mathematics. In order to be successful on a test, she had found it necessary to memorize a great deal of information.

> Memorizing is an important tool because in math you have a lot of concepts. You have a lot of different things to remember, whatever, and more or less in math classes you're not gonna be able to use your notes when you go in to take the test and whatever. So you have to remember a lot and know how to use it when you get your problems because if not, you're just gonna sit there, like "What's going on?" (Ke1, 14)

As she described solving problems in mathematics, Keona offered further insight into the experiences that had contributed to her counterproductive beliefs.

> You always have to go by a step-by-step process more or less when you're solving a mathematics problem, but it's always gonna be one answer to be found. I haven't come across a math problem yet that you could have more than just one specific answer for it, so maybe I'm wrong about that but for now this is what I think. (Ke2, 4)

Disagree
Before attempting a math problem, a person needs to know how to get the answer.
In order to do mathematics well, students must absorb as much information as possible.
Mathematics is made up of a lot of separate categories and unrelated rules.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should not invent their own way of doing things in mathematics.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
After you do math long enough it becomes very easy.
Learning math is often the result of a lot of trial and error.
Math is a step by step process where there is always a single solution to be found.
Memorization is the most important tool to use when doing mathematics.
Students should make discoveries in mathematics on their own, without it always being shown to them.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
There are many ways to do a math problem, but the answers should be the same.
There are some basic rules in mathematics, but there is always room for interpretation.

Figure 11. Results From Keona's Card Sorts of Mathematical Beliefs.

By believing that answers to mathematics problem were predictable, Keona agreed with the other informants who held that "There are many ways to do a math problem, but the answers should be the same." When it came to interpreting the rules in mathematics though, she seemed to focus on the teacher's interpretation.

> Yeah, I think that there are a lot of rules and different things in math, but you can – each person's gonna interpret it in a different way. Everyone's not gonna say, oh well this means that, that, that, you know? Their thinking might not be right, but they still might think well, this means – this theorem means this. And the teacher might come along and say, "Well, it doesn't mean that; It means this." (Ke1, 13)

Thus, although Keona agreed with several productive belief statements from the first card sort, her interpretation of these statements suggested that she viewed the mathematics teacher as the authority and mathematics answers as predictable.

Recent experiences in her mathematics education courses could explain why Keona's sorting suggested she held productive beliefs even when her explanations reflected counterproductive ones. In [*Number Concepts*], she had found that doing mathematics involved using trial and error.

> So you basically would have to go through a whole bunch of different concepts to see if this will work to solve this problem or if that will work, so you couldn't just come up with the answer. You have to give a reason. What concept did you use? Why did you use it? And so it was a lot of trial and error. (Ke1, 12)

Furthermore, a recent discussion in the study section had addressed the idea of allowing students to make their own discoveries in mathematics.

> My teacher in [*Probability and Statistics*], she was just talking about that. She's been talking about it actually since we started class – that in elementary school more students should be allowed to explore and not just be given the answers like they would ask the teacher is this right, you know, instead of letting the student figure it out if it's right or not and work more at it. You know, explore different options instead of just giving them the answer directly. (Ke1, 13-14)

This last description suggests that Keona may have adopted this belief simply because her teacher had suggested it, supporting the assumption that Keona relied on an outside authority concerning doing or teaching mathematics. By agreeing that

students ought to invent their own ways of solving problems, Keona appeared to contradict the view of teacher as authority, but during her explanation, she recanted.

> I don't think [students] should, but I guess it would depend on the teacher, you know, because more or less the teacher usually gives you the way in which you should do a problem and usually that's the way they want you to do it. They tell you to do a certain problem this way, then they usually don't want you to come up a different way and say well, I did it this way. They're like well, you should do it this way, something like that. So no, I don't think they should invent their way of doing mathematics unless the teacher tells them that it's okay to do it that way because more or less teachers say this is the way I was taught and do it this way. (Ke1, 13)

Based on her own mathematics classroom experiences, Keona maintained that in all things the teacher was the ultimate authority.

The four beliefs Keona selected as representing her strongest beliefs during the second interview (see Figure 11, the shaded statements) were rewritten as follows: (1) "Math problems can be done in many different ways, but only one answer can be found;" (2) "Math is done in a step-by-step process and only one answer can be found;" (3) "Memorization is the best way to learn math;" and (4) "There are many ways in which math can be interpreted." The first three statements in particular added further support that Keona's initial mathematical beliefs related to confidence and predictability were primarily counterproductive.

In each case, Keona reported that she had believed this since elementary school. She identified several generalities that had contributed in the development of this belief that mathematics is predictable.

> I've always been taught that a problem, specifically a math problem, has only one answer. It might be – you might have a dozen different ways you can do this math problem, but whoever does it and whatever way it's done it should

> still come out to the same answer. It should always come out to whatever the answer is. (Ke2, 6-7)

> ...where you're doing like the FOIL process with the algebraic problem. You always go in a certain process, you know, when you're doing this problem. You can't do it backwards. You can't do it forward. You have to do it in that specific way to get the answer, so I would say that that would be a good example of a process in which you use to get to an answer for that problem. (Ke2, 7)

> ... whatever the person reads – they interpret it in a certain way. Like oh, this theorem means I have to do this to that angle and do that, but the teacher might come along and say, "No, that's not the right way that you were supposed to do that." (Ke2, 8)

> I was always basically taught that you have to memorize, especially like in geometry. It was so many theorems and different things like that and the only way that I can think of, remember, the way to remember those theorems is to memorize them because you had to use them on the test or the quiz or whatever. So it was always a lot of memorization. Memorize your times' table. Memorize this. You have so much that you have to memorize. And one thing [in] college that we do [is] a lot of different things that you really don't need to memorize. It's just more or less application, just doing the problem or something like that. It wasn't like oh, this process – you have to know how to use this process with that problem or whatever. Because like in [*Probability and Statistics*] we really haven't come across anything that we really need to memorize. (Ke2, 9)

Yet, despite her recent mathematics experiences she maintained these old beliefs to the point where it interfered with her work in the study section.

> I feel like when someone asks a question, like she'll tell people to put their question up on the board that they were having problems with. It seems like to me the question just never gets answered. I'm just so in a routine that the teacher says this is the answer, you know? But she doesn't do that. It's more or less like a discussion and you have to figure out what the answer is. And I'm just sitting there like did she say the answer or was that the answer or what's the answer because she doesn't say, "Well students, this is the answer; this is how you do it," or whatever. She doesn't do that. (Ke2, 11)

Keona's vision of mathematics was not reflected in the instructional style found in the study section.

Keona had not experienced the same success in mathematics that Karen had achieved, but both clung to beliefs developed before taking any mathematics education courses. As a result of recent exposure to non-traditional teaching ideas, Keona could agree with elements of reform, but only because the teacher had supported them. In reality, she held onto her counterproductive beliefs, especially those concerning the authority of the mathematics teacher. Consequently, she became frustrated with and confused by an instructor who expected her students to think for themselves.

<u>Lucia's Entering Mathematical Beliefs and Related Experiences</u>

The data collected from Lucia's initial belief survey (see Appendix D, student 13) often presented conflicting mathematical beliefs. Although she was adamant about the need for understanding in mathematics, – "A student should know <u>why</u> the rules or reasons why a problem works, <u>not</u> just know the rule and carry on with a problem" – she selected 'sponge' as the best simile for a mathematics student. Perhaps she expected mathematics students to absorb the understanding behind the rules as well as the rules themselves. Lucia also seemingly embraced both counterproductive and productive beliefs when describing methods for solving a mathematics problem. "Sometimes its good to know step by step ways to do a problem, but sometimes it is good to explore and be able to do problems in other ways." My impression was that Lucia was attempting to adopt a fallibilist view of

mathematics but was finding it difficult to shed some of her counterproductive beliefs.

The discussion surrounding the first belief card sort confirmed that, at the very least, Lucia could verbalize productive beliefs concerning mathematics even if some of the statements she agreed with represented counterproductive beliefs (see Figure 12). Mathematics was not always predictable – "There's sometimes more than one solution" (Lu1, 12). The teacher was not always the absolute authority – "[I]f [students] see a different way of doing it and they think they understand it that way better, then why not" (Lu1, 13)? In addition, students ought to try to persevere when encountering a difficult mathematics problem – "Because when you look at a problem, you might get it right the first time, but if that way doesn't work then you just try another way until you really understand it... You just keep trying until you understand" (Lu2, 2). On the other hand, Lucia concluded this statement by saying, "You try all the different ways in the examples they give you in the book or examples the teacher gives you and then see which one you understand the most" (Lu2, 2). Perhaps, underneath all of the rhetoric Lucia still believed that outside authorities were the purveyors of the incontrovertible mathematical truths.

In fact, Lucia reported that any success she achieved in mathematics resulted from great teachers. All of these teachers shared a common trait, which she referred to when revising the belief about mathematics teachers (see the shaded statements in Figure 12). "A mathematics teacher should always teach new material in an organized, smooth and understanding fashion." She explained:

> I've had bad teachers and they probably still say one thing one way and then show it in another way and it's all confusing. And my good math teachers they always had everything – it seems like they always had everything planned out and they always did it in an organized way – certain ways to do it. (Lu2, 6)

Disagree
After you do math long enough it becomes very easy.
Before attempting a math problem, a person needs to know how to get the answer.
Math is a step by step process where there is always a single solution to be found.
Memorization is the most important tool to use when doing mathematics.
Students should not invent their own way of doing things in mathematics.
There are many ways to do a math problem, but the answers should be the same.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
In order to do mathematics well, students must absorb as much information as possible.
Learning math is often the result of a lot of trial and error.
Mathematics is made up of a lot of separate categories and unrelated rules.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should make discoveries in mathematics on their own, without it always being shown to them.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
There are some basic rules in mathematics, but there is always room for interpretation.

Figure 12. Results From Lucia's Card Sorts of Mathematical Beliefs.

A few specific teachers immediately came to Lucia's mind as examples of good and bad teachers.

> [M]y sixth grade teacher, Mr. [S], he had everything in a very specific way and how to do it and that's when I understood it. I always had to have everything organized, and this part is on the left side of the page and the answer is on the right with answer written and stuff like that. And then when I don't have a structure, I'm just sort of wandering by myself and not really seeing that, but Mr. [K] I remember he used to just – it seemed like he was out there and he never had - he'd assign homework problems and then never go

> over them. It seems like it happens at [this university], too. We always go over math problems and never cover them. (Lu2, 9)

The frustration she felt toward the lack of structure in her mathematics education courses suggested that she was not as confident in her ability to discover solution methods as she might lead one to believe.

Lucia's lack of confidence did not affect her belief that students had to persevere in order to be successful in mathematics. Recall that she identified herself as a Type II student and that she had to work hard to get good grades. Within her classes however, she encountered students who did not seem to struggle with the concepts being learned. Thus, she wrote "Strong math students always seem to know the material the first time they see it. Poorer math students need to put in more repetition, work, and concentration."

> [T]here's always those students in math class that can always raise their hand and know the answer. And then I feel like there's always the group that's struggling. I've seen that every single year since I had a math class. (Lu2, 6)

Because of her struggles, Lucia was determined to work hard – unlike the students who gave up after a few minutes on a tough problem. She found repetition helpful in her effort to succeed.

> I think I did better when I was given a lot of problems to do, like in elementary school and I always had a huge worksheet to do. When I came to the last problem, I always understood it. (Lu2, 9)

It is unclear whether she is referring to understanding the process used to complete the worksheet or the concept behind the process.

When repetition failed to help her understand, Lucia moved on to trial and error. All her life she had believed that, "Learning math is often the result of trial and error."

> [S]ince I have a hard time with math, it always seems like I'm trying something new and trying to figure out some way to understand it better, always crossing out and scratching out some of it. (Lu2, 6)

Her experiences with trial and error even extended to the methods teachers attempted to use to help her learn mathematics.

> I remember my [third grade] teacher always helping me. I always had to stay in for recess or something and she'd help me and I never would understand it. She tried a lot of different ways, but I just never got it. (Lu2, 8)

Again, Lucia's lack of confidence in her mathematical ability became evident; even when the mathematical authority "tried a lot of different ways," she still "never got it." No wonder Lucia believed her own success in mathematics depended on the teaching ability of the instructor.

The last belief Lucia rewrote read, "There are a lot of basic steps in math, but there is room for interpretation if needed." Interestingly, when describing experiences that related to his belief she concentrated on the first part, 'a lot of basic steps,' rather than the possibility for interpretation.

> Because it seems whenever I have a math class, there's always some sort of rule or theorem or anything that I have to remember – I have to memorize in order to understand the material. (Lu2, 5)

> Huge chunks that I remember in geometry… you could only understand this if you know this theorem, if you know how to do this, and then it may be enough. (Lu2, 8)

Although Lucia does not directly address the predictability of mathematics, these descriptions do allude to viewing mathematical rules as predictable.

By examining Lucia's beliefs, it became evident that at the core her mathematical belief system remained fairly counterproductive. Although she was able to intimate productive beliefs through her rationale and initial belief sort, the statements she felt strongest about resulted from early experiences and represented a traditional perspective. She was able to hear and repeat recent reform messages, but had not assimilated them into her core beliefs.

Lynn's Entering Mathematical Beliefs and Related Experiences

The data from Lynn's initial belief survey afforded more insight into her attitude toward doing mathematics than her beliefs about the subject (see Appendix D, student 20). Given the fact that Lynn identified herself as a Type III mathematics student, it was not surprising that 'hard' was a word she associated with mathematics. "[M]ath is hard to absorb like a sponge." "Math is a hard struggle that has a goal of attaining new information." "[D]ancing may be easy but math isn't." Thus, any examination of Lynn's beliefs was postponed until after the first card sort.

Based on Lynn's sorting of the belief statements alone, she seemed to hold counterproductive beliefs about perseverance in mathematics but a combination of counterproductive and productive beliefs regarding confidence and predictability (see Figure 13). As Lynn went on to explain why she had disagreed or agreed with each,

it became clear that traditional experiences in early mathematics classrooms had led her to hold counterproductive mathematical beliefs.

> I haven't experienced a lot of trial and error in trying to do math... I've had a lot of ways thrown at me at how you're supposed to do it and that's the way to do it. I'm trying to change that too, that thinking, too. (Ly1, 12)

Disagree
After you do math long enough it becomes very easy.
Before attempting a math problem, a person needs to know how to get the answer.
Learning math is often the result of a lot of trial and error.
Mathematics is made up of a lot of separate categories and unrelated rules.
Students should not invent their own way of doing things in mathematics.
Agree
A mathematics teacher should encourage and guide and not give rules to the student.
In order to do mathematics well, students must absorb as much information as possible.
Math is a step by step process where there is always a single solution to be found.
Memorization is the most important tool to use when doing mathematics.
Strong math students solve math problems quicker than weak mathematics students.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.
Students should make discoveries in mathematics on their own, without it always being shown to them.
The mathematics teacher needs to show how to solve math problems in a smooth fashion.
There are many ways to do a math problem, but the answers should be the same.
There are some basic rules in mathematics, but there is always room for interpretation.

Figure 13. Results From Lynn's Card Sorts of Mathematical Beliefs.

Lynn discussed her recent efforts to change her way of thinking about mathematics several times during the first interview.

> Interviewer: Students should not invent their own way of doing mathematics. You disagreed about that.
>
> Lynn: Well, to a degree. I mean, that just goes back to some of the experience I've had just recently that I've experienced in some

of classes that maybe students should be allowed to, so I'm still debating about that. (Ly1, 11)

Interviewer: Before attempting a math problem, a person needs to know how to get the answer.
Lynn: Partially, I put that in [the disagree pile] because it's like I keep thinking a person should know how to solve the problem and not worry about the answer... That's something I've had to switch in my thinking is to forget the answer and just try to figure how to solve the problem.
Interviewer: Would you agree with it if it said before attempting a math problem, a person needs to know the proper procedure?
Lynn: Yes. (Ly1, 12)

Interviewer: Math is a step-by-step process where there is always a single solution to be found.
Lynn: That was one of the iffy ones... Because I definitely see the step-by-step, but there have been times where I've done problems and I don't know if I've done them right or wrong, but I've had a different solution. And I'm now learning about probability and I even did in my science class I did a little oral report on probability, like the flipping of a coin. With that, you don't always get the same answer. (Ly1, 13-14)

This data suggests that Lynn entered the study section with a conflicted mathematical belief system. More recent experiences in mathematics demonstrated that "you don't always get the same answer" (unpredictable) and that "maybe students should be allowed to [invent their own ways of doing mathematics]" (confidence). Nonetheless, her modified beliefs hinted that the counterproductive axioms remained – "I keep thinking a person should know how to solve the problem [before attempting it]." After all, early experiences had reinforced that the teacher always imparted the proper procedure for solving a problem to the students and then the students practiced the procedure.

Since high school, Lynn had encountered several mathematics courses designed to model a different approach to teaching. Recall that she repeated *Number Concept* the maximum number of times (three) and was taking *Geometry* for the second time at the time of this research. Yet, these recent experiences related to only one of the four beliefs Lynn chose to rewrite (see the shaded statements in Figure 13): "There are many ways to do a math problem, but answers should be the same, and the student should also be able to say how they got their answer." Lynn felt that this belief resulted from her experiences in the second *Number Concepts* course.

> [W]hen we did our problems in our homework and we had to put them on the board, [the instructor] had every student who put the problem on the board explain what they did and it was a time where the whole class was watching. And she gave time for somebody else – if everybody solved the problem the same way we'd move on. But if somebody else had solved it a different way, she allowed that person to go up to the board, write it on the board, and explain how and what they did to get the same answer but not use the same method. And that helped a lot because the first time I took the class nobody did that. (Ly2, 6)

Based on this new experience, Lynn found that different methods could be successful when solving a mathematics problem, but the experience also reinforced the need "to get the same answer."

Lynn's second rewritten statement, "Some strong math students maybe will understand the material the first time, but poorer math students can also understand it the first time, too," resulted from various experiences throughout her entire life. While she reported that mathematics had always been a struggle, there were times when she experienced success.

> In consumer math in high school, I was shocked because I still felt poor because here I was taking this consumer math to fill a math credit because my

> scores in algebra weren't too good, and I was shocked at how well I was getting it the first time as compared to getting an algebraic equation. It made me feel good. (Ly2, 6)

Recently, even in failure Lynn had seen improvement in her mathematical ability.

> And then like now when I took *Geometry* this past summer, I compared it to when I took it in high school... I didn't understand a single thing in high school. And here I was going into it in college and I actually started understanding it. I was kind of shocked when I didn't pass the class... And it's like now taking it the second time I'm seeing what I grasped and what I didn't. So it helped out a lot and that's why I put that down because I don't think you should quote-unquote "label" a student poor and just think they're not gonna get it. They're gonna eventually get it or they might surprise you and get it and be ready to move on. (Ly2, 6-7)

Lynn's own experience as a poor mathematics student who could eventually succeed led her to believe that given time any student could persevere. Thus, my initial impression based on the first card sort that Lynn held counterproductive beliefs surrounding perseverance was in error.

According to Lynn, this perseverance followed certain stages: "To do mathematics well students need to absorb, understand, and put math into terms that they can use. With practice, they can build a basis of knowledge to make mathematics successful for them." She reported that recent experiences had influenced this rewrite, but that mostly it represented a lifelong belief.

> From the [*Geometry*] class I took this summer we had to read these journal articles that dealt with teaching math... And reading those articles it kind of reinforced a little bit of how I felt but it also gave me some new information... (Ly2, 6)

The articles and experiences in other education courses suggested that learning something new requires accessing students' prior knowledge. For Lynn, this meant focusing on memorization.

> Like when we were learning the areas of figures. I was using the algebraic equations I memorized in algebra back in high school. So I mean, if I hadn't memorized those, I wouldn't have recognized those equations or knowing hey, that's an area of a figure. I've got to figure that out. I wouldn't recognize it if I hadn't memorized it and if I hadn't put it into some kind of practice and built some knowledge on it. (Ly2, 6)

Lynn had taken the constructivist idea of building upon students' prior knowledge and used it to support her agreement with the belief that "Memorization is the most important tool to use when doing mathematics."

The final rewritten belief also represented a combination of productive and counterproductive beliefs. "There are some basic steps in mathematics, but there's always room for interpretation." As Lynn explained this position, she focused on the steps.

> In pre-algebra I learned the basic steps on like solving – let's see, an equation with variables. Like 2 times X equal 16. Well if you take the 2 from the one side and bring it over to the other to divide, you'll get the answer 8. So it's like knowing that little step and like the steps in adding and subtracting and now I'm learning that you're not supposed to call it borrowing anymore with the Chicago Math stuff. But knowing you have to carry the one, knowing that little basic step goes a long way. And that's where I think the basics – I don't really want to call them rules – but steps help build on, like I was saying, a basis on which you can put it into your own way and be able to solve problems. (Ly2, 8)

The basic steps represented the building blocks for future mathematics. However, Lynn's description of solving a simple linear equation and the steps used in adding and subtracting suggest procedural rather than conceptual knowledge. This raises the possibility that Lynn believed these basics reflect the predictable nature of mathematics and that it is the teacher's responsibility to present the basics, without mentioning borrowing, to the students. Once the basics had been learned, students

could begin trying to understand them. Several informants mentioned that they believed learning in mathematics represented this movement from memorizing facts to understanding them, which demonstrates how individuals attempt to maintain central beliefs (memorizing facts) by adding to the periphery of their belief systems instead.

Overall, Lynn's beliefs about the nature of mathematics were difficult to establish because of the length of her explanations and her proclivity to speak in generalities. While she spoke directly about the role perseverance plays in being mathematically successful, it was necessary to infer her beliefs concerning confidence and predictability based on her comments. Often, it seemed that her reported beliefs were both productive and counterproductive. Recently, she had heard and read about how instruction needed to change and she was willing to listen to the experts, but her own experiences suggested a different reality. Consequently, as was the case with most of the informants, Lynn entered the study section with conflicting views concerning the nature of mathematics, but mostly counterproductive beliefs.

Commonalities in Informants' Mathematical Beliefs and Experiences

During the first two interviews, each informant presented a unique perspective on the relationships between their view of mathematics and their experiences in mathematics classes. If one examines these findings as a whole, however, certain similarities become evident in the informants' beliefs regarding perseverance, confidence, and predictability in mathematics. Furthermore, while the experiences

related to these beliefs are personal to each informant, when prompted to examine the role early assessments played in the development of said beliefs, common relationships began to emerge among the informants. I conclude this section by addressing these two points.

Informants' Mathematical Beliefs

Figure 14 summarizes the mathematical beliefs of the seven informants (Anne, Cynthia, Janet, Karen, Keona, Lucia, and Lynn) based on the results presented in the preceding sections. In each of the belief areas – perseverance, confidence, and predictability – I analyzed the informants' belief data and determined whether my findings suggested they held 'CP' (all evidence suggested counterproductive beliefs), 'MCP' (most evidence suggested counterproductive beliefs), 'M' (the evidence was fairly evenly mixed for both counterproductive and productive), 'MP' (most evidence suggested productive beliefs), or 'P' (all evidence suggested productive beliefs) axioms. The results of this analysis suggested that most of the informants held the same general beliefs about the nature of mathematics.

	CP	MCP	M	MP	P
Perseverance			J^{II}	Ly^{III}	A^{II} C^{I} Ka^{I} Ke^{III} Lu^{II}
Confidence	A^{II} Ke^{III} Lu^{II} Ly^{III}	J^{II} Ka^{I}			C^{I}
Predictability	J^{II} Ke^{III}	A^{II} Ka^{I} Lu^{II} Ly^{III}	C^{I}		

Figure 14. Informant's General Mathematical Beliefs (Student Type Superscripted).

Of the seven informants, only Janet seemed to hold a mixture of productive and counterproductive beliefs concerning perseverance. The others agreed that being successful in mathematics often took time and thus would disagree with the statement, "A mathematics problem ought to be solved quickly or not at all." With the exception of Cynthia, all of the informants believed, to varying degrees, that the teacher represented a classroom's mathematical authority. This ranged from Keona's complete reliance on the teacher as expert to Karen's need for the teacher's assurances that she was doing a problem correctly. Cynthia alone reported believing students were capable of developing and evaluating the effectiveness of their own methods when solving problems, perhaps because of her own self-confidence concerning mathematics. Regarding predictability, each informant was willing to accept that there were numerous methods available for solving most mathematics problems, but this fact alone did not prove that they were thinking from the fallibilist perspective. Several informants continued to believe that there was only one correct answer to any mathematics problem. Furthermore, for all but one of the informants, these multiple methods represented pre-existing procedures that a teacher would demonstrate so that students could choose and perhaps memorize the one they liked best. Again, Cynthia's perspective was the lone exception; she believed the methods developed as students experimented in mathematics.

Therefore, it appears that all but two of the informants shared several common beliefs regarding mathematics. Paraphrased, these beliefs describe mathematics as containing a multitude of methods that an authority passes along to students who are

trying diligently to solve a mathematics problem. As the informants attempted to determine the experiences that contributed to the development of these beliefs, they reported a variety of experiences unique to each individual. The results presented in prior sections demonstrated that the majority of these formative experiences were traditional, especially related to assessments in mathematics.

The Effect of Traditional Mathematics Assessments on Beliefs

Further investigation into the experiences responsible for developing these core beliefs uncovered several significant connections between the informants' counterproductive mathematical beliefs and their early assessment experiences. These relationships began to emerge during a focused analysis of data related to assessments or beliefs collected during the first four interviews. The results from the fourth interviews' traditional testing vignette were especially helpful. Eight relationships connecting past assessment experiences to counterproductive beliefs emerged from the data. The informants confirmed or denied these relationships during the final interview. While none of the relationships garnered unanimous acceptance from the informants, four relationships did earn strong support (see Past Assessment Experiences in Appendix D).

The most frequently affirmed relationship was, "Problems on math tests had only one right answer so I developed the belief that there is only one right answer to any math problem." All but one of the informants reported that this relationship

reflected their own perception of how they developed this belief in an absolute answer. Janet's comments were typical.

