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A Study of the Knowledge Structure of Expert, Intermediate, and Novice Subjects in the Domain of Physics

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A STUDY OF THE KNOWLEDGE STRUCTURE OF EXPERT, INTERMEDIATE, AND NOVICE SUBJECTS IN THE DOMAIN OF PHYSICS

by

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A STUDY OF THE KNOWLEDGE STRUCTURE OF EXPERT, INTERMEDIATE, AND NOVICE SUBJECTS IN THE DOMAIN OF PHYSICS

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The objective of this research was to investigate the knowledge structure in physics by describing the knowledge of experts, intermediates, and novices. A review of the literature on expertise, physics knowledge, and conceptual structure provided two competing representations of physics knowledge: one defined as a hierarchy of models arranged from general to specific models, the second defined by theories which link knowledge together.

In order to investigate the two representations described above, a reiterative categorization task was employed. This task resulted in a hierarchical sort with larger piles at the top of the hierarchy and smaller piles at the bottom. The categories were classified in the study as either theory-, model- or mathematical model-based categories. The proportions of model-, theory-, and mathematical model-based categories at each level of categorization were compared by level of expertise in order to compare the subjects' categorizations with the competing representations defined by models and theories respectively. Proportions of the use of model- and theory-based sub-categories were compared at each level of categorization for each level of expertise in order to describe the structure more fully.

The categorizations that novices created was a hierarchy of models with motion as the most general category and object as a sub-category. The highest, most general level for both intermediates and experts was the theory used to solve the problems. The middle and lower levels combined model- and theory-based categories. The novices' hierarchy of models from motion to object in the problem was found to be a sub-set of the intermediates' representations. The motion of the object category was the lowest level category in the experts' representations.

These results support a combined representation of physics knowledge based on both theories and models. In this representation, the novices' hierarchy of models is a sub-set of the experts' and intermediates' hierarchies indicating that these models exist for experts and intermediates within the scope of the theory. The final representation is a hierarchy of models from general to specific connected and encompassed by a particular theory that can be used to create the models.

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Jennifer L. Discenna

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CHAPTER I

INTRODUCTION

Problem Statement

Currently in science education there is a constructivist perspective to learning and teaching. In this perspective, learners are active in their learning and knowledge acquisition is an individual process of adding new knowledge to existing knowledge in memory by creating and restructuring cognitive structures (Saunders, 1992). Cognitive structures may be thought of as an individual's beliefs, understandings, and explanations of the world that are theorized to have some organization or structure in memory (Saunders, 1992). As expertise in a certain discipline is reached, these cognitive structures become more complex and the elements of knowledge become more interconnected (Glaser, 1989). If expertise-like knowledge is the goal in science learning, then science teaching must involve assisting students in constructing inter-connected knowledge of the domain under inquiry. In order to teach the knowledge of the domain, it is necessary to understand the way in which cognitive structures in a discipline are arranged.

To investigate the knowledge structure in a particular domain, there are two important components. First, since knowledge structures consist of interconnected knowledge (Glaser, 1989), it is

important to understand what the individual elements of this knowledge are for a particular domain. Secondly, since knowledge becomes more interconnected with expertise (Glaser, 1989), it is necessary to understand how experts connect these elements of knowledge in order to understand the structure of the domain.

In this research, expertise studies are discussed to provide a description of experts' and novices' understanding of models and theories in physics. Next, theoretical and experimental research of how these models and theories may be arranged is discussed and existing models of the knowledge structure in physics are presented. Using this theoretical basis, a study was designed and conducted to further investigate models of the domain in terms of how they are arranged by subjects at various levels of expertise. Instead of using only experts, intermediates and novices were also used to provide a contrast to the experts' knowledge structure and to situate the study within the results of other studies of expert/novice differences in physics.

Knowledge in physics is primarily problem based (Hestenes, 1987); that is, the focus of a physics classroom is on solving particular problems in the discipline. Research on how individuals at various levels of expertise solve problems has been a concern of many researchers in the past decade (Larkin, 1981; 1983; Larkin et al, 1980a; 1980b). In these studies, individuals at various levels of expertise were compared in terms of the approaches they took to solving problems in physics (Larkin, 1981; 1983; Larkin et al, 1980a;

1980b). It was found that experts began a problem solution by choosing a particular theory and using that theory to create a representation of that problem (Larkin, 1981; 1983; Larkin et al, 1980a; 1980b). Novices, on the other hand, did not use theories but instead represented the problems in terms of the physical objects in the problems (Larkin, 1981; 1983; Larkin et al, 1980a; 1980b). In another study, subjects at various levels of expertise were asked to sort a set of physics problems and the categories that subjects made were analyzed (Chi et al, 1981; 1982). In this study, novices were found to categorize physics problems in terms of the surface features of the problems such as the objects contained in the problems; experts were found to organize the problems around principles used in the problem solution. In these studies (Chi et al, 1981;1982; Larkin, 1981; 1983; Larkin et al, 1980a; 1980b), the difference between experts and novices is the representations that they constructed of the problem situation. These findings may be used to suggest differences in the nature of experts' and novices' models of physical situations.

In Norman's (1983) description of models, individuals first consider a target system that consists of the objects and phenomenon in the situation to be modeled. A conceptual model is made of the target system that is a scientific representation of the phenomenon used by scientists and teachers to teach and understand a particular phenomenon. In essence, the difference between experts' and novices' models is a difference in how the target system (physics problem) is

represented. The experts represent and categorize the target system in terms of a conceptual model of the situation that aids them in problem solving, whereas the novices are only able to describe the target system in terms of real world objects (Chi et al, 1981;1982; Larkin, 1981; 1983; Larkin et al, 1980a; 1980b). Since the difference between experts and novices is the use of conceptual models (Chi et al, 1981;1982; Larkin, 1981; 1983; Larkin et al, 1980a; 1980b), the elements of knowledge in the domain of physics are proposed to consist of conceptualized models of physics problems. However, as highlighted by the results of Chi's (1981; 1982) categorization study, theories are also important to the knowledge of experts and therefore are important to knowledge structure in physics.

Giere (1988), a contemporary philosopher of science, proposes that theories in physics are composed of a family of conceptualized models and hypotheses linking these models to real world systems. In this framework, a theory can be thought of as a set of conceptualized models that are related to each other through a theory used to conceptualize the target domain (the physical world). The theory both defines and is defined by these models and the relationships between them and the physical world.

To elaborate further, Giere (1994) proposes that theories in physics are structured in terms of models similar to the way in which concepts are arranged (Klausmeier, 1990; Rosch, 1973; 1978; Rosch et al, 1976). Concepts are proposed to be arranged in a hierarchy with more general exemplars of a concept at the top and more specific

exemplars at the bottom of the hierarchy. The number of attributes that an exemplar exhibits determines whether an exemplar is more general or more specific. For example, a dog may be a more general exemplar of a pet than a spider because a dog exhibits more of the attributes of a pet than does a spider.

In the same way, Giere (1994) proposes that theories are a family of conceptualized models that are hierarchically arranged with the most general model at the top of the hierarchy and the most specific at the bottom. A general model is defined as a model that is conceptually simple. In the structure of concepts, the attributes of the exemplars define the exemplar's place on the hierarchy. In the structure of models, the conceptual attributes of the model define the model's place on the hierarchy. For example, a simple pendulum consisting of mass on a string is a more general pendulum model than a pendulum consisting of a mass on a string connected to a second mass on a second string. The double pendulum is more difficult conceptually than a single pendulum and therefore is lower on the hierarchy. Although this representation of a theory is more elaborate than Giere's (1988) initial representation of theories as a family of models, it is still vague in terms of how the theory is explicitly represented in this family and how a more conceptually difficult model is defined.

Giere (1994) used the results from Chi's (1981) categorization study to support his representation of mechanics models that is shown in Figure 1. This is a representation of mechanics models in

which conservative models are more general classical mechanics models and pendulums are more specific. In this representation, the novices in Chi's (1981) study are working at an intermediate level of abstraction where the models are represented by the objects that appear in them (Level 5). The experts in Chi's (1981) study understand physics more abstractly and work at a higher level on the hierarchy where the models are more general.

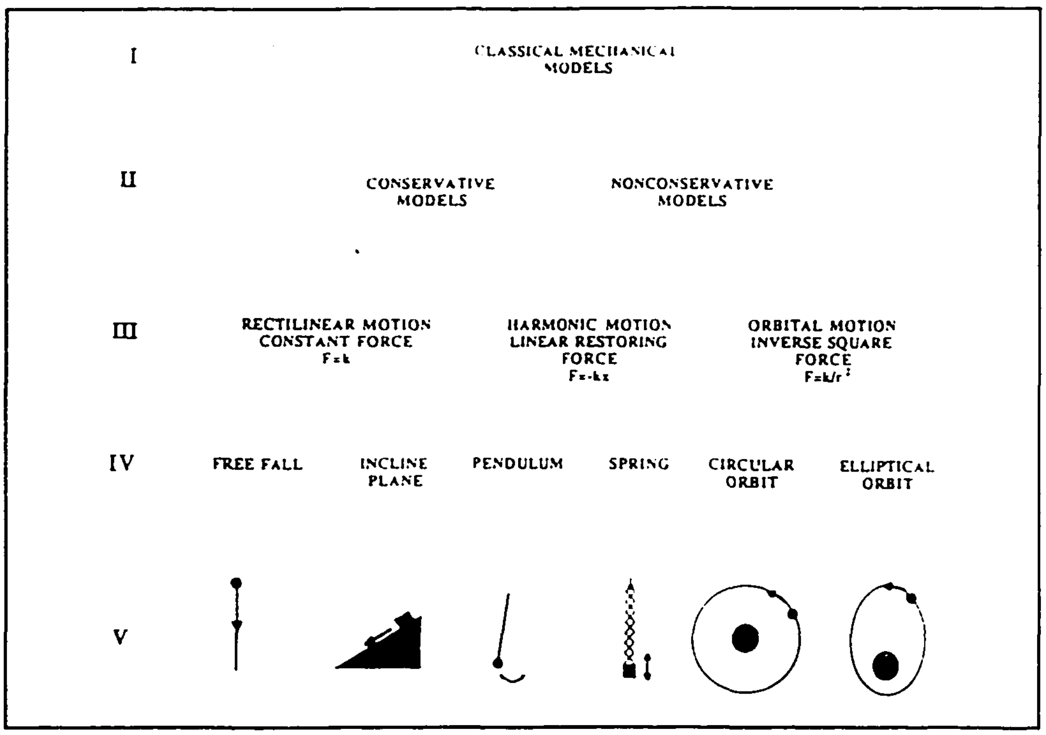

Figure 1. Giere's Representation of Models.

Source: Giere, R.N. (1994). The cognitive structure of scientific theories. Philosophy of Science, 61, p. 288.
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Giere's (1994) representation of physics knowledge, as shown in Figure 1, as a hierarchy of models has several problems. First, this hierarchy is not supported by the method used in Chi's (1981) study. Giere's (1991) hierarchy of models implies that each of the problems used in Chi's study could belong to more than one pile. Larger piles of problems, such as those belonging to the "conservative models" category in Figure 1, would have more general attributes in common to all the problems in that pile and would be higher on the hierarchy than smaller piles, such as "pendulums", whose members would share more specific attributes. Chi did not employ a reiterative categorization task and therefore, whether a hierarchical arrangement of models best describes the structure of physics knowledge cannot be addressed on the basis of Chi's data. In addition, the hierarchy in Figure 1 is not supported by the expert subjects' use of theories to categorize problems in Chi's study. The principles that experts used to categorize the problems such as Newton's 2nd law or Conservation of Energy are not explicitly stated in Giere's (1994) representation. In general, it is not clear in Giere's representation how these principles are used to define the hierarchy.

The limitation of not being able to account for the role of theories is also a criticism of the probabilistic view of concepts (Klausmeier, 1990; Rosch, 1973; 1978; Rosch et al, 1976) from which Giere (1994) borrows. Briefly, the probabilistic view holds that concepts are represented by exemplars and arranged by their attributes. Researchers have dismissed these attribute-defined

structures as being unable to account for the theories that individuals use to categorize knowledge (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). In particular, researchers argue that the studies used to substantiate the probabilistic view were done using basic level concepts that did not require higher-level knowledge of theories (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). That is, the concepts used in these studies were mostly familiar objects such as furniture or birds that may be defined by attributes explicit to the objects themselves. However, other types of concepts, such as those in physics, involve more than descriptive knowledge of a category of objects since physics is a semantically-rich domain (Bhaskar & Simon, 1977). This latter view of category construction, termed the coherent view, requires that theories provide links between the exemplars in a category (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). The coherent view of category construction is supported by the experts in Chi's (1981) study who used theories to categorize the problems. However, it is not supported by the novices in Chi's study who used surface features to categorize the problems.

At present, neither the probabilistic view of models, championed by Giere (1994) in which models are arranged by their attributes, or the coherent view (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) in which models are linked by theories is fully supported by both the novices and experts in Chi's

(1994) study. There are two ways to rectify this situation. The first is to claim that the knowledge of experts and novices in physics are completely different: Novice knowledge of physics is entirely based on the objects in models and their attributes, as in Giere's representation; and expert knowledge of physics is entirely based on theories used to solve problems, as in the coherent view of knowledge (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). The second way to alleviate this problem is to assert that knowledge in physics is neither entirely theory- nor model-driven. That is, at certain levels knowledge in physics is described by Giere's (1994) hierarchy of models and at other levels it is linked by the theories used to solve the problems.

In the present study, the latter option that physics is neither entirely theory- or model-driven, is hypothesized to be a better description of physics knowledge. It is suggested here that both models and theories play a role in physics knowledge. Models created by the same principles or theory are clustered into groups that may have some structure. Borrowing from Giere's (1994) representation, these theory clusters may be arranged in a hierarchy of general to specific models. Lastly, in the present research, it is hypothesized that these hierarchies of models are related to each other by specific principles used to solve the problems under inquiry.

This research investigates the combined theory- and model-based representation by employing a categorization study using experts, intermediates, and novices in which subjects were required to

categorize problems reiteratively. This means that the subjects were asked to categorize the problems and then to re-categorize until they were satisfied with their representation of the problem set in terms of the similarities and differences in the problems. These groupings were then classified as either model, theory, or mathematical model-based in order to determine the types of categories used in the subjects' representations at varying levels of the hierarchy.

The types of categories used across all levels of the hierarchy were compared by expertise in order to situate this study within previous literature on expert/novice differences in problem solving (Chi et al, 1981; 1982; Larkin, 1981; 1983; Larkin et al, 1980a; 1980b; de Jong & Ferguson-Hessler, 1986). Next, the use of model-, theory-, and mathematical model-based categories were compared by level of expertise to determine whether the knowledge that experts, intermediates, and novices used to categorize the problems was distinct or compatible. Finally, the theory- and model-based categories were sub-classified in terms of the attributes used to categorize (to be discussed further in the method section). Use of these attributes was compared for experts, intermediates, and novices providing a description of each level of the subjects' hierarchies. The specific research questions were as follows:

1. How do experts, intermediates, and novices compare in their use of model-, theory-, and mathematical model-based categories across all levels of categorization?

2. How do experts, intermediates, and novices compare in their use of model-, theory-, and mathematical model-based categories at each level of categorization?

3. Which model- or theory-based sub-categories do experts, intermediates, and novices use to define problems at each level of categorization?

Significance

By situating this research within previous studies on expertise in physics (Chi et al, 1981; 1982; Larkin, 1981; 1983; Larkin et al, 1980a; 1980b; de Jong & Ferguson-Hessler, 1986) this study may extend and have implications for this branch of research. In particular, this research uses intermediate subjects whose categorization may provide a comparison to both the experts' and novices' categorizations. The intermediates' categorization may describe the knowledge in transition from novice to expert. In addition, by employing a reiterative categorization task, this research may be able to describe more fully the knowledge structure of experts, intermediates, and novices and provide information about how these structures differ at various levels of expertise.

Also, by employing a reiterative categorization task this research may have implications for research on categorization in general. In much of the previous research on categorization (Rosch, 1973; 1978; Rosch et al, 1976) the items to be categorized were exemplars of everyday concepts that were described in terms of their physical

attributes. In this research, the items to be categorized are physics problems which can be described on many different levels. By asking subjects to categorize more conceptually difficult items, this research may contribute an understanding of how more complex and semantically rich knowledge is structured.

Furthermore, this research may have direct implications for understanding knowledge in physics in particular. By sub-classifying the model and theory categories, the knowledge used by experts, intermediates, and novices to categorize problems may be elaborated upon. Currently, the existing literature does not provide a thorough understanding of the relationship between models and theories in physics. Giere's (1988; 1994) representation of a theory as a family of models structured in a hierarchy is incomplete in terms of accounting for the physics knowledge of both experts and novices. The representation presented here may provide a more complete understanding of how experts, intermediates, and novices use theories and models to represent physics problems.

Finally, this research may have implications for physics pedagogy. The understanding of the connections between models and theories at different levels of expertise that may be gained from this study may be useful as a model of knowledge in physics. As discussed at the beginning of this section, this model of the structure of knowledge in physics may provide information regarding physics pedagogy which is more constructivist in nature. In particular, this model may be used to inform instruction designed to facilitate the

learning of models and theories in physics. By explicitly teaching the connections between models and theories in physics, physics learners may be able to understand physics at a deeper level.

CHAPTER II

REVIEW OF RELEVANT LITERATURE

The objective of this research was to investigate the knowledge structure of the domain of physics by studying the knowledge structures of experts, intermediates, and novices. The first task in this investigation was to situate this study within previous studies of expertise in physics in order to extend this body of literature. Towards this end, this review begins with a discussion of the importance of expertise in understanding the knowledge structure of a discipline and an overview of the expertise literature in physics. The second task was to use the expertise literature to provide evidence that physics knowledge is centered around models and theories and to discuss the role of models and theories in physics problem solving. Lastly, it was necessary to propose how knowledge may be structured in terms of these models and theories in physics for subjects at various levels of expertise. A review of the research on categorization and cognitive structure and its relationship to science concept learning will be given in order to provide a basis for the final proposed structures of knowledge in physics.

Studies of Expertise in Physics

The Importance of Expertise

As previously stated, the purpose of this study was to investigate the knowledge structure of the domain of physics by comparing the structure of knowledge of individuals at various levels of expertise. This section begins with a discussion of the importance of studying experts in order to investigate the knowledge structure of a domain. Following this discussion is a brief introduction of the means through which experts are studied, called the "Expertise Approach" (Ericsson & Smith, 1991) which is the approach adopted in this study.

First, it is important to explain why studies of expertise are important to understanding the knowledge structure of a domain. It is natural to assume that experts have more knowledge than novices in their domain. However, more importantly than quantity of knowledge, experts organize their knowledge more efficiently than novices (Ericsson & Smith, 1991; Glaser & Chi, 1988; Posner, 1988). An early study of expertise involved the study of chess players (de Groot, 1965) in which it was found that expert chess players could reconstruct more of a chess board than novice chess players could. This work in chess was extended upon by Chase and Simon (1973) who asked participants to study a chess position for five seconds and then reconstruct it (Chase & Simon, 1973). The researchers found that chess players with more expertise were able to reconstruct the

position of more of the pieces than novice chess players if the chess board was set up to represent real chess positions. If, however, the pieces were randomly positioned on the chess board, this difference in expertise was not observed (Chase & Simon, 1973). They concluded that the experts' superior performance on the memory task was dependent on meaningful relationships between the chess pieces in terms of how chess is played.

Next, Chase and Simon (1973) examined the data and noticed that there were periods of activity in which subjects placed pieces on the board. These periods were followed by a slight pause, then another period of activity. It was inferred that the subjects in this study placed pieces on the board that had meaningful relationships to each other, paused to recall another set of meaningful relationships, and then placed these on the board. This phenomenon was called "chunking" in the sense that the experts remembered the board in chunks that were meaningful in the domain of chess. When the board's pieces had no meaningful relationships, i.e., the pieces were randomly placed, their expertise was not helpful because there were no meaningful ways to "chunk" the positions on the board. This study exemplifies that expertise is not simply of matter of having a greater quantity of knowledge, but is rather a matter of having that knowledge arranged into meaningful chunks. The findings that expert knowledge is arranged into meaningful chunks and that as expertise is attained that these chunks become more complex has also been found by other researchers in various domains such as

baseball, computer programming, and electronic technology (Chi, Glaser & Rees, 1982).

Since knowledge becomes more interconnected with expertise, it is necessary to study how experts chunk knowledge in order to understand the structure of the discipline (Glaser, 1989). The "Expertise Approach" (Ericsson & Smith, 1991) permits one to describe and analyze expert performance in order to identify the aspects of the performance that make expert performance superior. This approach has three basic components. The first step in studying expertise is to design a series of tasks that will capture expert performance. The second task is to perform an analysis of the elicited performance. The third task is to account for the mechanisms underlying expert performance. In the next section, this approach will be discussed in terms of studies of expertise in physics.

Expertise in Physics

As previously stated, the first step of the expertise approach is to design tasks that will capture expert performance. Physics expertise studies have employed primarily problem solving tasks towards this end. Physics as a domain is centered around understanding and explaining physical phenomena in terms of the theories that explain the world (Anzai, 1981). Therefore, it is reasonable to use problem solving tasks to elicit superior performance in explaining the physical world.

The next step to utilizing the expertise approach in physics is to analyze the performance on problem solving tasks and then, lastly, to account for the processes that mediate expert performance. In order to analyze expert performance in problem solving, it is often useful to compare the experts' problem solving to that of novices. The research comparing experts' and novices' problem solving in physics has been reviewed by Maloney (1994). For the purposes of this discussion, problem solving research in physics can be divided into studies of problem representation (Larkin, 1983; 1985; Larkin, McDermott, Simon & Simon, 1980a, 1980b) and categorization (Chi, Feltovich, & Glaser, 1981; Chi, Glaser, & Rees, 1982; de Jong & Ferguson-Hessler, 1986; Veldhuis, 1986; Hardiman, Dufresne, & Mestre, 1989). There are also many studies of alternative conceptions in physics, but these have mostly been done with novice subjects and were used to understand the limitations of novice subjects' knowledge of specific physics concepts rather than to understand expertise (McDermott, 1984; Trowbridge & McDermott, 1981; Wandersee, Mintzes, & Novak, 1994).

In studies of expertise, researchers found that, in general, experts have knowledge that is organized more meaningfully than that of novices (Ericsson & Smith, 1991; Glaser & Chi, 1988; Posner, 1988). Evidence of this better representation of knowledge and for the notion of chunking was also found in physics problem solving research in the form of superior representation of problems (Larkin, 1983; 1985; Larkin, McDermott, Simon & Simon, 1980a, 1980b).

Research was conducted by Larkin and her colleagues (Larkin, 1983; 1985; Larkin, McDermott, Simon & Simon, 1980a, 1980b) who investigated how subjects at various levels of expertise represent physics problems. In one such study, problem representation was examined using problems with increasing difficulty; the experts were professors and graduate students in physics, and the novices were undergraduates who had completed a minimum of eight weeks of physics instruction (Larkin, 1983). An analysis of the think aloud protocols from the problem solving task showed that, in general, novices represented physics problems in terms of the objects or unknowns in the problems. This naive representation of the problem in terms of physical objects did not facilitate solving more difficult problems. Experts, on the other hand, created physical representations of the problems which included elements of a theory that were used to explain the phenomena and to facilitate problem solving. The experts' representations of problems recognized the physical schema involved in solving the problem such as Newton's Laws or Conservation of Energy. These schema were found to be useful in solving more difficult problems.

On the basis of this research, Larkin (1983) suggests that there are two qualitatively different kinds of representations constructed in problem solving. The first type of construction, used by novices, is a naive representation which is composed of objects that exist in the real world (blocks, pulleys, springs) and are developed through operators that correspond to events that occur in real time. The

second type of representation is a physical representation which experts construct. The physical representation extends the naive representation by including imagined entities such as forces and momenta and is developed by operators corresponding to the laws of physics. In contrast to the naive representation, the physical representation involves entities (forces, momenta) that have meaning only in the context of formal physics.

In the expertise research using categorization tasks, the results were very similar to those of the problem representation studies. Chi et al (1981) asked novice and expert subjects to sort problems from an introductory physics textbook into categories. The subjects were asked to sort the cards based on their similarities of solution; they were not allowed to use either paper or a pencil, nor were they allowed to actually solve the problems. The goal was to elicit the important attributes of the problem from the subjects on the basis of problem situation itself. After the subjects sorted the cards, they were asked to re-sort them as a measure of consistency and then to explain the rationale for their groupings. There were no differences found in the number of categories used by the novice and expert groups. Results of a cluster analysis demonstrated that novices, in general, used surface features to describe the problems, namely, objects that appeared in the problems, the literal physics terms used, or the physical configuration of the problem. For example, a pendulum problem asking students to find the velocity of the bob when the object is 30° might be categorized according to the pendulum bob, the

velocity that is asked for, or as a "pendulum" problem. Experts' categories, on the other hand, corresponded to the major principles involved in the solution of the problem. Chi (1981) termed this as the "deep structure" of the problem that relates to the underlying physics law applicable to the solution to the problem such as Newton s Third Law, Conservation of Energy, or Conservation of Momentum. There was little overlap between the experts and novices in the names or explanations of the categories used, with only five of the twenty categories utilized by both groups. In comparison to this study, Chi et al (1982) asked experts and novices to subdivide or combine their categories of physics problems into as many other categories as they could and found that the novices produced more categories with fewer members as compared to the experts who made larger, more encompassing categories.

In a second study (Chi et al, 1981), twenty problems were chosen in which surface features were crossed with physical laws, i.e., problems were chosen such that problems with the same surface features did not have the same deep structure. For example, there were two problems that involved pulleys; one problem required a solution using Conservation of Energy, the other used Newton's Laws. Here both problems have the same surface features, i.e., the pulley, but they have different deep structures, namely, Conservation of Energy and Newton's Laws. The results confirmed Chi et al's hypothesis: The novices used the surface features and the experts used the physical laws in order to categorize the problems.

Veldhuis (1990) repeated Chi et al's (1981) initial categorization task with a larger sample of novice subjects and found that these novice subjects used both surface features and principles to categorize problems. de Jong and Ferguson-Hessler (1986) proposed that perhaps Veldhuis's results were due to the difference between good and poor novice problem solvers. To determine this, de Jong and Ferguson-Hessler used good and poor problem solvers in their categorization task and found that good problem solvers used deep principles and poor problem solvers used surface features to categorize the problems. From Larkin's (1983) studies on problem representation and Chi et al's (1981; 1982) research, de Jong and Ferguson-Hessler concluded that the good problem solvers were able to organize their knowledge around the principles needed to solve the problems similar to that of experts. Thus, it was the organization of knowledge and not the quantity of knowledge that facilitated the superior problem-solving ability of the experts in the studies by Larkin, Chi et al, and de Jong and Ferguson-Hessler.

The consistent finding in these studies is that the problem representation is constructed by the novices and experts using their knowledge of the domain (Chi et al, 1981; 1982; de Jong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990). Neither group represented the problems exclusively on the basis of the literal statement of the problem. Even novices were able to generalize objects in the problems as belonging to a certain class. For example, novices might classify a problem involving

a playground swing as a pendulum since it has a mass on one end and is fixed on the other and allowed to rotate. Both novices and experts use their knowledge of physics to represent problems in terms of an understanding of the problem solution. However, the knowledge that experts and novices use to represent physics problems is qualitatively different. In the next sections, these differences will be described as differences in novices' and experts' understanding of physics models and theories.

Physics Problems, Models, and Theories

The preceding discussion of physics expertise focused on the differences between expert and novice performance in physics problem solving. The findings from these studies indicate that becoming an expert in physics involves becoming more adept at representing physics problems in terms of theories. In this section, it is argued that the differences between experts' and novices' representations of physics problems reflects a difference between their understanding of the problems in terms of the models and theories of the domain. This argument will be made by discussing the nature of problems in physics and then their relationship to the models and theories of the domain.

Physics Problems

To further interpret the results of the expertise studies in physics (Chi et al, 1981; 1982; de Jong & Ferguson-Hessler, 1986;

Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990), it is necessary to provide a brief background on the nature of problems in physics and how they are solved. Since the studies involved solving physics problems, this discussion is necessary to explain what a problem in physics is and how it may relate to the models and theories in physics. This discussion will be used to support the claim that the differences found in the expertise studies reflect a difference in the subjects' understanding of models and theories.

A problem, in general, is defined as a situation in which there is a specified goal state, but there is no clear set of actions that are apparent to get to that state (Newell & Simon 1972). A problem may also be defined as a set of constraints on the problem solution plus the demand that the solution be found (Nickles, 1981). This is the type of problems that researchers in physics investigate. In these problems, there are phenomena that need to be explained and descriptions of the phenomena are given as constraints on the solution of that problem. Next, the solution is found and the phenomena are explained through elaboration of a certain theory or model. For example, the problems of mechanics correspond to the question of how and why things move. A object is placed on an incline plane and it slides down the plane. The problem here concerns what is happening to the object in terms of the distance that it travels, the time that it takes it to travel, the velocity it travels at, and the acceleration that it achieves. The problem's solution involves

explaining why the object moves in the way that it does and how its movement corresponds to the movement of other objects.

The problems found in physics textbooks that novices solve do not conform to these definitions of a problem (Maloney, 1994). Problems in physics textbooks usually consist of a situation in which certain information is given and the task is to find the value of a certain variable in the problem (Maloney, 1994). In these problems, the theories and models to be used for its solution are embedded in the problem description and constrained by the variables given in the problem (c.f., Halliday & Resnick, 1988). Students very rarely are asked to create completely new solutions for phenomenon that are either causal or descriptive and textbook problems are very unlikely to be open-ended.

The importance of this discussion is to show that the physics problems that experts and novices solve are different qualitatively. The problems that experts solve in physics require an attempt to explain phenomena in the physical world through models and theories (Maloney, 1994). Textbook problems do not require novices to create models or theories, but to investigate models and theories that have been created already. In the studies of expertise in problem solving, both experts and novices used their knowledge of the domain to put the cards into categories (Chi et al, 1981; 1982; de Jong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990); however, the knowledge upon which each group had categorized was different. One reason for this may be that problems

are qualitatively different for each group. Experts know that problems involve the creation and elaboration of models and theories in physics. Novices, i.e., students in physics, know that problems come from the end of the chapter and that the answers are in the back of the book. From this, it is suggested that the differences in the categorizations of experts and novices may be explained by the experts' knowledge of models and theories and their role in problem solving.

Physics Models and Theories

As suggested in the preceding section, models and theories in physics likely play an important role in problem solving in the domain of physics. In this section, models and theories in physics will be discussed. Norman (1983) describes mental models in terms of four different components: the target system, the conceptual model of the target system, the mental model of the target system, and the scientist's conceptualization of the mental model. The target system consists of the objects and phenomenon in the situation to be modeled. A conceptual model is made of the target system that is a scientific representation of the phenomenon used by both scientists and teachers to teach and understand a particular phenomenon. In general, a mental model is an internal representation of the target system that is often dynamic, manipulable, and has explanatory and predictive power (Jones, 1995; Norman, 1983; Johnson-Laird, 1989; Collins & Gentner, 1987). Finally, the scientist's conceptualization is

a model of an individual's mental model that is created by a researcher.

Conceptualized models in physics are mostly mathematical and consist of descriptive and explanatory parts (Halloun, 1995; 1996; Halloun & Hestenes, 1985; 1987; Hestenes, 1987; 1992). The descriptive part of the model consists of names for the objects that appear in the model and any properties or variables that describe that object. The descriptive nature also contains any graphs or the time evolution of any variables in question. The agents which interact with the object and explain any changes in the variables are represented in the explanatory part of the model (Hestenes, 1987; 1992). The application of the interaction variables or agents results in relevant equations of the model which interpret the description of the phenomena. An example of a model is the motion of an object on a frictionless incline plane shown in Figure 2. In this model, the descriptive part states that the object has a mass, m, and is resting on a plane inclined with an angle, θ. Also shown are graphs of the time evolution of the variable of interest: distance (position), velocity, and acceleration. The motion is explained by using Newton's Laws and the interaction of forces. In this case, the forces are the normal force (N) and force due to gravity (F_g). Both forces are the agents that interact with the object to cause the change in the variables. The equations of motion (x(t), v(t) and a(t)) are explained by the application of these forces.

