Pedagogical Moves as Characteristics of One Instructor’s Instrumental Orchestrations with Tinkerplots and the TI-73 Explorer: A Case Study

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PEDAGOGICAL MOVES AS CHARACTERISTICS OF ONE INSTRUCTOR’S
INSTRUMENTAL ORCHESTRATIONS WITH TINKERPLOTS
AND THE TI-73 EXPLORER: A CASE STUDY

by

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PEDAGOGICAL MOVES AS CHARACTERISTICS OF ONE INSTRUCTOR’S INSTRUMENTAL ORCHESTRATIONS WITH TINKERPLOTS AND THE TI-73 EXPLORER: A CASE STUDY

James L. Kratky, Ph.D.
Western Michigan University, 2016

Those supporting contemporary reform efforts for mathematics education in the United States have called for increased use of technologies to support student-centered learning of mathematical concepts and skills. There is a need for more research and professional development to support teachers in transitioning their instruction to better meet the goals of such reform efforts.

Instrumental approaches to conceptualizing technology use in mathematics education, arising out of the theoretical and empirical work in France and other European nations, show promise for use to frame studies on school mathematics in the United States. Instrumental genesis is used to describe the bidirectional and influential relationship that develops between a mathematical user and the mathematical and technological tool—the tool shapes the user and the user shapes the tool. Instrumental orchestration is used to describe the teacher’s role in guiding and shaping students’ use of technology and their opportunities to engage in instrumental genesis. Although recent work has revealed different types of instrumental orchestrations, researchers have not yet unpacked the finer-grained characteristics of the performance phase of teachers’ instrumental orchestrations; doing so would help educators build their repertoires of teaching techniques from which they can draw when supporting technology-enhanced mathematical activities. Attending to teachers’ use of pedagogical moves during their
instrumental orchestrations is one way to investigate and reveal such finer-grained characteristics.

The case study presented here is used to discuss data collected from two sections of one college instructor’s class for preservice elementary teachers (PSETs). Instrumental approaches are elaborated by way of pedagogical moves related to the PSETs’ opportunities to engage in instrumental genesis with Tinkerplots and the TI-73 Explorer. The instructor’s use of different pedagogical moves is cross-referenced with her use of different types of instrumental orchestrations.

Results show two different categories of pedagogical moves related to the PSETs’ instrumental geneses—student-centered instrument moves and teacher-centered instrument moves. These pedagogical moves are viewed as technological variants of more general pedagogical moves that teachers might use during their instruction—including the notions of wait time, requesting students to participate in an aspect of the lesson, and think-pair-share. The participating instructor favored the use of both student-centered types of instrumental orchestrations and student-centered instrument moves.

One implication for teacher training and development programs is that teachers can be informed about different types of pedagogical moves they might choose to use during their instrumental orchestrations and that teachers should engage in the discussion of how and when to use different instrumental orchestrations and pedagogical moves.
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CHAPTER 1
INTRODUCTION

Background

Calls for the increased use of technological tools in school mathematics to support rich student learning have persisted over the past several decades (e.g., National Advisory Committee on Mathematics Education, 1975; National Council of Teachers of Mathematics [NCTM], 1989, 2000, 2014; National Research Council, 2001; Usiskin, 1985). Those making claims of this type often argue that digital and computerized technology is (everywhere) in contemporary society and that people use technology in the workplace and as a basic component of their daily lives. Further, technology can be leveraged to help students in “visualizing and understanding important mathematical concepts and to support students’ mathematical reasoning and problem solving” (NCTM, 2014, p. 82). The prevalence of technology changes what and how mathematics can be taught. Heid (1997) argued for the transformative power of technology, boldly claiming that “the single most important catalyst for today’s mathematics education reform movement is the continuing exponential growth in personal access to powerful computing technology” (p. 5). Such calls align with broader efforts to reconsider what students should learn and the types of classroom environments teachers can create to foster deeper student learning, and challenge teachers and other stakeholders to consider more of a balance between students’ development in mathematical skills and conceptual understanding (e.g., NCTM, 1989, 2000).

In addition to promoting new technologies, calls for reform have promoted problem-solving visions (e.g., Lester & Charles, 2003; Schoen & Charles, 2003) for school mathematics.
These visions are linked with requests for a revision of teacher and student roles in the classroom (i.e., Kieran, 2007; NCTM, 1991), as well as the type of tasks posed to students (e.g., NCTM, 1991; Stein, Grover, & Henningsen, 1996). For example, some teachers view their classrooms as student-centered mathematics communities, where the students engage in the shared enterprise of solving mathematical problems; discussions and mathematical debate may occur regularly in such classrooms, reflecting social and sociomathematical norms (Yackel & Cobb, 1996) that support a mathematical community of practice (Wenger, 1998) or a community of inquiry (Goos, 2004). These calls for reform promote views of mathematics instruction that are compatible with social constructivist views of learning (e.g., Ravitz, Becker, & Wong, 2000; Stiff, 2001) and are in contrast to typical teacher-centered classrooms with an emphasis on direct instruction, which includes the use of the initiation-reply-evaluation (IRE) interaction pattern (Mehan, 1979) and student practice of mathematical skills at the cost of mathematical concepts.

Technology is viewed as a means to reorganize one’s mind (Pea, 1985, 1987) and transform the learning opportunities in the classroom (e.g., Artigue, 2002; Heid, 1988, 1997; NCTM, 1989, 2000; Usiskin 1985) towards more meaningful learning opportunities in problem-solving environments. By this, proponents of technology also seek to find a balance between rote learning processes and more conceptually grounded approaches to helping students learn mathematics (e.g., Demana & Waits, 1997; NCTM, 1989). In their review of literature on access to digital technologies in mathematics education, Olive et al. (2010) underscored ways in which digital technologies have been used to transform the learning experiences for students in mathematics classes in ways that align with the goals of contemporary reform efforts.

There has been noticeable and substantial development of technologies for use in mathematics classrooms (Kaput & Thompson, 1994; Kelly, 2003; Zbiek, Heid, Blume, & Dick,
Among the first electronic tools used in classrooms, computer-assisted instruction applications served as tutorials supporting drill and practice activities (Kaput, 1992). The four-function calculator, another early tool, is still used in many elementary and middle school classrooms. Kaput (1992) described this tool as a general-purpose tool, one that is frequently used outside of the classroom, but has often been adapted for use in the classroom. Other general-purpose tools adapted for use in mathematics classes include computer spreadsheets, graphing utilities, and statistics applications. Some of the more specialized tools, created specifically for use in mathematics education, include Logo software (cf. Clements & Battista, 2001); microworld environments (see Kaput, 1992); graphing calculators, such as those produced by Texas Instruments (Texas Instruments, 2016); Geometer’s Sketchpad (McGraw-Hill, 2016) and other dynamic geometry applications; the applets featured on the NCTM Illuminations webpage (NCTM, 2016b); and tool suites such as Core Math Tools (NCTM, 2016a). In the current technological landscape, there is a plethora of technologies that teachers and students may use for mathematical purposes, each with its own affordances and constraints for mathematical computations and representations. For example, Tinkerplots allows dynamic manipulation of statistical plots, meaning data points can be grabbed and moved around directly. However, the TI-73 graphing calculator does not have this feature. With the TI-73 graphing calculator plots must be revised by typing in new data, typing in new plot constraints, activating a predefined zoom function, etc. While the TI-73 graphing calculator has easily accessible, predefined plots, Tinkerplots requires users to create their own. These tool-specific and other factors (such as technical knowledge, time and curricular materials used in the classroom) require teachers and teacher educators to carefully consider which tools to use in the classroom.
Research has provided evidence to reinforce the benefits of including technology in the mathematics classroom for student learning (Heid & Blume, 2008; Zbiek et al., 2007). Such benefits include increased student learning of concepts (e.g., Heid, 1988; Ruthven, 1990), students’ enhanced ability to complete tasks not otherwise feasible in the classroom (e.g., Doerr & Zangor, 2000), improved gender equity (e.g., Ruthven, 1990), and the co-emergence of students’ technology-related and symbolic proficiency (Kieran & Drijvers, 2006). Such studies add empirical support to the claim that students can learn as well or better with the use of technologies to support student thinking and learning than with instruction that does not make use of such technologies.

The CCSSM and Principles to Actions: Amplified Potential for Change

The release of the Common Core State Standards for Mathematics (CCSSM) by the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA & CCSSO, 2010) and its widespread adoption across the United States, has provided the nation’s education system with the impetus to help more students learn mathematics at higher levels in order to prepare them for college and career experiences (Gojak, 2013). The Conference Board of the Mathematical Sciences (2013), representing the views of leaders of fifteen organizations, released a unified statement supporting the potential of the CCSSM to help improve curriculum and learning experiences for all American students.

Included in the CCSSM is a set of student-centered\(^1\) statements called the Standards for Mathematical Practice (SMP), which embody ways in which student practitioners of mathematics should increasingly engage with mathematical ideas as they develop in their ________

\(^1\) In this document, the term student-centered refers to notions of student activity that are consistent with the Process Standards (NCTM, 2000) and the Standards for Mathematical Practice (NGA & CCSSO, 2010).
understanding over their K-12 years. The authors of the CCSSM drew explicit links between the SMP and student-centered contemporary reform efforts via the *Process Standards* (NCTM, 2000) and the strands of mathematical proficiency described in the National Research Council (NRC, 2001) report. The fifth SMP, *use appropriate tools strategically*, includes a request for students to learn how to “use technological tools to explore and deepen their understanding of concepts” (NGA & CCSSO, 2010, p. 7). This statement is twofold—students should learn while using tools and technology and that learning should involve understanding of concepts. The third SMP, *construct viable arguments and critique the reasoning of others*, calls for students to make conjectures, justify conclusions, and interact with others in mathematical and logical ways. When considered together, one may view the third and fifth SMPs as supporting social mathematical learning environments where students are encouraged to develop in conceptual understanding and mathematical discourse as they construct viable arguments and critique each other’s reasoning while strategically using appropriate mathematical tools and technologies. Two implications are that students may need ample time to talk and communicate about their mathematical arguments while utilizing different tools during class time and that teachers may need to alter their instruction to permit more student-student interactions in the classroom.

NCTM (2014) continues to promote both the *Process Standards* (NCTM, 2000) and the SMP as ways to ensure mathematical success for all students. The use of technology in mathematics education is viewed as a means to encourage students’ conceptual growth, particularly with the different mathematical representations that are easily-afforded by technology.
Instrumental Approaches to Mathematics Education with Technology

A promising line of research about teachers’ use of technological tools involves instrumental approaches to investigating the use of mathematical technologies in the classroom (e.g., Artigue, 2002; Drijvers, 2003; Guin & Trouche, 1998; Haspekian, 2005; Lagrange, 1999; Trouche, 2004, 2005). Such approaches start with the notion of an artifact, which is a (mathematical) tool by itself and separate from the user’s interaction with the tool (Guin & Trouche, 1998; Verillon & Rabardel, 1995). Instrumental approaches center on the construct of instrumental genesis—the iterative process by which the user and the artifact influence each other during activity when the user interacts with the tool (e.g., Guin & Trouche, 1998). Verillon and Rabardel (1995) marked the distinction between an artifact and an instrument as follows:

> It is important to stress the difference between two concepts: the artifact, as a manmade material object, and the instrument, as a psychological construct. The point is that no instrument exists in itself. A machine or a technical system does not immediately constitute a tool for the subject. Even explicitly constructed as a tool, it is not, as such, an instrument for the subject. It becomes so when the subject has been able to appropriate it for himself—has been able to subordinate it as a means to his ends—and, in this respect, has integrated it with his activity. Thus, an instrument results from the establishment, by the subject, of an instrumental relation with an artifact, whether material or not, whether produced by others or by himself. (pp. 84-85)

In other words, a technological tool can be viewed as an artifact or as an instrument, depending on whether the user’s thinking and mental schemes regarding the use of the tool are considered. Since an instrument is viewed as a psychological construct, the process of instrumental genesis occurs when the user develops or refines mental schemes for using a tool with their activity.

Whereas instrumental genesis concerns the learners’ experiences, the construct of instrumental orchestration can be viewed as the teacher’s external steering of students’ learning experiences related to the processes of instrumental genesis (Trouche, 2004, p. 296). Although
there is some research specifically related to this construct (e.g., Cayton, 2012; Drijvers, Doorman, Boon, Reid, & Gravemiejer, 2010; Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013; Erfjord, 2011; Kratky, 2013), more investigation is needed to further explore the nature of instrumental orchestrations within teachers’ practice of mathematics instruction. Drijvers and colleagues (2013) pointed to the need for more attention to the interactions within orchestrations.

As for the instrumental orchestration model, it provided a useful lens to identify and describe the observed orchestrations and teaching practices in the videotaped lessons. We admit, however, that this identification still has a somewhat superficial character; a better focus on the quality of interactions within orchestrations, which might be considered as part of the didactical performance, is needed. (p. 12)

In the literature on instrumental orchestrations, some researchers have suggested categorical differences concerning the teacher’s roles and actions related to the different types of instrumental orchestrations (Cayton, 2012; Drijvers et al., 2010; Drijvers et al., 2013; Erfjord, 2011). One theme relates to common pedagogical moves, which are verbal and nonverbal actions made by the teacher to guide, shape or influence students’ mathematical activity. Examples of pedagogical moves include initiation-reply-evaluation (IRE; Mehan, 1979) and the teacher’s use of closed or open questions during instrumental orchestrations. Some studies on instrumental orchestrations have also started to establish links to teacher discourse moves (Herbel-Eisenmann, Steele, & Cirillo, 2013) such as revoicing (Cayton, 2012; Drijvers et al., 2010) and wait time and probing (Cayton, 2012). However, some scholars (e.g., Forman & Ansell, 2002; Cazden, 2001; Herbel-Eisenmann et al., 2013) raised the issue that common pedagogical moves (such as the IRE sequence) seem more teacher-centered and may draw instruction away from the reform vision of NCTM (1989, 2000, 2014) when used too frequently. In light of this issue, there is a need for research to investigate how teachers’ instrumental orchestrations may support the
student-centered vision embodied in contemporary reform efforts. One way to address this need is to explore the pedagogical moves that teachers use during instrumental orchestrations.

**Preservice Teacher Education**

One significant hurdle to achieving success with contemporary reform efforts that promote mathematical practices, mathematical processes, and strands of mathematical proficiency, is the gap between ways in which preservice teachers learned mathematics and the instructional modes suggested by contemporary reform. Although there have been substantial efforts to promote both conceptually oriented activities and students’ use of technology as part of the process of doing mathematics, most U.S. students (including those who enter training for education) learned mathematics in classrooms where their teachers focused heavily on the practice of prescribed mathematical skills (Stigler & Hiebert, 1999). As a result, preservice teachers are often faced with reconceptualizing the practice of mathematics instruction during their teacher education coursework. Although many university and college programs incorporate recent reform efforts and the use of technology, not all are designed to prepare preservice teachers to learn how they might support their students’ use of technology while engaged in mathematical activity (Lederman & Niess, 2000; Russell, Bebell, O’Dwyer, & O’Connor, 2003). Investigation into technology-enhanced, reform-oriented teacher preparation programs is needed to inform the discussion of how to better prepare future educators to implement mathematics instruction that is compatible with the tenets of recent reform efforts.

**Research Questions**

Despite growth in research on technology in school mathematics, not enough is known about the relationship between the use of technological tools and teachers’ practice of
mathematics instruction (see Doerr, Arleback, & O’Neil, 2012; Zbiek et al., 2007; Zbiek & Hollebrands, 2008). The construct of instrumental orchestration provides one viable lens for studies in this domain (Zbiek et al., 2007), yet more research is needed to unpack the characteristics of teachers’ instrumental orchestrations (Drijvers et al., 2013). Studies framed in response to contemporary reform efforts may reveal characteristics of instrumental orchestrations that can be labeled, categorized, and used in the mathematics classroom to support deep and rich student engagement in mathematical concepts and mathematical practices. In an effort to illuminate some of the characteristics of teachers’ instrumental orchestrations, this study is guided by the following research questions:

1. What types of instrumental orchestrations does one instructor use in a technological-tools-enhanced, reform-oriented statistics course for pre-service elementary teachers?

2. What types of technology-related pedagogical moves does the instructor implement during her didactical performances in the course?

Given the framing of the study around the construct of instrumental orchestrations, the first research question is necessary to provide a backdrop within which the instructor’s pedagogical moves may be situated, since different instrumental orchestrations might naturally correlate with different types of pedagogical moves. The second research question focuses on pedagogical moves an instructor makes to influence the students’ interactions with the technology during the instructor’s enactment of instrumental orchestrations. The second research question stands to help illuminate different pedagogical moves that may help to support student-centered instruction or that may shift the focus of instruction from more teacher-centered towards more student-centered, or vice versa. Lastly, this study is framed to help stakeholders in
the field consider specific pedagogical moves that an instructor or teacher could make to promote
the processes of instrumental genesis between the students and the technology.
CHAPTER 2

REVIEW OF THEORY AND RESEARCH

This chapter provides a review of literature that is framed within the context of contemporary reform efforts to transform mathematics instruction. First, mathematical tasks are discussed, since the reform-seeking teacher needs to consider tasks that leverage the affordances of artifacts. This important step occurs during a teacher’s planning phase, before instruction. Next, the review of literature transitions to a chronological discussion of instrumental approaches to technology use during mathematics instruction, tracing a path through the artifact, instrumental genesis, the teacher and the artifact, and the teacher and instrumental orchestrations. The chapter closes with consideration of high-leverage pedagogical moves that can be used to promote conceptual aspects of students’ instrumental genesis. As characteristics of teachers’ didactical performance (their enactment of instrumental orchestrations), these pedagogical moves become the keystone to the design of the study that is discussed in Chapter 3. Figure 2.1 shows a visual representation for the structure of this chapter and is referenced frequently hereafter.
Mathematical Tasks

As mentioned in Chapter 1, those promoting the use of technology in mathematics education argued for its value in helping to transform learning experiences for students (Heid, 1997; NCTM 1989, 2000). Figure 2.1 reflects this notion and shows mathematical tasks at the heart of the model. Technology use does not occur in a vacuum, independent of particular mathematical tasks. Rather, tool use is in service to the practice of mathematics, which can include rich and challenging mathematics concepts and problems; problem solving is at the center of mathematical practice where students utilize tools and technology to support their learning (NCTM, 2000; NGA & CCSSO, 2010; Trouche, 2004; Vergnaud, 1996).

Technology use, along with other notions promoted in recent reform efforts, has the potential to be less powerful for users in school mathematics classrooms that place most of the
emphasis on direct use of prescribed algorithms and pencil and paper skills. Hiebert (2003) argued that the typical mathematics instructional model in the United States, focused on the practice of paper and pencil procedures, has helped students to develop in procedural knowledge with some understanding of concepts, but that students’ skills and concepts are often unconnected when they learn mathematics this way. Tasks used in these classrooms tend to focus on procedures, so there is little need for technology at all, other than perhaps to check one’s work. Theoretically and practically, it makes more sense to use technology in classrooms that favor multiple representations, multiple solution strategies, and learner engagement in a community of practice where social and sociomathematical norms (Yackel & Cobb, 1996) are explicitly negotiated to support calls for reform. A primary reason for this is that such environments encourage students to explicitly develop in their conceptual understandings and to make connections between mathematical concepts and procedures that one can use during mathematical activity (NCTM, 1991).

Reporting results from a constructivist teaching experiment, Simon (1995) advanced the notion of hypothetical learning trajectory (HLT), which relates to both the teacher’s planning efforts to select or design good mathematical tasks and the teacher’s “responsibility to be sensitive and responsive to the mathematical thinking of students” (p. 114). This notion of hypothetical learning trajectory is leveraged in some of the work in the area of instrumental orchestrations, as evidenced in the research by Drijvers and colleagues (2010).

Since mathematical technologies provide instant access to complex calculations and multiple representations of mathematical objects, many see these tools as opening new doors for student mathematical thinking and learning (e.g., Guin & Trouche, 1998; Heid, 1988; O’Callaghan, 1998; Ruthven, 1998; Trouche, 2004; Trouche & Drijvers, 2010). In the United
States, research has shown that mathematical technologies afford conceptually oriented tasks that place pencil and paper skills in the background or delay focus on such skills (e.g., Doerr & Zangor, 2000; Heid, 1988; O’Callaghan, 1998). As an example in algebra and functions, Kieran (2007) argued that the available tools have helped to advance functions-focused approaches to the teaching of algebra, which draw heavily on investigating and connecting the different representations of functions (e.g., numerical, tabular, graphical, and symbolic). This is mathematically important, given that the notion of function can be a unifying idea in mathematics, particularly when considering representations of mathematical ideas (Dick, 2011).

Recent work has also focused on the related area of representational fluency (e.g., Schwartz, 1995; Yerushalmy, 1997), arguing that technology supports students’ learning about and comparing of multiple representations of mathematical concepts. The assumption is that the multiple representations offer learners many access points to engage in mathematical activity and multiple connection points for constructing their knowledge. Given that mathematical technologies afford multiple representations and multiple command options, they also enable teachers to implement and support tasks of moderate or high cognitive demand (Stein et al., 1996). For example, tasks such as those suggested by Doorman and Drijvers (2011), Guin and Trouche (1998), and Lo and Kratky (2012) provided examples of open-ended and/or challenging tasks that target the affordances and constraints of mathematical technologies. There is no predefined path that students must take to solve such tasks. Hence, through multiple representations, students may construct their own path to use their own understandings and build an argument that leads to a solution to the task. In short, teachers should leverage mathematical tasks that promote students’ use of technology as learning tools to generate multiple representations and consider connections between them.
Following the metaphor of a jazz orchestra, the mathematical concepts and tasks posed to the students comprise the mathematical, musical scores (Drijvers & Trouche, 2008). Just as jazz musicians learn about particular songs and scores of music, they also adapt and insert themselves into the music being played via improvisation. The music is not static or prescribed, but comes to life with the influence of the players in the orchestra. In Figure 2.1, the students are the musicians and the teacher is the conductor in such an “orchestra.” This musician-focused and improvisational perspective related to a jazz orchestra parallels the theoretical views of radical and social constructivism (e.g., Confrey, 1995; Noddings, 1990; Von Glasersfeld, 1990, 1995), where learners construct their own individual understandings while engaging in social constructive processes. Thus, studies of instrumental orchestrations should take into account the nature of the tasks that the teacher selects and uses in the technologically enhanced mathematics classroom, as the nature and enactment of the tasks themselves may promote more student-centered or teacher-centered views of mathematical activity.

In the section that follows, the theoretical construct of artifact is discussed. The artifact becomes a critical element to both instrumental genesis and instrumental orchestrations, as shown in Figure 2.1. From this point on, in order to draw explicit focus to the theoretical components that set the foundation for the current study, the term artifact will be used throughout to communicate about mathematical technologies that are used as learning tools, even in cases when the referenced authors used different terminology.

**Artifacts**

The term *artifact*, as appropriated from the work of Verillon and Rabardel (1995), can be any physical, electronic, or symbolic tool or technology that influences the activity and thinking of the user. As often used in mathematics education literature, the term artifact may refer
specifically to a computer algebra system (e.g., Drijvers, 2003; Guin & Trouche, 1998), graphing calculators (e.g., Artigue, 2002; Kratky, 2013), dynamic geometry (e.g., Hollebrands, Laborde, & Straßer, 2008; Cayton, 2012), spreadsheet software (e.g., Haspekian, 2005), new forms of smart handheld devices (Trouche & Drijvers, 2010), or other software and applications for electronic technologies. Additionally, the notion of artifact relates to what Dick and Hollebrands (2011) called mathematical action technologies, which are technologies “that can perform mathematical tasks and/or respond to the user’s actions in mathematically defined ways” (p. xii). When the term artifact is used in this paper, the notion of mathematical action technology is implied. Adapting the argument of Drijvers and Trouche (2008), one can conceptualize an artifact in the following way: Artifact = Instrument – Scheme, which establishes that an artifact is considered separate from the user’s cognition and activity.

Taken generally, artifacts function as tools that may mediate, or influence, human activity and learning (Vygotsky, 1978). Zbiek et al. (2007) marked the connection to Vygotsky’s work when they stated, “We consider computer tools to be one of Vygotsky’s ‘psychological tools’ or ‘signs,’ and hence a means of mediation” (p. 1193). In other words, the learning that occurs with computer tools is different from learning without those tools due to such influence of the tools on the learner (Pea, 1985). Numerous authors use the term mediation in this sense as a reference to artifacts in the classroom (e.g., Aldon, 2010; Bussi & Mariotti, 2008; Lagrange, 1999; Ruthven, 2002). In Figure 2.1, the mediating influence of the artifact is represented by the arrow pointing from the artifact towards the user (such as the student or the teacher).

Although artifacts may stand to transform the learning opportunities in the classroom, they are not without limitations. In their work with computer algebra systems, Guin and Trouche (1998) reported three types of constraints with artifacts: internal, command, and organizational.
Internal constraints refer to the ability of artifacts to manage, represent, and process mathematical quantities and computations prior to displaying the results to the user. Tinkerplots, a computer program that can be used for studying statistics, includes the program code that manages and defines how the artifact generates and displays different plots of data, as well as its ability to handle dynamic alterations to data and the plots simultaneously. The TI-73 calculator, in comparison, does not include dynamic capabilities, which reflects a different set of internal constraints. Command constraints refer to the syntax or language structure that permits the user to communicate with the artifact. In computer algebra systems, this includes commands to solve an equation and compute the derivative of a function. In Tinkerplots and the TI-73, the user interacts with the artifact by selecting menu options and there is little command-level interaction between the user and these artifacts. The third factor, organization constraints, refers to the accessibility and structure of the artifact beyond that of a command-line interface. In the TI-73, this includes the Texas Instruments list editor and statistical plots menu, which permit and limit the user’s ability to enter and plot data using predefined plots (Figure 2.2). In Tinkerplots, this includes the many menu and toolbar options (Figure 2.3) for inputting, processing, and displaying data and statistics.
Figure 2.2. Statistical plot options for the TI-73.

Figure 2.3. Plot options for Tinkerplots.
Artifact constraints exist for all artifacts, regardless of their intended audience. Dick (2008) commented on three types of fidelity issues that may arise with artifacts that are designed for use in education: pedagogical, mathematical, and cognitive. First, pedagogical fidelity is the consideration of the level of transparency in the artifact for creating and acting on objects and showing evidence of those actions. A higher transparency level implies greater pedagogical fidelity. When an artifact has high pedagogical fidelity students have more access to information from the artifact, which can then be used to aid in learning concepts. Lower transparency indicates that the artifact acts as a black-box (Buchberger, 1989), hiding some of the processes. In regards to pedagogical fidelity, the dynamic nature of Tinkerplots allows the user to make real-time changes to data and/or plots and the artifact updates the display immediately. Although the TI-73 does not include dynamic features, it does allow the user to define window and plot parameters in a very popular interface. Dick suggested that the organization of menus and options (i.e., organizational constraints) plays a role in pedagogical fidelity, as well. Secondly, mathematical fidelity refers to how mathematically accurate an artifact is in computing and in representing mathematical quantities and displays. Hershkowitz and Kieran (2001) discussed a similar idea, noting that artifacts can produce false representatives, which are representations that fail to represent the key properties of a particular mathematical entity. As one example of a false representative, statistical software may be used to create a boxplot for a categorical dataset that uses numbers as labels for the different categories (e.g., group 1, color 1, etc.). A plot for numerical data, such as a boxplot, would not be useful to represent categorical data. The third type, cognitive fidelity, reflects the level of consistency between the artifact and ways in which the user would perform similar mathematical actions by hand. As Dick remarked, sometimes
Artifacts utilize more sophisticated, precise, or efficient methods than those a user would likely use in paper and pencil work.

The discussion of literature above on artifacts in mathematics highlights the important role that artifacts play in the mathematics classroom. Thus, studies on technology in mathematics education need to attend to the specific configurations, affordances and constraints of the artifacts in the classroom, since different artifacts may influence or mediate classroom activity in different ways.

The construct of artifact has influenced two related bodies of research: semiotic mediation and instrumental genesis. The similarities and distinctions between these two are offered in the sections that follow.

**Artifacts and Semiotic Mediation**

Vygotsky (1978) emphasized the view that *internalization*, or “the internal reconstruction of an external operation” (p. 56), reflects a perspective of higher importance on forces external to one’s mind over the internal workings of the human mind. In other words, there is no internalization without first some external (and hence, social) operation. These external influences are conceived of as *semiotic* processes, which include modes of communication involving the generation and interpretation of signs, such as words, language, gestures, tools, drawings, specialized symbols, and other human-created objects. Implicit in the use of the term mediation is that a person(s) may act as the mediator, something is mediated, someone is subjected to or receives that which is mediated, and that the context surrounding mediation plays a role, as well (Hasan, 2002).

Bussi and Mariotti (2008) extended Vygotskian theory to the specific case of semiotic mediation, which takes an explicitly educational perspective to consider the relationship between
artifact and learners. In their didactical cycle, they conceive of three phases. First, students engage in activities with the artifacts. Next, individual students produce signs, or records of their work and thinking. Then, there is a collective production of signs, whereby students acting in a social context generate records of their social activity related to the use of the artifact. Collective discussions are given prominence in this socially focused theory. From this perspective, “the main objective of a teacher’s action in a mathematical discussion is that of fostering the move towards mathematical signs, taking into account individual contributions and exploiting the semiotic potentialities coming from the use of the particular artifact” (Bussi & Mariotti, 2008, p. 755). From this perspective, the teacher designs and orchestrates mathematical activities with the goal of leading students to develop in their interpretation and generation of mathematical signs; the teacher acts as the mediator and the students are subjected to mediation through their activity with the artifacts. Since theories of semiotic mediation are focused more on the influence of the artifact over the user than vice versa, the mediating role of the artifact over the user’s activity is represented in Figure 2.1 by the arrow pointing from the artifact to the user than the arrow pointing from the user to the artifact. In contrast, theories of instrumental genesis are concerned with both arrows relating the artifact and the user.

Artifact and Instrumental Genesis

While semiotic mediation focuses on the influence of the artifact on learners and the signs that they create, the theory does not consider ways in which the learner may adapt or change the artifact. Theories of instrumental genesis, on the other hand, follow Rabardel’s (2002) assertion that students develop their own “instruments.” In this view, students constantly transform the artifact when they engage in activity with it—a form of mutual mediation between user and artifact takes place (shown by the arrows relating students and artifacts in Figure 2.1).
As a result, a higher status is given to students’ individual and socially generated schemes for working with the artifact, than in theories of mediation. This focus on the bi-directional relationship between artifact and learner is particularly important when considering students’ conceptual development. For example, Artigue (2002) argued for the need for theory to allow an intersection between technical and conceptual work and Drijvers (2003) confirmed that students’ conceptual and technical (or artifact-oriented) development show a close relation to each other. In the section that follows, the construct of instrumental genesis is further elaborated.

**Instrumental Genesis**

When technology is viewed as an artifact to be converted into an instrument (Guin & Trouche, 1998), the students’ cognitive processes are considered vitally important to their learning in a classroom with artifacts. These processes may take on individual or social forms, and constructivist theories consider the ways in which learners build upon their own understandings. As previously noted, an instrument cannot exist without the user’s mental schemes. To this point, Rabardel (2002) drew explicit influence from Piaget’s scheme theory when he discussed schemes that users use and develop when interacting with artifacts.

Any user interacting with an artifact may engage in instrumental genesis. This includes the teacher and the students as individuals or as social groups. Figure 2.1 reflects this via the arrows between the teacher and the artifact; the figure also reflects this via the arrows between the students (in different configurations) and the artifact. Much more research has focused on instrumental genesis between the student and the artifact than on the instrumental genesis between the teacher and the artifact or on the teachers’ actions in promoting students’ instrumental genesis. The research reported in this dissertation is one effort to help fill this gap in the research.
As soon as the user applies knowledge to use an artifact, that artifact becomes an instrument, at least on some level. As discussed above, artifacts pose the ability to mediate the users’ activity. However, the users also mediate the artifact during the activity. These mediations occur in iterative waves as users refine their activity in response to their interaction with the artifact (and now instrument). In a general sense, instrumental genesis occurs during these types of user activity; instrumental genesis reflects a relationship that develops between the user and the artifact (Artigue, 2002; Guin & Trouche, 1998; Verillon & Rabardel, 1995). Implicit in this construct is the fact that students need time to interact with, use, reflect on, and learn through the use of such artifacts in the classroom. The vital role of the user’s schemes in instrumental genesis implies that constructivist-compatible forms of instruction (Ravitz et al., 2000) may be particularly helpful to support students’ engagement in instrumental genesis with artifacts.

The following subsections are used to describe three aspects related to instrumental genesis. Specifically, instrumentation and instrumentalization, socially constructed schemes, and members of the “jazz orchestra.” Ideas related to each of these key components of instrumental approaches are reflected in Figure 2.1.

**Instrumentation and Instrumentalization**

The bidirectional relationship of instrumental genesis may be broken down into its constituent parts. Instrumentalization occurs when the user shapes the use of the artifact, and instrumentation occurs when the artifact shapes the user (Guin & Trouche, 1998; Zbiek et al., 2007). In Figure 2.1 above, the arrow pointing from students to the artifacts represent the former, whereas the arrow pointing from the artifacts to the students represents the latter.

Verillon and Rabardel (1995) used the example of a young child learning to use a spoon to elaborate on the relationship between user and artifact. On the one hand, the child may evoke
various mental schemes to grasp and manipulate a spoon (instrumentalization). However, as the child tries to use the spoon as a tool to aid in eating a food such as mashed potatoes, the child learns from the use of the spoon that some actions may work better than others (instrumentation). In this manner, the artifact effectively teaches the child something. Then, taking information gathered through the use of the spoon, the child revises their mental schemes used to govern interaction with the spoon (instrumentalization). In this case, when the child achieves a high level of precision with the use of the spoon for eating, the spoon has become a stable tool for such use in the mind of the child. In fact, older children and adults use a spoon almost without any conscious thought. At that point, the user (whether child or adult) may engage in instrumental genesis to use a spoon for new purposes or to use what they know about spoons to change how they use other “artifacts.”

Guin and Trouche (1998) discussed similar interactions with artifacts and Nijs, Lesaffre, and Leman (2009) described a relationship that develops between musicians and their musical instruments:

[We argue] that a symbiosis between musician and musical instrument results from a growing integration of instrumental and interpretative movements into a coherent whole that is compatible with the body of the musician and with the movement repertoire of daily life. Such integration leads to the transparency of the musical instrument that just like “natural” body parts disappears from consciousness.

The musical instrument has then become part of the stable background of every human experience and no longer inhibits an embodied interaction with the music. It has become a natural extension of the musician, thus allowing a spontaneous corporeal articulation of the music. (p. 132)

Returning to the orchestra metaphor, students can, through the process of instrumental genesis, learn to use artifacts in the service of mathematical activity. Just as the instrument becomes a natural extension of the musician, so many aspects of artifacts become natural extensions of those who engage in mathematical activity.
When thinking of the mental schemes (Vergnaud, 1996) that users may evoke during mathematical activity with the power of technology, Drijvers and Gravemeijer (2005) discussed two different types of utilization schemes, or schemes evoked by one who interacts with an artifact while engaged in activity: usage schemes and instrumented action schemes. Both schemes play a role in instrumental genesis (Drijvers & Gravemeijer, 2005), and marking the differences between these types of schemes helps researchers and teachers to more clearly reflect on potential student needs and ways to help students move forward in their use of artifacts.

*Usage schemes* (Drijvers & Gravemeijer, 2005) are schemes related to the operation of an artifact, such as turning the power on, navigating the artifact, and accessing different features included with the artifact. Drijvers and Gravemeijer (2005) noted that the cut-and-paste scheme, common to many computer applications, can be used in some computer algebra systems. Therefore, usage schemes represent necessary schemes for users to interact with the artifacts. However, such schemes are considered more basic and specific to the artifact and less specific to a particular activity (e.g., representing or computing with particular mathematical objects). In terms of the orchestra metaphor, a musician might evoke usage schemes when preparing a musical instrument prior to playing music with it.

In contrast to usage schemes, *instrumented action schemes* are those that the user evokes or modifies during or after engaging in instrumentation and instrumentalization. In other words, when a student begins to engage in mathematical activity with the use of an artifact, and interacts with the artifact, then that user engages in instrumental genesis and evokes (or refines) instrumented action schemes. When a learner evokes or modifies instrumented action schemes, we may alternatively say that the learner is engaged in instrumented activity. Aspects of instrumented action schemes are observable as the user interacts with the artifact and serve as
objects of investigation, as suggested by Drijvers (2003), Drijvers and Gravemeijer (2005), and Guin and Trouche (1998). However, since schemes reside in the mind of the user, one cannot study instrumented schemes directly, but only through the observable actions and communication of the user.

**Socially Constructed Schemes**

In review, instrumental approaches take the perspective that artifacts mediate the user’s mathematical activity, drawing influence from Vygotsky’s theories (Rabardel, 1995). In addition, these approaches hold that the user evokes cognitive schemes and refines those schemes, drawing from Piaget’s scheme theory (Rabardel, 1995). However, Trouche (2004) argued that instrumental genesis always includes both individual and social aspects, “The schemes thus constructed always have a social dimension (because of the social aspect of each artifact and/or because of the context of the schemes’ elaboration within a community of practice)” (p. 304). Thus, as students engage in collaborative problem-solving activity with the use of artifacts, such collaborations lead to the collective management of instrumental genesis. This is represented in Figure 2.1 by the arrows between the artifact and students within groups or the whole class as a community of practice. With multiple groups of students, each group may engage in social forms of instrumental genesis at different moments during class time. Additionally, the whole class may engage in a collective instrumental genesis. Studies drawing from instrumental approaches should take the individual-social dimension of instrumental genesis into consideration.