> This teacher was so specific that if you didn't have the answer, the numerical answer with the unit and then squared, cubed, whatever it was, that precise of an answer, it was wrong. So there were three things that you had to have there to make that answer right. There was no flexibility in that at all. (J4, 11)

Karen agreed that this type of experience was a contributing, but not sole, factor.

The other relationship that all seven agreed had at least reinforced a belief they held read, "Teachers wanted me to show my work so that they could be sure I was using the right method – the method they had shown me. Therefore, I began to believe that there was only one way to solve a problem." This statement was based on Keona's earlier explanation of why she did not think students should invent their own way to solve a mathematics problem.

> [Teachers] always gave us a specific way in which they wanted a problem solved and they wanted you to show your work that you did do it that way and not just give an answer. (Ke2, 2-3)

Five of the informants strongly agreed that this relationship between traditional testing and the counterproductive belief reflected their own belief development.

On the last two relationships, only Karen disagreed that traditional testing was in any way related to the development of the mathematical beliefs being discussed. The first concerned memorizing facts and rules: "The questions I saw on math tests were similar to problems I had seen solved before. As a result I came to believe that in order to do mathematics I had to memorize the method of solution." Cynthia strongly agreed and recalled learning her times' table in fourth grade.

> [W]e had a test every Friday on – first we did the one's, then we did the two's, then we did the three's, and it was just pure memorization... [I]t was, "This is how you do it. These are the correct answers. 4 x 3 is 12. That's the answer. 4 x 4 is 16. Now memorize them and we're gonna have a test on them on Friday." That was just the way [teachers] did things... I never visualized that '4 x 3' was 4 piles of 3 erasers in each or whatever. (C4, 15-16)

While this relationship stressed the predictability of mathematics, it also eluded to reliance on an outside authority, which was the focus of the last relationship that earned strong acceptance. "Since the teacher always showed the class how to solve problems similar to those on the test, I came to believe that in order to learn mathematics it had to be shown to me by the teacher." Although Anne could identify other factors that contributed to her lack of confidence in her ability to solve mathematics problems on her own, she did recognize how other students might interpret such an experience:

> [Students are] getting from [traditional testing experiences in mathematics] that the teacher stands before them and tells them how to do it and that's how you learn. (A4, 7)

Hence, to varying extents, six of the seven informants agreed that taking traditional mathematics tests had played a part in the development of counterproductive beliefs regarding predictability and confidence in mathematics.

While discussing the traditional test vignette, Janet described a representative picture of what the majority of the informants believed a student would learn from a typical pencil-and-paper mathematics test:

> That the test is absolute, that there's only one right answer and one way to do it, and that you've got to spit back what the teacher wants to see. The learning process is for the test and for the score and that's about it. (J4, 10-11)

In fact, many of the informants admitted that at some point they had held these very same counterproductive beliefs themselves. Although the informants reported that recent mathematical experiences had enabled them to view mathematics differently, in most cases their central beliefs remained counterproductive. For example, informants no longer believed that a problem in mathematics could be solved in only one way, but the multiple methods most now embraced still existed outside of the students' experiences and consequently would be considered counterproductive. The next section examines whether the alternative assessments employed in the study section succeeded in affecting any change in the informants' long-held central beliefs concerning the absolute nature of mathematics.

Pre-service Elementary Teachers' Exit Beliefs and Related Influences

On the last scheduled day of class, just before the final, the students from the study section of the *Probability and Statistics* course completed an exit survey similar to the one given at the start of the semester (see Appendix C). Again, the students selected the best and worst similes for teaching, learning, and doing mathematics, explaining each choice. A comparison of these results with students' original choices and rationale provided the first evidence of whether any changes had occurred in students' mathematical beliefs. Additionally, the exit questionnaire also asked the students to identify and order from greatest to least the experiences from the semester, within and outside the study section, which might have influenced their view of mathematics. The final set of interviews also attempted to examine shifts in

informants' mathematical beliefs over the course of the semester and determine the causes behind any variations. This section addresses data from the exit survey followed by the results from the informants' final interviews.

A Comparison of the Initial and Exit Survey Results

As I pointed out in the analysis of the initial survey results, the data provided by such an instrument is incapable of addressing completely the complexities of an individual's mathematical belief system. At best, the information gleaned from such sources offers a starting point for further research. Accordingly, the comparisons made within this section represent the first step in my attempts to identify changes in the mathematical beliefs of the students enrolled in the study section and the influences that might have contributed to these changes.

A comparison of the students' initial and exiting simile choices and responses led to five categories of movement from counterproductive to productive beliefs (see Appendix D for initial and exit results). 'Strong productive' movement meant a student's exit explanations in two or more of the simile choices demonstrated greater fallibilist thinking than his or her initial statements. Students identified as showing only 'slight productive' movement offered evidence of an increased fallibilist view in only one of the simile selections. As the label suggests, 'no movement' represents students whose rationale provided no indication of any change. Shifts toward counterproductive mathematical beliefs were labeled as 'slight counterproductive' or

'strong counterproductive' using the same rules as stated above, but this time toward the absolutist perspective.

Table 6 presents a breakdown of the thirty-one students who completed both surveys. (Informant's student identification numbers are in bold.) Although the results from the analysis of the students' exit belief surveys suggested that many of the students held more productive mathematical beliefs at the end of the study, unclear responses, lack of responses, and the misinterpretation of directions continued to be a problem. Informant interviews would clarify the true nature of any shift.

Table 6

Students' Shifts Between Counterproductive and Productive Beliefs
as Evidenced From a Comparison of Their Initial and Exit Belief Surveys

Movement	Students	Student Identification Numbers
Strong Productive	8	6, 7, **14**, **16**, 19, **20**, 24, and 25
Slight Productive	6	**1**, 5, 10, 13, 23, and 32
No Movement	12	2, 3, 4, 8, 11, 12, 15, 18, 21, 22, 27, and 31
Slight Counterproductive	4	9, **28**, 29, and 30
Strong Counterproductive	1	**26**

The final piece of the exit belief survey asked students to identify and order the recent experiences that influenced their simile choices and explanations (see Appendix C for this survey). Eight experiences received more than ten votes as one

of the top six influences (see Table 7). Notice that twelve students assigned assessments as the greatest influence on their simile selections. Another three put it as number two, and one each at five and six, meaning a total of 17 students felt that the course's assessments had affected their recent view of mathematics. While 19 students selected 'Interaction with Classmates' and 'Student Led Discussions' as influential, neither of these factors was ranked first by as many students. Therefore, the greatest number of the students identified the study section's alternative assessments as the most important influence on their view of teaching, learning, and doing mathematics.

Table 7

Students' Reported Influences on Simile Selections

Influence	1st	2nd	3rd	4th	5th	6th	Total
Interaction with classmates	4	2	5	2	6	0	19
Student led discussions	3	4	5	1	2	4	19
Assessments: projects & quizzes	12	3	0	0	1	1	17
Whole class discussions	0	2	2	4	3	5	16
Group activities: non-graded	2	3	4	5	0	1	15
Working with calculators	1	2	2	5	2	3	15
Interaction with instructor	1	4	1	3	3	1	13
Interaction with children	3	3	2	0	0	4	12

The relationships between students' assessment experiences in the study section and any changes in their mathematical beliefs were difficult to ascertain using the survey data alone. Afterall, the intent of this analysis was to offer a framework from which to examine the informant data. During the final interviews, a more complete picture of these relationships emerged.

Changes in Anne's Mathematical Beliefs and Related Influences

Anne represented the lone student from the study section whose exit survey indicated that her mathematical beliefs had become 'strongly counterproductive' during the semester. Although her choices for best and worst simile for a mathematics teacher still implied that teachers should refrain from "telling you everything to do," her choice for the best simile for a mathematics student represented a significant shift. Originally Anne had selected explorer, but on the exit survey she chose sponge and wrote, "absorbing everything." I asked her about this switch during the final interview and she explained,

> I still think math students need to just explore different ways, but I also think that they should absorb everything. And by absorbing everything, then they will be able to understand it better and to regurgitate it back to someone else. So I don't know – I switched, but I agree with both of them. (A5, 3)

I then suggested that these two similes represented distinct styles – explorer brings to mind a more active involvement while sponge a more passive one. Anne responded, "I think the active is probably what a math student should be" (A5, 3). While Anne's choice of sponge seemed a step backwards, her explanation during the final interview actually indicated a lack of movement.

The results of the belief card sort during the final interview verified the static nature of Anne's beliefs. She sorted the belief statements into exactly the same piles as she had during the first interview. As in the initial card sort, Anne even struggled with whether to agree or disagree with the belief that 'Learning math is often the result of a lot of trial and error,' finally placing it in the disagree pile.

> I think that there is an actual sequence of how to learn it. But I also agree that when you get something wrong, you're learning too. And so I think trial and error could be getting something wrong or getting it right. So it's kind of in the middle because – I don't know. I agree and I disagree. (A5, 9)

Recall that in earlier interviews, Anne had disagreed with 'trial and error' when it related to learning something new in mathematics, but agreed that it was helpful when doing mathematics. As she continued, it became apparent that this dual view remained:

> Yeah, because you might – the first time you do a math problem you might get it wrong, the answer wrong. Maybe you did two steps in different order or something. But the next time you do it, you're going to realize well, I did it wrong that time. I need to try something a little different. And then maybe you'll flip-flop the two steps and you'll get the right – the answer that you were told that it should be. So I don't know that that's considered trial and error. I'm not saying it's the only way to learn, but I think that it does happen. (A5, 10)

These comments offer evidence that Anne viewed mathematics as a sequence of prescribed steps ("Maybe you did two steps in different order or something") resulting in an answer that is judged correct or incorrect by some outside authority ("the answer that you were told that it should be"). Even after completing the *Probability and Statistics* course, Anne continued to hold counterproductive mathematical beliefs in regards to predictability and confidence.

As Anne revisited her rewritten belief statements from the start of the semester, the lack of movement was further confirmed. She still felt strongly about each of the four beliefs and chose not to make any changes. Despite the analysis of the exit survey data that suggested 'strong counterproductive' movement, it appeared that the study course had no affect on Anne's beliefs about the nature of mathematics whatsoever. The characterizations made in Figure 14 – perseverance (productive), confidence (counterproductive), and predictability (mostly counterproductive) – continued to apply with no perceptible shifts.

Because no movement occurred in Anne's mathematical beliefs, it would appear meaningless to address the influences that she identified as contributing to her simile choices and explanations on the exit survey. A discrepancy arose in the data, however, that required some attention. Although Anne did not select assessments as one of the top six factors from the semester (see Appendix D), during the relationship confirmation activity she strongly agreed with all but one of the statements concerning the impact the study section's alternative assessments had on her beliefs. Recall that strongly agreeing with a statement in this activity amounted to saying, "That's what happened to me." Anne had no explanation for the inconsistencies except to say that she had felt rushed when completing the survey and tired during the relationship confirmation activity. My attempts to clarify her perception of the relationships between her beliefs and the assessments in ***Probability and Statistics*** proved fruitless as she vacillated in her responses. Fortunately, there were six other

informants available to help provide insight into the effect that the alternative assessments had on their mathematical beliefs.

Changes in Cynthia's Mathematical Beliefs and Related Influences

Based on a comparison of her survey responses, it appeared that over the semester Cynthia had experienced a 'strong productive' shift in her beliefs about teaching and learning mathematics, while her view of doing mathematics remained constant. Although she selected 'coach' as the best simile for a mathematics teacher in both surveys, there was a difference in her interpretations. In her initial explanation, Cynthia discussed the "rules of math" (BMTS, 14), but the exit rationale suggested that teachers "should give the students the tools to discover what works" (BMTS2[5], 14). When choosing the best mathematics student simile, she switched from explorer to inventor. This represented a modest shift, especially since she spoke of discovering in the explanation for inventor, but it did provide a hint that Cynthia was beginning to see mathematics as something student constructed rather than a product they might uncover in their journeys. At the beginning of the study, I identified Cynthia as holding productive beliefs concerning perseverance and confidence and a mixture of productive and counterproductive beliefs in regards to predictability. The findings from the exit survey indicated that her view of the predictability nature of mathematics might now be more fallibilist.

[5] BMTS2 means the best mathematics teacher simile from the second (2) or exit survey. Any survey data that ends in two represents the exit belief survey. No number, BMTS, represents the initial belief survey.

Cynthia, however, was unwilling to accept that these differences represented any changes in her beliefs. When she discussed the comparisons in the fifth interview, she felt comfortable that what she had written in the initial and exit surveys was very close. Regarding the explorer to inventor shift she said that they were "kind of the same thing" (C5, 4) because the term discover was used to explain her choice of inventor. The exit belief card sort further supported Cynthia's perception that almost no movement had occurred within her mathematical belief system. Although two cards switched piles – "after you do math long enough it becomes very easy" and "in order to do mathematics well, students must absorb as much information as possible" moved from agree to disagree – Cynthia's explanation confirmed that, in actuality, there was no change. For example, her original explanation of the placement of the first statement was:

> I think things like adding and subtracting that we've all done since first and second grade is very easy just to whip off the answers. I think you get into other things that you don't do all the time on a daily basis and things that you don't do over and over again in every class and that are a part of everything else you do, those are more difficult because you don't do them all the time… (C1, 10)

In the later interview she explained:

> I guess that's when I was kind of thinking – that after you do math long enough – I guess probably I would agree if you're talking about addition and subtraction and multiplication. But if you keep going in math and it gets harder, then you're visiting new things and I guess that wouldn't be easy then. (C5, 6-7)

Similarly, her rationale for switching the 'absorbing' statement from the agree pile to the disagree pile represented a shift in how she interpreted the statement, not her view of mathematics.

As Cynthia reviewed the beliefs she had written, she decided only one needed any revising. It was probably no coincidence that this rewrite came from the belief statement concerning 'absorbing.'

> I guess this one, number three, listening to and processing a lot of different information is the key to understanding math. I guess the only thing I'd probably add to that would be listening to like experimenting with and processing a lot so you're not just listening but you're interacting more also. (C5, 7)

At this point in the interview, we had discussed the different interpretations of absorbing several times and Cynthia had become sensitive to wanting to present a more active approach to learning mathematics. Hence, it seems that this change is more representative of her attempt to clarify her thinking than evidence of any shift in her mathematical beliefs.

As was the case with Anne, the comparison of surveys implied that Cynthia's beliefs related to predictability had experienced a strong shift, although the interview results did not support this finding. Another similarity between the two informants was found during the analysis of reported influences. Both Anne and Cynthia had dismissed the study section's assessments as any kind of factor in their recent beliefs, yet they strongly agreed with several statements during the relationship confirmation related to alternative assessments and productive mathematical beliefs.

For Cynthia, this conflict seemed to stem from focusing on assessments other than those employed by the instructor in the study section. In regards to leaving 'assessments' off her list of top six influences, Cynthia explained:

> I'm not real hung up on the assessment part as I am the learning part of it. I don't know. Maybe they do play more of a role in my beliefs or whatever

> than I think they do, but when I think about math and stuff, I think about the way teachers taught it and the way we did stuff in class and homework and things like that. I don't really think about the one hour every three or four weeks that you took a test. It doesn't really stand out as much. (C5, 5)

Cynthia's comments indicate that she interpreted the survey to be referring to 'tests' in general and not the study section's alternative assessments. Statements made during earlier interviews confirmed that the teachers were an important influence on her beliefs.

> Everything that I was coming up with was stuff from [*Number Concepts*] and [*Geometry*] and things like that. And I think that kind of goes back to the students believing what their teachers believe and I just happen to have – now I have the beliefs of the teachers that I have now, not the teachers that I had before. If the teachers were doing the things that my professors are doing now before, then I'd have some things to fall back on but now my beliefs are coming from things I've only done in the past year and a half. (C4, 23)

Those things included alternative assessments similar to those done in the *Probability and Statistics* study section.

> Interviewer: And prior to this class, you said that you'd done this before, though, the writing assessments.
>
> Cynthia: In [*Geometry*], but that one was on our own. We wrote it on our own so I think we had some of that where you were asking yourself those questions, but somebody else wasn't critiquing your ideas at that time, so I think you get more of it when you're working with somebody else. (C3, 2)

This explains why Cynthia strongly agreed with the two statements relating working on assessments in groups to changes in mathematical beliefs (see Appendix D).

Evidently, Cynthia misunderstood the directions from the second part of the relationship confirmation and strongly agreed with statements related to earlier assessment experiences and not strictly the alternative assessments from the study section. Another statement she strongly agreed with involved no longer believing

memorization to be an integral part of doing mathematics; this supported my assumption that she misinterpreted the directions. Considering that she did not believe doing mathematics meant memorizing procedures upon entering the study section, the assessments that originally challenged this belief must have occurred before enrolling in the course. Therefore, it seems that the alternative assessments given in Cynthia's section of *Probability and Statistics* had little if any affect on how she viewed the nature of mathematics.

Changes in Janet's Mathematical Beliefs and Related Influences

Recall from Figure 14, that at the onset of this research Janet's beliefs regarding perseverance, confidence, and predictability in mathematics had been mixed, mostly counterproductive and counterproductive respectively. The analysis of her initial and exit belief surveys suggested that these beliefs had encountered a strong productive shift over the course of the study. This shift was most evident in her choices and rationale for best similes for a mathematics student and for doing mathematics. In the former, Janet switched from 'sponge' – "learn (or soak up) everything the instructor has to teach" (BMSS, 16) – to explorer – "the student should want to explore new concepts in the process of learning" (BMSS2, 16). Similarly, Janet went from feeling that doing mathematics was like 'playing a game' because "it should involve procedures for winning or getting the correct information" (BDMS, 16) to 'conducting an experiment.' Her reasoning for the latter, again stressed the process – "Math should be a trial and error to understand process" (BDMS2, 16). In

both explanations, Janet offered evidence that although she had at one time viewed mathematics as a predictable product to pass along [counterproductive], she now saw mathematics as a process of learning [productive].

Janet also demonstrated more productive beliefs concerning mathematical confidence. In explaining the differences between the initial and exit surveys, she described a change in her perspective of learning mathematics from passive observer to active participant.

> [T]he sponge is not a very active – just kind of laying there soaking it up. Whereas the explorer is actually doing a little bit of effort there and getting involved with the material. (J5, 4)
>
> I think that although they're both – like a game as an exploration and this is trial and error which is an exploration, I think that it's – I'm trying to think of the word – more involved in [conducting an experiment]. Otherwise, you're maybe looking for more instruction [with playing the game] where [with conducting an experiment] you're actually more involved. (J5, 2-3)

The emphasis in learning was no longer placed on the instructor's ability to present the material. Instead, the student became more involved in exploring the mathematics. While such statements did not eliminate the possibility that Janet continued to believe mathematics existed as truths beyond human experience, I would agree with her that "overall my thinking of math has improved tremendously this semester" (J5, 6).

Unlike Anne and Cynthia, Janet's changes in mathematical perspective were supported by the exit belief card sort. Four statements switched completely and three others moved to a more neutral position. Janet saw a connection between three of the statements that switched completely – 'There are some basic rules in mathematics,

but there is always room for interpretation' (disagree to agree); 'Students should not invent their own way of doing things in mathematics' (agree to disagree); and 'There are many ways to do a math problem, but the answers should be the same' (agree to disagree). When I asked her about the connection, she said, "That math is maybe not so finite, that there's always different – that there are different ways to do things and different ways to approach it" (J5, 9). The factor that Janet identified as being most influential in contributing to this change was the "classroom work, where we're working in a class and being told that [getting different answers is] okay. Well, you know, maybe this is a different idea, maybe this is a different approach" (J5, 9). Thus, experiencing mathematics as being more than pre-established facts and procedures had successfully challenged Janet's belief in the predictability of mathematics.

The discussion surrounding the four remaining statements that had shifted from Janet's original card sort provided further evidence that her beliefs were more productive. As she described why she felt more neutral towards statements related to teachers 'show[ing] how to solve problems in a smooth fashion' and students 'absorb[ing] as much information as possible,' her decreased reliance on the teacher as the mathematical authority was confirmed.

> I think that the teachers, when they're showing an example, it helps that they don't make a bunch of mistakes so it confuses you more. But I don't think that teachers always have to show how to do things for you to figure out how to do them. (J5, 9)

> You need to learn in order to do math well but it's not that it's an absolute that you have to do it well by absorbing. Some students work better by just teaching themselves. (J5, 9)

Furthermore, her slight shift of 'Mathematics is made up of a lot of separate categories and unrelated rules' to a neutral position demonstrated that she was starting to recognize relationships within the subject. She admitted, however, that:

> it's hard for me to make connections in math from one area to another, so it's like sometimes it seems like a mumbo jumbo that's not related at all, but I'm sure that it is somehow. I just haven't made all those connections. (J5, 9-10)

Finally, she no longer agreed that 'after you do math long enough it becomes very easy.' While I would assume that this represented a productive shift in how she saw perseverance, she never discussed why she now disagreed with the statement. In fact, she still agreed with statements related to strong mathematics students solving problems faster than poor mathematics students do. As a result, I am hesitant to suggest that Janet's strong productive shift applied as much to beliefs related to perseverance as it did to confidence and predictability.

The changes in these two areas of Janet's mathematical beliefs were confirmed as she revisited the strong belief statements written during her second interview. She explained:

> I wanted to make some alterations too. Number one is that "In order to do math well, students must absorb as much information as possible. You must be able to absorb the information or you can't learn the concepts." I just wanted to change that to – "You must be able to absorb to be able to understand it" to change it to "You must be able to understand." Somehow 'absorb' sounds like – again, we were talking about being passive and I think that instead of just having the information filtered into your brain, I think that maybe if you're working through it you may be able to do it – understand it better. And then the other one [I want to change] is "There are many ways to do the math problem but the answer should always be the same. It seems like there's always one right answer." I think that we were just talking about – I think there's more than one right answer depending on your approach to the problem and it doesn't always have to be done the exact same way all the time. (J5, 11)

No longer did she view mathematics as a pre-existing product that students passively absorbed from their teacher and textbooks. Thus, all the data supports the initial findings that Janet's counterproductive beliefs related to reliance on a mathematical authority figure and the predictability of mathematics had become more productive.

During an earlier interview, Janet identified several experiences from the study section that were challenging those ingrained counterproductive beliefs about what it meant to do mathematics.

> The biggest differences are probably the group work. And now that I've had Tuesday's class, I can tell you that also the assessment of working with notes to the extent that we did in our class on Tuesday for [*Evaluating Responses*] was a completely new experience. And evaluating sample work as part of an assessment was a different experience. Just coming up with brand new answers to questions was a new experience. So lots of different things in this class are coming up. (J4, 1)

In fact, on the exit survey, Janet named assessments as the most influential factor affecting her simile selections and explanations. Because the alternative assessments were so different from what she had experienced in previous mathematics classes, Janet explained that "I think it changed some of those really hard-and-fast thoughts that I've had all these years" (J5, 4-5). Therefore, it is not surprising that Janet strongly agreed with all of the relationship statements between altered beliefs and the assessments from the *Probability and Statistics* course except for the last one (see Appendix D). She placed herself between agree and strongly agree on this last one because of her experience with the final exam.

> Well, what I was thinking was we had two main tests in [*Probability and Statistics*] and one was a mid-term where we had notes with us to make evaluations, and then the final yesterday where we had just a little card. And I felt like yesterday's work I really couldn't rely on anything I had on the card.

> It really had to be memorization of different things that I needed to know how to process. So, although the assessments outside of class eliminated a lot of the pressure of tests and so I was able to get a deeper level of understanding, I still felt like I had to memorize certain things to do well on the test. (J5, 15)

Because the final exam resembled a traditional, paper-and-pencil mathematics test, Janet felt it reflected certain counterproductive beliefs.

Strong agreement with the next to the last relationship – 'Having time to do the assessment outside class eliminates the pressure I often feel during timed tests. I no longer believe that doing mathematics well means I have to do it quickly' – implies that Janet's counterproductive beliefs concerning perseverance might also have been swayed. There is simply not enough evidence, however, to support this assumption.

Overall, Janet considered the semester a positive experience and felt she had experienced growth in her perception of mathematics.

> I think my method of learning math has changed a lot this semester. Instead of just trying to sit in class and get everything I can and then go home and try and repeat it, do the homework problems, and come back to class, I think that I'm working through the material at a deeper level than what I used to and so I think that there's a different type of understanding. And so instead of just trying to memorize as much as I can, now I work through the material differently. (J5, 11)

Janet's data corroborates this growth. While the results related to shifts in her beliefs about perseverance in mathematics were inconclusive, her beliefs in regards to confidence and predictability had clearly become mostly productive. Janet reported that the course's alternative assessments were the number one contributor in these changes. Thus, at least in this case, the ability of assessments to influence certain mathematical beliefs was evident.

Changes in Karen's Mathematical Beliefs and Related Influences

Karen's explanations on the exit belief survey hinted at a 'slightly productive' shift, but only in the area of teaching mathematics. Originally, Karen selected 'gardener' as the best simile for a mathematics teacher "because they plant the seeds in your head and help them to grow" (BMTS, 1). On the exit survey, however, she felt that 'gardener' represented the worst simile "because they do not dig up information and plant the seed into your head" (WMTS2, 1). Karen explained that the change had resulted:

> from [the instructor] saying not to directly give the answers but give the questions that will help [students] to perceive the answer or give the information that will help them get to the answer itself… So I think the gardener kind of – they don't – they help, like the teachers help the information grow in your head but they don't necessarily plant it there. They help you to plant the information, to help it grow. (Ka5, 3)

These comments indicate that the study section had somewhat altered Karen's vision of teaching mathematics, but not of mathematics itself.