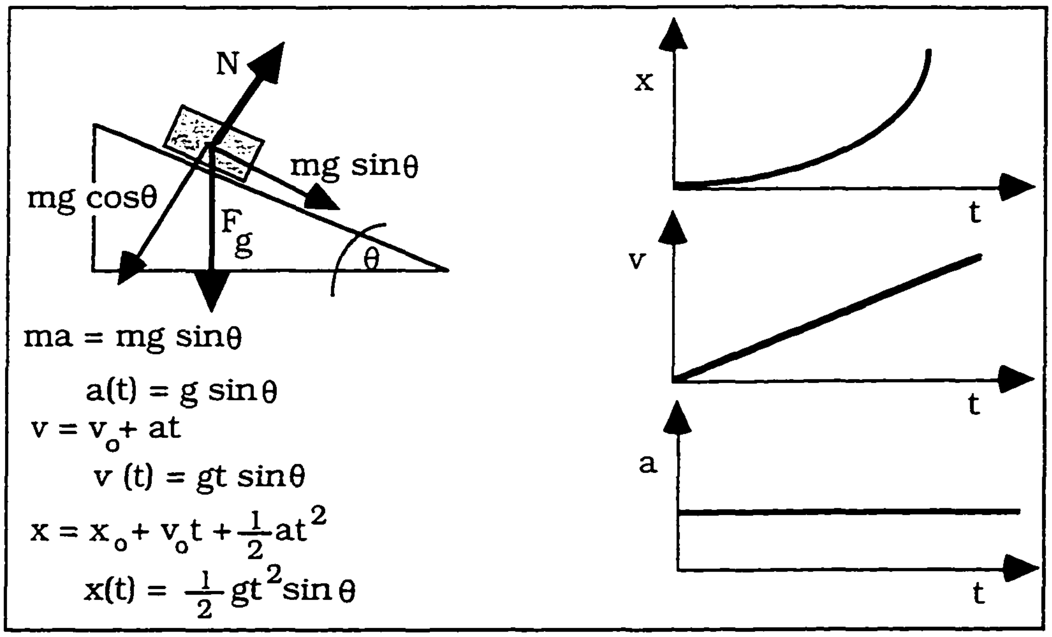


Figure 2. Model of Object on a Frictionless Incline Plane.

Larkin (Larkin, 1983; 1985; Larkin, McDermott, Simon & Simon, 1980a; 1980b) describes a schema as consisting of inferencing rules that are used by experts to construct physical representations or conceptualized models. These inferencing rules are of two types: (1) construction rules that act on the original (naive) problem representation to produce entities in the physical representation, and (2) extension rules that act on an existing physical representation to add new entities to that representation (Larkin, 1983). These schemata can be thought of as the laws, principles, definitions, theorems, or rules that allow experts to make conceptualized models (Larkin, 1983). The construction rules act on the descriptive element of the phenomenon to produce causal agents similar to the way that Newton's individual laws work to suggest different kinds of forces (Larkin, 1983). The extension rules extend the causal model to add

new entities that facilitate problem solving as when the equations of motion are derived from the various forces that were chosen (Larkin, 1983).

Theories (Halloun, 1995; 1996; Halloun & Hestenes, 1985; 1987; Hestenes, 1987; 1992) and schemata (Larkin, 1983) have both been referred to as the means by which the physical world is conceptualized. Schemata is often used as a more general word to describe the types of theories or explanations that individuals have about the world and their relationships to others. Theories in physics are more rigorous and refer to the principles that are used to solve problems and to make models (Halloun, 1995; 1996; Halloun & Hestenes, 1985; 1987; Hestenes, 1987; 1992). The schemata that physics experts use as referred to in Larkin's studies (1983; 1985; Larkin, McDermott, Simon, & Simon, 1980a, 1980b) can be thought of as physics theories used to make interpretations of physical phenomenon and create conceptual models such as the one in Figure 2.

The research on problem representation by Larkin (Larkin, 1983; 1985; Larkin et al 1980a; 1980b) can be interpreted in this study based on the descriptions of problems, models, and theories to suggest that the physics experts create conceptualized models of the problems using physics theories. Experts in physics were found to use theories of the domain to create representations of a problem situation. Furthermore, these representations are superior to novices in that they are able to represent abstract relations and properties

that are not in the situation itself (Johnson-Laird, 1989). Novice representations of problems relied on the physical situation itself in terms of the objects and processes that happen in real time (Larkin, 1983; 1985; Larkin et al 1980a; 1980b). In essence, it is interpreted in the present research that in terms of Norman's (1983) taxonomy of models, novices represented the problems in terms of the descriptive aspects of the conceptualized model that are dependent on the target system and that experts represented the problems in terms of conceptualized models that have superior explanatory power.

Categorization and Cognitive Structure

From the preceding discussion of expertise and knowledge in physics, it is suggested that the knowledge structure of the domain of physics is centered around conceptualized models and theories of the discipline. However, these are only the components of the structure of the discipline. It is also equally important to discuss how those theories and models might be organized.

The conceptual structure of a discipline and its relation to science concept learning has been well studied (Ausubel, Novak, & Hanesian, 1978; Chi, 1992; Chi et al, 1994; Klausmeier, 1990) and a discussion of this literature will provide a foundation for a representation of physics knowledge in terms of models and theories of the discipline. This section will begin with a discussion of knowledge structures in science disciplines and then discuss tasks

such as categorization that are used to understand the structures of individuals in these disciplines.

Conceptual Structure and Concept Learning

Interest in the study of cognitive structure increased with the publication of *A Study of Thinking* (Bruner, Goodnow, & Austin, 1956) which re-introduced the idea of concepts having structure and the process of categorization as a way of understanding that structure. Theories on how knowledge in general is represented and studies to support these theories have been summarized by Smith and Medin (1981). In this paper the positions of theorists on knowledge structure were placed into three basic approaches: the classical view, the probabilistic view, and the exemplar view (Smith & Medin, 1981). The exemplar view has not been used to represent science concepts, therefore only the classical and probabilistic views will be discussed. In addition, a third view linked to the classical view will be described, namely, the ontological view.

The classical view of concepts states that all members of a category of objects share common attributes and these properties are necessary and sufficient to define the concept (Smith & Medin, 1981). For an attribute to be necessary to define or categorize a particular concept or category, every member of that category must possess that attribute. For a set of attributes to be sufficient to define or classify a certain category, every example having that set of attributes must be a member of that category. An example of a concept in the classical

view is a square that can be defined as a four sided object with equal lengths and four ninety degree angles. The attributes "equal lengths", "four sided" and "ninety degree angles" are necessary to identify the category of squares in that all squares have these attributes. Also, these attributes are sufficient because every object that has these attributes is a square.

The classical view has been used to describe science concepts since their necessary and sufficient attributes are well known. Ausubel, Novak, and Hanesian (1978) take a classical view of science concepts by defining them as consisting of the generalized attributes of a given category of objects, events, or phenomena. That is, they define a particular concept in terms of the attributes that a class of objects, events, or phenomena have in common. For example, the concept "gas" as shown in Figure 3 may be defined by the molecules being far apart and in continuous motion. Concepts, in this framework, are hierarchically structured with more general attributes at the top of the hierarchy and more specific attributes at the bottom.

The notion that the structure of concepts is hierarchical and defined by the attributes is central to understanding how concepts are learned. One theory of learning in particular is Ausubel's meaningful verbal learning (Ausubel, 1963; 1966; 1967; Ausubel, Novak & Hanesian, 1978). Meaningful learning, in contrast to rote learning, occurs when the learner is presented with potentially meaningful material which can be subsumed into an existing cognitive structure. That is, it is assumed that a learner possesses a hierarchical

conceptual structure which consists of a variety of concepts related to one another by attributes that the student has built from experiences in his/her environment. When a student comes across a new concept, that concept is differentiated from other concepts in the structure in terms of its attributes. If the new concept gets connected into the existing structure, then the learning is said to be meaningful. Rote learning, on the other hand involves discrete and isolated entities (Ausubel, 1963). A new concept learned in this way exists by itself as a lone piece of information that is not attached to any other concept or idea and is easily forgotten.

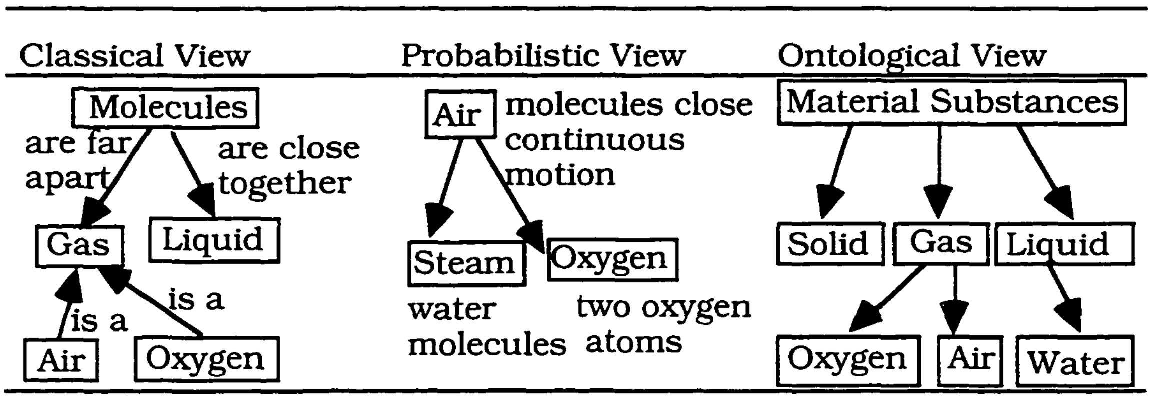


Figure 3. Three Views of a Concept.

The classical view describes concepts such as squares and other objects very well. However, there are concepts that are not described by the classical view of concepts (Smith & Medin, 1981) and for which there are no necessary and sufficient attributes. For example, consider the concept of pet. All pets have the necessary attributes of "is an animal" and "cared for by people". However, animals in the zoo also have these attributes, but are not pets. The

probabilistic view claims that there are no necessary and sufficient attributes to describe concepts like "pet". Instead, each member of a category varies in the degree to which they share certain attributes and therefore they vary in the degree to which they represent the concept (Smith & Medin, 1981). In this theory, there is an "ideal" description of a particular concept in which attributes in that description are true of most but not necessarily all members (Smith & Medin, 1981). This means that there are examples of a concept that are "better" examples in that they exhibit more of the common attributes of a particular category than other members of that category. In our example of pets, a dog may be a "better" example of a pet than a spider since many people regard spiders as pests rather than animals that they would nurture in their homes.

Most concepts in science can be defined in terms of the classical view, but the probabilistic view has the quality of also being able to describe the attributes of exemplars of objects explicitly. Klausmeier (1990) favors the probabilistic view as a model to represent science concepts. In this view, as depicted in Figure 3, examples of a concept are hierarchically structured with examples that idealize more general attributes of the concept at the top of the hierarchy and those with more specific attributes at the bottom. Concepts are defined in terms of examples which are either central or peripheral to the concept which they are exemplifying. The examples which exhibit the most attributes would be called central examples; the ones which exhibit less are called peripheral. In Figure 3, the concept of gas is

represented by air which is a very common gas that exhibits the properties of having molecules far apart and in continuous motion. Oxygen and steam are more peripheral examples of a gas since they are less common.

Klausmeier's (1990) theory of learning is related to the probabilistic conceptual structure. In Klausmeier's theory, the goal of concept learning is to create a conceptual structure that is identical to that of an expert. The first step in this process is for the instructor to define the expert conceptual structure in terms of its attributes. Learning then takes place at four successive levels of abstraction in which each level calls for different mental processes and each specifies a higher level of understanding of that concept. According to this theory, a person's representation of any given concept varies with the developmental level at which the concept is learned and the nature of the concept. For example, to learn the concept of a gas, the instructor must first define a gas in terms of its attributes. Next, appropriate examples of gases must be found to be presented to the learner. The learner interacts with these examples in order to differentiate the attributes of a gas from other substances and to find attributes similar to all gases. In this way, the learner builds a hierarchical structure of gases which is defined by their attributes.

A third view of conceptual structure which has been linked to the classical view of concepts is the ontological view (Chi, 1992; Chi, Slotta, and de Leeuw, 1994; Keil, 1979;1989). In this view, the only attributes used to categorize a concept are its ontological ones, e.g.

state of matter or process (Chi, 1992; Chi, Slotta, and de Leeuw, 1994; Keil, 1979;1989). Chi (1992) and Chi, Slotta, and de Leeuw (1994) use Keil's (1979;1989) work on concept formation in children to propose a theory of science concepts in which individual concepts are linked together and are hierarchically arranged according to the concept's ontology. Philosophically speaking, all concepts have some particular kind of ontology. This means that all concepts belong to a certain category defined by the "type" of knowledge it is. All conceptual knowledge belongs to one of three main categories: Material Substances, Processes, or Mental States (Chi 1992; Chi et al, 1994; Keil, 1979; 1989). Similar to Klausmeier s (1990) framework, the ontological categories that are higher on the tree structure are more general than those on the bottom. Concepts in the same tree have some of the same attributes. For example in Figure 3, oxygen and water both have the attributes of having weight and occupying space because they both belong to the higher level category of material substances. However, these same concepts also have different ontological attributes because they differ in position on the tree horizontally. Oxygen has the attribute molecules far apart because it belongs to the gas category; water does not because it belongs to the liquid category. Concepts that belong to different trees have none of the same attributes and are thus, ontologically distinct.

Learning in this paradigm is still mediated by the attributes of the object and their position on the hierarchy as suggested by Chi, Slotta, and de Leeuw (1994). However, the only attribute which

facilitates learning in Chi et al's theory is the concept's ontological attribute. Some concepts are not learned correctly because they have been incorrectly placed on one of the ontological trees. Thus, in this view a misconception is simply an ontological mis-categorization of that concept. For example, many young children believe that cars are living things because they are able to move on their own and eat gas (Osborne & Freyberg, 1985). In this case, they have mis-categorized a car into the "Living" category and attributed to it the descriptors is hungry and moves on its own . In order to enact a conceptual change in terms of Chi et al's theory, the child will have to remove these attributes from the concept of car and learn the correct attributes that would allow it to be categorized into the Non-Living category.

From this discussion there is an agreement that science concepts have some structure and that this structure is hierarchical. Also, placement of a concept within a hierarchy depends on the attributes of that hierarchy. Concepts having general attributes of a particular concept are placed higher on the hierarchy than those with attributes that are more specific. In each theory, there is agreement that learning is related to the way that concepts are structured. Briefly stated, for Ausubel (Ausubel, 1963; 1966; 1967; Ausubel, Novak & Hanesian, 1978) and Klausmeier (1990), learning is a process of differentiating the attributes which define a particular concept. For Chi, et al (Chi 1992; Chi et al, 1994; Keil, 1979; 1989)

learning is a process of correct placement of a concept within an ontological tree.

Conceptual Structure and Categorization

The preceding section described the structure of concepts as viewed by various researchers and the relationship between structure and how science concepts may be learned. This section addresses the research used to substantiate these representations of concepts and conceptual structure. Although the views of a concept discussed above are diverse, they are similar in that categorization plays a role in determining the organization of concepts.

Categorization is described as the process of identification or the act of placing an object, idea, or phenomenon into a certain class (Bruner, Goodnow, & Austin, 1956). There are many reasons that people may categorize objects, but the most compelling is their need to reduce the complexity of the world and to provide the most information with the smallest cognitive effort (Rosch, 1978). Categorization allows people to organize objects together by their attributes which is more efficient than recalling all the objects or stimuli which one encounters. Also, categorization reduces the necessity of constant learning (Bruner, Goodnow, & Austin, 1956). For example, once a category called "dogs" has been defined, each new dog encountered does not have to be treated as an entirely new object. Instead, the new dog can be identified as a dog simply by recalling the attributes of that category.

Categorization tasks give researchers a way of determining knowledge structure; that is, asking a subject how they would categorize certain items into classes provides some information about how their knowledge is structured (Smith & Medin, 1981). The answer to whether an object belongs to a certain class gives information about how objects are arranged in terms of hierarchy. This also provides information concerning the attributes which all the members of that category share. Lastly, the amount of time it takes for a subject to answer the question determines whether an exemplar is central (less time) or peripheral (more time) to a particular concept.

In summary, concepts provide us with a way of organizing information in the world. Concepts may be organized in a hierarchical structure in which the highest level category contains the most general attributes or examples and lower levels contain most specific. Categorization is the process through which concepts are developed (Bruner, Goodnow, & Austin, 1956). Finally, categorization tasks may be used experimentally to investigate these conceptual structures.

Structure of Knowledge in Physics

In the section on conceptual structure, a concept was defined as a set of hierarchically ordered exemplars belonging to that concept in which more specific exemplars of the concept were placed at the bottom of the hierarchy and more general exemplars were placed at the top. The individual concepts were linked together propositionally using linkage meanings (Novak, 1990; Novak, Gowin, & Johansen,

1983). The expertise literature has provided evidence to suggest that physics knowledge is centered around the theories and models used to solve problems in physics (Giere, 1994). However, this provides only a basic understanding of the elements of knowledge in physics. The goal of this research is to suggest and examine a more specific structure of the models and theories in physics similar to the elaborated structure of concepts (Ausubel, 1963; 1966; 1967; Ausubel, Novak & Hanesian, 1978; Klausmeier, 1990) .

It should be noted that physics as a domain may also be described by the concepts that underpin the models and theories of the domain (Halloun, 1995; 1996; Halloun & Hestenes, 1985; 1987). However, these conceptual structures might not be the best emphasis for a physics classroom as attempts to apply Ausubel's theory of concepts and concept learning (Ausubel, 1963; 1966; 1967; Ausubel, Novak & Hanesian, 1978) in physics classrooms have not been successful. In particular, a physics course applying Ausubel's (1963; 1966; 1967; Ausubel, Novak & Hanesian, 1978) theory of meaningful verbal learning was not successful for improving classroom learning (Moreira, 1978). Further, concept maps which are instructional tools in which concepts are connected by "linkage meanings" (Novak, 1990; Novak, Gowin, & Johansen, 1983) have not been shown to be effective in physics classrooms (Horton, et al, 1993; Prosser, Hazel, Trigwell, & Lyons, 1997; Roth & Roychoudhury, 1993). One reason why concept learning in physics has been unsuccessful may be due in part to assessments that did not measure concept learning, but instead

focused on traditional problem solving. Again, it is not suggested that there are no concepts in physics or that they have no structure. Instead, in light of results from the literature it is claimed that physics expertise involves superior knowledge of models and theories in physics. Therefore, the structure of the domain is depicted as a structure of these models and theories.

In his early work, Giere (1988) attempted to characterize theories in a more naturalistic way and proposed that a theory is a family of conceptualized models and hypotheses that link these models to systems in the real world. Giere formulated this theory of theories by inspecting advanced-level classical mechanics textbooks. Giere's assumption was that scientists learn about the nature of theories from these textbooks and that their notions of a theory are influenced by the authors. In addition, it is assumed that the experts writing the textbooks have a sufficient grasp on the nature of their discipline. From this inspection of textbooks, Giere noticed that in the case of classical mechanics, the force functions provide the chief organizing principles of the book. That is, treatments of classical mechanics tend to be centered around the introduction of objects with more and more complicated force functions. In addition, although the textbooks had some variation, there were a core group of topics and examples that were used in almost all the books such as an example of a falling object, a mass on a spring, or a pendulum (Giere, 1988). From this, Giere suggests that theories in classical mechanics are centered around these core examples and that these examples

actually form a group of conceptualized models. These models are then used to create more complex models through the addition of force functions.

The idea of theories as a family of models is only one aspect of Giere's (1988) interpretation of the textbook. The second idea is that theories are linguistic statements or hypotheses that claim a relationship between the model and a real system (Giere, 1988). The hypotheses that Giere proposes are not meant to be judged as either true or false, but rather express some degree of similarity between the conceptualized model and a real system. The question of whether a model is a true or false representation of a real world system is not as important as is the degree of similarity between the model and real world system (Giere, 1988).

To explain Giere's (1988) idea further, his representation of classical mechanics will be used as an example. In classical mechanics, such theories as Newton's Laws are used to create a family of models such as projectile, pendulum, or spring motion. These core models can be complicated by adding more force functions such as a damping force or air resistance. In essence, the family of models, projectile, pendulum, and spring, can be multiplied to create more models that are more complex creating an entire population of models created from Newton's Laws. These models also have some correspondence to "real" pendulums, projectiles, and springs. The theory is used to make statements of similarity between these

conceptualized models and the real world target system. An example of such a statement about the motion of a projectile would be:

> The velocity and position of a projectile in the earth-projectile system are very similar to those of a one-particle Newtonian model with a constant gravitational force.

This statement compares the model created of the projectile's motion using Newton's Laws and a constant gravitational force with the "real" motion of a projectile. This statement suggests that the model is only a "very similar" approximation to the conceptualized model.

In later work, Giere (1994) attempts to elaborate on the ways in which the population of models are structured by borrowing heavily from the work of cognitive scientists and particularly Rosch (1973) who inspired Klausmeier s (1990) probabilistic view of a concept. Recall that concepts in the probabilistic view reside in a graded structure in which examples of the concept that are more general appear at the top of the hierarchy and those that are more specific appear at the bottom. Giere (1994) applies this notion to models and suggests that a theory in physics is a family of conceptualized models and that these models are hierarchically arranged with more general models at the top of the hierarchy and more specific ones at the bottom. For example, a simple pendulum may be a very central model for all types of motion that can be described as moving back and forth with a restoring force. A peripheral pendulum model could be the physical pendulum because it has the same motion, but involves more complex forces. Models become more general, and less complex, as

one moves up the structure and become more specific and more complex as one moves down the structure (Giere, 1994).

Giere (1994) used the results from Chi et al's (1981) categorization study to support a representation of classical mechanics models shown in Figure 1 in the introduction section. In this interpretation, the novices of the Chi et al study are operating at level IV which is a "basic" level where models are less central and more specific. The idea of the "basic" level was introduced by Rosch (1978) who proposed that there existed an intermediate level of abstraction which is the most general category that still has a high degree of similarity among its members. Problems at this basic level have visual similarity, e.g., springs or pendulums to the other members of the category, but not to the more abstract category above it such as conservative models. Also, according to Rosch problems at the basic level are more likely to be learned first. Experts in Chi's (1981) study are interpreted by Giere as operating at a higher level of abstraction than the novices. According to Giere, the primary difference between experts and novices is essentially the place at which they are operating on the graded structure. In Giere's (1994) scheme, part of becoming an expert in physics is learning to categorize problems at a higher level of abstraction.

There are a few weaknesses to Giere's (1994) representation of knowledge that will be examined at this time. The first weakness is that Giere's structure of models is an analogy to the structure of concepts and this analogy may not be adequate. According to Giere,

physics knowledge can be thought of as consisting of models structured in a hierarchy in much the same way that concepts are structured in Klausmeier's (1990) view. However, this analogy may be only superficially true. Recall that in the case of conceptual knowledge, Klausmeier described in detail the criteria for establishing an example of a concept's place in the hierarchy. In this theory, each example of a concept is described by its attributes and these attributes define the position of the example in the conceptual structure. Similarly, Chi, Slotta, and de Leeuw (1994) offer a similar graded structure which is defined by the concept's ontological category. In order for the hierarchical structure of models proposed by Giere to be representative of knowledge in physics, something analogous to the attribute is needed to define its structure. In particular, Giere (1988; 1994) alludes to the notion that more peripheral models are more difficult conceptually. However, it is not clear what attributes define a more conceptually difficult model. In his first interpretation of physics textbooks (1988), Giere suggests that more peripheral models in classical mechanics are defined by more complex force functions. However, in the structure in Figure 1, these force functions ($F=k$, $F=-kx$, $F=k/r^2$) are listed at a higher level. It is not clear how these and other functions are used to define models at lower levels of abstraction.

In the research on problem classification, novices used the surface features in the problems to create their categories (Chi et al, 1981; 1982; de Jong & Ferguson-Hessler, 1986; Veldhuis, 1990).

These surface features are attributes of the models used to solve the problems. For example, the pendulum is an attribute of a model explaining its motion. Experts in these studies categorized the problems by the principles used to solve the problems (Chi et al, 1981; 1982; de Jong & Ferguson-Hessler, 1986; Veldhuis, 1990). Moreover, principles such as "Conservation of Energy" or "Newton's 2nd Law" are not attributes of the models used to solve the problems in that they do not appear explicitly in the models themselves. An attribute of a model is something that is a part of the model such as the objects in the model, the graphs of the time evolution of variables, or the resulting equations of motion (Hestenes, 1992; 1987). In the example of the incline plane shown in Figure 2, Newton s Laws were used to create the conceptualized model shown. This is not explicit in the finished model in that Newton's Laws are not explicitly represented in the model. However, the attributes "incline plane" or "frictionless" are always a part of the incline plane model itself. The hierarchy that Giere (1994) suggests describes the knowledge that novices have of the models in physics in terms of their surface features, but it does not explain experts' knowledge. The experts' knowledge is described by Giere to be at a higher level, such as the "conservative or nonconservative models" level, but this is not borne out in Chi et al's results.

The limitation in explaining the experts' use of principles to categorize the problems may also be a weakness of the probabilistic view of knowledge. Research on conceptual structure has found that

the probabilistic view of concepts is inadequate to explain the role that theories play in the construction of coherent categories (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). For example, dogs and cats go together in a category, but dogs and chairs do not. The argument is that the attributes dogs and cats have in common fit an encompassing category such as "pets" or "animals" that dogs and chairs do not. However, one could name many attributes that dogs and chairs have in common (found in houses, weigh less than 500 pounds, have four legs, etc.), but these attributes do not define a coherent category. Objects in a category may seem similar because they are in the same category and not because of the attributes that they have in common (Medin & Wattenmaker, 1984), i.e., a set of cups are similar because they are a category, but the attributes that they share do not necessarily define them. In order to account for the coherence of concepts, the coherent view of category construction requires that these concepts be linked by underlying principles that are common to the objects in the category (Murphy & Medin, 1985). Since experts classified problems by the principles used to solve the problems in Chi et al's (1981) study, it is suggested here that their categorization conforms to the coherent view of categorization rather than the probabilistic view in which the models are connected; that is, the models are connected not by their attributes, but by the principles used to solve the problems.

Another source from the literature that should be mentioned is the work by Reif and his colleagues (Eylon & Reif, 1984; Heller & Reif,

1984; Reif & Heller, 1982) who have done research on knowledge structure in physics and have developed a hierarchical model of knowledge to facilitate problem solving in physics. This structure is a prescriptive model (Heller & Reif, 1984; Reif & Heller, 1982), meaning that the model is not based on expert performance, but is based on how problems should be solved and how novice learners should learn to solve problems. However, the knowledge structure that Reif and colleagues present is a hierarchy of principles used to solve problems (Heller & Reif 1984; Reif & Heller, 1982). Studies using this hierarchical model of physics principles have been successful in facilitating problem solving in physics (Eylon & Reif, 1984). Although this is not a description of the knowledge structure that individuals have, it is supportive of the coherent view in which models are connected by the principles used to solve them.

The last weakness which both the structure that Giere (1994) proposes and the coherent view of categorization (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) share is that neither are supported entirely by the research findings. In particular, Chi et al's categorization study (1981) does not provide any support to the notion of a hierarchy. The subjects in Chi et al's study categorized the problems only once into only one set of piles. A hierarchical structure implies that the problems belong to more than one category where problems in larger categories have more general attributes in common and problems in smaller categories have more specific attributes in common. In order to

support the idea of a hierarchy, the subjects must be allowed to categorize problems more than once into bigger or smaller piles in order to determine which attributes are more general and which more specific.

The results of the previous research that novices use surface features to categorize problems (Chi et al, 1981) supports Giere's (1994) hierarchy of models and suggests that novices' knowledge is centered on attributes of models. However, the results that experts use theories to categorize problems (Chi et al, 1981) supports the coherent view of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) rather than the structure proposed by Giere. More specifically, the results with experts suggest that models are classified by the principles that are used to create them. Neither view, the coherent view nor Giere's view of the structure of knowledge can explain both the novices' and experts' categorizations.

There are two ways to rectify this situation. The first is to claim that the knowledge of experts and novices in physics are completely different with no relationship to each other; That is, novices' knowledge is entirely based on the objects in models and their attributes, as in Giere's (1994) representation and that experts' knowledge is entirely based on theories used to solve problems as in the coherent view of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996).

The second way to resolve this issue is to propose that knowledge structure in physics is a combination of these two views. That is, models may be structured in a hierarchy with more specific models at the bottom and more general models at the top as Giere (1994) suggests; This is supported by the results of novices' categorizations. These hierarchies are then clustered according to the theories used to create them as suggested by the coherent view of categorization (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) and supported by the experts' categorizations. This is the option hypothesized in the present study. More specifically, it is suggested here that both models and theories play a role in physics knowledge. Models created by the same principles or theory are clustered into groups that may have some structure. Borrowing from Giere's (1994) representation, these theory clusters may be arranged in a hierarchy of general to specific models; however, it is hypothesized that models do not build from one general model. In addition, it is hypothesized that these hierarchies of models are related to each other by specific principles used to solve the problems.

This research investigated the above mentioned structure for physics knowledge by employing a reiterative categorization task using experts, intermediates, and novices. Subjects were asked to categorize and recategorize the problems until they were satisfied with their representation of the problem set in terms of the similarities and differences in the problems. This resulted in a hierarchy of categories

in which categories with a large number of members have more general attributes in common than categories with a small number of cards. These categories were then classified as model-, theory-, or mathematical model-based categories in order to determine what type of categories are used at which levels of the hierarchy.

The objective of this study was to investigate the combined representation of clusters of models linked by theories in physics through a comparison of the categorizations of experts, intermediates, and novices. Specific hypotheses will be made in the Method chapter regarding a comparison of the types of categories used across all levels of the hierarchy in order to situate this study within previous literature on expert/novice differences in problem solving (Chi et al, 1981; 1982; Larkin, 1981; 1983; Larkin et al, 1980a, 1980b; de Jong & Ferguson-Hessler, 1986). Also, hypotheses will be made as to the comparison of the use of model-, theory-, and mathematical model-based categories by level of expertise using the proposed combined representation of physics knowledge. Finally, exploratory hypotheses will be made in terms of the use of model and theory sub-categories. Use of these attributes will be compared by level of categorization for experts, intermediates, and novices. This will provide a description of the specific categories used by experts, intermediates, and novices to define each level of their hierarchy.

CHAPTER III

METHOD

Subjects

The subjects for this study were novices, intermediates, and experts in physics. Novice subjects were students who had completed one semester of Classical Mechanics at the introductory level. All novices were recruited from the same college level physics course at a small midwestern liberal arts college. Students were recruited in their second term physics class with permission from the instructor. They were asked to participate, and it was explained that there would be no repercussions if they declined the invitation. Novice subjects were paid a small fee for their participation.

Intermediate subjects were first or second year graduate students who had completed a bachelor's degree in physics, but had not yet completed comprehensive examinations in physics. Intermediate subjects were drawn from a large midwestern research university and were contacted through e-mail. The e-mail addresses for the intermediate subjects were obtained from the Physics Graduate Student Organization president. A general message requesting volunteers was sent to the entire distribution list. Volunteers replied to the experimenter and dates and times for the interviews were arranged.

The expert subjects were university professors who had been involved in teaching and research in physics for at least 10 years. These experts were drawn from the same research university as the intermediate subjects. Experts were also contacted by e-mail through a distribution list compiled using the e-mail addresses of all faculty members in the Physics and Astronomy department. The same general message was sent to this list as was sent to the intermediates. Volunteers replied to the experimenter and dates and times for the interviews were arranged.

For all subjects, participation was confidential and anonymous. All subjects read and signed a release form for participation in the project. All protocols were given an identification code that identified the expertise level of the subject and a number (i.e., EX1 refers to the first expert interviewed). A list was not kept linking names or other information to these codes.