**Members of the Orchestra**

Figure 2.1 reflects the jazz orchestra metaphor (Drijvers & Trouche, 2008; Drijvers et al., 2010) and shows that students may work individually or interact with each other and the teacher in small groups as part of the larger classroom community. Students may at times work
individually (the *soloist* perspective—see Drijvers & Trouche, 2008), in pairs or triads, or within groups of other sizes. When students engage in instrumented actions collaboratively, one might view the students in the same small group as one section in the mathematical orchestra at any given moment. In other words, for a particular shared instrumented action scheme, the students are using the same (or very similar) instruments. However, as the students may evoke different schemes for other uses of the mathematical technology, they may not “play” the same instrument for all of the instrumented activity.

The jazz orchestra metaphor (Drijvers & Trouche, 2008) establishes the role that students’ constructive processes play to the mathematical activity (or *melodies*) that they create with their mathematical instruments. As the students engage in instrumental genesis and transform the artifacts into instruments that belong to them as individuals and as a group, the teacher guides the activity and permits the contributions, adaptations, and improvisations of the students in the activity at hand. Framed with specific attention to the students’ own knowledge and experiences, the jazz metaphor shows parallels to student-centered visions for school mathematics that are embodied in contemporary reform efforts.

Researchers drawing from instrumental approaches and using the orchestra metaphor, even those focused on the teacher, should also be mindful of the students in the classroom, their perspectives, the representations they may generate (particularly with the use of the artifacts) and the emergence of their action schemes related to the use of the artifacts. Figure 2.1 shows this aspect of instrumental orchestrations via the arrow that points from the students to the teacher. In other words, the teacher gathers information related to the students’ experiences and understandings, and then the teacher uses that information to customize their instrumental orchestrations to better meet the needs of their students. This is particularly important in
classrooms where teachers are striving to create student-centered learning environments that align with the tenants of contemporary reform efforts.

**Findings from Empirical Research on Instrumental Genesis**

Based on collecting data for a year in a classroom where students studied Calculus with the TI-92 calculator, Guin and Trouche (1998) reported on five work methods that the students displayed during their activity with the artifact. In the *random* work method, students hastily executed actions with the artifact in a guess-and-check sequence without demonstrating much thought beforehand to using the artifact purposefully. In the *mechanical* work method, students executed simple adjustments to the artifact and observed patterns in the results given by the artifact without demonstrating much mathematical thought prior to using the artifact. In the *resourceful* work method, students demonstrated reasoning as they considered all information (e.g., the artifact, paper/pencil work, etc.) and sometimes generated a wide range of solution strategies. In the *rational* work method, students mostly worked with the artifact in a systematic and mathematical manner. Lastly, in the *theoretical* work method, the students relied heavily on mathematical references and reason with analogies and interpretation of the outputs of the artifact. Guin and Trouche (1998) found that students spent less time during instances of the mechanical or rational work method when compared to instances of the theoretical work method. Also, they reported a gap where students with deeper mathematical understandings were more likely to persist with the artifact and adapt to mathematical work with the artifact; students with lower levels of mathematical understanding were more likely to give up and not progress in the ways in which they used the artifacts.

In his dissertation study, Drijvers (2003) studied students’ instrumented activity with the concept of parameter. He found that students’ instrumented techniques showed a close relation
with what he saw regarding students’ conceptual understandings of the concept of parameter. Due to this relationship, Drijvers found that instrumental genesis was necessarily a complex process as students had to simultaneously consider both technical and conceptual aspects related to their work on mathematical tasks. Further, he cited the significant amount of time consumed by helping students learning to use the complex artifact and how he did not observe a comparable level of algebraic insight gained by the students. His point was not that technology should be avoided, but that there are increased demands when integrating technology into mathematics classrooms in order to support both students’ technical and conceptual development.

After a series of teaching experiments designed to further explore aspects of instrumental genesis with computer algebra systems related to high school students’ successful growth, Drijvers and Gravemeijer (2005) unpacked three instrumented action schemes: one for solving parameterized equations, one for substituting expressions, and one for solving systems of equations. In each case, they suggest that the student needs a conceptual point of entrance on which to anchor the scheme, the student must have sufficient technical knowledge (e.g., of options and syntax) to successfully interact with the artifact, and the student must leverage both conceptual and technical understanding when interpreting the results given by the artifact.

The findings from research in the area of instrumental genesis underscore the complexities of artifacts used in the classroom and the corresponding complex interactions between student(s) and the artifacts. This suggests the importance of recognizing and attending to students’ technical and conceptual development in the classroom and the need for teachers to critically attend to the ways in which they structure opportunities—via their instrumental orchestrations—for students to engage in instrumental genesis (Drijvers et al., 2010; Trouche,
2005; Zbiek et al., 2007). One implication is that a teacher might choose to implement some instrumental orchestrations to facilitate students’ technical development, while other instrumental orchestrations might be used to focus on conceptual development or some balance between the two. In all cases, the research indicates that instrumental genesis depends on technical understanding, conceptual understanding and growth.

The Teacher

In Figure 2.1 above, the large arrow from the teacher to the students represents the ways in which the teacher influences students’ mathematical activity with artifacts. The teacher may draw upon her own experiences with using the artifact (the arrows between the teacher and the artifacts) and from her observations of students’ thoughts and actions with the artifacts as they engage in mathematical activity (the arrow pointing from the students back to the teacher). The latter fits with Drijvers and Trouche’s (2008) clarification of the teacher as a conductor of a jazz orchestra, where the teacher may make impromptu changes to a lesson or orchestration in response to the contributions of the students.

In the jazz orchestra metaphor, Drijvers and Trouche (2008) began with notions of instrumental genesis described in the sections above. An individual user, whether student or teacher, engages in instrumentalization when the user shapes the artifact. The user engages in instrumentation when the artifact shapes the user’s activity and development of mathematical concepts. Instrumental genesis is the ongoing interactions between artifact and user as the user grows in knowledge and understanding. When the teacher is viewed as the conductor of the orchestra, emphasis is placed on “a more collective, classroom-oriented teaching view” (Drijvers & Trouche, 2008, p. 376). In this case, individuals and the classroom community as a whole may engage in instrumental genesis, reflecting individual and social aspects of the construct. The
teacher structures the configurations of the artifacts in relation to tasks at different phases of a lesson and the teacher chooses how to exploit those configurations. Although implications for the enhanced role of mathematical discourse (as promoted by contemporary reform in the U.S.) may seem apparent when discussing a more collectively-oriented teaching view, Drijvers and Trouche only mentioned mathematical argumentation arising from one type of instrumental orchestration—one involving the so-called Sherpa student. Drijvers and Trouche say more about teachers functioning within communities of practice than about students functioning within communities of practice.

In the sections that follow, attention is drawn to teachers’ relationships with artifacts, teachers’ roles when artifacts are used in the mathematics classroom, and issues that arise when teachers seek to support students’ use of artifacts in the mathematics classroom. This discussion is used to set the stage for an elaboration on the construct of instrumental orchestration.

**The Teacher and the Artifacts**

As a user of the artifacts, the teacher engages in instrumental genesis. This is reflected in Figure 2.1 by the arrows pointing to and from the teacher and the artifacts. Not surprising, the teacher’s experiences with the artifacts seem to afford and constrain how the teacher supports students’ activity with artifacts. The assumption is that teachers also need to develop in both technical and conceptual ways when engaging in instrumental genesis. In a theoretical sense, this seems to suggest that teachers need to possess specific types of mathematical knowledge in order to successfully support students’ conceptual and technical growth with artifacts.

Mishra and Koehler (2006) provided a conceptual framework for Technological Pedagogical Content Knowledge (TPACK) to represent many different types of mathematical knowledge that teachers might need when implementing technology in educational settings.
Their framework extended Shulman’s (1986) construct of Pedagogical Content Knowledge in order to explicitly include technical forms of knowledge that teachers might need. As suggested by the TPACK label, there are three primary types of teacher knowledge considered:

*Technological Knowledge* specific to the use of technologies for educational purposes,

*Pedagogical Knowledge* relating to teaching in a particular classroom with specific students, and

*Content Knowledge* relating to a particular subject area, such as mathematics. Also extending Shulman’s argument, Mishra and Koehler stated that the overlaps between these types of knowledge reflect greater potential for a teacher to successfully implement and support technology use in the classroom. TPACK, or the overlapping of the three different types of knowledge, is viewed as the foundation for good teaching. Specifically, good teaching involves technology and requires that teachers understand particular pedagogical techniques to leverage technology; learners and difficulties they may face with content and technology; and how technologies represent concepts and can support students’ conceptual growth (Mishra & Koehler, 2006). Figure 2.4 shows the TPACK image (Koehler, 2016).
While commenting on ways in which mathematics teachers interact with and promote technology related to their instructional practice, Zbiek and Hollebrands (2008) elaborated on Beaudin and Bowers’ (1997) description of the five categories of the PURIA (Play, Use, Recommend, Incorporate, Assess) model, which is structured similar to a tiered or leveled model. At the first level, *Play* refers to the teacher’s use of the technology with no clear mathematical purpose. Because of this, one could say that a teacher need not have much TPACK knowledge to Play with an artifact. When a teacher progresses and *Uses* the technology, the teacher utilizes the artifact for one’s own mathematical purposes. For example, a teacher might Use an artifact, such as a spreadsheet application, to compute various statistics and to generate displays of student grades without letting the students interact with the artifact or seeing what the teacher generated with the artifact. The notions of play and use in the PURIA model reflect the
idea that theories of instrumental genesis apply to the teacher, as well as the students (as shown in Figure 2.1). Therefore, the teacher’s own mathematical understanding and activity is influenced by her use of artifacts. As soon as the teacher Recommends that students use technology, the teacher begins to support students’ engagement in instrumental genesis, at least at some level. In many cases, this means that the teacher allows, but does not require the use of, the technology. This sometimes occurs when teachers allow students to use technology to check work that they completed by hand. The teacher may then Incorporate technology into their classroom instruction, which may take different forms and the teacher may implement instruction and instrumental orchestrations that include teacher-centered or student-centered use of the technology (Drijvers et al., 2010; Drijvers et al., 2013). In the final stage of the PURIA model, the teacher places high status on the use of the mathematical technology by Assessing students’ instrumented mathematical activity. Historically, only a small minority of teachers has integrated technology to the level of assessing students when they use the artifacts, in both the U.S. (Zbiek et al., 2007) and France (Trouche, 2003). The PURIA model helps to show the level of commitment teachers have to supporting and promoting the use of artifacts in the classroom.

In their study of three teachers of introductory differential calculus, Kendal and Stacey (1999) identified the construct of teacher privileging—the notion that teachers may promote certain aspects or capabilities of artifacts, while avoiding or downplaying other aspects of the artifacts. They argued that teachers’ personal instructional styles and attitudes influenced student learning. These privileging decisions influenced both students’ access to the different features of the artifacts and their interactions with the artifacts.

Drijvers and colleagues (2013) used the construct of TPACK in relation to instrumental approaches to mathematics instruction. In their study of 12 high school teachers who had prior
experience in teaching with artifacts, they found correlations between evidence of teachers’ TPACK knowledge and the ways in which teachers guided and structured student activity with the use of artifacts. For example, they found that teachers who seemed to have less Pedagogical Knowledge were more likely to focus on technical and mathematical aspects of the artifacts and less on students’ conceptual development in the activities with the artifacts. As a result, Drijvers and colleagues found that TPACK and models related to instrumental approaches in mathematics education could be used together as lenses for considering teacher knowledge and teacher practice.

**Teacher Roles When Technology Is Used**

Zbiek and Hollebrands (2008) reviewed different types of teacher roles with respect to technology that have emerged in the research literature. They presented 11 distinct teacher roles, ranging from *allocator* of time and *task setter* to *manager* and *evaluator*. In short, the teacher may: allocate time for classroom activities; introduce and facilitate new problems for student work; serve as a collaborator with the students; assist and guide student work when they need help; assesses student progress; demonstrate ideas relevant to the current problems; manage classroom activity and time; implement activities; serve as a source of information; choose examples and strategies for public display; and serve as a technical assistant or expert with the artifacts. These 11 roles are specific to ways that teachers structure and support student work when students have access to artifacts in the classroom. In the jazz orchestra metaphor, Drijvers and Trouche (2008) directly connected these teacher roles to the construct of instrumental orchestration, as the construct considers the specific ways in which a teacher may configure and implement measures to support student learning with the use of artifacts.
Drijvers et al. (2010) demonstrated that different types of instrumental orchestrations favor different teacher roles. For example, in their study, Teacher A reported her preference to encourage student collaborations during mathematical activities and implemented many student-centered instrumental orchestrations. In contrast, Drijvers and colleagues reported that Teacher B was more focused on teacher-centered lesson structures and implemented instrumental orchestrations that allowed her to more directly control the flow of the activity (more teacher-centered instrumental orchestrations). As a result, these two teachers took on different roles during their instrumental orchestrations. Teacher A took on more facilitator-oriented roles, whereas Teacher B took on more authoritarian-oriented roles. The results in this study suggested a correlation between the type of instruction and the types of instrumental orchestrations used, in relation to the teacher-centered, student-centered spectrum.

Teacher Issues With Implementing Technology

Research has also documented issues that arise and must be overcome when teachers decide to use artifacts in the classroom. For example, Zbiek and Hollebrands (2008) reviewed literature that discusses teacher concerns when using technology. First, teachers have personal concerns regarding how technology will impact their teaching practice. Some fear that the use of technology will yield negative results in the classroom for their students’ learning. Secondly, teachers have classroom management concerns about issues that can arise during problem-solving activities that involve the use of artifacts. Lastly, Zbiek and Hollebrands discussed technology concerns that teachers often have, because of their lack of technical understanding and confidence. This point is reaffirmed in the TPACK model (Mishra & Koehler, 2006). In

\[\text{An elaboration on the different types of instrumental orchestrations follows in the section that is labeled Instrumental Orchestrations.}\]
other words, teachers need sufficient personal knowledge of the particular artifacts used in the class (Technological Knowledge), knowledge of methods to teach mathematics (Pedagogical Knowledge), knowledge of mathematics (Content Knowledge), and knowledge of the intersections of these three domains of knowledge. A lack of teacher knowledge in any area of TPACK may correlate with related challenges that a teacher may face while implementing technology in the classroom. Further, teachers with higher levels of TPACK knowledge might be better prepared to implement technology successfully in student-centered ways (Drijvers et al., 2013).

**Instrumental Orchestrations**

As the conductor of students’ mathematical work with artifacts, the teacher has the power to influence which artifacts the students may access and how they may use those artifacts. The notion of instrumental orchestration is represented by the wide arrow pointing from the teacher to the students and from the narrow arrow pointing from the students to the teacher in Figure 2.1. Teachers’ instrumental orchestrations are their efforts to steer students’ instrumental genesis and teachers may adapt their instrumental orchestrations in response to feedback that they receive from students, including feedback received through formative assessments of student thinking. As with the phenomenon of instrumental genesis, the ongoing implementation of instrumental orchestrations is viewed as an iterative and evolving process. The teacher uses pedagogical moves when she interacts with the students, which is encapsulated in the arrow pointing from the teacher to the students—pedagogical moves are one component of teachers’ instrumental orchestrations. The relational arrows in Figure 2.1 represent feedback loops since the teacher continually gathers information from the students and their use of the artifact during activity that centers on mathematical tasks and uses that information to guide instruction. In the metaphor of
the jazz orchestra, the relationships between the teacher and the artifact, the students and the artifact, and the teacher and the students are analogous to the conductor’s ongoing efforts to guide and shape the musicians’ use of their instruments as the orchestra plays and creates music during practice and performance. As Zbiek and Hollebrands (2008) noted, the teacher may act as the conductor in a variety of forms: whole-class, small-group, or individual settings. In the sections that follow, attention is given to recent work that develops typologies of instrumental orchestrations, which may occur in whole-class or small-group settings. The reason for focusing on instrumental orchestrations in these settings, rather than with individual students, is that the students are considered members of an orchestra and not one-man bands (Drijvers & Trouche, 2008).

Trouche (2004) introduced the notion of instrumental orchestration, citing two defining components: didactic configuration and exploitation modes. To plan a *didactic configuration*, the teacher selects the artifacts that students will use and arranges them (or plans for their arrangement) in the classroom. This includes any artifacts that students will use on their own or in groups, any artifacts that the teacher will use, and any artifacts or presentation technologies (such as a projector) that may be used during the activity. While designing the didactic configuration, the teacher also plans for the arrangement of students (in groups, as individuals, or in a whole-class setting). To plan an *exploitation mode*, the teacher decides how to leverage the didactic configuration with respect to her goals for the orchestration. She may demonstrate a particular artifact technique, establish a link between work done with the artifact and work done with paper and pencil, have a student present his work with the artifact, or initiate a discussion related to a representation generated by the artifact. Each of these examples represents different exploitation modes that the teacher may use.
Drijvers et al. (2010) extended Trouche’s (2004) defining components of instrumental orchestrations in the following two ways (see Figure 2.5). First, they added to the exploitation mode specific attention to the importance of the mathematical task (which is emphasized in Figure 2.1) and to Simon’s (1995) notion of hypothetical learning trajectory. Although they did not elaborate on the notion of hypothetical learning trajectory, the authors noted that the teacher plans the activity and anticipates how the students will interact with the artifacts while engaging in the tasks selected by the teacher. Secondly, Drijvers and colleagues amended Trouche’s description of instrumental orchestration to include a didactical performance, which includes the in-the-moment actions that the teacher makes in the midst of the instrumental orchestration. While the teacher may plan for and design the didactical configuration and exploitation modes for specific instrumental orchestrations, the teacher cannot fully plan her didactical performance, since it includes actions that a teacher makes in response to the students and activity within a particular lesson. More specifically, a teacher cannot completely anticipate students’ experiences, struggles, and successes when using artifacts to solve mathematical problems. Thus, a teacher makes in-the-moment decisions during a lesson, which make up her didactical performance. Both Cayton (2012) and Drijvers et al. noted how teachers often used multiple pedagogical moves during their didactical performances, which suggests that pedagogical moves are important elements of teachers’ didactical performances. The term didactical performance establishes the specific connection to a jazz orchestra, where the teacher/conductor may draw upon feedback from the students/musicians in the moment during an instrumental orchestration (Drijvers & Trouche, 2008).

3 Drijvers et al. (2010) changed from Trouche’s (2004) use of “didactic” to “didactical.”
As part of a project on a technological learning environment focused on the concept of function in eighth grade classrooms, Drijvers et al. (2010) and Drijvers and colleagues (2013) unveiled different types of instrumental orchestrations. Figure 2.6 shows Drijvers and colleagues’ (2013) overview of these different types of instrumental orchestrations, which was a modification of categories described by Drijvers et al. (2010). Descriptions of these instrumental orchestrations are provided later in Figure 2.7 and Figure 2.8. Although the researchers’ typology was not exhaustive, their categories have served as a point of reference in the literature for creating teacher profiles related to the types of instrumental orchestrations that teachers use (Cayton, 2012; Erfjord, 2011; Kratky, 2013). In the sections that follow, Drivers and colleagues’ (2013) types of instrumental orchestrations are described in whole-class and small-group settings, respectively.
Figure 2.6. Overview of whole-class and individual orchestrations.

**Instrumental Orchestrations in Whole-Class Settings**

In this section, results are presented chronologically from empirical studies on teachers’ instrumental orchestrations within the whole-class setting. One key theme across the studies is the distinction between teacher-centered and student-centered types of instrumental orchestrations as part of teachers’ instruction, which relates to the distinction between traditional-transmission and constructivist-compatible models of instruction (Ravitz et al., 2000).

As an important precursor to many whole-class instrumental orchestrations, Drijvers (2011) noted the teaching practice of the *Work-and-walk-by*. Essentially, the teacher may move about the classroom to observe and monitor students’ instrumented activity as the students engage in mathematical problem solving. This action allows the teacher to gather information on students’ approaches to the task; possible technical and mathematical hurdles that may interfere with the students’ ability to progress in their problem-solving efforts; and to observe particularly
interesting, unique, or advanced instrumented activity. Thus, the teacher may use the *Work-and-walk-by* teaching strategy to inform her didactical performance during her enactment of an instrumental orchestration.

Guin and Trouche (2002) presented an early example of a whole-class instrumental orchestration that makes use of a so-called *Sherpa*\(^4\) student. As a context for this instrumental orchestration, the students engage in instrumented activity that may involve both the artifact and work completed via paper and pencil. A particularly knowledgeable student (the Sherpa) operates the artifact in a publicly viewable manner (i.e., so that the screen is projected). The teacher interacts directly with the Sherpa, prompting the student to perform certain operations. The goal, according to Guin and Trouche, is to support collective management of instrumental genesis, as the rest of the students are encouraged to follow the lead of the student. Guin and Trouche remarked that, in this case, “The teacher thus fulfills the functions of an orchestra conductor rather than a one-man band.” They selected the term Sherpa to reflect a student who acts as a guide and carries the cognitive load during her presentation with artifacts (Trouche, 2004). The notion of Sherpa student was adopted into the work of others, such as Drijvers et al. (2010, 2013; see Figure 2.6).

Drijvers and colleagues (2010) observed 29 eighth grade Belgian and nine Dutch classrooms where teachers implemented the use of a customized java applet for investigations in the concept of function. The research team categorized six different instrumental orchestrations that they observed in terms of the observed didactical configurations and exploitation modes. Figure 2.7 provides summaries for the six types of instrumental orchestrations categorized by

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\(^4\) The notion of *Sherpa*, from the literature, is relabeled as *First-chair* in Chapter 3 as part of the conceptual framework. This was done to further strengthen the jazz orchestra metaphor.
Drijvers and colleagues. The “TC” indicates that a particular instrumental orchestration is more teacher-centered, and the “SC” indicates that a particular instrumental orchestration is more student-centered, as described by Drijvers and colleagues in relation to the respective didactical configurations and exploitation modes. In other words, some orchestration types are set up to be more teacher-centered, whereas others are set up to be more student-centered.

<table>
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<tr>
<th>Name</th>
<th>Description</th>
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<tbody>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>The teacher projects a screen image from the artifact and invites students to share their thinking and collaborate about what they see communicated on the screen. The image may come from the students’ activity and the teacher may make specific moves to encourage the students to make connections, raise questions, and build on their thinking as a group.</td>
</tr>
<tr>
<td>Sherpa-at-work (SC)</td>
<td>The teacher has a student present project, and explain his/her own work with the artifact. The other students follow the presentation and may ask questions or contribute to the presentation/discussion. The teacher may give directions for the Sherpa student to follow as the student presents.</td>
</tr>
<tr>
<td>Spot-and-show (SC)</td>
<td>The teacher accesses artifacts of student work during preparation of the lesson; She sees something novel or particularly relevant that a student did with the artifact and selects that work for use in class. The teacher explicitly shows an image of the artifact from student work as part of a discussion. The teacher has the students explain and discuss their artifact use and their peers’ artifact use related to the images shown.</td>
</tr>
<tr>
<td>Explain-the-screen (TC)</td>
<td>The teacher projects a screen image from the artifact in use. Rather than starting a conversation, the teacher leads the explanation about what is communicated on the screen.</td>
</tr>
<tr>
<td>Link-screen-board (TC)</td>
<td>Teacher and students have access to the artifacts and the teacher may use a projector. The teacher shows the screen and boardwork at the same time. The teacher leads and may dominate discussion to establish the connections between what the artifact generates and what one can do mathematically by hand on the board. The teacher may display student work, but she still does most of the talking.</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>The teacher shows how to do something. This is one way in which a teacher could lecture directly with the artifacts. The teacher and students may have access to the artifacts, but the teacher leads and projects her screen.</td>
</tr>
</tbody>
</table>

Figure 2.7. Whole-class instrumental orchestration types identified by Drijvers et al. (2010).

In one overarching finding, Drijvers and colleagues (2010) drew reference to the teacher-centered initiation-reply-evaluation interaction pattern (IRE; Mehan, 1979) that stands as a means to distinguish between teacher-centered and student-centered instrumental orchestrations:
A reflection on this set of six instrumental orchestration types led to the distinction of a teacher-student dimension. In the Technical-demo, Explain-the-screen and Link-screen-board instrumental orchestrations the teacher dominates the communication. Student input is restricted and the teacher guides the interactions in a direct manner. These orchestrations can be seen as teacher-centred \([sic]\). 

In the Discuss-the-screen, Spot-and-show and Sherpa-at-work instrumental orchestrations, students have the opportunity to react and have more input. Even though the teacher manages the orchestration, there is more interaction and students have more voice than in the first three orchestration types. These can therefore be seen as student-centred \([sic]\) orchestrations. (p. 220)

The authors suggested a spectrum with respect to teacher and student-centered instruction when they stated, “. . . the distinction of a teacher-student dimension.” As previously stated, there is a link between (1) teacher-centered and student-centered visions for mathematics education, and (2) traditional-transmission and constructivist-compatible instruction.

Although their typology drew explicit attention to specific instrumental orchestrations, Drijvers and colleagues (2010) also noted that instrumental orchestrations are not isolated, rather they are situated within the broader classroom activity, such as students working on tasks individually or in pairs—activities which often occur without the teacher’s immediate presence. They referred to these different phases of instruction as constituting orchestral \(sequences\).

Drijvers et al. (2013; see Figure 2.6) extended Drijvers and colleagues’ (2010; see Figure 2.7) typology of instrumental orchestrations to include two additional whole-class orchestrations: Guide-and-explain and Board-instruction. The researchers described the Guide-and-explain instrumental orchestration as a middle ground between the teacher-centered Technical-demo instrumental orchestration and the student-centered Discuss-the-screen instrumental orchestration type, since the teachers were observed incorporating some student feedback during these instrumental orchestrations:

On the one hand, a somewhat closed explanation based on what is on the screen is provided by the teacher. On the other hand, there are some, often closed, questions for
students, but this interaction is so limited and guided that it cannot be considered as an open discussion. (p. 999)

This finding was significant for two reasons. First, this new instrumental orchestration type supports the argument that the distinctions between teacher-centered and student-centered instrumental orchestrations may constitute a spectrum, rather than a dichotomy. Secondly, and related to the first point, the researchers argued that the teachers in the study shifted their emphasis from the *Technical-demo* instrumental orchestration to the *Guide-and-explain* instrumental orchestration type as a result of their collaborations with each other. The other newly added orchestration type, the *Board-instruction* orchestration, was used to refer to the traditional didactical configuration of the teacher at the front of the classroom, without any artifacts in use or referenced at that moment. The teacher typically exploits this didactical configuration by lecturing, but there are different degrees of possible student involvement and interaction (Drijvers et al., 2013). Drijvers and his colleagues noted that this orchestration was teacher-centered and there was no evidence of any goal to support instrumental genesis, but they included this as an orchestration type because they “felt the need to also include the regular teaching in our analysis” (p. 999). However, studies focusing on orchestrations that provide opportunities for students to engage in instrumental genesis might exclude instances of *Board-instruction* from the analysis.

Drawing from Drijvers et al. (2010), Erfjord (2011) reported similar results from a case study involving three eighth grade mathematics teachers at two different schools, “Austpark” and “Fjellet.” Specifically, Erfjord investigated the teachers’ initial instrumental orchestrations of the dynamic geometry software tool *Cabri* (www.cabri.com). The participating teachers were

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5 For this reason, the *Board-instruction* orchestration type was excluded from the data analysis for this dissertation.
involved with professional development projects designed to promote a focus on inquiry and student-centered instruction in the classroom. Also, all three teachers incorporated Cabri into their classrooms as an extension to the constructions that the students had completed on paper with a compass. Focused on the opportunities afforded for student engagement in mathematical activity related to geometric constructions, Erfjord noted the distinctions between the two teachers at the “Austpark” school and the teacher from the “Fjellet” school in the study. Erfjord (2011) stated that the teachers from Austpark did not seem to strive for inquiry or student-centered instrumental orchestrations in their instruction related to the students’ use of the artifacts or to the students’ learning of mathematics. The teacher from Fjellet, however, seemed to support some level of student inquiry with relation to the students’ use of Cabri.

In a study demonstrating that high cognitive demand tasks and rich mathematical discussions can occur in student-centered, technology-rich classrooms, Cayton (2012) investigated three high school mathematics teachers’ implementation of classroom discourse in the midst of student activity with Geometer’s Sketchpad (http://www.dynamicgeometry.com/). The three participating teachers were engaged in a larger professional development project and had training related to cognitively demanding tasks. These teachers implemented pre-constructed geometry mathematical tasks with high cognitive demand during the days when the researcher observed and collected data. Cayton documented the instrumental orchestrations that took place in the whole-class setting. She found that one teacher, Mrs. Lewis, favored the use of the Discuss-the-screen instrumental orchestration and was able to maintain cognitive demand of the tasks used during the study. Mrs. Lewis modeled student-centered instruction when she used questions to probe student thinking and support student exploration with the artifact. Cayton also reported an instance where Mrs. Lewis utilized questioning strategies (as part of her didactical
performance during instrumental orchestrations) and increased the cognitive demand of the slope-intercept task observed in the study.

The other two teachers in Cayton’s (2012) study, Mrs. Patterson and Mr. Phelps, frequently implemented the Discuss-the-screen and the Explain-the-screen instrumental orchestration types. The researcher found that, in connection with the teachers’ use of teacher-centered Explain-the-screen instrumental orchestrations, cognitive demand decreased during those activities. This matched the teacher-centered description that Drijvers and colleagues (2010) gave for the Explain-the-screen instrumental orchestration. When Mrs. Patterson and Mr. Phelps implemented the student-centered Discuss-the-screen instrumental orchestrations, cognitive demand remained high for the associated tasks. However, Cayton identified a trend where Mrs. Patterson started to implement the student-centered Discuss-the-screen instrumental orchestration, but struggled with supporting student engagement in the discussions. When this occurred, Mrs. Patterson transitioned into the Explain-the-screen instrumental orchestration and cognitive demand for the tasks decreased.

Umameh (2012) reported results from a graduate thesis study in a year 9 classroom in the United Kingdom. The participating teacher conducted instrumental orchestrations involving GeoGebra (https://web.geogebra.org/) and an interactive whiteboard. The researcher reported that the teacher used both teacher-centered and student-centered instrumental orchestrations. Lessons followed a consistent four-phase structure, where the teacher (1) led a whole-class introduction to the lesson and artifact use, similar to Drijvers’ et al. (2010) Technical-demo instrumental orchestration type; (2) the students engaged in a phase of small-group practice with the artifacts on geometric tasks similar to those demonstrated by the teacher (no instrumental orchestrations described during this phase); (3) the teacher implemented a whole-class
instrumental orchestration related to a completed task with the artifact, similar to the *Discuss-the-screen* instrumental orchestration type; (4) the teacher had students go back into small groups and the teacher evaluated the students’ use of the artifacts for the tasks at hand (no instrumental orchestrations described during this phase). The teacher’s four-phase lesson structure paralleled common models for direct instruction. However, the teacher did permit time for students to engage collaboratively with the artifacts, during the *Discuss-the-screen* instrumental orchestrations, which allowed for the collective management of instrumental genesis to occur.

The results reported from empirical studies on whole-class instrumental orchestrations have supported Drijvers’ and colleagues’ (2010) notion of the teacher-centered, student-centered dimension. Implicit in these results is that the teacher-centered instrumental orchestrations tend to favor technical aspects of instrumental genesis (such as being able to follow and understand a sequence of commands with an artifact), and the student-centered instrumental orchestrations tend to favor the collective management of conceptual aspects of instrumental genesis (due to the reliance on students sharing their instrumented activity and the opportunities for students to collaborate in problem-solving efforts with the artifacts). This relates to the discussion of constructivist-compatible instruction since constructivist-compatible instruction requires that the teacher incorporates more of the students’ interests, experiences and understandings into their instruction than traditional-transmission instruction (Ravitz et al., 2000, p. 4).

**Instrumental Orchestrations in Small-Group Settings**

In the previous section, instrumental orchestrations with students in whole class settings were described. This section attends to instrumental orchestrations that teachers conducted when students were configured in small-group or individual settings. As with the whole-class
instrumental orchestrations, attention is made to the teacher-centered, student-centered dimension.

Although Drijvers (2011) described the Work-and-walk-by action as related to whole-class instrumental orchestrations, it also serves to help the teacher to implement instrumental orchestrations in small-group settings (Drijvers et al., 2013). In addition, Drijvers and colleagues (2013) built on the existing typology of whole-class instrumental orchestrations by identifying examples of instrumental orchestrations that may be implemented in small-group settings, which they referred to as individual instrumental orchestrations (Figure 2.6). There are similarities between some of the whole-class and small-group instrumental orchestrations—some even bear the same name. Figure 2.8 shows summaries for the small-group instrumental orchestrations that Drijvers and colleagues described. As in Figure 2.7, a “TC” indicates that an instrumental orchestration is more teacher-centered, and an “SC” indicates that an instrumental orchestration is more student-centered.

In his study involving whole-class instrumental orchestrations, Erfjord (2011) did observe one type of instrumental orchestration in small-group settings for the teacher from Fjellet. Although Erfjord did not label this type of instrumental orchestration, he described the teacher as prompting the more clever students (who demonstrated technical and conceptual understanding) to orchestrate their peers’ activity with Cabri. This description shows similarities to the Sherpa-at-work instrumental orchestration in the whole-class setting, so one might consider this as a Sherpa-at-work instrumental orchestration in the small-group setting. This reflects the potential for students to engage in the collective management of instrumental genesis within the small-group setting (as in the case of the Sherpa-at-work instrumental orchestration in the whole-class setting—see Trouche, 2004). Erfjord’s finding of a Sherpa-at-work instrumental
orchestration in the small-group setting was novel in that this type of instrumental orchestration, as described, seems very student-centered, which contrasts from the results reported in other studies on instrumental orchestrations in small-group settings.

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<th>Description</th>
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<tr>
<td>Discuss-the-screen (SC)</td>
<td>The teacher notices something worthy of discussion on the (digital) artifact and initiates a discussion to encourage the students to consider, explain, or debate what they see on the screen. Alternatively, a student may pose a question that the teacher directs to the students for discussion. The teacher gives the students time and space to enter the conversation and the students have the ability to influence the direction of the conversation (related to their conceptions and what they share).</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>The teacher acts as an instructor to lead students in a small group with the artifact. The teacher explains concepts or methods based on what happens on the screen. The teacher may raise questions to check for student understanding.</td>
</tr>
<tr>
<td>Link-screen-paper (TC)</td>
<td>The teacher makes evident the connection(s) between what is represented by the artifact and what is represented in paper and pencil methods for a given task. In the small-group setting, the teacher should have the space to switch easily from the artifact to paper and pencil (or other handwritten media).</td>
</tr>
<tr>
<td>Technical-support (TC)</td>
<td>The teacher helps students overcome technical problems (e.g., access, login, power, etc.) in small groups.</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>The teacher demonstrates a clear, step-by-step method for using the artifact, in order to avoid obstacles that the students might face. Students in the small group follow along.</td>
</tr>
</tbody>
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*Figure 2.8. Small-group instrumental orchestration types identified by Drijvers et al. (2013).*

Kratky (2013) reported similar results to those offered by Drijvers et al. (2013), pertaining to teacher-centered instrumental orchestrations in small-group settings. In a study of one high school Algebra teacher’s instrumental orchestrations in small-group settings, the researcher found that the participating teacher favored three types of teacher-centered instrumental orchestrations. Kratky provided descriptions, but did not give labels for the types of instrumental orchestrations observed. However, Kratky’s descriptions of the instrumental orchestrations were consistent with the descriptions provided by Drijvers et al. (2013) for the *Technical-support, Technical-demo, and Guide-and-explain* instrumental orchestrations. In the
first type (*Technical-support*), Kratky observed the teacher fixing issues with the students’ calculators without showing the students the steps or keystrokes. In the second type (*Technical-demo*), the teacher demonstrated a particular action in such a way that the students could follow along and see what the teacher was doing. In the last type (*Guide-and-explain*), the teacher paused his demonstrations to ask *short closed questions* (Boaler, 1998) to assess students’ comprehension of the demonstrated artifact use. Kratky suggested that, although the latter instrumental orchestration type did not constitute a discussion that opened for the inclusion of student thinking, it might serve as a transitional type of instrumental orchestration towards more student-centered instrumental orchestrations.

**Discourse-Related Pedagogical Moves Observed in Teachers’ Didactical Performances**

The literature on instrumental orchestrations has begun to mark distinctions between teacher-centered and student-centered instances of such instrumental orchestrations, defined by their didactical configurations and exploitation modes (as shown in Figure 2.5). For example, Drijvers (2011) states that the *Technical-demo* and *Explain-the-screen* instrumental orchestrations are *strongly* teacher-centered, whereas the *Discuss-the-screen* and *Sherpa-at-work* instrumental orchestrations (Figure 2.6) are more student-centered. One issue with previous research is the lack of attention to teachers’ didactical performance—the ad-hoc decisions that a teacher makes in the midst of enacting an instrumental orchestration (Drijvers et al., 2013).

Cayton’s (2012) study presented some findings related to teachers’ didactical performances. Moreover, Cayton’s results supported an identifiable difference in interaction patterns between different types of instrumental orchestrations, particularly the *Discuss-the-screen* and *Explain-the-screen* instrumental orchestrations. In the former, participating teachers were observed promoting student contributions to the discussion and thus maintaining the
cognitive demand of the related tasks. In the latter, however, the participating teachers were observed talking more and the students participated less in the discussions—as a result, the cognitive demand of the related tasks decreased. Cayton noted the teacher’s use of probing, where teachers asked the students to share more of their thinking. Cayton also reported teachers’ use of revoicing, where teachers paraphrased what students said during an instrumental orchestration. Lastly, Cayton mentioned teachers’ use of wait time. Each of these three pedagogical moves is included in Herbel-Eisenmann et al.’s (2013) set of teacher discourse moves. Teacher discourse moves are described as student-centered pedagogical moves that teachers can use to promote student engagement in mathematical discussions, including those that follow a star pattern of interaction—where students engage in conversation directly with each other and responses are not routed through the teacher (Nathan & Knuth, 2003).

