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A Portrait of Assessment in Reformed Mathematics Classrooms

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A PORTRAIT OF ASSESSMENT IN REFORMED MATHEMATICS CLASSROOMS

by

James R. Kett

A Dissertation
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
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During the past decade, the National Council of Teachers of Mathematics published three documents (1989, 1991, 1995) which propose changes in content, instruction, and assessment. These documents call for instructional and assessment goals that support students as they construct their own meaning to mathematics. A next step is to develop assessment models consistent with these goals which would guide and support classroom teachers.

As a result of mathematics reform, many curriculum-development projects are underway. One such project is the Core-Plus Mathematics Project (CPMP). This research documents the assessment practices of four CPMP teachers working in 9th and 10th grade classrooms with diverse student populations. This information should prove useful as the research community develops assessment models consistent with a constructivist theory of learning.

The research used a case study design. Data included interviews, teaching observations, field journals, assessment documents, and teacher journals of a year-end Capstone assessment. Analysis involved two phases of coding
data. The first phase identified five major domains into which the teachers divided their assessment practices. The domains were: (1) group work, (2) the Checkpoint, (3) assignments, (4) tests and quizzes, and (5) projects. The second phase identified four themes that were used to compare participants. These themes were: (1) learning environment, (2) forms of communication, (3) teacher feedback, and (4) time.

The results of the research were the case studies of the 4 teachers. Each teacher was followed in each of the five domains as they planned assessments, collected data, analyzed the data, and used the results.

Comparison of the participants revealed that the learning environment was important for each teacher since much of the instruction took place in small groups. Teachers varied with the degree of success in producing that environment. Each teacher assessed students using both written and oral communication, although they differed on what they looked for with oral communication. Each teacher believed they had to provide feedback to students in all five domains, so time was a major issue. Even though the CPMP curriculum provided the assessment tasks, teachers required more time than what they had previously used to implement their assessment plan.
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James R. Kett
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CHAPTER I

THE PROBLEM

Introduction

During the past decade, the mathematics education community has been developing a more integrated, learner-centered curriculum where students are actively participating in their learning. Spearheaded by Everybody Counts (Mathematical Sciences Education Board, 1989), Reshaping School Mathematics (Mathematical Sciences Education Board, 1990), Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 1989), Professional Standards for Teaching School Mathematics (NCTM, 1991), and Assessment Standards for School Mathematics (NCTM, 1995), school mathematics in the United States is undergoing change. This change can be described as a shift from behaviorism to constructivism; from focusing on procedures and mastery of isolated facts to mathematics as problem solving, communication, reasoning, and making connections (NCTM, 1989).

The first phase of an agenda for school reform is complete. With the publication of the NCTM standards for curriculum, and teaching, and assessment, the mathematics
education community has agreed on some of the broad features of curriculum and instruction. The next phase involves setting performance standards, developing tasks, and devising assessment models aligned with the curriculum (Romberg & Wilson, 1995). To guide this phase, the NCTM developed the *Assessment Standards for School Mathematics* (1995).

As a result of the reform movement, many curriculum-development projects are under way (cf., Mathematics Teacher 88[8], 1995). The emerging curricula include instructional material and assessment tasks that attempt to capture the vision of the three volumes of the *Standards* (NCTM, 1989, 1991, 1995). A next step involves devising models that teachers can use to assist them in assessing student progress. These models would be a way of recommending to teachers what the professional community believes is important to assess, strategies that could be used to carry out the assessments, and examples of possible ways to merge new forms with traditional forms of assessment. The models should include strategies and techniques that have been tried and tested by real teachers in real classroom situations.

The reform movement in school mathematics has changed the domain of assessment from a relatively small collection of procedures to a larger realm that emphasizes conceptual
understanding, reasoning, discourse, and representation (Smith, 1996). A problem is that teachers often lack the knowledge, motivation, and experience to be able to develop and implement assessment techniques that target this new realm and that can be effectively and efficiently used in their classrooms (Nash, 1993). Teachers need assessment models that will guide them in this process. Although it may be impossible to prescribe an exact mix of techniques that fit all situations, research on possible mixes of strategies that might be used in typical classrooms are needed (Senk, Beckmann, & Thompson, 1997). The emerging curricula provide teachers with innovative tasks, suggestions on how to assess students, and how to use the data to inform instruction. The next step is to provide assessment models based on examples of teachers attempting to implement this vision in their classrooms (Smith, 1996). Experienced teachers, working with different student populations with a reform curriculum and striving to implement new forms of assessment provide a rich environment for exploring these needed examples. One such curriculum is the Core-Plus Mathematics Project (CPMP).

The CPMP Secondary Curriculum

CPMP commenced in 1992 with funding from the National Science Foundation with the objective of developing a 3-
year high school mathematics curriculum for all students and a 4th-year option continuing the preparation of students for college mathematics (Hirsch, Coxford, Fey, & Schoen, 1995). The curriculum builds upon the theme of mathematics as sense making and has the following five characteristics (CPMP, 1995):

1. Each year the curriculum is organized around four strands: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics.

2. The curriculum emphasizes mathematical modeling and modeling concepts of data collection, representation, interpretation, prediction, and simulation.

3. The curriculum is designed so that core topics are accessible to all students but accommodations are made for students who desire or need more depth and abstraction.

4. Graphics calculator technology is capitalized on so that instruction can focus on goals in which mathematical thinking is central.

5. Assessment is linked to instruction and promotes mathematical thinking by having students both individually and in groups investigate, conjecture, verify, apply, and communicate ideas.

The CPMP curriculum promotes active engagement of students in exploring problem situations. The design of the instructional materials enables teachers to use the follow-
ing four-phase cycle of classroom activities (CPMP, 1995):

1. Launch: The teacher directs a full class discussion of a problem situation. The teacher then poses related questions for students to think about.

2. Explore: Group investigations, using focused problems and questions related to the Launching situation, lead to gathering data, looking for patterns, constructing models and meanings, and making and verifying conjectures. The teacher circulates from group to group providing guidance.

3. Share/Summarize: The teacher moderates a full class discussion (referred to as the Checkpoint) of concepts and methods developed by different small groups. This leads to a class summary of important ideas.

4. Apply: A task is assigned for individual students to complete on their own to assess initial understanding of a concept or method. A sample On-Your-Own task is provided in Appendix A.

In addition to the classroom investigations, the CPMP curriculum provides sets of tasks designed to be completed by students outside of class. These tasks are divided into four categories collectively referred to as MORE activities. Sample MORE tasks are provided in Appendix A. MORE is an acronym which stands for Modeling, Organizing, Reflecting, and Extending. The Modeling tasks are related or new
contexts to which students can apply the ideas developed in the lesson. The Organizing tasks give students opportunities to integrate the mathematics and to make connections. The Reflecting tasks promote self-monitoring and evaluation of understanding. The Extending tasks enable students to pursue the topics at a deeper or more formal level.

The CPMP assessment program is much broader than the typical program and is consistent with recommendations of the NCTM Standards (1989, 1991, 1995). As students complete the investigations that make up the curriculum, the teacher informally assesses students in terms of process, content, and disposition. At the end of each investigation, the Checkpoint allows the teacher to assess the levels of understanding that each student obtained. The On-Your-Own problems and MORE tasks provide the teacher with another means of assessing student understanding. The program also provides quizzes, end-of-unit tests, and end-of-unit take-home activities that require from 1 to 5 days to complete. In addition, the teacher is encouraged to use classroom observations, portfolios, and student journals in his/her assessment plan. While it is recommended that teachers use these multiple methods to assess student growth, the selection and assessment process remains an individual decision.
Research Questions

The focus of this study centered on describing examples of teachers implementing classroom assessment based on recommendations in NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Assessment Standards for School Mathematics* (1995). In order to analyze these examples, this study was guided by NCTM's standards on assessment which view any assessment as being composed of four interrelated phases: (1) planning, (2) gathering evidence, (3) interpreting the evidence, and (4) using the results. This research explored these phases using 4 teachers involved in the field testing of CPMP materials. The specific questions about these 4 teachers that this research addressed are as follows:

1. What is their assessment plan?
2. How do they gather evidence needed to carry out their assessment plan?
3. How do they interpret the evidence?
4. How do they use the results?
5. What problems do they encounter when implementing their assessment plan?

Definitions

The terms "assessment" and "evaluation" are used throughout this document. For the purpose of clarification,
the following definitions are provided.

1. Assessment: "The process of gathering evidence about a student’s knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (NCTM, 1995, p.3).

2. Evaluation: "The process of determining the worth of, or assigning a value to, something on the basis of careful examination and judgment" (NCTM, 1995, p.3).

Framework for the Questions

This study is based on the framework described in the introduction to NCTM’s *Assessment Standards for School Mathematics* (1995). Figure 1 illustrates the four interrelated phases at which decisions are made. These phases

![Figure 1. Four Phases of Assessment (NCTM, 1995).](source)

are: (1) planning, (2) gathering evidence, (3) interpreting evidence, and (4) using the results.

Following are the kinds of questions that guide decisions made at each phase:

1. Planning the assessment
   • What purpose does the assessment serve?
   • What methods are used for gathering and interpreting the evidence?
   • What criteria are used for judging performances on activities?
   • What formats are used for summarizing judgments and reporting results?

2. Gathering evidence
   • How are activities and tasks created or selected?
   • How are procedures selected for engaging students in the activities?
   • How are methods for creating and preserving evidence of the performances to be judged?

3. Interpreting the evidence
   • How is the quality of the evidence determined?
   • How is an understanding of the performances to be inferred from the evidence?
   • What specific criteria are applied to judge the performances?
   • How will the judgments be summarized as results?

4. Using the results
   • How will the results be reported?
   • How should inferences from the results be made?
   • What action will be taken based on the inferences?
   • How can it be insured that these results will be incorporated in subsequent instruction and assessment? (NCTM, 1995, pp. 4-5).

In practice, the four phases do not have distinct divisions. They interact with each other and do not necessarily follow each other linearly. Teachers make decisions throughout the entire assessment process, but little is known about how these decisions are made and how teachers use the information they learn about students at
each stage of the assessment process. The current research begins to fill that void by investigating the assessment practices of 4 teachers who used multiple forms of assessment.

Theoretical Framework

Constructivist perspectives on learning have received increased attention in mathematics education since 1980 (Steffe & Kieren, 1994) and have helped shape the reform effort. For this reason, constructivism provides a theoretical framework for the study.

A constructivist theory of learning contends that we as human beings have no access to reality independent of our own experiences. Rather, we construct our knowledge from our own perceptions and experiences. Learning takes place as we make connections and adapt to our own experiences (von Glasersfeld, 1987). Learning is the process by which we adapt to our own experiential world. As new ideas arise, or what we experience differs from what is expected, our pre-existing knowledge is modified and learning occurs. Romberg and Wilson (1995) described learning as an "image that is gradually brought into sharper focus as the learner makes connections" (p.5).

In a constructivist view of learning, knowledge is unique to the learner. Starting at a young age, children...
come to school with their own ideas and beliefs (Piaget, 1953). The goal of education from a constructivist viewpoint is to find out what the student knows and then help the student construct or reconstruct knowledge.

One issue under discussion among constructivists is whether knowledge is primarily a cognitive process or social process. On the one hand are the radical constructivists who focus on the individual's construction of knowledge, and on the other are the social constructivists who focus on the social context of learning.

Von Glasersfeld is a contemporary defender of radical constructivism and has had a profound influence on mathematics education research (Steffe & Kieren, 1994). He argues that individuals construct their own knowledge on the basis of their own cognitive processes. This means that students do not study reality, but the construction of reality (Richards & von Glassersfeld, 1980).

On the other hand, researchers with a socio-cultural orientation see learning as social construction. Ernest (1991) is a leading proponent of the social constructivist position. In this philosophy, knowledge resides within culture and has no meaning outside of the culture. Social constructivists consider the "socially constructed world" to be the world and "persons in conversation" to be the mind (Ernest, 1991). Learning develops from social prac-
Cobb, Yackel, and Wood (1992) combined these two extremes and claimed that "a student's mathematical learning in a classroom should consider the development of both the taken-as-shared, communal meanings and practices and the individual student's personal meanings and practices" (p. 18). In order to more thoroughly understand student assessment, it is important to consider both the radical position, what happens within the individual student's mind; and the social position, what happens within the classroom setting (Cobb, Yackel, & Wood, 1992).

New Perspectives on Assessment

The process of assessment should be aligned with the learning it is designed to evaluate (Galbraith, 1991). Since constructivism has shaped the theory of learning on which some of the curriculum reforms are based, it should also shape the theory of assessment that accompanies such reforms. NCTM has proposed a framework for assessment which is based, at least in part, on a constructivist point of view. The first part of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), the Curriculum Standards, proposed changes in content and instruction. The second part, the Evaluation Standards, proposed changes in the way information is collected and
analyzed. Specifically, the Evaluation Standards proposed that:

(a) student assessment be integral with instruction, (b) multiple means of assessment methods be used, (c) all aspects of mathematical knowledge and its connections be assessed, and (d) instruction and curriculum be considered equally in judging the quality of a program (NCTM, 1989, p. 190).

Newmann and Archbalb (1992) elaborated on these ideas:

Our concern here is not mainly with the technical problem of designing assessments that measure more validly what schools try to teach. Instead we consider the more fundamental questions of what general forms of achievement ought to be promoted and assessed. Our inquiry arises in response to what we see as widespread disillusionment with the kinds of accomplishments that schools promote and assess.... Ultimately then, the quality and utility of assessment rest upon the extent to which the outcomes measured represent appropriate, meaningful, significant, and worthwhile forms of human accomplishment. (pp. 71-72)

Although all the details of an assessment model are not described in the Evaluation Standards, 14 standards are presented to help develop a model. The 14 standards are organized into three sections: (1) General Assessment, (2) Student Assessment, and (3) Program Evaluation.

The general-assessment standards discuss principles relevant to any form of assessment and program evaluation. The student-assessment standards consider aspects of mathematical knowledge that should be assessed, as derived from the Curriculum Standards. The program-evaluation standards examine the assessment of the extent to which a mathematics program is consistent with the Standards. (p. 190)

The three general-assessment standards are: (1) Alignment, (2) Multiple Sources of Information, and (3) Appropriate Assessment Methods and Uses.
The vision of mathematics education in the Standards places new demands on instruction and forces us to reassess the manner and methods by which we chart our students' progress. In an instructional environment that demands a deeper understanding of mathematics, testing instruments that call for only the identification of single correct responses no longer suffice. Instead our instruments must reflect the scope and intent of our instructional program to have students solve problems, reason, and communicate. Furthermore, the instruments must enable the teacher to understand students' perception of mathematical ideas and processes and their ability to function in a mathematical context. At the same time, they must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction. (p. 192)


The student assessment standards describe what is to be observed and measured in the process of understanding what mathematics students know. Teachers drawing meaning from their interactions with students is central to this process.... Assessment must be more than testing; it must be a continuous, dynamic, and often informal process.... Assessment is cyclic in nature, a process of observation, conjecture, and constant reformation of judgments about students' understanding.... These seven student-assessment standards focus on assessing students' understanding of, and disposition toward, mathematics. (pp. 203-204)

The four program-evaluation standards are: (1) Indicators for Program Evaluation, (2) Curriculum and Instructional Resources, (3) Instruction, and (4) Evaluation Team.

Evaluation can help determine a mathematics program status in relation to the Curriculum Standards and ensure that the pieces fit together.... In a certain sense, the Program Evaluation Standards are a guide to creating a program that meets the challenge of the Standards. (p. 237)
In summary, the 14 Evaluation Standards emphasize aspects of assessment and program evaluation that depart from current practice. They call for using multiple assessment techniques, including written, oral, and demonstration formats instead of only written tests and quizzes. They call for using assessment as an integral part of teaching with less emphasis on simply counting the number of correct answers. They take a holistic view of mathematics with decreased attention to focusing on isolated skills and procedures.

Research Methodology

The current study followed the format of naturalistic inquiry (Lincoln & Guba, 1985). This allows for collecting, organizing, and analyzing educational phenomena in depth and detail. It provides researchers opportunities for exploration, discovery, and interpretation rather than hypothesis testing (Merriam, 1988). This research used naturalistic inquiry to provide descriptions and analysis of teachers implementing assessment reform in the context of a reformed curriculum.

Four teachers from CPMP field test sites were selected for participation in the study. These 4 teachers were purposefully selected to represent a cross-section of student populations and because they were identified by the
CPMP administrative staff as using multiple assessment methods and applying the CPMP instructional model.

Data were collected over 20 weeks and included field notes, semi-ethnographic interviews, classroom observations, taped explanations of graded tests, evaluation forms used by participating teachers, and journals kept by the participating teachers of a year-end Capstone project. Two phases of coding were used for analysis of data. The first phase involved the examination of initial data for emerging domains of assessment. After all the data were collected and the domains were determined, a second phase of coding within each domain produced themes common to all participants which were used to compare participants. Member checking and triangulation of data collection methods promoted trustworthiness. A more thorough explanation of the methodology is provided in Chapter III.

Limitations of the Study

In conducting naturalistic research, one must be aware of certain limitations. Since conditions are not controlled and sampling is not random, one must avoid making generalizations. The study of a small number of cases simply produces descriptive information about those cases. It increases the understanding of the cases and situations studied, but the results should not be generalized beyond
the cases (Lincoln & Guba, 1985). The current study investigated the practices of 4 teachers in four different environments, all using the same curriculum. The intent of this research is not to generalize results to all teaching situations, but to provide examples of what is currently being done at the classroom level.

Since the researcher relies on his ability to interpret the data, subjectivity is inevitable. In order to address the validity of this research, Chapter III includes a brief account of the researcher's background and preparation for this research.
CHAPTER II

REVIEW OF THE LITERATURE

Introduction

In recent years, there has been a plethora of books and articles dealing with assessment. Most critique the state of assessment and offer suggestions for change (e.g., Gifford & O'Connor, 1992; Kulm, 1990; Lesh & Lamon, 1992; Romberg, 1989; Stake, 1995a). Some discuss frameworks that are needed to guide assessment reform (e.g., Goldin, 1992; Lajoie, 1995; NCTM, 1995; Romberg & Wilson, 1995; Zarinnia & Romberg, 1992). Many describe the nature of assessment activities that should be incorporated into a classroom (e.g., Kulm, 1994; Lesh & Lamon, 1992; Stenmark, 1989, 1991; Webb & Coxford, 1993). Another group argues that assessment and curriculum must be aligned and that teachers should integrate instruction and assessment (e.g., Chambers, 1993; Clarke, Stephens, & Waywood, 1992; Silver & Kenny, 1995).

Traditional Assessment System

Problems with our traditional assessment system are generally recognized. Bloom's Taxonomy of educational

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during the past half century. It assumed that learning could be classified into a linear sequence and broken up into mutually exclusive cells. This breakdown of knowledge affected the role of teachers, the content and structure of textbooks, and the monitoring and evaluation of students. Teachers viewed progress as mastery of simple steps which were monitored by frequent objective quizzes (Romberg, 1989). In addition, it was assumed that a single measure (e.g., letter grade) could be used to compare students on what was considered a general, fixed, unidimensional trait.

While Bloom’s Taxonomy and educational measurement are based on psychometric models with roots in behaviorist theory and are a product of the industrial revolution, cognitive scientists have come to view learning as construction of meaning in a social context (Ernest, 1991; NCTM, 1989; Resnick, 1987; Shephard, 1989). Learners gain understanding when they construct their own knowledge and develop their own cognitive maps of the interconnections among concepts and facts. In this view, the processes by which a response is produced are at least as important as the product.

As Galbraith (1991) suggested, there are inherent contradictions when constructivism drives curriculum design and knowledge construction, but positivism drives assessment practices. Most criticisms deal with large scale stan-
standardized tests, but they also carry over to classroom assessments as external assessments have a profound influence on the classroom (Romberg, Zarinnia, & Williams, 1989). Teachers, over the years, have primarily concentrated on assessing students' computational facility, mastery of discrete skills, and the ability to solve problems similar to those presented in the textbook (Cooney, 1992; Senk, Beckmann, & Thompson, 1997; Taylor, 1992). According to Webb and Romberg (1992), teachers need to expand their view of mathematics and embrace qualitative methods such as interviews, observations, and teacher opinion so that they can assess processes as well as answers.

Frameworks to Guide Reform

Based on the critiques and NCTM's standards for curriculum and evaluation, in 1995 NCTM published the Assessment Standards for School Mathematics. Designed to guide the development of assessment models, the standards for assessment focus on assessment of mathematical power. Mathematical power is characterized by:

(a) strong understanding of mathematical concepts and procedures, (b) a positive disposition toward mathematics, (c) the ability to apply knowledge to reason, analyze, and solve problems, (d) the use of mathematics to communicate ideas, and (e) confidence in one's ability to use mathematics (NCTM, 1989, p.205).

Fundamental to the formation of the NCTM's Assessment
Standards is the belief that "new assessment strategies and practices need to be developed that will enable teachers and others to assess students' performances in a manner that reflects the NCTM's reform vision for school mathematics" (NCTM, 1995, p.1). The following six standards were designed to complement NCTM's Evaluation Standards with the purpose of judging the quality of all kinds of mathematical assessments ranging from the individual classroom teacher to large scale standardized tests.

1. "Assessment should reflect the mathematics that all students need to know and be able to do" (p.11). The mathematics referred to is that which is contained in the NCTM's Curriculum and Evaluation Standards for School Mathematics (1989).

2. "Assessment should enhance mathematical learning" (p.13). Instruction and assessment should be closely linked. Teacher judgments, classwork, and student reflections are a valued part of assessment and help inform the teacher to make decisions about instruction and evaluation.

3. "Assessment should promote equity" (p. 15). Assessments should take into account differences in gender, physical condition, and ethnic, cultural, and social backgrounds.

4. "Assessment should be an open process" (p. 17). Information about how data will be gathered and how the
results will be interpreted should be available to all students.

5. "Assessment should promote valid inferences about mathematics learning" (p.19). The use of multiple sources of information can improve validity since the strengths in one type will compensate for the weaknesses in another.

6. "Assessment should be a coherent process" (p.21). A framework should exist so that all pieces of the assessment fit together and are aligned with the curriculum.