Further evidence that any change was limited to pedagogical beliefs rather than conceptual beliefs can be found in the remainder of Karen's exit survey. 'Sponge' continued to represent her best simile for a mathematics student, and although she no longer thought 'inventor' was the worst, her rationale behind selecting 'playing a game' as the worst simile for doing mathematics nevertheless implied predictability. "Playing a game because most of the time you make the rules up as you go along. While mathematics leaves room for adjustments there is no room to make your own way of doing things" (WDMS2, 1). Karen explained:

> I think basically what I meant was the room for adjustment is you can kind of alter the way things are going but don't maybe start from scratch and try to make up a completely new process. I know when I say people have their own way of doing things they still have the same concepts. They've just altered them to fit their needs. So it's the same basic way of doing it. They've just taken it and kind of done it a little different, like a detour... But yet there's always ways to alter it to fit people's needs. (Ka5, 3)

Karen continued to hold mostly counterproductive beliefs in relationship to confidence and predictability.

Changing her beliefs concerning instruction without altering her beliefs about mathematics in general continued to be a theme as Karen resorted the belief statements. All the cards remained in the same piles as the original sort with the exception of one: The mathematics teacher needs to show how to solve math problems in a smooth fashion. Initially, Karen agreed with this statement, but at the end of the semester she placed it in the disagree pile. Karen identified the instructor as influential in the switch, but that the 'student led discussions' also contributed.

> I think that if it's shown smoothly you never have room to catch someone's error – to know that you're doing it correctly. So if someone makes a mistake or an error on the board and then you catch it, you know that you have the right idea of what's going on instead of saying, "Oh yeah, okay, I know how to do that." (Ka5, 6)

Again, the methods were the focus of Karen's attention and not the mathematics.

Because none of Karen's rewritten beliefs dealt with teaching mathematics, it was not surprising that she did not wish to revise any of them. Hence, in regards to perseverance she persisted to exhibit productive beliefs, while her beliefs concerning confidence and predictability were still mostly counterproductive. During the fourth

interview and in a journal entry, Karen further documented that she would rely on an outside authority to present to her mathematical truths.

> My only thing was like we went to do – standard deviation was on it. We had problems before to prepare us to do them. Well, I know I've done it before but our book was no help to me and I wrote that in my journal that I found no help from the book when it came to that. I went to a high school book and got the information I needed to do it. And she wanted to know what the problem was and I haven't told her yet, but they have like a little paragraph and then they go onto problems about it. So they do underneath the problems and stuff, but for me I'd rather see it and then see it then see the problems after it. Instead of incorporating it all into the problem, explain it to me first and then show me the problem so I know what I'm doing as I see the problems instead of seeing a problem and saying oh, that's what they did. It just would make it a lot easier for me. (Ka4, 14)

> Journal Entry (10/13/98): Book needs a better description of standard deviation {*Why is that? What is missing? – Instructor's response*} I just couldn't follow. I need a better model to follow.

By virtue of this preponderance of evidence, it appears that Karen's mathematical beliefs remained static throughout the semester. Only her beliefs related to teaching mathematics had become more non-traditional.

While Karen selected assessment as one of the factors from the semester that affected her exiting view of teaching mathematics, it did not make the top six. She supported this during the relationship confirmation activity (see Appendix D). According to her, grades were not that important, so assessment experiences were not influential in developing her mathematical beliefs.

Karen:	I think that just from past math experiences – I've felt [experiences] create the beliefs rather than just the assessments I've done.
Interviewer:	Like what experiences?
Karen:	Just the teachers and how they explained things and how I perceived the way they do different things. (Ka5, 11)

Because Karen concentrated on mathematical pedagogy rather than concepts, it seems reasonable that the teacher would represent the most significant factor affecting her perception of teaching mathematics – what she considered her mathematical beliefs.

Even Karen's strong agreement with statements suggesting that working in groups on assessments had helped her to better understand the material and that memorization was less desirable than understanding because that was what the course assessments stressed was misleading. Considering all the evidence discussed in this section, I would propose that what Karen found attractive in these statements were the methods – working in groups and teaching for understanding rather than memorizing steps. Ultimately, Karen's counterproductive mathematical beliefs were impervious to the instructor's attempts to challenge them through non-traditional instruction and alternative assessments because she remained focused on pedagogical issues.

<u>Changes in Keona's Mathematical Beliefs and Related Influences</u>

Recall that many of Keona's responses on her initial belief survey focused on mathematical affect rather than her ideal view of teaching, learning, and doing mathematics. Certain responses on Keona's exit survey also stressed her attitude toward mathematics, not her beliefs, making it difficult to determine if any changes had occurred in her mathematical view. Hence, it seemed that the results from the comparison of the two surveys, which indicated a slight counterproductive shift, would carry little weight.

The lone piece of evidence, Keona replaced 'explorer' with 'sponge' as the best simile for a mathematics student, did suggest, however, that she ended the semester with even less confidence in students' ability to learn through exploration. When I asked Keona about the change she explained, "I feel it's a little of both. That's probably why one time I picked explorer and then one time I picked sponge because I feel like you can do both in mathematics, which you can" (Ke5, 4). She believed that both similes fit her view of mathematics.

> You have so many different areas of math. It's not just like one set thing. So that's why I said the math student is the explorer because you explore so many different topics of math... Well, with the sponge one ... I thought was most true for a math student because you do have a lot of different concepts that you have to learn and absorb. (Ke5, 2-3)

Although her explanation eliminated the possibility of a counterproductive shift, for Keona, mathematics continued to exist outside her own experiences – a product to absorb. In regards to confidence and predictability, she appeared to have retained the counterproductive beliefs she brought to the study section. This assumption was tested further using the activities conducted during the final interview.

The results from the exit belief card sort challenged the assumption that Keona's belief system had remained static because five statements switched piles. Nevertheless, her subsequent commentary confirmed that any changes regarding perseverance, confidence, and predictability were negligible. One of the statements that she now agreed with, describing students as absorbing information, mirrored her selection of sponge on the exit survey, but her explanation reflected our earlier discussion that mathematics involved both exploring and absorbing.

> The reason why I changed that one is because of the – back to the [survey discussion] that we just had. It's not always, like I said at the beginning, it's not always absorbing information whereas it's not always exploring. You have to do a little of both. (Ke5, 9)

Two of the statements presented conflicting data on Keona's position concerning perseverance in mathematics. She now disagreed with the statement, "After you do math long enough, it becomes very easy" and her explanation implied she continued to hold productive beliefs in this area.

> I don't think if I was to really go over geometry for a really long time that I would probably get the whole concepts much easier. But I wouldn't think geometry in general was easy because look how long it took me to get to the understanding of the whole concepts of geometry. (Ke5, 8)

Her recent acceptance that strong mathematics students solve problems quicker than weak students suggested that her beliefs were not as productive as originally presumed though.

> I don't really consider myself a really good math student so I don't really know if I would say that a strong math student would solve a problem quicker. I would think they would since they are very strong in math. (Ke5, 10)

The last two statements involved the idea that every mathematics problem has a single correct answer. Based on Keona's agreement with these statements during the first interview, she was identified as viewing mathematics as predictable, ergo absolute. She now disagreed with them both because of her experiences in the study section.

> Math is a step-by-step process where there is always a single solution to be found… That one, the reason I changed my mind with that is because I did realize it when we were doing one of our assessments that all math problems don't always have the same answer because there was one particular problem I do remember. She said it had more than one answer… (Ke5, 7)

> There are many ways to do a math problem but the answer should be the same… That's closely related to that one, going back to the assessment. That really brought out to me because when I saw that question, that was the first thing I thought about. When I was with David, I said that it's only one answer for every problem. No, it's not just one answer for every problem. Now I know that. (Ke5, 7-8)

Her experience meeting with me had made her aware of this counterproductive belief and thereby sensitive to an assessment experience where this belief proved untrue. The true factor in the experience had been the teacher, however, who Keona continued to consider the mathematical authority.

> I would have really liked if she had went over that one specific problem and showed us what were other answers for that problem because I couldn't figure it out. (Ke5, 8)

Keona believed that a mathematics problem could have more than one right answer only because the instructor had made a point of it. Thus, in actuality, no discernible change had occurred in Keona's central mathematical beliefs.

As Keona looked at the beliefs she had written at the beginning of the semester, the idea that mathematics problems are not restricted to a single correct answer persisted as her central focus. In the second interview, she had written two statements contradicting this perspective: "Math is done in a step-by-step process and only one answer can be found" and "Math problems can be done in many different ways but only one answer can be found." Keona no longer felt this way.

> I don't agree with that since we had that assessment where I found out – if she hadn't written that on the paper, I would of thought it was only one answer because I couldn't find any other way that I could get a different answer with the material I was given. So for this, I would probably say, "Math is the step-by-step process. Yet, in some cases more than one answer can be found. In some cases, not all." (Ke5, 11)

Again, the teacher's authority is evident as the major influence because "she couldn't find any other way." Looking at the second statement, Keona reported, "Basically it's like the same thing I said for that one. So these are the same. ...so I would just rewrite these as just one sentence saying the same thing" (Ke5, 11).

The only other statement she chose to rewrite involved learning mathematics by memorizing.

> It says, "Memorization is the best way to learn math." For that one I would – I don't still agree with that, that memorization is the best way to learn math because like, for instance, in [*Probability and Statistics*] we didn't do a lot of memorization. It was like applying your knowledge and just working with another person so you can get to know people in the class and see what they have to say and not just always work by yourself because when you're working with children you're not just gonna be all secluded. (Ke5, 11)

Here, actual experiences in the study section appeared to have affected how she viewed learning mathematics. She was able to see how people other than the instructor might help her to understand certain concepts, but her other comments suggested that the final authority remained with the teacher. Furthermore, she was not completely willing to abandon memorization considering that she agreed that 'memorization is the most important tool to use when doing mathematics' in the exit card sort. Keona's experiences in the *Probability and Statistics* course had caused her to reevaluate the place of memorization in mathematics though.

During the relationship confirmation activity, she agreed that the alternative assessments contributed to her new perspective, but they were not the major influence. The course as a whole presented a different picture of mathematics.

> I couldn't say that I strongly agree with it because I do believe in some courses you will have to do a lot more of memorization than just apply our

> knowledge. I don't know if I will have any more math classes where I would have to do that but in the past since I've had so many math classes where you have to do so much memorizing then I couldn't exactly say that I strongly agree with it. (Ke5, 18)

The study section represented an anomalous mathematics class based on Keona's prior mathematical experiences. Thus, while it appeared that Keona left the course holding more productive beliefs, she was hesitant to embrace them wholeheartedly.

The entire experience seemed so foreign to Keona that she was unwilling to posit that the alternative assessments represented the primary influence. In fact, assessment was not listed as a factor in her choices for the exit survey. When I asked her about this omission, she confessed that she had misunderstood 'assessment'.

> Because it was like when I got to this here, she was leaving. I was like okay; I have to circle something here. So I think maybe when I saw the word assessment, I really didn't pay attention to projects or quizzes. (Ke5, 6)

As far as she was concerned, the projects were still tests and the word assessment confused her. If tests had been one of the choices, then she said it probably would have been third on her list of influences. That the study section's assessments played only a minor role in Keona's overall experiences became evident when she strongly agreed with only one statement regarding the relationship between alternative assessments and changing beliefs (see Appendix D).

Surprisingly, this single statement seemingly contradicted all the previous data concerning Keona's counterproductive belief about mathematical confidence: 'When other students in my group helped me to understand the problems on an assessment, I became aware that I could learn how to do mathematics without always relying on the teacher and textbook.' It is possible, however, that this selection parallels the

presumption I made earlier that Keona came to believe that other students could help her understand the material, but the teacher remained the ultimate authority. Confirmation that Keona retained her reliance on the teacher came during a discussion surrounding the relationship statement describing the realization that mathematics problems might have more than one right answer:

Keona:	I know that the teacher says that it is another answer but how do I get to it? I don't know.
Interviewer:	So you still haven't seen that.
Keona:	I really haven't seen it yet. (Ke5, 17)

Although the study section, and in particular the instructor, had introduced fallibilist ideas to Keona and she was willing to accept them based on an authority's recommendation, her central mathematical beliefs remained unchanged from the start of the semester. Therefore, it is my contention that she had effectively built a completely new mathematical belief system around what she determined to be an isolated mathematical experience. The original system remained intact and generally counterproductive, however, and would apply again when mathematics returned to what she considered normal.

Changes in Lucia's Mathematical Beliefs and Related Influences

Lucia's exit survey provided evidence of a slightly productive shift in her mathematical beliefs (see Appendix D), but during the final interview, she explained that her choices and responses simply represented two different characteristics of mathematics. For example, she explained her switch from thinking that sponge was the best simile for a mathematics student to selecting explorer as the best as follows:

"Because I think you need both, not just – I couldn't just choose one" (Lu5, 4). Both similes represented qualities she felt were necessary to be successful in mathematics.

> The sponge because – I mean everything you learn you – I mean, it's something you're gonna remember until you learn other things and then – but an explorer you're out to try and learn them and you're trying to figure out things you don't understand. (Lu5, 5)

When I asked Lucia if she noticed any differences between her initial responses and her most recent ones she said no.

> Well, I've always – I don't know if I have always, but I've always thought that math should be taught differently, which I've been saying all year, all semester, but I don't think so, no. (Lu5, 6)

Because of her struggles with mathematics, Lucia felt that how it is taught could be improved. She continued to see the subject itself as predictable, however, as she described the student exploring "the surrounding [and] boundaries of rules" (WMSS2, 13).

The results from the exit belief sort also established a lack of change in Lucia's mathematical beliefs. All the cards were sorted identically to the first sort, with the exception of the belief regarding memorization. Whereas previously she had disagreed that memorization was an important part of learning, she now believed it had some merit. Consequently, she could not completely agree with the statement, but did feel as though it belonged more in the middle.

Interviewer:	So one of the reasons to memorize is because mathematics is building on this stuff, and if I don't memorize, I'm going to have to spend all the time to review.
Lucia:	And I'll just waste time.
Interviewer:	Do you still struggle with that part "is the most important?"

Lucia: Yeah. I don't think that's the most important. I think other things are too, like understanding it and not memorizing it. (Lu5, 10)

Memorization was important to Lucia, but not as important as understanding. This did not reflect any change in her core mathematical belief regarding predictability (mostly counterproductive) because she had shared this hierarchy from the beginning; mathematics continued to represent a body of outside knowledge to explore and absorb. Based on the card sort, there did not appear to be any major shift in her beliefs related to perseverance (productive) or confidence (counterproductive) either.

There were also no alterations to make in Lucia's written, strongly held beliefs. Nevertheless, she strongly agreed with each of the assessment-belief relationship statements (see Appendix D). While this implied that the course assessments had directly affected the stated beliefs, Lucia meant that the assessments had reinforced her pre-existing beliefs. This became clearer as we discussed the influences that she listed on the exit survey.

Interviewer: How did the assessments that you did in this class reinforce this idea of being the explorer, conducting experiments, and the teacher helping to fix the problem areas?

Lucia: Just because they're different kind of assessments, and so they allowed me to see deeper into what being an explorer – and maybe because I never had experienced going that far into – I mean, into substance way more… (Lu5, 6)

Lucia also mentioned the textbook, teacher directed instruction, and student led discussions as influences.

> Yeah, the textbook wasn't like a regular textbook either. You didn't just give the answers. I think if the answers were provided, though, it would be sort of better because then you could see if you were right and just make sure. (Lu5, 7)

> I think maybe [the instructor] could have taught more and not relied on the students unless – it got sort of confusing when some student would say something. Then it turns out they're completely wrong and so you're learning that way, and it's just frustrating. (Lu5, 11)

As she talked more about these experiences, it was apparent that she continued to hold counterproductive beliefs about learning mathematics – looking to an authority.

Overall, Lucia reported that *Probability and Statistics* had been a positive experience for her, especially the alternative assessments. The course reinforced her belief that there must be a better way to teach and assess mathematics. As was the case with Karen, Lucia concentrated on the pedagogical implications of the study section, which left her more central beliefs about the nature of mathematics unchanged. Lucia continued to believe that success in mathematics required persistence, while still lacking confidence in her ability to do mathematics without an authority available and viewing mathematics as a product rather than a process.

Changes in Lynn's Mathematical Beliefs and Related Influences

The comparison of Lynn's initial and exit belief survey indicated a strong productive shift in her mathematical beliefs. Although each of the 'worst' categories remained constant between the beginning and the end of the study, focusing on Lynn's struggles in mathematics, the 'best' similes and corresponding rationale implied that she had altered her position on confidence and predictability (see Appendix D). As Lynn discussed her own interpretation of the comparisons, it became evident that a misinterpretation of certain survey responses had occurred.

In terms of teaching, Lynn seemed to move from seeing the teacher as essential – "The doctor uses and practices procedures that are vital to the life of a patient" (BMTS, 20) – to more of a guide – "Helps to conduct the mathematical tasks we are given" (BMTS2, 20). The teacher appeared to remain center stage in both cases, but as Lynn discussed experiences from the study section, a more productive belief emerged.

> I started to look at a math teacher as – they've given you the book. They've given you tips in helping to understand it. And then I had to think about in this class where the teacher wasn't leading all the discussions. Students were leading the discussions. So it – as an orchestra conductor, in a way we're the ones who are doing the work and the orchestra conductor is leading. But also there are times where students were leading the orchestra and not the professor so that's how I looked at that one as in a way being a little bit different. (Ly5, 4)

Because of her experiences with 'student led discussions,' Lynn no longer relied entirely on the instructor to lead the way.

In the best mathematics student simile, initially Lynn's selected mechanic because "the student has a knowledge basis of math and adds to it with ways they understand" (BMSS, 20). Her rationale for choosing computer on the exit survey appeared nearly identical: "Gaining new data and processing it with old data (prior knowledge)" (BMSS2, 20). During the final interview, however, Lynn explained how for her these represented different perspectives.

> [Mechanics are] having to build upon what they know but they're not constantly building upon it... Whereas the computer, it's always got its own data but it's always getting new data in every day, every second. (Ly5, 3)

This suggests that Lynn was beginning to see learning as an ongoing process of constructing understanding, not something that happens occasionally.

Lynn's choices for doing mathematics went from 'climbing a mountain' to 'conducting an experiment.' While her 'experiment' rationale included "trying different things" (BDMS2, 20), when Lynn explained her perception of this change it seemed more absolutist.

> Well, with doing an experiment, you have a list of steps you need to do… I'm no longer struggling to get step to step… I've just got to make sure that I can go over it and I can go through it step by step by step. And in an experiment you've got these steps and you've got to do them in order to get through. And I'm thinking well, okay, if I get them done in order to get through, it might be better than climbing a mountain where you can go from here to there. There's no order. It's going straight up. (Ly5, 4-5)

For Lynn, doing mathematics consisted of following a list of predetermined steps.

Therefore, based on the analysis of Lynn's survey responses and subsequent commentary during the final interview, a more accurate picture of changes in her mathematical beliefs developed. It appeared that Lynn's beliefs had moved from counterproductive to mostly counterproductive in confidence and remained mostly counterproductive in predictability. This examination did not uncover any data on perseverance, but information did emerge in later activities.

The first evidence that any change had occurred in Lynn's beliefs related to perseverance emanated from the final belief card sort. Recall that at the beginning of the semester she agreed with two statements suggesting that being good at mathematics entailed doing it quickly and only a rewritten belief statement kept her from being categorized as mostly counterproductive. Now Lynn chose to disagree with both statements, implying more productive beliefs.

> "Strong math students solve math problems quicker than weak students." Well, my [assessment] partner was a strong math student and somehow I was

> able to solve the problems she couldn't, so that totally blew this one out of the water. (Ly5, 10)

> "Strong math students understand the material the first time they see it. Poorer math students need to put a lot of repetition into it." I don't feel that way any more because I still see myself as a poor student but I didn't do too much repetition this time... The experience – the repetition of redoing the assessments, yeah it was helpful but I didn't see a lot of repetition in going over and over and over. I didn't like cram for the final. (Ly5, 10)

Notice that in both cases, assessment experiences from the study section were used to justify the change in belief.

Assessment also caused Lynn to switch her position on "Memorization is the most important tool to use in mathematics" from agree to disagree, but it was not the only factor. Like Keona, Lynn believed that the entire course contributed to this change.

> Being able to have a note card on the final took away from the need to memorize equations and that. That helped out a lot and I think it was more of an experience level than a memorization level to get me through this class. If I didn't do the homework and if I didn't read the book, I wasn't gonna learn it. (Ly5, 10)

The structure of the study section emphasized experience rather than dissemination followed by repeated practice.

Trial and error emerged as one of the experiences that made the greatest impression on the last three belief statements that changed piles from Lynn's first interview to her last. Fittingly, the one statement that she moved from the disagree to the agree stack included the phrase 'trial and error.'

Interviewer	[L]ooking over that first transcript I remember you saying, "I've never experienced any trial and error in math."
Lynn	Well, now I've gotten to experience it... Especially the probability... We took a cup of tacks thing. That was like an

> experiment, too, but it was trial and error. Each time we did that, somebody else – there was always somebody around us who did the same thing but got totally different results. So it was strange seeing our results compared to somebody else's results. It showed that through some trial and error you could find the solution. It was really interesting. I enjoyed some of that stuff but it wasn't comfortable for a while because it was something new. (Ly5, 12)

As Lynn discussed the other two statements, which she had originally agreed with, her discomfort, her experience with trial and error in mathematics, and the role the assessments played remained evident.

> "In order to do mathematics well, the students must absorb as much information as possible." … I would have to say with the assessments and the information I learned this semester, it was more of a trial and error based and I felt very – I did feel very unconfident for a while but I still don't see where I absorbed it because the teacher wasn't there just giving it to me. I was getting it from my fellow students. I was going out and getting it. So in a way I was the teacher. (Ly5, 9)

Clearly, Lynn's confidence in her own ability to learn new mathematics had grown considerably over the course of the study. Her position on predictability was not so clear, however, and the commentary associated with the last statement only added to the confusion.

> "Math is a step-by-step process where there is always a single solution to be found." I look at this one and I look at where learning by trial and error is not following a step-by-step process and if you're trying to find your own way to understand the problems, you're still not doing this step-by-step process. It's like a jigsaw puzzle. You got all the pieces. If you get them all in the right places you solve it. That's where I started to disagree with that one. (Ly5, 10)

This last statement seemed to contradict her earlier justification for selecting 'conducting an experiment' as the best simile for doing mathematics. I asked her about the steps and she responded, "I still see it as steps that you have to follow but

there's got to be some leniency to go out of those bounds" (Ly5, 11). Evidently, Lynn still viewed mathematics itself as a predictable set of steps, but believed that the processes available to understanding a problem were more flexible. This distinction indicates that she still held mostly counterproductive beliefs about the predictability of mathematics.

Lynn chose to alter two of her original belief statements at the end of the study. The first dealt with the predictability of mathematics – in particular getting a single right answer.

> "There are many ways to do math but the answer should be the same and the student should be able to say how they got their answer." What's bothering me about this one is the answer should be the same. Probability – we saw the answers are not the same. So I would say that "The students should be able to answer the problems and not only say how they got their answer but explain their answer, explain what the answer means and find out if they feel confident about their answer." That's how I would probably rephrase this one. (Ly5, 13)

She identified student led discussions as the main influence here "because there was a problem we started to solve and we solved it, but she asked is there another way to solve it and boom the number of hands that went up in the air" (Ly5, 15). The other statement that Lynn altered reinforced her view of mathematics as being more than absorbing information and the role 'trial and error' played in altering that belief.

> "[M]ath students need to absorb, understand, and put math into terms that they can use. With practice they can build the basis of knowledge to make math successful for them." I would take out the word "absorb" and put in experience... [W]hat contributed to this belief was the trial and error, the probability experience I had. That would be the big thing. And then redoing of the assessments would be another factor. (Ly5, 13)

Both sets of statements reinforce that Lynn felt more confident in her ability to do mathematics, while the first statement also indicates that she saw mathematics as slightly less predictable than was originally assumed.

Hence, it appears that in all three areas, perseverance, confidence, and predictability, Lynn experienced a productive shift to varying degrees. At the end of the study she was much less likely to equate speed with ability in mathematics and much more likely to believe that she was capable of understanding new mathematical concepts. To a lesser extent, she began to view mathematics as less predictable than when she began the study. As Lynn reported the factors contributing to these shifts, assessment emerged as a primary cause. In fact, on Lynn's final survey she listed assessment as the number one influence from the semester, followed (in order) by interaction with instructor, reflecting in math journals, whole class discussion, working with calculators, and student led discussion.

Therefore, it was not surprising when Lynn strongly agreed with all but one of the relationships confirmation statements related to the study section's alternative assessments (see Appendix D). The one statement she agreed with, but not strongly, read "I am beginning to value understanding over memorization since these assessments asked me to demonstrate how I could apply what I'd learned rather than asking me to remember it." As before, Lynn explained that the assessments played a part, but the course offered several experiences that contributed to believing that mathematics involved more than memorization.

> The assessment, some of the discussions. After taking a quiz talking to my group about it... The journal problems making me think a lot. So this was experienced based. (Ly5, 17)

These experiences, especially those related to assessments, had challenged certain of Lynn's long held counterproductive beliefs about the nature of mathematics.

Consequently, of the seven informants only Lynn and Janet seemed to become more productive in their mathematical beliefs. Furthermore, only three informants recognized the alternative assessments as the primary influence on their beliefs, but Lynn and Janet represent two of these three. A complete summary of the informants' initial and exit beliefs and their reported influences is provided in the next section.

Summary of Changes in Informants' Mathematical Beliefs and Reported Factors

Figure 15 documents the changes to the informants' mathematical beliefs in the areas of perseverance, confidence, and predictability as presented in the previous sections. Notice that with two obvious exceptions, Janet and Lynn, the informants finished the semester holding essentially the same mathematical beliefs as they held on the study's first day. Anne, for example, sorted the belief statement into the exact same disagree and agree piles during the first and last interview, down to her uncertainty about where to place a particular statement. The similes and rationale provided by Cynthia on her initial and exit belief surveys were nearly identical. She even used phrases that were almost duplicates of one another, although she completed the surveys more than three months apart.