Drijvers et al. (2010) argued that, in order to successfully implement technology in school mathematics, teachers need to develop new repertoires of teaching techniques, but these new teaching techniques “are likely to be related to already existing [techniques]” (p. 214). Similarly, Drijvers et al. (2013) extended a call for more research in this area when they stated, “a better focus on the quality of interactions within orchestrations, which might be considered as part of the didactical performance, is needed” (p. 997). Given the preliminary evidence of teacher discourse moves showing up during instrumental orchestrations, it seems that teacher discourse moves might serve as a useful lens to examine teachers’ didactical performances and a means to help teachers develop new repertoires of teaching techniques.

IRE as a Typical Mathematics Teaching Technique

In this section, literature is reviewed related to pedagogical moves in relation to contemporary reform in school mathematics. As noted above, previous research has revealed
some student-centered pedagogical moves used during teachers’ didactical performances (Cayton, 2012). Unfortunately, the IRE sequence (Mehan, 1979) has been noted both as the predominant discourse structure in mathematics education and as insufficient for achieving mathematics education reform (Cazden, 2001; Herbel-Eisenmann et al., 2013; Wood, 1998). As a result, researchers have sought alternatives to IRE, such as the teacher discourse moves (Herbel-Eisenmann et al., 2013), as a means to support deeper student engagement in mathematical discourse during class time. Alternatives to IRE might also be useful in technology-enhanced classrooms, since technology use is viewed as an important component of contemporary reform. Before discussing alternatives to IRE, IRE and typical mathematics teaching techniques are elaborated.

Mehan’s (1979) elaboration of the initiation-reply-evaluation (IRE) sequence has been heavily referenced in education literature, which is not surprising due to the continued prevalence of this particular sequence in teachers’ instruction (e.g., Black & Wiliam, 2009; Cazden, 2001; Wagner, Herbel-Eisenmann, & Choppin, 2012). In short, IRE is a three-part sequence where a teacher initiates the interaction with a student (often by asking the student a question), the student replies with an answer to the question, and the teacher evaluates the student’s response. In a qualitative study, Mehan (1979) provided numerous examples from transcript data of teacher-student interaction sequences, with a primary focus on the words used by the teacher. Through the many examples, Mehan demonstrated both the prevalence of the IRE interaction pattern and variations of its enactment by teachers. Despite the argument for variation of enactment, Mehan’s commentary reflected a model for classroom interaction primarily focused on the teacher providing content information directly to students. When the teacher evaluates the students’ responses, the teacher assesses if students properly received and
understood the information presented by the teacher. The teacher then communicates that assessment to the students as feedback to confirm or deny whether they understand what the teacher is saying during instruction. This sort of exchange, although it does incorporate student participation in some form of conversation, does not typically constitute discussion where the students are permitted or expected to defend their answers, engage with other’s thoughts, make connections between (mathematical) ideas, or engage in the negotiation of sociomathematical norms.

Cazden, one of Mehan’s colleagues (e.g., Cazden & Mehan, 1989), characterized IRE as a central component to traditional (or more teacher-centered) instruction in her (Cazden, 2001) commentary on classroom discourse. Although IRE is not the only sequence of moves that teachers use in the classroom, it has served as a facet of many conversations related to supporting students’ understanding of concepts in mathematics (e.g., Cazden, 2001; Herbel-Eisenmann et al., 2013).

IRE becomes important in studies on instrumental orchestrations when considering teacher knowledge and practice related to the most dominant forms of instruction. To this point, Drijvers and colleagues (2010) remarked:

Lagrange and Monaghan (2009) argue that the availability of technology amplifies the complexity and, as a consequence, challenges the stability of teaching practices: techniques that are used in “traditional” settings can no longer be applied in a routine-like manner when technology is available. A new repertoire of teaching techniques, instrumented by the available tools, has to be developed. These new techniques are likely to be related to already existing ones as well as to the teachers’ underlying views on mathematics education (Pierce & Ball, 2009).

In order to help teachers to benefit from technology in everyday mathematics teaching, therefore, it is important to have more knowledge about the new teaching techniques that emerge in the technology-rich classroom . . . (p. 214)
Arguments such as this one demonstrate the need for alternatives to typical (or traditional-transmission) models of mathematics instruction.

**Alternatives to IRE Supporting a New Repertoire of Teaching Techniques**

Cazden (2001) argued that using alternatives to IRE can create more student-centered learning environments. Cazden then declared that a key element to nontraditional (or more student-centered) instruction involves the teacher’s use of variations of IRE, particularly the evaluation step. Some authors have referred to the *initiation-reply-follow-up* (IRF) sequence to show that small changes to IRE may increase potential for richer forms of student engagement in the discourse. For example, Hall and Walsh (2002) reviewed literature showing ways in which teachers in language-learning classrooms refrained from directly evaluating the students’ responses. Instead, the teachers followed up by “asking [the students] to expand on their thinking, justify or clarify their opinions, or make connections to their own experiences” (p. 190). Hall and Walsh also pointed out that the IRE and IRF occurred in the same classrooms. In other words, IRE was still present even in classrooms that seemed to display productive discussions where multiple students engaged in the conversation—the distinction was marked in the teachers’ use of alternatives, in addition to IRE.

Herbel-Eisenmann et al. (2013) underscored the notion that IRE need not be eliminated from the classroom. Rather, in the development of their professional development program designed to help teachers to engage students in mathematical discussions, they have encouraged teachers to consider when IRE might be useful and when other interaction sequences might be implemented to open doors for richer student engagement. This supports the view that, although total reliance on IRE would not give students many opportunities to engage as active collaborators in rich classroom discussions, there may be times when it can still be useful (as in
the case of helping to move the students along, particularly with an idea that is peripheral to the teacher’s main objectives for a specific lesson).

In terms of instrumental orchestrations, the use of IRE can be viewed as bad or good, depending on the context. On the one hand, student-centered instrumental orchestrations would seem to lose their student-centeredness if teachers rely too heavily on IRE as part of their didactical performances when enacting their instrumental orchestrations. On the other hand, as seems to be the case with the Guide-and-explain instrumental orchestration (Drijvers et al., 2013), IRE or the use of closed questions may be a natural component of the teachers’ didactical performance. Thus, as argued by Herbel-Eisenmann et al. (2013), caution should be used when discussing the role and use of IRE in the mathematics classroom, since there may be times when it can be useful. In light of this suggestion with regards to the use of IRE, educators and researchers might caution against labeling types of instrumental orchestrations as bad or good—instead, they might consider the benefits and shortcomings of each in relation to the specific instances in which a teacher uses them.

**Discourse Patterns**

In a study reporting on one middle school teacher’s professional development related to orchestrating mathematical discussions, Nathan and Knuth (2003) demonstrated two different whole-class interaction patterns that may occur during mathematical discussions in the classroom. The teacher, Ann, had expressed beliefs consistent with recent reform efforts (e.g., NCTM, 2000, 2014). During her first year of participating in the project, however, Ann facilitated discussions where she served as the so-called hub (as in the hub of a wheel) in a hub and spoke pattern of interaction (Nathan & Knuth, 2003). The vast majority of student contributions to the discussion were directed to (and routed through) her. As with IRE, there was
often a teacher-speak, student-speak, teacher-speak sequence that took place. Nathan and Knuth (2003) offered a *hub and spoke* diagram to show that the students rarely engaged with each other’s thinking during such discussions. Ann realized that this was inconsistent with her beliefs about mathematics instruction. During the second year of the project, she exhibited a different pattern of interaction when orchestrating mathematical discussions. Rather than serving as a hub, Ann was observed inviting and supporting student engagement in the discussion and with each other’s ideas. This was not by accident, as she reflected on her practice between Year 1 and Year 2 of the project and carefully considered ways to plan for mathematical discussions that allowed the students to participate in the conversation with each other. Nathan and Knuth (2003) provided a *star* pattern diagram to represent the interaction patterns that occurred in Year 2 of the study, where Ann physically moved away from the center of the classroom and intentionally stepped back from her central role during mathematical discussions. This helped to foster social norms where the students participated more and engaged in *peer-based scaffolding* (Nathan & Knuth, 2003)—students interacting with each other’s (mathematical) thoughts and building arguments from the conversation. Although Ann’s adjustments to her instruction promoted student participation in the discourse, she was less confident about the extent to which her students understood the mathematical concepts during the lessons; something was still lacking. The researchers speculated that Ann’s support of student engagement in the conversations brought new complexities to her instruction, as monitoring the students’ unprompted discourse might make it more difficult for the teacher to assess student knowledge. They concluded that Ann’s transformation of instruction was still in progress and they agreed with others who believe that the process to change instruction towards such reform visions takes years before its effects on students’ learning and mathematical practice are clearly visible. Thus, although the *star*
pattern of interaction stands as a contrast to IRE patterns of discourse, it might only be one step towards supporting the mathematical activity and instruction of recent visions for reform; teachers might benefit from additional frameworks to help them as they develop in their instructional practice to support student-centered classrooms.

The *hub and spoke* and the *star* pattern of discourse appear to help clarify some distinctions in different types of instrumental orchestrations. For example, the interaction pattern described in the *Guide-and-explain* instrumental orchestration seems very similar to the *hub and spoke* interaction pattern. In contrast, the interaction pattern described in the *Discuss-the-screen* instrumental orchestration seems more similar to the *star* pattern of interaction. These are consistent with the teacher-centered and student-centered designations that Drijvers and his team (2010) suggest.

In the section that follows, the *teacher discourse moves* (Herbel-Eisenmann et al., 2013) stand as student-centered alternatives to the IRE sequence that promote classroom interaction patterns that resemble the *star* pattern diagram discussed by Nathan and Knuth (2003). In terms of *new repertoires of teaching techniques* (Lagrange & Monaghan, 2009) related to teachers’ didactical performances, it seems helpful for teachers to know about these pedagogical moves so that they can choose and implement them as needed in order to engage students in the discussion, leverage their ideas, and create or sustain a *student-centered* classroom environment. As will be discussed, the previous work of Drijvers et al. (2010) and Cayton (2012) has begun to illustrate teachers’ use of teacher discourse moves as teaching techniques used during their didactical performances.
Teacher Discourse Moves

Herbel-Eisenmann et al. (2013) advanced a set of six pedagogical moves, referred to as teacher discourse moves (TDMs), that can be used to position the students and the mathematics in the classroom. In Figure 2.1, these and other pedagogical moves are represented within the arrow pointing from the teacher to the students. The researchers noted the prevalence of IRE in mathematics teachers’ instruction, even when teachers strive to refine their practice. The set of teacher discourse moves is an extension of Chapin, O’Connor, and Anderson’s (2003) “talk moves” and stands as alternatives to the IRE sequence. One goal of incorporating such pedagogical moves is to promote patterns of interaction that more closely resembled that of the star pattern described by Nathan and Knuth (2003). As will become clear later in the chapter, pedagogical moves represent specific actions that teachers may have in their repertoire, yet judiciously select during instruction as they respond to student engagement in classroom interactions. Taking this perspective fits with the improvisation that is embodied in the jazz metaphor for instrumental orchestrations (Drijvers & Trouche, 2008).

In the section that follows, each of Herbel-Eisenmann et al.’s (2013) six teacher discourse moves (TDMs) is described. The teacher discourse moves stand as alternatives to the IRE interaction pattern, particularly the Evaluation move, and position the students in different ways to support their agency as doers of mathematics. Figure 2.9 shows the list of teacher discourse moves, which are discussed in more detail below. These teacher discourse moves seem particularly helpful for when teachers enact the student-centered instrumental orchestrations (Figure 2.7), as evidenced by results emerging from previous studies (Cayton, 2012; Drijvers et al., 2010).
**Teacher Discourse Moves**

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<td>Inviting student participation</td>
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<td>Revoicing</td>
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<td>Asking students to revoice</td>
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<tr>
<td>Probing a student’s thinking</td>
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<td>Creating student-student opportunities</td>
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*Figure 2.9. Teacher discourse moves (Herbel-Eisenmann et al., 2013).*

**Waiting.** The notion of the waiting TDM can be linked to the ideas of *wait time* (Tobin, 1986), *think time* (Stahl, 1994) and other notions that reflect the teacher’s decision to pause for a moment during instruction, rather than to continue speaking. Tobin (1986) conducted a quantitative study where researchers randomly assigned 10 mathematics teachers to an experimental group and received guidance and feedback so that their average implementation of wait time lasted 3 to 5 seconds. Ten mathematics teachers assigned to a control (placebo) group did not receive the training or feedback related to implementing wait time lasting at least 3 seconds. Using analysis of variance, Tobin found that teachers in the experimental group achieved higher quality discourse with students and that the students showed higher mathematics achievement; the 10 indicators reported showed statistical significance at alpha = .05. These findings were further supported when Tobin replicated the study with language arts teachers.

Herbel-Eisenmann et al. (2013) referred to the waiting TDM as a specific teacher action that teachers may plan and reflect upon. In this *teacher discourse move*, the teacher explicitly provides or creates a space of time to allow students to engage in a question posed or to reflect on their thinking. The well-known concept of wait time is included as a part of this *teacher discourse move*. The teacher may use the waiting TDM to encourage student participation in the discussion (as a social norm) or to allow space for students to consider more sophisticated
expressions or representations or to contemplate the mathematical differences in different representations of ideas (moving towards sociomathematical norms). By posing a question in reference to a specific mathematical term or symbol (and providing space for student thinking), the teacher may also privilege the mathematical register or support students’ progression in their mathematical sophistication. Herbel-Eisenmann et al. stressed the importance of waiting before a student response and after an individual student shares her/his mathematical thinking. This is related to Stahl’s (1994) commentary on different types of wait time or think time.

Stahl (1994) highlighted positive findings related to wait time, student participation and student performance on tasks and promotes the notion of think time over wait time, where think time explicitly reflects the expectation that students are to take the silence as an opportunity to think carefully and possibly prepare their thoughts for inclusion into classroom discussion. Stahl established eight categories to describe different functions for periods of silence in the classroom: (1) a teacher waits after asking a question so that students can think before giving their responses, (2) a teacher remains silent when a student waits or hesitates while sharing her thinking, (3) a teacher remains silent after a student finishes sharing her thinking, (4) a teacher remains silent after a student poses a question, (5) a teacher remains silent to think about something that just took place in class, (6) a teacher waits during a lengthy presentation to allow students to process information that was just presented, (7) a teacher remains quiet so that students may complete a particular task, and (8) a teacher waits for dramatic impact to create a sense of suspense or enhanced importance. The presence of these eight categories demonstrates the usefulness of silent time or the waiting TDM.

This waiting TDM may be very important in technologically enhanced mathematics classrooms, since there are both technical and conceptual aspects of instrumental genesis.
(Drijvers et al., 2010). Although Cayton (2012) reported teacher’s use of wait time during instrumental orchestrations, she did not elaborate on what types of wait time were observed. Just as in teaching without artifacts, the teacher’s judicious use of wait time encourages students to pause, think, and/or reflect on their mathematical activity. In the technology-enhanced classroom, the students may also think and reflect on their interactions with the artifacts.

Teachers may plan for the use of the waiting TDM as they plan particular prompts and focus questions for a particular technology-enhanced lesson. They may also implement this teacher discourse move in a more impromptu manner in response to the students’ participation in the instrumental orchestration, as part of the teacher’s didactical performance.

**Inviting student participation.** In a broad sense, the inviting student participation TDM may be used to encourage student engagement in discourse in a variety of ways. For example, Herbel-Eisenmann et al. (2013) stated that a teacher may seek to elicit multiple solution paths used by students engaged in a rich task, or she may seek to have students explain their reasoning through their individual solution path. By inviting multiple solution strategies and paths, the teacher may provide space for students to discuss mathematical differences and/or mathematically sophisticated approaches and/or representations that arise in the students’ work. As described, this teacher discourse move stands to open the discussion and incorporate more than a brief response from a single student; it contrasts with the initiation component of IRE, where the teacher solicits a brief response from a single student.

A teacher may use the inviting student participation TDM by inviting students to explain their thoughts and reasoning or by asking students a more open-ended mathematical question. As shown in Figure 2.5, Drijvers and his team (2010) explicitly stated, “Firstly, [the teacher] evokes mathematical meaning in several ways, namely by posing open questions, by inviting students to
explain their reasoning, and by giving her own explanations” as part of a teacher’s didactical performance (her enactment of an instrumental orchestration).

Since teacher decisions to use teacher discourse moves in tandem may amplify their effectiveness (Herbel-Eisenmann et al., 2013), a teacher may utilize the waiting TDM together with the inviting student participation TDM. For example, the teacher may wish to invite a particular student to share her/his thinking, but also clearly communicate or provide time for the student to gather her/his thoughts before responding. As a second example, the teacher may pose a question to students and use wait time before inviting students to share their thinking.

**Revoicing.** When students share their mathematical thinking with the teacher and/or the rest of the classroom community, the teacher may choose to paraphrase what the students have said for multiple purposes. O’Connor and Michaels (1993) argued that teachers may use the revoicing TDM in response to students’ verbalized utterances while seeking to encourage students to: take an active role in forming and accepting their own identities in the mathematical community; connect the students’ thoughts with (perhaps more precise) mathematical ideas not explicitly communicated by the students; or support students who may struggle with utilizing the vernacular of the classroom culture (as non-native-language speakers). When the teacher asks if she correctly restated what the student said, this is called full-revoicing (O’Connor & Michaels, 1993). Herbel-Eisenmann et al. (2013) issued a common teacher concern that, if used too frequently, the revoicing TDM may discourage students from communicating with each other. Given the dependence on student contributions, the teacher may use the revoicing TDM as part of the didactical performance of instrumental orchestrations. The teacher decides who to revoice, what contribution to revoice, and the timing of this teacher discourse move.
The following is an example of one teacher’s didactical performance from Drijvers et al. (2010). During this performance, which occurred in a Discuss-the-screen instrumental orchestration, the teacher orchestrated a discussion of student work done with a custom-designed artifact for use with the concept of function. Here, the teacher has just asked one of the students for an explanation:

When the explanation turns out to be inappropriate, she invites another student, Kay, to give his explanation, and checks whether Florence understands it. When this is not the case, the teacher rephrases Kay’s explanation and once more checks with Florence, who now says she understands, although she does not actually give evidence of this. Throughout the didactical performance, the teacher uses the mouse to relate her oral expressions to the phenomena on the screen. (Drijvers et al., 2010, pp. 227-228)

In this didactical performance, the teacher used the revoicing TDM when she rephrased Kay’s explanation.

When considering a teacher’s instrumental orchestrations, the Sherpa-at-work, Spot-and-show, and Discuss-the-screen instrumental orchestrations seem fertile for the teacher discourse move of revoicing. The teacher may try to clarify the students’ utterances, implicitly or explicitly sanction particular aspects of the instrumented work shared by the presenting students, or focus the paraphrasing on technical items (specific to the artifacts being used) and/or mathematical concepts.

In her study of teacher’s use of pre-constructed dynamic geometry tasks, Cayton (2012) coded for instances when the participating teachers used the revoicing TDM to rephrase students’ mathematical comments. The teachers in the study implemented this move frequently in their instrumental orchestrations, often before asking probing questions to solicit deeper responses from the students or before inviting other students to enter the conversation in response to what the other student had shared. Cayton observed that this teacher discourse move contributed to a positive learning environment for the students.
**Asking students to revoice.** As an alternative to teacher revoicing, the teacher may implement the *asking students to revoice* TDM by asking a student to paraphrase what another student said. As with the *revoicing* TDM done by the teacher, a student may or may not provide an accurate account of what a peer had said. So, the teacher or a student may seek confirmation that the restatement of the ideas reflects what the student had originally said. The teacher may implement this *teacher discourse move* to assess student engagement with and understanding of each other’s ideas.

When considering the teacher’s positioning acts during instrumental orchestrations, the teacher seems more likely to consider the *asking students to revoice* TDM in the midst of more student-centered instrumental orchestrations such as the *Sherpa-at-work* or the *Discuss-the-screen* instrumental orchestrations. This *teacher discourse move* may be used to help support the collective management of instrumental genesis. For example, having students revoice another’s instrumented actions or thinking related to the use of the technology may help to create a history of a common language when talking about mathematical activity with the use of technology. Again, since the artifacts in the mathematics classroom afford rich and sometimes sophisticated representations, the *asking students to revoice* TDM may provide additional opportunities for the students to raise questions about what has been shared, challenge mathematical ideas, and push each other’s thinking. When a teacher capitalizes on these opportunities, a student-centered mathematical community of practice may be supported. This *teacher discourse move* seems to fit with teachers’ didactical performances (Figure 2.5) in that the teachers need to decide during their performance what utterance needs to be revoiced, which student(s) should engage by revoicing another students’ utterance, and the timing of implementing this *teacher discourse move*. 
**Probing a student’s thinking.** Herbel-Eisenmann et al. (2013) used the term *probing* in a manner consistent with Moyer and Milewicz (2002), where the teacher seeks to have a student further unpack his/her thinking. Although the *probing a student's thinking* TDM may be used to help the teacher formatively assess student thinking and knowledge (Black & William, 1998), it may also serve to encourage thoughtful reflection on the part of the presenting student.

In their work on formative assessment, Black and William (1998) argued that classroom dialogue should prompt reflection and exploration of student thinking, and allow students to think and share their ideas within the community. In order to amplify the impact of the *probing a student’s thinking* TDM, a teacher may attempt to further the mathematical development of other students by having someone revoice the ideas shared by the student who responds to the teacher’s probing.

Cayton (2012) coded for instances when the participating teachers used the *probing a student’s thinking* TDM during their instrumental orchestrations. She observed that the teachers used higher-level questions, which she described as being more likely to support the maintenance of cognitive demand, particularly in their student-centered *Discuss-the-screen* instrumental orchestrations. The researcher noted the correlation between this teacher discourse move and the maintenance of cognitive demand relative to the tasks at hand.

The *Spot-and-show, Discuss-the-screen, and Sherpa-at-work* instrumental orchestrations seem to present fertile opportunities for the teacher to probe a student to share more about her/his thinking through instrumented mathematical activity, given that the teacher solicits student thinking as part of the exploitation modes for these orchestration types. A teacher’s decision regarding when to use the *probing a student’s thinking* TDM depends on what the students
actually say during class. Hence, this teacher discourse move also fits with teachers’ didactical performances.

Creating student-student opportunities. As indicated earlier, a teacher may ask students to revoice what another student said; this may open a space for students to raise questions and discuss each other’s mathematical thinking. This engagement in each other’s reasoning may occur without explicit prompting from the teacher. In contrast, a teacher may purposefully use the creating student-student opportunities TDM by asking students to respond to each other’s mathematical reasoning. Further, as part of this teacher discourse move, the teacher may select and sequence students to share their ideas as part of a mathematically productive orchestration of a discussion (Stein & Smith, 2011). By purposefully choosing who shares their thinking and when, the teacher may add status to both the students’ ideas and the mathematics surrounding the task at hand. Herbel-Eisenmann et al. (2013) noted that, “Effective use of this discourse move could be enhanced by the prerequisite use of other discourse moves and works best when students are actively listening to each other” (p. 184).

As with the other teacher discourse moves, the creating student-student opportunities TDM seems to apply mostly to student-centered instrumental orchestrations such as Sherpa-at-work and Discuss-the-screen. One affordance of this teacher discourse move when technology is being used is that, as students engage with each other’s reasoning (and instrumented activity), the students may engage in the collective management of instrumental genesis. This type of peer-to-peer interaction, such as that embodied in the star pattern provided by Nathan and Knuth (2003), seems to be supported by this teacher discourse move.

The teachers decide when to implement the creating student-student opportunities TDM during an instrumental orchestration and which student ideas to consider connections between.
While preparing lessons, teachers may anticipate using *teacher discourse moves* or other pedagogical moves during their instrumental orchestrations, but the choice of when and how to use it occurs in the moment, during the teachers’ didactical performance (Figure 2.5). Therefore, teachers may have *teacher discourse moves* in their repertoires of possible pedagogical moves, but their implementation is ad hoc.

**Summary of teacher discourse moves.** The literature related to *teacher discourse moves* promotes such moves as alternatives to IRE in order to support students’ engagement in mathematical discussions in the classroom. Although the teacher may plan for some of these moves ahead of time, student contributions to the conversations cannot be fully anticipated. Therefore, the teacher implements *teacher discourse moves* in response to the conversation and students’ contributions as the conversation unfolds. The teacher decides which *teacher discourse move* to use, when to use it, and which students to engage with the use of the *teacher discourse move*. For this reason, all of the *teacher discourse moves* seem to fit as possible characteristics of teachers’ didactical performances within the enactment of their instrumental orchestrations, particularly the more student-centered ones. Hence, *teacher discourse moves* should belong to a new repertoire of teaching techniques to support mathematical discourse within contemporary reform in mathematics education. So, studies on instrumental orchestrations should build on the work of Cayton (2012) and Drijvers et al. (2013), which has begun to show how pedagogical moves—such as *teacher discourse moves*—might be used by teachers during their didactical performances to support student-centered instruction.

**Think-Pair-Share and Turn and Talk**

McTighe and Lyman (1988) offer an alternative to the traditional IRE sequence—“The Think-Pair-Share method (Lyman, 1981) combines the benefits of wait time and cooperative
learning” (p. 19). This instructional strategy has been promoted for use in classrooms to promote student engagement in classroom conversations. As the name suggests, this strategy occurs in three parts—the students first take a moment to think individually about a question or prompt from the teacher, then they tell their thoughts with the partner in their pair before sharing their thoughts with the whole class. McTighe and Lyman noted the importance of the teacher’s role in cueing the students during the instructional sequence. In his early field testing, Lyman (1981) found both an increase in the amount and quality of student contributions to discussions. Results such as these have been reported in mathematics education literature, as well (e.g., Thornton, 1991; Tyminski, Richardson, & Winarski, 2010).

Another well-known teaching strategy, the turn-and-talk, is similar to think-pair-share, yet does not always include reporting out to the whole class. Also, in this case, the focus is place on peer-to-peer discussion to help them construct knowledge in relation to comprehending a reading or solving a mathematical problem (e.g., Schuster, 2010). Therefore, teachers might use turn-and-talk for different purposes than think-pair-share.

Both of these instructional strategies stand as alternatives to IRE. Teachers seeking to promote student engagement in classroom conversation during their instrumental orchestrations might choose to use these strategies in order to get students thinking, talking, and building their ideas as they (the students) engage in instrumented activity.

**Chapter Summary**

This chapter addressed key elements related to instrumental approaches to mathematics education and contemporary reform efforts in the United States (Figure 2.1). The review of literature started with a discussion of tasks that enable technology to be used to transform students’ learning experiences. Next, literature on artifacts, instrumental genesis, the teacher and
instrumental genesis, and instrumental orchestrations was discussed. The notion of instrumental genesis embodies the user-artifact interaction—the artifact shapes the user’s understanding and the user shapes the artifact (Zbiek et al., 2007). The jazz orchestra metaphor (Drijvers & Trouche, 2008), where the teacher is the conductor, the students are the musicians, and the artifacts become mathematical *instruments* when the students’ mental schemes are activated, served as a construct to unify the major themes in the literature.

Empirical studies on teachers’ instrumental orchestrations have started to show distinctions between more teacher-centered and more student-centered instrumental orchestrations (Drijvers et al., 2010). Alternatives to typical models of instruction, which rely on IRE (Mehan, 1979), have begun to emerge in technology-enhanced mathematics classrooms. For example, Cayton (2012) and Drijvers et al. (2010) reported examples of teacher discourse moves (Herbel-Eisenmann et al., 2013) in teachers’ instrumental orchestrations. These observations draw attention to the notion of pedagogical moves that teachers may add to their repertoires of teaching techniques for use during their didactical performances. This may serve as a means to help teachers develop in their TPACK knowledge in order to implement more student-centered instrumental orchestrations. Since there are technical, conceptual, and social aspects of instrumental genesis, it seems that teachers need to use student-centered instrumental orchestrations in order to provide conditions favorable for instrumental genesis. Therefore, an explicit lens of pedagogical moves is an important component of studies of teachers’ instrumental orchestrations. Previous literature did not attend to instrument-specific pedagogical moves that teachers might use during their instrumental orchestrations. As Drijvers et al. (2010) stated, “A new repertoire of teaching techniques, instrumented by the available tools, has to be developed. These new techniques are likely to be related to already existing ones as well as to the
teachers’ underlying views on mathematics education” (p. 214). In the chapter that follows, the research methods used in this study—in the pursuit of examining one instructor’s teaching techniques related to her students’ instrumented activity—are explained, including attention to pedagogical moves during her didactical performances.
CHAPTER 3
METHODS

The methods used in the current study build on the exploratory case studies performed by others in the literature (Cayton, 2012; Drijvers et al., 2010; Erfjord, 2011; Kratky, 2013). The goal of this study was to investigate the nature of the interactions during one technologically experienced, reform-oriented instructor’s instrumental orchestrations, using a lens of pedagogical moves to focus attention on the nature of her didactical performances (see Figure 3.1). The methods employed in this study were chosen to address the research questions:

1. What instrumental orchestrations does one instructor use in a technological-tools-enhanced, reform-oriented statistics course for pre-service elementary teachers?

2. What types of technology-related pedagogical moves does the instructor implement during her didactical performances in the course?

This chapter begins with a discussion of the theory that supports the jazz orchestra metaphor. Next, the conceptual framework is unpacked, which relies on the notion of an instrumental orchestration separated into three components—the didactical configuration, the exploitation modes, and the didactical performance. The jazz orchestra metaphor is incorporated in the conceptual framework to reinforce the notions of improvisation and the ad hoc decisions that teachers make in mathematics classrooms. Then, the codes used and developed in the study are reported. From there, details related to the design of this case study are described, including participant selection, data collection and data analysis.
Theoretical Framework: Instrumental Genesis Compatible Instruction

The theoretical framework developed and used in this study was adapted from the notion of *constructivist-compatible instruction* (Ravitz et al., 2000) by incorporating theories related to instrumental approaches to mathematics education. The theory that set the foundation for the current study drew heavily from the jazz orchestra metaphor (Drijvers & Trouche, 2008; Drijvers et al., 2010). Students were viewed as “musicians” who were constructing their conceptual and technical understanding when they interacted with the artifacts. When the musician and artifact influence or shape each other, then an “instrument” is created. This is the essence of instrumental genesis, which has individual and social aspects (Trouche, 2005), and technical and conceptual aspects (Drijvers, 2003). When considering an individual student, one might consider a more radical constructivist approach (Ernest, 1994), and when considering students in groups or a whole-class setting, one might prefer a more social constructivist approach (Ernest, 1994), which is the perspective taken in this study. This was largely due to viewing the classroom community as *an orchestra*, rather than an environment of disconnected individuals. Also, the social-constructivist metaphor of *persons in conversation* (Ernest, 1994, 1996) was used to complement the orchestra metaphor.

In the jazz orchestra metaphor, the teacher is viewed as the conductor who plans for and adapts instruction in order to leverage the students’ own technical and conceptual understandings. As the conductor in a jazz orchestra, the teacher makes ad hoc decisions in order to improvise, while still staying true to the general essence of the music being played. The teacher’s instruction, when enacted in this manner, is taken to be *constructivist-compatible* (Ravitz et al., 2000), as opposed to the more rigid, prescribed, or closed activity that occurs in a symphony orchestra (Drijvers et al., 2010) or in a classroom that relies on *traditional-*
transmission instruction (Ravitz et al., 2000). Ravitz and colleagues note that constructivist-compatible instruction is based on a theory of learning that emphasizes prolonged student engagement in problem solving activities—traditional transmission models of instruction are based on a theory of learning that suggests that students learn best through repeated practice of prescribed methods following the teacher’s demonstrations.

In this metaphor of the jazz orchestra, the teacher’s primary goal is assumed to be to support students’ conceptual growth and development with regard to mathematical ideas and connections among mathematical ideas. Social forms of student activity are given priority in order to support all students’ conceptual growth. As a result, the researcher in this study defined instrumental genesis compatible instruction as constructivist-compatible instruction combined with the use of artifacts in the mathematics classroom. Framed in this manner, instrumental genesis compatible instruction is based on a similar theory of learning to that of constructivist-compatible instruction—students learn the best with technology when they engage in prolonged (and student-centered) problem solving activities with the assistance of artifacts. The jazz orchestra metaphor referenced in this section was used to set up the conceptual framework in the following section.

**Conceptual Framework: Instrumental Orchestration**

The conceptual framework that guided this study has roots in the literature referenced in Chapter 2 related to instrumental approaches to technology and to the jazz orchestra metaphor. The framework for this study was based upon Drijvers and colleagues’ (2010) components of an instrumental orchestration: the didactical configuration and exploitation modes (adopted from Trouche, 2004), and the didactical performance (see Figure 2.5). Further, this framework expanded two of the components: the didactical configuration and the didactical performance.
The didactical configuration was expanded to include student groupings (i.e., individual, pairs, small groups and whole class). The didactical performance was expanded to include pedagogical moves and the teacher-centered, student-centered spectrum. Because of this adaptation, one might consider the teacher’s didactical performance as being or containing a sequence of pedagogical moves. Additionally, the notion of a cue is introduced in this framework to signify the teacher’s role in initiating the didactical performance. This cue is assumed to be a verbal cue, where the teacher makes a statement that indicates the beginning of the orchestration. Characterized in this manner, the cue would be a pedagogical move. The conceptual framework for this study is represented in Figure 3.1 and discussed in the following subsections.

**Figure 3.1.** The conceptual framework, adapted from Drijvers et al. (2010).

**Didactical Configuration**

The didactical configuration includes the teacher’s choice for the arrangement of the classroom, the artifacts, and the students for instruction that explicitly includes or makes reference to the students’ use of the artifacts. The teacher considers the didactical configuration before performing the instrumental orchestration—this might be planned prior to the beginning of the class or the teacher may plan this during the class but before the performance of the
instrumental orchestration. For this study, an adaptation to Drijvers and colleagues’ (2010) work was made to specifically note the teacher’s role in configuring the general aspects of the classroom, which includes the configuration of the students’ desks, the use or nonuse of a digital projector, and other general classroom elements. The artifacts are not considered within the general classroom configuration in order to maintain the focus on instrumental genesis—which occurs between the user(s) and the artifact. As reported by Drijvers et al. (2010), the didactical configuration includes both the types and the arrangement of the artifacts in the classroom. In the current study, the artifacts included the TI-73 Explorer and Tinkerplots. For this study, another adaptation was made to the model from Drijvers et al. (2010) in order to include explicit reference to the students as part of the didactical configuration. In other words, when the teacher sets the didactical configuration for an instrumental orchestration, they also determine whether the students will be working alone, in pairs, in small groups, or in a whole-class structure (as shown in Figure 2.1, as well). Taken together, these three elements of the didactical configuration reflect the teacher’s goals for a lesson, activity, or instrumental orchestration. When students are arranged in groups while using the artifacts and the digital projector is used, for example, the didactical configuration suggests that a student-centered instrumental orchestration might take place.

**Exploitation Modes**

The exploitation mode is the teacher’s general plan for leveraging their chosen didactical configuration to order to structure their students’ opportunities to engage in instrumental genesis (Trouche, 2004). Drijvers et al. (2010) explicitly included the notion of hypothetical learning trajectory (Simon, 1995) as part of the anticipated means for exploiting the didactical configuration and this modification was preserved for the current study to underscore the
teacher’s role in judiciously selecting mathematical tasks (see Figure 2.1) and planning instruction to help students grow in their conceptual and technical understanding while using the artifacts. For this study, all of the teacher’s efforts to plan an instrumental orchestration, including their use of formative assessment strategies and the *work-and-walk-by* (Drijvers et al., 2010) instructional strategy, are considered to constitute the exploitation mode for a particular instance of an instrumental orchestration. As with the didactical configuration, the teacher plans or prepares for the exploitation mode prior to enacting or performing the instrumental orchestration.

Figure 3.1 shows that the didactical configuration and exploitation modes, taken as a set, signify the type of instrumental orchestration implemented (Drijvers et al., 2010; Drijvers et al., 2013; Trouche, 2004). This conceptualization is included in the current study, as well. For this study, only configurations and exploitation modes in which the artifacts were explicitly used or referenced are considered as instrumental orchestrations. Other types of instruction are considered separate from students’ opportunities to engage in instrumental genesis. This conceptualization is consistent with the report from Drijvers et al. (2010), but differs from the report from Drijvers et al. (2013), who included the typical *Board-instruction* type of instruction—where the teacher lectures at the front of the class while writing on the board—as part of their analysis. In the study presented here, the researcher framed the analysis to focus on orchestrations in which there was potential for student engagement in instrumental genesis.

**The Cue**

Just as a conductor makes a gesture to tell the orchestra when to start their performance (i.e., to start playing their instruments), the teacher must communicate verbally or nonverbally with their students to initiate the performance phase of an instrumental orchestration. This act of
communication is referred to as the cue, and is necessarily a pedagogical move. During the whole-class IOs, the teacher might introduce the task and/or the instrumental orchestration, make a statement or gesture to initiate a change in the didactical configuration, or tell the students to pay close attention and take notes. In this sense the notion of a cue is similar to the sense communicated by McTighe and Lyman (1988) in relation to the think-pair-share instructional sequence—they reported the teacher’s role in initiating and transitioning between the different parts of the sequence by way of cues. To initiate the performance of a whole-class First-chair IO, for example, the teacher might invite a student to serve as the First-chair and go up to the front of the classroom in order to demonstrate a particular instrumented technique; to initiate a whole-class Discuss-the-screen IO, the teacher might make a statement or use a teacher discourse move (Herbel-Eisenmann et al., 2013) to invite a student(s) to engage in the mathematical conversation involving their instrumented activity. During the small-group instrumental orchestrations, the teacher might include more subtle cues—for example, by walking near a group and observing the students’ activity, the teacher nonverbally communicates to the students that she is near and the students may respond in a variety of ways (such as asking questions to the teacher, continuing with their work, etc.). In each of these cases, the assumption is that teacher—serving as the conductor—acts in a manner to initiate the performance of the instrumental orchestration. The notion of the cue is an adaptation to the characterization of instrumental orchestrations provided by Drijvers et al. (2013).