Assessment Activities

Another topic that is prevalent in assessment literature deals with the kinds of activities that are needed to find out what students know and can do, and how teachers use these activities to assess students. The California Mathematics Council has prepared Assessment Alternatives in Mathematics (Stenmark, 1989), an overview of assessment techniques that promote learning. Several alternative assessments, such as portfolios, writing in mathematics, observations, interviews, open-ended questions, and student self-assessment, are explored. Examples and methods of record-keeping are provided. However, several important issues are listed as needing further research. These include how to deal with students with special needs, the equity of timed tests, the effect of technology, and more.
exploration on record-keeping.

*Mathematics Assessment: Myths, Models, Good Questions and Practical Solutions* (Stenmark, 1991) covers many important aspects of student assessment but gives emphasis to performance assessment. Performance assessment "involves presenting students with a mathematical task, project, or investigation, then observing, interviewing, and looking at their products to assess what they actually know and can do" (p. 13). This document provides a number of suggestions for recording, interpreting, and reporting alternative assessments.

Kulm (1994) discussed and illustrated different techniques for alternative assessments. A chapter on scoring and grading techniques emphasizes the holistic nature of evaluating students. However, Kulm provides limited examples of teachers actually using these alternative assessments in their classrooms.

The February, 1992 issue of *Arithmetic Teacher* focused on assessment. One article by Clarke (1992) advocates informal assessments such as annotated class lists and workfolios. It also encourages other alternative assessments such as practical tests, student-constructed tests, and student self-assessment. The article goes on to emphasize the use of better assessments, not just more techniques.

Researchers agree that teachers are the key to the
reform movement, but they often fail to realize that time is already at a premium and quality is sometimes sacrificed for quantity. Teachers are not able to find time to develop alternative assessments. With a typical schedule, teachers often come in early, stay late, and are still faced with 2 or more hours of work to do at home (Apple, 1992). Many teachers would like to use alternative assessments, but are constrained by lack of time and lack of expertise in developing these assessments (Taylor, 1992).

The CPMP curriculum provides teachers with an assortment of assessment tasks and activities which allow them to gather information from multiple sources. However it is still up to individual teachers to select the activities, collect and interpret the data, and use the results.

Assessment and Instruction

The final theme to be considered here is the integration of assessment and instruction. According to Webb and Briars (1990), assessment "must be an interaction between teacher and students, with the teacher continually seeking to understand what a student can do and then using this information to guide instruction" (p. 108). Traditionally, managing assessment information meant grading tests promptly and recording those grades in a grade book. This is still an important part of evaluation, but given the
multiple forms of assessment that are recommended, it does not seem reasonable to expect that results will fit into a typical grade book. The information that teachers need includes notes written during observations or interviews, drafts of projects and their final forms, checklists of skills shown during activities, and student reflections on their own progress. In addition, frequent review of the information is needed so teachers can analyze what growth is occurring and what is lacking. There are two ways that teachers can understand assessment information:

1. They can carry out an ongoing interpretation of information gathered during instruction.

2. They can score products that students produce.

Both are important but how is a teacher with 25 to 30 students in each of five classes able to do this, and how does the teacher make inferences about each student from the information obtained?

The CPMP curriculum addresses all of these dimensions of assessment reform. Using in-class questions, group work, journals, quizzes and tests that use open-ended questions, extended projects, and portfolios, teachers are able to continually assess students over a broad range of outcomes. In addition, teachers are continually asked to reflect on their practices. Teachers experienced with the CPMP curriculum provide a rich environment for exploring assessment
Research on Classroom Assessment

What do we actually know about classroom assessment in school mathematics? A review of literature in mathematics education reveals that we know very little about the practices of the classroom teacher. The Handbook of Research on Mathematics Teaching and Learning (Grouws, 1992) includes a section on this topic but a careful study of the section confirms the fact that there was limited research in this area, and that researchers are currently looking beyond testing to other assessment techniques. There has been some documentation of efforts at assessment reform in recent years (e.g., Cooney, 1992; Kulm, 1994; Nash, 1993; Senk, Beckmann, & Thompson, 1997; Taylor, 1992) but it is extremely limited. The current research adds to the body of knowledge of what teachers are using in their classrooms.

Testing and Evaluation

Classroom testing is one area that has received some attention. Cooney (1992) surveyed 201 secondary teachers in Georgia regarding their evaluation practices. The study consisted of three phases: (1) administration of a questionnaire about teachers’ evaluation practices, (2) a second questionnaire that requested teachers’ reactions to
open-ended assessment items, and (3) a limited number of teacher interviews. Results of the study suggest that teachers were reluctant to use open-ended questions because of time constraints. Cooney suggested that teachers might use a greater variety of assessment strategies if they were provided appropriate resources or given the opportunity to work with colleagues to create them. Many teachers did not see a connection between instruction and assessment and indicated that they would use open-ended questions during instruction but not for evaluation purposes.

Taylor (1992) surveyed 138 algebra and geometry teachers in Ohio to investigate their practices in evaluating students. She found that teachers relied mostly on tests, quizzes, and homework to determine grades. Teachers did not use publisher-provided test materials directly, but as a source of questions for teacher-made tests. She also found that most teachers tested at the knowledge/skill level when they thought they were testing a deep understanding. Few teachers used group projects, journals, individual projects, or portfolios in assessing their students. Taylor also concluded that many teachers wished to improve their assessment of higher-order thinking, but were constrained by time and lack of familiarity with alternative assessment techniques.

Research by Senk, Beckmann, and Thompson (1997)
support Taylor's conclusions and suggest that things have not changed substantially in the last few years. They surveyed 19 teachers of schools where newer forms of assessment were encouraged and where technology was used in the classrooms. Even with this sampling procedure, the researchers found little evidence of assessment reform. Performance on tests and quizzes was still the primary source of determining grades and virtually no teachers used open-ended questions on tests. Only one teacher used class participation and few teachers included oral communication in their evaluation plan.

Research by Cooney (1992), Taylor (1992), and Senk, Beckmann, and Thompson (1997) indicate that sampled classroom practices did not coincide with NCTM recommendations and that teachers need assistance on how to assess students more thoroughly and to use the results in class instruction.

When assistance is offered, research indicates that teachers were willing to alter their assessment plan and incorporate alternative techniques. Nash (1993) studied 3 teachers using different types of alternative assessment. She concluded that competent teachers can realistically implement alternative methods provided they have enough support. Enderson (1995) studied 3 student teachers and their assessment practices during student teaching. She
concluded that both knowledge of mathematical concepts and knowledge of methods of teaching were necessary to ensure valuable assessment techniques. If teachers have both kinds of knowledge and have assistance, then they may try new types of assessment.

Kulm's (1994) study of 18 teachers involved in a graduate course on classroom assessment supports this hypothesis. The results of the research indicate that when teachers are exposed to alternative assessment techniques, they will adjust their teaching activities to coincide with these techniques. Kulm concluded that information about new approaches to mathematics assessment at the classroom level are critical for further progress in reforming mathematics education. The current research extends Kulm's research by studying the entire assessment process of 4 teachers, each using multiple forms of assessment in the context of a reform curriculum.

**Impact on Students**

How students are evaluated has a profound effect on how they perceive and learn mathematics. Madaus, West, Harmon, Lomax, and Viator (1992) conducted an extensive nationwide survey and concluded that when teachers treat test results, instead of achievement, as the major goal of instruction, then students come to believe that mathematics
is simply the development of skills that are required by the teacher under test conditions. Teachers must learn ways to evaluate students other than simply using test results. Certainly the goal of mathematics education is more than getting a certain set of test questions correct (NCTM, 1989).

Crooks (1988) carried out a comprehensive survey on the effects of classroom evaluation on students. He concluded that tests are not the only motivating factor for many students. Teachers' comments on performance, use of checklists, and teacher questioning also impacted student motivation in the class.

Conclusion

This review of literature reveals limited classroom research on assessment reform in mathematics education. There are a few examples of teachers using isolated alternative assessment strategies in their classrooms, but research provides few examples of teachers' use of multiple techniques in their assessment plan. It also provides no examples of teachers using investigations where students construct their own meaning as the primary form of learning. In order to make significant progress towards developing assessment models based on NCTM's Standards (1989, 1991, 1995), it is necessary to provide rich
descriptive accounts of strategies that have been tried and tested in real classrooms by experienced teachers.
CHAPTER III

METHODOLOGY

Introduction

Significant changes have been recommended for school mathematics (NCTM, 1989, 1991, 1995). These recommendations include shifts in priority of mathematical content, in the nature of classroom instruction, and in the manner in which students are assessed. According to NCTM (1989), the ultimate goal of assessment is to improve instruction. Although NCTM’s Standards (1989, 1991, 1995) outline the broad goals for student assessment and provide isolated examples of multiple assessment techniques, there is little documented research on how these could be incorporated in a coherent way into a typical classroom and how teachers could use these methods to make decisions about students. The primary purpose of this research was to investigate how experienced teachers, using a reformed curriculum in different settings, planned, documented, interpreted, and used assessment.

This chapter describes the methodology of the study. The chapter begins with a description of naturalistic research. It then includes a description of the teachers.
involved in the research, background information about the researcher, the sequence of steps used to collect the data, and the methods used to analyze the data.

Naturalistic Inquiry

This study followed the format of naturalistic inquiry. In naturalistic inquiry the precise design cannot be specified in advance, but the following interconnected characteristics must be considered:

1. The research is carried out in a natural setting.
2. The researcher elects to use himself as well as other humans as the primary data-gathering instruments.
3. Tacit knowledge, in addition to propositional knowledge, is used.
4. The researcher uses qualitative methods.
5. Sampling is purposeful, not random.
6. The researcher prefers inductive over deductive data analysis.
7. The researcher negotiates meanings and interpretations with the human sources from which the data have been drawn.
8. A case study format is used to report the data.
9. The researcher interprets the data idiographically (in terms of the particulars of the case) rather than in terms of generalizations.
10. The researcher sets boundaries to the inquiry on the basis of the emergent focus.
11. Conventional criteria of validity, reliability, and objectivity are inconsistent with the procedures used. (Lincoln & Guba, 1985, pp. 39-43)
If the formulation of an assessment model is to be based on data, then this data must first be located and analyzed inductively. The current research has not developed a model, but has located and documented data needed for a model. Since the researcher could not specify the exact form of the data in advance, he used qualitative means to obtain the data. This caused the researcher to use seeing, hearing, reading, and tacit knowledge that led to interviews, observations, and analyzing documents. In this form of research, "no aggregations, no generalizations, no cause-effect statements can emerge, but only idiographic interpretations negotiated with knowledgeable respondents; hence an air of tentativeness surround any proposed application" (Lincoln & Guba, 1985, p.44).

Judgments about the trustworthiness of this process cannot be made with conventional criteria. Lincoln and Guba (1985) provide criteria to assure trustworthiness in a naturalistic study. Prolonged engagement, persistent observation, and member checking are activities that increase the credibility of the findings. Prolonged engagement and persistent observation were evidenced by the 20 weeks of time and the many contacts used in conducting this research. Each participant was visited at least five times to be interviewed or observed. In addition, participants were contacted by telephone to follow up on interviews and
observations. The researcher also spent considerable time familiarizing himself with the CPMP curriculum. However, he was not involved in the project other than to conduct this research, so there was no danger of biasing results to meet the expectations of the project directors (Lincoln & Guba, 1985). Member checking occurred at various stages of data collection. This means that participants verified data and interpretations of data as the research progressed. During each interview and following the last interview, participants were asked questions and read statements that confirmed previous observations and interpretations.

One valuable method for increasing the trustworthiness of a study is by triangulation of data (Lincoln & Guba, 1985). Triangulation of data refers to multiple methods of data collection and multiple sources of data. The primary sources of data for this study consisted of taped interviews with the participating teachers, assessment documents supplied by each teacher, notes taken during observations, taped descriptions of graded tests, and journals written by the participating teachers regarding the planning and implementing of a year-end project. The methods of data collection varied from direct responses of the participants which occurred during interviews to interpretations by the investigator which occurred during observations. Trustworthiness was also provided by member checking. This
occurred as the participants responded to the researcher’s interpretations of their actions and responses.

Selection of Teachers (Participants)

Four CPMP teachers were selected based on the following criteria: (a) the teacher had taught at least 2 full years using the CPMP curriculum, (b) the teacher used multiple methods of assessment, and (c) the teacher was willing to be involved in the project.

Each of the 4 teachers represented a different teaching environment. One teacher worked in a rural setting, another in a suburban setting, another in an urban setting, and a fourth in a setting that drew students from many different environments.

Jill is a pseudonym for a teacher who had taught for 15 years at a suburban district near a large city. Over 90% of the students were Caucasian. Most of the school’s students came from professional families that valued education. According to Jill, this attitude was changing. Because of the school’s reputation, families with problem kids were moving into the school district and asking the school to educate their children. According to Jill, these were people that “do not have much respect for education itself, but see it as a vehicle to get somewhere.” The CPMP curriculum had been used with a small number of students
since the school had not yet adopted it for all students. Jill and one other teacher were the only CPMP teachers. In addition, an honor’s program in mathematics started in the eighth grade. These students used a more traditional program and were not eligible for the CPMP program. Jill had piloted each of the courses of CPMP and had taught Course 1 for 3 years.

Karen is a pseudonym for an experienced teacher who had spent her entire 25 year career at the same high school in a medium sized rural school community. The student body was primarily Caucasian, with a few African-Americans and a few Hispanics. The school used block scheduling where classes met every day for 2 hours and full-year courses were taught in 18 weeks. All students were enrolled in the CPMP curriculum. Karen had taught Course 1 of CPMP for the past 3 years.

Mary is a pseudonym for a teacher who taught at a Math and Science Center for talented students. Students attended this school for only their math and science classes, returning to their regular school for all other courses. According to Mary, there was a nice blend of students from many different backgrounds including ethnicity. Many students came from the inner city, some students came from a nearby affluent professional school district, and the rest from nearby small rural school districts. Mary had
taught high school mathematics for 23 years. For the past few years she was also math coordinator in a large urban school district. Part of her responsibility had been to coordinate the implementation of CPMP in the school district. This was her first year at the Math and Science Center where she was a full time teacher and taught both Course 1 and Course 2.

Jack is a pseudonym for a teacher who had been teaching mathematics for 10 years at a medium sized urban school. The school's population was about 35% minority, most of which were Hispanic. The students came from predominantly blue collar families. About 50% of the students enrolled in college. Jack was a full time mathematics teacher and had taught each of the first three courses in CPMP. He had taught Course 1 for the past 3 years.

Background of the Researcher

Under the guidelines of naturalistic inquiry (Lincoln & Guba, 1985), the researcher served as the primary instrument in this study. Since the researcher relies on his ability to interpret the data, subjectivity is inevitable. In order to provide the reader with knowledge of potential biases in this research, I have included a brief account of my background and my preparation for this research.

I am a high school mathematics teacher with 27 years
of experience. Most of that career involved a traditional method of teaching where I began class by going over homework, then presented new material, and finally gave an assignment. Tests contained problems similar to the assignments and grades were determined strictly by percentages obtained from tests, quizzes, and homework. In recent years, I became dissatisfied with this limited view of assessment. It seemed that school mathematics was simply learning how to do a certain set of problems and a grade represented how well one learned how to solve those problems. As a result, I started using projects, presentations, and observations in my assessment plan.

In my graduate studies I concentrated on assessment and anticipated the publication of *The Assessment Standards for School Mathematics* (NCTM, 1995). Much has been written and suggested as to how we should change assessment in school mathematics, but I was not able to find extensive documentation of efforts in applying these changes in a typical classroom. During my graduate studies I became familiar with the CPMP curriculum and approached its director about studying classroom assessment using teachers in his program. My concern was that as the mathematics community develops curriculum, it is important that models for assessment be developed concurrently. The *Standards* (1989, 1991, 1995) have given direction, but the partic-
ulars are still to be developed. This research adds to the base from which these models can be formed.

Implementation

Data were collected over a 20-week period beginning in January of 1996. During the first 2 weeks, each teacher was interviewed in their classroom using a semi-ethnographic format. The intent was to gather as much information as possible by allowing each teacher to describe broadly how he or she viewed his or her assessment plan. A set of lead questions is provided in Appendix B. These questions provided a base from which the researcher branched out based on the responses of the participants. During the next 18 weeks the interviews became more focused as common domains emerged. During the last week of February and the first week of March, each teacher was observed conducting class. These observations confirmed statements that were made during the interviews and provided a focal point for future interviews. During the month of April, each teacher chose three students to focus on as he or she graded tests. The teachers were asked to choose a high performing student, one of medium performance, and one of low performance. The 4 teachers then audiotaped an analysis of their evaluation of those students' responses to test questions. The last formal interview for each teacher
occurred during the month of May, although follow-up phone conversations continued for member checking. During the last week of the school year, 3 of the 4 teachers conducted a year-end Capstone assessment. They kept a journal throughout this assessment that included entries for planning, documenting, interpreting, and grading the project.

Data Analysis and Interpretation

"The case study approach to qualitative analysis is a specific way of collecting, organizing, and analyzing data. The purpose is to gather comprehensive, systematic, and in-depth information about each case of interest" (Patton, 1990, p. 384). This study used a method of analysis that combines explicit coding and analytic procedures. To examine the data, two phases of coding were used. The first phase occurred during the interviews and involved a scheme that identified the teacher, interview, and assessment method. Analysis of the coded data produced domains or areas of assessment common to all participants. The second phase of coding occurred after all the data were collected and involved coding within the domains. Analysis produced common themes that were then used to compare participants.

Analysis started with the first interview. The primary method of analysis was direct interpretation of individual
interviews, observations, documents, and journals. The first two interviews were semi-ethnographic in nature where the participants were free to describe what they looked for on a day-to-day basis and how they used that information to assess learning. Immediately after each interview, the questions and responses were transcribed and coded. This provided the researcher with the opportunity to search for common domains that would help organize future interviews. Five domains common to all teachers emerged after the first two interviews. Subsequent interviews focused directly on those domains. Those domains were: (1) group work, (2) the Checkpoint, (3) tests and quizzes, (4) assignments, and (5) projects. Once these domains were discovered, all future responses that dealt with a particular domain were underlined in the same color. Letters were used to identify the teacher and numbers to identify the encounter in which the data were collected. A sample of this coding scheme is provided in Appendix B. Coding data was critical because it assisted in identifying the extent to which the interviews, observations, documents, and journals supported each other.

The second phase of coding occurred within each domain. The researcher started labeling ideas within each domain for each participant until common themes emerged. When a possible theme emerged, all responses relating to that theme were underlined in the same color. If a theme
for each participant had the same color then that was considered for a common theme. Close analysis of this type produced themes of: (a) learning environment, (b) forms of communication, (c) teacher feedback, and (d) time.

A descriptive method is used to report the findings of this research. This means reporting the strategies and procedures that each teacher used to implement their assessment plan. The investigator's intent was to record what each teacher was doing and to interpret the findings. Any problems or management issues that each teacher encountered were also reported.

The four chapters that follow are the case studies of the 4 teachers involved in the research. Each chapter provides descriptions and examples of these teachers assessing students in the five domains. The students that are mentioned in each case study were selected by the participants. Chapter VIII answers the questions addressed by this research and analyzes the four case studies using the common themes. Chapter IX summarizes the study, makes conclusions, and recommends further research.
CHAPTER IV

THE CASE OF JILL

Introduction

Jill was an experienced teacher of 25 years. For the past 15 years she had taught in a large suburban school district near a large city. She taught three CPMP classes in the morning and then worked at a local college in the afternoon. Her CPMP classes included two Course 1 classes and one Course 2 class. The school had not adopted the CPMP curriculum. However, Jill and one other teacher had arranged to use it on a trial basis. While most students in the school enrolled in a more traditional program, a few elected to sign up for CPMP courses. CPMP was not available for the honor students since they began the honors mathematics program in the eighth grade.

This chapter describes Jill’s assessment program. The information was drawn from her teaching of the second semester of CPMP Course 1. It starts with her description of a typical day in her classroom and then describes her assessment of students in each of the five domains.
A Typical Day

Students walked into class and sat down in groups of four. While they waited for the bell to ring, they talked with group members. When the bell rang, Jill welcomed them and reminded them about a project that was due in 2 days. She then said, "Turn to page 22 and follow along as I read." Page 22 contains an introduction to exponential decay and relates a real life situation referred to as a "Launch." Included in the Launch is a section titled "Think About This Situation" where students are asked to respond to questions related to the introduction. Figure 2 contains the Think About This Situation for exponential decay. The questions refer to a real life situation that involve an oil spill and environmental cleanup.

In describing a typical day in her class, Jill said,

Typically, I am implementing a launch, explore, (and) summary. I would motivate the kids to get started on something. If it is a new topic, then I need to assess where their background is, so there is a discussion of what they know. If we are in the middle of an ongoing investigation, I might just summarize what we did yesterday on the same page to start. Then we get to the "explore" when the students are in their small groups. The amount of time varies. This is when I sit down with the small groups and collect some group observations. The last part is the summary. At that time I will have students do oral reports. It is describing how your group made sense of a challenging part of the investigation or how they made sense of the mathematics in the investigation. It may be a single person, or a pair, or a whole group. I always call time and say that you will be asked for an oral report. So they have to clean up their act and make sure they have the information. They are graded on
Think About This Situation

The graphs below show two possible outcomes of the pollution and cleanup situation.

(a) What pattern of change is shown by each graph?
(b) Which graph shows the pattern of change that you would expect for this situation? Test your idea by running the experiment several times and plotting the (time, pollutant remaining) data.
(c) What sort of equation relating pollution $P$ and time $t$ would you expect to match your plot of data? Test your idea using a graphing calculator or computer.

Figure 2. Think About This Situation (Coxford et al, 1997).


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this so they must make sure they understand everything they have written down.

Group Work

To plan and prepare her students for group work, Jill spent the first 2 weeks of the school year in group training. She stated,

Each student was given a role to play in the group. One role was to be a quality control person and that person assigned a grade based on how well they were listening to each other. Did we do good work today? Did we check our answers? The quality control person does what I am doing later.

The Group Observation form (Appendix C) was developed after 2 weeks of practice. Students first discussed criteria for good group work and, based on that discussion, Jill developed the form.

As students were working their way through an investigation, Jill moved around the room and listened to the discussions and the group interaction. Often this researcher saw her stand next to a group and just listen to the discussion. She would then move to another group to listen. Once or twice a class-period Jill sat with a group and completed a Group Observation form. Appendix C contains two examples of completed Group Observation forms that Jill filled out while observing a group having trouble. Regarding this group, she said,

Pete and Mike do not work together very well. They feed off each other's worst qualities. The group was working on surface area and volume of prisms. Pete is
probably the weakest in the group as far as ability. His question was, "Do I multiply five by seven?" He should be asking, "Why do I want to multiply five by seven?" Barb’s answer is, "Yes and then add 70." Her answer should be, "Each base is five by seven so you multiply five by seven."