Materials

The materials consisted of 18 problems chosen from an intermediate level classical mechanics text (Marion & Thorton, 1988) printed on colored, laminated index cards (see Appendix B). The materials were chosen to represent typical physics problems in mechanics. Since the goal of this study was to investigate how subjects at different levels of expertise differed in their categorization of physics problems, the problems were chosen such that a coherent representation could be made using the model, mathematical model,

or theory used to solve the problems. The model and theory categories were chosen in order to situate this study within previous literature on expert/novice differences in problem solving (Chi et al, 1981; 1982; Larkin, 1981; 1983; Larkin et al, 1980a, 1980b; de Jong & Ferguson-Hessler, 1986). In this literature, experts and novices differed in their representations of problems in terms of the theories and models used in their solution. Experts used principles from a physical theory to represent the problems; novices used objects and attributes of models to represent the problems. The mathematical model category was chosen as a possible category for use by intermediates to classify problems. This category was included because the difference between intermediate and novice level mechanics problems is the mathematical sophistication and this may have been a concern for the intermediate level subjects.

The problems were chosen such that all could be solved using one of three theories: Newton s Laws, Conservation of Energy, or Conservation of Momentum. Also, all problems chosen involved two types of mathematical models, either differential equations or simultaneous equations, and involved forces proportional to a constant or to distance. Finally, all problems used only a few different kinds of objects in the models. The objects used were springs, pendulums, incline planes, pulleys, and projectiles. A complete representation of these problems using models, theories, or mathematical models are shown in Appendix D.

Task Analysis

In order to develop a reference frame upon which to base the coding scheme for the subjects' data, a task analysis was conducted. As stated in the preceding section, all the problems can be solved using a set of models, theories, and mathematical principles. The objective of this study was to describe the structure of knowledge in physics by comparing the categorization of subjects at different levels of expertise. The research questions stated at the end of Chapter I involve specifically comparing the use of model-, theory-, and mathematical model-based categories by expert, intermediate, and novice subjects at different levels of categorization. In order to accomplish this, an analysis of the problems in terms of possible categories that could be used to describe the problems was conducted by the researcher. These categories were then classified as either model-, theory-, or mathematical model-based. A table of these categories can be found in Appendix C.

The probable categories that subjects might use and their classification as either part of a theory, model, or mathematical model were determined by the experimenter in the following manner. First, the experimenter solved each problem by using the required theory and creating a model of the problem solution. The model was described using Chi's (1981) classification of surface features. The first surface feature classified was the object in the problem (M-OBJ), i.e., projectiles, pulleys, springs, or pendulums. This category includes both the name of the object in the problem and more

generalized names of that object (i.e., a ball could be more generally called a projectile). Next, the attributes of the object (M-ATT) described in the problem statement were listed. For example, an incline plane may have the attribute of having a frictionless surface. The unknown (M-UNK) in the problem was also listed directly from the problem statement, such as velocity or time. Finally, the motion (M-MOT) of the object in the problem was determined from information about the characteristics of the object and the situation in the problem statement. For example, a spring might be described as moving harmonically, moving up and down, or oscillating.

After the model characteristics were determined, the mathematical model (MTH-M) was determined from the researcher's problem solution. The name of the mathematical method used in the solution was entered along with the complexity of the force or energy used. Other solutions were considered and other usable mathematical models were recorded.

Finally, the researcher's solution of the problem was used to determine the theories that were used to solve the problem. The sub-categories in this class are described by Larkin's (1983) theories about expert problem solving. Experts in Larkin's study began by creating a physical representation or conceptual model by selecting an appropriate schema or theory for the problem and applying the construction and extension rules of that schema. Construction rules and extension rules in Larkin's (1983) theory are inferencing rules used primarily by experts. Construction rules act on the original

problem representation to produce causal agents such as specific forces, momenta, or energies (Larkin, 1983). According to Larkin, extension rules are used by experts to extend the causal model created from the application of construction rules to add new entities such as new equations of motion that facilitate problem solving. In summary, the theory-based attributes in this study are the name of the theory or schema used to solve the problems (THE), the construction rules (THE-CON), and the extension rules (THE-EXT) of that theory.

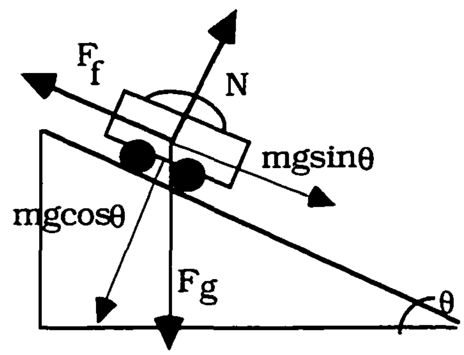
The problems were solved by the researcher through the application of Conservation of Energy, Conservation of Momentum, or Newton's Laws and the name of the theory (THE) used to solve each problem was recorded along with any other theories that could possibly be used. Next, the construction rules (THE-CON) and extension rules (THE-EXT) for that theory were listed for that problem. Construction rules (THE-CON) were the specific forces, energies, or momenta used in the problem solution such as spring force, tension, kinetic energy, or spring potential energy. The extension rules (THE-EXT) were primarily the equations of motion used to solve the problems.

An example of the physical representation for problem 16 as created by the researcher is shown in Table 1 along with the classifications used in that problem. The first row is the problem statement. The second row is the physical representation of that problem using Newton's Laws and the normal, frictional, and

Table 1

Classification of Problem Sixteen

16 An automobile driver traveling down an 8% grade slams on his brakes and skids 30 m before hitting a parked car. A lawyer hires an expert who measures the coefficient of kinetic friction between the tires and the road to be $\mu_k = 0.45$. Is the lawyer correct to accuse the driver of exceeding the 25 MPH speed limit? Explain.



$N = mg\cos\theta$	$v^2 = v_0^2 + 2ad$
$ma = \mu_k N - mg\sin\theta$	$v^2 = v_0^2 - 2dg(\mu_k \cos\theta - \sin\theta)$
$ma = \mu_k mg\cos\theta - mg\sin\theta$	v=29.96 MPH

Classification	Problem Attribute
M-OBJ	Car, Road, Incline, Surface, Slope
M-ATT	Coefficient of Friction, Angle
M-UNK	Velocity
M-MOT	Linear or Projectile Motion
MTH-M	Constant Force, F=c
THE	Newton's Laws
THE-CON	Gravitational, Normal, & Friction Force
THE-EXT	Equations of Motion

gravitational forces as construction rules. The third row is the final problem solution using the equation of motion ($v^2=v_0^2 + 2ad$) as an extension rule. Following this is the classification for this particular problem solution in which the object in the problem (M-OBJ) can be described as a car, road, incline, surface, or slope. The attributes of these objects (M-ATT) are the coefficient of friction and angle of the incline. The unknown (M-UNK) asked for in the problem is the velocity. The last model-based attribute is the motion of the object (M-MOT) which is linear or projectile motion. As described above, the name of the theory (THE) used here is Newton's Laws and the construction rules (THE-CON) were the normal, frictional, and gravitational forces. The extension rules (THE-EXT) were the equations of motion.

In order to determine the validity of the classifications of the attributes in the problems as model, theory, or mathematical model-based, the problems were given to a physics expert who did not participate in the study as a subject. The expert was also given Appendix C and the definitions of the theory, model, and mathematical model categories and their sub-categories. The expert was asked to read each problem and the categories as classified in the table for agreement. The percent agreement between the researcher and the expert was 97.8%. All disagreements were resolved by removing some categories from the table. For example, the category "center of mass" was classified as an extension rule in the original table. In a discussion with the expert, it was determined that this

category was more correctly classified as a definition. A definition was not a sub-category and therefore the "center of mass" category was deleted.

Procedure

Subjects were tested individually. Cards were randomly ordered and presented in a pile on a white piece of cardboard on a table in front of the subject. A video camera was placed on a tripod in front of the cardboard and was focused to include only the cards and the cardboard. It was explained that the task was part of a dissertation research project aimed at investigating physics knowledge at different levels of expertise. It was further explained that the camera would be focused on the white cardboard where the task would be performed and that objects and actions in that area would be recorded, but that their identities would not be recorded in any way. They were then asked to sign a consent form. The video camera was then turned on and subjects were given the following instructions:

> In front of you are randomly arranged cards. Each card has a physics problem written on it. Using your knowledge of physics, your task is to put the problems into groupings or categories based on their meaningful similarities. Once you have finished, please go back and make bigger piles or smaller piles until you have told me everything that you think is important to know about these problems.

The subjects then began reading through the cards one by one and putting the cards into piles. The subject was prompted, if necessary, to think aloud about the names of the categories. Once

the subjects had read all the cards, categorized them, and named their respective categories, they were prompted (if necessary) to make bigger or smaller categories if they felt it was necessary to understanding the problems. The subjects then either stopped at this point or combined piles or separated them. At the end of this pass, they were prompted again (if necessary) to make bigger or smaller categories. Again, the subjects either stopped or continued to recombine the cards. Other than this minimal communication between the experimenter and the subject, the experimenter limited herself to answering clarifying questions about the task and requesting explanations from the subject.

Protocol Analysis

An audio tape was made from the resulting videotapes of the task. These tapes were transcribed verbatim. Each subject was given a code (i.e., N1 for novice 1) and the experimenter was denoted by a "J" on the transcripts. The videotapes were viewed with the transcripts in order to add references to particular cards, the movement of cards into piles, removal of cards from piles, and references to particular piles.

The protocols were divided into segments reflecting each change in the piles. These changes included placing a card into a pile, removing a card from a pile, combining piles, or explaining the name of a pile if it had not already been explained. The first segment of each protocol contains the experimenter's explanation of the

instructions and is denoted by a 0.0. The task begins with segment 1.1 and continues until the subject has categorized all of the cards once. At that point, the following segment numbers begin with a 2.1 denoting that it is the second pass through the cards. An example of a protocol for expert 1 can be seen in Table 2.

Table 2

Protocol Segments 2.9 - 2.11 for Expert 1

Seg		Protocol	Action
2.9	EX1	simple harmonic oscillator	ref 4
		umm	
		consists of 100 g	
		at rest	
		mass is displaced	
		so that one has friction	ref 4,7,9,13,16
		so that's friction	ref 4
		uhhh.	
		that's got friction	ref 7
		two harmonic oscillators	
		with friction	ref 9
		umm	
		the block rests on a plane	ref 13
		the coefficient of friction	
		put these with these also	
		umm..	
		that's friction	ref 16
		these are all frictional kind of ones	ref 4,7,9,13,16
		so where are we now?	
		so we've got	
2.10	EX1	simple harmonic motion	ref 5,15,17,18
		small period	
2.11	EX1	stuff with friction	ref 4,7,9,13,16

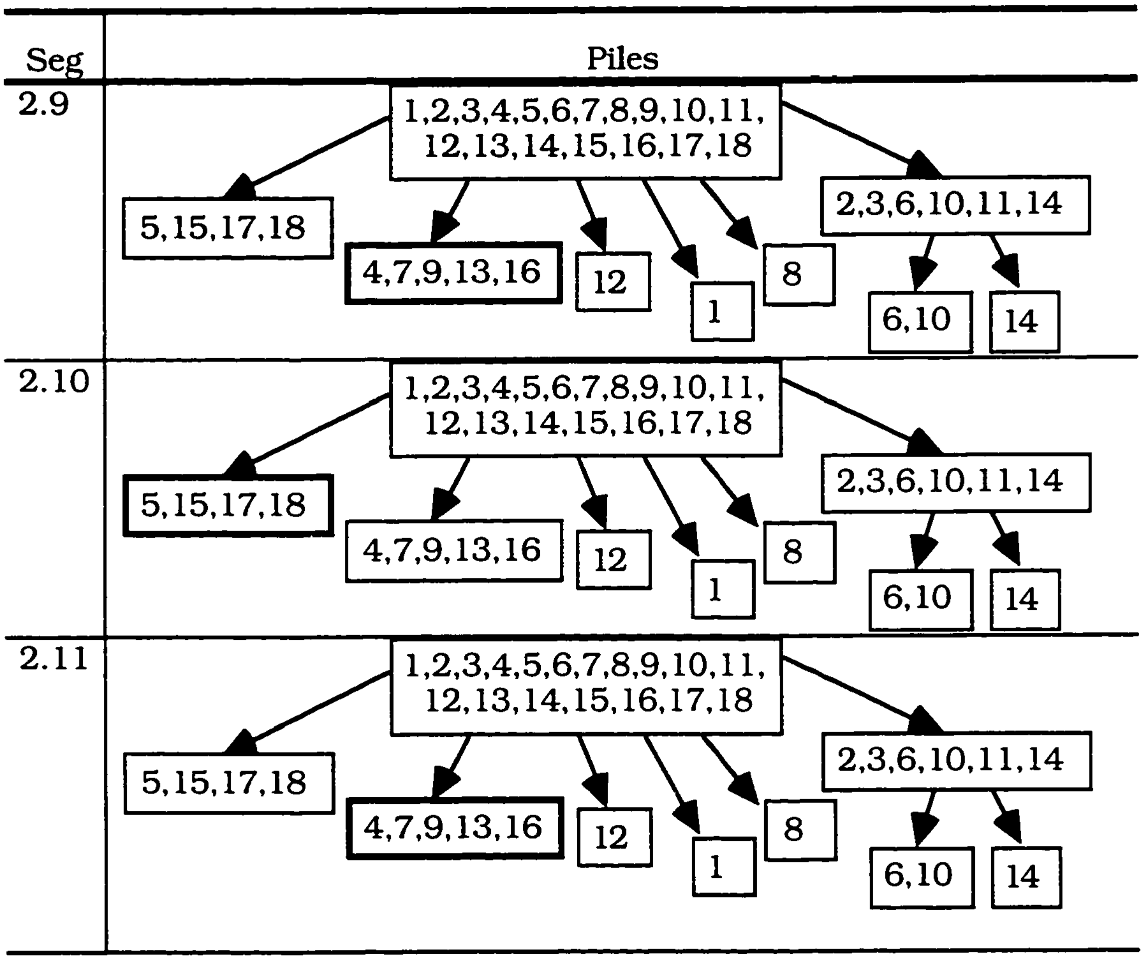
The protocol above contains three columns of information. The first column (Seg) indicates the segment number and the code of the person speaking. In this case, the subject speaking is indicated by the code EX1 (i.e., Expert 1). The second column (Protocol) includes the transcript of what was said during that segment. The third column (Action) describes the action that was taken during that segment or any references to any cards or piles; namely, placing a card with other cards, placing a card alone, removing a card, combining piles, or making a reference to a card or pile. In segment 2.9, Expert 1 begins by making a reference to card number four (ref 4). Similarly, segment 2.11 shows the expert making a reference to a pile that includes problems 4, 7, 9, 13, and 16.

A second set of information is attached to the end of each protocol and is again organized by segment numbers. This table provides a diagrammatic representation of the status of the piles during each segment. An example of this representation for the protocol for EX1 can be seen in Table 3. The diagrammatic representation consists of numbers surrounded by a box representing piles and their contents. Arrows were made between boxes to show that the contents of the pile in the lower box was once a part of the larger pile in the top box. Changes made during the segment are highlighted by showing the new card number in boldface and by a thickened perimeter of the box containing the new card. For example, in segment 2.10 the subject makes a reference to the pile including 5, 15, 17, and 18 (ref 5,15,17,18). In Segment 2.10 in Table 3, the box

that surrounds those problems is thicker than the others to indicate that it is being referred to by the subject. It can also be seen that it belongs to another pile that includes all the cards which is indicated by the arrow between the box with cards 5, 15, 17, and 18 and the box with all the cards listed.

Table 3

Diagrammatic for Segments 2.9-2.11 for Expert 1



A grid was then made for each subject. An example of part of the grid for EX1 is shown in Table 4. The grid is divided into three

columns to indicate the level (high, middle, or low) at which the cards were classified. In the task, the cards were sorted into increasingly larger or smaller piles as determined by the subject. These piles corresponded to a hierarchy that was arranged from general attributes to specific. For this reason, piles which had a general attribute in common were at a higher level than piles with more specific attributes. In other words, piles that were larger were more general and therefore at a higher level than piles that were smaller. The "High" level piles had the most members and therefore the attribute that these cards had in common was the most general. The "Low" level piles were the smallest and contained the fewest members. The attribute that these smaller number of cards had in common was more specific. The "Middle" level piles were larger than the "Low" level piles and the attribute that these cards shared was more general. The "Middle" level piles were smaller than the "High" level piles and therefore had more specific attributes in common.

In the data grids, the high, middle, and low level columns were divided into two sub-columns. An example of this is shown in Table 4. The larger of the two columns indicates the name given to the card at that level by the subject and the segment at which that card was classified into that level or when the category was given a name. For example, in segment 2.10 in Table 2, the pile that card 5 shares with cards 15,17, and 18 is called "Harmonic Motion" therefore, the name Harmonic Motion with the segment number 2.10 was recorded in the first column in the middle category. These names were then classified

in terms of the categories in Appendix C (model, theory, or mathematical model) and the code for the classification was recorded in the smaller of the two columns. For problem five, the name "Harmonic Motion" was found in Appendix C under the model sub-classification of the motion of the object and therefore the code M-MOT was recorded in the second column in the "Middle" category.

Table 4

Partial Data Grid for Expert 1

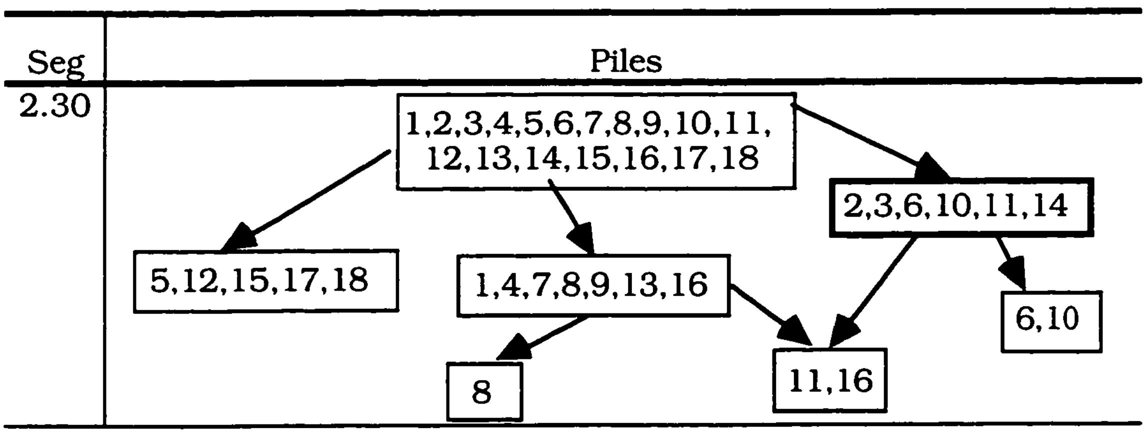
Subject	Expert 1					
Level Card #	High		Middle		Low	
5	Mechanics 1.3	THE	Harmonic Mot. 2.10	M-MOT		
6	Mechanics 1.3	THE	Kinematics 2.25	THE-EXT	Momentum 2.24	THE-CON
7	Mechanics 1.3	THE	Friction 2.9, 2,11	THE-CON		

The level of categorization was determined by inspecting the final diagrammatic representation constructed for each subject as described above. Most subjects made between two and three recombinations, meaning that they placed the card into two to three increasing larger piles. The number of levels was determined by

counting the number of arrows for a particular card. For example, the final diagrammatic representation for Expert 1 is shown in Table 5. In the representation in segment 2.11, card number six belongs to three different piles and the number six appears in three different boxes. Therefore, the number of levels for card six is three. In segment 2.11 in Table 5, card number six belongs to a high level pile that included all the cards and also belongs to a middle level pile that also included cards 2, 3, 10, 11, and 14. Finally, card number six belongs to a small, low level pile with card ten only. In the data grid in Table 4, card six has entries in the high, middle, and low level columns that refer to the names and segments where it was placed into the large, middle, and small piles referred to above.

Table 5

Final Diagrammatic Representation for Expert 1



The second task was to determine if the levels were high, middle, or low. As discussed above, it is expected that cards that belong to large piles have very general characteristics in common with

the other cards in that pile, and therefore it likely includes the most general attributes in the hierarchy. As the piles get smaller, the characteristics that the cards share become less general and more specific; a pile in the low level is the smallest, and therefore it likely includes the most specific features. In this research, it was necessary to compare these classifications at each level to determine which features of the problem are used to define general or specific characteristics with respect to expertise. In order to compare subjects' respective categorizations, a rule was developed to classify high, middle, and low levels. The rule stated that levels would be classified from low to high, unless the subject had a category that included the entire set of cards, then classification would be classified from high to low.

The rationale for the low to high rule stems from Rosch's (1978) work on categorization. Briefly, this theory holds that knowledge is categorized hierarchically with more specific attributes at the bottom of the hierarchy and more general ones at the top. In the middle there exists a "basic" level that is neither the most general nor the most specific. In Chi et al's (1982) study of hierarchical categorization, the novices were unable to make high level categories that the experts were able to make. It was hypothesized that the novices in Chi et al's (1981) study were working at Rosch's basic level and were unable to make higher levels because they had not learned to abstract the attributes of the members of the basic level (Chi et al, 1982). These data suggest that learning in this domain may require learning to

abstract the attributes and create higher level categories with more general attributes; therefore, the learning process starts from the most specific categories to the more general. In the present study, in order to show this progression from more specific categories to more general and compare the categorizations of subjects with different levels of expertise, the piles were ordered from low to high or from specific to general following the way in which they are perhaps learned.

The second part of the rule states that if subjects have a category that includes all the cards then the piles are ordered from high to low. This rule was needed in order to compare experts who had only two levels of categorization, but whose highest level included all the cards to novices who also had two levels, but whose piles were smaller. The assumption is that subjects created piles from the most specific attributes to the most general or low to high. However, a pile that included all the cards was necessarily at the highest level of abstraction, even if there are only two levels in the categorization. Therefore, the attribute which all the cards shared must have been the most general attribute for the set and must be compared to other subjects' high level categories. The second part of the convention attempts to assure that all piles that are high level piles are the most abstract. An example of this can be found in the representation for Expert 1 found in Table 5. Expert 1 categorized cards 5, 12, 15, 17, and 18 into only two levels, but the highest level included all the cards. The grid for Expert 1 for card five in Table 4 shows the card classified into two levels; high and middle. In the high level of

categorization, the card was placed into a pile with all of the cards. Again, this was the most general pile to which the card can belong. The middle level was the next smaller pile and included cards 12,15, 17, and 18.

Once the level for each card was determined for each subject, the protocol was searched for the segment in which that card or pile was named or classified at that level. For example, the partial data grid in Table 4 shows the data for problems five, six, and seven. In the final representation for Expert 1 in Table 5, problem seven was categorized at two levels. To find the segment in which the second level was named, the diagrammatic representations were searched to see which point in the protocol the pile with problem seven was named. In Table 3, the box that surrounded the pile at the middle level with problem seven was thickened in Segments 2.9 and 2.11. The Segment numbers 2.9 and 2.11 were then recorded in the grid for the middle level for problem seven. Next, the segment was read to determine the name of that pile. The name indicated for that pile was then recorded in the same column with the segment number. In Table 2, the protocol for segments 2.9 and 2.11 are shown. The name of the pile in which problem seven was a member was read in those segments. Segment 2.9 was more lengthy and indicated that the subject was checking all the cards to make sure that they belong to the group, "Friction". In segment 2.11, the subject again gave the group the name, "Friction".

Finally, the name that the subject gave was coded in terms of the matrix in Appendix C. That is, each name was classified as either model-based (M), mathematical model-based (MTH-M), theory-based (THE), or other (OTH). The model-based categories are sub-classified into the object in the model (M-OBJ), the attribute of that object (M-ATT), the unknown in the problem (M-UNK), or the motion of the object (M-MOT). The theory-based classifications also include the construction rules (THE-CON) and extension-rules (THE-EXT). For example, for problem number five in Table 4, the name "Harmonic Motion" was coded as a model-based attribute referring to the motion of the object (M-OBJ).

In order to establish reliability for this coding scheme, a second coder unfamiliar with the study was given approximately 30% (eight protocols) to analyze. A random number generator in Microsoft Excel™ for the Macintosh (Grey Matter International, Inc., 1994) was used to select the eight protocols. The coder was not given information regarding the expertise level of the subjects; all identifying marks on the protocols were also removed. The coder was given written instructions that separated the analysis task into three parts: finding the level, finding the names, and classifying the names. Table 6 shows the summary provided for each task.

The data grids that the coder created were compared to the data grids of the researcher. A percent agreement was obtained which compared the number of codings that were identical between the researcher and the second coder to the total number of codes used.

The inter-rater reliability obtained was 91% for agreement between the researcher and the coder.

Data Analysis

The model-, mathematical model-, and theory-based classifications from the grids were given numerical codes and entered into the spreadsheet program Microsoft Excel™ for the Macintosh (Grey Matter International, Inc., 1994). The numerical codes for the model-based attributes began with a one and the sub-categories were ordered according to the table in Appendix C. The object in the problem attribute (M-OBJ) was coded as 11, Attributes (M-ATT) as 12, unknown (M-UNK) as 13, and motion of the object (M-MOT) as 14. The mathematical model-based attributes (MTH-M) were coded as 20. The theory-based categories were numbered from 30-32 with the name of the theory (THE) as code 30, the construction rules (THE-CON) as code 31, and the extension rules (THE-EXT) as code 32. Finally, any categories that did not fit these classifications were given the code 40 to indicate that they belong to the Other (OTH) category. Summary data for each subject was calculated by the program.

There are three points of interest in this study. The first is a comparison of the general categories that the subjects used to classify their problems pooling over all levels i.e., high, middle, and low. This measure ignores the level at which the problem was categorized. At this point, there is no interest in the hierarchical nature of the subjects' categorizations. To find the general categories used,

proportions were calculated for each of the categories that subjects made that are classified as theory-based (THE), model-based (M), mathematical model-based (MTH-M), or other (OTH) over the total number of classifications made by each subject. These proportions were used to demonstrate which categories were more likely to be used by which group at all levels.

Table 6

Summary of Instructions for Coder

To Find the Level:	
A.	Look at the Final Representation to determine how many levels there are by counting the arrows.
B.	Place the cards into levels from the smallest pile to the largest UNLESS there is a category that has all of the cards in it. THEN place cards into categories from the largest to the smallest.
To Find the Names:	
A.	Look for a card in the final representation.
B.	Look for a hilighted box around the category that includes the card that you are looking for.
C.	Read the protocol for the segment number that has the hilighted box and jot down a summary name in the appropriate box in the data grid.
D.	Jot down the segment number you found the name in into the appropriate box in the data grid.
To classify the names:	
A.	Look in Appendix C for the card number in question and look for the name that you are trying to classify.
B.	Jot down the code for the classification that the name is in.

Next, the hierarchical nature of the data was addressed. The data was separated by the level at which the category was placed and classified as either model- (M), theory- (THE), mathematical model-based (MTH-M), or other (OTH). These values show the proportion of categories classified as theory-based, model-based, mathematical model-based, or other over the total number of categories made at each level. Inspecting these proportions provided information concerning which categories subjects used to create the different levels of their respective hierarchies.

Finally, all of the proportions were broken down by level and by individual sub-classifications. Proportions of the use of theory (THE), construction rule (THE-CON), and extension rule (THE-EXT) categories over all categories at each level were calculated. Similarly, calculations of the proportion of the use of the object in the problem (M-OBJ), the attributes of that object (M-ATT), the unknown in the problem (M-UNK), and the motion of the object (M-MOT) categories were calculated over all the classifications at each level. Proportions from this data were used to compare the sub-categories used by subjects of various levels of expertise at each level of categorization.

Method of Statistical Analyses

The independent variable used in this analysis was the level of expertise (expert, intermediate, or novice). Level of expertise was the only true independent variable since it was the only variable chosen by the researcher. The dependent variables were: classification

(theory-, model-, or mathematical model-based), theory sub-classification (theory name (THE), construction rules (THE-CON), or extension rules (THE-EXT)), and model sub-classification (object (M-OBJ), attribute (M-ATT), unknown (M-UNK), or motion (M-MOT)). These were dependent variables because they were chosen to vary with expertise and with level of categorization (high, middle, or low). Level of categorization was a stranger type of variable because its relationship to expertise is not clear. Since the researcher did not select the number of levels of categorization, it was not an independent variable like level of expertise. Therefore, in comparing the use of categories by level of expertise, the level of categorization was held constant and comparisons were made within each level of categorization. In this way, expertise was an independent variable at each level of categorization and classification was a dependent variable. However, in the present study, it was necessary to make comparisons of the sub-categories that each group created at each level of categorization. Since each subject chose the number of levels of categorization, for each subject, with expertise held constant, level of categorization was used as an independent variable.

In particular, this analysis answered the following general questions:

1. How do the classifications that experts, intermediates, and novices use compare across levels of categorization (i.e., pooling over levels of categorization)?

2. How do the classifications that experts, intermediates, and novices use compare by level of classification?

3. Which of the theory and model sub-categories do experts, intermediates, and novices use at which levels of categorization?

The order of the statistical analyses follows the order of the three questions above. That is, the statistical analyses begin with analyses that provide information on the general classifications used to categorize the problems across all levels of categorization and then progress to more specific information on the sub-classifications used by expert, intermediate, and novice subjects at different levels of categorization. The analyses begin with those that are predicted by the most existing literature and then progress with analyses that are more exploratory in nature.

The analyses begin with a comparison between the proportion of model-, theory-, and mathematical model-based categories over all categories made by experts, intermediates, and novices pooling over all levels of categorization. This provides a very general picture of the categories that subjects used for all the cards and how those categories differed by level of expertise. The second set of analyses took into account the level at which the cards were categorized. This set of analyses compares the categories made by experts, intermediates, and novices at each level of categorization. This analysis provides information on how experts, intermediates, and novices differed at each level of categorization in terms of the proportion of model-, theory-, and mathematical model-based

categories over all categories used at each level of categorization. The last set of analyses are the most specific. In this set of analyses, the theory- and model-based categories are separated into their specific sub-categories and then compared by level of categorization for experts, intermediates, and novices.

The following sections provide more specific information on the analyses conducted and the hypotheses tested.

Analysis of Categories Across All Levels

The first set of hypotheses involved the use of categories by subjects pooling over all levels of their categorization. This analysis provides a general description of the types of categories used by experts, intermediates, and novices. This is replication of the work of Chi (1981;1982) and Larkin (1983) who found that novice subjects attended to the surface features of a problem, whereas experts used the principles needed to solve the problems for their categorizations. In the present study, it is expected that the majority of categories that novices made are classified as model-based (M) attributes such as the object in the problem and that experts' categories are theory-based (THE) such as principles used to solve the problems. Due to the lack of research on intermediates' categorization, no literature-based hypotheses can be made for these subjects. However, it is intuitively plausible that the intermediates used mathematical model-based (M-MTH) categories since the difference between Introductory and Advanced Mechanics problems is the mathematical methods involved.

In summary, it was expected that intermediates would focus on the mathematical model-based attributes, such as the type of mathematical methods that were causing them the most difficulties.

A multivariate analyses of variance was used to find differences between levels of expertise and use of the different types of classifications.

Analysis 1: Analysis of Variance - Theory-, Model-, Mathematical Model-based Classifications

Between groups factors: Level of expertise (novice, intermediate, or expert, n=9)

Within-subjects factors: Proportions of frequencies of theory-, model-, and mathematical model-based classifications over all classifications made at all levels.

Analysis of Categories Separated by Level

The hierarchical nature of the classifications was investigated by comparing categorizations by experts, intermediates, and novices at each level of categorization. In this analysis, categories classified at higher levels are more general and indicate a deep understanding of the domain of physics and categories at the low level of categorization are more specific. Chi's (1982) comparison between novice and expert hierarchical categorization predicts that at the high level of categorization, experts will use more theory-based categories than novices. Conversely, at the low level of categorization, novices will use

more model-based categories. However, Giere's (1994) organization predicts that model-based categories are used at all levels of categorization. As previously discussed, the hypotheses put forth in the present research rely more heavily on the work of Chi (1982) rather than on Giere's (1994) organization of models. Thus, in terms of hypotheses, at the high and middle levels, experts are expected to use more categories classified as theory-based than novices; this is in accordance with Chi's (1982) results. Novices are expected to use model-based categories more than experts. Again, literature-based predictions can not be made for intermediates due to a lack of relevant literature, however they were expected to use categories classified as mathematical model-based more than experts and novices since this is the feature of the problems to which they were most likely attending in their level of study. At the lowest level (the most specific level of categorization), the categorizations of experts and novices were not expected to be distinct and all groups were expected to use categories classified as model-based in accordance with Giere's (1994) hierarchy of models.