Case	*Perseverance*	*Confidence*	*Predictability*
Anne **[exit]**	productive [no change]	counterproductive [no change]	mostly counterproductive [no change]
Cynthia **[exit]**	productive [no change]	productive [no change]	mixed [no change]
Janet **[exit]**	mixed [no change]	mostly counterproductive **[mostly productive]**	counterproductive **[mostly productive]**
Karen **[exit]**	productive [no change]	mostly counterproductive [no change]	mostly counterproductive [no change]
Keona **[exit]**	productive [no change]	counterproductive [no change]	counterproductive [no change]
Lucia **[exit]**	productive [no change]	counterproductive [no change]	mostly counterproductive [no change]
Lynn **[exit]**	mostly productive **[productive]**	counterproductive **[mostly productive]**	mostly counterproductive **[mixed]**

Figure 15. A Comparison of Informant's Entering and Exiting Mathematical Beliefs.

Certainly, changes did occur over the course of the semester. (The next section provides a discussion of the extent to which the informants' departing beliefs about assessing a student's ability in mathematics varied from the beginning of the study.) Furthermore, the *Probability and Statistics* course did challenge the traditional pedagogical beliefs of Karen, and to a lesser extent Lucia. Keona seemed to have developed a whole new separate belief system constructed around the study section experiences and the instructor's authority. Yet, only Janet and Lynn's core mathematical beliefs exhibited any noticeable shift.

There was nothing in these two informants' background that explains why they were the only ones to experience any movement in their beliefs. Janet identified herself as a Type II student while Lynn placed herself in the third quadrant. Janet was a non-traditional student recently returning to school and taking her first mathematics course for elementary education majors. Lynn, on the other hand, was a

traditional student who had taken several of the mathematics education courses, some of them more than once. Lynn had taken four years of high school mathematics to Janet's two. The few characteristics that they shared, race and gender, did not distinguish them from the other informants.

Another characteristic, not related to background, that they had in common was their choice of "Assessment: projects & quizzes" as the most significant factor influencing their simile choices on the exit survey. Lucia was the only other informant who placed this influence in the first position (see Figure 16). In fact, no other informant chose assessment whatsoever. (*Recall, however, that Keona admitted to being confused about the term assessment. If she had known it meant the study section's 'tests', she would have placed it third.) Moreover, notice that only Janet and Lynn included 'reflecting in math journals' in their list of influences. This is not to suggest that the reason behind the productive shift of these two informants lay in their attention to the alternative assessments and the mathematical journals, but it does raise certain questions for further research.

Although the intervention's attempt to alter the informants' counterproductive beliefs about the nature of mathematics proved somewhat ineffective, the alternative assessments were successful in challenging the informants' perception of assessing ability in mathematics. The next section describes in detail these findings.

Anne's Reported Influences	
1. Interaction with classmates	2. Group activities: non-graded
3. Interaction with children	4. Student led discussion
5. Whole class discussion	6. Interaction with instructor
Cynthia's Reported Influences	
1. Group activities: non-graded	2. Other courses and teachers
3. Student led discussion	4. Elementary classroom visits
5. Interaction with classmates	6. Whole class discussion
Janet's Reported Influences	
1. Assessments: projects & quizzes	2. Assignments: other than textbook
3. Group activities: non-graded	4. Reflecting in math journals
5. Student led discussion	6. Whole class discussion
Karen's Reported Influences	
1. Student led discussion	2. Working with calculators
3. Interaction with classmates	4. Individuals not in [this class]
5. Other courses and teachers	6. Interaction with children
Keona's Reported Influences	
1. Other courses and teachers	2. Working with calculators
3. Interaction with children*	4. Group activities: non-graded
5. Readings: other than textbook	6. Textbook: readings & assignments
Lucia's Reported Influences	
1. Assessments: projects & quizzes	2. Textbook: readings & assignments
3. Conferences attended	4. Group activities: non-graded
5. Teacher directed instruction	6. Student led discussion
Lynn's Reported Influences	
1. Assessments: projects & quizzes	2. Interaction with instructor
3. Reflecting in math journals	4. Whole class discussion
5. Working with calculators	6. Student led discussion

Figure 16. Informants' Reported Influences From the Belief Exit Survey.

How the Informants Viewed Mathematics Assessment After the Study Section

Recall that upon entering the study section all seven informants associated assessment in mathematics with giving a test. As mathematics tests represented the majority of their assessment experiences, this seemed a reasonable connection. After

exposure to the alternative assessments used in the study section, all the informants altered their perception of mathematical assessment to varying degrees.

Anne's Exiting View on Assessments in Mathematics

Upon completing *Probability and Statistics*, I asked Anne, "If I told you I was going to assess your ability in mathematics, would you still expect me to give you a test?" She responded:

> Probably not... Because now we've learned in [*Probability and Statistics*] that there's other ways to assess students and we've learned that this is – the other ways to do it, the three ways that we did it, are better ways for the students to understand or for the teacher to realize how much the student understands... Because it's not something where the student can just come in and say well, I'm ready because I memorized everything how to do everything. The student actually has to know how to do it and apply it to the situation they're given... I'll probably use tests, too, but I'll try to use other things, I think. (A5, 1-2)

Anne saw merit in including tests in her mathematical assessment framework, but also realized that other appropriate methods were available.

A journal entry written earlier in the year establishes that Anne was cautiously optimistic about using alternative forms of assessment in her mathematics classroom. When asked, "Would you ever use a task like 'Assessment Writing' to assess your students' mathematical ability? Please support your answer," she responded:

> I think if I were going to teach upper-El, middle school or high school I think I would use this type of assessment. I think it helps the student understand the material better than studying for a test and forgetting the process an hour after the test. (Journal entry, 10/15)

Anne had first-hand experience that other assessment methods, methods she preferred to traditional testing, existed and could be used effectively in the classroom, yet was

concerned that some of the methods might be inappropriate for younger students. Considering that she had only recently experienced these alternative forms as a college student, her hesitancy seems reasonable.

Cynthia's Exiting View on Assessments in Mathematics

Similarly, Cynthia found the alternative assessments enlightening, but was reluctant to eliminate traditional testing in mathematics. In her journal, she expressed concern with using the alternative methods modeled in the study section in the elementary classroom:

> I think for us as future teachers, writing and grading assessments is an important skill for us to master. However, I don't think I will do this with my own class someday... when it comes to assessing, I will probably stick to a more traditional form of testing. (Journal entry, 11/05)

During an interview before this entry, Cynthia made several similar statements supporting this position that these assessments were appropriate only because they were given in a course designed for future elementary teachers.

> Now I know nothing about engineering, but if they have to do this sort of a computation to figure out how much weight this wall will uphold, then have them do things that relate to that. Having them spending their time writing tests when they'll never do that for anybody else seems like a waste of time. It seems like you could focus their energies better than that. (C4, 5)

Consequently, Cynthia maintained this position throughout the final interview. When asked if she still felt that tests represented the best way to assess students' mathematical ability, she responded:

> We had a journal prompt on this and [the instructor] asked if we would still be using traditional tests in our classrooms, and I guess there's some modifications and things that you can do to them. You can have different

> parts to them and things like that, but I don't see the point in making my third or fourth grade students do things like making up an assessment and things like that. I guess maybe they could grade somebody else's, another assessment, and try to find the mistakes and things like – but the way we were doing it in this class, I don't see how that's really relevant to figuring out whether they know how to find the mean or whether they know how to do exactly things. … I think there are some modifications and stuff that you can do so it's not a straightforward what's 26 plus 25, you know, the traditional way. There are some things you can modify, so those aren't quite as traditional, but I think you still kind of go back to testing. … You don't want to test their grammar when it comes to writing tests and you don't want to test other things that are out there. You want to test what you've been working on and what you hope they gained. (C5, 1-2)

Cynthia seemed to accept that changes were necessary in the traditional mathematics test, but had difficulty seeing how modifications could be made to the alternative assessments found in the study section to make them age appropriate. As with Anne, it appeared to come down to lack of experience with using alternative methods.

When asked how she might modify the traditional testing format, Cynthia demonstrated that her tests would be more 'non-traditional.'

> Maybe have sections with manipulatives and things like that. That maybe you could test one-on-one or in a small group. Go to each table and test certain spots like that or their group working skills, things like that. (C5, 2)

Although she struggled with using the identical assessment tasks with her own future students, Cynthia was able to see how she might employ some of the formats; in this case, testing mathematical concepts in cooperative groups.

Janet's Exiting View on Assessments in Mathematics

Early on in the interview process, Janet shared her excitement about the alternative methods of assessing students in mathematics. As we compared her past

testing experiences to the alternative methods she explained, "The assessment shows what I can do. The traditional test shows you what I know this second" (J3, 6). She admitted that one of the reasons she liked these new forms of assessing was because of her own test anxiety:

> [T]aking a test is just so stressful that the anxiousness of taking a test – there just has to be other alternatives. And I think that's what I'm starting to see this semester is that there are other alternatives. And I just think that if you're in an environment or situation where you've got other factors playing into it and it's already damping what you know, I just think that's not the best way to assess your students' knowledge. (J4, 3)

Janet found these alternative assessments to be a completely new experience.

> It's just being able to work from your notes, being able to work from your class materials. Being able to have as much time as you need to. Being able to talk to somebody else when you're working on the work. Almost every aspect of the test seemed different than the standard typical written test that I've taken in the past… I think any of these things would help the traditional test. It's amazing how far apart they are and that both can be treated with the same subject matter. (J4, 13-14)

Exposure to something other than the traditional test meant Janet had experienced the benefits of using alternative assessment methods in mathematics first-hand.

Yet when I asked her about assessing her ability in mathematics, a test still came to mind "the majority of the time, but I now think there's other ways to do that as well" (J5, 1). These other ways included projects like those completed in her *Probability and Statistics* course that semester, especially

> … being able to work in a group on a different project and just being able to observe as the teacher. And I think you can do that, and I see myself doing that in the lower elementary classes, being able to watch the groups operate and then just being able to float around the room and observe what they're doing. I see that as being a lot less threatening to the students than the struggle of a test just because that's where I come from. I really struggle with tests. (J5, 1)

While Janet seemed more willing to embrace alternative assessment methods than either Anne or Cynthia, her reasoning did not appear related to selecting assessments that accurately represented the true nature of mathematics. Instead, her motive was to alleviating students' test anxiety, which is understandable given her own struggles with this malady.

Karen's Exiting View on Assessments in Mathematics

Like Cynthia, Karen viewed the study section's assessments as appropriate for the course only because they were all going to be teachers. Keep in mind that Karen concentrated her attention on pedagogical issues and not mathematics itself.

> I liked the fact that we had to make up the test. It's just a lot of experience to gain from it. Most teachers make up their own tests or they share their ideas throughout and kind of accumulate everything together to make one. I just thought it was a good experience. (Ka3, 5)

Karen, however, shared Anne's view regarding using such assessments in her own classroom, rather than Cynthia's desire to use paper-and-pencil tests.

> I think using the idea of [*Assessment Writing* is] great for older kids. Maybe something easier for lower levels. (Journal entry, 10/13)

Again, she was unsure of how to modify these new methods so that they would be appropriate in the early elementary classroom.

When I asked her to compare traditional testing to the projects completed in the *Probability and Statistics* course, she responded, "I think they're all a form of a test but I like the fact of making up your own questions, to perceive it that way, or

grading an actual test that somebody's already done to see if you can pick out the mistakes and errors and all that" (Ka5, 1). She went on to explain:

> I think if you can show your own way, it kind of shows more of what you know rather than saying okay, here's the question, answer it. Here you can make up something that you can relate to yourself, so you can better express it. In checking another person's test or one that's already been preset and done lets you know if you can catch someone else's mistake and helping catch your own if you were to make the same mistake. (Ka5, 1)

Karen seemed enthusiastic about using alternative assessment methods with her future students. As was the case with Janet, however, Karen's rationale suggested that her reasons related to the student issues rather than the nature of mathematics.

Keona's Exiting View on Assessments in Mathematics

In Keona's October 15th journal entry, she expressed her desire to use assessments like the 'Assessment Writing' task in the future "because it would help students to learn to develop problem writing [strategies]." It seems that Keona saw the goal of this assessment as learning to write problems and not as a means of assessing mathematical content. This would explain her responses to my queries regarding the best methods for evaluation in mathematics.

Interviewer	Is it still a test?
Keona	No, not necessarily. Since I've talked to you, since we've had our interviews and things like that, I would know that it could just be like maybe a survey evaluation or something like that. I don't know exactly how it might be set up but it could be something like that instead of a test or an assessment... It depends on – I don't know exactly what you're trying to find out about my math ability. Because we did so many different types of assessments it's really hard to say if you would use that or not because you might go to the approach of maybe just giving a regular test with just questions on it like we had for

> the final or you might give us little phrases and tell us to make up our own questions or something like that. (Ke5, 1-2)

For Keona, each assessment served a different purpose. The choice of assessment depended on "what [the teacher is] trying to find out about my math ability." Exposure to a variety of assessment styles had broadened her view of mathematical assessment methods, but she seemed to see the styles as disconnected from each other and sometimes from the subject of mathematics altogether.

Lucia's Exiting View on Assessments in Mathematics

When asked about mathematical assessments during the final interview, Lucia explained that she had two separate views – what she would want to experience and what she would expect. She described what she would want:

> Just maybe what we've been doing in [*Probability and Statistics*], just making up your own test or maybe giving a presentation on what you know maybe. Just maybe a paper, even in math, just other ways, other creative ways. Maybe a project or research or things like that. (Lu5, 2)

Yet, she explained that she continued to expect traditional tests in mathematics course, "Because I feel like professors or teachers aren't changing" (Lu5, 2). Despite the fact that she had recently completed a mathematics course employing various alternative approaches to testing, Lucia viewed the class as an exception. She was pessimistic that mathematics teachers as a whole were capable of changing their assessment methods.

Perhaps this frustration stemmed from the study section's final exam. As far as Lucia was concerned, an assessment ought to determine what a student knows and

not what he or she does not know. Because the final exam was closer to a traditional test than the other assessment methods used in *Probability and Statistics*, she felt it failed to meet this criterion.

> And then when you're working in groups and then you're expected to do this individual exam and I guess a lot of your grade is graded on individual – well, group and then your final is worth a lot of individual work, whether or not you understand the material a lot. But I don't think a final exam tests your individual achievement, what you've done in the class, because just the final – I might know a lot more than what was on the test in other areas, but I just don't think it's fair at all. (Lu5, 17)

Although this exam was not traditional, it represented aspects of traditional testing – individual accountability and timed – that made Lucia uncomfortable. The final exam experience appeared to strengthen her resolve that assessments in mathematics must change. Her rationale, however, like many of the other informants, stemmed from the idea of improved student achievement and not an attempt to represent the subject of mathematics accurately.

<u>Lynn's Exiting View on Assessments in Mathematics</u>

While Lynn did not provide as much data on this topic as the other informants did, her response to the question regarding mathematical assessments did offer some insight into her beliefs. I asked, "Would you still expect a test?" and she responded:

> Not exactly. I mean, it's still going to be a written form of something, but like [*Assessment Writing*]; that was an assessment… But I would have to say now it's more of our abilities and not just what we can put on a piece of paper, so it's like she's picking our brains in a way. (Ly5, 1)

Lynn saw how the study section assessments served a purpose different from those she had experienced in other mathematics courses. Instead of determining what the

student could remember and write on the paper, these alternative assessments evaluated students' abilities and their thinking. Due to a lack of data, however, it is unclear if Lynn's rationale for shifting assessment methods represented a pedagogical or conceptual shift.

Changes in Informants' Beliefs Related to Mathematics Assessment

Because of the various assessments modeled in the study section of *Probability and Statistics*, each of the informants ended the semester espousing that traditional tests represented only one of many ways to assess in mathematics. The extent to which alternative assessment methods would be used in the evaluation of their future students' mathematical ability depended on the particular informant, but each one recognized that the mathematics tests they had been given in elementary school were limited in their usefulness. As the informants explained the rationale behind their desire to improve on mathematics testing, most concentrated on issues related to teaching and learning such as test anxiety, group-work, and individual learning styles. Thus, even in assessment, when changes occurred they seemed limited to pedagogical beliefs and not mathematical concepts.

A Summary of the Research Results

In this section, I attempt to condense the relevant results in order to answer the questions from the beginning of this chapter. Each question is addressed in the order it was presented in the chapter. (Recall that the related research question is in

parenthesis at the end of each question.) The answers concentrate on my interpretation of the results from the data analysis. Readers requiring additional support are invited to return to the appropriate sections of this chapter.

Review of the Results

> What were the informants' past experiences relative to learning and assessment in mathematics (question one)?

All of the informants reported that their kindergarten through twelfth grade mathematical experiences were mostly traditional. In a typical mathematics classroom, the informants described sitting in rows listening to the teacher disseminate mathematical knowledge. The assessments that the informants encountered in these classrooms were also traditional – homework assignments, quizzes, and tests – and attempted to determine how well the students had assimilated the knowledge. It was not until enrolling in the mathematics courses for elementary education majors that most of the informants experienced non-traditional instruction or assessments with any consistency.

> What were the informants' initial beliefs regarding what it means to assess a student's mathematical understanding (question three)?

Because traditional pencil-and-paper test had dominated the informants' early assessment experiences in mathematics, it was understandable that they had come to believe that the way to assess mathematical understand was through questions on a test.

What were the initial mathematical beliefs of the study section as a whole and how did the informants perceive the relationships between their experiences in mathematics and their own beliefs (question two)?

Based on the results of the first belief survey, it appeared that nearly all the students enrolled in the study section held, to varying degrees, counterproductive beliefs about teaching, learning, and doing mathematics. The data collected during the first two interviews determined that the informants' mathematical beliefs regarding confidence and predictability tended to be counterproductive, but that they typically held more productive beliefs concerning perseverance. Informants reported that a wide range of experiences had influenced the development of their mathematical beliefs. These included, in alphabetical order, assessments, curricula, discussions, instruction, peers, readings, and teachers. In general, the evidence suggests that the experiences responsible for developing certain beliefs are personal in nature and thereby differ depending on the individual and the circumstances.

How and why did the mathematical beliefs of students in the study section change over the semester and what prompted any changes in the informants' beliefs in particular (question five)?

A comparison of the students' initial and exit belief surveys indicated that many of the students had experienced a positive shift in their thinking about mathematics. When asked to list the factors that contributed to any changes, the three most common responses were 'Interaction with classmates' (19), 'Student led discussions' (19), and 'Assessments: projects & quizzes' (17). Of these three, 'Assessments' was the overwhelming choice as the most influential factor with 12

first place votes, compared to four for 'Interaction with classmates' and three for 'Student led discussions.'

An analysis of the informants' data related to changes in beliefs suggested that the shifts indicated from the surveys might actually represent peripheral and not central movement in their beliefs. For five of the seven informants, no evidence emerged to suggest that any movement had occurred in their beliefs regarding perseverance, confidence, or predictability in mathematics. Only Janet and Lynn demonstrated noticeable shifts in central beliefs. There was nothing in the informants' background information to explain the reason some experienced change while others did not. Janet and Lynn did report that the study section's alternative assessments were a primary influence in their altered beliefs, along with the mathematics reflection journals that they were required to complete on a weekly basis.

How did experiences with alternative assessments affect informants' perceptions about assessing mathematical understanding (question four)?

Exposure to a variety of assessment methods enabled the informants to realize that a pencil-and-paper test represented only one possible way to assess students in mathematics. Informants' commitment to using alternatives to tests in their future classrooms varied, but all agreed that improvements could be made to the traditional testing methods. Most of these improvements dealt with pedagogy though, and did not attempt to align the assessments with the fallibilist view of mathematics.

Final Thoughts

The findings suggest that assessment does play a definite role in belief development and can challenge a student's pre-existing view of mathematics, but it is not the sole influence in either case. Furthermore, the results confirm findings from other research that describe pre-service teachers' central mathematical beliefs as resistant to change. An examination of how this study fits into the research related to mathematical beliefs and assessment is the major focus of the final chapter.

CHAPTER V

CONCLUSION

The purpose of this chapter is to consider the results of the research in three contexts. First, the findings or answers to the research questions, as suggested by the data presented in Chapter IV, are discussed. Second, limitations of the current research are assessed with the intention of clearly defining the scope of the research. Finally, implications of the study's findings are presented. In essence, the first two sections represent looking back, while the third section entails looking forward.

Looking Back at the Answers

This section discusses the connections between the existing research literature, the data collected during this study, and the grounded theories developed during the analysis of the data. The original research questions are used as a framework for this discussion.

Past Experiences

> What past experiences have pre-service elementary teachers had relative to learning and assessment in mathematics?

All seven informants reported encountering mostly traditional experiences in their mathematics classrooms prior to enrolling in the mathematics courses designed

specifically for elementary education majors; this reflects the findings of previous research (see Brown & Borko, 1992; Dossey, 1992; Thompson, 1992). These traditional experiences included both instruction and assessment. Neither is surprising considering the informants' dates of graduation from high school. Karen was the most recent graduate in 1996, meaning she would have been in fifth or sixth grade the year the NCTM (1989) published *Curriculum and Evaluation Standards for School Mathematics*. At the very least, this means that each informant had passed through elementary school mathematics instruction before any of the *Standards* documents were available.

Similarly, serious attention to alternative assessment methods in mathematics is a relatively recent phenomenon. An examination of the bibliography from *Assessment Standards for School Mathematics* (NCTM, 1995) found the earliest source to be dated 1987. Furthermore, research of assessment methods used in secondary mathematics classrooms conducted by Senk, Beckmann, and Thompson (1997) found that methods other than traditional tests, quizzes, or homework, accounted for less than 10% of students' grades. Because students value what is graded (L. Wilson, 1994), it was highly unlikely that any of the students enrolled in the study section of *Probability and Statistics*, including the seven informants, would recall encountering many alternative-style mathematical assessments before entering the mathematics education courses.

The informants' descriptions of their mathematics education courses were more likely to resemble the reform classroom vignette (see Appendix C) than their

earlier experiences. Instruction based on group work and a hands-on approach and assessments that included projects and written reports represented a relatively new mathematical learning experience for most informants. Until recently, nearly all of the informants had experienced traditional forms of teaching and testing in mathematics. Given the uniformity with which these experiences were reported, it would be reasonable to posit that the same holds true for a majority of pre-service elementary teachers.

Entering Mathematical Beliefs and Related Experiences

> How do students explain the relationship between their experiences in learning mathematics, with a focus on assessments, and their beliefs about the nature of mathematics?

The initial belief survey given to the entire study section suggested that nearly all the students entered the study with certain counterproductive beliefs often associated with an absolutist view of mathematics. These counterproductive beliefs confirmed certain results from earlier research on students' beliefs about mathematics (Borasi, 1990; Frank, 1988; Garofalo, 1989; McLeod, 1992; Schoenfeld, 1988a, 1992). Students' statements alluding to mathematics as a pre-existing product [predictable] presented by an outside authority [confidence] characterized this view. For example, 12 of the 31 students from the study section chose 'sponge' as the best simile for a mathematics student; this suggests that mathematics students were expected to "learn (or soak up) everything the instructor has to teach" (BMSS, 16). The next highest selection, 'explorer' at 11, appeared to represent a more fallibilist

view, but still indicated that these students perceived mathematics to be outside of their own experience and that exploring it might require a guide. Furthermore, none of the students selected inventor as the best mathematics student simile, thereby confirming that they were not confident in their own ability to originate mathematical ideas.

The first two interviews with the seven informants reinforced these perceptions. Although some informants espoused productive beliefs, subsequent probing uncovered that these beliefs were peripheral and not central to their mathematical belief systems. For example, Karen wrote that students should not invent their own ways of solving problems in mathematics, yet admitted they often do and that the classroom teacher should encourage this creativity. Cooney, Shealy, and Arvold (1998) suggest that such inconsistencies can result from examining different layers of beliefs. From a pedagogical [peripheral] standpoint, Karen believed that students learned better if they were active participants, but conceptually [central] she viewed mathematics knowledge as fixed. Based on previous research findings (Ball, 1990; Dossey, 1992; Middleton, 1999; Raymond, 1997; Thompson, 1992), Karen's central beliefs will most likely be the ones that ultimately influence her teaching of mathematics.

As the informants' mathematical beliefs were broken down, it became apparent that their beliefs were not counterproductive in all aspects. The informants' beliefs regarding perseverance in mathematics were to varying degrees productive. In other words, most did not believe that students should "stop working on a problem

after just a few minutes because, if they haven't solved it, they didn't understand the material (and therefore will not solve it)" (Schoenfeld, 1988a, p. 151). This mirrored earlier findings from the quantitative survey employed in the pilot study. Out of 33 students from the pilot, 22 students disagreed and 8 strongly disagreed with the statement, "If I can't do a math problem in a few minutes, I probably can't do it at all" (Kloosterman & Stage, 1992, p. 115). No evidence emerged during either the pilot study or the current study to explain why these pre-service elementary teachers had believed they could persevere when doing mathematics problems while students in earlier research studies had not. The seven informants simply explained that in their experiences they had found that perseverance was necessary in order to solve certain mathematics problems.

In the areas of confidence and predictability, the informants were more likely to exhibit the counterproductive mathematical beliefs frequently described in the research literature; one informant, Cynthia, stood out as an exception. Her beliefs related to confidence were considered productive because she believed in the autonomy of the learner, while she was identified as holding mixed beliefs about predictability due to her comments about students developing their own methods for solving a problem but arriving at an identical answer. She reported that class discussions in her previous mathematics education courses had developed these beliefs.

Other informants also described experiences in *Number Concepts* and *Geometry* as influencing their view of mathematics, but it appeared that their central beliefs somehow got in the way. Oaks (1987) found similar results in her research.

> Clearly, the indication is that they have been told understanding is important to doing well at mathematics, but their algorithmic conceptions do not allow them to define exactly what this 'understanding the concept' is. (p. 373)
>
> … to them understanding means knowing how to do the problems. (p. 378)

The informants in this study were also interested in why the concept worked and regularly toted 'understanding why' as the highest goal of learning mathematics. In truth, however, 'understanding why' often meant memorizing the set reasons behind some concept as presented by the instructor.