Jill read this report to the group at the end of the class period and gave them feedback on what they were doing and what they could do to improve. Her feedback centered around getting the students to ask more "why" questions and less "what" questions. She told the group that she would sit with them the next day to see if there was any improvement. The following day did not show much improvement. Jill reflected,

I sat with them again and it was almost as bad as the day before. They read the questions hurriedly. They weren’t listening to each other. Pete’s question was, "How did you get the volume of the cube?" Barb’s answer was, “eight times eight times eight.” That doesn’t help at all. So she was doing exactly what she was doing the day before. Her goal is to finish, not to understand.... Molly and Mike made a good effort. Molly tried to explain that there were two variables. (They were working on triangular prisms.) The variables are base and height. Others were thinking about just surface area of triangles. Molly said, "That’s not right." Finally Molly and Mike got into a good discussion. They were comparing volume of a rectangular prism and triangular prism. Mike said, "I don’t think that’s right because they both have the same height so the rectangular prism should have a larger volume than the triangular prism." So he was comparing answers as he went along, not just getting answers. That was good.

Jill assigned a grade to the quality of group work based on the items in the Group Observation form. According to Jill, “The students here are so grade conscious. Group work is so important that I even put a grade on it.” For
the first day, she gave the group a "D" and for the second
day she gave a "C." She returned these forms to the group
and went over them with the group to explain how they
needed to improve.

The Checkpoint

"Bill, you have 5 minutes to put together an oral
report." Thus began a part of the class referred to as the
Checkpoint. Figure 3 contains the Checkpoint questions for
the third investigation on exponential decay. Jill selected
a student or group of students to report on their findings
during the investigation.

J: They are not scary. It is describing how your
group made sense of the investigation or how you
made sense of the mathematics in the
investigation.

R: How do you make the selection?

J: Sometimes I want a particular group because they
came up with something different. Sometimes it is
because they got off track, but it is interesting.
Sometimes it blows up in my face because I think
they are prepared but they're not. Sometimes it
takes longer than I expected, but always something
useful comes out of it.

R: Do the students have some direction as to what
they should include in their oral report.

J: Yes, most of the time they go through the
questions in the Checkpoint section in the book.

While the students gave oral reports, Jill sat at her
desk and completed an Oral Report form. (Appendix C con-
tains two completed Oral Report forms.) Jill assigned a
grade based on the criteria in the form. During the report, other students in the class were encouraged to ask questions.

### Checkpoint

In this chapter you have seen that patterns of exponential change can be modeled by equations of the form \( y = a(b^x) \).

(a) What equation relates NOW and NEXT \( y \)-values of this model?

(b) What does the value of \( a \) tell about the situation being modeled? About the tables and graphs of \((x,y)\) values?

(c) What does the value of \( b \) tell about the situation being modeled? About the tables and graphs of \((x,y)\) values?

(d) How is the information provided by values of \( a \) and \( b \) in exponential equations like \( y = a(b^x) \) similar to, and different from, that provided by \( a \) and \( b \) in linear equations like \( y = a + bx \).

Figure 3. The Checkpoint (Coxford et al, 1997).


### Assignments

Jill put the week’s assignments for each class on the chalk board. She told the students which part of the
Assignment should be completed each day and then collected the work each Monday. Included in all assignments were problems from a section called On-Your-Own. Jill always evaluated these problems first, but allowed students to redo the On-Your-Own problems if they were done incorrectly. She then looked at the rest of the assignment and evaluated it on a scale of 1 to 10.

J: Some of the Organizing problems are so rich I prefer to do them in class. I will always assign some Modeling problems which are applications and I will always assign some Reflection questions which ought to be done individually.

R: How do you use the evidence gained from reading and evaluating the homework?

J: Mainly I see where we are as a class. I make notes to myself like, "everyone seems to be having trouble with number 5" or "Gary did this one real well." It is a chance for him to shine and put his solution on the board. Everybody will understand a little better and it gives Gary a chance to feel good about himself. That is what these are. (She referred to the chalkboard.) They got their work back today and students put solutions on the board.

Tests and Quizzes

Appendix C contains responses to a quiz by three of Jill's students along with Jill's evaluation of those responses. The quiz contained two tasks (Figure 4) that dealt with exponential growth and decay. Each task required the students to explain or describe their reasoning.

Fred, according to Jill, was an average student, but
Exponential Functions

1. Two exponential growth and decay situations are represented by graphs (1) and (2) and also by tables (A) and (B). For each graph there is a matching table.

(a) Write the number of the graph beside its corresponding table.

(1)  

\[
\begin{array}{c|cccc}
\text{x} & 1 & 2 & 3 & 4 \\
\hline
\text{y} & 6 & 12 & 24 & 48 \\
\end{array}
\]

(2)  

\[
\begin{array}{c|cccc}
\text{x} & 1 & 2 & 3 & 4 \\
\hline
\text{y} & 1.5 & 0.75 & 0.375 & 0.1875 \\
\end{array}
\]

(b) The equation of graph 1 in part (a) is of the form \( y = a(b)^x \). Will \( b \) be less than 1 or greater than 1? Explain your answer.

3. Consider the \((x,y)\) pattern produced by the rule, \( y = (1.3)^x \).

(a) Make a table of \((x,y)\) values to 1 decimal place, from 0 to 10 in steps of 1.

\[
\begin{array}{c|ccccccccc}
\text{x} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\text{y} & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ }
\end{array}
\]

(b) Mark the y-axis and plot the points.

(c) Describe the way that \( y \) changes as \( x \) increases.

Figure 4. Tasks From Jill's Quiz (Coxford et al, 1997a).

often "is off on a tangent, both in class as well as on test papers." Regarding his solutions, Jill said,

Fred has a good answer for question 1b. He refers to multiplying. He connects the vocabulary "decay" to what he is multiplying by is less than one. In his words, "when you multiply by a number less than one, then it is, he must mean the result is, less than the original number.

In question 3, the table is fine. In part c, I was hoping to read that when x increases by one, the y increases by multiplying by 1.3 making larger and larger increases. Fred starts off by saying, "as x increases." He doesn't say "by one." He seems to have misfired in the middle. That is fairly typical of Fred. I asked him to explain it to me (later) and his oral explanation was fine. I do that with Fred.

Barb, according to Jill, was above average in ability. She described Barb's responses as follows:

In lb she talks about the graph decreasing and then relates that to a number less than one, but she does not mention multiplication at all. I would like her to concentrate on when you multiply by a number less than one, you create some decreasing answers.

In part c of number three, her description is pretty good. She got the important distinction about the increasing rate of y although she doesn't say the x is increasing by one.

According to Jill, Molly was also "above average in ability. She can do well, but sometimes not so well." Regarding her test, Jill stated,

In lb Molly does not refer to multiplying by a number less than one. She says, "It will be less than one because the curve decreases." I'm not sure she is thinking about multiplication here. As I look further down the paper, I see she uses multiplication in other places, so a minimal amount off there.

In problem 3 she says, "As x increases by ones, the y increases by," and then she gets off into something concrete. She is perfectly correct but she is focusing too much on the concrete and not on the pattern of the change. I made a minimal deduction because she was not
focusing on the pattern.

Projects

Jill used one CPMP project per quarter. She believed that projects provided her with another way to assess students. According to Jill,

It gives another picture of the student when they have time to take stuff home and think about it. Some students who do not do well on quizzes or tests do better when they have more time.... They then make presentations of their project so everyone can see what everyone else did. They do that with partners. They always do projects with partners.

As a year-end assessment, Jill used the Capstone project provided by CPMP (Appendix G). Her school required her to give a written exam, so this project was not used as the final evaluation of students, but was included as part of the last marking period.

In planning for the project, Jill discussed a schedule with the other CPMP teacher at her school. They decided to spend 1 day doing Investigation 1 as a whole class activity. Groups of students would then spend 3 days doing two of the remaining six investigations. (She did not include Investigation 8.) Students would then prepare and present an oral report and complete a written report on their investigations. Jill noted,

This is a friendlier way to end the year than the final exam. (It) seems more in keeping with the focus of the year’s work. This should serve the purpose of students’ reflecting back on all the math they know, choose an appropriate idea/skill, apply it, and re-
port. The boundaries are less artificial than exam questions.

To begin the project, Jill read the introduction on the first page of the Capstone and asked the class for responses to the instructions in "Think About This Situation." The class then turned to page 2 of the project and used it to review the mathematics they had studied during the year. Jill said,

I asked "What was 'Patterns in Data' about? What activities do you recall? What ideas did you learn about?" After I got some responses, I then asked each group to come up with one way to use these ideas in the Carnival Project. We then noted these for Checkpoint part a. (We) repeated (this sequence) with the next five units. (I) had no problems in getting contributions for our list.

After discussing Investigation 1 as a full group, the small groups chose two investigations to complete over the next 3 days. During those days, Jill continued her method of observing groups and completing group evaluations. She tried not to help the groups too much but would limit her answers to suggestions. For example, in Investigation 4 two groups had difficulty agreeing on answers to question 2b. Students were asked to make an equation relating the cost of buying a given number of shirts and the cost of buying one more. Jill's help simply involved making a table that started at 150 shirts and looking at the pattern. She left it up to the students to find the equation.

After the oral reports and written reports were completed, Jill reflected on the project:
If I do this again, I'd want to start earlier because the students deserved feedback on all the work they had done. I would like to move the final exam up and have the oral reports and feedback done on the last day. To make this possible I would need to get students to help generate a quick way to report back on the oral presentations. I thought it was a positive closure for kids and I'll do it again, with modifications.

Summary

Jill believed that the CPMP curriculum allowed her to implement her philosophy of teaching and learning mathematics. Her classrooms were student focused and she worked hard at developing an environment where students could create their own meaning to mathematics. Jill commented on more than one occasion that students learn best when they are part of a group discussion and when they can communicate their understanding in words.

To assist her in assessing student learning, Jill developed a Group Observation form and an Oral Report form that she used on a daily basis. As students completed group investigations, she listened carefully to discussions and evaluated how groups involved all members and how well they concentrated on understanding, not just answers. As students led the class through the Checkpoint, she evaluated how well they understood the mathematics and could answer related questions that she or classmates asked.

Her assessment of students included their written
responses on MORE tasks as well as written responses to CPMP tests and quizzes. She took the time to read all responses to student assignments and used the results to inform instruction in class. In addition, Jill assigned projects which she believed allowed her to assess different skills than other parts of her assessment plan. While other parts of her plan had time restrictions, she used projects to give students more time to think and respond.
CHAPTER V

THE CASE OF KAREN

Introduction

Karen was an full time teacher with 25 years of experience, having taught her entire career in a rural school district. Karen was unsatisfied with grades as an evaluation tool and therefore used a five-point rubric which gave students more feedback. She used this rubric for group work, tests, quizzes, projects, and homework. The specific guidelines were developed for specific items but they all followed the general rubric contained in Appendix D.

This chapter describes Karen’s assessment system. The information was drawn from her teaching of Course 1 of the CPMP curriculum. It starts with a typical day in her classroom and then explains her method of assessment in each of the five domains.

A Typical Day

The school where Karen taught used block scheduling where each class met for 2 hours each day and a full-year course was taught in 18 weeks. Each class began with either a sharing session or launch of a new investigation.
K: A sharing session is where I start off by asking questions. What is nice about a block class, where I have the students for 2 hours, is students have an opportunity to speak every day. I try calling on every student every day.

R: Could you give me an example of a launch?

K: If we have finished an investigation, I will start class with a launch of a new investigation. This may range from using the launch in the book to a launch that we develop. For example, to launch the investigation about simulation models, we used a segment about population growth from the TV show 20/20. Then we had a couple who had gone to China to adopt a little girl come in and describe life in China. The first investigation involves a rule in China where families can only have one child.

After a sharing session or launch, the students got into their groups of four and began the investigation. Periodically, Karen stopped students and checked to see if they were on the right track. At the end of the investigation, Karen discussed with the class the Checkpoint questions in the book.

Group Work and Checkpoint

Karen called her class a "math, teamwork, communications class." To plan for this type of class, Karen had her students discuss "what good group work looks like and what we should hear if good group work is going on." On the basis of that discussion, Karen developed the following seven guidelines for assessing group work: (1) contributes appropriately to the group effort, (2) is willing to listen to others' ideas and to compromise when appropriate,
(3) asks questions about things when they are not understood, (4) works with the group until everyone understands and can explain the results, (5) encourages others to contribute to the project, (6) avoids or smoothes out friction, and (7) stays on task during class time.

R: When the students are working on the investigations, what are you doing?

K: Most of the time I am walking around listening to what the different groups are discussing. If I hear a misconception, I store that and make sure that, when we do the Checkpoint, I ask a question that brings out that misconception. If I see that same misconception happening in several groups, we will stop and talk as a class about the misconception.

R: Can you think of any examples to illustrate that.

K: Yes, yesterday we had a simulation where they had to model a family with two children. Three of the six groups were tallying every single flip of the coin with no purpose. So I stopped the class and we discussed what constitutes a trial.

R: If you notice behavior problems when walking around, what do you do?

K: Behavior to me means: What is it that you are doing? Is it appropriate and is it getting you to where you want? I am talking about a student who is just sitting there with his arms folded and just listening. I wouldn't just ask that person, I would also ask the others, "What are you doing? How can you draw Jeff into the conversation? How do you manage the situation where one person is uncooperative and will not contribute?" Those are the questions I ask.

While Karen observed students working in groups, she was often asked questions. The researcher observed that Karen would never directly answer the questions. Instead,
she would ask questions that would lead students to discover the answer to their own questions.

Karen used a group/participation score in her overall evaluation scheme and based that score on the seven categories listed earlier in this section. However, she did not keep a daily written record of what she observed. She was with her students for 2 hours every day, and she kept a mental record of how students were working. She stated,

I am a very literal, sensory person. My whole world is what I see and what I hear. When I am evaluating students every day, I evaluate them on what I see and hear. To have written documentation every time, that is hard and not worth the time.

Assignments

Assignments in Karen’s class were used to reinforce concepts, to help students make connections, to reflect on what was learned, and sometimes to extend ideas learned in class. Concerning the MORE exercises, she said,

The Modeling problems are similar to the investigations, but with a different context. The Organizing problems connect to different strands or are working backwards problems. The Reflecting problems are “what if” questions, or “investigate the history of,” or “look around your house for examples of,” or “how did you do that procedure?” So it gives them a chance to reflect on the math they have learned. The Extending problems are more difficult. They either have new information or they are more abstract.

Students were given 3- to 5-day assignments and were expected to complete them outside of class. On the day the assignment was due, Karen assessed understanding in one of
three ways.

K: On the day it is due, we discuss it, or turn it in, or they have a homework quiz where I spot check their work. It's not the same every time. Sometimes I will read through their work and grade their general understanding of the topic covered on a 0, 1, 2, 3, 4, 5 basis. Other times I will have them transfer onto a quiz page certain portions of certain problems.

R: You take a sample of their work?

K: Yes. Other times I have them go through their work and circle portions of it, especially graphs. Sometimes I will have them highlight what they want graded; what they think they did their best on.

If Karen was unsatisfied with certain portions of an assignment or on a homework quiz, then students had to complete "redos." This involved completing poorly done or incomplete problems. According to Karen,

A student could get three of them right (on a homework quiz) and get a pretty high score on the quiz. But if he had a problem that was completely off and he scored a 0 or 1 on that problem, then I would make him do a redo on that one problem before his homework quiz score is recorded.

No grade entered the gradebook until all redos on an assignment were completed.

Tests and Quizzes

Karen used the CPMP tests and quizzes with modifications. Sometimes she would add a problem or change the wording of a question to fit what had been emphasized in class.
K: A test in Core-Plus (CPMP) is usually no more than three or four questions. Each question may have several parts or aspects relating to practical applications. They are also asked to explain their reasoning, to compare, describe, (and) define, which gives them plenty of opportunity to have an incorrect numerical response but to show they understand the process. So I don’t get into a situation where a student does poorly simply because he made careless errors.

R: How do you prepare your students for a test?

K: One of the drawbacks, at least some parents think it is a drawback, of the program is there is no end of unit review or practice test to use to prepare for a test. There are no examples of problems for parents to look at to help the student at home.

R: Are there certain things you do the day before a test to get them ready?

K: Yes, we use some reading techniques where we make a formal outline as a class, or we may do a concept map, things like that where students come up with the outline or concept map. Also at the end of every book is a looking back section which is three or four problems that are representative of all the problems in the unit.

Appendix D contains a rubric that Karen used to evaluate her students on part of a test on Unit 3. It also contains responses of three students, Brenda, John, and Connie, on two questions of a test, and Karen’s evaluation of those responses. Before evaluating any test or quiz, Karen recorded what she would accept as a “5” response and a partial list of how a “4, 3, 2, or 1” response could appear.

The first question dealt with solving an equation and a corresponding inequality and applying it to different
circumstances. Karen described the students' responses as follows:

In part b, Brenda drew the graph and table correctly. She said you go down the y-column until you find 25 which is backwards. Twenty-five is the x-value. According to my rubric, she did not fulfill the requirements. She has major flaws and shows minimal understanding. I gave her a 0.5. On the second part, she says you would go to 25 on the graph. She has not said what axis to look on. My comment is: "How do you know to look for 25 when 25 is what you are finding?" I think she has shown no understanding.

Part c has two parts each worth two points on the rubric. On the first part she shows minimal understanding of finding the solution in the table. She does know that she has to look for 50 in the y column. She does not state that the solution is in the same row in the x column. She does say that whatever the number is beside 50 is the answer. Even though she knows where to look, she hasn't shown any kind of understanding that she is dealing with an inequality, and that there may be more than one solution. I gave her 0.5 out of 2 for minimal understanding.

On finding the solution on the graph she says to trace the graph until y=50 and then see what x is and anything after that would be greater than 50. This is true but it is unclear. It is poorly worded. I gave her 1 point out of 2 for having the basic idea with major flaws.

In part d her question is poorly worded. It seems that she has the idea she is working with more than 1/2 or less than 1/2, but she has shown minimal understanding of how to relate to the inequality with the way she worded the question, so I gave her 1 point out of 4.

John is a sophomore in a freshman class. In part b his graph is correct and also his table is correct. In his explanation on how to find a solution, he doesn't say where 35 appears in the y column or the x column. He doesn't say what to do after that. I gave him a 2 out of 5.

In part c the information that John has written down is totally unrelated to finding the solution in either the table or the graph. He got a 0.

In part d, his answer in no way relates to the inequality. I gave him a 0.

Connie came from a parochial school, not our middle school.... In part b both the graph and explanation are correct. She correctly explains how to find the
values. She did not use the specific numbers, but her explanations are correct. In part c she starts out OK, but then says to find the values (plural) of \( x \) where \( y \) is less-greater, she has both of those. She is maybe not sure of her sign, but she is aware that she has more than one solution. I gave her a 1.5 out of 2 indicating a minor flaw. On the next question, her procedure is correct, but her solution should be \( x < 17.5 \), so I gave her 1.5 indicating a minor flaw. Her question in part d is correct and well worded so she got a 4.

If students did poorly on parts of a test, Karen believed the reason involved students not participating in the investigations or not completing assignments. When asked what she did with students who do poorly on tests, she responded,

Most of the time when that is the case, it is because they had redos and did not do them before the test. As far as going back to go over the test with them, I could do that, but the other avenue is to get the redos done.... There ends up being some discussion or they do not make it.

Projects

Karen used the Capstone, Planning a Benefits Carnival Project (Appendix G), as an alternative to a year-end exam. This project was part of the CPMP material and according to Karen, "utilizes most of the mathematics covered throughout the course."

In gathering evidence for this project, she decided to work through Investigation 1 as a full class and to assign other investigations to groups of students. Each group prepared and presented an oral presentation for one
investigation and worked through a second investigation. Since she had six groups in her class and there were eight investigations, each of Investigations 2 through 7 were presented once and covered twice. If students wanted to do Investigation 8 in addition, she gave them extra-credit. She evaluated them daily according to the seven-item rubric mentioned earlier in this chapter. She also had all students take notes on the presentations and write a report using their notes. Instructions for this report are found in Appendix D. Finally, she decided to base the overall student evaluation by weighting it 40% on the presentation, 40% on the written report, and 20% on the group work.

Karen was pleased with the effort and positive comments she received about the project from the students. She noted,

The student comments were most encouraging. Most students commented that the Capstone was a fun way to review and a great way to finish the semester. From my perspective, the students invested more time and effort in this assessment than students in the past who took a traditional exam over the same material.

After Karen completed reading and evaluating the papers, she discussed results with other teachers. Because of these discussions, she decided to place more emphasis on certain topics in future years or revisit those topics at the next level.
Summary

Karen believed that both oral and written communication were the key to being successful in learning mathematics. She also believed that all students could be successful in her class and lack of success was directly linked to effort.

In evaluating students in all areas of her assessment plan, Karen used a five-point rubric which identified to what degree the students were meeting the expectations of what was being assessed. For group work, she developed seven guidelines and her rubric informed students of her assessment of how they were meeting her expectations. Her expectations focused on the degree to which they were willing to be involved in group investigations and the group interactions that they initiated.

MORE tasks were used for out-of-class assignments and Karen required students to complete all tasks in an assignment before they received credit. Karen also used CPMP tests and quizzes in her assessment plan. Her five-point rubric informed students of the amount of understanding they demonstrated on test and quiz questions and also encouraged students to go beyond what was asked.

Karen used the CPMP Capstone project as a culminating assessment of the course. Each student completed parts of
the project and used notes taken during the presentations to write a summary report.
CHAPTER VI

THE CASE OF MARY

Introduction

Mary was a full time teacher at a math and science center. Students came to this school for their mathematics and science classes and then returned to their regular school for all other courses. According to Mary, “These kids are very good. I can do much more than the average teacher.”

This chapter describes Mary’s assessment process. The information was drawn from her teaching of the second semester of CPMP Course 1. It starts with what she described as a typical day and then continues to discuss her assessment of students in the five domains.

A Typical Day

Students had a choice of three problems to do for homework. After they arrived to class, Mary divided them into three groups depending on which problem they had done. Students were instructed to compare and discuss their solutions. While this discussion took place, Mary walked around the room observing and listening to the discussions.
Joe walked in late and sat by himself in the back of class with his head on the desk. Mary went over to him and said, "Joe, it's time to get going." Joe said, "You're not paying any attention to me." So Mary asked him what problem he had worked on and led him to the appropriate group.