Three multivariate analyses of variance were used to find differences between levels of expertise and use of the different types of classifications at each level of categorization.

Analyses 2-4: Analysis of Variance - Theory-, Model-, Mathematical Model-based Classifications by Level

Between groups factors: Level of expertise (novice, intermediate or expert, n=9)

Within-subjects factors: Proportions of frequencies of theory-, model-, and mathematical model-based classifications for each of the high, middle, and low levels.

Analysis of Sub-classifications Separated by Level

Knowing about the general character of the classifications made provides only a general description of the knowledge structure of experts, intermediates, and novices. For example, it was expected that the novices would classify all cards using model-based classifications at all levels. However, the sub-classifications of these model-based categories were expected to have been different across each level of categorization. To further delineate the hierarchical structure of the data, investigations of the use of sub-classifications were necessary. These investigations were exploratory in the sense that there is no experimental literature upon which to predict which sub-category a subject would use at which level of categorization. To the author's knowledge, this study is the first of its kind in that it seeks to determine the categories that subjects use in terms of more specific sub-categories.

Since, it was expected that experts would use theory-based classifications at the high and middle levels, it was also expected that they would use more general theory sub-categories at the high level and more specific theory sub-categories at lower levels of categorization. The most general theory sub-category is the theory (THE) used to solve the problems. This is based on Larkin's (1983)

research which found that the first step experts completed in solving physics problems was to chose a theory in order to create a physical representation or conceptual model of the problems. It was assumed that the same theories would be used to solve many of the problems and therefore experts were expected to use this category at the high (most general) level of categorization. It was also expected that experts would use construction rules (THE-CON) at higher levels than extension rules (THE-EXT) since the application of construction rules precedes the application of extension rules in problem solving (Larkin, 1983). Extension rules are used to extend the causal model that has been created by the construction rules. Therefore, it was expected that the use of construction rules would precede the use of extension rules. It was also expected that the experts' structure would not be distinct from the novices such that the experts would use some model-based sub-categories. However, it was expected that experts would use only the most general of the model sub-categories since their understanding of the problems is more abstract.

Analysis 5: Analysis of Variance - Theory-based Sub-classifications by Experts

Between groups factors: Level of categorization (high, middle, or low, n=9)

Within-subjects factors: Proportions of frequencies of theory name, construction rule, and extension rule classifications over the total number of theory-based classifications made by experts.

Analysis 6: Analysis of Variance - Model-based Sub-Classifications by Experts

Between groups factors: Level of categorization (high, middle, or low, n=9)

Within-subjects factors: Proportions of frequencies of object, attribute, unknown, and motion model sub-classifications over the total number of model-based classifications made by experts.

Intermediates were not expected to have a completely distinct categorization from either novices or experts since intermediates are in transition from novice to expert. Therefore, it was expected that they would use a mixture of the mathematical model, model, and theory-based classifications. At low levels, they were expected to use model-based classifications similar to novices. At higher levels, the intermediates were expected to use theory-based categories in a manner similar to the experts. However, it was expected that intermediates should use mathematical model-based classifications at all levels of categorization to some extent. Again, an investigation of the sub-classifications was carried out to delineate these differences.

Analysis 7: Analysis of Variance - Theory-based Sub-classifications by Intermediates

Between groups factors: Level of categorization (high, middle, or low, n=9)

Within-subjects factors: Proportions of frequencies of theory name, construction rule, and extension rule classifications over all theory-based classifications made by intermediates.

Analysis 8: Analysis of Variance - Model-based Sub-classifications by Intermediates

Between groups factors: Level of categorization (high, middle, or low, n=9)

Within-subjects factors: Proportions of frequencies of object, attribute, unknown, and motion model sub-classifications over all model-based classifications made by intermediates.

Novices were not expected to use categories classified as theory-based to a significant extent at any level. This expectation is in accordance with previous research (Chi et al, 1981;1982; Larkin, 1983) in which novices used only surface features of the problem to categorize and represent problems. For this reason, an analysis of the theory-sub-classifications was not performed. However, from results of studies by Chi et al and Larkin, novices were expected to use a variety of model-based sub-classifications or surface features of the problems to categorize problems. In accordance with Giere's (1994) organization of models, it was expected that categories would be used from more general to more specific and that novices would use more general model-based sub-categories at higher levels of categorization and more specific sub-categories at lower levels of categorization. The most general model-based sub-category is the motion of the object (M-MOT). Many objects may have the same motion and thus, the motion

of the object is a general category for models. It is then predicted that novice subjects would use motion (M-MOT) classifications at the highest levels of categorization. The object in the problem (M-OBJ) is a more specific category than the motion (M-MOT) which would probably be used at lower levels. Finally, the attribute of the object (M-ATT) and the unknown in the problem (M-UNK) are the most specific of the model-based sub-categories and it was expected that these would be used at the lowest levels of categorization.

Analysis 9: Analysis of Variance - Model-based Sub-classifications by Novices

Between groups factors: Level of categorization (high, middle or low, n=9)

Within-subjects factors: Proportions of frequencies of object, attribute, unknown, and motion model sub-classifications over all model-based classifications made by novices.

A summary of the hypotheses are as follows:

1. Pooling across all levels of categorization, experts were expected to use theory-based (THE) categories, novices were expected to use model-based (M) categories, and intermediates were expected to use mathematical model-based (MTH-M) categories.

2. Expert, novice, and intermediate classifications were not expected to be distinct and some overlap was expected at the low level of categorization. At the high and middle levels of categorization, experts were expected to use theory-based (THE) categories; novices were expected to use model-based (M) categories; and intermediates

were expected to use a mix of theory- (THE) and mathematical model-based (MTH-M) categories. At the low level of categorization, experts, intermediates, and novices were expected to use model-based (M) categories.

3a. Experts were expected to use the most general theory-based sub-category which is the theory used to solve the problem (THE) at the high level of categorization. At lower levels of categorization, experts were expected to use the construction rule (THE-CON) sub-category before the extension rules (THE-EXT). At the low level of categorization, experts were expected to use the most general model-based category, motion of the object (M-MOT).

3b. Intermediates were expected to use mathematical model-based (MTH-M) categories at all levels of categorization. Intermediates were also expected to use some theory- and model-based categories at high and low levels of categorization.

3c. Novices were expected to use model-based (M) categories at all levels of categorization. Novices were also expected to use more general model-based (M) categories at higher levels, such as the motion of the object (M-MOT) or object in the problem (M-OBJ) and more specific categories at lower levels, such as the unknown in the problem (M-UNK) or the attributes of the objects (M-ATT).

CHAPTER IV

RESULTS

The purpose of this research was to investigate the knowledge structure in physics of individuals with different levels of expertise. Towards this end, individuals at different levels of expertise were given a categorization task as described in the preceding chapter. This task required the subjects to categorize a set of physics problems into as many categories as they needed in order to describe the similarities and differences between the problems. The task also allowed the subjects to re-categorize the cards into larger and smaller piles until they felt that they had given all the information about the problems that they thought was important in terms of their knowledge of physics. Further, they were asked to give these piles names to indicate what attributes the cards in the piles had in common. These names were then classified as either a model-, theory-, or mathematical model-based attributes (See Appendix C for a table of classifications). In order to describe the knowledge structure of experts, intermediates, and novices more specifically, the model- and theory-based attributes were sub-classified into the following categories: the object in the problem (M-OBJ), the attributes of the object (M-ATT), the unknown in the problem (M-UNK), the motion of the object (M-MOT), the construction rules (THE-CON) used to create the physical representation of the problem (i.e., gravity, friction force,

normal force, etc.), and the extension rules (THE-EXT) used to extend the physical model to add new entities that facilitate problem solving (i.e., equations of motion). Acronyms beginning with "M" are sub-categories of the model classification and those beginning with "THE" are sub-categories of the theory classification. Briefly, these categories were chosen based on a review of the research on problem representation and categorization (Chi et al, 1981; 1982; Larkin, 1981; 1983; Larkin et al, 1980a; 1980b; de Jong & Ferguson-Hessler, 1986). The classification of possible categories for each problem was created by the researcher through a task analysis described in the preceding chapter.

The tapes of the interviews were transcribed verbatim and protocols were divided into segments reflecting a single change in the piles. These changes included placing a card into a pile, removing a card from a pile, combining piles, or explaining the name of a pile if it had not already been explained. A diagrammatic representation of the status of the piles during each segment was also created from the protocols.

The data for this research consists of a grid for each subject which displays the classification for each card as either model-, theory-, or mathematical model-based and the level at which the card was categorized (This table can be seen in Appendix C). The level refers to the size of the pile or the generality of the attribute that the cards in that pile have in common. More general attributes were assigned to piles that were larger and thus at a higher level. Smaller

piles had more specific attributes in common and conversely are at the low level (for a justification of how a larger number of cards is equivalent to a higher level and how a smaller number of cards is equivalent to a lower level see p. 68).

Finally, proportions were calculated for each of the model-, theory-, and mathematical model-based classifications over both the total number of classifications as well as the number of classifications at each level. Further, proportions of the theory- and model-based sub-classifications of the number over the total number of theory- or model-based classifications at each level were calculated. These proportions comprise the data for these analyses.

The analyses begin with those that provide information on the general differences between experts, intermediates, and novices in terms of the model, theory, and mathematical model categories pooled over all levels of categorization. The second analysis compares the proportions of categories classified as either model-, theory-, or mathematical model-based over the total number of classifications made by experts, intermediates, and novices at each level of categorization. Next, proportions of model and theory sub-classifications over the total number of classifications made by experts, intermediates, and novices are compared by each level of categorization in order to more fully describe the categories used by subjects at different levels of categorization.

Statistical analyses of these data were conducted using SPSS™ for the Macintosh (Language Systems Corp., 1995). The results are

presented in the following order: the first section presents results on the classification of problems across all levels; the next section presents results on the analysis of the classification of problems by level of categorization; in the final sections, results are presented for the analysis of the theory and model sub-classifications, respectively.

Analysis of Classifications Across All Levels

The first analysis compared classifications made by subjects pooling over all levels of categorization. In this analysis, proportions of model-, theory-, and mathematical model-based classifications over the total number of classifications were calculated for each subject. The mean proportions for each classification type for experts, intermediates, and novices were calculated and graphed in Figure 4. This measure shows the categories used by subjects of varying levels of expertise in order to classify the problems. A multivariate analysis was conducted in order to determine differences in the use of theory-, model-, and mathematical model-based by subjects at different levels of expertise. In this analysis, level of expertise was used as the independent variable and theory-, model-, and mathematical model-based classifications as the dependent variables. The summary table of these results can be found in Table E.1 in Appendix E.

The multivariate test was significant ($F=4.48$, $p=0.001$) indicating that there was a difference in the mean proportions of theory, model, and mathematical model attributes used by experts, intermediates, and novices. Next, analyses of variance were

conducted by level of expertise for each of the dependent variables, i.e., model-, theory-, and mathematical model-based classifications. This was done in order to determine where the significant differences in the multivariate result lie. The results of these tests can be found in Table E.1 in Appendix E.

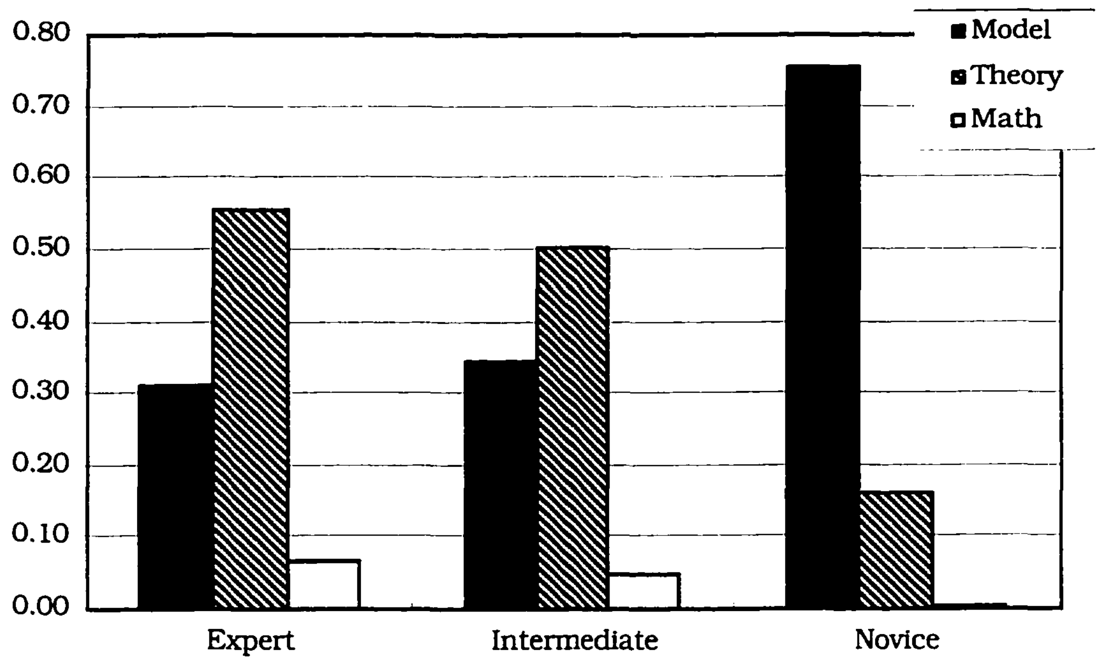


Figure 4. Classification Across All Levels.

The mathematical model classification was rarely used by any subjects and use was not significantly different across expertise (X_E=0.07, X_I=0.05, X_N=0.00, $\underline{F}$=1.03, $\underline{p}$=0.373, ns). However, significant differences were found on the ANOVA for proportions of theory-based classifications across the three levels of expertise (X_E=0.56, X_I=0.50, X_N=0.16, $\underline{F}$=11.22, $\underline{p}$=0.000). To determine which of the three groups were significant from each other, a Tukey-HSD

test with significance level at 0.05 was conducted. Here it was found that the proportions of theory-based classifications made by intermediates and experts was significantly greater than that of novices. No other differences were found. These results are summarized in Table 7.

Table 7

Theory-based Classifications Across All Levels

Mean		Expert	Intermediate	Novice
0.5556	Expert			
0.5044	Intermediate			
0.1622	Novice	*	*	

*$p<0.05$

The analysis of variance for the proportions of categories classified as model-based over all the categories used by experts, intermediates, and novices was also significant ($X_E=0.31$, $X_I=0.34$, $X_N=0.75$, $F=17.19$, $p=0.000$). A Tukey-HSD test with significance level at 0.05 was run to determine which groups were significantly different from each other. Here it was found that novices used a significantly larger proportion of model-based categories than did either experts or intermediates. No other differences were found. These results are summarized in Table 8.

Table 8

Model-based Classifications Across All Levels

Mean		Expert	Intermediate	Novice
0.3111	Expert			
0.3444	Intermediate			
0.7556	Novice	*	*	

*$p<0.05$

Analysis of Classifications Separated by Level of Categorization

Next, the hierarchical nature of the data was addressed. These analyses were conducted to provide information about how the categories used by experts, intermediates, and novices differed at each level of categorization, namely, high, middle, and low. Proportions of theory-, model-, and mathematical model-based classifications over the total number of classifications at each level of categorization were calculated and compared for experts, intermediates, and novices. The results of the categorizations by subjects at each of these levels will be addressed in turn.

Analysis of Classifications at the High Level of Categorization

The first analysis compared the proportions of the model-, theory-, and mathematical model-based categories over the total number of classifications at the high level of categorization made by

experts, intermediates, and novices as depicted in Figure 5. As this graph demonstrates, all of the classifications that were made at the high, i.e., most general level, were theory-based (X_E=0.76, X_I=0.44, X_N=0.00). There were no model- or mathematical model-based classifications made at this level (X_M=0.00, X_{MM}=0.00). A multivariate analysis was not appropriate for this data since there was only one dependent variable, i.e., proportion of theory-based classifications. Instead, an analysis of the variance was conducted with proportions of theory-based classifications over the total number of classifications at the high level of categorization as the dependent variable and level of expertise as the independent variable. This analysis was significant for the use of theory-based classifications at the high level of categorization by expert, intermediate, and novice subjects (X_E=0.76, X_I=0.44, X_N=0.00, F=8.45, p=0.000). A summary of this analysis is found in Table E.2 in Appendix E.

The analysis of the variance was significant which indicates a statistically significant difference between the proportion of categories classified as theory-based at the high level of categorization made by experts, intermediates, and novices. To determine which groups are statistically different, a Tukey-HSD test with significance level at 0.05 was executed and no significant differences were found between experts and intermediates on use of theory-based classifications at this level. However, it was found that proportions of categories classified as theory-based were greater at the high level of

categorization for experts than for novices. These results are summarized in Table 9.

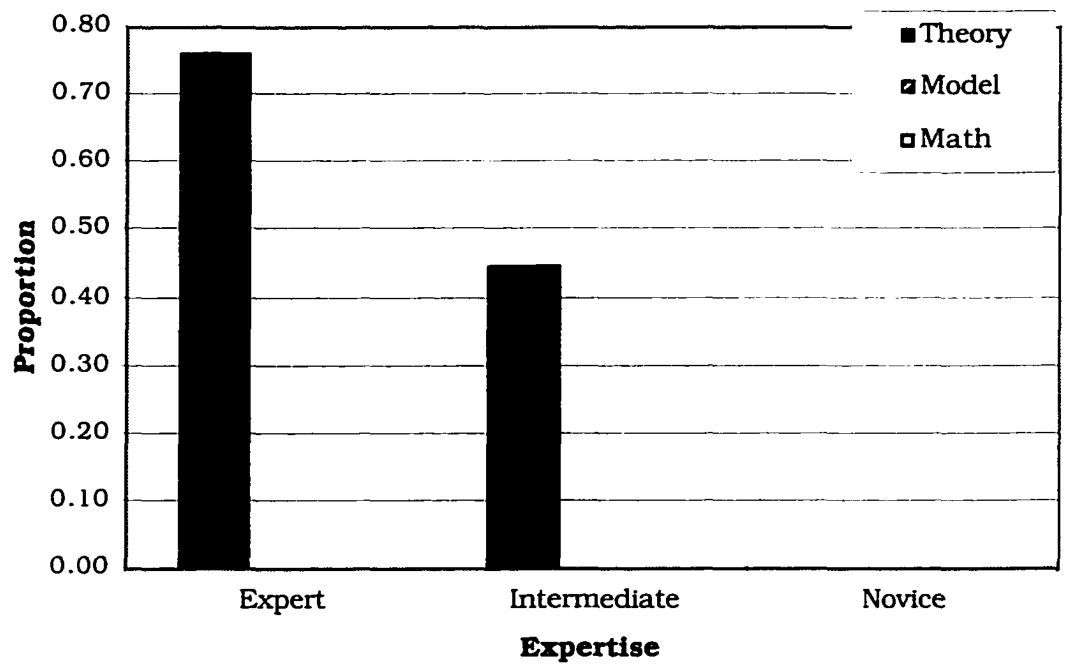


Figure 5. Classification at the High Level of Categorization.

Table 9

Theory-based Proportions at High Level

Mean		Expert	Intermediate	Novice
0.7578	Expert			
0.4444	Intermediate			
0.0000	Novice	*		

*p<0.05

Analysis of Classifications at the Middle Level of Categorization

The next analysis compared the proportions of categories classified as theory-, model, and mathematical model-based over the total number of classifications at the middle level of categorization made by experts, intermediates, and novices. The mean proportions of model-, theory-, and mathematical model-based classifications over all classifications made at the middle level are shown for each level of expertise in Figure 6. An overall multivariate analysis of variance was conducted with level of expertise as the independent variable and theory-, model-, and mathematical model-based classifications as the dependent variables in order to determine if there were significant differences between the proportions of categories used by subjects at different levels of expertise. The results can be found in Table E.3 in Appendix E. Tests of significance for this analysis were conducted and were significant ($F = 2.47$, $p = 0.038$) indicating a difference in the proportions of categories classified as theory-, model-, or mathematical model-based used by expert, intermediate, and novice subjects.

In order to determine which of the dependent variables were significantly different for expert, intermediate, and novice subjects, an analysis of variance was conducted for each of the dependent variables; namely, model-, theory-, and mathematical model-based classifications with level of expertise as the independent variable. The results of these tests can be found in Table E.3 in Appendix E. The use of model-based classifications was found to be significant at the

middle level categorization between experts, intermediates, and novices (X_E=0.44, X_I=0.37, X_N=0.79, F=5.22, p=0.013). To determine which of the three groups were significant from each other, a Tukey-HSD test with significance level at 0.05 was conducted with level of expertise as the independent variable and the proportion of model-based classifications as the dependent variable. Significant differences were found between intermediates and novices for model-based classifications indicating that proportions of model-based classifications were significantly greater for novices than for intermediates. No other significant differences were found for the proportion of model-based attributes used by experts, intermediates, and novices. These results are summarized in Table 10.

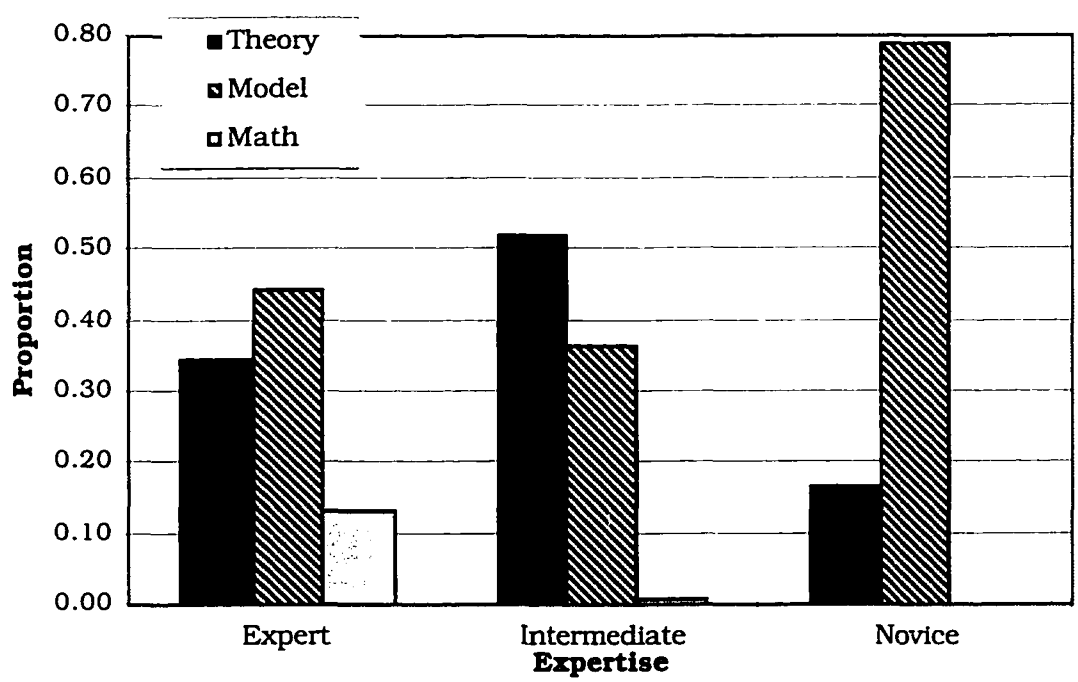


Figure 6. Classification at Middle Level of Categorization.

The analysis of variance using proportions of theory-based categories over the total number of categories at the middle level as the dependent variable and expertise as an independent variable was also found to be significant (X_E=0.34, X_I=0.52, X_N=0.17, F=4.11, p=0.029) indicating a difference in the use of theory-based classifications between experts, intermediates, and novices. To determine which of the three groups were different, a Tukey-HSD test with significance level at 0.05 was executed. No differences were found between experts and intermediates on use of theory-based classifications at this level; however, significant differences were found between intermediates and novices for theory-based classifications indicating that intermediates used theory-based classifications significantly more than novices. These results are summarized in Table 11.

Table 10

Model-based Proportions at Middle Level

Mean		Expert	Intermediate	Novice
0.4456	Expert			
0.3656	Intermediate			
0.7889	Novice		*	

*p<0.05

The analysis of variance using proportions of mathematical model-based categories over the total number of categories at the

middle level as a dependent variable and expertise as an independent variable was found to be not significant ($X_E=0.13$, $X_I=0.01$, $X_N=0.00$, $F=2.91$, $p=0.07$, *ns*) indicating no differences in the use of mathematical model-based classifications between experts, intermediates, and novices. However, the result of this test was very close to the 0.05 significance level and it might be expected that if more subjects were used (thus increasing statistical power), statistical significance may have been reached.

Table 11

Theory-based Proportions at Middle Level

Mean		Expert	Intermediate	Novice
0.3456	Expert			
0.5178	Intermediate			
0.1656	Novice		*	

*$p<0.05$

<u>Analysis of Classifications at the Low Level of Categorization</u>

In the last analysis, comparisons were made between the proportions of categories classified as theory-, model, and mathematical model-based over the total number of classifications at the low level of categorization by level of expertise. The mean proportions of model-, theory-, and mathematical model-based classifications over all classifications made at the low level of

categorization are shown for each level of expertise in Figure 7. This is the level in which the piles were the smallest and the attributes the cards had in common were the most specific. An overall multivariate analysis was conducted and a summary of this analysis is found in Table E.4 in Appendix E. Tests of significance for this analysis were conducted and were found to be not significant (F= 2.1948, p=0.061, ns) indicating no differences in the use of categories classified as theory-, model-, or mathematical model-based between experts, intermediates, and novices at this level of categorization.

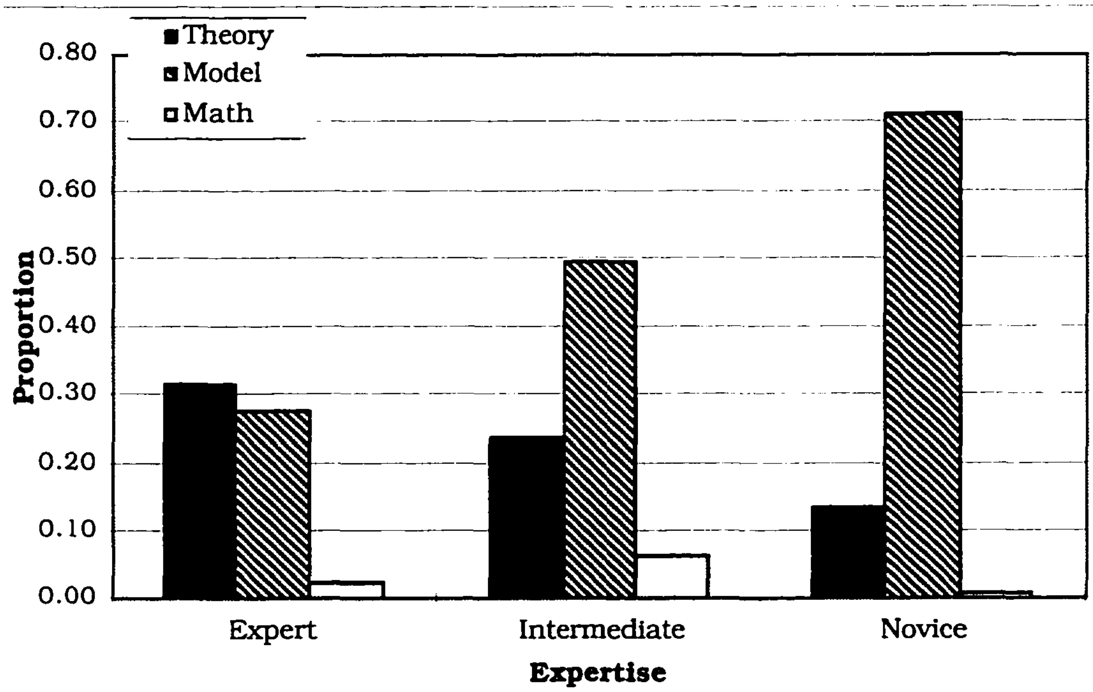


Figure 7. Classification at Low Level of Categorization.

Although tests of significance for the multivariate tests were not significant (F= 2.1948, p=0.061, ns), the probability was very close to the 0.05 significance level. Since the multivariate test is conservative

and that there were few subjects in this analysis (which decreases statistical power), an analysis of variance was conducted in order to find any possible trends between the use of model-, theory-, and mathematical model-based attributes between expert, intermediate, and novice subjects at the low level of categorization. The results of these tests can be found in Table E.4 in Appendix E. The use of model-based classifications was found to be significant at the univariate level for the low level of categorization between experts, intermediates, and novices (X_E=0.28, X_I=0.49, X_N=0.71, $\underline{F}$=6.08, $\underline{p}$=0.007) indicating a trend in the data towards a difference between these three levels of expertise on the use of model-based classifications. To determine which of the three groups were significant from each other, a Tukey-HSD test with significance level at 0.05 was conducted. Significant differences were found between experts and novices on the use of model-based classifications indicating that experts used model-based categories. No other significant differences were found for the proportion of model-based attributes used by experts, intermediates, and novices. These results are summarized in Table 12.

Summary of Analyses of Classifications at Each Level of Categorization

Table 13 provides a summary of the results of the previous analyses comparing the proportions of model-, theory-, and mathematical model-based classifications over the total number of

classifications made by experts, intermediates, and novices at each level of categorization.

Table 12

Model-based Proportions at Low Level

Mean		Expert	Intermediate	Novice
0.2800	Expert			
0.4956	Intermediate			
0.7144	Novice	*		

*$p<0.05$

The rows indicate the level of expertise of the subjects as either expert, intermediate, or novice. The columns indicate the level of categorization as either high, middle, or low. Recall that the high level of categorization is the level which contains the largest piles that the subjects made and thus the attribute common to all the cards in those piles is the most general one. The low level of categorization contain the smallest piles with the most specific attributes in common.

The columns are separated into theory (THE) and model (MOD) sub-headings. Under each sub-heading is the proportion of categories made by each group of subjects classified as model- or theory-based at each level of categorization. Also, the proportion is in bold to indicate that use of that classification was found to be statistically significant. In the last row the Table number where the statistical

finding is summarized is given. For example, at the middle level for intermediates the proportion of categories classified as theory-based (THE) is 0.52. This proportion of theory-based categories for intermediates is significantly higher compared to novices at this same level where only 0.16 of their classifications are theory-based. This information is summarized in Table 11. The proportions of mathematical model-based attributes was not included in this summary since that classification was seldomly used by any group and use of this classification was not significant in any of the analyses.

Table 13

Summary of Results of Analysis of Classifications
Separated by Level

	HIGH		MIDDLE		LOW	
	THE	MOD	THE	MOD	THE	MOD
EXPERT	**0.76**	0.00	0.35	0.45	0.32	0.28*
INTERMEDIATE	0.44	0.00	**0.52**	**0.37**	0.24	0.50
NOVICE	**0.00**	0.00	**0.16**	**0.79**	0.13	0.71*
(Table	9		11	10		12)

bold indicates $p < 0.05$. * indicates trends only.

Analysis of Sub-Classifications Separated by Level

The previous results, summarized in Table 13, provide a general description of the categories classified as model-, theory-, or mathematical model-based that individuals at different levels of expertise used at different levels of categorization. In this analysis, proportions of the theory- and model-based sub-classifications over the total number of classifications at each level used by experts, intermediates, and novices were analyzed in more detail in order to more fully describe the attributes that subjects at different levels of expertise used to categorize problems at different levels of categorization. The model-based category was sub-classified into the object in the problem (M-OBJ), the attribute of the object (M-ATT), the unknown in the problem (M-UNK), and the motion of the object (M-MOT). The theory-based classifications were broken down into the theory used to solve the problem (THE), the construction rules used to create the physical representation of the problem (THE-CON), and the extension rules used to solve for particular variables in question (THE-EXT).

As summarized in Table 13, experts and intermediates used a combination of theory- and model-based categories, therefore analyses of both the theory- and model-based sub-categories were performed. In these analyses, the level of expertise was held constant, and the proportions of theory- and model-based sub-categories over the total number of theory- or model-based categories made by experts or intermediates were analyzed at each level of categorization.