**Didactical Performance and Pedagogical Moves**

The didactical performance refers to the ad hoc decisions and actions a teacher might make in-the-moment during an instrumental orchestration (Drijvers et al., 2010; see Figure 2.5). Framed in this manner, the didactical performance can be viewed as the teacher’s enactment of
an instrumental orchestration, presumably in ways to support the anticipated hypothetical learning trajectory (Simon, 1995). The framework adapted Drijvers and colleagues’ (2010) component of didactical performance to explicitly include pedagogical moves related to classroom discourse and/or the students’ interaction with the artifacts. This was in response to evidence emerging from empirical studies (Cayton, 2012; Drijvers et al., 2010) that suggested that teacher discourse moves (TDMs; Herbel-Eisenmann et al., 2013) may play an important role in instrumental orchestrations. In fact, the conceptual framework used in this study marks the distinction between the pre-performance and performance phases of an instrumental orchestration with a teacher’s use of pedagogical moves. As previously stated, a teacher is viewed as initiating the performance phase of an instrumental orchestration with the use of a pedagogical move, whether verbal or nonverbal. Since teacher discourse moves are described as high-leverage pedagogical moves that teachers can use to support student-centered instruction (Herbel-Eisenman et al., 2013), they may be particularly helpful to support student-centered instrumental orchestrations in technology-enhanced mathematics classrooms.

By focusing on pedagogical moves as aspects of teachers’ didactical performances, the conceptual framework builds upon previous models of instrumental orchestrations. This framework also helps one to compare and contrast teachers’ actions with their use of the initiation-reply-evaluation sequence (IRE; Mehan, 1979) and discuss implications of such moves in relation to contemporary reform efforts. Notions of teacher and student-centeredness are connected to IRE in the literature (Cazden, 2001; Wood, 1998) and seem to suggest that the two constitute a dichotomy. This dichotomy view is apparent in the instrumental orchestrations described by Cayton (2012) and Erfjord (2011). However, there is an alternative perspective. For example, Drijvers and colleagues (2013) argued for the Guide-and-explain instrumental
orchestration as a middle-ground between the teacher-centered *Technical-demo* instrumental orchestrations and the student-centered *Discuss-the-screen* instrumental orchestrations. Also, Kratky (2013) noted small variations in a teacher’s instrumental orchestrations in the small-group settings, which seemed to reflect a spectrum of possible enactment. Further, *teacher discourse moves* may influence the locus in the classroom during a mathematical discussion, particularly when compared to a sole reliance on IRE (Herbel-Eisenmann et al., 2013). Framing the teacher’s didactical performance in terms of pedagogical moves also allowed the researcher to consider the teacher-centered, student-centered spectrum. One advantage of considering this spectrum is that specific attention could be made to particular moves that sparked changes in the nature or quality of the classroom discussion or activity. The conception of didactical performance, drawing from the notion of pedagogical moves, is also consistent with a jazz orchestra metaphor (Drijvers & Trouche, 2008), as teachers may both plan for and improvise with the particular pedagogical moves that they use during instruction and instrumental orchestrations.

As previously stated, the conceptual framework includes the three components of instrumental orchestrations: didactical configuration, exploitation modes, and didactical performance (including pedagogical moves as elements of a teacher’s didactical performance). These three components can be used to frame discussions about instrumental orchestrations in terms of the teacher-centered, student-centered spectrum. Such discussions are important when considering the student-centered focus in contemporary reform efforts. The section that follows is used to present the codebook that was developed during this study, which includes codes related to teachers’ student-centered and teacher-centered pedagogical moves.
Renaming the Sherpa as a First-chair

Drijvers and colleagues (2010) utilized Guin and Trouche’s (2002) Sherpa metaphor for one category of instrumental orchestrations in the whole-class setting (see Figure 2.7). Guin and Trouche mentioned that Sherpa “refers to the person who guides and who carries the load during expeditions in the Himalaya [sic] . . . [or] to diplomats who prepare international conferences” (p. 209). Here, it is argued that the label First-chair embodies a richer theoretical connection to the construct of instrumental orchestration, which was an adaptation made by the researcher. First, in an orchestra, the term First-chair is made with specific reference to a musical instrument, such as the First-chair violin or First-chair saxophone. In the classroom, a student might function at a high level while creating graphical representations and at a lower level with performing computations and other commands, for example. In this sense, this same student would be a First-chair candidate for creating graphical representations, but not for the other tasks. The label Sherpa, as used by Guin and Trouche (2002), referred primarily to the person and may downplay the user’s interaction with the artifacts. Secondly, although a Sherpa may carry the “load” (physical load in expeditions and cognitive load in the classroom), a Sherpa need not be under any other form of direct authority or guidance—she may be viewed as the authority during an expedition. In an orchestra, however, the conductor guides and leads the First-chairs’ activity (just as the teacher guides and leads students who serve as leaders in the classroom). This more clearly reflects the fact that the teacher holds the primary authority for shaping the activity in the classroom. Thirdly, there is a First-chair for each kind of instrument group in an orchestra (i.e., First-chair violin, a First-chair tuba, etc.). In the classroom, different students may serve as leaders (or First-chair students) with regards to different artifacts, different aspects of the same artifact, or the different instruments created with the inclusion of
their artifact-oriented schemes. Although something similar may be true for Sherpas with different skill sets with different artifacts, this fact is not as apparent with the Sherpa metaphor.

Fourthly, the term *First-chair* offers the potential for different students, even those seen as *weak* or *remedial* (Trouche, 2004), to rise in status with regards to their activity with the artifacts, just as members of the orchestra may have opportunities to rise in status. In colloquial use, the term Sherpa also refers to an ethnic group of people, where an outsider might not be able to become a Sherpa. Framed in this way, the notion of *First-chair* better fits with the orchestra metaphor and opens possibilities for the teacher to work towards increased equity concerning the students’ opportunities to increase their status as doers of mathematics in the classroom.

**Analytical Framework: The Codebook**

The codes drawn from the literature a priori and those arising during the study are reported in the figures that follow. Each of the three sets of figures corresponds to one of the two research questions.

**Codes for Types of Instrumental Orchestrations: Research Question 1**

All of the codes used in the study related to types of instrumental orchestrations are included below. Figure 3.2 contains the names, descriptions, and inferred goals for the student-centered instrumental orchestrations, and Figure 3.3 shows the same information for the more teacher-centered instrumental orchestrations. A key distinction between the two is made in the nature of the teacher’s role in the instrumental orchestrations. In the case of the student-centered instrumental orchestrations, the teacher requests that students engage in the conversation related
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Inferred Goals</th>
</tr>
</thead>
</table>
| Class-assist* (WC)            | DC: Access to the artifact. One student projecting their artifact publicly so all can see and discuss. EM: The teacher selects a presenter in order to stimulate discussion and support. The teacher requests that the other students give the presenting student help and guidance. | • The class tells the presenter what to do  
• The presenter encounters productive struggle with support  
• Collective management of instrumental genesis                                                                                           |
| Discuss-artifact-use* (WC, SG)| DC: Students need not have access to the artifact. All students positioned to participate in the discussion. The projector might not be used. EM: The teacher facilitates a discussion about the use of the artifacts and related mathematical concepts. | • A summarizing discussion that utilizes the students’ experiences  
• Students make comparisons between different artifacts  
• Collective management of instrumental genesis                                                                                           |
| Discuss-the-screen (WC, SG)   | DC: A projected screen image from the artifact. All students positioned to participate in the discussion. The teacher may make specific moves to encourage the students to make connections, raise questions, and build on their thinking as a group. | • Students share, discuss, participate  
• Technology is an important part of the discussion  
• Collective management of instrumental genesis (Drijvers et al., 2010)                                                                 |
| First-chair (WC, SG)          | DC: Access to the artifact. One student projecting their artifact publicly so all can see and discuss. EM: The teacher has a student present, project, and explain his/her own work with the artifact. The other students follow the presentation and may ask questions or contribute to the presentation/discussion. | • Teacher/Conductor and First-chair interaction. Other students receive/witness/observe or participate  
• Collective management of instrumental genesis (Guin & Trouche, 2002; Trouche, 2006)                                                                 |
| Spot-and-show (WC)            | DC: A projected screen image from the artifact. All students positioned to see and hear the teacher. The teacher may facilitate discussion to have students explain their work or what they see projected. | • Teacher spots an interesting technique and/or interpretation  
• Students share and discuss each other’s use of the artifacts  
• Collective management of instrumental genesis                                                                                           |
| Students-choose-tech* (SG, WC)| DC: Several different options are available to the students (e.g., different artifacts, graph paper, manipulatives, etc.). EM: The students are explicitly given the option to choose from the available artifacts one for use to help them solve a particular task. The teacher expects the students to justify the use of their selected artifact. | • Students consider different artifacts to address a mathematical problem, discuss pros and cons of different artifacts, and select and use artifacts with a mathematical purpose  
• Students used appropriate tools strategically                                                                                           |
| Talk-without-tech* (SG)       | DC: All students positioned to participate in the discussion. The projector and artifact are not used. EM: The teacher prompts or supports a discussion where the students discuss elements of a mathematical task without using an artifact (for the moment). The discussion might be to frame possible solution paths or to consider how to use the artifacts. | • Students engage in problem solving strategies without artifact use  
• Students engage in the mathematical discussion  
• Collective management of instrumental genesis                                                                                           |

*Note. This figure provides descriptions and goals for each of the student-centered instrumental orchestrations. WC stands for whole-class and SG stands for small-group. DC stands for didactical configuration and EM stands for exploitation mode. An asterisk designates a newly identified type of instrumental orchestration.*

*Figure 3.2. Codes for the more student-centered instrumental orchestrations.*
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Inferred Goals</th>
</tr>
</thead>
</table>
| Board-with-tech-reference*    | **DC:** No artifacts in use. Teacher uses blackboard or similar tool for writing. EM: The teacher explicitly references the artifact. The teacher may use this orchestration type to review key mathematical ideas or technical skills without the artifacts present. The teacher may call on students to report key ideas.                                                                                                                                    | • Regular, typical direct form of instruction  
• Deliver content to students  
• State, review or summarize technical procedures with the artifacts                                                                                                                                                                                                                           |
| Explain-the-screen            | **DC:** Publically viewable screen showing the artifact. EM: Rather than starting a conversation, the teacher leads the explanation about what is communicated on the screen from the artifact.                                                                                                                                                                                                                                                       | • Students receive an explanation  
• Students acquire knowledge about what happens on the screen, in terms of the underlying mathematics                                                                                                                                                                                                                             |
| Guide-and-explain            | **DC:** Teacher and students have access to the artifacts and the teacher uses a projector. EM: The teacher leads and dominates the conversation by explaining what happens on the screen. The teacher may ask several closed questions (such as those of the IRE pattern).                                                                                                                                                                                                                             | • Teacher guides explains student use of the artifact  
• Students receive/acquire the explanation on using the artifact  
• Minimal student participation                                                                                                                                                                                                                                           |
| Link-screen-board/task       | **DC:** Teacher and students have access to the artifacts and the teacher may use a projector. The teacher shows the screen and boardwork at the same time. The teacher leads and may dominate discussion to establish the connections between what the artifact generates and what one can do mathematically by hand on the board.                                                                                                                                                                                                                     | • The teacher makes the connections between what is written on the board or on paper with what is on the screen of the artifact  
• Students receive/acquire knowledge of the link between screen and board or paper                                                                                                                                                                                                                                   |
| Technical-demo               | **DC:** Teacher and students have access to the artifacts and the teacher may use a projector. The teacher projects their artifact. EM: The teacher demonstrates something with the artifact. The teacher leads and projects her screen.                                                                                                                                                                                                                                               | • Students acquire new techniques to be used for an upcoming task.  
• Help students to be efficient with artifacts                                                                                                                                                                                                                                              |
| Technical-support            | **DC:** The students are using the artifact in a small group. EM: The teacher helps the students overcome a technical issue (e.g., power, navigation, settings).                                                                                                                                                                                                                                                                   | • Help the students overcome technical hurdles so that they can engage in the mathematical activity  
• Students learn how to fix the technical issue in the future                                                                                                                                                                                                                                      |

Note. This figure provides descriptions and goals for each of the teacher-centered instrumental orchestrations. WC stands for whole-class and SG stands for small-group. DC stands for didactical configuration and EM stands for exploitation mode. An asterisk designates a newly identified type of instrumental orchestration.

*Figure 3.3.* Codes for the more teacher-centered instrumental orchestrations.
to the task at hand, and/or each other’s mathematical and instrument-related thinking. The assumption is that a common goal to these instrumental orchestrations is the “collective management of instrumental genesis,” a notion that Guin and Trouche (2002) used in reference to one such instrumental orchestration (Sherpa-at-work). In the more teacher-centered instrumental orchestrations, for example, the teacher dominates the conversation.

**Codes for Instrument-Related Pedagogical Moves: Research Question 2**

Figure 3.4 shows codes for special pedagogical moves referred to as *student-centered instrument moves* (SCIMs). The SCIMs are pedagogical moves that a teacher makes to open up opportunities for the students to engage in mathematical activity and solve problems with the artifact, explore the artifact, critically consider the use of the artifact, or share their thinking related to their use of the artifact. In order to qualify for this designation, a particular pedagogical move needed to exhibit specific attention to promoting student technical and/or conceptual growth with the use of the artifacts. In contrast to the literature on discourse-related pedagogical moves, previous literature did not identify pedagogical moves specific to the students’ interaction with the artifacts. The results presented in Chapter 5 are a response to this gap in previous research.

Figure 3.5 provides descriptions for the codes used for each of the *teacher-centered instrument moves* that were used during the study. None of these codes were anticipated prior to the study, but emerged during analysis.
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buddy up</td>
<td>The teacher asks the students to work in pairs or small groups with the artifact to solve a mathematical task.</td>
</tr>
<tr>
<td>Introducing</td>
<td>The teacher pauses part of the classroom activity or discussion to have the students take a moment to use the artifact for a specific purpose. The assumption is that the teacher will have students report on aspects of their experience.</td>
</tr>
<tr>
<td>Pressing for alternatives</td>
<td>The teacher asks the student(s) to consider different approaches to using the artifact to solve a given problem.</td>
</tr>
<tr>
<td>Pressing for scrutiny</td>
<td>The teacher asks the student(s) to carefully examine or critique an aspect of the artifact, using mathematical language.</td>
</tr>
<tr>
<td>Probing for thoughts</td>
<td>The teacher asks a student(s) to share more of their thoughts related to their use of the artifact.</td>
</tr>
<tr>
<td>Prompting vocabulary</td>
<td>The teacher invites the students to consider how aspects of the artifact relates to mathematical terminology. This prompt for reflection may involve menu options or the syntax used by the artifact.</td>
</tr>
<tr>
<td>Recalling</td>
<td>The teacher asks a student to publicly present aspects of their work with an artifact. This pedagogical move is likely to occur during the First-chair instrumental orchestration.</td>
</tr>
<tr>
<td>Requesting demonstration</td>
<td>The teacher asks a student to publicly present aspects of their work with an artifact. This pedagogical move is likely to occur during the First-chair instrumental orchestration.</td>
</tr>
<tr>
<td>Requesting technique</td>
<td>The teacher asks the student to explain, justify, or defend choices made with using the artifact. The student reveals aspects of his/her artifact-oriented schemes.</td>
</tr>
<tr>
<td>Requesting exploration</td>
<td>The teacher asks a student(s) to tinker with an artifact in order to build technical and/or conceptual understanding. The teacher expects students to experience issues, which provides opportunities for productive struggle and mathematical and technical discourse.</td>
</tr>
<tr>
<td>Requesting interpretation</td>
<td>The teacher asks a student to describe the output generated by the artifact.</td>
</tr>
<tr>
<td>Requesting prediction</td>
<td>The teacher asks the students to consider what will happen when they use a particular feature of the artifact prior to executing it. Students might write or state their predictions.</td>
</tr>
<tr>
<td>Requesting support</td>
<td>The teacher asks a student to serve as an aide for another student(s) with the artifact for technical and/or conceptual issues.</td>
</tr>
<tr>
<td>Take 30</td>
<td>The teacher pauses part of the classroom activity or discussion to have the students take a moment to use the artifact for a specific purpose. The assumption is that the teacher will have students report on aspects of their experience.</td>
</tr>
<tr>
<td>Voicing</td>
<td>The teacher verbalizes the actions that a student performed with an artifact, thus giving voice to the student and the actions.</td>
</tr>
<tr>
<td>Waiting</td>
<td>Similar to the waiting teacher discourse move, the instructor pauses, allowing the students to think during their activity with the artifacts.</td>
</tr>
</tbody>
</table>

**Figure 3.4.** Codes for the student-centered instrument moves.
### Research Design

Following other case study research in this domain (e.g., Cayton, 2012; Drijvers et al., 2010; Drijvers et al., 2013; Erfjord, 2011; Umameh, 2012) and building on a pilot study (Kratky, 2013; Appendix A), the researcher employed an *instrumental case study* design (Stake, 1995) in order to observe and document what occurred in the classroom. Stake used the term *instrumental* to signify that a particular case might be used to reveal something new about an issue or to revise a previously-held generalization—this was the researcher’s goal in conducting the present study. This decision was made as a deep investigation into one veteran instructor’s technological-tools-enhanced, reform-oriented instructional practice, which might illuminate particular pedagogical moves that teachers may consider to improve their success with instrumental orchestrations in the classroom.

Previous descriptions of theory related to instrumental orchestrations in the literature were adapted to frame the current study (as reflected in Figure 3.1 and discussed above). Identifying the types of instrumental orchestrations that the instructor used helped the researcher

<table>
<thead>
<tr>
<th>Name</th>
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<tbody>
<tr>
<td><strong>Diagnosing</strong></td>
<td>The teacher attempts to troubleshoot an issue with the artifact.</td>
</tr>
<tr>
<td><strong>Directing</strong></td>
<td>The teacher makes direct statements, rather than asking questions, in order</td>
</tr>
<tr>
<td></td>
<td>to facilitate the students’ use of the artifact. Student input is not</td>
</tr>
<tr>
<td></td>
<td>solicited.</td>
</tr>
<tr>
<td><strong>Documenting</strong></td>
<td>The teacher acts as a scribe to record ideas shared publicly.</td>
</tr>
<tr>
<td><strong>Explaining</strong></td>
<td>The teacher gives reasons related to work with the artifact, such as why the</td>
</tr>
<tr>
<td></td>
<td>artifact behaves in a particular manner.</td>
</tr>
<tr>
<td><strong>Guiding</strong></td>
<td>The teacher makes partial statements or asks short questions to help the</td>
</tr>
<tr>
<td></td>
<td>student(s) with a particular aspect of their work with the artifact, leaving</td>
</tr>
<tr>
<td></td>
<td>out some of the detail, as is sometimes done in scaffolding. Student input</td>
</tr>
<tr>
<td></td>
<td>is solicited.</td>
</tr>
<tr>
<td><strong>Mentioning pros and cons</strong></td>
<td>The teacher states strengths and/or weaknesses related to the use of an</td>
</tr>
<tr>
<td></td>
<td>artifact.</td>
</tr>
<tr>
<td><strong>Playing the instrument</strong></td>
<td>The teacher controls and/or demonstrates the use of the artifact in front of</td>
</tr>
<tr>
<td></td>
<td>the class.</td>
</tr>
</tbody>
</table>

*Figure 3.5. Codes for teacher-centered instrument moves.*
to answer the first research question, and unpacking the instructor’s ad hoc moves in the midst of her didactical performances helped the researcher to answer the second research questions. Emphasis was placed on the instructor’s words, so transcript data were vital to the study. Video data provided visual information related to the instructor, students, specific tasks, the artifacts in use, and information projected to the whole-class. The following subsections detail the participant selection, setting, data collection and data analysis of the research design for this study.

**Participant Selection**

The researcher wanted to select a participating teacher or instructor who exhibited the use of student-centered instrumental orchestrations in order to utilize *teacher discourse moves* as a lens to view the more reform-oriented interventions. In order to do so, the researcher employed a form of purposeful sampling referred to as *critical case sampling* (Creswell, 2007), where a participating educator might serve as a unique case of one who regularly and frequently implements student-centered instrumental orchestrations. The researcher visited the classrooms of several high school teachers who facilitated their lessons with the regular use of some form of mathematical technology (typically a graphing calculator). During these informal observations, most of the observed lessons and instrumental orchestrations were teacher-centered. It proved difficult to find an educator who would exhibit regular and frequent use of student-centered lessons and the use of technology at the same time.

Given the need to explore the potential for rich opportunities for students to interact with artifacts, few teachers (who were accessible to be observed) met the criteria that were set for participant selection. As a result, the researcher invited Jade, a college instructor of preservice elementary teachers (PSETs) with whose teaching he was familiar, to participate in the project.
Through informal conversations, Jade indicated that she had deliberately sought to refine her instruction towards the vision of NCTM (e.g., 2000, 2014) for more than a decade and she was conscious and intentional about leveraging student thinking during her lessons while attending to her mathematical objectives for her lessons. She exhibited pedagogical experience with orchestrating mathematical conversations similar to those that Stein, Engle, Smith, and Hughes (2008) suggested, and she had implemented mathematical tasks and technology in ways that reflected her commitment to students’ conceptual development. In the courses that she taught at the university, she both Incorporated and Assessed her students’ technology use in relation to tasks that leveraged the affordances of technology. In other words, she demonstrated the last two phases of the PURIA model (Zbiek & Hollebrands, 2008). Similarly, she was esteemed by her colleagues as someone who possessed a high level of TPACK (Mishra & Koehler, 2006), although no formal measurements of her TPACK were made. These characteristics of her philosophy, knowledge and practice were confirmed during the first few days of data collection.

**Setting**

The study took place in a probability and statistics content course for PSETs at a midsized research university. The PSETs completed a number course for PSETs as a prerequisite to the probability and statistics course, and were also required to take a geometry course for PSETs that also had the number course as a prerequisite. The university demographics included approximately 70% White, 10% Black, 5% Hispanic, 2% Asian, and 13% multiracial or other students. The PSETs in the classroom were predominantly college-aged White females.

Mathematics education faculty at the university had a decades-long tradition of seeking the vision of the NCTM (1989) *Standards*. By sitting in on courses, collecting data, and interacting with instructors, the researcher saw that the PSETs were given opportunities in their
coursework to engage in reasoning and sense-making—connecting multiple representations of mathematical concepts, problem-solving, the strategic use of artifacts, and mathematical conversations.

All sections of the probability and statistics content course for PSETs used a course text created by professors from the university. This text took a problem and concepts-based approach, using open-ended tasks that engaged learners in statistical contexts without front-loading the materials with procedures and terms to learn and memorize before engaging in applications. Both the TI-73 calculator and Tinkerplots software were used to support the development of ideas in the course.

The researcher observed two sections of the same course in order to help ensure that saturation—the state in analysis where no additional codes could be added (Creswell, 2007)—would occur. Figure 3.6 summarizes the general content and specific artifact use for the days observed in class A and class B of the same course. Note: On days A01, B01, and B06, no artifacts were used during class time. Also, class A met two fewer times than did class B, due to weather-related university closings.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Artifact Use</th>
<th>Class/Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Univariate Displays</td>
<td>Boxplot</td>
<td>B02</td>
</tr>
<tr>
<td></td>
<td>Circle Graph</td>
<td>A02, B03</td>
</tr>
<tr>
<td></td>
<td>Dot Plot</td>
<td>A03, B04</td>
</tr>
<tr>
<td></td>
<td>Histogram</td>
<td>B05</td>
</tr>
<tr>
<td>Bivariate Displays</td>
<td>Circle Graphs</td>
<td>A04, B07</td>
</tr>
<tr>
<td></td>
<td>Scatterplots</td>
<td>A05, B08</td>
</tr>
<tr>
<td></td>
<td>Line of Best Fit</td>
<td>A06, B09</td>
</tr>
<tr>
<td>Monte Carlo Simulations</td>
<td>Random Integers</td>
<td>A07, B10</td>
</tr>
<tr>
<td></td>
<td>Mixer/Sampler/Spinner</td>
<td>A08, B11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A09, B12</td>
</tr>
</tbody>
</table>

*Note.* The first two units indicated above occurred during the first several weeks of the course. The unit on Monte Carlo Simulations occurred during the last couple of weeks at the end of the course, prior to the semester exam.

*Figure 3.6.* Statistical unit and types of statistical representations addressed.
The Artifacts Used in the Classroom

Although each of the artifacts observed in this study permitted student work with univariate plots, bivariate plots, and statistical simulations, they differed in design and functionality. When viewed through a lens of instrumental genesis, the different design features may permit different forms of instrumentalization—the ways in which the technology shapes the students’ thinking and activity. Therefore, some of the key design features of the artifacts used in the classroom are discussed in this section.

The TI-73 handheld device was created specifically for middle school mathematics (Texas Instruments, 2016). In the area of statistics, it offers a list editor that is very similar to other TI graphing calculators, which permits common univariate displays, such as histograms, bar graphs, boxplots, and circle graphs. The TI-73 also permits scatterplots and the computation of simple statistics, such as measures of center. The types of plots permitted are fixed and accessed through a statistical plots menu (Figure 2.2). When creating plots, the user may adjust the viewing window manually or with one of several predefined zoom functions.

Although Tinkerplots permits the same types of displays as the TI-73, it was specifically designed to allow learners to explore (hence, tinker with) a wide range of statistical plots, including several that are uncommonly used or not used at all. For this reason, Bakker (2002) states that Tinkerplots is a landscape tool—that is, a tool that permits the user to make choices regarding which capabilities to use and when. In this sense, a landscape tool promotes user freedom and creativity. This design feature seems to encourage the conceptual aspects of instrumental genesis. For example, Tinkerplots boasts the affordance of dynamic manipulation, where the user may alter data values in tables (and the plots automatically adjust) or drag features of the plots (and the data values and plots automatically adjust). One might say that the
TI-73, given its lack of dynamic features and reliance on a smaller set of more conventional statistical plots, stands as less of a landscape tool than Tinkerplots.

Figure 3.7 shows an example of two unique representations permitted by Tinkerplots, each of which are non-standard and fail to meaningfully represent categorical data. As a result, these representations could be considered as false representatives (Hershkowitz & Kieran, 2001) that might suggest a lack of what Dick (2008) refers to as mathematical fidelity. With regards to instrumental genesis, this example of false representatives can be perceived from two perspectives. First, the affordance of false representatives in Tinkerplots may lead to technically oriented student struggles that could keep students from moving forward on tasks with the use of the artifact. In other words, false representatives could pose obstacles to students’ instrumental genesis. On the other hand, those linking instrumental genesis with constructivist-compatible models of instruction might note the affordances for deeper reasoning and the judicious use of artifacts that permit false representatives, which implies conceptual aspects of instrumental genesis. In either case, the implication is that the teacher needs to carefully guide students’ instrumental genesis to support both technical and conceptual aspects of instrumental genesis.

Figure 3.7. False representatives afforded by Tinkerplots with data on Titanic survivors.
Data Collection

Data collection occurred during the 18 days of instruction when the PSETs were using the artifacts. Even though the course met for 100 minutes per meeting, Jade did not have the PSETs use the artifacts for the entire time for each class session. On average, Jade led the PSETs through instrumented activity for 60 minutes on these days—thus, the total time observed in this study was 18 hours across the 18 days of observation. To examine aspects of Jade’s didactical performances (Figure 3.1) and respond to the research questions, data were collected during instructional time. Notes from direct observation, video data from three different camera angles, multiple sources of audio data, records of classroom items used, and member checks were used to form a palette of data to inform a response to the research questions. Note that the focus was on what was observable, rather than the teacher’s perceptions of her instruction. Figure 3.8 shows the configuration of the classroom and the devices used for data collection. The components represented in the figure will be discussed in the sections that follow.

The classroom was typically set up with large desks arranged in groups where eight students, forming two groups, sat near one another. At the front of the classroom, there was a projection screen, whiteboard, and instructor podium, which housed a computer, document camera, and equipment attached to the digital projector. A small, portable video camera, V1, was used to follow Jade as she moved about during class, particularly during her instrumental orchestrations with students working in small groups. A second, fixed, video camera, V2, was mounted from the ceiling to capture a dedicated video feed of information projected to the PSETs. Another video camera, V3, was fixed in place in the back of the classroom to capture a wide-angle view of the interactions that took place during class. Audio recorders were placed throughout the classroom to capture spoken utterances during instrumental orchestrations.
Figure 3.8. Classroom layout with data collection devices.

Video of the instructor: the conductor’s didactical performance. Given the critical role of the instructor’s actions in the process of instrumental orchestration, the researcher dedicated one video camera (V1 in Figure 3.8) to capturing Jade’s image and actions during the observations. In order to maintain focus on Jade and to document the artifacts being used during the small-group instrumental orchestrations, the researcher followed her with a video camera (V1). The researcher stayed to the side so that Jade could interact with PSETs who were working in the small groups. The researcher verbally confirmed with Jade that the observation process was not too intrusive. During whole-class instrumental orchestrations, the researcher stayed to the side or out of the way of student-student and teacher-student interactions. Video data
captured from this portable device provided information regarding Jade’s location in the classroom and her use of pedagogical moves during instrumental orchestrations (shown as part of the conceptual framework in Figure 3.1). Audio data captured from this device also served as an important source for verbal interactions between the PSETs and Jade during small-group instrumental orchestrations. Figure 3.9 shows an image collected from video camera V1 that was focused on Jade during one of the small-group instrumental orchestrations. In the image, Jade points at the PSET’s computer screen while discussing the different features of the artifact.

![Image of Jade pointing at the PSET's computer screen.](image)

*Figure 3.9. Jade during a small-group instrumental orchestration.*

**Video of the digital projector display.** At the front of the classroom, an instructor podium housed a computer and a document camera that were connected to the digital projector. The computer was used for Tinkerplots and the document camera was used to project images from the TI-73 Explorer. A second video camera (V2 in Figure 3.8) was mounted from the ceiling and was dedicated to capturing data presented on the digital projector during the instrumental orchestrations. Jade made frequent use of the projector during her whole-class
instrumental orchestrations, as suggested by Guin and Trouche (2002) and Drijvers et al. (2010), but she also used projected plots as representations for reference during small-group instrumental orchestrations. Additionally, Jade had groups of PSETs operate the projected artifacts publicly during some of the small-group activities. Data captured from this device was used to document the representations created and/or discussed during the instrumental orchestrations, which was particularly important during the First-chair instrumental orchestrations and Discuss-the-screen instrumental orchestrations, given their more student-centered nature. When Jade implemented artifact-specific pedagogical moves, it was critical to document the features and representations to which she or the PSETs were referring.

The projector video data also proved useful for capturing the in-the-moment dynamic changes that the PSETs and instructor made with Tinkerplots. Figure 3.10 shows an image taken from the camera focused on the digital projector during a moment when a PSET was making changes to the plots. Documenting the many representations that the PSETs generated was important as the whole-class had viewing access to the projector and all of the outputs displayed during any specific instrumental orchestration in the whole-class configurations. Also, since Jade used the outputs generated by the artifacts as objects for discussion, it was helpful to see which images Jade referenced.
Video of the didactical configurations. A third video camera (V3 in Figure 3.8) was placed on a tripod in the back of the classroom to capture a wide shot of the classroom. Data captured from this device were used primarily for documenting Jade’s didactical configurations (Figure 3.1) and changes in didactical configurations. This was important since changes in the didactical configurations signified the end of one instrumental orchestration (and sometimes the beginning of a new instrumental orchestration). Such changes included Jade changing her location in the classroom, changing the artifacts used, asking PSETs to join a particular group, and transitioning from small-group to whole-class and vice versa. This video data alleviated the need to repeatedly sketch classroom diagrams during the observations. Figure 3.11 shows an image taken during a transition from a First-chair instrumental orchestration in the whole-class setting to a Discuss-the-screen instrumental orchestration in the whole-class setting, as captured by the video camera in the back of the classroom (V3). A PSET (walking on the right of the screen) is shown leaving the podium, indicating a change in the didactical configuration and the end of the particular instance of the First-chair instrumental orchestration.
Documenting “cultural artifacts.” Qualitative research often includes consideration of cultural artifacts, or the items that people make and use (Creswell, 2007). The cultural artifacts present in the observed classroom included the textbook and tasks, artifact, and whiteboard. The video camera focused on Jade (V1, see Figure 3.8) was also used to capture video records of the tasks in the textbook, PSETs using Tinkerplots and the TI-73 in small groups, PSETs’ written work, and ideas written on the whiteboard. The wide shot camera captured transitions with the tasks and transitions with the technology. Only the PSET work that was completed or referenced during the instrumental orchestrations was considered as cultural artifacts for the study, whether recorded in their notebooks, on the whiteboard, or in another form. Concerning instrumental orchestrations and the conceptual framework used in the study, these artifacts were critically important in the Link-screen-and-board instrumental orchestrations and other instrumental orchestrations where the PSETs and/or Jade made references to the work done on paper or on the whiteboard. As with the images projected on the screen, Jade often referenced evidence of the PSETs’ written work, particularly in the small-group configurations. Selected frames from these videos, captured using the video camera V1 were extracted as images to document different
aspects of the cultural artifacts in the classroom. Figure 3.12 provides an example of an image that shows PSET work on paper, a specific task in the textbook, and Tinkerplots in use, as captured from the portable video camera (V1 in Figure 3.8).

![Figure 3.12. PSET work on Tinkerplots, a cultural artifact.](image)

**Audio of the instructor.** The third video camera (V3 in Figure 3.8), capturing a wide shot of the classroom, was equipped with a wireless lapel microphone receiver to capture high-quality audio data from Jade, regardless of where she was located in the classroom at any given moment. Jade wore a lapel microphone for each day of observation. This audio data was used as the primary audio for the transcription process, as Jade’s words proved paramount during analysis of Jade’s didactical performances during her instrumental orchestrations. Jade’s words almost always signaled the use of a particular pedagogical move related to discourse and/or the PSETs’ engagement in activity with the artifacts.

When Jade was located near PSETs, as during the small-group instrumental orchestrations, the PSETs’ voices were often also audible. This audio permitted transcription of both Jade’s audio and the PSETs’ audio, which helped to situate Jade’s use of different
pedagogical moves in the context of her didactical performances and also captured the PSETs’ contributions to those interactions.

**Audio of the First-chairs.** During *First-chair* instrumental orchestrations, the *First-chair* PSETs operated the artifact at Jade’s podium. In order to ensure accurate recordings of their audio, the researcher placed an audio recorder at the podium during the observation days. Data captured on this device were used to support the transcription process in moments when the *First-chair* PSET’s remarks were not clearly recorded by other devices. These data provided a clear reference point to what the *First-chair* PSETs said both before and after Jade’s use of particular pedagogical moves.

**Audio of the PSE Ts.** As a secondary source of audio data, the researcher placed four voice recorders in the classroom in the middle of each group of desks where the PSETs sat, worked collaboratively, and engaged in class discussions (see Figure 3.8). Data captured from these devices served as a backup when PSET voices were otherwise inaudible or when issues arose with the lapel microphone.

**Field notes.** Field notes were taken to document information related to Jade’s didactical configurations, exploitation modes, and didactical performances. The researcher used the classroom clock to record the times for different instrumental orchestrations. To help with synchronizing the notes with the video data, the researcher also noted the starting time of the video recorded during the observations. The field notes also documented shifts in the focus and pace of the lessons.

Even during times when no one was using the artifact, the researcher continued to note instances of Jade’s teacher moves and evidence of the classroom culture that was taking shape. This documenting served two purposes. First, these notes provided insight into whether Jade’s
teaching practice remained consistent with and without the active presence of the TI-73 and Tinkerplots. For instance, if there were discrepancies regarding Jade’s didactic configurations of the classroom and the PSETs when the artifact was used in comparison when the artifact was not used, they were worth noting and/or investigating. Concerning the conceptual framework (Figure 3.1), discrepancies with Jade’s exploitation of the didactical configurations and her didactical performances with or without the artifact present or being used were also noteworthy. In this respect, Jade’s teaching style appeared to be consistent as she incorporated discussions, both among students in small groups and within the whole-class environment; this occurred both when the artifact was not present and when it was present during instrumental orchestrations. Secondly, these additional observations provided additional potential data in the event that saturation was difficult to achieve. In the end, saturation occurred and the additional notes only served a minor role in the analysis process.

**Member checks.** To aid with trustworthiness of data collection and analysis (Creswell, 2007), the researcher regularly engaged in informal conversation with the participating instructor, typically at the end of a lesson, in order to verify what the researcher observed during the class. Thus, Jade was able to confirm, debate or clarify what the researcher had perceived related to her configurations of the artifacts and the students and the ways in which she exploited those configurations during her instrumental orchestrations.

**Data Analysis**

The researcher engaged in data analysis in order to focus on the characteristics of Jade’s instrumental orchestrations, including the types of instrumental orchestrations that Jade implemented and her use of different pedagogical moves as part of her didactical performances within her instrumental orchestrations (represented in Figure 3.1). The researcher sought to use
an ambitious amount of video and audio data. Previous studies on instrumental orchestrations have relied on transcript data to document and illustrate aspects of the interactions between the teacher and the students (Cayton, 2012; Drijvers et al., 2010). Also, the literature related to both IRE (Mehan, 1979) and teacher discourse moves (Herbel-Eisenmann et al., 2013) have made use of transcript data. For this study, transcriptions of vocalizations were necessary to capture nuances in details of what was said, while the video captured how it was said. In addition, the affordances provided by inexpensive audio and video recording, management, and production options have opened new doors for methods of analysis in qualitative research, which is paired with evermore-capable options in the form of qualitative data analysis tools (Creswell, 2007; Evers, 2011). In an effort to coordinate all of the audio and video data sources, the researcher devised a way to analyze transcript and video data simultaneously.