After 10 minutes, the class came together as a full group where Mary introduced dependent and independent variables. The students then got in their assigned small groups, opened their textbooks, and began working on an investigation. As the students completed this part of the day's work, Mary again walked around the room listening for ideas and misconceptions that she would use during the Checkpoint later in the class period.

Mary said, "Bill, your group is in charge of the Checkpoint today, so decide in your group who is doing what." Five minutes later, Bill and the other three members of his group went up to the front of class and presented their answers to the Checkpoint questions. During that time Mary recorded comments about their presentation on a seating chart and asked questions that helped clarify the misconceptions she observed during the investigations.

Group Work and Checkpoint

R: What are you doing when the students are involved in an investigation?

M: I'm watching. I'm roaming the room. I'm looking
for problems. If they have a group question, then I am available. Like today we had a big argument about dependent and independent variables. "Time," they thought, should always be independent. In the problem, time was dependent on speed so we had an argument.

Mary believed that following the CPMP curriculum, where students construct their own meaning through investigations, allowed her to assess student progress on a day-to-day basis.

M: Since doing CPMP I have a much better handle on how students are learning because I am watching them in different situations. I see them teaching each other and interacting in small groups. I make observations constantly, and I know so much more about them than I ever did before. As a result, I have a better handle on how the kids are learning.

R: What do you look for when you are listening to a group?

M: Ultimately I am looking for the Checkpoint. The Checkpoint is where everything that is going on comes together mathematically. Many times kids don't put it together. They can sometimes just go through the motions and not get anything out of it. So the Checkpoint is where I want to ask a lot of good questions. So I walk around the room and look for any misconceptions. That is where I pick up my cue questions that I use when we get together as a large group.

According to Mary, most of her students came from a traditional classroom where a typical class period began by the teacher going over homework problems, followed by the teacher teaching a lesson, and then students given time to begin their assignment. Mary believed that one of her main responsibilities with freshman was to get them to buy in to a new way of learning mathematics. She was convinced that
all her students would be highly successful if they would follow the curriculum as it was intended.

Mary insisted that students teach each other as much as possible. She spent part of two class periods at the beginning of the year instructing students on how to use the overhead projector and how to lead a Checkpoint.

M: The kids often lead the Checkpoint although I make sure misconceptions are brought out.

R: At the last interview you mentioned that whole groups often go up to make the presentations. You said that kids get ownership of what is presented even though they might not present.

M: Yes, and already they are getting better at that. Now, usually all of them have something to contribute, so they do not just stand there. Before they go up, they prepare who will do what.

R: Do you document what you hear?

M: What I do is check off the kids that do the report. Sometimes I make a comment in my book if it is particularly good. What I hope to do is get everyone some time in the week. Some kids like to teach more than others. I give extra credit for good explanations. I never take away points. This is always a positive experience.

Assignments

Mary assigned homework almost every day although she did not check it every day. She stated,

I do not spend much time going over homework. Most of it is self-checking. I can spot check it easily by going around the room and see if they did it or not. Then they must put it in their folder and I check it off on Friday. I do not push homework as much as I used to.
Mary used homework to reinforce what was learned in class and to reflect on that learning. Although she required students to do the daily assignments, she did not feel homework was necessary for her students to be successful. She said,

This is the first curriculum that is not heavily based on homework to be successful. Kids get enough out of class so they can perform reasonably well on tests. This would never happen in an old curriculum. It is not learning a procedure and then practicing it a number of times. It teaches kids that math is concepts and investigations, not just a bunch of procedures to be learned.

Tests and Quizzes

Mary used the test and quiz questions supplied by the CPMP curriculum although she often supplemented them with questions of her own design. She liked to use questions that caused students to extend themselves by applying the concepts to new situations. This allowed students to demonstrate their problem solving skills.

To prepare her students for a test, Mary sometimes used a "test ticket." The day before the test, students answered a series of questions relating to the unit. These questions reviewed all the main concepts that students had studied. On the day of the test, students were required to turn in their solutions before receiving a copy of the test.
Geometry Test

5. Kim is building a rectangular pen for her pet goat. One side of the pen is the back of the barn. She has 50 meters of fencing. Sketch and label the pen below. Make a table showing possible lengths, widths, and areas to find which rectangular area will give her the maximum area.

6. If she is able to build a pen of any shape against the back of the barn, which shape should she use to get maximum area? Sketch it below with dimensions. How much bigger is this than the rectangular pen?

Figure 5. Tasks From Mary’s Test.


Appendix E contains student responses to part of a test Mary gave on a geometry unit. During this unit students studied both 3-dimensional and 2-dimensional shapes, symmetry, tessellation’s, patterns, transformations, volume, surface area, and perimeter. Question 5 (Figure 5) involved a situation that students had not seen before. She wanted to see how they would respond to a familiar topic but in a new situation. Most students had trouble with question 6 because they could not make the
transition from rectangles to other shapes. Mary stated,

I want to see if they think inductively or deductively. It certainly helps me evaluate where my kids are at. When you ask a question like that, it is obvious you have kids at different levels. It’s great to see that kind of thing happen. I am not looking for anything specific. I am looking for how they develop their ideas. If they think inductively, I expect them to use more than one example.

The three students that wrote the responses vary in achievement. According to Mary, Joe was very bright and always did well, Melissa was a medium achieving student, and Brian often had trouble with reasoning. Explaining the student responses, Mary said,

On problem 3 Joe has good sketches and a good explanation. I can see he understands when a polygon tessellates.... On question 5 he did a good job of building the table although he did mess up on question 6. I was surprised at this. I expected him to get this one.

Melissa is a pretty nontraditional gifted student. Nontraditional in that she often questions why we have to do things and she tends to be quite creative.... On question 3 she tried to use a definition from the book. I wish she would feel more comfortable using her own language. Question 5 looks good and I was real pleased with her work on question 6.

Brian is a student who has come through a system where he memorized a lot. He is having trouble adjusting to the new curriculum. He has trouble applying math in new ways and in situations that are new to him. He tends to stay confined to one area rather than extending it into new realms. His paper, I am sure, was disappointing to him. We will have to go over it together.... In question 3 he had trouble with reasoning. Vocabulary seems to be giving him problems.... Question 5 is good although in question 6 he did not investigate shapes other than rectangles.

Mary used a point system to evaluate the tests. Question 3 was worth 5 points, question 5 was worth 10 points,
and question 6 was worth 5 points. Next to each response Mary wrote the points she determined each student received for each question. Students received no points if they did not demonstrate at least minimal understanding. Mary then used a sliding scale where the number of points students received was determined by the amount of understanding they demonstrated.

Projects

Mary was able to do most of the projects in the CPMP curriculum plus others that she adapted from other books. Projects were always done in groups and involved a presentation. When asked how she assessed student achievement using projects, she said,

Sometimes each kid gets 25 points. Each kid then secretly gave points to others in their group. The kids that did not do their share got nailed that way. You give points to yourself but only the other 3 count. I give the remaining 25 points.

Mary decided not to use the year-end Capstone assessment because she did not think it reviewed all the concepts of her course. She felt it would be more beneficial to her students to take a traditional paper and pencil exam.

Summary

Mary taught high-achieving students and believed she would be successful if she could convince her students to
accept learning as construction of meaning. Learning mathematics through group investigations was new to her students. Most of them had been highly successful with a traditional classroom, so she struggled with getting her students to accept a new way of learning.

Her classes were 75 minutes in length so she was able to use the entire CPMP curriculum plus student projects from other sources. Students were the focus of both learning and instruction. Mary introduced lessons, made sure misconceptions were discussed, and assessed learning on a daily basis, but students helped each other with investigations and led the Checkpoint.

Mary assigned MORE tasks, but felt students could be successful without completing those tasks. Her evaluation of assignments focused on completion. She left it up to students to ask questions about tasks they did not understand.

For tests and quizzes, Mary used questions from the CPMP curriculum but supplemented them with questions of her own. She wanted her students to extend ideas into unfamiliar areas, so she devised questions that would meet that need. In her words, this is "problem solving" and part of her assessment of students involved their problem solving ability.
CHAPTER VII

THE CASE OF JACK

Introduction

Jack came from a traditional background of teaching and learning. As many mathematics teachers, he found it difficult to relinquish control and allow students to learn by constructing their own meaning of the mathematics. He was in his 10th year teaching full time in an urban school district where many students had not been successful in mathematics. His classes were divided into small groups of three or four, and he was looking for good methods of assessing student progress and creating an atmosphere where students did not depend on him as the sole authority.

This chapter describes Jack’s assessment philosophy and how he attempted to implement that philosophy. Most of the information was drawn from his teaching of the second semester of CPMP Course 1. However, the test analysis was based on tasks from Course 2. The chapter starts with a typical day in his classroom as he described it and as observed by the researcher. It then continues with descriptions of his assessment practices in the five domains.
A Typical Day

Students walked into class and took their seats in groups of four. Written on the board in front of class were the instructions that the students were to follow for the first 30 minutes. These instructions told the students to answer three questions on specified pages in their textbooks. While the students tried to answer these questions, Jack walked around the room writing notes on a clipboard, sometimes answering questions, but mostly just listening. After 30 minutes of work, the students turned their desks to face the front of class. Jack asked, "Who has the formula?" One student went to the front and wrote "a² + b² = c²" on the board. Jack then asked, "For a 6x10 TV screen, what is the diagonal?"

According to Jack, there were two things that happened in class. One was small group work and the other was a full class discussion. The class started with students in small groups. After they had a chance to complete an investigation, they became a large group and discussed what they had discovered.

Group Work and Checkpoint

J: I start in groups and then move to a U-shape.

R: Do you have all your groups starting at the same place?
J: Yes. I keep them together that way.

R: When they are working in groups, what are you doing?

J: I am usually observing how well they are struggling with the questions. There will be a lot of kids asking questions. They get frustrated. They will want me to give them an answer, but I will not just give them an answer. They say "just tell us what to do." That part of the program was difficult for me. Sometimes I feel like answering their questions, but I try to respond to their questions by asking them questions that lead to answering their original question.

R: Do you record what you observe on a day-to-day basis?

J: I use a clipboard with a seating chart. I will look at a few students per day and try to give them a score of 1, 2, or 3 based on how well they are working in their groups. When we have class discussion, I make little marks on the sheet for people contributing to the discussion. At the end of 4 or 5 weeks, I look at the seating chart and also think back to what I observed and assign a participation grade. It is as much on my recollection as my notes.

After students had investigated and discussed in their groups, the class usually turned into a large group with Jack as the discussion leader.

R: After the investigation, what comes next?

J: Today, for example, we looked at some graph models to see if they had Euler circuits. If they could trace it and get back to where they started, that is an Euler circuit. We spent a good 20 to 25 minutes discussing some of the problems. In one class, we spent a lot of time on problem 1 (Figure 6). We tried to make it have an Euler circuit. One kid would try it, then another. We wanted to find a way by just looking at it and know. That led to a discussion of just looking at it and knowing. So we jumped from the first thing they did all the way down to the last question.... In 4th hour, we
went through the discussion more step by step. We all admitted the first one could not be done. When we got to the question "Is there a way to decide by just looking at it," there were some kids that knew the answer.

Making a Circuit

Shown below are graph models of the sidewalks along which parking meters are placed in two sections of a town.

East Town Model

West Town Model

(a) Why would it be helpful for a parking control officer to know if these graphs had Euler Circuits?

(b) Does the graph model of the east section of town have an Euler circuit? Explain your reasoning.

(c) Does the graph model of the west section of town have an Euler circuit? Explain.

Figure 6. Investigation Questions (Coxford et al, 1997).


Used with permission of Steve Mico, editor, Everyday Learning Corporation, 10-1-97.
Assignments

Jack spent little time going over homework problems in class. He put the responsibility on the students to complete assignments. Periodically, Jack would have a "homework day" where students were given the normal class period to work on unfinished assignments. At that time he was available to answer questions.

J: One of our goals as a district is to develop student responsibility. I also want students to work on solving some challenging problems. With those two things in mind, I give them a sheet with assignments and due dates. I do not give them all at once, but I update it periodically. They usually have 4 to 5 day assignments. It is the student’s responsibility to get them done and have them in their notebooks ready to go. Some students work on them a little at a time, others wait until the night before they are due. Then I collect them on unannounced occasions, so they have to make sure their homework book is kept up to date.

R: So you collect it on a due date?

J: Not on every due date. In fact I collected it five times last semester.

R: What do you look for when you collect them?

J: The first thing I look for is if they have attempted every problem. In order to get any credit they have to have attempted every problem. There may be parts a through q. They may have not done every part, but they have to have done some of it. Then I pick one in advance that I will look at closely. I have a rubric that will give them 8 out of 10, 9 out of 10, or 10 out of 10. Ten means it is done completely and well. Nine means there are parts that are correct but other parts incorrect, or it is sloppy. Eight means most are incorrect or quite sloppy. The only way to get fewer points is not to do the whole problem.
Tests and Quizzes

Appendix F contains parts of student responses to a test that Jack gave on power models. On this test, one of the major objectives that Jack had for his students was to relate the rule, the graph, and the table of a power model. To evaluate his students he included a question where the rule was given, another where the table was given, and a third that involved a graph. The sample in Appendix F contains two questions where the rule is given and a third question involving a graph. Before grading a set of tests, Jack would anticipate what kind of answers would be given, what kind of mistakes would be made, and how to weight the solutions. He then developed a rubric for each question to decide how to assign credit. In general he gave 40% credit for any reasonable effort at solving the problem and then divided up the remaining 60% depending on the amount of understanding the student demonstrated. For example, Jack decided to use 10 points for question 1b. He gave 4 points for a reasonable effort, 2 points for the equation, and 2 points for each of the times it would be 25 meters high.

Jack explained his evaluations of the three student papers in Appendix F as follows:

On question 1 part d, Laura had the wrong answer and her reasoning is incorrect. She got 4 out of 10 points for trying. Melissa says, “Because the numbers start getting negative.” I assume she is looking at a table. I gave her 2 out of 3 for the 6.5 and 1 point for an
attempt at an explanation. The reasoning should be something about when the t-value equals zero. I think she is looking at a table. So with 4 points for trying, 2 points for the 6.5 answer, and 1 point for her explanation, that makes 7 points. For Hillary, the only thing she has wrong is not rounding to the nearest 10th. This is merely a matter of not following directions, so she got 9 out of 10 points.

On question 1 part f, Jack expected the students to list at least two ways that the flight of the ball in part e differed from the flight of the ball in the original problem. On his rubric, Jack wrote down four ideas that he anticipated that his students would use. These were:

1. It starts higher.
2. It goes up at a slower rate.
3. It does not peak as high.
4. It hits the ground sooner.

For this question, students got 4 points for trying and 3 points for each correct response. Jack’s description of his evaluation follows:

For Laura, the part I underlined I counted as one thing. The other part does not answer the question. She got 4 points for trying and 3 points for one thing identified. The same with Melissa. I put a check mark by the one thing identified. For Hillary there are two checkpoints so she got 10 points.

Question 4 was very open-ended and had three parts: (1) the variable, (2) the rule, and (3) the graph. Jack decided to use 10 points for each part so the problem was worth 30 points. Jack said,

Laura has sketched a graph for which I gave her 4 points for trying, although it is not a power model. She has written a rule, so I gave her 4 points for
trying a rule. She did not try to identify the variables so she got a total of 8 points. Melissa got 10 points for the graph because it is a power model. This is one she made up herself because we did not do this one in class. I gave her 7 points for telling what the variables are but she really did not tell what x is and what y is. She does have them identified on the graph, but there is no rule. Hillary identified the variables, she does have a rule, and she does have the graph. She got 30 out of 30 points.

Jack used test results to inform what he taught. He did not use a checklist of specific items, but he tried to remember how well certain concepts were learned so that they would be emphasized in future instruction for future classes.

Projects

The only project that Jack used in Course 1 was the end-of-year Capstone (Appendix G) provided by the CPMP curriculum. He used the project as the students' final exam, and followed most of the suggestions contained in the directions with the exception of the amount of time. It was suggested that he use 10 days for the project, but Jack was able to use only the last 8 days of the school year.

As he planned for the project, he was concerned with the lack of structure to the investigations. He felt the activities might be too open-ended for freshman. However, he later reflected that the students responded extremely well to the structure and many did outstanding work. He developed a grading guide (Appendix F) that he handed out
to the students when he assigned the project.

To gather evidence he did Exploration 1 as a whole class and then assigned each group to be responsible for three other explorations. After completing the explorations, the groups then made oral presentations about one of their investigations. Finally, each student wrote a two-page report summarizing how the mathematics they learned in the course could be used to help plan a school carnival.

Jack felt he was very subjective in his evaluation of the project. He read each report and holistically assigned a grade. Since students did not receive any feedback other than a final grade, he did not feel it was necessary to be more structured in his evaluation.

Summary

Jack believed that students learn best when they construct their own meaning to mathematics, but he struggled with creating an environment where that could happen. Most days students completed group investigations, but often Jack felt his students wanted him to simply tell them what to do instead of discovering it on their own. Following group investigations, the class became one large group with Jack as leader. His assessment of students continued as he directed questions at individual students.

Jack gave individual assignments from the MORE tasks
in each unit of the CPMP curriculum. It was each student's responsibility to complete these assignments and Jack collected them on unannounced occasions.

Jack focused on effort as well as achievement when evaluating students in all five domains. He mentioned more than once that many of his students had been unsuccessful in previous mathematics classes, so he encouraged students by giving credit for effort. He continually was looking for techniques to assess individual learning and encourage student effort.
CHAPTER VIII

ANSWERING THE QUESTIONS AND ANALYSIS

Themes

Having presented each case study, the researcher returns to the research questions and compares the participants. As described in Chapter III, the first phase of coding generated five common domains in which the participants used assessment. This occurred early in the data collection process. After all the data were collected, the researcher then began a second phase of coding with the purpose of generating themes common to all participants. With this process the researcher was able to identify four themes that were used to compare the teachers. These themes are: learning environment, forms of communication, teacher feedback, and time.

Outline of the Chapter

Each of the five specific questions is posed, followed by answers for each participant. This is followed by a comparison of teachers for each question. Finally, a summary is provided which compares the participants across the four themes.
Revisiting the Research Questions

The study examined the following five questions about 4 experienced teachers using the CPMP curriculum.

1. What was their assessment plan?
2. How did they gather evidence needed to carry out their assessment plan?
3. How did they interpret the evidence?
4. How did they use the results?
5. What problems did they encounter when implementing their assessment plan?

Answering the Questions

Question 1: What Was Their Assessment Plan?

Question 1: The Case of Jill

Jill viewed assessment as an ongoing process, centered on obtaining a broad picture of what students can do. She believed her job was getting students to push themselves to their limits and to get them to continually ask questions that would further their learning. Paper and pencil tests were only a small part of the total picture. Important to Jill was how students communicated their understanding individually, in small groups, and in the large group.

To accomplish this, she daily completed group evaluation forms and oral presentation forms. These forms gave
immediate feedback to groups and individuals about how they were progressing in her assessment scheme.

Jill viewed grades as a motivating factor with her students. She assigned a grade to all areas of her assessment plan: (a) group work, (b) oral presentations, (c) projects, (d) tests and quizzes, and (e) homework. It was one of her ways of expressing to students what she valued. About grading group observations and presentations, Jill stated, "Kids are so grade conscious it's important to include it in their grade. It's that important, I am actually going to put a number on it."

Jill used assessment to inform instruction. The information she gained from observing students interact, from looking at assignments, and from results of written tests and quizzes determined the direction of instruction.

Question 1: The Case of Karen

Karen believed that communication was the gateway to learning mathematics and therefore the central ingredient of assessment. She stated,

I believe that students really don't understand what they are doing until they can explain it. Communication skills are very important. In order to communicate their ideas, students first have to organize them and then be able to understand them.

Karen was with her students for 2 hours every day and felt that this allowed her to more fully assess the
progress of her students. She was able to call on every student every day, and was also able to observe each student’s effort at learning the mathematics. Concerning her philosophy of assessment, she said,

I am a very literal, sensory person. My whole world is what I see and what I hear. I am not necessarily intuitive. When I evaluate students every day, I evaluate them on what I see and hear.

She viewed her job as getting all her students to become involved in each other’s learning. To her, problems arise when students are willing to write down answers without understanding. She said, “A problem to me is when one person is willing to just listen and write down results but is not willing to get involved in the conversation.”

Karen used assessment to inform instruction. Class discussion was always preceded by some assessment of what students knew. Sometimes it was based on what she heard during an investigation. Other times it followed what she read on homework assignments or how students performed on a test.

Grading was part of her assessment plan. She believed that grading involved much more than counting correct answers. Since she believed grades relate what teachers value, Karen assigned participation grades, assignment grades, project grades, as well as test and quiz grades. Karen did not use letter grades, but she assigned a number between 0 (no understanding) and 5 (understanding above and
beyond) for evaluation of all student work whether it was group work, written assignments, projects, tests, or quizzes. At the end of a term, those numbers were averaged and a final grade was determined.

**Question 1: The Case of Mary**

Mary had a broad picture of assessment. She believed that assessment and instruction were closely tied together. Her assessment of students was an ongoing process that involved close observations on a daily basis. She saw her main responsibilities as getting students to accept learning mathematics through investigations and discussions and to evaluate how these students learned and struggled with the mathematics. Her assessment of the entire program informed the changes she made for future years. She noted,

> As the local schools change their eighth grade math curriculums, we will have to make more adaptations. CPMP provides a super framework on which to hang our math curriculum. It provides an integrated structure whose content and process strands are adaptable and extendible.

Mary graded student work in each of the five domains, although she viewed grades as an unnecessary burden. She felt students were too concerned about grades and not concerned enough about learning. Concerning grades, she said,

> I do not ram grades down their throats. What is important is if you are learning. I do not want them to be worried about grades, but about learning.... I
don't think grades drive even the bright students. I do grades when I have to. I have to give grades in the middle of a marking period and at the end of a marking period.

**Question 1: The Case of Jack**

Jack's view of student assessment was quite narrow. He believed assessment of students was mostly evaluation. He was concerned that the grade he gave a student accurately reflected that student's ability to solve problems. He stated, "I have a problem with everyone walking out with an A or B when I know they don't know how to pass the (state's) proficiency test."