For example, a multivariate test was performed using the proportions of each of the model-based sub-categories over the total number of model-based categories used by experts as a dependent variable and level of categorization as an independent variable. Four separate similar multivariate tests were conducted for the theory- and model-based sub-categories used by experts and intermediates.

Also as indicated in Figure 4, the majority of the categories that novices made were classified as model-based (X_M=0.76). A minority of the classifications that novices made were theory-based (X_T=0.16) and since there are three levels of categorization and three theory-based sub-categories, the statistical power of these analyses would be too low to obtain statistical significance. In addition, a description of the theory-based sub-categories used by novices would be misleading since these attributes were used so rarely. In these analyses, multivariate tests were performed using the proportions of categories classified as either motion of the object (M-MOT), object in the problem (M-OBJ), unknown in the problem (M-UNK), or attribute of the object (M-ATT) over the total number of model-based classifications made by novices as a dependent variable and level of categorization as an independent variable.

Analysis of Sub Classifications by Experts

In the first analysis, the proportions of categories classified as theory name (THE), construction rules (THE-CON), or extension rules (THE-EXT) over the total number of categories made by experts were

compared at each level of categorization. Mean proportions for the use of classifications described as a theory name, construction rule, or extension rule are shown in Figure 8. A multivariate analysis was conducted using as dependent variables, the proportions of categories classified as theory, construction rules, and extension rules over the total number of categories that experts made at each level; the level of categorization was used as an independent variable. A summary of this analysis is found in Table E.5 in Appendix E. Tests of significance for this analysis were conducted and were found to be significant (F=0.48, p=0.009) indicating a difference in the use of these sub-categories by experts across levels of categorization.

Since the multivariate tests were significant, an analysis of variance was conducted for each theory-based sub-classification; a summary can be found in Table E.5 in Appendix E. Use of the theory name sub-classification by experts comparing levels of categorization was found to be significant (X_H=0.67, X_M=0.15, X_L=0.07, F=9.18, p=0.001) indicating that experts used the theory name sub-classification differently at different levels of categorization. A Tukey-HSD test with significance level at 0.05 was executed in order to determine which of the proportions of the theory name sub-category at which of the levels of categorization were significantly different. Differences were found for the use of theory name as a sub-classification between the high level and either the middle or low levels indicating that the use of the theory as a sub-category was significantly greater at the high level of categorization than at both the

middle and low levels. The results of the Tukey-HSD test are summarized in Table 14.

The analysis of the variance for proportions of categories classified as construction rules over the total number of categories made by experts compared at each level of categorization was not significant (X_H=0.04, X_M=0.12, X_L=0.23, F=2.02, p=0.155, ns) indicating that experts did not use construction rules to categorize the problems differently at different levels of categorization. Similarly, the analysis of variance for the use of extension rules was not significant (X_H=0.00, X_M=0.11, X_L=0.07, F=2.33, p=0.119, ns) indicating that experts did not use extension rules to categorize the problems differently at different levels of categorization. These results are summarized in Table E.5 in Appendix E.

Next, the proportions of each category classified as object in the problem (M-OBJ), attribute of the object (M-ATT), unknown in the problem (M-UNK), and motion of the object (M-MOT) over the total number of categories made by experts were compared at each level of categorization. Mean proportions for the use of these model-based sub-categories by experts at each level of categorization are shown in Figure 9. Since experts did not use the unknown in the problem as a model-based sub-classification, a multivariate analysis was conducted using only the categories classified as either object in the problem, attribute of the object, or motion of the object as dependent variables compared by level of categorization. A summary of this analysis is found in Table E.6 in Appendix E. Tests of significance for this

analysis were conducted and were found to be not significant (F=1.81, p=0.119, ns). The lack of significance indicates that there are no differences in the proportions of object, attributes, unknown, or motion classifications used by experts across levels of categorizations. Therefore, no univariate tests were conducted.

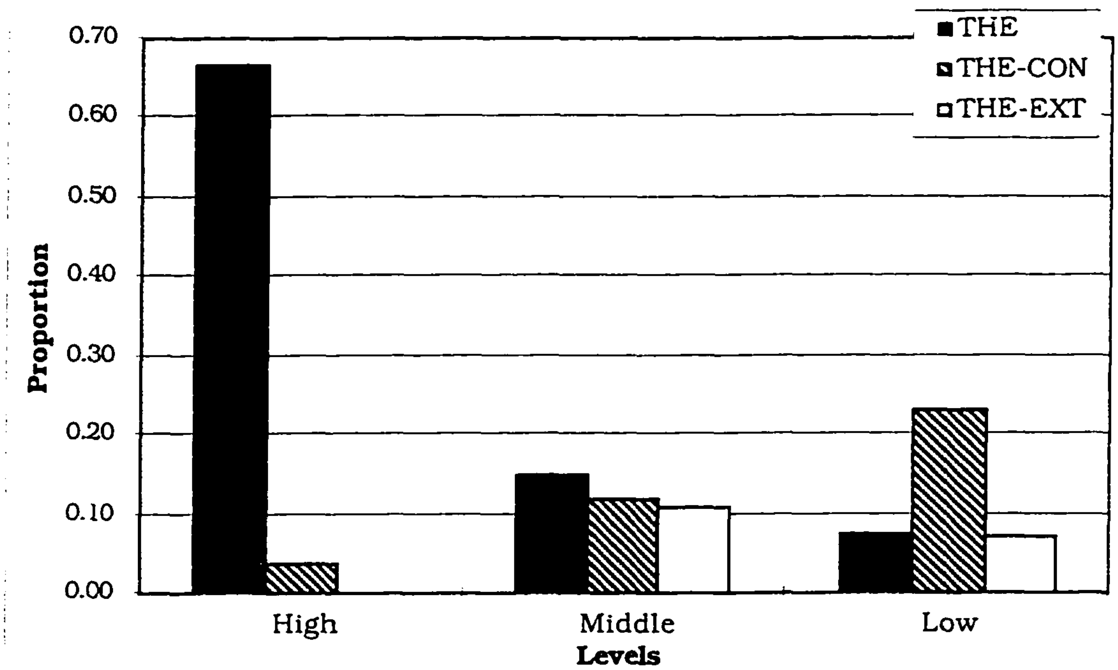


Figure 8. Theory Sub Classification by Experts.

Analysis of Sub Classifications by Intermediates

Next, an analysis of the categorization of intermediates was conducted comparing the use of model- and theory-based sub-categories. In the first analysis, the proportions of categories classified as the name of the theory used to solve the problem (THE), construction rules (THE-CON), or extension rules (THE-EXT) over the

total number of categories made by intermediates were compared at each level of categorization. Mean proportions of theory-based sub-categories used by intermediates at each level of categorization are shown in Figure 10. A multivariate analysis was conducted using the proportions of use of the theory, construction rules, and extension rules by intermediates as dependent variables and level of categorization as an independent variable. A summary of this analysis is found in Table E.7 in Appendix E. Tests of significance for this analysis were conducted and were found to be not significant (F=2.29, p=0.052, ns) indicating that there were no differences in the proportions of categories classified as theory, construction rules, or extension rules made by intermediates at different levels of categorization.

Table 14

Use of Theory Name Sub Classification by Experts

Mean		High	Middle	Low
0.6667	High			
0.1478	Middle	*		
0.0733	Low	*		

*p<0.05

Although tests of significance for the multivariate tests were not significant, the probability was very close to the 0.05 significance

level. Since the multivariate test is conservative and there were few subjects in this analysis (thus statistical power was low), an analysis of variance was conducted in order to determine whether there were any trends between the use of theory, construction rules, or extension rules as categories used by intermediates at each level of categorization. The results of these analyses can be found in Table E.7 in Appendix E. The analysis of variance for the proportion of categories classified as construction rules over all categories made by intermediates at each level of categorization was significant (X_H=0.00, X_M=0.19, X_L=0.20, F=3.62, p=0.042) indicating a difference in the proportion of the use of the category between levels of categorization. In order to determine which of the levels of categorization were significantly different from each other in terms of the proportions of use of the construction rules category, a Tukey-HSD test was conducted with a significance level of 0.05. No statistically significant differences were found between any of the levels of categorization indicating no significant differences for the use of construction rules by intermediate subjects.

The analysis of the variance for proportions of categories classified as theory name over the total number of categories made by intermediates compared at each level of categorization was not significant (X_H=0.44, X_M=0.29, X_L=0.07, F=2.45, p=0.107, ns) indicating that intermediates did not use theory name to categorize the problems differently at different levels of categorization. Similarly, the analysis of variance for the use of extension rules was not

significant (X_H=0.00, X_M=0.09, X_L=0.01, $\underline{F}$=1.25, $\underline{p}$=0.304, ns) indicating that intermediates did not use extension rules to categorize the problems differently at different levels of categorization. These results are summarized in Table E.7 in Appendix E.

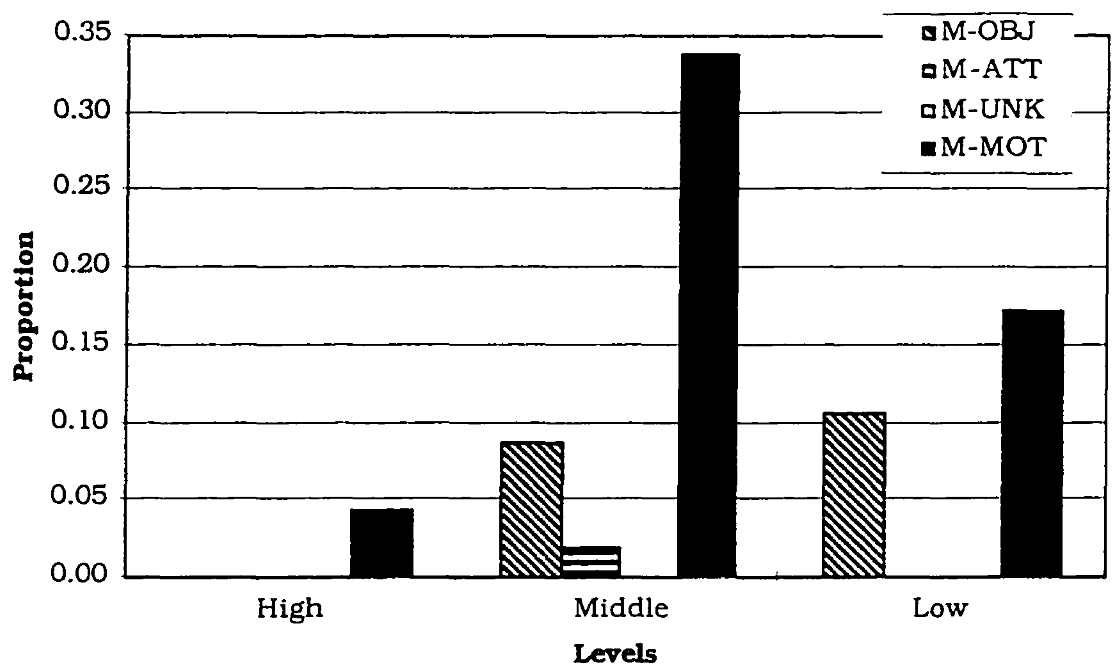


Figure 9. Model Sub Classifications by Experts.

In the second analysis of intermediate categorization, the proportions of categories classified as object in the problem (M-OBJ), attribute of the object (M-ATT), unknown in the problem (M-UNK), and motion of the object (M-MOT) over the total number of categories made by intermediates were compared at each level of categorization. The mean proportions of these model-based sub-classifications over the total number of classifications made by intermediates at each level of categorization is summarized in the graph in Figure 11.

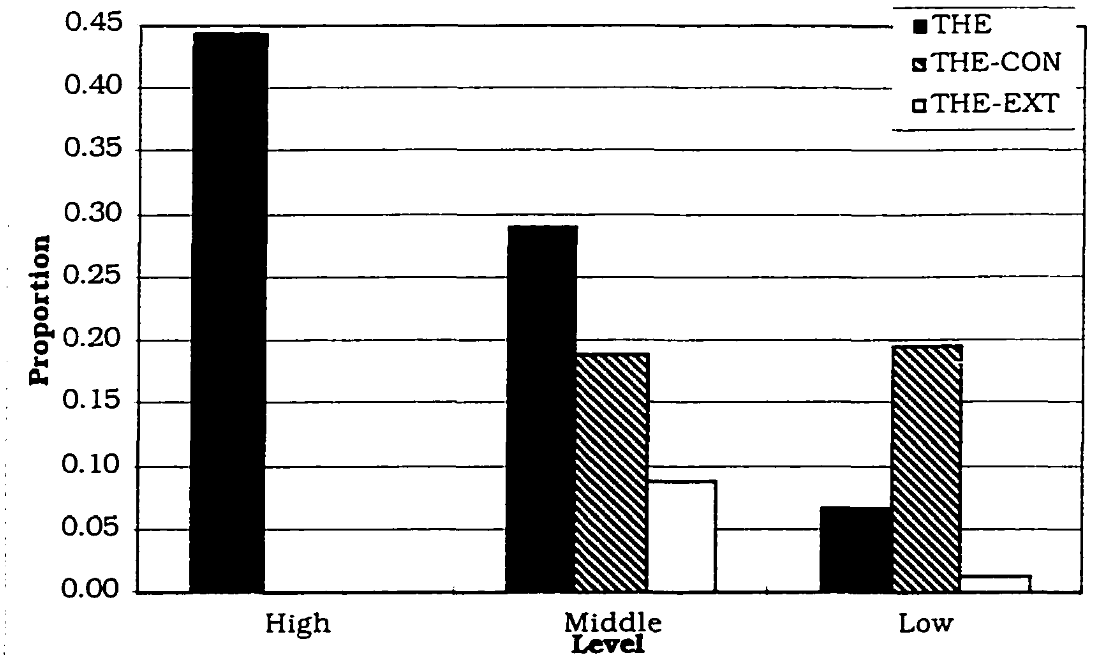


Figure 10. Theory Sub-classification by Intermediates.

A multivariate analysis was conducted using proportions of the use of the object in problem, attributes of the object, unknown in the problem, and motion of the object by intermediates as dependent variables at the different levels of categorization. Tests of significance for this analysis were conducted and were found to be significant (F=0.24, p=0.000) indicating a difference in the use of object, attribute, unknown, and motion attributes by intermediates at different levels of categorization. A summary of this analysis can be found in Table E.8 in Appendix E.

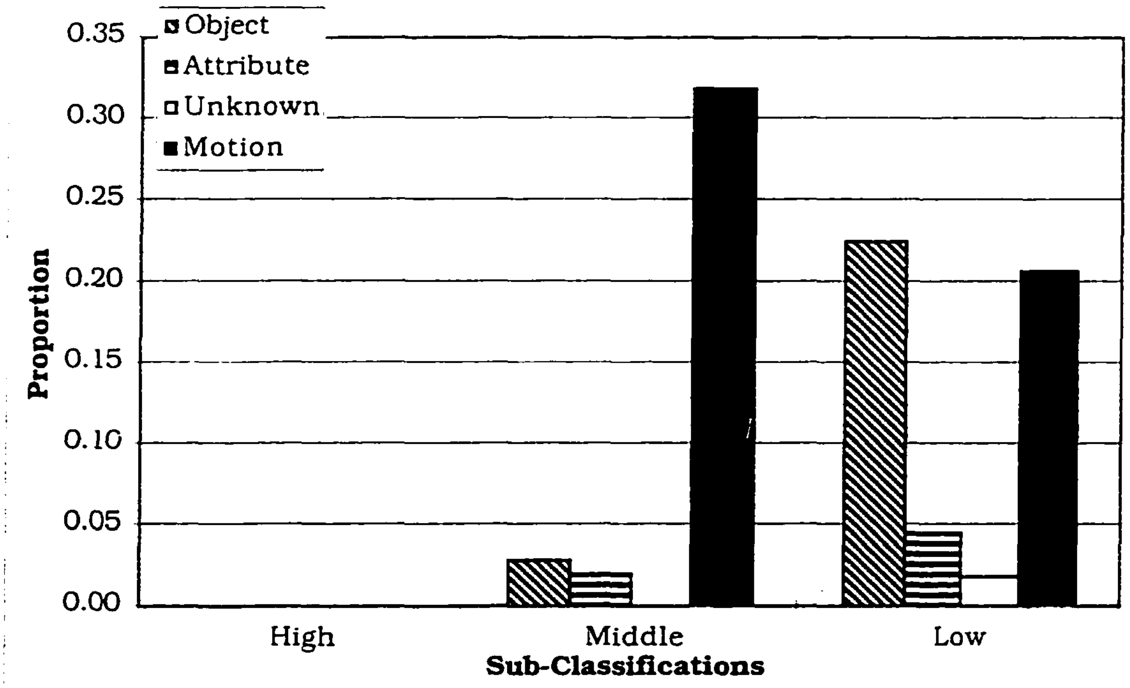


Figure 11. Model Sub Classifications by Intermediates.

Since the multivariate tests were significant, univariate tests were conducted for each of the model-based sub-categories as dependent variables at different levels of categorization. The results of these analyses are summarized in Table E.8 in Appendix E. The analysis of variance for the proportion of categories classified as the motion of the object (M-MOT) over the total number of categories made by intermediates at each level of categorization was found to be significant (X_H=0.00, X_M=0.32, X_L =0.21, F=4.76, p=0.018) indicating a difference in the use of the motion of the object attribute at different levels of categorization. A Tukey-HSD test with significance level at 0.05 was executed in order to determine which of the proportions of the motion sub-category at which of the levels of categorization were

significantly different. The use of the motion attribute (M-MOT) between the high to the middle level was found to be significant indicating that the use of the motion of the object to categorize the problems at the middle level was significantly greater than at the high level of categorization. No other significant differences were found. These results are summarized in Table 15.

Table 15

Use of Motion Sub Classification by Intermediates

Mean		High	Middle	Low
0.000	High			
0.3189	Middle	*		
0.2078	Low			

*$p<0.05$

The analysis of variance of the proportion of categories classified as the object in the problem (M-OBJ) over all categories made by intermediates compared by level of categorization was also found to be significant ($X_H=0.00$, $X_M=0.03$, $X_L=0.22$, $F=14.75$, $p=0.001$) indicating a difference in the use of the object in the problem as an attribute to categorize the problems between levels of categorization. A Tukey-HSD test with significance level at 0.05 was executed in order to determine which of the proportions of the object in the problem sub-category at which of the levels of categorization were significantly different. The use of the object in the problem as an

attribute for categorization by intermediates was found to be statistically significant between the low level of categorization and both the middle and high levels indicating that the use of the object in the problem to categorize the problems at the low level was significantly greater than at both the high and middle levels of categorization. No other significant differences were found. These results are summarized in Table 16.

Table 16

Use of Object Sub Classification by Intermediates

Mean		High	Middle	Low
0.0000	High			*
0.0278	Middle			*
0.2233	Low			

*$p<0.05$

The analysis of the variance for proportions of categories classified as the attribute of the object in the problem over the total number of categories made by intermediates compared at each level of categorization was not significant ($X_H=0.00$, $X_M=0.02$, $X_L=0.04$, $F=2.02$, $p=0.154$, ns) indicating that intermediates did not use the attributes of the object to categorize the problems differently at different levels of categorization. Similarly, the analysis of variance for the use of the unknown in the problem was not significant ($X_H=0.00$, $X_M=0.00$, $X_L=0.02$, $F=2.06$, $p=0.150$, ns) again, indicating

that intermediates did not use the unknown in the problem to categorize the problems differently at different levels of categorization. These results are summarized in Table E.8 in Appendix E.

Analysis of Sub Classifications by Novices

As previously explained, only the proportions of model-based sub-classifications were analyzed. In this analysis, the proportions of categories classified as object in the problem (M-OBJ), attribute of the object (M-ATT), unknown in the problem (M-UNK), or motion of the object (M-MOT) over the total number of categories made by novices were compared at each level of categorization. The mean proportions of these model-based sub-classifications over the total number of classifications made by novices at each level of categorization is summarized in the graph in Figure 12. A multivariate analysis was conducted using proportions of the use of the object in the problem, attributes of the object, unknown in the problem, and motion of the object by novices as dependent variables. A summary of this analysis can be found in Table E.9 in Appendix E. Tests of significance for this analysis were conducted and were found to be significant ($F=11.36$, $p=0.000$) indicating a difference in the use of object, attribute, unknown, and motion attributes by novices at different levels of categorization.

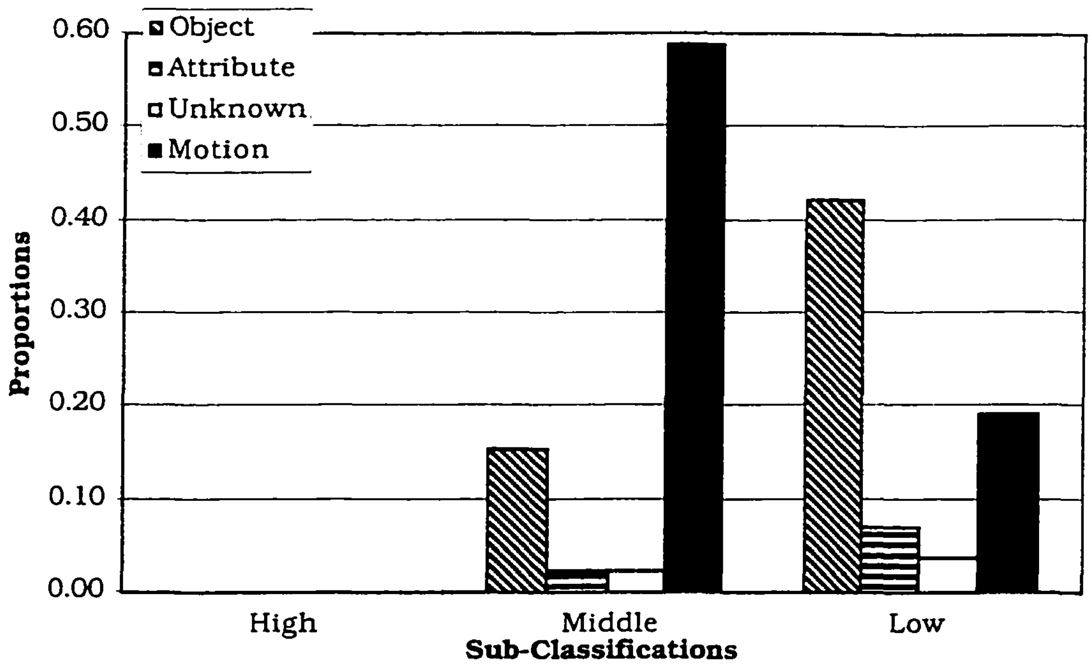


Figure 12. Model Sub Classifications by Novices.

Since the multivariate tests were significant, analyses of variance were conducted using each of proportions of categories classified as object, attribute, unknown, and motion over the total number of classifications made by novices at each level of categorization. A summary of these analyses can be found in Table E.9 in Appendix E.

The univariate analysis of the proportions of the use of the motion of the object (M-MOT) attribute was found to be significant (X_H=0.00, X_M=0.59, X_L=0.19, $\underline{F}$=20.22, $\underline{p}$=0.00) indicating that novices used the motion of the object to categorize the problems differently at different levels of categorization. A Tukey-HSD test with significance

level of 0.05 was conducted to determine which levels of categorization are significant for use of the motion attribute. The use of the motion of the object (M-MOT) as an attribute used to categorize problems at the middle level was found to be statistically different from both the low and high levels; that is, the motion of the object was used to categorize the problems significantly more at the middle level than both the low and high levels of categorization. No other significant results were found. These results are summarized in Table 17.

Table 17

Use of Motion Sub Classification by Novices

Mean		High	Middle	Low
0.0000	High		*	
0.5878	Middle			
0.1933	Low		*	

*$p<0.05$

The univariate test of the proportion of categories classified as the object in the problem (M-OBJ) over the total number of classifications that novices made compared by each level of categorization was found to be significant (X_H=0.00, X_M=0.15, X_L=0.42, F=9.99, p=0.001) indicating that novices used the object in the problem to categorize the problems differently at different levels of categorization. A Tukey-HSD test with significance level of 0.05 was

conducted to determine which levels of categorization were significant for use of the object attribute. The proportion of categories classified as the object in the problem (M-OBJ) at the low level was found to be statistically significant from both the high and middle levels indicating that the proportion of categories classified as object in the problem by novices was significantly greater at the low level than at either the high or middle levels of categorization. No other significant results were found. These results are summarized in Table 18.

Table 18

Use of Object Sub Classification by Novices

Mean		High	Middle	Low
0.0000	High			*
0.1533	Middle			*
0..4233	Low			

*$p<0.05$

The analysis of the variance for proportions of categories classified as the attribute of the object in the problem over the total number of categories made by novices at each level of categorization was not significant ($X_H=0.00$, $X_M=0.02$, $X_L=0.07$, $F=1.83$, $p=0.182$, ns) indicating that novices did not use the attributes of the object to categorize the problems differently at different levels of categorization. Similarly, the analysis of variance for the use of the unknown in the

problem was not significant (X_H=0.00, X_M=0.02, X_L=0.04, F=0.55, p=0.586, ns) indicating that novices did not use the unknown in the problem to categorize the problems differently at different levels of categorization. These results are summarized in Table E.9 in Appendix E.

There were many analyses conducted here in order to support the hypotheses made in Chapter III. These analyses will be further summarized and discussed in Chapter V in terms of the hypotheses made concerning the knowledge structure of experts, intermediate, and novice subjects in physics.

CHAPTER V

DISCUSSION

The main objective in this study was to describe the knowledge structure in physics by comparing the knowledge of individuals with varying levels of expertise. Towards this end, subjects were asked to categorize a set of physics problems and were then asked to re-categorize the problems more than once into bigger or smaller categories. In this way, the subjects created a hierarchy of categories to which the problems belonged. The placement of cards into categories and the explanations that the subjects gave of these categories were recorded on videotape. The videotapes were transcribed and the actions by the subjects were recorded. The categorization task resulted in a hierarchical sort which was recorded on grids along with the name of the categories made by each subject. In order to make comparisons between the types of categories subjects made and the level of expertise of the subjects, a classification system was created and can be seen in Appendix C.

The classification system for all the cards was developed from the literature on categorization and problem representation in physics. These studies showed that experts categorized and represented problems in terms of the principles or theories needed to solve them, whereas novices used the surface features of the problems

(Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990). In light of this, the classification system used in the present research is composed of categories belonging to the theories and the models used to the solve the problems. The mathematical model classification was added as a possible set of categories to be used by intermediate subjects. The model-based categories were composed of attributes of the conceptual model used to represent the problems such as the objects or attributes of the objects. The theory-based categories included principles that were used to create the conceptual model used to solve the problems. Finally, the mathematical model-based attributes were defined as the mathematical sophistication or methods necessary to solve the problems.

In order to further elaborate on the categories that subjects used to categorize the problems, the theory and model categories were sub-divided based on results from the literature on expertise in physics. This represents a more exploratory aspect of the present research. In previous categorization studies, novices were found to use surface features of the problem in order to categorize the a set of problems; this suggests that novices may be focused on attributes of the models used to solve the problems (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990). A model may have the components of the object in the model, relationships to that object, and the action the objects undergo. Therefore, in this study, the model class was sub-divided according to the object in the problem,

i.e., "incline plane", "spring", the attributes of those objects, i.e., "frictionless", and the motion of those objects, i.e., "two dimensional". In addition, the unknown in the problem, i.e., "time", "final velocity", was included as one of the model-based sub-categories. This was done to reflect previous findings which demonstrated that novice physics students use the unknown from the problem statement and then search for an equation involving that unknown (Larkin, 1983; 1985; Larkin et al 1980a; 1980b). The unknown in the problem statement, an important aspect of the novices' problem solving, was included as a sub-category. In summary, the model classification was sub-classified into four attributes: (1) the object in the problem, (2) the attributes of that object, (3) the unknown in the problem, and (4) the motion of the object.

The theory classification was used by experts and this classification was sub-divided according to the approach that experts take in problem solving tasks (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990). In these studies, experts began the problem solving process by creating a physical representation of the problem. In order to do this, the expert selected an appropriate schema or theory that could be used to solve the problem (Larkin, 1983; 1985; Larkin et al 1980a; 1980b). Next, the expert used inferencing rules within this theory to construct the problem representation and solve the problem. The first set of inferencing rules used are the construction rules that act on the original (naive) problem

representation to create a causal model. Next, the extension rules are used to extend this causal model to add new entities which facilitate problem solving, e.g., when the equations of motion are derived from the various forces that are chosen. Based on those previous findings (Larkin, 1983; 1985; Larkin et al 1980a; 1980b), the theory category was sub-divided into three categories in order to reflect the experts' reasoning. The categories are: (1) the name of the theory, (2) the construction rules, and (3) the extension rules used to create the model and solve the problems.

Sub classes were not made for the mathematical model category since it was not a category derived from the existing literature and therefore there was no existing literature on how this classification might be used by the subjects.

In studying the knowledge structure of experts, intermediates, and novices, there were three main objectives. The first objective was to describe the differences between the classifications by experts, intermediates, and novices across all levels of categorization and compare it to the existing literature. This will substantiate the findings of other research and situate the current study as an extension of these results. Since the major differences between experts and novices in the categorization literature stemmed from their knowledge of models and theories (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990), the comparisons between the levels of expertise will be made in terms of model-, theory-, and mathematical model classifications. The second objective was to

compare the theory-, model-, and mathematical model-based classifications made by experts, intermediates, and novices at each level of categorization. These findings are used to support and extend theoretical and empirical accounts of knowledge in physics in terms of two competing views of knowledge from the literature (Giere, 1988; 1994; Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). The first of these views, presented by Giere (1994), represents knowledge in terms of a hierarchy of conceptualized models. The second view, referred to as the coherent view (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996), describes knowledge as connected according to an individual's theories of the world. The third objective was to describe the knowledge structure of experts, intermediates, and novices more fully in terms of the model- and theory-based sub-classifications used at different levels of categorization. This was an exploratory aspect of the study for which there is little existing empirical data.

The results in the previous chapter showed how experts, intermediates, and novices differed statistically in terms of the way they categorized the problems. These results will be discussed in terms of the objectives stated above, the research questions from Chapter I, and the hypotheses from Chapter III. In addition to this, there will be a discussion of implications for further research and teaching in physics.

Classifications Across All Levels

The first objective was to compare the proportions of model-, theory-, and mathematical model-based classifications over all the classifications made by pooling over all the levels of categorization. The rationale for this objective was to situate the current research within previous research on expert/novice categorization and problem representation (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990). The research question for this analysis and the hypotheses were as follows:

Question 1: How do experts, intermediates, and novices compare in their use of model-, theory-, and mathematical model-based categories across all levels of categorization?

Hypothesis 1: Pooling across all levels of categorization, experts were expected to use theory-based (THE) categories, novices were expected to use model-based (M) categories, and intermediates were expected to use mathematical model-based (MTH-M) categories.

The results confirm the hypotheses regarding both the novices' and experts' categorizations. The results validate previous research findings in that experts used theory-based classifications more than did the novices (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990). These theory-based attributes include the theory and the inferencing rules used to solve the problems.

It is also not surprising that the novices in this study used model-based attributes to categorize the problems more than did the

experts because the model-based attributes include the surface features of the problems. These results are in accordance with those of other categorization experiments (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990) and investigations of problem representation (Larkin, 1983; 1985; Larkin et al 1980a; 1980b) in which experts represent and categorize problems in terms of physical quantities used to solve problems and novices represent and categorize problems in terms of the actual objects in the problems.

The results of the categorization of the intermediate subjects was different than expected in terms of the use of mathematical model-based classifications. In general, the result was that the mathematical model-based category, i.e., the mathematical methods or sophistication, was used less often by intermediates than was the "other" category. It was expected that intermediates would use this category because the difference between Introductory and Advanced Mechanics lies in the sophistication of the mathematical methods involved. In Introductory Mechanics, the mathematical sophistication is limited to algebra, geometry, and basic calculus. Advanced Mechanics requires the use of the linear algebra, multivariable calculus, and differential equations. It was expected then that the intermediates would categorize the problems according to the attribute of the problems that caused them the most difficulty (the mathematical method). Yet, this was not the case. The mathematical model-based category was seldomly used by any group of subjects. Instead, intermediates categorized in generally the same way as

experts in terms of using theories to categorize the problems more than did the novices.