**Coordination of Data Sources**

By way of internet research, tinkering with video playback and screen recording via version 10.1 of the *Quicktime* application (Apple, 2011b), and script writing efforts via the *AppleScript* application (Apple, 2011a), the researcher devised an efficient scheme to synchronize the three video feeds with a transcript and render the data as a single video file for coding. In the upper-left quadrant, the transcript plays in synch with the three video feeds—the projector video (V2) in the upper-right quadrant, Jade and up-close video (V1) in the bottom right quadrant, and the wide shot video (V3) in the lower-left quadrant (see Figure 3.8 for classroom setup). The product of this scheme, which allowed the researcher to examine all the audio and video data sources simultaneously, is represented in Figure 3.13 below. The three phases of coding, discussed below, were performed on this composite data.
Unit of Analysis

In line with the conceptual framework (Figure 3.1), the unit of analysis chosen for this study is the instrumental orchestration (Drijvers et. al, 2010). This decision was made in response to Drijvers’ (personal communication, October 16, 2013) suggestion to identify each instance of an instrumental orchestration in the classroom in terms of its didactical configuration and exploitation mode (i.e., Drijvers et al., 2010; Drijvers et al., 2013). Thus, any time a change occurred in the didactical configuration or the exploitation mode, that change marked the beginning of a separate instrumental orchestration. Even when Jade implemented multiple, brief instrumental orchestrations as a sequence, the focus in this study was at the level of individual instrumental orchestrations. To this end, the researcher utilized the video data to identify the instrumental orchestrations in their relation to the timeline for each observed lesson. Figure 3.14 shows the timecodes for the beginnings of nine whole-class instrumental orchestrations that
occurred in a sequence on Day A03. During this sequence, there were clear changes in the didactical configuration—either asking a PSET to serve as the First-chair and go up to the front of the classroom to present or a First-chair PSET returned to their seat and Jade orchestrated a conversation about what was displayed on the projection screen. Therefore, each change in the didactical configuration during this sequence denoted the ending of one instrumental orchestration and the beginning of a new instrumental orchestration.

<table>
<thead>
<tr>
<th>Type of whole-class instrumental orchestration</th>
<th>Time during class when the instrumental orchestration began (h:mm:ss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-chair</td>
<td>0:40:08</td>
</tr>
<tr>
<td>Discuss-the-screen</td>
<td>0:40:56</td>
</tr>
<tr>
<td>First-chair</td>
<td>0:41:38</td>
</tr>
<tr>
<td>Discuss-the-screen</td>
<td>0:43:24</td>
</tr>
<tr>
<td>First-chair</td>
<td>0:43:53</td>
</tr>
<tr>
<td>Discuss-the-screen</td>
<td>0:44:35</td>
</tr>
<tr>
<td>First-chair</td>
<td>0:46:13</td>
</tr>
<tr>
<td>Discuss-the-screen</td>
<td>0:47:39</td>
</tr>
<tr>
<td>First-chair</td>
<td>0:48:45</td>
</tr>
</tbody>
</table>

Figure 3.14. Timecodes for a sequence of instrumental orchestrations.

In the subsections that follow, the three phases of coding are described. The researcher made a full pass through the entire data set during each phase and often recoded and revised the codebook at different points during each phase of coding. The iterative process of coding was drawn from methods of analytic induction and constant comparison (Creswell, 2007) as the researcher refined the codebook and new codes were added.

Coding phase 1: Instances of instrumental orchestrations. The video data were initially coded to identify instances of instrumental orchestrations. This facilitated the latter phases of coding and helped with sorting the data. During the first pass of coding for the
instances of instrumental orchestrations, the researcher used HyperResearch (ResearchWare, 2015), a qualitative data analysis tool, to code the synchronized video and transcript data, using a “whole-class instrumental orchestration” or a “small-group instrumental orchestration” code, to identify instrumental orchestrations that occurred in the whole-class or in small-group configurations, respectively. During this coding phase, the researcher watched the 18 hours of video/transcript data that were collected over the course of 18 days of instruction in which the participating instructor implemented instrumental orchestrations. While engaged in the preliminary coding, the researcher began to make use of the annotation feature in HyperResearch. Aspects of the instrumental orchestrations that indicated a particular type of instrumental orchestration were documented, which were pre-coding for the instrumental orchestration types (e.g., First-chair, Discuss-the-screen, Guide-and-explain). Also, aspects of Jade’s pedagogical moves were included in the annotations as a means to document some of the pedagogical moves that seemed most apparent (such as Jade’s use of wait time). By including these annotations, the researcher provided information to refer to when conducting additional rounds of coding. The researcher continually referred to the annotations while coding in an effort to promote internal consistency.

Coding phase 2: Instrumental orchestration types and pedagogical moves. During the second phase of coding, the researcher made another full pass through the 18 hours of video/transcript data. One emphasis during the second phase of coding was to categorize each instance of an instrumental orchestration by the type established by Drijvers et al. (2013, see Figure 2.6). The researcher annotated justifications for the use of each code to allow for comparison of codes and refinement of the codebook. Tentative new codes were added when Jade appeared to exhibit a previously unidentified type of instrumental orchestration. This
generative aspect of the coding process resembled what some qualitative researchers refer to as open coding (Creswell, 2007).

**Coding phase 3: Refining and finalizing the codes.** Following the second full round of coding, the researcher began to run coding reports and look for inconsistencies and other issues with the codes. The codes generated during the second phase were organized, refined, and reduced to help the researcher focus explicitly on types of instrumental orchestrations (Research Question 1) and pedagogical moves related to the PSETs’ use of the technologies (Research Question 2). When inconsistencies were identified with a code, the researcher went back through all instances of that code and made changes when necessary. The researcher continued this process of constant comparison until no inconsistencies remained among the instances of the codes. The annotations served as a critical referent to aid in identifying and rectifying inconsistencies. This process of constant comparison to manage and reduce the emergent codes was informed by methods often employed in grounded theory research (Creswell, 2007), which helped to set conditions for the researcher to develop an emergent theory related to the pedagogical moves observed (discussed in Chapter 5). The final sets of codes, which served to inform a response to each of the two research questions, are provided in Figure 3.2 and Figure 3.3 (Research Question 1) and Figure 3.4 and Figure 3.5 (Research Question 2). The new codes created as part of this study are described more fully in Chapters 4 and 5.

**Memos and Reports**

A running memo document was kept during data collection and analysis. During data collection, the researcher reflected on the items mentioned in the field notes, emerging trends that were noticed across days of observation, and progress towards saturation with the data. After the first few days of data collection, it was unclear whether the researcher would achieve
saturation. By the end of the observations, however, the researcher had documented several trends in the data and had noticed that Jade’s actions seemed to be stable and unchanging—hence, saturation was achieved.

During analysis, the researcher memoed while transcribing, coding, running coding reports, and drafting intermediate reports on the results from the study. When transcribing, the researcher memoed about emerging patterns and began an internal conversation with regards to conceptualizing and characterizing Jade’s actions related to the artifact that were observed in the data. While coding, the researcher memoed on emerging patterns within and across the codes.

When running coding reports and reflecting on this stage of analysis, the researcher compared the codes to the characterization of the *teacher discourse moves* that Herbel-Eisenmann et al. (2013) provided, and raised the question of fidelity in terms of characterizing pedagogical moves. As the researcher drafted reports on the findings related to the pedagogical moves in the data, the process of memoing continued to inform the work of analysis.

Once coding was complete, the researcher began writing reports about the different instrumental orchestrations and pedagogical moves observed. In the reports, the researcher drafted descriptions of the characteristics of the observed instrumental orchestrations and added excerpts from the transcripts and video data to provide examples to show the alignment between the descriptions and the actual data. These reports were revised and developed into the three results chapters that follow.

**Chapter Summary**

The conceptual framework (Figure 3.1) was explicitly designed as a lens to focus attention on Jade’s pedagogical moves during her instrumental orchestrations. This design helped set the conditions to address the research questions posed for the current study. Multiple video
and audio recording devices were employed during data collection to capture what Jade said, did, and referenced during her instrumental orchestrations. Transcripts of utterances were generated and embedded in video files that also featured the three video angles documenting the teacher, images projected on the screen at the front of the class, and a wide-angle perspective of the classroom space (Figure 3.8). The data compiled in this manner permitted the researcher to examine the didactical configurations in the classroom and Jade’s exploitation modes, vocalizations, gestures, and pedagogical moves during her instrumental orchestrations.

Analysis began with codes for the instrumental orchestrations, drawn from Drijvers and colleagues (2010, 2013). New codes were added as new themes related to Jade’s instrumental orchestrations and instrument-related pedagogical moves emerged. Through methods of constant comparison and analytical induction, the researcher combined codes and refined the codebook until stability was achieved (no new codes added, deleted, or modified). The process of annotating code instances supported constant comparison and memoing was used to help the researcher achieve saturation with the data and refine the codebook. The final codes were placed into three categories: types of instrumental orchestrations (to inform a response to Research Question 1) and pedagogical moves related to the PSETs’ use of the artifacts (to inform a response to Research Question 2). Analysis progressed as the researcher drafted and revised reports on the data associated with the research questions in order to provide descriptions of the observed phenomena related to pedagogical moves during Jade’s didactical performances. The revised reports were then added to the results, which follow. Each of the following two chapters provides a response to one of the two research questions investigated by the study, respectively.
CHAPTER 4
JADE’S ORCHESTRATION TYPES

This chapter serves as a response to the first research question: What types of instrumental orchestrations does one instructor use in a technological-tools-enhanced, reform-oriented statistics course for pre-service elementary teachers? In order to answer this question, the didactical configuration and exploitation modes of each instrumental orchestration, given in Figure 3.2 and Figure 3.3, were used to identify the different types of instrumental orchestrations implemented by Jade, the instructor of a Tinkerplots and TI-73-enhanced, reform-oriented statistics course for preservice elementary teachers (PSETs). The results presented in this chapter show the types of instrumental orchestrations Jade used, how she typically used them, and how often they were used.

This chapter is divided into three main sections: one section for instrumental orchestrations in the whole-class configuration, one section for instrumental orchestrations in the small-group configuration, and one section to discuss student-centered versus teacher-centered instrumental orchestrations. In addition to six of the eight whole-class instrumental orchestrations and four of the five small-group instrumental orchestrations identified by Drijvers and colleagues (2013), three new whole class instrumental orchestrations and two new small-group instrumental orchestrations are reported and described (see Figure 3.2 and Figure 3.3 for the full list of types of instrumental orchestrations).
Whole-Class Instrumental Orchestrations

Eight different types of whole-class instrumental orchestrations were observed in total. The five types of whole-class instrumental orchestrations (IOs) that were observed in this study and recognized by Drijvers and colleagues (2013) were the First-chair, Discuss-the-screen, Spot-and-show, Guide-and-explain, and Technical-demo IOs. The researcher used Drijvers and colleagues’ labels for instrumental orchestrations that had didactical configurations and exploitation modes consistent with those identified by Drijvers et al. There were several instances when Jade implemented a whole-class instrumental orchestration that had a didactical configuration or exploitation mode that did not fit with any of Drijvers and colleagues’ previously identified instrumental orchestrations. In these cases, a new type of instrumental orchestration was identified. There were three newly identified types of whole-class instrumental orchestrations: Board-with-tech-reference, Class-assist, and Discuss-artifact-use.

Over the 18 days of observation in this study, Jade implemented a total of 164 whole-class instrumental orchestrations.

Table 4.1 provides the frequency, minimum, maximum, mean, and total time for each of the whole-class instrumental orchestrations that Jade implemented, sorted by student-centered then teacher-centered instrumental orchestrations, then ordered by frequency. The whole-class instrumental orchestrations ranged from 12 seconds to 21 minutes and 57 seconds with an overall mean time of approximately 4 minutes, and the total time spent in whole-class instrumental orchestrations was about 11 hours 44 minutes and 50 seconds out of 30 hours of instruction. The frequencies of whole-class instrumental orchestrations (about 9 on average per day) seem attributed to two primary factors. First, Jade sometimes implemented short whole-class instrumental orchestrations (lasting less than 5 minutes). Secondly, when Jade changed the
didactical configuration or the exploitation mode, she transitioned from one instrumental orchestration into another.

This section is divided into three subsections. The first two subsections are used to present the student-centered whole-class instrumental orchestrations and the teacher-centered whole-class instrumental orchestrations that were observed, respectively. The last section shows the analysis for the whole-class instrumental orchestrations across the units of univariate statistics, bivariate statistics, and Monte Carlo simulations.

### Table 4.1

**Frequencies and Times for the Whole-Class Instrumental Orchestrations**

<table>
<thead>
<tr>
<th>Instrumental Orchestration Type</th>
<th>Frequency</th>
<th>Shortest Time</th>
<th>Longest Time</th>
<th>Mean Time</th>
<th>Total Time</th>
<th>% of Time&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-chair (SC)</td>
<td>63</td>
<td>0:00:24</td>
<td>0:21:57</td>
<td>0:04:11</td>
<td>4:23:52</td>
<td>37.44%</td>
</tr>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>34</td>
<td>0:00:30</td>
<td>0:18:16</td>
<td>0:03:59</td>
<td>2:15:32</td>
<td>19.23%</td>
</tr>
<tr>
<td>Class-assist&lt;sup&gt;a&lt;/sup&gt; (SC)</td>
<td>9</td>
<td>0:01:24</td>
<td>0:17:25</td>
<td>0:07:59</td>
<td>1:11:48</td>
<td>10.19%</td>
</tr>
<tr>
<td>Discuss-artifact-use&lt;sup&gt;a&lt;/sup&gt; (SC)</td>
<td>7</td>
<td>0:00:56</td>
<td>0:12:21</td>
<td>0:05:12</td>
<td>0:36:25</td>
<td>5.17%</td>
</tr>
<tr>
<td>Spot-and-show (SC)</td>
<td>1</td>
<td>0:05:30</td>
<td>0:05:30</td>
<td>0:05:30</td>
<td>0:05:30</td>
<td>0.78%</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>18</td>
<td>0:00:12</td>
<td>0:13:07</td>
<td>0:02:57</td>
<td>0:53:14</td>
<td>7.55%</td>
</tr>
<tr>
<td>Board-with-tech-reference&lt;sup&gt;a&lt;/sup&gt; (TC)</td>
<td>17</td>
<td>0:01:33</td>
<td>0:18:35</td>
<td>0:05:37</td>
<td>1:35:27</td>
<td>13.54%</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>15</td>
<td>0:00:27</td>
<td>0:07:20</td>
<td>0:02:52</td>
<td>0:43:02</td>
<td>6.11%</td>
</tr>
<tr>
<td>Total</td>
<td>164</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>11:44:50</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Note.** TC stands for teacher-centered and SC stands for student-centered. Drijvers and colleagues’ (2013) *Link-screen-board* and instrumental orchestrations were not observed. The total time represents the sum of the time spent implementing whole-class instrumental orchestrations, which was less than the 18 hours of video data collected. More than three hours was spent on small-group instrumental orchestrations and the remaining time was spent without instrumental orchestrations.

<sup>a</sup> This is a newly identified type of instrumental orchestration.

<sup>b</sup> These percentages are out of the total time spent in whole-class instrumental orchestrations.

### Student-Centered Whole-Class Instrumental Orchestrations

In this section, five student-centered whole-class instrumental orchestrations are discussed in order from most frequently to least frequently occurring. Two of the five student-
centered, whole-class instrumental orchestrations presented here are newly identified—the 
Class-assist IO and the Discuss-artifact-use IO.

**First-chair.** The student-centered *First-chair* IO was used in 63 (38.4%)\(^6\) of Jade’s whole-class instrumental orchestrations, which accounted for over four hours of instructional time (see Table 4.1). The didactical configuration for the *First-chair* IO included an artifact projected publicly with a *First-chair* PSET at the projection cart (see Figure 3.8) controlling the artifact in order to demonstrate a particular instrumented technique. The exploitation mode for the *First-chair* IO included Jade interacting with the *First-chair* PSET in order to set up what was done with the artifact, followed by Jade facilitating discussion among the PSETs related to the *First-chair* PSET’s instrumented activity.

Before implementing the *First-chair* IO, Jade typically observed PSET activity on a task while they were using Tinkerplots or the TI-73, in a way similar to what Drijvers and colleagues (2010) call the Work-and-walk-by instructional technique. Then, Jade identified the PSET or a group of PSETs to serve as the *First-chair* PSET(s) and informed them to prepare to go up to the front and share what they were able to do with Tinkerplots or the TI-73. Other times, while in the midst of a whole-class discussion, Jade invited PSETs to come up and show the class what they were talking about. In both of these cases, Jade had PSETs share their own thinking in a public setting, rather than Jade guiding the PSETs through a sequence of artifact actions. Also, some of the PSETs seemed more facile with their use of the artifacts. Although no formal measures were taken, it was clear that some PSETs were more likely to serve as *First-chairs*. PSETs less facile with the artifacts were more likely to be asked to share their instrumented activity during a

\(^6\) The percentages reported in reference to the total numbers of instrumental orchestrations is done in addition to the percentages in Table 4.1, which were based on duration. In general, the percentages were approximately the same.
Class-assist IO (described in a section below). As the First-chair PSETs demonstrated what they could do with the artifacts, Jade refrained from speaking until the PSETs finished their demonstration. On rare exceptions, Jade interrupted the presenting PSETs and asked them to rephrase or show again what they did, or asked polling questions to ensure that the other PSETs were following the presentation. Jade also invited the other students to ask questions directly to the First-chair PSETs. To conclude the instrumental orchestration, Jade would restate or summarize what the PSETs said. In sum, Jade’s actions placed emphasis on the First-chair PSETs’ thinking and provided opportunities for the other PSETs to engage in the conversation. These findings supported Drijvers and colleagues’ suggestion that “The goal [of this type of instrumental orchestration] is to enhance collective instrumental genesis” (p. 999).

**Discuss-the-screen.** The student-centered Discuss-the-screen IO was used in 34 (20.7%) of Jade’s whole-class instrumental orchestrations, which accounted for over 2 hours of instructional time (see Table 4.1). The didactical configuration for the Discuss-the-screen IO included Tinkerplots or the TI-73 projected publicly so that all PSETs could see it. The exploitation mode for the Discuss-the-screen IO included Jade orchestrating a discussion related to the projection screen. Jade invited PSETs to enter the conversation and engage with each other’s thinking.

The student-centered emphasis in the whole-class First-chair IOs was also evident in the Jade’s use of the Discuss-the-screen IO. In fact, 19 of the 34 instances of the whole-class Discuss-the-screen IO (55.9%), were preceded with a whole-class First-chair IO. Typically, Jade invited the PSETs to interpret what they saw on the screen, to respond to the First-chair PSET’s work represented on the screen, and/or to connect the projected representation to the task at hand. As a result, Jade used these instrumental orchestrations to focus on the PSETs thinking in
two dimensions. First, Jade initiated the conversation by referencing a PSET’s thinking and work. Second, the *Discuss-the-screen* IO provided a forum for the PSETs to enter the conversation and talk about each other’s work. The resulting interaction patterns in these instrumental orchestrations resembled the *star* pattern described by Nathan and Knuth (2003). Thus, not only were the PSETs engaged in statistical conversation and interaction with each other in the whole-class setting, but the seed for the conversation itself was rooted in a peer’s thinking and work. The way Jade used the *Discuss-the-screen* IO was consistent with the description provided by Drijvers et al. (2013), who stated that the goal of the *Discuss-the-screen* IO is the collective management of instrumental genesis (p. 209).

**Class-assist (newly identified).** The student-centered *Class-assist* IO was used in 9 (5.5%) of Jade’s whole-class instrumental orchestrations, which accounted for over 1 hour of instructional time (see Table 4.1). The didactical configuration for the *Class-assist* IO included an artifact projected publicly with a PSET controlling the artifact at the projection cart (see Figure 3.8), which is the same as the didactical configuration for the student-centered *First-chair* IO. The exploitation mode, however, is different, but still student-centered. The exploitation mode for the *Class-assist* IO included Jade having the PSETs tell their classmate at the projector what to do with the artifact, rather than Jade saying what to do with it. The assisting PSETs also provided rationale for why to use the artifacts in certain ways. Thus, Jade facilitated collaborative efforts regarding the technical and conceptual use of the artifact. Since the didactical configuration and exploitation mode for this instrumental orchestration represent a combination that was not previously identified, the Class-assist IO, is a newly identified type of student centered, whole-class instrumental orchestration.
The Class-assist IO represents an inversion of the roles within the First-chair IO. In the First-chair IO, the PSETs presenting their work were, in some way, identified as experts with regards to the artifacts and the authority transferred from Jade to the First-chair PSETs. In the Class-assist IO, however, the non-presenting PSETs shared the authority to help guide and instruct the presenting PSET.

The Class-assist IO seems to support productive struggle with the use of the artifacts (particularly for the presenting student). During five instances of this instrumental orchestration, Jade remarked that she wanted to be free from the projector and artifacts so that she could “float around” and give the PSETs space to carry the weight of the discussion or the instrumented activity. These remarks conveyed Jade’s intention that the Class-assist IO should be student-centered. As Jade orchestrated the discussion that ensued, she frequently paused, or refrained from speaking, reinforcing the expectation that the PSETs had to carry the conversation and help the presenting PSETs. As in the whole-class First-chair, Discuss-the-screen, and Spot-and-Show IOs, the goal of the Class-assist IO appears to be the collective management of instrumental genesis.

Discuss-artifact-use (newly identified). The student-centered Discuss-artifact-use IO was used in 7 (4.3%) of Jade’s whole-class instrumental orchestrations, which accounted for over 30 minutes of instructional time (see Table 4.1). The didactical configuration for the Discuss-artifact-use IO included Jade moving about the classroom with the PSETs facing each other. The exploitation mode for the Discuss-artifact-use IO included Jade facilitating a discussion about the use of the artifacts without directly using the artifacts. These discussions often related to what the PSETs would need to do before using the artifacts for a particular task or to considering the affordances and constraints of the different artifacts. The didactical
configuration and exploitation mode described here represent a combination that yields a student-centered whole-class instrumental orchestration not previously discussed in the literature. Thus, the *Discuss-artifact-use* IO is a newly identified type of instrumental orchestration.

During the *Discuss-artifact-use* IO, Jade emphasized key statistical ideas when using Tinkerplots or the TI-73 calculators and required the PSETs to discuss and communicate understanding of when, how, and why to select different plots. By framing the interactions in this manner, Jade was able to formatively assess student understanding and help the PSETs see and make connections between different types of plots. The focus was on the PSETs’ conceptual understanding related to the use of the artifacts, so the inferred goal of the *Discuss-artifact-use* IO is the collective management of instrumental genesis.

**Spot-and-show.** The student-centered *Spot-and-show* IO was used in 1 (0.6%) of Jade’s whole-class instrumental orchestrations, which accounted for over 5 minutes of instructional time (see Table 4.1). The didactical configuration for the single instance of the *Spot-and-show* IO included one PSET’s TI-84 graphing calculator at the projection cart in the front of the class, projected for all the PSETs to see. The exploitation mode for the *Spot-and-show* IO included Jade showing the artifact display publicly.

In this occurrence, Jade showed an error message that occurred when a PSET tried to plot a scatterplot, but the two lists of data did not have the same number of values (needed for making ordered pairs). Jade talked about the “dimension mismatch” error message and solicited some input from the PSETs. Jade mentioned how the error message was a common one, and then invited the PSETs to discuss and figure out how to resolve the issue, rather than directly telling the students how to remedy the error. By implementing the *Spot-and-show* IO in this manner, Jade used this whole-class instrumental orchestration in a way similar to her use of the *Discuss-
the-screen IOs and the First-chair IOs. For example, Jade paused multiple times to allow the PSETs time to think and add to the discussion and she probed for student thinking without giving away relevant terminology. Jade guided the discussion to a point where a PSET suggested the importance of “ordered pair” in the context of the bivariate data and the scatterplots that the class was trying to generate.

Teacher-Centered Whole-Class Instrumental Orchestrations

In this section, four teacher-centered whole-class instrumental orchestrations are discussed in order from most frequently to least frequently occurring. One of the four teacher-centered whole-class instrumental orchestrations presented here is newly identified—the Board-with-tech-reference.

Guide-and-explain. The teacher-centered Guide-and-explain IO was used in 18 (11.0%) of Jade’s whole-class instrumental orchestrations, which accounted for almost one hour of instructional time (see Table 4.1). The didactical configuration for the Guide-and-explain IO included access to Tinkerplots or the TI-73 and sometimes included an image of an artifact projected in the front of the classroom (see Figure 3.8). As the label suggests, the exploitation mode for the Guide-and-explain IO included Jade making guiding statements and explaining statements about the artifact. At times, Jade’s statements seemed to scaffold the PSETs’ activity. Jade did most of the talking and asked predominantly closed questions, rather than prompting the PSETs to discuss, argue, or share their thoughts or predictions.

In two-thirds of the instances of this teacher-centered, whole-class instrumental orchestration, Jade guided and gave instructions. In the other third of the instances, Jade guided, explained, and solicited PSET engagement with the use of questions, typically in the form of IRE. In both cases, the emphasis was on Jade’s efforts to guide the PSETs towards particular
uses of the artifacts. In other words, the focus was on technical aspects more than on conceptual aspects of the instrumented activity.

**Board-with-tech-reference (newly identified).** The teacher-centered Board-with-tech-reference IO was used in 17 (10.4%) of Jade’s whole-class instrumental orchestrations, which accounted for over 1 hour and 30 minutes of instructional time (see Table 4.1). The didactical configuration for the Board-with-tech-reference IO was similar to the didactical configuration for the Board-instruction orchestration type (Drijvers et al., 2013), and included Jade next to the whiteboard at the front of the classroom, while the PSETs remained in their seats, facing Jade. The exploitation mode for the Board-with-tech-reference IO included Jade talking with limited contributions from the PSETs.

Although the didactical configuration for the Board-with-tech-reference IO was the same as that for the Board-instruction IO, the exploitation mode was different because it included direct references to the artifact, whereas the Board-instruction IO (Drijvers et al., 2013) did not. Therefore, the Board-with-tech-reference IO is a newly identified type of instrumental orchestration. Additionally, the exploitation mode for the Board-with-tech-reference IO was in between those for the Board-instruction and the Discuss-the-screen IOs, and the pattern of interaction resembled a *hub and spoke pattern* (Nathan & Knuth, 2003), where the PSETs talked with Jade, but not each other. Thus, Jade’s Board-with-tech-reference was teacher-centered.

During the Board-with-tech-reference IO, Jade regularly went to the front of the class to help the PSETs summarize what they had done with Tinkerplots and/or the TI-73. During these instances, Jade wrote notes on the whiteboard and solicited PSET input about what they were able to do with the artifacts. Emphasis was placed on distilling the actions needed to successfully use the artifacts, discussing the pros and cons of the artifacts, or relating the use of the artifacts to
solving particular statistical problems. Jade also used the whole-class *Board-with-tech-reference* IO when she invited the PSETs to engage in the summarizing activity by sharing key ideas from the activities they had completed.

**Technical-demo.** The teacher-centered *Technical-demo* IO was used in 15 (9.1%) of Jade’s whole-class instrumental orchestrations, which accounted for over 40 minutes of instructional time (see Table 4.1). The didactical configuration for the *Technical-demo* IO included Tinkerplots or the TI-73 projected publicly from the projection cart (see Figure 3.8). Jade stood at the projection cart and controlled the artifact. The exploitation mode included Jade demonstrating a particular instrumented technique and talking about the technique during the demonstration. PSET interaction was limited during the instrumental orchestration. As was the case with the teacher-centered *Guide-and-explain* IO, the *Technical-demo* IO was more focused on technical aspects of the artifact use than on conceptual aspects of the PSETs’ work with the artifacts.

**Whole-Class Instrumental Orchestrations Across the Three Units**

Table 4.2 provides the distribution of whole-class instrumental orchestrations for each section of the observed course over the three statistical units of univariate displays, bivariate displays, and Monte Carlo simulations. These units are in chronological order, with the Monte Carlo simulations unit occurring at the end of the semester.

As can be seen in Table 4.2, three discrepancies are apparent with regards to the Monte Carlo simulations, as compared to the other two units. First, there is a noticeable decline for both classes in the frequency of the *First-chair* IO. Secondly, the frequency of Jade’s *Discuss-the-screen* IO drops to zero during the unit on Monte Carlo simulations. Thirdly, Jade used the *Guide-and-explain* IO more frequently during the unit on Monte Carlo simulations than she did
during the first two units combined. Amid the Monte Carlo simulations unit, Jade indicated, during member checks, the pressure that she felt to finish the content and help the PSETs prepare for the final exam. Additionally, Jade stated that, based on her previous experiences with the course, the PSETs would need more direct guidance in order to complete the tasks in the Monte Carlo simulations unit. It seems that the pressure. Jade felt with the impending semester end and her previous experiences influenced her use of instrumental orchestrations. As a result, Jade used fewer student-centered instrumental orchestrations, such as First-chair and Discuss-the-screen IOs, and she used more teacher-centered Guide-and-explain IOs.

Table 4.2 also shows that Jade used more whole-class First-chair IOs and Discuss-the-screen IOs with Class A than she did with Class B. Moreover, Jade’s whole-class First-chair IOs and Discuss-the-screen IOs were shorter, on average, in Class A than in Class B. Specifically, these instrumental orchestrations lasted for an average duration of 3 minutes in Class A and over 5 minutes in Class B. In other words, Jade used more of these shorter instrumental orchestrations in Class A, which underscores jade’s perception that the PSETs in Class A were less likely to engage in the conversations—so, it seemed that jade had to implement more instrumental orchestrations in Class A in order to keep the PSET conversations going. This finding suggests that teachers may need to implement different of more instrumental orchestrations with different sections of the same class, since the students’ participation and engagement can vary between different sections of a course.
Table 4.2

Frequencies of Whole-Class Instrumental Orchestrations by Unit

<table>
<thead>
<tr>
<th>Instrumental Orchestration Type</th>
<th>Class</th>
<th>Univariate Statistics</th>
<th>Bivariate Statistics</th>
<th>Monte Carlo Simulations</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-chair (SC)</td>
<td>A</td>
<td>12</td>
<td>16</td>
<td>8</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>10</td>
<td>12</td>
<td>5</td>
<td>27</td>
</tr>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>A</td>
<td>12</td>
<td>8</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Class-assist(^a) (SC)</td>
<td>A</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Discuss-artifact-use(^a) (SC)</td>
<td>A</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Spot-and-show (SC)</td>
<td>A</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>A</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Board-with-tech-reference(^a) (TC)</td>
<td>A</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>A</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>–</td>
<td>63</td>
<td>65</td>
<td>36</td>
<td>164</td>
</tr>
</tbody>
</table>

Note. TC stands for teacher-centered and SC stands for student-centered. The Link-screen-board instrumental orchestrations (Drijvers et al., 2013) were not observed.

\(^a\) This is a newly identified type of instrumental orchestration.

Small-Group Instrumental Orchestrations

Six different types of small-group instrumental orchestrations were observed in this study, two of which are considered newly identified. The four types of small-group instrumental orchestrations (IOs) that were observed in this study and recognized by Drijvers and colleagues (2013) were the Discuss-the-screen, Guide-and-explain, Technical-support, and Technical-demo IOs. As with the whole-class instrumental orchestrations, the researcher used Drijvers and colleagues’ labels for small-group instrumental orchestrations that had didactical configurations...
and exploitation modes consistent with those identified by Drijvers et al. When Jade implemented a small-group instrumental orchestration that had a didactical configuration or exploitation mode that did not fit with any of Drijvers and colleagues’ previously identified instrumental orchestrations, a new type of instrumental orchestration was identified. The two newly identified small-group instrumental orchestrations were the Students-choose-tech IO and the Talk-without-tech IO.

Over the 18 days of observation in this study, Jade implemented a total of 153 small-group instrumental orchestrations. Table 4.3 provides the frequency, minimum, maximum, mean, and total time for each of the small-group instrumental orchestrations that Jade implemented. The total time spent in small-group instrumental orchestrations was approximately 3.5 hours. The small-group instrumental orchestrations ranged from 12 seconds to 5 minutes and 39 seconds with an overall mean time of approximately 1 minute and 30 seconds. The overall mean time for the small-group instrumental orchestrations was less than the overall mean time for the whole-class instrumental orchestrations, which was approximately 4 minutes. These results confirmed Jade’s teaching style in that she wanted to engage her students in problem-solving activities and where she served as a guide to support her students’ activity with the use of the artifacts. Additionally, these results suggest that teachers in contemporary reform oriented mathematics classrooms may spend more time implementing whole-class instrumental orchestrations than small-group instrumental orchestrations, particularly when the teacher seeks to orchestrate whole-class discussions that include the students’ use of the artifacts.

This section is divided into three subsections. The first two subsections are used to present the student-centered small-group instrumental orchestrations and the teacher-centered small-group instrumental orchestrations that were observed, respectively. The last section shows
the analysis for the small-group instrumental orchestrations across the units of univariate statistics, bivariate statistics, and Monte Carlo simulations.

Table 4.3

*Frequencies and Times for the Small-Group Instrumental Orchestrations*

<table>
<thead>
<tr>
<th>Instrumental Orchestration Type</th>
<th>Frequency</th>
<th>Shortest Time</th>
<th>Longest Time</th>
<th>Mean Time</th>
<th>Total Time</th>
<th>% of Time&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Discuss-the-screen</em> (SC)</td>
<td>54</td>
<td>0:00:12</td>
<td>0:04:10</td>
<td>0:01:20</td>
<td>1:12:00</td>
<td>33.9%</td>
</tr>
<tr>
<td><em>Students-choose-tech</em>&lt;sup&gt;a&lt;/sup&gt; (SC)</td>
<td>5</td>
<td>0:00:33</td>
<td>0:02:45</td>
<td>0:01:20</td>
<td>0:06:40</td>
<td>3.1%</td>
</tr>
<tr>
<td><em>Talk-without-tech</em>&lt;sup&gt;a&lt;/sup&gt; (SC)</td>
<td>3</td>
<td>0:00:46</td>
<td>0:02:47</td>
<td>0:01:38</td>
<td>0:04:54</td>
<td>2.3%</td>
</tr>
<tr>
<td><em>Guide-and-explain</em> (TC)</td>
<td>77</td>
<td>0:00:12</td>
<td>0:05:39</td>
<td>0:01:31</td>
<td>1:56:47</td>
<td>55.1%</td>
</tr>
<tr>
<td><em>Technical-support</em> (TC)</td>
<td>12</td>
<td>0:00:13</td>
<td>0:02:27</td>
<td>0:00:38</td>
<td>0:07:36</td>
<td>3.6%</td>
</tr>
<tr>
<td><em>Technical-demo</em> (TC)</td>
<td>2</td>
<td>0:01:15</td>
<td>0:02:52</td>
<td>0:02:04</td>
<td>0:04:08</td>
<td>1.9%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>153</strong></td>
<td><strong>–</strong></td>
<td><strong>–</strong></td>
<td><strong>–</strong></td>
<td><strong>3:32:05</strong></td>
<td><strong>100.0%</strong></td>
</tr>
</tbody>
</table>

*Note.* SC stands for student-centered and TC stands for teacher-centered. Drijvers and colleagues’ (2013) *Link-screen-task* instrumental orchestration was not observed.

<sup>a</sup> This is a newly identified type of instrumental orchestration.

<sup>b</sup> These percentages are out of the total time spent in small-group instrumental orchestrations.

**Student-Centered Small-Group Instrumental Orchestrations**

In this section, three student-centered small-group instrumental orchestrations are discussed in order from most frequently to least frequently occurring. Two of the three student-centered, small-group instrumental orchestrations presented here is newly identified—the *Students-choose-tech IO* and the *Talk-without-tech IO*.

**Discuss-the-screen.** The student-centered *Discuss-the-screen IO* was used in 54 (35.3%) of Jade’s small-group instrumental orchestrations, which accounted for over one hour of instructional time (see Table 4.3). The didactical configuration for the *Discuss-the-screen IO* included the PSETs sitting in groups with access to the artifacts while engaged in the assigned tasks for a given day (see Figure 3.8). The exploitation mode for the *Discuss-the-screen IO* included Jade facilitating a conversation between members of a single group related to their
instrumented activity, rather than telling the PSETs what they needed to know or do to complete a task.

The nature of the discussions during small-group Discuss-the-screen IOs was slightly different than the whole-class Discuss-the-screen IOs. For example, in the whole-class setting Jade facilitated discussions and solicited multiple solution paths and/or uses of the artifacts that the PSETs had developed or used in their small groups. In the small-group setting, however, Jade spent time observing the PSETs during their collaborative activity and then facilitated discussions and probed into the PSETs’ thinking and actions in real-time. While observing the small groups, Jade frequently paused and stood quietly as the PSETs talked amongst themselves or operated the artifacts. In those moments, Jade did not interject, but allowed the PSETs to continue their work almost as if she was not present. Thus, by pausing and listening, Jade left the authority, decision-making, and problem solving up to the PSETs—this reflected the student-centered nature of the Discuss-the-screen IO.