Jack did assess student learning in each of the domains. On a daily basis, he used classroom observations to evaluate how students worked at their assigned tasks and answered the Checkpoint questions. Weekly quizzes and end-of-unit tests were used to evaluate student achievement. Assignments were periodically collected and a small sample was used to evaluate student progress. Also, he used the Capstone project to evaluate student achievement for the entire course.

Jack used grades as a motivating factor and therefore included effort as a major component in all five domains. By doing so, he believed students would be encouraged to work harder at completing their work and therefore learn more mathematics.
Question 1: Comparison of Participants

Revisiting the theoretical framework of constructivism, which was used in this research, it is evident that each teacher believed that students learn best when constructing their own meaning to mathematics. The teachers planned to develop learning environments where they could observe, listen, and direct students as they learn, and then have students demonstrate their understanding in written form. Of the 4 teachers, Jill was the most successful in creating this environment, where Jack was the least successful. Many of Jack’s students had been unsuccessful in mathematics during previous courses and lacked motivation, which was the reason, he believed, his class environment was not as rich as the others. Both Jill and Karen trained their students to discuss and work in groups where Mary and Jack simply assumed students were able to learn this way. Mary struggled with the environment of her class. Most of her students had been successful with a more traditional curriculum, so they often objected to learning mathematics by constructing their own meaning.

All participants believed that assessment was the result of observing students as they completed the investigations, listening to student responses to the Checkpoint questions, and reading student responses to assigned tasks. As will be seen later in this chapter, the 4 teachers
differed in what they listened for when students discussed and what they looked for when reading student responses to assigned tasks.

All participants included multiple forms of assessment in their assessment plan. A priority was placed on test and quiz results, but the 4 teachers were similar in their attempts to include teacher observations of student involvement as another major component. For Jill and Karen, student involvement included the types of questions that students asked and how other students responded to those questions. On the other hand, student involvement for Mary and Jack simply meant student activity. As long as students were actively working their way through assigned tasks, they were satisfied.

Jill and Mary's assessment plan included student presentations and student-led class discussions, whereas Karen and Jack felt that class discussions should be led by the teacher. In this way, Jill and Mary gave more responsibility to their students.

Question 2: How Did They Gather the Evidence Needed to Carry Out Their Assessment Plan?

Question 2: The Case of Jill

The activities and tasks that Jill selected to carry out her assessment plan are contained in the CPMP curricu-
lum. The school where Jill taught had not adopted the CPMP curriculum system-wide. Only Jill and one other teacher were using this curriculum. Jill found CPMP to be consistent with her view of teaching and learning, so she had convinced her school system to try CPMP on a trial basis with two teachers.

To engage students in the investigations, Jill divided her class into groups of four. Her intent was to get all students involved and to have groups develop their own meaning to the mathematics. Jill would not allow students to just listen to others and write down their answers. She insisted on all group members participating in a discussion. Most days Jill filled out an evaluation form (Appendix C) on one or two groups to give them feedback on how she saw their progress.

Daily assignments were given to help Jill assess how students were progressing both individually and as a class. Jill required all students to complete the On-Your-Own problems to demonstrate their understanding. She also assigned MORE tasks each week which helped her assess how the class was progressing. If she noticed from reading the assignments that many students had problems with certain concepts, she would revisit those concepts and assign additional tasks.

Weekly quizzes and end-of-unit tests were used to
provide information on how students had achieved in the areas of content and process. These tests and quizzes were selected from the CPMP materials and included questions that had an "explain" component which gave students the opportunity to demonstrate their understanding.

Jill selected one project per quarter and the year-end Capstone project for students to complete in groups. Jill explained that this "gives me another picture of the student.... Some students who do not do well on tests or quizzes do better when they have more time." These projects were selected from the CPMP curriculum, and some class time was used to complete the projects.

**Question 2: The Case of Karen**

To gather evidence about student learning, Karen divided students into groups of four. These groups were selected randomly and changed after each unit. The groups’ main responsibilities involved working through the CPMP investigations and discussing solutions to the questions. Karen saw her responsibilities as monitoring student activity during the investigations and orchestrating class discussions during the Checkpoint.

Individual achievement was evaluated using projects, quizzes, tests, and assignments of MORE tasks in the CPMP curriculum. Although projects were done in groups there was
always an individual part that required each student to
demonstrate an understanding of the mathematics. Tests and
quizzes that assessed student understanding were taken by
all students. Assignments were evaluated each week, and
Karen required students to redo problems if assignments
were incomplete or incorrect.

Karen ended the school year by having students
complete parts of the CPMP Capstone assessment. She did
this as an alternative to a major semester exam because it
fit her philosophy of learning better than a written exam.
Part of the project involved students writing individual
reports which provided Karen with evidence of their under­
standing of the main concepts studied during the year.

Question 2: The Case of Mary

To gather evidence about students, Mary used the CPMP
curriculum and supplemented it with a variety of projects.
Most days students worked in groups of four as they went
through the investigations. Meanwhile, Mary monitored their
work, answering questions only if all the students in a
group were unable to answer them. Mary believed communica­
tion was the key to learning and she looked at her job as
getting all students to learn by communicating with each
other.

Assignments were given from the CPMP curriculum each
day. She supplemented CPMP tests and quizzes with questions of her own and supplemented CPMP projects with projects from other curricula. Class periods at her school were 75 minutes in length, and students were highly motivated so Mary was able to require more of her students than is the case in most schools. She pushed her students by asking more challenging questions and assigning more challenging projects that were not included in the CPMP curriculum.

Question 2: The Case of Jack

To gather evidence about students, Jack evaluated his students in three categories: (1) daily observations, (2) student notebooks, and (3) tests and quizzes. He divided his students into groups of three with the purpose of having them read the investigations and answer the questions. As they worked in their groups, Jack walked around the room often focusing on two or three students per day. Sometimes he wrote a number between 1 and 3 next to a student's name on a seating chart to indicate how well that student was working.

Each week, Jack gave the class an assignment selected from the MORE tasks. He followed the suggestions in the CPMP curriculum guide when selecting the problems. Periodically, he collected the student work, which was kept in a notebook, and evaluated it. He said,
My goals with homework are to develop student responsibility and to get students to work at solving rather challenging problems. With those two things in mind, I give them a sheet with the assignments and due dates. I update it periodically.... It is the student's responsibility to get them done and have them in their notebooks.... I collect them on unannounced occasions, so they have to make sure that homework is kept up to date.

CPMP quizzes were given weekly and CPMP tests were given at the end of each unit. In place of a paper and pencil exam, Jack used the CPMP Capstone as the final assessment of the year.

**Question 2: Comparison of Participants**

All participants used the CPMP curriculum to gather evidence about student learning. Only Mary used additional sources for gathering evidence. Much of this additional evidence was gathered using student projects that were not part of the CPMP curriculum.

The five domains of group work, the Checkpoint, assigned tasks, tests and quizzes, and projects were the common divisions that each teacher chose to categorize the information. Karen, Mary, and Jack combined group work and the Checkpoint, whereas Jill separated those two areas.

Teachers differed in what type of information they gathered during the first two domains. Mary and Jack simply wanted student participation, whereas Jill and Karen wanted student involvement. Jill and Karen's assessment of
student’s understanding included a student’s willingness to ask “why” questions and of other students to respond with more than simple numerical answers. They assessed students’ willingness and ability to explain answers to other students.

Teachers differed on how they gathered information through projects. On the one hand, Mary used multiple projects throughout the course where Jack only used the year-end Capstone project. Jill and Karen were between these two extremes and used two projects each semester with their classes.

Teachers also differed in how they recorded results. Jill was the only participant who completed reports on how well students participated in group investigations and led class discussions. Karen, Mary, and Jack evaluated student involvement, but based it mostly on recollection, not documented evidence. They did not believe time allowed them to document all their observations.

**Question 3: How Did They Interpret the Evidence?**

**Question 3: The Case of Jill**

The evidence that Jill used to make decisions about students was a combination of what she heard, what she saw, and what she read. Each day she observed students interacting and discussing, many days she observed students pre-
senting oral reports, and often she read evidence of their learning from tests, quizzes, and notebooks.

The criteria for group work was specified on her Group Observation form (Appendix C). Using this form, Jill was able to assess students in terms of process, content, and disposition. As students read through and discussed the group investigations, Jill could see how they connected the new mathematics with what they had learned previously and how well they had learned the content. By listening to their discussions, she could observe to what degree students were willing to become involved and discuss mathematics with other students. While listening to the discussions, her main concern was to get students to ask probing questions and other students to explain their answers to those questions. According to Jill, "Everyone should be listening to the ideas of everyone in the group. The group should be open to new ideas. Communication has to be clear. The group should be getting results.... Everyone must participate." If this was happening, then Jill rewarded students with high grades and positive comments. If it was not happening, then she assigned lower grades and wrote negative comments.

The results of each group evaluation were summarized by Jill on a Group Evaluation form and returned to the group. It contained written comments that indicated areas
that needed improvement and areas where group discussion was effective or ineffective. It also contained a grade that Jill assigned based on how well the discussion met the criteria as stated in the form. If the discussion was lacking in certain areas, Jill made comments on the form as to how the group could improve.

Appendix C contains two completed Group Observation forms that Jill chose to explain how she assessed students who were having difficulty with group work. Normally, Jill was positive and encouraging in her comments on the form, but this group did not discuss and interact in a manner that Jill expected. Her comments on the first form indicate that she expected all students in the group to be listening to each other and to contribute to the discussion. In this group, Pete was mainly interested in getting answers, not in understanding the mathematics. Jill’s comments indicate that Pete was not asking the kinds of questions that promote understanding. She also criticized Barb for answering Pete’s question directly instead of explaining why she performed a certain multiplication. At the end of the class period, Jill spoke with the group about her observation and indicated that she would observe them the next day, expecting improvement. The next day Jill again listened to the group and completed a Group Evaluation form (Appendix C). This time the group was more actively participating,
but some group members were still not listening to the ideas of others. Her comments to Pete and Barb indicate that they were still only interested in writing answers instead of understanding the mathematics.

During the Checkpoint, Jill assessed student understanding of the mathematics they were expected to learn during group investigations. Class discussion of the Checkpoint questions was normally led by students. She expected students to be well prepared and to give clear explanations to the class. They were judged by Jill according to how well students demonstrated understanding the mathematics and how well they expressed themselves. However, Jill did not intimidate the students by expecting a professional presentation. According to Jill, "It is not scary. It is describing how your group made sense of the mathematics in the investigation."

To summarize results of a presentation, Jill completed an Oral Report Grade Sheet during the presentation. Appendix C contains two examples of completed Oral Report Grade Sheets. The first one has all positive comments and Jill determined this to be an "A" presentation. The second example evaluated an oral presentation by a group of students who struggled with understanding the mathematics. Jill indicated on the form that the students did not give a clear explanation, they did not have correct information,
and they could not answer questions from the class or teacher. As a result she gave them a grade of "C-/D+.

Jill evaluated written evidence of student learning, such as tests and quizzes, by assigning a certain number of points to each question and awarding partial credit as she read the solutions. If there was no evidence of understanding, then she awarded no points. If there was some evidence, then she used a sliding scale according to how well, in her opinion, the student demonstrated understanding. At the end of the test, she added up points and used a percentage to report a final score.

Occasionally Jill was not able to determine from student responses on a test the degree of understanding the student had regarding a certain topic and, in special cases, would ask that student to explain his or her answer. Appendix C contains a copy of a test written by Fred where this was true. Fred's answer to part c of question 3 seems to indicate that Fred was confused. However, Jill knew that Fred was a special needs student and that he often had trouble expressing himself in writing. After asking him to verbally explain his answer, she decided to give him full credit for his answer. However, she did this only with students that had difficulty expressing themselves in writing and when she felt it was fair to the student.

Jill used a computer program to keep track of her
written evaluations. (A sample printout is contained in Appendix C.) Each entry has a code that put it into one of eight categories that Jill used in evaluating students. Every 3 or 4 weeks Jill printed out a report for each student to show them in writing how they were progressing. Jill stated that her record keeping was extremely time consuming, but necessary for her to implement her assessment plan.

**Question 3: The Case of Karen**

To interpret and evaluate student achievement in all areas of her assessment plan, Karen used a five point rubric. The following list provided her with a guide for specific assessments:

0 - Shows no understanding.
1 - Shows some understanding with major errors.
2 - Clear evidence of understanding with errors.
3 - Good understanding with minor errors.
4 - Complete understanding with explanation.
5 - Above and beyond a complete response.

**Question 1 (Figure 7)** provides a sample written assessment task used by Karen. An equation is put in context and students are asked to make a table and graph to represent the equation. Part b asks the students to solve the equation using the table and graph. The correct
Unit 3 Task

1. Almost everything costs more today than it did a few years ago. An equation estimating the median cost $C$ in cents of a 12-ounce soft drink $x$ years ago (for $x$ up to 40 years) is $C = -2x + 85$.

(a) Solve the equation below for $x$. Show or explain your work, and explain what the solution means in the above situation.

$$35 = -2x + 85$$

(b) Use your graphing calculator to make a table and a graph of the relation between number of years ago and cost for $0 \leq x \leq 40$. Show how to find the solution of the equation in part (a) in the table and on the graph.

Figure 7. Task From Karen’s Test on Linear Models (Coxford et al., 1997a).


numerical answer is 25, but this question involved explaining how to get that answer. Appendix D contains a sample rubric for this question and how Karen applied it to three students. Brenda’s response indicates that she had minimal understanding because she used 25 in her explanation. Karen felt she had minimal understanding of how to answer the question so she gave her 0.5 out of a possible 5 points. John’s response indicated more understanding but
his explanation is incomplete, so Karen gave him 2 out of 5 points. Connie's response to both parts are correct so she received a 4 out of 5. To receive 5 points she would have had to discuss changing the increment in the table.

Weekly assignments were often evaluated using homework quizzes. Karen selected parts of a few problems that were assigned and instructed the students to copy their work onto a quiz sheet. As she did with test questions, Karen developed a rubric for the selected problems and used that rubric to grade the homework quiz. If individual responses received a 2 or less, then those students were required to redo those problems before the homework quiz grade was recorded.

For evaluation of group work and class discussion, Karen focused mainly on student disposition. She used the following seven categories: (1) contributes appropriately to the group effort, (2) is willing to listen to others' ideas and to compromise when appropriate, (3) asks questions about things when they are not understood, (4) works with the group until everyone understands and can explain the results, (5) encourages others to contribute to the project, (6) avoids or smoothes out friction, and (7) stays on task during class time. Karen did not have a daily log of student group work, but every 2 weeks she reflected on how students participated in group work and
class discussions and assigned each student a number
between 0 and 5. She said, "When I am evaluating students
every day, I evaluate them on what I see and hear. To have
written documentation every time, that is hard and not
worth the time."

Karen used a grade program to record and average all
evaluations. Students received a printout of her record
every 3 weeks. For grading purposes, Karen converted an
average of 5 to an A+, an average of 4 to an A, an average
of 3 to a B, etc.

**Question 3: The Case of Mary**

The students that Mary taught were selected based on
achievement tests so she did not have low ability students
in her class. She also did not have serious discipline
problems and was able to focus on student achievement. She
had high expectations for student work.

Although students worked in groups to learn the
mathematics, Mary assessed students individually. Each week
she looked closely at each student's work and assigned a
notebook grade. A notebook included all the work the
student did during the week. It had to be neat, organized,
and complete. To emphasize the importance of the notebook,
she counted it as 25% of the overall grade.

Tests and quizzes together counted for 55% of the
grade. These tests included questions that took familiar concepts and extended them into unfamiliar settings. This is what Mary called “problem solving” and she wanted her students to have that experience. In that regard, Mary viewed tests and quizzes as a learning experience as well as a demonstration of what students knew.

The tests in Appendix E provide a good example of Mary’s expectations and how she evaluated student work. Task 3 (Figure 8) assessed student understanding about polygons that tessellate. Mary expected a clear explanation of why the student’s example did or did not tessellate. This topic had been discussed in class and all three students were able to give an acceptable solution to the first part of the question. On the second part, only Joe used the idea that the sum of the measures of the angles around a point must be 360.

Tasks 5 and 6 (Figure 8) exemplify how Mary assessed each student’s ability to take a familiar topic and extend it into unfamiliar areas. All of her students were able to give a correct solution to question 5, but only a few were able to make the transition from rectangles to other shapes. If students investigated other shapes besides rectangles either inductively or deductively, then Mary believed they were progressing the way she expected. Of the three students, only Melissa was able to make the
transition.

Geometry Tasks

3. (a) Draw a polygon that tessellates. Use mathematical reasoning to explain why it tessellates.

(b) Draw a polygon that does not tessellate. Use mathematical reasoning to explain.

5. Kim is building a rectangular pen for her goat. One side of the pen is the back of the barn. She has 50 meters of fencing. Make a table showing possible lengths, widths, and areas to find which rectangular area will give her the maximum area.

6. If she is able to build a pen of any shape against the back of the barn, which shape should she use to get maximum area? Sketch it below with dimensions. How much bigger is this than the rectangular pen?

Figure 8. Tasks From Mary's Test on Patterns in Space and Visualization (Coxford et al, 1997a).


Projects were used to assess persistence as well as achievement. About projects, Mary said,
**Question 3: The Case of Jack**

To interpret the evidence, Jack used a combination of subjectivity and objectivity. Part of his evaluation of students involved their day-to-day participation in group work. Every 4 or 5 weeks, he looked at his notes and recalled how students worked in class and assigned a participation grade for each student. His criteria for this part of his evaluation was primarily effort. If students worked hard each day they received a high grade, but if students did not put forth much effort, they received a low grade.

Jack worked in a school environment where few students had experienced much success in mathematics, so he also included effort when grading student tests and quizzes. He gave students 40% credit on a problem if they made an attempt at solving it. The other 60% depended on the degree to which their attempt was correct, or the amount of understanding they demonstrated.

Appendix F contains student responses to part of a test that Jack chose to demonstrate how he evaluated students. Task 4 (Figure 9) contains three parts: (1) stating the variables, (2) giving a rule, and (3) sketching a graph. Jack used 10 points for each part. Laura’s answer is incorrect, but Jack awarded her 4 points.
for writing a rule and 4 points for drawing a graph, because she attempted the task. Jack awarded Melissa 10 points for drawing a correct power model, 7 points for labeling the variables on the graph, and 0 points for not even trying a rule. Jack awarded Hillary full credit even though her answer represents an inverse power model, because he believed she demonstrated an excellent understanding of variables, rules, and graphs.

**Task - Power Models**

4. From your homework, recalling what we did in class, or creating one of your own, give an example of a simple power model. Tell what the variables are, give a rule that represents the relation between the variables, and give a sketch of the graph (label the graph, however, you do not need to scale the axis).

**Figure 9. Task From Jack’s Test on Power Models.**


Effort was also a central ingredient in evaluating daily assignments. Jack awarded 80% if all the problems were attempted. The remaining 20% was determined by looking at a few problems and evaluating the amount of understanding the student demonstrated.
Question 3: Comparison of Participants

For all participants, interpreting the evidence was an ongoing process throughout a unit. It included both oral and written communication, and interpretation occurred in each of the five domains. Initial impressions occurred as teachers observed students interacting during investigations. Misconceptions were discussed during the Checkpoint, and understanding was further assessed using assignments, tests, quizzes, and projects.

Participants differed on how they interpreted the evidence within domains. As discussed earlier, Jill and Karen had different criteria than Mary and Jack in the domain of group work. Jill and Karen focused on each group's progress whereas Jack and Mary focused on the individuals within the group. Jill and Karen listened for the types of questions being asked and the way students responded to those questions. Their expectations included total group involvement and that students would ask and answer questions that led to understanding, not just answers. Mary and Jack, on the other hand, tended to be satisfied with student participation.

Jill and Mary had students lead class discussion during the Checkpoint, so they were able to concentrate on student understanding while listening to the presentations and discussions. Jill and Mary would often ask questions
during the Checkpoint that would further assess student understanding. On the other hand, Karen and Jack felt they could better assess student understanding if they led the class through the Checkpoint questions themselves.

Assessment of students in the domain of assignments differed considerably. Only Jill read all the work of each student to get a holistic impression of student understanding. The other 3 participants took a sampling of the work and based their assessment of student understanding on the sample. Karen required individual students to redo assignments if she was not satisfied, whereas the other participants assigned a grade to the work and returned it to the students.

Participants were similar in how they interpreted tests and quizzes. All 4 teachers looked for clear explanations and correct answers. Evaluation of tests and quizzes involved much more than counting the number of correct answers. Before reading student responses, each teacher wrote what was expected as a solution that demonstrated full understanding and what might demonstrate partial understanding. As they read student responses, they assigned numbers that represented the degree of understanding. Only Jack included effort in the numbers. He believed for his students, that attempting a task is better than not attempting the task. Mary was the only participant
who included tasks on a test that were extensions of the work done in class. Using these she was able to evaluate how students could apply ideas studied in class in new situations.

Of the 4 participants, Karen was the only one who used a system other than letter grades to evaluate the quality of work. She used a system that gave feedback about the amount of understanding a student demonstrated on a particular task. This system included a category of "above and beyond," which encouraged students to do more than was asked. Close analysis of this system reveals that it is similar to a point system with the added dimension of describing meaning to the points. With her system, students understood what the numbers represented, whereas other participants did not have a definite meaning to the number of points a student would receive on a task.

**Question 4: How Did They Use the Results?**

**Question 4: The Case of Jill**

Jill used a data base to record all numerical data. The data were coded into the following eight categories: (1) tests, (2) quizzes, (3) On-Your-Own problems, (4) group work, (5) oral reports, (6) notebook, (7) homework, and (8) projects. Every 3 or 4 weeks a report including all eight categories was printed and given to each student.
The information that Jill gained from her data informed instruction. In fact Jill indicated that all instruction in her course was preceded by some assessment. Even when she started a new unit, she would first ask questions to see what students knew. Most of the instruction was done during group work or during the Checkpoint. The focus of this instruction was always based on what Jill observed as she listened to groups complete the investigations.

During group work and oral presentations, Jill completed evaluation forms which included comments and a grade. At the end of the class period, Jill would return the completed evaluation forms to the students and, if necessary, talk to them about improvement.

Jill used results of homework assignments and On-Your-Own problems to assess the progress of the class. Once a week she collected assignments and read them carefully. When asked how she used assignments, Jill responded,

Mainly I see where we are as a class. I make notes to myself like “everyone seems to be having trouble with this one” or “Gary did this one real well.” It is a chance to have him shine and put his solution on the board. Everyone will understand a little better and it gives Gary a chance to feel good about himself.