The informants often found it difficult to pinpoint the conception of their mathematical beliefs. Some spoke in generalities about their experiences and others described experiences that had reinforced their beliefs, but few were able to identify an exact causal experience. In retrospect, the informants realized that traditional tests in mathematics might be behind certain beliefs associated with absolutist thinking, yet only Janet, Lucia, and Lynn discerned the impact tests had on their own mathematical beliefs.

Whereas the data collected related to the second research question suggests that the pre-service elementary teachers serving as informants generally held similar beliefs about mathematics in regards to perseverance, confidence, and predictability, the personal nature of specific beliefs and experiences renders further speculation moot. As with so many students from other research (Brown & Borko, 1992; Dossey,

1992; Thompson, 1992), the informants in this study exhibited counterproductive mathematical beliefs. Each informants' belief system appeared to be unique, however, and thereby not generalizable.

Prior Beliefs About Assessments on Mathematics

> Prior to completing a sequence of alternative assessments, what were pre-service elementary teachers' beliefs about what it means to assess a student's mathematical understanding?

Recall from the findings for the first research question that before taking any mathematics education courses the informants had chiefly experienced traditional mathematical assessments – homework assignments, quizzes, and tests. Accordingly, it was not surprising when the informants chose a test or described questions that might be found on a test as the best method for assessing students in mathematics. "Because that's how we had been tested. That's all what we've experienced" (Ly5, 1)

Lynn's statement was inaccurate, however, because several of the informants, herself included, reported encountering alternative forms of assessment in their *Number Theory* or *Geometry* courses. Perhaps the informants dismissed these experiences as anomalies because they occurred outside of what they thought of as a typical mathematics course. Regardless of the reasons why the informants ignored these new assessment forms, the research on mathematical assessments suggests that pre-service teachers need to experience various styles of assessments if these future teachers are to become comfortable using such methods in their classrooms

(Raymond, 1998; Romberg & Wilson, 1995; Senk et al., 1997; Wilcox & Lanier, 1999). Data from this study assert that such alternative assessment experiences must be numerous enough that they will overcome the beliefs formed on the basis of the pre-service teachers' copious traditional testing encounters.

Exiting Beliefs Concerning Mathematical Assessments

> Does experience with a sequence of alternative assessments affect students' perceptions about what it means to assess mathematical understanding? If so, how and why?

At the close of the study, the informants had come to recognize that tests embody only one of the ways to assess students in mathematics. Even the informants, like Cynthia, who admitted that they would still use mathematics tests in their future classrooms, furnished details suggesting that these would not be traditional tests. She described tests where students solved problems using manipulatives or working in small groups. In this way, the informants described non-traditional assessments that reflected certain assessment environments modeled in the study section if not the assessments themselves.

In fact, many of the informants expressed concerns about using the *Assessment Writing* or *Evaluating Responses* projects in the elementary classroom, questioning whether they would be appropriate. It was not readily apparent to the informants how they could modify these assessments for their future students. This paralleled the findings of other researchers that teachers often see themselves as being poorly prepared to develop alternative assessments (Cooney et al., 1993; Crooks,

1988; Lambdin, 1993; Long & Benson, 1998; Senk et al., 1997). While exposure to alternative assessments increases pre-service teachers awareness of such methods, there is no guarantee that these methods will be adopted in the future teachers' classrooms.

As the informants discussed what mathematics assessments would look like when they became teachers, the focus was on pedagogical issues. The informants' greatest concerns were that they select assessments that would provide accurate information about students' mathematical understanding and allow the students to feel comfortable. Six of the seven women had reported experiencing test anxiety in mathematics; hence, student comfort was an issue for them. None of the informants indicated that they would use alternative forms of assessment in order to reflect the fallible nature of mathematics.

Changes in Students' Mathematical Beliefs

> After completing a course using a sequence of alternative assessments, do students report any change in their mathematical beliefs? If so, to what do they ascribe the change?

A comparison of the initial and exit belief surveys completed by thirty-one students from the study section found that fourteen of the students wrote statements on the exit survey indicating that their mathematical beliefs had become more productive. Eight of these fourteen students reported that the study section's alternative assessments were most influential in such changes to their beliefs. Although this data implied that alternative forms of assessment might indeed be able

to challenge pre-service teachers' counterproductive beliefs, these results required a great deal of interpretation on the part of the researcher. The final interviews with the informants demonstrated that the survey findings provided only a partial picture of the alterations, if any, that occurred within the students' mathematical belief systems.

The comprehensive evidence suggests that five of the seven informants held fast to the mathematical beliefs they had communicated to me during the first two interviews. Whereas I saw discrepancies in certain responses implying change, the informants often reported that they saw their comments as meaning "kind of the same thing" (C5, 4). These results confirm Pajares (1992) when he writes, "the more central the belief, the more it will resist change" (p. 318). Even when the informants were willing to admit that changes had occurred, very rarely were they to core beliefs. These pseudo-changes fell into one of two categories: peripheral changes or creating an entirely new belief system.

In the first category, changes in peripheral beliefs, there is an apparent shift in the belief system, but it is limited to those beliefs found along the periphery. For example, Karen seemingly changed her view that the teacher represents the mathematical authority during the course of the semester based on her survey responses (see BMTS, 1 and WMTS2, 1). In her commentary on these changes, however, it became apparent that Karen had altered her beliefs about teaching mathematics and not the nature of mathematics. The shift in peripheral, or pedagogical, beliefs appeared as a change in the entire belief system, but in truth, the central beliefs remained constant (see Figure 17). Cooney and Shealy (1995) and

Raymond (1997) described similar examples of cases where peripheral beliefs about classroom practices shift without there being any changes to the central beliefs about mathematics.

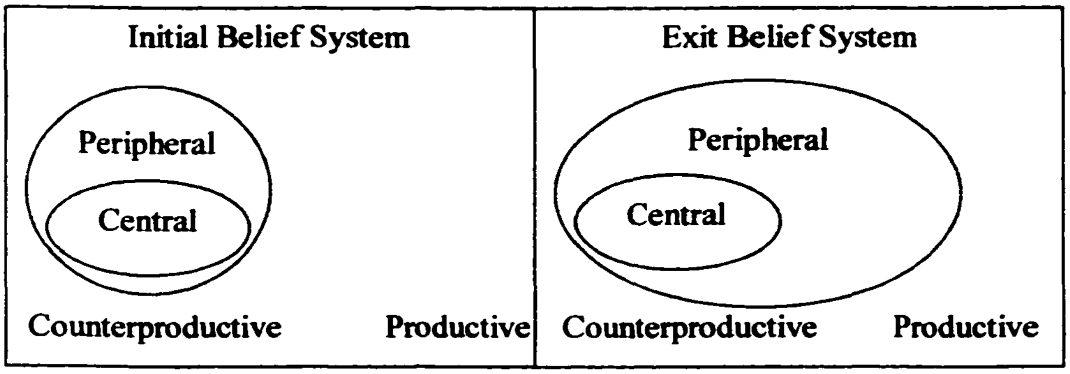


Figure 17. Example of a Peripheral Shift.

Keona, on the other hand, was a prime example of student who evidently created a new belief system in order to accommodate any experiences in the *Probability and Statistics* course that ran counter to her initial central beliefs. According to Keona's comments, the instructor's authority was responsible for the development of an entirely new belief cluster. As a result of something the instructor said about a solution to one of the problems from *Evaluating Responses*, Keona reported no longer believing that all mathematics problems had only one correct answer, but she was not totally convinced. The fact that she based this new belief solely on the instructor's statement because she could not validate it herself indicates that this belief reflected the view held by the instructor and not a belief based on her experience. Moreover, Keona indicated reluctance in accepting that the experiences encountered in *Probability and Statistics* represented mathematics as a whole. Thus,

this new system of beliefs seemed to refer only to the study section (see Figure 18). Because her initial beliefs and exit beliefs were contained in separate systems, or clusters, Keona was able to hold both sets of beliefs even though some contradicted one another (Thompson, 1992).

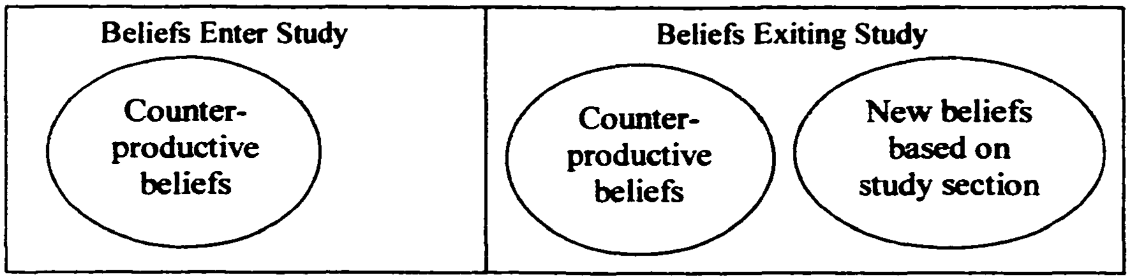


Figure 18. Example of Belief Clusters.

Janet and Lynn represented the lone informants to demonstrate what was determined to be genuine shifts in their central beliefs about mathematics from counterproductive to productive. Furthermore, both of these informants reported that the alternative assessments from the study section were the most instrumental factors in their shifts in beliefs. This supports the findings from the comparisons of students' initial and exit belief surveys that suggested a relationship between alternative assessments and the perceived productive shifts in fourteen students' beliefs about mathematics. Nothing in the data explains, however, why these assessments affected only certain students' central mathematical beliefs and not all the students from the study section. Herein lies one of the limitations of this study.

Looking Back at the Study's Limitations

The limitations of this study can be broken into two categories. First, there are the limitations associated with the study's design; these include the focus of the study, the selection of informants, and the collection of data. Limitations resulting from the implementation of the study's design make up the second category. In this set, I address possible intervening factors that occurred during the implementation of the research plan. Both design and implementation limitations are discussed here.

Design Limitations

Over the course of the study, interesting questions arose pertaining to certain aspects of the data. For example, it was not clear why the students in the pilot study and the informants embraced the belief that they should persevere when encountering difficult mathematics problems. Other researchers had reported that students were likely to dismiss a problem as too hard if they worked on it for more than five minutes. Although this discrepancy was intriguing, the purpose of this research was to determine the effect of alternative assessments on pre-service elementary teachers' beliefs about mathematics. Similarly, it was not the intention of this research to determine why assessments were influential in affecting some students' central mathematical beliefs and not others, only if the assessments were at all responsible. Thus, the focus of the study limited the questions that could be explored.

A further limitation in the design of the study was the method chosen for the selection of informants. The use of convenience sampling to gather a pool of possible

informants assured that they would be willing participants in the study and open to sharing their perceptions. On the other hand, this technique also requires the researcher to recognize the qualities that accompany a volunteer. In the case of this research, for example, the students signing-up for the study were probably interested in the idea of exploring the relationships between mathematical experiences and beliefs (see the Oral Script for Recruiting Informants in Appendix C). While the research design called for a more representative sample to be selected from the pool of volunteers, the very fact that they showed interest in the study when others did not must be kept in mind. Therefore, the sampling restricts the generalizability of the results.

The research methodology utilized in the collection and analysis of data represents the final design limitation. Pajares (1992) writes, "beliefs cannot be directly observed or measured but must be inferred from what people say, intend, and do – fundamental prerequisites that educational researchers have seldom followed" (p. 314). While I attempted to determine the exact nature of each informant's mathematical beliefs using a variety of methods in an attempt to triangulate my findings, the results presented in this report, at best, represent my interpretation of what informants said and did in the interviews. The fact that individuals are often unaware of their own beliefs or the reasons behind them further complicated this issue (McEntire & Kitchens, 1984; Pajares, 1993). Activities such as the card sorts and vignettes attempted to access these beliefs and experiences, but always left open the possibility for erroneous interpretations. The informants sometimes

misunderstood instructions and, at times, I found myself making assumptions about certain statements without checking the validity of my interpretation with the informant. Whereas both of these behaviors are indicative of human error and common in most research, one cannot dismiss them as limitations.

Implementation Limitations

The remaining limitations resulted from circumstances arising during the implementation of the study's design. Issues related to the selection of informants, their awareness of the researcher's philosophical position, the instructor chosen for the study, and extrinsic characteristics of the study section's assessments were all possible factors influencing the research findings. One must consider these issues when examining the results.

First, almost any research benefits from including more subjects and ensuring that the informants are representative of the population as a whole. Because only seven students agreed to take part in the study, it was not possible to select a more purposeful sample. While seven informants represented a sizable percentage of the study section's thirty-two students, other students' perspective would have certainly enhanced the research results. Along those same lines, having a more representative group of students would allow for greater insight into how the study section as a whole reacted to the alternative assessments. It was not a great concern that no males were accounted for in the study for a variety of reasons (including low male enrollment in the elementary education program and no evidence that gender

contributed to differing beliefs during the pilot study). Still, without the male perspective it is difficult to maintain that these findings are indicative of all pre-service elementary teachers. Furthermore, redundancy is a desirable attribute in qualitative research. Although the seven informants offered supporting evidence on several research questions, additional collaborating or alternative results would have strengthened the findings reported here.

That the informants were probably aware of my philosophy regarding the teaching and learning of mathematics presents a second issue. Certainly, every attempt was made to ensure that the interview questions were objective in nature and allowed the informants to express their reality without the influence of my personal bias. Furthermore, throughout the interview process the informants assured me that I did not lead them in any way. Still, it is unrealistic to suggest that the informants were unaware of my position that the methods used to assess students in mathematics affects their view of the subject. There is no indication that this awareness influenced any of the informants' responses, but it is a consideration.

Three possible limitations related to the implementation of the study's design surround the instructor of the study section. First, she was my dissertation advisor and had access to all my background research. Having read much of the same literature on beliefs and assessment and being directly involved in the development of my research methods, my advisor was probably more connected to this project and attuned to my expectations than any arbitrary instructor would have been. The alternative options, placing myself as combination teacher and researcher or

involving an instructor who did not share my vision, could prove just as limiting though; thereby the choice of my advisor seemed the most appropriate of the three. Second, as my advisor, it was possible for her to ascertain the identities of certain informants based on their descriptions despite the use of pseudonyms. On occasion, the informants themselves broke their own anonymity by referring to me in their journal responses. Therefore, although the instructor attempted to avoid any knowledge of the informants' identities, once it became apparent it is possible that it influenced the instructor-student relationship and thereby contributed to the intervention. Finally, the instructor's expertise in assessment and beliefs certainly affected each student's experience in the study section, making it difficult to limit any shifts to productive mathematical beliefs to the assessments alone. The instructor's command of mathematical assessment and understanding of belief development may have contributed to changes in students' view of mathematics. In fact, I contend that one of the implications of this study is that teachers must possess just such knowledge in order to effectively and accurately teach students about the nature of mathematics.

A final pair of considerations concerns the format of the first two assessments and the setting for the final exam. Because *Assessment Writing* and *Evaluating Responses* attempted to mirror teacher activities, when reflecting on the assessments the students in the study section tended to focus more on pedagogy rather than mathematical content. This seemed to affect the informants' ability to see how such assessments could be used to evaluate elementary students' content understanding. It is also possible that presenting the assessments in a pedagogical format interfered

with the study's attempt to challenge to the students' counterproductive mathematical beliefs, because this format allowed the students to focus on peripheral beliefs instead. The fact that the study section's final exam resembled a traditional, paper-and-pencil mathematics test (it was timed and students took it individually) was also an issue for some informants. Both Janet and Lucia felt it was reminiscent of earlier tests and thus represented a view of mathematics associated with certain counterproductive beliefs. Perhaps the final exam affected the informants' exiting beliefs more than anyone realized; in essence, it could have undone any productive shifts by reinforcing earlier counterproductive beliefs. As was the case with the instructor, the possible limitations associated with the study section's assessments and final exam result in implications as well.

Looking Forward to the Study's Implications

While the previous section describes several limitations inherent in the study, this does not indicate that the study's findings do not have certain far-reaching implications. The research demonstrates that assessment tasks can affect an individual's beliefs about the nature of mathematics. This fact alone should be of interest to mathematics teacher educators, mathematics education researchers, and anyone who teaches mathematics.

Alternative Assessments in the K-12 Mathematics Curriculum

The findings presented here have shown that traditional pencil-and-paper testing experiences in mathematics can foster and further support counterproductive beliefs in students. For example, Keona reported that she had never seen a test question with more than one right answer before the study and that consequently she viewed answers to mathematical problems as absolute. Romberg (1992) makes a case that the traditional mathematics test exemplifies "an essentialist view of knowledge, a behavioral absorption theory of learning, and a dispensary approach to learning" (p. 47). Other researchers agree that traditional mathematics tests that require students to follow an algorithm taught by the instructor to achieve a predetermined answer do not reflect the true nature of mathematics (M. Wilson, 1992, L. Wilson, 1994b). Yet, for many of the informants, memorizing facts and formulas provided by the teacher or the textbook was synonymous with learning mathematics because that was what was necessary to succeed on a test.

Consequently, in order for assessments to align with reform efforts in mathematics instruction and curricula, K-12 teachers need to become more proficient at creating and/or choosing assessment methods that reflect the complexities of mathematics. Unfortunately, as was discussed earlier, teachers are rarely confident in the development, utilization, or evaluation of alternative assessments (Cooney et al., 1996; Cooney & Shealy, 1995; Long & Benson, 1998; Senk et al., 1997; Wilcox & Lanier, 1999). This suggests a second implication of this study.

Alternative Assessments and Pre-service Teachers

Mathematics education programs must prepare pre-service teachers to use alternative assessment methods. Based on this research, exposure to new assessment tasks is not enough to ensure that pre-service elementary teachers will embrace the use of alternatives to tests in assessing mathematics. The exposure increases their awareness that various assessment methods exist, but most are dubious about their ability to design their own; a finding supported in numerous studies (Cooney et al., 1993; Crooks, 1988; Lambdin, 1993; Long & Benson, 1998). Recommendations to provide pre-service teachers plenty of opportunities to create, critique, employ, and evaluate the results of alternative assessments in their mathematics courses are hardly new (see Senk et al., 1997; Raymond, 1998). Certainly, such experiences improve the likelihood that a pre-service teacher will choose to assess mathematics using alternative methods, but only if the teacher values the mathematics inherent within such tasks (Cooney et al., 1993). The amount of time and effort involved when using alternative assessments necessitates teachers committed to reform ideals. Hence, the problem of pre-service teachers' counterproductive mathematical beliefs returns to center stage.

Changing Pre-service Teachers' Absolutist View of Mathematics

Because a teacher's central mathematical beliefs affect not only instructional choices but also decisions involving assessment, teacher preparation programs must make challenging counterproductive beliefs about mathematics a priority. Education

majors must encounter fallibilist ideas in all areas of their mathematics courses – instruction, curricula, and assessment. The findings presented in this document indicate that experiences with alternative assessments can send a powerful message about the nature of mathematics to some students, but these assessments alone do not bring about changes to central beliefs.

An individual's central beliefs tend to exert tremendous control over the person's behavior, including how he or she interprets a particular experience (Benbow, 1995; McDiarmid, 1990; Pajares, 1992). Two examples of this principle are evident within my findings. First, several informants exhibited a reluctance to let go of the belief that memorization was important in mathematics even though recent experiences had stressed understanding, especially through assessment. Instead, they modified their belief; memorization of the basic facts was necessary in elementary school so students could later develop understanding. Shaw (1989), Schoenfeld (1992), and Vacc and Bright (1999) all described encountering similar findings. The second instance of protecting central beliefs occurred because two of the assessments selected for the study section reflected teacher activities – making a test and grading student work. As a result, several informants focused primarily on changing their pedagogical beliefs, which exist on the periphery of a student's belief system.

Therefore, if mathematics teacher educators are going to challenge pre-service teachers' counterproductive beliefs, then perhaps the attack must be more direct. This will require intentional discussion about different mathematical beliefs systems and how these beliefs are reflected in the various assessment methods. While the

instructor in this study did discuss various mathematical beliefs, even assigning the students a journal entry asking about their perception of assessments and beliefs in mathematics, the design of the study was to focus on the effect of the assessments themselves. A different design would be required to determine whether a direct attack on counterproductive beliefs is worthwhile.

Recommendations for Further Study

Besides studying the effectiveness of confronting students' misconceptions about the nature of mathematics head on, this study raises several other questions for researchers in mathematics education to examine. These questions include, but are not limited to the following:

1. How do elementary students develop their beliefs concerning mathematics? The informants from this study found it difficult to pinpoint the causal experiences related to their earliest mathematical beliefs. It was much easier for them to identify recent factors influencing newer beliefs. Therefore, research with elementary age school children would enable a clearer picture of the relationship between mathematical beliefs and instructional, assessment, and ancillary experiences.

2. What effect will recent reform efforts in mathematics have on the mathematical beliefs of future elementary education majors? All of the informants in this study reported encountering traditional mathematical experiences in both instruction and assessment. Many of their counterproductive beliefs could be traced

to these experiences. It would be interesting to investigate if this longstanding trend will continue or if students will experience enough non-traditional mathematics prior to college so that their beliefs will be more productive.

3. Why did the alternative assessments from the study section affect some students' mathematical beliefs and not others? All the necessary information is available within this document to allow a researcher to replicate this study, but add to the framework methodology that would analyze this discrepancy. If it is possible to determine the reason why students value assessment experiences differently, then teacher educators can use this information to plan assessment tasks accordingly.

4. Would authentic assessments not related to teaching be more likely to challenge pre-service teachers' absolutist view of mathematics? Two of the three assessments dealt with tasks of teaching, and this led some students to concentrate on pedagogical issues rather than their mathematical beliefs. (Because of lack of time and low informant interest in the third assessment, the *Monte Carlo Task*, there was not enough data to analyze its effect.) A researcher could follow the procedures outlined in this report but select tasks not associated with teaching and then investigate the changes in students' beliefs.

5. What are the long-term effects of alternative assessments on teachers' mathematical beliefs? Previous research has shown that when a teacher's non-traditional methods meet challenges, the teacher often reverts back to teaching as they were taught, which typically means in a traditional manner (Ball, 1990; Foss, 1997; Raymond, 1997). It would be of interest to follow students, like Janet and Lynn, who

reported that alternative assessments played a major role in their more productive mathematical beliefs. Thereby, researchers could determine whether these teachers experienced the same difficulty holding to their productive beliefs as the teachers from prior studies.

Concluding Thoughts

> Tests also must change because they are one way of communicating what is important for students to know. ... In this way tests can effect change. (NCTM, 1989, p189-190)

Whereas the results from this study confirmed the NCTM's position that under certain circumstances assessments can be the agent of change, the resistance that accompanies any attempt to alter a person's core beliefs was also evident. Although assessment cannot be considered the Holy Grail in the fight against counterproductive beliefs, that does not diminish the need to align assessment, curriculum, and instruction so that they represent more productive beliefs. Such an alignment may ultimately result in a community of learners who, when encountering a difficult mathematics problem, are willing to persevere because they are confident in their own ability to create reasonable, mathematical solutions.

Appendix A

Alternative Assessment Descriptions and Rubrics

Assessment Writing

OBJECTIVE: to assess your ability to construct and draw inferences from plots, tables, and graphs that present data from real-world situations and to determine the depth of your understanding

METHOD: Pairs of students will design assessment questions that could be used to evaluate their peer's understanding of the topics presented in class. The assessments should adequately represent the main concepts and procedural information found in chapters one and two of the [*Probability and Statistics*] course pack and additional information discussed during class. The assessment questions turned into the instructor will also need a correction key. It is assumed that all students "taking" your exam will have access to a TI-73 calculator. Assigning point values is not required or expected. It is expected that your group's work will demonstrate your understanding and mastery of the topics.

NEEDED: Read the following criteria several times. The final product **must** include the following in order for the project to be acceptable:

- unique questions that attend to main concepts and procedural information (one question can and should attend to several rubric criteria).
- a key to the assessment that shows possible correct solutions for each question.
- a rubric of the assessment task thoughtfully completed by the pair.
- an overall evaluation of your small group (including yourself) using the rubric provided.

GRADE: The instructor will use the rubric provided on the back of this sheet to evaluate the projects. A scale for each criteria is presented below.

5 (*exemplary*) Questions AND responses clearly demonstrate deep understanding of the material and of the intent of the assessment. The developed questions and responses are clear in their purpose, are of a high quality, and provide sufficient intellectual challenge to the test-taker. The task highlights the group's creativity.

4 (*above average*) Many of the qualities from the Exemplary category are evident but not all. There must be evidence of a good understanding of the material and intent of the assessment.

3 (*minimally met*) Some of the qualities from the Exemplary category are evident.

2 (*needs work*) Unacceptable level of understanding is demonstrated.
(i.e.: all work is not shown)

1 (*attempted*) Project reveals a lack of understanding of the task and/or the material.

0 (*not evident*) No evidence provided.

Each group member should assess all work before it is submitted and complete the following rubric.

DUE DATE: Oct 1st (any work submitted after this date will receive a 0 for submission points).

Assessment Writing Rubric

"Instruction guide" for using the rubric:

- **READ** the rubric through carefully.
- **DEVELOP** and **ASK** questions about the scoring and expectations of using the rubric.
- **WORK** on the task, **KEEPING** the rubric close at hand.
- Critically **EVALUATE** your work using the rubric.
- **COMPARE** your evaluation of your work using the rubric with your groupmates.
- **REWRITE** your work based upon the critical evaluations.
- **SUBMIT** the rubric below, with your group's brief but thoughtful comments on how each expectation was met, at what level you met the expectation, and why you believe the work is at that level.

NCTM Standards

_____ Questions ask students to apply knowledge and truly use problem solving skills, not simply recite answers.

_____ Questions make use of "real-world" data the [*Probability and Statistic* students] would find interesting and relevant to their future teaching profession.

_____ Questions provide opportunities for communicating student understanding in a variety of ways vs. simply a computational or single sentence response.

Constructing Graphs

_____ Graphs correctly represent the information being presented.

_____ A circle graph has been chosen for the appropriate data and I can clearly determine that it is correctly based on raw data provided.