**Students-choose-tech (newly identified).** The student-centered Students-choose-tech IO was used in 5 (3.3%) of Jade’s small-group instrumental orchestrations, which accounted for just over 6 minutes of instructional time (see Table 4.3). The didactical configuration for the Students-choose-tech IO included the PSETs sitting in groups with access to the artifacts while engaged in the assigned tasks for a given day. The exploitation mode for the Students-choose-tech IO included Jade inviting the PSETs to choose which artifact to use or paper and pencil. Jade expected the PSETs to think critically about which options would help the PSETs solve the problem at hand. Since the PSETs chose the artifact, this instrumental orchestration was considered student-centered.
Although the *Students-choose-tech* IO occurred infrequently, it seemed noteworthy because Jade explicitly invited the PSETs to choose which artifact(s) to use. Also, the Common Core State Standards for Mathematics (NGA & CCSSO, 2010) includes a Standard for Mathematical Practice, *Use appropriate tools strategically*, that specifically calls for students to strategically use appropriate tools, implying that students should learn how to choose from different tools at their disposal during their problem solving activities.

**Talk-without-tech (newly identified).** The student-centered *Talk-without-tech* IO was used in 3 (2.0%) of Jade’s small-group instrumental orchestrations, which accounted for approximately 5 minutes of instructional time (see Table 4.3). The didactical configuration for the *Talk-without-tech* IO included the PSETs sitting in groups with access to the artifacts while engaged in the assigned tasks for a given day. The artifacts were not used during this instrumental orchestration even though the PSETs had access to them. The exploitation mode for the *Talk-without-tech* IO included Jade facilitating small-group discussions about conceptual elements of the tasks at hand, which the PSETs needed to consider before utilizing the artifacts. Jade encouraged the PSETs to work together and build on each other’s thinking, rather than implementing the initiation-reply-evaluation sequence (IRE; Mehan, 1979) or other teacher-centered instructional techniques. This instrumental orchestration is considered student-centered since the PSETs interacted with each other in a *star* pattern (Nathan & Knuth, 2003) of discourse. Even though the PSETs did not directly use the artifacts during these moments, this form of instruction is included as an instrumental orchestration because Jade used it in a way that helped set the stage for the PSETs’ use of the artifacts, which is viewed as setting the stage for instrumental genesis to occur. For example, two of the instances occurred during the unit on Monte Carlo simulations. During the unit, Jade had the PSETs start by carefully describing the
context for each problem and setting of the model for running each simulation before reaching
for an artifact, since Jade has prior experience with students reaching for the artifacts too soon
before figuring out how they would use the artifacts. By using this instrumental orchestration,
Jade reinforced the necessity of her students’ engagement in conceptual aspects of their
mathematical work. Additionally, the PSETs referenced the models that they created when they
later used their artifacts to actually run the simulation—Jade helped the PSETs to avoid careless
use of the artifacts.

**Teacher-Centered Small-Group Instrumental Orchestrations**

In this section, three teacher-centered small-group instrumental orchestrations are
discussed in order from most frequently to least frequently occurring. All three were types of
instrumental orchestrations previously identified by Drijvers et al. (2013).

**Guide-and-explain.** The teacher-centered *Guide-and-explain* IO was used in 77 (50.3%)
of Jade’s small-group instrumental orchestrations, which accounted for almost two hours of
instructional time (see Table 4.3). The didactical configuration for the *Guide-and-explain* IO
included the PSETs sitting in groups with access to the artifacts (see Figure 3.8). The
exploitation mode for the *Guide-and-explain* IO included Jade making guiding and explaining
statements, typically to help the PSETs in the group move through and past an issue related to
the use of the artifact for the task at hand.

During the instances of the *Guide-and-explain* IO, Jade typically favored one of two
implementation schemes or some combination of the two. Jade either addressed an issue that
PSETs in a group were experiencing and guided them to overcome their mistake(s) with the
artifacts or she observed their work and guided them to consider alternative or more efficient
uses for the artifact. Since Jade guided the PSETs through their instrumented work, her Guide-and-explain IOs were teacher-centered.

**Technical-support.** The Technical-support IO was used in 12 (7.8%) of Jade’s small-group instrumental orchestrations, which accounted for 38 minutes of instructional time (see Table 4.3). The didactical configuration for the Technical-support IO included the PSETs sitting in groups with access to the artifacts while engaged in the assigned tasks for a given day. The exploitation mode for the Technical-support IO included Jade helping the PSETs in the group with a technical issue that was preventing them from fully engaging in the task at hand.

During all of the Technical-support IOs, a PSET from a group requested that Jade help them with an issue with Tinkerplots. Some of the technical issues included computer operating system questions, issues with downloading and installing Tinkerplots, and issues with locating the Tinkerplots files needed for the activities. The PSETs did not seem to have any technical issues with the TI-73 during the observed days, as Jade did not implement any Technical-support IOs in relation to the TI-73. Although the PSETs made the requests for help, Jade played an authoritative role in diagnosing and helping the PSETs overcome the technical issues. Because of this interaction structure, the Technical-support orchestrations were teacher-centered and focused almost exclusively on technical aspects of the PSETs’ interactions with the artifacts.

**Technical-demo.** The Technical-demo IO was used in 2 (1.3%) of Jade’s small-group instrumental orchestrations, which accounted for just over four minutes of instructional time (see Table 4.3). The didactical configuration for the Technical-demo IO included the PSETs sitting in groups with access to the artifacts while engaged in the assigned tasks for a given day. The exploitation mode for the Technical-demo IO included Jade operating the PSETs’ artifacts in order to demonstrate how to perform a particular instrumented technique. As Drijvers et al.
(2013) suggest, the Technical-demo IO in the small-group setting was essentially the same as the Technical-demo IO in the whole-class setting.

**Small-Group Instrumental Orchestrations Across the Three Units**

Table 4.4 provides the distribution of small-group instrumental orchestrations for each section of the observed course over the three statistical units of univariate displays, bivariate displays, and Monte Carlo simulations. Two discrepancies stand out in Table 4.4. First, Jade used the Discuss-the-screen IO in Class A 26 more times than she did in Class B, as can be seen from the Total column in the table. This finding is more pronounced since Class A met two times less than did Class B. Secondly, Jade used the Guide-and-explain IO in Class B 25 more times than she did in Class A. In both cases, this difference in Jade’s use of these instrumental orchestrations is the most prominent during the middle unit on bivariate statistics. During this unit, it appeared that the PSETs in Class A were generally more successful with the artifacts than the PSETs in Class B. In response to the PSETs’ success in Class A, Jade used the Discuss-the-screen IO with the small groups in order to listen and learn about the PSETs’ thinking and instrumented techniques. In contrast, since the PSETs in Class B struggled more with the artifacts during the unit on bivariate statistics, Jade used the Guide-and-explain IO to help direct the PSETs towards more effective use of the artifacts. Essentially, when the PSETs were having more trouble with the artifacts, Jade had to help them more so that they could move forward, and when the PSETs were doing well with the artifacts, Jade could spend more time engaging the PSETs in conversations about their work.

Similar to what was observed with the whole-class Guide-and-explain IO, there was an increase in Jade’s use of the small-group Guide-and-explain IO in Class A during the Monte Carlo simulations unit, the last unit of the semester. As noted in the whole-class instrumental
orchestration section, this observation makes sense given that Jade expressed feeling pressured to finish the material. Also, from previous experiences, she expected that students would need more guidance during the Monte Carlo simulations.

Table 4.4

Frequencies of Small-Group Instrumental Orchestrations by Unit

<table>
<thead>
<tr>
<th>Instrumental Orchestration Type</th>
<th>Class</th>
<th>Unit</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Univariate Statistics</td>
<td>Bivariate Statistics</td>
<td>Monte Carlo Simulations</td>
<td>Total</td>
</tr>
<tr>
<td><strong>Discuss-the-screen (SC)</strong></td>
<td>A</td>
<td>8</td>
<td>24</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td><strong>Students-choose-tech&lt;sup&gt;a&lt;/sup&gt; (SC)</strong></td>
<td>A</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Talk-without-tech&lt;sup&gt;a&lt;/sup&gt; (SC)</strong></td>
<td>A</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Guide-and-explain (TC)</strong></td>
<td>A</td>
<td>4</td>
<td>5</td>
<td>17</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5</td>
<td>23</td>
<td>23</td>
<td>51</td>
</tr>
<tr>
<td><strong>Technical-support (TC)</strong></td>
<td>A</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td><strong>Technical-demo (TC)</strong></td>
<td>A</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>–</td>
<td>32</td>
<td>68</td>
<td>53</td>
<td>153</td>
</tr>
</tbody>
</table>

*Note. TC stands for teacher-centered and SC stands for student-centered.

<sup>a</sup> This is a newly identified type of instrumental orchestration.

**Student-Centered Versus Teacher-Centered Instrumental Orchestrations**

In this section, results related to student-centered and teacher-centered instrumental orchestrations are discussed and compared. Jade’s use of whole-class instrumental orchestrations is summarized first, followed by her small-group instrumental orchestrations.

Jade favored the student-centered instrumental orchestrations over the teacher-centered instrumental orchestrations in the whole-class setting (see Table 4.1). Of the 164 whole-class
instrumental orchestrations, Jade implemented 114 (69.5%) student-centered instrumental orchestrations. Of the 11 hours and 44 minutes of whole-class instrumental orchestrations, Jade used the student-centered instrumental orchestrations for more than twice the total time that she spent in small-group instrumental orchestrations. The two whole-class instrumental orchestration types that she used the most were the student-centered *First-chair* and *Discuss-the-screen* IOs. Thus, not only did Jade use more student-centered, whole-class instrumental orchestrations, she spent more than twice as much time using them compared to teacher-centered, whole-class instrumental orchestrations. These findings align with what Jade had informally shared with the researcher regarding her perception of the importance of eliciting and supporting student thinking in the mathematics classroom.

In contrast to Jade’s preference for student-centered instrumental orchestrations in the whole-class setting, Jade favored the teacher-centered instrumental orchestrations in the small-group setting (see Table 4.3). Of the 153 small-group instrumental orchestrations, Jade implemented 91 (59.5%) teacher-centered instrumental orchestrations. Specifically, 77 (84.6%) of the 91 small-group, teacher-centered instrumental orchestrations were the *Guide-and-explain* IO. Of the nearly 3 hours and 32 minutes of small-group instrumental orchestrations, Jade spent about 2 hours and 8 minutes implementing teacher-centered, small-group instrumental orchestrations and only about 1 hour and 24 minutes implementing student-centered, small-group instrumental orchestrations. In other words, about 60% of the time Jade spent implementing small-group instrumental orchestrations she was using teacher-centered, small-group instrumental orchestrations. Thus, not only did Jade use more teacher-centered, small-group instrumental orchestrations, she spent more time using them compared to student-centered, small-group instrumental orchestrations. At the surface level, this finding seems to contradict
Jade’s student-centered learning goals. However, a closer look at Jade’s most frequently implemented teacher-centered, small-group instrumental orchestration shows how Jade used teacher-centered instrumental orchestrations to support a student-centered environment. The implication is that instruction and instrumental orchestrations may be perceived as teacher-centered or student-centered based on one’s perspective. Zoomed in on the small-group instrumental orchestrations themselves, Jade’s instruction appeared teacher-centered. However, when zooming out and situating Jade’s small-group instrumental orchestrations in the context of Jade’s efforts to engage the PSETs in whole-class discussions, one may perceive of her small-group instrumental orchestrations as helping to set the stage for the student-centered conversations. This seemed to be the case when Jade served as a guide to help the PSETs navigate technical hurdles so that they could continue with their conceptual and technical work with the artifacts.

The single most frequently occurring small-group instrumental orchestration was the *Guide-and-explain* IO, which accounted for 77 or 50.3% of the small-group instrumental orchestrations and a total time of over 1 hour and 56 minutes. Jade’s preference for the *Guide-and-explain* IO in the small-group setting paralleled one of Drijvers and colleagues’ (2013) findings, “The data show that the Guide-and-explain orchestration accounts for the majority of the observations . . .” (p. 993). Although the “guiding” nature of this instrumental orchestration rendered it teacher-centered, the *Guide-and-explain* IO seemed less teacher-centered than the *Technical-demo* IO or the *Explain-the-screen* IO. One reason for this is that, during the *Guide-and-explain* IOs, the PSETs continued to operate their artifacts. A second reason is that Jade used the *Guide-and-explain* IO in ways similar to those described by Drijvers and colleagues, who stated, “The exploitation mode [for the *Guide-and-explain* IO], however, holds the middle
between [the teacher-centered] Explain-the-screen and [the student-centered] Discuss-the-screen . . .” (p. 999). Jade incorporated some of the PSETs’ thinking and she asked questions during the Guide-and-explain IOs—she was not simply telling or lecturing nor was she facilitating a rich, PSET-led mathematical conversation.

Overall, Jade’s use of instrumental orchestrations was consistent with her student-centered views and beliefs about mathematics education. In the whole-class setting, Jade took time to orchestrate discussions that drew heavily on the PSETs’ thinking and interactions with each other. Although not as frequently, Jade also implemented student-centered instrumental orchestrations in the small-group settings, promoting peer-to-peer interactions. When Jade implemented teacher-centered instrumental orchestrations, whether in the whole-class or small group settings, she helped the PSETs to move past technical issues or obstacles related to the tasks at hand. This allowed the PSETs’ to return to their student-centered activities. Therefore, Jade’s teacher-centered instrumental orchestrations seemed to complement the student-centered activities in the classroom.

Chapter Summary

In her technological-tools-enhanced, reform-oriented statistics course for pre-service elementary teachers, Jade implemented a total of nine types of whole-class instrumental orchestrations, three of which were newly identified, and six types of small-group instrumental orchestrations with two being newly identified. The following summarizes key outcomes related to the types of instrumental orchestrations Jade implemented.

In the whole-class setting, Jade used more student-centered instrumental orchestrations, including the Discuss-the-screen IO and the First-chair IO (Table 4.1), than teacher-centered instrumental orchestrations. During these instrumental orchestrations, Jade conducted statistical
conversations where she encouraged the PSETs to enter the discussion and engage with each other’s thinking. As a result, multiple instrumented techniques were often discussed for the tasks that the PSETs were solving.

The three newly identified whole-class instrumental orchestrations were the student-centered Class-assist IO, the student-centered Discuss-artifact-use IO, and the teacher-centered Board-with-tech-reference IO. Since the Class-assist IO and the Discuss-artifact-use IO are considered student-centered with respect to the PSETs’ active roles in the discussions, these instrumental orchestrations may be useful for educators seeking to implement or promote student-centered instruction. The Class-assist IO and the Discuss-artifact-use IO each seemed to favor the collective activity with the artifacts in ways not previously identified in the literature. In particular, the Class-assist IO favored collective engagement by having the PSETs tell a classmate what actions to take with the artifact and why; the Discuss-artifact-use IO favored broader discussions about how to use the artifacts to solve the tasks at hand. Jade also implemented the Board-with-tech-reference IO, although structured more as a teacher-centered intervention, in ways that solicited and incorporated the PSETs’ thoughts and experiences. Much like the Guide-and-explain IO, the Board-with-tech-reference IO seemed to hold the middle between a student-centered and a teacher-centered instrumental orchestration. Jade’s authoritative role as a guide is the characteristic that distinguishes the Board-with-tech-reference IO as teacher-centered.

In the small-group setting, Jade used more teacher-centered instrumental orchestrations, including the Guide-and-explain IO (Table 4.3), than student-centered instrumental orchestrations. During the Guide-and-explain IO, Jade often guided the PSETs in their instrumented activity to help them move forward with their work. When Jade finished the small-
group instrumental orchestrations, she left the groups so that the PSETs could continue on with their investigations. Hence, Jade was able to use teacher-centered instrumental orchestrations to promote student-centered activity with the artifacts.

The two newly identified small-group instrumental orchestrations were the student-centered *Students-choose-tech IO* and the student-centered *Talk-without-tech IO*. In the *Students-choose-tech IO*, the PSETs were challenged to think critically, discuss, and choose appropriate tools for the tasks at hand. In the *Talk-without-tech IO*, Jade conducted a discussion related to the artifacts without actually using the artifacts—often focusing on conceptual components that would help the PSETs use the artifacts more successfully. As with the newly identified instrumental orchestrations in the whole-class setting, these small-group instrumental orchestrations show new options for teachers to consider when they seek to implement student-centered instruction with artifacts.

The language of instrumental orchestrations may be used to enrich discussions regarding teacher-centered and student-centered learning environments or traditional-transmission versus *constructivist-compatible* instruction, even though there does not appear to be a direct correspondence between teacher-centered instrumental orchestrations and traditional transmission models of instruction. For example, Jade used seemed to use teacher-centered instrumental orchestrations in ways that fit with her student-centered instruction. The data do seem to suggest that the student-centered instrumental orchestrations fit well with constructivist-compatible instruction. The results presented in this chapter show different instrumental orchestrations that teachers may have in their repertoires of teaching techniques for supporting technology as artifacts in the mathematics classroom. While Chapter 4 focused on Jade’s choices for her didactical configurations and exploitation modes during the preparation phase, Chapter 5
focuses on the pedagogical moves that Jade used during her didactical performances as she enacted her instrumental orchestrations (see Figure 3.1). Emphasis is placed on the nature of these pedagogical moves in relation to teacher-centered and student-centered instruction.
CHAPTER 5

JADE’S ARTIFACT-RELATED PEDAGOGICAL MOVES

This chapter serves as a response to the second research question: *What types of technology-related pedagogical moves does the instructor implement during her didactical performances in the course?* In order to answer this question, Jade’s didactical performances—the enactment of her instrumental orchestrations—were analyzed to identify the different types of instrument-related pedagogical moves that she implemented. As state previously, an instrument includes both an artifact and the user’s mental schemes for working with the artifact—the artifacts used in the classroom were the TI-73 and Tinkerplots. Also, instrumental genesis occurs when the artifact shapes the user’s understanding and the user shapes the artifact (Guin & Trouche, 2002; Drijvers & Trouche, 2008). This chapter focuses on evidence of the types of instrument-related pedagogical moves observed in Jade’s classroom, which includes coded transcript excerpts, notes on the codes applied, and frequencies of Jade’s use of the different pedagogical moves.

The results in this chapter are framed with a more narrow scope than the results in Chapter 4, since the instrument-related pedagogical moves used during instruction are viewed as components of Jade’s didactical performances within her instrumental orchestrations (see Figure 3.1). These results are divided into two sections pertaining to the types of instrument-related pedagogical moves that Jade used—one section contains the *student-centered instrument moves* and another section contains the *teacher-centered instrument moves*. Student-centered instrument
moves (SCIMs)\(^7\) are pedagogical moves that a teacher uses to open up opportunities for students to critically examine or judiciously use an artifact. Teacher-centered instrument moves (TCIMs)\(^8\) are pedagogical moves that a teacher uses to directly lead or restrict students in their use of an artifact and student decisions and explorations with the artifacts are not emphasized. This chapter concludes with a reflection on these instrument-related pedagogical moves and profiles of Jade’s didactical performances within her instrumental orchestrations. Before moving to the results related to Jade’s didactical performances, general patterns related to Jade’s use of cues to initiate her didactical performances are briefly mentioned in the section that follows.

**Jade’s Use of Cues**

Although not a focal point in the analysis, the researcher noted general patterns concerning Jade’s use of cues to transition from her preparation phase into her performance of her instrumental orchestrations. Often, Jade made a verbal remark to signify the transition into the performance phase. For the *First-chair-at-work* and the *Class-assist* IOs, Jade’s cue was a request to have a PSET go to the front of the class and interact with the artifact so that the rest of the PSETs could observe and or discuss what their peer and the artifact were doing and the representations they were creating. For the *Discuss-the-screen* IOs, Jade made a statement to denote the beginning of the conversation or an invitation to have a PSET share their thoughts about what the artifact was showing on the screen. To cue her teacher-centered instrumental orchestrations, Jade cued the didactical performance when she started talking to the PSETs. As part of the cue for the *Board-with-tech-reference* IOs, Jade walked to the whiteboard at the front

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\(^7\) As a convention, this acronym is used when paired with a specific student-centered instrument move, as in the case of the requesting SCIM. Elsewhere, the entire phrase is used.

\(^8\) As a convention, this acronym is used when paired with a specific teacher-centered instrument move, as in the case of the directing TCIM. Elsewhere, the entire phrase is used.
of the classroom and to cue a *Technical-demo* IO Jade walked to the front of the classroom and began to interact with a publicly-viewable artifact.

In the small-group settings, Jade first moved close to a particular group, implementing the *work-and-walk-by* (Drijvers et al., 2010). Jade cued the performances by asking questions related to the PSETs’ instrumented activity, responding to PSETs’ requests for guidance or assistance, or by making requests to have the PSETs interact with the artifacts and/or each other while she continued to observe their activity.

**Student-Centered Instrument Moves**

Results reported in Chapter 4 underscore Jade’s persistent efforts to support student-centered mathematical activity. Her two most frequently used instrumental orchestrations (IOs) in the whole-class setting were the student-centered *First-chair* IO and the student-centered *Discuss-the-screen* IO (Table 4.2), reflecting her efforts to support a classroom environment consistent with the tenants of contemporary reform efforts in mathematics education. Further analysis of Jade’s didactical performances revealed that she used at least one *student-centered instrument move* in 66.5% of her instrumental orchestrations. In this section, the 10 types of *student-centered instrument moves* observed in Jade’s classroom are discussed in relation to the different types of instrumental orchestrations in order from most frequently to least frequently used (see Figure 5.1). This section concludes with a discussion of the types and frequencies of *student-centered instrument moves* (SCIMs) that were used in each of the three statistics units that were observed.
Student-centered instrument moves

<table>
<thead>
<tr>
<th>Instrumental Orchestration</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Requesting</td>
</tr>
<tr>
<td>First-chair (SC)</td>
<td>63</td>
</tr>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>34</td>
</tr>
<tr>
<td>Class-assist (SC)</td>
<td>9</td>
</tr>
<tr>
<td>Discuss-artifact-use (SC)</td>
<td>7</td>
</tr>
<tr>
<td>Spot-and-show (SC)</td>
<td>1</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>18</td>
</tr>
<tr>
<td>Board-with-tech-reference (TC)</td>
<td>17</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>15</td>
</tr>
</tbody>
</table>

Types of Instrumental Orchestrations

| Whole-class Total           | 164       | 102     | 51      | 27      | 26       | 33       | 17       | 24       | 17       | 14       | 16       |
| Discuss-the-screen (SC)     | 54        | 28      | 19      | 10      | 7        | 1        | 2        | 1        | 1        | 3        | 1       |
| Students-choose-tech (SC)   | 5         | –       | 1       | –       | –        | –        | –        | –        | –        | –        | 1       |
| Talk-without-tech (SC)      | 3         | 1       | –       | –       | –        | –        | –        | –        | –        | –        | –       |
| Guide-and-explain (TC)      | 78        | 19      | 17      | 2       | 4        | –        | 2        | 1        | 5        | 2        | –       |
| Technical-support (TC)      | 12        | 2       | 1       | 1       | –        | –        | 6        | –        | –        | –        | –       |
| Technical-demo (TC)         | 1         | –       | –       | –       | –        | –        | –        | –        | –        | –        | –       |

Small-group Total

| 153 | 50 | 38 | 13 | 11 | 1 | 10 | 2 | 6 | 6 | 1 |

Total

| 317 | 152 | 89 | 40 | 37 | 34 | 27 | 26 | 23 | 20 | 17 |

Note. The values indicate that a particular student-centered instrument move occurred at least once in an instance of an instrumental orchestration. For example, the requesting SCIM occurred at least once during 48 of the 63 whole-class First-chair instrumental orchestrations. TC stands for teacher-centered and SC stands for student-centered.

Figure 5.1. Frequencies of student-centered instrument moves.

Requesting

The requesting SCIM occurred when Jade invited the PSETs to interact with, reflect on, or talk about their use of the artifacts. Since this pedagogical move focused on the PSETs’ experiences with the artifacts, it was student-centered and looked similar to the inviting TDM discussed by Herbel-Eisenmann et al. (2013)—the distinction is made with respect to the PSETs’
experiences with the artifacts instead of invitations for the PSETs to communicate with each other. The researcher chose to use the label *requesting*, rather than *inviting*, in order to specifically convey Jade’s authority and presumed intention—viewed as the conductor of the jazz orchestra, her *requests* were seen as polite alternatives to demands that the PSETs were expected to follow. Jade used the *requesting* SCIM more than any other student-centered *instrument move*. Jade implemented the *requesting* SCIM during 102 (62.2%) of the whole-class instrumental orchestrations and 50 (32.7%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the *requesting* SCIM every 12 minutes or almost eight times per class session, on average.

Jade used the *requesting* SCIM in several ways and it appeared that she had different goals for each variation of this pedagogical move. This finding seemed to parallel Herbel-Eisenmann et al.’s (2013) discussion of the *inviting* teacher discourse move, which may “take on multiple forms and address many goals” (p. 183). During the different *requesting* SCIMs, Jade requested that a PSET: show or demonstrate their work publicly and talk about their experience (*requesting demonstration*); interpret the output represented on the screen of the artifact (*requesting interpretation*); explore or tinker with aspects of an artifact (*requesting exploration*); offer artifact-related support to a peer (*requesting support*); explain their actions with the artifact (*requesting technique*); and make a prediction about what the artifact would create when a particular command is executed (*requesting prediction*). Figure 5.2 shows the frequencies of the requesting SCIMs. In the subsections that follow, the variations of the *requesting* SCIM are described in order from most frequently to least frequently used in Jade’s classroom.

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9 The class met for 100 minutes per day, twice per week. This includes time not spent in instrumental orchestrations.
### Requesting SCIMs

<table>
<thead>
<tr>
<th>Types of Instrumental Orchestrations</th>
<th>Instrumental Orchestration Frequency</th>
<th>Requesting demonstration</th>
<th>Requesting interpretation</th>
<th>Requesting exploration</th>
<th>Requesting support</th>
<th>Requesting technique</th>
<th>Requesting prediction</th>
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<tbody>
<tr>
<td><strong>Whole-class</strong></td>
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<td>First-chair (SC)</td>
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<td>Technical-support (TC)</td>
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<tr>
<td><strong>Total</strong></td>
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<td>72</td>
<td>65</td>
<td>33</td>
<td>14</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

*Note.* The values indicate that a particular type of *requesting* SCIM occurred at least once in an instance of an instrumental orchestration. For example, the *requesting demonstration* SCIM occurred at least once during 33 of the 63 whole-class *First-chair instrumental* orchestrations. For this reason, the totals for each type of instrumental orchestration in this figure may be greater than the totals shown in Figure 5.1. TC stands for teacher-centered and SC stands for student-centered.

*Figure 5.2.* Frequencies of the *requesting* SCIMs.
**Requesting demonstration.** The *requesting demonstration* SCIM occurred when Jade asked a PSET to show one of their instrumented techniques to their peers. Jade used the *requesting demonstration* SCIM during 52 (31.7%) of the whole-class instrumental orchestrations and 20 (13.1%) of the small-group instrumental orchestrations (Figure 5.2). Essentially, Jade implemented an instrumental orchestration that included the *requesting demonstration* SCIM every 25 minutes or 4 times per class session, on average. Figure 5.2 shows that Jade used the *requesting demonstration* SCIM in more than 50% of the whole-class *First-chair* IOs and more than 44% of the whole-class *Class-assist* IOs.

When implementing the *requesting demonstration* SCIM, Jade sometimes invited volunteers—“Do we have any volunteers who are willing to come up and help us do some things that they’ve discovered [with the artifact]?” (Day B08, 1:32:04). At other times, Jade called on someone in particular—“[To a particular PSET] I want you to remember this . . . on Monday, I want you to share how you were able to come up with this [by using the artifact]. . . . If you need to jot down some little notes that you can remember” (Day A02, 1:30:40).

**Requesting interpretation.** The *requesting interpretation* SCIM occurred when Jade asked a PSET to explain the meaning of the objects shown on the artifact screen. Jade used the *requesting interpretation* SCIM during 46 (28.0%) of the whole-class instrumental orchestrations and 19 (12.4%) of the small-group instrumental orchestrations (Figure 5.2). Essentially, Jade implemented an instrumental orchestration that included the *requesting interpretation* SCIM every 27 minutes or three times per class session, on average. Figure 5.2 shows that Jade used the *requesting interpretation* SCIM in more than 34% of the whole-class *First-chair* IOs and 35% of the whole-class *Discuss-the-screen* IOs.
In the transcript excerpt that follows, Jade initiated a small-group *Discuss-the-screen* IO by using the *requesting interpretation* SCIM to have the PSETs interpret what they saw from Tinkerplots. In order to explicitly support the discussion, Jade requested that the PSETs interpret the screen generated with Tinkerplots. In this case, a single utterance by Jade, “What are we seeing?” was used to mark the transition into a *Discuss-the-screen* IO. During this exchange, the PSETs were examining data about survivors on the Titanic. Multiple PSETs verbally responded, after Jade’s request, with an interaction pattern that resembled the *star* pattern discussed by Nathan and Knuth (2003).

Jade:  What are we seeing [on the screen]? [*requesting interpretation* SCIM]

(Jade pauses) [*waiting* SCIM]

Jon:  More female out of the female population survived. And more males.

Lucy:  There were males on the ship.

Jon:  Yeah, but they died.

Lucy:  – survived.

Kay:  (from another group) More – of the total number of females, a majority of them survived. Versus the total number of males.

(Day A05, 0:17:44)

**Requesting exploration.** The *requesting exploration* SCIM occurred when Jade asked a PSET to engage in unscripted activity with an artifact. Jade used the *requesting exploration* SCIM during 22 (13.4%) of the whole-class instrumental orchestrations and 11 (7.2%) of the small-group instrumental orchestrations (Figure 5.2). Essentially, Jade implemented an instrumental orchestration that included the *requesting interpretation* SCIM every 54 minutes or nearly twice per class session, on average.
In the example that follows, Jade implemented a whole-class Board-with-tech-reference IO and ended it by using the requesting exploration SCIM to get the PSETs use Tinkerplots to display the data for the hours worked by high school students.

Jade: Play a little bit. [requesting exploration SCIM]. Show your new group mate what you are able to create for the variable of “hours worked.” How many hours per week did these high school students work? . . . or if there are any other displays that we can create that might be useful for displaying this variable of “hours worked.”  

(Day A03, 12:00)

In another example, as shown in the transcript excerpt below, Jade used the requesting exploration SCIM in response to a PSET’s efforts to solicit her guidance. By turning the PSET’s question back around, Jade may have maintained the cognitive demand of the activity and the PSET-artifact interaction. This echoes one of the findings in Cayton’s (2012) study related to cognitive demand and technology-enhanced tasks. Jade could have simply implemented the Initiation-Reply-Evaluation sequence (IRE; Mehan, 1979) and evaluated the PSET’s response. Instead, however, Jade encouraged the PSET to explore the idea on her own.

Trinity: You need four children.

Neo: You need four.

Jade: We need four. So, how are we going to change this to get a family of four kids? (Jade waits)

Trinity: Add more pairs?

Jade: You can try it and see what happens. [requesting exploration SCIM]

Trinity: Okay.

(Day B11, 0:33:10)

Other instances of requesting. Although infrequent, Jade exhibited three more types of requesting SCIMs related to the PSETs’ experiences with the artifacts. First, she used the
requesting support SCIM in ten (6.1%) of the whole-class instrumental orchestrations and four (2.6%) of the small-group instrumental orchestrations. With the requesting support SCIM, Jade asked for a volunteer to help a peer with the use of the artifact, and this pedagogical move occurred in more than 55% of Jade’s whole-class class-assist IOs (Figure 5.2). Secondly, during nine (5.5%) of the whole-class instrumental orchestrations and three (2.0%) of the small-group instrumental orchestrations, Jade used the requesting technique SCIM when she asked a PSET to elaborate on how to perform a particular instrumented technique. Lastly, during three (1.8%) of the whole-class instrumental orchestrations and 0 of the small-group instrumental orchestrations, Jade used the requesting prediction SCIM to ask a PSET to make an educated guess as to what the artifact would show if a particular action was taken.

Waiting

The waiting SCIM occurred when Jade created a quiet moment of time in which the PSETs could engage in either instrumented activity or peer discussion that explicitly related to their instrumented techniques. Often, Jade implemented this student-centered instrument move by simply refraining from talking. This pedagogical move is called waiting because it functioned similarly to the pedagogical move of wait time (Stahl, 1994) or the waiting teacher discourse move (Herbel-Eisenmann et al., 2013), but with explicit attention to the PSETs’ activity with the artifacts. Jade used the waiting SCIM during 51 (31.1%) of the whole-class instrumental orchestrations and 38 (24.8%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the waiting SCIM every 20 minutes or almost five times per class session, on average.

Jade sometimes asked the PSETs a question before she implemented the waiting SCIM. At times, Jade implemented this pedagogical move by simply refraining from interrupting the
PSETs’ activity. For example, Jade implemented the waiting SCIM when PSET discussion led the PSETs to interact with the artifacts and Jade paused to let the PSETs continue their discussion and work with the artifacts.

To illustrate the potential power of the waiting SCIM, one of Jade’s First-chair IOs is described and exemplified by the transcript below. Figure 5.3 and Figure 5.4 show the histograms that the PSETs generated and discussed. During this instrumental orchestration, a pair of PSETs operated Tinkerplots on the computer that was publically projected and the First-chair PSETs generated histograms for the U.S. Students Work Hours sample data set that is included with Tinkerplots. In previous lessons, the PSETs learned about and discussed some of the conventions for creating histograms. Through discussions orchestrated by Jade, the PSETs arrived at a consensus that the lower boundary value for each bin in a histogram was inclusive and the upper boundary value for each bin (including the final bin within the viewing window) was exclusive. Although this choice would not have made a difference if the PSETs were using the TI-73, Tinkerplots is designed differently. The upper boundary on the final bin is inclusive by design (C. Konold, personal communication, April 18, 2015).

Jade:  Okay, so let’s listen to the ladies up there. [waiting SCIM]

Renee:  So, on my screen up there, it looks like this (Figure 5.3). And, I think Karley’s point is, though, that the data that we’re given goes up to 40, so that was the maximum. So, I mean, if you’re making a histogram – like if you are a student making a histogram, would you really go to 45 so there really is that gap between 35 and 40? Because, 40 is the maximum number in your list, so I don’t know how, if a student would make a graph that goes all the way to 45 to show that gap. I guess it depends. [waiting SCIM]

Jamie:  I think we’re talking about the gap between 30 and 40. Because, there’s like, the way you have it, you can see there’s no data there. But then, like the way we had it, originally, it looks like all of the bars were touching between 30 and 40 (Figure 5.4). I don’t think it’s between 40 and 45. Yeah, like that. So, I think it’s just—
[waiting SCIM]

Karley: Because, the way we learned how to make histograms, it is saying 35 to 40. Because, the boundary of 30 goes to the right. So, it’s going to be in the next bin. So, there’s really only the one bin that doesn’t have anything. And this way, Tinkerplots is throwing us off because they’re putting the boundary line into the bin to the left.

Jade: So, does everybody see what Karley is saying about Tinker – are you saying that Tinkerplots is getting this wrong? [voicing SCIM]

(Day B04, 1:04:45)

Figure 5.3. A histogram created with Tinkerplots (Day B04, 1:04:48).
Depending on how the PSETs configured the window settings, they ended up with two different looking histograms for the *U.S. Students Work Hours* data, which contained three data points with the value of 40, as discussed in the transcript above. Figure 5.3 shows a histogram with a gap in one of the intervals and the second image, Figure 5.4, shows a histogram without a gap in the same interval. The first histogram had a maximum value of 40 and the second had a maximum value of 45 while using the same bin width of 5. Tinkerplots included the values of 40 in the final bin for values in the range [35,40], instead of [35,40). Some of the PSETs noticed this apparent contradiction, and Jade capitalized on this as an opportunity for the whole-class to consider. Jade paused, implementing the *waiting* SCIM, and allowed the *First-chair* PSETs to share their instrumented thinking with the other students, both verbally and through the use of Tinkerplots (projected to the whole-class). Jade could have intervened, but she chose not to,
keeping the emphasis on the PSETs’ conceptual development and collaborations while working with the artifacts.

The transcript excerpt included above shows Jade promoting the status of the First-chair PSETs and their thinking, “Okay, so let’s listen to the ladies up there.” What followed was an exchange between the First-chair PSETs and another PSET (Jamie), where they discussed the relationship between the data and the resulting histograms. In other words, Jade’s use of waiting served two purposes: as a teacher discourse move (TDM; Herbel-Eisenmann et al., 2013) and as a student-centered instrument move. Using the waiting pedagogical move in this way provided the conditions favorable for PSET-PSET discourse and for the collective management of instrumental genesis. Jade was able to help the students lead themselves into a statistical conversation, with a pattern of discourse similar to the star pattern described by Nathan and Knuth (2003), rather than a hub and spoke pattern of interaction. This example seemed powerful in that the PSETs arrived at a discrepancy between Tinkerplots and histograms and Jade’s use of the waiting SCIM allowed the PSETs time to work through the issue.

Voicing

The voicing SCIM occurred when Jade stated or summarized one PSET’s instrumented activity—she gave “voice” to the PSETs activity. In effect, Jade seemed to raise or maintain the status of the PSETs’ thinking when she used this student-centered instrument move. Jade used the voicing SCIM during 27 (16.5%) of the whole-class instrumental orchestrations and 13 (8.5%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the voicing SCIM every 45 minutes or twice every class session, on average.
Jade seemed to use the voicing SCIM as an analog to the revoicing teacher discourse move (TDM)—the voicing SCIM was used to state something the PSETs did with the artifacts and the revoicing TDM was used to restate something that the PSETs had said during a conversation. Rather than expressing her own procedures or procedures dictated by curricular materials during the voicing SCIM, Jade illuminated the steps taken and/or interpreted what a PSET was thinking, providing potential insights into a PSET’s instrumented action schemes. When Jade restated instrumented work that PSETs showed and narrated, she effectively used the revoicing TDM and the voicing SCIM simultaneously. As with some of the other pedagogical moves, one pedagogical move served as both a teacher discourse move (Herbel-Eisenmann et al., 2013) and a student-centered instrument move. This student-centered pedagogical move seemed to help provide favorable conditions for the collective management of instrumental genesis.