Tests and quizzes were used to evaluate student achievement but also to inform instruction. If students did poorly on certain concepts, Jill would revisit them either with individual students or with the entire class.
Question 4: The Case of Karen

Karen used results to inform instruction and to evaluate students. Information from investigations informed the class discussions at the Checkpoint. Information from tests, quizzes, and homework informed what remedial work had to be done. Information from the end-of-year Capstone project informed what should be emphasized in future years.

Each day Karen observed students completing the investigations and listened to them discussing solutions. During the investigations, her role was to monitor and facilitate learning. At the Checkpoint her role changed to that of a moderator. She used the information gained during the group work to direct the discussion at the Checkpoint. She said, "My role (at the Checkpoint) is to make sure through questioning that all the essential skills from an investigation come out in the discussion."

Karen used grading to show what she valued as important in her class. Communication of understanding, both written and verbal, was an essential ingredient in her assessment plan, so she put an emphasis on communication in all aspects of her grading scheme.

Karen continued to adjust her teaching as she became more experienced with students struggling with certain concepts. For example, after teaching the Simulation Models
unit three times, she realized that students often had the same misunderstanding. To eliminate this, she came up with five questions students should answer whenever they encountered a simulation problem. The Capstone project indicated to her that many students had not understood total surface area and volume to the degree expected, so Karen and other teachers at her school decided to place more emphasis on those topics in future years and to incorporate them in the next course for their current students.

**Question 4: The Case of Mary**

Mary used results of assessment to inform instruction, to evaluate students, and to adapt course content in future years. Information she gained during investigations determined the cue questions she brought out at the Checkpoint. In Mary’s opinion, the Checkpoint was the focal point of instruction in her course. She said, “At a Checkpoint, one group saw it one way and another saw it another way. I am looking for themes, comparing and contrasting of ideas, and I focus on that when the kids come up and lead discussions.”

Mary also used assessment to evaluate students. Although she felt students were too concerned about grades, her school required her to provide a grade for each
student. To evaluate her students, Mary based 25% on their notebook, 55% on tests and quizzes, 10% on presentations, and 10% on classroom participation.

At the end of the year, Mary and one other teacher discussed how to adapt CPMP to their program in future years. They decided that their students did not need to complete all the investigations since they generalize quickly. They felt they should add more abstraction to some units and to require a research paper. Finally, they would require students to spend more time outside of class on reading and completing assignments.

**Question 4: The Case of Jack**

Jack used results of assessment to evaluate students and to inform instruction. Group observations always preceded instruction. The information gained from observing and listening to group discussions gave focus to the instruction that followed. Jack stated,

> I assess on a daily basis how much time should be spent on investigations, and how much on class discussion, and where to go with discussion.... Part of my observation is seeing what they are wrestling with. Sometimes they have no clue and we must stop and make some clarification and give them some hints.

Information gained from daily observations, test and quiz results, notebook checks, and the end-of-year project were used to evaluate the students. With regard to how he used results, Jack responded,
I am trying to get a measure of what they got out of this class. I think I do a fairly decent job of that. If a student gets an A or B, that means they have learned enough to pass the (state's) proficiency test. If a student gets a D or E, which many of them do, they have some problems and might repeat the course.

**Question 4: Comparison of Participants**

All participants used results to provide feedback to students and to inform instruction. Often, feedback included a grade. Grades were determined by all participants in the domains of group work, assignments, tests and quizzes, and projects. In addition, Jill determined grades for presentations during the Checkpoint. Every 3 or 4 weeks, teachers informed students what their grades were in each of the graded domains. All teachers believed this kind of feedback motivated students to improve. Mary indicated that grades do not motivate students, but her actions did not support this claim. She graded each of the domains and used the grades as feedback to the students.

Jill was the only participant that provided daily feedback to students in written form. This feedback was in the domains of group work and the Checkpoint where she daily completed observation forms.

For each participant, the assessment during group work, which included observing and questioning students, was directly linked with class instruction that occurred at the Checkpoint. It provided each teacher with opportunities
to identify problems and misconceptions that were then discussed during the Checkpoint. In addition, Jill and Karen used the results of reading student responses on assigned MORE tasks to identify problems and correct misconceptions.

**Question 5: What Problems Did They Encounter When Implementing Their Assessment Plan?**

**Question 5: The Case of Jill**

The main problem that Jill saw with her assessment plan was lack of time. She observed,

There is a large volume of stuff and I tend to get submerged under it and forget why I'm doing it. Sometimes I get a good idea on how to make the groups work better, but something else comes up. A month later I remember I was going to do something and I never tried it.

Jill also felt she had problems with how her third-year students were reacting to the investigations and Checkpoints. She remarked,

The juniors are becoming very hohum about investigations and Checkpoints. Their agenda has become more and more "lets get this finished" instead of listening to each other and delving into it.... They know very well what good group work looks like, and they act like it when I'm there. But when my back is turned they go off and just try to get finished. (It's the) same thing with a Checkpoint. They know how to do a good job but sometimes feel they do not have to listen. So I'm trying to find a way to get them back on track.

In spite of the time commitment and monitoring problems with her juniors, Jill was convinced that students
learned far more and processed ideas much more completely under her assessment program than they had using a traditional program. She believed that communication was the central ingredient to student learning and that her method of assessment maximized communication.

Question 5: The Case of Karen

Karen believed that a major problem implementing her assessment system was lack of time. She would have liked to document what she observed each day and discuss her observations with students, but time did not allow it.

Evaluating students using paper-and-pencil tests caused tension for Karen. She occasionally encountered the situation where students demonstrated poor understanding on a written test, but those same students had demonstrated good understanding during class observations. Her system of evaluation called for averaging numbers that were based on both oral and written communication. However, written communication counted more than oral communication so these students received lower grades than what she felt they deserved. So balancing written work with oral work was a problem for Karen.

Question 5: The Case of Mary

Mary indicated two major problems with her assessment
process: (1) lack of time to individually assess each student and (2) lack of success at convincing her students to accept learning by constructing their own meaning to the mathematics.

Mary was able to observe each of her students as they constructed meaning to the investigations. She stated,

Since doing CPMP, I have a much better handle on how students are learning because I am watching them in different situations than I ever saw before. I am seeing them teaching (each other) in small groups and interacting. I am making observations constantly and I know so much more about them than I ever did before. As a result I have a better handle on how the kids are learning.

In spite of the preceding statement, Mary indicated that all the reading of explanations that students wrote on tests, quizzes, projects, and homework required a great deal of time and often she was not able to give feedback as soon as she wished.

At the end of the year, Mary noted that her students had mixed feelings about studying and learning mathematics through investigations. She said,

About a third of the students want to go back to learning the old way.... They want more structure, more examples, and stick to learning one thing at a time. Another third are unsure about the change. The last third really like the change and have found more success with the new than the old.

Mary was convinced this type of learning was best for all students, but she struggled with convincing her students.
**Question 5: The Case of Jack**

Jack mentioned two main problems he had with implementing assessment reform. Lack of time was his biggest concern, and developing a better system of assessing student understanding on a day-to-day basis was also a concern.

In the urban environment in which he taught, many students were unsuccessful in his mathematics class. He attributed that to lack of motivation, but also because he was not able to spend enough time answering their questions.

Jack was struggling with a system of documenting daily observations. He indicated that he might be more successful if he could document students' efforts at learning and then give immediate feedback to the students on how they could improve. He was looking for ideas for how to fit this into an already demanding schedule.

**Question 5: Comparison of Participants**

Participants agreed that time was the main problem they encountered when implementing their assessment plan. They all wanted to spend more time working with individual students and assessing their achievement. They all professed to having students who had difficulties in certain areas, but they did not have the time to help each one to
the degree they wanted. Participants would like to listen or be part of the discussions that students had while completing the investigations, but time did not allow it. They also did not have the time to read all the student responses to assigned tasks. Jill was the only participant able to read all responses to MORE tasks, but even she stated that it would be impossible if she had more students.

It was also time consuming for teachers to evaluate tests and quizzes because they looked for more than correct answers. Since most questions had an "explain" component, they read explanations and used those to evaluate understanding. All this took more time than what they had previously used to evaluate tests.

Comparison of Participants

The major component of this study was to provide examples of teachers implementing assessment reform within a mathematics reform curriculum. In this section, the researcher compares the 4 case studies and summarizes the analysis using the themes of learning environment, forms of communication, teacher feedback, and time.

Each participant viewed assessment as an ongoing process where the teacher observes students on a daily basis. They believed students learn best by communicating
in both small and large groups. For this reason, each teacher used group work both as a tool of instruction and an opportunity for assessment. In order for students to learn constructively and for teachers to assess that learning, it was necessary to have a learning environment where all students were actively involved in group discussions. Teachers struggled to different degrees with creating this class environment. Jill and Karen trained their students how to work in group situations, where Mary and Jack assumed their students did not need specific instruction for working in groups. While training their students for group learning, Jill and Karen emphasized that all students should not only participate, but that their questions and answers should promote conceptual understanding, not simply getting answers. While listening to group discussions, they assessed student’s willingness to ask good questions and other’s ability to explain how they arrived at certain answers. On the other hand, Jack and Mary were satisfied with student involvement.

All participants used both oral and written communication as part of their assessment plan. They formed initial impressions of individuals or groups by listening to their discussions during the investigations. At the Checkpoint, they further checked understanding and clarified ideas if students were not clear on concepts or were not making the
intended connections. The assessment process continued as they assigned and evaluated On-Your-Own problems and MORE tasks to be done outside of class. By allowing students to redo incorrect MORE tasks, Karen further directed and reassessed learning. Finally, quizzes and tests were used to formally evaluate students.

While each teacher observed group discussions carefully, only Jill completed written evaluations of group discussions. Karen and Jill looked at how well the students worked as a group, while Jack and Mary focused on the individuals in the group. Jill and Karen listened for the types of questions and responses that students gave whereas Mary and Jack were satisfied with participation. On tests and quizzes, only Jack gave students credit for effort. Jill, Karen, and Mary required students to show achievement, not just effort. All teachers indicated that the combination of observing students involved in investigations and reading their responses to assigned tasks allowed them to more fully assess student learning. Prior to using the CPMP curriculum, all participants had used teacher-directed lectures and teacher-led discussions as the primary forms of instruction. Now they felt they had a better handle on student learning and were able to provide help when needed.

Each participant believed that students are motivated
to learn when they are provided feedback on their progress. They also believed that one of their primary responsibilities was to provide that feedback. In most cases this feedback was in the form of grades. Each participant felt it necessary to grade students in each of the domains. This caused each participant to focus on grades when interpreting the information. Karen had devised a rubric that interpreted what each grade meant, so she focused on the rubric when interpreting information about students. Jill was the only participant who provided feedback in the form of written comments in addition to grades. This was done on her group observation reports and her presentation reports. In addition, she often wrote positive comments on assigned tasks and often encouraged students by having them explain creative solutions to the entire class.

The use of time while assessing students was a major concern for all participants. All 4 teachers stated that implementation of their assessment plan required a great deal of time. Three teachers taught a full schedule of CPMP classes while Jill taught three CPMP classes and was employed by a local college in the afternoon. They all were using multiple forms of assessment that included observations, projects, and written explanations, and time did not allow them to be as complete as they would like.

Participants agreed that creating an environment
where students learn mathematics through constructing their own meaning was more time consuming than using a more traditional approach. Therefore teachers had difficulty completing the curriculum in the normal school year. Both Jill and Karen would use more projects if time allowed. Mary did not have this difficulty, but her classes met for 75 minutes each day for the entire year which was considerably longer than the other 3 teachers.

For assigned tasks, only Jill read all student responses and she indicated that she was able to do that because she had a smaller number of students than other teachers. She also indicated that if she had a normal class load, she would sample student responses and not read everything. The other 3 teachers agreed that reading more student work would be valuable, but time did not allow this.

Preview to Chapter IX

Summary, results, and recommendations are included in Chapter IX. In the summary section, the researcher provides a brief description of the study. Following the summary are the results or findings of the study, and finally connections to previous research and recommendations for further research are made.
CHAPTER IX

SUMMARY, RESULTS, AND RECOMMENDATIONS

Purpose of the Study

Current reforms in school mathematics view learning as the construction of knowledge and assessment as gathering and interpreting evidence of this process. This study documents the efforts of 4 teachers who used a reform curriculum and multiple techniques in their assessment of students. Investigations of this nature are useful when attempting to develop assessment models for classroom teachers. They extend the knowledge of what is known about classroom assessment and document how teachers interpret assessment reform. Such research may also influence teachers to reflect on their own practices and conceptions related to student learning and student assessment in school mathematics. This, in turn, may cause them to strengthen their assessment practices and help them learn more about how their students think and understand mathematics.

This research produced four case studies using the four phases of classroom assessment as described in NCTM’s Assessment Standards for School Mathematics (1995). These
phases are: (1) planning, (2) gathering evidence, (3) interpreting the evidence, and (4) using the results.

Research Methodology

Design and Participants

The research design used a case study format and focused on 4 teachers who taught a reform curriculum in diverse school environments. The reform curriculum was part of the CPMP project and the four environments were: (1) a suburban school, (2) a rural school, (3) an urban school, and (4) a math and science center. Although the participants had similar ideas about assessment, each worked in different environments and therefore offered things unique to the study.

Data Collection and Analysis

Data were collected over a 20-week period beginning in January of 1996. Taped Interviews, observations, teacher journals, and class documents were collected and analyzed for the study. Almost all data involved Course 1 of the CPMP curriculum. The only exception was a taped analysis of a test by one of the participants. This analysis involved Course 2 of the CPMP curriculum.

Analysis involved coding data with the purpose of identifying common domains and common themes. Common
domains or divisions of assessment emerged after the first two interviews. These domains were: (a) group work, (b) the Checkpoint, (c) assignments, (d) tests and quizzes, and (e) projects. After the second interview, questions became more focused as teachers described their assessment system in each domain. Participants were also encouraged to share any concerns or problems they had with their assessment system. After all the data were collected a second phase of coding was used. Data within each domain were coded with the purpose of identifying common themes. The common themes were then used to compare the 4 participants. The themes that emerged with this process were: (1) learning environment, (2) forms of communication, (3) teacher feedback, and (4) time.

Results of the Study

This research provided data that identified efforts of classroom teachers implementing mathematics assessment strategies from multiple sources in a reform curriculum. The 4 participants were experienced teachers who taught in diverse school environments but were all using the same reform curriculum. With the exception of one participant, this curriculum provided all the material needed for them to carry out their assessment plan.

The research identified four themes that the mathe-
mathematics education community should consider as it forms assessment models for classroom teachers. These themes are: (1) class environment, (2) forms of communication, (3) feedback to students, and (4) time.

All teachers used group investigations as the primary mode of learning. For this reason, teachers continually worked at producing a class environment conducive to discussion. To be effective, the classroom environment must allow students to actively participate in investigations and ask questions if explanations are not clear. Two of the teachers, Jill and Karen, listened carefully to the types of questions students asked and the types of answers other students gave. Conceptual understanding was important for them. They wanted their students to help each other understand the mathematics, not just write down correct answers.

Both oral and written communication were part of each participant's assessment plan. Oral communication occurred during group work, the Checkpoint, and projects. Written communication occurred on assigned tasks, projects, quizzes, and tests. Prior to using the CPMP curriculum, each teacher agreed that they had not put as much emphasis on oral communication. Assessment of students had primarily occurred on tests and quizzes. Now they daily assessed student learning. Assessing a combination of oral and written work gave them a much better understanding of their...
The 4 participants believed feedback to students was important for students to progress. For the most part this feedback involved grades and was periodically determined in all domains. In addition, Jill often completed group observation forms and oral presentation forms which gave students written feedback and suggestions on how to improve. Karen was the only participant who developed a system other than grades. She believed her system gave students more information on achievement than grades. However at the end of each making period she used her system to assign grades.

The final theme, time, was an important issue for each teacher. Implementing their assessment plan required a great deal of time. They all agreed with Jill when she stated, “There is so much stuff I would like to do, but time does not allow it.” The assessment models that are formed will have to fit an already demanding schedule for the classroom teacher.

Research Related to This Study

Assessment is an important component of teaching and learning. Teachers often need help in planning assessments, in using techniques for gathering data, in ways of interpreting levels of understanding from the data, and in using
the results to improve instruction.

Cooney (1992) and Taylor (1992) conducted research that found some teachers classified students as having a high level of understanding if they could work multiple-step problems or problems with difficult computations. The 4 teachers of the current research not only wanted students to solve problems, but also to explain concepts and demonstrate their understanding through reasoning. For them, assessment was more than testing. It was an ongoing process and involved close observation on a daily basis.

Research by Nash (1993), Enderson (1995), and Kulm (1994) indicate that when teachers are exposed to alternative assessment techniques and assistance is offered, they will adjust their teaching and assessment practices to coincide with these techniques. This particular research supports those findings. The CPMP curriculum provided assessment tasks and the teachers received support by attending inservice meetings on assessment techniques. The 4 teachers of this research were able to alter their previous practices and incorporate multiple methods in their assessment plan.

The Assessment Standards for School Mathematics (NCTM, 1995) calls for "a shift toward using multiple and complex assessment tasks, projects, writing assignments, oral demonstrations, and portfolios, and away from sole reliance
on answers to brief questions on quizzes and tests" (p. 29). Recent research indicates that this shift is starting to occur in limited cases. Senk, Beckmann, and Thompson (1997) surveyed the assessment practices of 19 teachers from schools using newer forms of assessment. The teachers in their research were using textbooks consistent with recent recommendations for reform and had opportunities for professional development. They found that, even with this reform-minded sample, only about two-thirds of the teachers had begun to make the shift. They suggest that future curriculums include appropriate assessment tasks and that teachers be inserviced on how to use those tasks.

The current research addresses these same issues. The CPMP curriculum provides teachers with appropriate assessment tasks and teachers are inserviced on how to use them. Teachers in this research were able to make a major shift in their assessment practices. All 4 teachers started assessing oral communication as well as written communication. They included projects and oral demonstrations in what they expected from students. They used CPMP test and quiz tasks that required students to have written explanations as well as answers.

Recommendations for Further Research

This research presented case studies of how 4 teachers
planned and implemented assessment strategies within the context of a reform curriculum. Studying their assessment process was important since it documented evidence that can be used to form models of assessment. This research also helped to identify areas that need further research.

Before assessment models are formed, it would be beneficial to research the assessment practices of teachers using other reform curricula and compare those with CPMP teachers or teachers of other reform curricula. Once more examples are documented, then models can be formed which should be tested by teachers using different programs.

Current reforms view assessment as "cyclic in nature, a process of observation, conjecture, and constant reformation of judgments about students' understanding" (NCTM, 1989, p. 203). Assessment models should fit this cyclic process. This study investigated the cycle from a broad perspective. It would be beneficial to focus on one or two students and study how a teacher such as Jill continually reformulates her judgments about students throughout the assessment process.

More research is needed on the third stage of the assessment cycle: interpreting the evidence. Certainly a goal of the reform efforts in school mathematics is to improve learning. Since teachers are being asked to assess conceptual understanding, reasoning, discourse, and repre-
sentation, research is needed on how teachers could evaluate different levels within each domain. In addition, research is needed to investigate which forms of assessment best evaluate the different realms teachers are asked to evaluate.

Closing Remarks

This study documented the assessment practices of 4 teachers using a reform curriculum in their classrooms. These four case studies suggest that assessment reform can be realized by dedicated teachers who are willing to create an environment in their classrooms where students become involved in their learning and construct their own meanings through discussions with other students and the teacher. Assessment becomes more than testing and questioning where right answers are the only important criteria. It is ongoing where the teacher continually reformulates what students know and can do. More research on this reformation process is necessary. Hopefully, this research will stimulate further research on developing this process with the goal of developing assessment models for school mathematics.
Appendix A

On Your Own and MORE Tasks


Used with permission of Steve Mico, editor, Everyday Learning Corporation, 10-1-97.
Recall that the Census Bureau estimates U.S. population growth based on birth, death, and immigration data. The 1990 U.S. population was 248 million, with a birth rate of 1.6% a death rate of 0.9% and about 0.9 million people immigrating to the U.S. each year.

(a). Use this calculator procedure to estimate the population in the year 2000 and the year 2010. Compare the population estimates for the years 1990, 2000, and 2010.

(b). Explain what calculations are being done at each step of the calculator procedure for tracking population change in the U.S.

(c). Suppose the U.S. immigration rate increased to 2 million per year. Calculate the population in the years 2000 and 2010 based on this assumption.

(d). How would the U.S. population change if we had 2 million people each year leave, rather than enter, the country? Compare the results for the years 2000 and 2010 in this case to those in part (c).
Modeling — Organizing — Reflecting — Extending

Modeling:

2. The People's Republic of China is the country with the largest population in the world, over 1.1 billion in 1990. Despite efforts to limit families to one child, the population of China is still growing at a rate of 1.3% per year.

(a) Predict the population of China for each of the next ten years and record your predictions in a data table.

(b) When will the population of China reach the 2 billion mark?

(c) Using the word NOW to stand for the population in any year, write an expression that shows how to calculate the population in the NEXT year.

(d) Suppose that China allows 7 million people each year to leave for other countries. How would this affect the growth of the population of China over the next 10 years? (0.007 billion = 7 million)

(e) Using NOW to stand for the population in any year, write an expression that shows how to calculate the population in the NEXT year, assuming that the 7 million Chinese leave annually.

(f) Search for a balance of growth rate and the number of people leaving China each year that will lead to zero population growth in China.

Organizing:

1. The studies of populations changing over time can be represented with graphs if you form ordered pairs of (year, population) data. Recall that the 1990 population of Brazil was 145 million people and that the growth rate each year is about 1.9%. Use your calculator to plot (year, population) data for each ten-year period from 1990 to 2050.

(a) Make a sketch of the plot and write a brief description summarizing the pattern of the plotted data. Note: Sketches for (b) and (c) should be made on the same set of axes.

(b) Sketch the pattern of (year, population) data you would expect in Brazil if the birth rates increased.

(c) Sketch the pattern of (year, population) data you would expect in Brazil if the birth and death rates were equal.
Reflecting:

1. The models of population change studied in this lesson are somewhat different from those involved in the Bungee Jumping investigation. Look back over the examples of the investigations in this lesson and see if you can figure out the meaning of the lesson title "What’s Next?" What do the prediction models have in common that involve the word NEXT? How does the word NOW get involved in describing these situations?