_____ A histogram has been chosen for the appropriate data and I can clearly determine that it is correctly based on raw data provided.

Constructing Tables

_____ A contingency table has been chosen for the appropriate data and I can clearly determine that it is correctly based on raw data provided.

_____ A frequency table has been chosen for the appropriate data and I can clearly determine that it is correctly based on raw data provided.

Constructing Plots

_____ A stem-and-leaf plot has been chosen for the appropriate data and I can clearly determine that it is correctly based on raw data provided.

Assessment Writing Rubric
(continued)

Interpreting Graphs

_____ A good distribution of read, derive, and interpret questions are provided (this does not mean that every question must have a read, derive, and interpret component nor that other "types" of questions should not be included!) These need to be labeled on the key.

_____ Questions involving a misleading (vs. just inappropriate) graph are provided.

_____ Questions involving a picture graph are provided.

_____ Questions involving a bar graph are provided.

_____ Questions involving a circle graph are provided.

_____ Questions involving a line graph are provided.

_____ Questions involving a scatter plot are provided.

_____ Questions involving a histogram are provided.

Interpreting Tables

_____ Questions involving a contingency table are provided.

Interpreting Plots

_____ Questions involving a stem-and-leaf plot are provided.

Technology Use

_____ At least two questions are developed that make appropriate use of the TI-73 and its graphical/tabular features.

Overall Quality

_____ Assessment questions are word processed, clearly written, and error free (the key may be hand written but needs to be legible).

_____ Assessment task is submitted in time.

_____ Assessment rubric has been thoughtfully responded to and submitted with the task (worth double points).

Evaluating Responses

OBJECTIVE: to make and evaluate arguments that are based on data interpretation;

to understand and apply concepts of central tendency, spread, and variability;

to describe, in general terms, the normal curve and its properties to answer questions about sets of data that are assumed to be normally distributed;

to construct and draw inferences from tables and graphs that represent data;

METHOD: **In pairs**, students will create a key for a pre-existing [*Probability and Statistics*] exam. The exam questions are based on material presented in chapters two through four from the course pack, and supplemental materials presented in class.

Individually, students will use their "pair" key to evaluate the answers from an exam completed by a student who was selected randomly from a large pool of imaginary [*Probability and Statistics*] students existing in [the instructor's] head.

This last piece of the assessment will be completed in class on October 22 and will involve the whole class period.

NEEDED: The final product **must** include the following in order for this assessment to be acceptable:

- a key for the original exam from each pair (one key is needed from each pair; during the assessment, everyone will need a copy of their key to refer to making a total of 3 copies);

- a copy of the evaluated exam from each individual in the pair;

GRADE: I will use the rubric provided on the back of these instructions to evaluate the assessment. You should self-assess your key and each pair should assess their evaluation of the answers before it is turned in to the instructor.

DUE DATE: Oct 22 (work submitted after this date will incur a 10% penalty).

Evaluating Responses Rubric

Key *(0-40 points)*

Point values have been assigned to each problem and are provided on the exam itself next to each question. I will be "grading" your key using the following criteria as I evaluate each response:

100% **Exemplary** – all work is shown and correct. Construction of plots are correct. Answers requiring an explanation are clear, concise, and correct.

80% **Above average** – all work is shown and the student is headed in the right direction, but some steps are incorrect. Construction of plots contain an error. Explanation is not always clear and concise.

60% **Minimal** – all work is shown but method chosen to solve the problem is not appropriate, or some work is missing. Construction of plots contain two errors. An explanation has some elements of understanding but result is unclear.

40% **Needs work** – no work is shown, but answer is correct. Construction of plots contains three errors. The explanation has one point of understanding but the result is incorrect.

20% **Attempted** – the problem has been tried, but contains few elements of understanding.

0% **Blank or Attempted** – the problem has not even been attempted or the problem has been tried, but contains no elements of understanding.

EVALUATION and SUMMARY of the Imaginary [*Probability and Statistics*] Student Work *(0-60 points)*

In class, on October 22nd, each of you will use the key you created to evaluate [a Probability and Statistics] student's exam. You are to use the above scale as you evaluate the work of the student. I will then assess your evaluation work based on the following:

The point evaluation of each response is reasonable based on the number of errors present. In other words, all mistakes have been identified. *(0-25 points)*

If less than full credit is given to an answer, then a justification (using the rubric) for the score is provided for the student on the exam itself. *(0-20 points)*

Possible rationale for student mistakes are provided for the appropriate questions. *(0-15 points)*

Math 266 Fall 1998 Exam II

Name: Meryl Streep

When responding to items on this test, please show all work, or steps taken, so that you will be eligible for partial credit.

1. **The stem-and-leaf plot given at the left below shows the grade level reading scores of several high school students. Note that the leaves have not been ordered within each stem.**

Stem	Leaves
8	1
9	56597
10	9168273405988
11	89721643017565
12	3213

s	l
8	1
9	5679
10	0234579
11	0123456789
12	123

Key: 10 | 3 represents a grade level of 10.3

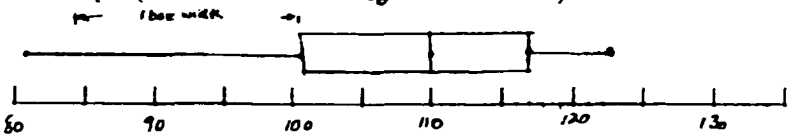
10 pts

a. **Determine the five critical numbers needed for constructing a box-and-whisker plot of these scores. Name and give the value of each of these five numbers:**

Name:	Lower extreme	Lower Quartile	Median	Upper Quartile	Upper Extreme
Value:	81	101	110	116.5	123

b. **Draw a box-and-whisker plot of the scores using the given line segment to define and label a scale for the plot (use as much of the line segment as is feasible).**

5 pts



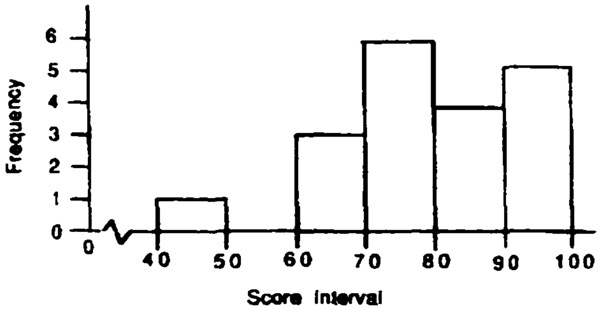
c. **Determine if any of the scores in this data set should be considered outliers. Justify your response.**

5 pts

To be an outlier, the data value has to be more than 1.5 times the width of the box. 123 is no way an outlier. To close to the upper quartile. Now 81 could be but I measured it out and it isn't an outlier.

2. The histogram below shows a distribution for a set of test scores (all scores were whole numbers less than 100).

Histogram of Test Scores



a. Create a grouped frequency distribution of the data as presented in the histogram.

	Score Interval	Frequency
$\frac{40+50}{2} = 45$	40 - 50	1
$\frac{50+60}{2} = 55$	50 - 60	0
65	60 - 70	3
75	70 - 80	6
85	80 - 90	4
95	90 - 100	5

8 pts

You are to use your graphing calculator to approximate the mean and standard deviation of the original set of test scores from the grouped data provided above.

b. First, you need to enter the data into two lists into your graphing calculator. Provide the "title" in the header of each list (eg L_1, L_3, Interval, etc). Also write out the contents you entered into each list.

Header for 1st List: L_1 Contents: 45, 55, 65, 75, 85, 95

8 pts

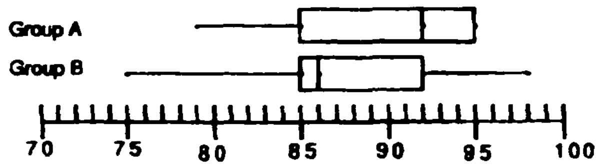
Header for 2nd List: L_2 Contents: 1, 0, 3, 6, 4, 5

c. Use a graphing calculator to approximate the mean and standard deviation of this set of data. State the specific (not rounded) values of each of these statistics as shown on the calculator display.

Mean: 79.21052632 Standard Deviation: 13.10515747

8 pts

3. Box-and-whisker plots of the exam scores for two different groups are given below. Assume that each group has approximately 1000 subjects. Answer each of the given questions based upon the information provided. If more information is needed to answer a question, then respond by answering MIN (more information needed).



a. What is the interquartile range for Group B? 7

3 pts

b. Complete this sentence: Approximately 50 % of the subjects from Group B scored at, or below, the median score in Group A.

3 pts

c. Approximately what percent of scores from the two groups are at or above 95 on the exam?

4 pts Group A: 0% Group B: 12.5%

d. Assume now that Group A consists of only 15 subjects. Give a list of 15 scores (ordered from low to high) that will "fit" the box-and-whisker plot for Group A given above.

79	80	81	85	85	86	87	92	93	94	95	95	95	95	95

5 pts

4. Use the definition of standard deviation as the square root of the average of the squared deviations from the mean (not your calculator's statistical memory capabilities) to determine the standard deviation of the numbers 18, 16, 15, and 11. (Note that the mean of this small data set is 15.) Organize your paper and pencil computations into an appropriate table with column labels.

6 pts

X	$x-\bar{x}$	$(x-\bar{x})^2$
18	3	9
16	1	1
15	0	0
11	−4	16
		$\Sigma = 26$

$\frac{26}{4} = 6.5$

$\sigma \approx 2.5$

5. Complete the following set of six data values so that the mean, median, and range of the set are all equal to 36. Assume that the values (Data 1, Data 2, etc.) are ordered from low to high. Explain how you know each answer without having to use a trial-and-error approach. (Note: more than one solution exists for this problem.)

Data 1	Data 2	Data 3	Data 4	Data 5	Data 6
21	24	32	40	42	57

Explanations: (That is describe how you arrived at the four unlisted data values Data 3, Data 5, and Data 6. You may need to determine these data values "out of order.")

Data 3 must equal 32 because $\frac{32+40}{2} = 36$ the median

10 pts Data 5 must equal 42 because $\frac{21+24+32+40+42+57}{6} = 36$

Data 6 must equal 57 because $57-21=36$ 36 is the range

6. One of your students is working a problem from the text that required him to determine the median of the following frequency distribution. Reasoning that "median" means "middle" (refer to student's arrow in table) he determines that the required median is 30.

Data	Frequency
15	6
20	15
25	10
--> 30	4
35	3
40	2
45	1

a. How might you get the student to understand the error in his thinking without just telling him the "right way" to get the answer? Be specific.

I would remind him that he needs to add everything up first

6 pts

b. Now, tell me how this student should have determined the median from the given frequency distribution (without the aid of a calculator's statistical capabilities).

6 pts add everything up and divide by 41.

7. The results on a job aptitude exam taken by 250 applicants gave an overall mean of 78 and a standard deviation of 6. (Assume the distribution of exam scores is normal.)

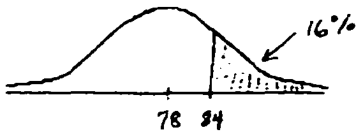
a. One applicant earned a score of 75 on the exam. Z-scores were computed for all applicants. What z-score did this applicant earn?

$$\frac{78-75}{6} = \frac{3}{6} = .5$$

4 pts

b. Another applicant earned a score of 84 on the exam. Approximately what percent of those taking the exam scored lower than this applicant?

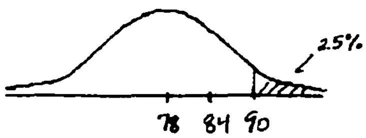
Label and shade a diagram of a normal curve to indicate the "region" of the solution and then provide a solution. (Recall that about 68% of the scores fall within ±1 SD of the mean, 95% within ±2 SDs of the mean, and 99.7% within ±3 SDs of the mean.)



$$\begin{array}{r} 100 \\ -68 \\ \hline 32 \end{array}$$

5 pts

c. If a z-score of +2 or higher was necessary to be selected for a follow-up interview, about how many of the 250 applicants would be called in for an interview?



$$\begin{array}{r} 100 \\ -95 \\ \hline 5 \end{array}$$

≈ 6.25 people

5 pts

Monte Carlo Task

OBJECTIVE: to model situations by devising and conducting simulations to estimate probabilities;

to construct and draw inferences from tables and graphs that represent data;

to solve problems that involve collecting, organizing, and interpreting data;

to make arguments that are based on data interpretation;

to understand and apply concepts of central tendency and variability;

to use experimental probability to represent and solve problems involving uncertainty.

METHOD: Students will each receive a problem that can be solved using a simulation model (the Monte Carlo Technique). Each student team/pair will design a model, carry out a simulation, and analyze the results. A presentation of the team's findings may be made using any method that meets the prerequisites.

NEEDED: The final product **must** include the following in order for this assessment to be acceptable:

- an introduction to the planning process used in solving the problem;
- an organized representation of the simulation (including at least 100 trials);
- a statistical analysis of the simulation;
- a summary of the findings including graphical representations.

Further details on what should be included in each of these broad categories are provided on the rubric.

GRADE: I will use the rubric provided on the back of these instructions to evaluate the project. Students should perform both self and peer assessment of their work prior to submitting it to the instructor.

DUE DATE: Dec. 1st. No late work can be accepted after Dec. 1st.

Monte Carlo Rubric

Given the original problem statements:

Planning

12 **Predict** the outcome of your questions and explain your reasoning.

Describe a model that matches problem characteristics to the generation of random outcomes.

9 Create a TI-73/82/83 program that will run the simulation for you

Experiment

6 Define and conduct **one** trial, just one, and record the observation of interest.

3 Then properly conduct and display at least 100 trials using the model defined during the planning portion of the project.

6 Organize the data using a histogram and describe what information the display provides about the distribution of the data.

6 Represent the spread of the data using a box plot. Describe differences between the box plot and histogram in terms of what you can or cannot say about the data depending upon which display you choose.

Statistics

6 Find the mean, median, and mode of the data and explain what each represents **in this situation, not in general**.

3 Find the range of the data and explain what it represents **in this situation, not in general**.

3 Find the standard deviation of the data and explain what it represents **in this situation, not in general**.

3 Find the inter-quartile range of the data and explain what it represents **in this situation, not in general**.

6 Find the upper and lower quartile of the data and explain what each represents **in this situation, not in general**.

Summary

9 Use the data from the Monte Carlo simulation to answer the situation's questions

9 Which central tendency best answers the "average question" and why (its pros and others' con).

3 Include a circle graph that displays the results of the "probability question".

Keys

Use the key below for the above criteria		Use this key for the Quality Control criteria	
100%	Correct, Complete, and Clear (the 3 Cs)	100%	Exemplary
$66\frac{2}{3}$%	Two out of the three Cs are met.	80%	Above average
$33\frac{1}{3}$%	One out of the three Cs are met.	60%	Minimally met
0%	None of the three Cs are met.	40%	Needs work
		20%	Attempted
		0%	Not evident

Correct: mathematically appropriate and true; **Complete**: attended to each part of the criteria; **Clear**: response are well written and anyone in [*Probability and Statistics*] could understand what you are saying.

Quality Control

4 Work done is of a professional quality, carefully compiled and completed and submitted on time.

5 The format for the presentation of the project was creative.

6 There is evidence of peer assessment (other than those in the pair) of the work completed.

Seven Monte Carlo Situations

1 In the United States, 45% of the people have type 'O' blood. These people are called universal donors since their blood can be used in transfusions to people of any blood type. Assume that donors arrive at the local blood bank in a random fashion. How many donors would need to stop at the blood bank in a day, if those running the bank want 5 type 'O' blood donors each day? What is the probability that the bank will achieve their goal of 5 type 'O' donors with 10 or fewer donors?

2 A father was given twenty new ties for Christmas. So as not to offend anyone he chooses a tie at random to wear to work each day. After wearing a tie he places it back on the rack so it might be picked again. How many different ties do you expect that he will wear before picking a tie he has already worn? What are the chances that he will wear 5 or more different ties before picking one that he wore before?

3 Lounge-lizard Larry is at it again. He's trying to strike up a conversation with single women at a local watering hole by using the oldest line in the book. He is asking them their zodiacal sign. What are the chances that Larry will come across two women with the same sign before asking six women? How many women will Larry have to ask before he comes across two women with the same sign?

4 A multiple-choice test consists of 10 questions, and each question has three possible answers, only one of which is correct. Chris has not studied for the test and must guess on each question. How many right answers would you expect Chris to make on the test? What are the chances that Chris will get 50% or better?

5 Joe drives a minibus in his town. The bus has ten seats. People buy tickets in advance, but on the average each person has a 25% chance of not showing up. Trying to fill the bus, Joe sells 15 tickets for each trip. Sometimes more than 10 people show up. What percent of the time will there be more people than seats? On the average, how many people will show up with a ticket?

6 Suppose the probability that an exploratory oil well will strike oil is about 0.3 and that each well costs $5,000 to drill. Assume that the outcome of each drilling is random. How much will it cost before two wells find oil? What are the chances that it will cost $35,000 or less to drill two successful wells?

7 Entering the final game before graduation, high school quarterback Ace Passer is 5 completions away from completing 100 passes in his career. Over four years he has steadily completed 55% of his passes. How many passes will he need to attempt in order to reach exactly 100 completions? If the coach has a strict rule against passing the ball more that 10 times in a game, what are the chances that Ace will reach 100 completions.

Appendix B

Protocol Clearance From the Human Subjects Institutional Review Board

Human Subjects Institutional Review Board

Kalamazoo, Michigan 49008-3899

WESTERN MICHIGAN UNIVERSITY

Date: 10 August 1998

To: Christine Browning, Principal Investigator
David Coffey, Student Investigator

From: Richard Wright, Chair Richard A. Wright

Re: HSIRB Project Number 98-08-05

This letter will serve as confirmation that your research project entitled "The Connection Between Alternative Assessment in a Mathematics Education Content Course and the Mathematical Beliefs of Pre-Service Elementary Teachers" has been **approved** under the **exempt** category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note that you may **only** conduct this research exactly in the form it was approved. You must seek specific board approval for any changes in this project. You must also seek reapproval if the project extends beyond the termination date noted below. In addition if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: 10 August 1999

WESTERN MICHIGAN UNIVERSITY
H. S. I. R. B.
Approved for use for one year from this date:
AUG 10 1998
HSIRB Chair

Western Michigan University
Department of Mathematics and Statistics

The Connection Between Alternative Assessment in a Mathematics Education Content Course and the Mathematical Beliefs of Pre-service Elementary Teachers
Advisor: Christine Browning
Student Investigator: David Coffey

I have been invited to take part in a research project intended to study the relationship between mathematical beliefs and assessment in elementary education majors. I further understand that this project is a study for David Coffey's dissertation project.

My consent to participate in this study indicates that I am willing to meet with David five times during the course of the semester to discuss my mathematical background and what is currently going on in mathematics class. Each of these meetings will last between forty-five minutes and an hour and will be tape-recorded after I give my consent at each meeting. All meetings will occur at a mutually convenient place and time. I also understand that David will make copies of all my course work. This includes any written work connected to this class - assessments, homework, journals, etc.

I understand that there are no anticipated risks for participating in this study. As in all research, there may be unforeseen risks to the participant. If an accidental injury occurs, appropriate emergency measures will be taken; however, no compensation or additional treatment will be made available to the subject except as otherwise stated in this consent form.

One way in which I may benefit from participating in this study is to become more aware of my beliefs about mathematics and how they are related to past and present assessments. I also understand that this information will help mathematics educators in their attempts to better prepare future elementary educators.

I understand that all information collected will be treated as confidential. That means that my name will not appear on any papers on which this information is recorded. The forms will all be coded, and David Coffey will keep a separate master list with the names of all participants and corresponding code numbers in a locked file. Once the data are collected and analyzed, the master list will be destroyed. All other forms and tapes will be retained for three years but will serve no other purpose besides providing information for this study.

I understand that I may refuse to participate or quit at any time during the study without prejudice or penalty. If I have any questions about this study I may contact either Dr. Christine Browning at 616-387-4561 or David Coffey at 616-387-4579. I may also contact the Chair of the Human Subjects Institutional Review Board at 616-387-8293 or the Vice President for Research at 616-387-8298 if questions arise during the course of this study. My signature below indicates that I understand the purpose of this study and that I agree to participate.

_________________________ __________
Signature Date

This consent document has been approved for use for one year by the Human Subjects Institutional Review Board (HSIRB) as indicated by the stamped date and signature of the board chair in the upper right corner. Subjects should not sign this document if the corner does not show a stamped date and signature.

Appendix C

Data Collection Instruments

Oral Script for Recruiting Informants

Hello, my name is David Coffey, and I am a Ph.D. student… This semester I am conducting a study investigating the relationships between students' experiences in mathematics and their beliefs about what it means to do mathematics. Information from this research will be used to help better prepare elementary education majors to teach mathematics.

In order to conduct this study, I need you help. I will be conducting interviews and collecting copies of completed course work in an effort to explore the relationships between mathematical experiences and beliefs. At least twelve participants are needed. Signing up today does not necessarily mean that you will take part in the study – only that you are interested.

Taking part in the study will require that we meet together five times during the semester. Each meeting should last between forty-five minutes and an hour. During these meetings we will talk about your view of mathematics, reflect on past experiences in mathematics class and explore your perception of the assessments in this course.

What's in it for you? That depends – all participants will gain insight into their own mathematical belief system. Research has shown that such insight can help an individual to become a better teacher. While other perks are negotiable – based on need and availability – I am looking for people who simply want to share their thoughts and experiences, and so they will be given first priority.

Does anyone have any questions about this study?

(Consent forms will be passed out to all students and discussed.)

[Probability and Statistics] Survey

Name ________________

High school graduated from ______________________ Year _________

High school mathematics classes:

______________ ______________ ______________

______________ ______________ ______________

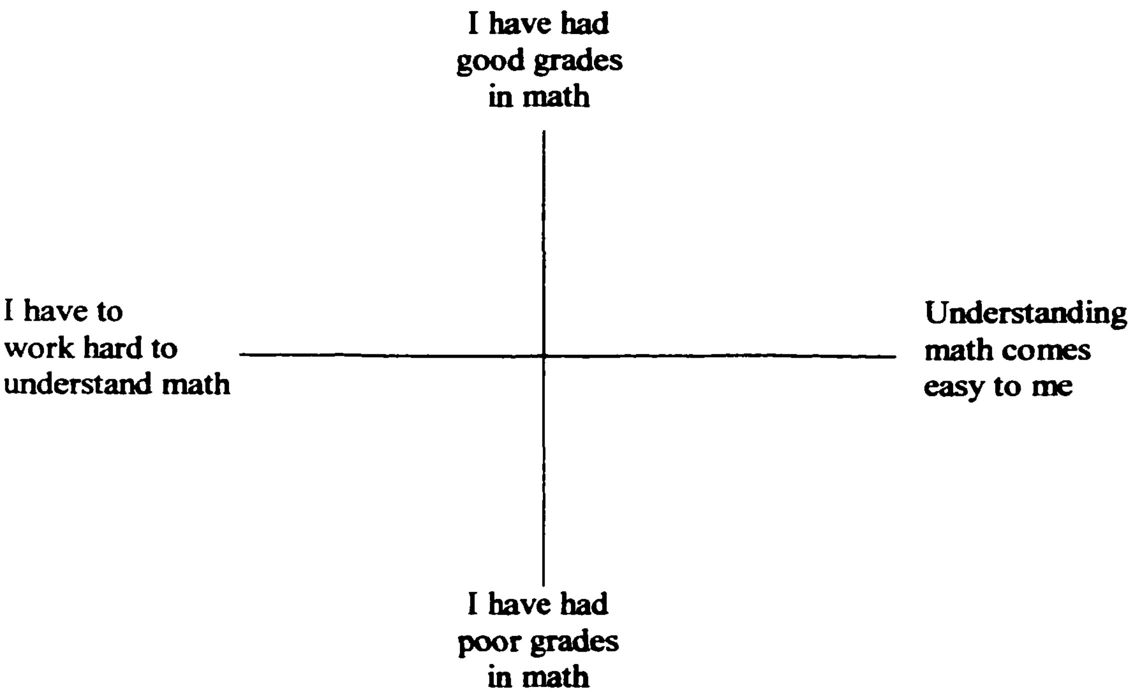
College mathematics courses (please indicate what school they were taken at):

______________ ______________ ______________

______________ ______________ ______________

______________ ______________ ______________

Where do you think you belong on the graph below?



Consider the following similes:

Ideally, a mathematics teacher is like a(n)

news broadcaster	entertainer	doctor
orchestra conductor	gardener	coach

- ➢ Choose the simile that you believe best describes a mathematics teacher and explain your choice.

- ➢ Choose the simile that does the worst job of describing a mathematics teacher and explain your choice.

Ideally, a mathematics student is like a(n)

inventor	sponge	mechanic
computer	reporter	explorer

- ➢ Choose the simile that you believe best describes a mathematics student and explain your choice.

- ➢ Choose the simile that does the worst job of describing a mathematics student and explain your choice.

Ideally, doing mathematics is like

cooking a meal	conducting an experiment	doing a dance
playing a game	doing a puzzle	climbing a mountain

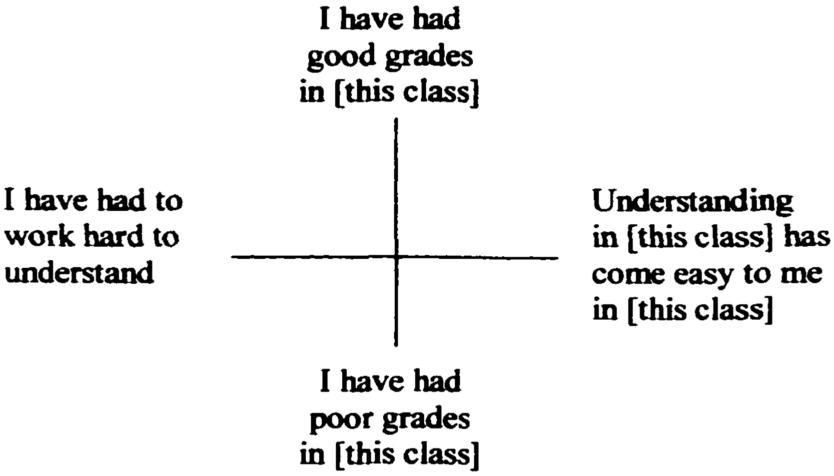
- ➢ Choose the simile that you believe best describes doing mathematics and explain your choice.