On Day A05, while monitoring PSET work in small groups on a task comparing gender and survival rate on the Titanic, Jade noticed that Katie was attempting to combine variables on her circle graphs of the data (Figure 5.5). Later, during a whole-class instrumental orchestration with Rebecca serving as a First-chair PSET, Jade publicly referenced Katie’s attempt to display plots representing three variables (class, gender, and survival rates). Then, Jade used the voicing SCIM, by referring to Katie’s work, to transition into a new whole-class First-chair IO with Katie serving as a First-chair PSET to demonstrate her instrumented activity. Katie showed her method for generating her plot, and Autumn and another PSET engaged in a discussion related to Katie’s method and the resulting plot. Below is a transcript excerpt of this interaction.

Jade: And, I mean, there’s a very strong relationship that went all the way down. That we see that trend. And some people even combined both of these variables. Katie, you did that, right? Do you think you can show us what you did, up there (Katie moves to the front of the class)? [voicing SCIM, requesting demonstration SCIM] I shouldn’t say both—I should say all three—looking at both the impact of class and gender on survival rate (Katie manipulates projected computer—see
Figure 5.5) So, what did you think this would help us to see now, Katie?

(requesting prediction SCIM)

Katie: Well, like how many females to males survived. Depending on like what class they were in (Katie returns to her seat).

(Day A05, 0:42:44)

Figure 5.5. Circle graphs with the Titanic survivor data.

Probing

The probing SCIM occurred when Jade asked a PSET to elaborate on their instrumented techniques. In other words, the probing SCIM was a variant of the probing teacher discourse move (Herbel-Eisenmann et al., 2013) that specifically included the artifacts. Jade used the probing SCIM during 26 (15.9%) of the whole-class instrumental orchestrations and 11 (7.2%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the probing SCIM every 49 minutes or twice per class session, on average. Figure 5.1 shows that Jade used the probing SCIM across a variety of student-centered and teacher-centered instrumental orchestrations.
When using the probing SCIM, as in the case of the probing TDM, Jade often asked the question “why?” in order to request more information from a PSET. Examples of Jade’s use of the probing SCIM include the following: “So, why is it that, when you did it that way?” “So, what’s different, do you think?” “Why does [the current plot] bother you?” “What did you have to do to do that?” (Day A03, 0:18:10); “But, it looks like you’re—what made you think to look at ‘Job Hours’ and ‘Job Pay’?” (Day A03, 0:21:30); “So, can you tell me a little bit about how you’ve got this set up here?” (Day A08, 0:15:02); “Now, how—tell me a little bit about what’s happening when [Tinkerplots] runs [the simulation]?” (Day A08, 0:16:58); “But, I’m also seeing some differences in the kinds of results you’re pulling up. I think you might have changed some things. Do you want to talk about what you changed?” (Day B11, 0:20:20).

**Prompting Vocabulary**

The prompting vocabulary SCIM occurred when Jade asked the preservice elementary teachers (PSETs) to consider the meaning of mathematical terms that were used by the artifacts. Jade used the prompting vocabulary SCIM during 33 (20.1%) of the whole-class instrumental orchestrations and 1 (0.7%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the prompting vocabulary SCIM every 53 minutes or twice per class session, on average.

Since Jade was utilizing a problem and concept-oriented textbook, she had the PSETs engage their thinking in problem contexts before providing skills and terminology directly. Although no formal measures were taken, it seemed as though many of the tasks included in the textbook would be considered as high cognitive demand tasks (Stein et al., 1996). Along with this, Jade encouraged the PSETs to use their own words to talk about the concepts in univariate and bivariate statistics and Monte Carlo simulations. By using the prompting vocabulary SCIM
to get the PSETs to specifically consider and discuss the meaning of statistical terms related to their use of the artifacts, Jade promoted the collective management of concepts related to the artifacts. For this reason, the *prompting vocabulary* pedagogical move is considered student-centered. Jade had the PSETs use the artifacts to explore concepts as they came up in the problems that the PSETs worked through, which is also consistent with maintaining cognitive demand. This allowed the PSETs to work with the artifacts without needing to use specific mathematical or statistical terms.

In the example that follows, Jade used the *prompting vocabulary* SCIM by prompting the PSETs to deepen their conceptions of the term “circle graph” as part of a *Discuss-the-screen* IO. Jade prompted the PSETs to reflect on a univariate plot generated by Tinkerplots as they considered whether that particular plot should be considered a “circle graph.” Although Jade could have generated a similar plot by hand, she leveraged Tinkerplots’ ability to create nonstandard plots. The PSETs drew upon the affordances of Tinkerplots that permitted them to use a circle graph to represent a set of quantitative, rather than categorical, data. Jade opened the floor for discussion with a question.

*Jade*: Take 30 seconds, chat with each other [about the projected plot]. [*take 30 SCIM*]

(Jade waits to allow PSET-PSET conversation)

*Jade*: Thank you. You can go ahead and take a seat (Kim returns to her seat).

*Jade*: So, again, what—we’re trying to tease out these thoughts on—how does this measure up as a numerical display, compared to the others we have written in our dictionary? Are we gonna want to add “circle graph” to our dictionaries for numerical displays? [*prompting vocabulary SCIM*]

*PSETs*: No.

*Jade*: Oh, a lot of people are saying “no.”
In this example, the term *circle graph* was already in use in the classroom community. However, Jade pushed the PSETs to be more precise in their conceptions of the term—“Are we gonna want to add ‘circle graph’ to our dictionaries for numerical displays?” Rather than telling the PSETs that circle graphs work better for categorical data, Jade used the *prompting vocabulary* SCIM to explore the PSETs’ thoughts and allow them to defend their perspectives. By using the *prompting vocabulary* SCIM, Jade seemed to support the PSETs’ conceptual development related to the distinctions between categorical and numerical data sets and their respective displays. In this case, some PSETs noticed the issue with plotting quantitative data with a representation designed for use with categorical data. Jamie’s response reveals this understanding, which centered on the limitation that occurred—forcing quantitative data into a plot for categorical data can result in so many categories that the display loses meaning.

As the excerpt above shows, Jade utilized the *prompting vocabulary* SCIM as part of the ending component of the *take 30* pedagogical move. Since the PSETs were talking with each other about conceptual aspects of their work with Tinkerplots, supporting both student-centered discussion and the collective management of instrumental genesis. In this case, the source for the conversation came from the PSETs’ work and thinking with the use of the artifact. Here is a case where a *student-centered instrument* seemed to promote both the PSETs’ interactions with the artifacts and the PSETs’ engagement in the mathematical discussions. By drawing attention to specific vocabulary and concepts with this particular pedagogical move, Jade demonstrated one way that teachers can implement student-centered instrumental orchestrations and still guide the activity and discussion towards key concepts. Returning to the conceptual framework (Figure
3.1), the prompting vocabulary SCIM seems to be a technique that can be used, particularly when a teacher plans for students to build their thinking towards refined conceptions of key terms in a class. Jade’s use of this student-centered instrument move was noticeably different than if she had simply told the PSETs the difference between categorical and numerical data and between different plots that are used to represent data—doing so would have reflected an aspect of a traditional-transmission model of instruction (Ravitz et al., 2000).

**Buddy Up**

The buddy up SCIM occurred when Jade gave a direct, verbal statement to have PSETs form pairs in order to engage in instrumented activity together. Often, Jade used the explicit phrase “buddy up” when utilizing this student-centered instrument move. Jade used the buddy up SCIM during 17 (10.4%) of the whole-class instrumental orchestrations and 10 (6.5%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the buddy up SCIM every 67 minutes or approximately once per class session, on average.

From a general perspective, the buddy up SCIM can be used to configure the students in the classroom in a manner to promote discourse and collaboration, even without artifacts present. However, Jade implemented this student-centered instrument move to ensure that all PSETs were either using the artifacts directly or following along with a peer who was using the artifacts. Since the PSETs needed to turn to work with a peer or get up and move to a different location in the room in order to “buddy up,” Jade used this pedagogical move as a way to help set (or reset) the didactical configuration during her didactical performances. One apparent expectation of the buddy up SCIM was that the PSETs would work productively together and engage in conversation and collaboration while using their artifacts. Because of this expectation, the buddy
up SCIM seemed to promote the collective management of instrumental genesis in the small-group setting. The emphasis on collaboration was evident in the types of interactions that the PSETs had with each other while working in small groups. They shared their thinking, and worked through issues with the artifacts and with the investigation types of tasks they had to complete.

The data suggest that Jade also used the buddy up SCIM as an ad-hoc response to unanticipated events or to make adjustments during her didactical performances. Jade made statements such as, “Look around, if you didn’t bring a computer with Tinkerplots, find someone who did, and buddy up—to know some new people” (Day A02, 1:24:48), “So, you need to open up your computers, or buddy up . . .” (Day A09, 0:08:48), or “So, Heather, do you want to go work over there? That way it won’t be three of you on one [computer]” (Day B09, 0:12:21).

Introducing

The introducing SCIM occurred when Jade stood in front of the PSETs and gave a monologue about what would occur during the upcoming instrumental orchestration. As part of the introducing SCIM, Jade often referred back to the PSETs’ previous work and thoughts that they had shared in whole-class discussions, which underscored the student-centered nature of the classroom activities. For this reason, the introducing SCIM is considered student-centered.

Jade used the introducing SCIM during 24 (14.6%) of the whole-class instrumental orchestrations and 2 (1.3%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the introducing SCIM every 69 minutes or once every class session, on average. During 16 of the First-chair IOs, Jade introduced what the presenting PSETs would be sharing (e.g., sharing a method of generating a particular plot), thereby using the introducing SCIM. Jade also used the introducing SCIM by
telling the PSETs what the *First-chair* PSETs would be doing as they moved to the computer and document camera at the front of the classroom.

On day B03, during the univariate statistics unit, Jade had the PSETs work in small groups on Tinkerplots to explore different plots that they could generate with the artifact. While monitoring the PSETs, Jade *selected* and *sequenced* (Stein et al., 2008) who would present. Then, she transitioned into a whole-class discussion, composed of a string of three *First-chair* IOs. The transcript below shows Jade’s use of the *introducing* SCIM when she gave an introduction to this string of whole-class *First-chair* IOs.

Jade: Okay, so I’ll ask everybody to do that [reset their data plots]. Sorry, I know you had some nice displays. And then, together—several of you will [serve as *First-chair* PSETs and] lead us through some ways that we can organize this data. Some of these ways are gonna look familiar, some of them might be things that you’re—you haven’t seen before. Um, but please make sure you’re asking questions as they’re going and following along. So, at this place is everyone buddied up so that everyone has—like you’re not trying to look like five of you on one computer. [*introducing SCIM, buddy up SCIM*]

(Day B03, 1:21:56)

**Recalling**

The *recalling* SCIM occurred when Jade referenced the PSETs’ instrumented activity from a previous lesson or task. As in the *voicing* SCIM, the *recalling* SCIM was student-centered because Jade spoke about the PSETs’ experiences and instrumented techniques, rather than techniques she had prescribed. Jade used the *recalling* SCIM during 17 (10.3%) of the whole-class instrumental orchestrations and 6 (3.9%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the *recalling* SCIM every 78 minutes or once every class session, on average.

When using the *recalling* SCIM, Jade recalled both what the PSETs had done with the artifacts and the major takeaways from the prior activities and discussions. On Day A03, for
example, Jade referenced different univariate displays that the PSETs had generated. She used general statements and invited the PSETs to add in specific recollections and to reconnect their activity with relevant statistical terminology. By recalling the PSETs’ work and thinking, Jade used the recalling SCIM to support the student-centered learning environment. Jade also used the recalling SCIM as a means to establish a connection to the previous day of instruction and to introduce what the PSETs would do for the beginning of the current class session. In the following transcript excerpt, Jade used the recalling SCIM to build on the PSETs’ work from the previous day.

Jade: So, last time we took a look at these four different plots. I think we even started that the time before, but ran out of time. Four different displays that people had created about the pulse rate data. They were labeled A, B, C, D and you brought up points of things that you thought were effective about those and things that were bothering you about certain graphs and that helped us to define a new type of display that we can use with our numerical data. Does anybody remember here what that was called? [recalling SCIM]

Pressing

The pressing SCIM occurred when Jade challenged the PSETs to a deeper level of thinking about the use of the artifacts. Stein et al. (1996) remarked that “the presence in the environment of a sustained press for explanation, meaning, and understanding” is one key factor in maintaining cognitive demand during the enactment of mathematical tasks (p. 462). Although used infrequently, the pressing SCIMs seemed particularly well-suited to support cognitive demand with regards to the artifacts. For this reason, special attention is paid to the pressing SCIMs. These student-centered instrument moves are distinct from the probing SCIMs in that Jade was asking for more than an elaboration of the PSETs’ thoughts when she used the pressing SCIMs—Jade was prompting the PSETs to extend their thinking with the artifacts. Since this
pedagogical move focused on the PSETs’ experiences with the artifacts, it was student-centered.

Jade implemented the **pressing** SCIM during 14 (8.5%) of the whole-class instrumental orchestrations and six (3.9%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the **pressing** SCIM every 90 minutes or once per class session, on average.

Jade used the **pressing** SCIM in two ways. During the different **pressing** SCIMs, Jade challenged the PSETs to: critically consider the representations generated by the artifact (**pressing for scrutiny**) or to consider other instrumented techniques (**pressing for alternatives**). These two types of the **pressing** SCIM are discussed in the subsections that follow. Figure 5.6 shows the frequencies of the **pressing** SCIMs.

**Pressing for scrutiny.** The **pressing for scrutiny** SCIM was observed when Jade challenged the PSETs to explore and question the results provided by an artifact. Jade used the **pressing for scrutiny** SCIM during 9 (5.5%) of the whole-class instrumental orchestrations and 3 (2.0%) of the small-group instrumental orchestrations (Figure 5.6). Essentially, Jade implemented an instrumental orchestration that included the **pressing for scrutiny** SCIM every 150 minutes or once every other class session, on average.

In the transcript excerpt below, Jade invited PSETs to explore Tinkerplots’ ability to represent univariate data. Then, Jade used the **pressing for scrutiny** SCIM when she challenged the PSETs to scrutinize an output afforded by Tinkerplots. One group of PSETs investigated the **fuse circular** command, which permits a range of variants to circle graphs (see Figure 5.7). Next, Jade initiated a **First-chair** IO where one purpose was to have PSETs share what they found by exploring circle graphs. Jade knew that their plot, although afforded by Tinkerplots, was not useful since their data were numerical and not categorical. The presenting PSET demonstrated
Note. The values indicate that a particular pressing SCIM occurred at least once in an instance of an instrumental orchestration. TC stands for teacher-centered and SC stands for student-centered.

Figure 5.6. Frequencies of the pressing SCIMs.
Tinkerplots’ ability to dynamically manipulate a single circle graph into multiple circle graphs utilizing subsets of a univariate data set. Jade used the pressing for scrutiny SCIM when she asked PSETs to reflect on the costs and benefits of generating the plots that were presented. Specifically, by posing the question, “What do you think about using a circle graph to represent numerical data . . . ?” By using the pressing for scrutiny SCIM, Jade pushed the PSETs to critically examine the circle graph display that was afforded by Tinkerplots, but not available on the TI-73. This interaction is shown in the transcript below and Figure 5.7 shows the First-chair PSET’s circle graphs that she dynamically altered with Tinkerplots.

Jade: So, I’m gonna ask you to chat with each other. What do you think about using a circle graph to represent numerical data, as opposed to the other three plots that we’ve used so far? I mean, compare how the circle graph displays our numerical data, compared to our other plots that we have encountered so far. [pressing for scrutiny SCIM]

Kim: Yeah, that label don’t work.

Jade: Take 30 seconds, chat with each other. [take 30 SCIM]

(Jade waits to allow PSET-PSET conversation) [waiting SCIM]

Jade: Thank you. You can go ahead and take a seat.

(Kim returns to her seat as Jade transitions into a Discuss-the-screen IO)

(Day B4, 1:19:30)

Figure 5.7. A PSET's circle graphs with Tinkerplots (Day B4, 1:19:30).
When Jade stated, “You can go ahead and take a seat,” in the above transcript, she effectively used a cue to transition out of the First-chair IO and into a Discuss-the-screen IO. This transition occurred because the didactical configuration changed (Drijvers et al., 2010). As a result, Jade took the focus off of the First-chair PSET’s thinking and turned it toward the rest of the class and the goal of building consensus regarding the use of circle graphs. In the discussion that followed, the PSETs arrived at the conclusion that the plot was not helpful for displaying numerical data. Jade’s pedagogical moves here had the potential to be powerful because she was able to get the PSETs to struggle through the issue for themselves, rather than telling them about the issue and why it was an issue.

**Pressing for alternatives.** The pressing for alternatives SCIM occurred when Jade challenged the PSETs to not settle for just one way to use an artifact and to consider alternatives that might have more affordances for the PSETs’ mathematical activity. Jade used the pressing for alternatives SCIM during 5 (3.0%) of the whole-class instrumental orchestrations and 3 (2.0%) of the small-group instrumental orchestrations (Figure 5.6). Essentially, Jade implemented an instrumental orchestration that included the pressing for alternatives SCIM every 225 minutes or once every three class sessions, on average.

On the last three days of data collection in each of the classes, Jade had the PSETs explore concepts relating to Monte Carlo simulations. Jade permitted PSETs to develop their own ways of using the artifacts, rather than prescribing specific actions. The transcript excerpt below, from Day B11, demonstrates the pressing for alternatives SCIM in the social contexts of small-group work and whole-class discussions. The PSETs were investigating ways to run a simulation on a True or False test of ten questions to see the likelihood of someone earning a 60% or better by chance alone. A straightforward solution, which might be prescribed in a
typical mathematics classroom, would focus on the chances of getting each question correct. Instead, when engaged in the task, PSETs in some of the small groups used Tinkerplots and focused on simulating the correct answers for each test item (e.g., True or False) rather than focusing directly on whether the individual answered a particular question on the test correctly. This PSET-generated solution path, although valid if done correctly, added unnecessary complexity to the simulation. As shown below, in response to this, Jade initiated a discussion with the PSETs in order to push the PSETs’ thinking and challenge them to revise their simulations.

Jade: Like, how often did “zero” come up? I mean, granted, we could count them, one by one. But, it should be nice if there’s a more efficient way of doing that, especially since we have this tool. Do you have a way that we could get that? [pressing for alternatives SCIM] Rick?

Rick: Well, I was just gonna say that you could count the dots. But, also you can see how the first dot starts at the bottom of “0,” so if that—obviously is the first one. So, if the last dot starts at the bottom of 25, then there’s 25, which is kind of how I looked—how I saw it.

Jade: Do you want to come show us what you mean? I don’t know if I totally followed. [requesting demonstration SCIM]

(A03, 29:28)

Take 30

The take 30 SCIM occurred when Jade paused the activity in order to have the PSETs take a moment to interact with the artifact in a particular way, such as creating a certain type of plot. Jade used the take 30 SCIM during 16 (9.8%) of the whole-class instrumental orchestrations and 1 (0.7%) of the small-group instrumental orchestrations (Figure 5.1). Essentially, Jade implemented an instrumental orchestration that included the take 30 SCIM every 106 minutes or almost once per class session, on average.
Jade used the *take 30 SCIM* to request that the PSETs take a moment of time to explore a particular function of an artifact in such a way that they could share their experience or what they learned in the process. This was consistent with the notions of *wait time* (Stahl, 1994) and maintaining cognitive demand (Stein et al., 1996, p. 462). Jade’s use of the *take 30 SCIM* seemed to function in ways that were similar to the *think-pair-share* teaching strategy (Lyman, 1981), as the PSETs interacted with the artifacts and/or each other, then reported their experience to the whole class. The *take 30 SCIM* was identified as a single utterance, such as, “Why don’t you take a second and play with some of these things [using the artifact]” (Day A03, 0:09:12).

Following Jade’s statement to have the PSETs take a moment to tinker, explore, or work with an artifact, she paused and gave the PSETs a moment of time. By providing this time, Jade allowed the PSETs to make mistakes (and learn from them) and critically consider how the artifacts worked. As a result, Jade reinforced the collective management of conceptual aspects of instrumental genesis.

On Day A03, Jade initiated a discussion on the features of histograms. Then, she instructed the PSETs to work in dyads or small groups to tinker with different displays and see what they could generate with Tinkerplots. In this case, the primary function of the *take 30 SCIM* is for the PSETs to explore making plots with Tinkerplots—“I’ll give you a couple minutes to [tinker with the artifact].” However, since some of the PSETs were working in pairs and Jade returned back to the whole-class in order for the PSETs to report back to the big group, the *take 30 SCIM* also served to motivate the discussion. In the transcript that follows, Jade’s statement, “. . . and then see what else you could create,” suggests the expectation that the PSETs should have explored the artifacts and their capabilities. Thus, Jade used the *take 30 SCIM* to open up the PSETs’ possible uses of the artifacts, rather than limit particular aspects of the
artifacts. By promoting exploration and discovery, Jade seemed to use the *take 30 SCIM* as an element of her *constructivist-compatible* instruction (Ravitz et al., 2000).

Jade: . . . I’d be curious to see if any of those are things that we can create, or if there are any other displays that we can create that might be useful for displaying this variable of “hours worked.” I think they wrote it like that. So, let me give you just a couple of minutes to try to get back into that. I’m gonna come around, and I’m gonna have a few people, in particular, share some of the things that we’ve developed. In particular, I think I had asked you to see if you could create a dotplot, and then see what else you could create. So, that would be a good thing to start from, as you’re working with your group. If you need to move a little bit to be with somebody who has a computer—I see a lot of computers on this side of the room, maybe not quite as many over there. Feel free to kind of roll over and buddy up with somebody who has a computer. Um, if it’s not working or you haven’t downloaded it yet, we just don’t have time to do that right now—just buddy up [buddy up SCIM] with someone who has it, and then you can do that on your own. All right, I’ll give you a couple minutes to play. [*take 30 SCIM, requesting exploration SCIM*]

(Day A03, 0:21:10)

**Student-Centered Instrument Moves Across the Three Units**

Table 5.1 shows the distribution of Jade’s *student-centered instrument moves* across the three units in the course. Perhaps the most striking pattern is that Jade’s use of *student-centered instrument moves* seemed nearly the same between the two classes, as reflected in the last column of the table. One noticeable discrepancy was that Jade used the *probing SCIM* almost twice as often in Class B as she did in Class A. This may have occurred because the PSETs in Class B were more willing to engage in the discussions and Jade may have had more time and opportunities to probe for more information from them. Another noticeable discrepancy was that Jade used the *prompting vocabulary SCIM* almost twice as often in Class A as she did in Class B. It seemed that the PSETs in Class B, who were more willing to engage in the discussions, arrived at some discussions about vocabulary on their own, without Jade using the *prompting vocabulary SCIM*. One example of this was their discussion of the definition of *histogram*,...
which Jade orchestrated as a whole-class, *First-chair* IO on Day B04—this was discussed earlier in the section on the *waiting* SCIM, which includes the PSET-generated histograms shown in Figure 5.3 and Figure 5.4. Jade used the *student-centered instrument moves* less frequently during the Monte Carlo simulations unit when compared to the first two units. This may have been attributed to Jade’s efforts to get through the material at the end of the semester, as previously indicated.

Table 5.1

*Frequencies of Student-Centered Instrument Moves by Unit*

<table>
<thead>
<tr>
<th>Student-Centered Instrument Moves</th>
<th>Class</th>
<th>Unit</th>
<th></th>
<th></th>
<th></th>
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</tr>
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<tr>
<td></td>
<td></td>
<td>Class</td>
<td>Univariate Statistics</td>
<td>Bivariate Statistics</td>
<td>Monte Carlo Simulations</td>
<td>Total</td>
<td></td>
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<tr>
<td><em>Waiting</em></td>
<td></td>
<td>A</td>
<td>14</td>
<td>18</td>
<td>10</td>
<td>42</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>B</td>
<td>13</td>
<td>24</td>
<td>10</td>
<td>47</td>
<td></td>
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<tr>
<td><em>Requesting</em></td>
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<td>32</td>
<td>30</td>
<td>19</td>
<td>81</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>B</td>
<td>27</td>
<td>29</td>
<td>15</td>
<td>71</td>
<td></td>
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<tr>
<td><em>Voicing</em></td>
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<td>A</td>
<td>6</td>
<td>12</td>
<td>4</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>12</td>
<td>3</td>
<td>3</td>
<td>18</td>
<td></td>
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<tr>
<td><em>Probing</em></td>
<td></td>
<td>A</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>11</td>
<td>4</td>
<td>10</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td><em>Prompting vocabulary</em></td>
<td></td>
<td>A</td>
<td>11</td>
<td>8</td>
<td>3</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>12</td>
<td></td>
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<tr>
<td><em>Buddy up</em></td>
<td></td>
<td>A</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>8</td>
<td>3</td>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td><em>Introducing</em></td>
<td></td>
<td>A</td>
<td>5</td>
<td>2</td>
<td>6</td>
<td>13</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>B</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>13</td>
<td></td>
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<tr>
<td><em>Recalling</em></td>
<td></td>
<td>A</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td><em>Take 30</em></td>
<td></td>
<td>A</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>B</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td><em>Pressing</em></td>
<td></td>
<td>A</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>–</td>
<td>182</td>
<td>187</td>
<td>129</td>
<td>465</td>
<td></td>
</tr>
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</table>

*Note.* There were more opportunities for pedagogical moves during the bivariate statistics unit since there were more instrumental orchestrations during this unit (see Tables 4.2 and 4.4).
Teacher-Centered Instrument Moves

When Jade assumed a more direct role in the classroom, she implemented teacher-centered instrument moves. At times, such moves may have prevented richer opportunities for the PSETs to engage in instrumented activity. At other times, Jade’s interventions seemed to help the PSETs overcome technical issues in an efficient manner, which saved time for more student-centered, conceptually oriented work. Taken as a set, these moves did not seem to directly promote conceptual aspects of instrumental genesis, because Jade was the one interacting with the artifacts or Jade’s comments could have bypassed opportunities for the PSETs to learn something on their own. In this section, the seven types of teacher-centered instrument moves observed in Jade’s classroom are discussed in relation to the different types of instrumental orchestrations in order from most frequently to least frequently used (see Figure 5.8). This section concludes with a discussion of the types and frequencies of teacher-centered instrument moves that were used in each of the three statistics units that were observed.

Directing

The directing TCIM occurred when Jade gave the PSETs artifact-related instructions. Jade used the directing TCIM during 50 (30.5%) of the whole-class instrumental orchestrations and 44 (28.8%) of the small-group instrumental orchestrations (Figure 5.8). Essentially, Jade implemented an instrumental orchestration that included the directing TCIM every 19 minutes or five times per class session, on average. The transcript excerpt shows one instance when Jade used the directing TCIM.

Jade: Yes, Trinity?

Trinity: Do we start off with a blank thing. Like, we don’t even know how to get to—
Jade: Yes, I will tell you how to get to this. I will need to get you to go to “File.” Everybody ready? “File,” “Open Sample Document,” “Social Studies,” across from “Curriculum.” [directing TCIM]

Trinity: I clicked on it.

Jade: Double-click it.

Trinity: I did.

(Day B08, 0:51:40)

<table>
<thead>
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<th>Types of Instrumental Orchestrations</th>
<th>Teacher-centered instrument moves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Instrumental Orchestration Frequency</td>
</tr>
<tr>
<td>Whole-class</td>
<td></td>
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<tr>
<td>First-chair (SC)</td>
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</tr>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>34</td>
</tr>
<tr>
<td>Class-assist (SC)</td>
<td>9</td>
</tr>
<tr>
<td>Discuss-artifact-use (SC)</td>
<td>7</td>
</tr>
<tr>
<td>Spot-and-show (SC)</td>
<td>1</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>18</td>
</tr>
<tr>
<td>Board-with-tech-reference (TC)</td>
<td>17</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>15</td>
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<tr>
<td>Whole-class Total</td>
<td>164</td>
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<tr>
<td>Small-group</td>
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<tr>
<td>Discuss-the-screen (SC)</td>
<td>54</td>
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<tr>
<td>Students-choose-tech (SC)</td>
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<tr>
<td>Talk-without-tech (SC)</td>
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<tr>
<td>Guide-and-explain (TC)</td>
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<td>Technical-demo (TC)</td>
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</tr>
<tr>
<td>Small-group total</td>
<td>153</td>
</tr>
</tbody>
</table>

| Total                                | 317                                | 94        | 73      | 22         | 18                     | 10         | 9          | 6          |

Note. The values indicate that a particular teacher-centered instrument move occurred at least once in an instance of an instrumental orchestration. For example, the directing TCIM occurred at least once during 17 of the 63 whole-class First-chair instrumental orchestrations. TC stands for teacher-centered and SC stands for student-centered.

Figure 5.8. Frequencies of teacher-centered instrument moves.
Guiding

The **guiding** TCIM occurred when Jade made short, leading, statements or questions in order to guide the PSETs’ instrumented activity. The **guiding** TCIM often seemed to resemble either the *funneling* or *focusing pattern* discussed by Wood (1998). Jade used the **guiding** TCIM during 20 (12.2%) of the whole-class instrumental orchestrations and 53 (34.6%) of the small-group instrumental orchestrations (Figure 5.8). Essentially, Jade implemented an instrumental orchestration that included the **guiding** TCIM every 25 minutes or four times per class session, on average.

When describing the *Guide-and-explain* IO, Drijvers et al. (2013) discuss the presence of guiding questions that teachers might use during that particular instrumental orchestration (p. 999). Thus, it was not surprising that the **guiding** TCIM was used during more than half (56.4%) of Jade’s small-group *Guide-and-explain* IOs (see Figure 5.8). However, it was surprising that Jade only used the **guiding** TCIM in 5.6% of the whole-class *Guide-and-explain* IOs. Rather than using the **guiding** TCIM during the whole-class *guide-and-explain* IOs, Jade seemed to prefer to use the **directing** TCIM. This suggests that her use of the whole-class *guide-and-explain* IOs was more closed to PSET contributions than were her small-group *guide-and-explain* IOs, since the **directing** TCIM is less-open to student contributions than the **guiding** TCIM.

When using the **guiding** TCIM, Jade often gave hints rather than explicitly telling the PSETs exactly what to do with the artifacts. The act of giving hints might be viewed as a form of scaffolding. Examples of Jade’s artifact-oriented guiding statements include the following: “So, maybe ‘height’ and ‘weight,’ and see what you get [as an attempt to plot a scatterplot]” (Day A05, 0:52:26); “so, you need to play around with this ‘inspect sampler.’ Go over to the ‘History’
tab, and see if you can find how you can set it up” (Day A08, 1:31:05); or “if you put it in ‘Table’ mode . . .” (Day B 1:40:03).

Generally, Jade’s guiding statements were not as explicit as when she gave directions. Also, Jade’s guiding statements were not as supportive of student-centered exploration and investigation as the more student-centered pedagogical moves. As a result, Jade’s use of the guiding TCIM was more teacher-centered than student-centered, as was the case for the Guide-and-explain IOs.

**Explaining**

Even though Jade did not implement the whole-class Explain-the-screen IO during data collection, she did implement the explaining TCIM. The explaining TCIM occurred when Jade provided details about how to use the artifacts for the tasks at hand. Jade used the explaining TCIM during 5 (3.0%) of the whole-class instrumental orchestrations and 17 (11.1%) of the small-group instrumental orchestrations (Figure 5.8). Essentially, Jade implemented an instrumental orchestration that included the explaining TCIM every 82 minutes or about once per class session, on average. The lack of this teacher-centered pedagogical move is further evidence of Jade’s consistently student-centered instruction.

When Jade used the explaining TCIM, she often focused on items related to the design of the artifacts. As shown in the transcript below, Jade noted the six preset lists and the ability to add other lists with user-defined names on the TI-84, which some of the PSETs were using instead of the TI-73.

Jade: Okay, so those of you who have an “84” you’re gonna hit “STAT,” and then hit “EDIT.” And then, it will look like this. Now, for those of you who want to name your lists—These calculators are really attached to “L1” to “L6”—they love those names. They will not let you rename it. So, right now, if they try to rename it, it will give them an error. But, if you keep arrowing over further, there, now you can name it, if you’d like. So, if you want to call this “GEORGE”—you can, as
Anthony or somebody over here said. If you want to name it “PULSE RATE”—well, you won’t be able to fit “PULSE RATE”—I think it’s only like five or size letters. But, we could call it “PULSE” or “HEART.” [explaining TCIM, playing TCIM]

(Day B04, 1:28:31)

Playing the Instrument

The playing the instrument TCIM occurred when Jade operated one of the artifacts in front of the PSETs, often as a means to demonstrate a particular instrumented technique. Jade used the playing the instrument TCIM during 17 (10.4%) of the whole-class instrumental orchestrations and 1 (0.7%) of the small-group instrumental orchestrations (Figure 5.8). Essentially, Jade implemented an instrumental orchestration that included the playing the instrument TCIM every 100 minutes or once per class session, on average.

In 100% of the instances of the teacher-centered Technical-demo IO, Jade controlled the featured artifact and implemented the playing the instrument TCIM. However, Jade also used the playing the instrument TCIM in a few of the more student-centered instrumental orchestrations. For example, on Day B05, Jade adjusted the plot on the TI-73 in response to the PSETs discussion about the “trace” button and she made window adjustments to the projected calculators before starting a Discuss-the-screen IO. These two adjustments are shown in Figure 5.9.
Figure 5.9. Jade operates the TI-73 to show the trace feature and to adjust window settings.

Other Teacher-Centered Instrument Moves

Although infrequent, Jade exhibited three more types of teacher-centered instrument moves related to the PSETs’ experiences with the artifacts. First, she used the documenting TCIM in 10 (6.1%) of the whole-class instrumental orchestrations and zero of the small-group instrumental orchestrations. With the documenting TCIM, Jade acted as a scribe to write notes publicly related to the use of the artifacts. Secondly, during zero of the whole-class instrumental orchestrations and nine (5.9%) of the small-group instrumental orchestrations, Jade used the diagnosing TCIM when she identified the nature of an issue that a PSET was having with their artifact. While Jade was more likely to understand the issue quickly and offer suggestions as scaffolds to help the PSETs continue on with their work, there were instances when the issue appeared to be less obvious to her and she had to work to diagnose the error. In these instances, Jade drew from her own technical knowledge of the artifacts and was able to fix the issue. Also, Jade was more direct when helping the PSETs in these cases. Lastly, during five (3.0%) of the whole-class instrumental orchestrations and one (0.7%) of the small-group instrumental
orchestrations, Jade used the _mentioning pros and cons_ TCIM when she stated advantages or disadvantages to using an artifact.

**Teacher-Centered Instrument Moves Across the Three Units**

Table 5.2 shows the distribution of Jade’s _teacher-centered instrument moves_ across the three units in the course. Jade used a consistent number of _teacher-centered instrument moves_ for each unit in the course. One of the more striking patterns is that Jade used the _directing_ TCIM more frequently in Class A and the _guiding_ TCIM more frequently in Class B. This finding seems attributed to Jade’s perception about the difference between the PSETs in the two classes. As previously stated, Jade thought that the PSETs in Class A were less willing to engage in collaborative activity and discussions than the PSETs in Class B. Thus, Jade might have taken a more direct role when implementing instrumental orchestrations with PSETs from Class A—and she may have been more likely to use the _directing_ TCIM. In contrast, Jade may have utilized the _guiding_ TCIM with the PSETs from Class B as a means to less-directly influence their activity and discussions, which is consistent with Jade’s more frequent use of the small-group _Guide-and-explain_ IO in Class B (see Table 4.4).
Table 5.2

*Frequencies of Teacher-Centered Instrument Moves by Unit*

<table>
<thead>
<tr>
<th>Teacher-Centered Instrument Moves</th>
<th>Class</th>
<th>Unit</th>
<th>Monte Carlo Simulations</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Univariate Statistics</td>
<td>Bivariate Statistics</td>
<td></td>
</tr>
<tr>
<td>Directing</td>
<td>A</td>
<td>17</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>12</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>Guiding</td>
<td>A</td>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>15</td>
<td>25</td>
<td>21</td>
</tr>
<tr>
<td>Explaining</td>
<td>A</td>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>3</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Playing the instrument</td>
<td>A</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Documenting</td>
<td>A</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Diagnosing</td>
<td>A</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Mentioning pros and cons</td>
<td>A</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>74</td>
<td>76</td>
<td>82</td>
</tr>
</tbody>
</table>

*Note.* There were more opportunities for pedagogical moves during the bivariate statistics unit since there were more instrumental orchestrations during this unit (see Tables 4.2 and 4.4).

**Jade’s Predominant Instrument Moves**

In this section, profiles are given of Jade’s instrumental orchestrations in terms of her most frequently used instrument-related pedagogical moves. The information from Figure 5.1 and Figure 5.8 show the student-centered and teacher-centered instrument moves that Jade used during her instrumental orchestrations, respectively. The frequencies of Jade’s instrument-related pedagogical moves were used to generate Figure 5.10, below, which shows a profile for each type of instrumental orchestration in terms of the predominant pedagogical moves that Jade used.

Of the student-centered instrument moves that Jade used, she favored the requesting SCIMs the most—this pedagogical move was predominant during her whole-class First-chair,
Discuss-the-screen, Class-assist, Guide-and-explain, and Technical-Demo IOs, as well as her small-group Discuss-the-screen IOs. Moreover, Jade actually used the requesting SCIMs during more than 75% of the whole-class First-chair IOs (see Figure 5.1). Also, the waiting SCIM was predominant in Jade’s whole-class First-chair and Class-assist IOs.