2. Compare the use of the calculator ANS function to the equation relating NOW and NEXT for calculating the total population at a particular point in time.

Extending:

1. The kinds of models of change used in studying populations are sometimes quite different from the ones you have investigated so far. For example, many psychologists study the way people learn and remember information. Suppose that when school closes in June you know the meaning of 500 Spanish words, but you don’t study or speak Spanish during the summer vacation.

(a). One model of memory suggests that during each week of the summer you will forget 5% of the words you know at the beginning of that week. Make a table showing (weeks, words in memory) for 10 weeks and describe the pattern of data in the table.

(b). A second model suggests that you will forget 20 words each week. Make a table showing (weeks, words in memory) data for 10 weeks following this model and describe the pattern of data in that table.

(c). Graph the data from the two models and describe the patterns of data in those graphs.

(d). How would answers to parts (a) - (c) be different if you knew only 300 words at the start of summer?

(e). Which model do you think best represents memory loss? Explain your reasoning.

(f). Suppose 10 weeks of summer are gone and you decide to do an intensive vocabulary review for the remaining 2 weeks before school starts. If you are able to regain 20% of your vocabulary each week, how many words will you know when school begins? Which model of memory loss did you assume for the first 10 weeks?
Appendix B

Interview Questions and Coding Data
Lead Questions for Interview 1

(Name), could you please give me some information about yourself? How long have you been teaching? What school district(s)? What are you teaching this year? How long have you been involved with Core-Plus?

This is the first time I have ever been to your school. Could you describe the community: the economics of the community, the ethnicity, home environment, size of the community, etc.

We are going to focus on your Course 1 class. Would you say that the kids in that class are representative of the whole population?

I have never been in your classroom. If there were such a thing as a typical day in your classroom, would you give an overview of what would go on during that day?

I will use the term “assessment” to include all the ways you use to determine what your students know, their learning strategies, their ability to do math. Remember, my goal is to understand how you do this. How you determine grades is only a small percentage of the whole assessment process. I want to learn what you do on a daily basis. Today I am mainly interested in the grand tour, the big picture. I am going to list a few areas that are associated with assessment. Feel free to expand on what you do in these different areas.

1. Classroom observations
2. Journals
3. Open-ended questions
4. Projects
5. Portfolios
6. Self-assessment
7. Tests and quizzes
8. Assignments
9. Any others?

You seem to have a very well thought-out philosophy of assessment. If you had to capsulize it in a sentence or two what would be the first thing you would say?
Coding of Data Examples

Key:  J - Jill  K - Karen  M - Mary  JA - Jack
1, 2, 3, 4, 5 - Interview number
6 - classroom observation 1
7 - classroom observation 2
8 - audiotaped description of test evaluation
9 - project journal
10 - phone conversation

K3:  They get a group grade once every 2 weeks. Acutely, it’s a participation grade because it is a large group and small group grade (Underlined in yellow to indicate Group Work).

K4:  They are hard to set up and hard to grade (underlined in green to indicate Projects).

J4:  Sometimes I want a particular group up there because they have come up with something very different. Sometimes I want a group up there because they are off track, but it is interesting that they got off track (underlined in blue to indicate Checkpoint).

JA3:  One of our goals as a district is to develop student responsibility. I also want students to work on solving some challenging problems (underlined in red to indicate Assignments).

J9:  This is a friendlier way to end the year than the final exam. (It) seems more in keeping with the focus of the year’s work. This should serve the purpose of students reflecting back on all the math they know, chose an appropriate idea/skill, apply it, and report. The boundaries are less artificial than exam questions (underlined in green to indicate Projects).

M8:  I want to see if they think inductively or deductively. It certainly helps me evaluate where my kids are at.... I am looking for how they develop their ideas. If they think inductively, I expect them to use more then one example (underlined in orange to indicate tests and quizzes).
Appendix C

Jill’s Documents

Source of quiz tasks:
Group Observation

Date: 5/13

These are the criteria agreed upon for excellent groupwork:

- Everyone is listening carefully to the ideas of others in the group.
- Everyone contributes something to the discussion.
- The group is open to new ideas being voiced.
- Communication is clear.
- The group gets results.
- Group members are actively offering each other help as needed.
- Everyone is encouraged to participate.

See comments below for specific observations.

<table>
<thead>
<tr>
<th>Students</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pete</td>
<td>Don't agree. Need to keep going.</td>
</tr>
<tr>
<td>Barb</td>
<td>Not listening to Pete's method!</td>
</tr>
<tr>
<td>Mike</td>
<td>Good explanation - unique idea.</td>
</tr>
<tr>
<td>Molly</td>
<td></td>
</tr>
</tbody>
</table>

General Comments:

Too much "What do I do next?" - too little "Why?"

Pete's question was "Do I multiply 5 by 7?" It should have been "Why do I multiply 5 by 7?"

Barb's answer to Pete was "Now add 70." It should have been "Now you need to find the area.

2. The base of each case is 5 by 7 so....

This was not good evidence of communicating how you reasoned. You were making superficial explanation.
These are the criteria agreed upon for excellent groupwork:

- Everyone is listening carefully to the ideas of others in the group.
- Everyone contributes something to the discussion.
- The group is open to new ideas being voiced.
- Communication is clear.
- The group gets results.

Group members are actively offering each other help as needed.

- Everyone is encouraged to participate.

See comments below for specific observations.

<table>
<thead>
<tr>
<th>Students</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barb</td>
<td>&quot;Exe6&quot; is not an explanation!</td>
</tr>
<tr>
<td>Mike</td>
<td>Good ideas—try to stick to the topic.</td>
</tr>
<tr>
<td>Pete</td>
<td>Kept working till you are satisfied</td>
</tr>
<tr>
<td>Molly</td>
<td>Ponder! You have good answers and ideas, so speak up!</td>
</tr>
</tbody>
</table>

General Comments: Better than yesterday, but still not efficient.

Pete's question: "How did you get the volume of the cube?"

Barb - You ask good questions

- But your explanations to date don't help.

Molly - Good explanation about base area, how to use clue to calculate it. "That's an important...

Mike - I don't think that's right because all the others...

- Good observation. I have good discussion with...
Oral Report Grade Sheet

An excellent report
- Good eye contact
- Speaks clearly to class
- Expressive and confident presentation
- Clear explanation using correct vocabulary
- Good, correct information
- Can answer questions from class or teacher
- Asks class if they have questions
- Gives enough information, a full and detailed report
- Gives examples

No penalty for minor prompting from group.

A good report:
- As above, but missing one element from each group
- or
- As above, but minor help from group does not get the speaker back on track

A fair report:
- Missing two elements from each list above

A poor report:
- Missing three elements from each list above

Zero points if unprepared to make report.
Oral Report Grade Sheet

An excellent report
- Good eye contact
- Speaks clearly to class
- Expressive and confident presentation
- Clear explanation using correct vocabulary
- Good, correct information
- Can answer questions from class or teacher
- Asks class if they have questions
- Gives enough information, a full and detailed report
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No penalty for minor prompting from group.

A good report:
- As above, but missing one element from each group
- As above, but minor help from group does not get the speaker back on track

A fair report:
- Missing two elements from each list above

A poor report:
- Missing three elements from each list above

Zero points if unprepared to make report.

Dec 6
Meghan M

*good model for
discussion

Peter

This made no sense
Disappointing how
you apparently all
forgot how to tackle?
Next = Next

Incorrect E = mile wide
Lesson 2:
Quiz, Form A

Name: Fred

1. Two exponential growth and decay situations are represented by graphs (1) and (2) and also by tables (A) and (B). For each graph there is a matching table.

   (a). Write the number of the graph beside its corresponding table.

   ![Graphs](image)

   |   |   |   |
   ---|---|---|
   (1) |   |   |
   (2) |   |   |

   (b). The equation of graph 1 in part (a) is of the form \( y = a \cdot b^x \). Will \( b \) be less than 1 or greater than 1? Explain your answer.

   Greater __ Less __

   Explanation:

2. Consider the \((x,y)\) pattern produced by the rule \( y = 1.3^x \).

   (a). Make a table of \((x,y)\) values, to 1 decimal place, for \( x \) from 0 to 10 in steps of 1.

   |   |   |
   ---|---|
   \( x \) | \( y \) |
   0 | 1.0 |
   1 | 1.3 |
   2 | 1.7 |
   3 | 2.3 |
   4 | 3.0 |
   5 | 3.9 |
   6 | 5.0 |
   7 | 6.3 |
   8 | 8.1 |
   9 | 10.6 |
   10 | 13.8 |

   (b). Mark the \( y \)-axis, and plot the points.

   (c). Describe the way that \( y \) changes as \( x \) increases.

   Description:

   As \( x \) increases, \( y \) jumps in larger intervals than it would if \( y \) were constant. It reads down, then up, down, then up, and the jumps get bigger as it goes.
Lesson 2: Quiz, Form A

1. Two exponential growth and decay situations are represented by graphs (1) and (2) and also by tables (A) and (B). For each graph there is a matching table.

   (a). Write the number of the graph beside its corresponding table.

   \[
   \begin{array}{c|cccc}
   \text{(1)} & x & 1 & 2 & 3 & 4 \\
   \hline
   y & 6 & 12 & 24 & 48 \\
   \end{array}
   \]

   \[
   \begin{array}{c|cccc}
   \text{(2)} & x & 1 & 2 & 3 & 4 \\
   \hline
   y & 1.5 & 0.75 & 0.375 & 0.1875 \\
   \end{array}
   \]

   (b). The equation of graph 1 in part (a) is of the form \(y = a(b)^x\). Will \(b\) be less than 1 or greater than 1? Explain your answer.

   Greater \( \Box \) Less \( \Box \)

   Explanation: Because when the graph is decreasing, the exponential growth is less than 1 so the graph is decending.

3. Consider the \((x, y)\) pattern produced by the rule, \(y = 1.3^x\).

   (a). Make a table of \((x, y)\) values, in steps of 1, for \(x\) from 0 to 10.

   \[
   \begin{array}{c|cccccccccc}
   x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \hline
   y & 1 & 1.3 & 1.7 & 2.2 & 2.9 & 3.7 & 4.8 & 6 & 8 & 10 & 13 & 8 \\
   \end{array}
   \]

   (b). Mark the \(y\)-axis, and plot the points.

   (c). Describe the way that \(y\) changes as \(x\) increases.

   Description:

   \(y\) increases at a slow rate up into about the \(y\) axis then the \(y\) axis begins to increase at a faster rate each time the \(x\) axis increases by 1.
Lesson 2: Quiz, Form A

Name: Molly

1. Two exponential growth and decay situations are represented by graphs (1) and (2) and also by tables (A) and (B). For each graph there is a matching table.

(a). Write the number of the graph beside its corresponding table.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b). The equation of graph 1 in part (a) is of the form \( y = a(b)^x \). Will \( b \) be less than 1 or greater than 1? Explain your answer.

Greater    Less

Explaination: It will be less because the curve on the graph decreases, so it has to be less than 1.

3. Consider the \((x, y)\) pattern produced by the rule, \( y = 1.3^x \). Make a table of \((x, y)\) values, to three decimal places, for \( x \) from 0 to 10 in steps of 1.

<p>| | | | | | | | | | |</p>
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</tr>
</tbody>
</table>

(b). Mark the \( y \)-axis and plot the points.

c. Describe the way that \( y \) changes as \( x \) increases.

Description: As \( x \) increases by 1, \( y \) increases by a smaller amount each time.

Example: from 0 to 1, the difference is 3. From 1 to 2, it is 3. Then 4, 7.3, 11.2, 13.8, and so on.
### Progress Report

**ID:** Misc. 1: Misc. 2:

<table>
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<tr>
<th>#</th>
<th>Assignment</th>
<th>Category</th>
<th>Score</th>
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<td>10</td>
<td>7.1</td>
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<tr>
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<td>5</td>
<td>2.9</td>
</tr>
<tr>
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<td>OYO p.30</td>
<td>OYO</td>
<td>5.0</td>
<td>5</td>
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<td>6</td>
<td>OYO p.36</td>
<td>Quiz</td>
<td>100.0</td>
<td>100</td>
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</tr>
<tr>
<td>7</td>
<td>QUIZ3 (change)</td>
<td>Homework</td>
<td>10.0</td>
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<tr>
<td>8</td>
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<td>Homework</td>
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<td>4.0</td>
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<tr>
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<td>100</td>
<td>38.4</td>
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<tr>
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<td>Test</td>
<td>94.0</td>
<td>100</td>
<td>78.3</td>
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<tr>
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<td>TEST (linear)</td>
<td>Notebook</td>
<td>4.5</td>
<td>5</td>
<td>4.2</td>
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<td>Quiz</td>
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<td>100</td>
<td>37.5</td>
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<tr>
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<td>9.3</td>
</tr>
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<td>5</td>
<td>2.0</td>
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<td>19</td>
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<td>5</td>
<td>3.9</td>
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<td>94.0</td>
<td>100</td>
<td>38.3</td>
</tr>
<tr>
<td>21</td>
<td>QUIZ2 (linear)</td>
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<td>10</td>
<td>9.1</td>
</tr>
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<td>10.0</td>
<td>10</td>
<td>9.1</td>
</tr>
<tr>
<td>23</td>
<td>Group (term2)</td>
<td>Homework</td>
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<td>7.8</td>
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<td>H.W. (1/8)</td>
<td>OYO</td>
<td>5.0</td>
<td>5</td>
<td>3.3</td>
</tr>
</tbody>
</table>

**Quarter 2:** 606.0/640.0 = 94.7% A (Class average = 83.4%)

- **Test** (20% of grade): 94.0% A (Class average = 79.3%)
- **Quiz** (20% of grade): 95.5% A (Class average = 87.4%)
- **OYO** (15% of grade): 92.2% A (Class average = 70.4%)
- **Group** (10% of grade): 100.0% A+ (Class average = 96.7%)
- **Oral** (10% of grade): 100.0% A+ (Class average = 91.1%)
- **Notebook** (5% of grade): 90.0% A- (Class average = 84.4%)
- **Homework** (15% of grade): 91.4% A- (Class average = 81.7%)

**Overall Grade:** 606.0/640.0 = 94.7% A (Class average = 83.4%)

**Key:** _Incomplete dr-Dropped ex-Excused_

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Appendix D

Karen’s Documents

Source of unit 3 tasks:

156
Math General Scoring Procedure

0 - No response at all, question left blank. Bluff response; totally wrong response. Shows no understanding of the concept.

1 - Begins problem but omits parts of the response. Has major errors or uses inappropriate strategies. Shows some understanding of the basic concept.

2 - Contains a complete response, but the explanation is muddled. Presents incomplete arguments. No diagrams or diagrams are inappropriate. Clear evidence of some understanding but has significant errors.

3 - Contains a good solid response with some but not all of the characteristics of a "4" response. Shows good understanding, but has minor errors.

4 - Contains a complete response with clear, coherent, and unambiguous explanation. Includes appropriate pictures, graphs, diagrams, and/or examples (or counterexamples). Uses appropriate mathematical terminology, answers all parts of the question, and shows total understanding of concept.

5 - Contains all the elements of a complete and competent response and includes additional information which pertains to the concept. Response is "above and beyond" a complete and competent response.
Scoring Rubric For Unit 3 Test

1. Almost everything costs more today than it did a few years ago. An equation estimating the median cost \( C \) in cents of a 12-ounce soft drink \( x \) years ago (for \( x \) up to 40 years) is \( C = -2x + 85 \).

   (a) Solve the equation for \( x \).
       \[ 35 = -2x + 85 \]

   (b) Use your graphing calculator to make a table and a graph of the relation between number of years ago and cost for \( 0 \leq x \leq 40 \). Show how to find the solution of the equation in part (a) in the table and on the graph. (5 point rubric)

   
   **How to find solution in the table:** (2.5 point rubric)
   2.5 - Table set up \( \text{min} = 0 \), \( \Delta t \text{bl} = 10 \). Look for 35 in y column. Change \( \Delta t \text{bl} \) to 5. Look for 35 again. Read x value corresponding to that y value. \( x = 25 \).
   2.0 - correct without the \( \Delta t \text{bl} \).
   1.0 - correct but not completed.
   0.5 - Concept correct but minimal understanding such as columns reversed.
   0.0 - no understanding.

   **How to find solution on graph:** (2.5 point rubric)
   2.5 - Trace the graph until you find \( y = 35 \). Zoom in for more accuracy and trace until \( y = 35 \). Look at x value for that point.
   2.0 - Correct but no zoom.
   1.0 - Correct idea but not complete.
   0.5 - Concept correct but axes reversed or minimal understanding.
   0.0 - No understanding.
Scoring Rubric For Unit 3 (cont)

(c) Explain how both the graph and the table on the previous page can be used to solve the inequality \(-2x + 85 > 50\).

**How to find the solution in table:** (2.5 point rubric)

2.5 - Find 50 in the \(y\) column of the table. All \(x\) values which would correspond to a \(y\) value under 50 are solutions for \(x\). Change \(\Delta tbl\) to smaller jumps until 50 appears in the \(y\) column. Substitute numbers from both sides of 50 to see which works.

2.0 - Correct but no \(\Delta tbl\).
1.5 - Correct with minor error.
1.0 - Correct idea but not complete.
0.5 - Minimal understanding.
0.0 - No understanding

(d) What question related to a 12-ounce soft drink can be answered by the inequality and its solution in part (c)?

(5 point rubric).

5 - Above and beyond
4 - Correct
3 - Correct concept, but unclear or minor flaws.
2 - Correct concept but major flaw.
1 - Minimal understanding.
0 - No understanding

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Almost everything costs more today than it did a few years ago. An equation estimating the median cost $C$ in cents of a 12-ounce soft drink $x$ years ago (for $x$ up to 40 years) is $C = -2x + 85$.

(a). Solve the equation below for $x$. Show or explain your work, and explain what the solution means in the above situation.

$35 = -2x + 85$

(b). Use your graphing calculator to make a table and a graph of the relation between number of years ago and cost for $0 \leq x \leq 40$. Show how to find the solution of the equation in part (a) in the table and on the graph.

(c). Explain how the table on the previous page can be used to solve the inequality $-2x + 85 > 50$. Give the solution.

(d). What question related to the cost of a 12-ounce soft drink can be answered by the inequality and its solution in part (c)?

Would 16 years ago cost more than 50¢?
1. Almost everything costs more today than it did a few years ago. An equation estimating the median cost $C$ in cents of a 12-ounce soft drink $x$ years ago (for $x$ up to 40 years) is $C = -2x + 85$.

(a). Solve the equation below for $x$. Show or explain your work, and explain what the solution means in the above situation.

$$35 = -2x + 85$$

(b). Use your graphing calculator to make a table and a graph of the relation between number of years ago and cost for $0 \leq x \leq 40$. Show how to find the solution of the equation in part (a) in the table and on the graph.

<table>
<thead>
<tr>
<th>Years Ago</th>
<th>Cost (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td>65</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
</tr>
<tr>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>

(c). Explain how both the graph and the table on the previous page can be used to solve the inequality $-2x + 85 > 50$. Give the solution.

How to find solution in table: Enter data into y1, and follow down to 50. Then what? Find x-intercept. Where is it? Solution: $x < 50$

How to find solution on graph: Look for where the line crosses the x-axis.

(d). What question related to the cost of a 12-ounce soft drink can be answered by the inequality and its solution in part (c)?

How much would a can of soda cost 100 years ago?
1. Almost everything costs more today than it did a few years ago. An equation estimating the median cost \( C \) in cents of a 12-ounce soft drink \( x \) years ago (for \( x \) up to 40 years) is \( C = -2x + 85 \).

(a). Solve the equation below for \( x \). Show or explain your work, and explain what the solution means in the above situation.

\[ 35 = -2x + 85 \]

(b). Use your graphing calculator to make a table and a graph of the relation between number of years ago and cost for \( 0 \leq x \leq 40 \). Show how to find the solution of the equation in part (a) in the table and on the graph.

<table>
<thead>
<tr>
<th>Years Ago</th>
<th>Cost (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td>65</td>
</tr>
<tr>
<td>20</td>
<td>45</td>
</tr>
<tr>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
</tr>
</tbody>
</table>

How to find solution in table:

1. Find the y-value in the y-column, then look at the corresponding x-value in the x-column.
2. To the line and look down for the value on the x-axis.

(c). Explain how both the graph and the table on the previous page can be used to solve the inequality \(-2x + 85 > 50\). Give the solution.

(d). What question related to the cost of a 12-ounce soft drink can be answered by the inequality and its solution in part (c)?

"How many years ago did a 12 ounce soda become more than 50 cents?"
Appendix E

Mary's Documents

Source of Geometry tasks:
3. Draw a polygon that tessellates. Use mathematical reasoning to explain why it tessellates.

4. Draw a polygon that does not tessellate. Use mathematical reasoning to explain why it does not tessellate.

5. Kim is building a rectangular pen for her pet goat named Gwen. One side of the pen is the back of the barn. She has 50 meters of fencing. Sketch and label the pen below. Make a table showing possible lengths, widths, and areas to find which rectangular area will give her the maximum area.

6. If she is able to build a pen of any shape against the back of the barn, which shape should she use to get maximum area? Sketch it below with dimensions. How much bigger is this than the rectangular pen?
3. Draw a polygon that tesselates.  
   Use mathematical reasoning to explain why it tesselates.

   - It tesselates because the sum of the angles is 360°, which means it will fit together without gaps.

5. Kim is building a rectangular pen for her pet goat named Gwen. One side of the pen is the back of the barn. She has 50 meters of fencing. Sketch and label the pen below. Make a table showing possible lengths, widths, and areas to find which rectangular area will give her the maximum area?

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td>48 m²</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>200 m²</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>300 m²</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>300 m²</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>400 m²</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>48 m²</td>
</tr>
</tbody>
</table>

   - The width times the length equals the area.
   - The maximum area occurs when the width is 12 meters and the length is 26 meters.

6. If she is able to build a pen of any shape against the back of the barn, which shape should she use to get maximum area? Sketch it below with dimensions. How much bigger is this than the rectangular pen?

   - The area of a circle is $A = \pi r^2$.
   - The area of the circle is $\approx 312 m^2$.
   - It is about 2.5 times bigger than the rectangular pen.
3. Draw a polygon that tessellates. Use mathematical reasoning to explain why it tessellates. All sides match up to each other and form no gaps. The lesser amount of sides the more likely it will tessellate.

5. Kim is building a rectangular pen for her pet goat named Gwen. One side of the pen is the back of the barn. She has 50 meters of fencing. Sketch and label the pen below. Make a table showing possible lengths, widths, and areas to find which rectangular area will give her the maximum area?