Many of the previous categorization studies had graduate students as experts participants (Chi et al, 1981; 1982) . In this research, graduate students were not classified as experts, rather were used as intermediates. This was done in accordance with the expertise literature which defines an expert as an individual with approximately 10 years of experience in a particular field (Ericsson & Smith, 1991). In addition, the problems used in the current study were at the intermediate level, rather than at the introductory level as used in other studies (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990). In the expertise literature it has been found that as the difficulty of a task increases, observed expert performance decreases (Bereiter & Scardamalia, 1993; Johnston & Afflerbach, 1985). For these reasons, it might be expected that graduate students are experts at solving introductory level problems, however, some differences might have been expected between intermediates and experts in using harder problems that required the subjects to use their advanced knowledge of physics.

These results replicate those of other studies and support the existing literature in that experts and novices use their knowledge of physics to represent and categorize the problems. However, it is suggested here that the knowledge of physics that the subjects used to categorize the problems was qualitatively different. More specifically, from these data it appears that the knowledge that the

novices used is dependent on the attributes of the models used to solve the problems; whereas the knowledge that the experts and intermediates used is centered on the principles. Principles compose theories that are used to solve the problems and to create the models. Models refer to the representations that are made of a problem situation. The result that experts and intermediates used principles to categorize the problems in this study may indicate a better understanding of how the problems are solved; whereas the novices' use of model-based attributes suggest that the knowledge that novices have is centered around problem solutions. This research extends the existing literature by suggesting that the knowledge that novices use to categorize problems is based on knowledge of physics in terms of the problem solutions.

Classifications Separated by Level

The first part of the data analysis allowed for an investigation of the general nature of all the classifications that subjects made in terms of theory-, model-, and mathematical model-based attributes used to solve the problems. In this analysis, these results were extended to include the hierarchical nature of the subjects' categorizations. In these analyses, comparisons were made between novices', intermediates', and experts' use of model-, theory-, and mathematical model-based attributes at each level of categorization. This was done in order to investigate the competing models of knowledge in physics. As discussed earlier, Giere (1994) proposed

that physics knowledge consists of a hierarchy of models that are structured with more general models at the top of the hierarchy and more specific ones at the bottom. Medin and Murphy (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) suggest that in order to account for the coherence of category construction for knowledge the role of theories in knowledge structure must be taken into account. In this latter view, models are structured according to the theories used to create them. The research question for this analysis and the hypotheses were as follows:

Question 2: How do experts, intermediates, and novices compare in their use of model-, theory-, and mathematical model-based categories at each level of categorization?

Hypothesis 2: Expert, novice, and intermediate classifications were not expected to be distinct and some overlap was expected at the low level of categorization. At the high and middle levels of categorization, experts were expected to use theory-based (THE) categories, novices were expected to use model-based (M) categories, and intermediates were expected to use a mix of theory- (THE) and mathematical model-based (MTH-M) categories. At the low level of categorization, experts, intermediates, and novices were expected to use model-based (M) categories.

In the Results chapter, the categorizations that subjects made were compared by level of expertise at each level of categorization. Table 13 Chapter IV provides a summary of the results of that investigation and can be found on page102. The categorizations of

the novices in this study was well predicted by the hypotheses and by Giere's (1994) hierarchical representation of models. The result that the novices' hierarchies were entirely composed of model-based attributes as compared to experts and intermediates whose categorizations were mixed between models and theories at lower levels supports a representation of the novices' knowledge as a structure of models. The result that novices were unable to make "High" level categories has also been found by Chi et al (1982). These results may also support Giere's (1994) assertion that novices are acting at a "Basic Level" where models have the most visual similarity to each other. It could be explained that the novices in the present study were focused on the visual similarity between the models, such as the objects in the problems, and were not able to abstract any further than the Basic level. This would explain why novices in this study used more model-based attributes to categorize the problems and create their hierarchy as compared to experts and intermediates.

Results of the classifications that intermediates used in the present study were not predicted by the hypothesis in that mathematical model-based categories were not used to classify the problems at any level of categorization. The experts' categorizations were predicted by the hypothesis above in that experts used theory-based categories at all levels of categorization and model-based categories at lower levels of categorization.

One possible reason why novices did not use many theory-based attributes may have been that problems in physics, such as the

ones used in the study, are not real problems to students in that they do not represent theories of how the world works. As discussed earlier, the physics problems that students are presented with are based on models that are ready-made. Novices do not have many opportunities to engage in the interpretation of phenomenon or to use any theories, naive or otherwise. These students may view problems as artificial in that they do not relate to real situations. Since they do not see explanation as part of the problem solving or representation phase in solving physics problems, they may not use their naive theories to explain the problems. The question of whether novices have theories can not be answered by this research because it did not ask them directly about their understanding of the phenomena underlying the problems. However, it appears from these data that novices do not use these naive theories to categorize problems since they did not use categories that were not described by the models or theories in physics in Appendix C. Experts and intermediates, in contrast, realize that problems relate to "real" situations. They understand the formal physics theories used to solve problems and therefore use theory-based classifications.

Neither Giere's (1994) nor the coherent views of physics knowledge (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) are sufficient to explain the results of all the categorizations in this study. The novices' categorizations were accurately predicted by Giere (1994) in that they categorized the problems into a hierarchy of models. The intermediates' and experts'

categorizations also had elements of this hierarchy in that model-based categories were used at the middle and low levels. However, experts and intermediates also used theory-attributes to categorize the problems. The use of theory-based attributes is predicted by the coherent view of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996).

Recall that there were two proposals regarding how to resolve the issue of whether knowledge in physics is organized by a hierarchy of models (Giere, 1994) or by theories (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). The first was to propose that the knowledge of experts, intermediates, and novices is distinct in terms of their use of models and theories to represent problems. This was not the option expected and it is not supported by the results of this study. Specifically, in this study, novices used model-based attributes to categorize the problems at all levels of categorization and the experts and intermediates also used model-based attributes at lower levels of categorization. It is suggested from these results that the categories used by experts, intermediates, and novices overlap at lower levels of categorization.

There is a second proposal presented in this study to resolve the problem that knowledge in physics is neither entirely model- nor theory-driven; That is, to suggest that knowledge in physics may be best described as a combined view driven by both theories and models. In this view, there are clusters of hierarchies of models connected by the theories used to create them. The results found

here that the intermediates and experts used both theory-and model-based categorizations provide support for this proposal. However, it is not clear whether these theory- and model-based attributes are hierarchically arranged such that more general model- or theory-based categories are used at higher levels than at lower levels. This issue will be explored in the following sections which discuss the results of the use of the sub-classifications used by experts, intermediates, and novices in this study.

Sub-Classifications Separated by Level

The objectives of the first two analyses were to compare the categorizations of experts, intermediates, and novices in terms of the general model- and theory-based classifications. These results were discussed in terms of the literature. In this section, the objective was to further describe the criteria that expert, intermediate, and novice subjects used to create their hierarchies more fully in terms of the model- and theory-based sub-classifications used at different levels of categorization. The model-based sub-categories used are related to the descriptive and dynamic elements of the models used to solve the problems. There are four of these sub-categories; namely (1) object in the problem (M-OBJ), (2) attributes (M-ATT) of these objects, (3) the unknown (M-UNK) in the problem, and (4) the motion (M-MOT) of the object. The three theory-based sub-categories were developed from studies of problem representation by expert subjects (Larkin, 1983; 1985; Larkin et al 1980a; 1980b) and are (1) the theory (THE) used to

solve the problem, (2) the construction rules (THE-CON) used to create the causal model, and (3) the extension rules (THE-EXT) used to extend the causal model and create entities to solve specific problems. One of the goals of this study was to further elaborate on the categories used by experts, intermediates, and novices at different levels of categorization. Specifically, these sub-categories were used to investigate the use of hierarchies in the categorization task.

The results of the analyses of the comparisons of the use of the model- and theory-based sub-categories between levels of categorization for each level of expertise will be discussed in terms of the hypotheses made in the following sections. The discussions and hypotheses will be separated by level of expertise in order to elaborate on the structure of novice, intermediate, and expert subjects. The research question relating to this section was as follows:

Question 3: Which model- or theory-based sub-categories do experts, intermediates, and novices use to define problems at each level of categorization?

Representation of Novices' Categorizations

The novices' categorizations will be discussed first since their categorizations were the most straight-forward, and thus open to clearer interpretation. The hypotheses for the novices' categorizations were:

Hypothesis 3c: Novices were expected to use model-based (M) categories at all levels of categorization. Novices were also expected to

use more general model-based (M) categories at higher levels, such as the motion of the object (M-MOT) or object in the problem (M-OBJ), and more specific categories at lower levels, such as the unknown in the problem (M-UNK) or the attributes of the objects (M-ATT).

The results of the novices' categorizations supported the hypothesis above. The novices used only model-based classifications to categorize the problems and only categorized at the middle and low levels of categorization. There was a difference between the model-based attributes used to categorize the problems at the low level and the attributes used at the middle level. At the middle level, novices used the motion of the object (M-MOT) to categorize the problems; at the low level, novices used the object in the problem (OBJ) to categorize the problems. An example of this type of arrangement would be to categorize of all the objects that oscillate (Motion) together at the middle level of categorization and then to separate these into pendulums and springs (Objects) at the low level of categorization. Figure 13 shows a representation of the results of the novices' categorizations at the middle and low levels of categorization.

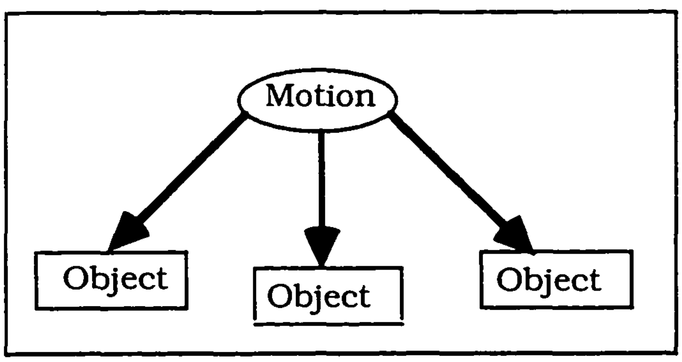


Figure 13. Representation of Novices' Categorizations.

The representation that Novice 1 created will be used as an example to represent the knowledge structure of the novices in this study. The representation in Figure 14 is part of the representation that Novice 1 created. This representation is the ideal novice representation for these particular problems. The problems are categorized into a hierarchy with two levels. The most general category that Novice 1 created for these cards is a motion category called "Harmonic Motion". In order to get more insight into what this category means, Table 19 shows how Novice 1 is trying to categorize an unfamiliar problem that has attributes of other harmonic oscillator problems. The problem in question, number 8, consists of a mass under the influence of a strange force that will cause the mass to oscillate. This is not a very typical harmonic motion problem. As can be seen in Table 19, Novice 1 thinks aloud about what category is appropriate.

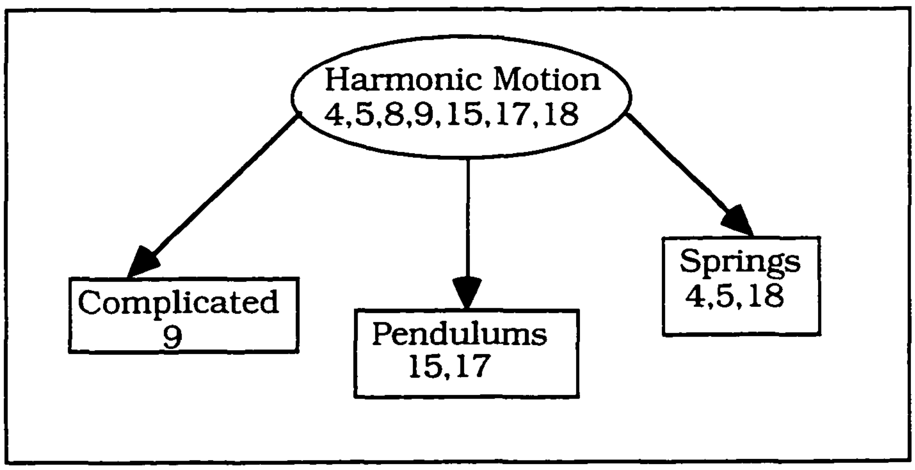


Figure 14. Representation From Novice 1.

Table 19

Segment 2.1 for Novice 1

Seg	Protocol	Action
N1	Ok We have harmonic motion over here and I think this has something to do with harmonic motion because of the period and the amplitude and things like that but I'm not really sure	8 w/(4,5,9,18), (15,17)

In this segment, Novice 1 encountered a problem that was unfamiliar, but categorized it with other harmonic motion problems because of "the period and the amplitude and things like that". Novice 1 categorized a problem that had amplitude and period as things that move with harmonic motion. "Harmonic Motion" is a very general category that included other problems in which the object moves with period or amplitude.

After placing the cards into general motion categories, i.e., "Harmonic Motion", Novice 1 began to differentiate the objects that move with Harmonic Motion. Table 20 shows segments 2.2-2.4 in which Novice 1 separated the Harmonic Motion category into problems which involve pendulums (Seg 2.2), complicated systems (2.3), and springs (2.4). Novice 1 started out with a general category that includes all things that move with Harmonic Motion and then separated them into Springs (Seg 2.4), Pendulums (Seg 2.2), and a Complicated system (Seg 2.3). The complicated system that Novice 1 refers to is a set of two spring harmonic oscillators. In this case, all of

the sub-categories: springs, pendulums, and complicated system, are more specific than the general category of harmonic motion. It is interesting to note that problem 8 was not separated into a sub-category. The reason for this may be that the harmonic motion exhibited in problem 8 was not due the object itself, but to a strange force that is positive when the position is greater than zero, and negative when position is less than zero. Categorizing this problem according to the object in the problem would not be appropriate since the motion is not inherent to the object, but to an outside force.

Table 20

Segments 2.2-2.4 for Novice 1

Seg		Protocol	Action
2.2	N1	Let me see If I can break them down I thought when we talked about harmonic motion but I couldn't remember These are pendulums	 ref 15,17
2.3		This is like a complicated system about harmonics with the two of them against each other	ref 9
2.4		And then this these are all harmonic oscillators with springs	 4,5,18

In summary, the novices categorized the problems into a hierarchy based on attributes of the conceptualized models of the problems. More abstract attributes, i.e., the motion of the objects

were used to categorize problems at higher levels and more specific attributes, i.e., the objects in the problems, were used at lower levels of categorization.

The hierarchical categorization of problems by novices described above supports Giere's (1994) representation of knowledge in physics in terms of a hierarchy of models. Again, Giere (1994) proposed that knowledge in physics is composed of a hierarchy of models in which models at higher levels were more abstract than models at lower levels. He also proposed that novices work at the "Basic Level" within this hierarchy. Specifically, in support of Giere's representation, the novices in this study used the motion of the object (M-MOT) as a general attribute of a model and therefore used it as a category at a higher level. The object in the problem (M-OBJ) was used as a more specific or lower level attribute. The motion of the object is a more general category because many different objects such as pendulums and springs can have the same oscillating motion as an attribute. The hierarchy here is one in which the motion of the object is higher on the hierarchy and the object in the problem is lower.

Representation of Intermediates' Categorizations

The intermediates' categorization is not as clear as the hierarchy of models created by the novices. The intermediates categorized the problems in this study using a combination of model- and theory-based categories. The hypotheses for the intermediates' categorization were:

Hypothesis 3b: Intermediates were expected to use mathematical model-based (MTH-M) categories at all levels of categorization. Intermediates were also expected to use some theory- and model-based categories at high and low levels of categorization.

The hypotheses for the intermediates' categorizations were not supported by the data. As has been discussed previously, the intermediates were expected to use mathematical model-based classifications to categorize the problems. This did not occur. Intermediates were also expected to use some theory- and model-based categories; this was supported by the results of this study. However, the objective of this part of the current study was to describe fully the knowledge structure of intermediates. In order to do this, it was necessary to examine their use of both model- and theory-based sub-categories.

A representation of the intermediates' categorizations of the problems in this study is shown in Figure 15. All classifications made at the high level of categorization by intermediates were theory-based. The lines extending from the top level category, "Theory" are highlighted to indicate that there is a difference between the high and middle levels of categorization. At the low and middle levels of categorization, both model- and theory-based categories were used. For this reason, the representation of the intermediates' categorizations has two branches belonging to the model- and theory-based sub-categories. These two branches will be discussed separately.

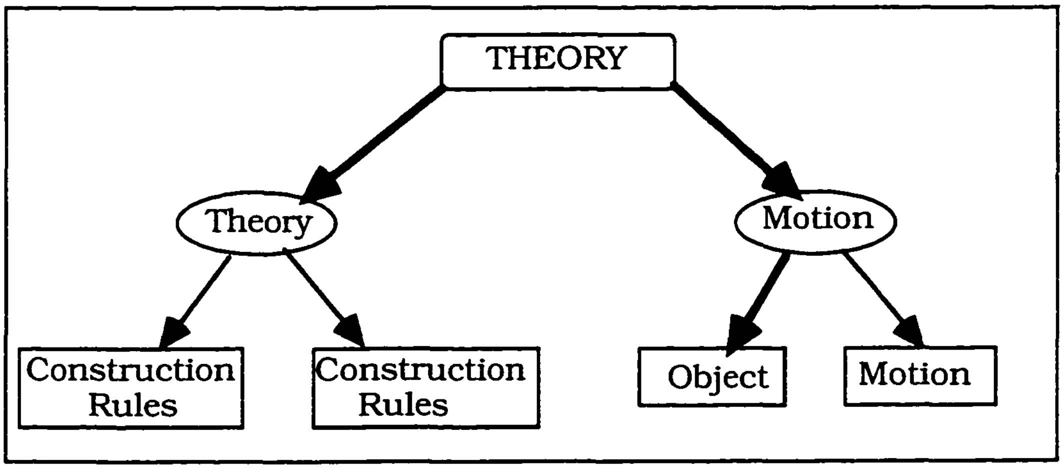


Figure 15. Representation of Intermediates' Categorizations.

There were no differences in the proportions of the use of any of the theory-based sub-categories at any level of categorization. This means that although theories were used to categorize the problems, a particular sub-category, i.e., construction rules or extension rules, was not used at a particular level of categorization. At the high level of categorization, the theory used to solve the problems was used to categorize all the cards. These theories are a set of all the laws, physical entities, and relationships needed to create the models and solve the problems. The theory-based attributes used most at the low level were the construction rules used to construct the physical representation. Use of the sub-categories was mixed at the middle level.

These results indicate that the intermediates in this study did not use theories in a hierarchical fashion to categorize the problems. Also, the intermediates in this study did not use the construction rules as a more general or more specific category as compared to

extension rules or theory. Construction rules and extension rules were chosen in this study based on previous research (Larkin, 1983; 1985; Larkin et al 1980a; 1980b) in which experts first chose a particular theory to be used, applied the construction rules and then the extension rules. These theory-based sub-categories were used to describe the nature of different principles that could be used in the problem categorization task. The results of the intermediates' categorizations suggest that theory selection, application of construction rules and extension rules are steps to be used and that none of them are more specific or general than another.

The model-based sub-categories used by intermediates, namely, object in the problem and motion of the object, are shown in the right branch of the tree. There were no model sub-categories used at the high level of categorization. Below the high level, the intermediates' model-based categorizations were very similar to that of the novices. At the middle level, the category used most was the motion of the object. The proportion of the use of this sub-category at the middle level was different from use at the high level and the low level and is indicated by a highlighted connecting line. In addition, use of the object in the problem as an attribute was also used more at the low level than at the high or middle levels. This appears to be the same hierarchy as found in the novices' categorizations. The only difference between the novices' and intermediates' classifications in terms of the use of model-based attributes is that the novices used a majority of motion categories at the middle level and a majority of object

categories at the low level of categorization. The categories classified by intermediates at the low level of categorization do not largely rely on either motion or object categories. This may indicate that intermediates are in a state of transition between novice to expert and therefore their categorizations are not as consistent.

What is interesting in terms of the knowledge structure of intermediates is that there are no differences between the use of model- or theory-based sub-categories between the middle and low levels. This may indicate that the structure at the middle and low levels of categorization is not hierarchical. If the intermediates' representations of knowledge were hierarchical, then it would be expected that there would be differences in the frequency of the use of some attributes between the different levels of categorization. Since this was not the case, there is reason to suspect that the intermediates' representations are not hierarchical at the middle and low levels.

The representation that Intermediate 6 created is typical of the intermediates' categorizations. Figure 16 shows this representation for a few of the cards categorized. The top level category is "Mechanics" which is the name of a set of theories used to solve problems. The next level is divided between the Conservation of Energy category and the Harmonic Motion category. The Harmonic Motion category is a model-based sub-category relating to the motion of the object in the problem. It is similar to the novices' category in that it is comprised of things that oscillate. However, the objects that

the intermediates used to sub-categorize this category are different. Table 21 gives two segments showing the subject's reasoning while creating these categories. In this segment, Intermediate 6 is separating the Harmonic Motion category into two kinds of oscillators: single and double.

Table 21

Segments 2.19-2.20 for Intermediate 6

Seg	Protocol	Action
2.19	two of them that are coupled type of harmonic oscillators basically two oscillators	ref 9, 17
2.20	two cards with one oscillator	ref 5, 15

In Novice 1's categorization, the Harmonic Motion category was divided into pendulums and springs which are two different objects that oscillate. However, Intermediate 6 (above) divides the category into coupled oscillators (where there are two or more springs or pendulums) and single oscillators (where there are only one). In general, problems that have coupled oscillators require different methods of solution than single oscillators. The novices would not have known this since the coupled oscillator is typically an intermediate level problem. Although both novices and intermediates used the object in the problem as an attribute used to categorize the problems, intermediates have a higher level of understanding because

they recognize that the problem solution is difference because there are two oscillators.

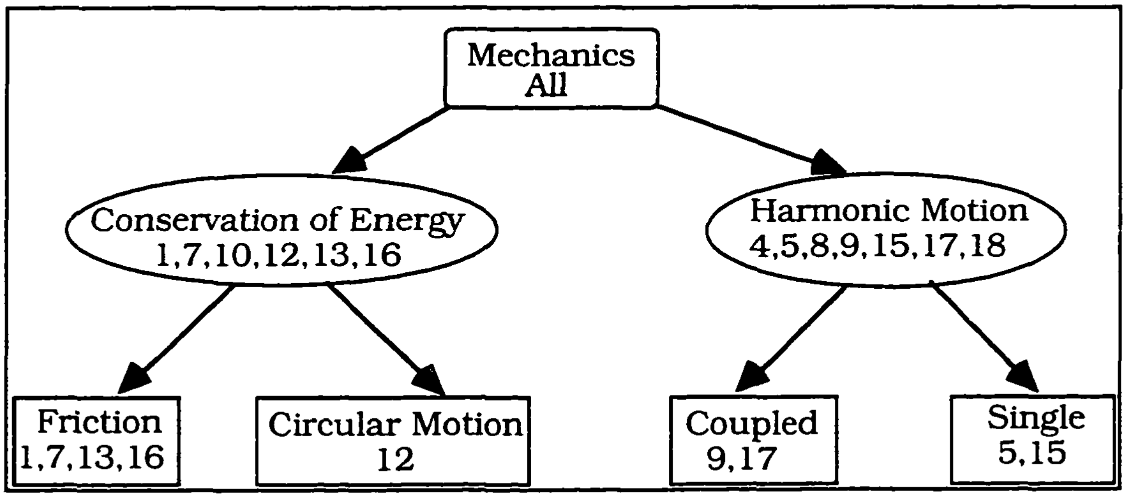


Figure 16. Representation From Intermediate 6.

The right branch of the tree in the representation of the categorization of Intermediate 6 is very similar to that of Novice 1 in the preceding discussion in that the top level category is the motion of the object and the bottom level is the objects that exhibit that motion. The second tree in the representation belongs to the theories used to solve the problems. Beneath the Mechanics category is the Conservation of Energy category and Friction and Circular Motion sub-categories. In these segments, Intermediate 6 begins with the large category of Conservation of Energy or equating forces and then separates the cards in this category by the forces present in each problem. Conservation of Energy is the name of a theory used to solve the problems. Circular motion belongs to the model-based sub-category motion of the object. Friction is a force that adds

complication to using conservation of energy and is a construction rule used to create the physical representation. Table 22 shows the segments in which the Conservation of Energy (Seg. 2.8), Friction (end of Seg 2.8), and Circular Motion (Seg 2.9) categories were explained.

Table 22

Segments 2.8-2.9 for Intermediate 6

Seg	Protocol	Action
2.8	and then problems that involve either conservation of energy or simply just equating forces doing free body diagrams and that sort of thing So those were the three bigger categories that I had And out of the three body diagram type of division I split things into ones dealing with friction or retarding force and those that were frictionless	ref (12), (10), (1,3,7,13,16) ref 1, 3, 7, 13, 16
2.9	and then there was one that was circular motion which is a little different view different forces types of forces	ref 12

In segment 2.8, Intermediate 6 begins with all the cards in the Conservation of Energy category. The subject discusses "equating forces" and "doing free-body diagrams" as steps needed to be taken to solve the problems. These statements are taken to be equivalent to

the steps needed to create the physical representation that Larkin (Larkin, 1983; 1985; Larkin et al 1980a; 1980b) refers to in her research. The subject in Larkin's study chose a theory, in this case conservation of energy or equating forces (Newton's Laws), and then used that theory's construction rules to create a representation of the problem (free body diagram). The first sub-category is the Friction category. Friction is a construction rule for these problems in that it is one of the forces that the could be used to make a free body diagram. Finally, the last category is a model-based sub-category belonging to the motion of the object, or in this case, rotational motion. The subject explains that rotational motion is different because there are different forces involved. Basically, the friction and rotational motion categories are similar in that there are different forces at play in representing the problem.

As discussed briefly in a previous section, the result that the intermediates used both model- and theory-based attributes provides a problem for views of knowledge representation in the literature. In general, Giere's (1994) representation of a hierarchy of models and the coherent view of knowledge connected by theories (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) only fit part of the results of the intermediates' categorizations. There is a part of the intermediates' categorizations that are similar to that of the novices' hierarchy of motion to object categories. Intermediates also used theory-based attributes in combination with these model-based attributes. As described in the previous section,

there is some indication that the intermediates' structures in terms of theories may not be hierarchical at the middle and low levels of categorization.

Representation of Experts' Categorizations

The categorizations made by experts were similar to those made by intermediates in that the experts used a combination of model- and theory-based classifications to categorize the problems in this study. The hypotheses for the experts' categorizations were:

Hypothesis 3a: Experts were expected to use the most general theory-based sub-category to solve the problem (THE) at the high level of categorization. At lower levels of categorization, experts were expected to use the construction rule (THE-CON) sub-category before using extension rules (THE-EXT). At the low level of categorization, experts were expected to use the most general model-based category, motion of the object (M-MOT).

Some of the hypotheses for the experts' categorizations were supported by the results of the study. Figure 17 shows a representation of the categories used by experts to categorize the problems in this study. Similar to the intermediates' categorizations, this representation is a mix of theory- and model-based classifications at the middle and low levels. At the top level, all of the categories were classified as theory-based categories. As hypothesized above, the theory-based sub-category most often used here is the theory used to solve the problems. There was a difference between use of this

theory-based sub-category at the high level of categorization and use at the middle or low levels. This suggests that there is a hierarchy in the experts' categorizations between classifications made at the high level and those at the middle and low levels of categorization. There were no differences in the use of any of the other sub-categories at any other level of categorization. The two branches of the representation in Figure 17 will be discussed separately.

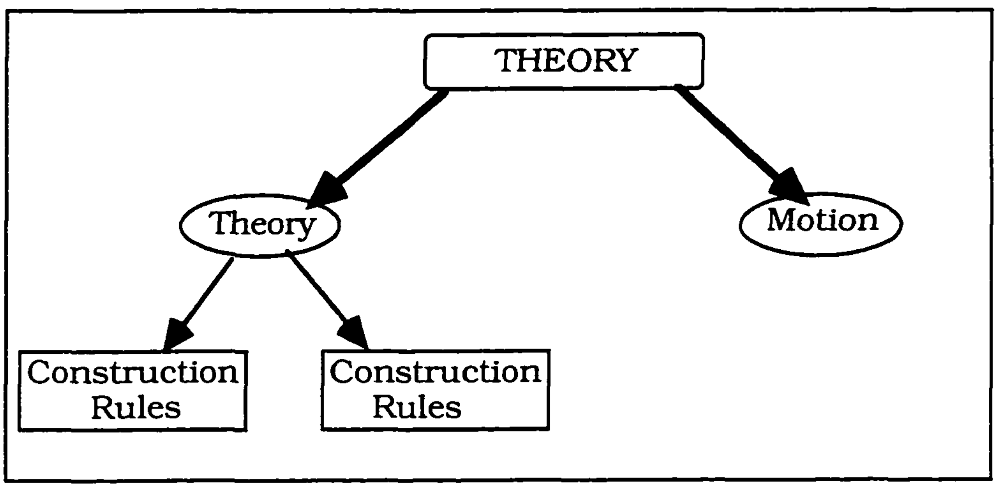


Figure 17. Representation of Experts' Categorizations.

On the theory tree, at the middle level there was no distinct preference for any of the three theory-based sub-categories: theory, construction rules, or extension rules. At the low level, the most used category was the construction rules which are used to create the physical representation of the problem. The use of construction rules was not different in frequency at the low level than at either the middle or high levels of categorization. The theory-based branch of the representation of the categorization that experts created is very

similar to that of the intermediates. There were also no preferences for any of the theory-based sub-categories at the middle and low levels of categorization. Earlier, it was suggested that the theory-based sub-categories were not hierarchical and instead were used at all levels of categorization. In the experts' categorizations, there is an indication that the theory used to solve the problems is a very general category at the top of the experts' hierarchies. This is because the name of the theory as a theory-based sub-category was used by experts more at the high level of categorization than at the middle or low levels. Beyond this highest level, there is no indication of a hierarchy. This means that the theory used to solve the problems was the most important attribute of the problems for the expert subjects in this study, but that this was where the hierarchy ended.

The most frequently used model-based sub-category by experts was the motion of the object which supports the hypothesis above. This is the same category that was used at the middle level of categorization by intermediates and novices. In the experts' categorizations, there was no difference in the use of the motion of the object as a category between the middle and low levels of categorization. Therefore, all categorizations made by experts that were model-based at all levels of categorization were most likely to have been based on the motion of the object. There was no hierarchy to the model-based branch of the categorizations made by experts as there was for novices.

The representation of the problems that Expert 9 created is exemplary of the experts' representations. Expert 9 categorized only at the high and middle levels, but there were not many distinctions between the these levels of categorization for experts as a group. The top level is the name of the theory used to solve all the problems. Table 23 shows parts of segment 2.10 for Expert 9 in which he explains the rationale for the category at the top level. This segment comes at the end of the categorization task, at which point Expert 9 combines all the cards into one large pile and explains the rationale for this combination.

Table 23

Part of Segment 2.10 for Expert 9

Seg		Protocol	Action
2.10		these problems tend to be hard for students in general for them they don't see the fact that they all are about the same thing	
	J	*yup*	
	EX9	they don't see the fact this situation looks like a brand new situation and they don't really see that by and large there are mathematical methods for solution but they all I think every one of them is the application of Newton's second law	ref ALL

Expert 9 classified all the cards in terms of the name of the theory used to solve the problems. Expert 9 explains that every one of the cards "is the application of Newton's second law". It is also interesting to note that Expert 9 makes a comment about students and how they approach each situation as a new one, i.e., they do not understand that every problem could be solved by Newton's second law (according to Expert 9). This segment provides support that the category made by Expert 9 at the high level of categorization refers to the mathematical methods for solution that all the problems have in common. Also, each problem is not a new situation, but is just another application of Newton's 2nd Law.