Of the teacher-centered instrument moves that Jade used, she favored the directing TCIM the most—this pedagogical move was predominant in her whole-class First-chair, Class-assist, Guide-and-explain, and Technical-demo IOs. Also, the guiding TCIM was predominant in the small-group Guide-and-explain IOs.

<table>
<thead>
<tr>
<th>Instrumental Orchestrations</th>
<th>Predominant Pedagogical Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole-Class</strong></td>
<td></td>
</tr>
<tr>
<td>First-chair (SC)</td>
<td>SCIMs: requesting, waiting, introducing TCIMs: directing</td>
</tr>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>SCIMs: requesting, prompting vocabulary</td>
</tr>
<tr>
<td>Class-assist (SC)</td>
<td>SCIMs: requesting, waiting, voicing TCIMs: directing, guiding</td>
</tr>
<tr>
<td>Discuss-artifact-use (SC)</td>
<td>SCIMs: requesting, voicing, pressing TCIMs: guiding</td>
</tr>
<tr>
<td>Spot-and-show (SC)</td>
<td>n/a*</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>SCIMs: requesting, buddy up TCIMs: directing</td>
</tr>
<tr>
<td>Board-with-tech-reference (TC)</td>
<td>SCIMs: prompting vocabulary, requesting, voicing, recalling</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>SCIMs: requesting, buddy up, introducing, take 30 TCIMs: playing the instrument, directing</td>
</tr>
<tr>
<td><strong>Small-Group</strong></td>
<td></td>
</tr>
<tr>
<td>Discuss-the-screen (SC)</td>
<td>SCIMs: requesting, waiting</td>
</tr>
<tr>
<td>Students-choose-tech (SC)</td>
<td>TCIMs: directing</td>
</tr>
<tr>
<td>Talk-without-tech (SC)</td>
<td>n/a*</td>
</tr>
<tr>
<td>Guide-and-explain (TC)</td>
<td>TCIMs: guiding, directing</td>
</tr>
<tr>
<td>Technical-support (TC)</td>
<td>SCIMs: buddy up TCIMs: guiding</td>
</tr>
<tr>
<td>Technical-demo (TC)</td>
<td>n/a*</td>
</tr>
</tbody>
</table>

Note. Pedagogical moves in boldface text were present in 50% or more of the instances of the corresponding instrumental orchestration, whereas those in regular text were present in 25-49%. An asterisk indicates that there were fewer than five occurrences of that instrumental orchestration so predominant moves were not determined.

Figure 5.10. Jade’s predominant instrument-related pedagogical moves.
**Chapter Summary**

Instrument-related pedagogical moves were considered components or characteristics of Jade’s didactical performances, as represented in the conceptual framework shown in Figure 3.1. In this chapter, data related to Jade’s use of *student-centered instrument moves* and *teacher-centered instrument moves* during her didactical performances were discussed, and the results as a whole confirm Jade’s preference for student-centered aspects of her instrumental orchestrations. On average, Jade used each of the *student-centered instrument moves* at least once per day. Of these pedagogical moves, Jade most frequently implemented the *requesting* and *waiting* SCIMs. Moreover, Jade used both of these pedagogical moves during instrumental orchestrations where she also implemented other types of *student-centered instrument moves*, as shown in Figure 5.10. Similar to the argument that Herbel-Eisenmann et al. (2013) made in reference to the use of multiple teacher discourse moves in student-centered instruction, there seemed to be an amplified power supporting the collective management of instrumental genesis when Jade implemented multiple *student-centered instrument moves* within a single instrumental orchestration. Furthermore, *student-centered instrument moves* occurred very frequently during Jade’s *First-chair IOs*, *Discuss-the-screen IOs*, and *Class-assist IOs*, particularly in the whole-class configurations (see Figure 5.1). Also, Jade’s *student-centered instrument moves* sometimes seemed to serve two purposes, promoting both the conversation and the PSETs’ engagement in instrumented activity. The student-centered nature of the *student-centered instrument moves*, as well as Jade’s constant use of these pedagogical moves, seemed to provide and help sustain conditions favorable for the collective management of instrumental genesis.

Jade’s *teacher-centered instrument moves* were also discussed in this chapter. Jade did not use as many types of *teacher-centered instrument moves* compared to *student-centered
instrument moves, nor were they as frequently used (see Figure 5.1 and Figure 5.8). Taken as a set, these teacher-centered instrument moves did not seem to promote conceptual aspects of instrumental genesis, but they did seem to promote technical aspects of instrumental genesis. As with IRE (Mehan, 1979) in the literature on discourse (e.g., Herbel-Eisenmann et al., 2013), the results suggest that the teacher-centered pedagogical moves need not be judged as bad or as necessarily preventing instrumental genesis.

Nevertheless, Jade’s use of the directing TCIM and the guiding TCIM played a noticeable role in both the teacher-centered and student-centered instrumental orchestrations (see Figure 5.10). Jade frequently used teacher-centered instrument moves to help the PSETs efficiently acquire specific instrumented techniques. While this might suggest that Jade’s teacher-centered instrument moves limited the PSETs’ opportunities for engaging in instrumental genesis, the results show how Jade utilized these pedagogical moves in ways that still seemed to provide the PSETs with clear opportunities to shape and be shaped by the artifacts.

Both the student-centered instrument moves and the teacher-centered instrument moves can be considered performance-related teaching techniques that Jade had in her repertoire and chose from during her didactical performances. Although the different types of instrumental orchestrations are presented as student-centered or teacher-centered (Table 4.1 and Table 4.3), Jade used student-centered instrument moves during teacher-centered instrumental orchestrations (Figure 5.1) and she used teacher-centered instrument moves during student-centered instrumental orchestrations (Figure 5.8). Moreover, in the case of technical issues that could become roadblocks to conceptual thinking, for example, Jade’s use of guiding, explaining, diagnosing, or other teacher-centered instrument moves may have freed up classroom time for
more student-centered, collaborative and conceptual work with the use of the artifacts. These findings further support the notion that teachers’ didactical performances occur on a spectrum. Thus, teachers may use *student-centered instrument moves* during teacher-centered instrumental orchestrations in order to make those instrumental orchestrations a little less teacher-centered.
CHAPTER 6
DISCUSSION

Student-centered, contemporary reform efforts in mathematics education (e.g., NCTM, 2014; NGA & CCSSO, 2010) continue to push for the increased and judicious use of technology in the classroom. However, the premise for including technology is not to just have students use technology with the “same” old lesson plans and activities—technology is viewed as a catalyst to help transform learning experiences for mathematics students (e.g., Heid, 1988, 1997; Pea, 1985, 1987). Instrumental approaches have been used to demonstrate aspects of how students engage in instrumental genesis—where the artifact shapes the user’s understanding and the user shapes the artifact (e.g., Artigue, 2002; Drijvers, 2003; Guin & Trouche, 1998; Haspekian, 2005) and the types of instrumental orchestrations that teachers might use when supporting the students’ strategic use of such artifacts (Drijvers et al., 2010; Drijvers et al., 2013). To investigate the characteristics of one instructor’s didactical performances in a statistics course for preservice elementary teachers (PSETs), I sought to answer the following research questions:

1. What types of instrumental orchestrations does one instructor use in a technological-tools-enhanced, reform-oriented statistics course for preservice elementary teachers?
2. What types of technology-related pedagogical moves does the instructor implement during her didactical performances in the course?
Summary of the Results

Research Question #1

In the whole-class configurations, Jade favored the use of student-centered instrumental orchestration (IO) types—particularly the *Discuss-the-screen* IO and the *First-chair* IO (Table 4.1). Jade chose the *Discuss-the-screen* IO for 20.7% of the whole-class instrumental orchestrations, and she chose the *First-chair* IO for 38.4% of the whole-class instrumental orchestrations. Together, Jade utilized about 6 hours and 39 minutes of instructional time for these two instrumental orchestration types during the 18 hours of observation. During these instrumental orchestrations, the PSETs spent much of the time discussing the mathematical tasks and their use of the artifacts. The pattern of interaction often resembled a *star* pattern, which may be viewed *constructivist-compatible* (Ravitz et al., 2000) and is more in line with contemporary reform oriented approaches than the hub and spoke pattern of interaction (Nathan & Knuth, 2003, p. 91).

The nature of Jade’s whole-class instruction shifted in the direction of teacher-centered instrumental orchestrations during the last unit on Monte Carlo simulations (see Table 4.2 and Table 4.4). Specifically, there was an unanticipated decrease in the student-centered *First-chair* and *Discuss-the-screen* IOs and an increase in the use of the teacher-centered *Guide-and-explain* IOs. Jade mentioned the influence of the approaching end of the semester and the need to finish the last unit in time to allow the PSETs an opportunity to study for the final exam. This shift in Jade’s instruction serves as a reminder of the practical issues and pressures that teachers face when they seek to support a student-centered classroom environment. In other words, we are reminded of the push-and-pull tension that exists with the issue of time (Drijvers et al., 2010). On the one hand, teachers need to invest more time when implementing student-centered
instrumental orchestrations compared to teacher-centered instrumental orchestrations. On the other hand, by implementing more student-centered instrumental orchestrations, teachers may have less time to use for covering content. Drijvers et al. (2010, p. 224) noted similar remarks from their “Teacher C” who avoided the use of student-centered instrumental orchestrations because of the perceived time it takes to implement student-centered instrumental orchestrations compared to teacher-centered instrumental orchestrations. From the perspective of contemporary reform, the sacrifice of coverage is worth the deeper gains in student conceptual understanding and student understanding of connections between mathematical concepts (NCTM, 1989, 2000, 2014). The value of student conceptual understanding is affirmed by the newer high-stakes tests, including the Smarter Balanced Assessment Consortium assessment and the Partnership for Assessment of Readiness for College and Careers assessment.

In the small-group configurations, Jade favored the use of the teacher-centered Guide-and-explain IO and the student-centered Discuss-the-screen IO (Table 4.3). Jade chose the Guide-and-explain IO for 50.3% of the small-group instrumental orchestrations and the Discuss-the-screen IO for 35.3% of the small-group instrumental orchestrations. In the former, Jade served as a resource to help the PSETs move past technical or conceptual hurdles. In the latter, Jade facilitated conversation between PSETs working within the same group. Although the finding concerning Jade’s frequent use of the teacher-centered Guide-and-explain IO was unanticipated, a deeper look into the instances in which Jade utilized this teacher-centered instrumental orchestration type indicates that Jade’s student-centered instructional goals remained paramount. When Jade had the PSETs work in small groups, emphasis was placed on the PSETs interacting and solving tasks. Thus, while monitoring the PSETs’ work in the small-group setting, Jade typically gave the groups space and time to work. When the PSETs
encountered issues that seemed to hinder their forward progress, whether technical or conceptual in nature, Jade often stepped in as a guide to help the PSETs move past the issue. When the PSETs seemed to get back on track with their work, Jade transitioned out of the instrumental orchestration so that the PSETs could re-engage in their collaborative problem-solving activity. As a result, Jade’s instruction during the small-group activities and instrumental orchestrations is still viewed as being heavily student-centered.

The results from this study extend the findings reported by Drijvers and colleagues (2013) by identifying three types of whole-class instrumental orchestrations that were not discussed in previous literature—the Class-assist, the Discuss-artifact-use, and the Board-with-tech-reference IOs, and two new types of small-group instrumental orchestrations, the Students-choose-tech and the Talk-without-tech IOs. All but one of these, the Board-with-tech-reference IO, are considered student-centered. For educators seeking to discuss or implement student-centered instrumental orchestrations, these results provide additional types of instrumental orchestrations that may be added to their repertoires of teaching strategies.

**Research Question #2**

To answer the second research question, I analyzed the instrument-related pedagogical moves that Jade used within her didactical performances. Since these artifact moves appeared to be either more student-centered or more teacher-centered, in terms of opening up or limiting opportunities for the PSETs to explore and learn through the use of the artifacts, they became known as the student-centered instrument moves and the teacher-centered instrument moves, respectively. These results extend the work of Drijvers et al. (2013) by offering instructional techniques that mathematics educators may use during their didactical performances to promote students’ opportunities to use artifacts and engage in instrumental genesis—where the artifact
shapes the user’s understanding and the user shapes the artifact. Jade was more than twice as likely to use a student-centered instrument move than a teacher-centered instrument move (Figure 5.1 and Figure 5.8). Jade’s most frequently used student-centered instrument moves (SCIMs) were the requesting SCIM and the waiting SCIM—these pedagogical moves were used in 47.2%, and 27.6% of all instrumental orchestrations, respectively. Also, Jade often used the student-centered instrument moves in combinations with other student-centered instrument moves. Herbel-Eisenmann and colleagues (2013) claimed that combinations of teacher discourse moves have amplified power to promote student engagement in mathematical discussions—similarly, it seems that combinations of student-centered instrument moves might have amplified power to promote both students’ conceptual understandings and instrumental geneses. For teachers seeking to implement instrumental orchestrations, they may want to consider how they may utilize combinations of student-centered instrument moves during their didactical performances. Teachers might anticipate such combinations during the preparation phase and they may also make ad hoc decisions in-the-moment during their performance phase (see Figure 3.1).

As mentioned previously, Jade implemented wait time (Stahl, 1994) frequently each day of observation, and she used it in different ways. The term waiting SCIM refers to a variant of wait time that is specific to a teacher pausing for students to think about or interact with the artifacts—this is viewed as allowing and promoting instrumental genesis. The transcript data showed that Jade sometimes implemented the waiting SCIM in ways that also seemed to promote PSET-PSET mathematical discourse. In terms of the literature supporting the notion of wait time in relation to student thinking, the waiting SCIM seems particularly vital for teachers to consider and implement within their instrumental orchestrations.
The Conceptual Framework

Implicit in the second research question is the notion of framing and discussing instrument-related pedagogical moves, and I drew from aspects of grounded theory research in the methods I used via open coding and a form of constant comparison during analysis. A primary goal in grounded theory studies is to generate theory and related framings. I did not predict that one of the results from this study would be the conceptual framework. But, forged out of the many iterations of coding, writing reports, and synthesizing results, the conceptual framework (Figure 3.1), by itself, is an important result from this study. Specifically, my adaptations to Drijvers and colleagues’ (2010) framing of instrumental orchestrations adds depth (by attending specifically to pedagogical moves as aspects of teachers’ didactical performances) and clarity (by noting the configuration of the classroom, introducing the notion of the cue to initiate a didactical performance, and the relabeling of the Sherpa as a First-chair). The conceptual framework can serve as the anchor to frame discussions concerning the use of instrumental orchestrations in mathematics classrooms.

Interpretation of Results

The results of this study can be described in two complementary models of instruction: constructivist-compatible instruction and instrumental genesis compatible instruction. These two models are described below, where the former is drawn from Ravitz et al. (2000) and the latter comes from the theoretical framework used in the current study.

Constructivist-Compatible Instruction

Jade’s use of student-centered instrumental orchestrations and pedagogical moves seemed to fit with her efforts to promote mathematical discourse and sense making in her classroom. Jade’s classroom seemed to function as a community engaged in the practice of
statistical problem solving, which was analogous to a jazz orchestra working towards the shared enterprise of playing jazz music. Jade’s persistent use of the student-centered instructional strategies suggests that this classroom environment was not happenstance. Rather, Jade regularly took direct action to promote and sustain this student-centered learning environment. Jade often used the student-centered instrument moves in combinations, which seemed to amplify the opportunities and the expectation for the PSETs to use the artifacts as learning tools during their collaborative activities — the effect seemed similar to Herbel-Eisenmann et al.’s (2013) statement about the amplifying effect of using multiple teacher discourse moves in combinations (p. 183). Thus, Jade’s reliance on student-centered instructional strategies reflects a constructivist-compatible model of instruction, as opposed to a traditional-transmission model of instruction (Ravitz et al., 2000).

Wait time, by itself, seemed very important in relation to constructivist-compatible instruction. It is perhaps the most student-centered move that a teacher can make—simply pause and wait without intervention. This is not to say that teachers should never say or do anything, but that the act of waiting opens doors for student thinking and collaboration. In this study, I observed an artifact-specific form of wait time, labeled as the waiting SCIM, where Jade simply paused for several seconds or a minute to allow the PSETs to continue with their work with the artifacts. On the surface, this was very similar to Stahl’s (1994) description for the Student Task-Completion Work-Time. However, when viewing these instances of wait time related to the use of the artifacts through a lens of instrumental approaches and the jazz orchestra metaphor, the artifact-specific label, “waiting SCIM,” can be used to help mathematics educators develop and use a common language in the areas of discourse, instrumental orchestration and instrumental genesis. Given the prevalence of the waiting SCIM in Jade’s didactical performances, this
student-centered instrument move seems very useful for teachers to have in their repertoires of instructional strategies. Further, it seems important that teachers understand that using the waiting SCIM gives students opportunities to engage in instrumental genesis with artifacts — this goes beyond the notion of giving students a moment to think.

As a complement to Stahl’s argument for eight forms of wait time, Herbel-Eisenmann and colleagues (2013) state, “Inviting student participation takes on multiple forms and addresses many goals” (p. 183). In Chapter 5, I noted six different types of requesting SCIMs, which could be viewed as instrument-related analogs to the inviting teacher discourse moves. Viewing these requesting SCIMs as a set can make them easier to remember, use, and discuss in the future. For example, a teacher may make different types of requests relating to the students’ interactions with the artifacts, and it may be helpful for teachers to remember the big idea—requesting implies that the teacher is requesting that the students engage in some form of instrumented activity or discussion about instrumented activity. Also, by explicitly describing each variant of the pedagogical move, I provide a finer level of detail that can help teachers and teacher educators to be precise in their selection of different pedagogical moves to serve their in-the-moment goals during their didactical performances.

Advancing the Notion of Instrumental Genesis Compatible Instruction

Returning to the theoretical framework, I interpret Jade’s predominant use of the whole-class First-chair IO and the whole-class Discuss-the-screen IO (see Table 4.1) as indicating an environment conducive for the collective management of instrumental genesis, where the artifact shapes the student and the student shapes the artifact. I make this interpretation following Trouche’s (2004, p. 298) argument that the sherpa student instrumental orchestration (called First-chair in my study) favors the collective management of aspects of instrumental genesis,
since traces of the sherpa student’s activity are publicly viewable, which gives the teacher and other students information related to the sherpa’s instrumented action schemes. This interpretation is also consistent with Drijvers et al. (2010), who argue that “the goal [for the Discuss-the-screen IO] is to enhance collective instrumental genesis” (p. 999). One assumption is that the collective management of instrumental genesis must include conceptual development on the part of the students, since the construct of instrumental genesis relies on the notions of utilization schemes and instrumented action schemes, which are both focused on the user’s experiences.

 Whereas Cayton (2012) reported on teachers’ use of wait time, probing, and revoicing pedagogical moves, and Drijvers et al. (2013) reported on teachers’ use of revoicing during instrumental orchestrations, I reported on pedagogical moves that were used specifically in the context of the user-artifact interactions. The instrument-related pedagogical moves suggests a variety of specific moves that teachers may utilize to promote different aspects of instrumental genesis—from the individual and social use of artifacts to the technical and conceptual aspects of instrumented activity. For example, the student-centered instrument moves seem to promote the collective management of instrumental genesis.

 Since Jade’s instrumental orchestrations seemed to regularly promote the collective management of instrumental genesis, I refer to Jade’s instruction as being instrumental genesis compatible. One way of identifying this type of instruction is by a teacher’s prevalent use of student-centered instrumental orchestrations and student-centered pedagogical moves. A second way of recognizing instrumental genesis compatible instruction, as noted in the theoretical framework, is constructivist-compatible instruction in a classroom environment that includes regular use of artifacts. In either case, emphasis is placed on students constructing knowledge
through their interactions with the artifacts while they also collaborate with one another. While these characterizations might seem ordinary in light of the literature on instrumental approaches, I draw attention to scholars and researchers, including Ravitz et al. (2000), who carefully characterized the notion of *constructivist-compatible instruction*. In their view, *constructivist-compatible instruction* places emphasis on opportunities for students to build on their own knowledge—their thinking, their efforts in problem-solving, and their opportunities to learn from their own mistakes are given status in the classroom. In a similar sense, I use the term *instrumental genesis compatible instruction* in order to explicitly emphasize opportunities for students to build on their own knowledge as they interact with an artifact— their thinking, their efforts in instrumented activity, and their opportunities to learn from their own mistakes with the artifacts are given status in the classroom. Thus, student-centered instrumental orchestrations and *student-centered instrument moves* become vital in a classroom where a teacher seeks to implement instruction that promotes rich opportunities for students to engage in instrumental genesis.

**Implications**

**General Implications**

In their review of literature of technology in mathematics education, Zbiek et al. (2007) declared, “Careful attention in research work to the orchestrations that teachers establish . . . may inform practice as well as help researchers and theoreticians to unravel the complexity of teaching with technology” (p. 1189). The study reported in this dissertation has helped the efforts in unraveling this complexity, particularly in the careful attention to pedagogical moves. Terms such as *student-centered instrument moves* help advance the discussion in relation to instrumental approaches to mathematics education — there is also an opportunity for advancing
a shared language for discussing both instrumental orchestrations and instrument-related pedagogical moves. Researchers, scholars, and teacher educators would benefit from adopting and using a common language such as the one used in this dissertation—this includes the different types of student-centered instrument moves and teacher-centered instrument moves and the language used in framing the study, particularly the conceptual framework (Figure 3.1).

The notion of student-centered instrument moves may be used to help teacher educators and teachers to think critically about the different types of pedagogical moves that teachers might use while implementing instrumental orchestrations (when guiding and shaping students’ opportunities to engage in instrumental genesis). Teachers can and should use student-centered instrument moves — they should be knowledgeable of the different moves (including pedagogical moves not discussed in this dissertation), anticipate which moves they might use frequently (as part of the preparation phase of their instrumental orchestrations), and reflect on what types of moves they use in-the-moment, during their didactical performances. Teacher educators should explicitly include the notions of instrumental orchestrations and pedagogical moves when training future mathematics educators in the use of artifacts.

**Implications for the Early Use of Student-Centered Instrumental Orchestrations**

Some may caution against the overemphasis of student-centered instrumental orchestrations, specifically when students are new to an artifact or a particular feature of an artifact, since the technical aspects of using the artifacts are important, just as the conceptual aspects of instrumental genesis are important (Drijvers, 2003). Some technical aspects, such as navigating the artifact and understanding its setup, are necessary before a student can use the artifact for mathematical and conceptual purposes. Therefore, some might advocate that teachers should start a semester, term, unit, or lesson with the prevalent use of teacher-centered
instrumental orchestrations, such as the whole-class *Technical-demo* IO, where the teacher takes the lead and directly performs a specific instrumented action with the artifact for the students to follow. While this suggestion is not necessarily a bad thing, Jade’s prevalent use of the whole-class *First-chair* IO seems to stand as a *student-centered* alternative.

Instead of the teacher demonstrating, as in the teacher-centered *Technical-demo* IO, the *First-chair* student may demonstrate technical aspects while also sharing insights or discussing the conceptual elements of the work with the artifact. Again, there is potential for the collective management of aspects of instrumental genesis in the *First-chair* IO (or the *Sherpa* IO discussed by Trouche, 2004). One caveat to being able to use the *First-chair* IO early on in a semester, term, unit, or lesson is that there needs to be at least one student with enough prior knowledge that they can stand up in front of their peers and demonstrate a particular instrumented technique in order to serve as the *First-chair* student. This example demonstrates that a teacher could start the year with *student-centered instrumental orchestrations*, with the explicit goal of fostering student-centered classroom norms that correlate with *constructivist-compatible instruction*. In the case of Jade’s instruction, she utilized PSETs who seemed to learn how to use the artifacts quickly\(^\text{10}\) and Jade did not work with them ahead of time to train them to serve as a *First-chair* PSET, whereas teachers might also work specifically with a student to prepare that student to act as a *First-chair* student. To reinforce the goal of fostering student-centered classroom norms, a teacher could then implement combinations of *student-centered instrument moves* to support the collective engagement in the mathematical discussion and the collective management of instrumental genesis.

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\(^{10}\) Some of the PSETs always seemed to understand a little more with the artifacts than their peers. These more knowledgeable PSETs were more likely to serve as *First-chairs*. 
In the case where there is no student to serve as the First-chair student, the teacher has a couple of choices. First, the teacher might use the newly-identified and student-centered Class-assist IO to encourage community discussion and interaction with the artifacts. Given the nature of this instrumental orchestration type, it could take some time to implement. Alternatively, the teacher may implement a teacher-centered instrumental orchestration, but make use of student-centered instrument moves to promote student-centered classroom norms during the teacher’s didactical performance. For example, when implementing the teacher-centered Guide-and-explain and Technical-demo IOs, Jade used several student-centered instrument moves (Figure 5.1), which suggests that she made measures to support student-centered norms during her more teacher-centered instrumental orchestrations.

When thinking in terms of fostering a classroom environment conducive to student-centered interactions and instrumental genesis, it seems that teachers should seek to use student-centered instrumental orchestrations and student-centered pedagogical moves as early as possible in a course. Doing so might increase the chances of establishing productive norms to help the teacher implement and sustain instrumental genesis compatible instruction.

**Limitations**

The study presented above was a case study involving a single participant, with the goal of investigating and describing aspects of her didactical performances. Having only one participant in this study means that the specific results relating to the types and frequencies of the instrumental orchestrations and pedagogical moves that Jade used are not generalizable.

Different teachers may use different instrumental orchestrations and/or different pedagogical moves, I could have observed additional characteristics of teachers’ didactical performances had I observed more teachers. Despite this limitation, the purposeful selection of an educator such as
Jade proved very worthwhile in this study, given the many different types of pedagogical moves that Jade utilized during her didactical performances. By choosing an instrumental case study (Stake, 1995), the results provide evidence of a kind of instruction that includes both instrumental orchestrations and the orchestration of mathematical discussions.

A second limitation in this study is that Jade was a college instructor and not a K-12 teacher. Due to this, the results might not reflect the actual practice of in-service K-12 teachers. Future studies should be done in K-12 classrooms to provide more insights and generalizable results.

A third limitation in this study is that several of the newly labeled pedagogical moves, such as the guiding TCIM, the explaining TCIM, the directing TCIM, the requesting SCIMs, seem commonplace in many classrooms. In this sense, many of the actual pedagogical moves, by themselves, might not seem to add much to discussions about instrumental orchestrations. However, by adopting the conceptual framework used in this study, teachers can discuss commonly used pedagogical moves in terms of the implications for student activity and the social and sociomathematical norms that might develop when a teacher uses them. For example, teachers might consider the norms that may be fostered when a teacher frequently implements the guiding TCIM compared to the use of the directing TCIM.

**Significance of the Study**

One significant outcome from this exploratory, qualitative study is the contribution to theory and framing for instrumental orchestrations. For example, by relabeling Trouche’s (2004) sherpa student as a First-chair student, I have added even more substance to the jazz orchestra metaphor (Drijvers et al., 2010; Drijvers & Trouche, 2008). I added the arrangement of the students to the didactical configuration in the preparation phase of instrumental orchestrations.
because it was clear that Jade had planned for small-group activity and this should be reflected in the preparatory phase of the instrumental orchestrations (Figure 3.1). As a result, we can explicitly talk about the teacher’s plans for instrumental orchestrations in the different student configurations (particularly the whole-class and small-group arrangements). This distinction of the arrangement of the PSETs was important because Jade implemented student-centered instrumental orchestrations in more than half of the whole-class instances, but she implemented teacher-centered instrumental orchestrations in more than half of the small-group instances. Also, the explicit addition of student-centered and teacher-centered pedagogical moves to the didactical performance (Figure 3.1) could help to advance discussions of teacher development and teacher training related to contemporary reform efforts, particularly the Process Standards (NCTM, 2000) and the Standards for Mathematical Practice (NGA & CCSSO, 2010). Applying the teacher-centered, student-centered spectrum to the didactical performance permits a finer-grained discussion of the locus during a teacher’s instrumental orchestrations. Observing a teacher and documenting her pedagogical moves may give a sense for the ebb and flow of a lesson, as relates to the students’ progression in their problem-solving activities.

Viewed through the lens of the jazz orchestra metaphor, the data reported in Chapter 5 tell a story of Jade’s frequent and persistent use of student-centered pedagogical moves, which is a significant finding in that it shows how teachers, whether novice or veteran, might be persistent with such pedagogical moves. These moves are considered student-centered as they open up opportunities for students to engage in mathematical discourse and/or to engage in the judicious and reflective use of artifacts, which are key aspects of the Principles and Process Standards (NCTM, 2000) and the Standards for Mathematical Practice (NGA & CCSSO, 2010). When examining the actual pedagogical moves themselves, we see how many of these pedagogical
moves are not brand-new, but may be moves that teachers may already utilize. In fact, educators reading this dissertation might easily state, “I use this pedagogical move—it’s not new,” “I see this all the time,” or “I could easily use this pedagogical move.” When thinking in terms of reform and teacher development, this lack of novelty can be viewed as a good thing. This is a significant result because it demonstrates easily accessible instructional strategies that teachers may add to their repertoires and select and utilize during their didactical performances with respect to the students’ use of artifacts.

Another significant finding from the results of this study is Jade’s persistent use of a multiple student-centered pedagogical moves within an individual instrumental orchestration. This contributes to the stance taken by Herbel-Eisenmann et al. (2013) with regards to their discussion of teacher discourse moves:

. . . when the talk moves were used in combination (rather than individually), the impact on the quality and quantity of student contributions was quite substantial. Thus, we shifted focus from addressing each talk move individually and instead organized the materials based on what the talk moves together seemed to accomplish. (p. 183)

This finding is significant in that Jade used the student-centered instrument moves in combinations, and there seemed to be an amplified effect—similar to the one that Herbel-Eisenmann et al. (2013) noted—on the nature of the PSETs’ interactions with the artifacts. Specifically, the PSETs typically explored, tinkered, and engaged in collaborative activity while using the artifacts as learning tools to help them in the pursuit of solving mathematical problems.

My study was designed as a response to the work of Drijvers and colleagues (2013), and I drew heavily from the jazz orchestra metaphor (Drijvers & Trouche, 2008). All of the instrument-related pedagogical moves from Chapter 5 may be viewed as techniques to be included as “part of the teacher’s repertoire of teaching techniques” related to teachers’ didactical performances (Drijvers et al., 2010, p. 215). As with the different types of instrumental
orchestrations, teachers may learn about and acquire these pedagogical moves (and others not identified in this study) as instructional tools from which they might choose to implement during their didactical performances.

**Directions for Future Research**

Researchers may employ design research studies that attend to teachers’ didactical performances and why they choose to use certain pedagogical moves instead of others. The results from such studies could contribute to grounding additional instructional strategies that teachers may utilize during their didactical performances. The instructional strategies could provide valuable data to shed more light on K-12 teachers’ instruction and possible areas for a focused professional development program relating to teachers’ instrumental orchestration. The different discourse-related and instrument-related pedagogical moves may inform the initial development of such a program, which might also build off of the work of Drijvers et al. (2013) and utilize the TPACK framework in relation to teacher knowledge. This type of research would directly involve K-12 teachers, which was not done in the study reported in this dissertation.

Studies focused on instructional quality may be used to investigate the maintenance of cognitive demand across teachers’ instrumental orchestrations in relation to the different pedagogical moves that teachers use during their didactical performances. One conjecture could be that the prevalence of the teacher-centered pedagogical moves would correlate with a decrease in cognitive demand, but higher use of the student-centered pedagogical moves during teachers’ didactical performances would correlate with the maintenance of cognitive demand. This type of study would also involve K-12 teachers, which would have the potential of producing more generalizable results.
Lastly, future studies may be used to more closely examine connections between teachers’ instrumental orchestrations and the students’ use of artifacts to solve mathematical problems. Particularly, researchers could examine teachers’ instrumental orchestrations and observe corresponding changes in the student’s instrumented techniques, changes in their process of solving mathematical problems, and changes in how the students interact with each other while engaged in technology-enhanced mathematical tasks. These types of studies could be done in K-12 classrooms contribute to discussions that more explicitly and deeply link the notions of instrumental genesis and instrumental orchestration.

Conclusions

My first conclusion is that the results in this study reinforce the notion of a teacher-centered and student-centered spectrum, rather than a dichotomy (see Figure 3.1). Drijvers et al. (2013) seemed to suggest this when they added the Guide-and-explain IO to the inventory of instrumental orchestrations from Drijvers et al. (2010), noting that it belongs in between the more teacher-centered and student-centered instrumental orchestrations. Jade sometimes implemented student-centered pedagogical moves, such as the waiting or requesting SCIMs, during teacher-centered instrumental orchestrations, like the Guide-and-explain IOs (Figure 5.1), which made the instrumental orchestration seem less teacher-centered.

At times, Jade used a single commonly-used pedagogical move that encouraged the PSETs to enter the conversation and share aspects of their instrumented techniques or to reflect on their use of the artifacts—thus, this pedagogical move served two purposes. One example is the waiting pedagogical move, which might be used to support discussion and/or opportunities for students to learn with the use of artifacts. My second conclusion is that these pedagogical moves, which may serve two purposes, seem helpful in discussions related to contemporary
reform efforts in the sense that the NCTM Principles and Process Standards (NCTM, 2000) are not meant to be isolated and separate from each other—these principles and standards reflect interconnected sets of actions that students can engage in that are consistent with the ways in which citizens in the contemporary world do and use mathematics in their work and in their lives. In other words, mathematical discourse and the judicious use of artifacts should not be thought of as mutually exclusive, but as interconnected aspects of mathematical work. This finding and framing seems to connect Herbel-Eisenmann et al.’s (2013) focus on discourse with Drijvers and colleagues’ (2013) focus on aspects related to the student-artifact interactions. Teachers, researchers, and curriculum designers may benefit from explicitly considering the connections between supporting student-centered discourse and instrumental genesis.

I end with a final thought—contemporary reform in the United States supports the transformative power of technology for the teaching and learning of mathematics (NCTM, 1989, 2000, 2014). Pierce and Ball (2009) underscore this stance, arguing that “teaching mathematics with technology requires a marked change in behaviour [sic] for practising [sic] mathematics teachers who have taught, and been taught, in traditional mathematics classrooms dominated by working with pen and paper” (p. 300). Therefore, “A new repertoire of teaching techniques, instrumented by the available tools, has to be developed” (Drijvers et al., 2010, p. 214). Drijvers and colleagues (2013) offer a repertoire of instrumental orchestration types that teachers can learn and implement in their classrooms to support deep mathematical learning that is instrumented by available tools, but they note that their repertoire exists in what I call the preparation phase of instrumental orchestration. The results presented in this study contribute to the discussion of repertoires of teaching techniques, characterized in terms of the newly identified or newly labeled instrument-related pedagogical moves, that teachers can anticipate.
and use in-the-moment during their didactical performances as they implement *instrumental genesis compatible* instruction.
REFERENCES


Appendix A

Pilot Investigation
Pilot Investigation

The study reported in Kratky (2013) served as a pilot investigation to the study presented in this dissertation. In the spring of 2013, the researcher conducted a four-day study of one high school teacher’s instrumental orchestrations in small-group settings. The study documented and reported types of small-group instrumental orchestrations that occurred in one teacher’s Algebra II classroom, where students were allowed to use graphing calculators as they encountered mathematical problems related to quadratic functions. The students generated graphs and tables for quadratic functions and performed quadratic regressions to fit quadratic models to given data sets.

The investigation was designed as a single case study. During data collection, the researcher set up two video cameras to capture two different viewing angles, took field notes, and conducted brief semi-structured post-lesson interviews with the teacher to get a sense for his perspective on what he did to guide and shape his students’ interactions with the graphing calculators. During analysis, the researcher reviewed the classroom videos and generated a spreadsheet to document what happened in the class and when it occurred. The results from the investigation reflected the teacher’s reliance on more teacher-centered lessons, instruction, and instrumental orchestrations. The teacher primarily assisted students when they faced technical issues and demonstrated ways to use the graphing calculators.

The pilot investigation provided initial insights regarding instrumental orchestrations in small-group settings and paralleled the findings reported by Drijvers et al. (2013) concerning the Technical Assistant and Technical-demo IOs in small-group settings. However, given the teacher’s reliance on teacher-centered lesson structures and skill-focused activities, the data were limited to teacher-centered interventions. As a result of this pilot project, the researcher refined
the strategy for selecting participants for future research efforts in order to increase the chances of observing student-centered instrumental orchestrations.

The pilot investigation informed the methods used in this dissertation study in two additional ways. Although the researcher noted how the use of two video cameras noticeably increased what could be seen, since neither was fixed with a focus on the digital projector, the focus of a video camera had to be changed to switch to the projector. Dedicating a video camera to the projector would allow continuous and focused records of data projected to the whole-class. Also, audio data from the video cameras was difficult to analyze at times because of the noise level when students were working in small groups. Audio recorders strategically placed in the classroom would help to capture higher quality audio data.
Appendix B

Human Subjects Institutional Review Board Approval Letter
Date: January 15, 2014

To: Jon Davis, Principal Investigator
    James Kratky, Student Investigator for dissertation

From: Amy Naugle, Ph.D., Chair

Re: HSIRB Project Number 14-01-01

This letter will serve as confirmation that your research project titled “Investigating the Nature of Mathematics Teachers’ Instrumental OrCHEstrations” has been approved under the expedited category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note: This research may only be conducted exactly in the form it was approved. You must seek specific board approval for any changes in this project (e.g., you must request a post approval change to enroll subjects beyond the number stated in your application under “Number of subjects you want to complete the study”). Failure to obtain approval for changes will result in a protocol deviation. In addition, if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

Reapproval of the project is required if it extends beyond the termination date stated below.

The Board wishes you success in the pursuit of your research goals.

Approval Termination: January 15, 2015
Appendix C

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