- ➢ Choose the simile that does the worst job of describing doing mathematics and explain your choice.

[Probability and Statistics] Exit Questionnaire

High school graduated from ______________________ Year __________

Where do you think you belong on the graph below?



Consider the following similes:

Ideally, a mathematics teacher is like a(n)

news broadcaster	entertainer	doctor
orchestra conductor	gardener	coach

- Choose the simile that you believe best describes a mathematics teacher and explain your choice.

- Choose the simile that does the worst job of describing a mathematics teacher and explain your choice.

Ideally, a mathematics student is like a(n)

inventor	sponge	mechanic
computer	reporter	explorer

- Choose the simile that you believe best describes a mathematics student and explain your choice.

- Choose the simile that does the worst job of describing a mathematics student and explain your choice.

Ideally, doing mathematics is like

cooking a meal	conducting an experiment	doing a dance
playing a game	doing a puzzle	climbing a mountain

- Choose the simile that you believe best describes doing mathematics and explain your choice.

- Choose the simile that does the worst job of describing doing mathematics and explain your choice.

From the alphabetical list provided below, please circle the experiences from this semester that influenced the choices you made in the simile section.

From [Probability and Statistics]	Outside [Probability and Statistics]
Assessments: projects & quizzes	Conferences attended
Assignments: other than textbook	Elementary classroom visits
Group activities: non-graded	Individuals not in [this class]
Interaction with classmates	Interactions with children
Interaction with instructor	Other courses and teachers
Readings: other than textbook	Readings not required for courses
Reflecting in Math Journals	Professional education groups
Student led discussions	*OTHER* (Please list)
Teacher directed instruction	
Textbook: reading & assignments	
Whole class discussions	
Working with calculators	
OTHER (Please list)	

Please pick the top six experiences you chose above in order from most significant in influencing your choices to least significant. Please list *OTHER* experiences as individual experiences.

1. ____________________ 2. ____________________

3. ____________________ 4. ____________________

5. ____________________ 6. ____________________

Interview One Protocol

First, we need to pick a pseudonym. What name would you like to be known as for this study?

(begin taping) Is it all right if I tape record our conservation today?

As I explained to you that first day and over the phone, this is part of a study to determine the nature of relationships between students' experiences and their beliefs about mathematics. Your responses cannot be right or wrong, so please just try to be honest.

EXPERIENCES
How would you describe yourself as a math student? How have you arrived at this description?

I'd like you to read through these two descriptions of math class (vignette activity). Now read them through again. Outline in yellow the experiences within each math class that you identify as being close to your own experiences. Outline in blue the experiences most foreign to your own.

What would you need to add to the experiences outlined in yellow to make a complete word picture of your experiences in mathematics?

ASSESSMENTS
If I told you I was going to assess you on your ability in mathematics, what would you expect me to do?

Here are some cards listing ways students remembered being assessed in the past. Please make three piles. One with those methods that you remember being used 'a lot;' one with those that were used 'sometimes;' and one with those that were 'never used.'

Are there any other methods that you remember being used to assess your math ability?

BELIEFS
Briefly, what do you think mathematics is all about?

Here are some cards with other students' ideas. Please make two piles – one with cards you disagree with and the other with cards you agree with.

Please explain how you decided where this card belongs (do for each card).

[Traditional Classroom Vignette]

The desks are arranged in rows, one student per desk, each facing the front backboard. The teacher begins the math lesson by addressing the problems assigned yesterday. A few students offer problems they had difficulties with and the teacher shows how each should have been solved using the methods that the teacher demonstrated the previous day.

Once all the problems have been addresses, the teacher directs the students to turn in their textbooks to the next page. While they follow along the teacher reads the text and demonstrates the examples provided. A few extra examples are included so students can see the process developed first hand.

Some similar practice problems are assigned so the teacher can determine whether or not the students understand the material. The students work quietly at their desks. Every once in awhile a student has a question and raises a hand until the teacher can offer assistance.

After a short period of time, the teacher calls several students up to reproduce their solutions on the board. The teacher observes if the correct method has been used to obtain the right answer. If an answer or method is incorrect, then the teacher re-works the problem in an effort to eliminate the error.

Once all the practice problems have been successfully completed at the board, the teacher assigns problems from the book that will be discussed the next day. Students are to work on those problems individually, without help from others or calculators so that the teacher knows exactly what each individual is capable of doing. This practice follows the same format that students will encounter when taking a test.

[Reform Classroom Vignette]

Students are seated in small groups at tables placed around the room discussing a problem written on the board. The problem is challenging and some of the students are using manipulatives to reason through it and to help explain their thinking. After a brief period of time, the teacher interrupts for a whole class discussion. In an effort to make connections to previous lessons, the teacher asks several questions. *What are the key parts to this problem – Why? How is this like other problems you've done?*

Once the students have shared their ideas, the teacher asks them to continue exploring methods of solving the problem. As the teacher observes each group, notes are taken on the students' progress. Not only will these notes help the teacher to determine which groups will lead the next discussion, but they also provide observational evidence that can be placed in the students' portfolios along with samples of their work.

After a sufficient amount of time the teacher calls the students back into a whole class discussion and prompts certain groups for their methods of solution. During this time the groups not presenting their solutions are expected to critically evaluate their peers' ideas to determine whether or not the methods will work and how their own methods compare.

Since some groups arrived at significantly different answers or used seemingly different methods a discussion develops. Students talk directly with one another trying to arrive at an acceptable solution. The teacher only interjects to keep the discussion on tract and point out some interesting ideas as they surface.

Near the end of the time the teacher suggests that students get out their math journals and write about which method they like best and why. As the students write in their journals the teacher puts a few more problems that build on the one solved in class up on the board as an assignment.

Assessment List used for Assessment Card Sort

ASSIGNMENTS Problems from the book Worksheet problems other assigned problems	ORAL REPORTS	TEST (no partial credit) multiple choice test true/false test fill in the blank
	PARTICIPATION	
CUMULATIVE FINAL	PORTFOLIOS	TEST W/ CHEAT SHEET
EXTRA CREDIT	QUIZZES	WRITTEN WORK journals essay questions reports
GROUP TEST	TAKE HOME TEST	
MATH PROJECTS	TEST (credit for work)	

Mathematical Statements used for Belief Card Sort

Perseverance	Confidence	Predictability
Strong math students solve math problems quicker than weak math students.	In order to do mathematics well, students must absorb as much information as possible.	There are some basic rules in mathematics, but there is always room for interpretation.
Strong math students understand the material the first time they see it. Poorer math students need to put in a lot of repetition to get it.	Students should make discoveries in mathematics on their own, without it always being shown to them.	Math is a step-by-step process where there is always a single solution to be found.
After you do math long enough it becomes very easy.	The mathematics teacher needs to show how to solve math problems in a smooth fashion.	Mathematics is made up of a lot of separate categories and unrelated rules.
Before attempting a math problem, a person needs to know how to get the answer.	A mathematics teacher should encourage and guide and not give rules to the student.	Memorization is the most important tool to use when doing mathematics.
Learning math is often the result of a lot of trial and error.	Students should not invent their own way of doing things in mathematics.	There are some basic rules in mathematics, but there is always room for interpretation.

Shaded statements were removed during the second card sort.

Interview Two Protocol

Is it all right to tape this section?

I have questions about some of the things you brought up during the first interview. Can you clarify what you meant by … (insert questions here).

Has anything else come up for you related to what we talked about last time?

Last time we began to look at your experiences in math classes and your beliefs about mathematics. Today I want to get into both areas more deeply and see what relationships you see between your beliefs and experiences. Again, in order to get an accurate mapping of this relationship for you, it is important that you re as honest as possible.

BELIEFS
Here again are the beliefs you looked through last time minus three statements I remover because you all shared identical beliefs on them. This time I would like you to choose the four statements you feel strongest about – either positive or negative. [*Student selects four statements.*]

Now, try to rewrite these statements in your own words on the back of the card so that you would strongly agree with each of them. [*Student rewrites each statement in her own words.*]

Here is one last card. Please pick the word in the curly brackets that would create the statement you would agree with most. Rewrite this statement on the back using your own words. [*Student rewrites her beliefs about her ability in math.*]

Ask about all five rewritten statements: Why did you choose *this* statement? How long would you say that you've held *this* belief?

EXPERIENCE – MAPPING
Theory suggests (McEntire & Kitchens, 1984) that experiences lead to the formation of beliefs and thus affect behavior. If I told you that I avoid dogs (behavior) because they bite (belief), what might be my reasons (experiences)? In other words, how might my belief that dogs bite developed?

Now go through your rewritten statements and try to determine what experiences you would credit with developing or reinforcing these statements. Be as thorough as possible – providing any experiences that apply. Try to be very specific about how the belief and experience are related. [*Interviewer will briefly write experiences on an index card.*]

Please try to order, from most significant to least significant, all the experiences for each belief statement.

Do you recall any experiences that you would describe as an exception to this belief? For example, if I had an aunt who had a very friendly dog that never bit any one. Why do you think these experiences had no affect on you belief statement?

Take this copy of your rewritten beliefs and over the next two weeks think of any experiences that may have created or reinforced these beliefs and write them down.

CONCLUSION
What are your thoughts about this interview?

Interview Three Protocol

Is it all right to tape this section?

I have questions about some of the things you brought up during the previous interview. Can you clarify what you meant by … (insert questions here).

EXPERIENCE – BELIEFS MAPPING
After our last meeting I asked you to look over your belief statements and try to remember experiences that might have created or reinforced these beliefs and any experiences that were exceptions to these beliefs. So what have you come up with?

ASSESSMENTS
Tell me about the assessment (writing a test) you've just completed in [*Probability and Statistics*].
How do you think you did?
What did you like about the assessment?
What did you dislike about the assessment?

Compare this assessment to past math tests:
Which would you put more time into – this assessment or preparing for and taking a paper-pencil test?
Which would you learn more from?
Which gives a better indication of what you know?
Which type of assessment would you rather do?

Almost all of you have told me that tests were something you experienced a lot in earlier math classes. Compare the test you've written to the tests you've taken before in math classes.

Tell me a little more about your past experiences with math tests:
How did you prepare for a math test? (Was there a review or practice test?)
How many problems would typically be on a test?
What were the problems like?
How long did you typically have to complete a math test?
What type of setting was it taken in?
What else sticks out in your mind about these past math tests?

CONCLUSION
What are your thoughts about today's interview? In particular, do you feel that I am leading you to any conclusions – if so which?

ANOTHER BELIEF SURVEY
For next time, I'd like you to complete these two brief belief surveys. Just place an 'X' on each line at the spot that represents what you believe to be true about mathematics and learning mathematics.

Interview Four Protocol

Is it all right if I tape this interview?

I have a couple of questions from the last interview that I'd like to clear up before we go on.
(insert questions here)
Has anything come up regarding the first assessment (writing a test) that you think is important and you'd like to share with me?
Have you ever had opportunities to redo tests before? Do you like it? Why?

BELIEFS
Last time I asked you to mark your beliefs about mathematics and learning mathematics on the continuums on this index card. Tell me about what you came up with and why.

EXPERIENCES IN ASSESSMENT
Read through this vignette about taking a test. Like we did before, mark those experiences you remember best in yellow and those most different from your experience in blue.

Do you recall the theory we've been working with relating experiences to beliefs?
Based on the test given by the teacher (his or her action), what type of beliefs do you think she has about *mathematics* and *mathematics learning*?
Based on this experience in math testing what sort of *mathematical* beliefs might the students have developed?
Compare these experiences with your beliefs (have original and rewritten beliefs handy).

ALTERNATIVE ASSESSMENT
Tell me about the assessment (evaluating responses) that you've just taken in [*Probability and Statistics*]:
How do you think you did?
What did you like about the assessment?
What did you dislike about the assessment?

Compare this assessment to past math tests:
Which would you put more time into – this assessment or preparing for and taking a paper and pencil test?
Which would you learn more from?
Which gives a better indication of what you know?
Which type of assessment would you rather do?

CONCLUSION
Have any other experiences related to your beliefs about mathematics come up since we spoke last?
What things have come up for you during today's interview? Did you feel that I was leading you in any way during today's interview?

Mathematics Test Vignette

The class period before the math test is spent reviewing the chapter. The teacher begins by highlighting what will be on the test. "Is there anything you want to go over or anything you don't understand 100 percent?" The students ask for help with a few points and these are addressed. If time allows, the teacher gives practice problems to work on. These problems may be from a worksheet or the review at the end of the chapter.

When the student studies for the test, she begins by going over the points stressed by the teacher in the review and looking over her notes from the chapter. Whenever she comes across something she knows how to do she says, "All I need to do is go over that once." She spends a little bit more time on the parts she isn't as sure about. She looks through her notes and textbook for help. If she still doesn't get it she might call some other students from the class and ask for their assistance.

The student also tries to do any practice problems the teacher has assigned to make sure she knows how to do them. If there are no assigned practice problems, then she goes through the textbook and tries to do problems from the chapter. She picks problems with the answers in the back of the book so that she can determine if she's doing them right. Recent math quizzes offer another study tool. She looks them over and tries to do the problems over again to see if she can get the right answer.

On the day of the math test the teacher passes out the test almost immediately so that everyone will have nearly the whole period to work on it. Leafing through the test the student finds about 20 problems. Those that ask her to perform computations give room so she can show her work. The remaining problems only leave space for the answer. Some of the test problems involve only numbers and symbols. Others use words in an attempt to connect the math to the real world. She's seen problems similar to most of these before. Some of the problems are modified versions of problems from homework assignments and quizzes. The room is quiet as each student works on his or her own. Some are writing on the test or on scratch paper. Others are using their calculators to help them with the problems. And still other students are reading through a problem trying to find the important information that will enable them to solve it.

The students who finish their tests early turn them in after checking over their work. The rest work until the teacher says, "Time's up." Of those who are left, a few hand in what they've done while others rush to finish the question they've been working on. The students leave asking one another how they think they did on the test. They may get the results the next class period. Otherwise, their teacher will start on something new.

Interview Five Protocol

Is it all right if I tape this interview?

***[POBABILITY AND STATISTICS]* ASSESSMENTS**

During the first interview I asked you what you would expect if I told you I was going to assess your ability in mathematics. You all said, "A test."

1. Why do you think you all picked a test? Did it have anything to do with the fact that tests made up the majority of your assessment experiences in mathematics?
2. Has your perception of what it means to assess mathematical understanding changed at all since that first interview?
3. If so, how and why?

Beliefs

I have your pre- and post-questionnaires from [*Probability and Statistics*]. These asked you to choose similes that did the best and worst job of describing math teachers, math students and what it means to do mathematics. Tell me a little bit about your original choices and your most recent choices. What experiences from this semester influenced any change?

Here, written in your own words, were your strongest held beliefs about learning and doing mathematics from the start of the semester. Looking at them now, would you make any changes in the selection or the wording? If so, what changes would you make and why would you make them?

Here are the beliefs about doing and learning mathematics that we looked at in the first interview. Again, please sort them into two piles, agree and disagree. Now compare the sorted piles to your original choices. Are there any changes? If so, what are they and to what would you attribute the changes?

Relationships

These are the relationships connecting assessment experiences and mathematical beliefs that I believe I have uncovered in my reading and through our interviews. Read through each statement and choose the level of agreement that most accurately represents your position.

- ➢ SD (strongly disagree): I've never held that belief.
- ➢ D (disagree): I've held that belief, but I don't think that it arose from the experience given.
- ➢ A (agree): I've held that belief and the experience makes sense, but it's not the first one to come to mind.
- ➢ SA (strongly agree) Yes that connection between belief and experience is right on for me.
- ➢ Tell me a little bit about your choices.

Conclusion

1. What things have come up for you during today's interview?
2. Did you feel that I was leading you in any way during today's interview?
3. Give me a brief idea of what this entire experience has meant to you?
4. Remember, this study is an attempt to look at the relationship between mathematical beliefs and assessment. So very briefly, tell me about your last three assessments in [*Probability and Statistics*] in regards to this relationship.

- ✓ Oral re-take
- ✓ Monte Carlo
- ✓ Final exam

5. Briefly talk to me about your overall impressions of your [*Probability and Statistics*] class.

Relationship Confirmations

Valuing assessment

Grades are very important in math class. Therefore, my assessment experiences have been some of the most influential in the creation of my beliefs about mathematics.	SD	D	A	SA

Past assessment experiences:

The grades I got in math class determined my perception of my ability in math. Good grades meant I was a good math student and poor grades mean I was a poor math student.	SD	D	A	SA
Because math tests were timed I came to believe that to be good at math I had to solve math problems quickly.	SD	D	A	SA
The questions I saw on math tests were similar to problems I had seen solved before. As a result I came to believe that in order to do mathematics I had to memorize the method of solution.	SD	D	A	SA
Since the teacher always showed the class how to solve problems similar to those on the test I came to believe that in order to learn mathematics it had to be shown to me by the teacher.	SD	D	A	SA
Problems on math tests had only one right answer so I developed the belief that there is only one right answer to any math problem.	SD	D	A	SA
Teachers wanted me to show my work so that they could be sure I was using the right method – the method they had shown me. Therefore, I began to believe that there was only one way to solve a problem.	SD	D	A	SA
Questions on math tests often break the material into easy to assess pieces. This helped me to develop the belief that math could be separated into many different categories and unrelated rules.	SD	D	A	SA
Students need to memorize the basic facts in mathematics and then later understand how it works. This belief resulted because earlier timed tests stressed memorization while recent tests stressed understanding.	SD	D	A	SA
Space for a connection an informant may feel is missing.				SA

Extra Credit

Because extra credit work was often simply doing extra work on things we had already learned I came to believe that being good at math (getting good grades) was the result of hard work.	SD	D	A	SA
Sometimes extra credit problems reinforced my belief that only a few special people could do math that hadn't been shown to them because the extra credit was on something that we hadn't covered in class.	SD	D	A	SA
Space for a connection an informant may feel is missing.				SA

Relationship Confirmations
(continued)

Assessments in [*Probability and Statistics*]

Working on the assessments in groups allowed me to see different people correctly solving the problems in different ways. I, therefore, became aware that there is more than one way to solve a math problem.	SD	D	A	SA
As we worked on the assessments in groups I found that sometimes I had to explain my thinking to other students. I found out that I understood the material better when I had to explain it in my own words.	SD	D	A	SA
When other students in my group helped me to understand the problems on an assessment, I became aware that I could learn how to do mathematics without always relying on the teacher and textbook.	SD	D	A	SA
I am beginning to value understanding over memorization since these assessments asked me to demonstrate how I could apply what I'd learned rather than asking me to remember it.	SD	D	A	SA
Having time to do the assessment outside class eliminates the pressure I often feel during timed tests. I no longer believe that doing mathematics well means I have to do it quickly.	SD	D	A	SA
Working on the assessments outside class eliminates the pressure I often feel to memorize material. I no longer believe that doing mathematics means I have to memorize steps and formulas.	SD	D	A	SA
The assessments in this class were appropriate since we are teachers in training. Using them outside the math-ed classroom would be a waste since they focus on skills unnecessary to people outside education.	SD	D	A	SA
Because the assessments in this class were written to prepare us to be teachers of elementary school mathematics I learned more about teaching math than I did about doing math.	SD	D	A	SA
Space for a connection an informant may feel is missing.				SA

Appendix D

Selected Data

Students' Initial Similes and Rationale for Mathematics Teachers

Student	*Best Math Teacher Simile*	*Worst Math Teacher Simile*
1 *Karen*	I chose entertainer because a math teacher should be educated as well as making math fun. Or a gardener because they plant the seeds in your head and help them to grow.	I choose coach because a math teacher should be calm and understanding not strict.
2	Coach, because I need so much help, I usually get it from my professor. Therefore, my teacher is a coach.	News broadcaster, because the teacher doesn't just throw facts, they get the students involved.
3	Coach – they teach you how to do something through techniques but don't do the actual task for you.	News broadcaster – they just give the facts and don't really work with you.
4	A math teacher is like a coach, because they train you then they set you free to execute what you've learned.	A math teacher is least like a doctor, because you only go to the doctor when there's something wrong.
5	A coach, because they should not only teach you what you're supposed to know, but they should encourage you and support you.	I believe a news broadcaster, because it doesn't help a student if you're just telling them what to do and not being interactive.
6	A gardener plants and knows about many different flowers, vegetables etc… and how to plant them.	Orchestra conductor because orchestra music is boring to me.
7	Gardener – you plant the seeds for the trees the students grow.	Entertainer – math is fun but your purpose is not entertain.
8	Coach – because you have to help everyone a great amount and try to get people excited who are loosing interest in math.	News broadcaster – because you can't just state something and expect people to know off hand what your talking about.
9	A mathematics teacher is like a coach, works with you to beat an equation.	Worst job: entertainer although math can be fun, there a reason why we are here, to take teaching children seriously and learn as we go.
10	Doctor needs math to do the dosing right.	Entertainers don't know why he would need a lot of math.

Students' Initial Similes and Rationale for Mathematics Teachers (continued)

Student	*Best Math Teacher Simile*	*Worst Math Teacher Simile*
11	Coach – A math teacher should encourage his students and show them how to execute problem in a smooth fashion.	News broadcaster – A math teacher should not preach and bore his students in a monotone voice.
12	If I can choose only one, it would be coach. A coach ideally is a teacher who guides enthusiastically but lets the 'player' truly learn on the field of play.	Doctor – It assumes something is wrong.
13 *Lucia*	Doctor – Trying to make them better in whatever they need help with.	News broadcaster – Filling a students mind with new information everyday.
14 *Cynthia*	Coach – helpful, good tips, rules of math, encouragement, clarity when things are going wrong, team building.	News broadcaster – someone who stands up and lectures, no real interaction.
15	Coach – because a good math teacher can see talents in students and provides the correct environment and stimulation for the student to perform up to their potential.	News broadcaster – bad teacher just states the facts.
16 *Janet*	Coach – A mathematics teacher will hopefully encourage and guide us when we need the help. A teacher that works *with* us.	News broadcaster – a teacher that just throws info at us without the time to discuss questions, isn't helping me learn.
17	*no response - absent*	*no response - absent*
18	Gardener because a math teacher should provide what a student needs to grow mathematically.	Entertainer because a math teacher should educate as well as entertain – of course it is bonus! ☺
19	Coach – I like someone to tell me how to do work, and then cheer me on when I do it right.	Entertainer – I need to have someone *teach* me, I can entertain myself.
20 *Lynn*	The doctor uses and practices procedures that are vital to the life of the patient.	An entertainer because entertaining can take away from the primary goal of learning math.
21	Coach – because in math you usually have one correct answer. The teacher has to coach you to the right answers and make sure you do it right.	Orchestra conductor – because there are only hand motions with this job.

Students' Initial Similes and Rationale for Mathematics Teachers (continued)

Student	*Best Math Teacher Simile*	*Worst Math Teacher Simile*
22	Coach – I believe a math teacher is here to help students along the way and make suggestions in the way we learn different techniques.	Entertainer – I am not here to be entertained. I am here to learn math and to better my skills.
23	I believe that a math teacher should make learning math fun (entertainer) as well as being easy to talk to (coach) when problems in understanding arise.	A math teacher should never just state facts like a news broadcaster. It makes math harder to learn. A teacher should never be like a doctor and just give you an answer to difficult problems.
24	Orchestra conductor – The teacher teaches the material and guides the students as everything comes together.	A teacher should not be like a news broadcaster because the information should not be given to students as lectured material.
25	Doctor – because they know and have to follow very specific procedures in order to accomplish their goal.	Entertainer – because math teachers do not entertain you like an athlete does, they just teach you how to do things.
26 *Anne*	Coach – Math teachers should teach the fundamentals and give time to practice them and then be there for guidance.	Gardener – Math teachers don't need to guide students along and "feed" them at all times.
27	Coach – Most math teachers give you the homework assignments and shows you how it is done. No more no less.	Entertainer – Teachers doesn't entertain (make the students to enjoy math while they are teaching).
28 *Keona*	Coach – They give the orders and you do it.	Gardener – Because gardening seems to be relaxing. Math is not.
29	I believe a mathematics teacher is like a coach because they help teach you math like a coach teaches you how to play the game. They both try to motivate the individuals.	I would have to say an entertainer. A math teacher is there to help you learn and not there to entertain the class.
30	Doctor – out of the choices given, a doctor seem to be a profession which involves a tremendous amount of studying.	An entertainer …
31	Coach – You are coaching students and helping them learn about math.	News broadcaster – Math is the learning of numbers where teachers teach us, a broadcaster tells the world the news, I don't see them as the same. ☺
32	A mathematics teacher is like a coach because they have to try different methods and strategies to get across to all the students.	A news broadcaster- this is because there is a lot more to teaching math than simply relaying information.

Students' Initial Similes and Rationale for Mathematics Students

Student	*Best Math Students Simile*	*Worst Math Students Simile*
1 *Karen*	I chose sponge because ideally a math student should be absorbing every bit of information.	I choose inventor because students should not be inventing their own way doing things.
2	Reporter, because the student gets all the facts and then puts them together to solve a problem.	Sponge, because everything one learns in math will not be remembered unless a refresher course is given.
3	Sponge – they absorb info and can spit it back.	*no response*
4	A student is like a sponge because they retain knowledge until they need to squeeze it out.	Students are not like inventors because they are taught, they don't teach themselves.
5	I believe a math student is most like a mechanic, because you have all of these different things you have to take apart and be able to put back together.	A reporter is the worst description because all you are doing is reading back what someone has given you.
6	Explorer, learning how math works using trial/error.	Sponge because a sponge to me only does two things.
7	Explorer, you are the one who explore and are responsible of you learning.	If you are mechanic, you are not understanding what you learn.
8	You have to soak up so much information, & sometimes it can be overwhelming. (Sponge)	Computer – because students usually don't automatically know what's going on in math.
9	A mathematics student is like an explorer, lots of roads will get you to the same spot, but how you get there is the interesting thing.	A computer *does not* reason!
10	Sponge – they have to absorb every angle.	Reporter – they just repeat what they are told.