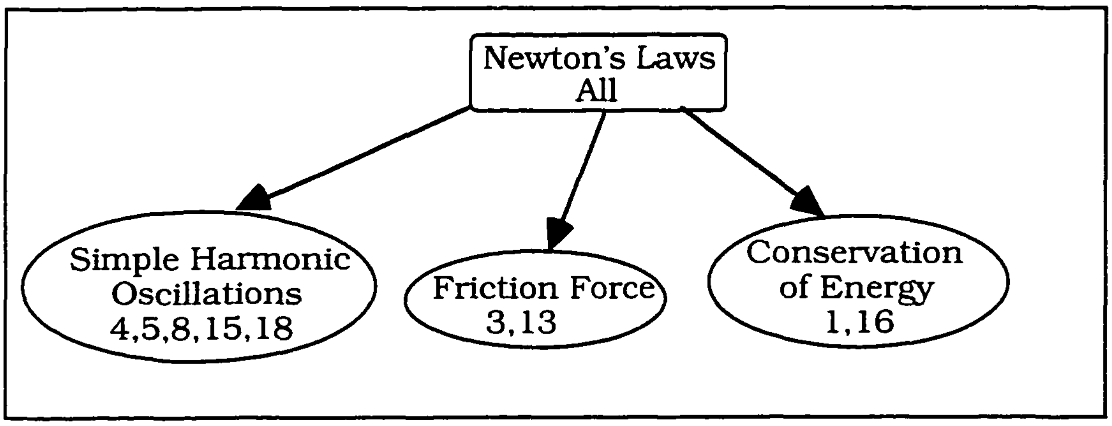


Figure 18. Representation From Expert 9.

The middle level that Expert 9 created was a combination of theory sub-categories and the motion sub-category. The Simple Harmonic Oscillations category is similar to the Harmonic Motion category that both Novice 1 and Intermediate 6 used in their

representations. The Simple Harmonic Oscillations category includes all objects that oscillate in a certain way. The Conservation of Energy category is also identical to the category that Intermediate 6 chose to categorize the problems at the middle level. It is the name of a theory used to solve the problems in that category. The category Friction Force is a construction rule which is a theory-based sub-category. The friction force is one of the forces used to create the physical representation (free body diagram) necessary for solving the problem. In Segment 1.9 in Table 24, Expert 9 first defines this category in relation to problem 13.

Table 24

Segment 1.9 for Expert 9

Seg	Protocol	Action
1.9	block of mass two m rests on a plane whose coefficient of friction is mu kay so this is a problem which is also related to forces and coefficients of frictions and um that is a new concept because you have to worry about at one point you have to worry about the forces	ref 13

Expert 9 noticed this problem was related to "forces and coefficients of frictions". This was important to him since this was "a new concept" in which you had to "worry about the forces". Therefore, because there was a coefficient of friction listed in the problem description,

Expert 9 said that friction would be needed to solve the problem. The fact that there was friction as a component of this problem made it different from others in that a new force would be needed to solve the problem. In a later segment, this category, with card number three, was called Friction Force to distinguish it from problems in which the friction force was not important to solving the problem. This new force changes the physical representation of the problem and is an important construction rule used in this and other problems.

The experts' categorizations were difficult to interpret in terms of Giere's (1994) representation of knowledge in physics as a hierarchy of models and the coherent view of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) which represents knowledge in general as connected by theories. The experts' representations do not support Giere's (1994) hierarchy in that there is no indication of a hierarchy of models. Experts used only the most general attribute of the conceptualized models and only at the lower levels of categorization, namely, the motion of the object. This could be due to the nature of the task requested of the subjects. That is, in previous research, categorization tasks used to investigate knowledge structure asked subjects to list <u>any</u> attributes that may be associated with a particular concept (Rosch, 1973; 1978; Rosch et al, 1976). The task in the present study was different in that it required subjects to make categories until they had told everything that they thought was <u>important</u> to know about the problems, rather than <u>any</u> attributes.

The expert subjects may realize that there are other categories that could be made, but those categories are not necessarily "important" in terms of the problem solution. This result has been found in the expertise literature in general, and has been called the "intermediate peak effect" (Anderson, Greeno, Kline & Neves, 1981; Neves & Anderson, 1981). This effect has been used to explain the result that when expert computer programmers were asked to describe their knowledge of a particular program, they listed only high level, more abstract knowledge; whereas intermediates described all their knowledge of a particular program without discerning what was important to know. In the present study, the experts' categorizations were not as detailed as the intermediates' categorizations. This may be because of the "peak effect" in which the intermediates did not know what information was important to know and thus reported more information than did the experts.

Combined Representation of Knowledge in Physics

In the preceding sections, the categorizations of novices, intermediates, and experts were represented in terms of the results of the analyses conducted to compare the use of model-, and theory-based categories and sub-categories. Novices categorized the problems using the more general model-based category, i.e., motion of the object, at the middle level of categorization and the more specific category, i.e., object in the problem, at the low level of categorization. Novices did not categorize at the high level of categorization and did

not use theory-based categories to a significant degree. This categorization supports Giere's (1994) representation of physics knowledge in which models are hierarchically arranged with more general attributes defining models at the top of the hierarchy and more specific attributes defining those at the bottom.

At the high level of categorization, experts and intermediates categorized the problems by the theory used to solve the problems. Experts and intermediates used a combination of model- and theory-based categories at the middle and low levels of categorization. Intermediates used the motion of the object to categorize problems at the middle level of categorization and the object in the problem to categorize problems at the low level of categorization. This also appears to support Giere's (1994) hierarchy of models. However, intermediates and experts both used theory-based attributes at the low and middle levels with no preference for either of the three theory-based sub-categories: theory, construction rules, and extension rules. These results suggest that there is no hierarchical ordering of the theory-based sub-categories at the middle and low levels of categorization. The use of theory-based categories to classify the problems in this study supports the coherent view of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) in which knowledge in general is connected by theories.

The problem here is that neither Giere's (1994) representation of a hierarchy of models nor Medin and Murphy's view of coherent

categories(Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) can explain the results obtained in the categorization task in this study. For this reason, it is suggested that a combination of these representations is needed to capture the structure of knowledge in physics. The representation in Figure 19 shows a combination of the representations of the novices', intermediates', and experts' categorizations. The highest level of this representation is the theory used to solve the problem which came from the experts' representations in which the theory that provided a set of principles used to solve the problems related all the problems in the set. There are then two branches from the theory category for the model- and theory-based attributes used by the subjects in this study.

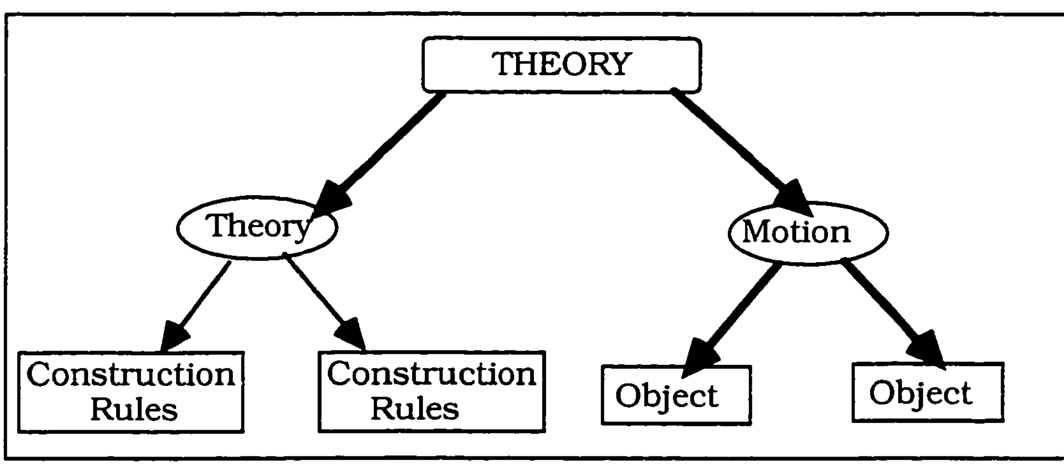


Figure 19. Representation of Combined Categorizations.

The branch on the right side of Figure 19 represents the model-based attributes used by experts, intermediates, and novices. The higher (more general) category in this tree is the motion of the object.

The lower (more specific) category is described by the object in the problem. The branch extending from the left of the theory category represents the theory-based categorizations made by experts and intermediates. At the higher level of this categorization, a combination of all the theory-based categories are used. The lower level on the theory tree is described by the construction rules which are used to construct the model of the problem.

The two branches of the representation in Figure 19 can be represented separately by Giere (1994) and Medin and Murphy (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). The first (right) branch is the model branch that is represented as a hierarchy of models with more general models at the top of the hierarchy and more specific at the bottom. However, the top attribute in the hierarchy is the theory used to solve the problems. The name of a theory represents a set a principles, not a model. Therefore, the hierarchy of models starts with the motion model as the most general model represented. The theory used to solve the problems will be represented as a circle encompassing the hierarchy of models. The problems, through this theory lens, are represented in terms of models. Figure 20 shows a combination of Giere's (1994) hierarchy of models and the theory used to create them.

This representation is only a slight modification of Giere's (1994) representation of models in which physics knowledge is represented as a hierarchy of models with more specific models at the bottom of the hierarchy and more general models at the top. In this

representation, a more general model is defined by the motion of the object in the model. A more specific model is defined by the object in the model. This hierarchy has a highest point at which there are no more abstract attributes to use in order to categorize the models. The representation of the theory used to solve the problems is not contradictory to Giere's representation. Giere (1988) proposes that a theory is defined as: (a) a family of models which are created by that particular theory, and (b) a set of statements which relate those models to real world systems. This representation is not far from what Giere intended for the hierarchy of models, the only difference is that the idea that these models represent a theory is made more clear. In essence, this representation allows for individual theories to be represented by hierarchies of models.

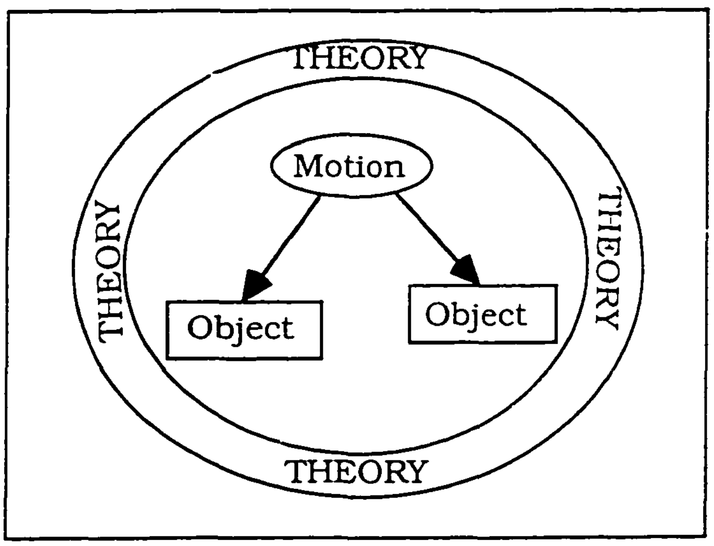


Figure 20. Representation of the Model Tree.

The left branch of the tree in Figure 19 consists of the theory-based categories used by experts and intermediates in this study. In this branch there are no differences between the use of the theory-based sub-categories at the middle and low levels. This suggests that the use of theory-based attributes to categorize the problems is not hierarchical. This means that once the name of the theory used to solve the problems is used to describe all the cards, there is no preference for any of the theory-based sub-categories to categorize the problems. This representation is predicted by Medin and Murphy (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) who propose that concepts are connected in a network by principles of a theory. The most general category is still the name of the theory used to solve the problem and will be represented as a circle as it was in the model representation. The middle and low level classifications used to categorize the cards were the principles of the theory used to solve the problems. These principles are represented by the connections between the concepts in the representation. The resulting representation is shown in Figure 21.

The representations in Figure 20 and 21 portray only part of the categorizations made by the expert, intermediate, and novice subjects in this study. In the representation of the model tree, which supports Giere's (1994) representation of knowledge in physics, there is a hierarchy of models, but there are no connections between these models. In the representation of the theory tree, which supports the coherent view (Medin & Wattenmaker, 1984; Murphy & Medin, 1985;

Spalding & Murphy, 1996) of knowledge as connected by theories, there are connectors without entities to be connected. The reason that there are no entities to be connected in the coherent view of category construction could be that Murphy and Medin's theory was developed for general knowledge as opposed to domain specific knowledge. It does not specify what kind of knowledge is connected by theories, only that knowledge in general is connected.

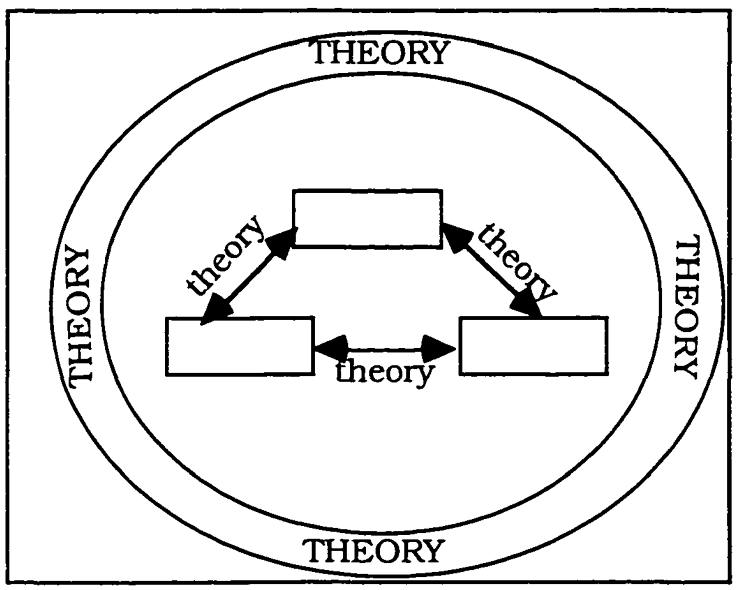


Figure 21. Representation of Theory Tree.

It is suggested in the present study that physics knowledge, as depicted through the categorizations in this study, is best described as a combination of these two representations. In this representation in Figure 22, the theory used to solve the problems is shown as a circle which encompasses all of the models. The circle represents the theory used to solve the problems and is the most general attribute of

physics problems that all the cards in this study have in common. In the literature on problem representation (Larkin, 1983; 1985; Larkin et al 1980a; 1980b) it was found that in order to solve a problem, an expert first chose an appropriate theory and then created a physical representation of that problem. In Figure 22, the problems are represented by the models shown within the name of the theory used to create those models. Those models are arranged in a hierarchy with the more general model described by the motion of the object at the top of the hierarchy and the more specific model described by the objects in the problem at the bottom. The motion models are connected by any of the principles named in the theory-based subcategories. The object models are connected to the motion models through the principles used in solving the problem.

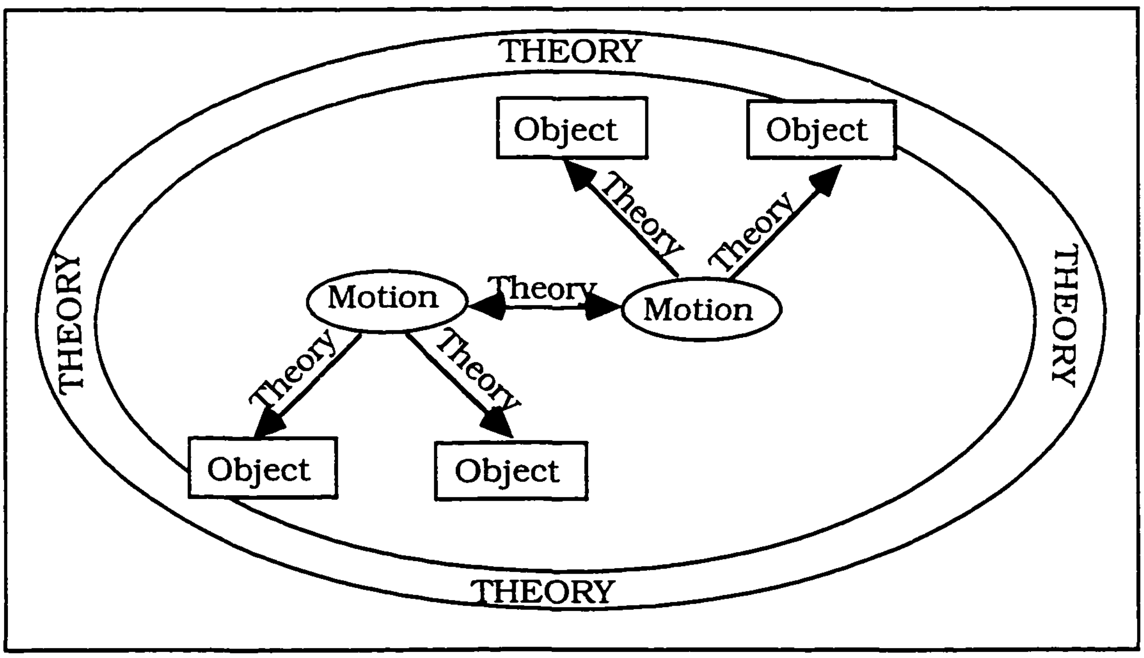


Figure 22. Final Combined Representation.

This representation of physics knowledge is a combination of the theories of Giere (1994) and of Medin and Murphy (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). This representation is an extension of these theories of knowledge representation in terms of the current study of the knowledge structure of expert, intermediate, and novice subjects in physics. Prior to this study, there was no empirical evidence to support either representation. The evidence Giere (1994) used to support his representation had methodological limitations that did not allow for this hierarchical representation to be supported (Chi et al, 1982). The subjects in Chi's study were not allowed to categorize problems more than once which is required in order to obtain a possible hierarchical set of categories. The evidence used to support the coherent view of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996) was based on more basic concepts, i.e., furniture, birds, which are not as complex as the problems used in this study. In the present research study, a model of knowledge structure for physics, a semantically-rich domain, is presented. This study has alleviated both these limitations and provided a combined view of physics knowledge in terms of models and theories. Further elaboration and support of this model may be obtained through the use of other areas in physics or other semantically complex domains.

Implications for Research

By situating this research within previous studies (Chi et al, 1981; 1982; Larkin, 1981; 1983; Larkin et al, 1980; de Jong & Ferguson-Hessler, 1986) this study extends and has implications for the study of expertise in physics. In particular, this research extends the results of previous research by providing more information about the structure of novices' knowledge. In previous studies, novices were described as using surface features to categorize problems which implies that novices' knowledge is superficial (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990). However, in the present study, the novices were able to identify more general attributes of the problems than just the surface features. The object in the problem was used by novices as an attribute to classify the problems at the low level of categorization. These results are a replication of the categorization found in many of these studies (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990). However, at the middle level of categorization, novices in this study chose the motion of the object as a more general category than the different objects in the problems.

The result that novices can make coherent categories such as motion reveals a deeper understanding of the problems than allowed by the "surface feature" result of many other studies (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Veldhuis, 1990). It might also be argued that the novices in this study are not acting at the basic level as Giere (1994) predicts. Recall that the basic level is a

term used by Rosch (1973; 1978; Rosch et al, 1976) to describe an intermediate level in a hierarchy of concepts in which members have more visual similarity with each other and their members than those at a higher level. Giere (1994) suggests that novices in Chi's (1981) categorization study were working at this intermediate level where models were not the most specific or the most general. This notion of novices working at the basic level was not borne out by the results. The highest and most general category novices used in this study was the motion of the object which was not a basic level category. Consider the categories: harmonic motion, oscillator, spring, pendulum. In this list, the object category "oscillator" has visual similarity to the spring and pendulum categories. An oscillator is a more general category that includes all things that oscillate including both springs and pendulums. However, harmonic motion is not visually similar to oscillators, springs, or pendulums; rather, harmonic motion is the way in which oscillators, springs, and pendulums move. Therefore, the object models, rather than the motion models, are probably at the basic level in this study. The motion models such as harmonic motion are at a more general level. It is at this more general level that the novices in this study are situated. However, this research was not designed to determine the basic level in physics knowledge and further research would be warranted to make a stronger claim about the basic level in physics knowledge.

By employing a reiterative categorization task of physics problems, this research may have implications for research on categorization in general. In much of the previous research on categorization (Rosch, 1973; 1978; Rosch et al, 1976), the items to be categorized were exemplars of everyday concepts that were described in terms of their physical attributes. In this research, the items to be categorized are physics problems which can be described on many different levels including the models and theories used to solve problems. The results of the categorization of the problems in the current study were more fully described by a combination of the probabilistic (Rosch, 1973; 1978; Rosch et al, 1976) and coherent views of category construction (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). At low levels, categories in this study were probabilistic and constructed using attributes of the exemplars of the categories. At higher levels, these exemplars were categorized according to theories. It is not likely that physics knowledge is unique in this respect. There may be other domains which include knowledge that is better represented by this combined view since there are many other domains that contain more semantically rich concepts than the everyday concepts used in earlier studies.

This research also has implications for research on knowledge in physics. The categorizations in this study were represented by a combined view of physics knowledge in which theories are represented by clusters of models connected by principles of those theories. The

attribute of the models used to define a more abstract model was the motion of the object; the object in the problem defined a more specific model. This is probably not the case outside of the topic area of Mechanics from which the problems in this study were taken. An investigation of the attributes of models in other topic areas in physics such as Electricity and Magnetism that are used to categorize general and specific models may be fruitful. In addition, reiterative categorization studies in other topic areas may also contribute support to the combined view of physics knowledge which has thus far only be substantiated by the present study for the area of Mechanics.

Implications for Teaching

This research also has implications for physics pedagogy. The understanding of the connections between models and theories at different levels of expertise gained from this study may be useful as a model of knowledge in physics to be used in the classroom. Recall that the classical and probabilistic views of concepts were used to explain how concepts were learned both in Klausmeier's (1990) and Ausubel's work (1963) and to explain conceptual change in work by Chi, Slotta and de Leeuw (1994). Briefly, in Klausmeier's scheme, learning a concept depended on learning the attributes of the concept, and differentiating these attributes. For Chi et al (1994), conceptual change is the process of re-categorizing a concept from its incorrect to correct ontological tree. In physics, learning may be viewed in terms

of the knowledge structure elaborated on in the present research. In particular, learning a physics theory may depend on learning the conceptual models that compose the hierarchies by differentiating them from each other in terms of their attributes.

Many researchers have used models in order to teach theories in physics. Hestenes and Halloun (Halloun, 1995; 1996; Halloun & Hestenes, 1985; 1987; Hestenes, 1987; 1992) provide a very detailed description for instruction using modeling techniques. Brown and Clement (Brown, 1993; Brown & Clement, 1989) use teaching models or analogies to facilitate students' conceptual change toward more scientific models. Finally, White and Frederiksen (White, 1993; White & Frederiksen, 1990) have developed a computer tool to help students investigate model development in physics.

Researchers have suggested that novices not only need knowledge of physics models, but they also need to know how to connect these models into coherent theories (deJong & Ferguson-Hessler, 1986; diSessa, 1983; 1993; Robertson, 1990; Smith, diSessa, & Roschelle, 1994). This suggestion is also substantiated by the results of this study. As the results of this study indicate, novice subjects have been successful in structuring the conceptualized models in physics into hierarchies. However, these subjects were not successful in connecting these models together in terms of a theory. In order to change this, it may be possible to use the representation of knowledge described in this research to make students aware of the

role of theories in model-making and in problem-solving. A description of such an attempt is described in the following section.

Physical Science in Preservice Elementary Education - SCI 180

The representation of models organized into families connected by the theories that created them has been used to facilitate meaningful learning experiences in the physics course for pre-service Elementary Education students at Western Michigan University. The representation described earlier was used to create a "Model Map." A Model Map is a set of models chosen to represent a particular theory. These models are connected by the principles used to create them and clustered into hierarchies defined by their attributes. A sample Model Map for the section on Mechanics can be seen in Figure 23. The Model Map was created in order to organize and structure the content of the course. The course itself is organized around activities in which phenomena are presented to students as problems. The students are given the task of describing and explaining the phenomena, or making models. Problems or phenomena are chosen to represent the most inclusive models and the key parts of the theory. For example, the Model Map for the motion section consists of four basic models which compose the "object" level of the hierarchies: Stomper Car, Incline Car, Hot Wheels Car, and Free Fall. These are basic objects that illustrate the basic models that can be made using Newton's Laws. The Stomper Car is a battery operated car that has a constant velocity and is an example of an object that is in motion, but whose motion

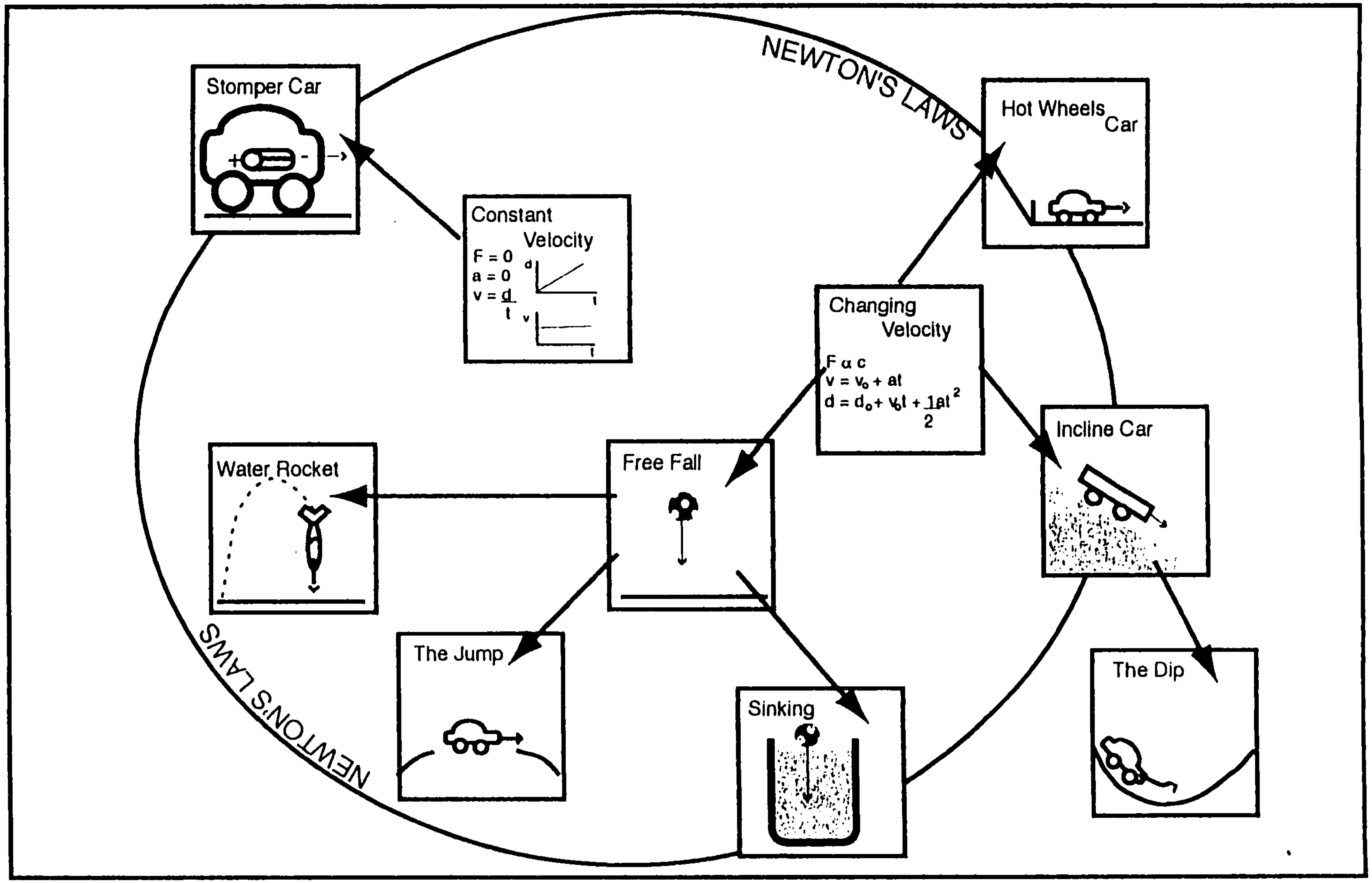


Figure 23. Model Map for Mechanics.

does not change. The Free Fall object is acted on only by gravity and is a simple example of Newton's 2nd Law. The Incline Car adds the normal force to the force of gravity. Many students infer the normal force from some wording of Newton's 3rd Law. Finally, the Hot Wheels Car adds friction to the model.

The four basic models have both higher and lower level models attached to them which are more abstract and more specific, respectively. The two higher level models describe the two kinds of motion that are exhibited in these four objects. One is the case of an object with no external forces where the velocity is constant. The second is the case where the external force is non-zero and constant, and the velocity is changing. The lower level models are more conceptually difficult situations that use the four basic level models. For example, the model for The Jump is identical to the Free Fall model except that it involves an object rising as well as falling. Also, the object in The Jump moves in two dimensions. However, the model itself only involves the force of gravity as in the Free Fall model. All of the models in the motion section combine to illustrate Newton's Laws.

This organization is constructed by the learners through inquiry experiences. During the first weeks of the course, students work together in groups of four and as a class on describing the motion of the objects in the four main models. When the students have become comfortable with the concepts of distance, time, velocity, and acceleration and particularly with the graphs associated with these concepts, they are asked to ponder the question of why things move

the way that they do. Many of these students have had some prior experience with the idea of force and with Newton's Laws and many suggest this as the reason for the motion. However, knowing that Newton's Laws explain the motion is much easier than knowing how!

After the students have created the complete models for the four main types of motion, they are given a problem solving activity. The activity required students to make models of a toy car traveling along The Dip, The Jump, and then Sinking. The students are asked to describe and explain the motion of the object in groups of four and present their model to the class. One model that causes particular problems is The Dip. The velocity vs. time graph shows a changing acceleration. This is the first time that acceleration has not been constant. The students usually decide that The Dip is just two Incline Cars front to front. However, this usually causes them to model the acceleration as being constant on the way up and constant on the way down. Instead of having a curved dip, the students' models are usually of a "V" shaped curve.

All students were able to choose procedures to describe the phenomenon and to propose a model for solution. The largest difficulties in this activity arose from interpreting the data in relationship to a theory. Some groups were still uncomfortable using theories to interpret data and were employing a more empiricist notion of science that did not allow them to understand the nature of the assignment. In general, students understood the Model Map and its connections to the new situations and often explained the new

model in relation to the other more well described and understood models. For example, most students were able to extend the Free Fall model to the case of The Jump.

In general, the Model Maps were a useful tool in organizing content in terms of the explicit connections between the models and theories of physics. Students were able to extend their understanding of simple physics models to more complicated phenomena. In addition to content knowledge, the students developed an understanding of the nature of models and theories.

Summary and Conclusions

The objective of this research was to investigate the knowledge structure in the domain of physics by describing the knowledge of experts, intermediates, and novices. A review of the literature on expertise, knowledge in physics, and conceptual structure provided two competing representations of physics knowledge. The first representation of knowledge, proposed that physics knowledge is a hierarchy of models with more general models at the top of the hierarchy and more specific models at the bottom (Giere, 1994). The second representation of knowledge suggested that knowledge in physics is connected by theories (Medin & Wattenmaker, 1984; Murphy & Medin, 1985; Spalding & Murphy, 1996). Results of categorization and problem representation studies with novices supported the model-based representation and results of studies with experts supported the theory-based representation (Chi et al, 1981;

1982; deJong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990).

In order to investigate these two representations, a reiterative categorization task was employed. This task resulted in a hierarchy of categories with larger piles having more general attributes in common at the top of the hierarchy and smaller piles having more specific attributes in common at the bottom. The categories were classified in the study as either theory-, model- or mathematical model-based categories. The proportions of model-, theory- and mathematical model-based categories pooling over all levels of categorization were calculated and compared by expertise. Similar results were obtained in the present study as in other studies of categorization (Chi et al, 1981; 1982; deJong & Ferguson-Hessler, 1986; Larkin, 1983; 1985; Larkin et al 1980a; 1980b; Veldhuis, 1990). Experts and intermediates used theory-based categories more than novices pooling over all levels of categorization. Novices used model-based categories more than did both experts and intermediates.

In order to extend of these results, the proportions of model-, theory-, and mathematical model-based categories at each level of categorization were compared by level of expertise. These comparisons were also done in order to compare the subjects' categorization with the competing representations defined by models and theories, respectively. Again, novices used model-based categories at all levels of categorization. Experts and intermediates used a combination of model- and theory-based categories at the

middle and low levels and all theory-based categories at the highest level of categorization.