6. If she is able to build a pen of any shape against the back of the barn, which shape should she use to get maximum area? Sketch it below with dimensions. How much bigger is this than the rectangular pen? Kim is unable to make a different shaped pen with more area. A rectangle will give her max area for her fencing.
Source of tasks for Power Models Test:
1. A baseball hit (on earth) with initial upward velocity of 30 meters per second will reach height $h$ meters after $t$ seconds as estimated by the rule $h = 0.5 + 30t - 4.9t^2$.

(d). When does the baseball hit the ground? Explain how you found your answer.

40 seconds added 4.9 with 4.9.

(e). A little leaguer swung his bat 0.8 meters above the ground and hit a baseball with an initial upward velocity of 10 meters per second. Write an equation to model the height of the baseball as a function of time.

8/10 $h = 0.8 + 10t[-4.9t^2]$ (teacher addition)

(f). Explain how the flight of the baseball in part (e) differs from the one described originally.

7/10 The numbers are different and so the one in $e$ the numbers are 0.8 and 10 and the baseball in part $e$ flies much slower than the original one.

4. From your homework, recalling what we did in class, or creating one on your own, give an example of a simple power model. Tell what the variables are, give the rule that represents the relation between the variables, and give a sketch of the graph (label the graph, however, you do not need to scale the axis).
A baseball hit (on earth) with initial upward velocity of 30 meters per second will reach height \( h \) meters after \( t \) seconds as estimated by the rule \( h = 0.5 \times 30t - 4.9t^2 \).

4. From your homework: recalling what we did in class, or invent one on your own, give an example of a simple power model. Tell what the variables are, give the rule that represents the relation between the variables, and give a sketch of the graph (label the graph, however, you do not need to scale the axes).

\[ \text{Rule?} \]

\( \frac{7}{30} \)
1. A baseball hit (on earth) with initial upward velocity of 30 meters per second will reach height \( h \) meters after \( t \) seconds as estimated by the rule \( h = 0.5 \times 30t - 4.9t^2 \).

   (d). When does the baseball hit the ground? Explain how you found your answer.
   - The ball hits the ground between 0.1 and 0.2 seconds. I found this by looking at the table made by the rule. I just found where the height went from a low positive number to a low negative number.

   (e). A little leaguer swung his bat 0.8 meters above the ground and hit a baseball with an initial upward velocity of 10 meters per second. Write an equation to model the height of the baseball as a function of time.

   \[ h = 0.8 + 10t - 4.9t^2 \]

   (f). Explain how the flight of the baseball in part (e) differs from the one described originally. The baseball in part (e) starts at a height of 0.8 meters while the baseball in part (a) starts at a height of 0.5 meters. Also, the baseball in part (a) goes higher than the baseball in part (e) because (a) has an upward velocity of 30 m/s and (e) only has an upward velocity of 30 m/s. The baseball in part (a) stays in the air longer, goes higher and goes faster than the baseball in part (e).

4. From your homework, recalling what we did in class, or creating one on your own, give an example of a simple power model. Tell what the variables are, give the rule that represents the relation between the variables, and give a sketch of the graph (label the graph, however, you do not need to scale the axis).

   \[ y = \frac{1}{x} \quad (A \text{ is only 1 number, in this case}) \]

   - The variables are: \( y = \text{Time} / x = \text{Speed} / A = \text{Distance} \)

   - Time
   - Speed

   \[ \text{TIME} \]

   \[ \text{SPEED} \]
The final exam in Math 1 will have three components. Your exam grade will be determined by evaluating each of the following three areas:

- participation and work on the project during the scheduled class times
- quality (neatness, accuracy, etc.) of your written reports
- quality (presentation, accuracy, etc.) of your oral report.

Your exam grade will count as 10% of your final semester grade.
Appendix G

The Capstone Assessment

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Capstone: Planning a Benefits Carnival

In this course you have built useful mathematical models – including linear, exponential, geometric, simulation, and graph models. You have used these models to solve important problems in many different settings. You have investigated patterns in data, change, chance, and shape. And you have learned how to make sense of situations by representing them in different ways using physical representations, words, graphs, tables, and symbols. In this final unit, you will pull together many of the important concepts, techniques, and models that you have learned, and use them to analyze one big project.

Think About This Situation

Many schools organize fund raising events to raise money for improving their programs. The event might be a dance, a bike-a-thon, a book sale, or anything else that gets the community involved in raising money for the school. One event that is common for elementary schools is a benefits carnival. Suppose that a local elementary school is considering such a carnival, and your class has offered to plan the event and prepare a full report for the school's principal. (In return, your class will get part of the proceeds to use for yourselves!)

(a). Make a list of all the things that need to be done to plan, carry out, and clean up following such a carnival.

(b). Make a list of the kinds of booths or activities that might be good to have at the carnival. Think of as many as you can.
INVESTIGATION 1: Lots of Math

Mathematics can be used in many different ways to help you organize the carnival. Think about the mathematics you have studied in each of the units in this course. The units are listed below. As a group, brainstorm and then write down two ways the mathematics in each unit could be used in the carnival project.

1. **Patterns in Data**
2. **Patterns of Change**
3. **Linear Models**
4. **Graph Models**
5. **Patterns in Space and Visualization**
6. **Exponential Models**
7. **Simulation Models**

√ Checkpoint

(a). For each unit, compare and discuss the ideas from different groups.

(b). Are there any big mathematical ideas or topics from this course that have not been applied to the carnival project? If so, is there any way they could be applied?

Be prepared to share your group’s thinking with the whole class.

At the end of this unit, you will write an individual report for the principal of the elementary school explaining how you used mathematics to help plan the carnival. To assist in the preparation of the reports, your group will complete three of the following investigations and then will present an oral report to the class on one of them. Guidelines for the group report are given on page 14.

As a group, examine Investigations 2 through 8 and choose three to complete. Confirm your choices with your teacher, and then start investigating!
INVESTIGATION 2: Careful Planning

Careful planning is necessary to make the carnival a success. An important part of planning is identifying and scheduling all the tasks that need to be done.

1. Suppose that some of the tasks for the carnival project, along with task times, are: building the booths (5 days), laying out the floor plan (2 days), designing a carnival logo (1 day), publicity (10 days), and finding a location (2 days). Write down at least two more tasks that will need to be done and their times.

2. Using the tasks from activity 1:
   (a). Construct a prerequisite table showing the tasks, the task times, and the immediate prerequisites.
   (b). Draw a project digraph.
   (c). Find the critical tasks and the earliest finish time for the whole project.
   (d). Set up a schedule for completing all the tasks. Your schedule should show earliest start times, earliest finish times, and slack times.

3. Committees must be formed to work on each of the tasks. Since some students will be on more than one committee, it is impossible for all the committees to meet at the same time. Assume there are five committees, each of which has a member in common with at least two of the other committees.
   (a). Construct a vertex-edge graph model that shows the five committees and which committees share members (you decide which committees share members).
   (b). What is the fewest number of meeting times needed so that all committees can meet? Explain how you can use the graph model from part (a) to answer this question.

4. Make a neat copy of your project digraph, showing the critical tasks, and earliest finish time. Also make a copy of the graph model for the committee scheduling problem, showing the graph coloring.
   (a). File these two graphs at the location in the classroom designated by your teacher. Check out the graphs filed by other groups in the class and compare their graphs to those from your group.
   (b). Write a question to at least one group asking them to explain something about their work that you found interesting or that you did not understand. Answer any questions your group receives.
INVESTIGATION 3: Booths and Floor Plans

In this investigation you will sketch a floor plan for the carnival and build a scale model of a booth.

1. Assume that the carnival will be held in a rectangular-shaped gym with dimensions 40 meters by 30 meters. Using centimeter graph paper, with one centimeter corresponding to 1 meter, sketch a floor plan for the carnival. Your floor plan should show the placement of the following items:

- The ticket/information booth is hexagonal-shaped so that during peak times customers can line up at six windows to get information or tickets. Two sides of the booth have length 1 m and the other sides have length 2.24 m. The booth is to be placed in the center of the gym.
- There are ten game booths to be arranged along the sides of the gym. Eight of them are U-shaped, 1.5 m x 2 m x 1.5 m. The 2 m side faces out. The other two are triangular-shaped - two sides are the same length and the third side, which faces out, is 2 m long.
- Six tables, each of which has a 2 m x 1 m table top, are placed in a U-shape to serve as the area where food is sold.
- Decide on one other feature of the floor plan, describe it, and add it to your floor plan.

2. (a). Build a model of the ticket booth frame using a scale of 10 cm to 1 m. The ticket booth has base dimensions as described in activity 1. The vertical walls of the booth are 2 m high.
   (b). Design a tent-like canopy for the booth.

3. (a). Make a careful sketch of the ticket booth.
   (b). How much canvas will be needed to cover the booth, including the canopy?

4. File a copy of your floor plan and your ticket booth model and sketch at the location in the classroom designated by your teacher. Study the floor plans, models, and sketches filed by other groups in the class and compare to your group's work. Write a question to at least one group asking them to explain something about their work that you found interesting or that you did not understand. Answer any questions your group receives.
INVESTIGATION 4: Carnival Tee-Shirts

As part of the promotion and fund-raising for the carnival, tee-shirts will be designed and sold.

1. Design a logo for the carnival, to be used on the tee-shirts as well as on other promotional materials. The logo might include the name of the school or a message of some sort. It could be an abstract design or a picture of something related to the carnival. It can be whimsical or serious. Be creative! The only requirement is that the logo must be symmetrical in some way.

   (a). Describe the symmetry shown in your logo.

   (b). Briefly explain why you chose your particular design and explain its meaning.

2. The price list for a local tee-shirt shop is shown below.

   **Midwest Athletic Supply and Screen Printing**

   Set-up
   $17.50 for one color
   $12.50 for each additional color

   Art
   $25 per hour

   Tees
   Prices for shirts with one color:
   1-15       $6.55 ea.
   16-31      $6.25 ea.
   32-62      $5.95 ea.
   63-147     $5.75 ea.
   148-       $5.40 ea.

   Add $0.30 per shirt for each additional color

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INVESTIGATION 5: Money Made and Money Spent

The main purpose of the carnival is to make money in a fun way. Besides having fun, both organizers and customers are concerned about the money aspect. The organizers want to know how much money the carnival will make, and the customers want to know how much money they will spend.

1. Customers at the carnival will buy tickets from a ticket booth and then pay for the games using one or more tickets per game. Each ticket costs 25¢. This year, parents who buy tickets in advance can specify that the money paid goes to equipment in their child's classroom. Parents are asking, "About how many tickets will I use at the carnival?" For the last three years, 25 parents were randomly chosen and asked how many tickets they used. These data are shown in the table below.

| Number of Game Tickets Used by Sample of Parents at the Last Three Carnivals |
|---|---|---|
| 3 Years Ago | 2 Years Ago | Last Year |
| 26 | 20 | 33 |
| 20 | 30 | 15 |
| 20 | 18 | 10 |
| 16 | 25 | 15 |
| 4 | 8 | 10 |
| 21 | 15 | 44 |
| 20 | 14 | 40 |
| 20 | 48 | 8 |
| 26 | 16 | 12 |
| 10 | 10 | 8 |
| 16 | 16 | 12 |
| 20 | 35 | 10 |
| 34 | - | - |

(a). Make a box plot of each year's data. Divide up the work among your group. Draw all three box plots on the same axis.
(b). Write a brief summary of the information shown in each box plot and how patterns of ticket buying vary from year to year.
(c). What other graphs or statistics might be useful in helping you answer a parent's question of "How many tickets will I use?"
(d). When a parent asks you, "How many tickets will I use?", what will you say? Explain the reasoning behind your response.

2. This year you are not only going to make money to support the school for the current year, you are also going to set up a fund that will grow and support the school in future years. Your goal is to put $800 of the carnival profits into a savings account for the school that pays 5% interest compounded yearly.

(a). How much money will be in the account when you graduate, assuming no withdrawals?
(b). Write equations that will allow you to calculate the balance of this account:
   • For any year, given the balance for the year before.
   • After any number of years x.
(c). Use the equations from part (b) to make a table and a graph showing the growth of this account.
(d). Describe the pattern of growth in the savings account over a 10-year period.
(e). Think about how fast the account grows:
   • Is the account growing faster in year 5 or year 10? How can you tell?
   • What would the graph look like if the account was growing at a constant rate of change?
(f). Suppose that the school decides to cash in the account when there is enough money in it to buy a new computer for the library, which is expected to cost about $1000. How long will it be before they can buy the computer?

3. File a neat copy of your work on this investigation, including graphs, plots, tables, and explanations, at the location in the classroom designated by your teacher. Check the solutions filed by other groups and compare to your group's work. Write a question to at least one group asking them to explain something about their work that you found interesting or that you did not understand. Answer any questions your group receives.
(a). Suppose you decide to go with a deluxe four-color design that requires two hours of artwork from the shop's designers. Complete a table like the one below.

<table>
<thead>
<tr>
<th>Number of Tee-shirts Purchased</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>201</td>
<td></td>
</tr>
<tr>
<td>202</td>
<td></td>
</tr>
</tbody>
</table>

(b). Suppose that you buy more than 150 of the shirts. If NOW is the cost to buy a given number of shirts and NEXT is the cost to buy one more, write an equation showing the relationship between NOW and NEXT.

(c). If \( T \) is the number of tee-shirts purchased and \( C \) is the total cost, write an equation showing the relationship between \( T \) and \( C \) for any number \( T > 150 \).

3. You plan to sell the tee-shirts and make a profit. What price should you set for the shirts? Write a brief analysis justifying your choice of selling price. Your analysis should include the following:

- An equation and graph showing the relationship between profit \( P \) and number of tee-shirts \( T \) sold, where \( T > 150 \);
- An explanation of how to use the profit equation and graph to find the number of shirts you must sell to break even;
- An estimate of the profit you expect to make;
- A summary of why you chose your selling price.

4. File a copy of your tee-shirt logo, with an explanation of the symmetry it exhibits, along with your profit analysis from activity 3 at the location in the classroom designated by your teacher. Examine the logos and solutions filed by other groups and compare to your group's work. Write a question to at least one group asking them to explain something about their work that you found interesting or that you did not understand. Answer any questions your group receives.
INVESTIGATION 6: Ring-Toss Game

The most important part of a carnival is, of course, the games. Suppose that you are setting up a ring-toss game. A number of two-liter bottles of soda are lined up and the goal of the game is to toss a ring around the top of one of them. If a player hooks one of the bottles (which is called getting a “ringer”), then she gets to keep it. Your task is to design this game, by considering how large the rings should be, how far back the players should stand, and what the cost to play the game should be.

1. Examine the following data collected from a ring tossing experiment. The data show the average number of ringers per 100 tosses at a distance of 1 meter from the bottles, for rings of varying diameters.

<table>
<thead>
<tr>
<th>Diameter of Ring (in centimeters)</th>
<th>Average Number of Ringers per 100 Tosses</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
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<tr>
<td>8</td>
<td>14</td>
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<td>9</td>
<td>19</td>
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<tr>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>27</td>
</tr>
<tr>
<td>12</td>
<td>32</td>
</tr>
</tbody>
</table>

(a). Make a scatterplot of the data.
(b). Describe and explain any patterns or unusual features of the data.
(c). Notice that when the ring diameter doubles, the number of ringers increases by about a factor of four. Can you suggest any explanation for this pattern in terms of the size of the circular rings?
2. Examine the data in the following table showing the average number of ringers per 100 tosses with rings of diameter 12 cm, for players standing at varying distances from the bottles.

<table>
<thead>
<tr>
<th>Distance from the Bottles (in meters)</th>
<th>Average Number of Ringers per 100 Tosses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>32</td>
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<tr>
<td>2</td>
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<td>8</td>
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<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

(a). Make a scatterplot of the data.
(b). Describe any patterns you see in the data.
(c). If NOW is the average number of ringers at one of the distances in the table and NEXT is the average number of ringers for a distance 1 meter farther away, write an equation that approximates the relationship between NOW and NEXT.

3. It is important to charge the right amount of money to play the game, so that the income from ticket sales for the game is greater than the cost of the prizes given away. For this activity, assume that a ring with diameter 12 cm is used and players stand 2 meters away from the bottles.

(a). Suppose that a local merchant offers to support the carnival by loaning you all the soda you need to set up your game, and then charging you 60c per bottle for every bottle you give away as a prize. You decide to charge 25c for a toss, and, based on the data in the tables, you expect 15% of the tosses to be ringers. If T is the number of tosses by customers and P is your profit from the game, write an equation that shows the relationship between P and T.

(b). Given the arrangement with the local merchant in part (a), what is the least you can charge for a toss and still make a profit?

(c). Based on the data in activity 2, how would your profit-modeling equation change if players tossed rings at a distance of 3 meters?

   • In this situation, what is the least you can charge for a toss and still make a profit?
   • As designers of fun, profitable games, would your group recommend the option in part (a) or in part (c)? Explain your reasoning.
Suppose you have no sponsor and must pay the usual retail price for the soda. For stores near where you live, what is a reasonable price for a two-liter bottle of soda? In this situation, what is the least you can charge for a toss at a distance of 2 meters and still make a profit?

4. Make a neat copy of your work on this investigation, showing graphs, equations, and other answers. File this “solution sheet” at the location in the classroom designated by your teacher. Check out the solutions filed by other groups in the class and compare to your solutions. Write a question to at least one group asking them to explain something about their work that you found interesting or that you did not understand. Answer any questions your group receives.

INVESTIGATION 7: Free-Throw Game: Beat the Pro

Games of skill, especially those involving sports, are always popular at carnivals. Suppose that you are in charge of setting up a basketball free-throw game where a challenger pits his or her skill against a pro. In this case, the pro is the top free-throw shooter from the girls basketball team. The challenger and the pro each shoot ten free-throws. If the challenger makes more baskets than the pro, then he or she wins a prize. Your job is to figure out how much to charge for a challenger to play the game and what prizes should be awarded to winners.

1. You know from the basketball season’s statistics that the pro makes about 85% of her free-throws. But what about the challenger’s percentage of successful free-throws? Although you cannot find out the shooting percentage of every challenger that might play the game, it would be helpful to get some information to help you decide on the price to charge and prizes to award. One of your friends, who likes basketball and is a pretty good shooter, agrees to help you gather some data by being a sample challenger. He completes 50 trials of 10 free-throws and records the number of baskets for each trial. These data are shown in the table at the top of the next page.
Number of Free-Throws Made (Out of 10) for 50 Trials by One Sample Challenger

<p>| | | | | | | | | | |</p>
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<td>6</td>
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</table>

(a). Construct a histogram for these data. Describe and explain any patterns in the distribution.

(b). Based on the data, about what percent of free-throws does the sample challenger make? Explain using a statistical measure.

(c). Does he appear to be a fairly consistent shooter? Justify your answer by using an appropriate plot and summary statistics.

2. To help make good decisions about the amount you should charge to play the game and the value of prizes to give away, you can simulate a game between the sample challenger and the pro.

(a). What is a reasonable estimate for the probability that the pro will make a particular free-throw?

(b). Explain why 0.75 is a reasonable estimate for the probability that the sample challenger will make a particular free-throw.

(c). Using the probabilities above, design a simulation to estimate the probability that the challenger wins the game.

(d). Conduct 5 trials of your simulation and add your results to the table.

Simulated Beat The Pro Game, so there is a total of 100 trials.

3. Now examine the simulation table you completed in activity 2.

(a). Construct a histogram of the difference of the number of free-throws made by the challenger and the number made by the pro. Describe the shape of the distribution. Interpret the shape in terms of outcomes of the “Beat The Pro” game.

(b). What is your estimate of the probability that the challenger wins? Remember that the challenger wins only if he makes more of the 10 free throws than the pro does.
4. A local sporting goods store will support the carnival by selling you top-quality basketballs for winning prizes at a cost of $8 per basketball.

(a). Based on the simulation data and your analysis in activity 3, determine how much you should charge the sample challenger to play the game in order to keep the game affordable and yet ensure a profit over the course of many games.

(b). The actual game will be played with many different challengers, not just the one sample challenger. What do you think is a good price to charge for the actual game?

5. Would the pro be more likely to beat a 75% free-throw shooter in a game with 20 shots or one with 10 shots? Explain your reasoning.

6. Make a neat copy of your work on this investigation, and file it at the location in the classroom designated by your teacher. Examine the work filed by other groups in the class and compare to yours. Write a question to at least one group asking them to explain something about their work that you found interesting or that you did not understand. Answer any questions your group receives.

INVESTIGATION 8: Further Analysis

There are many other factors you might consider as you plan the carnival. Choose one of your ideas from Investigation 1 or from the Think About This Situation at the beginning of this unit. Carry out a brief mathematical analysis of the idea. Specifically, you should formulate and answer at least two questions related to your idea. For example, you might design and analyze another game, as is done in Investigations 6 and 7, or you might collect and analyze data on what kinds of games are most popular or how much money customers typically spend at a carnival. File a copy of your analysis at the location designated by your teacher.
REPORTS: Putting It All Together

Finish this Capstone by preparing two reports – one oral group report and one individual written report as described below.

1. Your group should prepare a brief oral report that meets the following guidelines:
   • Choose one of the investigations you have completed to report on. Confirm your choice with your teacher before beginning to prepare your report.
   • Present the report as if you are reporting to the principal of the elementary school that is planning to have the carnival. Your teacher will play the role of the principal.
   • Examine the work that other groups have filed on your chosen investigation. Compare your work to theirs, discuss any differences with them, and modify your solutions, if you think you should.
   • Begin your presentation with a brief summary of your work on the investigation.
   • Convince the principal that your solutions are correct and should be adopted.
   • Be prepared to discuss alternative solutions, particularly those proposed by other groups that also worked on the same investigation.
   • Be prepared to answer any questions from the “principal” or your classmates.

2. On your own, write a two-page report summarizing how the mathematics you have learned in this course can be used to help plan a school carnival.
Appendix H

Human Subjects Institutional Review Board Approval
Date: November 28, 1995
To: James Kett
From: Richard Wright, Chair
Re: HSIRB Project Number 95-12-13

This letter will serve as confirmation that your research project entitled "A portrait of assessment in reformed mathematics classrooms" has been approved under the exempt category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note that you must seek specific approval for any changes in this design. You must also seek reapproval if the project extends beyond the termination date. In addition if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: November 28, 1996

xc: Christian Hirsch, MATH
BIBLIOGRAPHY