Students' Initial Similes and Rationale for Mathematics Students (continued)

Student	*Best Math Students Simile*	*Worst Math Students Simile*
11	Reporter – a math student should take in the information then interpret it on his own in order to solve a problem.	Sponge – a math student should not only be absorbing the information, but should also be able use it on his own.
12	Explorer – with help from the instructor, the student ought to be intrigued by the material & want to find out more.	Reporter – nothing is learned, only related.
13 *Lucia*	Sponge – because in mathematics, it's good to hold onto the info for future knowledge.	Computer - a student should know *why* the rules or reasons why a problem works, *not* just know the rules and carry on with a problem.
14 *Cynthia*	Explorer – free to try things out to see what works the best and what makes the most sense.	Sponge – just sits back and absorbs things. No experimentation or trial and error goes on. Basic memorization with no knowledge of the how or why.
15	Computer – because a good math student will accept information and use it.	Sponge – because a sponge just absorbs and doesn't produce anything.
16 *Janet*	Sponge – The spongy student will learn (or soak up) everything the instructor has to teach.	Reporter – A student that just feeds back the numbers without interpreting it, hasn't really learned.
17	*no response - absent*	*no response - absent*
18	Mechanic – they take knowledge they already have to fix a problem.	Reporter - because a reporter just presents what info is given to them. They don't necessarily need to learn it.
19	Sponge – I soak in information and later someone wrings it out of me.	Inventor – I think math is already set up for you, you just need to do it.
20 *Lynn*	Mechanic – the student has a knowledge basic of math and adds to it with ways they understand.	A sponge, math is hard to absorb like a sponge.
21	Sponge – because students soak up information and hold as much knowledge as they can.	Mechanic – because mechanics usually have one fixed way of doing things. Students explore different ways to figure out problems.

Students' Initial Similes and Rationale for Mathematics Students (continued)

Student	*Best Math Students Simile*	*Worst Math Students Simile*
22	Explorer – a student is like an explorer taking different routes to arrive at the same destination.	A reporter – as a student I feel I am not a reporter – I am not here to report answers, but to find the ways to arrive at those answers.
23	Math students ideally should be like a sponge. He/she should be able to absorb all that is said and taught before applying it.	A computer is the worst way a math student should act because computers give answers without reason.
24	A mechanic – they learn and then apply the knowledge how it relates to their lifestyles.	A reporter – students should not just spit back formulas. He/she should understand.
25	Explorer – because they are always trying new way to solve problems.	Reporter – because they rarely ever stand up and give reports to people.
26 *Anne*	Explorer – Math students need to "explore" different ways to go about doing, solving, and explaining a math problem.	Reporter – reporters tend to want all kinds of information that may or may not be related to the subject. Math students need to know exactly the info for the math problem.
27	Computer – They are told how to do things and what to do.	Explorer –
28 *Keona*	I believe a student is an explorer because you usually do a lot of figuring out concepts.	Reporter – Because in math you usually don't do a lot of reporting.
29	I believe a mathematics student is like an explorer. Through learning about math, students can learn how to explore math and use it in any kind of situation.	To me sponge is the worst simile to describe a mathematics student. A student shouldn't just sit back and absorb the information but be able to use it as well.
30	Explorer – Math involves a lot of exploring different ways of solving individual problems.	Computer – a computer doesn't involve thinking it is programmed.
31	Sponge – Absorbing all the information you can until you can't take anymore!	Reporter – Reporters tell about news, etc. Students learn about the news.
32	A mathematics student is like a sponge because they soak up as much information as possible from their instructor.	A reporter- because just reporting facts does not prove that the student understands a concept.

Students' Initial Similes and Rationale for Doing Mathematics

Student	*Best Doing Math Simile*	*Worst Doing Math Simile*
1 *Karen*	I chose doing a puzzle because math should be just piecing all of the information together to make it work.	I choose cooking a meal because everyone cooks meals differently and if everyone did math differently there would be no need for this course.
2	Climbing a mountain, going up is a struggle but once you get to the top its flat ground.	Cooking a meal because all the ingredients don't add up into one big whole. Math is many categories and strategies that shouldn't be generalized.
3	*no response*	*no response*
4	It's like playing a game, first you learn it then it becomes easier as you play more.	*no response*
5	Doing a puzzle – You have to put all of the pieces together in order to make the final product.	Cooking a meal – This is the worst because all you do is follow instructions. Unless you repeat it, you won't learn how to cook a certain meal off the top of your head.
6	Doing a puzzle, sometimes is easy and other times hard to solve	Cooking a meal because I like to cook and usually it is easy to cook with directions.
7	Climbing a mountain, you make an effort, but finally you reach your goal and you enjoy climbing at the same time.	*no response*
8	Doing a puzzle – you have to put all the pieces together for the final answer.	Playing a game – because it's not that fun and exciting to me.
9	Doing math is like doing a puzzle, sort out all the similar pieces, and work through the problem the way that best fits.	Worst job: Climbing mountain; assuming you are climbing *up* the mountain that is the hardest way to go. Eventually things (math) becomes easier and then you are all downhill, if that makes sense ?!
10	Climbing a mountain – I get half-way and can't continue or see the top.	Playing a game – What's fun about it.☺

Students' Initial Similes and Rationale for Doing Mathematics (continued)

Student	*Best Doing Math Simile*	*Worst Doing Math Simile*
11	Conducting an experiment – There are many different ways to do mathematics, and no one way is always the best way.	*no response*
12	Doing a puzzle – There's always a solution to be found.	Doing a dance – I think math is more analytical; a step-by-step process as opposed to a fluid group of movements.
13 *Lucia*	Conducting an experiment – experimenting with math is fun and discovering ways to …	Cooking a meal – sometimes its good to know step-by-step ways to do a problem, but sometimes it is good to explore and be able to do problems in other ways.
14 *Cynthia*	Playing a game – there are a few basic rules but there is room for a lot of interpretations, and different ways of reaching the final end.	*no response*
15	Cooking a meal – because doing mathematics is like starting with some things and using them together to produce something else.	Playing a game – I just can not see why that would relate.
16 *Janet*	Playing a game – Math should be fun, like playing a game, and it should involve procedures for winning or getting the correct information.	If math were like climbing a mountain, I would think of it like the near impossible task of climbing a Mt. Everest or K-2.
17	*no response - absent*	*no response - absent*
18	Cooking a meal – taking what you already know tastes good (works) and applying it to a problem for a finished product.	Actually – all of the similes can be related to math in a certain way. No answer seems to be worst/best – they all fit.
19	Climbing a mountain – I work hard at getting to the top and it is a bit scary sometimes.	Playing a game – I think playing as fun and math as doing work; hard work!
20 *Lynn*	Climbing a mountain – Math is a hard struggle that has a goal of attaining new information.	Doing a dance, dancing may be easy but math isn't.
21	Conducting an experiment – because sometimes it works and sometimes it doesn't.	Cooking a meal – because sometimes you add or subtract certain ingredients that you may think are necessary or not.

Students' Initial Similes and Rationale for Doing Mathematics (continued)

Student	*Best Doing Math Simile*	*Worst Doing Math Simile*
22	Cooking a meal – because you start out with the proper ingredients and instructions – you should come out with the same finished product.	Conducting an experiment – I think while conducting an experiment your results are not always the same.
23	Math should be like doing a puzzle. You need to take many little pieces of information and be able to put them together and apply it to a problem.	Math should never be like climbing a mountain because it is the job of the teacher and student to make learning math easier.
24	Climbing a mountain …	*no response*
25	Cooking a meal – because after you do it enough it becomes very easy to you.	Climbing a mountain – because mathematics is not that much of a continuous struggle.
26 *Anne*	Doing a puzzle – there are many ways to do math as there are a puzzle but the answer should end up similar.	Cooking a meal – Math needs to be done in a somewhat controlled order where cooking a meal you can start in whatever order you choose.
27	Doing a puzzle – doing it once all while.	Playing a game – never in my life I done math for fun, but as an assignment.
28 *Keona*	Conducting an experiment was my choice because I find math to be hard and you always have some type of conclusion.	Doing a dance – Because dancing is fun; and math is not for me.
29	I think doing math is like doing a puzzle. Some puzzles are easy to figure out while some are hard just like math problems. Plus some math can be fun to encounter.	Climbing mountain because math should not be hard to achieve like climbing a mountain may be.
30	Doing a puzzle …	Doing a dance …
31	Conducting an experiment – you find all the facts, test them, and find a conclusion.	Doing a dance – is fun and easy.
32	Climbing a mountain – I have always struggled with math.	Playing a game – The games I prefer to play are usually fun, easy, and stress free – math is not that simple for me

Relationship Confirmation
Counterproductive Beliefs and Traditional Mathematics Tests

Past assessment experiences	A	C	J	Ka	Ke	Lu	Ly
The grades I got in math class determined my perception of my ability in math. Good grades meant I was a good math student and poor grades mean I was a poor math student.	SA	A	SA	D	SA	SA	SA
Because math tests were timed I came to believe that to be good at math I had to solve math problems quickly.	A	A	SA	SD	D	SA	SD
The questions I saw on math tests were similar to problems I had seen solved before. As a result I came to believe that in order to do mathematics I had to memorize the method of solution.	SA	SA	SA	D	A	A	SA
Since the teacher always showed the class how to solve problems similar to those on the test I came to believe that in order to learn mathematics it had to be shown to me by the teacher.	A	SA	SA	D	A	A	SA
Problems on math tests had only one right answer so I developed the belief that there is only one right answer to any math problem.	SA	SA	SA	A	SA	SA	SA
Teachers wanted me to show my work so that they could be sure I was using the right method – the method they had shown me. Therefore, I began to believe that there was only one way to solve a problem.	SA	SA	SA	A	SA	SA	A
Questions on math tests often break the material into easy to assess pieces. This helped me to develop the belief that math could be separated into many different categories and unrelated rules.	SD	D	SA	SD	D	SA	D
Students need to memorize the basic facts in mathematics and then later understand how it works. This belief resulted because earlier timed tests stressed memorization while recent tests stressed understanding.	SA	D	SD	D	A	SA	SA

Students' Exit Similes and Rationale for Mathematics Teachers

Student	*Best Math Teacher Simile*	*Worst Math Teacher Simile*
1 *Karen*	I believe a coach is the best choice because they want you to do your best and will help you in any way possible.	A gardener because they do not dig up the info and plant the seed into your head. They try to help you learn.
2	Orchestra conductor – shows students the steps it takes to get to the final product.	Entertainer – you're not there *just* for entertainment. If you're a good teacher, it just comes naturally.
3	Like a coach because he can get you in the right direction but he sits on the sideline while you do the work in practice and game time. He can give you ideas on how to improve your game.	News broadcaster, all they do is relate info as they see it. Their description does not vary from the L or R; it is just as it is told to them. Very little adaptability.
4	A coach, because they train you to get ready for "the big game" (life).	An entertainer is the worst, because they just keep you awake. You usually don't get anything from them.
5	Coach because they introduce something new to you and then you take it from there and try to perfect it.	News Broadcaster – because they just relay a message to you and sometimes it's not very accurate.
6	Entertainer: Because a mathematics teacher needs to know how to present material in interesting, fun ways.	News broadcaster: A news broadcaster I think of as just plainly describing the news. A good math teacher lets the students investigate and describes.
7	Coach – you prepare your students for the game that is the life. For me, all has something similar. Gardener – you make grow the seeds (knowledge).	Maybe the worst is orchestra conductor; it is the way it use to be in today's schools.
8	Gardener – teachers have to plant the info in you and help you expand your knowledge.	Entertainer – You do more than just watch a teacher.
9	Coach – you have to work with your students to find the best way for *them* to understand.	News broadcaster – just tells how it is, no communication otherwise one on one.
10	Entertainer/Gardener – You need to make math fun and interesting and you need to plant math skills into your kids.	News Broadcaster/Conductor – You're not just talking at the kids or just standing around in front of them pointing a stick at them.

Students' Exit Similes and Rationale
for Mathematics Teachers
(continued)

Student	*Best Math Teacher Simile*	*Worst Math Teacher Simile*
11	Coach – he encourages and guides student to understanding.	News broadcaster – math should be fun and interesting, not monotone and boring.
12	Entertainer – make it fun!	News broadcaster – boring!
13 *Lucia*	A coach leads you in the right direction. They help when problems arise.	Gardener plants things into the ground, like a math teacher would plant thing into students' head and not explain why.
14 *Cynthia*	Coach because they should give students the tools to discover what works and what doesn't, but they should not give them the answers.	News broadcaster because they sit and spew out information telling the student what is true. There is no interaction.
15	Coach – the teacher needs to see talents and strengths and encourage students to pursue them.	News broadcaster – because they simply state information. Teachers need to so more than that.
16 *Janet*	Coach – the teacher should encourage and help guide you through the physical pain of learning math (just kidding dave). Should help encourage learning.	News broadcaster – I don't want a teacher to just yell the information @ me and not hear me.
17	Coach – They are there to guide and help their students.	News broadcaster – instead of just speaking, they need to teach.
18	Gardener: Taking the math/resources we may already have and tending them, enriching them w/ what we need.	News broadcaster – This promotes no interaction. I think a big part of learning math is interaction – especially between teachers and students.
19	Coach gives information, but team has o work together to achieve success.	News broadcaster simply gives facts. This would not let kids think on their own and it would be boring.
20 *Lynn*	Orchestra conductor – helps us conduct the mathematical tasks we are given.	Entertainer – Math is not all play. It is very hard work.
21	Coach – a coach guides you through but doesn't help you play the game. Teacher helps you through -> doesn't give answers.	News broadcaster – gives you stories, doesn't help w/ anything except *telling* you what goes on throughout the day.

Students' Exit Similes and Rationale
for Mathematics Teachers
(continued)

Student	*Best Math Teacher Simile*	*Worst Math Teacher Simile*
22	Coach because a coach is there to help you along the way.	Entertainer – I am not here to be entertained. I am here to learn.
23	Coach – they should walk you through math and help to develop your skills as well as someone who makes themselves available for student help.	News broadcaster – someone who just reads things without really explaining them and doesn't help or give any interaction with students would not be a good teacher.
24	Coach, guiding students with ideas and techniques.	Orchestra conductor – shouldn't lead, should allow students to learn and move on their own.
25	Gardener because they just get you going on the right tract and then let you go on your own, but always stay by and give the student help when they need it.	News broadcaster because all they do it just tell you the info you need with really explaining it.
26 *Anne*	Coach – explains plays and is there to run practices (class and homework) and games (tests).	Doctor – helps you along – telling you everything to do.
27	Orchestra conductor – Most math teachers during high school years only gave homework assign and went over if problem was correct or incorrect.	Entertainer – you would never think any kind of teacher would be an entertainer. Entertainer makes most people stay awake and understand what is happening.
28 *Keona*	Coach – I think of math teacher as a coach because they really train you for what you need to know and they're there to guide you in the right direction.	Entertainer – A math teacher is not an entertainer at all. I find nothing about math to be amusing.
29	A mathematics teacher should be like a coach because they teach you how to do the math and help you do better with their support.	A news broadcaster because all they have to do is give you the facts and don't explain things very well.
30	Doctor seems to be a very complicated profession that requires a well developed mind as does understanding math.	Orchestra conductor – it doesn't have as much to do with math as the other professions do.
31	Coach –Coaching kid into understanding numbers like a coach does athletes.	Entertainer – Making big bucks!! ☺ Not here.
32	Coach – Because a math teacher has to come up with many techniques and approaches to teaching math to different students.	News broadcaster – because teaching math is more than relaying information. Students need to be engaged in learning activities.

Students' Exit Similes and Rationale for Mathematics Students

Student	*Best Math Students Simile*	*Worst Math Students Simile*
1 *Karen*	A sponge because they need to gather all the information possible to help them succeed.	A computer because we cannot be programmed to know this information. We need to learn and absorb it.
2	Computer, storing information and plugging it in the correct times.	Reporter – doesn't get facts and just write them down, you have to understand the concepts too.
3	Sponge – they take in all the info and when squeezed they spit out what they can give you.	Computer – has data and spits it out; cannot compensate for variation by themselves.
4	Sponge, because they retain knowledge until they get squeezed.	An explorer, because kids need direction in math.
5	Sponge/mechanic – A student needs to soak up the information and put into his own context. One needs to be able to disassemble and reassemble procedures.	Computer – you just put in exactly what you want not giving the student a chance to make it his/her own work.
6	Explorer: learning math involves time and trial and error.	Inventor: Math really does not revolve around only one way to solve the problem.
7	Explorer, you explore as far as you want, but the farther you get, the richer you get.	Computer. You have a capacity to know if something is wrong. The computer don't. you store a big amount of information w/o adding memory to your head.
8	Sponge: you have to absorb so much info to go anywhere in your classes.	Inventor: you have too many guidelines to follow.
9	Explorer – no one way is correct, there are many ways to solve a problem.	Computer – it has no reasoning, logic.
10	Mechanic/explorer – they need to pull things apart and put things together. They need to ask why and go where they have not gone before.	Inventor/computer

Students' Exit Similes and Rationale for Mathematics Students (continued)

Student	*Best Math Students Simile*	*Worst Math Students Simile*
11	Reporter – takes in info and uses it for his own good.	Sponge – a student cannot just absorb the info. It must also be used on his own.
12	Explorer – enthusiasm	Mechanic – tries to fix only what is wrong; no interest or adventure.
13 *Lucia*	Explorer – Everyday, we learn something new in math. We explore the surrounding and boundaries of rules, etc…	Computer, because computer only read programs, rules, etc… they are not reasons why it happens.
14 *Cynthia*	Inventor – they discover what works and what doesn't and make modifications accordingly.	Sponge – they just sit back and absorb what other people are saying without any feedback or information.
15	Computer – students should take in information and use it.	Sponge – students should not just soak in information.
16 *Janet*	Explorer – The student should want to explore new concepts in the process of learning.	Reporter – just spitting back information does not equate gaining knowledge.
17	Sponge – Students absorb what is being taught.	*no response*
18	Explorer – taking the skills we acquire, we venture out to learn new things.	Reporter – this gives the impression that we are just regurgitating what has been told to us.
19	Explorer learns new things and take knowledge above and beyond to use later.	Reporter simply gives back facts! Boring, not creative!
20 *Lynn*	Computer – gaining new data and processing it with old data (prior knowledge).	Sponge – one cannot learn from just absorbing information. You need to understand it.
21	Sponge – soaks in information from teacher. Holds some of it in, some info slips away.	Reporter gives out information to others.

Students' Exit Similes and Rationale for Mathematics Students (continued)

Student	*Best Math Students Simile*	*Worst Math Students Simile*
22	Explorer – I am here it explore the different approaches to math situations.	Reporter – I just want to learn how to do it, not just report it.
23	Explorer – I think a student should be able to discover new things that will help them w/ math.	Reporter – Student should not just report answers; they should be able to understand and explain.
24	Mechanic, working w/ tools.	Sponge – shouldn't just soak up material. must learn to use material.
25	Mechanic because they're always working on problems and trying to figure them out.	Computer, because they can only think one way about how to do a problem.
26 *Anne*	Sponge – absorbing everything.	Computer – memorizes everything but can't explain how to do it.
27	Reporter – we just sit and listen.	Explorer usually deals with find new and exciting things. Where in math we might learn new stuff, but it might not be very exciting.
28 *Keona*	Sponge – I basically feel that a math student is a sponge because you always seem to have a lot of info to absorb.	Reporter – I would say that a student is not a reporter for math because I usually do not have much to report.
29	A sponge because they should absorb they knowledge and information they learn.	Computer because they just give the answers and don't understand the whole reasoning behind the process.
30	Computer – because it is ideally. Students are still assumed to be able to shoot out info like a computer when it comes to math.	Reporter – the other choices seem to be more closely related to what is looked for in a math student.
31	Sponge – trying to absorb all this info until there is too much and can't absorb any more!	Computer – knows everything.
32	Explorer – the student has to take information and explore how the information can be used in different situations.	Reporter – it is easy to simply state facts and figures w/out grasping the concepts.

Students' Exit Similes and Rationale for Doing Mathematics

Student	*Best Doing Math Simile*	*Worst Doing Math Simile*
1 *Karen*	Doing a puzzle. You have to find the right pieces to make everything match accordingly.	Playing a game because most of the time you make the rules ^ as you go along. While mathematics leaves room for adjustments, there is no room to make your own way of doing things.
2	Cooking a meal – all ingredients form together to make the end product.	Doing a dance – If your doing it correctly, you don't have to move around to get the end product.
3	Cooking a meal	Playing a game
4	Doing a puzzle because you start to understand the whole picture when the pieces start coming together.	Doing a dance, because you should understand the work, not just follow steps.
5	Doing a puzzle – You have to construct your answers based on what you know.	*no response*
6	Doing a puzzle: need to explore, use trial and error.	Playing a game: must follow the rules and when doing mathematics sometimes go out of order.
7	Climbing a mountain because you so it by steps and little by little.	Conducting an experiment – in math, you are never done.
8	Doing a puzzle – you have to put pieces together for an answer.	Doing a dance – to much structure to be a dance.
9	Doing a puzzle – usually start with the edge and work in to solve.	Doing a dance – it takes two to tango, one to add, sub…
10	Conducting an experiment/ Doing a puzzle – you are solving a problem.	Doing a dance/climbing a mountain

Students' Exit Similes and Rationale for Doing Mathematics (continued)

Student	*Best Doing Math Simile*	*Worst Doing Math Simile*
11	Doing a puzzle – you try to find the best way to solve the problem.	*no response*
12	Doing a puzzle – fun, figuring out solution.	Doing a dance – routine, same procedure.
13 *Lucia*	Conducting an experiment b/c in experiments, you learn by trial and error. Like math, there is a lot of trial and error but also discoveries.	Cooking a meal – Step by step process are too organized, not allowing room…
14 *Cynthia*	Conducting an experiment – there are several important steps, yet many different options to conduct the experiment.	*no response*
15	Cooking a meal – we start with something and use mathematics to make something else.	Doing a puzzle – puzzles do not require much thinking and there are not many options.
16 *Janet*	Conducting an experiment – Math should be a trial and error to understand process.	Climbing a mountain – sometimes it feels that way! *Too tough!*
17	Doing a puzzle – You have to do a bunch of things to make a problem fit.	Doing a dance – It's not fun.
18	Conducing an experiment – using the skills we have, and certain ideas we have about the problems, we test our hypotheses.	none of them are really bad. They all explain some way or another of doing math.
19	Doing a puzzle – Sometimes fun, sometimes *very* frustrating!	Cooking a meal – cooking is relaxing. Math is far from it!
20 *Lynn*	Conducting an experiment – one way to learn is through trying different things.	Doing a dance – Math is not that easy. You need more than practice.
21	Climbing a mountain – at first it may seem difficult, but while you keep going you learn more.	Doing a puzzle – there is only one way that the pieces can fit as a whole.

Students' Exit Similes and Rationale for Doing Mathematics (continued)

Student	*Best Doing Math Simile*	*Worst Doing Math Simile*
22	Conducting an experiment because if you follow directions you will come out right.	Playing a game – ??
23	Cooking a meal – It should be like taking parts from every area of math that you know and using it to solve on large problem.	Climbing a mountain – math shouldn't be that hard.
24	Cooking a meal, since you work with numbers. Depending on which step you take first, you may get different answers.	Doing a dance – a specific outline should not be followed, there must be a chance for change and adaptation.
25	Doing a puzzle, because once you figure out how to put all the little pieces together you have the complete project.	Climbing a mountain, because once you get to the top of the mountain it isn't all down hill after that.
26 *Anne*	Climbing a mountain – start at the bottom and have to be sure of yourself as you climb to the top.	Doing a puzzle– guess and choose what goes where.
27	Doing a puzzle – comes easy to me and due it when I have free time.	Conducting an experiment – I don't think math is a hard subject to understand.
28 *Keona*	Conducting an experiment – Math is like conducting an experiment because you can take it to so many levels and always learn new info.	Playing a game – Math is no where near plying a game for me because I usually do not have fun when doing math..
29	*no response*	*no response*
30	Conducting an experiment because at times it requires a lot of trial and error.	Doing a dance – doing a dance is not nearly as complicated as math.
31	Climbing a mountain – I feel like I will never reach the top until I am way exhausted.	Doing a dance – Is fun, easy, math takes work.
32	Cooking a meal – because if you miss one step the whole problem will suffer.	Doing a puzzle

Relationship Confirmation
Fallibilist Beliefs and Alternative Assessment Experiences

Probability and Statistics Assessment Experiences	**A**	**C**	**J**	**Ka**	**Ke**	**Lu**	**Ly**
Working on the assessments in groups allowed me to see different people correctly solving the problems in different ways. I, therefore, became aware that there is more than one way to solve a math problem.	SA	A	SA	A	A	SA	SA
As we worked on the assessments in groups I found that sometimes I had to explain my thinking to other students. I found out that I understood the material better when I had to explain it in my own words.	SA	SA	SA	SA	A	SA	SA
When other students in my group helped me to understand the problems on an assessment, I became aware that I could learn how to do mathematics without always relying on the teacher and textbook.	SA	SA	SA	A	SA	SA	SA
I am beginning to value understanding over memorization since these assessments asked me to demonstrate how I could apply what I'd learned rather than asking me to remember it.	SA	A	SA	SA	A	SA	A
Having time to do the assessment outside class eliminates the pressure I often feel during timed tests. I no longer believe that doing mathematics well means I have to do it quickly.	SA	A	SA	D	A	SA	SA
Working on the assessments outside class eliminates the pressure I often feel to memorize material. I no longer believe that doing mathematics means I have to memorize steps and formulas.	A	SA	SA	D	A	SA	SA

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