Finally, proportions of the use of model- and theory-based sub-categories were compared at each level of categorization for each level of expertise in order to describe the structure more fully. The novices' categorizations can be described as a hierarchy of models with motion as the most general category and object as a sub-category. The highest, most general, level for both intermediates and experts was the theory used to solve the problems. The middle and lower levels combined model- and theory-based categories. The novices' hierarchy of models from motion to object in the problem was found to be a sub-set of the intermediates' representations. The motion of the object category was the lowest level category in the experts' representation.

The novices' categorizations supported Giere's (1994) representation of a hierarchy of models. However, the experts' and intermediates' categorizations were not fully described by this model. Instead, experts and intermediates used theories to describe the problems at all levels of categorization.

To alleviate this situation, a combined representation of physics knowledge based on both theories and models was proposed. In this representation, a hierarchy of models described from the novices' categorization is from general to specific and is connected by the principles of a theory used to create them. These clusters of models are encompassed by a particular theory which was used to create all the models.

This study has implications for research in that it extends the results of previous research by providing more information about the structure of novices' knowledge. In particular, this research suggests that novices' knowledge is not as superficial as suggested. In addition, by using a semantically rich domain such as physics, this research has extended the research in categorization in general. This research also has implications for research about knowledge in physics and how it may be structured. Finally, this research also has implications for instruction in suggesting a model for teaching physics in a deeper way.

Appendix A

Protocol Clearance From the Human Subjects Institutional Review Board

Human Subjects Institutional Review Board

Kalamazoo, Michigan 49008-3899

WESTERN MICHIGAN UNIVERSITY

Date: 31 March 1997

To: Janice Gobert, Principal Investigator
Jennifer Discenna, Student Investigator

From: Richard Wright, Chair

Re: HSIRB Project Number 97-03-16

This letter will serve as confirmation that your research project entitled "A Study of Knowledge Structure in Physics Among Experts, Intermediates and Novices" has been **approved** under the **expedited** category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note that you may **only** conduct this research exactly in the form it was approved. You must seek specific board approval for any changes in this project. You must also seek reapproval if the project extends beyond the termination date noted below. In addition if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: 31 March 1998

Appendix B

Problems Used in Categorization Task

Card #	Problem Narrative
1	A skier weighing 90 kg starts from rest down a hill inclined at 17°. He skies 100 m down the hill and then coasts for 70 m along level snow until he stops. Find the coefficient of kinetic friction between the skis and the snow. What velocity does the skier have at the bottom of the hill?
2	A projectile is fired with an initial velocity v_0 at an elevation angle of a up a hill of slope b (a > b). (a) How far up the hill will the projectile land? (b) At what angle a will the range be a maximum? (c) What is the maximum range?
3	Two blocks, each of mass *M*, are connected by an extensionless string of length *L* One block is placed on a smooth horizontal surface, and the other block hangs over the side, the string passing over a frictionless pulley. Describe the motion of the system (a) when the mass of the string is negligible and (b) when the string has a mass *m*.
4	A simple harmonic oscillator consists of a 100g mass attached to a spring whose force constant is 10^4 N/m and which is sitting in a resisting medium. The mass is displaced 3 cm and released from rest. After 10s, the maximum amplitude decreases to half of the initial value. Calculate the frequency n_1 (compare with the undamped frequency n_0).
5	Two masses m_1 and m_2 slide freely on a horizontal frictionless surface and are connected by a spring whose force constant is *k*. Find the frequency of oscillatory motion for this system.
6	A rocket has an initial mass of 7 x 10^4 kg and upon firing burns its fuel at a rate of 250 kg/s. The exhaust velocity is 2500 m/s. If the rocket has a vertical ascent from resting on the earth, how long after the rocket engines fire will the rocket lift off? What is wrong with the design of this rocket?
7	Find the displacement and velocity of a particle undergoing vertical motion in a medium having a retarding force proportional to the velocity.

Card #	Problem Narrative
8	A mass *m* moves in one dimension and is subject to a constant force $+F_o$ when $x < 0$ and to a constant force, $-F_o$ when $x > 0$. Describe the motion by constructing a phase diagram. Calculate the period of the motion in terms of *m*, F_o and the amplitude *A* (disregard damping).
9	Two identical harmonic oscillators are placed such that the two masses slide against one another. The frictional force provides a coupling of the motions proportional to the instantaneous relative velocity. Discuss the coupled oscillations of the system.
10	A projectile of mass *M* explodes while in flight into three fragments. One mass ($m_1 = M/2$) travels in the original direction of the projectile, mass m_2 (= $M/6$) travels in the opposite direction, and mass m_3 (= $M/3$) comes to rest. The energy *E* released in the explosion is equal to five times the projectile's kinetic energy at the explosion. What are the velocities of m_1, m_2 and m_3?
11	A student drops a water filled balloon from the roof of the tallest building in town trying to hit her roommate (who is too quick). The first student ducks back but hears the water splash 4.021 s after dropping the balloon. If the speed of sound is 331 m/s, find the height of the building, neglecting air resistance.
12	A light string of length *a* has bobs of mass m_1 and m_2 ($m_2 > m_1$) on its ends. The end with m_1 is held and whirled vigorously by hand above the head and released. Describe the subsequent motion, and find the tension in the string after release.
13	Two blocks of unequal mass are connected by a string over a smooth pulley. One mass is set on a plane whose coefficient of friction is m $_k$. What angle q of the incline allows the masses to move at a constant speed?
14	A particle is projected with an initial velocity v_0 up a slope that makes an angle a with the horizontal. Assume frictionless motion and find the time required for the particle to return to its starting position.

15	A simple pendulum of length *b* and bob with mass *m* is attached to a massless support moving horizontally with constant acceleration *a*. Determine the period for small oscillations.
16	An automobile driver traveling down an 8% grade slams on his brakes and skids 30 m before hitting a parked car. A lawyer hires an expert who measures the coefficient of kinetic friction between the tires and the road to be m $_k$ = 0.45. Is the lawyer correct to accuse the driver of exceeding the 25 MPH speed limit? Explain.

Card #	Problem Narrative
17	A simple pendulum consists of a bob of mass *m* suspended by an inextensible (and massless) string of length *l*. From the bob of the pendulum is suspended a second, identical pendulum. Consider the case of small oscillations (such that sinq @ q), and calculate the characteristic frequencies. Describe also the normal modes of the system.
18	A simple harmonic oscillator consists of a 100 g mass attached to a spring whose force constant is 10^4 N/m. The mass is displaced 3 cm and released from rest. Calculate (a) the natural frequency n_0 and the period t_0.

Appendix C

Category Codes for All Problems

card #	MODEL				Mathematical Model MTH-M	THEORY		
	Object M-OBJ	Object Attributes M-ATT	Unknown M-UNK	Motion M-MOT		Theory THE	Construction rules THE-CON	Extension Rules THE-EXT
1	Incline plane Skier Surface	Distance Slope 1-dimension	Coefficient of kinetic friction Velocity	Linear, projectile, downhill or dimensional motion Kinetics	Constant Force F = c Simultaneous equations	Newton's Laws/F=ma Mechanics Conservation of energy	Gravitational , frictional, normal or nonconservative force Friction Work Kinetic or potential energy	Equations of motion Kinematics
2	Projectile Incline plane	Velocity Elevation Slope 1-dimension	Distance Range Incline	Trajectory Projectile, dimensional, or linear motion Kinetics	Constant Force F = c Simultaneous equations	Newton's Laws/F=ma Mechanics Conservation of energy	Gravitational or conservative force Work Kinetic or potential energy	Equations of motion Kinematics
3	Blocks on a string Pulley Incline plane	Mass Length 1-dimension Velocity Frictionless	Description of motion	Horizontal, projectile, linear, or dimensional motion Kinetics Hanging	Changing Force $F \alpha x$ Simultaneous Equations Differential EQ	Newton's Laws/F=ma Mechanics Conservation of energy	Gravitational , normal, tension or conservative force Work Kinetic or potential energy	Equations of motion Kinematics Lagrange's equation
4	Simple harmonic oscillator Spring Resisting medium	Force constant Mass Displacet	Frequency	Oscillatory, up/down or harmonic motion Harmonic oscillator	Force proportional to velocity $F \alpha v$ Differential eq	Newton's Laws Mechanics Wave mechanics	Spring, damping, frictional, gravitational or nonconservative force Friction	Equations of Motion

	MODEL				Mathematical	THEORY		
card #	Object M-OBJ	Object Attributes M-ATT	Unknown M-UNK	Motion M-MOT	Model MTH-M	Theory THE	Construction rules THE-CON	Extension Rules THE-EXT
5	Masses Surface Spring Simple oscillator	Mass Coeficient of friction Force constant Frictionless	Frequency	Oscillatory, side to side or harmonic motion Harmonic oscillator	Force proportional to x F α x Differential eq Linear algebra	Newton's Laws/F=ma Mechanics Conservation of energy Wave mech	Spring, normal , gravitational or conservative force Elastic or potential energy	Lagrange's equations Normal modes
6	Rocket Fuel	Mass Burn rate Exhaust velocity Variable mass Retarded acceleration	Time	Trajectory Vertical , projectile or dimensional motion Kinetics	Simple Differential Equation	Newton's Laws/F=ma Mechanics Conservation of momentum	Momentum of Fuel and Rocket Change in momentum	Equations of motion Kinematics
7	Particle Retarded force	Direction Retarded acceleration	Displace- ment Velocity	Projectile, vertical, dropping or dimensional motion Kinetics	Force proportional to velocity plus a constant F α v + c Differential eq	Newton's Laws/F=ma Mechanics Conservation of energy	Gravitational , frictional, damping or nonconservative force Work Friction Kinetic/Potential Energy	Equations of motion Kinematics
8	Particle Mass Force Simple oscillator	Mass Amplitude	Phase diagram Period	Oscillatory, sideto side, or harmonic motion Harmonic oscillator	Constant force changing in distance F α x Differential eq Time dependent	Newton's Laws/F=ma Mechanics Conservation of energy Wave mechanics	Nonconservative force	Equations of motion Kinematics

card #	MODEL				Mathematical Model MTH-M	THEORY		
	Object M-OBJ	Object Attributes M-ATT	Unknown M-UNK	Motion M-MOT		Theory THE	Construction rules THE-CON	Extension Rules THE-EXT
9	Coupled oscillator Surface Spring	Coeficient of friction	Coupled oscillations of the system	Oscillatory, up & down, or harmonic motion Harmonic oscillator	Force proportional to x F α x Differential equation Linear algebra	Newton's Laws/F=ma Mechanics Conservation of energy Wave mech	Spring, frictional, normal or nonconservative force Work Kinetic or potential energy	Equations of motion Kinematics Lagrange's eq Normal modes
10	Projectile	Mass	Velocity	Trajectory Projectile or dimensional motion kinetics	Constant Force F = c Differential equation	Conservation of energy Conservation of momentum Newton's Laws/F=ma Mechanics	Work Kinetic or potential energy Change in momentum	Equations of motion Kinematics
11	Balloon Building Sound	Mass Height Speed 1-dimension Velocity	Height Distance Travelled	Linear, dropping, vertical, dimensional, or projectile motion Kinetics	Constant force F = c Differential equation Simultaneous eq	Newton's Laws/F=ma Mechanics Conservation of energy	Gravitational or conservative force Work Kinetic or potential energy	Equations of motion Kinematics
12	String Bobs Coupled pen-dulum Coupled osscill.	Length Mass	Tension	Circular, rotational, angular or oscillatory motion Kinetics Harmonic oscillator	Constant force F=c	Newton's Laws/F=ma Mechanics Conservation of energy or angular momentum	Tension, conservative, gravitational or centripetal force Kinetic or potential Energy	Equations of motion Kinematics

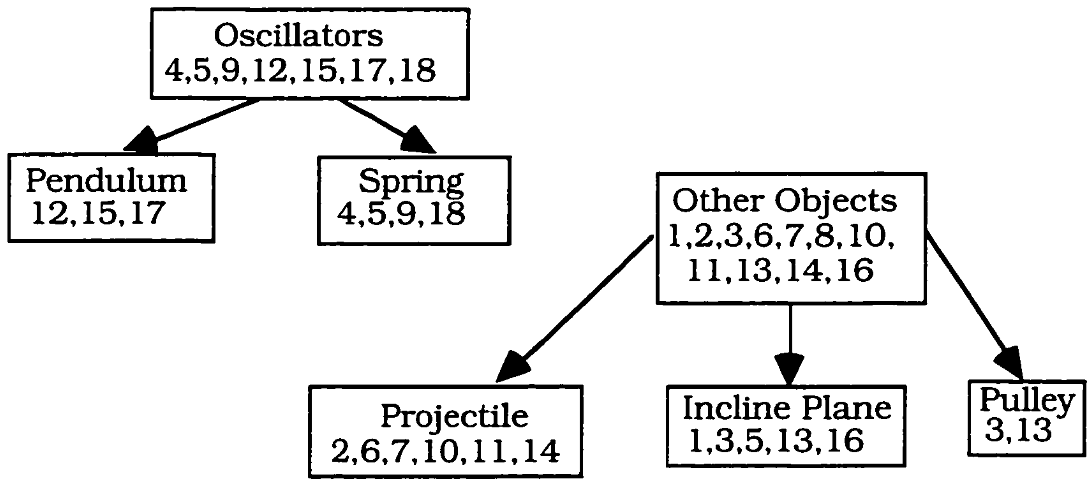
card #	MODEL				Mathematical Model MTH-M	THEORY		
	Object M-OBJ	Object Attributes M-ATT	Unknown M-UNK	Motion M-MOT		Theory THE	Construction rules THE-CON	Extension Rules THE-EXT
13	Pulley Blocks on a string Incline plane	Mass Length Coeficient of friction Velocity Angle/slope	Angle of incline	Dimensional, linear or projectile motion Kinetics	Constant Force F=c	Newton's Laws/F=ma Mechanics Conservation of energy	Tension, normal, frictional, gravitational or nonconservative force Friction Kinetic or potential Energy	Equations of motion Kinematics
14	Particle Incline plane Surface	Velocity Angle/slope Frictionless 2-dimension Range	Time	Trajectory dimensional, shooting or projectile motion Kinetics	Constant Force F=c Differential eq Simultaneous eq	Newton's Laws/F=ma Mechanics Conservation of energy	Gravitational, frictional, normal or conservative force Kinetic or potential Energy Friction	Equations of motion Kinematics
15	Pen-dulum Bob Single oscillator	Length Mass Accleration	Period for small oscillations	Oscillatory, harmonic or side to side motion Harmonic oscillator	Force proportional to x $F \alpha x$ Differential equation	Newton's Laws/F=ma Mechanics Conservation of energy Wave mechanics	Tension, constant or gravitational force Work Kinetic or potential energy	Equations of motion Kinematics Lagrange's equation Normal modes
16	Auto Road Incline Surface Slope	coefficient of friction angle	Velocity	Linear, projectile, or dimensional motion Kinetics	Constant Force F=c	Newton's Laws/F=ma Mechanics Conservation of momentum Conservation of energy	Gravitational, normal, friction, or nonconservative force Work Kinetic or potential energy Friction	Equations of motion Kinematics

card #	MODEL				Mathematical Model MTH-M	THEORY		
	Object M-OBJ	Object Attributes M-ATT	Unknown M-UNK	Motion M-MOT		Theory THE	Construction rules THE-CON	Extension Rules THE-EXT
17	Two pendula Bob String Coupled oscillator	Mass Mass Length	Char-acteristic frequencies	Oscillatory, harmonic, or up-down motion Harmonic oscillator	Force proportional to x $F \alpha x$ Differential equation	Newton's Laws/F=ma Mechanics Conservation of energy Wave mechanics	Gravitational force Tension Work Kinetic or potential Energy	**Lagrange's equation** **Normal modes**
18	Mass Spring Single oscillator	Mass Force constant Displace-ment	Natural frequency Period	Oscillatory, harmonic, or up-down motion Harmonic oscillator	Force proportional to x $F \alpha x$ Differential equation	Newton's Laws/F=ma Mechanics Wave mechanics	Gravitational or spring force Elastic, kinetic or potential energy	**Equations of motion** **Kinematics**

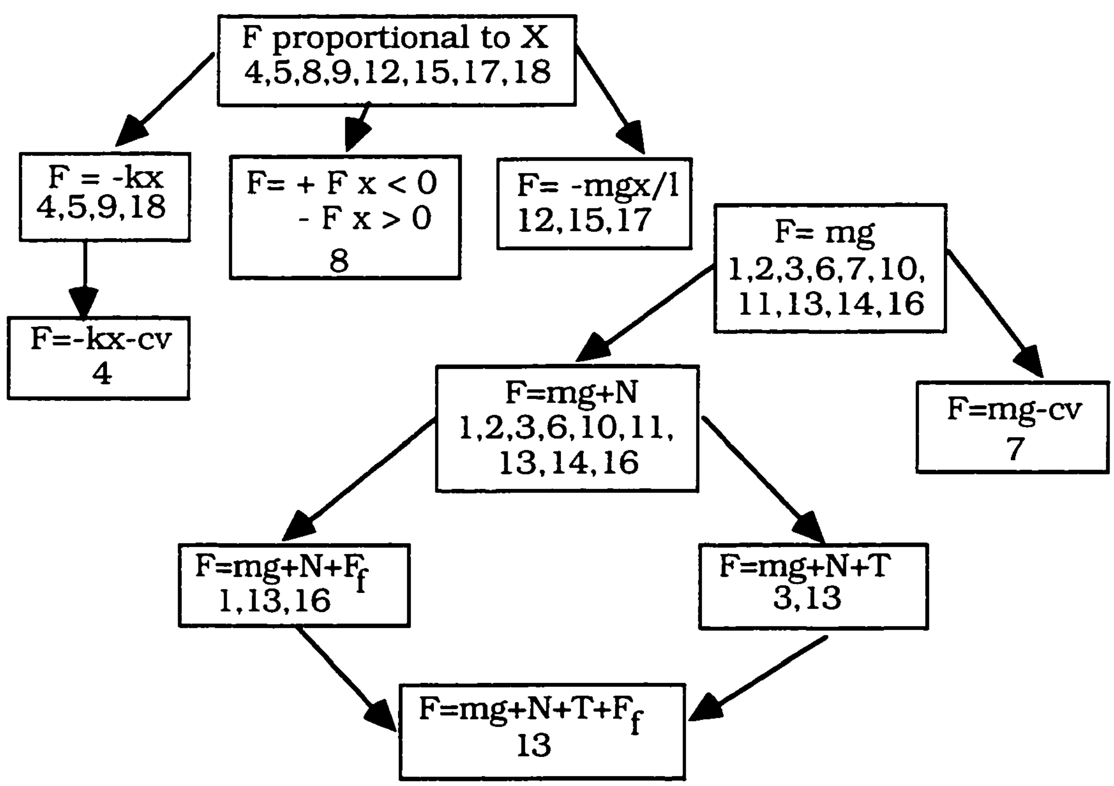
Appendix D

Examples of Possible Representations for Novice, Intermediate, and Expert Subjects

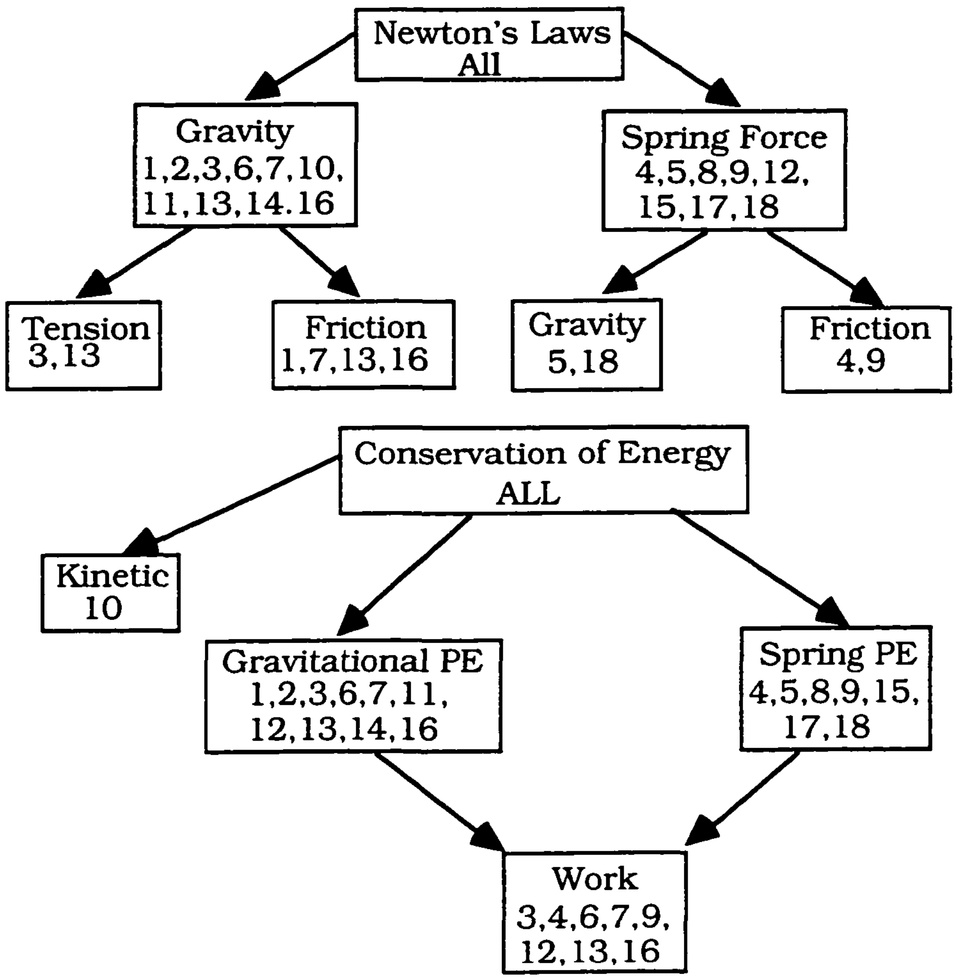
1) Example of possible representation of problems by the objects in the problem:



2) Example of possible representation of problems by Mathematical Model:



2) Example of possible representation of problems by Theory:



Appendix E

Summary of Statistics

Table E.1

Summary Analysis for Proportions of Model-, Theory-, and Mathematical Model-Based Classifications Across all Levels

Multivariate Analysis

Test Name	Value	df_B	df_W	F	Sig. of F
Pillais	0.6404	6	46	3.6113	0.005
Hotellings	1.5293	6	42	5.3528	0.000
Wilks	0.3852	6	44	4.4821	0.001

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
MODEL					
Expertise	1.1030	2	0.5515	17.1861	0.0000
Within Groups	0.7701	24	0.0321		
THEORY					
Expertise	0.8233	2	0.4117	11.2220	0.0004
Within Groups	0.8804	24	0.0367		
MATH MODEL					
Expertise	0.0190	2	0.0095	1.02883	0.3730
Within Groups	0.2220	24	0.0092		

Table E.2

Summary Analysis of Variance for Proportions of Model-, Theory-, and Mathematical Model-Based Classifications at the High Level of Categorization

Univariate Analysis					
Source of variation	SS	df	MS	F	Sig. of F
THEORY					
Expertise	0.8233	2	0.4117	11.2220	0.0004
Within Groups	0.8804	24	0.0367		

*No Entries for Model or Math Model at this Level of Categorization

Table E.3

Summary Analysis of Variance for Proportions of Model-, Theory-, and Mathematical Model-Based Classifications at the Middle Level of Categorization

Multivariate Analysis

Test Name	Value	df_B	df_W	F	Sig. of F
Pillais	0.4999	6	46	2.5550	0.032
Hotellings	0.6826	6	42	2.3891	0.045
Wilks	0.5592	6	44	2.4733	0.038

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
MODEL					
Expertise	0.9015	2	0.4552	5.2287	0.0130
Within Groups	2.0895	24	0.0871		
THEORY					
Expertise	0.5584	2	0.2792	4.1096	0.0292
Within Groups	1.6304	24	0.0679		
MATH MODEL					
Expertise	0.0982	2	0.0491	2.9079	0.074
Within Groups	0.4051	24	0.0169		

Table E.4

Summary Analysis of Variance for Proportions of Model-, Theory-, and Mathematical Model-Based Classifications at the Low Level of Categorization

Multivariate Analysis

Test Name	Value	dfB	dfW	F	Sig. of F
Pillais	0.4312	6	46	2.1078	0.070
Hotellings	0.6483	6	42	2.2689	0.055
Wilks	0.5924	6	44	2.1948	0.061

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
MODEL					
Expertise	0.8494	2	0.4247	6.0804	0.007
Within Groups	1.6762	24	0.694		
THEORY					
Expertise	0.1500	2	0.0750	1.9986	0.157
Within Groups	0.9008	24	0.0375		
MATH MODEL					
Expertise	0.0146	2	0.0073	0.7304	0.492
Within Groups	0.2404	24	0.0100		

Table E.5

Summary Analysis of Variance for Proportions of Theory-Based Sub Classifications used by Experts at Each Level of Categorization

Multivariate Analysis

Test Name	Value	dfB	dfW	F	Sig. of F
Pillais	0.5639	6	46	3.0103	0.014
Hotellings	1.0102	6	42	3.5355	0.006
Wilks	0.4771	6	44	3.2837	0.009

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
Theory Name					
Level	1.8805	2	0.9402	9.1778	0.0011
Within Groups	2.4588	24	0.1024		
Construction Rule					
Level	0.1719	2	0.0860	2.0169	0.155
Within Groups	1.0229	24	0.0426		
Extension Rule					
Level	0.0558	2	0.0279	2.3331	0.119
Within Groups	0.2870	24	0.0120		

Table E.6

Summary Analysis of Variance for Proportions of Model-Based Sub Classifications used by Experts at Each Level of Categorization

Multivariate Analysis					
Test Name	Value	df_B	df_W	F	Sig. of F
Pillais	0.3750	6	46	1.7691	0.127
Hotellings	0.5266	6	42	1.8430	0.114
Wilks	0.6432	6	44	1.8107	0.119

Table E.7

Summary Analysis of Variance for Proportions of Theory-Based Sub Classifications used by Intermediates at Each Level of Categorization

Multivariate Analysis

Test Name	Value	dfB	dfW	F	Sig. of F
Pillais	0.4625	6	46	2.3056	0.050
Hotellings	0.6485	6	42	2.2696	0.055
Wilks	0.5805	6	44	2.2913	0.052

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
Theory Name					
Level	0.6453	2	0.3227	2.4513	0.107
Within Groups	3.1592	24	0.1316		
Construction Rule					
Level	0.2258	2	0.1129	3.6202	0.042
Within Groups	0.7486	24	0.0312		
Extension Rule					
Level	0.4027	2	0.0210	1.2535	0.304
Within Groups	0.3856	24	0.0161		

Table E.8

Summary Analysis of Variance for Proportions of Model-Based Sub Classifications used by Intermediates at Each Level of Categorization

Multivariate Analysis

Test Name	Value	df_B	df_W	F	Sig. of F
Pillais	0.9650	8	44	5.1275	0.000
Hotellings	2.2404	8	40	5.6010	0.000
Wilks	0.2441	8	42	5.3763	0.000

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
ATTRIBUTE					
Level	0.0094	2	0.0047	2.0224	0.154
Within Groups	0.0555	24	0.0023		
MOTION					
Level	0.4716	2	0.2358	4.7613	0.0181
Within Groups	1.1886	24	0.0495		
OBJECT					
Level	0.2667	2	0.1333	14.7500	0.0001
Within Groups	0.2170	24	0.0090		
UNKNOWN					
Level	0.0021	2	0.0011	2.05694	0.150
Within Groups	0.1249	24	0.0005		

Table E.9

Summary Analysis of Variance for Proportions of Model-Based Sub Classifications used by Novices at Each Level of Categorization

Multivariate Analysis

Test Name	Value	dfB	dfW	F	Sig. of F
Pillais	1.2696	8	44	9.5606	0.000
Hotellings	5.3084	8	40	13.2710	0.000
Wilks	0.1000	8	42	11.3572	0.000

Univariate Analysis

Source of variation	SS	df	MS	F	Sig. of F
ATTRIBUTE					
Level	0.0229	2	0.0114	1.8317	0.182
Within Groups	0.1498	24	0.0062		
MOTION					
Level	1.6153	2	0.8077	20.218	0.0000
Within Groups	0.9588	24	0.0399		
OBJECT					
Level	0.8269	2	0.4134	9.9883	0.0007
Within Groups	0.9934	24	0.0414		
UNKNOWN					
Level	0.0062	2	0.0031	0.54706	0.586
Within Groups	0.1360	24	0.0057		

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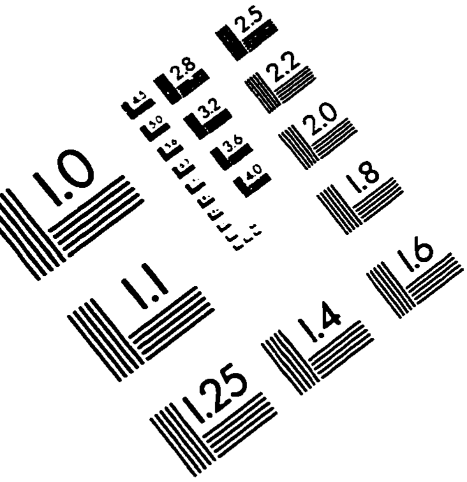
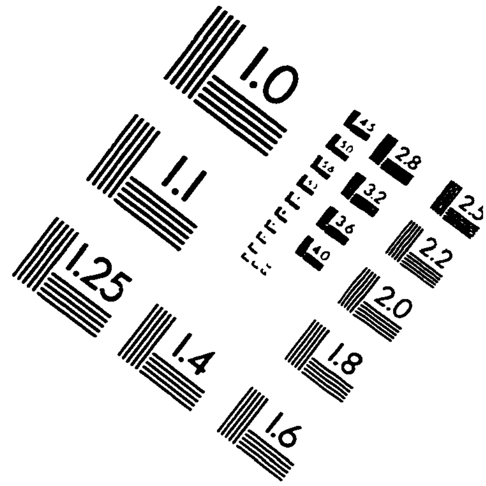
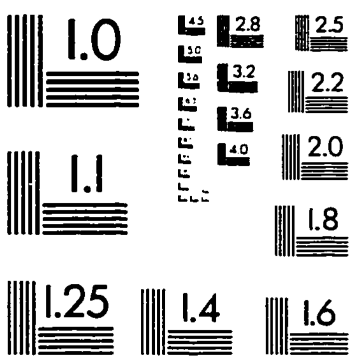
IMAGE EVALUATION TEST TARGET (QA-3)







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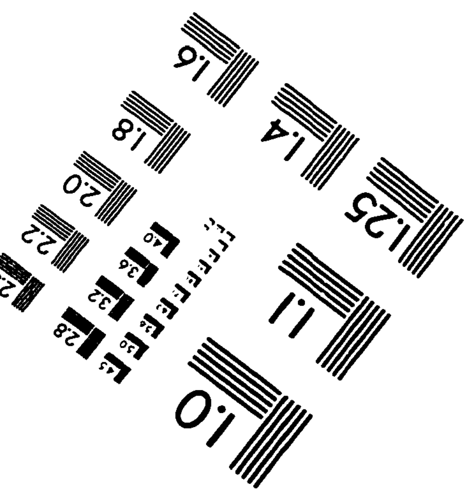

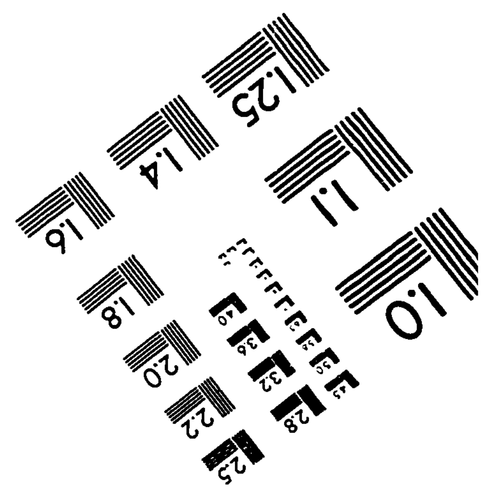

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